Lecture 4: Random Variables

ECON 480 - Econometrics - Fall 2018

Ryan Safner

September 10, 2018

Random Variables

Random Variables

- An experiment is any procedure that can (in principle) be repeated infinitely and has a well-defined set of outcomes

Example
Flip a coin 10 times

- A random variable (RV) takes on values that are unknown in advance, but determined by an experiment

Example
The number of Heads from 10 coin flips

- A random variable (RV) takes on values that are unknown in advance, but determined by an experiment
- A R.V. is a numerical summary of a random outcome

Example
The number of Heads from 10 coin flips

Random Variables: Notation

- Random variable X takes on individual values (X_i) from a set of possible values

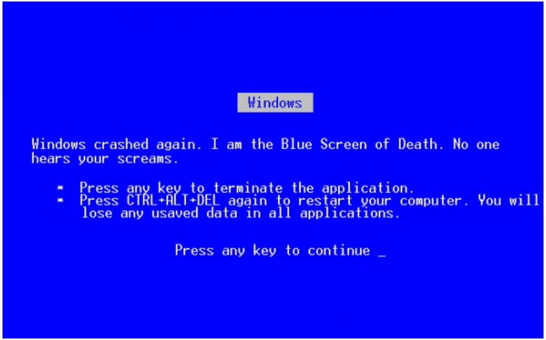
Example
Let X be the number of Heads from 10 coin flips. $x_i \in \{0, 1, 2, ..., 10\}$

- Random variable X takes on individual values (X_i) from a set of possible values
- Often capital letters to denote RV's

Example
Let X be the number of Heads from 10 coin flips. $x_i \in \{0, 1, 2, \ldots, 10\}$

- A discrete random variable takes on a finite/countable set of possible values

Example
Let X be the number of times your computer crashes this semester, $x_i \in \{0, 1, 2, 3, 4\}$



- The probability distribution of a R.V. fully lists all the possible values of X and their associated probabilities

Example

x_i	$P(X = x_i)$
0	0.80
1	0.10
2	0.06
3	0.03
4	0.01

- The probability distribution function (pdf) summarizes the possible outcomes of X and their probabilities

$$f_X = p_i, i = 1, 2, ..., k$$

- The probability distribution function (pdf) summarizes the possible outcomes of X and their probabilities
- Notation: f_X is the pdf of X:

$$f_X = p_i, i = 1, 2, ..., k$$

- The probability distribution function (pdf) summarizes the possible outcomes of X and their probabilities
- Notation: f_X is the pdf of X:

$$f_X = p_i, i = 1, 2, ..., k$$

- For any real number x_i, $f(x_i)$ is the probablity that $X = x_i$

Example

x_i	$P(X = x_i)$
0	0.80
1	0.10
2	0.06
3	0.03
4	0.01

Example

x_i	$P(X = x_i)$
0	0.80
1	0.10
2	0.06
3	0.03
4	0.01

- What is $f(0)$?

Example

x_i	$P(X = x_i)$
0	0.80
1	0.10
2	0.06
3	0.03
4	0.01

- What is $f(0)$?
- What is $f(3)$?

Discrete Random Variables: PDF Graph

```
crashes<-c(0,1,2,3,4)
prob<-c(0.8,0.1,0.06,0.03,0.01)
crashes<-data.frame(crashes, prob)

crashes.pdf<-ggplot(crashes, aes(x=crashes, y=prob))+
  geom_bar(stat="identity", fill="#0072B2")+xlab("Crashes")+ylab("Probability")

crashes.pdf
```

- The *cumulative* density function (cdf) describes the probability that X will be *at most* (less than or equal to) a given value x_i

Example

x_i	$f(x)$	$F(X)$
0	0.80	0.80
1	0.10	0.90
2	0.06	0.96
3	0.03	0.99
4	0.01	1.00

- The *cumulative* density function (cdf) describes the probability that X will be *at most* (less than or equal to) a given value x_i
- Notation: $F_X = P(X \leq x_i)$

Example

x_i	$f(x)$	$F(X)$
0	0.80	0.80
1	0.10	0.90
2	0.06	0.96
3	0.03	0.99
4	0.01	1.00

- The *cumulative* density function (cdf) describes the probability that X will be *at most* (less than or equal to) a given value x_i
- Notation: $F_X = P(X \leq x_i)$

Example

x_i	$f(x)$	$F(X)$
0	0.80	0.80
1	0.10	0.90
2	0.06	0.96
3	0.03	0.99
4	0.01	1.00

- What is the probability your computer will crash at most once? $F(1)$?

- The *cumulative* density function (cdf) describes the probability that X will be *at most* (less than or equal to) a given value x_i
- Notation: $F_X = P(X \leq x_i)$

Example

x_i	$f(x)$	$F(X)$
0	0.80	0.80
1	0.10	0.90
2	0.06	0.96
3	0.03	0.99
4	0.01	1.00

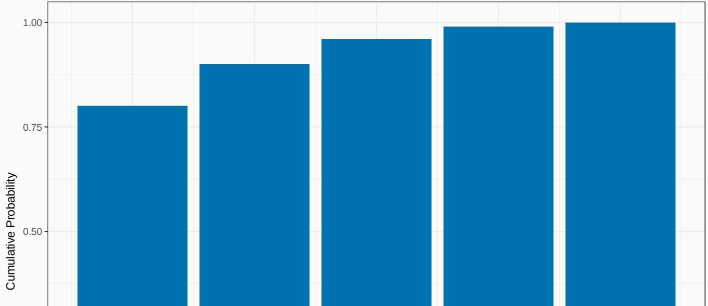
- What is the probability your computer will crash at most once? $F(1)$?

Discrete Random Variables: CDF Graph

```
crashes$cprob<-(cumsum(crashes$prob))

crashes.cdf<-ggplot(crashes, aes(x=crashes, y=cprob))+
  geom_bar(stat="identity", fill="#0072B2")+xlab("Crashes")+ylab("Cumulative Pro

crashes.cdf
```



- The expected value of a random variable X, written $E(X)$, is the long-run average value of X "expected" after many repetitions

- The expected value of a random variable X, written $E(X)$, is the long-run average value of X "expected" after many repetitions

Expected Value of a Random Variable

- The expected value of a random variable X, written $E(X)$, is the long-run average value of X "expected" after many repetitions

$$E(X) = p_1 x_1 + p_2 x_2 + \ldots + p_k x_k$$

- The expected value of a random variable X, written $E(X)$, is the long-run average value of X "expected" after many repetitions

$$E(X) = p_1 x_1 + p_2 x_2 + \ldots + p_k x_k = \sum_{i=1}^{k} p_i x_i$$

- A probability-weighted average of X, with each x_i weighted by its associated probability p_i
- Also called the mean or expectation of X, also denoted μ_X

Example

Suppose you lend your friend \$100 at 10% interest. If the loan is repaid, you receive \$110 . Your friend is 99% likely to repay, but there is a default risk of 1% where you get nothing. What is the expected value of repayment?

Expected Value: Example II

Example

Let X be a random variable that is described by the following pdf:

x_i	$P(X = x_i)$
1	0.50
2	0.25
3	0.10
4	0.05

- What is $E(X)$?

Variance of a Random Variable

- The variance of a random variable X, denoted $var(X)$ or σ_X^2 is:

$$\sigma_X^2 = \sum_{i=1}^{n} (x_i - \mu_X)^2 p_i$$

Variance of a Random Variable

- The variance of a random variable X, denoted $var(X)$ or σ_X^2 is:

$$\sigma_X^2 = \sum_{i=1}^{n} (x_i - \mu_X)^2 p_i$$

- The variance of a random variable X, denoted $var(X)$ or σ_X^2 is:

$$\sigma_X^2 = \sum_{i=1}^{n} (x_i - \mu_X)^2 p_i = E\left[(X - \mu_X)^2\right]$$

- This is the expected value of the squared deviations from the mean
    - i.e. the probability-weighted average of the squared deviations

- The standard deviation of a random variable X, denoted $sd(X)$ or σ_X is:

$$\sigma_X = \sqrt{\sigma_X^2}$$

Example
What is the standard deviation of computer crashes?

	Number of Computer Crashes				
x_i	0	1	2	3	4
$P(X = x_i)$	0.80	0.10	0.06	0.03	0.01

Example
What is the standard deviation of computer crashes?

	Number of Computer Crashes				
x_i	0	1	2	3	4
$P(X = x_i)$	0.80	0.10	0.06	0.03	0.01

- First, calculate the mean:

$$\begin{aligned} E(X) &= 0(0.80) + 1(0.10) + 2(0.06) + 3(0.03) + 4(0.01) \\ &= 0 + 0.1 + 0.12 + 0.09 + 0.04 \\ &= 0.35 \end{aligned}$$

Example
What is the standard deviation of computer crashes?

Example

What is the standard deviation of computer crashes?

- Next, find the deviations from the mean and square them:

	Number of Crashes ($E(X) = 0.35$)				
x_i	0	1	2	3	4
$P(X = x_i)$	0.80	0.10	0.06	0.03	0.01
$(x_i - E(X))$	-0.35	0.65	1.65	2.65	3.65
$(x_i - E(X)$	0.1225	0.4225	2.7225	7.0225	13.3225

Example
What is the standard deviation of computer crashes?

- Next, find the deviations from the mean and square them:

	Number of Crashes ($E(X) = 0.35$)				
x_i	0	1	2	3	4
$P(X = x_i)$	0.80	0.10	0.06	0.03	0.01
$(x_i - E(X))$	-0.35	0.65	1.65	2.65	3.65
$(x_i - E(X)$	0.1225	0.4225	2.7225	7.0225	13.3225

- Take the expectation of the squared deviations to get variance

$$
\begin{aligned}
\sigma_X^2 &= 0.1225(0.80) + 0.4225(0.10) + 2.7225(0.06) + 7.0225(0.03) + 13.3225(0.01) \\
&= 0.098 + 0.04225 + 0.16335 + 0.210675 + 0.133225 \\
&= 0.6475
\end{aligned}
$$

Example
What is the standard deviation of computer crashes?

- To get standard deviation, σ, take the square root:

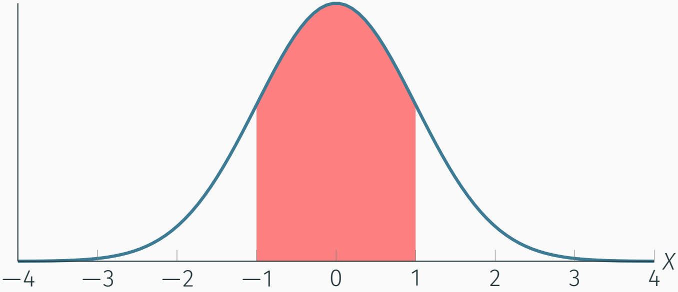
$$\begin{aligned}\sigma &= \sqrt{\sigma^2} \\ &= \sqrt{0.6475} \\ &= 0.8047\end{aligned}$$

- *Continuous* random variables can take on an uncountable (infinite) number of values

- *Continuous* random variables can take on an uncountable (infinite) number of values
- So many values that the probability of any specific value is infinitely small $\rightarrow 0$.

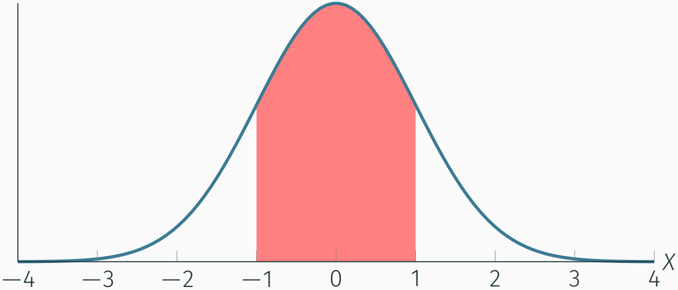
- *Continuous* random variables can take on an uncountable (infinite) number of values
- So many values that the probability of any specific value is infinitely small $\longrightarrow 0$.
- Instead, we focus on a *range* of values it might take on

- The probability *density* function (pdf) of a continuous variable represents the probability between two values as the area under a curve



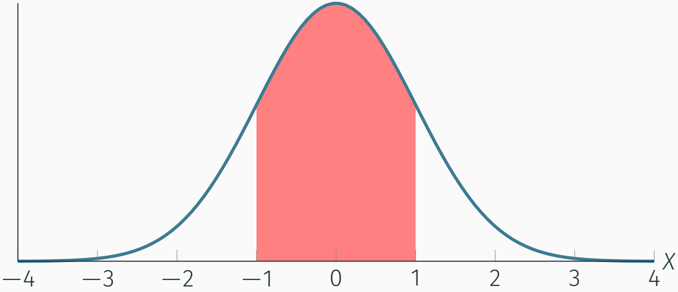
$P(-1 \leq X \leq 1)$: area under the curve between -1 and 1.

- The probability *density* function (pdf) of a continuous variable represents the probability between two values as the area under a curve
- The total area under the curve is 1



P($-1 \leq X \leq 1$): area under the curve between -1 and 1.

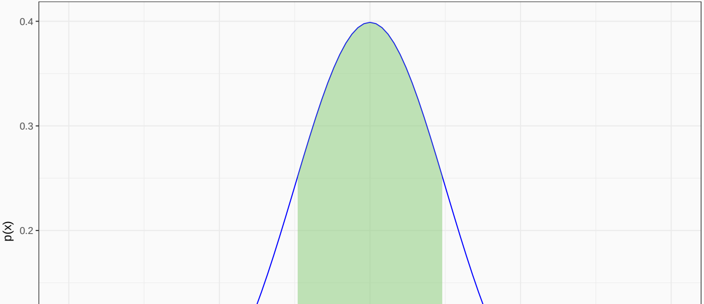
- The probability *density* function (pdf) of a continuous variable represents the probability between two values as the area under a curve
- The total area under the curve is 1
- Since $P(a) = 0$ and $P(b) = 0$, $P(a < X < b) = P(a \le X \le b)$



P($-1 \le X \le 1$): area under the curve between -1 and 1.

- FYI using calculus:

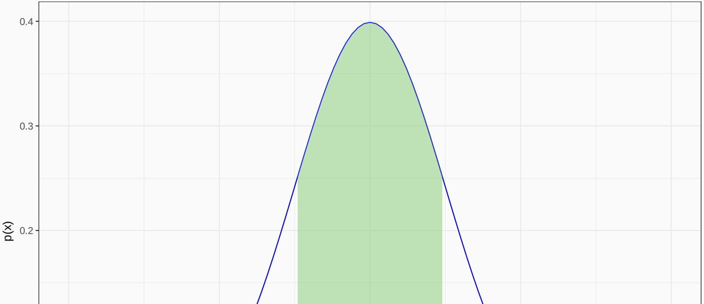
$$P(a \leq X \leq b) = \int_a^b f(x)dx$$



- FYI using calculus:

$$P(a \leq X \leq b) = \int_a^b f(x)dx$$

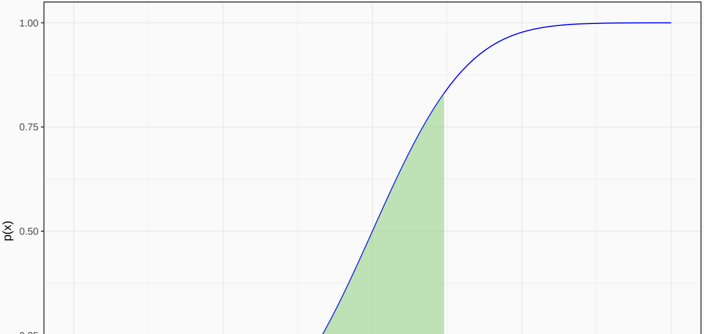
- These functions are complicated: we have software or (old fashioned!) probability tables to calculate



- The *cumulative* density function (cdf) describes the area under the pdf for all values less than or equal to (i.e. to the left of) a given value, k

$$P(X \leq k)$$

cdf shaded L-1.bb cdf shaded L-1.bb

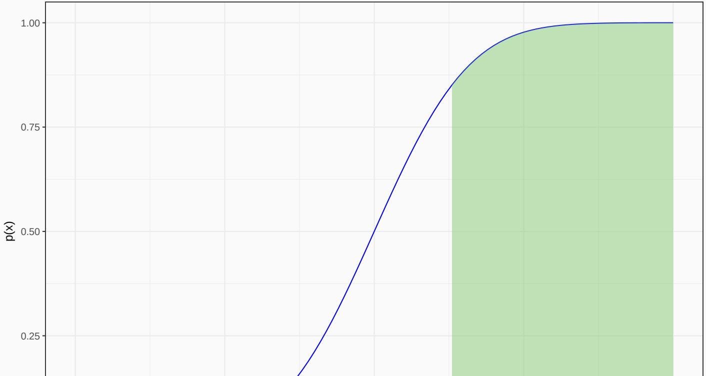


Continuous Random Variable CDF II

- Note: to find the probability of values *greater* than or equal to (to the right of) a given value k:

$$P(X \geq k) = 1 - P(X \leq k)$$

cdf shaded R-1.bb cdf shaded R-1.bb



- The Gaussian or normal distribution is the most common type of probability distribution, and we make extensive use of it

$$X \sim N(\mu, \sigma)$$

- The Gaussian or normal distribution is the most common type of probability distribution, and we make extensive use of it

$$X \sim N(\mu, \sigma)$$

- Continuous, symmetric, unimodal, with mean μ and standard deviation σ

- The pdf of $X \sim N(\mu, \sigma)$ is

$$P(X = k) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{(k-\mu)}{\sigma}\right)^2}$$

```r
# Find probability that a student earns at least an 80 if
#the average grade is a 75 and standard deviation is 10

#lower.tail is TRUE if calculating area to LEFT of 80,
#FALSE if to the RIGHT
pnorm(80, mean=72, sd=10, lower.tail=FALSE)
```

```
## [1] 0.2118554
```

- The pdf of $X \sim N(\mu, \sigma)$ is

$$P(X = k) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{(k-\mu)}{\sigma}\right)^2}$$

- **Do not try and learn this**, we have software and (previously tables) to calculate pdfs and cdfs

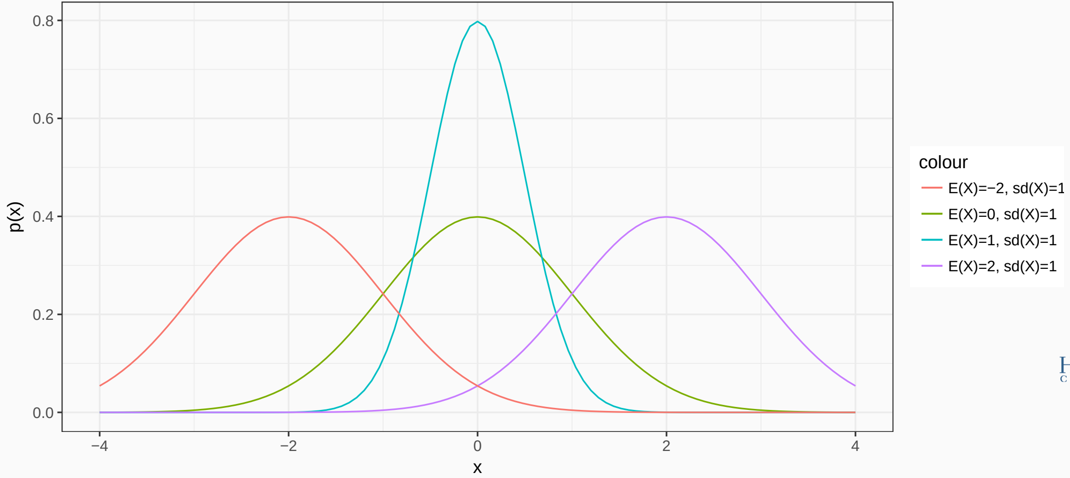
```
# Find probability that a student earns at least an 80 if
#the average grade is a 75 and standard deviation is 10

#lower.tail is TRUE if calculating area to LEFT of 80,
#FALSE if to the RIGHT
pnorm(80, mean=72, sd=10, lower.tail=FALSE)
```

```
## [1] 0.2118554
```

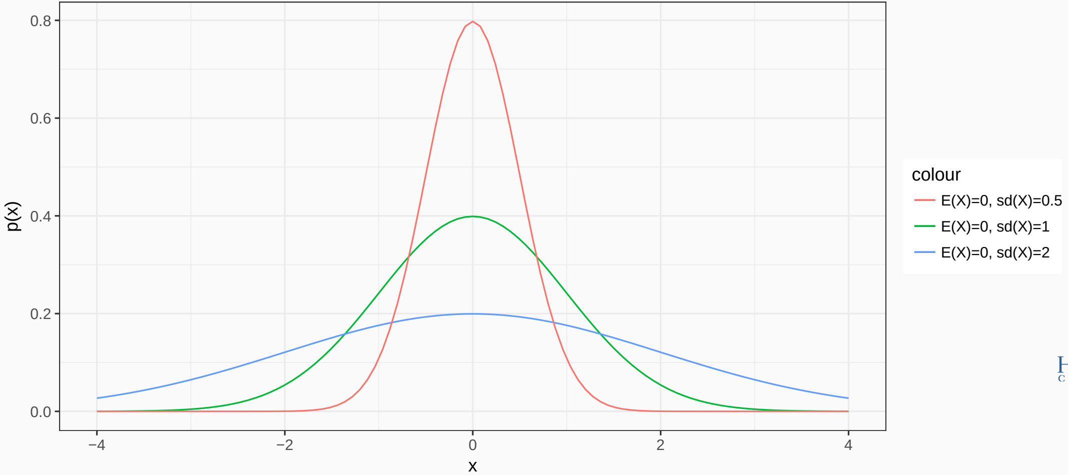
- The pdf moves left/right based on μ

pdf mu changes-1.bb pdf mu changes-1.bb

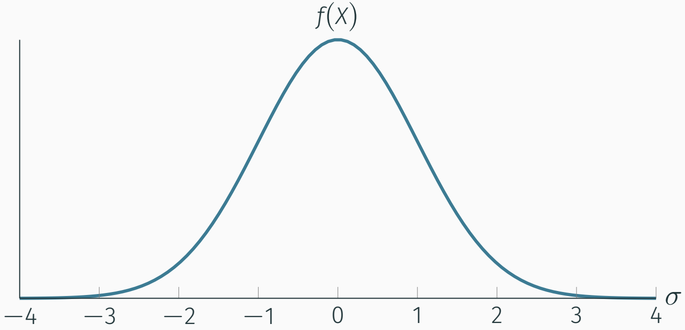


- The pdf gets fatter/skinnier based on σ

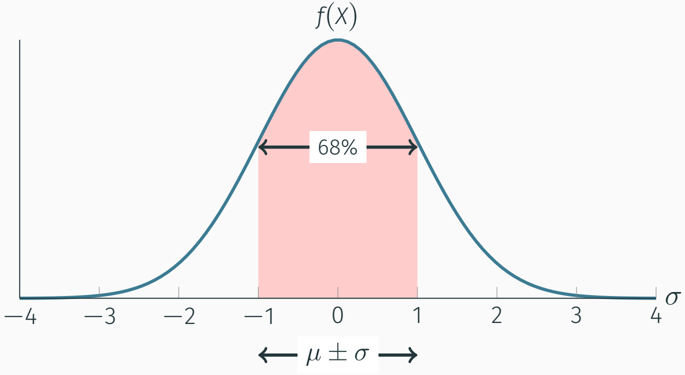
pdf sigma changes-1.bb pdf sigma changes-1.bb



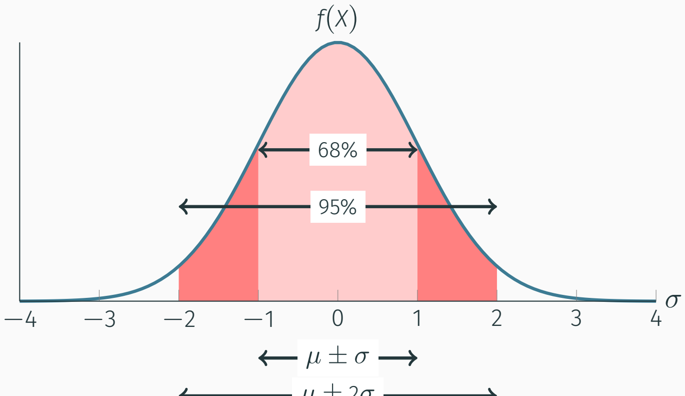
- The 68-95-99.7% (empirical) rule: for a normal distribution:



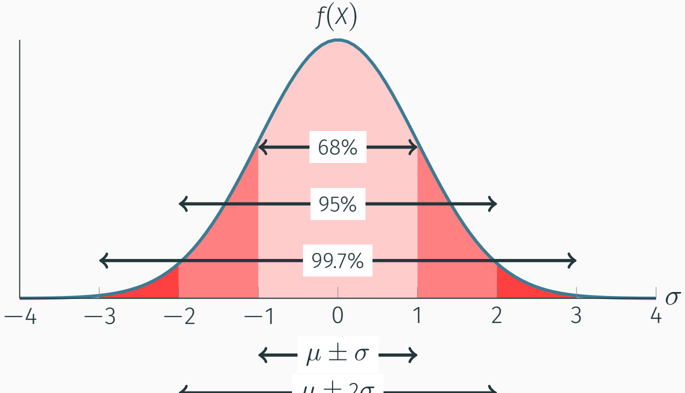
- The 68-95-99.7% (empirical) rule: for a normal distribution:
    - $P(\mu - 1\sigma \leq X \leq \mu + 1\sigma) \approx 68\%$



- The 68-95-99.7% (empirical) rule: for a normal distribution:
  - $P(\mu - 1\sigma \leq X \leq \mu + 1\sigma) \approx 68\%$
  - $P(\mu - 2\sigma \leq X \leq \mu + 2\sigma) \approx 95\%$



- The 68-95-99.7% (empirical) rule: for a normal distribution:
    - $P(\mu - 1\sigma \leq X \leq \mu + 1\sigma) \approx 68\%$
    - $P(\mu - 2\sigma \leq X \leq \mu + 2\sigma) \approx 95\%$
    - $P(\mu - 3\sigma \leq X \leq \mu + 3\sigma) \approx 99.7\%$



- We standardize a variable by calculating its Z-score and converting to the standard normal distribution

$$Z = \frac{x - \mu}{\sigma}$$

- We standardize a variable by calculating its Z-score and converting to the standard normal distribution

$$Z = \frac{x - \mu}{\sigma}$$

- Z is the number of standard deviations a value is above/below of its mean

- We standardize a variable by calculating its Z-score and converting to the standard normal distribution

$$Z = \frac{x - \mu}{\sigma}$$

- Z is the number of standard deviations a value is above/below of its mean
- Can compare distributions of variables with very different units!

- The standard normal distribution has mean 0 and standard deviation 1

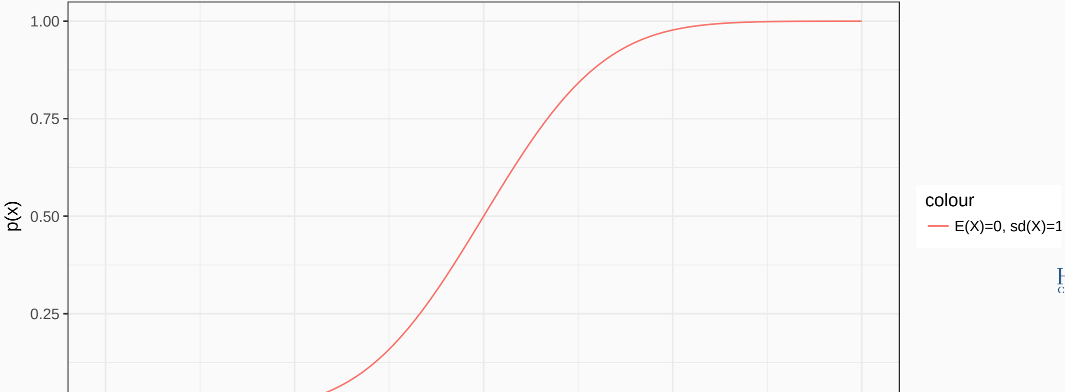
$$Z \sim N(0, 1)$$

normal-1.bb normal-1.bb

- Standard normal cdf:

$$\Phi(k) = P(Z \leq k)$$

normal cdf-1.bb normal cdf-1.bb



Example

On August 8, 2011, the Dow dropped 634.8 points, sending shock waves through the financial community. Assume that during mid-2011 to mid-2012 the daily change for the Dow is normally distributed, with the mean daily change of 1.87 points and a standard deviation of 155.28 points. What is the Z-score?

Example

On August 8, 2011, the Dow dropped 634.8 points, sending shock waves through the financial community. Assume that during mid-2011 to mid-2012 the daily change for the Dow is normally distributed, with the mean daily change of 1.87 points and a standard deviation of 155.28 points. What is the Z-score?

- Convert to Z-score:

$$Z = \frac{X - \mu}{\sigma}$$

Example

On August 8, 2011, the Dow dropped 634.8 points, sending shock waves through the financial community. Assume that during mid-2011 to mid-2012 the daily change for the Dow is normally distributed, with the mean daily change of 1.87 points and a standard deviation of 155.28 points. What is the Z-score?

- Convert to Z-score:

$$Z = \frac{X - \mu}{\sigma} = \frac{634.8 - 1.87}{155.28}$$

Example

On August 8, 2011, the Dow dropped 634.8 points, sending shock waves through the financial community. Assume that during mid-2011 to mid-2012 the daily change for the Dow is normally distributed, with the mean daily change of 1.87 points and a standard deviation of 155.28 points. What is the Z-score?

- Convert to Z-score:

$$Z = \frac{X - \mu}{\sigma} = \frac{634.8 - 1.87}{155.28} = -4.1$$

- This is 4.1 standard deviations (σ) beneath the mean.

Example

In the last quarter of 2015, a group of 64 mutual funds had a mean return of 2.4% with a standard deviation of 5.6%. These returns can be approximated by a normal distribution.

Example

In the last quarter of 2015, a group of 64 mutual funds had a mean return of 2.4% with a standard deviation of 5.6%. These returns can be approximated by a normal distribution.

- What percent of the funds would you expect to be earning between -3.2% and 8.0% returns?

- Convert to standard normal to find Z-scores for 8 and -3.2.

- Convert to standard normal to find Z-scores for 8 and -3.2.

$$P(-3.2 < X < 8)$$

- Convert to standard normal to find Z-scores for 8 and -3.2.

$$P\left(-3.2 < X < 8\right)$$

$$P(\frac{-3.2 - 2.4}{5.6} < \frac{X - 2.4}{5.6} < \frac{8 - 2.4}{5.6})$$

Standardizing Variables: Example III

- Convert to standard normal to find Z-scores for 8 and -3.2.

$$P\left(-3.2 < X < 8\right)$$

$$P\left(\frac{-3.2 - 2.4}{5.6} < \frac{X - 2.4}{5.6} < \frac{8 - 2.4}{5.6}\right)$$

$$P\left(-1 < Z < 1\right)$$

- Convert to standard normal to find Z-scores for 8 and -3.2.

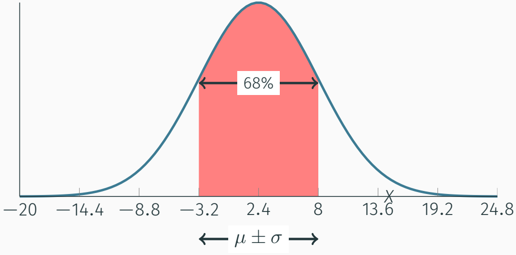
$$P(-3.2 < X < 8)$$

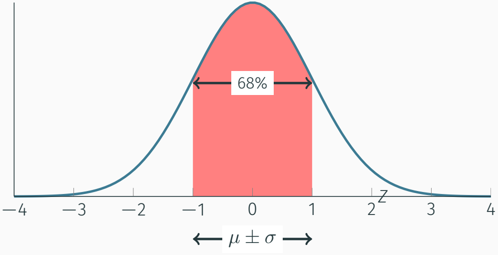
$$P(\frac{-3.2 - 2.4}{5.6} < \frac{X - 2.4}{5.6} < \frac{8 - 2.4}{5.6})$$

$$P(-1 < Z < 1)$$

- $P(X \pm 1\sigma) = 0.68$

Standardizing Variables: Example IV





Example

In the last quarter of 2015, a group of 64 mutual funds had a mean return of 2.4% with a standard deviation of 5.6%. These returns can be approximated by a normal distribution.

- What percent of the funds would you expect to be earning between -3.2% and 8.0% returns?

Example

In the last quarter of 2015, a group of 64 mutual funds had a mean return of 2.4% with a standard deviation of 5.6%. These returns can be approximated by a normal distribution.

- What percent of the funds would you expect to be earning between -3.2% and 8.0% returns?

Example

In the last quarter of 2015, a group of 64 mutual funds had a mean return of 2.4% with a standard deviation of 5.6%. These returns can be approximated by a normal distribution.

- What percent of the funds would you expect to be earning between -3.2% and 8.0% returns?
- What percent of the funds would you expect to be earning 2.4% or less?

Example

In the last quarter of 2015, a group of 64 mutual funds had a mean return of 2.4% with a standard deviation of 5.6%. These returns can be approximated by a normal distribution.

- What percent of the funds would you expect to be earning between -3.2% and 8.0% returns?
- What percent of the funds would you expect to be earning 2.4% or less?
- What percent of the funds would you expect to be earning between -8.8% and 13.6%?

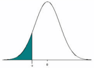
Example

In the last quarter of 2015, a group of 64 mutual funds had a mean return of 2.4% with a standard deviation of 5.6%. These returns can be approximated by a normal distribution.

- What percent of the funds would you expect to be earning between -3.2% and 8.0% returns?
- What percent of the funds would you expect to be earning 2.4% or less?
- What percent of the funds would you expect to be earning between -8.8% and 13.6%?
- What percent of the funds would you expect to be earning returns greater than 13.6%?

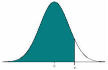
- How do we actually find the probabilities for Z—scores?

Table of Standard Normal Probabilities for Negative Z-scores



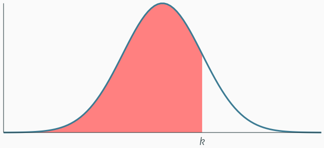
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Table of Standard Normal Probabilities for Positive Z-scores

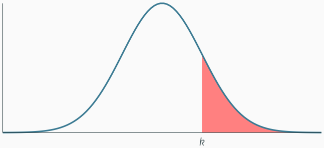


z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

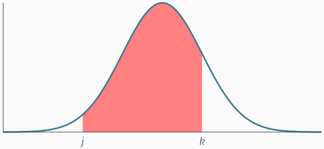
Note that the probabilities given in this table represent the area to the LEFT of the z-score.
The area to the RIGHT of a z-score = 1 – the area to the LEFT of the z-score



$$P(Z \leq k) = \Phi(k)$$



$$P(Z \geq k) = 1 - \Phi(k)$$



$$P(j \geq Z \geq k) = \Phi(k) - \Phi(j)$$

Finding Z-Score Probabilities with *R*

- Let the distribution of grades be normal, with mean 75 and standard deviation 10.

```r
pnorm(80, mean=75, sd=10, lower.tail=FALSE)
```

```
## [1] 0.3085375
```

```r
pnorm(80, mean=75, sd=10, lower.tail=TRUE)
```

```
## [1] 0.6914625
```

Finding Z-Score Probabilities with *R*

- Let the distribution of grades be normal, with mean 75 and standard deviation 10.
- Note: `lower.tail=TRUE` if calculating area to LEFT of value k, `=FALSE` if to the RIGHT

```r
pnorm(80, mean=75, sd=10, lower.tail=FALSE)
```

```
## [1] 0.3085375
```

```r
pnorm(80, mean=75, sd=10, lower.tail=TRUE)
```

```
## [1] 0.6914625
```

Finding Z-Score Probabilities with *R*

- Let the distribution of grades be normal, with mean 75 and standard deviation 10.
- Note: `lower.tail=TRUE` if calculating area to LEFT of value k, `=FALSE` if to the RIGHT
- Probability a student gets at least an 80:

```r
pnorm(80, mean=75, sd=10, lower.tail=FALSE)
```

```
## [1] 0.3085375
```

```r
pnorm(80, mean=75, sd=10, lower.tail=TRUE)
```

```
## [1] 0.6914625
```

Finding Z-Score Probabilities with *R*

- Let the distribution of grades be normal, with mean 75 and standard deviation 10.
- Note: `lower.tail=TRUE` if calculating area to LEFT of value k, `=FALSE` if to the RIGHT
- Probability a student gets at least an 80:

```r
pnorm(80, mean=75, sd=10, lower.tail=FALSE)
```

```
## [1] 0.3085375
```

- Probability a student gets at most an 80:

```r
pnorm(80, mean=75, sd=10, lower.tail=TRUE)
```

```
## [1] 0.6914625
```

- Probability a student gets between a 65 and 85:

```
pnorm(85, mean=75, sd=10, lower.tail=TRUE)-pnorm(65, mean=75, sd=10, lower.tail=
```

```
## [1] 0.6826895
```