Enrollment No.: _____

Darshan UNIVERSITY
योगः कर्मसु कौशलम्

Darshan Institute of Engineering & Technology
B.Tech. | Sem-3 | Summer-2023

Course Code	:	2101HS302	**Date**	:	26-04-2023
Course Name	:	Discrete Mathematics	**Duration**	:	150 Minutes
			Total Marks	:	70

Instructions:
1. Attempt all the questions.
2. Figures to the right indicates maximum marks.
3. Make suitable assumptions wherever necessary.

Q.1 **(A)** If $A = \{x; x \in R, x^2 - 3x - 4 = 0\}$, $B = \{x; x \in Z, x^2 = x\}$, then find $A \cup B$, $A \cap B$, $A - B$ and $A \Delta B$. **4**

(B) Find $n(A' \cap B')$. where, A and B are two subsets of universal set U, such that $n(A) = 20, n(B) = 30, n(U) = 80, n(A \cap B) = 10$. **3**

OR

Find g. c. d. (a, b) and l. c. m(a, b). where, $a = 7254$ and $b = 10025$.

(C) (i) Give the power set of the following. **7**
(a) $A = \{x; x$ is multiple of 4, $x \in N$ and $x \leq 16\}$
(b) $B = \{x; x$ is aprime number and $x < 8\}$

(ii) If $f(x) = x^2$ and $g(x) = 2x + 1$ be the real functions, then find $(f + g)(x), (f - g)(x), \left(\frac{f}{g}\right)(x)$.

OR

(i) If $f = \{(1,2), (2,3), (3,1)\}$, $g = \{(1,2), (2,1), (3,3)\}$, then find $f \circ g$, $g \circ f$, $(f \circ g)^{-1}$.

(ii) Write the matrix of R and sketch its graph. Where $X = \{1,2,3,4\}$ and $R = \{(1,1), (1,4), (4,1), (4,4), (2,2), (2,3), (3,2), (3,3)\}$.

Q.2 **(A)** Let $X = \{1,2,3 \ldots \ldots, 7\}$ & $R = \{(x, y): x - y$ is even$\}$. Show that R is an equivalence relation. **4**

(B) Solve the recurrence relation: $a_n - 3a_{n-1} = 2$; $n \geq 1$ using method of generating function. **3**

OR

Solve the recurrence relation: $a_n = 2a_{n-1} - a_{n-2}$; $a_1 = 1.5$, $a_2 = 3$
Using undetermined coefficient method.

(C) (i) Draw Hasse Diagram of S_{12}. **7**

(ii) Let $A = \{a, b, c\}$ then show that the poset $(P(A), \subseteq)$ is a lattice.

OR

(i) Let $R = \{(a, b), (c, d), (b, b)\}$ and $S = \{(d, b), (b, e), (c, a), (a, c)\}$ be a relation on a set $A = \{a, b, c, d, e\}$. Obtain matrix of $R \circ S$, $\overline{R}$, S^2.

(ii) Draw Hasse Diagram of S_{75}.

Q.3 **(A)** Define the following terms: **4**
(i) Null graph, (ii) Multi graph, (iii) Mixed graph, (iv) Simple graph.

(B) Obtain the dnf and cnf of $(P \longrightarrow Q) \wedge (P \vee \urcorner Q)$. **3**

OR

Find truth table of $(P \longrightarrow Q) \Rightarrow (\urcorner P \vee Q)$.

(C) Using truth table check whether each of the following implication is tautology or contradiction. **7**
(i) $(P \vee R) \rightarrow P$
(ii) $(P \vee Q) \rightarrow (P \longrightarrow Q)$
(iii) $P \rightarrow (P \wedge Q)$

OR

(i) Prove that $(P \longrightarrow Q) \Leftrightarrow (\urcorner P \vee Q)$.

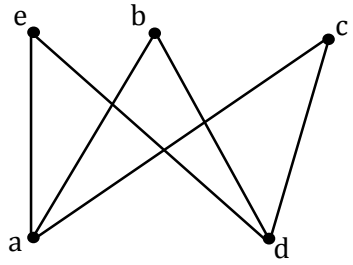
(ii) Indicate the variables that are free and bound. Also show that scope of quantifiers for $(x)(P(x) \wedge (\exists x)Q(x))) \vee ((x)\big(P(x) \rightarrow Q(x)\big)$.

Q.4 **(A)** Determine the number of edges in a graph with 6 vertices, 2 of degree 4 and 4 of degree 2. Draw two such graphs. **4**

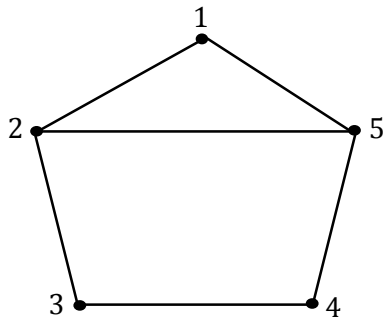
(B) State and Prove First fundamental theorem. **3**

OR

Is $G_1 \cong G_2$? If they are isomorphic, give the isomorphism. If not, explain.



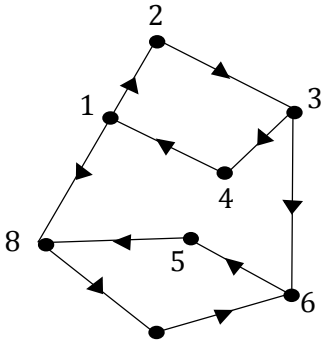
Graph G_1



Graph G_2

(C) (i) How many vertices are necessary to construct a graph with exactly 6 edges in which each vertex is of degree 2. **7**

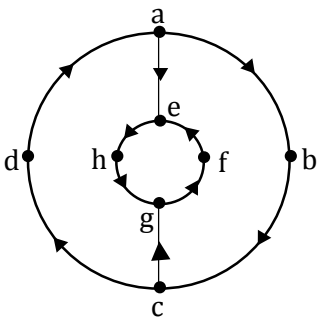
(ii) Find the strong, weak and unilateral component for the given diagraph



OR

(i) How many vertices are necessary to construct a graph with 12 edges in which each vertex is degree 4.

(ii) Find the strong, weak and unilateral component for the given diagraph.



Q.5 **(A)** Prove that $(Z_5, +_5)$ is Group. **4**

(B) Let the permutations of the elements of {1,2,3,4,5,6} be given by **3**

$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 6 & 1 & 2 & 4 & 3 \end{pmatrix}$, $g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 3 & 1 & 5 & 6 & 2 \end{pmatrix}$

Find $f \cdot g$, $g \cdot f$, f^{-1}.

OR

Let $f = (1\ 2\ 3)(1\ 4\ 5) \in S_5$ then find f^{99} .

(C) Find the left cosets of $H = \{p_1, p_5, p_6\}$ in the group $G = (S_3, \circ)$. **7**

OR

Find all normal subgroup of the $(Z_{15}, +_{15})$

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