Enrollment No.: _____

Darshan Institute of Engineering & Technology

B.Tech. | Sem-3 | Winter-2023

Course Code	:	2101HS302	**Date**	:	07-11-2023
Course Name	:	Discrete Mathematics	**Duration**	:	150 Minutes
			Total Marks	:	70

Instructions:
1. Attempt all the questions.
2. Figures to the right indicates maximum marks.
3. Make suitable assumptions wherever necessary.

Q.1 **(A)** For $A = \{ x : x \text{ is a divisor of } 42 \}, B = \{ x : x \text{ is a divisor of } 21 \}$ and $C = \{ x : x \text{ is a divisor of } 7 \}$, Verify the following identities. **4**

(1) $A - (B \cap C) = (A - B) \cup (A - C)$

(2) $A - (B \cup C) = (A - B) \cap (A - C)$

(B) Draw Venn Diagram for identity: $B - A = B \cap A'$. **3**

OR

State and prove De Morgan's law without using Venn diagram.

(C) (i). In a survey it was found that 21 people liked product A, 26 liked product B and 29 liked product C. If 14 people liked products A and B, 12 people liked products C and A, 14 people liked products B and C and 8 liked all the three products. Find how many liked product C only. **7**

(ii). Find gcd ($2260,\ 816$) by using Euclidean Algorithm.

OR

(i) Let $f : \mathbb{R} \to \mathbb{R}$ defiend as $f(x) = \frac{x+2}{4}$ for all $x \in \mathbb{R}$.
State whether the function f is bijective or not. If yes, then find f^{-1}.

(ii) Using mathematical induction, prove that the sum of the squares of the first n positive integers is $\frac{n(n+1)(2n+1)}{6}$.

Q.2 **(A)** Define complemented lattice. Is $\langle S_{30},\ D \rangle$ complemented lattice? **4**

(B) Solve the recurrence relation: $a_n = 3a_{n-1} + 2 ; n \geq 1$ with $a_0 = 1$ using method of generating function. **3**

OR

Solve the recurrence relation: $a_n = 3a_{n-1} - 2a_{n-2} ;\ a_1 = -2,\ a_2 = 4$ using undetermined coefficient method.

(C) Let $X = \{1,2,3,4,5,6\}$ and $R = \{(x, y): x - y \text{ is divisible by } 2\}$. Show that R is an equivalence relation. Also draw the graph of R. **7**

OR

Prove that $\langle A = \{ 1,\ 2,\ 3,\ 6,\ 12,\ 18 \}, D \rangle$ is a poset, Where D is relation of division and draw Hasse Diagram.

Q.3 **(A)** Construct the truth table for the compound proposition $(p \to q) \rightleftarrows (\sim p \vee \sim q)$. **4**

(B) Define simple graph, directed graph and mixed graph. **3**

OR

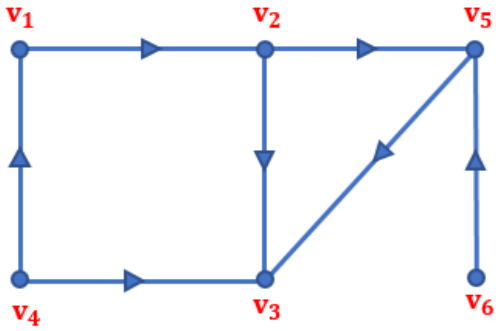
Define compound proposition, biconditional and predicate formula.

(C) (i) Check whether $(p \to q) \equiv (\sim p \vee q)$ or not? **7**
(ii) Find DNF for the compound proposition $\sim(p \to (\sim p \wedge q))$.

OR

Check whether the compound propositions $(p \to q) \wedge (q \to r)$ and $(p \to r)$ are tautologically imply or not?

Q.4 **(A)** Find the (i) an adjacency matrix (ii) path matrix (iii) geodesic and distance of from v_1 to v_5. **4**

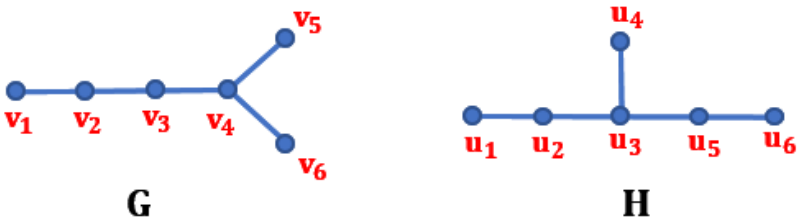


(B) A graph G has 13 edges, 2 vertices of degree 4 and other vertices of degree 3 then find the number of vertices in graph G. **3**

OR

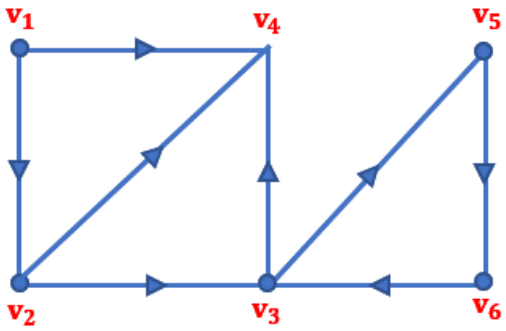
Draw the graph with 6 vertices which is bipartite graph but not complete graph.

(C) (i) Check whether the following graphs G & H are isomorphic or not? Justify **7**



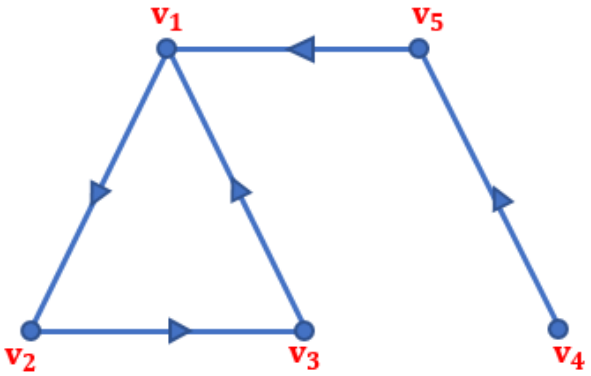
G H

(ii) Find the node base of the following diagraph.



OR

(i) Apply Warshall's algorithm to produce a path matrix for given digraph.



Q.5 **(A)** Write the composition table for binary operation (i) additive modulo 5 and (ii) multiplicative modulo 5 for the set $\{0,\ 1,\ 2,\ 3,\ 4\}$. **4**

(B) Let $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 3 & 4 & 1 & 6 & 2 \end{pmatrix} \in S_6$ & **3**

$\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 5 & 1 & 4 & 6 & 2 \end{pmatrix} \in S_6$, then find $\sigma^{-1},\ \sigma\tau,\ o(\tau)$.

OR

Find all subgroups of $(\mathbb{Z}_{12}, +_{12})$.

(C) (i) Show that fourth root of unity with multiplication is group. **7**

(ii) if the binary operation $*$ is defined on $R \times R$ by
$(a,\ b) * (c,\ d) = (ad + bc,\ bd)$ & $*$ is associative then find the value of
$(1,\ 2) * (3,\ 5) * (3,\ 4)$.

OR

(i) Consider the binary operation $*$ on the set $G = \{a,\ b,\ c,\ d\}$ defined by following composition table

$*$	a	B	c	d
a	a	C	b	d
b	d	A	b	c
c	c	D	a	b
d	d	B	a	c

Is it commutative? justify
Is it associative? justify

(ii) Show that cube root of unity with multiplication is group.

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