Lecture 7: Goodness of Fit and Bias

ECON 480 - Econometrics - Fall 2018

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Goodness of Fit

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- Recall OLS estimators are chosen specifically to minimize SSE $\left(\sum_{i=1}^{n} \hat{\epsilon}_i^{\,2}\right)$

- Primary measure[1] is regression R^2, the fraction of variation in Y explained by variation in predicted values

$$R^2 = \frac{\text{variation in } \widehat{Y_i}}{\text{variation in } Y_i}$$

[1] Sometimes called the "**coefficient of determination**"

Goodness of Fit: R^2 Formula

$$R^2 = \frac{ESS}{TSS}$$

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$$ESS = \sum_{i=1}^{n} (\hat{Y}_i - \bar{Y})^2$$

- Total Sum of Squares (TSS): sum of squared deviations of actual values from their mean[3]

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- Equivalently, the square of the correlation coefficient between X and Y:

$$R^2 = (r_{X,Y})^2$$

- The Standard Error of the Regression[4], $\hat{\sigma}$ or $\hat{\sigma}_\epsilon$ is an estimator of the standard deviation of ϵ_i

$$\hat{\sigma}_\epsilon = \sqrt{\frac{SSE}{n-2}}$$

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  - Note R tells you it calculates this with a df of $n-2$

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Goodness of Fit: Looking at R

```r
summary(school.regression)
```

```
##
## Call:
## lm(formula = testscr ~ str, data = CASchool)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -47.727 -14.251   0.483  12.822  48.540
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 698.9330     9.4675  73.825  < 2e-16 ***
## str          -2.2798     0.4798  -4.751 2.78e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 18.58 on 418 degrees of freedom
## Multiple R-squared:  0.05124,    Adjusted R-squared:  0.04897
## F-statistic: 22.58 on 1 and 418 DF,  p-value: 2.783e-06
```

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- Indicates there are other important factors that also influence test scores
- **Note: it is very rare in econo(metr)ics that we get very high R^2 values**
  - Lots of unobserved variables affecting economic outcomes
  - **Don't get discouraged!** We care about **marginal (causal) effects**, not R^2!

- A lot of regression diagnostics have to do with exploring the residuals a bit more

Measures of Fit: Looking at Residuals

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```
# Save the residuals as a vector called 'res'
CASchool$res <- residuals(school.regression) # use 'res()' function
summary(CASchool$res) # get summary stats of residuals
```

```
##     Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
## -47.7267 -14.2507   0.4826   0.0000  12.8222  48.5404
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```r
# Save the predicted values of the regression as a vector called 'yhat'
CASchool$yhat <- predict(school.regression) # use 'predict()' function
summary(CASchool$yhat) # get summary stats of predictions
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   640.1   651.3   654.0   654.2   656.6   667.0
```

Measures of Fit: Looking at Residuals II

```r
# Remake the scatterplot and point out the residuals
scatterplot.res<-ggplot(CASchool, aes(x=str, y=testscr))+
  geom_point(color="blue")+ # plot original points blue
  geom_point(aes(y=yhat),color="red")+ # plot predicted yhat in red
  geom_segment(aes(xend=str,yend=yhat),linetype=2, alpha=0.5)
# last line connects predicted (yhat) and actual points with dashed line
```

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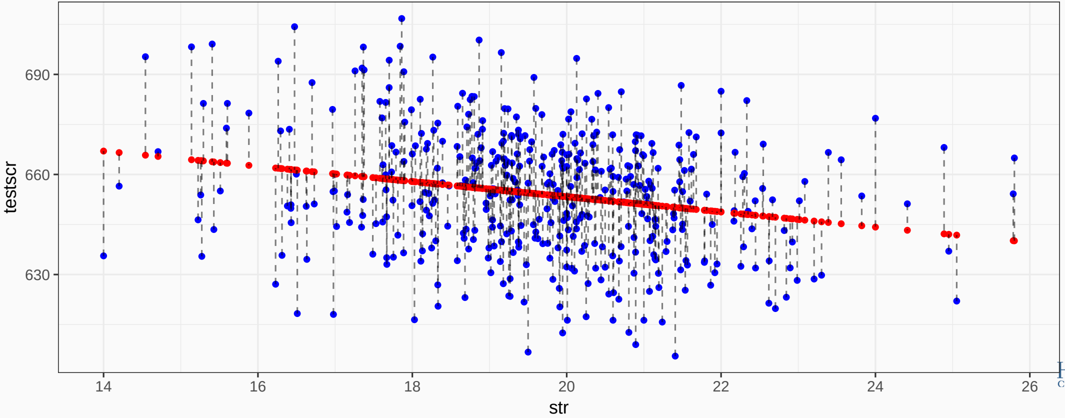
Measures of Fit: Looking at Residuals III

```
scatterplot.res
```



Measures of Fit: Looking at Residuals IV

```r
# A vector of all residuals for each observation is stored in the reg object:
head(school.regression$residuals) #look at first 6 obs residuals
```

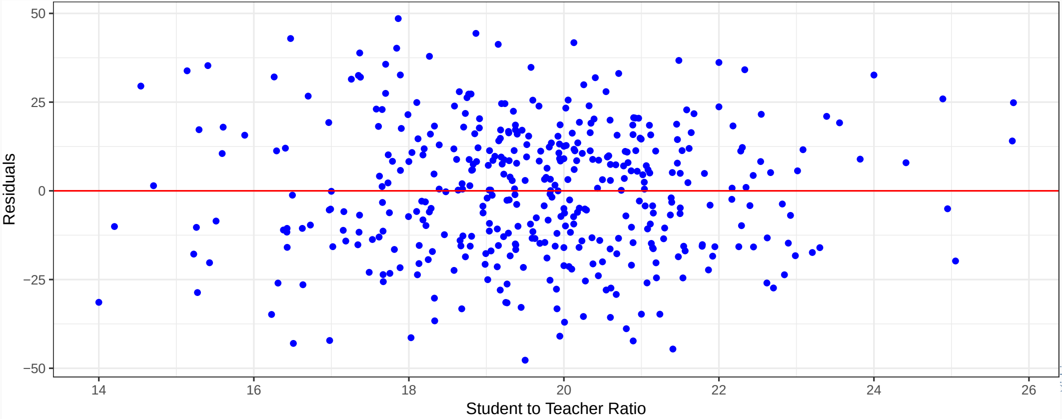
```
##         1         2         3         4         5         6
##  32.65260  11.33917 -12.70689 -11.66198 -15.51593 -44.58076
```

Measures of Fit: Residual Plot

- We often plot the residuals against X in a residual plot

```
# Create a scatterplot with the residuals.
# Same as before, but instead of testscr, we will use residuals (res)
school.resplot<-ggplot(CASchool, aes(str,school.regression$residuals))+
  geom_point(color="blue",fill="blue")+
  xlab("Student to Teacher Ratio")+
  ylab("Residuals")+theme_bw()+
  geom_hline(yintercept=0, color="red") #add horizontal line at y=0 to graph
```

```
school.resplot
```



The Sampling Distributions of the OLS Estimators

Recall: The Two Big Problems with Data

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2. Problem for **inference**: randomness
    - Data is random due to **natural sampling variation**
    - Taking one sample of a population will yield slightly different information than another sample of the same population

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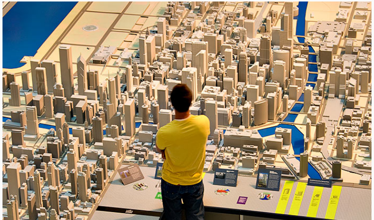
- Thus, $\hat{\beta}_0, \hat{\beta}_1$ are also random variables, with their own **sampling distribution**

- **Inferential statistics** analyzes a **sample** to make inferences about a much larger (unobservable) **population**



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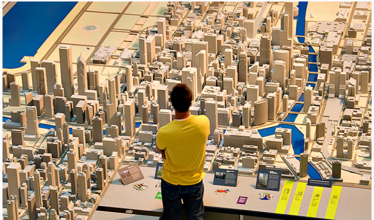

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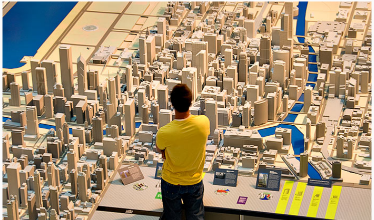

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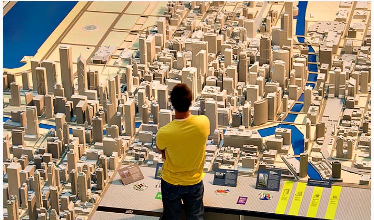

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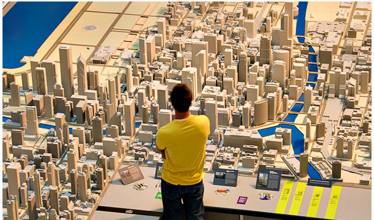

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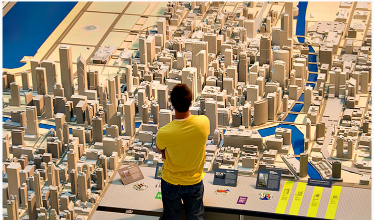
- We almost never can directly study the population, so we *model* it with our samples



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    - Sampling variability describes the effect of a statistic varying somewhat from sample to sample
    - This is normal, not the result of any error or bias

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    - Each sample comes from the *identical* underlying population distribution

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  - Taking different samples will create different values of $\hat{\beta}_0, \hat{\beta}_1$
  - Therefore, $\hat{\beta}_0, \hat{\beta}_1$ have sampling distributions across different samples

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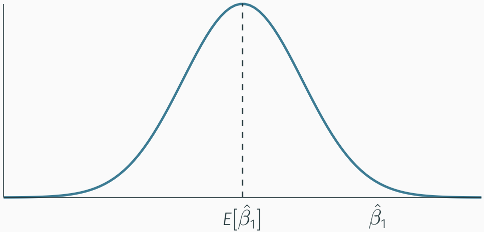
- If neither of these are true, we have other methods (coming shortly!)
- One of the most fundamental principles in all of statistics
    - Allows for virtually all testing of statistical hypotheses $\rightarrow$ estimating probabilities of values on a normal distribution

- The CLT allows us to approximate the sampling distributions of $\hat{\beta}_0$ and $\hat{\beta}_1$ as normal

$$\hat{\beta}_1 \sim N(E[\hat{\beta}_1], \sigma_{\hat{\beta}_1})$$



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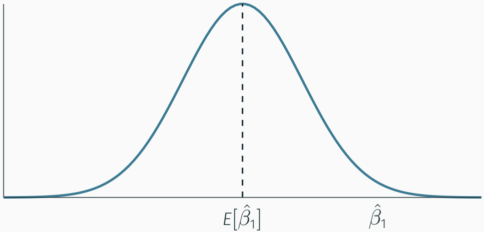
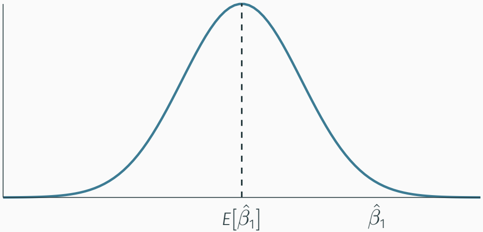


The Sampling Distribution of $\hat{\beta}_1$

- The CLT allows us to approximate the sampling distributions of $\hat{\beta}_0$ and $\hat{\beta}_1$ as normal
  - Generally agreed for $n > 100$
- We care about $\hat{\beta}_1$ (slope) since it has economic meaning, rarely about $\hat{\beta}_0$ (intercept)

$$\hat{\beta}_1 \sim N(E[\hat{\beta}_1], \sigma_{\hat{\beta}_1})$$



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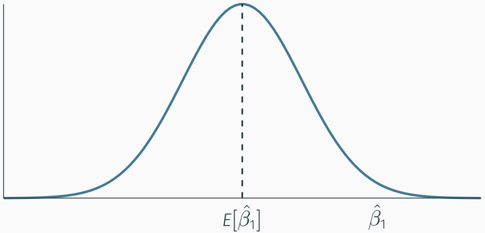

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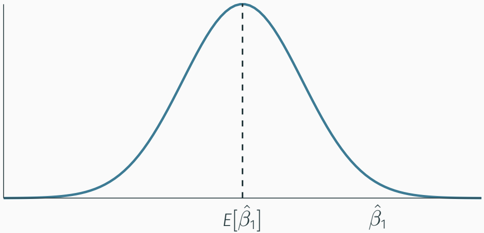

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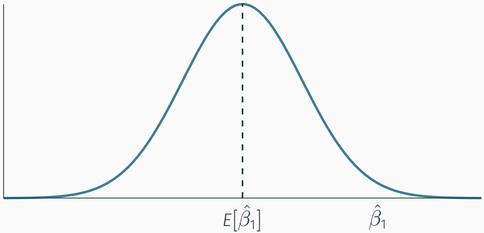

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  - This *would* be a problem (for inference), but we have a simple fix for this (coming shortly)

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  - We have fixes to deal with autocorrelation (coming much later)

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- "Does knowing X give me any useful information about ϵ?"
    - If yes, your model is endogenous, biased and **not-causal**!

- We want to see if $\hat{\beta}_1$ is unbiased: there is no systematic difference, on average, between sample values of $\hat{\beta}_1$ and the true population β_1, i.e.

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Sidenote: Estimators of Statistics

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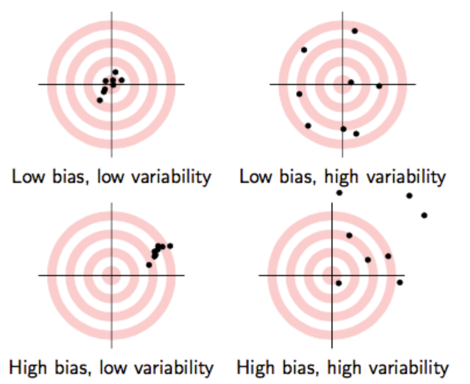
  - There are many estimators we *could* use to estimate μ_H
    - How about using the first value in our sample: H_1

- What makes one estimator (e.g. $\bar{H}$) better than another (e.g. H_1)?[5]



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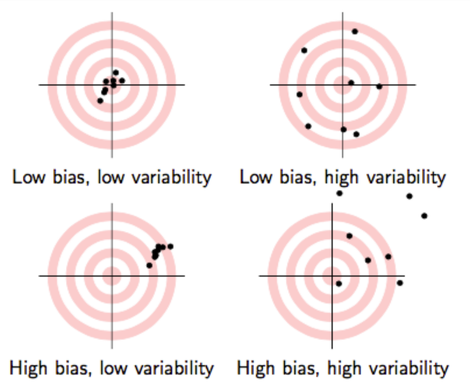

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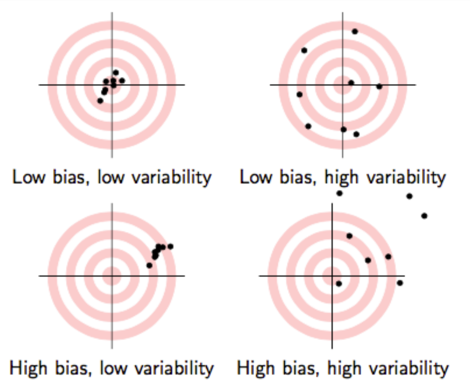

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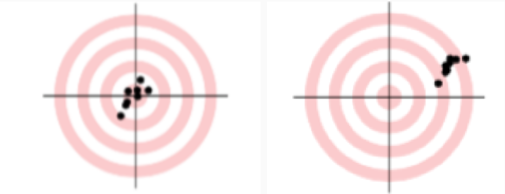

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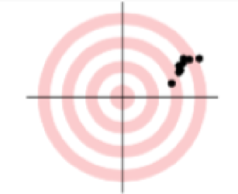

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