Contents

1 Basic

1.1 vimrc

```
colo torte
syn on
se ai ar sm nu rnu is
se mouse=a bs=2 ww+=<,>,[,] so=6 ts=4 sw=4 ttm=100
se makeprg=g++\ -Wall\ -Wshadow\ -O2\ -std=c++0x\ -o\
    %<\ %
au BufNewFile *.cpp 0r ~/default.cpp | :1,$-6 fo
filetype indent on

map <F7> <ESC>:wa<CR>:make!<CR>
imap <F7> <ESC>:wa<CR>:make!<CR>
map <C-F7> <ESC>:tabe %<.in<CR>
map <F8> :cope <CR>
map <S-F8> :ccl <CR>
map <F9> :!./%< <CR>
map <C-F9> :!./%< < %<.in <CR>
```

1.2 IncreaseStackSize

```cpp
//stack resize
asm( "mov %0,%%esp\n" ::"g"(mem+10000000) );
//change esp to rsp if 64-bit system

//stack resize (linux)
#include <sys/resource.h>
void increase_stack_size() {
  const rlim_t ks = 64*1024*1024;
  struct rlimit rl;
  int res=getrlimit(RLIMIT_STACK, &rl);
  if(res==0){
    if(rl.rlim_cur<ks){
      rl.rlim_cur=ks;
      res=setrlimit(RLIMIT_STACK, &rl);
    }
  }
}
```

1.3 Default Code

```cpp
#include<bits/stdc++.h>
#include<unistd.h>
using namespace std;
#define FZ(n) memset((n),0,sizeof(n))
#define FMO(n) memset((n),-1,sizeof(n))
#define F first
#define S second
#define PB push_back
#define ALL(x) begin(x),end(x)
#define SZ(x) ((int)(x).size())
#define IOS ios_base::sync_with_stdio(0); cin.tie(0)
#define REP(i,x) for (int i=0; i<(x); i++)
#define REP1(i,a,b) for (int i=(a); i<=(b); i++)
#ifdef ONLINE_JUDGE
#define FILEIO(name) \
    freopen(name".in", "r", stdin); \
    freopen(name".out", "w", stdout);
#else
#define FILEIO(name)
#endif
template<typename A, typename B>
ostream& operator <<(ostream &s, const pair<A,B> &p) {
    return s<<"("<<p.first<<","<<p.second<<")";
}
template<typename T>
ostream& operator <<(ostream &s, const vector<T> &c) {
    s<<"[ ";
    for (auto it : c) s << it << " ";
    s<<"]";
    return s;
}
// Let's Fight!
```

```cpp
int main() {

    return 0;
}
```

2 Data Structure

2.1 Bigint

```cpp
struct Bigint{
  static const int LEN = 60;
  static const int BIGMOD = 10000;

  int s;
  int vl, v[LEN];
  //  vector<int> v;
  Bigint() : s(1) { vl = 0; }
  Bigint(long long a) {
    s = 1; vl = 0;
    if (a < 0) { s = -1; a = -a; }
    while (a) {
      push_back(a % BIGMOD);
      a /= BIGMOD;
    }
  }
  Bigint(string str) {
    s = 1; vl = 0;
    int stPos = 0, num = 0;
    if (!str.empty() && str[0] == '-') {
      stPos = 1;
      s = -1;
    }
    for (int i=SZ(str)-1, q=1; i>=stPos; i--) {
      num += (str[i] - '0') * q;
      if ((q *= 10) >= BIGMOD) {
        push_back(num);
        num = 0; q = 1;
      }
    }
    if (num) push_back(num);
  }

  int len() const {
    return vl;
    //    return SZ(v);
  }
  bool empty() const { return len() == 0; }
  void push_back(int x) {
    v[vl++] = x;
    //    v.PB(x);
  }
  void pop_back() {
    vl--;
    //    v.pop_back();
  }
  int back() const {
    return v[vl-1];
    //    return v.back();
  }
  void n() {
    while (!empty() && !back()) pop_back();
  }
  void resize(int nl) {
    vl = nl;
    fill(v, v+vl, 0);
    //    v.resize(nl);
    //    fill(ALL(v), 0);
  }

  void print() const {
    if (empty()) { putchar('0'); return; }
    if (s == -1) putchar('-');
    printf("%d", back());
    for (int i=len()-2; i>=0; i--) printf("%.4d",v[i]);
  }
  friend std::ostream& operator << (std::ostream& out,
      const Bigint &a) {
    if (a.empty()) { out << "0"; return out; }
    if (a.s == -1) out << "-";
    out << a.back();
    for (int i=a.len()-2; i>=0; i--) {
      char str[10];
      snprintf(str, 5, "%.4d", a.v[i]);
      out << str;
    }
```

```cpp
    return out;
  }

  int cp3(const Bigint &b)const {
    if (s != b.s) return s - b.s;
    if (s == -1) return -(*this).cp3(-b);
    if (len() != b.len()) return len()-b.len();//int
    for (int i=len()-1; i>=0; i--)
      if (v[i]!=b.v[i]) return v[i]-b.v[i];
    return 0;
  }

  bool operator < (const Bigint &b)const{ return cp3(b)
    <0; }
  bool operator <= (const Bigint &b)const{ return cp3(b
    )<=0; }
  bool operator == (const Bigint &b)const{ return cp3(b
    )==0; }
  bool operator != (const Bigint &b)const{ return cp3(b
    )!=0; }
  bool operator > (const Bigint &b)const{ return cp3(b)
    >0; }
  bool operator >= (const Bigint &b)const{ return cp3(b
    )>=0; }

  Bigint operator - () const {
    Bigint r = (*this);
    r.s = -r.s;
    return r;
  }
  Bigint operator + (const Bigint &b) const {
    if (s == -1) return -(-(*this)+(-b));
    if (b.s == -1) return (*this)-(-b);
    Bigint r;
    int nl = max(len(), b.len());
    r.resize(nl + 1);
    for (int i=0; i<nl; i++) {
      if (i < len()) r.v[i] += v[i];
      if (i < b.len()) r.v[i] += b.v[i];
      if(r.v[i] >= BIGMOD) {
        r.v[i+1] += r.v[i] / BIGMOD;
        r.v[i] %= BIGMOD;
      }
    }
    r.n();
    return r;
  }
  Bigint operator - (const Bigint &b) const {
    if (s == -1) return -(-(*this)-(-b));
    if (b.s == -1) return (*this)+(-b);
    if ((*this) < b) return -(b-(*this));
    Bigint r;
    r.resize(len());
    for (int i=0; i<len(); i++) {
      r.v[i] += v[i];
      if (i < b.len()) r.v[i] -= b.v[i];
      if (r.v[i] < 0) {
        r.v[i] += BIGMOD;
        r.v[i+1]--;
      }
    }
    r.n();
    return r;
  }
  Bigint operator * (const Bigint &b) {
    Bigint r;
    r.resize(len() + b.len() + 1);
    r.s = s * b.s;
    for (int i=0; i<len(); i++) {
      for (int j=0; j<b.len(); j++) {
        r.v[i+j] += v[i] * b.v[j];
        if(r.v[i+j] >= BIGMOD) {
          r.v[i+j+1] += r.v[i+j] / BIGMOD;
          r.v[i+j] %= BIGMOD;
        }
      }
    }
    r.n();
    return r;
  }
  Bigint operator / (const Bigint &b) {
    Bigint r;
```

```cpp
    r.resize(max(1, len()-b.len()+1));
    int oriS = s;
    Bigint b2 = b; // b2 = abs(b)
    s = b2.s = r.s = 1;
    for (int i=r.len()-1; i>=0; i--) {
      int d=0, u=BIGMOD-1;
      while(d<u) {
        int m = (d+u+1)>>1;
        r.v[i] = m;
        if((r*b2) > (*this)) u = m-1;
        else d = m;
      }
      r.v[i] = d;
    }
    s = oriS;
    r.s = s * b.s;
    r.n();
    return r;
  }
  Bigint operator % (const Bigint &b) {
    return (*this)-(*this)/b*b;
  }
};
```

2.2 unordered_map

```cpp
struct Key {
  int first,second;
  Key () {}
  Key (int _x, int _y) : first(_x),  second(_y) {}
  bool operator == (const Key &b) const {
    return tie(F,S) == tie(b.F,b.S);
  }
};
struct KeyHasher {
  size_t operator()(const Key& k) const {
    return k.first + k.second*100000;
  }
};

typedef unordered_map<Key,int,KeyHasher> map_t;

int main(int argc, char** argv){
  map_t mp;
  for (int i=0; i<10; i++)
    mp[Key(i,0)] = i+1;
  for (int i=0; i<10; i++)
    printf("%d\n", mp[Key(i,0)]);

  return 0;
}
```

2.3 extc_balance_tree

```cpp
#include <bits/extc++.h>
using namespace __gnu_pbds;
typedef tree<int,null_type,less<int>,rb_tree_tag,
    tree_order_statistics_node_update> set_t;

int main()
{
  // Insert some entries into s.
  set_t s;
  s.insert(12);
  s.insert(505);

  // The order of the keys should be: 12, 505.
  assert(*s.find_by_order(0) == 12);
  assert(*s.find_by_order(3) == 505);

  // The order of the keys should be: 12, 505.
  assert(s.order_of_key(12) == 0);
  assert(s.order_of_key(505) == 1);

  // Erase an entry.
  s.erase(12);

  // The order of the keys should be: 505.
```

```cpp
  assert(*s.find_by_order(0) == 505);

  // The order of the keys should be: 505.
  assert(s.order_of_key(505) == 0);

}
```

2.4 Disjoint Set

```cpp
struct DisjointSet {
  // save() is like recursive
  // undo() is like return
  int n, fa[MXN], sz[MXN];
  vector<pair<int*,int>> h;
  vector<int> sp;
  void init(int tn) {
    n=tn;
    for (int i=0; i<n; i++) {
      fa[i]=i;
      sz[i]=1;
    }
    sp.clear(); h.clear();
  }
  void assign(int *k, int v) {
    h.PB({k, *k});
    *k=v;
  }
  void save() { sp.PB(SZ(h)); }
  void undo() {
    assert(!sp.empty());
    int last=sp.back(); sp.pop_back();
    while (SZ(h)!=last) {
      auto x=h.back(); h.pop_back();
      *x.F=x.S;
    }
  }
  int f(int x) {
    while (fa[x]!=x) x=fa[x];
    return x;
  }
  void uni(int x, int y) {
    x=f(x); y=f(y);
    if (x==y) return ;
    if (sz[x]<sz[y]) swap(x, y);
    assign(&sz[x], sz[x]+sz[y]);
    assign(&fa[y], x);
  }
}djs;
```

2.5 Treap

```cpp
const int MEM = 16000004;
struct Treap {
  static Treap nil, mem[MEM], *pmem;
  Treap *l, *r;
  char val;
  int size;
  Treap () : l(&nil), r(&nil), size(0) {}
  Treap (char _val) :
    l(&nil), r(&nil), val(_val), size(1) {}
} Treap::nil, Treap::mem[MEM], *Treap::pmem = Treap::
    mem;

int size(const Treap *t) { return t->size; }
void pull(Treap *t) {
  if (!size(t)) return;
  t->size = size(t->l) + size(t->r) + 1;
}
Treap* merge(Treap *a, Treap *b) {
  if (!size(a)) return b;
  if (!size(b)) return a;
  Treap *t;
  if (rand() % (size(a) + size(b)) < size(a)) {
    t = new (Treap::pmem++) Treap(*a);
    t->r = merge(a->r, b);
  } else {
    t = new (Treap::pmem++) Treap(*b);
    t->l = merge(a, b->l);
```

```cpp
  }
  pull(t);
  return t;
}
void split(Treap *t, int k, Treap *&a, Treap *&b) {
  if (!size(t)) a = b = &Treap::nil;
  else if (size(t->l) + 1 <= k) {
    a = new (Treap::pmem++) Treap(*t);
    split(t->r, k - size(t->l) - 1, a->r, b);
    pull(a);
  } else {
    b = new (Treap::pmem++) Treap(*t);
    split(t->l, k, a, b->l);
    pull(b);
  }
}

int nv;
Treap *rt[50005];

void print(const Treap *t) {
  if (!size(t)) return;
  print(t->l);
  cout << t->val;
  print(t->r);
}

int main(int argc, char** argv) {
  IOS;
  rt[nv=0] = &Treap::nil;
  Treap::pmem = Treap::mem;
  int Q, cmd, p, c, v;
  string s;
  cin >> Q;
  while (Q--) {
    cin >> cmd;
    if (cmd == 1) {
      // insert string s after position p
      cin >> p >> s;
      Treap *tl, *tr;
      split(rt[nv], p, tl, tr);
      for (int i=0; i<SZ(s); i++)
        tl = merge(tl, new (Treap::pmem++) Treap(s[i]))
            ;
      rt[++nv] = merge(tl, tr);
    } else if (cmd == 2) {
      // remove c characters starting at position
      Treap *tl, *tm, *tr;
      cin >> p >> c;
      split(rt[nv], p-1, tl, tm);
      split(tm, c, tm, tr);
      rt[++nv] = merge(tl, tr);
    } else if (cmd == 3) {
      // print c characters starting at position p, in
      //     version v
      Treap *tl, *tm, *tr;
      cin >> v >> p >> c;
      split(rt[v], p-1, tl, tm);
      split(tm, c, tm, tr);
      print(tm);
      cout << "\n";
    }
  }
  return 0;
}
```

2.6 Heavy Light Decomposition

```cpp
// only one segment tree / no 0/1 base issue
// getPathSeg return the segment in order u->v
// fa[root] = root
typedef pair<int,int> pii;

int N,fa[MXN],belong[MXN],dep[MXN],sz[MXN],que[MXN];
int step,line[MXN],stPt[MXN],edPt[MXN];
vector<int> E[MXN], chain[MXN];

void DFS(int u){
  vector<int> &c = chain[belong[u]];
  for (int i=c.size()-1; i>=0; i--){
```

```cpp
    int v = c[i];
    stPt[v] = step;
    line[step++] = v;
  }
  for (int i=0; i<(int)c.size(); i++){
    u = c[i];
    for (auto v : E[u]){
      if (fa[u] == v || (i && v == c[i-1])) continue;
      DFS(v);
    }
    edPt[u] = step-1;
  }
}
void build_chain(int st){
  int fr,bk;
  fr=bk=0; que[bk++] = 1; fa[st]=st; dep[st]=0;
  while (fr < bk){
    int u=que[fr++];
    for (auto v : E[u]){
      if (v == fa[u]) continue;
      que[bk++] = v;
      dep[v] = dep[u]+1;
      fa[v] = u;
    }
  }
  for (int i=bk-1,u,pos; i>=0; i--){
    u = que[i]; sz[u] = 1; pos = -1;
    for (auto v : E[u]){
      if (v == fa[u]) continue;
      sz[u] += sz[v];
      if (pos==-1 || sz[v]>sz[pos]) pos=v;
    }
    if (pos == -1) belong[u] = u;
    else belong[u] = belong[pos];
    chain[belong[u]].PB(u);
  }
  step = 0;
  DFS(st);
}
int getLCA(int u, int v){
  while (belong[u] != belong[v]){
    int a = chain[belong[u]].back();
    int b = chain[belong[v]].back();
    if (dep[a] > dep[b]) u = fa[a];
    else v = fa[b];
  }
  return sz[u] >= sz[v] ? u : v;
}
vector<pii> getPathSeg(int u, int v){
  vector<pii> ret1,ret2;
  while (belong[u] != belong[v]){
    int a = chain[belong[u]].back();
    int b = chain[belong[v]].back();
    if (dep[a] > dep[b]){
      ret1.PB({stPt[a],stPt[u]});
      u = fa[a];
    } else {
      ret2.PB({stPt[b],stPt[v]});
      v = fa[b];
    }
  }
  if (dep[u] > dep[v]) swap(u,v);
  ret1.PB({stPt[u],stPt[v]});
  reverse(ret2.begin(), ret2.end());
  ret1.insert(ret1.end(),ret2.begin(),ret2.end());
  return ret1;
}
// Usage
void build(){
  build_chain(1); //change root
  init(0,step,0); //init segment tree
}
int get_answer(int u, int v){
  int ret = -2147483647;
  vector<pii> vec = getPathSeg(u,v);
  for (auto it : vec)
    ; // check answer with segment [it.F, it.S]
  return ret;
}
```

2.7 Link-Cut Tree

```cpp
const int MXN = 100005;
const int MEM = 100005;

struct Splay {
  static Splay nil, mem[MEM], *pmem;
  Splay *ch[2], *f;
  int val, rev, size;
  Splay () : val(-1), rev(0), size(0) {
    f = ch[0] = ch[1] = &nil;
  }
  Splay (int _val) : val(_val), rev(0), size(1) {
    f = ch[0] = ch[1] = &nil;
  }
  bool isr() {
    return f->ch[0] != this && f->ch[1] != this;
  }
  int dir() {
    return f->ch[0] == this ? 0 : 1;
  }
  void setCh(Splay *c, int d) {
    ch[d] = c;
    if (c != &nil) c->f = this;
    pull();
  }
  void push() {
    if (rev) {
      swap(ch[0], ch[1]);
      if (ch[0] != &nil) ch[0]->rev ^= 1;
      if (ch[1] != &nil) ch[1]->rev ^= 1;
      rev=0;
    }
  }
  void pull() {
    size = ch[0]->size + ch[1]->size + 1;
    if (ch[0] != &nil) ch[0]->f = this;
    if (ch[1] != &nil) ch[1]->f = this;
  }
} Splay::nil, Splay::mem[MEM], *Splay::pmem = Splay::
    mem;
Splay *nil = &Splay::nil;

void rotate(Splay *x) {
  Splay *p = x->f;
  int d = x->dir();
  if (!p->isr()) p->f->setCh(x, p->dir());
  else x->f = p->f;
  p->setCh(x->ch[!d], d);
  x->setCh(p, !d);
  p->pull(); x->pull();
}

vector<Splay*> splayVec;
void splay(Splay *x) {
  splayVec.clear();
  for (Splay *q=x;; q=q->f) {
    splayVec.push_back(q);
    if (q->isr()) break;
  }
  reverse(begin(splayVec), end(splayVec));
  for (auto it : splayVec) it->push();
  while (!x->isr()) {
    if (x->f->isr()) rotate(x);
    else if (x->dir()==x->f->dir()) rotate(x->f),rotate
        (x);
    else rotate(x),rotate(x);
  }
}

Splay* access(Splay *x) {
  Splay *q = nil;
  for (;x!=nil;x=x->f) {
    splay(x);
    x->setCh(q, 1);
    q = x;
  }
  return q;
}
void evert(Splay *x) {
  access(x);
```

```
  splay(x);
  x->rev ^= 1;
  x->push(); x->pull();
}
void link(Splay *x, Splay *y) {
//  evert(x);
  access(x);
  splay(x);
  evert(y);
  x->setCh(y, 1);
}
void cut(Splay *x, Splay *y) {
//  evert(x);
  access(y);
  splay(y);
  y->push();
  y->ch[0] = y->ch[0]->f = nil;
}

int N, Q;
Splay *vt[MXN];

int ask(Splay *x, Splay *y) {
  access(x);
  access(y);
  splay(x);
  int res = x->f->val;
  if (res == -1) res=x->val;
  return res;
}
int main(int argc, char** argv) {
  scanf("%d%d", &N, &Q);
  for (int i=1; i<=N; i++)
    vt[i] = new (Splay::pmem++) Splay(i);
  while (Q--) {
    char cmd[105];
    int u, v;
    scanf("%s", cmd);
    if (cmd[1] == 'i') {
      scanf("%d%d", &u, &v);
      link(vt[v], vt[u]);
    } else if (cmd[0] == 'c') {
      scanf("%d", &v);
      cut(vt[1], vt[v]);
    } else {
      scanf("%d%d", &u, &v);
      int res=ask(vt[u], vt[v]);
      printf("%d\n", res);
    }
  }

  return 0;
}
```

3   Graph

3.1   BCC Edge

```
struct BccEdge {
  static const int MXN = 100005;
  struct Edge { int v,eid; };
  int n,m,step,par[MXN],dfn[MXN],low[MXN];
  vector<Edge> E[MXN];
  DisjointSet djs;
  void init(int _n) {
    n = _n; m = 0;
    for (int i=0; i<n; i++) E[i].clear();
    djs.init(n);
  }
  void add_edge(int u, int v) {
    E[u].PB({v, m});
    E[v].PB({u, m});
    m++;
  }
  void DFS(int u, int f, int f_eid) {
    par[u] = f;
    dfn[u] = low[u] = step++;
    for (auto it:E[u]) {
```

```
      if (it.eid == f_eid) continue;
      int v = it.v;
      if (dfn[v] == -1) {
        DFS(v, u, it.eid);
        low[u] = min(low[u], low[v]);
      } else {
        low[u] = min(low[u], dfn[v]);
      }
    }
  }
  void solve() {
    step = 0;
    memset(dfn, -1, sizeof(int)*n);
    for (int i=0; i<n; i++) {
      if (dfn[i] == -1) DFS(i, i, -1);
    }
    djs.init(n);
    for (int i=0; i<n; i++) {
      if (low[i] < dfn[i]) djs.uni(i, par[i]);
    }
  }
}graph;
```

3.2   BCC Vertex

```
struct BccVertex {
  int n,nScc,step,dfn[MXN],low[MXN];
  vector<int> E[MXN],sccv[MXN];
  int top,stk[MXN];
  void init(int _n) {
    n = _n;
    nScc = step = 0;
    for (int i=0; i<n; i++) E[i].clear();
  }
  void add_edge(int u, int v) {
    E[u].PB(v);
    E[v].PB(u);
  }
  void DFS(int u, int f) {
    dfn[u] = low[u] = step++;
    stk[top++] = u;
    for (auto v:E[u]) {
      if (v == f) continue;
      if (dfn[v] == -1) {
        DFS(v,u);
        low[u] = min(low[u], low[v]);
        if (low[v] >= dfn[u]) {
          int z;
          sccv[nScc].clear();
          do {
            z = stk[--top];
            sccv[nScc].PB(z);
          } while (z != v);
          sccv[nScc].PB(u);
          nScc++;
        }
      } else {
        low[u] = min(low[u],dfn[v]);
      }
    }
  }
  vector<vector<int>> solve() {
    vector<vector<int>> res;
    for (int i=0; i<n; i++) {
      dfn[i] = low[i] = -1;
    }
    for (int i=0; i<n; i++) {
      if (dfn[i] == -1) {
        top = 0;
        DFS(i,i);
      }
    }
    REP(i,nScc) res.PB(sccv[i]);
    return res;
  }
}graph;
```

3.3   Strongly Connected Components

```cpp
struct Scc{
  int n, nScc, vst[MXN], bln[MXN];
  vector<int> E[MXN], rE[MXN], vec;
  void init(int _n){
    n = _n;
    for (int i=0; i<MXN; i++){
      E[i].clear();
      rE[i].clear();
    }
  }
  void add_edge(int u, int v){
    E[u].PB(v);
    rE[v].PB(u);
  }
  void DFS(int u){
    vst[u]=1;
    for (auto v : E[u])
      if (!vst[v]) DFS(v);
    vec.PB(u);
  }
  void rDFS(int u){
    vst[u] = 1;
    bln[u] = nScc;
    for (auto v : rE[u])
      if (!vst[v]) rDFS(v);
  }
  void solve(){
    nScc = 0;
    vec.clear();
    FZ(vst);
    for (int i=0; i<n; i++)
      if (!vst[i]) DFS(i);
    reverse(vec.begin(),vec.end());
    FZ(vst);
    for (auto v : vec){
      if (!vst[v]){
        rDFS(v);
        nScc++;
      }
    }
  }
};
```

3.4   DMST_with_sol

```cpp
const int INF = 1029384756;

struct edge_t{
  int u,v,w;
  set< pair<int,int> > add, sub;
  edge_t() : u(-1), v(-1), w(0) {}
  edge_t(int _u, int _v, int _w) {
    u = _u; v = _v; w = _w;
    add.insert({u, v});
  }
  edge_t& operator += (const edge_t& obj) {
    w += obj.w;
    FOR (it, obj.add) {
      if (!sub.count(*it)) add.insert(*it);
      else sub.erase(*it);
    }
    FOR (it, obj.sub) {
      if (!add.count(*it)) sub.insert(*it);
      else add.erase(*it);
    }
    return *this;
  }
  edge_t& operator -= (const edge_t& obj) {
    w -= obj.w;
    FOR (it, obj.sub) {
      if (!sub.count(*it)) add.insert(*it);
      else sub.erase(*it);
    }
    for (auto it : obj.add) {
      if (!add.count(it)) sub.insert(it);
      else add.erase(it);
    }
    return *this;
  }
}eg[MXN*MXN],prv[MXN],EDGE_INF(-1,-1,INF);
```

```cpp
int N,M;
int cid,incyc[MXN],contracted[MXN];
vector<int> E[MXN];

edge_t dmst(int rt){
  edge_t cost;
  for (int i=0; i<N; i++){
    contracted[i] = incyc[i] = 0;
    prv[i] = EDGE_INF;
  }
  cid = 0;
  int u,v;
  while (true){
    for (v=0; v<N; v++){
      if (v != rt && !contracted[v] && prv[v].w == INF)
        break;
    }
    if (v >= N) break; // end
    for (int i=0; i<M; i++){
      if (eg[i].v == v && eg[i].w < prv[v].w)
        prv[v] = eg[i];
    }
    if (prv[v].w == INF) // not connected
      return EDGE_INF;
    cost += prv[v];
    for (u=prv[v].u; u!=v && u!=-1; u=prv[u].u);
    if (u == -1) continue;
    incyc[v] = ++cid;
    for (u=prv[v].u; u!=v; u=prv[u].u){
      contracted[u] = 1;
      incyc[u] = cid;
    }
    for (int i=0; i<M; i++){
      if (incyc[eg[i].u] != cid && incyc[eg[i].v] ==
          cid){
        eg[i] -= prv[eg[i].v];
      }
    }
    for (int i=0; i<M; i++){
      if (incyc[eg[i].u] == cid) eg[i].u = v;
      if (incyc[eg[i].v] == cid) eg[i].v = v;
      if (eg[i].u == eg[i].v) eg[i--] = eg[--M];
    }
    for (int i=0; i<N; i++){
      if (contracted[i]) continue;
      if (prv[i].u>=0 && incyc[prv[i].u] == cid)
        prv[i].u = v;
    }
    prv[v] = EDGE_INF;
  }
  return cost;
}

void solve(){
  edge_t cost = dmst(0);
  for (auto it : cost.add){ // find a solution
    E[it.F].PB(it.S);
    prv[it.S] = edge_t(it.F,it.S,0);
  }
}
```

3.5   Maximum Clique

```cpp
class MaxClique {
public:
    static const int MV = 210;

    int V;
    int el[MV][MV/30+1];
    int dp[MV];
    int ans;
    int s[MV][MV/30+1];
    vector<int> sol;

    void init(int v) {
        V = v; ans = 0;
        FZ(el); FZ(dp);
    }

    /* Zero Base */
```

```cpp
    void addEdge(int u, int v) {
        if(u > v) swap(u, v);
        if(u == v) return;
        el[u][v/32] |= (1<<(v%32));
    }

    bool dfs(int v, int k) {
        int c = 0, d = 0;
        for(int i=0; i<(V+31)/32; i++) {
            s[k][i] = el[v][i];
            if(k != 1) s[k][i] &= s[k-1][i];
            c += __builtin_popcount(s[k][i]);
        }
        if(c == 0) {
            if(k > ans) {
                ans = k;
                sol.clear();
                sol.push_back(v);
                return 1;
            }
            return 0;
        }
        for(int i=0; i<(V+31)/32; i++) {
            for(int a = s[k][i]; a ; d++) {
                if(k + (c-d) <= ans) return 0;
                int lb = a&(-a), lg = 0;
                a ^= lb;
                while(lb!=1) {
                    lb = (unsigned int)(lb) >> 1;
                    lg ++;
                }
                int u = i*32 + lg;
                if(k + dp[u] <= ans) return 0;
                if(dfs(u, k+1)) {
                    sol.push_back(v);
                    return 1;
                }
            }
        }
        return 0;
    }

    int solve() {
        for(int i=V-1; i>=0; i--) {
            dfs(i, 1);
            dp[i] = ans;
        }
        return ans;
    }
};
```

3.6 MinimumMeanCycle

```cpp
/* minimum mean cycle */
const int MAXE = 1805;
const int MAXN = 35;
const double inf = 1029384756;
const double eps = 1e-6;
struct Edge {
  int v,u;
  double c;
};
int n,m,prv[MAXN][MAXN], prve[MAXN][MAXN], vst[MAXN];
Edge e[MAXE];
vector<int> edgeID, cycle, rho;
double d[MAXN][MAXN];
inline void bellman_ford() {
  for(int i=0; i<n; i++) d[0][i]=0;
  for(int i=0; i<n; i++) {
    fill(d[i+1], d[i+1]+n, inf);
    for(int j=0; j<m; j++) {
      int v = e[j].v, u = e[j].u;
      if(d[i][v]<inf && d[i+1][u]>d[i][v]+e[j].c) {
        d[i+1][u] = d[i][v]+e[j].c;
        prv[i+1][u] = v;
        prve[i+1][u] = j;
      }
    }
  }
}
```

```cpp
double karp_mmc() {
    // returns inf if no cycle, mmc otherwise
    double mmc=inf;
    int st = -1;
    bellman_ford();
    for(int i=0; i<n; i++) {
        double avg=-inf;
        for(int k=0; k<n; k++) {
            if(d[n][i]<inf-eps) avg=max(avg,(d[n][i]-d[k][i])
                /(n-k));
            else avg=max(avg,inf);
        }
        if (avg < mmc) tie(mmc, st) = tie(avg, i);
    }
    FZ(vst); edgeID.clear(); cycle.clear(); rho.clear();
    for (int i=n; !vst[st]; st=prv[i--][st]) {
        vst[st]++;
        edgeID.PB(prve[i][st]);
        rho.PB(st);
    }
    while (vst[st] != 2) {
        int v = rho.back(); rho.pop_back();
        cycle.PB(v);
        vst[v]++;
    }
    reverse(ALL(edgeID));
    edgeID.resize(SZ(cycle));
    return mmc;
}
```

4 Flow

4.1 ISAP

```cpp
struct Isap{
  static const int MXN = 10000;
  struct Edge{ int v,f,re; };
  int n,s,t,h[MXN],gap[MXN];
  vector<Edge> E[MXN];
  void init(int _n, int _s, int _t){
    n = _n; s = _s; t = _t;
    for (int i=0; i<n; i++) E[i].clear();
  }
  void add_edge(int u, int v, int f){
    E[u].PB({v,f,SZ(E[v])});
    E[v].PB({u,0,SZ(E[u])-1});
  }
  int DFS(int u, int nf, int res=0){
    if (u == t) return nf;
    for (auto &it : E[u]){
      if (h[u]==h[it.v]+1 && it.f>0){
        int tf = DFS(it.v,min(nf,it.f));
        res += tf; nf -= tf; it.f -= tf;
        E[it.v][it.re].f += tf;
        if (nf == 0) return res;
      }
    }
    if (nf){
      if (--gap[h[u]] == 0) h[s]=n;
      gap[++h[u]]++;
    }
    return res;
  }
  int flow(int res=0){
    FZ(h); FZ(gap);
    gap[0] = n;
    while (h[s] < n) res += DFS(s,2147483647);
    return res;
  }
}flow;
```

4.2 Dinic

```cpp
struct Dinic{
  static const int MXN = 10000;
  struct Edge{ int v,f,re; };
```

```
int n,s,t,level[MXN];
vector<Edge> E[MXN];
void init(int _n, int _s, int _t){
  n = _n; s = _s; t = _t;
  for (int i=0; i<n; i++) E[i].clear();
}
void add_edge(int u, int v, int f){
  E[u].PB({v,f,SZ(E[v])});
  E[v].PB({u,0,SZ(E[u])-1});
}
bool BFS(){
  FMO(level);
  queue<int> que;
  que.push(s);
  level[s] = 0;
  while (!que.empty()){
    int u = que.front(); que.pop();
    for (auto it : E[u]){
      if (it.f > 0 && level[it.v] == -1){
        level[it.v] = level[u]+1;
        que.push(it.v);
      }
    }
  }
  return level[t] != -1;
}
int DFS(int u, int nf){
  if (u == t) return nf;
  int res = 0;
  for (auto &it : E[u]){
    if (it.f > 0 && level[it.v] == level[u]+1){
      int tf = DFS(it.v, min(nf,it.f));
      res += tf; nf -= tf; it.f -= tf;
      E[it.v][it.re].f += tf;
      if (nf == 0) return res;
    }
  }
  if (!res) level[u] = -1;
  return res;
}
int flow(int res=0){
  while ( BFS() )
    res += DFS(s,2147483647);
  return res;
}
}flow;
```

4.3   Cost Flow

```
typedef pair<long long, long long> pll;
struct CostFlow {
  static const int MXN = 205;
  static const long long INF = 102938475610293847LL;
  struct Edge {
    int v, r;
    long long f, c;
  };
  int n, s, t, prv[MXN], prvL[MXN], inq[MXN];
  long long dis[MXN], fl, cost;
  vector<Edge> E[MXN];
  void init(int _n, int _s, int _t) {
    n = _n; s = _s; t = _t;
    for (int i=0; i<n; i++) E[i].clear();
    fl = cost = 0;
  }
  void add_edge(int u, int v, long long f, long long c)
      {
    E[u].PB({v, SZ(E[v])   , f,  c});
    E[v].PB({u, SZ(E[u])-1, 0, -c});
  }
  pll flow() {
    while (true) {
      for (int i=0; i<n; i++) {
        dis[i] = INF;
        inq[i] = 0;
      }
      dis[s] = 0;
      queue<int> que;
      que.push(s);
      while (!que.empty()) {
```

```
        int u = que.front(); que.pop();
        inq[u] = 0;
        for (int i=0; i<SZ(E[u]); i++) {
          int v = E[u][i].v;
          long long w = E[u][i].c;
          if (E[u][i].f > 0 && dis[v] > dis[u] + w) {
            prv[v] = u; prvL[v] = i;
            dis[v] = dis[u] + w;
            if (!inq[v]) {
              inq[v] = 1;
              que.push(v);
            }
          }
        }
      }
      if (dis[t] == INF) break;
      long long tf = INF;
      for (int v=t, u, l; v!=s; v=u) {
        u=prv[v]; l=prvL[v];
        tf = min(tf, E[u][l].f);
      }
      for (int v=t, u, l; v!=s; v=u) {
        u=prv[v]; l=prvL[v];
        E[u][l].f -= tf;
        E[v][E[u][l].r].f += tf;
      }
      cost += tf * dis[t];
      fl += tf;
    }
    return {fl, cost};
  }
}flow;
```

4.4   Bipartite Matching (Augmenting Path)

```
bool DFS(int u){
  for (auto v : E[u]){
    if (!vst[v]){
      vst[v]=1;
      if (match[v] == -1 || DFS(match[v])){
        match[v] = u; match[u] = v;
        return true;
      }
    }
  }
  return false;
}
int DoMatch(int res=0){
  memset(match,-1,sizeof(match));
  for (int i=1; i<=N; i++){
    if (match[i] == -1){
      memset(vst,0,sizeof(vst));
      DFS(i);
    }
  }
  for (int i=1; i<=N; i++)
    if (match[i] != -1) res++;
  return res;
}
```

4.5   Kuhn Munkres

```
struct KM{
// Maximum Bipartite Weighted Matching (Perfect Match)
  static const int MXN = 650;
  static const int INF = 2147483647; // long long
  int n,match[MXN],vx[MXN],vy[MXN];
  int edge[MXN][MXN],lx[MXN],ly[MXN],slack[MXN];
  // ^^^^ long long
  void init(int _n){
    n = _n;
    for (int i=0; i<n; i++)
      for (int j=0; j<n; j++)
        edge[i][j] = 0;
  }
  void add_edge(int x, int y, int w){ // long long
    edge[x][y] = w;
  }
```

```
  bool DFS(int x){
    vx[x] = 1;
    for (int y=0; y<n; y++){
      if (vy[y]) continue;
      if (lx[x]+ly[y] > edge[x][y]){
        slack[y] = min(slack[y], lx[x]+ly[y]-edge[x][y
            ]);
      } else {
        vy[y] = 1;
        if (match[y] == -1 || DFS(match[y])){
          match[y] = x;
          return true;
        }
      }
    }
    return false;
  }
  int solve(){
    fill(match,match+n,-1);
    fill(lx,lx+n,-INF);
    fill(ly,ly+n,0);
    for (int i=0; i<n; i++)
      for (int j=0; j<n; j++)
        lx[i] = max(lx[i], edge[i][j]);
    for (int i=0; i<n; i++){
      fill(slack,slack+n,INF);
      while (true){
        fill(vx,vx+n,0);
        fill(vy,vy+n,0);
        if ( DFS(i) ) break;
        int d = INF; // long long
        for (int j=0; j<n; j++)
          if (!vy[j]) d = min(d, slack[j]);
        for (int j=0; j<n; j++){
          if (vx[j]) lx[j] -= d;
          if (vy[j]) ly[j] += d;
          else slack[j] -= d;
        }
      }
    }
    int res=0;
    for (int i=0; i<n; i++)
      res += edge[match[i]][i];
    return res;
  }
}graph;
```

4.6 SW-Mincut

```
struct SW{ // O(V^3)
  static const int MXN = 514;
  int n,vst[MXN],del[MXN];
  int edge[MXN][MXN],wei[MXN];
  void init(int _n){
    n = _n;
    FZ(edge);
    FZ(del);
  }
  void add_edge(int u, int v, int w){
    edge[u][v] += w;
    edge[v][u] += w;
  }
  void search(int &s, int &t){
    FZ(vst); FZ(wei);
    s = t = -1;
    while (true){
      int mx=-1, cur=0;
      for (int i=0; i<n; i++)
        if (!del[i] && !vst[i] && mx<wei[i])
          cur = i, mx = wei[i];
      if (mx == -1) break;
      vst[cur] = 1;
      s = t;
      t = cur;
      for (int i=0; i<n; i++)
        if (!vst[i] && !del[i]) wei[i] += edge[cur][i];
    }
  }
  int solve(){
    int res = 2147483647;
```

```
    for (int i=0,x,y; i<n-1; i++){
      search(x,y);
      res = min(res,wei[y]);
      del[y] = 1;
      for (int j=0; j<n; j++)
        edge[x][j] = (edge[j][x] += edge[y][j]);
    }
    return res;
  }
}graph;
```

4.7 Maximum Simple Graph Matching

```
struct GenMatch { // 1-base
  static const int MAXN = 250;
  int V;
  bool el[MAXN][MAXN];
  int pr[MAXN];
  bool inq[MAXN],inp[MAXN],inb[MAXN];
  queue<int> qe;
  int st,ed;
  int nb;
  int bk[MAXN],djs[MAXN];
  int ans;
  void init(int _V) {
    V = _V;
    FZ(el); FZ(pr);
    FZ(inq); FZ(inp); FZ(inb);
    FZ(bk); FZ(djs);
    ans = 0;
  }
  void add_edge(int u, int v) {
    el[u][v] = el[v][u] = 1;
  }
  int lca(int u,int v) {
    memset(inp,0,sizeof(inp));
    while (1) {
      u = djs[u];
      inp[u] = true;
      if(u == st) break;
      u = bk[pr[u]];
    }
    while (1) {
      v = djs[v];
      if(inp[v]) return v;
      v = bk[pr[v]];
    }
    return v;
  }
  void upd(int u) {
    int v;
    while(djs[u] != nb) {
      v = pr[u];
      inb[djs[u]] = inb[djs[v]] = true;
      u = bk[v];
      if(djs[u] != nb) bk[u] = v;
    }
  }
  void blo(int u,int v) {
    nb = lca(u,v);
    memset(inb,0,sizeof(inb));
    upd(u); upd(v);
    if(djs[u] != nb) bk[u] = v;
    if(djs[v] != nb) bk[v] = u;
    for(int tu = 1; tu <= V; tu++)
      if(inb[djs[tu]]) {
        djs[tu] = nb;
        if(!inq[tu]){
          qe.push(tu);
          inq[tu] = 1;
        }
      }
  }
  void flow() {
    memset(inq,false,sizeof(inq));
    memset(bk,0,sizeof(bk));
    for(int i = 1; i <= V;i++)
      djs[i] = i;

    while(qe.size()) qe.pop();
```

```
      qe.push(st);
      inq[st] = 1;
      ed = 0;
      while(qe.size()) {
        int u = qe.front(); qe.pop();
        for(int v = 1; v <= V; v++)
          if(el[u][v] && (djs[u] != djs[v]) && (pr[u] !=
              v)) {
            if((v == st) || ((pr[v] > 0) && bk[pr[v]] >
                0))
              blo(u,v);
            else if(bk[v] == 0) {
              bk[v] = u;
              if(pr[v] > 0) {
                if(!inq[pr[v]]) qe.push(pr[v]);
              } else {
                ed = v;
                return;
              }
            }
          }
      }
    }
    void aug() {
      int u,v,w;
      u = ed;
      while(u > 0) {
        v = bk[u];
        w = pr[v];
        pr[v] = u;
        pr[u] = v;
        u = w;
      }
    }
    int solve() {
      memset(pr,0,sizeof(pr));
      for(int u = 1; u <= V; u++)
        if(pr[u] == 0) {
          st = u;
          flow();
          if(ed > 0) {
            aug();
            ans ++;
          }
        }
      return ans;
    }
};

int main() {
  gp.init(V);
  for(int i=0; i<E; i++) {
    int u, v;
    cin >> u >> v;
    gp.edge(u, v);
  }
  cout << gp.solve() << endl;
}
```

4.8 Minimum Weight Matching (Clique version)

```
struct Graph {
  // Minimum General Weighted Matching (Perfect Match)
  static const int MXN = 105;

  int n, edge[MXN][MXN];
  int match[MXN],dis[MXN],onstk[MXN];
  vector<int> stk;

  void init(int _n) {
    n = _n;
    FZ(edge);
  }
  void add_edge(int u, int v, int w) {
    edge[u][v] = edge[v][u] = w;
  }
  bool SPFA(int u){
    if (onstk[u]) return true;
    stk.PB(u);
    onstk[u] = 1;
```

```
    for (int v=0; v<n; v++){
      if (u != v && match[u] != v && !onstk[v]){
        int m = match[v];
        if (dis[m] > dis[u] - edge[v][m] + edge[u][v]){
          dis[m] = dis[u] - edge[v][m] + edge[u][v];
          onstk[v] = 1;
          stk.PB(v);
          if (SPFA(m)) return true;
          stk.pop_back();
          onstk[v] = 0;
        }
      }
    }
    onstk[u] = 0;
    stk.pop_back();
    return false;
  }

  int solve() {
    // find a match
    for (int i=0; i<n; i+=2){
      match[i] = i+1;
      match[i+1] = i;
    }
    while (true){
      int found = 0;
      FZ(dis); FZ(onstk);
      for (int i=0; i<n; i++){
        stk.clear();
        if (!onstk[i] && SPFA(i)){
          found = 1;
          while (SZ(stk)>=2){
            int u = stk.back(); stk.pop_back();
            int v = stk.back(); stk.pop_back();
            match[u] = v;
            match[v] = u;
          }
        }
      }
      if (!found) break;
    }
    int ret = 0;
    for (int i=0; i<n; i++)
      ret += edge[i][match[i]];
    ret /= 2;
    return ret;
  }
}graph;
```

4.9 2-Commodity Flow

```
const int MAXN = 64;
const int INF = 1029384756;

int N;
int s1, s2, t1, t2, d1, d2, S, T;
int edge[MAXN][MAXN];
int cap[MAXN][MAXN];

int h[MAXN], gap[MAXN];
bool vis[MAXN];

int isap(int v, int f)
{
    if(v == T)return f;

    if(vis[v])return 0;
    vis[v] = true;

    for(int i=0; i<N+2; i++)
    {
        if(cap[v][i] <= 0)continue;
        if(h[i] != h[v] - 1)continue;
        int res = isap(i, min(cap[v][i], f));
        if(res > 0)
        {
            cap[v][i] -= res;
            cap[i][v] += res;
            return res;
        }
    }
```

```
        }

        gap[h[v]]--;
        if(gap[h[v]] <= 0)h[S] = N + 4;
        h[v]++;
        gap[h[v]]++;

        return 0;
}

int get_flow()
{
        for(int i=0; i<MAXN; i++)
        {
                h[i] = gap[i] = 0;
        }
        gap[0] = N + 2;

        int flow = 0;

        while(h[S] <= N + 3)
        {
                for(int i=0; i<N+2; i++)
                {
                        vis[i] = false;
                }

                int df = isap(S, INF);
                flow += df;
        }

        return flow;
}

int main()
{
        ios_base::sync_with_stdio(0);

        int TT;
        cin>>TT;
        while(TT--)
        {
                cin>>N;
                cin>>s1>>t1>>d1>>s2>>t2>>d2;

                for(int i=0; i<MAXN; i++)
                {
                        for(int j=0; j<MAXN; j++)
                        {
                                edge[i][j] = 0;
                        }
                }

                for(int i=0; i<N; i++)
                {
                        string s;
                        cin>>s;
                        for(int j=0; j<N; j++)
                        {
                                if(s[j] == 'X')edge[i][j] = 0;
                                else if(s[j] == 'O')edge[i][j] = 1;
                                else if(s[j] == 'N')edge[i][j] = INF;
                        }
                }

                int ans = 0;

                S = N;
                T = N + 1;

                //first
                for(int i=0; i<MAXN; i++)
                {
                        for(int j=0; j<MAXN; j++)
                        {
                                cap[i][j] = edge[i][j];
                        }
                }

                cap[S][s1] = cap[t1][T] = d1;
                cap[S][s2] = cap[t2][T] = d2;
```

```
                ans = get_flow();

                //second
                for(int i=0; i<MAXN; i++)
                {
                        for(int j=0; j<MAXN; j++)
                        {
                                cap[i][j] = edge[i][j];
                        }
                }

                cap[S][s1] = cap[t1][T] = d1;
                cap[S][t2] = cap[s2][T] = d2;

                ans = min(ans, get_flow());

                cout<<(ans == d1 + d2 ? "Yes" : "No")<<endl;
        }

        return 0;
}
```

4.10 (+1) SW-mincut $O(NM)$

```
// {{{ StoerWagner
const int inf=1000000000;
// should be larger than max.possible mincut
class StoerWagner {
  public:
    int n,mc; // node id in [0,n-1]
    vector<int> adj[MAXN];
    int cost[MAXN][MAXN];
    int cs[MAXN];
    bool merged[MAXN],sel[MAXN];
    // --8<-- include only if cut is explicitly needed
    DisjointSet djs;
    vector<int> cut;
    //--8<------------------------------------------
    StoerWagner(int _n):n(_n),mc(inf),djs(_n) {
      for(int i=0;i<n;i++)
        merged[i]=0;
      for(int i=0;i<n;i++)
        for(int j=0;j<n;j++)
          cost[i][j]=cost[j][i]=0;
    }
    void append(int v,int u,int c) {
      if(v==u) return;
      if(!cost[v][u]&&c) {
        adj[v].PB(u);
        adj[u].PB(v);
      }
      cost[v][u]+=c;
      cost[u][v]+=c;
    }
    void merge(int v,int u) {
      merged[u]=1;
      for(int i=0;i<n;i++)
        append(v,i,cost[u][i]);
      // --8<-- include only if cut is explicitly
      //      needed
      djs.merge(v,u);
      //
          --8<------------------------------------------

    }
    void phase() {
      priority_queue<pii> pq;
      for(int v=0;v<n;v++) {
        if(merged[v]) continue;
        cs[v]=0;
        sel[v]=0;
        pq.push({0,v});
      }
      int v,s,pv;
      while(pq.size()) {
        if(cs[pq.top().S]>pq.top().F) {
          pq.pop();
          continue;
        }
        pv=v;
```

```
        v=pq.top().S;
        s=pq.top().F;
        pq.pop();
        sel[v]=1;
        for(int i=0;i<adj[v].size();i++) {
          int u=adj[v][i];
          if(merged[u]||sel[u]) continue;
          cs[u]+=cost[v][u];
          pq.push({cs[u],u});
        }
      }
      if(s<mc) {
        mc=s;
        // --8<-- include only if cut is explicitly
        needed ------
          cut.clear();
        for(int i=0;i<n;i++)
          if(djs.getrep(i)==djs.getrep(v)) cut.PB(i);
        //--8<------------------------------------
      }
      merge(v,pv);
    }
    int mincut() {
      if(mc==inf) {
        for(int t=0;t<n-1;t++)
          phase();

      return mc;
    }
    // --8<-- include only if cut is explicitly needed
    ------
      vector<int> getcut() { // return one side of the
        cut
        mincut();
        return cut;
      }
    //--8<------------------------------------------
};
// }}}
```

5 Math

5.1 ax+by=gcd

```
typedef pair<int, int> pii;

pii gcd(int a, int b){
  if(b == 0) return make_pair(1, 0);
  else{
    int p = a / b;
    pii q = gcd(b, a % b);
    return make_pair(q.second, q.first - q.second * p);
  }
}
```

5.2 Chinese Remainder

```
int pfn; // number of distinct prime factors
int pf[MAXNUM]; // prime factor powers
int rem[MAXNUM]; // corresponding remainder
int pm[MAXNUM];
inline void generate_primes() {
  int i,j;
  pnum=1;
  prime[0]=2;
  for(i=3;i<MAXVAL;i+=2) {
    if(nprime[i]) continue;
    prime[pnum++]=i;
    for(j=i*i;j<MAXVAL;j+=i) nprime[j]=1;
  }
}
inline int inverse(int x,int p) {
  int q,tmp,a=x,b=p;
  int a0=1,a1=0,b0=0,b1=1;
  while(b) {
    q=a/b; tmp=b; b=a-b*q; a=tmp;
    tmp=b0; b0=a0-b0*q; a0=tmp;
    tmp=b1; b1=a1-b1*q; a1=tmp;
  }
  return a0;
}
inline void decompose_mod() {
  int i,p,t=mod;
  pfn=0;
  for(i=0;i<pnum&&prime[i]<=t;i++) {
    p=prime[i];
    if(t%p==0) {
      pf[pfn]=1;
      while(t%p==0) {
        t/=p;
        pf[pfn]*=p;
      }
      pfn++;
    }
  }
  if(t>1) pf[pfn++]=t;
}
inline int chinese_remainder() {
  int i,m,s=0;
  for(i=0;i<pfn;i++) {
    m=mod/pf[i];
    pm[i]=(long long)m*inverse(m,pf[i])%mod;
    s=(s+(long long)pm[i]*rem[i])%mod;
  }
  return s;
}
```

5.3 Fast Fourier Transform

```cpp
// const int MAXN = 262144;
// (must be 2^k)

typedef long double ld;
typedef complex<ld> cplx;
const ld PI = acosl(-1);
const cplx I(0, 1);

cplx omega[MAXN+1];
void pre_fft()
{
  for(int i=0; i<=MAXN; i++)
    omega[i] = exp(i * 2 * PI / MAXN * I);
}
void fft(int n, cplx a[], bool inv=false)
{
  int basic = MAXN / n;
  int theta = basic;
  for (int m = n; m >= 2; m >>= 1) {
    int mh = m >> 1;
    for (int i = 0; i < mh; i++) {
      cplx w = omega[inv ? MAXN-(i*theta%MAXN) : i*
          theta%MAXN];
      for (int j = i; j < n; j += m) {
        int k = j + mh;
        cplx x = a[j] - a[k];
        a[j] += a[k];
        a[k] = w * x;
      }
    }
    theta = (theta * 2) % MAXN;
  }
  int i = 0;
  for (int j = 1; j < n - 1; j++) {
    for (int k = n >> 1; k > (i ^= k); k >>= 1);
    if (j < i) swap(a[i], a[j]);
  }
  if (inv)
    for (i = 0; i < n; i++)
      a[i] /= n;
}
```

5.4 Fast Linear Recurrence

```cpp
typedef long long ll;

const int MAXN = 514;
const int LG = 61;
const ll MOD = 1000000007;

ll N;
int M;
ll f[LG][MAXN];
ll tmp[MAXN*2];
ll g[MAXN];

void calc_f(ll f0[MAXN], ll f1[MAXN], ll f2[MAXN]) {
  FZ(tmp);
  for(int i=0; i<M; i++)
    for(int j=0; j<M; j++)
      tmp[i+j] = (tmp[i+j] + f0[i] * f1[j]) % MOD;
  for(int i=2*M-2; i>=M; i--) {
    ll mrat = (MOD - tmp[i]) % MOD;
    tmp[i] = 0;
    for(int j=0; j<M; j++)
      tmp[i+j-M+1] = (tmp[i+j-M+1] + mrat * f[0][j]) %
          MOD;
    tmp[i-M] = (tmp[i-M] + mrat * (MOD-1)) % MOD;
  }
  for(int i=0; i<M; i++)
    f2[i] = tmp[i];
}
void make() {
  for(int i=1; i<LG; i++)
    calc_f(f[i-1], f[i-1], f[i]);

  for(int i=0; i<10; i++) {
```

```cpp
    for(int j=0; j<M; j++)
      cout<<f[i][j]<<" ";
    cout<<endl;
  }
}
ll calc(ll x) {
  bool any = false;
  for(int i=0; i<LG; i++) {
    if(x&(1LL<<i)) {
      if(!any) {
        any = true;
        for(int j=0; j<M; j++)
          g[j] = f[i][j];
      } else {
        calc_f(g, f[i], g);
      }
    }
  }

  return g[0];
}
int main() {
  cin>>N>>M;
  for(int i=0; i<M; i++)
    cin>>f[0][i];

  // f[n] = \sum_{0 <= i < M} f[n-i-1]

  make();
  ll ans = calc(N);

  cout<<ans<<endl;

  return 0;
}
```

5.5 (+1) ntt

```cpp
int P=605028353,root=3,MAXNUM=262144;
// Remember coefficient are mod P
/*
p=a*2^n+1
```

```
n    2^n         p           a      root
5    32          97          3      5
6    64          193         3      5
7    128         257         2      3
8    256         257         1      3
9    512         7681        15     17
10   1024        12289       12     11
11   2048        12289       6      11
12   4096        12289       3      11
13   8192        40961       5      3
14   16384       65537       4      3
15   32768       65537       2      3
16   65536       65537       1      3
17   131072      786433      6      10
18   262144      786433      3      10 (605028353,
     2308, 3)
19   524288      5767169     11     3
20   1048576     7340033     7      3
21   2097152     23068673    11     3
22   4194304     104857601   25     3
23   8388608     167772161   20     3
24   16777216    167772161   10     3
25   33554432    167772161   5      3 (1107296257, 33,
     10)
26   67108864    469762049   7      3
27   134217728   2013265921  15     31
*/
```

```cpp
int bigmod(long long a,int b){
  if(b==0)return 1;
  return (bigmod((a*a)%P,b/2)*(b%2?a:1ll))%P;
}
int inv(int a,int b){
  if(a==1)return 1;
  return (((long long)(a-inv(b%a,a))*b+1)/a)%b;
}
std::vector<long long> ps(MAXNUM);
std::vector<int> rev(MAXNUM);
struct poly{
```

```cpp
  std::vector<unsigned int> co;
  int n;//polynomial degree = n
  poly(int d){n=d;co.resize(n+1,0);}
  void trans2(int NN){
    int r=0,st,N;
    unsigned int a,b;
    while((1<<r)<(NN>>1))++r;
    for(N=2;N<=NN;N<<=1,--r){
      for(st=0;st<NN;st+=N){
        int i,ss=st+(N>>1);
        for(i=(N>>1)-1;i>=0;--i){
          a=co[st+i]; b=(ps[i<<r]*co[ss+i])%P;
          co[st+i]=a+b; if(co[st+i]>=P)co[st+i]-=P;
          co[ss+i]=a+P-b; if(co[ss+i]>=P)co[ss+i]-=P;
        }
      }
    }
  }
  void trans1(int NN){
    int r=0,st,N;
    unsigned int a,b;
    for(N=NN;N>1;N>>=1,++r){
      for(st=0;st<NN;st+=N){
        int i,ss=st+(N>>1);
        for(i=(N>>1)-1;i>=0;--i){
          a=co[st+i]; b=co[ss+i];
          co[st+i]=a+b; if(co[st+i]>=P)co[st+i]-=P;
          co[ss+i]=((a+P-b)*ps[i<<r])%P;
        }
      }
    }
  }
  poly operator*(const poly& _b)const{
    poly a=*this,b=_b;
    int k=n+b.n,i,N=1;
    while(N<=k)N*=2;
    a.co.resize(N,0); b.co.resize(N,0);
    int r=bigmod(root,(P-1)/N),Ni=inv(N,P);
    ps[0]=1;
    for(i=1;i<N;++i)ps[i]=(ps[i-1]*r)%P;
    a.trans1(N);b.trans1(N);
    for(i=0;i<N;++i)a.co[i]=((long long)a.co[i]*b.co[i
      ])%P
      ;
    r=inv(r,P);
    for(i=1;i<N/2;++i)std::swap(ps[i],ps[N-i]);
    a.trans2(N);
    for(i=0;i<N;++i)a.co[i]=((long long)a.co[i]*Ni)%P;
    a.n=n+_b.n; return a;
  }
};
```

5.6 Mod

```cpp
/// _fd(a,b)  floor(a/b).
/// _rd(a,m)  a-floor(a/m)*m.
/// _pv(a,m,r) largest x s.t x<=a && x%m == r.
/// _nx(a,m,r) smallest x s.t x>=a && x%m == r.
/// _ct(a,b,m,r) |A| , A = { x : a<=x<=b && x%m == r }.

int _fd(int a,int b){ return a<0?(-~a/b-1):a/b; }
int _rd(int a,int m){ return a-_fd(a,m)*m; }
int _pv(int a,int m,int r)
{
    r=(r%m+m)%m;
    return _fd(a-r,m)*m+r;
}
int _nt(int a,int m,int r)
{
    m=abs(m);
    r=(r%m+m)%m;
    return _fd(a-r-1,m)*m+r+m;
}
int _ct(int a,int b,int m,int r)
{
    m=abs(m);
    a=_nt(a,m,r);
    b=_pv(b,m,r);
    return (a>b)?0:((b-a+m)/m);
}
```

5.7 (+1) Miller Rabin

```cpp
// n < 4,759,123,141       3 :  2, 7, 61
// n < 1,122,004,669,633   4 :  2, 13, 23, 1662803
// n < 3,474,749,660,383       6 :  pirmes <= 13
// n < 2^64                    7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
// Make sure testing integer is in range [2, n-2] if
// you want to use magic.
long long power(long long x,long long p,long long mod){
  long long s=1,m=x;
  while(p) {
    if(p&1) s=mult(s,m,mod);
    p>>=1;
    m=mult(m,m,mod);
  }
  return s;
}
bool witness(long long a,long long n,long long u,int t)
  {
  long long x=power(a,u,n);
  for(int i=0;i<t;i++) {
    long long nx=mult(x,x,n);
    if(nx==1&&x!=1&&x!=n-1) return 1;
    x=nx;
  }
  return x!=1;
}
bool miller_rabin(long long n,int s=100) {
  // iterate s times of witness on n
  // return 1 if prime, 0 otherwise
  if(n<2) return 0;
  if(!(n&1)) return n==2;
  long long u=n-1;
  int t=0;
  // n-1 = u*2^t
  while(!(u&1)) {
    u>>=1;
    t++;
  }
  while(s--) {
    long long a=randll()%(n-1)+1;
    if(witness(a,n,u,t)) return 0;
  }
  return 1;
}
```

5.8 Pollard Rho

```cpp
// does not work when n is prime
long long modit(long long x,long long mod) {
  if(x>=mod) x-=mod;
  //if(x<0) x+=mod;
  return x;
}
long long mult(long long x,long long y,long long mod) {
  long long s=0,m=x%mod;
  while(y) {
    if(y&1) s=modit(s+m,mod);
    y>>=1;
    m=modit(m+m,mod);
  }
  return s;
}
long long f(long long x,long long mod) {
  return modit(mult(x,x,mod)+1,mod);
}
long long pollard_rho(long long n) {
  if(!(n&1)) return 2;
  while (true) {
    long long y=2, x=rand()%(n-1)+1, res=1;
    for (int sz=2; res==1; sz*=2) {
      for (int i=0; i<sz && res<=1; i++) {
        x = f(x, n);
        res = __gcd(abs(x-y), n);
      }
      y = x;
    }
    if (res!=0 && res!=n) return res;
```

```
    }
}
```

5.9 Algorithms about Primes

```
/*
 * 12721
 * 13331
 * 14341
 * 75577
 * 123457
 * 222557
 * 556679
 * 999983
 * 1097774749
 * 1076767633
 * 100102021
 * 999997771
 * 1001010013
 * 1000512343
 * 987654361
 * 999991231
 * 999888733
 * 98789101
 * 987777733
 * 999991921
 * 1010101333
 * 1010102101
 * 1000000000039
 * 1000000000000037
 * 2305843009213693951
 * 4611686018427387847
 * 9223372036854775783
 * 18446744073709551557
 */

int mu[MX],p_tbl[MX];
vector<int> primes;
void sieve() {
  mu[1] = p_tbl[1] = 1;
  for (int i=2; i<MX; i++) {
    if (!p_tbl[i]) {
      p_tbl[i] = i;
      primes.PB(i);
      mu[i] = -1;
    }
    for (auto p : primes) {
      int x = i*p;
      if (x >= M) break;
      p_tbl[x] = p;
      mu[x] = -mu[i];
      if (i%p==0) {
        mu[x] = 0;
        break;
      }
    }
  }
}

vector<int> factor(int x) {
  vector<int> fac{1};
  while (x > 1) {
    int fn=SZ(fac), p=p_tbl[x], pos=0;
    while (x%p == 0) {
      x /= p;
      for (int i=0; i<fn; i++)
        fac.PB(fac[pos++]*p);
    }
  }
  return fac;
}
```

5.10 (+1) PolynomialGenerator

```
class PolynomialGenerator {
  /* for a nth-order polynomial f(x), *
   * given f(0), f(1), ..., f(n) *
   * express f(x) as sigma_i{c_i*C(x,i)} */
```

```
public:
  int n;
  vector<long long> coef;
  // initialize and calculate f(x), vector _fx should
      be
  // filled with f(0) to f(n)
    PolynomialGenerator(int _n,vector<long long> _fx)
        :n(_n
        ),coef(_fx) {
      for(int i=0;i<n;i++)
        for(int j=n;j>i;j--)
          coef[j]-=coef[j-1];
    }
  // evaluate f(x), runs in O(n)
  long long eval(int x) {
    long long m=1,ret=0;
    for(int i=0;i<=n;i++) {
      ret+=coef[i]*m;
      m=m*(x-i)/(i+1);
    }
    return ret;
  }
};
```

5.11 Gauss Elimination

```
const int MAX = 300;
const double EPS = 1e-8;

double mat[MAX][MAX];
void Gauss(int n) {
  for(int i=0; i<n; i++) {
    bool ok = 0;
    for(int j=i; j<n; j++) {
      if(fabs(mat[j][i]) > EPS) {
        swap(mat[j], mat[i]);
        ok = 1;
        break;
      }
    }
    if(!ok) continue;

    double fs = mat[i][i];
    for(int j=i+1; j<n; j++) {
      double r = mat[j][i] / fs;
      for(int k=i; k<n; k++) {
        mat[j][k] -= mat[i][k] * r;
      }
    }
  }
}
```

5.12 Simplex

```cpp
const int maxn = 111;
const int maxm = 111;
const double eps = 1E-10;

double a[maxn][maxm], b[maxn], c[maxm], d[maxn][maxm];
double x[maxm];
int ix[maxn + maxm]; // !!! array all indexed from 0
// max{cx} subject to {Ax<=b,x>=0}
// n: constraints, m: vars !!!
// x[] is the optimal solution vector
//
// usage :
// value = simplex(a, b, c, N, M);
double simplex(double a[maxn][maxm], double b[maxn],
    double c[maxm], int n, int m) {
    ++m;
    int r = n, s = m - 1;
    memset(d, 0, sizeof(d));
    for (int i = 0; i < n + m; ++i) ix[i] = i;
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < m - 1; ++j) d[i][j] = -a[i
            ][j];
        d[i][m - 1] = 1;
        d[i][m] = b[i];
        if (d[r][m] > d[i][m]) r = i;
    }
    for (int j = 0; j < m - 1; ++j) d[n][j] = c[j];
    d[n + 1][m - 1] = -1;
    for (double dd;; ) {
        if (r < n) {
            int t = ix[s]; ix[s] = ix[r + m]; ix[r + m]
                = t;
            d[r][s] = 1.0 / d[r][s];
            for (int j = 0; j <= m; ++j) if (j != s) d[
                r][j] *= -d[r][s];
            for (int i = 0; i <= n + 1; ++i) if (i != r
                ) {
                for (int j = 0; j <= m; ++j) if (j != s
                    ) d[i][j] += d[r][j] * d[i][s];
                d[i][s] *= d[r][s];
            }
        }
        r = -1; s = -1;
        for (int j = 0; j < m; ++j) if (s < 0 || ix[s]
            > ix[j]) {
            if (d[n + 1][j] > eps || (d[n + 1][j] > -
                eps && d[n][j] > eps)) s = j;
        }
        if (s < 0) break;
        for (int i = 0; i < n; ++i) if (d[i][s] < -eps)
             {
            if (r < 0 || (dd = d[r][m] / d[r][s] - d[i
                ][m] / d[i][s]) < -eps || (dd < eps &&
                ix[r + m] > ix[i + m])) r = i;
        }
        if (r < 0) return -1; // not bounded
    }
    if (d[n + 1][m] < -eps) return -1; // not
        executable
    double ans = 0;
    for(int i=0; i<m; i++) x[i] = 0;
    for (int i = m; i < n + m; ++i) { // the missing
        enumerated x[i] = 0
        if (ix[i] < m - 1)
        {
            ans += d[i - m][m] * c[ix[i]];
            x[ix[i]] = d[i-m][m];
        }
    }
    return ans;
}
```

5.13 Theorem

5.13.1 Lucas' Theorem

For non-negative integer n, m and prime p, $\binom{m}{n} \equiv \prod_{i=0}^{k} \binom{m_i}{n_i} \pmod{p}$
where m_i is the i-th digit of m in base p.

5.13.2 Sum of Two Squares Thm (Legendre)

For a given positive integer n, let
$D_1 = $ (# of positive integers d dividing N that $1 \equiv d \pmod 4$)
$D_3 = $ (# of positive integers d dividing N that $3 \equiv d \pmod 4$)
then n can be written as a sum of two squares in exactly
$R(n) = 4(D_1 - D_3)$ ways.

5.13.3 Difference of D1-D3 Thm

let $n = 2^t \cdot (p_1^{e_1} \cdot \ldots \cdot p_r^{e_r}) \cdots (q_1^{f_1} \cdot \ldots \cdot q_s^{f_s})$
where p_i, q_i are primes and $1 \equiv p_i \pmod 4, 3 \equiv q_i \pmod 4$
then $D_1 - D_3 = \begin{cases} (e_1 + 1)(e_2 + 1)\ldots(e_r + 1), & \text{if } (f_i)\text{s all even} \\ 0, & \text{if any } f_i \text{ is odd} \end{cases}$

5.13.4 Krush–Kuhn–Tucker Conditions

Stationarity
For maximizing $f(x)$: $\nabla f(x^*) = \sum_{i=1}^{m} \mu_i \nabla g_i(x^*) + \sum_{j=1}^{l} \lambda_j \nabla h_j(x^*)$
For minimizing $f(x)$: $-\nabla f(x^*) = \sum_{i=1}^{m} \mu_i \nabla g_i(x^*) + \sum_{j=1}^{l} \lambda_j \nabla h_j(x^*)$

Primal feasibility
$g_i(x^*) \leq 0$, for all $i = 1, \ldots, m$
$h_j(x^*) = 0$, for all $j = 1, \ldots, l$

Dual feasibility
$\mu_i \geq 0$, for all $i = 1, \ldots, m$

Complementary slackness
$\mu_i g_i(x^*) = 0$, for all $i = 1, \ldots, m$

6 Geometry

6.1 Point operators

```cpp
#include<bits/stdc++.h>
using namespace std;

#define _x first
#define _y second
typedef pair<double, double> pdd;

pdd operator + (const pdd p1, const pdd p2){
  return pdd(p1._x + p2._x, p1._y + p2._y);
}
pdd operator - (const pdd p1, const pdd p2){
  return pdd(p1._x - p2._x, p1._y - p2._y);
}

pdd operator * (const double c, const pdd p){
  return pdd(p._x * c, p._y * c);
}
pdd operator - (const pdd p){
  return (-1.0) * p;
}
double operator * (const pdd p1, const pdd p2){
  return p1._x * p2._x + p1._y * p2._y;
}
double operator % (const pdd p1, const pdd p2){
  return p1._x * p2._y - p2._x * p1._y;
}
```

6.2 Intersection of two circles

Let $\mathbf{O_1} = (x_1, y_1), \mathbf{O_2} = (x_2, y_2)$ be two centers of circles, r_1, r_2 be the radius. If:
$d = |\mathbf{O_1} - \mathbf{O_2}|$ $\mathbf{u} = \frac{1}{2}(\mathbf{O_1} + \mathbf{O_2}) + \frac{(r_2^2 - r_1^2)}{2d^2}(\mathbf{O_1} - \mathbf{O_2})$
$\mathbf{v} = \frac{\sqrt{(r_1 + r_2 + d)(r_1 - r_2 + d)(r_1 + r_2 - d)(-r_1 + r_2 + d)}}{2d^2}(y_1 - y_2, -x_1 + x_2)$ then $\mathbf{u} + $
$\mathbf{v}, \mathbf{u} - \mathbf{v}$ are the two intersections of the circles, provided that $d < r_1 + r_2$.

```cpp
vector<pdd> interCircle(pdd o1, double r1, pdd o2,
    double r2) {
  ld d2 = (o1 - o2) * (o1 - o2);
  ld d = sqrt(d2);
  if (d > r1+r2) return {};
  pdd u = 0.5*(o1+o2) + ((r2*r2-r1*r1)/(2*d2))*(o1-o2);
  double A = sqrt((r1+r2+d) * (r1-r2+d) * (r1+r2-d) *
      (-r1+r2+d));
  pdd v = A / (2*d2) * pdd(o1.S-o2.S, -o1.F+o2.F);
  return {u+v, u-v};
}
```

6.3 Intersection of two lines

```cpp
#include<bits/stdc++.h>

using namespace std;
const double EPS = 1e-9;

pdd interPnt(pdd p1, pdd p2, pdd q1, pdd q2){
  double f1 = (p2 - p1) % (q1 - p1);
  double f2 = (p2 - p1) % (p1 - q2);
  double f = (f1 + f2);

  if(fabs(f) < EPS) return pdd(nan(""), nan(""));

  return (f2 / f) * q1 + (f1 / f) * q2;
}
```

6.4 Half Plane Intersection

```cpp
#include<bits/stdc++.h>

using namespace std;

#define PB push_back
#define SZ(x) ((int)(x).size())
#define ALL(x) begin(x), end(x)
#define F first
#define S second
#define x first
#define y second
#define IOS ios::sync_with_stdio(0);cin.tie(0)

const double EPS = 1E-9;

struct point : public pair<double, double> {

    point() {}
    point(double _x, double _y) : pair<double, double>(
        _x, _y) {
    }

    point operator + (const point he) const {
        return {x+he.x, y+he.y};
    }
    point operator - (const point he) const {
        return {x-he.x, y-he.y};
    }
    friend point operator * (const double c, const
        point &p) {
        return {c*p.x, c*p.y};
    }
    point operator * (const double c) const {
        return {c*x, c*y};
    }
    double operator * (const point he) const {
        return x*he.x + y*he.y;
    }
    double cross (const point &he) const {
        return x*he.y - y*he.x;
    }
    friend ostream& operator << (ostream& o, const
        point &p) {
        cout << "(" << p.x << ", " << p.y << ")";
        return o;
    }
};
```

```cpp
typedef pair<point, point> Line;

point interPnt(Line l1, Line l2, bool &res){
    point p1, p2, q1, q2;
    tie(p1, p2) = l1;
    tie(q1, q2) = l2;
  double f1 = (p2 - p1).cross(q1 - p1);
  double f2 = (p2 - p1).cross(p1 - q2);
  double f = (f1 + f2);

    if(fabs(f) < EPS) {
        res = false;
        return {0, 0};
    }

    res = true;
  return (f2 / f) * q1 + (f1 / f) * q2;
}

bool isin(Line l0, Line l1, Line l2) {
    // Check inter(l1, l2) in l0
    bool res;
    point p = interPnt(l1, l2, res);
    return (l0.S - l0.F).cross(p - l0.F) > 1e-9;
}

/* If no solution, check: 1. ret.size() < 3
 * Or more precisely, 2. interPnt(ret[0], ret[1])
 * in all the lines. (use (l.S - l.F).cross(p - l.F) >
     0
 */
vector<Line> halfPlaneInter(vector<Line> lines) {
    int sz = lines.size();
    vector<double> ata(sz), ord(sz);
    for (int i=0; i<sz; i++) {
        ord[i] = i;
        point d = lines[i].S - lines[i].F;
        ata[i] = atan2(d.y, d.x);
    }
    sort(ALL(ord), [&](int i, int j) {
        return ata[i] < ata[j];
    });
    vector<Line> fin;
    for (int i=0; i<sz; i++) {
        if (i and fabs(ata[ord[i]] - ata[ord[i-1]]) <
            EPS) {
            Line li = lines[ord[i]];
            Line lj = lines[ord[i-1]];
            if ((li.S - li.F).cross(lj.S - li.F) >= 0)
                continue;
            else
                fin.back() = li;
        } else {
            fin.PB(lines[ord[i]]);
        }
    }

    deque<Line> dq;
    for (int i=0; i<SZ(fin); i++) {
        while(SZ(dq) >= 2 and
            not isin(fin[i], dq[SZ(dq)-2], dq[SZ(dq)
                -1])) {
            dq.pop_back();
        }
        while(SZ(dq) >= 2 and
            not isin(fin[i], dq[0], dq[1])) {
            dq.pop_front();
        }
        dq.push_back(fin[i]);
    }

    while (SZ(dq) >= 3 and
        not isin(dq[0], dq[SZ(dq)-2], dq[SZ(dq)-1]))
        {
        dq.pop_back();
    }

    while (SZ(dq) >= 3 and
        not isin(dq[SZ(dq)-1], dq[0], dq[1])) {
        dq.pop_front();
    }
```

```
  vector<Line> res(SZ(dq));
  copy(ALL(dq), res.begin());
  return res;
}
```

6.5   Convex Hull

```
double cross(pdd o, pdd a, pdd b){
  return (a-o) % (b-o);
}
vector<pdd> convex_hull(vector<pdd> pt){
  sort(pt.begin(),pt.end());
  int top=0;
  vector<pdd> stk(2*pt.size());
  for (int i=0; i<(int)pt.size(); i++){
    while (top >= 2 && cross(stk[top-2],stk[top-1],pt[i
        ]) <= 0)
      top--;
    stk[top++] = pt[i];
  }
  for (int i=pt.size()-2, t=top+1; i>=0; i--){
    while (top >= t && cross(stk[top-2],stk[top-1],pt[i
        ]) <= 0)
      top--;
    stk[top++] = pt[i];
  }
  stk.resize(top-1);
  return stk;
}
```

6.6   Minimum Covering Circle

```
struct Mcc{
  // return pair of center and r^2
  static const int MAXN = 1000100;
  int n;
  pdd p[MAXN],cen;
  double r2;

  void init(int _n, pdd _p[]){
    n = _n;
    memcpy(p,_p,sizeof(pdd)*n);
  }
  double sqr(double a){ return a*a; }
  double abs2(pdd a){ return a*a; }
  pdd center(pdd p0, pdd p1, pdd p2) {
    pdd a = p1-p0;
    pdd b = p2-p0;
    double c1=abs2(a)*0.5;
    double c2=abs2(b)*0.5;
    double d = a % b;
    double x = p0.x + (c1 * b.y - c2 * a.y) / d;
    double y = p0.y + (a.x * c2 - b.x * c1) / d;
    return pdd(x,y);
  }

  pair<pdd,double> solve(){
    random_shuffle(p,p+n);
    r2=0;
    for (int i=0; i<n; i++){
      if (abs2(cen-p[i]) <= r2) continue;
      cen = p[i];
      r2 = 0;
      for (int j=0; j<i; j++){
        if (abs2(cen-p[j]) <= r2) continue;
        cen = 0.5 * (p[i]+p[j]);
        r2 = abs2(cen-p[j]);
        for (int k=0; k<j; k++){
          if (abs2(cen-p[k]) <= r2) continue;
          cen = center(p[i],p[j],p[k]);
          r2 = abs2(cen-p[k]);
        }
      }
    }
    return {cen,r2};
  }
}mcc;
```

6.7   KDTree (Nearest Point)

```
const int MXN = 100005;

struct KDTree {
  struct Node {
    int x,y,x1,y1,x2,y2;
    int id,f;
    Node *L, *R;
  }tree[MXN];
  int n;
  Node *root;

  long long dis2(int x1, int y1, int x2, int y2) {
    long long dx = x1-x2;
    long long dy = y1-y2;
    return dx*dx+dy*dy;
  }
  static bool cmpx(Node& a, Node& b){ return a.x<b.x; }
  static bool cmpy(Node& a, Node& b){ return a.y<b.y; }
  void init(vector<pair<int,int>> ip) {
    n = ip.size();
    for (int i=0; i<n; i++) {
      tree[i].id = i;
      tree[i].x = ip[i].first;
      tree[i].y = ip[i].second;
    }
    root = build_tree(0, n-1, 0);
  }
  Node* build_tree(int L, int R, int dep) {
    if (L>R) return nullptr;
    int M = (L+R)/2;
    tree[M].f = dep%2;
    nth_element(tree+L, tree+M, tree+R+1, tree[M].f ?
        cmpy : cmpx);
    tree[M].x1 = tree[M].x2 = tree[M].x;
    tree[M].y1 = tree[M].y2 = tree[M].y;

    tree[M].L = build_tree(L, M-1, dep+1);
    if (tree[M].L) {
      tree[M].x1 = min(tree[M].x1, tree[M].L->x1);
      tree[M].x2 = max(tree[M].x2, tree[M].L->x2);
      tree[M].y1 = min(tree[M].y1, tree[M].L->y1);
      tree[M].y2 = max(tree[M].y2, tree[M].L->y2);
    }

    tree[M].R = build_tree(M+1, R, dep+1);
    if (tree[M].R) {
      tree[M].x1 = min(tree[M].x1, tree[M].R->x1);
      tree[M].x2 = max(tree[M].x2, tree[M].R->x2);
      tree[M].y1 = min(tree[M].y1, tree[M].R->y1);
      tree[M].y2 = max(tree[M].y2, tree[M].R->y2);
    }

    return tree+M;
  }
  int touch(Node* r, int x, int y, long long d2){
    long long dis = sqrt(d2)+1;
    if (x<r->x1-dis || x>r->x2+dis || y<r->y1-dis || y>
        r->y2+dis)
      return 0;
    return 1;
  }
  void nearest(Node* r, int x, int y, int &mID, long
      long &md2) {
    if (!r || !touch(r, x, y, md2)) return;
    long long d2 = dis2(r->x, r->y, x, y);
    if (d2 < md2 || (d2 == md2 && mID < r->id)) {
      mID = r->id;
      md2 = d2;
    }
    // search order depends on split dim
    if ((r->f == 0 && x < r->x) ||
        (r->f == 1 && y < r->y)) {
      nearest(r->L, x, y, mID, md2);
      nearest(r->R, x, y, mID, md2);
    } else {
      nearest(r->R, x, y, mID, md2);
      nearest(r->L, x, y, mID, md2);
    }
  }
```

```
  int query(int x, int y) {
    int id = 1029384756;
    long long d2 = 102938475612345678LL;
    nearest(root, x, y, id, d2);
    return id;
  }
}tree;
```

6.8 (+1) MinkowskiSum

```
/* convex hull Minkowski Sum*/
#define INF 100000000000000LL
class PT{ public:
  long long x,y;
  int POS(){
    if(y==0) return x>0?0:1;
    return y>0?0:1;
  }
};
PT pt[300000],qt[300000],rt[300000];
long long Lx,Rx;
int dn,un;
inline bool cmp(PT a,PT b){
  int pa=a.POS(),pb=b.POS();
  if(pa==pb) return (a^b)>0;
  return pa<pb;
}
int minkowskiSum(int n,int m){
  int i,j,r,p,q,fi,fj;
  for(i=1,p=0;i<n;i++){
    if(pt[i].y<pt[p].y || (pt[i].y==pt[p].y && pt[i].x<
        pt[p].x)) p=i; }
  for(i=1,q=0;i<m;i++){
    if(qt[i].y<qt[q].y || (qt[i].y==qt[q].y && qt[i].x<
        qt[q].x)) q=i; }
  rt[0]=pt[p]+qt[q];
  r=1; i=p; j=q; fi=fj=0;
  while(1){
    if((fj&&j==q) || ((!fi||i!=p) && cmp(pt[(p+1)%n]-pt
        [
            p],qt[(q+1)%m]-qt[q]))){
      rt[r]=rt[r-1]+pt[(p+1)%n]-pt[p];
      p=(p+1)%n;
      fi=1;
    }else{
      rt[r]=rt[r-1]+qt[(q+1)%m]-qt[q];
      q=(q+1)%m;
      fj=1;
    }
    if(r<=1 || ((rt[r]-rt[r-1])^(rt[r-1]-rt[r-2]))!=0)
        r
    ++;
    else rt[r-1]=rt[r];
    if(i==p && j==q) break;
  }
  return r-1;
}
void initInConvex(int n){
  int i,p,q;
  long long Ly,Ry;
  Lx=INF; Rx=-INF;
  for(i=0;i<n;i++){
    if(pt[i].x<Lx) Lx=pt[i].x;
    if(pt[i].x>Rx) Rx=pt[i].x;
  }
  Ly=Ry=INF;
  for(i=0;i<n;i++){
    if(pt[i].x==Lx && pt[i].y<Ly){ Ly=pt[i].y; p=i; }
    if(pt[i].x==Rx && pt[i].y<Ry){ Ry=pt[i].y; q=i; }
  }
  for(dn=0,i=p;i!=q;i=(i+1)%n){ qt[dn++]=pt[i]; }
  qt[dn]=pt[q]; Ly=Ry=-INF;
  for(i=0;i<n;i++){
    if(pt[i].x==Lx && pt[i].y>Ly){ Ly=pt[i].y; p=i; }
    if(pt[i].x==Rx && pt[i].y>Ry){ Ry=pt[i].y; q=i; }
  }
  for(un=0,i=p;i!=q;i=(i+n-1)%n){ rt[un++]=pt[i]; }
  rt[un]=pt[q];
}
inline int inConvex(PT p){
  int L,R,M;
  if(p.x<Lx || p.x>Rx) return 0;
  L=0;R=dn;
  while(L<R-1){ M=(L+R)/2;
    if(p.x<qt[M].x) R=M; else L=M; }
    if(tri(qt[L],qt[R],p)<0) return 0;
    L=0;R=un;
    while(L<R-1){ M=(L+R)/2;
      if(p.x<rt[M].x) R=M; else L=M; }
      if(tri(rt[L],rt[R],p)>0) return 0;
      return 1;
}
int main(){
  int n,m,i;
  PT p;
  scanf("%d",&n);
  for(i=0;i<n;i++) scanf("%I64d %I64d",&pt[i].x,&pt[i].
      y);
  scanf("%d",&m);
  for(i=0;i<m;i++) scanf("%I64d %I64d",&qt[i].x,&qt[i].
      y);
  n=minkowskiSum(n,m);
  for(i=0;i<n;i++) pt[i]=rt[i];
  scanf("%d",&m);
  for(i=0;i<m;i++) scanf("%I64d %I64d",&qt[i].x,&qt[i].
      y);
  n=minkowskiSum(n,m);
  for(i=0;i<n;i++) pt[i]=rt[i];
  initInConvex(n);
  scanf("%d",&m);
  for(i=0;i<m;i++){
    scanf("%I64d %I64d",&p.x,&p.y);
    p.x*=3; p.y*=3;
    puts(inConvex(p)?"YES":"NO");
  }
}
```

7 Stringology

7.1 Suffix Array

```cpp
const int MAX = 1020304;
int ct[MAX], he[MAX], rk[MAX], sa[MAX], tsa[MAX], tp[
    MAX][2];

void suffix_array(char *ip){

  int len = strlen(ip);
  int alp = 256;

  memset(ct, 0, sizeof(ct));
  for(int i=0;i<len;i++) ct[ip[i]+1]++;
  for(int i=1;i<alp;i++) ct[i]+=ct[i-1];
  for(int i=0;i<len;i++) rk[i]=ct[ip[i]];

  for(int i=1;i<len;i*=2){
    for(int j=0;j<len;j++){
      if(j+i>=len) tp[j][1]=0;
      else tp[j][1]=rk[j+i]+1;

      tp[j][0]=rk[j];
    }
    memset(ct, 0, sizeof(ct));
    for(int j=0;j<len;j++) ct[tp[j][1]+1]++;
    for(int j=1;j<len+2;j++) ct[j]+=ct[j-1];
    for(int j=0;j<len;j++) tsa[ct[tp[j][1]]++]=j;

    memset(ct, 0, sizeof(ct));
    for(int j=0;j<len;j++) ct[tp[j][0]+1]++;
    for(int j=1;j<len+1;j++) ct[j]+=ct[j-1];
    for(int j=0;j<len;j++) sa[ct[tp[tsa[j]][0]]++]=tsa[
        j];

    rk[sa[0]]=0;
    for(int j=1;j<len;j++){
      if( tp[sa[j]][0] == tp[sa[j-1]][0] &&
        tp[sa[j]][1] == tp[sa[j-1]][1] )
        rk[sa[j]] = rk[sa[j-1]];
      else
        rk[sa[j]] = j;
    }
  }

  for(int i=0,h=0;i<len;i++){
    if(rk[i]==0) h=0;
    else{
      int j=sa[rk[i]-1];
      h=max(0,h-1);
      for(;ip[i+h]==ip[j+h];h++);
    }
    he[rk[i]]=h;
  }
}
```

7.2 Suffix Array (SAIS TWT514)

```cpp
struct SA{
#define REP(i,n) for ( int i=0; i<int(n); i++ )
#define REP1(i,a,b) for ( int i=(a); i<=int(b); i++ )
    static const int MXN = 300010;
    bool _t[MXN*2];
    int _s[MXN*2], _sa[MXN*2], _c[MXN*2], x[MXN], _p[
        MXN], _q[MXN*2], hei[MXN], r[MXN];
    int operator [] (int i){ return _sa[i]; }
    void build(int *s, int n, int m){
        memcpy(_s, s, sizeof(int) * n);
        sais(_s, _sa, _p, _q, _t, _c, n, m);
        mkhei(n);
    }
    void mkhei(int n){
        REP(i,n) r[_sa[i]] = i;
        hei[0] = 0;
        REP(i,n) if(r[i]) {
            int ans = i>0 ? max(hei[r[i-1]] - 1, 0) :
                0;
            while(_s[i+ans] == _s[_sa[r[i]-1]+ans]) ans
                ++;
            hei[r[i]] = ans;
        }
    }
    void sais(int *s, int *sa, int *p, int *q, bool *t,
         int *c, int n, int z){
        bool uniq = t[n-1] = true, neq;
        int nn = 0, nmxz = -1, *nsa = sa + n, *ns = s +
            n, lst = -1;
#define MS0(x,n) memset((x),0,n*sizeof(*(x)))
#define MAGIC(XD) MS0(sa, n); \
        memcpy(x, c, sizeof(int) * z); \
        XD; \
        memcpy(x + 1, c, sizeof(int) * (z - 1)); \
        REP(i,n) if(sa[i] && !t[sa[i]-1]) sa[x[s[sa[i
            ]-1]]++] = sa[i]-1; \
        memcpy(x, c, sizeof(int) * z); \
        for(int i = n - 1; i >= 0; i--) if(sa[i] && t[
            sa[i]-1]) sa[--x[s[sa[i]-1]]] = sa[i]-1;
        MS0(c, z);
        REP(i,n) uniq &= ++c[s[i]] < 2;
        REP(i,z-1) c[i+1] += c[i];
        if (uniq) { REP(i,n) sa[--c[s[i]]] = i; return;
             }
        for(int i = n - 2; i >= 0; i--) t[i] = (s[i]==s
            [i+1] ? t[i+1] : s[i]<s[i+1]);
        MAGIC(REP1(i,1,n-1) if(t[i] && !t[i-1]) sa[--x[
            s[i]]]=p[q[i]=nn++]=i);
        REP(i, n) if (sa[i] && t[sa[i]] && !t[sa[i]-1])
             {
            neq=lst<0||memcmp(s+sa[i],s+lst,(p[q[sa[i
                ]]+1]-sa[i])*sizeof(int));
            ns[q[lst=sa[i]]]=nmxz+=neq;
        }
        sais(ns, nsa, p + nn, q + n, t + n, c + z, nn,
            nmxz + 1);
        MAGIC(for(int i = nn - 1; i >= 0; i--) sa[--x[s
            [p[nsa[i]]]]] = p[nsa[i]]);
    }
}sa;

void suffix_array(int* ip, int len) {
    // should padding a zero in the back
    // s is int array, n is array length
    // s[0..n-1] != 0, and s[n] = 0
    // resulting SA will be length n+1
    ip[len++] = 0;
    sa.build(ip, len, 128);
    // original 1-base
    for (int i=0; i<l; i++) {
        hei[i] = sa.hei[i + 1];
        sa[i] = sa._sa[i + 1];
    }
}
```

7.3 Aho-Corasick Algorithm

```cpp
struct ACautomata{
  struct Node{
    int cnt,dp;
    Node *go[26], *fail;
    Node (){
      cnt = 0;
      dp = -1;
      memset(go,0,sizeof(go));
      fail = 0;
    }
  };

  Node *root, pool[1048576];
  int nMem;

  Node* new_Node(){
    pool[nMem] = Node();
    return &pool[nMem++];
  }
  void init(){
    nMem = 0;
    root = new_Node();
  }
  void add(const string &str){
    insert(root,str,0);
  }
  void insert(Node *cur, const string &str, int pos){
    if (pos >= (int)str.size()){
      cur->cnt++;
      return;
    }
    int c = str[pos]-'a';
    if (cur->go[c] == 0){
      cur->go[c] = new_Node();
    }
    insert(cur->go[c],str,pos+1);
  }
  void make_fail(){
    queue<Node*> que;
    que.push(root);
    while (!que.empty()){
      Node* fr=que.front();
      que.pop();
      for (int i=0; i<26; i++){
        if (fr->go[i]){
          Node *ptr = fr->fail;
          while (ptr && !ptr->go[i]) ptr = ptr->fail;
          if (!ptr) fr->go[i]->fail = root;
          else fr->go[i]->fail = ptr->go[i];
          que.push(fr->go[i]);
        }
      }
    }
  }
};
```

7.4 KMP

```cpp
#include<bits/stdc++.h>
using namespace std;

void build_fail_function(string B, int *fail) {
    int len = B.length(), pos;
    pos = fail[0] = -1;
    for (int i = 1; i < len; i ++) {
        while (pos != -1 and B[pos + 1] != B[i])
            pos = fail[pos];
        if (B[pos + 1] == B[i]) pos ++;
        fail[i] = pos;
    }
}

void match(string A, string B, int *fail) {
    int lenA = A.length(), lenB = B.length();
    int pos = -1;
    for (int i = 0; i < lenA; i ++) {
        while (pos != -1 and B[pos + 1] != A[i])
            pos = fail[pos];
        if (B[pos + 1] == A[i]) pos ++;

        if (pos == lenB - 1) {
            // Match ! A[i - lenB + 1, i] = B
            pos = fail[pos];
        }
    }
}
```

7.5 Z value

```cpp
void Zval(const char *s, int len, int *z) {
    z[0] = len;
    for (int b=0, i=1; i<len; i++) {
        z[i] = max(min(z[i-b], z[b] + b - i), 0);
        while (s[i + z[i]] == s[z[i]]) z[i] ++;
        if (i+z[i] > b+z[b]) b=i;
    }
}
```

7.6 Z value (palindrome ver.)

```cpp
void Zpal(const char *s, int len, int *z) {
    // Only odd palindrome len is considered
    // z[i] means that the longest odd palindrom
        centered at
    // i is [i-z[i] .. i+z[i]]
    z[0] = 0;
    for (int b=0, i=1; i<len; i++) {
        if (z[b] + b >= i) z[i] = min(z[2*b-i], b+z[b]-
            i);
        else z[i] = 0;
        while (i+z[i]+1 < len and i-z[i]-1 >= 0 and
                s[i+z[i]+1] == s[i-z[i]-1]) z[i] ++;
        if (z[i] + i > z[b] + b) b = i;
    }
}
```

7.7 palindromic tree

```cpp
int len[maxn];
int suffLink[maxn];
int to[maxn][2];
int cnt[maxn];
int numV;
char str[maxn];

int v;

void addLetter(int n)
{
    while (str[n - len[v] - 1] != str[n] )
        v = suffLink[v];
    int u = suffLink[v];
    while (str[n - len[u] - 1] != str[n] )
        u = suffLink[u];
    int u_ = to[u][str[n] - 'a'];
    int v_ = to[v][str[n] - 'a'];
    if (v_ == -1)
    {
        v_ = to[v][str[n] - 'a'] = numV;
        len[numV++] = len[v] + 2;
        suffLink[v_] = u_;
    }
    v = v_;
    cnt[v]++;
}

void init()
{
    memset(to, -1, sizeof to);
    str[0] = '#';
    len[0] = -1;
    len[1] = 0;
```

```
    len[2] = len[3] = 1;
    suffLink[1] = 0;
    suffLink[0] = 0;
    suffLink[2] = 1;
    suffLink[3] = 1;
    to[0][0] = 2;
    to[0][1] = 3;
    numV = 4;
}
```

7.8 Lexicographically Smallest Rotation

```
string mcp(string s){
  int n = s.length();
  s += s;
  int i=0, j=1;
  while (i<n && j<n){
    int k = 0;
    while (k < n && s[i+k] == s[j+k]) k++;
    if (s[i+k] <= s[j+k]) j += k+1;
    else i += k+1;
    if (i == j) j++;
  }
  int ans = i < n ? i : j;
  return s.substr(ans, n);
}
```

7.9 Suffix Automaton

```
// par : fail link
// val : a topological order ( useful for DP )
// go[x] : automata edge ( x is integer in [0,26) )

struct SAM{
  struct State{
    int par, go[26], val;
    State () : par(0), val(0){ FZ(go); }
    State (int _val) : par(0), val(_val){ FZ(go); }
  };
  vector<State> vec;
  int root, tail;

  void init(int arr[], int len){
    vec.resize(2);
    vec[0] = vec[1] = State(0);
    root = tail = 1;
    for (int i=0; i<len; i++)
      extend(arr[i]);
  }
  void extend(int w){
    int p = tail, np = vec.size();
    vec.PB(State(vec[p].val+1));
    for ( ; p && vec[p].go[w]==0; p=vec[p].par)
      vec[p].go[w] = np;
    if (p == 0){
      vec[np].par = root;
    } else {
      if (vec[vec[p].go[w]].val == vec[p].val+1){
        vec[np].par = vec[p].go[w];
      } else {
        int q = vec[p].go[w], r = vec.size();
        vec.PB(vec[q]);
        vec[r].val = vec[p].val+1;
        vec[q].par = vec[np].par = r;
        for ( ; p && vec[p].go[w] == q; p=vec[p].par)
          vec[p].go[w] = r;
      }
    }
    tail = np;
  }
};
```

8 Problems

8.1 Find the maximun tangent (x,y is increasing)

```
typedef long long LL;
const int MAXN = 100010;
struct Coord{
  LL x, y;
  Coord operator - (Coord ag) const{
    Coord res;
    res.x = x - ag.x;
    res.y = y - ag.y;
    return res;
  }
}sum[MAXN], pnt[MAXN], ans, calc;

inline bool cross(Coord a, Coord b, Coord c){
  return (c.y - a.y) * (c.x - b.x) > (c.x - a.x) * (c.y
      - b.y);
}

int main(){
  int n, l, np, st, ed, now;
  scanf("%d %d\n", &n, &l);
  sum[0].x = sum[0].y = np = st = ed = 0;
  for (int i = 1, v; i <= n; i++){
    scanf("%d", &v);
    sum[i].y = sum[i - 1].y + v;
    sum[i].x = i;
  }
  ans.x = now = 1;
  ans.y = -1;
  for (int i = 0; i <= n - l; i++){
    while (np > 1 && cross(pnt[np - 2], pnt[np - 1],
        sum[i]))
      np--;
    if (np < now && np != 0) now = np;
    pnt[np++] = sum[i];
    while (now < np && !cross(pnt[now - 1], pnt[now],
        sum[i + l]))
      now++;
    calc = sum[i + l] - pnt[now - 1];
    if (ans.y * calc.x < ans.x * calc.y){
      ans = calc;
      st = pnt[now - 1].x;
      ed = i + l;
    }
  }
  double res = (sum[ed].y-sum[st].y)/(sum[ed].x-sum[st
      ].x);
  printf("%f\n", res);
  return 0;
}
```

8.2 Painter

```
#include<bits/stdc++.h>
using namespace std;
#define F first
#define S second
#define PB push_back
#define IOS ios_base::sync_with_stdio(0); cin.tie(0);
#define SZ(x) ((int)((x).size()))
#define ALL(x) begin(x),end(x)
#define REP(i,x) for (int i=0; i<(x); i++)
#define REP1(i,a,b) for (int i=(a); i<=(b); i++)

typedef long long ll;
typedef pair<ll,ll> pll;

typedef pll Point;
const int MXN = 100005;

Point operator + (const Point &a, const Point &b) {
    return Point(a.F+b.F, a.S+b.S); }
Point operator - (const Point &a, const Point &b) {
    return Point(a.F-b.F, a.S-b.S); }
ll operator * (const Point &a, const Point &b) { return
    a.F*b.F + a.S*b.S; }
ll operator % (const Point &a, const Point &b) { return
    a.F*b.S - a.S*b.F; }

struct Segment {
  int v,id;
```

```
  Point p,q;
  Segment () {}
  Segment (int _v, int _id, Point _p, Point _q) :
    v(_v), id(_id), p(_p), q(_q) {}
};
bool operator < (const Segment &a, const Segment &b) {
  if (a.p == b.q) return false;
  if (a.q == b.p) return true;
  if (a.p == b.p) return (a.q-a.p) % (b.q-a.p) > 0;
  if (a.q == b.q) return (a.p-a.q) % (b.p-a.q) < 0;
  if (a.p.F == b.p.F) return a.p.S < b.p.S;
  if (a.q.F == b.q.F) return a.q.S < b.q.S;
  if (a.p.F < b.p.F) return (a.q-a.p) % (b.p-a.p) > 0;
  else return (b.q-b.p) % (a.p-b.p) < 0;
}
bool operator == (const Segment &a, const Segment &b) {
  return tie(a.v,a.id,a.p,a.q) == tie(b.v,b.id,b.p,b.q)
    ;
}
struct Triangle {
  Point pt[3];
}ip[MXN];

const int MEM = 350004;
struct Treap {
  static Treap nil, mem[MEM], *pmem;
  Treap *l, *r;
  int sum,presum,size;
  Segment seg;
  Treap () : l(&nil), r(&nil), sum(0), presum(0), size
      (0), seg() {}
  Treap (Segment _val) :
    l(&nil), r(&nil), sum(_val.v), presum(max(_val.v,0)
      ), size(1), seg(_val) {}
} Treap::nil, Treap::mem[MEM], *Treap::pmem = Treap::
    mem;

int size(const Treap *t) { return t->size; }
void pull(Treap *t) {
  if (!size(t)) return;
  t->size = size(t->l) + size(t->r) + 1;
  t->sum = t->l->sum + t->seg.v + t->r->sum;
  t->presum = max(t->l->presum, t->l->sum + t->seg.v);
  t->presum = max(t->presum, t->l->sum + t->seg.v + t->
    r->presum);
}
Treap* merge(Treap *a, Treap *b) {
  if (!size(a)) return b;
  if (!size(b)) return a;
  Treap *t;
  if (rand() % (size(a) + size(b)) < size(a)) {
    t = a;
    t->r = merge(a->r, b);
  } else {
    t = b;
    t->l = merge(a, b->l);
  }
  pull(t);
  return t;
}
void split(Treap *t, int k, Treap *&a, Treap *&b) {
  if (!size(t)) a = b = &Treap::nil;
  else if (size(t->l) + 1 <= k) {
    a = t;
    split(t->r, k - size(t->l) - 1, a->r, b);
    pull(a);
  } else {
    b = t;
    split(t->l, k, a, b->l);
    pull(b);
  }
}
int get_rank(Treap *t, Segment x) {
  if (!size(t)) return 0;
  if (x < t->seg) return get_rank(t->l, x);
  return get_rank(t->r,x) + size(t->l) + 1;
}
Treap* find_leftist(Treap *t) {
  while (size(t->l)) t = t->l;
  return t;
}
Treap* find_rightist(Treap *t) {
```

```
    while (size(t->r)) t = t->r;
    return t;
}

int N;
vector<int> allx;
vector<Segment> _seg[3*MXN];
#define seg(x) _seg[(x)+100000]

inline void add_seg(Segment s) {
  seg(s.p.F).PB(s);
  if (s.q.F != s.p.F) seg(s.q.F).PB(s);
}
void predo() {
  allx.clear();
  REP(i,N) REP(j,3) {
    seg(ip[i].pt[j].F).clear();
    allx.PB(ip[i].pt[j].F);
  }
  sort(ALL(allx));
  allx.resize(unique(ALL(allx))-begin(allx));
  REP(i,N) {
    sort(ip[i].pt, ip[i].pt+3);
    Point *pt = ip[i].pt;
    Segment seg1 = Segment(1,i,pt[0],pt[1]);
    Segment seg2 = Segment(1,i,pt[0],pt[2]);
    Segment seg3 = Segment(1,i,pt[1],pt[2]);
    if (seg2 < seg1) seg1.v = -1;
    else seg2.v = -1;
    seg3.v = seg1.v;
    add_seg(seg1);
    add_seg(seg2);
    add_seg(seg3);
  }
}
inline int sgn(ll x) { return x < 0 ? -1 : x > 0; }
bool interPnt(Point p1, Point p2, Point q1, Point q2){
  ll c1 = (p2-p1)%(q1-p1), c2 = (p2-p1)%(q2-p1);
  ll c3 = (q2-q1)%(p1-q1), c4 = (q2-q1)%(p2-q1);
  return sgn(c1) * sgn(c2) <= 0 and sgn(c3) * sgn(c4)
      <= 0;
}
bool check_error(Segment a, Segment b) {
  if (a.id == b.id) return false;
  return interPnt(a.p,a.q,b.p,b.q);
}
int solve() {
  Treap::pmem = Treap::mem;
  Treap *rt = &Treap::nil;
  int res = 0;
  for (auto i:allx) {
    for (auto l:seg(i)) {
      int k = get_rank(rt, l);
      Treap *t,*tl,*tm,*tr;
      split(rt,k,tl,tr);
      t = find_rightist(tl);
      if (size(t) and check_error(t->seg,l)) return -1;
      t = find_leftist(tr);
      if (size(t) and check_error(t->seg,l)) return -1;
      rt = merge(tl,tr);
      if (l.p.F == i and l.p.F != l.q.F) {
        k = get_rank(rt, l);
        split(rt,k,tl,tr);
        tm = new (Treap::pmem++) Treap(l);
        rt = merge(merge(tl,tm),tr);
      }
    }
    for (auto l:seg(i)) {
      if (l.q.F == i and l.p.F != l.q.F) {
        Treap *tl,*tm,*tr;
        int k = get_rank(rt, l);
        split(rt,k-1,tl,tm);
        split(tm,1,tm,tr);
        Treap *t1=find_rightist(tl),*t2=find_leftist(tr
          );
        if (size(t1) and size(t2) and check_error(t1->
            seg,t2->seg)) return -1;
        rt = merge(tl,tr);
      }
    }
    res = max(res, rt->presum);
  }
```

```cpp
    res++;
    return res;
}
int main() {
    IOS;
    int cas = 0;
    while (cin >> N) {
        if (N == -1) break;
        REP(i,N) {
            REP(j,3) cin >> ip[i].pt[j].F >> ip[i].pt[j].S;
        }
        predo();
        int ans = solve();
        cas++;
        cout << "Case " << cas << ": ";
        if (ans == -1) cout << "ERROR\n";
        else cout << ans << " shades\n";
    }

    return 0;
}
```