Lecture 6: Correlation and Linear Regression Basics

ECON 480 - Econometrics - Fall 2018

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Covariance and Correlation

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    - Immediate aim is to explore associations between variables, quantified with **correlation** and **linear regression**
    - Later we want to develop more sophisticated tools to argue for **causation**

```r
econfreedom<-read.csv("~/Dropbox/Teaching/Hood College/ECON 480 - Econometrics/D
head(econfreedom)
```

```
##   ISO.Code         Country Economic.Freedom.Summary.Index GDP.Per.Capita
## 1      AGO          Angola                           5.08       4153.146
## 2      ALB         Albania                           7.40       4543.088
## 3      ARE Unit. Arab Em.                            7.98      39313.274
## 4      ARG       Argentina                           4.81      10501.660
## 5      ARM         Armenia                           7.71       3796.517
## 6      AUS       Australia                           7.93      54688.446
```

- **Rows** are individual observations

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Bivariate Data: Spreadsheets

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```

- **Rows** are individual observations
- **Columns** are variables on all individuals
- Let X be Economic Freedom and Y be GDP per capita

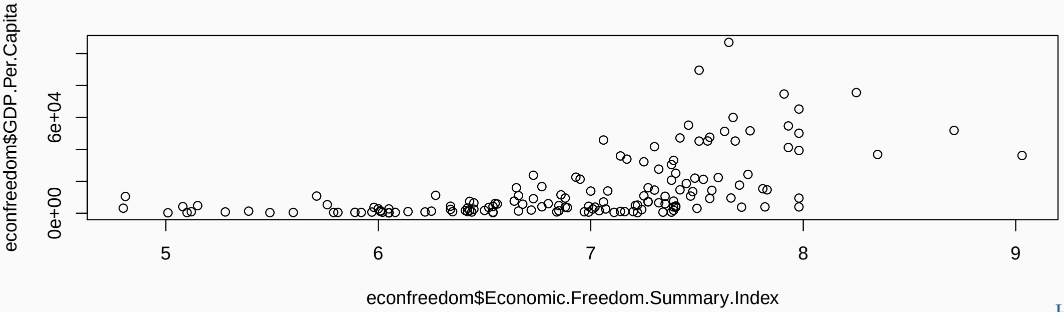
```r
str(econfreedom)
```

```
## 'data.frame':    152 obs. of  4 variables:
##  $ ISO.Code                        : Factor w/ 152 levels "AGO","ALB","ARE",..:
##  $ Country                         : Factor w/ 152 levels "Albania","Algeria",.
##  $ Economic.Freedom.Summary.Index: num  5.08 7.4 7.98 4.81 7.71 7.93 7.56 6.5
##  $ GDP.Per.Capita                  : num  4153 4543 39313 10502 3797 ...
```

```r
summary(econfreedom)
```

```
##      ISO.Code          Country      Economic.Freedom.Summary.Index
##  AGO     :  1   Albania   :  1   Min.   :4.800
##  ALB     :  1   Algeria   :  1   1st Qu.:6.430
##  ARE     :  1   Angola    :  1   Median :7.050
##  ARG     :  1   Argentina:  1   Mean   :6.909
##  ARM     :  1   Armenia   :  1   3rd Qu.:7.428
##  AUS     :  1   Australia:  1   Max.   :9.030
##  (Other):146   (Other)   :146
##  GDP.Per.Capita
##  Min.   :   206.7
##  1st Qu.:  1588.3
##  Median :  5719.3
```

```r
# syntax for plotting is similar to hist() and boxplot()
# just tell R "plot(df$x,df$y)"
plot(econfreedom$Economic.Freedom.Summary.Index, econfreedom$GDP.Per.Capita)
```



- The best way to visualize an association between two variables is with a scatterplot

```r
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# just tell R "plot(df$x,df$y)"
plot(econfreedom$Economic.Freedom.Summary.Index, econfreedom$GDP.Per.Capita)
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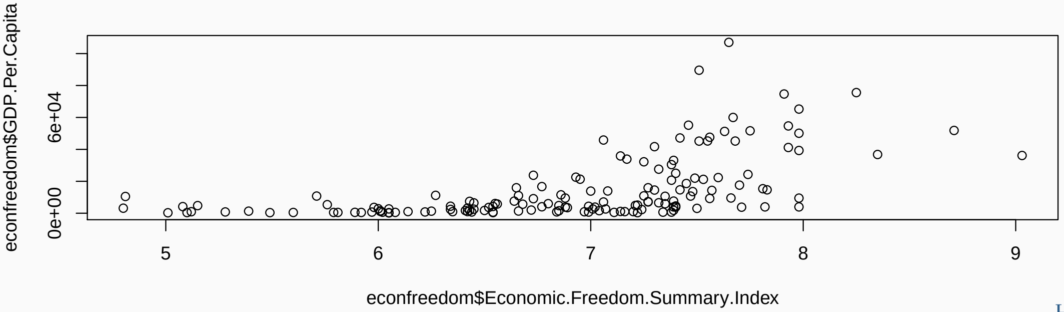
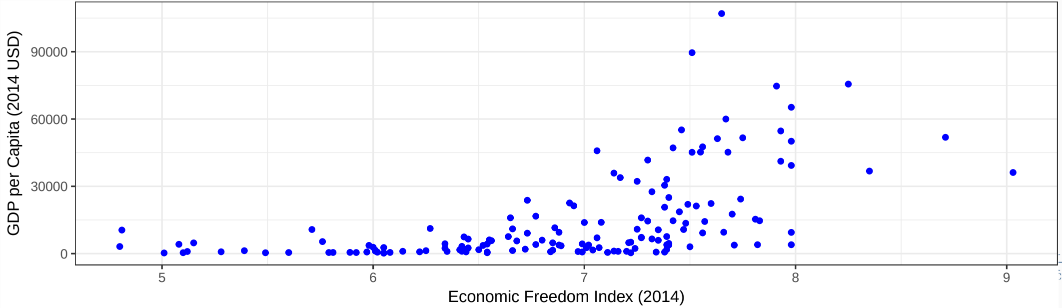


- Each point is a pair of variable values (X_i, Y_i) for observation i

Bivariate Data: A Better-Looking Scatterplot (with ggplot2)

```
library("ggplot2")
ggplot(econfreedom, aes(x=Economic.Freedom.Summary.Index,y=GDP.Per.Capita))+
  geom_point(color="blue")+theme_bw()+
  xlab("Economic Freedom Index (2014)")+ylab("GDP per Capita (2014 USD)")
```



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    2. *Form*: is the trend linear, quadratic, something else, or no pattern?
    3. *Strength*: is the association strong or weak?
    4. *Outliers*: do any observations break the trends above?

- For any two variables, we can measure their sample covariance, $cov(X, Y)$ or $s_{X,Y}$ to quantify how they vary *together*[1]

$$s_{X,Y} = E\left[(X - \bar{X})(Y - \bar{Y})\right]$$

[1] Henceforth we limit to samples, for convenience. Population covariance is denoted $\sigma_{X,Y}$

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    - to be *above* its mean also (X and Y covary *positively*)
    - to be *below* its mean (X and Y covary *negatively*)
- Covariance is a common measure, but the units are meaningless, thus we rarely need to use it so **don't worry about learning the formula**

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- More convenient to standardize covariance into a more intuitive concept: correlation (ρ or r), normalized to be between -1 and 1

$$r_{X,Y} = \frac{s_{X,Y}}{s_X s_Y} = \frac{cov(X,Y)}{sd(X)sd(Y)}$$

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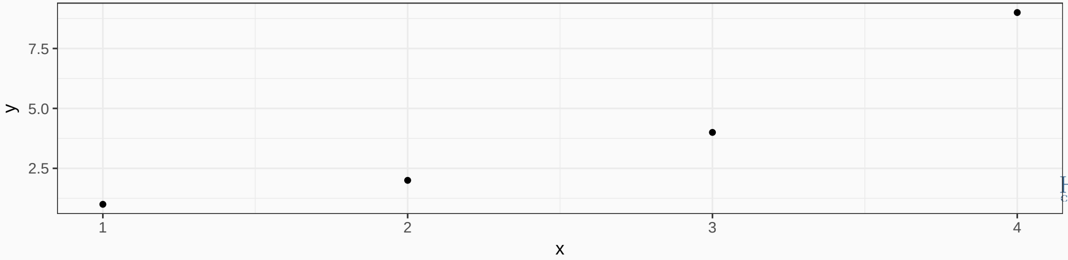
- Alternatively, sample correlation can be found by standardizing (finding the Z-score) X and Y and multiplying, for each (X,Y) pair, and then averaging (over $n-1$, due to sampling df, again):

$$r = \frac{1}{n-1}\sum_{i=1}^{n}\left(\frac{X_i-\bar{X}}{s_X}\right)\left(\frac{Y_i-\bar{Y}}{s_Y}\right)$$
$$= \frac{1}{n-1}\sum_{i=1}^{n} Z_X Z_Y$$

Example

$$(1,1),(2,2),(3,4),(4,9)$$

```
corr.example<-data.frame(x=c(1,2,3,4),
                         y=c(1,2,4,9))
ggplot(corr.example,aes(x=x,y=y))+geom_point()
```



```r
mean(corr.example$x) #find mean of x
```

```
## [1] 2.5
```

```r
mean(corr.example$y) #find mean of y
```

```
## [1] 4
```

```r
sd(corr.example$x) #find sd of x
```

```
## [1] 1.290994
```

```r
sd(corr.example$y) #find sd of y
```

```
## [1] 3.559026
```

Correlation: Example Calculation III

```r
#take z score of x,y for each pair and multiply them
corr.example$z.product<-(((corr.example$x-2.5)/1.291)*
                              ((corr.example$y-4)/3.559))

corr.example
```

```
##   x y z.product
## 1 1 1 0.9793959
## 2 2 2 0.2176435
## 3 3 4 0.0000000
## 4 4 9 1.6323265
```

```r
(sum(corr.example$z.product)/3) #average z products over n-1
```

```
## [1] 0.943122
```

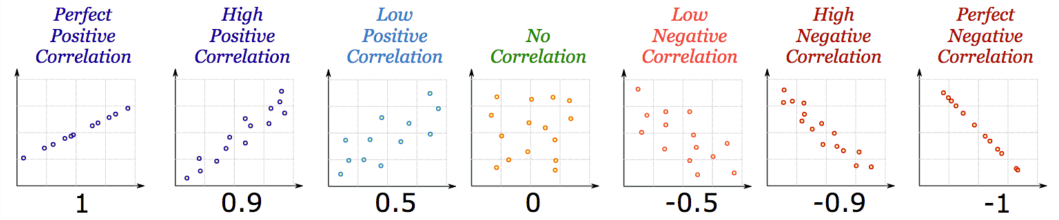
```r
cor(corr.example$x, corr.example$y) #compare our answer to cor() command
```

```
## [1] 0.9431191
```

```r
cov(corr.example$x, corr.example$y) #just for kicks - covariance
```

```
## [1] 4.333333
```

- Correlation is standardized to $-1 \leq r \leq 1$



Correlation: Interpretation

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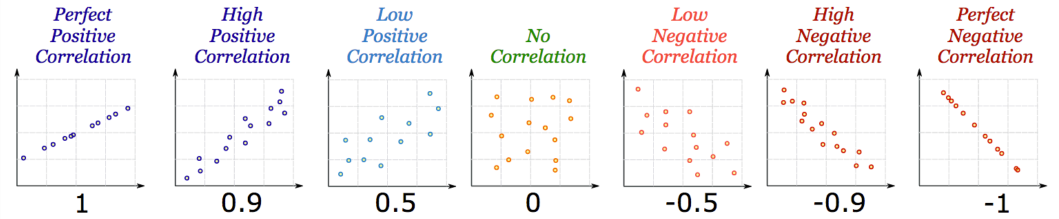
  - Negative values $\implies$ negative association



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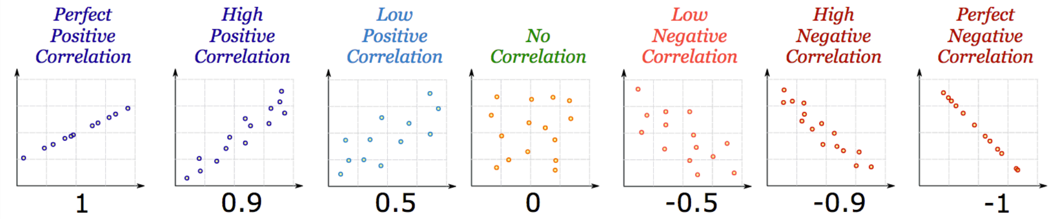

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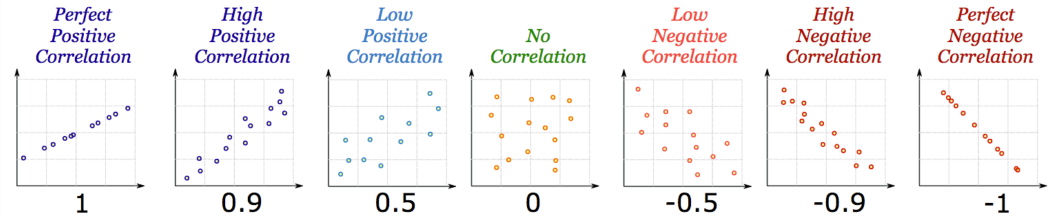

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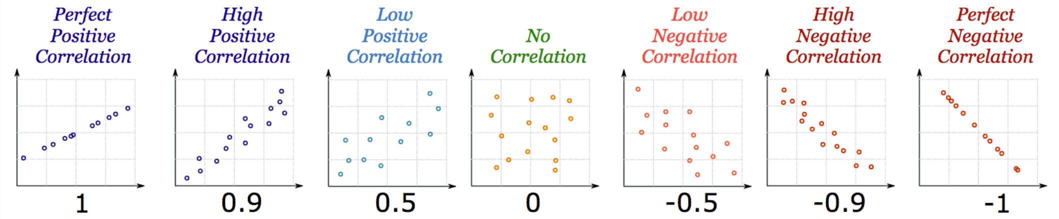

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  - Negative values $\implies$ negative association
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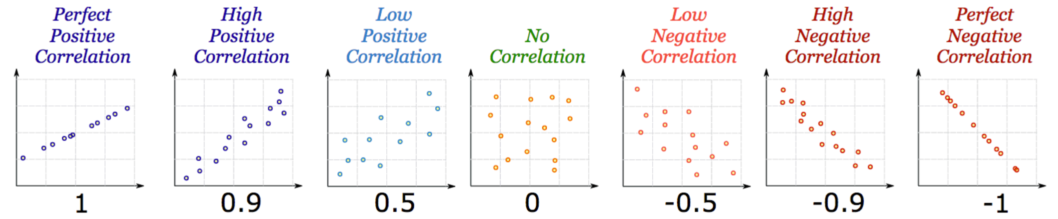
  - As $|r| \to 1 \implies$ the stronger the association
  - Correlation of $|r| = 1 \implies$ a perfect linear relationship



Guess The Correlation Game

- Reminder: Correlation does not imply causation!

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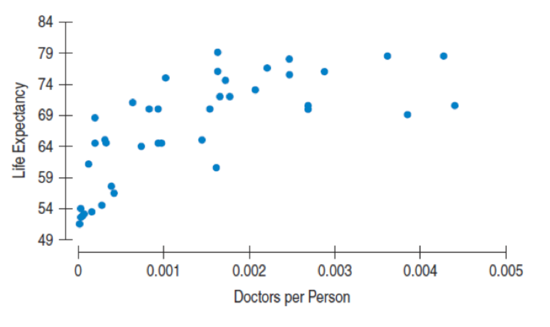
- See the **Handout** for more on Covariance and Correlation

Example



- The correlation between Life Expectancy and Doctors Per Person is 0.705.

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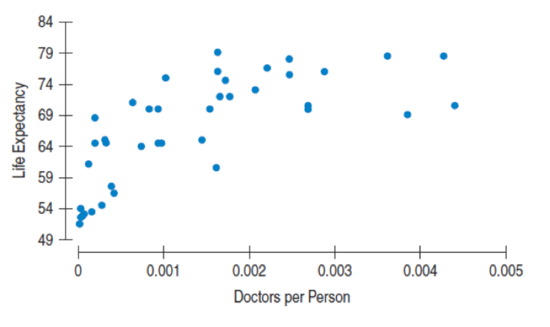

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- So should we send more doctors to developing countries to increase their life expectancy?

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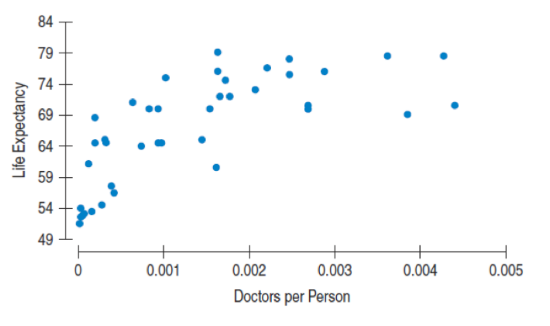

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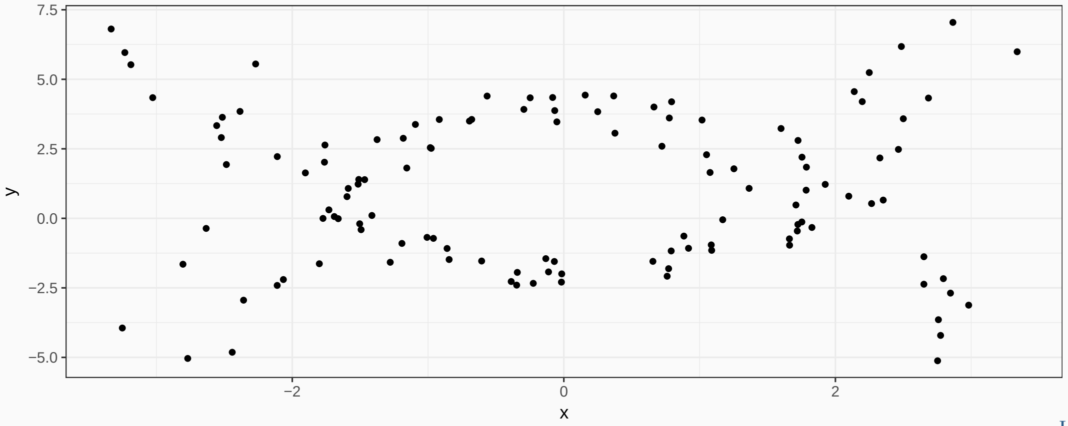
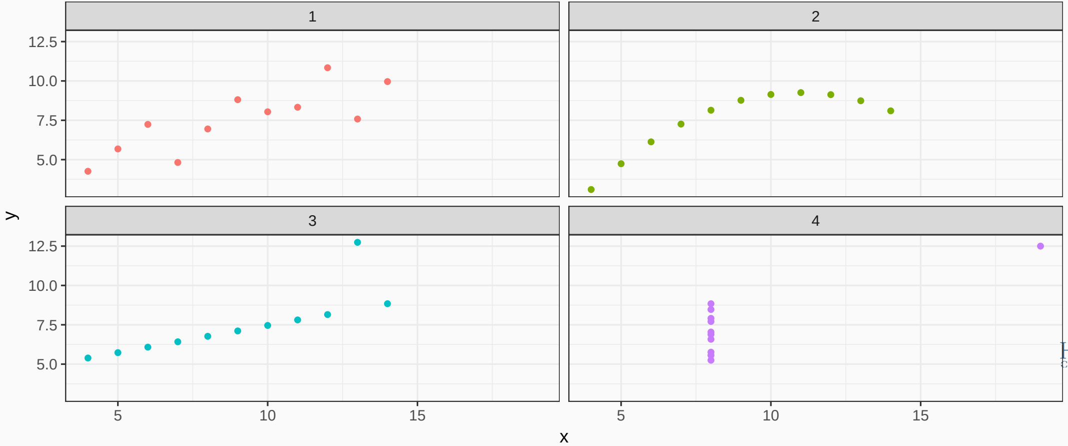
- So should we send more doctors to developing countries to increase their life expectancy?
- Properly interpreting relationships requires both statistical *and* economic intuition!

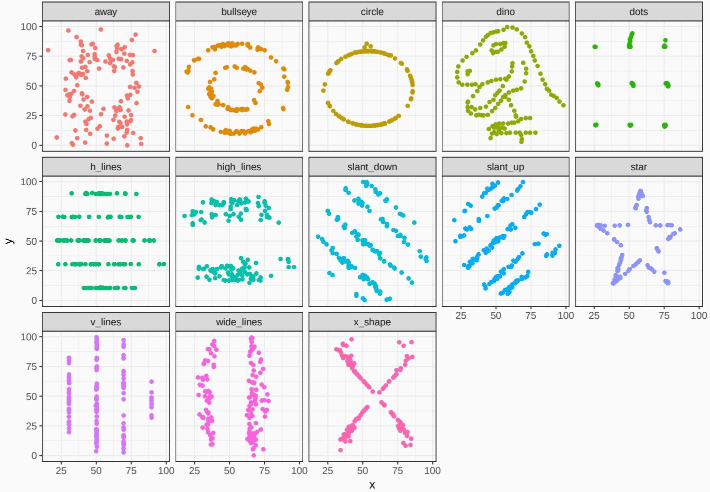
Always Plot Your Data!



```
##     Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##      4.0     6.5     9.0     9.0    11.5    14.0
```



```
##    dataset                   x                y
##  Length:1846        Min.   :15.56   Min.   : 0.01512
##  Class :character   1st Qu.:41.07   1st Qu.:22.56107
##  Mode  :character   Median :52.59   Median :47.59445
##                     Mean   :54.27   Mean   :47.83510
##                     3rd Qu.:67.28   3rd Qu.:71.81078
##                     Max.   :98.29   Max.   :99.69468
```



See the Datasaurus

Population Linear Regression Model

- If an association appears linear, we can estimate the equation of a line that would “fit” the data

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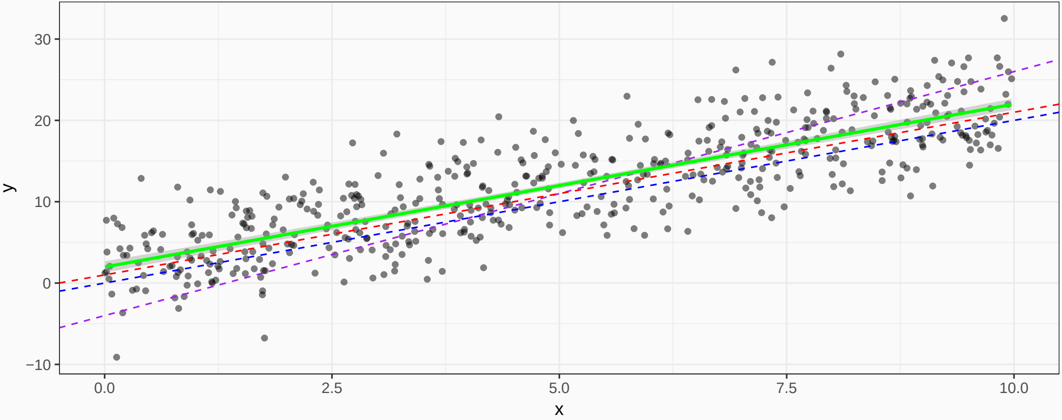
$$Y = a + bX$$

- Recall a linear equation describing a line contains: - a: vertical intercept - b: slope - Note we will use different symbols for a and b, in line with standard econometric notation



- How do we choose the equation that best fits the data? Process is called linear regression

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- We can make **inferences** about the population slope coefficient
  - eventually, a causal interpretation
  - slope $= \frac{\Delta Y}{\Delta X}$: for a 1-unit change in X, how many units will this *cause* Y to change?

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1. Estimation of the marginal effect of X on Y (slope of population regression line)
2. Hypothesis Testing of the value of the marginal effect (slope)
3. Confidence Interval construction of a range for the true effect (slope)

Example
What is the relationship between class size and educational performance?

- Policy question: What is the effect of reducing class sizes by 1 student per class on test scores? 10 students?

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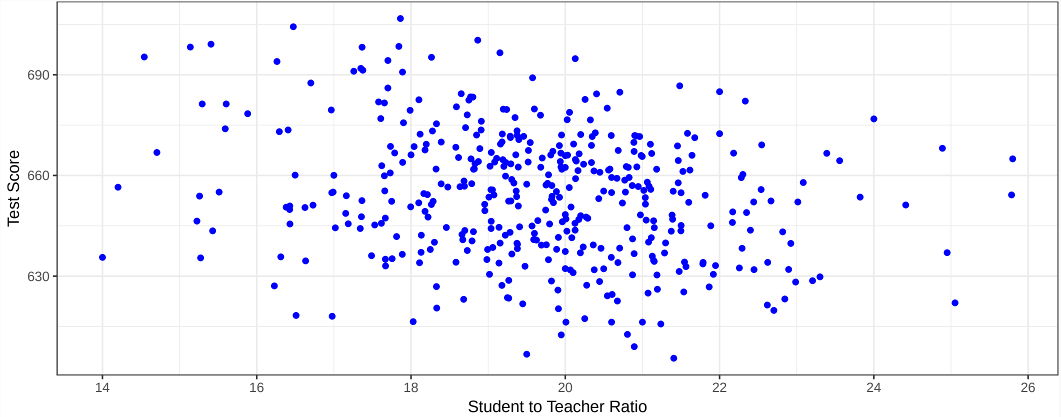
```r
library("foreign") #for importing .dta files
CASchool<-read.dta("~/Dropbox/Teaching/Hood College/ECON 480 - Econometrics/Data

ca.scatter<-ggplot(CASchool, aes(str,testscr))+
  geom_point(color="blue",fill="blue")+
  xlab("Student to Teacher Ratio")+
  ylab("Test Score")+theme_bw()
```

An Extended Example: Scatterplot II



- If we *change* (Δ) the class size by an amount, what would we expect the *change* in test scores to be?

$$\beta_{ClassSize} = \frac{\text{change in test score}}{\text{change in class size}} = \frac{\Delta \text{test score}}{\Delta \text{class size}}$$

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$$\beta_{ClassSize} = \frac{\text{change in test score}}{\text{change in class size}} = \frac{\Delta \text{test score}}{\Delta \text{class size}}$$

- If we knew $\beta_{ClassSize}$, we could say that changing class size by 1 student will change test scores by $\beta_{ClassSize}$

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$$\Delta \text{test score} = \beta_{ClassSize} \times \Delta \text{class size}$$

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An Extended Example: Slope II

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$$\Delta \text{test score} = -0.6$$
$$\Delta \text{test score} = \times - 2 = 1.2$$

An Extended Example: Slope III

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- The line relating class size and test scores has the above equation
    - β_0 is the vertical-intercept, test score where class size is 0
    - $\beta_{ClassSize}$ is the slope of the regression line
- This relationship only holds **on average** for all districts in the population, individual districts are also affected by other factors

- To get an equation that holds for *each* district, we need to include other factors

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- Thus, $\beta_0 + \beta_{ClassSize} \times$ class size gives the **average effect** of class sizes on scores
- Later, we will want to estimate the **marginal effect** (**causal effect**) of each factor on an individual district's test score, holding all other factors constant

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \epsilon$$

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- y is the dependent variable of interest
  - AKA “response variable,” “regressand,” “Left-hand side (LHS) variable”

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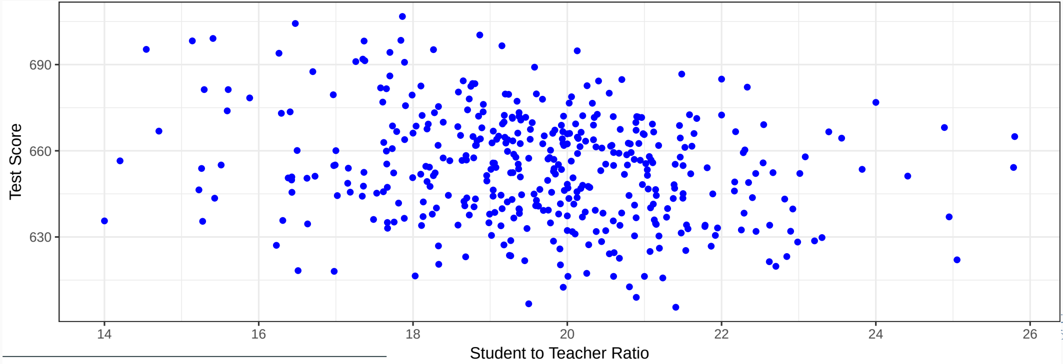
- ϵ is the error term
  - It is **stochastic** (random)
  - We can never measure the error term

The Population Regression Model

- How do we draw a line through the scatterplot? We do not know the true $\beta_{ClassSize}$



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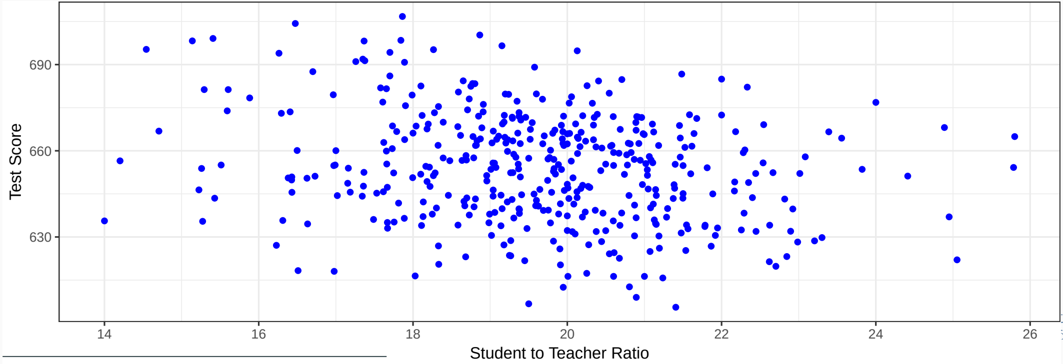

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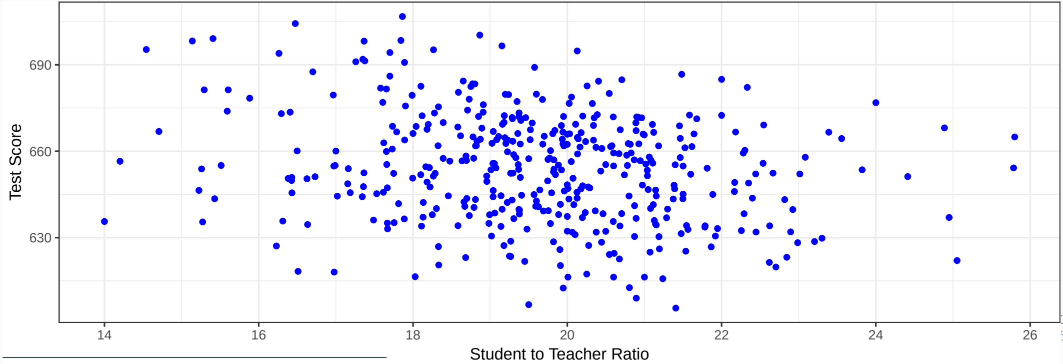
- We do have data from a *sample* of class sizes and test scores[2]
- So the real question is, **how can we estimate β_0 and β_1**?



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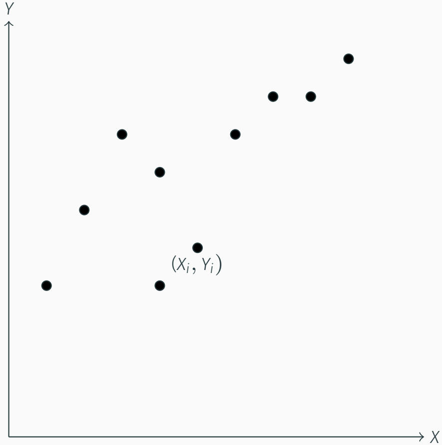
OLS Estimators and Sample Regression Model



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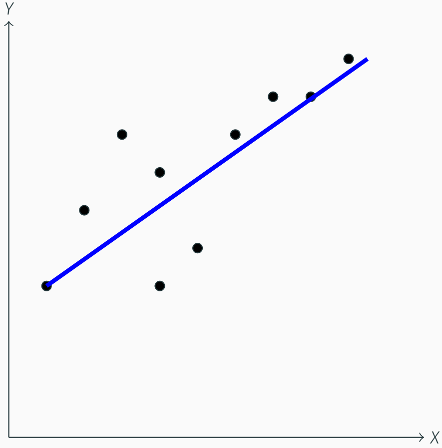

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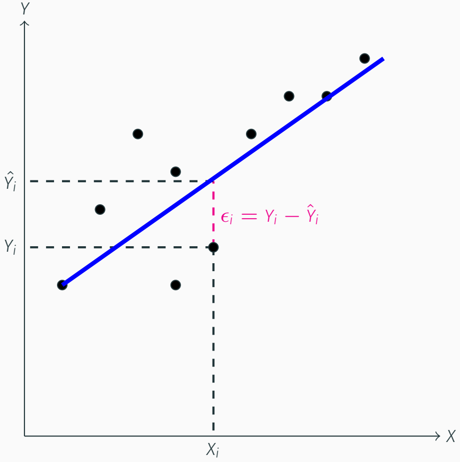

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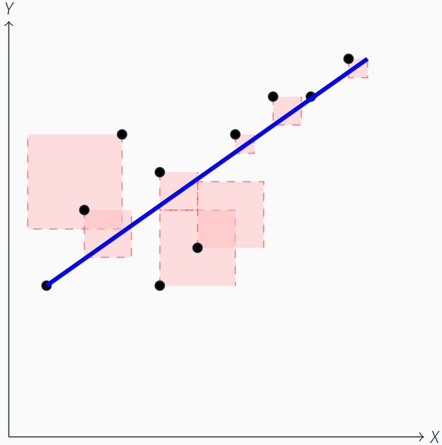
$$\epsilon_i = Y_i - \hat{Y}_i$$



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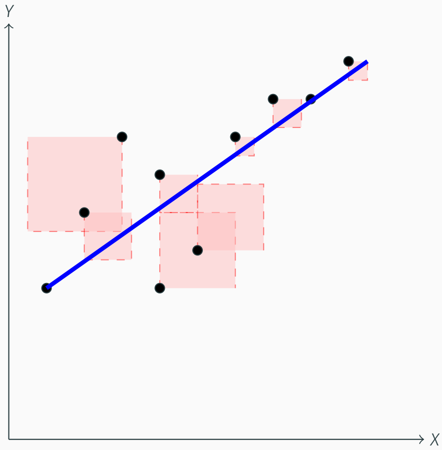

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- The line of best fit **minimizes SSE**

- I coded an example (using an application of R called `shiny`) to demonstrate how OLS tries to solve the problem by picking optimal line parameters

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$$\min_{\beta_0,\beta_1} \sum_{i=1}^{n} [Y_i - \underbrace{(\beta_0 + \beta_1 X_i)}_{\hat{Y}_i}]^2$$

- OLS estimators minimize the average squared distance between the actual values (Y_i) and the predicted values ($\hat{Y}_i$) along the estimated regression line

- The OLS regression line or sample regression line is the linear function constructed using the OLS estimators:

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- The **residual** or **prediction error** for the i^{th} observation is the difference between observed Y_i and its predicted value, $\hat{\epsilon}_i = Y_i - \hat{Y}_i$

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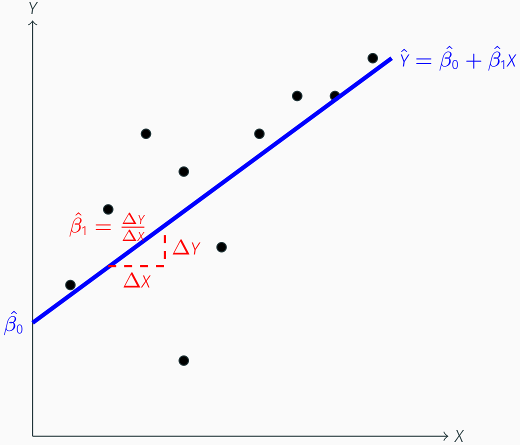
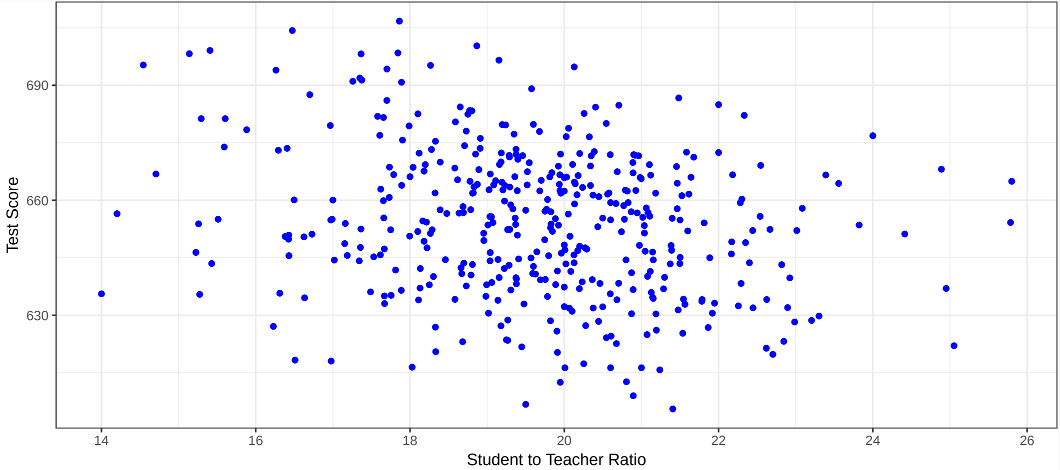
The OLS Regression Estimators II



OLS Example: Class Size



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$$\text{Test Score} = \beta_0 + \beta_1 \times STR$$

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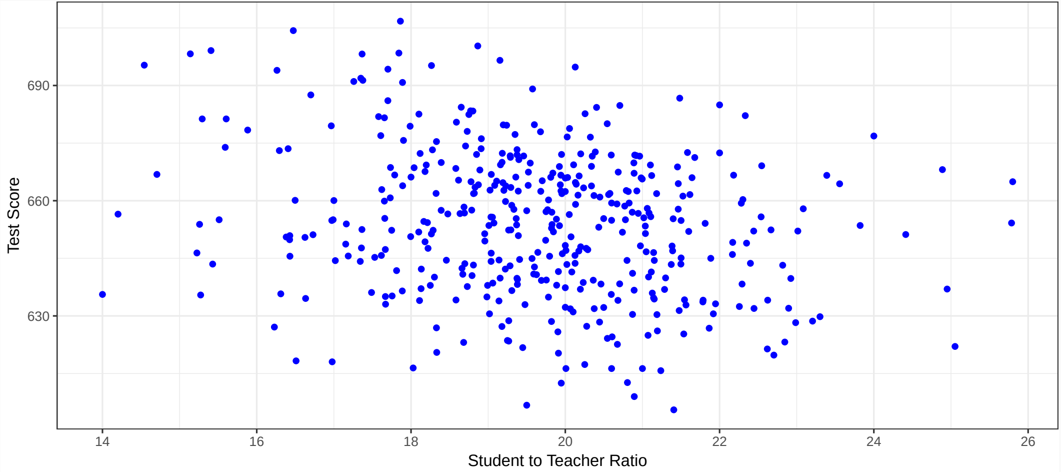

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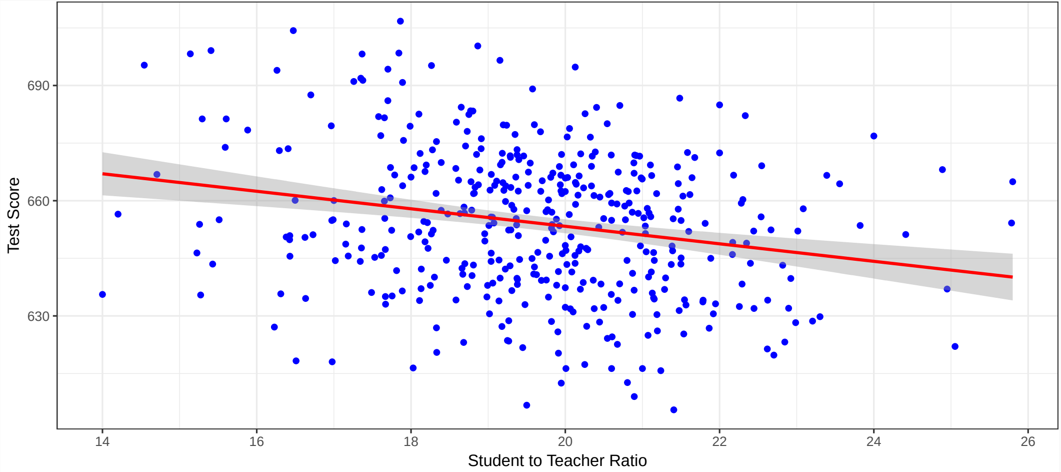
OLS Example: Class Size: OLS Estimation



- Using OLS, we find:

$$\widehat{\text{Test Score}} = 689.9 - 2.28 \times STR$$

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    - Not always economically meaningful
    - Literally: “districts with 0 students have a predicted test score of 689.9”

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- We can now make simple predictions with our model:
  - For a district with 20 students per teacher, the predicted test score is:

$$689.9 - 2.28(20) = 644.3$$

- Is this big or small? How **economically** meaningful is 644?

- Syntax for running a regression in R is simple:

```
# name an object e.g. "regression.name", "lm" stands for "linear model"
regression.name<-lm(y~x, data=data.frame.name)

# get simple (beta) coefficients by calling the object
regression.name

# get more detailed information with summary()
summary(regression.name)
```

```r
school.regression<-lm(testscr~str, data=CASchool)
school.regression
```

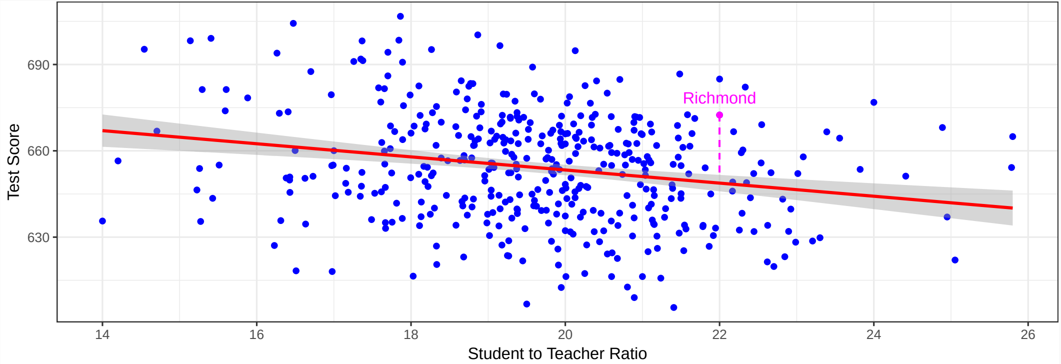
```
##
## Call:
## lm(formula = testscr ~ str, data = CASchool)
##
## Coefficients:
## (Intercept)          str
##      698.93        -2.28
```

OLS Example: Class Size: In R II

```r
summary(school.regression)
```

```
##
## Call:
## lm(formula = testscr ~ str, data = CASchool)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -47.727 -14.251   0.483  12.822  48.540
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 698.9330     9.4675  73.825  < 2e-16 ***
## str          -2.2798     0.4798  -4.751 2.78e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 18.58 on 418 degrees of freedom
## Multiple R-squared:  0.05124,    Adjusted R-squared:  0.04897
## F-statistic: 22.58 on 1 and 418 DF,  p-value: 2.783e-06
```

The Sample OLS Regression Model: A Data Point



- One district in our sample is Richmond, CA with STR=22, Test Score=672

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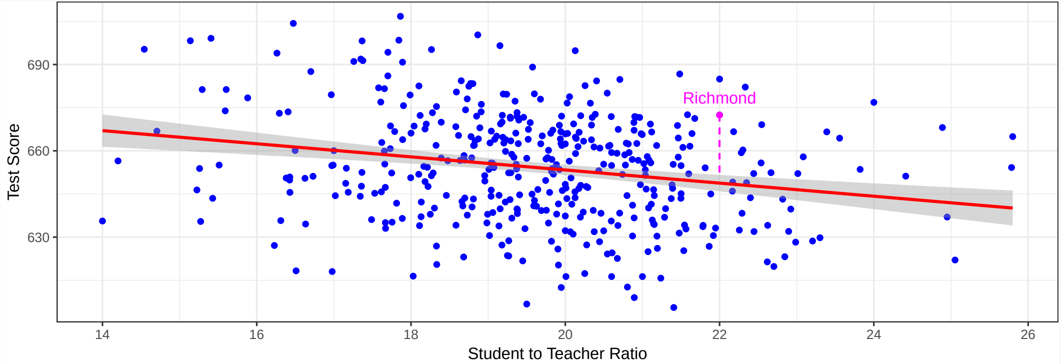

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HOOD COLLEGE

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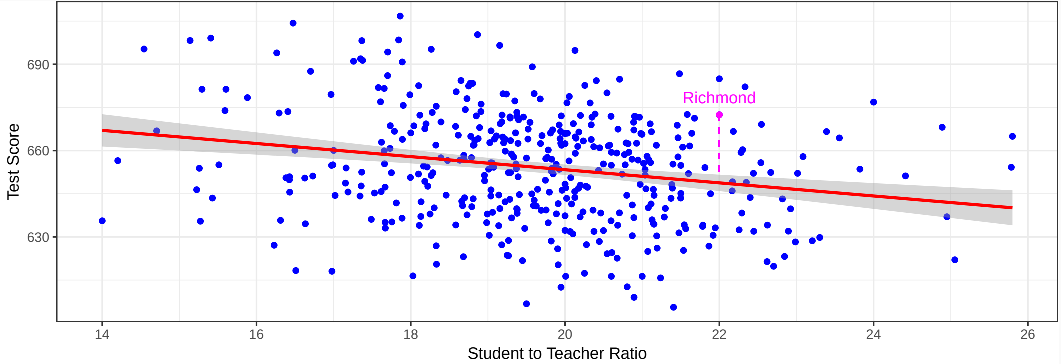

- One district in our sample is Richmond, CA with STR=22, Test Score=672
- Predicted value: $\widehat{\text{Test Score}}_{Richmond} = 698 - 2.28(22) \approx 647$
- Residual: $\widehat{\epsilon_{Richmond}} = 672 - 647 = 25$