Lecture 8: Precision of OLS and Hypothesis Testing

ECON 480 - Econometrics - Fall 2018

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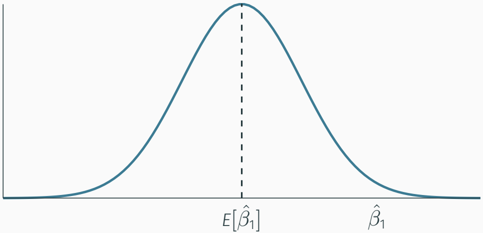
September 26, 2018

The Precision of OLS

$$\hat{\beta}_1 \sim N(E[\hat{\beta}_1], \sigma_{\hat{\beta}_1})$$

- We want to know:



Recall: The Sampling Distribution of $\hat{\beta}_1$

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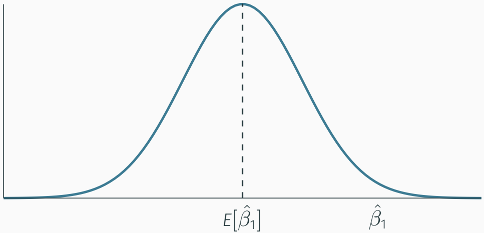

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- We want to know:
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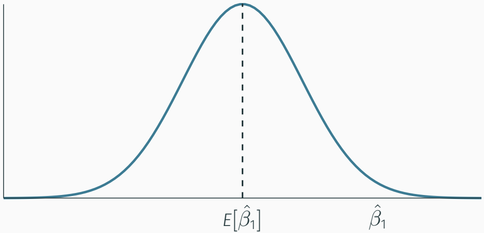
    - $\sigma_{\hat{\beta}_1}$; how precise is our estimate?



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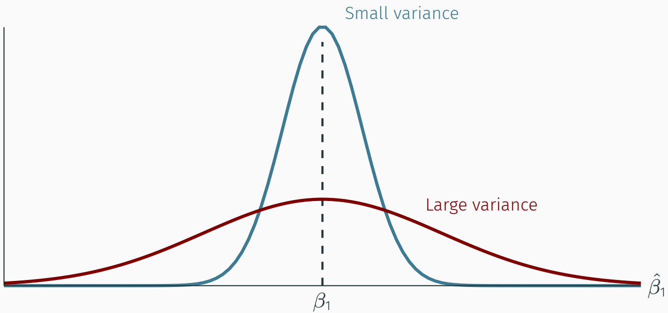
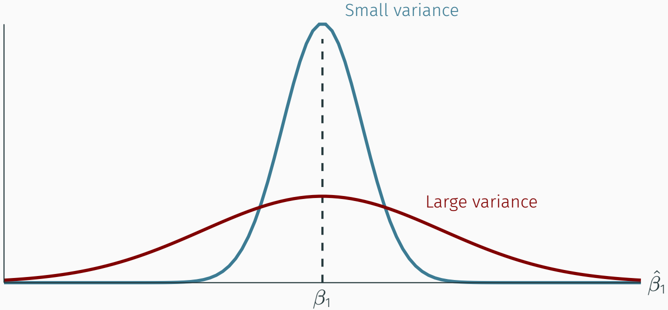


[1]The "standard **error**" is the analogue of standard *deviation* for a sample statistic's sampling distribution. Recall the sampling distribution is the distribution of a statistic, like $\bar{X}$ or $\hat{\beta}_1$ over many potential samples.

- How precise is our estimate $\hat{\beta}_1$?
- We can talk of the variance $(\sigma^2_{\hat{\beta}_1})$ or the standard error $(\sigma_{\hat{\beta}_1})$ of $\hat{\beta}_1$[1]



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$$var(\hat{\beta}_1) = \frac{(SER)^2}{n \times var(X)}$$

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- The standard error of $\hat{\beta}_1$ is simply the square root of the variance

$$se(\hat{\beta}_1) = \sqrt{var(\hat{\beta}_1)}$$

What Affects Variance

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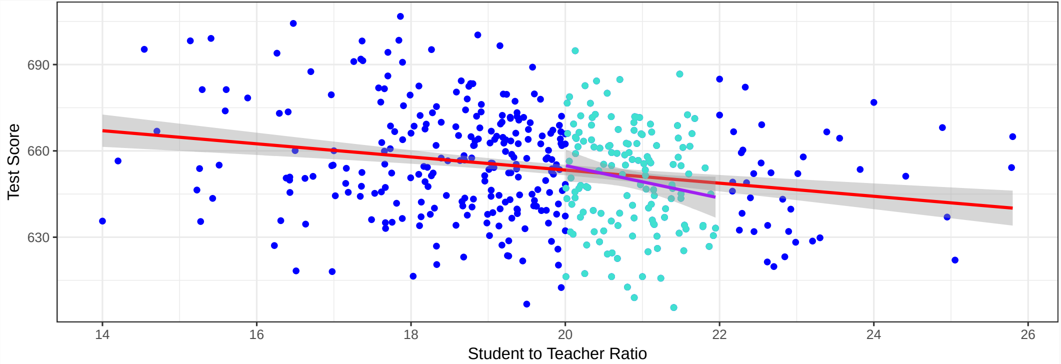
THE RELATION BETWEEN VARIANCE OF X AND VARIANCE OF $\hat{\beta}_1$



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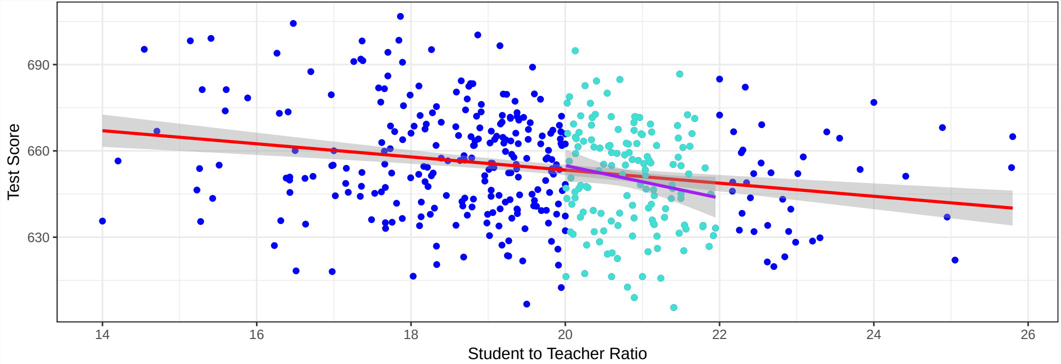

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- Larger $var(X)$ (all dots) $\implies$ smaller $var(\hat{\beta}_1)$: easier to determine precise slope!

Hypothesis Testing About Regression

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- **All modern science is built upon statistical hypothesis testing, so understand it well!**

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  - i.e. if/when we've done our model right, the causal effect of X on Y

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    - Note: the test is *always* about H_0! See if we have sufficient evidence to reject the status quo

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    - Beyond the critical value is the "rejection region", sufficient evidence to reject H_0
4. A **conclusion** whether or not to reject H_0 in favor of H_a

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- **We cannot distinguish between these two possibilities with any certainty**

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    - Believing we found nothing when there was truly a relationship to find

	H_0 is True	H_0 is False
Reject H_0	Type I Error False Positive	Correct Outcome True Positive
Don't Reject H_0	Correct Outcome True Negative	Type II Error False Negative

Type I and Type II Errors IV

	Defendant is Innocent	Defendant is Guilty
Convict "I think he's guilty"	Type I Error False Positive	Correct Outcome True Positive
Don't Convict "I think he's innocent"	Correct Outcome True Negative	Type II Error False Negative

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- Common law *presumes* the defendant is innocent and a jury judges whether the evidence presented against the defendant would be plausible *if the defendant were in fact innocent*

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- H_0: a rock climber's equipment is safe
- H_0: a woman is not pregnant
- H_0: a highway project will cost no more than \$10 million
- H_0: an experimental cancer drug has a cure rate of at least 75%

- The significance level, α, is the probability of a **Type I error**

$$\alpha = P\big(\text{Reject } H_0 \big| H_0 \text{ is true}\big)$$

$$\beta = P\big(\text{Don't reject} H_0 \big| H_0 \text{ is false}\big)$$

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- The probability of a **Type II error** is defined as β:

$$\beta = P(\text{Don't reject} H_0 | H_0 \text{ is false})$$

α AND β

	H_0 is True	H_0 is False
Reject H_0	Type I Error α	Correct Outcome $(1-\beta)$
Don't Reject H_0	Correct Outcome $(1-\alpha)$	Type II Error β

- The statistical power of the test is $1 - \beta$, the probability of correctly rejecting H_0 when H_0 is in fact false (e.g. not convicting an innocent person)

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- The p-value or significance probability is the probability that, given the null hypothesis is true, the test statistic from a random sample will be at least as extreme as the test statistic of our sample

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- The statistical power of the test is $1-\beta$, the probability of correctly rejecting H_0 when H_0 is in fact false (e.g. not convicting an innocent person)

$$\text{Power} = 1-\beta = P(\text{Reject } H_0 | H_0 \text{ is false})$$

- The p-value or significance probability is the probability that, given the null hypothesis is true, the test statistic from a random sample will be at least as extreme as the test statistic of our sample
- If $p < \alpha$, the results are "**statistically significant**"

HOOD COLLEGE

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    - Note this does **not** mean H_0 is true! We merely have *failed* to *reject* H_0

Digression: p-Values and the Philosophy of Science

"The null hypothesis is never proved or established, but is possibly disproved, in the course of experimentation. Every experiment may be said to exist only in order to give the facts a chance of disproving the null hypothesis."

(1931). *The Design of Experiments*

Sir Ronald A. Fisher

(1890-1962)

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- Note: economics is a very different kind of "science" with a different methodology!

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- p tells us how significant our finding is (p tells us nothing about the *size* or the *real world significance* of any effect deemed "statistically significant")

- Again, **p is the probability that, assuming the null hypothesis is true, we obtain (by pure random chance) a test statistic at least as extreme as the one we estimated for our sample**
    - This will make more sense in context, when we discuss the nature of our test statistics

Statistical Significance: Cautions II

- Again, **p is the probability that, assuming the null hypothesis is true, we obtain (by pure random chance) a test statistic at least as extreme as the one we estimated for our sample**
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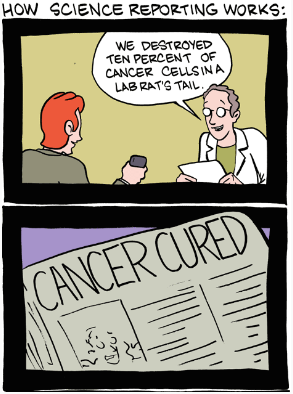
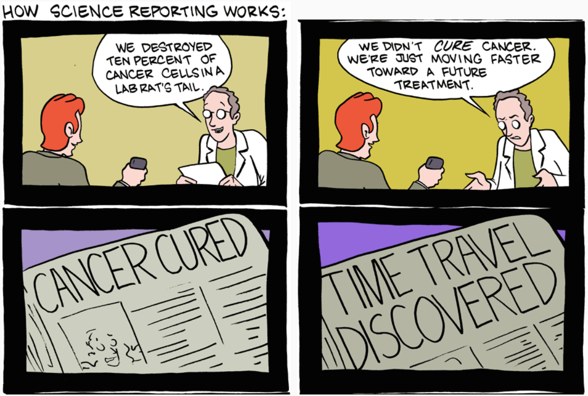
- Remember a low p-value means **either** that the null hypothesis is true and a highly improbable event has occurred or that the null hypothesis is false (we don't know which!)


HOW SCIENCE REPORTING WORKS:
WE DESTROYED TEN PERCENT OF CANCER CELLS IN A LAB RAT'S TAIL.
CANCER CURED

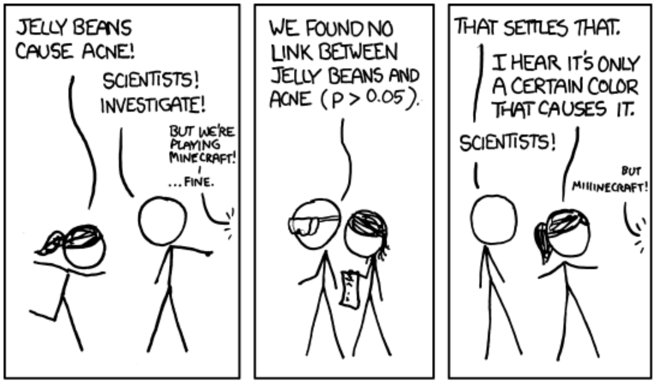
SMBC 1623

Statistical Significance and p-Values



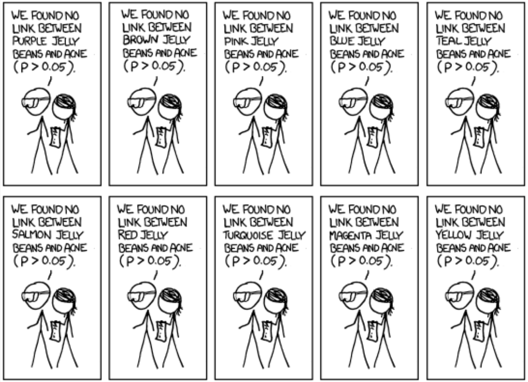
SMBC 1623

Statistical Significance and p-Values II



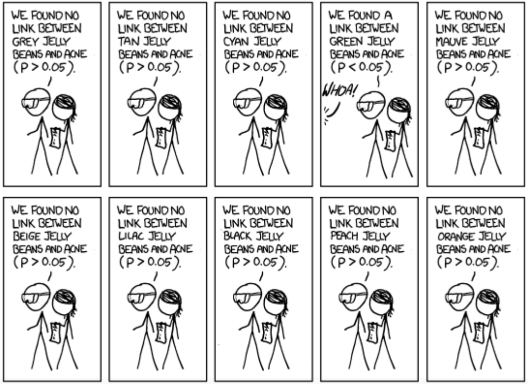
XKCD 882

Statistical Significance and p-Values III

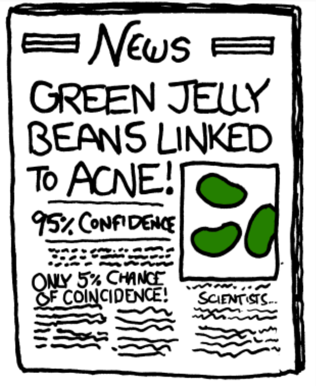


XKCD 882

Statistical Significance and p-Values IV

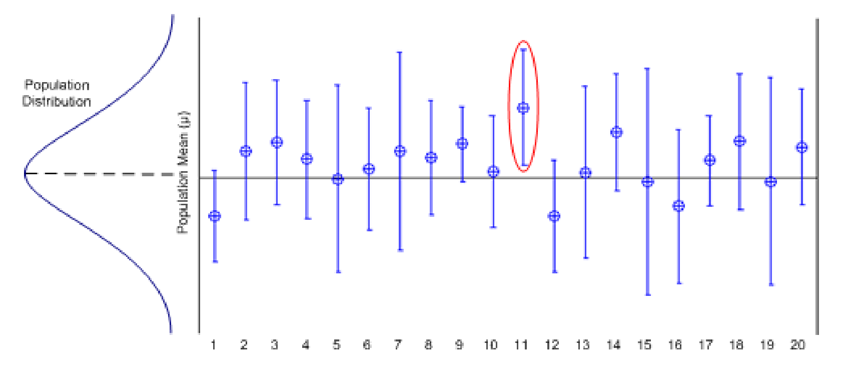


XKCD 882

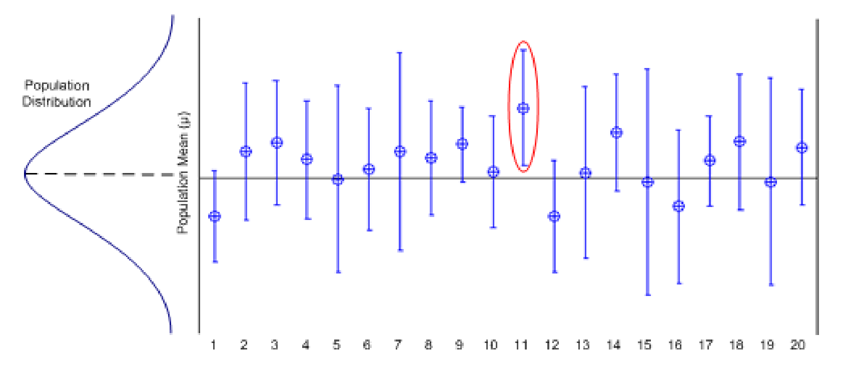

NEWS
GREEN JELLY BEANS LINKED TO ACNE!
95% CONFIDENCE
ONLY 5% CHANCE OF COINCIDENCE!
SCIENTISTS...

XKCD 882

- Consider what "95% significance" or $\alpha = 0.05$ means:



- Consider what "95% significance" or $\alpha = 0.05$ means:
    - If we repeat a procedure 20 times, we should *expect* 1/20 (5%) to produce a fluke result!



"The widespread use of "statistical significance" (generally interpreted as ($p \leq 0.05$) as a license for making a claim of a scientific finding (or implied truth) leads to considerable distortion of the scientific process."

Wasserstein, Ronald L. and Nicole A. Lazar, (2016). "The ASA's Statement on p-Values: Context, Process, and Purpose" *The American Statistician* 30(2): 129-133.

Morning Mix

How, and why, a journalist tricked news outlets into thinking chocolate makes you thin

By Sarah Kaplan May 28, 2015

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Our Online Games

Play right from this page

Washington Post: How, and why, a journalist tricked news outlets into thinking chocolate makes you thin

Back to Our Hypothesis Test: The Test-Statistic

- We next consider the population distribution **assuming H_0 is true** and calculate a test statistic, which takes the following form:

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- We then compare our test statistic against a **critical value** to determine if we can reject H_0
- Essentially: test to see how likely a sample statistic at least as extreme as our discovery is if H_0 were true

Distribution of H_0 II

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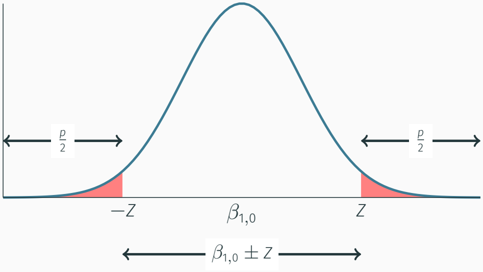
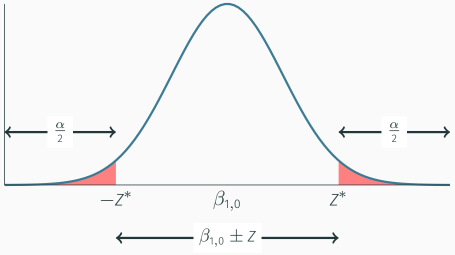
- *p*-**value**: area in the tail(s) of the distr. of $\hat{\beta}_1$ under H_0 beyond our Z score



Distribution of β_1 under $H_0 : \beta_1 = 0$; p-value is the area in red (two-tailed test)

- The **critical value** Z^* is determined by our α level (e.g. 0.05)



Critical values of Z^* with rejection regions in red

[2] As you can see, the empirical 68-95-99.7% rule is very close, but not perfect!

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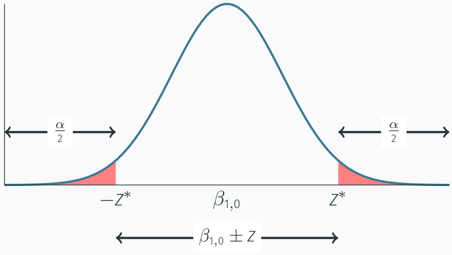

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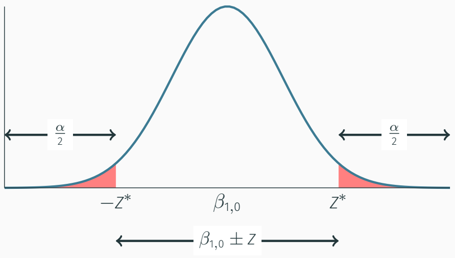

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Distribution of H_0 IV

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- We almost never know them...

- Worked at Guinness testing beer quality

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(1876-1937)

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t-DISTRIBUTION

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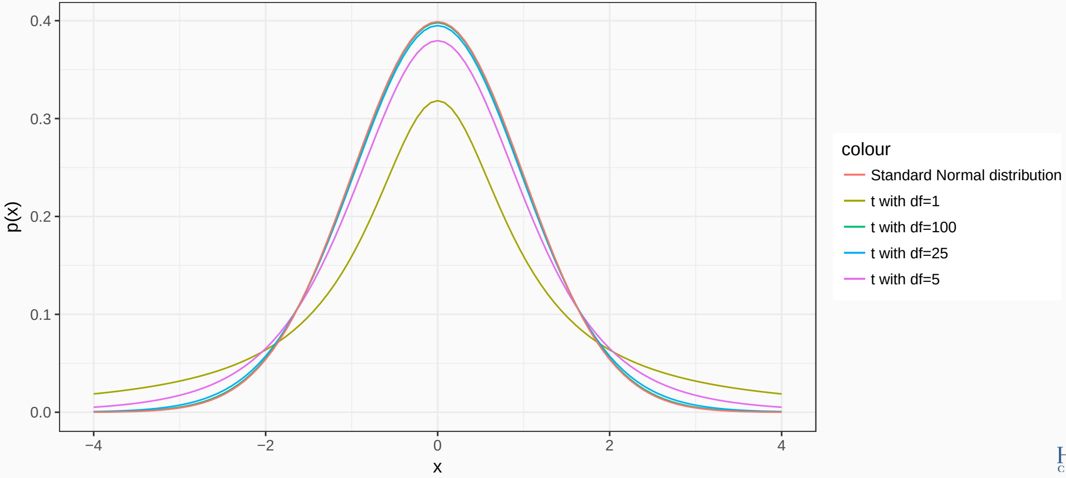
- Exact shape of t depends on df: as $\uparrow df$, $t \rightarrow$ Normal distribution

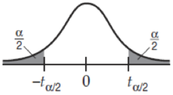


0.4
0.3
0.2
0.1
0.0
p(x)
−4
−2
0
2
4
x
colour
Standard Normal distribution
t with df=1
t with df=100
t with df=25
t with df=5

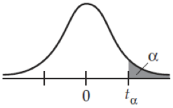
Calculating t-scores: Old-Fashioned Way

Table T

Values of t_α



Two tails



One tail

Two tail probability	0.20	0.10	0.05
One tail probability	0.10	0.05	0.025
df			
1	3.078	6.314	12.706
2	1.886	2.920	4.303
3	1.638	2.353	3.182
4	1.533	2.132	2.776
5	1.476	2.015	2.571
6	1.440	1.943	2.447
7	1.415	1.895	2.365
8	1.397	1.860	2.306
9	1.383	1.833	2.262
10	1.372	1.812	2.228
11	1.363	1.796	2.201
12	1.356	1.782	2.179
13	1.350	1.771	2.160
14	1.345	1.761	2.145
15	1.341	1.753	2.131
16	1.337	1.746	2.120
17	1.333	1.740	2.110
18	1.330	1.734	2.101
19	1.328	1.729	2.093
⋮	⋮	⋮	⋮
∞	1.282	1.645	1.960
Confidence levels	80%	90%	95%

Calculating t-scores: In R

```r
# use pt() command, needs t value and df
pt(2,df=5) #probability of t>2 with 5 df
```

```
## [1] 0.9490303
```

```r
pt(2,df=40)# probability of t>2 with 40 df
```

```
## [1] 0.9738388
```

```r
pt(2, df=100) # probability of t>2 with 100 df
```

```
## [1] 0.9758939
```

```r
pnorm(2, mean=0, sd=1) # compare to normal distribution!
```

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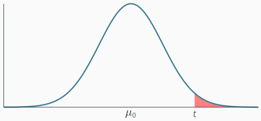
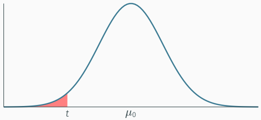
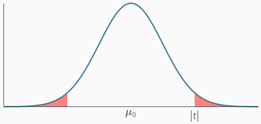
- p-value$= P(t < T)$
- Reject H_0 if p-value $< \alpha$

HOOD COLLEGE

Hypothesis Testing with t-distribution II

Depending on the desired alternative hypothesis:

Alternative	p-value	PDF
$H_a : \beta_1 > \beta_{1,0}$	$P(T \geq t)$	
$H_a : \beta_1 < \beta_{1,0}$	$P(T \leq t)$	
$H_a : \beta_1 \neq \beta_{1,0}$	$2P(T \geq \lvert t \rvert)$	

Example

We have an estimated regression line:

$$\widehat{\text{Test Score}} = \underset{(9.47)}{689.93} - \underset{(0.48)}{2.28}\ \text{STR}$$

- Regression reporting format: Coefficients with their (standard errors) beneath them

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Example

We have an estimated regression line:

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```
# calculate p-value for t=-4.75, df=418
2*pt(-4.75,df=418) # x2 because we want both tails!
```

```r
summary(school.regression)
```

```
##
## Call:
## lm(formula = testscr ~ str, data = CASchool)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -47.727 -14.251   0.483  12.822  48.540
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 698.9330     9.4675  73.825  < 2e-16 ***
## str          -2.2798     0.4798  -4.751 2.78e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 18.58 on 418 degrees of freedom
## Multiple R-squared:  0.05124,    Adjusted R-squared:  0.04897
## F-statistic: 22.58 on 1 and 418 DF,  p-value: 2.783e-06
```

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- Since essentially $t = \frac{\hat{\beta}_k}{SE(\hat{\beta}_k)}$ and we roughly want $t \geq 2$ for 95% confidence level (α=0.05)