Enrollment No.: ______________________

Darshan UNIVERSITY

Darshan Institute of Engineering & Technology
B.Tech. | Sem-3 | Winter-2022

Course Code	:	2101HS302	**Date**	:	17-10-2022
Course Name	:	Discrete Mathematics	**Duration**	:	150 Minutes
			Total Marks	:	70

Instructions:
1. Attempt all the questions.
2. Figures to the right indicates maximum marks.
3. Make suitable assumptions wherever necessary.

Q.1 **(A)** Let, $U = \{1, 2, 3, \dots, 15\}, A = \{3, 8, 9, 11, 12\}$ and $B = \{1, 3, 8, 9, 10\}$. Find $A \cup B, A - B, A \Delta B$ and $A' \cup B'$. **4**

(B) Draw Venn Diagram for identity: $A - B = A \cap B'$. **3**

OR

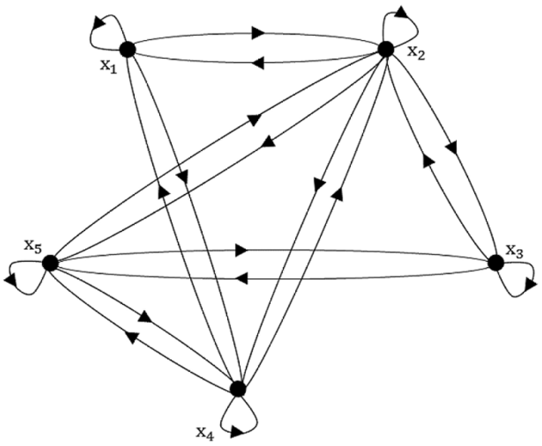
Find GCD and LCM of $a = 4606$ and $b = 13912$.

(C) For set $A = \{1, 2, 3, 4, 8\}$ and $R = \{(1, 1), (1,3), (2,4), (2, 2), (3, 3), (3,8), (2,8), (4, 4), (3,1), (8,8), (4,2)\}$. **7**

i. Find the properties of relation R.
ii. Draw graph of relation R.
iii. Write relation matrix for relation R.

OR

Check the properties of given graph. Also, Write relation matrix for it.



Q.2 **(A)** Define: Cover of a Set. Check following sets are cover of set A or not. Where, $A = \{1, 2, 3, 4, 5\}$. **4**

i. $A_1 = \{\{1,2\}, \{2,4\}, \{5\}\}$
ii. $A_2 = \{\{1,3\}, \{2\}, \{5\}\}$
iii. $A_3 = \{\{1\}, \{2\}, \{3\}, \{4\}, \{5\}\}$

(B) Solve the recurrence relation: $a_n = 7a_{n-1}\ ; n \geq 1$ using method of generating function. **3**

OR

Solve the recurrence relation: $a_n = 11a_{n-1} - 24a_{n-2}$; $n \geq 2$ using undetermined coefficient method.

(C) i. Show that the relation R in the set Z of integers given by $R = \{(a,b) : |a - b| \text{ is multiple of } 2\}$ is an equivalence relation. **7**

ii. Draw Hasse Diagram of S_{75}.

OR

i. Let $R = \{(a,b),(c,d),(b,b)\}$ and $S = \{(d,b),(b,e),(c,a),(a,c)\}$ be a relation on a set $A = \{a,b,c,d,e\}$. Obtain matrix of $R \circ S$, $\widetilde{R}$, S^2.

ii. Draw Hasse Diagram of S_{45}.

Q.3 **(A)** Consider the graph $G = (V,E)$ with $V = \{1,2,3,4,5\}$ and $E = \{(1,2),(1,3),(2,3),(2,4),(3,4),(4,5)\}$. Then, **4**

i. Draw Graph G.

ii. Draw $G - (3,4)$.

iii. Draw $G + (2,5)$.

iv. Find an Adjacency Matrix of G.

(B) Obtain the dnf and cnf of $\neg Q \vee (P \wedge Q)$. **3**

OR

Construct the truth table for formula $(Q \wedge (P \rightarrow Q)) \rightleftarrows P$.

(C) Check whether the following formulae are Tautology, Contradiction **OR** None. **7**

i. $(P \rightarrow Q) \rightleftarrows (Q \vee \neg P)$

ii. $(P \wedge (\neg P \vee Q)) \wedge (\neg Q)$

iii. $(Q \wedge P) \vee (Q \wedge \neg P)$

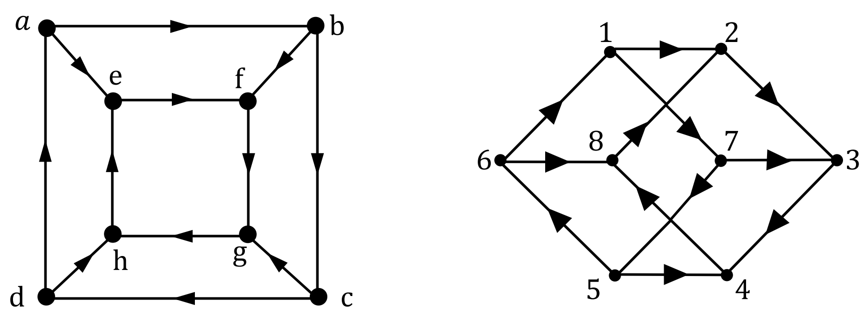
OR

Let $B(x)$ be " x is a boy ". Let $S(x)$ be " x is a student ". Let $P(x)$ be " x is a player ". Do as directed.

i. Translate into a formula: "Every boy is a player".

ii. Verify: "$(\exists(x))(S(x) \wedge (\neg(B(x)))$".

Q.4 **(A)** Is $G \cong H$? If they are isomorphic, give the isomorphism. If not, explain. **4**



Graph G

Graph H

(B) Define: Path Matrix, Order of Graph, Isolated Vertex **3**

OR

Define: Regular Graph, Adjacency Matrix, Simple Graph.

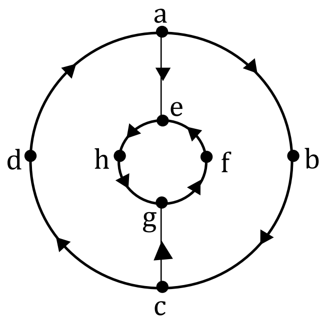
(C) i. How many vertices are necessary to construct a graph with 12 edges in which each vertex is degree 4. **7**

ii. Prove that in any undirected graph G, number of odd vertices must be even.

OR

i. Suppose a simple graph has 15 vertices, 3 vertices of degree 4 and all other vertices of degree 3. Find the number of edges in the graph.

ii. Find the strong, weak and unilateral component for the given diagraph.



Q.5 **(A)** Show that $(\mathbb{Z}_{13}^{*}, \times_{13})$ is an abelian group. **4**

(B) Let, $G = (\mathbb{Z}, +)$ and $H = 5\mathbb{Z}$ is a subgroup of G. then, show that $\left(\frac{G}{H}, +\right)$ is a group. **3**

OR

Find all subgroups of symmetric group S_3.

(C) Answer the following questions: **7**

i. If $f = (1\ 7\ 3\ 4\ 2)(5\ 6\ 1) \in S_7$. Find f^{-1}.

ii. What is the identity element of $(a, b) \oplus (c, d) = (ac, bc + d)$.

iii. What are the subgroup of group $(\mathbb{Z}, +)$?

iv. What are the generators of the group $(\mathbb{Z}_5, +_5)$.

v. If $G = \mathbb{Z}$ and $H = n\mathbb{Z}$ then, $\frac{G}{H} \cong$ .... ?

vi. Give an example of a Ring which has no zero divisor.

vii. Find the units of $\{1, 3, 7, 5\}$ in $\mathbb{Z}_8$.

OR

i. Show that cube root of unity forms an abelian group.

ii. Show that the set $S = \{ f: \mathbb{R} \longrightarrow \mathbb{R} \ / \ f \text{ is continuous } \}$ is a group under binary operation matrix addition.

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