Lecture 3: Data and Descriptive Statistics

ECON 480 - Econometrics - Fall 2018

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Data Basics

Basic Statistics

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- Metadata describe the process about how data is collected

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        - e.g. we can have separate observations on the same individual over time

- Categorial variables place an individual into one of several possible categories

Question	Categories or Responses
Do you invest in the stock market?	__ Yes __ No
What kind of advertising do you use?	__ Newspapers __ Internet __ Direct mailings
What is your class at school?	__ Freshman __ Sophomore __ Junior __ Senior
I would recommend this course to another student.	__ Strongly Disagree __ Slightly Disagree __ Slightly Agree __ Strongly Agree
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  - can be quantitative (e.g. age, zip code)

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Count	1610	4906	12082	13791	21551
Proportion	0.030	0.091	0.224	0.256	0.400

Cut characteristics of 53,940 diamonds

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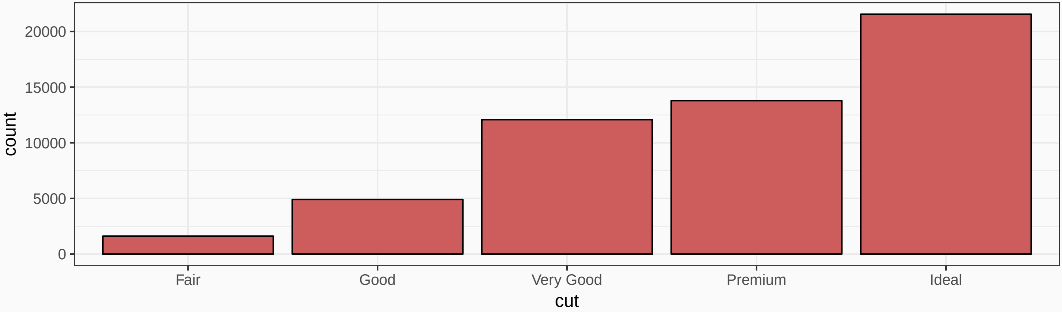
- **Proportion**: *relative* frequency (percentage of all individuals) in a category

Representing Categorical Variables II

```
ggplot(diamonds, aes(x=cut))+
  geom_bar(fill="indianred", color="black")
```



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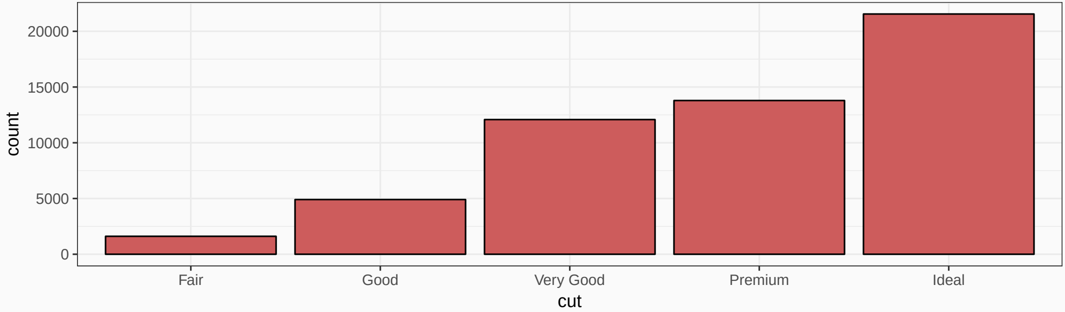

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- A bar chart represents categories as bars, with lengths proportional to the count or relative frequency fo each category

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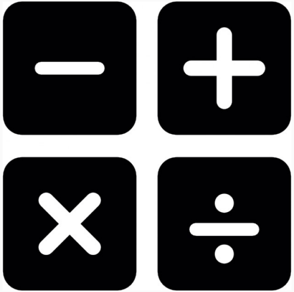

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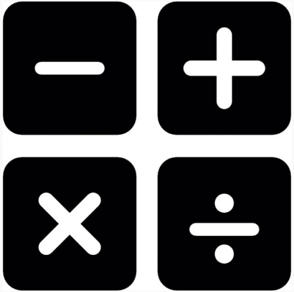

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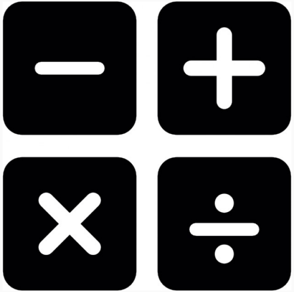

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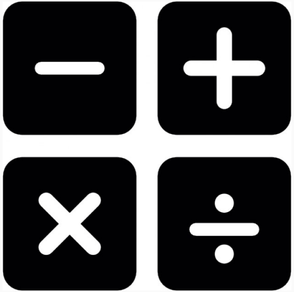

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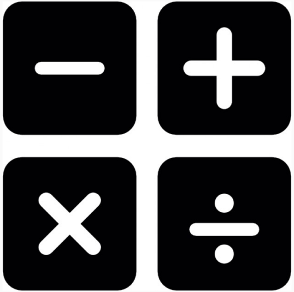
    - e.g. sum, average, standard deviation



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Context!

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Example

- *Age*, measured in years (quantitative) vs. categories of child, adult, senior, etc.

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8. Number of correct answers on an exam

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  - Quantitative: integers, e.g. SAT Score, number of children

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1. Weight in pounds

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What kind of data (discrete or continuous) does each variable describe ?

1. Weight in pounds
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Discrete or Continous?

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- It is good practice to have an **ID variable** to count and keep track of each observation

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  - Simplest and most common data
  - A "snapshot" to compare differences across individuals

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## 1 1950  8.2         0.06 100
## 2 1960  9.9         0.04 118
## 3 1970 10.2         0.08 130
## 4 1980 12.4         0.08 190
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    - A “moving picture” to see how individuals change over time

Datasets: Panel Data

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## 2    Philadelphia 1990       8      4.200          7.2    471
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    - A combination of “snapshot” comparisons and differences over time

Basic Statistics

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- We want to *visualize* and *analyze* distributions to search for meaningful patterns using **statistics**

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        1.2 Inferential Statistics: uses a sample in order to infer properties about a larger population

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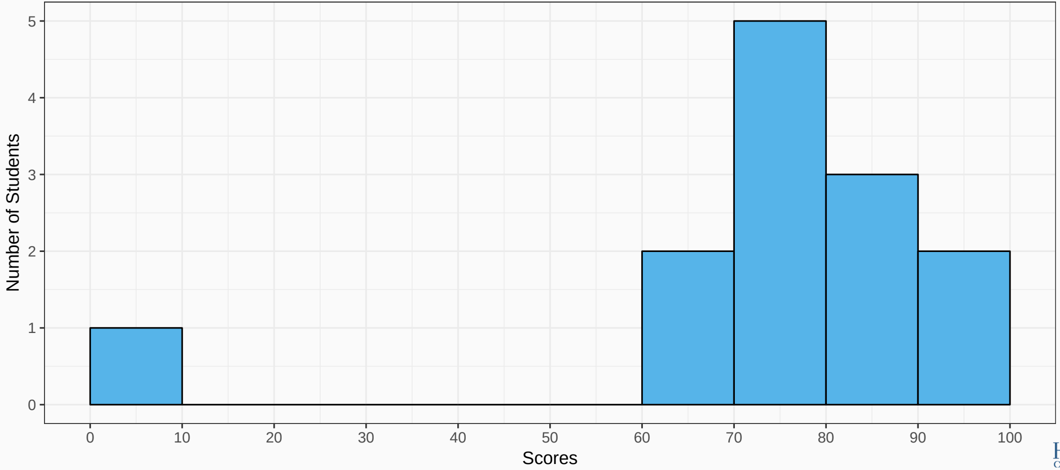
  - The quantitative analog to the bar graph for a categorical variable
- We divide up the data values into **bins** of a certain size, and count the number of values falling within each bin, representing them visually as bars

Example

A class of 13 students takes a quiz (out of 100 points) with the following results:

$$\{0, 62, 66, 71, 71, 74, 76, 79, 83, 86, 88, 93, 95\}$$

Histogram: Example



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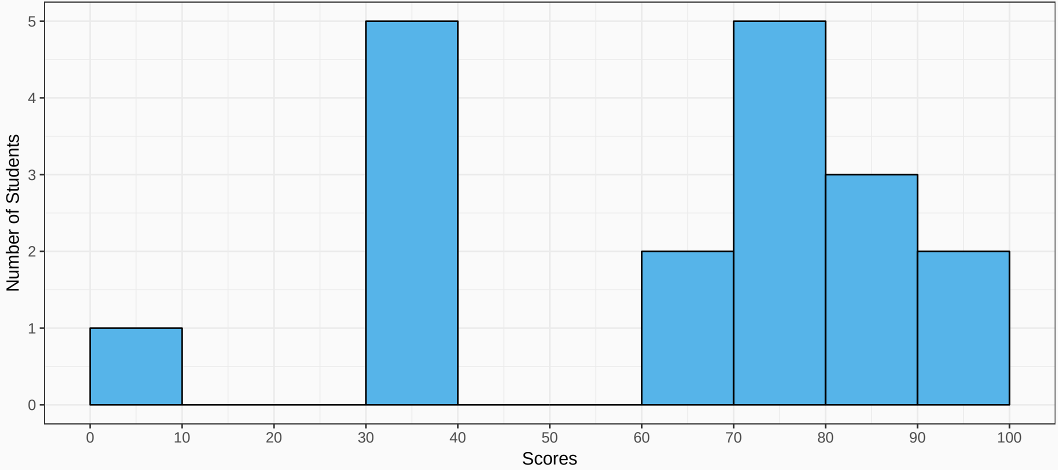
- The mode of a variable is simply its most frequent value
- A variable can have multiple modes

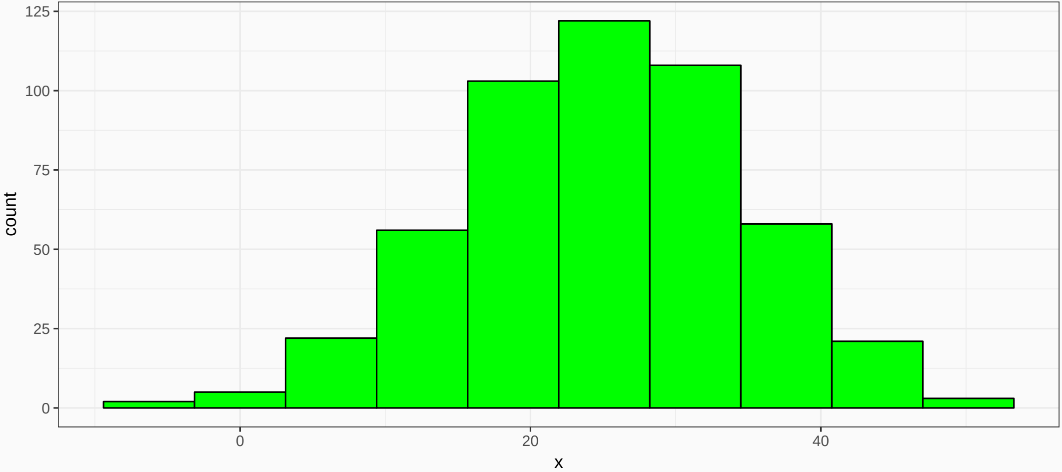
Example
{0, 62, 66, 71, 71, 74, 76, 79, 83, 86, 88, 93, 95}

Multi-Modal Distributions



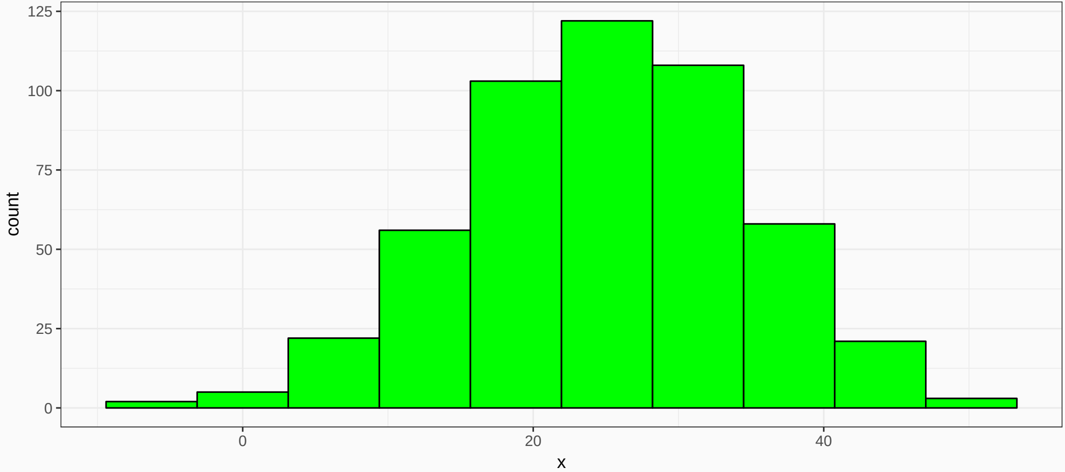
- Looking at a histogram, the modes are often the "peaks" of the distribution - May be **unimodal**, **bimodal**, **trimodal**, etc

Symmetry

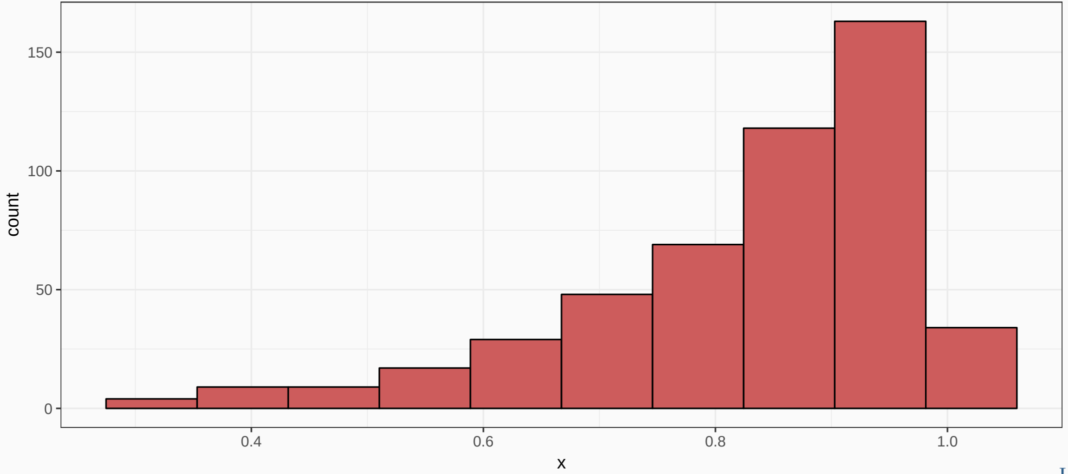


- A distribution is **symmetric** if it looks roughly the same on either side of the "center"

Symmetry

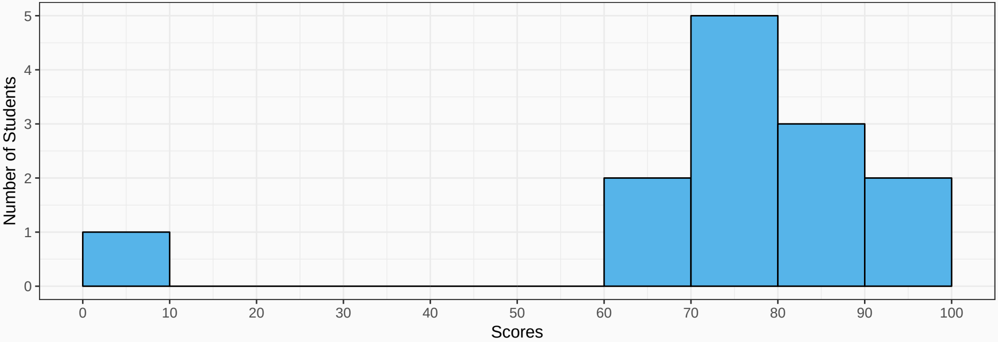


- A distribution is **symmetric** if it looks roughly the same on either side of the "center"
- The thinner ends (far left and far right) are called the **tails** of a distribution



- If one tail stretches farther than the other, distribution is **skewed** in the direction of the longer tail

- An extreme value that does not appear part of the general pattern of a distribution is an outlier



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- Outliers could be the result of errors
- Outliers should always be discussed in presentations about data

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Arithmetic Mean (Population)

- The natural measure of the center of a *population*'s distribution is its "average" or arithmetic mean (μ)

$$\mu = \frac{x_1 + x_2 + \ldots + x_N}{N} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

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$$\mu = \frac{x_1 + x_2 + \ldots + x_N}{N} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

- For N values of variable x, "mu" is the sum of all individual x values (x_i) from 1 to N, divided by the N number of values

Arithmetic Mean (Sample)

- When we have a *sample*, we compute the sample mean ($\bar{x}$)

$$\bar{x} = \frac{x_1 + x_2 + \ldots + x_n}{n} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

-For n values of variable x, “x-bar” is the sum of all individual x values (x_i) from 1 to n, divided by the n number of values

Sample Mean Example

$$\{0, 62, 66, 71, 71, 74, 76, 79, 83, 86, 88, 93, 95\}$$

$$\begin{aligned}
\bar{x} &= \frac{1}{13}(0 + 62 + 66 + 71 + 71 + 74 + 76 + 79 + 83 + 86 + 88 + 93 + 95) \\
&= \frac{944}{13} \\
&= 72.61
\end{aligned}$$

- Note the mean need not be an actual value of the data!

Sample Mean and Outliers

$$\{62, 66, 71, 71, 74, 76, 79, 83, 86, 88, 93, 95\}$$

- If we drop the outlier (0)

$$\begin{aligned}\bar{x} &= \frac{1}{12}(62 + 66 + 71 + 71 + 74 + 76 + 79 + 83 + 86 + 88 + 93 + 95) &= \frac{944}{12} \\ &= 78.67\end{aligned}$$

Median

$$\{0, 62, 66, 71, 71, 74, 76, 79, 83, 86, 88, 93, 95\}$$

- The median is the midpoint of the distribution

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- Arrange values in numerical order

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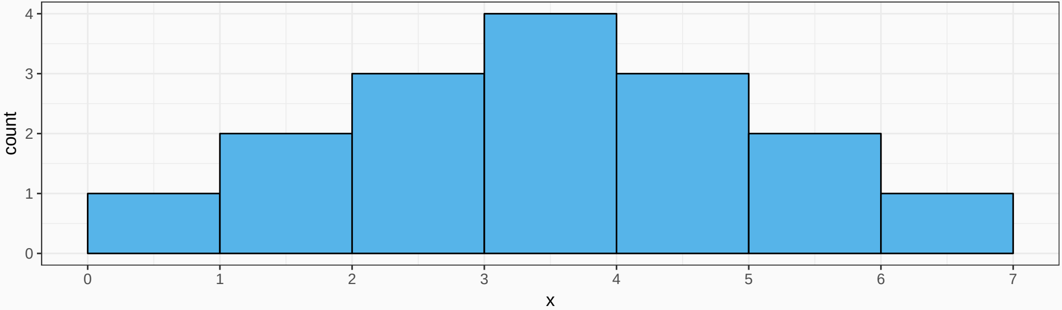
- 50% to the left of the median, 50% to the right of the median
- Arrange values in numerical order
    - For odd n: median is middle observation
    - For even n: median is average of two middle observations

$$\{0, 62, 66, 71, 71, 74, 76, 79, 83, 86, 88, 93, 95\}$$

- The median is *robust* to outliers (if 0 changes to 62)

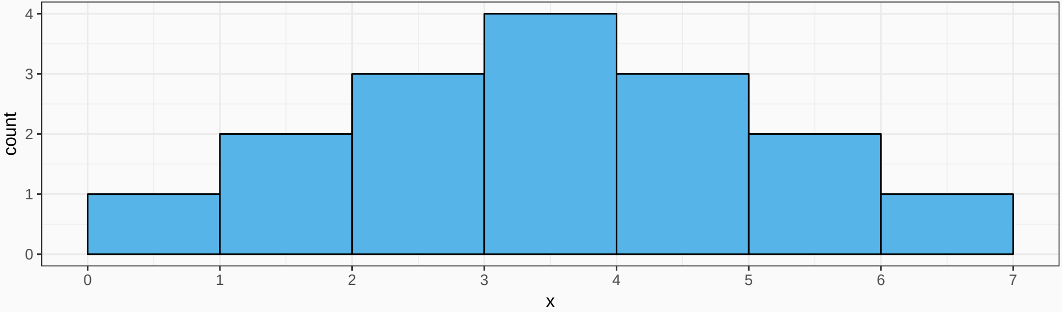
$$\{62, 62, 66, 71, 71, 74, 76, 79, 83, 86, 88, 93, 95\}$$

$$\{1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 6, 6, 7\}$$



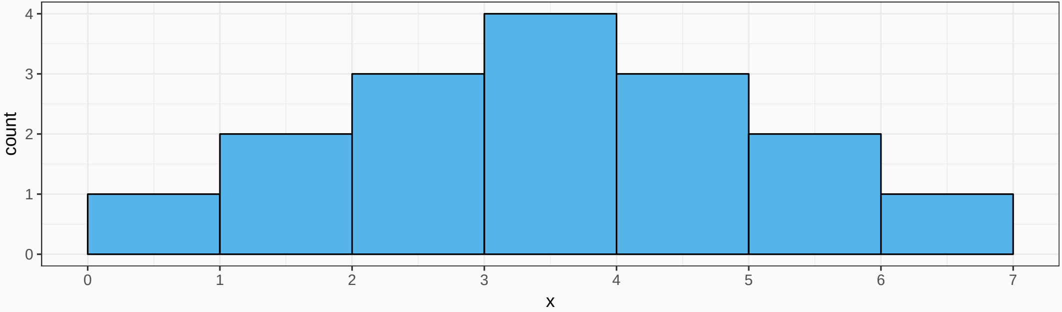
- Mean: $\frac{64}{16} = 4$

$$\{1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 6, 6, 7\}$$



- Mean: $\frac{64}{16} = 4$
- Median: 4

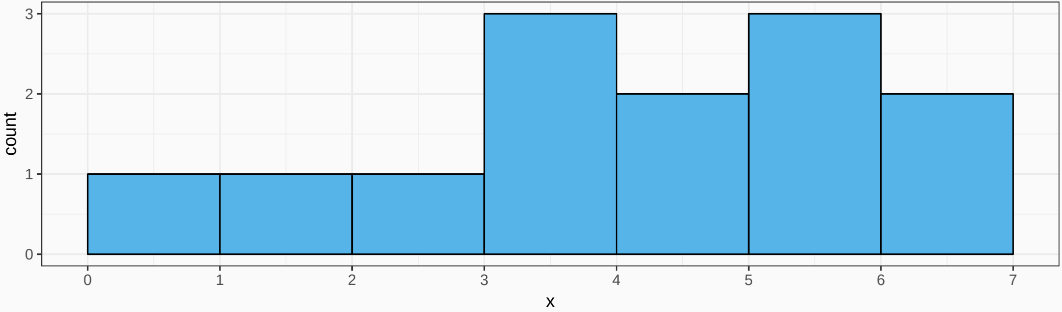
$$\{1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 6, 6, 7\}$$



- Mean: $\frac{64}{16} = 4$
- Median: 4
- For a symmetric distribution, mean=median

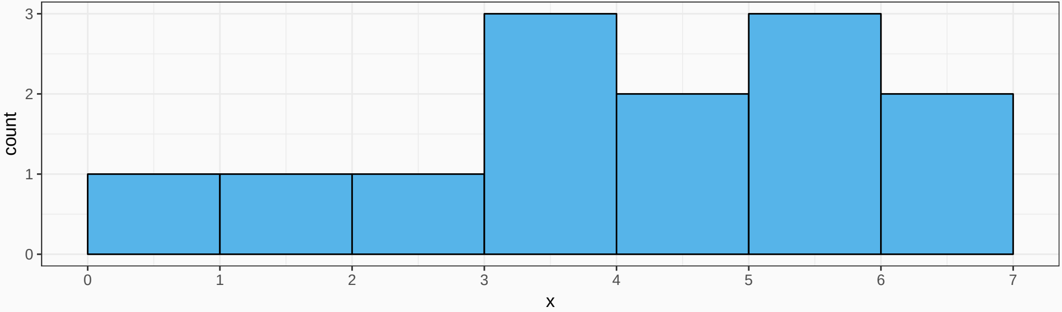
Mean, Median, Symmetry, Skew II

$$\{1, 2, 3, 4, 4, 4, 5, 5, 6, 6, 6, 7, 7\}$$



- Mean: $\frac{60}{13} = 4.6$

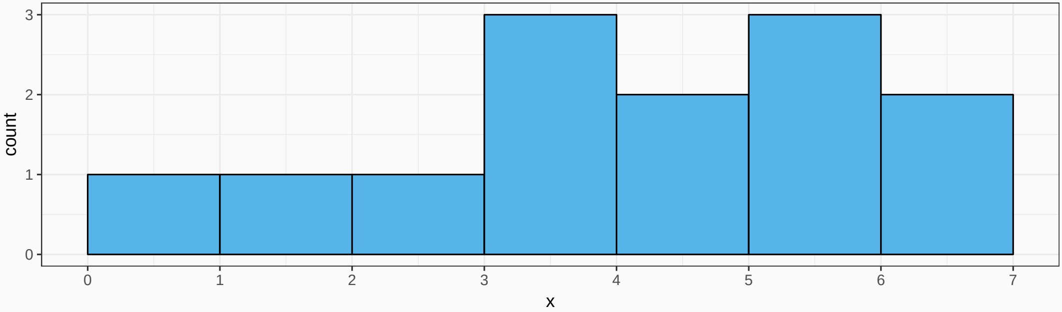
$$\{1, 2, 3, 4, 4, 4, 5, 5, 6, 6, 6, 7, 7\}$$



- Mean: $\frac{60}{13} = 4.6$
- Median: 5

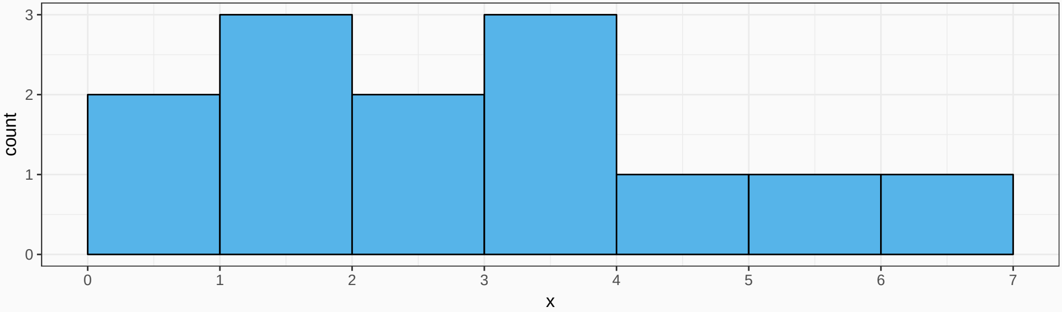
Mean, Median, Symmetry, Skew II

$$\{1, 2, 3, 4, 4, 4, 5, 5, 6, 6, 6, 7, 7\}$$



- Mean: $\frac{60}{13} = 4.6$
- Median: 5
- For a left-skewed distribution, mean$<$median

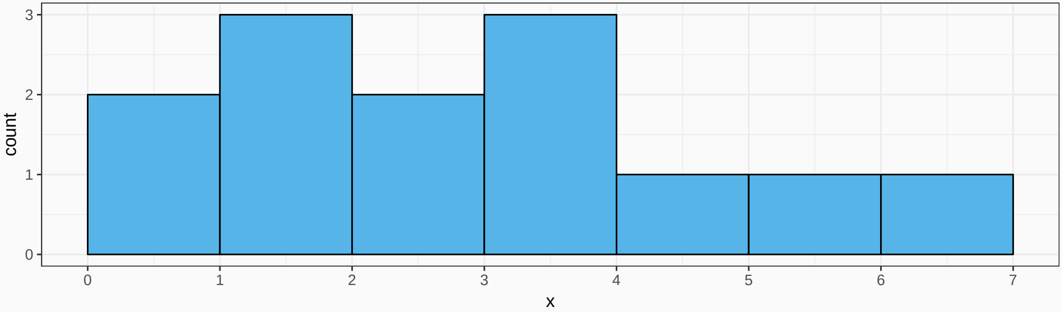
$$\{1, 1, 2, 2, 2, 3, 3, 4, 4, 4, 5, 6, 7\}$$



- Mean: $\frac{44}{13} = 3.4$

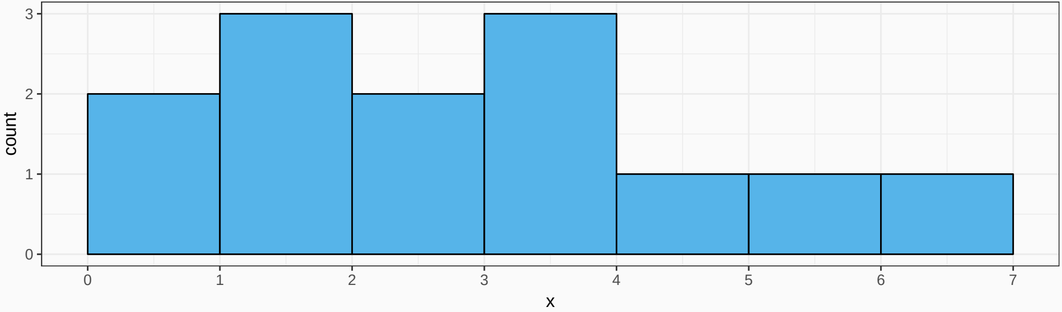
Mean, Median, Symmetry, Skew III

$$\{1, 1, 2, 2, 2, 3, 3, 4, 4, 4, 5, 6, 7\}$$



- Mean: $\frac{44}{13} = 3.4$
- Median: 3

$$\{1, 1, 2, 2, 2, 3, 3, 4, 4, 4, 5, 6, 7\}$$



- Mean: $\frac{44}{13} = 3.4$
- Median: 3
- For a right-skewed distribution, mean$>$median

- The more *variation* in the data, the less helpful a measure of central tendency will tell us

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- Beyond just the center, we also want to measure the spread

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- Beyond just the center, we also want to measure the spread
- Simplest metric is range=max-min

Once we know the values of the quartiles, we can construct a five-number summary of a distribution, including: 1. Minimum 2. Q_1 (25%) 3. Median (50%) 4. Q_3 (75%) 5. Maximum

Five Number Summary II

```r
summary(quizzes$scores)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##    0.00   71.00   76.00   72.62   86.00   95.00
```

- Graphical way to visualize five number summary is a boxplot

Boxplots

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  - The length of the box is the IQR (Q1-Q3)

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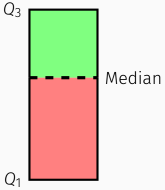
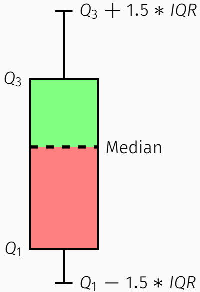
    - The line within the box is the median

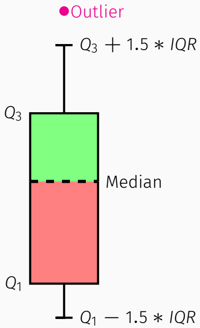


- Graphical way to visualize five number summary is a boxplot
    - The length of the box is the IQR (Q1-Q3)
    - The line within the box is the median
    - The "whiskers" identify data within $1.5 \times IQR$



Boxplots

- Graphical way to visualize five number summary is a boxplot
  - The length of the box is the IQR (Q1-Q3)
  - The line within the box is the median
  - The “whiskers” identify data within $1.5 \times IQR$
  - Points beyond the whiskers are outliers



Boxplots II

Quiz 1: {0, 62, 66, 71, 71, 74, 76, 79, 83, 86, 88, 93, 95}

Quiz 2: {50, 62, 72, 73, 79, 81, 82, 82, 86, 90, 94, 98, 99}

Quiz 1: {0, 62, 66, 71, 71, 74, 76, 79, 83, 86, 88, 93, 95}

Quiz 2: {50, 62, 72, 73, 79, 81, 82, 82, 86, 90, 94, 98, 99}

Quiz 1

Min	Q_1	Median	Q_3	Max
0	71	76	86	95

Boxplots II

Quiz 1: {0, 62, 66, 71, 71, 74, 76, 79, 83, 86, 88, 93, 95}

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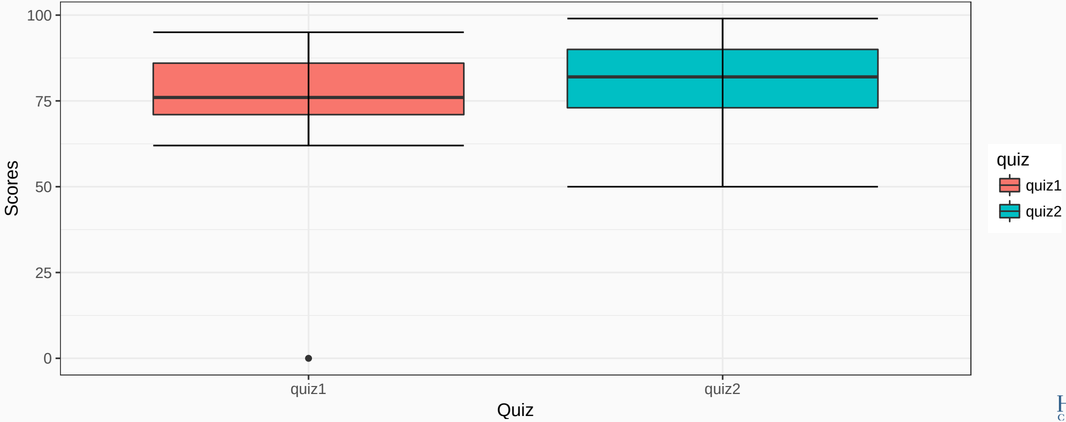
Quiz 1

Min	Q_1	Median	Q_3	Max
0	71	76	86	95

Quiz 2

Min	Q_1	Median	Q_3	Max
50	73	82	90	99

quiz-1.bb quiz-1.bb



Deviations

- Each observation deviates from the mean of the data:

$$deviation = x_i - \mu$$

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- There are as many deviations as there are data points (n)

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- There are as many deviations as there are data points (n)
- We can measure the *average* or standard deviation from the mean

- The population variance (σ^2) of a *population* distribution measures the average of the *squared* deviations from the population mean

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2$$

Variance

- The population variance (σ^2) of a *population* distribution measures the average of the *squared* deviations from the population mean

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2$$

- Why do we square deviations?

-Square root the variance to get the population standard deviation (σ), the average deviation from the mean (in x units)

$$\sigma = \sqrt{\sigma^2} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2}$$

- The sample variance (s^2) of a *sample* distribution measures the average of the *squared* deviations from the sample mean

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

-Why divide by $n - 1$?

Sample Standard Deviation

- Square root the variance to get the sample standard deviation (s), the average deviation from the mean (in x units)

$$s = \sqrt{s^2} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

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Example
Calculate the sample standard deviation for the following series:

$$\{2, 4, 6, 8, 10\}$$

Population Parameters

- Population Size: N
- Mean: μ
- Variance:
  $$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2$$
- Standard Deviation:
  $$\sigma = \sqrt{\sigma^2}$$

Sample Statistics

- Sample Size: n
- Mean: $\bar{x}$
- Variance:
  $$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$
- Standard Deviation: $s = \sqrt{s^2}$