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### 1 Basic

### 1.1 vimrc

```
//stack resize
asm( "mov %0,%%esp\n" ::"g"(mem+10000000) );
//change esp to rsp if 64-bit system

//stack resize (linux)
#include <sys/resource.h>
void increase_stack_size() {
   const rlim_t ks = 64*1024*1024;
   struct rlimit rl;
   int res=getrlimit(RLIMIT_STACK, &rl);
   if(res==0){
      if(rl.rlim_cur<ks){
        rl.rlim_cur=ks;
        res=setrlimit(RLIMIT_STACK, &rl);
   }
   }
}</pre>
```

#### 1.3 Default Code

```
#include<bits/stdc++.h>
#include<unistd.h>
using namespace std;
#define FZ(n) memset((n),0,sizeof(n))
#define FMO(n) memset((n),-1,sizeof(n))
#define F first
#define S second
#define PB push_back
#define ALL(x) begin(x),end(x)
#define SZ(x) ((int)(x).size())
#define IOS ios_base::sync_with_stdio(0); cin.tie(0)
#define REP(i,x) for (int i=0; i<(x); i++)</pre>
#define REP1(i,a,b) for (int i=(a); i<=(b); i++)
#ifdef ONLINE_JUDGE
#define FILEIO(name) \
    freopen(name".in", "r", stdin); \
    freopen(name".out", "w", stdout);
#else
#define FILEIO(name)
#endif
template<typename A, typename B>
ostream& operator <<(ostream &s, const pair<A,B> &p) {
    return s<<"("<<p.first<<","<<p.second<<")";</pre>
template<typename T>
ostream& operator <<(ostream &s, const vector<T> &c) {
    s<<"[ ";
    for (auto it : c) s << it << " ";</pre>
    s<<"]";
    return s;
// Let's Fight!
int main() {
    return 0;
```

### 2 Data Structure

# 2.1 Bigint

```
struct Bigint{
  static const int LEN = 60;
  static const int BIGMOD = 10000;
  int s;
  int vĺ, v[LEN];
  // vector<int> v;
  Bigint() : s(1) { vl = 0; }
  Bigint(long long a) {
    s = 1; vl = 0;
    if (a < 0) { s = -1; a = -a; }
    while (a) {
      push_back(a % BIGMOD);
      a /= BIGMOD;
  Bigint(string str) {
    s = 1; vl = 0;
    int stPos = 0, num = 0;
    if (!str.empty() && str[0] == '-') {
      stPos = 1;
      s = -1;
    for (int i=SZ(str)-1, q=1; i>=stPos; i--) {
  num += (str[i] - '0') * q;
      if ((q *= 10) >= BIGMOD) {
        push_back(num);
        num = 0; q = 1;
    if (num) push_back(num);
    n();
  }
  int len() const {
    return vl;
          return SZ(v);
  bool empty() const { return len() == 0; }
  void push_back(int x) {
    v[vl++] = x;
    //
         v.PB(x);
  }
  void pop_back() {
    //
          v.pop_back();
  int back() const {
    return v[vl-1];
          return v.back();
  void n() {
    while (!empty() && !back()) pop_back();
  void resize(int nl) {
    vl = nl;
    fill(v, v+vl, 0);
         v.resize(nl);
    //
    //
          fill(ALL(v), 0);
  void print() const {
    if (empty()) { putchar('0'); return; }
    if (s == -1) putchar('-');
    printf("%d", back());
    for (int i=len()-2; i>=0; i--) printf("%.4d",v[i]);
  friend std::ostream& operator << (std::ostream& out,</pre>
      const Bigint &a) {
    if (a.empty()) { out << "0"; return out; }</pre>
    if (a.s == -1) out << "-";</pre>
    out << a.back();
    for (int i=a.len()-2; i>=0; i--) {
      char str[10];
      snprintf(str, 5, "%.4d", a.v[i]);
      out << str;</pre>
```

```
return out;
int cp3(const Bigint &b)const {
  if (s != b.s) return s - b.s;
  if (s == -1) return -(-*this).cp3(-b);
  if (len() != b.len()) return len()-b.len();//int
  for (int i=len()-1; i>=0; i--)
    if (v[i]!=b.v[i]) return v[i]-b.v[i];
  return 0:
}
bool operator < (const Bigint &b)const{ return cp3(b)</pre>
bool operator <= (const Bigint &b)const{ return cp3(b</pre>
    )<=0; }
bool operator == (const Bigint &b)const{ return cp3(b
    )==0; }
bool operator != (const Bigint &b)const{ return cp3(b
    )!=0; }
bool operator > (const Bigint &b)const{ return cp3(b)
    >0; }
bool operator >= (const Bigint &b)const{ return cp3(b
    )>=0; }
Bigint operator - () const {
  Bigint r = (*this);
  r.s = -r.s;
  return r;
Bigint operator + (const Bigint &b) const {
  if (s == -1) return -(-(*this)+(-b));
  if (b.s == -1) return (*this)-(-b);
  Bigint r;
  int nl = max(len(), b.len());
  r.resize(nl + 1);
  for (int i=0; i<nl; i++) {</pre>
    if (i < len()) r.v[i] += v[i];</pre>
    if (i < b.len()) r.v[i] += b.v[i];</pre>
    if(r.v[i] >= BIGMOD) {
      r.v[i+1] += r.v[i] / BIGMOD;
      r.v[i] %= BIGMOD;
    }
  }
  r.n();
  return r;
Bigint operator - (const Bigint &b) const {
  if (s == -1) return -(-(*this)-(-b));
  if (b.s == -1) return (*this)+(-b);
  if ((*this) < b) return -(b-(*this));</pre>
  Bigint r;
  r.resize(len());
  for (int i=0; i<len(); i++) {</pre>
    r.v[i] += v[i];
    if (i < b.len()) r.v[i] -= b.v[i];</pre>
    if (r.v[i] < 0) {</pre>
      r.v[i] += BIGMOD;
      r.v[i+1]--;
    }
  r.n();
  return r;
Bigint operator * (const Bigint &b) {
  r.resize(len() + b.len() + 1);
  r.s = s * b.s;
  for (int i=0; i<len(); i++) {</pre>
    for (int j=0; j<b.len(); j++) {</pre>
      r.v[i+j] += v[i] * b.v[j];
      if(r.v[i+j] >= BIGMOD) {
        r.v[i+j+1] += r.v[i+j] / BIGMOD;
        r.v[i+j] %= BIGMOD;
      }
    }
  }
  r.n();
  return r;
Bigint operator / (const Bigint &b) {
```

```
Bigint r:
  r.resize(max(1, len()-b.len()+1));
  int oriS = s:
  Bigint b2 = b; // b2 = abs(b)
  s = b2.s = r.s = 1;
  for (int i=r.len()-1; i>=0; i--) {
    int d=0, u=BIGMOD-1;
    while(d<u) {</pre>
      int m = (d+u+1)>>1;
      r.v[i] = m;
      if((r*b2) > (*this)) u = m-1;
      else d = m;
    }
   r.v[i] = d;
  }
  s = oriS;
  r.s = s * b.s;
  r.n();
  return r:
Bigint operator % (const Bigint &b) {
 return (*this)-(*this)/b*b;
```

### 2.2 unordered\_map

```
struct Kev {
  int first, second;
  Key () {}
  Key (int _x, int _y) : first(_x), second(_y) {}
  bool operator == (const Key &b) const {
    return tie(F,S) == tie(b.F,b.S);
 }
};
struct KevHasher {
 size_t operator()(const Key& k) const {
    return k.first + k.second*100000;
};
typedef unordered_map<Key,int,KeyHasher> map_t;
int main(int argc, char** argv){
  map_t mp;
  for (int i=0; i<10; i++)</pre>
   mp[Key(i,0)] = i+1;
  for (int i=0; i<10; i++)</pre>
    printf("%d \ n", mp[Key(i,0)]);
  return 0;
}
```

### 2.3 extc\_balance\_tree

```
#include <bits/extc++.h>
typedef __gnu_pbds::priority_queue<int> heap_t;
heap_t a,b;
int main() {
  a.clear();
  b.clear();
  a.push(1);
  a.push(3);
  b.push(2);
  b.push(4);
  assert(a.top() == 3);
  assert(b.top() == 4);
  // merge two heap
  a.join(b);
  assert(a.top() == 4);
  assert(b.empty());
  return 0;
```

### 2.4 extc heap

```
#include <bits/extc++.h>
using namespace __gnu_pbds;
typedef tree<int,null_type,less<int>,rb_tree_tag,
    tree_order_statistics_node_update> set_t;
int main()
 // Insert some entries into s.
 set_t s;
 s.insert(12);
 s.insert(505);
 // The order of the keys should be: 12, 505.
 assert(*s.find_by_order(0) == 12);
 assert(*s.find_by_order(3) == 505);
 // The order of the keys should be: 12, 505.
 assert(s.order_of_key(12) == 0);
 assert(s.order_of_key(505) == 1);
 // Erase an entry.
 s.erase(12);
 // The order of the keys should be: 505.
 assert(*s.find_by_order(0) == 505);
 // The order of the keys should be: 505.
 assert(s.order_of_key(505) == 0);
```

# 2.5 Disjoint Set

```
struct DisjointSet {
  // save() is like recursive
  // undo() is like return
 int n, fa[MXN], sz[MXN];
 vector<pair<int*,int>> h;
 vector<int> sp;
 void init(int tn) {
   n=tn;
    for (int i=0; i<n; i++) {</pre>
      fa[i]=i;
      sz[i]=1;
    sp.clear(); h.clear();
  void assign(int *k, int v) {
   h.PB({k, *k});
    *k=v;
 void save() { sp.PB(SZ(h)); }
  void undo() {
   assert(!sp.empty());
    int last=sp.back(); sp.pop_back();
    while (SZ(h)!=last) {
      auto x=h.back(); h.pop_back();
      *x.F=x.S;
   }
  int f(int x) {
   while (fa[x]!=x) x=fa[x];
    return x;
 void uni(int x, int y) {
    x=f(x); y=f(y);
    if (x==y) return ;
    if (sz[x]<sz[y]) swap(x, y);</pre>
    assign(\&sz[x], sz[x]+sz[y]);
    assign(&fa[y], x);
}djs;
```

### 2.6 Treap

```
const int MEM = 16000004:
struct Treap {
  static Treap nil, mem[MEM], *pmem;
  Treap *l, *r;
  char val;
  int size;
  Treap () : l(&nil), r(&nil), size(0) {}
  Treap (char _val) :
    l(&nil), r(&nil), val(_val), size(1) {}
} Treap::nil, Treap::mem[MEM], *Treap::pmem = Treap::
int size(const Treap *t) { return t->size; }
void pull(Treap *t) {
  if (!size(t)) return;
  t\rightarrow size = size(t\rightarrow l) + size(t\rightarrow r) + 1;
Treap* merge(Treap *a, Treap *b) {
  if (!size(a)) return b;
  if (!size(b)) return a;
  Treap *t;
  if (rand() % (size(a) + size(b)) < size(a)) {</pre>
    t = new (Treap::pmem++) Treap(*a);
    t->r = merge(a->r, b);
  } else {
    t = new (Treap::pmem++) Treap(*b);
    t->l = merge(a, b->l);
  }
  pull(t);
  return t;
void split(Treap *t, int k, Treap *&a, Treap *&b) {
  if (!size(t)) a = b = &Treap::nil;
  else if (size(t->l) + 1 <= k) {
    a = new (Treap::pmem++) Treap(*t);
    split(t->r, k - size(t->l) - 1, a->r, b);
    pull(a);
  } else {
    b = new (Treap::pmem++) Treap(*t);
    split(t->l, k, a, b->l);
    pull(b);
  }
}
int nv;
Treap *rt[50005];
void print(const Treap *t) {
  if (!size(t)) return;
  print(t->l);
  cout << t->val;
  print(t->r);
int main(int argc, char** argv) {
  rt[nv=0] = &Treap::nil;
  Treap::pmem = Treap::mem;
  int Q, cmd, p, c, v;
  string s;
  cin >> Q;
  while (Q--) {
    cin >> cmd;
    if (cmd == 1) {
      // insert string s after position p
      cin >> p >> s;
      Treap *tl, *tr;
      split(rt[nv], p, tl, tr);
for (int i=0; i<SZ(s); i++)</pre>
        tl = merge(tl, new (Treap::pmem++) Treap(s[i]))
      rt[++nv] = merge(tl, tr);
    } else if (cmd == 2) {
      // remove c characters starting at position
      Treap *tl, *tm, *tr;
      cin >> p >> c;
      split(rt[nv], p-1, tl, tm);
      split(tm, c, tm, tr);
      rt[++nv] = merge(tl, tr);
    } else if (cmd == 3) {
      // print c characters starting at position p, in
           version v
```

```
Treap *tl, *tm, *tr;
    cin >> v >> p >> c;
    split(rt[v], p-1, tl, tm);
    split(tm, c, tm, tr);
    print(tm);
    cout << "\n";
    }
}
return 0;
}</pre>
```

# 2.7 Heavy Light Decomposition

```
// only one segment tree / no 0/1 base issue
// getPathSeg return the segment in order u->v
// fa[root] = root
typedef pair<int,int> pii;
int N,fa[MXN],belong[MXN],dep[MXN],sz[MXN],que[MXN];
int step,line[MXN],stPt[MXN],edPt[MXN];
vector<int> E[MXN], chain[MXN];
void DFS(int u){
  vector<int> &c = chain[belong[u]];
  for (int i=c.size()-1; i>=0; i--){
    int v = c[i];
    stPt[v] = step;
    line[step++] = v;
  for (int i=0; i<(int)c.size(); i++){</pre>
   u = c[i];
    for (auto v : E[u]){
      if (fa[u] == v || (i && v == c[i-1])) continue;
      DFS(v);
    edPt[u] = step-1;
 }
void build chain(int st){
  int fr,bk;
  fr=bk=0; que[bk++] = 1; fa[st]=st; dep[st]=0;
 while (fr < bk){</pre>
    int u=que[fr++];
    for (auto v : E[u]){
  if (v == fa[u]) continue;
      que[bk++] = v;
      dep[v] = dep[u]+1;
      fa[v] = u;
  for (int i=bk-1,u,pos; i>=0; i--){
   u = que[i]; sz[u] = 1; pos = -1;
    for (auto v : E[u]){
      if (v == fa[u]) continue;
      sz[u] += sz[v];
      if (pos==-1 || sz[v]>sz[pos]) pos=v;
    if (pos == -1) belong[u] = u;
    else belong[u] = belong[pos];
    chain[belong[u]].PB(u);
  step = 0;
 DFS(st);
int getLCA(int u, int v){
 while (belong[u] != belong[v]){
    int a = chain[belong[u]].back();
    int b = chain[belong[v]].back();
    if (dep[a] > dep[b]) u = fa[a];
    else v = fa[b];
 }
  return sz[u] >= sz[v] ? u : v;
vector<pii> getPathSeg(int u, int v){
  vector<pii> ret1,ret2;
  while (belong[u] != belong[v]){
    int a = chain[belong[u]].back();
    int b = chain[belong[v]].back();
    if (dep[a] > dep[b]){
      ret1.PB({stPt[a],stPt[u]});
```

```
u = fa[a];
    } else {
     ret2.PB({stPt[b],stPt[v]});
      v = fa[b];
  if (dep[u] > dep[v]) swap(u,v);
  ret1.PB({stPt[u],stPt[v]});
  reverse(ret2.begin(), ret2.end());
  ret1.insert(ret1.end(),ret2.begin(),ret2.end());
  return ret1:
// Usage
void build(){
  build_chain(1); //change root
  init(0,step,0); //init segment tree
int get_answer(int u, int v){
  int ret = -2147483647;
  vector<pii> vec = getPathSeg(u,v);
  for (auto it : vec)
    ; // check answer with segment [it.F, it.S]
  return ret;
```

#### 2.8 Link-Cut Tree

```
const int MXN = 100005;
const int MEM = 100005:
struct Splay {
  static Splay nil, mem[MEM], *pmem;
  Splay *ch[2], *f;
  int val, rev, size;
  Splay () : val(-1), rev(0), size(0) {
    f = ch[0] = ch[1] = &nil;
  Splay (int _val) : val(_val), rev(0), size(1) {
    f = ch[0] = ch[1] = &nil;
  bool isr() {
    return f->ch[0] != this && f->ch[1] != this;
  int dir() {
    return f->ch[0] == this ? 0 : 1;
  void setCh(Splay *c, int d) {
    ch[d] = c;
    if (c != &nil) c->f = this;
    pull();
  void push() {
    if (rev) {
      swap(ch[0], ch[1]);
      if (ch[0] != &nil) ch[0]->rev ^= 1;
      if (ch[1] != &nil) ch[1]->rev ^= 1;
      rev=0;
    }
  void pull() {
    size = ch[0]->size + ch[1]->size + 1;
    if (ch[0] != &nil) ch[0]->f = this;
    if (ch[1] != &nil) ch[1]->f = this;
} Splay::nil, Splay::mem[MEM], *Splay::pmem = Splay::
    mem;
Splay *nil = &Splay::nil;
void rotate(Splay *x) {
  Splay *p = x->f;
  int d = x->dir();
  if (!p->isr()) p->f->setCh(x, p->dir());
  else x->f = p->f;
  p->setCh(x->ch[!d], d);
  x->setCh(p, !d);
  p->pull(); x->pull();
vector<Splay*> splayVec;
void splay(Splay *x) {
```

```
splayVec.clear();
  for (Splay *q=x;; q=q->f) {
    splayVec.push_back(q);
    if (q->isr()) break;
  reverse(begin(splayVec), end(splayVec));
  for (auto it : splayVec) it->push();
  while (!x->isr()) {
    if (x->f->isr()) rotate(x);
    else if (x->dir()==x->f->dir()) rotate(x->f),rotate
    else rotate(x),rotate(x);
  }
}
Splay* access(Splay *x) {
  Splay *q = nil;
  for (;x!=nil;x=x->f) {
    splay(x);
    x \rightarrow setCh(q, 1);
    q = x;
  }
  return q;
void evert(Splay *x) {
  access(x);
  splay(x);
  x->rev ^= 1;
  x->push(); x->pull();
void link(Splay *x, Splay *y) {
// evert(x);
 access(x);
  splay(x);
  evert(y);
  x->setCh(y, 1);
void cut(Splay *x, Splay *y) {
// evert(x);
 access(y);
  splay(y);
  y->push();
  y - ch[0] = y - ch[0] - f = nil;
int N, Q;
Splay *vt[MXN];
int ask(Splay *x, Splay *y) {
  access(x);
  access(y);
  splay(x);
  int res = x->f->val;
  if (res == -1) res=x->val;
  return res;
int main(int argc, char** argv) {
  scanf("%d%d", &N, &Q);
for (int i=1; i<=N; i++)
    vt[i] = new (Splay::pmem++) Splay(i);
  while (Q--) {
    char cmd[105];
    int u, v;
scanf("%s", cmd);
    if (cmd[1] == 'i') {
      scanf("%d%d", &u, &v);
      link(vt[v], vt[u]);
    } else if (cmd[0] == 'c') {
      scanf("%d", &v);
      cut(vt[1], vt[v]);
    } else {
      scanf("%d%d", &u, &v);
      int res=ask(vt[u], vt[v]);
      printf("%d \ n", res);
  }
  return 0:
}
```

# 3 Graph

# 3.1 BCC Edge

```
struct BccEdge {
  static const int MXN = 100005;
  struct Edge { int v,eid; };
  int n,m,step,par[MXN],dfn[MXN],low[MXN];
  vector<Edge> E[MXN];
  DisjointSet djs;
  void init(int _n) {
    n = _n; m = 0;
    for (int i=0; i<n; i++) E[i].clear();</pre>
    djs.init(n);
  void add_edge(int u, int v) {
    E[u].PB({v, m});
    E[v].PB({u, m});
  void DFS(int u, int f, int f_eid) {
    par[u] = f;
    dfn[u] = low[u] = step++;
    for (auto it:E[u]) {
      if (it.eid == f_eid) continue;
      int v = it.v;
      if (dfn[v] == −1) {
        DFS(v, u, it.eid);
        low[u] = min(low[u], low[v]);
      } else {
        low[u] = min(low[u], dfn[v]);
      }
    }
  }
  void solve() {
    step = 0;
    memset(dfn, -1, sizeof(int)*n);
    for (int i=0; i<n; i++) {</pre>
      if (dfn[i] == -1) DFS(i, i, -1);
    djs.init(n);
    for (int i=0; i<n; i++) {</pre>
      if (low[i] < dfn[i]) djs.uni(i, par[i]);</pre>
  }
}graph;
```

### 3.2 BCC Vertex

```
struct BccVertex {
  int n,nScc,step,dfn[MXN],low[MXN];
  vector<int> E[MXN],sccv[MXN];
  int top,stk[MXN];
  void init(int _n) {
    n = _n;
    nScc = step = 0;
    for (int i=0; i<n; i++) E[i].clear();</pre>
  void add_edge(int u, int v) {
    E[u].PB(v);
    E[v].PB(u);
  void DFS(int u, int f) {
    dfn[u] = low[u] = step++;
    stk[top++] = u;
    for (auto v:E[u]) {
      if (v == f) continue;
      if (dfn[v] == -1) {
        DFS(v,u);
        low[u] = min(low[u], low[v]);
        if (low[v] >= dfn[u]) {
          int z;
          sccv[nScc].clear();
          do {
            z = stk[--top]:
            sccv[nScc].PB(z);
          } while (z != v);
          sccv[nScc].PB(u);
```

```
nScc++:
      } else {
        low[u] = min(low[u],dfn[v]);
    }
  }
  vector<vector<int>> solve() {
    vector<vector<int>> res;
    for (int i=0; i<n; i++) {</pre>
      dfn[i] = low[i] = -1;
    for (int i=0; i<n; i++) {</pre>
      if (dfn[i] == -1) {
        top = 0;
        DFS(i,i);
      }
    REP(i,nScc) res.PB(sccv[i]);
    return res;
  }
}graph;
```

# 3.3 Strongly Connected Components

```
struct Scc{
  int n, nScc, vst[MXN], bln[MXN];
  vector<int> E[MXN], rE[MXN], vec;
  void init(int _n){
    n = _n;
for (int i=0; i<MXN; i++){</pre>
      E[i].clear();
      rE[i].clear();
    }
  void add_edge(int u, int v){
    E[u].PB(v);
    rE[v].PB(u);
  void DFS(int u){
    vst[u]=1;
    for (auto v : E[u])
      if (!vst[v]) DFS(v);
    vec.PB(u);
  void rDFS(int u){
    vst[u] = 1;
    bln[u] = nScc;
    for (auto v : rE[u])
      if (!vst[v]) rDFS(v);
  void solve(){
    nScc = 0;
    vec.clear();
    FZ(vst);
    for (int i=0; i<n; i++)</pre>
      if (!vst[i]) DFS(i);
    reverse(vec.begin(),vec.end());
    FZ(vst);
    for (auto v : vec){
      if (!vst[v]){
        rDFS(v);
        nScc++;
      }
    }
  }
};
```

# 3.4 DMST\_with\_sol

```
const int INF = 1029384756;
struct edge_t{
  int u,v,w;
  set< pair<int,int> > add, sub;
  edge_t() : u(-1), v(-1), w(0) {}
  edge_t(int _u, int _v, int _w) {
    u = _u; v = _v; w = _w;
```

```
add.insert({u, v});
  edge_t& operator += (const edge_t& obj) {
    w += obj.w;
    FOR (it, obj.add) {
      if (!sub.count(*it)) add.insert(*it);
      else sub.erase(*it);
    FOR (it, obj.sub) {
      if (!add.count(*it)) sub.insert(*it);
      else add.erase(*it);
    return *this;
  edge_t& operator -= (const edge_t& obj) {
    w -= obj.w;
    FOR (it, obj.sub) {
      if (!sub.count(*it)) add.insert(*it);
      else sub.erase(*it);
    for (auto it : obj.add) {
      if (!add.count(it)) sub.insert(it);
      else add.erase(it);
    return *this;
}eg[MXN*MXN],prv[MXN],EDGE_INF(-1,-1,INF);
int N,M;
int cid,incyc[MXN],contracted[MXN];
vector<int> E[MXN];
edge_t dmst(int rt){
  edge_t cost;
  for (int i=0; i<N; i++){</pre>
    contracted[i] = incyc[i] = 0;
    prv[i] = EDGE_INF;
  cid = 0;
  int u,v;
  while (true){
    for (v=0; v<N; v++){
      if (v != rt && !contracted[v] && prv[v].w == INF)
    if (v >= N) break; // end
    for (int i=0; i<M; i++){</pre>
      if (eg[i].v == v && eg[i].w < prv[v].w)</pre>
        prv[v] = eg[i];
    if (prv[v].w == INF) // not connected
      return EDGE_INF;
    cost += prv[v];
    for (u=prv[v].u; u!=v && u!=-1; u=prv[u].u);
    if (u == -1) continue;
    incyc[v] = ++cid;
    for (u=prv[v].u; u!=v; u=prv[u].u){
      contracted[u] = 1;
      incyc[u] = cid;
    for (int i=0; i<M; i++){</pre>
      if (incyc[eg[i].u] != cid && incyc[eg[i].v] ==
        eg[i] -= prv[eg[i].v];
      }
    for (int i=0; i<M; i++){</pre>
      if (incyc[eg[i].u] == cid) eg[i].u = v;
      if (incyc[eg[i].v] == cid) eg[i].v = v;
      if (eg[i].u == eg[i].v) eg[i--] = eg[--M];
    for (int i=0; i<N; i++){</pre>
      if (contracted[i]) continue;
      if (prv[i].u>=0 && incyc[prv[i].u] == cid)
        prv[i].u = v;
    prv[v] = EDGE_INF;
  return cost;
}
void solve(){
 edge_t cost = dmst(0);
```

```
for (auto it : cost.add){ // find a solution
    E[it.F].PB(it.S);
    prv[it.S] = edge_t(it.F,it.S,0);
}
```

# 3.5 Maximum Clique

```
class MaxClique {
public:
    static const int MV = 210;
    int V;
    int el[MV][MV/30+1];
    int dp[MV];
    int ans;
    int s[MV][MV/30+1];
    vector<int> sol;
    void init(int v) {
         V = v; ans = 0;
         FZ(el); FZ(dp);
     /* Zero Base */
    void addEdge(int u, int v) {
         if(u > v) swap(u, v);
         if(u == v) return;
         el[u][v/32] |= (1<<(v%32));
    bool dfs(int v, int k) {
         int c = 0, d = 0;
         for(int i=0; i<(V+31)/32; i++) {</pre>
             s[k][i] = el[v][i];
             if(k != 1) s[k][i] &= s[k-1][i];
             c += __builtin_popcount(s[k][i]);
         if(c == 0) {
             if(k > ans) {
                 ans = k;
                 sol.clear();
                 sol.push_back(v);
                 return 1;
             }
             return 0;
         for(int i=0; i<(V+31)/32; i++) {</pre>
             for(int a = s[k][i]; a; d++) {
                 if(k + (c-d) <= ans) return 0;</pre>
                 int lb = a&(-a), lg = 0;
                 a ^= lb;
                 while(lb!=1) {
                     lb = (unsigned int)(lb) >> 1;
                 int u = i*32 + lg;
                 if(k + dp[u] <= ans) return 0;</pre>
                 if(dfs(u, k+1)) {
                     sol.push_back(v);
                     return 1;
                 }
             }
         return 0;
    }
    int solve() {
         for(int i=V-1; i>=0; i--) {
             dfs(i, 1);
             dp[i] = ans;
         return ans;
    }
};
```

```
3.6 MinimumMeanCycle
```

```
/* minimum mean cvcle */
const int MAXE = 1805;
const int MAXN = 35;
const double inf = 1029384756;
const double eps = 1e-6;
struct Edge {
  int v,u;
  double c;
int n,m,prv[MAXN][MAXN], prve[MAXN][MAXN], vst[MAXN];
Edge e[MAXE];
vector<int> edgeID, cycle, rho;
double d[MAXN][MAXN];
inline void bellman_ford() {
  for(int i=0; i<n; i++) d[0][i]=0;</pre>
  for(int i=0; i<n; i++) {</pre>
    fill(d[i+1], d[i+1]+n, inf);
for(int j=0; j<m; j++) {</pre>
      int v = e[j].v, u = e[j].u;
      if(d[i][v]<inf && d[i+1][u]>d[i][v]+e[j].c) {
        d[i+1][u] = d[i][v]+e[j].c;
        prv[i+1][u] = v;
        prve[i+1][u] = j;
      }
    }
  }
}
double karp_mmc() {
  // returns inf if no cycle, mmc otherwise
  double mmc=inf;
  int st = -1;
  bellman_ford();
  for(int i=0; i<n; i++) {</pre>
    double avg=-inf;
    for(int k=0; k<n; k++) {</pre>
      if(d[n][i]<inf-eps) avg=max(avg,(d[n][i]-d[k][i])</pre>
           /(n-k));
      else avg=max(avg,inf);
    if (avg < mmc) tie(mmc, st) = tie(avg, i);</pre>
  FZ(vst); edgeID.clear(); cycle.clear(); rho.clear();
  for (int i=n; !vst[st]; st=prv[i--][st]) {
    vst[st]++;
    edgeID.PB(prve[i][st]);
    rho.PB(st);
  while (vst[st] != 2) {
    int v = rho.back(); rho.pop_back();
    cycle.PB(v);
    vst[v]++;
  }
  reverse(ALL(edgeID));
  edgeID.resize(SZ(cycle));
  return mmc;
```

# 4 Flow

#### **4.1 ISAP**

```
struct Isap{
    static const int MXN = 10000;
    struct Edge{ int v,f,re; };
    int n,s,t,h[MXN],gap[MXN];
    vector<Edge> E[MXN];
    void init(int _n, int _s, int _t){
        n = _n; s = _s; t = _t;
        for (int i=0; i<n; i++) E[i].clear();
    }
    void add_edge(int u, int v, int f){
        E[u].PB({v,f,SZ(E[v])});
        E[v].PB({u,0,SZ(E[u])-1});
    }
    int DFS(int u, int nf, int res=0){
        if (u == t) return nf;
        for (auto &it : E[u]){
            if (h[u]==h[it.v]+1 && it.f>0){
```

```
int tf = DFS(it.v,min(nf,it.f));
        res += tf; nf -= tf; it.f -= tf;
        E[it.v][it.re].f += tf;
        if (nf == 0) return res;
    if (nf){
      if (--gap[h[u]] == 0) h[s]=n;
      gap[++h[u]]++;
    return res;
  int flow(int res=0){
    FZ(h); FZ(gap);
    gap[0] = n;
    while (h[s] < n) res += DFS(s,2147483647);
    return res:
}flow;
```

#### 4.2 Dinic

```
struct Dinic{
  static const int MXN = 10000;
  struct Edge{ int v,f,re; };
  int n,s,t,level[MXN];
  vector<Edge> E[MXN];
  void init(int _n, int _s, int _t){
  n = _n;  s = _s;  t = _t;
    for (int i=0; i<n; i++) E[i].clear();</pre>
  void add_edge(int u, int v, int f){
    E[u].PB({v,f,SZ(E[v])});
    E[v].PB({u,0,SZ(E[u])-1});
  bool BFS(){
    FMO(level);
    queue<int> que;
    que.push(s);
    level[s] = 0;
    while (!que.empty()){
      int u = que.front(); que.pop();
      for (auto it : E[u]){
        if (it.f > 0 && level[it.v] == -1){
          level[it.v] = level[u]+1;
          que.push(it.v);
        }
      }
    return level[t] != -1;
  int DFS(int u, int nf){
    if (u == t) return nf;
    int res = 0;
    for (auto &it : E[u]){
      if (it.f > 0 && level[it.v] == level[u]+1){
        int tf = DFS(it.v, min(nf,it.f));
        res += tf; nf -= tf; it.f -= tf;
        E[it.v][it.re].f += tf;
        if (nf == 0) return res;
      }
    if (!res) level[u] = -1;
    return res;
  int flow(int res=0){
    while ( BFS() )
      res += DFS(s,2147483647);
    return res:
}flow;
```

### 4.3 Cost Flow

```
typedef pair<long long, long long> pll;
struct CostFlow {
 static const int MXN = 205;
 static const long long INF = 102938475610293847LL;
```

```
struct Edge {
    int v, r;
    long long f, c;
  };
  int n, s, t, prv[MXN], prvL[MXN], inq[MXN];
  long long dis[MXN], fl, cost;
  vector<Edge> E[MXN];
  void init(int _n, int _s, int _t) {
    n = _n; s = _s; t = _t;
for (int i=0; i<n; i++) E[i].clear();</pre>
    fl = cost = 0;
  void add_edge(int u, int v, long long f, long long c)
    E[u].PB(\{v, SZ(E[v]), f, c\});
    E[v].PB({u, SZ(E[u])-1, 0, -c});
  pll flow() {
    while (true) {
      for (int i=0; i<n; i++) {</pre>
        dis[i] = INF;
        inq[i] = 0;
      dis[s] = 0;
      queue<int> que;
      que.push(s);
      while (!que.empty()) {
        int u = que.front(); que.pop();
         inq[u] = 0;
         for (int i=0; i<SZ(E[u]); i++) {</pre>
          int v = E[u][i].v
           long long w = E[u][i].c;
           if (E[u][i].f > 0 && dis[v] > dis[u] + w) {
             prv[v] = u; prvL[v] = i;
             dis[v] = dis[u] + w;
             if (!inq[v]) {
               inq[v] = 1;
               que.push(v);
            }
          }
        }
      if (dis[t] == INF) break;
      long long tf = INF;
      for (int v=t, u, l; v!=s; v=u) {
        u=prv[v]; l=prvL[v];
        tf = min(tf, E[u][l].f);
      for (int v=t, u, l; v!=s; v=u) {
        u=prv[v]; l=prvL[v];
        E[u][l].f -= tf;
        E[v][E[u][l].r].f += tf;
      cost += tf * dis[t];
      fl += tf;
    return {fl, cost};
}flow;
4.4 Kuhn Munkres
```

```
struct KM{
// Maximum Bipartite Weighted Matching (Perfect Match)
  static const int MXN = 650;
  static const int INF = 2147483647; // long long
  int n,match[MXN],vx[MXN],vy[MXN];
  int edge[MXN][MXN],lx[MXN],ly[MXN],slack[MXN];
  // ^^^ long long
  void init(int _n){
    n = _n;
    for (int i=0; i<n; i++)</pre>
      for (int j=0; j<n; j++)</pre>
        edge[i][j] = 0;
  void add_edge(int x, int y, int w){ // long long
    edge[x][y] = w;
  bool DFS(int x){
    vx[x] = 1;
```

```
for (int y=0; y<n; y++){</pre>
       if (vy[y]) continue;
      if (lx[x]+ly[y] > edge[x][y]){
         slack[y] = min(slack[y], lx[x]+ly[y]-edge[x][y
      } else {
         vy[y] = 1;
         if (match[y] == -1 || DFS(match[y])){
           match[y] = x;
           return true;
      }
    }
    return false;
  int solve(){
    fill(match, match+n, -1);
    fill(lx,lx+n,-INF);
    fill(ly,ly+n,0);
    for (int i=0; i<n; i++)</pre>
       for (int j=0; j<n; j++)</pre>
        lx[i] = max(lx[i], edge[i][j]);
    for (int i=0; i<n; i++){</pre>
      fill(slack,slack+n,INF);
      while (true){
         fill(vx,vx+n,0);
         fill(vy,vy+n,0);
         if ( DFS(i) ) break;
         int d = INF; // long long
         for (int j=0; j<n; j++)
  if (!vy[j]) d = min(d, slack[j]);</pre>
         for (int j=0; j<n; j++){</pre>
           if (vx[j]) lx[j] -= d;
           if (vy[j]) ly[j] += d;
           else slack[j] -= d;
        }
      }
    int res=0;
    for (int i=0; i<n; i++)</pre>
      res += edge[match[i]][i];
    return res;
}graph;
```

### 4.5 SW-Mincut

```
struct SW{ // O(V^3)
  static const int MXN = 514;
  int n,vst[MXN],del[MXN];
  int edge[MXN][MXN],wei[MXN];
  void init(int _n){
    n = _n;
    FZ(edge);
    FZ(del);
  void add_edge(int u, int v, int w){
    edge[u][v] += w;
    edge[v][u] += w;
  void search(int &s, int &t){
    FZ(vst); FZ(wei);
    s = t = -1;
    while (true){
      int mx=-1, cur=0;
      for (int i=0; i<n; i++)
  if (!del[i] && !vst[i] && mx<wei[i])</pre>
          cur = i, mx = wei[i];
      if (mx == -1) break;
      vst[cur] = 1;
      s = t;
      t = cur;
      for (int i=0; i<n; i++)</pre>
        if (!vst[i] && !del[i]) wei[i] += edge[cur][i];
    }
  int solve(){
    int res = 2147483647;
    for (int i=0,x,y; i<n-1; i++){</pre>
      search(x,y);
```

```
res = min(res,wei[y]);
  del[y] = 1;
  for (int j=0; j<n; j++)
      edge[x][j] = (edge[j][x] += edge[y][j]);
  }
  return res;
}
}graph;</pre>
```

### 4.6 Maximum Simple Graph Matching

```
struct GenMatch { // 1-base
  static const int MAXN = 250;
  int V;
  bool el[MAXN][MAXN];
  int pr[MAXN];
  bool inq[MAXN],inp[MAXN],inb[MAXN];
  queue<int> qe;
  int st,ed;
  int nb;
  int bk[MAXN],djs[MAXN];
  int ans;
  void init(int _V) {
    V = V;
    FZ(el); FZ(pr);
    FZ(inq); FZ(inp); FZ(inb);
    FZ(bk); FZ(djs);
    ans = 0;
  void add_edge(int u, int v) {
    el[u][v] = el[v][u] = 1;
  int lca(int u,int v) {
    memset(inp,0,sizeof(inp));
    while(1) {
      u = djs[u];
      inp[u] = true;
      if(u == st) break;
      u = bk[pr[u]];
    while(1) {
      v = djs[v];
      if(inp[v]) return v;
      v = bk[pr[v]];
    return v;
  void upd(int u) {
    int v;
    while(djs[u] != nb) {
      v = pr[u];
      inb[djs[u]] = inb[djs[v]] = true;
      u = bk[v];
      if(djs[u] != nb) bk[u] = v;
    }
  void blo(int u,int v) {
    nb = lca(u,v);
    memset(inb,0,sizeof(inb));
    upd(u); upd(v);
    if(djs[u] != nb) bk[u] = v;
    if(djs[v] != nb) bk[v] = u;
    for(int tu = 1; tu <= V; tu++)</pre>
      if(inb[djs[tu]]) {
        djs[tu] = nb;
        if(!inq[tu]){
          qe.push(tu);
          inq[tu] = 1;
        }
      }
  void flow() {
    memset(inq, false, sizeof(inq));
    memset(bk,0,sizeof(bk));
    for(int i = 1; i <= V;i++)</pre>
      djs[i] = i;
    while(qe.size()) qe.pop();
    qe.push(st);
    inq[st] = 1;
```

```
ed = 0:
    while(qe.size()) {
      int u = qe.front(); qe.pop();
for(int v = 1; v <= V; v++)</pre>
        if(el[u][v] && (djs[u] != djs[v]) && (pr[u] !=
             v)) {
           if((v == st) || ((pr[v] > 0) && bk[pr[v]] >
               0))
             blo(u,v);
           else if(bk[v] == 0) {
             bk[v] = u;
             if(pr[v] > 0) {
               if(!inq[pr[v]]) qe.push(pr[v]);
             } else {
               ed = v;
               return;
          }
        }
    }
  void aug() {
    int u,v,w;
    u = ed;
    while(u > 0) {
      v = bk[u];
      w = pr[v];
      pr[v] = u;
      pr[u] = v;
      u = w;
    }
  int solve() {
    memset(pr,0,sizeof(pr));
    for(int u = 1; u <= V; u++)</pre>
      if(pr[u] == 0) {
        st = u;
        flow();
        if(ed > 0) {
          aug();
          ans ++:
        }
    return ans;
 }
};
int main() {
 gp.init(V);
  for(int i=0; i<E; i++) {</pre>
    int u, v;
    cin >> u >> v;
    gp.edge(u, v);
  cout << gp.solve() << endl;</pre>
```

### 4.7 Minimum Weight Matching (Clique version)

```
struct Graph {
 // Minimum General Weighted Matching (Perfect Match)
  static const int MXN = 105;
  int n, edge[MXN][MXN];
 int match[MXN],dis[MXN],onstk[MXN];
 vector<int> stk;
 void init(int _n) {
   n = _n;
   FZ(edge);
 void add_edge(int u, int v, int w) {
    edge[u][v] = edge[v][u] = w;
 bool SPFA(int u){
   if (onstk[u]) return true;
   stk.PB(u);
    onstk[u] = 1;
    for (int v=0; v<n; v++){</pre>
      if (u != v && match[u] != v && !onstk[v]){
```

```
int m = match[v];
        if (dis[m] > dis[u] - edge[v][m] + edge[u][v]){
          dis[m] = dis[u] - edge[v][m] + edge[u][v];
          onstk[v] = 1;
          stk.PB(v);
          if (SPFA(m)) return true;
          stk.pop_back();
          onstk[v] = 0;
        }
      }
    onstk[u] = 0;
    stk.pop_back();
    return false;
  int solve() {
    // find a match
    for (int i=0; i<n; i+=2){</pre>
      match[i] = i+1;
      match[i+1] = i;
    while (true){
      int found = 0;
      FZ(dis); FZ(onstk);
      for (int i=0; i<n; i++){</pre>
        stk.clear();
        if (!onstk[i] && SPFA(i)){
          found = 1;
          while (SZ(stk)>=2){
            int u = stk.back(); stk.pop_back();
            int v = stk.back(); stk.pop_back();
            match[u] = v;
            match[v] = u;
        }
      if (!found) break;
    int ret = 0;
    for (int i=0; i<n; i++)</pre>
      ret += edge[i][match[i]];
    ret /= 2;
    return ret;
  }
}graph;
```

### 4.8 2-Commodity Flow

```
const int MAXN = 64;
const int INF = 1029384756;
int N;
int s1, s2, t1, t2, d1, d2, S, T;
int edge[MAXN][MAXN];
int cap[MAXN][MAXN];
int h[MAXN], gap[MAXN];
bool vis[MAXN];
int isap(int v, int f)
{
    if(v == T)return f;
    if(vis[v])return 0;
    vis[v] = true;
    for(int i=0; i<N+2; i++)</pre>
        if(cap[v][i] <= 0)continue;</pre>
        if(h[i] != h[v] - 1)continue;
        int res = isap(i, min(cap[v][i], f));
        if(res > 0)
        {
            cap[v][i] -= res;
            cap[i][v] += res;
            return res;
        }
    }
```

```
gap[h[v]]--;
    if(gap[h[v]] <= 0)h[S] = N + 4;</pre>
    gap[h[v]]++;
    return 0;
}
int get_flow()
    for(int i=0; i<MAXN; i++)</pre>
        h[i] = gap[i] = 0;
    gap[0] = N + 2;
    int flow = 0;
    while(h[S] \le N + 3)
        for(int i=0; i<N+2; i++)</pre>
        {
             vis[i] = false;
        int df = isap(S, INF);
        flow += df;
    return flow;
}
int main()
    ios_base::sync_with_stdio(0);
    int TT;
    cin>>TT;
    while(TT--)
        cin>>N;
        cin>>s1>>t1>>d1>>s2>>t2>>d2;
        for(int i=0; i<MAXN; i++)</pre>
             for(int j=0; j<MAXN; j++)</pre>
                 edge[i][j] = 0;
        }
        for(int i=0; i<N; i++)</pre>
             string s;
             cin>>s:
             for(int j=0; j<N; j++)</pre>
             {
                 if(s[j] == 'X')edge[i][j] = 0;
                 else if(s[j] == '0')edge[i][j] = 1;
                 else if(s[j] == 'N')edge[i][j] = INF;
        }
        int ans = 0;
        S = N;
        T = N + 1;
        //first
        for(int i=0; i<MAXN; i++)</pre>
        {
             for(int j=0; j<MAXN; j++)</pre>
                 cap[i][j] = edge[i][j];
        }
        cap[S][s1] = cap[t1][T] = d1;
        cap[S][s2] = cap[t2][T] = d2;
        ans = get_flow();
```

```
12
         //second
         for(int i=0; i<MAXN; i++)</pre>
             for(int j=0; j<MAXN; j++)</pre>
                 cap[i][j] = edge[i][j];
             }
         }
         cap[S][s1] = cap[t1][T] = d1;
        cap[S][t2] = cap[s2][T] = d2;
        ans = min(ans, get_flow());
         cout<<(ans == d1 + d2 ? "Yes" : "No")<<endl;</pre>
    return 0;
}
     (+1) SW-mincut O(NM)
4.9
// {{{ StoerWagner
const int inf=10000000000;
// should be larger than max.possible mincut
class StoerWagner {
  public:
    int n,mc; // node id in [0,n-1]
vector<int> adj[MAXN];
    int cost[MAXN][MAXN];
    int cs[MAXN];
    bool merged[MAXN],sel[MAXN];
    // --8<-- include only if cut is explicitly needed
      DisjointSet djs;
    vector<int> cut;
    //--8<--
      StoerWagner(int _n):n(_n),mc(inf),djs(_n) {
        for(int i=0;i<n;i++)</pre>
          merged[i]=0;
         for(int i=0;i<n;i++)</pre>
           for(int j=0;j<n;j++)</pre>
             cost[i][j]=cost[j][i]=0;
    void append(int v,int u,int c) {
      if(v==u) return:
      if(!cost[v][u]&&c) {
        adj[v].PB(u);
        adj[u].PB(v);
      cost[v][u]+=c;
      cost[u][v]+=c;
    void merge(int v,int u) {
      merged[u]=1;
      for(int i=0;i<n;i++)</pre>
        append(v,i,cost[u][i]);
      // --8<-- include only if cut is explicitly
           needed
        djs.merge(v,u);
```

void phase() {

cs[v]=0; sel[v]=0;

int v,s,pv;
while(pq.size()) {

}
pv=v;

priority\_queue<pii> pq;
for(int v=0;v<n;v++) {</pre>

pq.push({0,v});

pq.pop();

continue;

v=pq.top().S; s=pq.top().F;

if(merged[v]) continue;

if(cs[pq.top().S]>pq.top().F) {

```
pq.pop();
        sel[v]=1;
        for(int i=0;i<adj[v].size();i++) {</pre>
          int u=adj[v][i];
          if(merged[u]||sel[u]) continue;
          cs[u]+=cost[v][u];
          pq.push({cs[u],u});
        }
      if(s<mc) {</pre>
        // --8<-- include only if cut is explicitly
        needed -----
          cut.clear();
        for(int i=0;i<n;i++)</pre>
          if(djs.getrep(i)==djs.getrep(v)) cut.PB(i);
      merge(v,pv);
    }
    int mincut() {
      if(mc==inf) {
        for(int t=0;t<n-1;t++)</pre>
          phase();
      return mc;
    // --8<-- include only if cut is explicitly needed
      vector<int> getcut() { // return one side of the
          cut
        mincut();
        return cut:
    //--8<-----
};
// }}}
```

### 5 Math

### 5.1 ax+by=gcd

```
typedef pair<int, int> pii;

pii gcd(int a, int b){
   if(b == 0) return make_pair(1, 0);
   else{
      int p = a / b;
      pii q = gcd(b, a % b);
      return make_pair(q.second, q.first - q.second * p);
   }
}
```

### 5.2 Fast Fourier Transform

```
// const int MAXN = 262144;
// (must be 2^k)
typedef long double ld;
typedef complex<ld> cplx;
const ld PI = acosl(-1);
const cplx I(0, 1);
cplx omega[MAXN+1];
void pre_fft()
{
  for(int i=0; i<=MAXN; i++)</pre>
    omega[i] = exp(i * 2 * PI / MAXN * I);
void fft(int n, cplx a[], bool inv=false)
  int basic = MAXN / n;
  int theta = basic;
  for (int m = n; m >= 2; m >>= 1) {
    int mh = m >> 1;
for (int i = 0; i < mh; i++) {</pre>
```

```
cplx w = omega[inv ? MAXN-(i*theta%MAXN) : i*
           theta%MAXN];
      for (int j = i; j < n; j += m) {</pre>
        int k = j + mh;
        cplx x = a[j] - a[k];
        a[j] += a[k];
        a[k] = w * x;
      }
    }
    theta = (theta * 2) % MAXN;
  int i = 0;
  for (int j = 1; j < n - 1; j++) {
    for (int k = n >> 1; k > (i ^= k); k >>= 1);
    if (j < i) swap(a[i], a[j]);</pre>
  if (inv)
    for (i = 0; i < n; i++)</pre>
      a[i] /= n;
}
```

### 5.3 Fast Linear Recurrence

```
ll n,m,dp[N+N];
void pre_dp(){
  dp[0]= 1;
  ll bdr = min(m+m,n);
  for(ll i=1; i<=bdr; i++)</pre>
     for(ll j=i-1; j>=max(0ll,i-m); j--)
       dp[i]= add(dp[i],dp[j]);
vector<ll> Mul(const vector<ll>& v1,const vector<ll>&
    v2){
  int sz1 = (int)v1.size();
  int sz2 = (int)v2.size();
  assert(sz1 == m and sz2 == m);
  vector<ll> _v(m+m);
for(int i=0; i<m+m; i++) _v[i]= 0;</pre>
  // expand
  for(int i=0; i<sz1; i++)</pre>
     for(int j=0; j<sz2; j++)</pre>
       _v[i+j+1]= add(_v[i+j+1],mul(v1[i],v2[j]));
  // shrink
  for(int i=0; i<m; i++)</pre>
    for(int j=1; j<=m; j++)</pre>
       _v[i + j]= add(_v[i + j],_v[i]);
  for(int i=0; i<m; i++)</pre>
    _v[i]= _v[i + m];
  _v.resize(m);
  return _v;
vector<ll> I,A;
ll solve(){
  pre dp();
  if(n <= m+m)return dp[n];</pre>
  I.resize(m);
  A.resize(m);
  for(int i=0; i<m; i++) I[i]=A[i]=1;</pre>
  // dp[n]= /Sum_{i=0}^{m-1} A_i * dp[n - i - 1]
  ll dlt = (n - m) / m;
  ll rdlt = dlt * m;
  while(dlt){
    if(dlt & 1ll) I = Mul(I,A);
    A = Mul(A,A);
    dlt >>= 1;
  ll ans = 0;
  for(int i=0; i<m; i++)</pre>
    ans = add(ans,mul(I[i],dp[n-i-1-rdlt]));
  return ans;
}
```

### 5.4 (+1) ntt

```
int P=605028353,root=3,MAXNUM=262144;
// Remember coefficient are mod P
/*
p=a*2^n+1
```

```
2^n
                                       root
                 97
5
    32
                                3
    64
                 193
                                3
6
                                       5
7
                 257
                                2
                                       3
    128
    256
8
                 257
                                1
                                       3
    512
                 7681
                                15
                                       17
10
                 12289
                                12
                                                              };
   1024
                                       11
    2048
                 12289
                                6
                                       11
11
   4096
                 12289
12
                                3
                                       11
                                5
13
   8192
                 40961
                                       3
                                                               5.5
                                       3
14
    16384
                 65537
                                4
15
   32768
                 65537
                                2
                                       3
16
   65536
                 65537
                                1
                                       3
17
    131072
                 786433
                                6
                                       10
                                       10 (605028353,
18
   262144
                 786433
                                3
    2308, 3)
19
   524288
                 5767169
                                       3
                                11
20
   1048576
                 7340033
                                       3
                 23068673
                                11
   2097152
                                       3
                 104857601
                                       3
22
   4194304
                                25
23
   8388608
                 167772161
                                20
                                       3
   16777216
                 167772161
                                10
                                       3 (1107296257, 33,
25
                 167772161
   33554432
    10)
   67108864
                 469762049
                                15
27
   134217728
                 2013265921
                                       31
                                                               {
int bigmod(long long a,int b){
  if(b==0)return 1;
  return (bigmod((a*a)%P,b/2)*(b%2?a:1ll))%P;
int inv(int a,int b){
  if(a==1)return 1;
                                                               {
  return (((long long)(a-inv(b%a,a))*b+1)/a)%b;
std::vector<long long> ps(MAXNUM);
std::vector<int> rev(MAXNUM);
struct poly{
                                                               }
  std::vector<unsigned int> co;
  int n;//polynomial degree = n
  poly(int d){n=d;co.resize(n+1,0);}
  void trans2(int NN){
    int r=0,st,N;
    unsigned int a,b;
    while((1<<r)<(NN>>1))++r;
    for (N=2; N<=NN; N<<=1, --r) {</pre>
      for(st=0;st<NN;st+=N){</pre>
        int i,ss=st+(N>>1);
        for(i=(N>>1)-1;i>=0;--i){
          a=co[st+i]; b=(ps[i<<r]*co[ss+i])%P;</pre>
          co[st+i]=a+b; if(co[st+i]>=P)co[st+i]-=P;
          co[ss+i]=a+P-b; if(co[ss+i]>=P)co[ss+i]-=P;
        }
      }
   }
  void trans1(int NN){
    int r=0,st,N;
    unsigned int a,b;
    for (N=NN; N>1; N>>=1,++r) {
      for(st=0;st<NN;st+=N){</pre>
        int i,ss=st+(N>>1);
        for(i=(N>>1)-1;i>=0;--i){
           a=co[st+i]; b=co[ss+i];
          co[st+i]=a+b; if(co[st+i]>=P)co[st+i]-=P;
          co[ss+i]=((a+P-b)*ps[i<<r])%P;
        }
      }
   }
  poly operator*(const poly& _b)const{
    poly a=*this,b=_b;
    int k=n+b.n,i,N=1;
    while (N \le k) N = 2;
    a.co.resize(N,0); b.co.resize(N,0);
    int r=bigmod(root, (P-1)/N), Ni=inv(N,P);
    ps[0]=1;
    for(i=1;i<N;++i)ps[i]=(ps[i-1]*r)%P;</pre>
    a.trans1(N);b.trans1(N);
                                                                   u>>=1;
    for(i=0;i<N;++i)a.co[i]=((long long)a.co[i]*b.co[i</pre>
                                                                   t++;
        ])%P
                                                                 while(s--) {
```

```
r=inv(r,P):
    for(i=1;i<N/2;++i)std::swap(ps[i],ps[N-i]);</pre>
    a.trans2(N):
    for(i=0;i<N;++i)a.co[i]=((long long)a.co[i]*Ni)%P;</pre>
    a.n=n+_b.n; return a;
     Mod
/// _fd(a,b) floor(a/b).
/// _rd(a,m) a-floor(a/m)*m.
/// _pv(a,m,r) largest x s.t x<=a && x%m == r.
/// _nx(a,m,r) smallest x s.t x>=a && x%m == r.
/// \_ct(a,b,m,r) |A|, A = \{ x : a <= x <= b && x %m == r \}.
int _fd(int a,int b){ return a<0?(-~a/b-1):a/b; }</pre>
int _rd(int a,int m){ return a-_fd(a,m)*m; }
int _pv(int a,int m,int r)
    r = (r\%m + m)\%m;
    return _fd(a-r,m)*m+r;
int _nt(int a,int m,int r)
    m=abs(m):
    r=(r%m+m)%m;
    return _fd(a-r-1,m)*m+r+m;
int _ct(int a,int b,int m,int r)
    m=abs(m);
    a=_nt(a,m,r);
    b = pv(b, m, r)
    return (a>b)?0:((b-a+m)/m);
5.6 (+1) Miller Rabin
// n < 4,759,123,141
                              3: 2, 7, 61
// n < 1,122,004,669,633
                              4:
                                   2, 13, 23, 1662803
// n < 3,474,749,660,383
                                    6 : pirmes <= 13
// n < 2^64
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
// Make sure testing integer is in range [2, n-2] if
// you want to use magic.
long long power(long long x,long long p,long long mod){
  long long s=1,m=x;
  while(p) {
    if(p&1) s=mult(s,m,mod);
    p>>=1;
    m=mult(m,m,mod);
  return s;
bool witness(long long a,long long n,long long u,int t)
  long long x=power(a,u,n);
for(int i=0;i<t;i++) {</pre>
    long long nx=mult(x,x,n);
    if(nx==1&&x!=1&&x!=n-1) return 1;
    x=nx;
  return x!=1;
bool miller_rabin(long long n,int s=100) {
  // iterate s times of witness on n
    return 1 if prime, 0 otherwise
  if(n<2) return 0;</pre>
  if(!(n&1)) return n==2;
  long long u=n-1;
  int t=0;
  // n-1 = u*2^t
  while(!(u&1)) {
```

```
long long a=randll()%(n-1)+1;
if(witness(a,n,u,t)) return 0;
}
return 1;
}
```

#### 5.7 Pollard Rho

```
// does not work when n is prime
long long modit(long long x,long long mod) {
  if(x>=mod) x-=mod;
  //if(x<0) x+=mod;
  return x;
long long mult(long long x,long long y,long long mod) {
  long long s=0, m=x%mod;
  while(y) {
    if(y&1) s=modit(s+m,mod);
    y>>=1;
    m=modit(m+m,mod);
  return s;
long long f(long long x,long long mod) {
  return modit(mult(x,x,mod)+1,mod);
long long pollard_rho(long long n) {
  if(!(n&1)) return 2;
  while (true) {
    long long y=2, x=rand()%(n-1)+1, res=1;
    for (int sz=2; res==1; sz*=2) {
      for (int i=0; i<sz && res<=1; i++) {</pre>
        x = f(x, n);
        res = \_gcd(abs(x-y), n);
      y = x;
    if (res!=0 && res!=n) return res;
  }
}
```

### 5.8 Algorithms about Primes

```
* 12721
* 13331
* 14341
* 75577
* 123457
* 222557
* 556679
* 999983
* 1097774749
* 1076767633
  100102021
* 999997771
* 1001010013
  1000512343
* 987654361
 * 999991231
* 999888733
* 98789101
 * 987777733
* 999991921
* 1010101333
 * 1010102101
* 10000000000039
 * 1000000000000037
* 2305843009213693951
* 4611686018427387847
* 9223372036854775783
* 18446744073709551557
int mu[MX],p_tbl[MX];
vector<int> primes;
void sieve() {
 mu[1] = p_tbl[1] = 1;
```

```
for (int i=2; i<MX; i++) {</pre>
    if (!p_tbl[i]) {
      p_tbl[i] = i;
      primes.PB(i);
      mu[i] = -1;
    for (auto p : primes) {
      int x = i*p;
      if (x >= M) break;
      p_{thl}[x] = p;
      mu[x] = -mu[i];
      if (i%p==0) {
        mu[x] = 0;
        break;
      }
    }
  }
}
vector<int> factor(int x) {
  vector<int> fac{1};
  while (x > 1) {
    int fn=SZ(fac), p=p_tbl[x], pos=0;
    while (x%p == 0) {
      x /= p;
      for (int i=0; i<fn; i++)</pre>
        fac.PB(fac[pos++]*p);
    }
  }
  return fac;
}
```

# 5.9 (+1) PolynomialGenerator

```
class PolynomialGenerator {
  /* for a nth-order polynomial f(x), *
   * given f(0), f(1), ..., f(n) *
   * express f(x) as sigma_i\{c_i*C(x,i)\} */
  public:
    int n;
    vector<long long> coef;
    // initialize and calculate f(x), vector _fx should
          be
    // filled with f(0) to f(n)
      PolynomialGenerator(int _n,vector<long long> _fx)
           ),coef(_fx) {
         for(int i=0;i<n;i++)</pre>
           for(int j=n;j>i;j--)
             coef[j]-=coef[j-1];
    // evaluate f(x), runs in O(n)
    long long eval(int x) {
      long long m=1,ret=0;
      for(int i=0;i<=n;i++) {</pre>
         ret+=coef[i]*m;
        m=m*(x-i)/(i+1);
       return ret;
};
```

# 5.10 Pseudoinverse of Square matrix

```
Mat pinv(Mat m)
{
   Mat res = I;

FZ(used);
   for(int i=0; i<W; i++)
   {
      int piv = -1;
      for(int j=0; j<W; j++)
      {
        if(used[j]) continue;
        if(abs(m.v[j][i]) > EPS)
      {
            piv = j;
      }
}
```

```
break:
      }
    if(piv == -1)
      continue;
    used[i] = true;
    swap(m.v[piv], m.v[i]);
    swap(res.v[piv], res.v[i]);
    ld rat = m.v[i][i];
    for(int j=0; j<W; j++)</pre>
    {
      m.v[i][j] /= rat;
      res.v[i][j] /= rat;
    for(int j=0; j<W; j++)</pre>
      if(j == i) continue;
      rat = m.v[j][i];
       for(int k=0; k<W; k++)</pre>
        m.v[j][k] -= rat * m.v[i][k];
        res.v[j][k] -= rat * res.v[i][k];
    }
  }
  for(int i=0; i<W; i++)</pre>
    if(used[i]) continue;
    for(int j=0; j<W; j++)</pre>
      res.v[i][j] = 0;
  return res;
}
```

# 5.11 Theorom

#### 5.11.1 Lucas' Theorem

For non-negative integer n,m and prime  $p,\binom{m}{n}\equiv\prod_{i=0}^k\binom{m_i}{n_i}\pmod{p}$  where  $m_i$  is the i-th digit of m in base p.

### 5.11.2 Sum of Two Squares Thm (Legendre)

For a given positive integer n, let  $D_1=$  (# of positive integers d dividing N that  $1\equiv d\pmod 4$ )  $D_3=$  (# of positive integers d dividing N that  $3\equiv d\pmod 4$ ) then n can be written as a sum of two squares in exactly  $R(n)=4(D_1-D_3)$  ways.

### 5.11.3 Difference of D1-D3 Thm

```
\begin{array}{l} \text{let } n=2^t\cdot (p_1^{e_1}\cdot\ldots\cdot p_r^{e_r})\cdots (q_1^{f_1}\cdot\ldots\cdot q_s^{f_s})\\ \text{where } p_i,q_i \text{ are primes and } 1\equiv p_i\pmod 4, 3\equiv q_i\pmod 4\\ \text{then } D_1-D_3=\begin{cases} (\mathbf{e}_1+1)(e_2+1)...(e_r+1), & \text{if } (\mathbf{f}_i)\text{s all even}\\ 0, & \text{if any } \mathbf{f}_i \text{ is odd} \end{cases}
```

### 5.11.4 Krush-Kuhn-Tucker Conditions

```
Stationarity
```

```
Stationarity For maximizing f(x): \nabla f(x^*) = \sum_{i=1}^m \mu_i \nabla g_i(x^*) + \sum_{j=1}^l \lambda_j \nabla h_j(x^*) For minimizing f(x): -\nabla f(x^*) = \sum_{i=1}^m \mu_i \nabla g_i(x^*) + \sum_{j=1}^l \lambda_j \nabla h_j(x^*) Primal feasibility g_i(x^*) \leq 0, for all i=1,\ldots,m h_j(x^*) = 0, for all j=1,\ldots,l Dual feasibility \mu_i \geq 0, for all i=1,\ldots,m Complementary slackness \mu_i g_i(x^*) = 0, for all i=1,\ldots,m
```

# 5.11.5 Chinese remainder theorem

```
\begin{array}{c} x \equiv r_i \mod p_i \\ N = \prod p_i \end{array}
```

```
N_i = N/p_i
x \equiv \sum r_i N_i (N_i)_{p_i}^{-1} \mod N
```

# 5.12 Simplex

```
const int maxn = 111;
const int maxm = 111;
const double eps = 1E-10;
double a[maxn][maxm], b[maxn], c[maxm], d[maxn][maxm];
double x[maxm];
int ix[maxn + maxm]; // !!! array all indexed from 0
// max{cx} subject to {Ax<=b,x>=0}
// n: constraints, m: vars !!!
// x[] is the optimal solution vector
//
// usage :
// value = simplex(a, b, c, N, M);
double simplex(double a[maxn][maxm], double b[maxn],
    double c[maxm], int n, int m) {
    ++m:
    int r = n, s = m - 1;
    memset(d, 0, sizeof(d));
    for (int i = 0; i < n + m; ++i) ix[i] = i;</pre>
    for (int i = 0; i < n; ++i) {
    for (int j = 0; j < m - 1; ++j) d[i][j] = -a[i]</pre>
             ][j];
        d[i][m - 1] = 1;
d[i][m] = b[i];
         if (d[r][m] > d[i][m]) r = i;
    for (int j = 0; j < m - 1; ++j) d[n][j] = c[j];</pre>
    d[n + 1][m - 1] = -1;
    for (double dd;; ) {
         if (r < n) {
             int t = ix[s]; ix[s] = ix[r + m]; ix[r + m]
             d[r][s] = 1.0 / d[r][s];
             for (int j = 0; j <= m; ++j) if (j != s) d[</pre>
                 r][j] *= -d[r][s];
             for (int i = 0; i <= n + 1; ++i) if (i != r
                 for (int j = 0; j <= m; ++j) if (j != s</pre>
                      ) d[i][j] += d[r][j] * d[i][s];
                 d[i][s] *= d[r][s];
             }
        }
         r = -1; s = -1;
         for (int j = 0; j < m; ++j) if (s < 0 || ix[s]</pre>
             > ix[j]) {
             if (d[n + 1][j] > eps || (d[n + 1][j] > -
                 eps && d[n][j] > eps)) s = j;
         if (s < 0) break;</pre>
         for (int i = 0; i < n; ++i) if (d[i][s] < -eps)</pre>
             if (r < 0 || (dd = d[r][m] / d[r][s] - d[i</pre>
                 ][m] / d[i][s]) < -eps || (dd < eps &&
                 ix[r + m] > ix[i + m])) r = i;
        if (r < 0) return -1; // not bounded</pre>
    if (d[n + 1][m] < -eps) return -1; // not
         executable
    double ans = 0;
    for(int i=0; i<m; i++) x[i] = 0;</pre>
    for (int i = m; i < n + m; ++i) { // the missing</pre>
         enumerated x[i] = 0
         if (ix[i] < m - 1)</pre>
             ans += d[i - m][m] * c[ix[i]];
             x[ix[i]] = d[i-m][m];
    }
    return ans;
}
```

# 6 Geometry

### 6.1 Point operators

```
#include<bits/stdc++.h>
using namespace std;
#define _x first
#define _y second
typedef pair<double, double> pdd;
pdd operator + (const pdd p1, const pdd p2){
 return pdd(p1._x + p2._x, p1._y + p2._y);
pdd operator - (const pdd p1, const pdd p2){
 return pdd(p1._x - p2._x, p1._y - p2._y);
pdd operator * (const double c, const pdd p){
 return pdd(p._x * c, p._y * c);
pdd operator - (const pdd p){
 return (-1.0) * p;
double operator * (const pdd p1, const pdd p2){
  return p1._x * p2._x + p1._y * p2._y;
double operator % (const pdd p1, const pdd p2){
 return p1._x * p2._y - p2._x * p1._y;
```

### 6.2 Intersection of two circles

### 6.3 Intersection of two lines

```
#include<bits/stdc++.h>
using namespace std;
const double EPS = 1e-9;

pdd interPnt(pdd p1, pdd p2, pdd q1, pdd q2){
    double f1 = (p2 - p1) % (q1 - p1);
    double f2 = (p2 - p1) % (p1 - q2);
    double f = (f1 + f2);

if(fabs(f) < EPS) return pdd(nan(""), nan(""));

return (f2 / f) * q1 + (f1 / f) * q2;
}</pre>
```

#### 6.4 Half Plane Intersection

```
typedef pair<point, point> Line;
ostream& operator << (ostream& o, const Line &p) {
    return o << p.F << " - " << p.S;
}</pre>
```

```
template<tvpename T>
ostream& operator << (ostream& o, const vector<T> &v) {
    o << "\[";
    for (auto x: v) o << x << ", ";</pre>
    return o << "]";
}
point interPnt(Line l1, Line l2, bool &res){
    point p1, p2, q1, q2;
    tie(p1, p2) = l1;
  tie(q1, q2) = l2;

double f1 = (p2 - p1).cross(q1 - p1);
  double f2 = (p2 - p1).cross(p1 - q2);
  double f = (f1 + f2);
    if(fabs(f) < EPS) {</pre>
        res = false;
        return {0, 0};
    res = true;
  return (f2 / f) * q1 + (f1 / f) * q2;
bool isin(Line l0, Line l1, Line l2) {
    // Check inter(l1, l2) in l0
    bool res;
    point p = interPnt(l1, l2, res);
    return (l0.S - l0.F).cross(p - l0.F) > 1e-9;
/* If no solution, check: 1. ret.size() < 3</pre>
 * Or more precisely, 2. interPnt(ret[0], ret[1])
 * in all the lines. (use (l.S - l.F).cross(p - l.F) >
 */
vector<Line> halfPlaneInter(vector<Line> lines) {
    int sz = lines.size();
    vector<double> ata(sz), ord(sz);
    for (int i=0; i<sz; i++) {
        ord[i] = i;
        point d = lines[i].S - lines[i].F;
        ata[i] = atan2(d.y, d.x);
    sort(ALL(ord), [&](int i, int j) {
        if (abs(ata[i] - ata[j]) < EPS) {</pre>
            return (lines[i].S - lines[i].F).cross(
                 lines[j].S - lines[i].F) < 0;</pre>
        return ata[i] < ata[j];</pre>
    vector<Line> fin;
    for (int i=0; i<sz; i++) {</pre>
        if (!i or fabs(ata[ord[i]] - ata[ord[i-1]]) >
             EPS) {
            fin.PB(lines[ord[i]]);
        }
    }
    deque<Line> dq;
    for (int i=0; i<SZ(fin); i++) {</pre>
        while(SZ(dq) >= 2 and
               not isin(fin[i], dq[SZ(dq)-2], dq[SZ(dq)
                   -1])) {
            dq.pop_back();
        while(SZ(dq) >= 2 and
              not isin(fin[i], dq[0], dq[1])) {
            dq.pop_front();
        dq.push_back(fin[i]);
    }
    while (SZ(dq) >= 3 and
           not isin(dq[0], dq[SZ(dq)-2], dq[SZ(dq)-1]))
        dq.pop_back();
    while (SZ(dq) >= 3 and
           not isin(dq[SZ(dq)-1], dq[0], dq[1])) {
        dq.pop_front();
```

```
}
vector<Line> res(ALL(dq));
return res;
}
```

#### 6.5 Convex Hull

```
double cross(pdd o, pdd a, pdd b){
 return (a-o) % (b-o);
vector<pdd> convex_hull(vector<pdd> pt){
 sort(pt.begin(),pt.end());
  int top=0;
 vector<pdd> stk(2*pt.size());
 for (int i=0; i<(int)pt.size(); i++){</pre>
    while (top >= 2 && cross(stk[top-2],stk[top-1],pt[i
       ]) <= 0)
      top--;
    stk[top++] = pt[i];
  for (int i=pt.size()-2, t=top+1; i>=0; i--){
    while (top >= t && cross(stk[top-2],stk[top-1],pt[i
       ]) <= 0)
      top--;
   stk[top++] = pt[i];
 stk.resize(top-1);
  return stk;
}
```

### 6.6 Minimum Covering Circle

```
struct Mcc{
  // return pair of center and r^2
  static const int MAXN = 1000100;
  int n;
  pdd p[MAXN],cen;
  double r2;
  void init(int _n, pdd _p[]){
    n = _n;
    memcpy(p,_p,sizeof(pdd)*n);
  double sqr(double a){ return a*a; }
  double abs2(pdd a){ return a*a; }
  pdd center(pdd p0, pdd p1, pdd p2) {
    pdd a = p1-p0;
    pdd b = p2-p0;
    double c1=abs2(a)*0.5;
    double c2=abs2(b)*0.5;
    double d = a % b;
    double x = p0.x + (c1 * b.y - c2 * a.y) / d;
double y = p0.y + (a.x * c2 - b.x * c1) / d;
    return pdd(x,y);
  }
  pair<pdd,double> solve(){
    random_shuffle(p,p+n);
    for (int i=0; i<n; i++){</pre>
      if (abs2(cen-p[i]) <= r2) continue;</pre>
      cen = p[i];
      r2 = 0;
       for (int j=0; j<i; j++){</pre>
         if (abs2(cen-p[j]) <= r2) continue;</pre>
         cen = 0.5 * (p[i]+p[j]);
         r2 = abs2(cen-p[j]);
         for (int k=0; k<j; k++){</pre>
           if (abs2(cen-p[k]) <= r2) continue;</pre>
           cen = center(p[i],p[j],p[k]);
           r2 = abs2(cen-p[k]);
         }
    return {cen,r2};
  }
}mcc;
```

# 6.7 KDTree (Nearest Point)

```
const int MXN = 100005;
struct KDTree {
  struct Node {
    int x,y,x1,y1,x2,y2;
    int id,f;
    Node *L, *R;
  }tree[MXN];
  int n;
  Node *root;
  long long dis2(int x1, int y1, int x2, int y2) {
    long long dx = x1-x2;
    long long dy = y1-y2;
    return dx*dx+dy*dy;
  static bool cmpx(Node& a, Node& b){ return a.x<b.x; }</pre>
  static bool cmpy(Node& a, Node& b){ return a.y<b.y; }</pre>
  void init(vector<pair<int,int>> ip) {
    n = ip.size();
    for (int i=0; i<n; i++) {
  tree[i].id = i;</pre>
      tree[i].x = ip[i].first;
      tree[i].y = ip[i].second;
    root = build_tree(0, n-1, 0);
  Node* build_tree(int L, int R, int dep) {
    if (L>R) return nullptr;
    int M = (L+R)/2;
    tree[M].f = dep%2;
    nth_element(tree+L, tree+M, tree+R+1, tree[M].f ?
         cmpy : cmpx);
    tree[M].x1 = tree[M].x2 = tree[M].x;
    tree[M].y1 = tree[M].y2 = tree[M].y;
    tree[M].L = build_tree(L, M-1, dep+1);
    if (tree[M].L) {
      tree[M].x1 = min(tree[M].x1, tree[M].L->x1);
      tree[M].x2 = max(tree[M].x2, tree[M].L->x2);
tree[M].y1 = min(tree[M].y1, tree[M].L->y1);
       tree[M].y2 = max(tree[M].y2, tree[M].L->y2);
    tree[M].R = build_tree(M+1, R, dep+1);
    if (tree[M].R) {
      tree[M].x1 = min(tree[M].x1, tree[M].R->x1);
      tree[M].x2 = max(tree[M].x2, tree[M].R->x2);
      tree[M].y1 = min(tree[M].y1, tree[M].R->y1);
tree[M].y2 = max(tree[M].y2, tree[M].R->y2);
    return tree+M;
  int touch(Node* r, int x, int y, long long d2){
    long long dis = sqrt(d2)+1;
    if (x<r->x1-dis || x>r->x2+dis || y<r->y1-dis || y>
         r->y2+dis)
      return 0;
    return 1:
  void nearest(Node* r, int x, int y, int &mID, long
      long &md2) {
    if (!r || !touch(r, x, y, md2)) return;
    long long d2 = dis2(r->x, r->y, x, y);
    if (d2 < md2 || (d2 == md2 && mID < r->id)) {
      mID = r -> id;
      md2 = d2;
    // search order depends on split dim
    if ((r->f == 0 && x < r->x) ||
         (r->f == 1 \&\& y < r->y)) {
      nearest(r\rightarrow L, x, y, mID, md2);
      nearest(r\rightarrow R, x, y, mID, md2);
    } else {
      nearest(r\rightarrow R, x, y, mID, md2);
       nearest(r->L, x, y, mID, md2);
    }
  }
```

```
National Taiwan University bcw0x1bd2
  int query(int x, int y) {
                                                                  int L,R,M;
    int id = 1029384756;
                                                                  if(p.x<Lx || p.x>Rx) return 0;
    long long d2 = 102938475612345678LL;
                                                                  L=0:R=dn:
                                                                  while (L<R-1) { M=(L+R)/2;
    nearest(root, x, y, id, d2);
    return id;
                                                                    if(p.x<qt[M].x) R=M; else L=M; }</pre>
  }
                                                                    if(tri(qt[L],qt[R],p)<0) return 0;</pre>
}tree;
                                                                    L=0; R=un;
                                                                    while (L<R-1) { M=(L+R)/2;
                                                                      if(p.x<rt[M].x) R=M; else L=M; }</pre>
                                                                      if(tri(rt[L],rt[R],p)>0) return 0;
      (+1) MinkowskiSum
/* convex hull Minkowski Sum*/
                                                               int main(){
#define INF 1000000000000000LL
                                                                  int n,m,i;
                                                                  PT p;
class PT{ public:
  long long x,y;
                                                                  scanf("%d",&n);
  int POS(){
                                                                  for(i=0;i<n;i++) scanf("%I64d %I64d",&pt[i].x,&pt[i].</pre>
    if(y==0) return x>0?0:1;
                                                                  scanf("%d",&m);
    return y>0?0:1;
                                                                  for(i=0;i<m;i++) scanf("%I64d %I64d",&qt[i].x,&qt[i].</pre>
  }
                                                                      y);
PT pt[300000],qt[300000],rt[300000];
                                                                  n=minkowskiSum(n,m);
                                                                  for(i=0;i<n;i++) pt[i]=rt[i];</pre>
long long Lx,Rx;
                                                                  scanf("%d",&m);
int dn,un;
inline bool cmp(PT a,PT b){
                                                                  for(i=0;i<m;i++) scanf("%I64d %I64d",&qt[i].x,&qt[i].</pre>
  int pa=a.POS(),pb=b.POS();
                                                                      y);
  if(pa==pb) return (a^b)>0;
                                                                  n=minkowskiSum(n,m);
                                                                  for(i=0;i<n;i++) pt[i]=rt[i];</pre>
  return pa<pb;</pre>
                                                                  initInConvex(n);
                                                                  scanf("%d",&m);
int minkowskiSum(int n,int m){
                                                                  for(i=0;i<m;i++){</pre>
  int i,j,r,p,q,fi,fj;
  for(i=1,p=0;i<n;i++){</pre>
                                                                    scanf("%I64d %I64d",&p.x,&p.y);
                                                                    p.x*=3; p.y*=3;
    if(pt[i].y<pt[p].y || (pt[i].y==pt[p].y && pt[i].x<</pre>
                                                                    puts(inConvex(p)?"YES":"NO");
           pt[p].x)) p=i; }
  for(i=1,q=0;i<m;i++){</pre>
    if(qt[i].y<qt[q].y || (qt[i].y==qt[q].y && qt[i].x<</pre>
           qt[q].x)) q=i; }
  rt[0]=pt[p]+qt[q];
  r=1; i=p; j=q; fi=fj=0;
  while(1){
    if((fj&&j==q) || ((!fi||i!=p) && cmp(pt[(p+1)%n]-pt
             p],qt[(q+1)%m]-qt[q]))){
      rt[r]=rt[r-1]+pt[(p+1)%n]-pt[p];
      p=(p+1)%n;
       fi=1;
    }else{
      rt[r]=rt[r-1]+qt[(q+1)%m]-qt[q];
      q=(q+1)%m;
      fj=1;
    if(r<=1 || ((rt[r]-rt[r-1])^(rt[r-1]-rt[r-2]))!=0)</pre>
    else rt[r-1]=rt[r];
    if(i==p && j==q) break;
  }
  return r-1;
void initInConvex(int n){
  int i,p,q;
  long long Ly,Ry;
  Lx=INF; Rx=-INF;
  for(i=0;i<n;i++){</pre>
    if(pt[i].x<Lx) Lx=pt[i].x;
if(pt[i].x>Rx) Rx=pt[i].x;
```

Ly=Ry=INF;

for(i=0;i<n;i++){</pre>

for(i=0;i<n;i++){</pre>

rt[un]=pt[q];

qt[dn]=pt[q]; Ly=Ry=-INF;

inline int inConvex(PT p){

if(pt[i].x==Lx && pt[i].y<Ly){ Ly=pt[i].y; p=i; }</pre> if(pt[i].x==Rx && pt[i].y<Ry){ Ry=pt[i].y; q=i; }</pre>

if(pt[i].x==Lx && pt[i].y>Ly){ Ly=pt[i].y; p=i; } if(pt[i].x==Rx && pt[i].y>Ry){ Ry=pt[i].y; q=i; }

for(un=0,i=p;i!=q;i=(i+n-1)%n){ rt[un++]=pt[i]; }

for(dn=0,i=p;i!=q;i=(i+1)%n){ qt[dn++]=pt[i]; }

# 7 Stringology

### 7.1 Suffix Array

```
const int MAX = 1020304;
int ct[MAX], he[MAX], rk[MAX], sa[MAX], tsa[MAX], tp[
     MAX][2];
void suffix_array(char *ip){
  int len = strlen(ip);
  int alp = 256;
  memset(ct, 0, sizeof(ct));
  for(int i=0;i<len;i++) ct[ip[i]+1]++;</pre>
  for(int i=1;i<alp;i++) ct[i]+=ct[i-1];</pre>
  for(int i=0;i<len;i++) rk[i]=ct[ip[i]];</pre>
  for(int i=1;i<len;i*=2){</pre>
    for(int j=0;j<len;j++){</pre>
      if(j+i>=len) tp[j][1]=0;
      else tp[j][1]=rk[j+i]+1;
      tp[j][0]=rk[j];
    memset(ct, 0, sizeof(ct));
    for(int j=0;j<len;j++) ct[tp[j][1]+1]++;</pre>
     for(int j=1;j<len+2;j++) ct[j]+=ct[j-1];</pre>
    for(int j=0;j<len;j++) tsa[ct[tp[j][1]]++]=j;</pre>
    memset(ct, 0, sizeof(ct));
    for(int j=0;j<len;j++) ct[tp[j][0]+1]++;</pre>
     for(int j=1;j<len+1;j++) ct[j]+=ct[j-1];</pre>
     for(int j=0;j<len;j++) sa[ct[tp[tsa[j]][0]]++]=tsa[</pre>
    rk[sa[0]]=0;
    for(int j=1;j<len;j++){</pre>
       if( tp[sa[j]][0] == tp[sa[j-1]][0] &&
         tp[sa[j]][1] == tp[sa[j-1]][1] )
         rk[sa[j]] = rk[sa[j-1]];
       else
         rk[sa[j]] = j;
    }
  }
  for(int i=0,h=0;i<len;i++){</pre>
    if(rk[i]==0) h=0;
     else{
       int j=sa[rk[i]-1];
      h=max(0,h-1);
       for(;ip[i+h]==ip[j+h];h++);
    he[rk[i]]=h;
  }
}
```

### 7.2 Suffix Array (SAIS TWT514)

```
struct SA{
#define REP(i,n) for ( int i=0; i<int(n); i++ )</pre>
#define REP1(i,a,b) for ( int i=(a); i<=int(b); i++ )
    static const int MXN = 300010;
    bool _t[MXN*2];
    int _s[MXN*2], _sa[MXN*2], _c[MXN*2], x[MXN], _p[
        MXN], _q[MXN*2], hei[MXN], r[MXN];
    int operator [] (int i){ return _sa[i]; }
    void build(int *s, int n, int m){
        memcpy(_s, s, sizeof(int) * n);
        sais(_s, _sa, _p, _q, _t, _c, n, m);
        mkhei(n);
    void mkhei(int n){
        REP(i,n) r[\_sa[i]] = i;
        hei[0] = 0;
        REP(i,n) if(r[i]) {
            int ans = i>0 ? max(hei[r[i-1]] - 1, 0) :
                0;
```

```
while(_s[i+ans] == _s[_sa[r[i]-1]+ans]) ans
             hei[r[i]] = ans;
        }
    void sais(int *s, int *sa, int *p, int *q, bool *t,
          int *c, int n, int z){
         bool uniq = t[n-1] = true, neq;
         int nn = 0, nmxz = -1, *nsa = sa + n, *ns = s +
              n, lst = -1;
#define MSO(x,n) memset((x),0,n*sizeof(*(x)))
#define MAGIC(XD) MS0(sa, n); \
         memcpy(x, c, sizeof(int) * z); \
         XD; \
         memcpy(x + 1, c, sizeof(int) * (z - 1)); \
         REP(i,n) if (sa[i] \&\& !t[sa[i]-1]) sa[x[s[sa[i]-1]])
             ]-1]]++] = sa[i]-1; \
        memcpy(x, c, sizeof(int) * z); \
for(int i = n - 1; i >= 0; i--) if(sa[i] && t[
             sa[i]-1]) sa[--x[s[sa[i]-1]]] = sa[i]-1;
         MSO(c, z);
         REP(i,n) uniq \&= ++c[s[i]] < 2;
         REP(i,z-1) c[i+1] += c[i];
         if (uniq) { REP(i,n) sa[--c[s[i]]] = i; return;
         for(int i = n - 2; i >= 0; i--) t[i] = (s[i]==s
             [i+1] ? t[i+1] : s[i] < s[i+1]);
         MAGIC(REP1(i,1,n-1) if(t[i] && !t[i-1]) sa[--x[
             s[i]]]=p[q[i]=nn++]=i);
         REP(i, n) if (sa[i] && t[sa[i]] && !t[sa[i]-1])
             neq=lst<0||memcmp(s+sa[i],s+lst,(p[q[sa[i</pre>
                  ]]+1]-sa[i])*sizeof(int));
             ns[q[lst=sa[i]]]=nmxz+=neq;
         sais(ns, nsa, p + nn, q + n, t + n, c + z, nn,
             nmxz + 1);
         MAGIC(for(int i = nn - 1; i >= 0; i--) sa[--x[s]]
             [p[nsa[i]]]] = p[nsa[i]]);
}sa;
void suffix_array(int* ip, int len) {
    // should padding a zero in the back
    // s is int array, n is array length
// s[0..n-1] != 0, and s[n] = 0
    // resulting SA will be length n+1
    ip[len++] = 0;
    sa.build(ip, len, 128);
// original 1-base
    for (int i=0; i<l; i++) {</pre>
         hei[i] = sa.hei[i + 1];
         sa[i] = sa.\_sa[i + 1];
}
```

### 7.3 Aho-Corasick Algorithm

```
struct ACautomata{
  struct Node{
    int cnt.dp:
    Node *go[26], *fail;
    Node (){
      cnt = 0;
      dp = -1;
      memset(go,0,sizeof(go));
      fail = 0;
  };
  Node *root, pool[1048576];
  int nMem;
  Node* new_Node(){
    pool[nMem] = Node();
    return &pool[nMem++];
  void init(){
    nMem = 0;
    root = new_Node();
```

```
void add(const string &str){
    insert(root,str,0);
  void insert(Node *cur, const string &str, int pos){
    if (pos >= (int)str.size()){
      cur->cnt++;
      return;
    int c = str[pos]-'a';
    if (cur->go[c] == 0){
      cur->go[c] = new_Node();
    insert(cur->go[c],str,pos+1);
  7
  void make_fail(){
    queue<Node*> que;
    que.push(root);
    while (!que.empty()){
      Node* fr=que.front();
      que.pop();
      for (int i=0; i<26; i++){</pre>
        if (fr->go[i]){
          Node *ptr = fr->fail;
          while (ptr && !ptr->go[i]) ptr = ptr->fail;
          if (!ptr) fr->go[i]->fail = root;
          else fr->go[i]->fail = ptr->go[i];
          que.push(fr->go[i]);
        }
      }
   }
 }
};
```

### 7.4 KMP

```
#include<bits/stdc++.h>
using namespace std;
void build_fail_function(string B, int *fail) {
    int len = B.length(), pos;
    pos = fail[0] = -1;
    for (int i = 1; i < len; i ++) {
    while (pos != -1 and B[pos + 1] != B[i])</pre>
             pos = fail[pos];
         if (B[pos + 1] == B[i]) pos ++;
         fail[i] = pos;
    }
void match(string A, string B, int *fail) {
    int lenA = A.length(), lenB = B.length();
    int pos = -1;
    for (int i = 0; i < lenA; i ++) {</pre>
         while (pos != -1 and B[pos + 1] != A[i])
             pos = fail[pos];
         if (B[pos + 1] == A[i]) pos ++;
         if (pos == lenB - 1) {
             // Match ! A[i - lenB + 1, i] = B
             pos = fail[pos];
    }
}
```

### 7.5 Z value

```
void Zval(const char *s, int len, int *z) {
   z[0] = len;
   for (int b=0, i=1; i<len; i++) {
        z[i] = max(min(z[i-b], z[b] + b - i), 0);
        while (s[i + z[i]] == s[z[i]]) z[i] ++;
        if (i+z[i] > b+z[b]) b=i;
   }
}
```

# 7.6 Z value (palindrome ver.)

# 7.7 palindromic tree

```
int len[maxn]:
int suffLink[maxn];
int to[maxn][2];
int cnt[maxn];
int numV;
char str[maxn];
int v;
void addLetter(int n)
    while (str[n - len[v] - 1] != str[n] )
        v = suffLink[v];
    int u = suffLink[v];
    while (str[n - len[u] - 1] != str[n] )
        u = suffLink[u];
    int u_ = to[u][str[n] - 'a'];
    int v_ = to[v][str[n] - 'a'];
    if (v_ == -1)
    {
        v_{-} = to[v][str[n] - 'a'] = numV;
        len[numV++] = len[v] + 2;
        suffLink[v_] = u_;
    v = v_{-};
    cnt[v]++;
}
void init()
    memset(to, -1, sizeof to);
str[0] = '#';
    len[0] = -1;
    len[1] = 0;
    len[2] = len[3] = 1;
    suffLink[1] = 0;
    suffLink[0] = 0;
    suffLink[2] = 1;
    suffLink[3] = 1;
    to[0][0] = 2;
    to[0][1] = 3;
    numV = 4;
}
```

### 7.8 Lexicographically Smallest Rotation

```
string mcp(string s){
  int n = s.length();
  s += s;
  int i=0, j=1;
  while (i<n && j<n){
    int k = 0;
    while (k < n && s[i+k] == s[j+k]) k++;
    if (s[i+k] <= s[j+k]) j += k+1;
    else i += k+1;
    if (i == j) j++;
}</pre>
```

```
int ans = i < n ? i : j;
return s.substr(ans, n);
}</pre>
```

#### 7.9 Suffix Automaton

```
// par : fail link
// val : a topological order ( useful for DP )
// go[x] : automata edge ( x is integer in [0,26) )
struct SAM{
  struct State{
    int par, go[26], val;
    State () : par(0), val(0){ FZ(go); }
    State (int _val) : par(0), val(_val){ FZ(go); }
  vector<State> vec;
  int root, tail;
  void init(int arr[], int len){
    vec.resize(2);
    vec[0] = vec[1] = State(0);
    root = tail = 1;

for (int i=0; i<len; i++)
      extend(arr[i]);
  void extend(int w){
    int p = tail, np = vec.size();
    vec.PB(State(vec[p].val+1));
    for ( ; p && vec[p].go[w]==0; p=vec[p].par)
      vec[p].go[w] = np;
    if (p == 0){
      vec[np].par = root;
    } else {
      if (vec[vec[p].go[w]].val == vec[p].val+1){
        vec[np].par = vec[p].go[w];
      } else {
        int q = vec[p].go[w], r = vec.size();
        vec.PB(vec[q]);
        vec[r].val = vec[p].val+1;
        vec[q].par = vec[np].par = r;
        for ( ; p && vec[p].go[w] == q; p=vec[p].par)
           vec[p].go[w] = r;
      }
    tail = np;
  }
|};
```

### 8 Problems

### 8.1 Find the maximum tangent (x,y is increasing)

```
typedef long long LL;
const int MAXN = 100010;
struct Coord{
  LL x, y;
  Coord operator - (Coord ag) const{
    Coord res;
    res.x = x - ag.x;
    res.y = y - ag.y;
    return res;
}sum[MAXN], pnt[MAXN], ans, calc;
inline bool cross(Coord a, Coord b, Coord c){
  return (c.y - a.y) * (c.x - b.x) > (c.x - a.x) * (c.y)
       - b.y);
int main(){
  int n, l, np, st, ed, now;
scanf("%d %d\n", &n, &l);
  sum[0].x = sum[0].y = np = st = ed = 0;
  for (int i = 1, v; i <= n; i++){</pre>
    scanf("%d", &v);
```

```
sum[i].y = sum[i - 1].y + v;
    sum[i].x = i;
  ans.x = now = 1;
  ans.y = -1;
  for (int i = 0; i <= n - l; i++){</pre>
    while (np > 1 && cross(pnt[np - 2], pnt[np - 1],
        sum[i]))
      np--;
    if (np < now && np != 0) now = np;
    pnt[np++] = sum[i];
    while (now < np && !cross(pnt[now - 1], pnt[now],</pre>
         sum[i + l]))
      now++;
    calc = sum[i + l] - pnt[now - 1];
    if (ans.y * calc.x < ans.x * calc.y){</pre>
      ans = calc;
      st = pnt[now - 1].x;
      ed = i + l;
    }
  double res = (sum[ed].y-sum[st].y)/(sum[ed].x-sum[st
      1.x);
  printf("\%f \ n", res);
  return 0:
}
```

#### 8.2 Painter

```
#include<bits/stdc++.h>
using namespace std;
#define F first
#define S second
#define PB push_back
#define IOS ios_base::sync_with_stdio(0); cin.tie(0);
#define SZ(x) ((int)((x).size()))
#define ALL(x) begin(x),end(x)
#define REP(i,x) for (int i=0; i<(x); i++)</pre>
#define REP1(i,a,b) for (int i=(a); i<=(b); i++)
typedef long long ll;
typedef pair<ll,ll> pll;
typedef pll Point;
const int MXN = 100005;
Point operator + (const Point &a, const Point &b) {
    return Point(a.F+b.F, a.S+b.S); }
Point operator - (const Point &a, const Point &b) {
    return Point(a.F-b.F, a.S-b.S); }
ll operator * (const Point &a, const Point &b) { return
     a.F*b.F + a.S*b.S; }
ll operator % (const Point &a, const Point &b) { return
     a.F*b.S - a.S*b.F: }
struct Segment {
  int v, id;
  Point p,q;
  Segment () {}
  Segment (int _v, int _id, Point _p, Point _q) :
    v(_v), id(_id), p(_p), q(_q) {}
bool operator < (const Segment &a, const Segment &b) {</pre>
  if (a.p == b.q) return false;
  if (a.q == b.p) return true;
  if (a.p == b.p) return (a.q-a.p) % (b.q-a.p) > 0;
  if (a.q == b.q) return (a.p-a.q) % (b.p-a.q) < 0;</pre>
  if (a.p.F == b.p.F) return a.p.S < b.p.S;</pre>
  if (a.q.F == b.q.F) return a.q.S < b.q.S;</pre>
  if (a.p.F < b.p.F) return (a.q-a.p) % (b.p-a.p) > 0;
  else return (b.q-b.p) % (a.p-b.p) < 0;
bool operator == (const Segment &a, const Segment &b) {
  return tie(a.v,a.id,a.p,a.q) == tie(b.v,b.id,b.p,b.q)
struct Triangle {
  Point pt[3];
}ip[MXN];
```

```
const int MEM = 350004;
struct Treap {
 static Treap nil, mem[MEM], *pmem;
  Treap *l, *r;
  int sum,presum,size;
  Segment seg;
 Treap () : l(&nil), r(&nil), sum(0), presum(0), size
      (0), seg() {}
 Treap (Segment _val) :
    l(&nil), r(&nil), sum(_val.v), presum(max(_val.v,0)
         ), size(1), seg(_val) {}
} Treap::nil, Treap::mem[MEM], *Treap::pmem = Treap::
int size(const Treap *t) { return t->size; }
void pull(Treap *t) {
 if (!size(t)) return;
 t\rightarrow size = size(t\rightarrow l) + size(t\rightarrow r) + 1;
  t->sum = t->l->sum + t->seg.v + t->r->sum;
  t\rightarrow presum = max(t\rightarrow l\rightarrow presum, t\rightarrow l\rightarrow sum + t\rightarrow seg.v);
  t\rightarrow presum = max(t\rightarrow presum, t\rightarrow l\rightarrow sum + t\rightarrow seg.v + t\rightarrow
      r->presum);
Treap* merge(Treap *a, Treap *b) {
  if (!size(a)) return b;
  if (!size(b)) return a;
  Treap *t;
  if (rand() % (size(a) + size(b)) < size(a)) {</pre>
    t->r = merge(a->r, b);
  } else {
    t = b;
    t->l = merge(a, b->l);
 pull(t);
  return t;
void split(Treap *t, int k, Treap *&a, Treap *&b) {
 if (!size(t)) a = b = &Treap::nil;
  else if (size(t->l) + 1 <= k) {
    a = t;
    split(t->r, k - size(t->l) - 1, a->r, b);
    pull(a);
  } else {
    b = t;
    split(t->l, k, a, b->l);
    pull(b);
  }
int get_rank(Treap *t, Segment x) {
  if (!size(t)) return 0;
  if (x < t->seg) return get_rank(t->l, x);
  return get_rank(t->r,x) + size(t->l) + 1;
Treap* find_leftist(Treap *t) {
 while (size(t->l)) t = t->l;
  return t;
Treap* find_rightist(Treap *t) {
 while (size(t->r)) t = t->r;
  return t;
int N;
vector<int> allx;
vector<Segment> _seg[3*MXN];
#define seg(x) _seg[(x)+100000]
inline void add_seg(Segment s) {
  seg(s.p.F).PB(s);
  if (s.q.F != s.p.F) seg(s.q.F).PB(s);
void predo() {
 allx.clear();
  REP(i,N) REP(j,3) {
    seg(ip[i].pt[j].F).clear();
    allx.PB(ip[i].pt[j].F);
  sort(ALL(allx));
  allx.resize(unique(ALL(allx))-begin(allx));
  REP(i,N) {
    sort(ip[i].pt, ip[i].pt+3);
```

```
Point *pt = ip[i].pt;
    Segment seg1 = Segment(1,i,pt[0],pt[1]);
    Segment seg2 = Segment(1,i,pt[0],pt[2]);
    Segment seg3 = Segment(1,i,pt[1],pt[2]);
    if (seg2 < seg1) seg1.v = -1;
    else seg2.v = -1;
    seg3.v = seg1.v;
    add_seg(seg1);
    add_seg(seg2);
    add_seg(seg3);
inline int sgn(ll x) { return x < 0 ? -1 : x > 0; }
bool interPnt(Point p1, Point p2, Point q1, Point q2){
  ll c1 = (p2-p1)\%(q1-p1), c2 = (p2-p1)\%(q2-p1);
  ll c3 = (q2-q1)\%(p1-q1), c4 = (q2-q1)\%(p2-q1);
  return sgn(c1) * sgn(c2) <= 0 and sgn(c3) * sgn(c4)</pre>
      <= 0;
bool check_error(Segment a, Segment b) {
  if (a.id == b.id) return false;
  return interPnt(a.p,a.q,b.p,b.q);
int solve() {
  Treap::pmem = Treap::mem;
  Treap *rt = &Treap::nil;
  int res = 0;
  for (auto i:allx) {
    for (auto l:seg(i)) {
      int k = get_rank(rt, l);
      Treap *t,*tl,*tm,*tr;
      split(rt,k,tl,tr);
      t = find_rightist(tl);
      if (size(t) and check_error(t->seg,l)) return -1;
      t = find_leftist(tr);
      if (size(t) and check_error(t->seg,l)) return -1;
      rt = merge(tl,tr);
      if (l.p.F == i and l.p.F != l.q.F) {
        k = get_rank(rt, l);
        split(rt,k,tl,tr);
        tm = new (Treap::pmem++) Treap(l);
        rt = merge(merge(tl,tm),tr);
    for (auto l:seg(i)) {
      if (l.q.F == i and l.p.F != l.q.F) {
        Treap *tl,*tm,*tr;
        int k = get_rank(rt, l);
        split(rt,k-1,tl,tm);
        split(tm,1,tm,tr);
        Treap *t1=find_rightist(tl),*t2=find_leftist(tr
            );
        if (size(t1) and size(t2) and check_error(t1->
            seg,t2->seg)) return -1;
        rt = merge(tl,tr);
      }
    }
    res = max(res, rt->presum);
  }
  res++;
  return res;
int main() {
  IOS;
  int cas = 0;
  while (cin >> N) {
    if (N == -1) break;
    REP(i,N) {
      REP(j,3) cin >> ip[i].pt[j].F >> ip[i].pt[j].S;
    predo();
    int ans = solve();
    cout << "Case " << cas << ": ";
    if (ans == -1) cout << "ERROR\\dot{n}";
    else cout << ans << " shades\n";</pre>
  return 0;
}
```

### 8.3 Mo-Algorithm on Tree

```
//bcw0x1bd2 {{{
#include<bits/stdc++.h>
#include<unistd.h>
using namespace std;
#define F first
#define S second
#define MP make_pair
#define PB push_back
#define IOS ios_base::sync_with_stdio(0); cin.tie(0);
#define SZ(x) ((int)((x).size()))
#define ALL(x) begin(x),end(x)
#define REP(i,x) for (int i=0; i<(x); i++)
#define REP1(i,a,b) for (int i=(a); i<=(b); i++)
typedef long long ll;
typedef pair<int,int> pii;
typedef pair<ll,ll> pll;
typedef long double ld;
#ifdef DARKHH
#define FILEIO(name)
#else
#define FILEIO(name) \
  freopen(name".in", "r", stdin); \
freopen(name".out", "w", stdout);
#endif
#ifdef DARKHH
template<typename Iter>
ostream& _out(ostream &s, Iter b, Iter e) { s << "[ "; }
    for ( auto it=b; it!=e; it++ ) s << *it << " ";</pre>
    s << "]";
    return s;
template<typename A, typename B>
ostream& operator << (ostream &s, const pair<A,B> &p) {
   return s<<"("<<p.first<<","<<p.second<<")"; }</pre>
template<typename T>
ostream& operator << (ostream &s, const vector<T> &c) {
     return _out(s,ALL(c)); }
template<typename T, size_t N>
ostream& operator << (ostream &s, const array<T,N> &c)
    { return _out(s,ALL(c)); }
template<typename T>
ostream& operator << (ostream &s, const set<T> &c) {
    return _out(s,ALL(c)); }
template<typename A, typename B>
ostream& operator << (ostream &s, const map<A,B> &c) {
    return _out(s,ALL(c)); }
#endif
// }}}
// Let's Fight! ~OAO~~
const int MX = 500005;
const int SQ = 1400;
const int LOG = 17;
struct BIT {
  int bit[MX];
  int lb(int x) { return x & -x; }
  void add(int p, int v) {
    for (int i=p; i<MX; i+=lb(i))</pre>
      bit[i] += v;
  int qry() {
    int v = 0;
    for (int i=1<<LOG; i>0; i>>=1) {
      if ((v|i) < MX \text{ and } bit[v|i]==i) v |= i;
    }
    return v;
  }
}bit;
struct Query {
  int l,r,qid;
}qry[MX];
struct Edge {
```

```
int v,x;
int N,Q,timestamp[MX],ans[MX];
int in[MX],cnt[MX];
vector<Edge> E[MX];
vector<Edge> seq;
void DFS(int u, int f) {
  timestamp[u] = SZ(seq);
  for (auto it:E[u]) {
    if (it.v == f) continue;
    seq.PB(it);
    DFS(it.v,u);
    seq.PB(it);
void poke(int id) {
  int v = seq[id].v;
  int x = seq[id].x;
  in[v] ^= 1;
  cnt[x] += in[v] ? 1 : -1;
  if (in[v] \text{ and } cnt[x] == 1) bit.add(x, 1);
  if (!in[v] \text{ and } cnt[x] == 0) bit.add(x, -1);
int main() {
  IOS;
  cin >> N >> Q;
  REP(_,N-1) {
    int u,v,x;
    cin >> u >> v >> x;
    x = min(x,N);
    E[u].PB(\{v,x\});
    E[v].PB(\{u,x\});
  DFS(1,1);
  REP1(i,1,Q) {
    int u,v;
    cin >> u >> v;
    int l = timestamp[u], r = timestamp[v];
    if (l > r) swap(l,r);
    r--
    qry[i] = {l,r,i};
  sort(qry+1,qry+1+Q, [](Query a, Query b) {
      return make_pair(a.l/SQ,a.r) < make_pair(b.l/SQ,b</pre>
           .r);
      });
  int curL = 1, curR = 0;
  REP1(i,1,Q) {
    int ql=qry[i].l,qr=qry[i].r;
    while (curL > ql) poke(--curL);
    while (curR < qr) poke(++curR);</pre>
    while (curL < ql) poke(curL++);</pre>
    while (curR > qr) poke(curR--);
    ans[qry[i].qid] = bit.qry();
  REP1(i,1,Q) {
    cout << ans[i] << "\n";</pre>
  return 0;
```

# 9 YAKELI

# 9.1 Periodic Table

ic lable									7			6			5			4			ω			2			1		
Name	Symbol	Z mass	☐ Noble Gas☐ Lanthanide/Actinide	☐ Halogen	Metalloid	Alkaline Earth Metal     Metal	Alkali Metal	Francium	Ţ	87 223	Caesium	Cs	55 132.91	Rubidium	Rb	37 85.468	Potassium	~	19 39.098	Sodium	Na	11 22.990	Lithium	_	3 6.941	Hydrogen	I	1 1.0079	1 IA
	man-made		tinide			Metal		Radium	Ra	88 226	Barium	Ba	56 137.33	Strontium	Sr	38 87.62	Calcium	Ca	20 40.078	Magnesium	Μg	12 24.305	Beryllium	Be	4 9.0122	2 IIA			
	de						. e e e e e e e e e e e e e e e e e e e	Actinide	Ac-Lr	89-103	Lanthanide	La-Lu	57-71	Yttrium	<b>~</b>	39 88.906	Scandium	Sc	21 44.956	3 IIIA									
Actinium	Ac	89 227		Lanthanum	Ь	57 138.91		Rutherfordium	Rf	104 261	Halfnium	Ŧ	72 178.49	Zirconium	Zr	40 91.224	Titanium	=!	22 47.867	4 IVB									
Thorium	Ŧ	90 232.04		Cerium	Ce	58 140.12		Dubnium	DЬ	105 262	Tantalum	Ta	73 180 95	Niobium	N <sub>P</sub>	41 92.906	Vanadium	<	23 50.942	5 VB									
Protactinium	Pa	91 231.04		Praseodymium	Pr	59 140.91		Seaborgium	Sg	106 266	Tungsten	\$	74 183.84	Molybdenum	Mo	42 95.94	Chromium	Ć	24 51.996	6 VIB									
Uranium	_	92 238.03		Neodymium	M	60 144.24		Bohrium	뫄	107 264	Rhenium	Re	<b>75</b> 186.21	Technetium	Tc	<b>43</b> 96	Manganese	Mn	<b>25</b> 54.938	7 VIIB									
Neptunium	Np	93 237		Promethium	Pm	61 145		Hassium	Нѕ	108 277	Osmium	o <sub>s</sub>	<b>76</b> 190.23	Ruthenium	Ru	44 101.07	Iron	Fe	<b>26</b> 55.845	8 VIIIB									
Plutonium	Pu	94 244		Samarium	Sm	62 150.36		Meitnerium	Mt	<b>109</b> 268	Iridium	₹	77 192.22	Rhodium	Rh	<b>45</b> 102.91	Cobalt	င	27 58.933	9 VIIIB									
Americium	Am	<b>95</b> 243		Europium	Ē	63 151.96		Darmstadtium	Ds	110 281	Platinum	Ρţ	78 195.08	Palladium	Pd	46 106.42	Nickel	Z.	28 58.693	10 VIIIB									
Curium	Cm	96 247		Gadolinium	ស	64 157.25		Roentgenium	Rg	111 280	Gold	Au	79 196.97	Silver	Ag	<b>47</b> 107.87	Copper	U.	29 63.546	11 IB									
Berkelium	Ŗ	97 247		Terbium	ТЬ	65 158.93		Copernicium	C.	112 285	Mercury	Hg	80 200.59	Cadmium	С	48 112.41	Zinc	Zn	<b>30</b> 65.39	12 IIB									
Californium	Çf	98 251		Dysprosium	Dy	<b>66</b> 162.50		Ununtrium	Uut	113 284	Thallium	=	<b>81</b> 204.38	Indium	<u>-</u>	49 114.82	Gallium	Ga	31 69.723	Aluminium	≥	13 26.982	Boron	В	5 10.811	13 IIIA			
Einsteinium	Es	99 252		Holmium	Но	67 164.93		Flerovium	2	114 289	Lead	РЬ	<b>82</b> 207.2	Tin	Sn	50 118.71	Germanium	Ge	32 72.64	Silicon	Si	14 28.086	Carbon	C	6 12.011	14 IVA			
Fermium	Fm	100 257		Erbium	ф	68 167.26		Ununpentium	Qup	115 288	Bismuth	<u>B</u> :	83 208.98	Antimony	dS	51 121.76	Arsenic	As	33 74.922	Phosphorus	P	15 30.974	Nitrogen	z	7 14.007	15 VA			
Mendelevium	M	101 258		Thulium	Tm	69 168.93		Livermorium	۲	116 293	Polonium	Po	84 209	Tellurium	Te	52 127.6	Selenium	Se	34 78.96	Sulphur	s	<b>16</b> 32.065	Oxygen	0	8 15.999	16 VIA			
Nobelium	N <sub>o</sub>	102 259		Ytterbium	<b>¥</b>	70 173.04		Unurseptium	Uus	117 292	Astatine	Ą	<b>85</b> 210	lodine	-	53 126.9	Bromine	Br	35 79.904	Chlorine	Ω	<b>17</b> 35,453	Flourine	п	9 18.998	17 VIIA			
Lawrencium	Ę	103 262		Lutetium	٦	71 174.97		Ununoctium	Uuo	118 294	Radon	Rn	86 222	Xenon	Χe	<b>54</b> 131.29	Krypton	ᅐ	<b>36</b> 83.8	Argon	Ą	18 39.948	Neon	Ne	<b>10</b> 20.180	Helium	He	<b>2</b> 4.0025	18 VIIIA