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1 Basic

1.1 vimrc

```
colo torte
syn on
se ai ar sm nu rnu is
se mouse=a bs=2 ww+=<,>,[,] so=6 ts=4 sw=4 ttm=100
se makeprg=g++\ -Wall\ -Wshadow\ -O2\ -std=c++0x\ -o\
    %<\ %
au BufNewFile *.cpp Or ~/default.cpp | :1,$-6 fo
filetype indent on
map <F7> <ESC>:wa<CR>:make!<CR>
imap <F7> <ESC>:wa<CR>:make!<CR>
map <C-F7> <ESC>:tabe %<.in<CR>
map <F8> :cope <CR>
map <S-F8> :ccl <CR>
map <F9> :!./%< <CR>
map <C-F9> :!./%< < %<.in <CR>
    IncreaseStackSize
1.2
//stack resize
```

```
//stack resize
asm( "mov %0,%%esp\n" ::"g"(mem+10000000) );
//change esp to rsp if 64-bit system

//stack resize (linux)
#include <sys/resource.h>
void increase_stack_size() {
   const rlim_t ks = 64*1024*1024;
   struct rlimit rl;
   int res=getrlimit(RLIMIT_STACK, &rl);
   if(res==0) {
      if(rl.rlim_cur<ks) {
        rl.rlim_cur=ks;
        res=setrlimit(RLIMIT_STACK, &rl);
   }
   }
}</pre>
```

1.3 Default Code

```
#pragma GCC optimize ("02")
#include<bits/stdc++.h>
#include<unistd.h>
using namespace std;
#define FZ(n) memset((n),0,sizeof(n))
#define FMO(n) memset((n),-1,sizeof(n))
#define F first
#define S second
#define PB push_back
#define ALL(x) begin(x),end(x)
#define SZ(x) ((int)(x).size())
#define IOS ios_base::sync_with_stdio(0); cin.tie(0)
#define REP(i,x) for (int i=0; i<(x); i++)
#define REP1(i,a,b) for (int i=(a); i<=(b); i++)
#ifdef ONLINE_JUDGE
#define FILEIO(name) \
    freopen(name".in", "r", stdin); \
freopen(name".out", "w", stdout);
#else
#define FILEIO(name)
#endif
template<typename A, typename B>
ostream& operator <<(ostream &s, const pair<A,B> &p) {
    return s<<"("<<p.first<<","<<p.second<<")";</pre>
template<typename T>
ostream& operator <<(ostream &s, const vector<T> &c) {
    s<<"/";
    for (auto it : c) s << it << " ";</pre>
    s<<"]";
    return s;
// Let's Fight!
int main() {
    return 0;
}
```

2 Data Structure

2.1 Bigint

```
struct Bigint{
  static const int LEN = 60;
  static const int BIGMOD = 10000;
  int s;
  int vĺ, v[LEN];
  // vector<int> v;
  Bigint() : s(1) { vl = 0; }
  Bigint(long long a) {
    s = 1; vl = 0;
    if (a < 0) { s = -1; a = -a; }
    while (a) {
      push_back(a % BIGMOD);
      a /= BIGMOD;
  Bigint(string str) {
    s = 1; vl = 0;
    int stPos = 0, num = 0;
    if (!str.empty() && str[0] == '-') {
      stPos = 1;
      s = -1;
    for (int i=SZ(str)-1, q=1; i>=stPos; i--) {
  num += (str[i] - '0') * q;
      if ((q *= 10) >= BIGMOD) {
        push_back(num);
        num = 0; q = 1;
    if (num) push_back(num);
    n();
  }
  int len() const {
    return vl;
          return SZ(v);
  bool empty() const { return len() == 0; }
  void push_back(int x) {
    v[vl++] = x;
    //
          v.PB(x);
  }
  void pop_back() {
    //
          v.pop_back();
  int back() const {
    return v[vl-1];
          return v.back();
  void n() {
    while (!empty() && !back()) pop_back();
  void resize(int nl) {
    vl = nl;
    fill(v, v+vl, 0);
         v.resize(nl);
    //
    //
          fill(ALL(v), 0);
  void print() const {
    if (empty()) { putchar('0'); return; }
    if (s == -1) putchar('-');
    printf("%d", back());
    for (int i=len()-2; i>=0; i--) printf("%.4d",v[i]);
  friend std::ostream& operator << (std::ostream& out,</pre>
      const Bigint &a) {
    if (a.empty()) { out << "0"; return out; }</pre>
    if (a.s == -1) out << "-";</pre>
    out << a.back();
    for (int i=a.len()-2; i>=0; i--) {
      char str[10];
      snprintf(str, 5, "%.4d", a.v[i]);
      out << str;</pre>
```

```
return out;
int cp3(const Bigint &b)const {
  if (s != b.s) return s - b.s;
  if (s == -1) return -(-*this).cp3(-b);
  if (len() != b.len()) return len()-b.len();//int
  for (int i=len()-1; i>=0; i--)
    if (v[i]!=b.v[i]) return v[i]-b.v[i];
  return 0:
}
bool operator < (const Bigint &b)const{ return cp3(b)</pre>
bool operator <= (const Bigint &b)const{ return cp3(b</pre>
    )<=0; }
bool operator == (const Bigint &b)const{ return cp3(b
    )==0; }
bool operator != (const Bigint &b)const{ return cp3(b
    )!=0; }
bool operator > (const Bigint &b)const{ return cp3(b)
    >0; }
bool operator >= (const Bigint &b)const{ return cp3(b
    )>=0; }
Bigint operator - () const {
  Bigint r = (*this);
  r.s = -r.s;
  return r;
Bigint operator + (const Bigint &b) const {
  if (s == -1) return -(-(*this)+(-b));
  if (b.s == -1) return (*this)-(-b);
  Bigint r;
  int nl = max(len(), b.len());
  r.resize(nl + 1);
  for (int i=0; i<nl; i++) {</pre>
    if (i < len()) r.v[i] += v[i];</pre>
    if (i < b.len()) r.v[i] += b.v[i];</pre>
    if(r.v[i] >= BIGMOD) {
      r.v[i+1] += r.v[i] / BIGMOD;
      r.v[i] %= BIGMOD;
    }
  }
  r.n();
  return r;
Bigint operator - (const Bigint &b) const {
  if (s == -1) return -(-(*this)-(-b));
  if (b.s == -1) return (*this)+(-b);
  if ((*this) < b) return -(b-(*this));</pre>
  Bigint r;
  r.resize(len());
  for (int i=0; i<len(); i++) {</pre>
    r.v[i] += v[i];
    if (i < b.len()) r.v[i] -= b.v[i];</pre>
    if (r.v[i] < 0) {</pre>
      r.v[i] += BIGMOD;
      r.v[i+1]--;
    }
  r.n();
  return r;
Bigint operator * (const Bigint &b) {
  r.resize(len() + b.len() + 1);
  r.s = s * b.s;
  for (int i=0; i<len(); i++) {</pre>
    for (int j=0; j<b.len(); j++) {</pre>
      r.v[i+j] += v[i] * b.v[j];
      if(r.v[i+j] >= BIGMOD) {
        r.v[i+j+1] += r.v[i+j] / BIGMOD;
        r.v[i+j] %= BIGMOD;
      }
    }
  }
  r.n();
  return r;
Bigint operator / (const Bigint &b) {
```

```
Bigint r:
  r.resize(max(1, len()-b.len()+1));
  int oriS = s:
  Bigint b2 = b; // b2 = abs(b)
  s = b2.s = r.s = 1;
  for (int i=r.len()-1; i>=0; i--) {
    int d=0, u=BIGMOD-1;
    while(d<u) {</pre>
      int m = (d+u+1) >> 1;
      r.v[i] = m;
      if((r*b2) > (*this)) u = m-1;
      else d = m;
    }
   r.v[i] = d;
 }
  s = oriS;
 r.s = s * b.s;
  r.n();
 return r:
Bigint operator % (const Bigint &b) {
 return (*this)-(*this)/b*b;
```

2.2 unordered_map

```
struct Kev {
  int first, second;
  Key () {}
  Key (int _x, int _y) : first(_x), second(_y) {}
  bool operator == (const Key &b) const {
    return tie(F,S) == tie(b.F,b.S);
 }
};
struct KevHasher {
 size_t operator()(const Key& k) const {
    return k.first + k.second*100000;
};
typedef unordered_map<Key,int,KeyHasher> map_t;
int main(int argc, char** argv){
  map_t mp;
  for (int i=0; i<10; i++)</pre>
   mp[Key(i,0)] = i+1;
  for (int i=0; i<10; i++)</pre>
    printf("%d \ n", mp[Key(i,0)]);
  return 0;
}
```

2.3 extc_heap

```
#include <bits/extc++.h>
typedef __gnu_pbds::priority_queue<int> heap_t;
heap_t a,b;
int main() {
  a.clear();
  b.clear();
  a.push(1);
  a.push(3);
  b.push(2);
  b.push(4);
  assert(a.top() == 3);
  assert(b.top() == 4);
  // merge two heap
  a.join(b);
  assert(a.top() == 4);
  assert(b.empty());
  return 0;
```

2.4 extc balance tree

```
#include <ext/pb_ds/assoc_container.hpp>
using namespace std;
using namespace __gnu_pbds;
typedef tree<int,null_type,less<int>,rb_tree_tag,
    tree_order_statistics_node_update> set_t;
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
typedef cc_hash_table<int,int> umap_t;
int main()
  // Insert some entries into s.
  set_t s;
  s.insert(12):
  s.insert(505);
 // The order of the keys should be: 12, 505.
 assert(*s.find_by_order(0) == 12);
 assert(s.find_by_order(2) == end(s));
 // The order of the keys should be: 12, 505.
 assert(s.order_of_key(12) == 0);
 assert(s.order_of_key(505) == 1);
 // Erase an entry.
 s.erase(12);
 // The order of the keys should be: 505.
 assert(*s.find_by_order(0) == 505);
  // The order of the keys should be: 505.
 assert(s.order_of_key(505) == 0);
}
```

2.5 Disjoint Set

```
struct DisjointSet {
 // save() is like recursive
  // undo() is like return
  int n, fa[MXN], sz[MXN];
  vector<pair<int*,int>> h;
  vector<int> sp;
 void init(int tn) {
   n=tn;
    for (int i=0; i<n; i++) {</pre>
      fa[i]=i;
      sz[i]=1;
    sp.clear(); h.clear();
 void assign(int *k, int v) {
   h.PB({k, *k});
    *k=v;
  void save() { sp.PB(SZ(h)); }
  void undo() {
    assert(!sp.empty());
    int last=sp.back(); sp.pop_back();
    while (SZ(h)!=last) {
      auto x=h.back(); h.pop_back();
      *x.F=x.S;
    }
  int f(int x) {
    while (fa[x]!=x) x=fa[x];
    return x;
  void uni(int x, int y) {
    x=f(x); y=f(y);
    if (x==y) return ;
    if (sz[x] < sz[y]) swap(x, y);
    assign(&sz[x], sz[x]+sz[y]);
    assign(&fa[y], x);
}djs;
```

2.6 Treap

```
const int MEM = 16000004;
struct Treap {
  static Treap nil, mem[MEM], *pmem;
  Treap *l, *r;
  char val;
  int size;
  Treap () : l(&nil), r(&nil), size(0) {}
  Treap (char _val) :
    l(&nil), r(&nil), val(_val), size(1) {}
} Treap::nil, Treap::mem[MEM], *Treap::pmem = Treap::
    mem;
int size(const Treap *t) { return t->size; }
void pull(Treap *t) {
  if (!size(t)) return;
  t\rightarrow size = size(t\rightarrow l) + size(t\rightarrow r) + 1;
Treap* merge(Treap *a, Treap *b) {
  if (!size(a)) return b;
  if (!size(b)) return a;
  Treap *t;
  if (rand() % (size(a) + size(b)) < size(a)) {</pre>
    t = new (Treap::pmem++) Treap(*a);
    t->r = merge(a->r, b);
  } else {
    t = new (Treap::pmem++) Treap(*b);
    t->l = merge(a, b->l);
  pull(t);
  return t;
void split(Treap *t, int k, Treap *&a, Treap *&b) {
  if (!size(t)) a = b = &Treap::nil;
  else if (size(t->l) + 1 <= k) {
    a = new (Treap::pmem++) Treap(*t);
    split(t->r, k - size(t->l) - 1, a->r, b);
    pull(a);
  } else {
    b = new (Treap::pmem++) Treap(*t);
    split(t->l, k, a, b->l);
    pull(b);
}
int nv;
Treap *rt[50005];
void print(const Treap *t) {
  if (!size(t)) return;
  print(t->l);
  cout << t->val;
  print(t->r);
int main(int argc, char** argv) {
  IOS;
  rt[nv=0] = &Treap::nil;
  Treap::pmem = Treap::mem;
  int Q, cmd, p, c, v;
  string s;
  cin >> Q;
  while (Q--) {
    cin >> cmd;
    if (cmd == 1) {
      // insert string s after position p
      cin >> p >> s;
      Treap *tl, *tr;
      split(rt[nv], p, tl, tr);
for (int i=0; i<SZ(s); i++)</pre>
        tl = merge(tl, new (Treap::pmem++) Treap(s[i]))
      rt[++nv] = merge(tl, tr);
    } else if (cmd == 2) {
      // remove c characters starting at position
      Treap *tl, *tm, *tr;
      cin >> p >> c;
      split(rt[nv], p-1, tl, tm);
      split(tm, c, tm, tr);
      rt[++nv] = merge(tl, tr);
```

2.7 Heavy Light Decomposition

```
// only one segment tree / 0-base
// should call init after input N
 getPathSeg return the segment in order u->v
// fa[root] = root
typedef pair<int,int> pii;
int N, fa[MXN], belong[MXN], dep[MXN], sz[MXN], que[MXN];
int step,line[MXN],stPt[MXN],edPt[MXN];
vector<int> E[MXN], chain[MXN];
void init() {
 REP(i,N) {
   E[i].clear();
    chain[i].clear();
 }
void DFS(int u){
 vector<int> &c = chain[belong[u]];
 for (int i=c.size()-1; i>=0; i--){
   int v = c[i];
    stPt[v] = step;
    line[step++] = v;
  for (int i=0; i<(int)c.size(); i++){</pre>
   u = c[i];
    for (auto v : E[u]){
      if (fa[u] == v || (i && v == c[i-1])) continue;
      DFS(v);
    edPt[u] = step-1;
 }
void build_chain(int st){
 int fr,bk;
  fr=bk=0; que[bk++]=st; fa[st]=st; dep[st]=0;
 while (fr < bk){</pre>
    int u=que[fr++];
    for (auto v : E[u]){
      if (v == fa[u]) continue;
      que[bk++] = v;
      dep[v] = dep[u]+1;
      fa[v] = u;
   }
  for (int i=bk-1,u,pos; i>=0; i--){
   u = que[i]; sz[u] = 1; pos = -1;
    for (auto v : E[u]){
      if (v == fa[u]) continue;
      sz[u] += sz[v];
      if (pos==-1 || sz[v]>sz[pos]) pos=v;
   if (pos == -1) belong[u] = u;
    else belong[u] = belong[pos];
   chain[belong[u]].PB(u);
  }
 step = 0;
 DFS(st);
int getLCA(int u, int v){
 while (belong[u] != belong[v]){
    int a = chain[belong[u]].back();
    int b = chain[belong[v]].back();
    if (dep[a] > dep[b]) u = fa[a];
```

```
else v = fa[b]:
  return sz[u] >= sz[v] ? u : v;
}
vector<pii> getPathSeg(int u, int v){
  vector<pii> ret1,ret2;
  while (belong[u] != belong[v]){
    int a = chain[belong[u]].back();
    int b = chain[belong[v]].back();
    if (dep[a] > dep[b]){
      ret1.PB({stPt[a],stPt[u]});
      u = fa[a];
    } else {
      ret2.PB({stPt[b],stPt[v]});
      v = fa[b];
  if (dep[u] > dep[v]) swap(u,v);
  ret1.PB({stPt[u],stPt[v]});
  reverse(ret2.begin(), ret2.end());
  ret1.insert(ret1.end(),ret2.begin(),ret2.end());
  return ret1;
// Usage
void build(){
  build_chain(0); //change root
  init(0,step,0); //init segment tree
int get_answer(int u, int v){
  int ret = -2147483647;
  vector<pii> vec = getPathSeg(u,v);
  for (auto it : vec)
    ; // check answer with segment [it.F, it.S]
  return ret;
```

2.8 Link-Cut Tree

```
const int MXN = 100005;
const int MEM = 100005:
struct Splay {
  static Splay nil, mem[MEM], *pmem;
  Splay *ch[2], *f;
  int val, rev, size;
  Splay () : val(-1), rev(0), size(0) {
    f = ch[0] = ch[1] = &nil;
  Splay (int _val) : val(_val), rev(0), size(1) {
    f = ch[0] = ch[1] = &nil;
  bool isr() {
    return f->ch[0] != this && f->ch[1] != this;
  int dir() {
    return f->ch[0] == this ? 0 : 1;
  void setCh(Splay *c, int d) {
    ch[d] = c;
    if (c != &nil) c->f = this;
    pull();
  void push() {
    if (rev) {
      swap(ch[0], ch[1]);
      if (ch[0] != &nil) ch[0]->rev ^= 1;
      if (ch[1] != &nil) ch[1]->rev ^= 1;
      rev=0;
    }
  void pull() {
    size = ch[0] -> size + ch[1] -> size + 1;
    if (ch[0] != &nil) ch[0]->f = this;
    if (ch[1] != &nil) ch[1]->f = this;
} Splay::nil, Splay::mem[MEM], *Splay::pmem = Splay::
    mem:
Splay *nil = &Splay::nil;
void rotate(Splay *x) {
```

```
Splay *p = x -> f;
  int d = x->dir();
  if (!p->isr()) p->f->setCh(x, p->dir());
  else x->f = p->f;
  p->setCh(x->ch[!d], d);
  x->setCh(p, !d);
  p->pull(); x->pull();
vector<Splay*> splayVec;
void splay(Splay *x) {
  splayVec.clear();
  for (Splay *q=x;; q=q->f) {
    splayVec.push_back(q);
    if (q->isr()) break;
  reverse(begin(splayVec), end(splayVec));
for (auto it : splayVec) it->push();
  while (!x->isr()) {
    if (x->f->isr()) rotate(x);
    else if (x->dir()==x->f->dir()) rotate(x->f),rotate
         (x);
    else rotate(x),rotate(x);
}
Splay* access(Splay *x) {
  Splay *q = nil;
  for (;x!=nil;x=x->f) {
    splay(x);
    x->setCh(q, 1);
    q = x;
  }
  return q;
void evert(Splay *x) {
  access(x);
  splay(x);
  x->rev ^= 1;
  x->push(); x->pull();
void link(Splay *x, Splay *y) {
// evert(x);
 access(x);
  splay(x);
  evert(y);
  x->setCh(y, 1);
void cut(Splay *x, Splay *y) {
// evert(x);
  access(y);
  splay(y);
  y->push();
  y->ch[0] = y->ch[0]->f = nil;
int N, Q;
Splay *vt[MXN];
int ask(Splay *x, Splay *y) {
 access(x);
  access(y);
  splay(x);
  int res = x->f->val;
  if (res == -1) res=x->val;
  return res;
int main(int argc, char** argv) {
  scanf("%d%d", &N, &Q);
  for (int i=1; i<=N; i++)</pre>
    vt[i] = new (Splay::pmem++) Splay(i);
  while (Q--) {
    char cmd[105];
    int u, v;
scanf("%s", cmd);
if (cmd[1] == 'i') {
      scanf("%d%d", &u, &v);
      link(vt[v], vt[u]);
    } else if (cmd[0] == 'c') {
      scanf("%d", &v);
      cut(vt[1], vt[v]);
```

} **else** {

```
scanf("%d%d", &u, &v);
    int res=ask(vt[u], vt[v]);
    printf("%d\n", res);
    }
}
return 0;
}
```

3 Graph

3.1 BCC Edge

```
struct BccEdge {
   static const int MXN = 100005;
   struct Edge { int v,eid; };
   int n,m,step,par[MXN],dfn[MXN],low[MXN];
   vector<Edge> E[MXN];
   DisjointSet djs;
   void init(int _n) {
    n = _n; m = 0;
for (int i=0; i<n; i++) E[i].clear();
     djs.init(n);
   void add_edge(int u, int v) {
     E[u].PB({v, m});
     E[v].PB({u, m});
     m++;
   void DFS(int u, int f, int f_eid) {
     par[u] = f;
     dfn[u] = low[u] = step++;
     for (auto it:E[u]) {
       if (it.eid == f_eid) continue;
       int v = it.v;
       if (dfn[v] == -1) {
         DFS(v, u, it.eid);
         low[u] = min(low[u], low[v]);
         low[u] = min(low[u], dfn[v]);
    }
   void solve() {
     step = 0;
     memset(dfn, -1, sizeof(int)*n);
     for (int i=0; i<n; i++) {</pre>
      if (dfn[i] == -1) DFS(i, i, -1);
     djs.init(n);
     for (int i=0; i<n; i++) {</pre>
       if (low[i] < dfn[i]) djs.uni(i, par[i]);</pre>
  }
}graph;
```

3.2 BCC Vertex

```
struct BccVertex {
  int n,nBcc,step,root,dfn[MXN],low[MXN];
  vector<int> E[MXN], ap;
  vector<pii> bcc[MXN];
  int top;
  pii stk[MXN];
  void init(int _n) {
    n = _n;
    nBcc = step = 0;
    for (int i=0; i<n; i++) E[i].clear();</pre>
  void add_edge(int u, int v) {
    E[u].PB(v);
    E[v].PB(u);
  void DFS(int u, int f) {
    dfn[u] = low[u] = step++;
    int son = 0;
```

```
for (auto v:E[u]) {
      if (v == f) continue;
      if (dfn[v] == -1) {
        son++;
        stk[top++] = \{u,v\};
        DFS(v,u);
        if (low[v] >= dfn[u]) {
          if(v != root) ap.PB(v);
          do {
            assert(top > 0);
            bcc[nBcc].PB(stk[--top]);
          } while (stk[top] != pii(u,v));
        low[u] = min(low[u], low[v]);
        if (dfn[v] < dfn[u]) stk[top++] = pii(u,v);</pre>
        low[u] = min(low[u],dfn[v]);
    }
    if (u == root && son > 1) ap.PB(u);
  }
  // return the edges of each bcc;
  vector<vector<pii>>> solve() {
    vector<vector<pii>>> res;
    for (int i=0; i<n; i++) {</pre>
      dfn[i] = low[i] = -1;
    ap.clear();
    for (int i=0; i<n; i++) {</pre>
      if (dfn[i] == -1) {
        top = 0;
        root = i;
        DFS(i,i);
    REP(i,nBcc) res.PB(bcc[i]);
    return res;
  }
}graph;
```

3.3 Strongly Connected Components

```
struct Scc{
  int n, nScc, vst[MXN], bln[MXN];
  vector<int> E[MXN], rE[MXN], vec;
  void init(int _n){
    n = _n;
for (int i=0; i<n; i++){</pre>
      E[i].clear();
      rE[i].clear();
    }
  void add_edge(int u, int v){
    E[u].PB(v);
    rE[v].PB(u);
  void DFS(int u){
    vst[u]=1;
    for (auto v : E[u])
      if (!vst[v]) DFS(v);
    vec.PB(u);
  }
  void rDFS(int u){
    vst[u] = 1;
    bln[u] = nScc;
    for (auto v : rE[u])
      if (!vst[v]) rDFS(v);
  void solve(){
    nScc = 0;
    vec.clear();
    for (int i=0; i<n; i++) vst[i] = 0;</pre>
    for (int i=0; i<n; i++)
      if (!vst[i]) DFS(i);
    reverse(vec.begin(),vec.end());
for (int i=0; i<n; i++) vst[i] = 0;</pre>
    for (auto v : vec){
      if (!vst[v]){
        rDFS(v);
```

```
nScc++;
}
}
};
```

3.4 DMST_with_sol

```
const int INF = 1029384756;
struct edge_t{
  int u,v,w;
  set< pair<int,int> > add, sub;
  edge_t() : u(-1), v(-1), w(0) {}
  edge_t(int _u, int _v, int _w) {
    u = _u;    v = _v;    w = _w;
    add.insert({u, v});
  edge_t& operator += (const edge_t& obj) {
    w += obj.w;
    FOR (it, obj.add) {
      if (!sub.count(*it)) add.insert(*it);
      else sub.erase(*it);
    FOR (it, obj.sub) {
      if (!add.count(*it)) sub.insert(*it);
      else add.erase(*it);
    return *this;
  edge_t& operator -= (const edge_t& obj) {
    w -= obj.w;
    FOR (it, obj.sub) {
      if (!sub.count(*it)) add.insert(*it);
      else sub.erase(*it);
    for (auto it : obj.add) {
      if (!add.count(it)) sub.insert(it);
      else add.erase(it);
    return *this;
}eg[MXN*MXN],prv[MXN],EDGE_INF(-1,-1,INF);
int N,M;
int cid,incyc[MXN],contracted[MXN];
vector<int> E[MXN];
edge_t dmst(int rt){
  edge_t cost;
  for (int i=0; i<N; i++){</pre>
    contracted[i] = incyc[i] = 0;
    prv[i] = EDGE_INF;
  cid = 0;
  int u,v;
  while (true){
    for (v=0; v<N; v++){</pre>
      if (v != rt && !contracted[v] && prv[v].w == INF)
           break;
    if (v >= N) break; // end
    for (int i=0; i<M; i++){</pre>
      if (eg[i].v == v && eg[i].w < prv[v].w)</pre>
        prv[v] = eg[i];
    if (prv[v].w == INF) // not connected
      return EDGE_INF;
    cost += prv[v];
    for (u=prv[v].u; u!=v && u!=-1; u=prv[u].u);
    if (u == -1) continue;
    incyc[v] = ++cid;
    for (u=prv[v].u; u!=v; u=prv[u].u){
      contracted[u] = 1;
      incyc[u] = cid;
    for (int i=0; i<M; i++){</pre>
      if (incyc[eg[i].u] != cid && incyc[eg[i].v] ==
           cid){
        eg[i] -= prv[eg[i].v];
```

```
for (int i=0; i<M; i++){</pre>
      if (incyc[eg[i].u] == cid) eg[i].u = v;
      if (incyc[eg[i].v] == cid) eg[i].v = v;
      if (eg[i].u == eg[i].v) eg[i--] = eg[--M];
    for (int i=0; i<N; i++){</pre>
      if (contracted[i]) continue;
      if (prv[i].u>=0 && incyc[prv[i].u] == cid)
        prv[i].u = v;
    prv[v] = EDGE_INF;
  }
  return cost;
}
void solve(){
  edge_t cost = dmst(0);
  for (auto it : cost.add){ // find a solution
    E[it.F].PB(it.S);
    prv[it.S] = edge_t(it.F,it.S,0);
}
```

3.5 Dominator Tree

```
// idom[n] is the unique node that strictly dominates n
     but does
// not strictly dominate any other node that strictly
    dominates n.
// idom[n] = 0 if n is entry or the entry cannot reach
struct DominatorTree{
  static const int MAXN = 200010;
  int n,s;
  vector<int> g[MAXN],pred[MAXN];
  vector<int> cov[MAXN];
  int dfn[MAXN],nfd[MAXN],ts;
  int par[MAXN];
  int sdom[MAXN],idom[MAXN];
  int mom[MAXN],mn[MAXN];
  inline bool cmp(int u,int v) { return dfn[u] < dfn[v</pre>
      ]; }
  int eval(int u) {
    if(mom[u] == u) return u;
    int res = eval(mom[u]);
    if(cmp(sdom[mn[mom[u]]),sdom[mn[u]]))
      mn[u] = mn[mom[u]];
    return mom[u] = res;
  void init(int _n, int _s) {
    n = _n;
    s = _s;
    REP1(i,1,n) {
      g[i].clear();
      pred[i].clear();
      idom[i] = 0;
    }
  void add_edge(int u, int v) {
    g[u].push_back(v);
    pred[v].push_back(u);
  void DFS(int u) {
    ts++;
    dfn[u] = ts;
    nfd[ts] = u;
    for(int v:g[u]) if(dfn[v] == 0) {
      par[v] = u;
      DFS(v);
    }
  void build() {
    ts = 0;
    REP1(i,1,n) {
      dfn[i] = nfd[i] = 0;
      cov[i].clear();
```

```
mom[i] = mn[i] = sdom[i] = i;
    DFS(s);
    for (int i=ts; i>=2; i--) {
      int u = nfd[i];
      if(u == 0) continue ;
      for(int v:pred[u]) if(dfn[v]) {
        eval(v);
        if(cmp(sdom[mn[v]],sdom[u])) sdom[u] = sdom[mn[
      cov[sdom[u]].push_back(u);
      mom[u] = par[u];
      for(int w:cov[par[u]]) {
        eval(w);
        if(cmp(sdom[mn[w]],par[u])) idom[w] = mn[w];
        else idom[w] = par[u];
      cov[par[u]].clear();
    REP1(i,2,ts) {
      int u = nfd[i];
      if(u == 0) continue
      if(idom[u] != sdom[u]) idom[u] = idom[idom[u]];
  }
}dom;
```

3.6 Maximum Clique

```
class MaxClique {
public:
    static const int MV = 210;
    int el[MV][MV/30+1];
    int dp[MV];
    int ans;
    int s[MV][MV/30+1];
    vector<int> sol;
    void init(int v) {
        V = v; ans = 0;
        FZ(el); FZ(dp);
    /* Zero Base */
    void addEdge(int u, int v) {
        if(u > v) swap(u, v);
        if(u == v) return;
        el[u][v/32] |= (1<<(v%32));
    bool dfs(int v, int k) {
        int c = 0, d = 0;
        for(int i=0; i<(V+31)/32; i++) {</pre>
            s[k][i] = el[v][i];
            if(k != 1) s[k][i] &= s[k-1][i];
            c += __builtin_popcount(s[k][i]);
        if(c == 0) {
             if(k > ans) {
                 ans = k;
                 sol.clear();
                 sol.push_back(v);
                 return 1;
            return 0;
        for(int i=0; i<(V+31)/32; i++) {</pre>
             for(int a = s[k][i]; a; d++) {
                 if(k + (c-d) <= ans) return 0;</pre>
                 int lb = a&(-a), lg = 0;
                 a ^= lb;
                 while(lb!=1) {
                     lb = (unsigned int)(lb) >> 1;
                     lg ++;
                 int u = i*32 + lg;
                 if(k + dp[u] <= ans) return 0;</pre>
```

3.7 MinimumMeanCycle

```
/* minimum mean cvcle */
const int MAXE = 1805;
const int MAXN = 35;
const double inf = 1029384756;
const double eps = 1e-6;
struct Edge {
  int v,u;
  double c;
int n,m,prv[MAXN][MAXN], prve[MAXN][MAXN], vst[MAXN];
Edge e[MAXE];
vector<int> edgeID, cycle, rho;
double d[MAXN][MAXN];
inline void bellman_ford() {
  for(int i=0; i<n; i++) d[0][i]=0;</pre>
  for(int i=0; i<n; i++) {</pre>
    fill(d[i+1], d[i+1]+n, inf);
    for(int j=0; j<m; j++) {</pre>
      int v = e[j].v, u = e[j].u;
      if(d[i][v]<inf && d[i+1][u]>d[i][v]+e[j].c) {
        d[i+1][u] = d[i][v]+e[j].c;
        prv[i+1][u] = v
        prve[i+1][u] = j;
      }
    }
  }
double karp_mmc() {
  // returns inf if no cycle, mmc otherwise
  double mmc=inf;
  int st = -1;
  bellman_ford();
  for(int i=0; i<n; i++) {</pre>
    double avg=-inf;
    for(int k=0; k<n; k++) {</pre>
      if(d[n][i]<inf-eps) avg=max(avg,(d[n][i]-d[k][i])</pre>
           /(n-k);
      else avg=max(avg,inf);
    if (avg < mmc) tie(mmc, st) = tie(avg, i);</pre>
  for(int i=0; i<n; i++) vst[i] = 0;</pre>
  edgeID.clear(); cycle.clear(); rho.clear();
  for (int i=n; !vst[st]; st=prv[i--][st]) {
    vst[st]++;
    edgeID.PB(prve[i][st]);
    rho.PB(st);
  while (vst[st] != 2) {
    int v = rho.back(); rho.pop_back();
    cycle.PB(v);
    vst[v]++;
  reverse(ALL(edgeID));
  edgeID.resize(SZ(cycle));
  return mmc;
}
```

4 Flow

4.1 Dinic

```
struct Dinic{
  static const int MXN = 10000;
  struct Edge{ int v,f,re; };
  int n,s,t,level[MXN];
  vector<Edge> E[MXN];
  void init(int _n, int _s, int _t){
    n = _n; s = _s; t = _t;
    for (int i=0; i<n; i++) E[i].clear();</pre>
  void add_edge(int u, int v, int f){
    E[u].PB({v,f,SZ(E[v])});
    E[v].PB({u,0,SZ(E[u])-1});
  bool BFS(){
    for (int i=0; i<n; i++) level[i] = -1;</pre>
    queue<int> que;
    que.push(s);
    level[s] = 0;
    while (!que.empty()){
      int u = que.front(); que.pop();
      for (auto it : E[u]){
        if (it.f > 0 && level[it.v] == -1){
          level[it.v] = level[u]+1;
          que.push(it.v);
        }
      }
    }
    return level[t] != -1;
  int DFS(int u, int nf){
    if (u == t) return nf;
    int res = 0;
    for (auto &it : E[u]){
      if (it.f > 0 && level[it.v] == level[u]+1){
        int tf = DFS(it.v, min(nf,it.f));
        res += tf; nf -= tf; it.f -= tf;
        E[it.v][it.re].f += tf;
        if (nf == 0) return res;
      }
    if (!res) level[u] = -1;
    return res;
  int flow(int res=0){
    while ( BFS() )
      res += DFS(s,2147483647);
    return res;
}flow;
```

4.2 Cost Flow

```
typedef pair<long long, long long> pll;
struct CostFlow {
 static const int MXN = 205;
 static const long long INF = 102938475610293847LL;
 struct Edge {
   int v, r;
   long long f, c;
 int n, s, t, prv[MXN], prvL[MXN], inq[MXN];
 long long dis[MXN], fl, cost;
 vector<Edge> E[MXN];
 for (int i=0; i<n; i++) E[i].clear();</pre>
   fl = cost = 0;
 void add_edge(int u, int v, long long f, long long c)
   E[u].PB(\{v, SZ(E[v]), f, c\});
   E[v].PB({u, SZ(E[u])-1, 0, -c});
 pll flow() {
```

```
while (true) {
      for (int i=0; i<n; i++) {</pre>
        dis[i] = INF;
        inq[i] = 0;
      dis[s] = 0;
      queue<int> que;
      que.push(s);
      while (!que.empty()) {
         int u = que.front(); que.pop();
         inq[u] = 0;
        for (int i=0; i<SZ(E[u]); i++) {</pre>
          int v = E[u][i].v;
           long long w = E[u][i].c;
          if (E[u][i].f > 0 && dis[v] > dis[u] + w) {
             prv[v] = u; prvL[v] = i;
             dis[v] = dis[u] + w;
             if (!inq[v]) {
               inq[v] = 1;
               que.push(v);
          }
        }
      if (dis[t] == INF) break;
      long long tf = INF;
      for (int v=t, u, l; v!=s; v=u) {
        u=prv[v]; l=prvL[v];
        tf = min(tf, E[u][l].f);
      for (int v=t, u, l; v!=s; v=u) {
        u=prv[v]; l=prvL[v];
        E[u][l].f -= tf;
        E[v][E[u][l].r].f += tf;
      cost += tf * dis[t];
      fl += tf;
    return {fl, cost};
}flow;
```

4.3 Kuhn Munkres

```
struct KM{
// Maximum Bipartite Weighted Matching (Perfect Match)
 static const int MXN = 650;
static const int INF = 2147483647; // long long
  int n,match[MXN],vx[MXN],vy[MXN];
 int edge[MXN][MXN],lx[MXN],ly[MXN],slack[MXN];
  // ^^^ long long
  void init(int _n){
   n = _n;
    for (int i=0; i<n; i++)</pre>
      for (int j=0; j<n; j++)</pre>
        edge[i][j] = 0;
  void add_edge(int x, int y, int w){ // long long
    edge[x][y] = w;
 bool DFS(int x){
    vx[x] = 1;
    for (int y=0; y<n; y++){</pre>
      if (vy[y]) continue;
      if (lx[x]+ly[y] > edge[x][y]){
        slack[y] = min(slack[y], lx[x]+ly[y]-edge[x][y
             ]);
      } else {
        vy[y] = 1;
        if (match[y] == -1 || DFS(match[y])){
          match[y] = x;
          return true;
      }
    return false;
  int solve(){
    fill(match, match+n, -1);
    fill(lx,lx+n,-INF);
```

```
fill(ly,ly+n,0);
      for (int i=0; i<n; i++)</pre>
        for (int j=0; j<n; j++)</pre>
           lx[i] = max(lx[i], edge[i][j]);
      for (int i=0; i<n; i++){</pre>
        fill(slack,slack+n,INF);
        while (true){
           fill(vx,vx+n,0);
           fill(vy,vy+n,0);
           if ( DFS(i) ) break;
          int d = INF; // long long
for (int j=0; j<n; j++)</pre>
             if (!vy[j]) d = min(d, slack[j]);
           for (int j=0; j<n; j++){
  if (vx[j]) lx[j] -= d;</pre>
             if (vy[j]) ly[j] += d;
             else slack[j] -= d;
       }
     }
      int res=0;
     for (int i=0; i<n; i++)</pre>
       res += edge[match[i]][i];
      return res;
   7
}graph;
```

4.4 SW-Mincut

```
struct SW{ // O(V^3) 0-base
  static const int MXN = 514;
  int n,vst[MXN],del[MXN];
  int edge[MXN][MXN],wei[MXN];
  void init(int _n){
    n = _n;
for (int i=0; i<n; i++) {</pre>
      for (int j=0; j<n; j++)</pre>
         edge[i][j] = 0;
      del[i] = 0;
    }
  }
  void add_edge(int u, int v, int w){
    edge[u][v] += w;
    edge[v][u] += w;
  void search(int &s, int &t){
    for (int i=0; i<n; i++)</pre>
      vst[i] = wei[i] = 0;
    s = t = -1:
    while (true){
      int mx=-1, cur=0;
for (int i=0; i<n; i++)</pre>
         if (!del[i] && !vst[i] && mx<wei[i])</pre>
           cur = i, mx = wei[i];
      if (mx == -1) break;
      vst[cur] = 1;
      s = t;
      t = cur;
      for (int i=0; i<n; i++)</pre>
         if (!vst[i] && !del[i]) wei[i] += edge[cur][i];
    }
  }
  int solve(){
    int res = 2147483647;
    for (int i=0,x,y; i<n-1; i++){</pre>
      search(x,y);
      res = min(res,wei[y]);
      del[y] = 1;
      for (int j=0; j<n; j++)</pre>
         edge[x][j] = (edge[j][x] += edge[y][j]);
    return res;
  }
}graph;
```

4.5 Maximum Simple Graph Matching

```
struct GenMatch { // 1-base
  static const int MAXN = 514;
 bool el[MAXN][MAXN];
  int pr[MAXN];
 bool inq[MAXN],inp[MAXN],inb[MAXN];
  queue<int> qe;
  int st,ed;
  int nb:
  int bk[MAXN],djs[MAXN];
  int ans;
  void init(int V) {
    V = V;
    for(int i = 0; i <= V; i++) {</pre>
      for(int j = 0; j <= V; j++) el[i][j] = 0;</pre>
      pr[i] = bk[i] = djs[i] = 0;
      inq[i] = inp[i] = inb[i] = 0;
    }
    ans = 0:
  }
  void add_edge(int u, int v) {
   el[u][v] = el[v][u] = 1;
  int lca(int u,int v) {
    for(int i = 0; i <= V; i++) inp[i] = 0;</pre>
    while(1) {
      u = djs[u];
      inp[u] = true;
      if(u == st) break;
      u = bk[pr[u]];
    while(1) {
      v = djs[v];
      if(inp[v]) return v;
      v = bk[pr[v]];
    return v;
  void upd(int u) {
    int v;
    while(djs[u] != nb) {
      v = pr[u];
      inb[djs[u]] = inb[djs[v]] = true;
      u = bk[v];
      if(djs[u] != nb) bk[u] = v;
  }
  void blo(int u,int v) {
   nb = lca(u,v);
    for (int i=0; i<=V; i++) inb[i] = 0;</pre>
    upd(u); upd(v);
    if(djs[u] != nb) bk[u] = v;
    if(djs[v] != nb) bk[v] = u;
    for(int tu = 1; tu <= V; tu++)</pre>
      if(inb[djs[tu]]) {
        djs[tu] = nb;
        if(!inq[tu]){
          qe.push(tu);
          inq[tu] = 1;
        }
      }
  void flow() {
    for(int i = 1; i <= V; i++) {</pre>
      inq[i] = 0;
      bk[i] = 0;
      djs[i] = i;
    while(qe.size()) qe.pop();
    ge.push(st);
    inq[st] = 1;
    ed = 0;
    while(qe.size()) {
      int u = qe.front(); qe.pop();
      for(int v = 1; v <= V; v++)
        if(el[u][v] && (djs[u] != djs[v]) && (pr[u] !=
            v)) {
          if((v == st) || ((pr[v] > 0) && bk[pr[v]] >
              0))
            blo(u,v);
          else if(bk[v] == 0) {
```

```
bk[v] = u;
if(pr[v] > 0) {
                if(!inq[pr[v]]) qe.push(pr[v]);
             } else {
               ed = v;
               return;
             }
           }
         }
    }
  void aug() {
    int u,v,w;
    u = ed;
    while(u > 0) {
      v = bk[u];
      w = pr[v];
      pr[v] = u;
      pr[u] = v;
      u = w;
    }
  int solve() {
    for(int i = 0; i <= V; i++) pr[i] = 0;</pre>
    for(int u = 1; u <= V; u++)</pre>
      if(pr[u] == 0) {
         st = u;
         flow();
         if(ed > 0) {
           aug();
           ans ++;
         }
    return ans;
}G;
int main() {
  G.init(V);
  for(int i=0; i<E; i++) {</pre>
    int u, v;
    cin >> u >> v;
    G.add_edge(u, v);
  cout << G.solve() << endl;</pre>
```

4.6 Minimum Weight Matching (Clique version)

```
struct Graph {
  // Minimum General Weighted Matching (Perfect Match)
      0-base
  static const int MXN = 105;
  int n, edge[MXN][MXN];
  int match[MXN],dis[MXN],onstk[MXN];
  vector<int> stk;
  void init(int _n) {
    n = _n;
for (int i=0; i<n; i++)</pre>
      for (int j=0; j<n; j++)</pre>
        edge[i][j] = 0;
  void add_edge(int u, int v, int w) {
    edge[u][v] = edge[v][u] = w;
  bool SPFA(int u){
    if (onstk[u]) return true;
    stk.PB(u);
    onstk[u] = 1;
    for (int v=0; v<n; v++){</pre>
      if (u != v && match[u] != v && !onstk[v]){
        int m = match[v];
        if (dis[m] > dis[u] - edge[v][m] + edge[u][v]){
          dis[m] = dis[u] - edge[v][m] + edge[u][v];
          onstk[v] = 1;
          stk.PB(v);
          if (SPFA(m)) return true;
          stk.pop_back();
```

```
onstk[v] = 0:
        }
      }
    }
    onstk[u] = 0;
    stk.pop_back();
    return false;
  int solve() {
    // find a match
    for (int i=0; i<n; i+=2){
      match[i] = i+1;
      match[i+1] = i;
    while (true){
      int found = 0;
      for (int i=0; i<n; i++)</pre>
        dis[i] = onstk[i] = 0;
      for (int i=0; i<n; i++){</pre>
        stk.clear();
        if (!onstk[i] && SPFA(i)){
          found = 1;
          while (SZ(stk)>=2){
             int u = stk.back(); stk.pop_back();
             int v = stk.back(); stk.pop_back();
             match[u] = v;
             match[v] = u;
          }
        }
      if (!found) break;
    int ret = 0;
    for (int i=0; i<n; i++)</pre>
      ret += edge[i][match[i]];
    ret /= 2;
    return ret;
  }
}graph;
```

4.7 (+1) SW-mincut O(NM)

```
// {{{ StoerWagner
const int inf=10000000000;
// should be larger than max.possible mincut
class StoerWagner {
  public:
    int n,mc; // node id in [0,n-1]
    vector<int> adj[MAXN];
    int cost[MAXN][MAXN];
    int cs[MAXN];
    bool merged[MAXN],sel[MAXN];
    // --8<-- include only if cut is explicitly needed
      DisjointSet dis;
    vector<int> cut;
    //--8<--
      StoerWagner(int _n):n(_n),mc(inf),djs(_n) {
        for(int i=0;i<n;i++)</pre>
          merged[i]=0;
        for(int i=0;i<n;i++)</pre>
          for(int j=0;j<n;j++)</pre>
            cost[i][j]=cost[j][i]=0;
    void append(int v,int u,int c) {
      if(v==u) return;
      if(!cost[v][u]&&c) {
        adj[v].PB(u);
        adj[u].PB(v);
      cost[v][u]+=c;
      cost[u][v]+=c;
    void merge(int v,int u) {
      merged[u]=1;
      for(int i=0;i<n;i++)</pre>
        append(v,i,cost[u][i]);
      // --8<-- include only if cut is explicitly
          needed
        djs.merge(v,u);
```

```
void phase() {
       priority_queue<pii> pq;
       for(int v=0;v<n;v++) {</pre>
         if(merged[v]) continue;
         cs[v]=0;
         sel[v]=0;
         pq.push({0,v});
       int v,s,pv;
       while(pq.size()) {
         if(cs[pq.top().S]>pq.top().F) {
           pq.pop();
           continue:
         pv=v;
         v=pq.top().S;
         s=pq.top().F;
         pq.pop();
         sel[v]=1:
         for(int i=0;i<adj[v].size();i++) {</pre>
           int u=adj[v][i];
           if(merged[u]||sel[u]) continue;
           cs[u]+=cost[v][u];
           pq.push({cs[u],u});
         }
       if(s<mc) {</pre>
         mc=s;
         // --8<-- include only if cut is explicitly
         needed -----
           cut.clear();
         for(int i=0;i<n;i++)</pre>
           if(djs.getrep(i)==djs.getrep(v)) cut.PB(i);
       }
       merge(v,pv);
     int mincut() {
       if(mc==inf) {
         for(int t=0;t<n-1;t++)</pre>
           phase();
       return mc:
     // --8<-- include only if cut is explicitly needed
       vector<int> getcut() { // return one side of the
           cut
         mincut();
         return cut;
     //--8<----
};
// }}}
```

5 Math

5.1 ax+by=gcd

```
typedef pair<int, int> pii;

pii gcd(int a, int b){
   if(b == 0) return make_pair(1, 0);
   else{
      int p = a / b;
      pii q = gcd(b, a % b);
      return make_pair(q.second, q.first - q.second * p);
   }
}
```

5.2 Fast Fourier Transform

```
// const int MAXN = 262144;
// (must be 2^k)
typedef long double ld;
typedef complex<ld> cplx;
const ld PI = acosl(-1);
const cplx I(0, 1);
cplx omega[MAXN+1];
void pre_fft()
  for(int i=0; i<=MAXN; i++)</pre>
    omega[i] = exp(i * 2 * PI / MAXN * I);
void fft(int n, cplx a[], bool inv=false)
  int basic = MAXN / n;
  int theta = basic;
  for (int m = n; m >= 2; m >>= 1) {
    int mh = m >> 1;
    for (int i = 0; i < mh; i++) {</pre>
      cplx w = omega[inv ? MAXN-(i*theta%MAXN) : i*
           theta%MAXN];
      for (int j = i; j < n; j += m) {</pre>
        int k = j + mh;
        cplx x = a[j] - a[k];
        a[j] += a[k];
        a[k] = w * x;
      }
    theta = (theta * 2) % MAXN;
  int i = 0;
  for (int j = 1; j < n - 1; j++) {
    for (int k = n >> 1; k > (i ^= k); k >>= 1);
    if (j < i) swap(a[i], a[j]);</pre>
  if (inv)
    for (i = 0; i < n; i++)</pre>
      a[i] /= n;
}
```

5.3 Fast Linear Recurrence

```
ll n,m,dp[N+N];
void pre_dp(){
  dp[0] = 1;
  ll bdr = min(m+m,n);
  for(ll i=1; i<=bdr; i++)</pre>
    for(ll j=i-1; j>=max(0ll,i-m); j--)
      dp[i]= add(dp[i],dp[j]);
vector<ll> Mul(const vector<ll>& v1,const vector<ll>&
    v2){
  int sz1 = (int)v1.size();
  int sz2 = (int)v2.size();
  assert(sz1 == m and sz2 == m);
  vector<ll> _v(m+m);
  for(int i=0; i<m+m; i++) _v[i]= 0;</pre>
  // expand
  for(int i=0; i<sz1; i++)</pre>
    for(int j=0; j<sz2; j++)</pre>
      _v[i+j+1]= add(_v[i+j+1],mul(v1[i],v2[j]));
  // shrink
  for(int i=0; i<m; i++)</pre>
    for(int j=1; j<=m; j++)</pre>
       _v[i + j]= add(_v[i + j],_v[i]);
  for(int i=0; i<m; i++)</pre>
    _{v[i]} = _{v[i + m]};
  _v.resize(m);
  return _v;
vector<ll> I,A;
ll solve(){
  pre_dp();
  if(n <= m+m)return dp[n];</pre>
  I.resize(m):
  A.resize(m);
  for(int i=0; i<m; i++) I[i]=A[i]=1;</pre>
  // dp[n] = /Sum_{i=0}^{m-1} A_i * dp[n - i - 1]
```

```
ll dlt = (n - m) / m;
ll rdlt = dlt * m;
while(dlt){
   if(dlt & 1ll) I = Mul(I,A);
   A = Mul(A,A);
   dlt >>= 1;
}
ll ans = 0;
for(int i=0; i<m; i++)
   ans = add(ans,mul(I[i],dp[n-i-1-rdlt]));
return ans;
}</pre>
```

5.4 (+1) ntt

```
int P=605028353, root=3, MAXNUM=262144;
// Remember coefficient are mod P
/*
p=a*2^n+1
   2^n
n
                р
                               а
                                     root
                97
.5
    32
                               .3
                                     .5
                193
6
                                     5
   128
                257
                                     3
8
   256
                257
                               1
                                     3
9
    512
                7681
                               15
                                     17
10 1024
                12289
                               12
                                     11
11 2048
                12289
                               6
                                     11
12
   4096
                12289
                               3
                                     11
13 8192
                40961
                               -5
                                     .3
14 16384
                65537
15
   32768
                65537
                               2
                                     3
16 65536
                65537
                               1
                                     3
17 131072
                786433
                                     10
                786433
18 262144
                               3
                                     10 (605028353,
    2308, 3)
19 524288
                5767169
                               11
20 1048576
                7340033
                               7
                                     .3
21
   2097152
                23068673
                               11
                                     3
22 4194304
                104857601
                               25
                                     3
23 8388608
                167772161
                               20
                                     3
24
   16777216
                167772161
                               10
25 33554432
                167772161
                                     3 (1107296257, 33,
                               5
    10)
26
   67108864
                469762049
                                     3
27
   134217728
                2013265921
                               15
                                     31
int bigmod(long long a,int b){
  if(b==0)return 1;
  return (bigmod((a*a)%P,b/2)*(b%2?a:1ll))%P;
int inv(int a,int b){
  if(a==1)return 1;
  return (((long long)(a-inv(b%a,a))*b+1)/a)%b;
std::vector<long long> ps(MAXNUM);
std::vector<int> rev(MAXNUM);
struct poly{
  std::vector<unsigned int> co;
  int n;//polynomial degree = n
  poly(int d) {n=d;co.resize(n+1,0);}
  void trans2(int NN){
    int r=0,st,N;
    unsigned int a,b;
    while((1<<r)<(NN>>1))++r;
    for(N=2;N<=NN;N<<=1,--r){</pre>
      for(st=0;st<NN;st+=N){</pre>
        int i,ss=st+(N>>1);
        for(i=(N>>1)-1;i>=0;--i){
          a=co[st+i]; b=(ps[i<<r]*co[ss+i])%P;
          co[st+i]=a+b; if(co[st+i]>=P)co[st+i]-=P;
          co[ss+i]=a+P-b; if(co[ss+i]>=P)co[ss+i]-=P;
        }
      }
    }
  void trans1(int NN){
    int r=0,st,N;
    unsigned int a,b;
    for (N=NN; N>1; N>>=1,++r) {
      for(st=0;st<NN;st+=N){</pre>
```

```
int i,ss=st+(N>>1);
         for(i=(N>>1)-1;i>=0;--i){
           a=co[st+i]; b=co[ss+i];
           co[st+i]=a+b; if(co[st+i]>=P)co[st+i]-=P;
           co[ss+i]=((a+P-b)*ps[i<< r])%P;
         }
      }
    }
  }
  poly operator*(const poly& _b)const{
    poly a=*this,b=_b;
    int k=n+b.n,i,N=1;
    while(N<=k)N*=2;</pre>
    a.co.resize(N,0); b.co.resize(N,0);
    int r=bigmod(root, (P-1)/N), Ni=inv(N,P);
    ps[0]=1;
    for(i=1;i<N;++i)ps[i]=(ps[i-1]*r)%P;</pre>
    a.trans1(N);b.trans1(N);
     for(i=0;i<N;++i)a.co[i]=((long long)a.co[i]*b.co[i</pre>
         ])%P
    r=inv(r,P);
    for(i=1;i<N/2;++i)std::swap(ps[i],ps[N-i]);</pre>
    for(i=0;i<N;++i)a.co[i]=((long long)a.co[i]*Ni)%P;</pre>
    a.n=n+_b.n; return a;
};
```

5.5 Mod

```
/// _fd(a,b) floor(a/b).
/// _{rd(a,m)} a-floor(a/m)*m.
/// _pv(a,m,r) largest x s.t x<=a && x%m == r.
/// _nx(a,m,r) smallest x s.t x>=a && x%m == r.
/// \_ct(a,b,m,r) |A|, A = \{ x : a <= x <= b && x %m == r \}.
int _fd(int a,int b){ return a<0?(-~a/b-1):a/b; }</pre>
int _rd(int a,int m){ return a-_fd(a,m)*m; }
int _pv(int a,int m,int r)
{
    r = (r\%m + m)\%m:
    return _fd(a-r,m)*m+r;
int _nt(int a,int m,int r)
    m=abs(m):
    r=(r%m+m)%m;
    return _fd(a-r-1,m)*m+r+m;
int _ct(int a,int b,int m,int r)
    m=abs(m);
    a=_nt(a,m,r);
    b=pv(b,m,r);
    return (a>b)?0:((b-a+m)/m);
```

5.6 (+1) Miller Rabin

```
// n < 4,759,123,141
                            3 :
                                 2, 7, 61
// n < 1,122,004,669,633
                            4:
                                 2, 13, 23, 1662803
// n < 3,474,749,660,383
                                  6 : pirmes <= 13
// n < 2^64
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
// Make sure testing integer is in range [2, n-2] if
// you want to use magic.
long long power(long long x,long long p,long long mod){
 long long s=1,m=x;
  while(p) {
    if(p&1) s=mult(s,m,mod);
    p>>=1:
    m=mult(m,m,mod);
  return s:
bool witness(long long a,long long n,long long u,int t)
    {
```

```
long long x=power(a,u,n);
  for(int i=0;i<t;i++) {</pre>
    long long nx=mult(x,x,n);
    if(nx==1&&x!=1&&x!=n-1) return 1;
    x=nx;
  return x!=1;
bool miller_rabin(long long n,int s=100) {
  // iterate s times of witness on n
  // return 1 if prime, 0 otherwise
  if(n<2) return 0;</pre>
  if(!(n&1)) return n==2;
  long long u=n-1;
  int t=0;
  // n-1 = u*2^t
  while(!(u&1)) {
    u>>=1;
    t++:
  while(s--) {
    long long a=randll()%(n-1)+1;
    if(witness(a,n,u,t)) return 0;
  return 1:
}
```

5.7 Pollard Rho

```
// does not work when n is prime
long long modit(long long x,long long mod) {
 if(x \ge mod) x = mod;
  //if(x<0) x+=mod;
  return x;
long long mult(long long x,long long y,long long mod) {
  long long s=0, m=x%mod;
  while(y) {
    if(y&1) s=modit(s+m,mod);
    v>>=1;
    m=modit(m+m, mod);
  return s;
long long f(long long x,long long mod) {
  return modit(mult(x,x,mod)+1,mod);
long long pollard_rho(long long n) {
  if(!(n&1)) return 2;
  while (true) {
    long long y=2, x=rand()%(n-1)+1, res=1;
    for (int sz=2; res==1; sz*=2) {
      for (int i=0; i<sz && res<=1; i++) {</pre>
        x = f(x, n);
        res = \_gcd(abs(x-y), n);
      }
      y = x;
    if (res!=0 && res!=n) return res;
}
```

5.8 Algorithms about Primes

```
/*
    * 12721
    * 13331
    * 14341
    * 75577
    * 123457
    * 222557
    * 556679
    * 999983
    * 1097774749
    * 1076767633
    * 100102021
    * 999997771
    * 1001010013
```

```
* 1000512343
 * 987654361
 * 999991231
* 999888733
* 98789101
 * 987777733
* 999991921
 * 1010101333
 * 1010102101
* 1000000000039
 * 1000000000000037
 * 2305843009213693951
 * 4611686018427387847
 * 9223372036854775783
* 18446744073709551557
int mu[MX],p_tbl[MX];
vector<int> primes;
void sieve() {
  mu[1] = p_tbl[1] = 1;
  for (int i=2; i<MX; i++) {</pre>
    if (!p_tbl[i]) {
      p_tbl[i] = i;
      primes.PB(i);
      mu[i] = -1;
    for (auto p : primes) {
      int x = i*p;
      if (x >= M) break;
      p_{tbl}[x] = p;
      mu[x] = -mu[i];
      if (i%p==0) {
        mu[x] = 0;
        break;
      }
   }
 }
}
vector<int> factor(int x) {
  vector<int> fac{1};
  while (x > 1) {
    int fn=SZ(fac), p=p_tbl[x], pos=0;
    while (x\%p == 0) {
      x /= p;
      for (int i=0; i<fn; i++)</pre>
        fac.PB(fac[pos++]*p);
 }
  return fac;
```

5.9 (+1) PolynomialGenerator

```
class PolynomialGenerator {
 /* for a nth-order polynomial f(x), *
  * given f(0), f(1), ..., f(n) *
   * express f(x) as sigma_i\{c_i*C(x,i)\} */
 public:
    vector<long long> coef;
    // initialize and calculate f(x), vector _fx should
    // filled with f(0) to f(n)
      PolynomialGenerator(int _n,vector<long long> _fx)
          ),coef(_fx) {
        for(int i=0;i<n;i++)</pre>
          for(int j=n;j>i;j--)
            coef[j]-=coef[j-1];
    // evaluate f(x), runs in O(n)
    long long eval(int x) {
      long long m=1,ret=0;
      for(int i=0;i<=n;i++) {</pre>
        ret+=coef[i]*m;
        m=m*(x-i)/(i+1);
      }
      return ret;
```

```
}
};
```

5.10 Pseudoinverse of Square matrix

```
Mat pinv(Mat m)
  Mat res = I;
  FZ(used);
  for(int i=0; i<W; i++)</pre>
    int piv = -1;
    for(int j=0; j<W; j++)</pre>
       if(used[j]) continue;
       if(abs(m.v[j][i]) > EPS)
         piv = j;
         break;
      }
    if(piv == −1)
      continue;
    used[i] = true;
    swap(m.v[piv], m.v[i]);
    swap(res.v[piv], res.v[i]);
    ld rat = m.v[i][i];
    for(int j=0; j<W; j++)</pre>
      m.v[i][j] /= rat;
       res.v[i][j] /= rat;
    for(int j=0; j<W; j++)</pre>
      if(j == i) continue;
rat = m.v[j][i];
       for(int k=0; k<W; k++)</pre>
         m.v[j][k] -= rat * m.v[i][k];
         res.v[j][k] -= rat * res.v[i][k];
  }
  for(int i=0; i<W; i++)</pre>
    if(used[i]) continue;
    for(int j=0; j<W; j++)</pre>
       res.v[i][j] = 0;
  return res;
```

5.11 Theorem

5.11.1 Lucas' Theorem

For non-negative integer n, m and prime $p, \binom{m}{n} \equiv \prod_{i=0}^k \binom{m_i}{n_i} \pmod{p}$ where m_i is the i-th digit of m in base p.

5.11.2 Sum of Two Squares Thm (Legendre)

For a given positive integer n, let $D_1=$ (# of positive integers d dividing N that $1\equiv d\pmod 4$)) $D_3=$ (# of positive integers d dividing N that $3\equiv d\pmod 4$)) then n can be written as a sum of two squares in exactly $R(n)=4(D_1-D_3)$ ways.

5.11.3 Difference of D1-D3 Thm

```
\begin{array}{l} \det n=2^t\cdot (p_1^{e_1}\cdot\ldots\cdot p_r^{e_r})\cdots (q_1^{f_1}\cdot\ldots\cdot q_s^{f_s})\\ \text{where }p_i,q_i\text{ are primes and }1\equiv p_i\pmod 4, 3\equiv q_i\pmod 4\\ \\ \tanh D_1-D_3=\begin{cases} (e_1+1)(e_2+1)...(e_r+1), & \text{if }f_i\text{ all even}\\ 0, & \text{if any }f_i\text{ is odd} \end{cases}
```

5.11.4 Krush-Kuhn-Tucker Conditions

```
\begin{split} & \textbf{Stationarity} \\ & \text{For maximizing } f(x) \colon \nabla f(x^*) = \sum_{i=1}^m \mu_i \nabla g_i(x^*) + \sum_{j=1}^l \lambda_j \nabla h_j(x^*) \\ & \text{For minimizing } f(x) \colon -\nabla f(x^*) = \sum_{i=1}^m \mu_i \nabla g_i(x^*) + \sum_{j=1}^l \lambda_j \nabla h_j(x^*) \\ & \textbf{Primal feasibility} \\ & g_i(x^*) \leq 0, \text{ for all } i = 1, \dots, m \\ & h_j(x^*) = 0, \text{ for all } j = 1, \dots, l \\ & \textbf{Dual feasibility} \\ & \mu_i \geq 0, \text{ for all } i = 1, \dots, m \\ & \textbf{Complementary slackness} \\ & \mu_i g_i(x^*) = 0, \text{ for all } i = 1, \dots, m \end{split}
```

5.11.5 Chinese remainder theorem

```
\begin{split} x &\equiv r_i \mod p_i \\ N &= \prod p_i \\ N_i &= N/p_i \\ x &\equiv \sum r_i N_i (N_i)_{p_i}^{-1} \mod N \end{split}
```

5.12 Simplex

```
const int maxn = 111;
const int maxm = 111;
const double eps = 1E-10;
double a[maxn][maxm], b[maxn], c[maxm], d[maxn][maxm];
double x[maxm];
int ix[maxn + maxm]; // !!! array all indexed from 0
// \max\{cx\} \text{ subject to } \{Ax \le b, x \ge 0\}
// n: constraints, m: vars !!!
// x[] is the optimal solution vector
//
// usage :
// value = simplex(a, b, c, N, M);
double simplex(double a[maxn][maxm], double b[maxn],
    double c[maxm], int n, int m) {
    ++m;
    int r = n, s = m - 1;
    memset(d, 0, sizeof(d));
    for (int i = 0; i < n + m; ++i) ix[i] = i;</pre>
    for (int i = 0; i < n; ++i) {</pre>
        for (int j = 0; j < m - 1; ++j)
            d[i][j] = -a[i][j];
        d[i][m - 1] = 1;
        d[i][m] = b[i];
        if (d[r][m] > d[i][m]) r = i;
    for (int j = 0; j < m - 1; ++j) d[n][j] = c[j];</pre>
    d[n + 1][m - 1] = -1;
    for (double dd;; ) {
        if (r < n) {
             int t = ix[s];
             ix[s] = ix[r + m]; ix[r + m] = t;
             d[r][s] = 1.0 / d[r][s];
             for (int j = 0; j <= m; ++j)</pre>
                 if (j != s) d[r][j] *= -d[r][s];
             for (int i = 0; i <= n + 1; ++i)
                 if (i != r) {
                     for (int j = 0; j <= m; ++j)</pre>
                          if (j != s)
                              d[i][j] += d[r][j]*d[i][s];
                     d[i][s] *= d[r][s];
                 }
        r = -1; s = -1;
        for (int j = 0; j < m; ++j)</pre>
             if (s < 0 || ix[s] > ix[j]) {
                 if (d[n + 1][j] > eps || (d[n + 1][j] >
                       -eps && d[n][j] > eps)) s = j;
        if (s < 0) break;</pre>
        for (int i=0; i<n; ++i) if (d[i][s] < -eps) {</pre>
             if (r < 0 || (dd = d[r][m] / d[r][s] - d[i</pre>
                 ][m] /d[i][s]) < -eps || (dd < eps &&
                 ix[r + m] > ix[i + m])) r = i;
        if (r < 0) return -1; // not bounded</pre>
    }
```

6 Geometry

6.1 Point operators

```
#define x first
#define y second
#define cpdd const pdd
struct pdd : pair<double, double> {
    using pair<double, double>::pair;
    pdd operator + (cpdd &p) const {
        return {x+p.x, y+p.y};
    }
    pdd operator - () const {
        return {-x, -y};
    }
    pdd operator - (cpdd &p) const {
        return (*this) + (-p);
    pdd operator * (double f) const {
        return {f*x, f*y};
    double operator * (cpdd &p) const {
        return x*p.x + y*p.y;
    }
};
double abs(cpdd &p) { return hypot(p.x, p.y); }
double arg(cpdd &p) { return atan2(p.y, p.x); }
double cross(cpdd &p, cpdd &q) { return p.x*q.y - p.y*q
    .x; }
double cross(cpdd &p, cpdd &q, cpdd &o) { return cross(
    p-o, q-o); }
pdd operator * (double f, cpdd &p) { return p*f; } //
    !! Not f*p !!
```

6.2 Intersection of two circles

6.3 Intersection of two lines

```
const double EPS = 1e-9;

pdd interPnt(pdd p1, pdd p2, pdd q1, pdd q2, bool &res)
      {
        double f1 = cross(p2, q1, p1);
        double f2 = -cross(p2, q2, p1);
        double f = (f1 + f2);

        if(fabs(f) < EPS) {
            res = false;
            return {};
        }

        res = true;
        return (f2 / f) * q1 + (f1 / f) * q2;
}</pre>
```

6.4 Half Plane Intersection

const double EPS = 1e-9;

```
pdd interPnt(Line l1, Line l2, bool &res){
    pdd p1, p2, q1, q2;
    tie(p1, p2) = l1;
    tie(q1, q2) = l2;
  double f1 = cross(p2, q1, p1);
    double f2 = -cross(p2, q2, p1);
  double f = (f1 + f2);
    if(fabs(f) < EPS) {</pre>
        res = false:
        return {0, 0};
    res = true;
  return (f2 / f) * q1 + (f1 / f) * q2;
}
bool isin(Line l0, Line l1, Line l2) {
    // Check inter(l1, l2) in l0
    bool res;
    pdd p = interPnt(l1, l2, res);
    return cross(l0.S, p, l0.F) > EPS;
/* If no solution, check: 1. ret.size() < 3</pre>
 * Or more precisely, 2. interPnt(ret[0], ret[1])
 * in all the lines. (use (l.S - l.F).cross(p - l.F) >
 */
vector<Line> halfPlaneInter(vector<Line> lines) {
    int sz = lines.size();
    vector<double> ata(sz), ord(sz);
    for (int i=0; i<sz; i++) {</pre>
        ord[i] = i;
        pdd d = lines[i].S - lines[i].F;
        ata[i] = atan2(d.y, d.x);
    sort(ALL(ord), [&](int i, int j) {
        if (abs(ata[i] - ata[j]) < EPS) {</pre>
            return cross(lines[i].S, lines[j].S, lines[
                 i].F) < 0;
        }
        return ata[i] < ata[j];</pre>
    });
    vector<Line> fin;
    for (int i=0; i<sz; i++) {</pre>
        if (!i or fabs(ata[ord[i]] - ata[ord[i-1]]) >
            FPS) {
            fin.PB(lines[ord[i]]);
        }
    }
    deque<Line> dq;
    for (int i=0; i<SZ(fin); i++) {</pre>
        while (SZ(dq) >= 2 \text{ and }
              not isin(fin[i], dq[SZ(dq)-2], dq[SZ(dq)
                   -1])) {
            dq.pop_back();
        }
```

```
while(SZ(dq) >= 2 and
              not isin(fin[i], dq[0], dq[1])) {
            dq.pop_front();
        }
        dq.push_back(fin[i]);
    }
    while (SZ(dq) >= 3 and
           not isin(dq[0], dq[SZ(dq)-2], dq[SZ(dq)-1]))
        dq.pop_back();
    }
    while (SZ(dq) >= 3 and
           not isin(dq[SZ(dq)-1], dq[0], dq[1])) {
        dq.pop_front();
    vector<Line> res(ALL(dq));
    return res:
}
```

6.5 Convex Hull

```
vector<pdd> convex_hull(vector<pdd> pt){
  sort(pt.begin(),pt.end());
  int top=0;
  vector<pdd> stk(2*pt.size());
  for (int i=0; i<(int)pt.size(); i++){</pre>
    while (top >= 2 && cross(stk[top-1],pt[i],stk[top
        -2]) <= 0)
      top--:
    stk[top++] = pt[i];
  for (int i=pt.size()-2, t=top+1; i>=0; i--){
    while (top >= t && cross(stk[top-1],pt[i], stk[top
        -2]) <= 0)
      top--;
    stk[top++] = pt[i];
  stk.resize(top-1);
  return stk;
```

6.6 Minimum Covering Circle

```
struct Mcc{
  // return pair of center and r^2
  static const int MAXN = 1000100;
  pdd p[MAXN],cen;
  double r2;
  void init(int _n, pdd _p[]){
    n = _n;
    memcpy(p,_p,sizeof(pdd)*n);
  double sqr(double a){ return a*a; }
  double abs2(pdd a){ return a*a; }
  pdd center(pdd p0, pdd p1, pdd p2) {
    pdd a = p1-p0;
    pdd b = p2-p0;
    double c1=abs2(a)*0.5;
    double c2=abs2(b)*0.5;
    double d = a % b;
    double x = p0.x + (c1 * b.y - c2 * a.y) / d;
    double y = p0.y + (a.x * c2 - b.x * c1) / d;
    return pdd(x,y);
  pair<pdd,double> solve(){
    random_shuffle(p,p+n);
    r2=0;
    for (int i=0; i<n; i++){</pre>
      if (abs2(cen-p[i]) <= r2) continue;</pre>
      cen = p[i];
      r2 = 0;
      for (int j=0; j<i; j++){</pre>
        if (abs2(cen-p[j]) <= r2) continue;</pre>
```

```
cen = 0.5 * (p[i]+p[j]);
        r2 = abs2(cen-p[j]);
         for (int k=0; k<j; k++){</pre>
          if (abs2(cen-p[k]) <= r2) continue;</pre>
           cen = center(p[i],p[j],p[k]);
          r2 = abs2(cen-p[k]);
        }
      }
    }
    return {cen,r2};
}mcc;
```

KDTree (Nearest Point)

```
const int MXN = 100005;
struct KDTree {
  struct Node {
    int x,y,x1,y1,x2,y2;
    int id,f;
    Node *L, *R;
  }tree[MXN];
  int n;
  Node *root;
  long long dis2(int x1, int y1, int x2, int y2) {
    long long dx = x1-x2;
long long dy = y1-y2;
    return dx*dx+dy*dy;
  static bool cmpx(Node& a, Node& b){ return a.x<b.x; }</pre>
  static bool cmpy(Node& a, Node& b){ return a.y<b.y; }</pre>
  void init(vector<pair<int,int>> ip) {
    n = ip.size();
    for (int i=0; i<n; i++) {</pre>
      tree[i].id = i;
      tree[i].x = ip[i].first;
      tree[i].y = ip[i].second;
    root = build_tree(0, n-1, 0);
  Node* build_tree(int L, int R, int dep) {
    if (L>R) return nullptr;
    int M = (L+R)/2;
    tree[M].f = dep%2;
    nth_element(tree+L, tree+M, tree+R+1, tree[M].f ?
         cmpy : cmpx);
    tree[M].x1 = tree[M].x2 = tree[M].x;
    tree[M].y1 = tree[M].y2 = tree[M].y;
    tree[M].L = build_tree(L, M-1, dep+1);
    if (tree[M].L) {
      tree[M].x1 = min(tree[M].x1, tree[M].L->x1);
      tree[M].x2 = max(tree[M].x2, tree[M].L->x2);
      tree[M].y1 = min(tree[M].y1, tree[M].L->y1);
      tree[M].y2 = max(tree[M].y2, tree[M].L->y2);
    tree[M].R = build_tree(M+1, R, dep+1);
    if (tree[M].R) {
      tree[M].x1 = min(tree[M].x1, tree[M].R->x1);
      tree[M].x2 = max(tree[M].x2, tree[M].R->x2);
tree[M].y1 = min(tree[M].y1, tree[M].R->y1);
      tree[M].y2 = max(tree[M].y2, tree[M].R->y2);
    return tree+M;
  int touch(Node* r, int x, int y, long long d2){
    long long dis = sqrt(d2)+1;
    if (x<r->x1-dis || x>r->x2+dis || y<r->y1-dis || y>
         r->y2+dis)
      return 0;
    return 1;
  void nearest(Node* r, int x, int y, int &mID, long
      long &md2) {
    if (!r || !touch(r, x, y, md2)) return;
    long long d2 = dis2(r->x, r->y, x, y);
```

```
if (d2 < md2 || (d2 == md2 && mID < r->id)) {
      mID = r \rightarrow id;
      md2 = d2;
     // search order depends on split dim
    if ((r->f == 0 && x < r->x) ||
         (r->f == 1 \&\& y < r->y)) {
       nearest(r->L, x, y, mID, md2);
      nearest(r->R, x, y, mID, md2);
    } else {
      nearest(r\rightarrow R, x, y, mID, md2);
      nearest(r\rightarrow L, x, y, mID, md2);
  int query(int x, int y) {
    int id = 1029384756;
    long long d2 = 102938475612345678LL;
    nearest(root, x, y, id, d2);
    return id:
  }
}tree;
```

Triangulation 6.8

```
bool inCircle(pdd a, pdd b, pdd c, pdd d) {
    b = b - a;
    c = c - a;
    d = d - a;
    if (cross(b, c) < 0) swap(b, c);
    double m[3][3] = {
        \{b.x, b.y, b*b\},
        \{c.x, c.y, c*c\},\
        \{d.x, d.y, d*d\}
    };
    double det = m[0][0] * (m[1][1]*m[2][2] - m[1][2]*m
        [2][1])
                + m[0][1] * (m[1][2]*m[2][0] - m[1][0]*m
                    [2][2])
                + m[0][2] * (m[1][0]*m[2][1] - m[1][1]*m
                    [2][0]);
    return det < 0;</pre>
}
bool intersect(pdd a, pdd b, pdd c, pdd d) {
    return cross(b, c, a) * cross(b, d, a) < 0 and
        cross(d, a, c) * cross(d, b, c) < 0;
}
const double EPS = 1e-12;
struct Triangulation {
    static const int MXN = 1e5+5;
    int N:
    vector<int> ord;
    vector<pdd> pts;
    set<int> E[MXN];
    vector<vector<int>> solve(vector<pdd> p) {
        N = SZ(p);
        ord.resize(N);
        for (int i=0; i<N; i++) {</pre>
            E[i].clear();
            ord[i] = i;
        sort(ALL(ord), [&p](int i, int j) {
            return p[i] < p[j];</pre>
        pts.resize(N);
        for (int i=0; i<N; i++) pts[i] = p[ord[i]];</pre>
        go(0, N);
        vector<vector<int>> res(N);
        for (int i=0; i<N; i++) {</pre>
             int o = ord[i];
             for (auto x: E[i]) {
                 res[o].PB(ord[x]);
        }
```

```
return res:
}
void add_edge(int u, int v) {
    E[u].insert(v);
    E[v].insert(u);
}
void remove_edge(int u, int v) {
    E[u].erase(v);
    E[v].erase(u);
void go(int l, int r) {
    int n = r - l;
    if (n <= 3) {
        for (int i=l; i<r; i++)</pre>
            for (int j=i+1; j<r; j++) add_edge(i, j</pre>
        return;
    int md = (l+r)/2;
    go(l, md);
    go(md, r);
    int il = l, ir = r-1;
    while (1) {
        int nx = -1;
        for (auto i: E[il]) {
            double cs = cross(pts[il], pts[i], pts[
                 ir]);
            if (cs > EPS ||
                 (abs(cs) < EPS and abs(pts[i]-pts[</pre>
                     ir]) < abs(pts[il]-pts[ir]))) {</pre>
                 nx = i;
                 break;
            }
        if (nx != -1) {
            il = nx;
            continue;
        for (auto i: E[ir]) {
            double cs = cross(pts[ir], pts[i], pts[
                 il]);
            if (cs < -EPS ||
                 (abs(cs) < EPS and abs(pts[i]-pts[</pre>
                     il]) < abs(pts[ir]-pts[il]))) {</pre>
                 nx = i;
                 break;
            }
        }
        if (nx != -1) {
            ir = nx;
        } else break;
    add_edge(il, ir);
    while (1) {
        int nx = -1;
        bool is2 = false;
        for (int i: E[il]) {
             if (cross(pts[il], pts[i], pts[ir]) < -</pre>
                 EPS and
                 (nx == -1 or inCircle(pts[il], pts[
                     ir], pts[nx], pts[i]))) nx = i;
        }
        for (int i: E[ir]) {
             if (cross(pts[ir], pts[i], pts[il]) >
                 EPS and
                 (nx == -1 or inCircle(pts[il], pts[
                     ir], pts[nx], pts[i]))) nx = i,
                      is2 = 1;
        }
```

```
if (nx == -1) break;
            int a = il, b = ir;
            if (is2) swap(a, b);
             for (auto i: E[a]) {
                 if (intersect(pts[a], pts[i], pts[b],
                     pts[nx])) {
                     remove_edge(a, i);
                 }
             if (is2) {
                 add_edge(il, nx);
                 ir = nx;
            } else {
                 add_edge(ir, nx);
                 il = nx;
            }
        }
    }
} tri;
```

6.9 (+1) MinkowskiSum

```
/* convex hull Minkowski Sum*/
#define INF 1000000000000000LL
class PT{ public:
  long long x,y;
int POS(){
    if(y==0) return x>0?0:1;
    return y>0?0:1;
 }
PT pt[300000],qt[300000],rt[300000];
long long Lx,Rx;
int dn,un;
inline bool cmp(PT a,PT b){
  int pa=a.POS(),pb=b.POS();
  if(pa==pb) return (a^b)>0;
  return pa<pb;</pre>
int minkowskiSum(int n,int m){
  int i,j,r,p,q,fi,fj;
  for(i=1,p=0;i<n;i++){</pre>
    if(pt[i].y<pt[p].y || (pt[i].y==pt[p].y && pt[i].x<</pre>
          pt[p].x)) p=i; }
  for(i=1,q=0;i<m;i++){</pre>
    if(qt[i].y<qt[q].y || (qt[i].y==qt[q].y && qt[i].x<</pre>
          qt[q].x)) q=i; }
  rt[0]=pt[p]+qt[q];
  r=1; i=p; j=q; fi=fj=0;
  while(1){
    if((fj&&j==q) || ((!fi||i!=p) && cmp(pt[(p+1)%n]-pt
             p],qt[(q+1)%m]-qt[q]))){
      rt[r]=rt[r-1]+pt[(p+1)%n]-pt[p];
      p=(p+1)%n;
      fi=1;
    }else{
      rt[r]=rt[r-1]+qt[(q+1)%m]-qt[q];
      q=(q+1)\%m;
      fj=1;
    if(r<=1 || ((rt[r]-rt[r-1])^(rt[r-1]-rt[r-2]))!=0)</pre>
      ++;
    else rt[r-1]=rt[r];
    if(i==p && j==q) break;
  return r-1;
void initInConvex(int n){
  int i,p,q;
  long long Ly,Ry;
  Lx=INF; Rx=-INF;
  for(i=0;i<n;i++){</pre>
    if(pt[i].x<Lx) Lx=pt[i].x;</pre>
    if(pt[i].x>Rx) Rx=pt[i].x;
  Ly=Ry=INF;
```

```
for(i=0;i<n;i++){</pre>
    if(pt[i].x==Lx && pt[i].y<Ly){ Ly=pt[i].y; p=i; }</pre>
     if(pt[i].x==Rx && pt[i].y<Ry){ Ry=pt[i].y; q=i; }</pre>
  for(dn=0,i=p;i!=q;i=(i+1)%n){ qt[dn++]=pt[i]; }
  qt[dn]=pt[q]; Ly=Ry=-INF;
  for(i=0;i<n;i++){</pre>
    if(pt[i].x==Lx && pt[i].y>Ly){ Ly=pt[i].y; p=i; }
    if(pt[i].x==Rx && pt[i].y>Ry){ Ry=pt[i].y; q=i; }
  for(un=0,i=p;i!=q;i=(i+n-1)%n){ rt[un++]=pt[i]; }
  rt[un]=pt[q];
inline int inConvex(PT p){
  int L,R,M;
  if(p.x<Lx || p.x>Rx) return 0;
  L=0:R=dn:
  while(L<R-1){ M=(L+R)/2;</pre>
    if(p.x<qt[M].x) R=M; else L=M; }</pre>
    if(tri(qt[L],qt[R],p)<0) return 0;</pre>
    L=0; R=un;
    while(L<R-1){ M=(L+R)/2;
       if(p.x<rt[M].x) R=M; else L=M; }</pre>
       if(tri(rt[L],rt[R],p)>0) return 0;
int main(){
  int n,m,i;
  PT p;
  scanf("%d",&n);
  for(i=0;i<n;i++) scanf("%I64d %I64d",&pt[i].x,&pt[i].</pre>
  scanf("%d",&m);
  for(i=0;i<m;i++) scanf("%I64d %I64d",&qt[i].x,&qt[i].</pre>
  n=minkowskiSum(n,m);
  for(i=0;i<n;i++) pt[i]=rt[i];</pre>
  scanf("%d",&m);
  for(i=0;i<m;i++) scanf("%I64d %I64d",&qt[i].x,&qt[i].</pre>
  n=minkowskiSum(n,m);
  for(i=0;i<n;i++) pt[i]=rt[i];</pre>
  initInConvex(n);
  scanf("%d",&m);
  for(i=0;i<m;i++){</pre>
    scanf("%I64d %I64d",&p.x,&p.y);
    p.x*=3; p.y*=3;
    puts(inConvex(p)?"YES":"NO");
  }
}
```

7 Stringology

7.1 Suffix Array

```
const int MAX = 1020304;
int ct[MAX], he[MAX], rk[MAX], sa[MAX], tsa[MAX], tp[
    MAX][2];
void suffix_array(char *ip){
  int len = strlen(ip);
  int alp = 256;
 memset(ct, 0, sizeof(ct));
  for(int i=0;i<len;i++) ct[ip[i]+1]++;</pre>
  for(int i=1;i<alp;i++) ct[i]+=ct[i-1];</pre>
  for(int i=0;i<len;i++) rk[i]=ct[ip[i]];</pre>
  for(int i=1;i<len;i*=2){</pre>
    for(int j=0;j<len;j++){</pre>
      if(j+i>=len) tp[j][1]=0;
      else tp[j][1]=rk[j+i]+1;
      tp[j][0]=rk[j];
    memset(ct, 0, sizeof(ct));
    for(int j=0;j<len;j++) ct[tp[j][1]+1]++;</pre>
```

```
for(int j=1;j<len+2;j++) ct[j]+=ct[j-1];</pre>
    for(int j=0;j<len;j++) tsa[ct[tp[j][1]]++]=j;</pre>
    memset(ct, 0, sizeof(ct));
    for(int j=0;j<len;j++) ct[tp[j][0]+1]++;</pre>
     for(int j=1;j<len+1;j++) ct[j]+=ct[j-1];</pre>
    for(int j=0;j<len;j++) sa[ct[tp[tsa[j]][0]]++]=tsa[</pre>
    rk[sa[0]]=0;
     for(int j=1;j<len;j++){</pre>
       if( tp[sa[j]][0] == tp[sa[j-1]][0] &&
         tp[sa[j]][1] == tp[sa[j-1]][1] )
         rk[sa[j]] = rk[sa[j-1]];
       else
         rk[sa[j]] = j;
    }
  }
  for(int i=0,h=0;i<len;i++){</pre>
    if(rk[i]==0) h=0;
    else{
      int j=sa[rk[i]-1];
      h=max(0,h-1);
      for(;ip[i+h]==ip[j+h];h++);
    he[rk[i]]=h;
  }
}
```

7.2 Suffix Array (SAIS TWT514)

```
struct SA{
#define REP(i,n) for ( int i=0; i<int(n); i++ )</pre>
#define REP1(i,a,b) for ( int i=(a); i<=int(b); i++ )
    static const int MXN = 300010;
    bool _t[MXN*2];
    int _s[MXN*2], _sa[MXN*2], _c[MXN*2], x[MXN], _p[
        MXN], _q[MXN*2], hei[MXN], r[MXN];
    int operator [] (int i){ return _sa[i]; }
    void build(int *s, int n, int m){
        memcpy(_s, s, sizeof(int) * n);
        sais(_s, _sa, _p, _q, _t, _c, n, m);
        mkhei(n);
    void mkhei(int n){
        REP(i,n) r[\_sa[i]] = i;
        hei[0] = 0;
        REP(i,n) if(r[i]) {
            int ans = i>0 ? max(hei[r[i-1]] - 1, 0) :
            while(_s[i+ans] == _s[_sa[r[i]-1]+ans]) ans
                ++:
            hei[r[i]] = ans;
        }
    void sais(int *s, int *sa, int *p, int *q, bool *t,
         int *c, int n, int z){
        bool uniq = t[n-1] = true, neq;
        int nn = 0, nmxz = -1, *nsa = sa + n, *ns = s +
             n, lst = -1;
#define MSO(x,n) memset((x),0,n*sizeof(*(x)))
#define MAGIC(XD) MSO(sa, n); \
        memcpy(x, c, sizeof(int) * z); \
        memcpy(x + 1, c, sizeof(int) * (z - 1)); \
        REP(i,n) \ \textbf{if}(sa[i] \ \&\& \ !t[sa[i]-1]) \ sa[x[s[sa[i]-1]])
            ]-1]]++] = sa[i]-1; \
        memcpy(x, c, sizeof(int) * z);
        for(int i = n - 1; i >= 0; i--) if(sa[i] && t[
            sa[i]-1]) sa[--x[s[sa[i]-1]]] = sa[i]-1;
        MSO(c, z);
        REP(i,n) uniq &= ++c[s[i]] < 2;
        REP(i,z-1) c[i+1] += c[i];
        if (uniq) { REP(i,n) sa[--c[s[i]]] = i; return;
        for(int i = n - 2; i >= 0; i--) t[i] = (s[i]==s
            [i+1] ? t[i+1] : s[i] < s[i+1]);
        MAGIC(REP1(i,1,n-1) if(t[i] && !t[i-1]) sa[--x[
            s[i]]]=p[q[i]=nn++]=i);
```

```
REP(i, n) if (sa[i] && t[sa[i]] && !t[sa[i]-1])
            neq=lst<0||memcmp(s+sa[i],s+lst,(p[q[sa[i</pre>
                 ]]+1]-sa[i])*sizeof(int));
            ns[q[lst=sa[i]]]=nmxz+=neq;
        sais(ns, nsa, p + nn, q + n, t + n, c + z, nn,
            nmxz + 1);
        MAGIC(for(int i = nn - 1; i >= 0; i--) sa[--x[s]]
             [p[nsa[i]]]] = p[nsa[i]]);
}sa;
void suffix_array(int* ip, int len) {
    // should padding a zero in the back
    // s is int array, n is array length
    // s[0..n-1] != 0, and s[n] = 0
    // resulting SA will be length n+1
    ip[len++] = 0;
    sa.build(ip, len, 128);
    // original 1-base
    for (int i=0; i<l; i++) {</pre>
        hei[i] = sa.hei[i + 1];
        sa[i] = sa.\_sa[i + 1];
}
```

7.3 Aho-Corasick Algorithm

```
struct ACautomata{
 struct Node{
    int cnt,dp;
    Node *go[26], *fail;
    Node (){
      cnt = 0;
      dp = -1;
      memset(go,0,sizeof(go));
      fail = 0:
   }
 };
 Node *root, pool[1048576];
 int nMem;
 Node* new_Node(){
   pool[nMem] = Node();
    return &pool[nMem++];
 void init(){
   nMem = 0;
    root = new_Node();
 void add(const string &str){
    insert(root,str,0);
  void insert(Node *cur, const string &str, int pos){
    if (pos >= (int)str.size()){
     cur->cnt++;
      return;
    int c = str[pos]-'a';
    if (cur->go[c] == 0){
     cur->go[c] = new_Node();
    insert(cur->go[c],str,pos+1);
  void make_fail(){
    queue<Node*> que;
    que.push(root);
    while (!que.empty()){
      Node* fr=que.front();
      que.pop();
      for (int i=0; i<26; i++){</pre>
        if (fr->go[i]){
          Node *ptr = fr->fail;
          while (ptr && !ptr->go[i]) ptr = ptr->fail;
          if (!ptr) fr->go[i]->fail = root;
          else fr->go[i]->fail = ptr->go[i];
          que.push(fr->go[i]);
```

```
}
| }
| };
```

7.4 KMP

```
#include<bits/stdc++.h>
using namespace std;
void build_fail_function(string B, int *fail) {
    int len = B.length(), pos;
    pos = fail[0] = -1;
    for (int i = 1; i < len; i ++) {</pre>
        while (pos != -1 and B[pos + 1] != B[i])
            pos = fail[pos];
        if (B[pos + 1] == B[i]) pos ++;
        fail[i] = pos;
    }
}
void match(string A, string B, int *fail) {
    int lenA = A.length(), lenB = B.length();
    int pos = -1;
    for (int i = 0; i < lenA; i ++) {</pre>
        while (pos != -1 and B[pos + 1] != A[i])
            pos = fail[pos];
        if (B[pos + 1] == A[i]) pos ++;
        if (pos == lenB - 1) {
            // Match ! A[i - lenB + 1, i] = B
            pos = fail[pos];
        }
    }
}
```

7.5 Z value

```
void Zval(const char *s, int len, int *z) {
    z[0] = 0;
    for (int b=0, i=1; i<len; i++) {
        z[i] = max(min(z[i-b], z[b] + b - i), 0);
        while (s[i + z[i]] == s[z[i]]) z[i] ++;
        if (i+z[i] > b+z[b]) b=i;
    }
}
```

7.6 Z value (palindrome ver.)

7.7 palindromic tree

```
//bcw0x1bd2 {{{
#include<bits/stdc++.h>
#include<unistd.h>
using namespace std;
#define F first
```

```
#define S second
#define MP make_pair
#define PB push_back
#define IOS ios_base::sync_with_stdio(0); cin.tie(0);
#define SZ(x) ((int)((x).size()))
#define ALL(x) begin(x),end(x)
#define REP(i,x) for (int i=0; i<(x); i++)
#define REP1(i,a,b) for (int i=(a); i<=(b); i++)
typedef long long ll;
typedef pair<int,int> pii;
typedef pair<ll,ll> pll;
typedef long double ld;
#ifdef DARKHH
#define FILEIO(name)
#else
#define FILEIO(name) \
 freopen(name".in", "r", stdin); \
freopen(name".out", "w", stdout);
#endif
#ifdef DARKHH
template<typename T>
void _dump( const char* s, T&& head ) { cerr<<s<<"="<<</pre>
    head<<endl; }
template<typename T, typename... Args>
void _dump( const char* s, T&& head, Args&&... tail ) {
 while ( *s!=',' || c!=0 ) {
  if ( *s=='(' || *s=='[' || *s=='{' }) c++;
    if ( *s==')' || *s==']' || *s=='}') c--;
    cerr<<*s++;
  cerr<<"="<<head<<", ";
  _dump(s+1,tail...);
#define dump(...) do { \
  fprintf(stderr, "%s:%d - ", __PRETTY_FUNCTION__,
        _LINE__); \
  _dump(#__VA_ARGS__, __VA_ARGS__); \
} while (0)
template<typename Iter>
ostream& _out( ostream &s, Iter b, Iter e ) {
  s<<"[";
  for ( auto it=b; it!=e; it++ ) s<<(it==b?"":" ")<<*it</pre>
  s<<"j́";
  return s;
template<typename A, typename B>
ostream& operator <<( ostream &s, const pair<A,B> &p )
    { return s<<"("<<p.first<<","<<p.second<<")"; }
template<typename T>
ostream& operator <<( ostream &s, const vector<T> &c )
    { return _out(s,ALL(c)); }
template<typename T, size_t N>
ostream& operator <<( ostream &s, const array<T,N> &c )
     { return _out(s,ALL(c)); }
template<typename T>
ostream& operator <<( ostream &s, const set<T> &c ) {
    return _out(s,ALL(c)); }
template<typename A, typename B>
ostream& operator <<( ostream &s, const map<A,B> &c ) {
     return _out(s,ALL(c)); }
#else
#define dump(...)
#endif
// }}}
struct palindromic_tree{
  struct node{
    int next[26],fail,len;
    int cnt,num,st,ed;
    node(int l=0):fail(0),len(l),cnt(0),num(0){
      for(int i=0;i<26;++i)next[i]=0;</pre>
```

};

```
vector<node> state:
  vector<char> s;
  int last,n;
  void init(){
    state.clear();
    s.clear();
    last=1;
    n=0:
    state.push_back(0);
    state.push_back(-1);
    state[0].fail=1;
    s.push_back(-1);
  int get_fail(int x){
    while(s[n-state[x].len-1]!=s[n])x=state[x].fail;
    return x:
  void add(int c){
    s.push_back(c-='a');
    ++n;
    int cur=get_fail(last);
    if(!state[cur].next[c]){
      int now=state.size();
      state.push_back(state[cur].len+2);
      state[now].fail=state[get_fail(state[cur].fail)].
          next[c];
      state[cur].next[c]=now;
      state[now].num=state[state[now].fail].num+1;
    last=state[cur].next[c];
    ++state[last].cnt;
  int size(){
    return state.size()-2;
}pt;
int main() {
  string s;
  cin >> s;
  pt.init();
  for (int i=0; i<SZ(s); i++) {</pre>
    int prvsz = pt.size();
    pt.add(s[i]);
    if (prvsz != pt.size()) {
      int r = i;
      int l = r - pt.state[pt.last].len + 1;
      cout << "Find pal @ [" << l << " " << r << "] : "
           << s.substr(l,r-l+1) << endl;
  }
  return 0;
}
```

7.8 Lexicographically Smallest Rotation

```
string mcp(string s){
  int n = s.length();
  s += s;
  int i=0, j=1;
  while (i<n && j<n){
    int k = 0;
    while (k < n && s[i+k] == s[j+k]) k++;
    if (s[i+k] <= s[j+k]) j += k+1;
    else i += k+1;
    if (i == j) j++;
  }
  int ans = i < n ? i : j;
  return s.substr(ans, n);
}</pre>
```

7.9 Suffix Automaton

```
// par : fail link
// val : a topological order ( useful for DP )
// go[x] : automata edge ( x is integer in [0,26) )
```

```
struct SAM{
  struct State{
    int par, go[26], val;
    State () : par(0), val(0){ FZ(go); }
    State (int _val) : par(0), val(_val){ FZ(go); }
  vector<State> vec;
  int root, tail;
  void init(int arr[], int len){
    vec.resize(2);
    vec[0] = vec[1] = State(0);
    root = tail = 1;
    for (int i=0; i<len; i++)</pre>
      extend(arr[i]);
  void extend(int w){
    int p = tail, np = vec.size();
    vec.PB(State(vec[p].val+1));
    for ( ; p && vec[p].go[w]==0; p=vec[p].par)
      vec[p].go[w] = np;
    if (p == 0){
      vec[np].par = root;
    } else {
      if (vec[vec[p].go[w]].val == vec[p].val+1){
        vec[np].par = vec[p].go[w];
      } else {
        int q = vec[p].go[w], r = vec.size();
        vec.PB(vec[q]);
        vec[r].val = vec[p].val+1;
        vec[q].par = vec[np].par = r;
        for ( ; p && vec[p].go[w] == q; p=vec[p].par)
          vec[p].go[w] = r;
    }
    tail = np;
  }
};
```

8 Problems

8.1 Painter

```
#include<bits/stdc++.h>
using namespace std;
#define F first
#define S second
#define PB push_back
#define IOS ios_base::sync_with_stdio(0); cin.tie(0);
#define SZ(x) ((int)((x).size()))
#define ALL(x) begin(x),end(x)
#define REP(i,x) for (int i=0; i<(x); i++)
#define REP1(i,a,b) for (int i=(a); i<=(b); i++)
typedef long long ll;
typedef pair<ll,ll> pll;
typedef pll Point;
const int MXN = 100005;
Point operator + (const Point &a, const Point &b) {
    return Point(a.F+b.F, a.S+b.S); }
Point operator - (const Point &a, const Point &b) {
    return Point(a.F-b.F, a.S-b.S); }
ll operator * (const Point &a, const Point &b) { return
     a.F*b.F + a.S*b.S; }
ll operator % (const Point &a, const Point &b) { return
     a.F*b.S - a.S*b.F; }
struct Segment {
  int v,id;
  Point p,q;
  Segment () {}
  Segment (int _v, int _id, Point _p, Point _q) :
    v(_v), id(_id), p(_p), q(_q) {}
bool operator < (const Segment &a, const Segment &b) {</pre>
```

```
if (a.p == b.q) return false;
   if (a.q == b.p) return true;
   if (a.p == b.p) return (a.q-a.p) % (b.q-a.p) > 0;
   if (a.q == b.q) return (a.p-a.q) % (b.p-a.q) < 0;</pre>
   if (a.p.F == b.p.F) return a.p.S < b.p.S;</pre>
   if (a.q.F == b.q.F) return a.q.S < b.q.S;</pre>
   if (a.p.F < b.p.F) return (a.q-a.p) % (b.p-a.p) > 0;
   else return (b.q-b.p) % (a.p-b.p) < 0;
bool operator == (const Segment &a, const Segment &b) {
   return tie(a.v,a.id,a.p,a.q) == tie(b.v,b.id,b.p,b.q)
struct Triangle {
  Point pt[3];
}ip[MXN];
const int MEM = 350004;
struct Treap {
  static Treap nil, mem[MEM], *pmem;
   Treap *l, *r;
   int sum,presum,size;
   Segment seg;
   Treap () : l(&nil), r(&nil), sum(0), presum(0), size
       (0), seg() {}
   Treap (Segment _val) :
     l(&nil), r(&nil), sum(_val.v), presum(max(_val.v,0)
         ), size(1), seg(_val) {}
} Treap::nil, Treap::mem[MEM], *Treap::pmem = Treap::
 int size(const Treap *t) { return t->size; }
void pull(Treap *t) {
  if (!size(t)) return;
   t\rightarrow size = size(t\rightarrow l) + size(t\rightarrow r) + 1;
   t->sum = t->l->sum + t->seg.v + t->r->sum;
   t\rightarrow presum = max(t\rightarrow l\rightarrow presum, t\rightarrow l\rightarrow sum + t\rightarrow seg.v);
   t\rightarrow presum = max(t\rightarrow presum, t\rightarrow l\rightarrow sum + t\rightarrow seg.v + t\rightarrow
       r->presum);
Treap* merge(Treap *a, Treap *b) {
   if (!size(a)) return b;
   if (!size(b)) return a;
   Treap *t;
   if (rand() % (size(a) + size(b)) < size(a)) {
     t = a;
     t->r = merge(a->r, b);
   } else {
     t = b;
     t->l = merge(a, b->l);
  pull(t);
   return t;
void split(Treap *t, int k, Treap *&a, Treap *&b) {
   if (!size(t)) a = b = &Treap::nil;
   else if (size(t->l) + 1 <= k) {
     a = t;
     split(t->r, k - size(t->l) - 1, a->r, b);
     pull(a);
   } else {
     b = t;
     split(t->l, k, a, b->l);
     pull(b);
int get_rank(Treap *t, Segment x) {
   if (!size(t)) return 0;
   if (x < t->seg) return get_rank(t->l, x);
   return get_rank(t->r,x) + size(t->l) + 1;
Treap* find_leftist(Treap *t) {
  while (size(t->l)) t = t->l;
   return t;
Treap* find_rightist(Treap *t) {
   while (size(t->r)) t = t->r;
   return t;
 int N;
vector<int> allx;
```

```
vector<Segment> _seg[3*MXN];
#define seg(x) _seg[(x)+100000]
inline void add_seg(Segment s) {
  seg(s.p.F).PB(s);
  if (s.q.F != s.p.F) seg(s.q.F).PB(s);
void predo()
 allx.clear():
  REP(i,N) REP(j,3) {
    seg(ip[i].pt[j].F).clear();
    allx.PB(ip[i].pt[j].F);
  sort(ALL(allx));
  allx.resize(unique(ALL(allx))-begin(allx));
  REP(i,N) {
    sort(ip[i].pt, ip[i].pt+3);
    Point *pt = ip[i].pt;
    Segment seg1 = Segment(1,i,pt[0],pt[1]);
    Segment seg2 = Segment(1,i,pt[0],pt[2]);
    Segment seg3 = Segment(1,i,pt[1],pt[2]);
    if (seg2 < seg1) seg1.v = -1;</pre>
    else seg2.v = -1;
    seg3.v = seg1.v;
    add_seg(seg1);
    add_seg(seg2);
    add_seg(seg3);
  }
inline int sgn(ll x) { return x < 0 ? -1 : x > 0; }
bool interPnt(Point p1, Point p2, Point q1, Point q2){
 ll c1 = (p2-p1)\%(q1-p1), c2 = (p2-p1)\%(q2-p1);
  ll c3 = (q2-q1)%(p1-q1), c4 = (q2-q1)%(p2-q1);

return sgn(c1) * sgn(c2) <= 0 and sgn(c3) * sgn(c4)
      <= 0;
bool check_error(Segment a, Segment b) {
  if (a.id == b.id) return false;
  return interPnt(a.p,a.q,b.p,b.q);
int solve() {
 Treap::pmem = Treap::mem;
  Treap *rt = &Treap::nil;
  int res = 0;
  for (auto i:allx) {
    for (auto l:seg(i)) {
      int k = get_rank(rt, l);
      Treap *t,*tl,*tm,*tr;
      split(rt,k,tl,tr);
      t = find_rightist(tl);
      if (size(t) and check_error(t->seg,l)) return -1;
      t = find_leftist(tr);
      if (size(t) and check_error(t->seg,l)) return -1;
      rt = merge(tl,tr);
      if (l.p.F == i and l.p.F != l.q.F) {
        k = get_rank(rt, l);
        split(rt,k,tl,tr);
        tm = new (Treap::pmem++) Treap(l);
        rt = merge(merge(tl,tm),tr);
    for (auto l:seg(i)) {
  if (l.q.F == i and l.p.F != l.q.F) {
        Treap *tl,*tm,*tr;
        int k = get_rank(rt, l);
        split(rt,k-1,tl,tm);
        split(tm,1,tm,tr);
        Treap *t1=find_rightist(tl),*t2=find_leftist(tr
        if (size(t1) and size(t2) and check_error(t1->
            seg,t2->seg)) return -1;
        rt = merge(tl,tr);
    }
    res = max(res, rt->presum);
 }
  res++;
  return res;
int main() {
  IOS;
  int cas = 0;
```

```
while (cin >> N) {
    if (N == -1) break;
    REP(i,N) {
        REP(j,3) cin >> ip[i].pt[j].F >> ip[i].pt[j].S;
    }
    predo();
    int ans = solve();
    cas++;
    cout << "Case " << cas << ": ";
    if (ans == -1) cout << "ERROR\n";
    else cout << ans << " shades\n";
}

return 0;
}</pre>
```

8.2 Mo-Algorithm on Tree

```
#include<bits/stdc++.h>
using namespace std;
#define IOS ios_base::sync_with_stdio(0); cin.tie(0);
#define SZ(x) ((int)((x).size()))
const int MX = 500005;
const int SQ = 1400;
const int LOG = 17;
struct BIT {
  int bit[MX];
  int lb(int x) { return x & -x; }
  void add(int p, int v) {
    for (int i=p; i<MX; i+=lb(i)) bit[i] += v;</pre>
  int qry() {
    int v = 0;
    for (int i=1<<LOG; i>0; i>>=1) {
      if ((v|i) < MX \text{ and } bit[v|i]==i) v |= i;
    return v;
  }
}bit;
struct Query {
 int l,r,qid;
}qry[MX];
struct Edge {
  int v,x;
int N,Q,timestamp[MX],ans[MX];
int in[MX],cnt[MX];
vector<Edge> E[MX];
vector<Edge> seq;
void DFS(int u, int f) {
  timestamp[u] = SZ(seq);
  for (auto it:E[u]) {
    if (it.v == f) continue;
    seq.push_back(it);
    DFS(it.v,u);
    seq.push_back(it);
  }
void poke(int id) {
  int v = seq[id].v;
  int x = seq[id].x;
  in[v] ^= 1;
  cnt[x] += in[v] ? 1 : -1;
  if (in[v] \text{ and } cnt[x] == 1) bit.add(x, 1);
  if (!in[v] \text{ and } cnt[x] == 0) \text{ bit.add}(x, -1);
int main() {
  IOS;
  cin >> N >> Q;
  for (int i=0; i<N-1; i++) {</pre>
    int u,v,x;
    cin >> u >> v >> x;
    x = min(x,N);
    E[u].push_back({v,x});
```

```
E[v].push_back({u,x});
DFS(1,1);
for (int i=1; i<=Q; i++) {</pre>
  int u,v;
  cin >> u >> v;
  int l = timestamp[u], r = timestamp[v];
  if (l > r) swap(l,r);
  qry[i] = {l,r,i};
sort(qry+1,qry+1+Q, [](Query a, Query b) {
    return make_pair(a.l/SQ,a.r) < make_pair(b.l/SQ,b</pre>
         .r);
    });
int curL = 1, curR = 0;
for (int i=1; i<=Q; i++) {</pre>
  int ql=qry[i].l,qr=qry[i].r;
  while (curL > ql) poke(--curL);
  while (curR < qr) poke(++curR);</pre>
  while (curL < ql) poke(curL++);</pre>
  while (curR > qr) poke(curR--);
  ans[qry[i].qid] = bit.qry();
for (int i=1; i<=Q; i++) cout << ans[i] << "\n";</pre>
return 0;
```

8.3 Manhattan MST

```
#include<bits/stdc++.h>
#define REP(i,n) for(int i=0;i<n;i++)</pre>
using namespace std;
typedef long long LL;
const int N=200100;
int n.m:
struct PT {int x,y,z,w,id;}p[N];
inline int dis(const PT &a,const PT &b){return abs(a.x-
    b.x)+abs(a.y-b.y);}
inline bool cpx(const PT &a,const PT &b){return a.x!=b.
    x? a.x>b.x:a.y>b.y;}
inline bool cpz(const PT &a,const PT &b){return a.z<b.z</pre>
    ;}
struct E{int a,b,c;}e[8*N];
bool operator<(const E&a,const E&b){return a.c<b.c;}</pre>
struct Node{
 int L,R,key;
}node[4*N];
int s[N];
int F(int x){return s[x]==x?x:s[x]=F(s[x]);}
void U(int a,int b){s[F(b)]=F(a);}
void init(int id,int L,int R) {
  node[id]=(Node)\{L,R,-1\};
  if(L==R)return;
  init(id*2,L,(L+R)/2);
  init(id*2+1,(L+R)/2+1,R);
void ins(int id,int x) {
  if(node[id].key==-1 || p[node[id].key].w>p[x].w)node[
      id].key=x;
  if(node[id].L==node[id].R)return;
  if(p[x].z<=(node[id].L+node[id].R)/2)ins(id*2,x);</pre>
  else ins(id*2+1,x);
int Q(int id,int L,int R){
  if(R<node[id].L || L>node[id].R)return -1;
  if(L<=node[id].L && node[id].R<=R)return node[id].key</pre>
  int a=Q(id*2,L,R),b=Q(id*2+1,L,R);
  if(b==-1 || (a!=-1 && p[a].w<p[b].w)) return a;
  else return b;
void calc() {
 REP(i,n) {
    p[i].z=p[i].y-p[i].x;
    p[i].w=p[i].x+p[i].y;
```

```
sort(p,p+n,cpz);
  int cnt=0,j,k;
  for(int i=0;i<n;i=j){</pre>
    for(j=i+1;p[j].z==p[i].z && j<n;j++);</pre>
    for(k=i,cnt++;k<j;k++)p[k].z=cnt;</pre>
  init(1,1,cnt);
  sort(p,p+n,cpx);
  REP(i,n) {
    j=Q(1,p[i].z,cnt);
    if(j!=-1)e[m++]=(E){p[i].id,p[j].id,dis(p[i],p[j])
        };
    ins(1,i);
  }
LL MST() {
  LL r=0;
  sort(e,e+m);
  REP(i,m) {
    if(F(e[i].a) == F(e[i].b)) continue;
    U(e[i].a,e[i].b);
    r+=e[i].c;
  }
  return r;
int main(){
  int ts;
  scanf("%d", &ts);
  while (ts--) {
    m = 0;
    scanf("%d",&n);
    REP(i,n) {
      scanf("%d%d",&p[i].x,&p[i].y);
      p[i].id=s[i]=i;
    calc();
    REP(i,n)p[i].y= -p[i].y;
    calc();
    REP(i,n)swap(p[i].x,p[i].y);
    calc();
    REP(i,n)p[i].x=-p[i].x;
    calc():
    printf("%lld\n",MST()*2);
  return 0;
```

9 YAKELI

9.1 Periodic Table

ic Table									7			6			ъ			4		ω			2			<u> </u>			
Name	Symbol	Z mass	☐ Noble Gas☐ Lanthanide/Actinide	Non-metal		Alkaline Earth Metal	Alkali Metal	Francium	Ţ	87 223	Caesium	Cs	2.91	Rubidium	Rb	37 85.468	Potassium	~	19 39.098	Sodium	Na	11 22.990	Lithium	Ξ.	3 6.941	Hydrogen	I	1 1.0079	1 IA
	man-made		ctinide			Metal		Radium	Ra	88 226	Barium	Ва	56 137.33	Strontium	Sr	38 87.62	Calcium	Ca	20 40.078	Magnesium	Δ	12 24.305	Beryllium	Be	4 9.0122	2 IIA			
	de							Actinide	Ac-Lr	89-103	Lanthanide	La-Lu	57-71	Yttrium	~	39 88.906	Scandium	Sc	21 44.956	3 IIIA									
Actinium	Ac	89 227		Lanthanum	La	57 138.91		Rutherfordium	Rf	104 261	Halfnium	Ŧ	72 178.49	Zirconium	Zr	40 91.224	Titanium	=	22 47.867	4 IVB									
Thorium	Τh	90 232.04		Cerium	Ce	58 140.12		Dubnium	Db	105 262	Tantalum	Ta	73 180.95	Niobium	В	41 92.906	Vanadium	<	23 50.942	5 VB									
Protactinium	Pa	91 231.04		Praseodymium	Pr	59 140.91		Seaborgium	Sg	106 266	Tungsten	\$	74 183.84	Molybdenum	M _o	42 95.94	Chromium	Ç	24 51.996	6 VIB									
Uranium	_	92 238.03		Neodymium	Nd	60 144.24		Bohrium	뫄	107 264	Rhenium	Re	75 186.21	Technetium	Tc	43 96	Manganese	Μn	25 54.938	7 VIIB									
Neptunium	Νp	93 237		Promethium	Pm	61 145		Hassium	Нѕ	108 277	Osmium	0s	76 190.23	Ruthenium	Ru	44 101.07	Iron	Fe	26 55.845	8 VIIIB									
Plutonium	Pu	94 244		Samarium	Sm	62 150.36		Meitnerium	Mt	109 268	Iridium	₹	77 192.22	Rhodium	Rh	45 102.91	Cobalt	င	27 58.933	9 VIIIB									
Americium	Am	95 243		Europium	Ē	63 151.96		Darmstadtium	Ds	110 281	Platinum	Ρţ	78 195.08	Palladium	Pd	46 106.42	Nickel	<u>z</u>	28 58.693	10 VIIIB									
Curium	Cm	96 247		Gadolinium	ଜ	64 157.25		Roentgenium	Rg	111 280	Gold	Au	79 196.97	Silver	Ag	47 107.87	Copper	C	29 63.546	11 IB									
Berkelium	Вķ	97 247		Terbium	ТЬ	65 158.93		Copernicium	Ç	112 285	Mercury	Hg	80 200.59	Cadmium	С	48 112.41	Zinc	Zn	30 65.39	12 IIB									
Californium	Çf	98 251		Dysprosium	Dy	66 162.50		Ununtrium	Uut	113 284	Thallium	⊒	81 204.38	Indium	Б	49 114.82	Gallium	Ga	31 69.723	Aluminium	≥	13 26.982	Boron	В	5 10.811	13 IIIA			
Einsteinium	Es	99 252		Holmium	Но	67 164.93		Flerovium		114 289	Lead	РЬ	82 207.2	Tin	Sn	50 118.71	Germanium	Ge	32 72.64	Silicon	Si	14 28.086	Carbon	С	6 12.011	14 IVA			
Fermium	Fm	100 257		Erbium	ĒΓ	68 167.26		Ununpentium	Uup	115 288	Bismuth	<u>B:</u>	83 208.98	Antimony	dS	51 121.76	Arsenic	As	33 74.922	Phosphorus	P	15 30.974	Nitrogen	z	7 14.007	15 VA			
Mendelevium	Md	101 258		Thulium	Tm	69 168.93		Livermorium	r	116 293	Polonium	Po	84 209	Tellurium	Te	52 127.6	Selenium	Se	34 78.96	Sulphur	s	16 32.065	Oxygen	0	8 15.999	16 VIA			
Nobelium	No	102 259		Ytterbium	ት	70 173.04		Unurseptium	Uus	117 292	Astatine	At	85 210	lodine	-	53 126.9	Bromine	Br	35 79.904	Chlorine	Ω	17 35.453	Flourine	'n	9 18.998	17 VIIA			
Lawrencium	Ę	103 262		Lutetium	<u>-</u>	71 174.97		Ununoctium	Uuo	118 294	Radon	R	86 222	Xenon	Xe	54 131.29	Krypton	ᅐ	36 83.8	Argon	Ąr	18 39.948	Neon	Ne	10 20.180	Helium	He	2 4.0025	18 VIIIA