

# Danger of using fully homomorphic encryption, a look at Microsoft SEAL

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**360 INTERNET SECURITY CENTER**

# Who we are

**Zhiniang Peng**

**Ph.D. in cryptography**

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## **Research areas:**

Software security

Applied cryptography

Threat hunting

# About the topic

**Introduction to homomorphic encryption**

**Introduction to SEAL**

**Security pitfalls of SEAL**

- CCA attack on BFV

- Data recovery against FPSI

- Circuit privacy of SEAL

- Information leakage

- Countermeasures

**Other issues**

**Conclusion**

# Computing on encrypted data

**Data leakage become more serious nowadays**

Data security, privacy become a public concern

**It will be nice to be able to....**

Encrypt my data before send to the cloud

While still allowing the cloud to search/sort/edit the data on my behalf

Keep the data in cloud in encrypted form

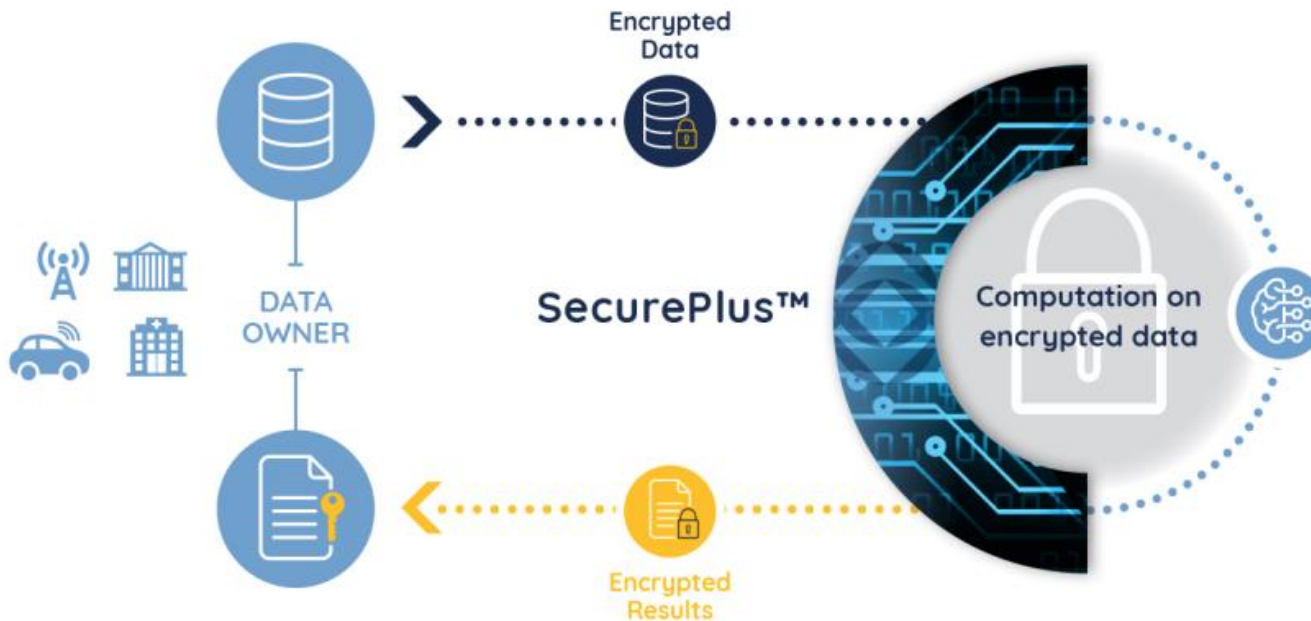
# Computing on encrypted data

**It will be nice to be able to....**

Encrypt my query to the cloud

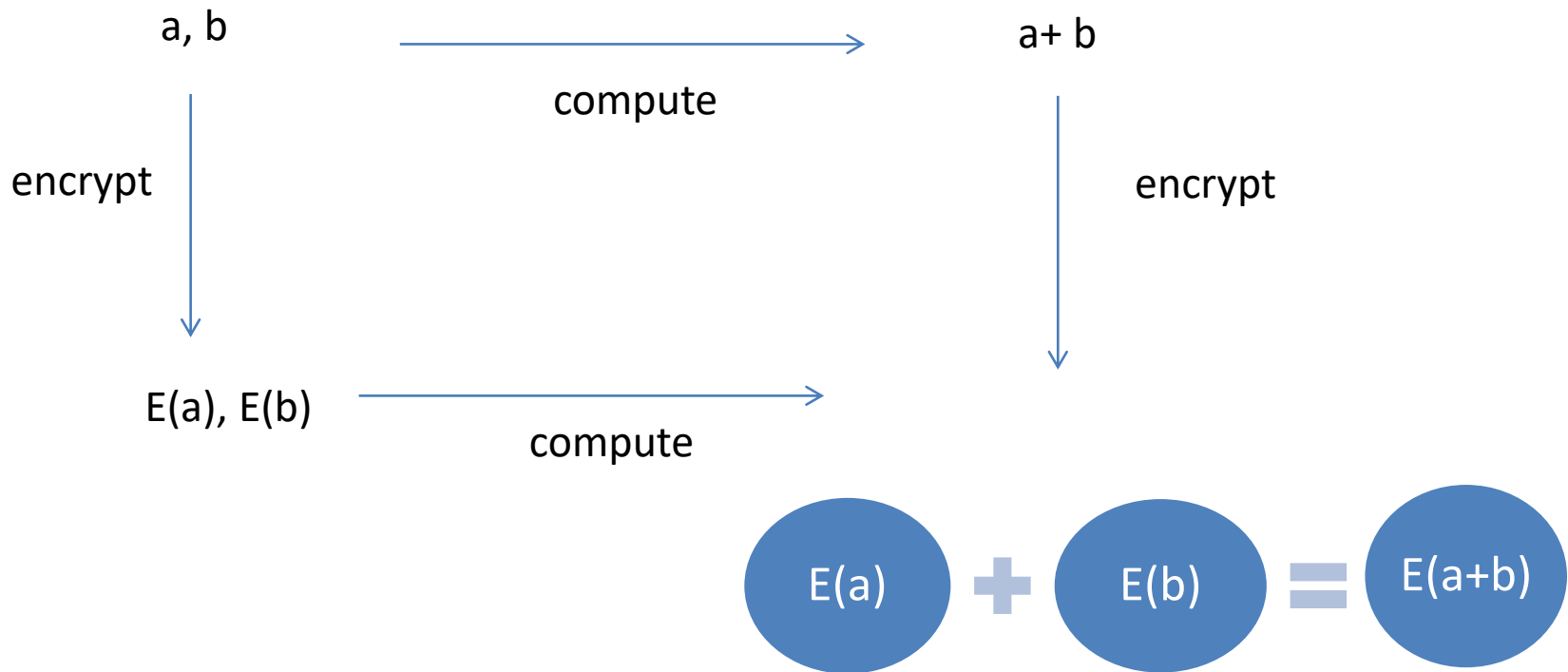
While still allowing the cloud to process them

Cloud returns encrypted answers  
that I can decrypt



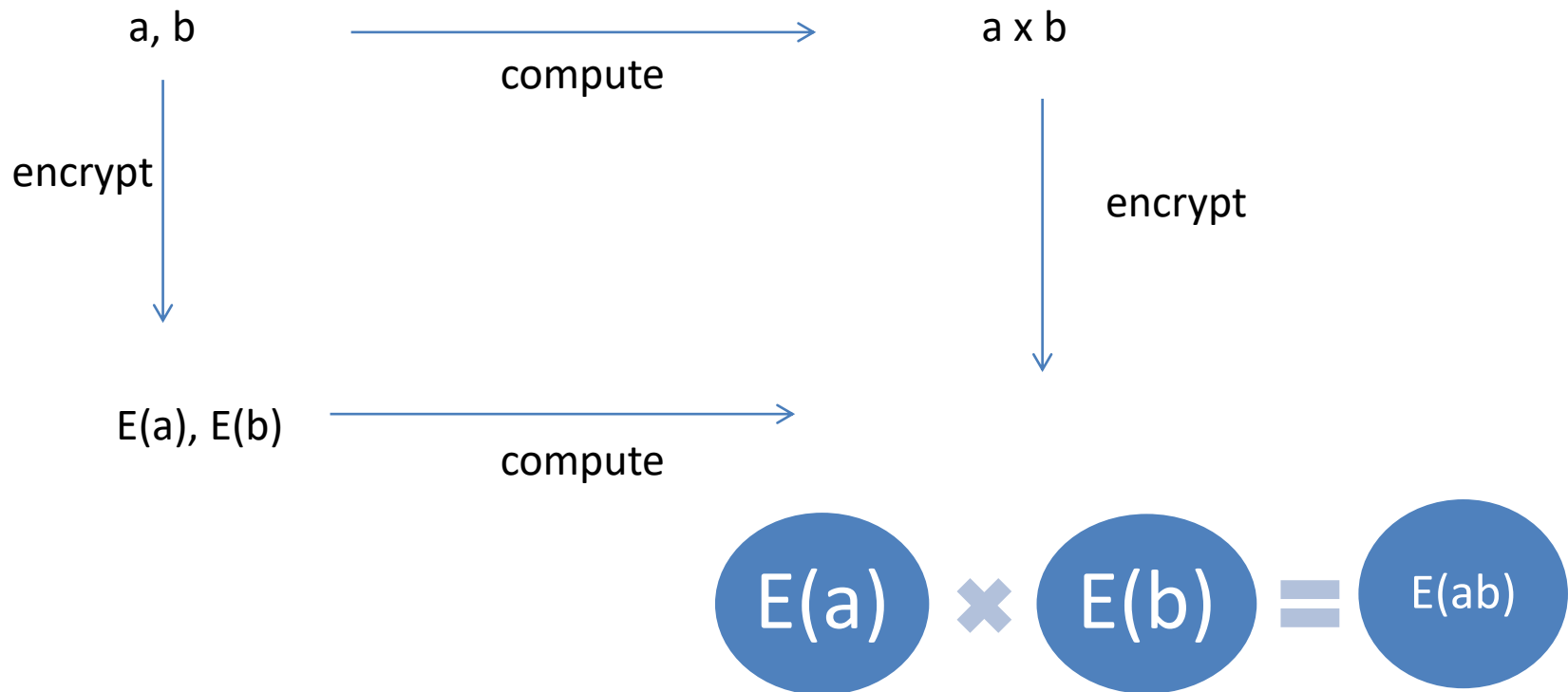
This picture is from [duality.cloud](https://duality.cloud)

# Homomorphic addition



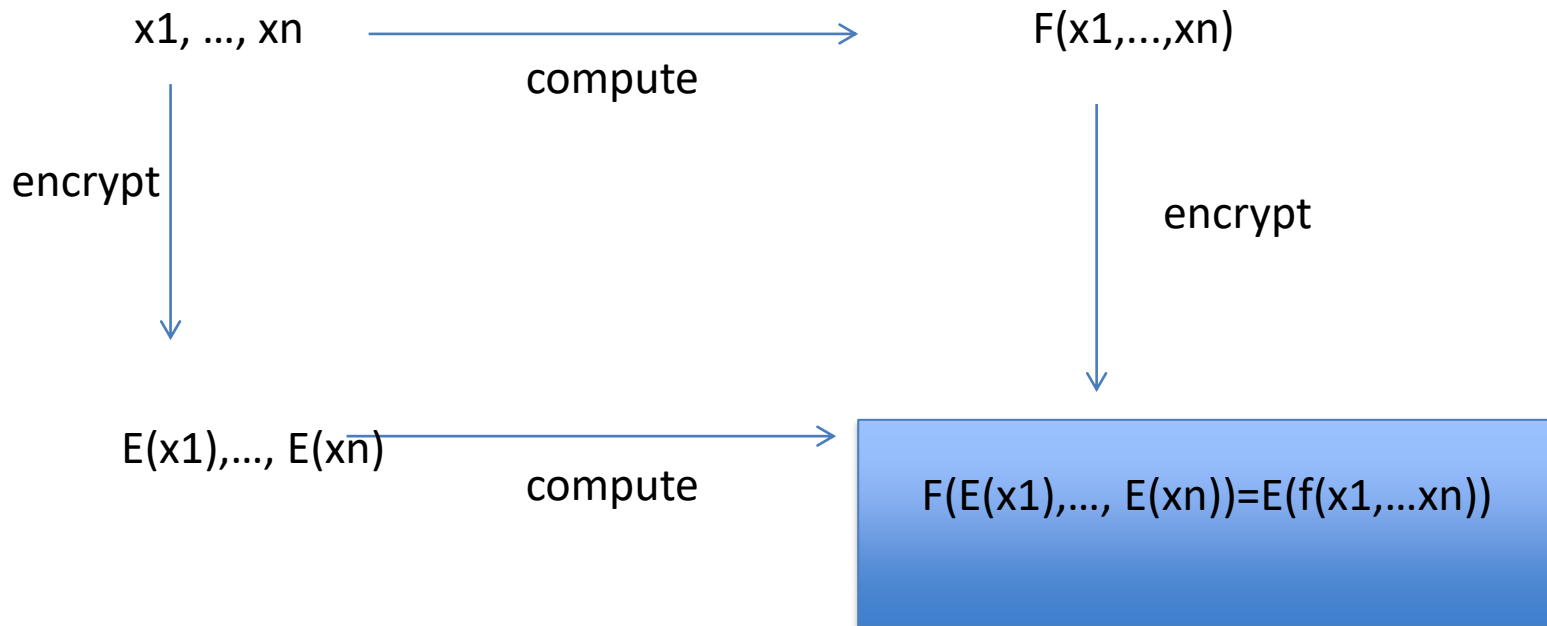
**Pure RSA support homomorphic addition!**

# Homomorphic multiplication



**Pure Elgamal support homomorphic multiplication!**

# Fully homomorphic encryption



**Craig Gentry proposed the first fully homomorphic encryption in 2009**



# Protecting Data via Encryption

## A famous metaphor



1. Put your gold in the locked box
2. Keep your key
3. Let the worker work on it through a glove box
4. Unlock the box and get the jeweler

# **A p p l i c a t i o n s   o f   H E**

**Outsourcing computation**

**Machine learning on encrypted data**

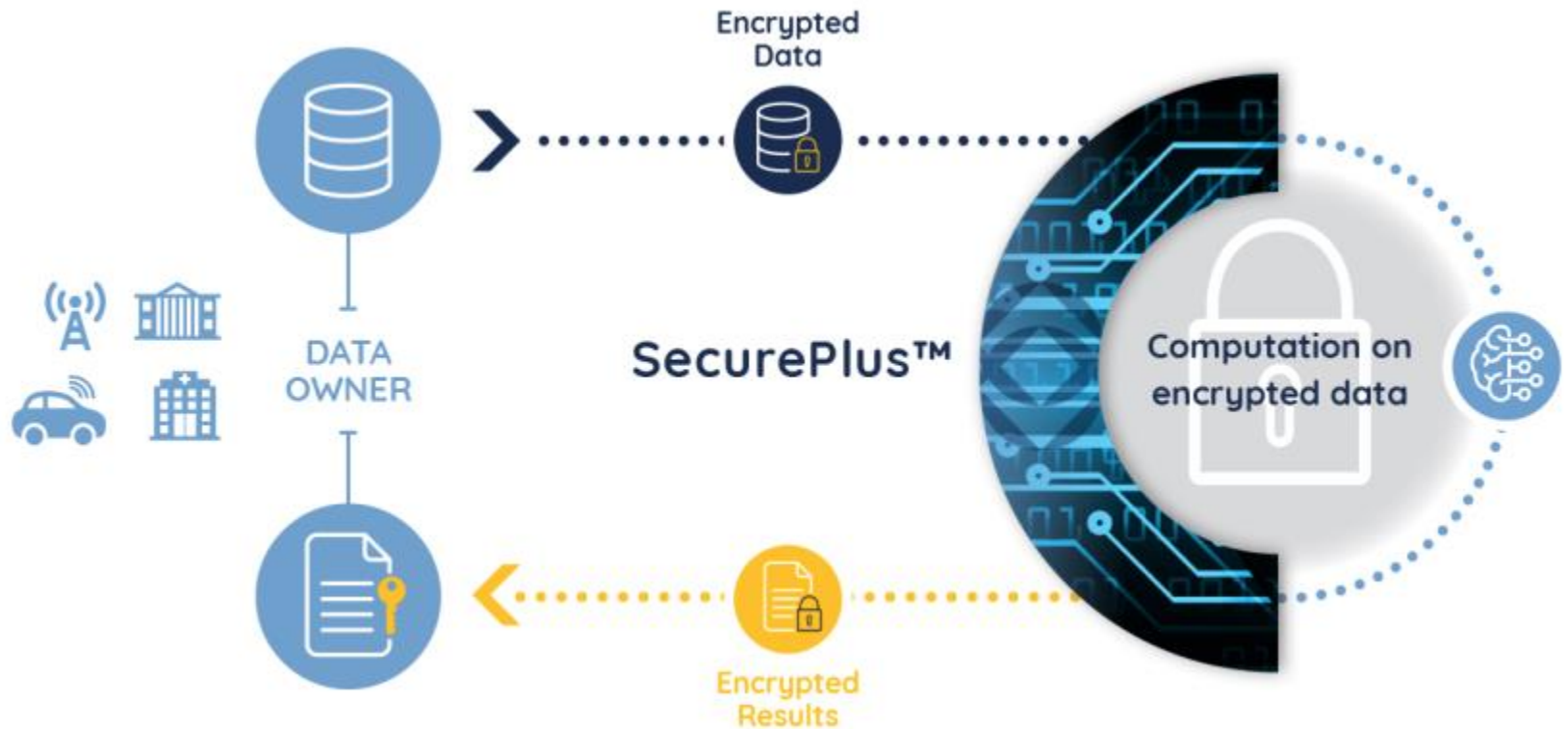
**Cloud service without knowing clients' privacy**

**There are two kinds of applications:**

private data , public function

private data , private function

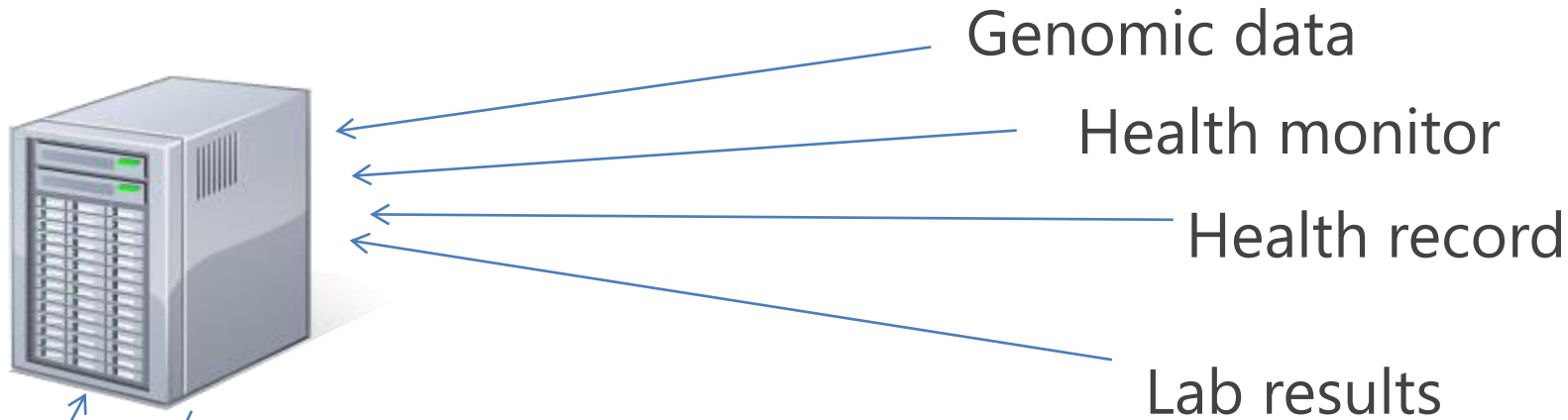
# Private data , Public function



This picture is from [duality.cloud](https://duality.cloud)

Data should be keep secret.  
The function  $f$  can be public.

# Disease prediction



## private data , public function

- All data uploaded to the server encrypted under patient's public or private key
- Cloud operates on encrypted data and returns encrypted predictive results



# Private data , Private function



This picture is from [duality.cloud](https://duality.cloud)

Both data and the model should be keep secret.

## Circuit privacy:

An additional requirement in many FHE applications is that the evaluated ciphertext should also hide the function  $f$ .



# SEAL

Simple Encrypted Arithmetic Library

# Quick Background

**Homomorphic Encryption library from Microsoft Research**

**First version released in 2015; SEAL 3.1 just released**

**Developed in standard C++**

**Implements BFV and CKKS schemes**

**Simple and easy to use**

**Comes with detailed examples**

# Performance of SEAL

## **CryptoNets:**

**MIST handwritten digital picture recognition**

**60,000 predictions per hour**

**99% correct rate**

**16 encrypted pictures per second**

## **Our experiment:**

**logistic regression prediction**

**10,000 pieces of data in 5 minutes**

**300 times slower than using sklearn directly on plaintext**

**Seems reasonable**



HOW  
DOES IT  
WORK



# Ring-LWE problem

Ring  $R = \mathbb{Z}_q[x]/(x^{n+1})$

Given:

To make it simply to understand

You can think all elements here are integer

$$a_1, b_1 = a_1 \cdot s + e_1$$

$$a_2, b_2 = a_2 \cdot s + e_2$$

...

$$a_k, b_k = a_k \cdot s + e_k$$

Find:  $s$

$s$  is random in  $R$

$e_i$  are “small” (distribution symmetric around 0)

# Decision Ring-LWE problem

Ring  $R = \mathbb{Z}_q[x]/(x^{n+1})$

Given:

$a_1, b_1$

$a_2, b_2$

...

$a_k, b_k$

Question: Does there exist an  $s$  and “small”

$e_1, \dots, e_k$  such that  $b_i = a_i \cdot s + e_i$

or are all  $b_i$  uniformly random in  $R$ ?

# BFV key pair

**SecretKeyGen():**

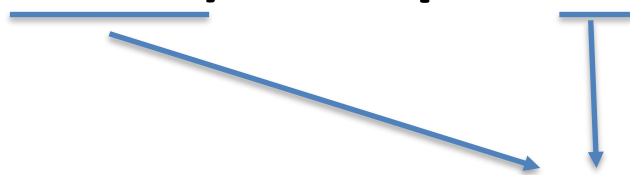
sample secret key  $s \in \chi$

**PublicKeyGen(s):**

sample  $a \in R_q, e \in \chi$

$pk_0 = -(a \cdot s + e)$

$pk_1 = a$



**Ring-LWE pair**

$s$  cannot be recovered

**Over simplified!**

**You can think all these are integer**

# BFV encryption

Encrypt(m): sample  $u \in R_q$ ,  $e_1, e_2 \in \chi$

$$c_0 = pk_0 \cdot u + e_1 + \Delta \cdot m, \quad c_1 = pk_1 \cdot u + e_2$$

Replace pk with  $(-(a \cdot s + e), a)$

$$c_0 = -(a \cdot s + e) \cdot u + e_1 + \Delta \cdot m, \quad c_1 = a \cdot u + e_2$$

$$c_0 = -w \cdot \mathbf{s} + e_1 + e \cdot u + \Delta \cdot m, \quad c_1 = w + e_2$$

Decision Ring-LWE pair (cannot be distinguished with random value)  
Message  $m$  is encrypted with a random pad

Ciphertext can be considered as a polynomial:

$$f(x) = c_0 + c_1 \cdot x$$

# BFV decryption

Decrypt(c):

$$f(x) = c_0 + c_1 \cdot x$$

substitute x with s

$$f(s) = c_0 + c_1 \cdot s$$

Replace c with  $([-w \cdot s + e_1 + e \cdot u + \Delta \cdot m]_q, [w + e_2]_q)$

$$f(s) = -w \cdot s + e_1 + e \cdot u + \Delta \cdot m + (w + e_2) \cdot s$$

$$= \underline{e_1 + e \cdot u + e_2 \cdot s} + \Delta \cdot m$$

Much smaller than  $\Delta$   
Then we can recover m

We can think that:

$$f(s) = v + \Delta \cdot m$$

where v is much smaller than  $\Delta$

# Homomorphic addition

## Homomorphic addition:

Ciphertext1:  $f_1(x)$

Ciphertext2:  $f_2(x)$

Compute:  $f_3(x) = f_1(x) + f_2(x)$

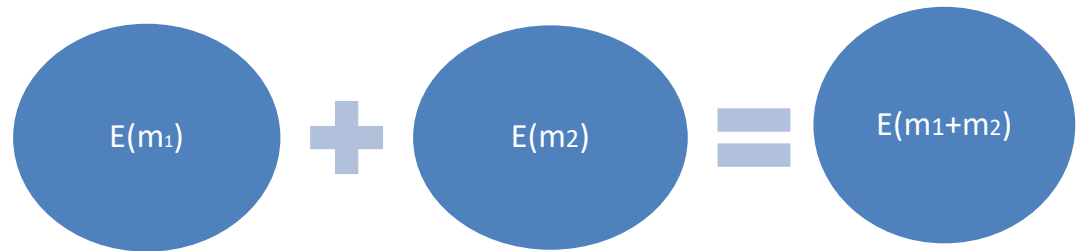
We have:

$$f_1(s) = v_1 + \Delta \cdot m_1$$

$$f_2(s) = v_2 + \Delta \cdot m_2$$

**Then decrypt  $f_3(x)$ :**

$$\begin{aligned} f_3(s) &= f_1(s) + f_2(s) = v_1 + v_2 + \Delta \cdot (m_1 + m_2) \\ &= v_3 + \Delta \cdot (m_1 + m_2) \end{aligned}$$



# Homomorphic multiplication

## Homomorphic multiplication:

Ciphertext1:  $f_1(x)$

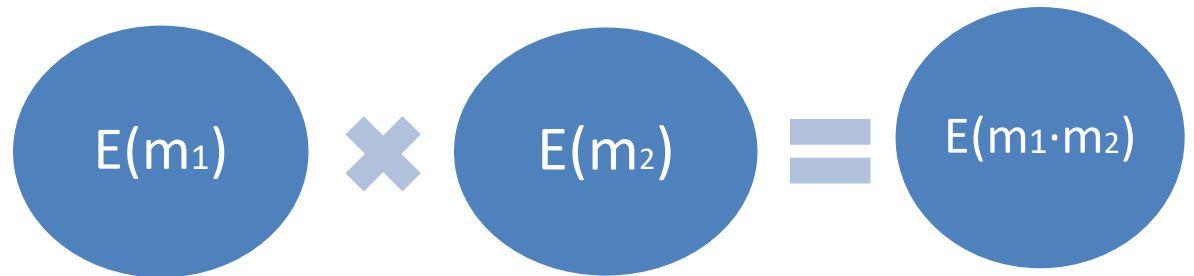
Ciphertext2:  $f_2(x)$

Compute:  $f_3(x) = f_1(x) * f_2(x)$

We have:

$$f_1(s) = v_1 + \Delta \cdot m_1$$

$$f_2(s) = v_2 + \Delta \cdot m_2$$



## Then decrypt $f_3(x)$ :

$$f_3(s) = f_1(s) * f_2(s) = v_1 \cdot v_2 + \Delta \cdot (v_1 \cdot m_2 + v_2 \cdot m_1) + \Delta \cdot m_1 \cdot m_2$$

## Divide by $\Delta$ , we can get:

$$f_3(s) / \Delta = v_3 + \Delta \cdot (m_1 \cdot m_2)$$



# Security of BFV schemes

**Encrypt message  $m$  to an polynomial  $f(x)$**

**Decrypt by substitute  $x$  with  $s$**

get  $f(s)=v+\Delta \cdot m$ , can recover  $m$  easily

**Message is “blind” by Ring-LWE pair + noise**

Distinguish ciphertext  $\rightarrow$  distinguish Ring-LWE pair

**Provable security: IND-CPA  $\rightarrow$  Ring-LWE**

Chosen plaintext attack

If someone break IND-CPA, he can break Ring-LWE

Ring-LWE is suppose to be a hard math problem

# IND-CCA?

## **chosen ciphertext attack**

attacker is given access to a *decryption oracle*

## **BFV doesn't have IND-CCA security**

## **All practical FHE cannot guarantee IND-CCA**

homomorphic property seem conflict with CCA

## **Theoretical research on CCA FHE**

Chosen-Ciphertext Secure Fully Homomorphic Encryption

## **All FHE implementation cannot guarantee security in the IND-CCA scenario**

# IND-CCA Scenario

## Why need IND-CCA

attacker can always ask for a decryption in real scenario

IND-CCA is a standard requirement for encryption scheme

## Scenarios that require HE often require IND-CCA

rich data flows between data-owner and cloud

multi-party's cooperation and data exchange

If certain decrypted data is leaked to the cloud

break the CPA model, need CCA security

# One query attack

**Suppose attacker can query decryption oracle 1 time**

Realistic in many scenarios

**Ask to decrypt a malicious ciphertext  $f(x)$**

$f(x) = c_0 + c_1x$  with  $c_0 = 0$ ,  $c_1 = \Delta$

Decryption substitutes  $x$  with  $s$

We have :  $f(s) = \Delta s$

then the decrypted message equal to  $s$  (private key)

**Recover private key with only one query**

extremely dangerous

other FHE face the same problem

# Countermeasures

**Do not use HE in any scenario that decrypted result may leak to evaluator.**

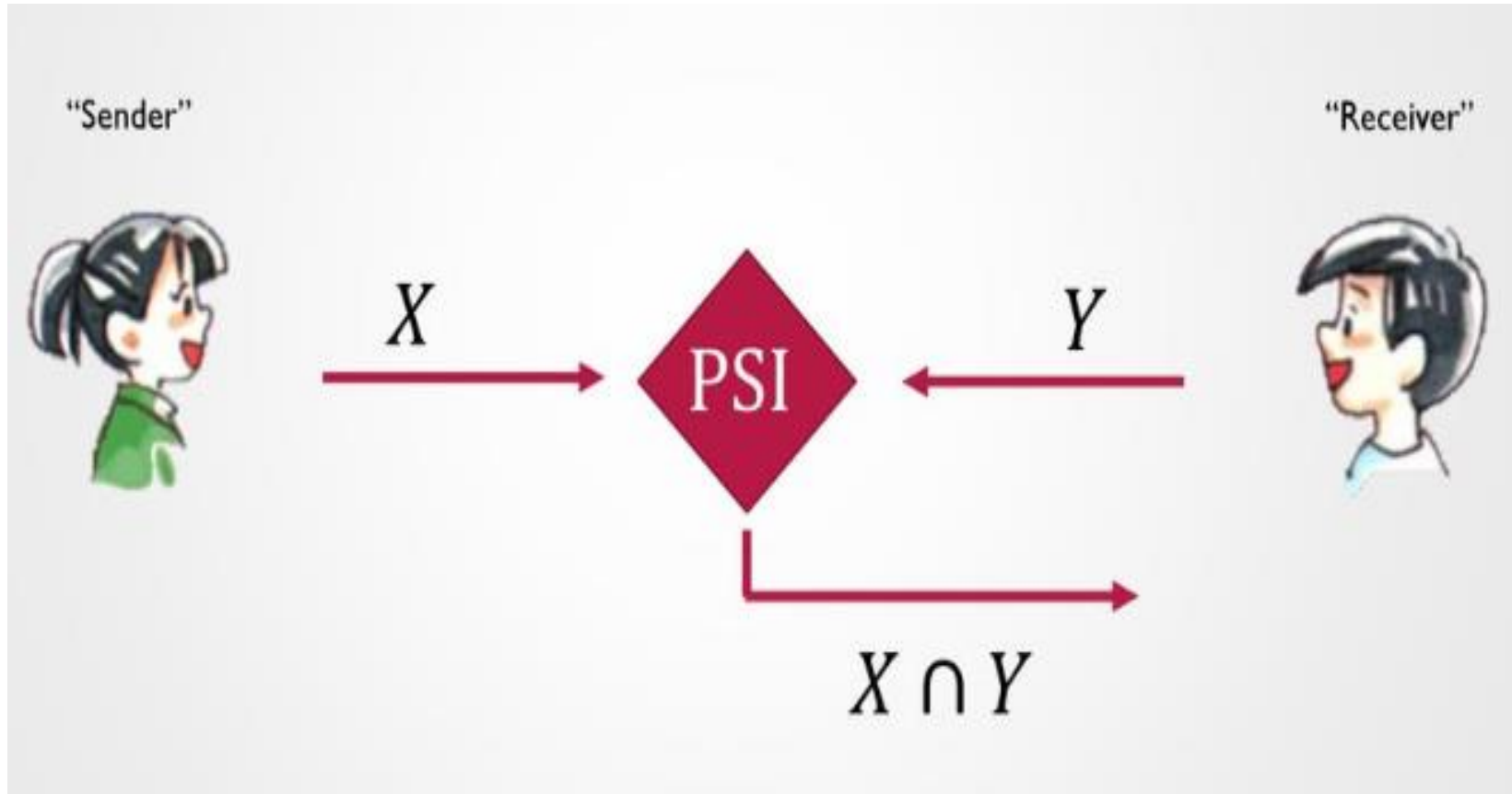
**Otherwise, there is no encryption at all.**

The decrypted result may leak to evaluator in many scenarios, with or without being noticed.

**But how can We make sure there is no leakage?**

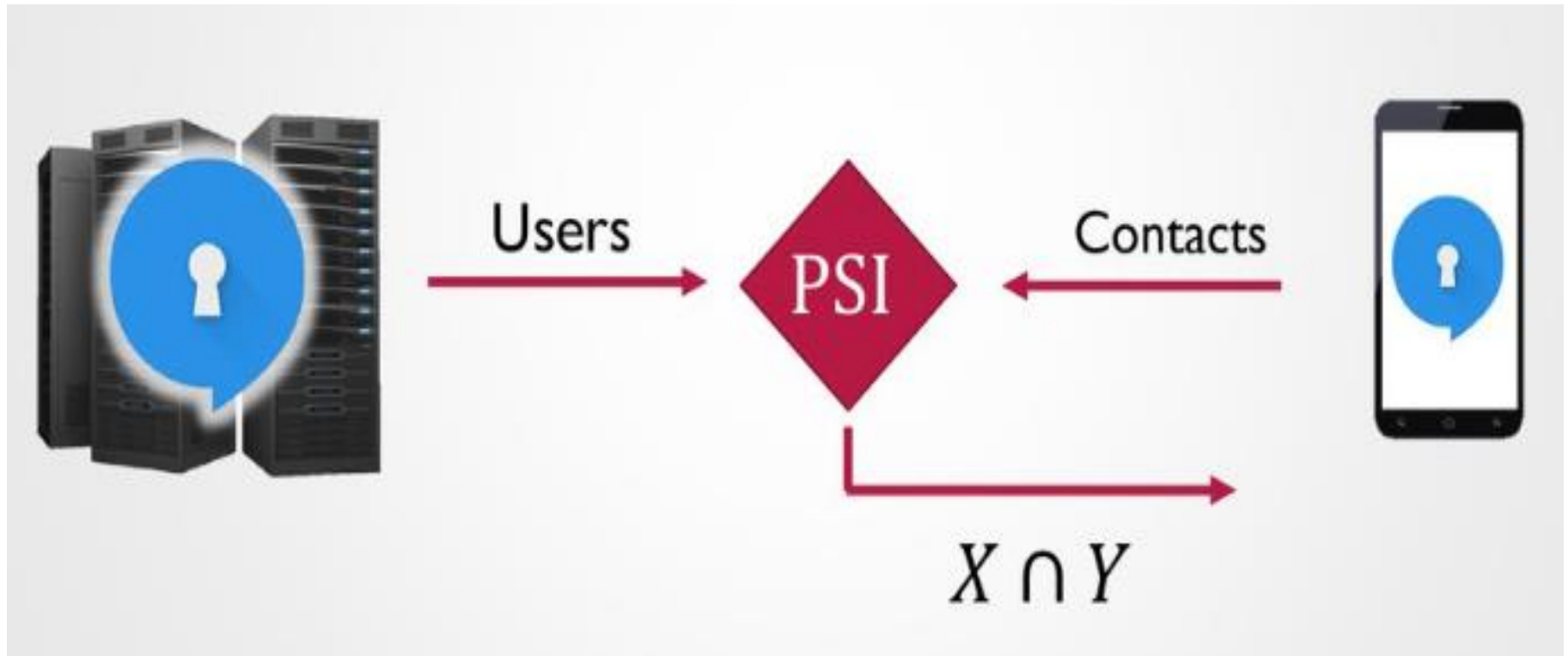
Currently, SEAL team is working on building mitigations on this problem. To detect malicious ciphertext before decrypt it.

# Private Set Intersection (PSI)



**Without leaking anything else**

# App: Contact discovery



private contact discovery on E2EE IM (signal...)

# Using HE to build PSI FPSI in CCS17 (oversimplified)



local database Y



Encrypt(X) with HE

Send Encrypted X to Server



Compute with local database Y,  
get the encrypted  $X \cap Y$

Send Encrypted  $X \cap Y$  to Client



Decrypt the result,  
Get the  $X \cap Y$



# CCA attack on this scenario



After client get  $X \cap Y$ .

He found out that  $X \cap Y$  are also using signal.

Then he add them as friends

$X \cap Y$  in plaintext



**Information leakage**

**To server.**

**Server can launch CCA attack!**

Lesson learned:

There are always unexpected data flows between data-owner and cloud.  
Be extremely careful when using homomorphic encryption.

# Another attack



local database Y



Encrypt(X) with HE

Send Encrypted X to Server



Compute with local database Y,  
get the encrypted  $X \cap Y$ .

But most of the HE have no circuit privacy.  
Other information except  $X \cap Y$  may also leak.

Send Encrypted  $X \cap Y$  to Client



Client decrypt the result,  
Get the  $X \cap Y$

Also get other information on Y

More details can be found on our whitepaper

# Circuit Privacy of SEAL

## **SEAL doesn't provide circuit privacy on default**

Addressed in SEAL handbook

Best practice is “noise flooding”

adding an encrypted 0 to the final result, with “enough” noise

But there is no standard interface of “noise flooding” in SEAL

normal software developer definitely can't play with the magic

## **Hardness of providing “noise flooding”**

Need to know how much noise is needed, this is also some kind of information we need to protect. :(

## **All practical FHE lib seems have the circuit privacy problem**

The crypto community need to solve this

# Countermeasures

**An improved PSI protocol is published in CCS18**

<https://eprint.iacr.org/2018/787>

Secure against malicious party

**As for circuit privacy of SEAL and HE**

You need an crypto expert to review your implementation

You need professional knowledge on lattice-based crypto

SEAL team is working on provide a standard interface to this problem

# **Info leakage of Encoder**

**HE is working on a polynomial ring based on finite field**

plaintext is integer, float or string

we need convert them to the ring

**IntegerEncoder of SEAL**

**encode an integer to a polynomial**

**many to one mapping**

**Information leakage!**

More details can be found on our [whitepaper](#).

I think you don't want the mathematical formula here.

# Countermeasures

## **Coding problem may also happen in other HE libs**

Be careful when using encode functionality provided by HE lib

The crypto part have a security proof, but the encoding may not

## **Don't use IntegerEncoder, FloatEncoder in SEAL**

They should considered primarily as a demonstrative tool

BatchEncoder is safe in SEAL

# Other security issues

## **HE does not provide the security features as the commonly known encryption algorithms**

- HE is not an Authenticated Encryption

- cannot guarantee the integrity of the data

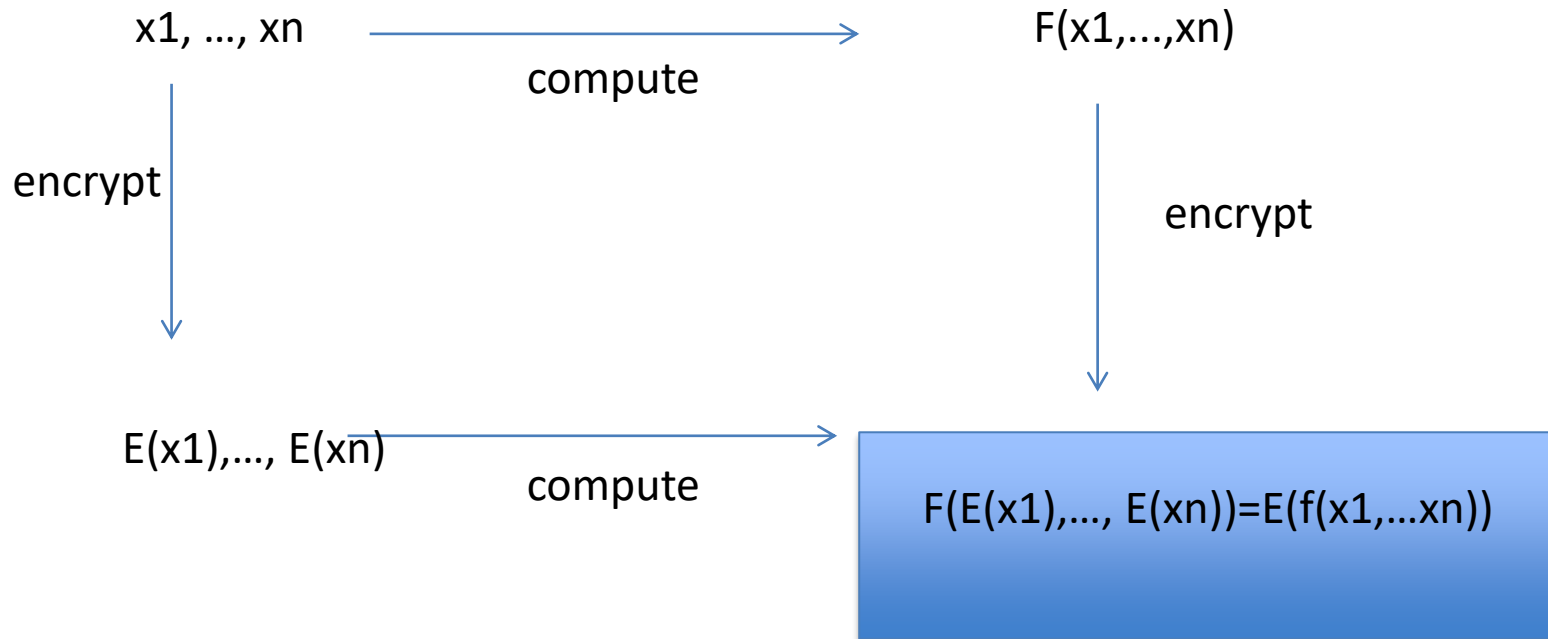
- attacker can use the homomorphic nature of HE to modify the ciphertext

- don't use HE for storage and data transmission directly

## **We need a standard documentation for HE**

- Microsoft is currently leading the development of standard for HE

# Is FHE really can compute arbitrary function?



Arbitrary function in FHE means arbitrary addition and multiplication here.

Arbitrary addition and multiplication does not mean you can run arbitrary program

**You can't do comparison directly**  
(if branch is not support here)



# Update the famous metaphor



1.Put your gold in the locked box

2.Keep your key

3.Let the worker work on it through a glove box **with eyeshade**

4.Unlock the box and get the jeweler

**This box should be opaque!**

# Conclusion

**HE is a useful in many scenario**

Its performance is improving and acceptable

**HE is not omnipotent**

It can not really run arbitrary program

**HE has many security pitfalls**

It's extremely dangerous to use HE without an crypto expert for now

Still need a long way to go

# Acknowledgments

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# Thanks



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**SAFETY FIRST**