DSA HW3 Report

Zhongze Tang (zt67)

Q1

In Line 7, you can change the number of random input. Then you will see the output of in-order traversal of this tree.

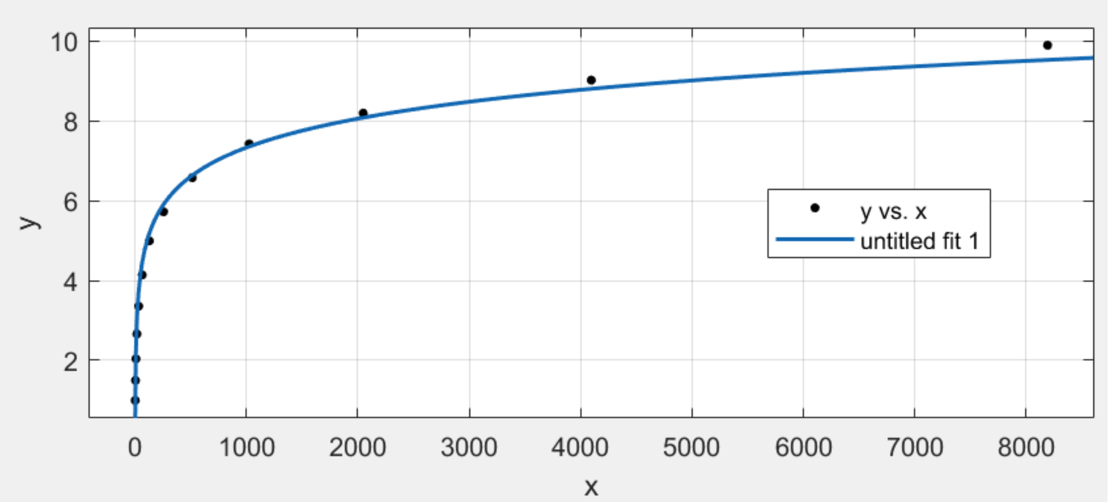
Q2

The experiment is based on the tree written in Q1.
If we consider the tree as a 2-3 tree (skip the red node when calculate the path length), the result is:

N	Ave path len(ordered)	Ave path len(random)
1	1.0	1.0
2	1.0	1.0
4	1.5	1.5
8	2.5	2.03875
16	4.5	2.665
32	8.5	3.35625
64	16.5	4.14390625
128	32.5	4.995859375
256	64.5	5.723359375
512	128.5	6.57669921875
1024	256.5	7.422119140625
2048	512.5	8.19423828125
4096	1024.5	9.0209716796875
8192	2048.5	9.893995361328125

I use Curve Fitting Tool in MATLAB to fit the number and the average length (with random input), and the result is:

*f(x) = 0.7237*log2(x) – 0.1194*



As for the ordered input,

the average path length = 1 (N<4) or N / 4 + 0.5 (N>=4).

At the same time, I also test the situation when we count the red nodes. I think this situation is the same as the BST’s. The result is shown in the table:

N	Ave path len(ordered)	Ave path len(random)
1	1.0	1.0
2	1.5	1.5
4	2.5	2.225
8	4.5	3.165
16	8.5	4.188125
32	16.5	5.3846875
64	32.5	6.496875
128	64.5	8.089921875
256	128.5	9.2400390625
512	256.5	10.5942578125
1024	512.5	12.00736328125
2048	1024.5	13.3846533203125
4096	2048.5	14.75482177734375
8192	4096.5	16.236561279296875

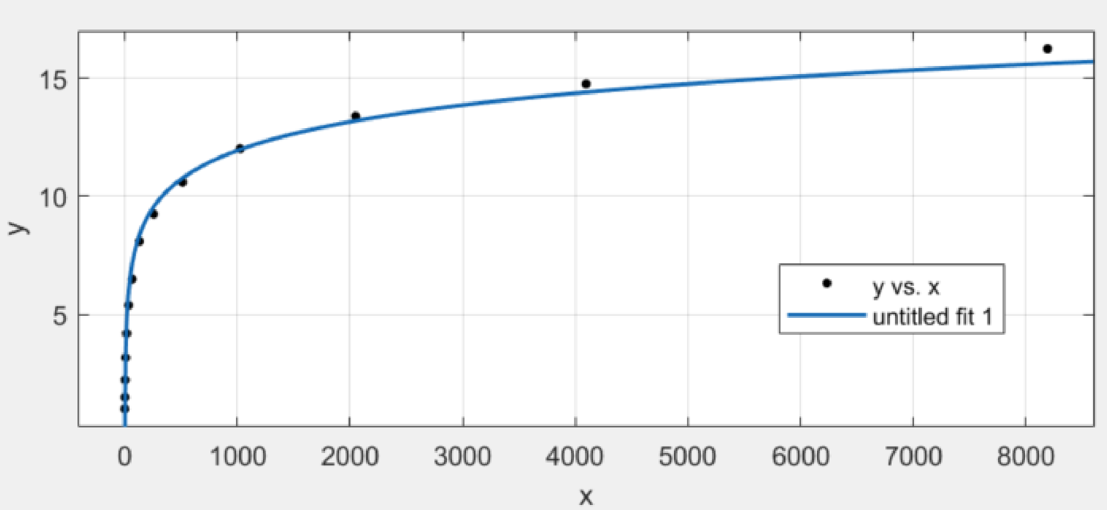
Adding the depths of all nodes, we get a quantity known as the internal path length of the tree. So, I use this formula to calculate the internal path length P_N:

$$P_N = N + P_L + P_R$$

where N is the size of Node P, and P_L and P_R are its two subtrees.

I use Curve Fitting Tool in MATLAB to fit the number and the average length (with random input), and the result is:

$$f(x) = 1.212*log2(x) - 0.1438$$



As for the ordered input,

the average path length = (N / 2 + 0.5).

The curve fitting file is Q2/Q2withoudred.sfit (first one) and Q2/Q2curvefit.sfit (second one).

The program now is calculating the path length skipping the red node. If you want to check the result of the one with red node, please take place of *getLenSumWithoutRed()* in Line 17 and 19 to *getLenSum()*.

Q3

The results are:

```
Result of 10000 is: 0.25411100000000003
Result of 100000 is: 0.2539944
Result of 1000000 is: 0.25390362999999994
```

We can safely conclude that the average percentage of red nodes in a random-input red-black tree is 25.4%.

The code of Red-Black BST is from *Algorithms* book website.

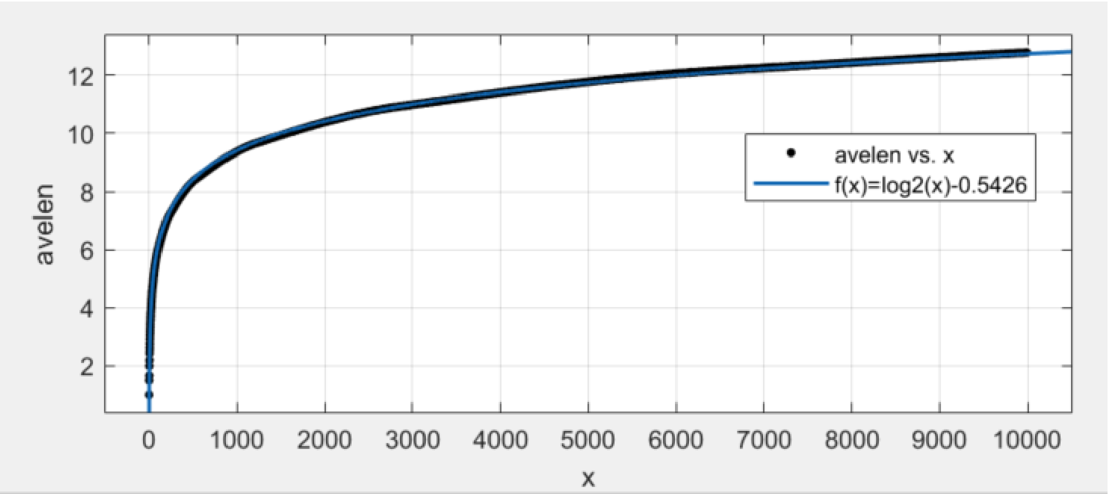
Q4

The calculation method of internal path length is the same in Q2:

$$P_N = N + P_L + P_R$$

I use Curve Fitting Tool in MATLAB to fit the number and the average length, and the result is:

$$f(x) = log2(x) - 0.5426$$



At the same time, the standard deviation is quite small, and its average value is 0.077.

All the results are stored in Q4/result.csv, and the curve fitting file is Q4/curvefit.sfit. The code of Red-Black BST is from *Algorithms* book website.

Note that it took almost 3 hours on my computer (i7-4710MQ, 8G RAM) to work out all the results.

Q5

(i) The value of *select(7)* is 8.

(ii) The value of *rank(7)* is 6.