Enrollment No.: _____

Darshan UNIVERSITY

योगः कर्मसु कौशलम्

Darshan Institute of Engineering & Technology

B.Tech. | Sem-3 | Summer-2024

Course Code	:	2101HS302	**Date**	:	15-05-2024
Course Name	:	Discrete Mathematics	**Duration**	:	150 Minutes
			Total Marks	:	70

Instructions:

1. Attempt all the questions.
2. Figures to the right indicates maximum marks.
3. Make suitable assumptions wherever necessary.

Q.1 **(A)** Verify $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. **4**

Where, $A = \{4, 5, 7, 8, 10\}$

$B = \{4, 5, 9\}$

$C = \{1, 4, 6, 9\}$

(B) Find $P(A)$. Where, $A = \{a, b, c\}$. How many elements are there are in $P(P(A))$? **3**

OR

Show that, $A \times B \neq B \times A$. Where, $A = \{1, x, k\}$ & $B = \{2, 3, d, g\}$.

(C) a. If $f(x) = 2x, g(x) = x^2, h(x) = x + 1$, then find $(f \circ g) \circ h$ and $f \circ (g \circ h)$. **7**

b. Let N be the set of natural numbers. Let R be a relation on N defined by xRy if and only if $x + 3y = 12$. Check the relation for reflexive, symmetric and transitive.

OR

Prove that proposition P, the sum of the cubes of the first n positive integers is $\frac{n^2(n+1)^2}{4}$.

Q.2 **(A)** Let $X = \{1, 2, \ldots\ldots, 7\}$ and $R = \{(x, y) : x - y \text{ is divisible by } 3\}$. Draw graph of relation R. **4**

(B) Is relation R is an equivalence relation? Explain. **3**

$R = \{(1,1), (1,2), (2,1), (2,2), (3,3), (3,4), (4,3), (4,4)\}$ on set $A = \{1, 2, 3, 4\}$.

OR

Is relation R is anti-symmetric relation, transitive or irreflexive? Explain.

$R = \{(x,x), (x,y), (y,x), (y,y), (z,z), (z,w), (w,z), (w,w)\}$ on set $A = \{x, y, z, w\}$.

(C) Draw the Hasse–diagram of $\langle A, \leq \rangle$. Where, $a \leq b$ if a divides b, for set $A = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$. Carry out cover of each element of set A. **7**

OR

Solve the recurrence relation $a_n - 7\,a_{n-1} + 12\,a_{n-2} = n \cdot 4^n$ using the method of undetermined coefficients.

Q.3 **(A)** Let, $K(x)$: x is student, $L(x)$: x is clever, $N(x)$: x is successful. **4**

Express the following using quantifiers.

a. There exists a student
b. Some students are cleaver
c. All students are not cleaver
d. Some students are not successful

(B) Is following are propositions? **3**

a. India got freedom in 1947
b. Open the door!
c. $3 * 4 = 12$

OR

Is following well-formed formulae?

a. $(p \rightarrow q) \rightarrow (\wedge q)$.
b. $\big((p \rightarrow q) \wedge (q \rightarrow r)\big)$.
c. $\neg p \wedge q \rightarrow r$.

(C) Show that, $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$. **7**

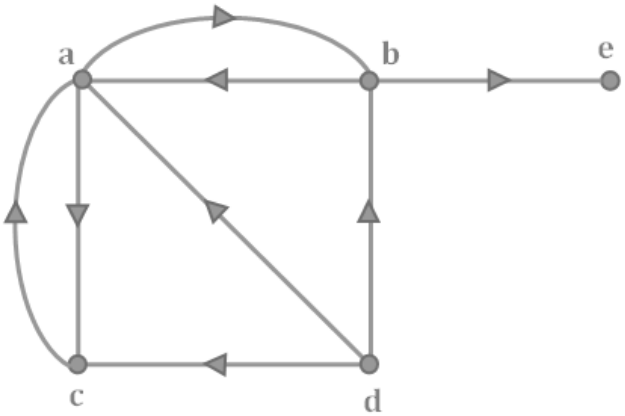
OR

Find cnf and dnf for $(\sim (p \wedge q) \vee r) \rightarrow \sim p$.

Q.4 **(A)** Define the terms: **4**

a. Bipartite Graph
b. Path
c. Geodesic
d. Node Base

(B) Discuss Indegree and outdegree of all vertices for below graph. **3**



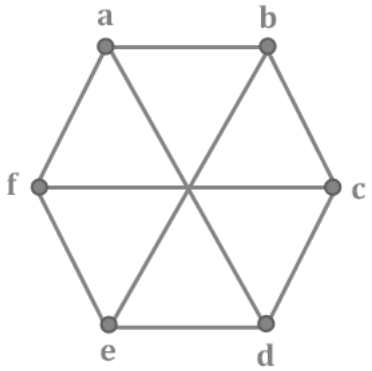
OR

Express a graph from below adjacency matrix of graph.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

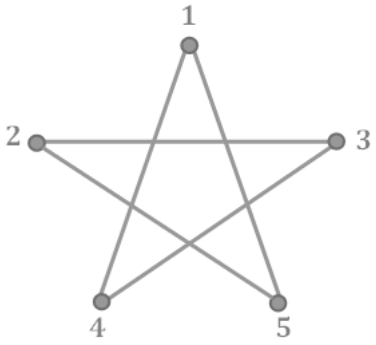
(C) a. Does a 3 – regular graph with 5 vertices exist? Explain. **7**

b. Show that handshaking theorem satisfies for below graph.



OR

a. Prepare 3 – regular graph with 10 vertices.
b. Sketch K_6.
c. Sketch one isomorphic graph of below graph.



Q.5 **(A)** Show that a group $(G, +_6)$ is a cyclic group, where $G = \{0, 1, 2, 3, 4, 5\}$. **4**

(B) Write definition of Group and give 2 different examples of it. **3**

OR

Write definition of Ring and give 2 different examples of it.

(C) Show that an algebraic structure $(G, \times)$ is an abelian group. **7**

Where, $G = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \right\}$.

OR

Show that $S = \{ a + b\sqrt{2} : a, b \in Z \}$ for the operations usual $+$, $\times$ is an integral domain.

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