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## 1 Basic

### 1.1 vimrc

```

colo torte
syn on
se ai ar sm nu rnu is
se mouse=a bs=2 ww+=<, >, [, ] so=6 ts=4 sw=4 tt=100
se makeprg=g++\ -Wall\ -Wshadow\ -O2\ -std=c++0x\ -o\
    %<\ %
au BufNewFile *.cpp Or ~/default.cpp | :1,$-6 fo
filetype indent on

map <F7> <ESC>:wa<CR>:make!<CR>
imap <F7> <ESC>:wa<CR>:make!<CR>
map <C-F7> <ESC>:tabe %<.in<CR>
map <F8> :cope <CR>
map <S-F8> :ccl <CR>
map <F9> :!./%< <CR>
map <C-F9> :!./%< < %<.in <CR>

```

### 1.2 IncreaseStackSize

```

//stack resize
asm( "mov %0,%esp\n" ::"g"(mem+100000000) );
//change esp to rsp if 64-bit system

//stack resize (linux)
#include <sys/resource.h>
void increase_stack_size() {
    const rlim_t ks = 64*1024*1024;
    struct rlimit rl;
    int res=getrlimit(RLIMIT_STACK, &rl);
    if(res==0){
        if(rl.rlim_cur<ks){
            rl.rlim_cur=ks;
            res=setrlimit(RLIMIT_STACK, &rl);
        }
    }
}

```

### 1.3 Default Code

```
#include<bits/stdc++.h>
#include<unistd.h>
using namespace std;
#define FZ(n) memset((n),0,sizeof(n))
#define FMO(n) memset((n),-1,sizeof(n))
#define F first
#define S second
#define PB push_back
#define ALL(x) begin(x),end(x)
#define SZ(x) ((int)(x).size())
#define IOS ios_base::sync_with_stdio(0); cin.tie(0)
#define REP(i,x) for (int i=0; i<(x); i++)
#define REPI(i,a,b) for (int i=(a); i<=(b); i++)
#ifdef ONLINE_JUDGE
#define FILEIO(name) \
    freopen(name".in", "r", stdin); \
    freopen(name".out", "w", stdout);
#else
#define FILEIO(name)
#endif
template<typename A, typename B>
ostream& operator <<(ostream &s, const pair<A,B> &p) {
    return s<<" "<<p.first<<" "<<p.second<<" ";
}
template<typename T>
ostream& operator <<(ostream &s, const vector<T> &c) {
    s<<"[";
    for (auto it : c) s << it << " ";
    s<<"]";
    return s;
}
// Let's Fight!

int main() {
    return 0;
}
```

## 2 Data Structure

### 2.1 Bigint

```
struct Bigint{
    static const int LEN = 60;
    static const int BIGMOD = 10000;

    int s;
    int vl, v[LEN];
    // vector<int> v;
    Bigint() : s(1) { vl = 0; }
    Bigint(long long a) {
        s = 1; vl = 0;
        if (a < 0) { s = -1; a = -a; }
        while (a) {
            push_back(a % BIGMOD);
            a /= BIGMOD;
        }
    }
    Bigint(string str) {
        s = 1; vl = 0;
        int stPos = 0, num = 0;
        if (!str.empty() && str[0] == '-') {
            stPos = 1;
            s = -1;
        }
        for (int i=SZ(str)-1, q=1; i>=stPos; i--) {
            num += (str[i] - '0') * q;
            if ((q *= 10) >= BIGMOD) {
                push_back(num);
                num = 0; q = 1;
            }
        }
        if (num) push_back(num);
        n();
    }

    int len() const {
        return vl;
        // return SZ(v);
    }
    bool empty() const { return len() == 0; }
    void push_back(int x) {
        v[vl++] = x;
        // v.PB(x);
    }
    void pop_back() {
        vl--;
        // v.pop_back();
    }
    int back() const {
        return v[vl-1];
        // return v.back();
    }
    void n() {
        while (!empty() && !back()) pop_back();
    }
    void resize(int nl) {
        vl = nl;
        fill(v, v+vl, 0);
        // v.resize(nl);
        // fill(ALL(v), 0);
    }

    void print() const {
        if (empty()) { putchar('0'); return; }
        if (s == -1) putchar('-');
        printf("%d", back());
        for (int i=len()-2; i>=0; i--) printf("%.4d",v[i]);
    }
    friend ostream& operator << (std::ostream& out,
        const Bigint &a) {
        if (a.empty()) { out << "0"; return out; }
        if (a.s == -1) out << "-";
        out << a.back();
        for (int i=a.len()-2; i>=0; i--) {
            char str[10];
            snprintf(str, 5, "%.4d", a.v[i]);
            out << str;
        }
    }
}
```

```

    }
    return out;
}

int cp3(const Bigint &b) const {
    if (s != b.s) return s - b.s;
    if (s == -1) return -(*this).cp3(-b);
    if (len() != b.len()) return len() - b.len(); //int
    for (int i=len()-1; i>=0; i--)
        if (v[i] != b.v[i]) return v[i] - b.v[i];
    return 0;
}

bool operator < (const Bigint &b) const { return cp3(b) < 0; }
bool operator <= (const Bigint &b) const { return cp3(b) <= 0; }
bool operator == (const Bigint &b) const { return cp3(b) == 0; }
bool operator != (const Bigint &b) const { return cp3(b) != 0; }
bool operator > (const Bigint &b) const { return cp3(b) > 0; }
bool operator >= (const Bigint &b) const { return cp3(b) >= 0; }

Bigint operator - () const {
    Bigint r = (*this);
    r.s = -r.s;
    return r;
}

Bigint operator + (const Bigint &b) const {
    if (s == -1) return -(-(*this) + (-b));
    if (b.s == -1) return (*this) - (-b);
    Bigint r;
    int nl = max(len(), b.len());
    r.resize(nl + 1);
    for (int i=0; i<nl; i++) {
        if (i < len()) r.v[i] += v[i];
        if (i < b.len()) r.v[i] += b.v[i];
        if (r.v[i] >= BIGMOD) {
            r.v[i+1] += r.v[i] / BIGMOD;
            r.v[i] %= BIGMOD;
        }
    }
    r.n();
    return r;
}

Bigint operator - (const Bigint &b) const {
    if (s == -1) return -(-(*this) - (-b));
    if (b.s == -1) return (*this) + (-b);
    if ((*this) < b) return -(b - (*this));
    Bigint r;
    r.resize(len());
    for (int i=0; i<len(); i++) {
        r.v[i] += v[i];
        if (i < b.len()) r.v[i] -= b.v[i];
        if (r.v[i] < 0) {
            r.v[i] += BIGMOD;
            r.v[i+1]--;
        }
    }
    r.n();
    return r;
}

Bigint operator * (const Bigint &b) {
    Bigint r;
    r.resize(len() + b.len() + 1);
    r.s = s * b.s;
    for (int i=0; i<len(); i++) {
        for (int j=0; j<b.len(); j++) {
            r.v[i+j] += v[i] * b.v[j];
            if (r.v[i+j] >= BIGMOD) {
                r.v[i+j+1] += r.v[i+j] / BIGMOD;
                r.v[i+j] %= BIGMOD;
            }
        }
    }
    r.n();
    return r;
}

Bigint operator / (const Bigint &b) {

```

```

    Bigint r;
    r.resize(max(1, len() - b.len() + 1));
    int oriS = s;
    Bigint b2 = b; // b2 = abs(b)
    s = b2.s = r.s = 1;
    for (int i=r.len()-1; i>=0; i--) {
        int d=0, u=BIGMOD-1;
        while(d<u) {
            int m = (d+u+1)>>1;
            r.v[i] = m;
            if((r*b2) > (*this)) u = m-1;
            else d = m;
        }
        r.v[i] = d;
    }
    s = oriS;
    r.s = s * b.s;
    r.n();
    return r;
}

Bigint operator % (const Bigint &b) {
    return (*this) - (*this) / b * b;
}
};

```

## 2.2 unordered\_map

```

struct Key {
    int first, second;
    Key () {}
    Key (int _x, int _y) : first(_x), second(_y) {}
    bool operator == (const Key &b) const {
        return tie(F,S) == tie(b.F,b.S);
    }
};

struct KeyHasher {
    size_t operator()(const Key& k) const {
        return k.first + k.second*1000000;
    }
};

typedef unordered_map<Key,int,KeyHasher> map_t;

int main(int argc, char** argv){
    map_t mp;
    for (int i=0; i<10; i++)
        mp[Key(i,0)] = i+1;
    for (int i=0; i<10; i++)
        printf("%d\n", mp[Key(i,0)]);

    return 0;
}

```

## 2.3 extc\_balance\_tree

```

#include <bits/extc++.h>
typedef __gnu_pbds::priority_queue<int> heap_t;
heap_t a,b;

int main() {
    a.clear();
    b.clear();
    a.push(1);
    a.push(3);
    b.push(2);
    b.push(4);
    assert(a.top() == 3);
    assert(b.top() == 4);
    // merge two heap
    a.join(b);
    assert(a.top() == 4);
    assert(b.empty());

    return 0;
}

```

## 2.4 extc\_heap

```
#include <bits/extc++.h>
using namespace __gnu_pbds;
typedef tree<int, null_type, less<int>, rb_tree_tag,
    tree_order_statistics_node_update> set_t;

int main()
{
    // Insert some entries into s.
    set_t s;
    s.insert(12);
    s.insert(505);

    // The order of the keys should be: 12, 505.
    assert(*s.find_by_order(0) == 12);
    assert(*s.find_by_order(3) == 505);

    // The order of the keys should be: 12, 505.
    assert(s.order_of_key(12) == 0);
    assert(s.order_of_key(505) == 1);

    // Erase an entry.
    s.erase(12);

    // The order of the keys should be: 505.
    assert(*s.find_by_order(0) == 505);

    // The order of the keys should be: 505.
    assert(s.order_of_key(505) == 0);
}
```

## 2.5 Disjoint Set

```
struct DisjointSet {
    // save() is like recursive
    // undo() is like return
    int n, fa[MXN], sz[MXN];
    vector<pair<int*, int*>> h;
    vector<int> sp;
    void init(int tn) {
        n=tn;
        for (int i=0; i<n; i++) {
            fa[i]=i;
            sz[i]=1;
        }
        sp.clear(); h.clear();
    }
    void assign(int *k, int v) {
        h.PB({k, *k});
        *k=v;
    }
    void save() { sp.PB(SZ(h)); }
    void undo() {
        assert(!sp.empty());
        int last=sp.back(); sp.pop_back();
        while (SZ(h)!=last) {
            auto x=h.back(); h.pop_back();
            *x.F=x.S;
        }
    }
    int f(int x) {
        while (fa[x]!=x) x=fa[x];
        return x;
    }
    void uni(int x, int y) {
        x=f(x); y=f(y);
        if (x==y) return;
        if (sz[x]<sz[y]) swap(x, y);
        assign(&sz[x], sz[x]+sz[y]);
        assign(&fa[y], x);
    }
}djs;
```

## 2.6 Treap

```
const int MEM = 16000004;
struct Treap {
    static Treap nil, mem[MEM], *pmem;
    Treap *l, *r;
    char val;
    int size;
    Treap () : l(&nil), r(&nil), size(0) {}
    Treap (char _val) :
        l(&nil), r(&nil), val(_val), size(1) {}
} Treap::nil, Treap::mem[MEM], *Treap::pmem = Treap::
    mem;

int size(const Treap *t) { return t->size; }
void pull(Treap *t) {
    if (!size(t)) return;
    t->size = size(t->l) + size(t->r) + 1;
}
Treap* merge(Treap *a, Treap *b) {
    if (!size(a)) return b;
    if (!size(b)) return a;
    Treap *t;
    if (rand() % (size(a) + size(b)) < size(a)) {
        t = new (Treap::pmem++) Treap(*a);
        t->r = merge(a->r, b);
    } else {
        t = new (Treap::pmem++) Treap(*b);
        t->l = merge(a, b->l);
    }
    pull(t);
    return t;
}
void split(Treap *t, int k, Treap *&a, Treap *&b) {
    if (!size(t)) a = b = &Treap::nil;
    else if (size(t->l) + 1 <= k) {
        a = new (Treap::pmem++) Treap(*t);
        split(t->r, k - size(t->l) - 1, a->r, b);
        pull(a);
    } else {
        b = new (Treap::pmem++) Treap(*t);
        split(t->l, k, a, b->l);
        pull(b);
    }
}

int nv;
Treap *rt[50005];

void print(const Treap *t) {
    if (!size(t)) return;
    print(t->l);
    cout << t->val;
    print(t->r);
}

int main(int argc, char** argv) {
    IOS;
    rt[nv=0] = &Treap::nil;
    Treap::pmem = Treap::mem;
    int Q, cmd, p, c, v;
    string s;
    cin >> Q;
    while (Q--) {
        cin >> cmd;
        if (cmd == 1) {
            // insert string s after position p
            cin >> p >> s;
            Treap *tl, *tr;
            split(rt[nv], p, tl, tr);
            for (int i=0; i<SZ(s); i++)
                tl = merge(tl, new (Treap::pmem++) Treap(s[i]));
            rt[++nv] = merge(tl, tr);
        } else if (cmd == 2) {
            // remove c characters starting at position
            Treap *tl, *tm, *tr;
            cin >> p >> c;
            split(rt[nv], p-1, tl, tm);
            split(tm, c, tm, tr);
            rt[++nv] = merge(tl, tr);
        } else if (cmd == 3) {
            // print c characters starting at position p, in
            // version v

```

```

    Treap *tl, *tm, *tr;
    cin >> v >> p >> c;
    split(rt[v], p-1, tl, tm);
    split(tm, c, tm, tr);
    print(tm);
    cout << "\n";
}
}
return 0;
}

```

## 2.7 Heavy Light Decomposition

```

// only one segment tree / no 0/1 base issue
// getPathSeg return the segment in order u->v
// fa[root] = root
typedef pair<int,int> pii;

int N,fa[MXN],belong[MXN],dep[MXN],sz[MXN],que[MXN];
int step,line[MXN],stPt[MXN],edPt[MXN];
vector<int> E[MXN],chain[MXN];

void DFS(int u){
    vector<int> &c = chain[belong[u]];
    for (int i=c.size()-1; i>=0; i--){
        int v = c[i];
        stPt[v] = step;
        line[step++] = v;
    }
    for (int i=0; i<(int)c.size(); i++){
        u = c[i];
        for (auto v : E[u]){
            if (fa[u] == v || (i && v == c[i-1])) continue;
            DFS(v);
        }
        edPt[u] = step-1;
    }
}

void build_chain(int st){
    int fr,bk;
    fr=bk=0; que[bk++] = 1; fa[st]=st; dep[st]=0;
    while (fr < bk){
        int u=que[fr++];
        for (auto v : E[u]){
            if (v == fa[u]) continue;
            que[bk++] = v;
            dep[v] = dep[u]+1;
            fa[v] = u;
        }
    }
    for (int i=bk-1,u,pos; i>=0; i--){
        u = que[i]; sz[u] = 1; pos = -1;
        for (auto v : E[u]){
            if (v == fa[u]) continue;
            sz[u] += sz[v];
            if (pos==-1 || sz[v]>sz[pos]) pos=v;
        }
        if (pos == -1) belong[u] = u;
        else belong[u] = belong[pos];
        chain[belong[u]].PB(u);
    }
    step = 0;
    DFS(st);
}

int getLCA(int u, int v){
    while (belong[u] != belong[v]){
        int a = chain[belong[u]].back();
        int b = chain[belong[v]].back();
        if (dep[a] > dep[b]) u = fa[a];
        else v = fa[b];
    }
    return sz[u] >= sz[v] ? u : v;
}

vector<pii> getPathSeg(int u, int v){
    vector<pii> ret1,ret2;
    while (belong[u] != belong[v]){
        int a = chain[belong[u]].back();
        int b = chain[belong[v]].back();
        if (dep[a] > dep[b]){
            ret1.PB({stPt[a],stPt[u]});

```

```

        u = fa[a];
    } else {
        ret2.PB({stPt[b],stPt[v]});
        v = fa[b];
    }
}
if (dep[u] > dep[v]) swap(u,v);
ret1.PB({stPt[u],stPt[v]});
reverse(ret2.begin(), ret2.end());
ret1.insert(ret1.end(),ret2.begin(),ret2.end());
return ret1;
}

// Usage
void build(){
    build_chain(1); //change root
    init(0,step,0); //init segment tree
}

int get_answer(int u, int v){
    int ret = -2147483647;
    vector<pii> vec = getPathSeg(u,v);
    for (auto it : vec)
        ; // check answer with segment [it.F, it.S]
    return ret;
}

```

## 2.8 Link-Cut Tree

```

const int MXN = 100005;
const int MEM = 100005;

struct Splay {
    static Splay nil, mem[MEM], *pmem;
    Splay *ch[2], *f;
    int val, rev, size;
    Splay () : val(-1), rev(0), size(0) {
        f = ch[0] = ch[1] = &nil;
    }
    Splay (int _val) : val(_val), rev(0), size(1) {
        f = ch[0] = ch[1] = &nil;
    }
    bool isr() {
        return f->ch[0] != this && f->ch[1] != this;
    }
    int dir() {
        return f->ch[0] == this ? 0 : 1;
    }
    void setCh(Splay *c, int d) {
        ch[d] = c;
        if (c != &nil) c->f = this;
        pull();
    }
    void push() {
        if (rev) {
            swap(ch[0], ch[1]);
            if (ch[0] != &nil) ch[0]->rev ^= 1;
            if (ch[1] != &nil) ch[1]->rev ^= 1;
            rev=0;
        }
    }
    void pull() {
        size = ch[0]->size + ch[1]->size + 1;
        if (ch[0] != &nil) ch[0]->f = this;
        if (ch[1] != &nil) ch[1]->f = this;
    }
} Splay::nil, Splay::mem[MEM], *Splay::pmem = Splay::
mem;
Splay *nil = &Splay::nil;

void rotate(Splay *x) {
    Splay *p = x->f;
    int d = x->dir();
    if (!p->isr()) p->f->setCh(x, p->dir());
    else x->f = p->f;
    p->setCh(x->ch[!d], d);
    x->setCh(p, !d);
    p->pull(); x->pull();
}

vector<Splay*> splayVec;
void splay(Splay *x) {

```

```

splayVec.clear();
for (Splay *q=x;; q=q->f) {
    splayVec.push_back(q);
    if (q->isr()) break;
}
reverse(begin(splayVec), end(splayVec));
for (auto it : splayVec) it->push();
while (!x->isr()) {
    if (x->f->isr()) rotate(x);
    else if (x->dir()==x->f->dir()) rotate(x->f), rotate(x);
    else rotate(x), rotate(x);
}
}

Splay* access(Splay *x) {
    Splay *q = nil;
    for (;x!=nil;x=x->f) {
        splay(x);
        x->setCh(q, 1);
        q = x;
    }
    return q;
}

void evert(Splay *x) {
    access(x);
    splay(x);
    x->rev ^= 1;
    x->push(); x->pull();
}

void link(Splay *x, Splay *y) {
    // evert(x);
    access(x);
    splay(x);
    evert(y);
    x->setCh(y, 1);
}

void cut(Splay *x, Splay *y) {
    // evert(x);
    access(y);
    splay(y);
    y->push();
    y->ch[0] = y->ch[0]->f = nil;
}

int N, Q;
Splay *vt[MXN];

int ask(Splay *x, Splay *y) {
    access(x);
    access(y);
    splay(x);
    int res = x->f->val;
    if (res == -1) res=x->val;
    return res;
}

int main(int argc, char** argv) {
    scanf("%d%d", &N, &Q);
    for (int i=1; i<=N; i++)
        vt[i] = new (Splay::pmem++) Splay(i);
    while (Q--) {
        char cmd[105];
        int u, v;
        scanf("%s", cmd);
        if (cmd[1] == 'i') {
            scanf("%d%d", &u, &v);
            link(vt[v], vt[u]);
        } else if (cmd[0] == 'c') {
            scanf("%d", &v);
            cut(vt[1], vt[v]);
        } else {
            scanf("%d%d", &u, &v);
            int res=ask(vt[u], vt[v]);
            printf("%d\n", res);
        }
    }

    return 0;
}

```

## 3 Graph

### 3.1 BCC Edge

```

struct BccEdge {
    static const int MXN = 100005;
    struct Edge { int v, eid; };
    int n, m, step, par[MXN], dfn[MXN], low[MXN];
    vector<Edge> E[MXN];
    DisjointSet djs;
    void init(int _n) {
        n = _n; m = 0;
        for (int i=0; i<n; i++) E[i].clear();
        djs.init(n);
    }
    void add_edge(int u, int v) {
        E[u].PB({v, m});
        E[v].PB({u, m});
        m++;
    }
    void DFS(int u, int f, int f_eid) {
        par[u] = f;
        dfn[u] = low[u] = step++;
        for (auto it:E[u]) {
            if (it.eid == f_eid) continue;
            int v = it.v;
            if (dfn[v] == -1) {
                DFS(v, u, it.eid);
                low[u] = min(low[u], low[v]);
            } else {
                low[u] = min(low[u], dfn[v]);
            }
        }
    }
    void solve() {
        step = 0;
        memset(dfn, -1, sizeof(int)*n);
        for (int i=0; i<n; i++) {
            if (dfn[i] == -1) DFS(i, i, -1);
        }
        djs.init(n);
        for (int i=0; i<n; i++) {
            if (low[i] < dfn[i]) djs.uni(i, par[i]);
        }
    }
}graph;

```

### 3.2 BCC Vertex

```

struct BccVertex {
    int n, nScc, step, dfn[MXN], low[MXN];
    vector<int> E[MXN], sccv[MXN];
    int top, stk[MXN];
    void init(int _n) {
        n = _n;
        nScc = step = 0;
        for (int i=0; i<n; i++) E[i].clear();
    }
    void add_edge(int u, int v) {
        E[u].PB(v);
        E[v].PB(u);
    }
    void DFS(int u, int f) {
        dfn[u] = low[u] = step++;
        stk[top++] = u;
        for (auto v:E[u]) {
            if (v == f) continue;
            if (dfn[v] == -1) {
                DFS(v, u);
                low[u] = min(low[u], low[v]);
                if (low[v] >= dfn[u]) {
                    int z;
                    sccv[nScc].clear();
                    do {
                        z = stk[--top];
                        sccv[nScc].PB(z);
                    } while (z != v);
                    sccv[nScc].PB(u);
                }
            }
        }
    }
}

```

```

        nScc++;
    }
    } else {
        low[u] = min(low[u], dfn[v]);
    }
}
}
vector<vector<int>> solve() {
    vector<vector<int>> res;
    for (int i=0; i<n; i++) {
        dfn[i] = low[i] = -1;
    }
    for (int i=0; i<n; i++) {
        if (dfn[i] == -1) {
            top = 0;
            DFS(i, i);
        }
    }
    REP(i, nScc) res.PB(sccv[i]);
    return res;
}
}graph;

```

### 3.3 Strongly Connected Components

```

struct Scc{
    int n, nScc, vst[MXN], bln[MXN];
    vector<int> E[MXN], rE[MXN], vec;
    void init(int _n){
        n = _n;
        for (int i=0; i<MXN; i++){
            E[i].clear();
            rE[i].clear();
        }
    }
    void add_edge(int u, int v){
        E[u].PB(v);
        rE[v].PB(u);
    }
    void DFS(int u){
        vst[u]=1;
        for (auto v : E[u])
            if (!vst[v]) DFS(v);
        vec.PB(u);
    }
    void rDFS(int u){
        vst[u] = 1;
        bln[u] = nScc;
        for (auto v : rE[u])
            if (!vst[v]) rDFS(v);
    }
    void solve(){
        nScc = 0;
        vec.clear();
        FZ(vst);
        for (int i=0; i<n; i++)
            if (!vst[i]) DFS(i);
        reverse(vec.begin(), vec.end());
        FZ(vst);
        for (auto v : vec){
            if (!vst[v]){
                rDFS(v);
                nScc++;
            }
        }
    }
};

```

### 3.4 DMST\_with\_sol

```

const int INF = 1029384756;

struct edge_t{
    int u, v, w;
    set< pair<int, int> > add, sub;
    edge_t() : u(-1), v(-1), w(0) {}
    edge_t(int _u, int _v, int _w) {
        u = _u; v = _v; w = _w;
    }
};

```

```

    add.insert({u, v});
}
edge_t& operator += (const edge_t& obj) {
    w += obj.w;
    FOR (it, obj.add) {
        if (!sub.count(*it)) add.insert(*it);
        else sub.erase(*it);
    }
    FOR (it, obj.sub) {
        if (!add.count(*it)) sub.insert(*it);
        else add.erase(*it);
    }
    return *this;
}
edge_t& operator -= (const edge_t& obj) {
    w -= obj.w;
    FOR (it, obj.sub) {
        if (!sub.count(*it)) add.insert(*it);
        else sub.erase(*it);
    }
    for (auto it : obj.add) {
        if (!add.count(it)) sub.insert(it);
        else add.erase(it);
    }
    return *this;
}
}eg[MXN*MXN], prv[MXN], EDGE_INF(-1, -1, INF);
int N, M;
int cid, incyc[MXN], contracted[MXN];
vector<int> E[MXN];

edge_t dmst(int rt){
    edge_t cost;
    for (int i=0; i<N; i++){
        contracted[i] = incyc[i] = 0;
        prv[i] = EDGE_INF;
    }
    cid = 0;
    int u, v;
    while (true){
        for (v=0; v<N; v++){
            if (v != rt && !contracted[v] && prv[v].w == INF)
                break;
        }
        if (v >= N) break; // end
        for (int i=0; i<M; i++){
            if (eg[i].v == v && eg[i].w < prv[v].w)
                prv[v] = eg[i];
        }
        if (prv[v].w == INF) // not connected
            return EDGE_INF;
        cost += prv[v];
        for (u=prv[v].u; u!=v && u!=-1; u=prv[u].u);
        if (u == -1) continue;
        incyc[v] = ++cid;
        for (u=prv[v].u; u!=v; u=prv[u].u){
            contracted[u] = 1;
            incyc[u] = cid;
        }
        for (int i=0; i<M; i++){
            if (incyc[eg[i].u] != cid && incyc[eg[i].v] == cid){
                eg[i] -= prv[eg[i].v];
            }
        }
        for (int i=0; i<M; i++){
            if (incyc[eg[i].u] == cid) eg[i].u = v;
            if (incyc[eg[i].v] == cid) eg[i].v = v;
            if (eg[i].u == eg[i].v) eg[i--] = eg[--M];
        }
        for (int i=0; i<N; i++){
            if (contracted[i]) continue;
            if (prv[i].u >= 0 && incyc[prv[i].u] == cid)
                prv[i].u = v;
        }
        prv[v] = EDGE_INF;
    }
    return cost;
}

void solve(){
    edge_t cost = dmst(0);
}

```



```

for (auto it : cost.add){ // find a solution
    E[it.F].PB(it.S);
    prv[it.S] = edge_t(it.F,it.S,0);
}
}

```

### 3.5 Dominator Tree

```

// idom[n] is the unique node that strictly dominates n
// but does
// not strictly dominate any other node that strictly
// dominates n.
// idom[n] = 0 if n is entry or the entry cannot reach
// n.
struct DominatorTree{
    static const int MAXN = 200010;
    int n,s;
    vector<int> g[MAXN],pred[MAXN];
    vector<int> cov[MAXN];
    int dfn[MAXN],nfd[MAXN],ts;
    int par[MAXN];
    int sdom[MAXN],idom[MAXN];
    int mom[MAXN],mn[MAXN];

    inline bool cmp(int u,int v) { return dfn[u] < dfn[v]; }

    int eval(int u) {
        if(mom[u] == u) return u;
        int res = eval(mom[u]);
        if(cmp(sdom[mn[mom[u]]],sdom[mn[u]]))
            mn[u] = mn[mom[u]];
        return mom[u] = res;
    }

    void init(int _n, int _s) {
        n = _n;
        s = _s;
        REP1(i,1,n) {
            g[i].clear();
            pred[i].clear();
            idom[i] = 0;
        }
    }

    void add_edge(int u, int v) {
        g[u].push_back(v);
        pred[v].push_back(u);
    }

    void DFS(int u) {
        ts++;
        dfn[u] = ts;
        nfd[ts] = u;
        for(int v:g[u]) if(dfn[v] == 0) {
            par[v] = u;
            DFS(v);
        }
    }

    void build() {
        ts = 0;
        REP1(i,1,n) {
            dfn[i] = nfd[i] = 0;
            cov[i].clear();
            mom[i] = mn[i] = sdom[i] = i;
        }
        DFS(s);
        for (int i=ts; i>=2; i--) {
            int u = nfd[i];
            if(u == 0) continue;
            for(int v:pred[u]) if(dfn[v]) {
                eval(v);
                if(cmp(sdom[mn[v]],sdom[u])) sdom[u] = sdom[mn[v]];
            }
            cov[sdom[u]].push_back(u);
            mom[u] = par[u];
            for(int w:cov[par[u]]) {
                eval(w);
                if(cmp(sdom[mn[w]],par[u])) idom[w] = mn[w];
                else idom[w] = par[u];
            }
        }
    }
}

```

```

        cov[par[u]].clear();
    }
    REP1(i,2,ts) {
        int u = nfd[i];
        if(u == 0) continue;
        if(idom[u] != sdom[u]) idom[u] = idom[idom[u]];
    }
}
}dom;

```

### 3.6 Maximum Clique

```

class MaxClique {
public:
    static const int MV = 210;

    int V;
    int el[MV][MV/30+1];
    int dp[MV];
    int ans;
    int s[MV][MV/30+1];
    vector<int> sol;

    void init(int v) {
        V = v; ans = 0;
        FZ(el); FZ(dp);
    }

    /* Zero Base */
    void addEdge(int u, int v) {
        if(u > v) swap(u, v);
        if(u == v) return;
        el[u][v/32] |= (1<<(v%32));
    }

    bool dfs(int v, int k) {
        int c = 0, d = 0;
        for(int i=0; i<(V+31)/32; i++) {
            s[k][i] = el[v][i];
            if(k != 1) s[k][i] &= s[k-1][i];
            c += __builtin_popcount(s[k][i]);
        }
        if(c == 0) {
            if(k > ans) {
                ans = k;
                sol.clear();
                sol.push_back(v);
                return 1;
            }
            return 0;
        }
        for(int i=0; i<(V+31)/32; i++) {
            for(int a = s[k][i]; a; a >> 1) {
                if(k + (c-d) <= ans) return 0;
                int lb = a&(-a), lg = 0;
                a ^= lb;
                while(lb!=1) {
                    lb = (unsigned int)(lb) >> 1;
                    lg++;
                }
                int u = i*32 + lg;
                if(k + dp[u] <= ans) return 0;
                if(dfs(u, k+1)) {
                    sol.push_back(v);
                    return 1;
                }
            }
        }
        return 0;
    }

    int solve() {
        for(int i=V-1; i>=0; i--) {
            dfs(i, 1);
            dp[i] = ans;
        }
        return ans;
    }
};

```





```

    }
}
}
if (dis[t] == INF) break;
long long tf = INF;
for (int v=t, u, l; v!=s; v=u) {
    u=prv[v]; l=prvL[v];
    tf = min(tf, E[u][l].f);
}
for (int v=t, u, l; v!=s; v=u) {
    u=prv[v]; l=prvL[v];
    E[u][l].f -= tf;
    E[v][E[u][l].r].f += tf;
}
cost += tf * dis[t];
fl += tf;
}
return {fl, cost};
}
}f_low;

```

### 4.3 Kuhn Munkres

```

struct KM{
// Maximum Bipartite Weighted Matching (Perfect Match)
static const int MXN = 650;
static const int INF = 2147483647; // long long
int n, match[MXN], vx[MXN], vy[MXN];
int edge[MXN][MXN], lx[MXN], ly[MXN], slack[MXN];
// ^^^^ long long
void init(int _n){
    n = _n;
    for (int i=0; i<n; i++)
        for (int j=0; j<n; j++)
            edge[i][j] = 0;
}
void add_edge(int x, int y, int w){ // long long
    edge[x][y] = w;
}
bool DFS(int x){
    vx[x] = 1;
    for (int y=0; y<n; y++){
        if (vy[y]) continue;
        if (lx[x]+ly[y] > edge[x][y]){
            slack[y] = min(slack[y], lx[x]+ly[y]-edge[x][y]);
        }
        else {
            vy[y] = 1;
            if (match[y] == -1 || DFS(match[y])){
                match[y] = x;
                return true;
            }
        }
    }
}
return false;
}
int solve(){
    fill(match, match+n, -1);
    fill(lx, lx+n, -INF);
    fill(ly, ly+n, 0);
    for (int i=0; i<n; i++)
        for (int j=0; j<n; j++)
            lx[i] = max(lx[i], edge[i][j]);
    for (int i=0; i<n; i++){
        fill(slack, slack+n, INF);
        while (true){
            fill(vx, vx+n, 0);
            fill(vy, vy+n, 0);
            if (DFS(i)) break;
            int d = INF; // long long
            for (int j=0; j<n; j++)
                if (!vy[j]) d = min(d, slack[j]);
            for (int j=0; j<n; j++){
                if (vx[j]) lx[j] -= d;
                if (vy[j]) ly[j] += d;
                else slack[j] -= d;
            }
        }
    }
}
int res=0;

```

```

    for (int i=0; i<n; i++)
        res += edge[match[i]][i];
    return res;
}
}graph;

```

### 4.4 SW-Mincut

```

struct SW{ // O(V^3)
static const int MXN = 514;
int n, vst[MXN], del[MXN];
int edge[MXN][MXN], wei[MXN];
void init(int _n){
    n = _n;
    FZ(edge);
    FZ(del);
}
void add_edge(int u, int v, int w){
    edge[u][v] += w;
    edge[v][u] += w;
}
void search(int &s, int &t){
    FZ(vst); FZ(wei);
    s = t = -1;
    while (true){
        int mx=-1, cur=0;
        for (int i=0; i<n; i++)
            if (!del[i] && !vst[i] && mx<wei[i])
                cur = i, mx = wei[i];
        if (mx == -1) break;
        vst[cur] = 1;
        s = t;
        t = cur;
        for (int i=0; i<n; i++)
            if (!vst[i] && !del[i]) wei[i] += edge[cur][i];
    }
}
int solve(){
    int res = 2147483647;
    for (int i=0, x, y; i<n-1; i++){
        search(x, y);
        res = min(res, wei[y]);
        del[y] = 1;
        for (int j=0; j<n; j++)
            edge[x][j] = (edge[j][x] += edge[y][j]);
    }
    return res;
}
}graph;

```

### 4.5 Maximum Simple Graph Matching

```

struct GenMatch { // 1-base
static const int MAXN = 250;
int V;
bool el[MAXN][MAXN];
int pr[MAXN];
bool inq[MAXN], inp[MAXN], inb[MAXN];
queue<int> qe;
int st, ed;
int nb;
int bk[MAXN], djs[MAXN];
int ans;
void init(int _V) {
    V = _V;
    FZ(el); FZ(pr);
    FZ(inq); FZ(inp); FZ(inb);
    FZ(bk); FZ(djs);
    ans = 0;
}
void add_edge(int u, int v) {
    el[u][v] = el[v][u] = 1;
}
int lca(int u, int v) {
    memset(inp, 0, sizeof(inp));
    while(1) {
        u = djs[u];
        inp[u] = true;
    }
}

```

```

    if(u == st) break;
    u = bk[pr[u]];
}
while(1) {
    v = djs[v];
    if(inp[v]) return v;
    v = bk[pr[v]];
}
return v;
}
void upd(int u) {
    int v;
    while(djs[u] != nb) {
        v = pr[u];
        inb[djs[u]] = inb[djs[v]] = true;
        u = bk[v];
        if(djs[u] != nb) bk[u] = v;
    }
}
void blo(int u, int v) {
    nb = lca(u, v);
    memset(inb, 0, sizeof(inb));
    upd(u); upd(v);
    if(djs[u] != nb) bk[u] = v;
    if(djs[v] != nb) bk[v] = u;
    for(int tu = 1; tu <= V; tu++)
        if(inb[djs[tu]]) {
            djs[tu] = nb;
            if(!inq[tu]){
                qe.push(tu);
                inq[tu] = 1;
            }
        }
}
void flow() {
    memset(inq, false, sizeof(inq));
    memset(bk, 0, sizeof(bk));
    for(int i = 1; i <= V; i++)
        djs[i] = i;

    while(qe.size()) qe.pop();
    qe.push(st);
    inq[st] = 1;
    ed = 0;
    while(qe.size()) {
        int u = qe.front(); qe.pop();
        for(int v = 1; v <= V; v++)
            if(el[u][v] && (djs[u] != djs[v]) && (pr[u] != v)) {
                if((v == st) || ((pr[v] > 0) && bk[pr[v]] > 0))
                    blo(u, v);
                else if(bk[v] == 0) {
                    bk[v] = u;
                    if(pr[v] > 0) {
                        if(!inq[pr[v]]) qe.push(pr[v]);
                    } else {
                        ed = v;
                        return;
                    }
                }
            }
    }
}
void aug() {
    int u, v, w;
    u = ed;
    while(u > 0) {
        v = bk[u];
        w = pr[v];
        pr[v] = u;
        pr[u] = v;
        u = w;
    }
}
int solve() {
    memset(pr, 0, sizeof(pr));
    for(int u = 1; u <= V; u++)
        if(pr[u] == 0) {
            st = u;
            flow();
            if(ed > 0) {

```

```

                aug();
                ans ++;
            }
        }
    }
    return ans;
}
};

int main() {
    gp.init(V);
    for(int i=0; i<E; i++) {
        int u, v;
        cin >> u >> v;
        gp.edge(u, v);
    }
    cout << gp.solve() << endl;
}

```

#### 4.6 Minimum Weight Matching (Clique version)

```

struct Graph {
    // Minimum General Weighted Matching (Perfect Match)
    static const int MXN = 105;

    int n, edge[MXN][MXN];
    int match[MXN], dis[MXN], onstk[MXN];
    vector<int> stk;

    void init(int _n) {
        n = _n;
        FZ(edge);
    }

    void add_edge(int u, int v, int w) {
        edge[u][v] = edge[v][u] = w;
    }

    bool SPFA(int u) {
        if (onstk[u]) return true;
        stk.PB(u);
        onstk[u] = 1;
        for (int v=0; v<n; v++){
            if (u != v && match[u] != v && !onstk[v]){
                int m = match[v];
                if (dis[m] > dis[u] - edge[v][m] + edge[u][v]){
                    dis[m] = dis[u] - edge[v][m] + edge[u][v];
                    onstk[v] = 1;
                    stk.PB(v);
                    if (SPFA(m)) return true;
                    stk.pop_back();
                    onstk[v] = 0;
                }
            }
        }
        onstk[u] = 0;
        stk.pop_back();
        return false;
    }

    int solve() {
        // find a match
        for (int i=0; i<n; i+=2){
            match[i] = i+1;
            match[i+1] = i;
        }
        while (true){
            int found = 0;
            FZ(dis); FZ(onstk);
            for (int i=0; i<n; i++){
                stk.clear();
                if (!onstk[i] && SPFA(i)){
                    found = 1;
                    while (SZ(stk)>=2){
                        int u = stk.back(); stk.pop_back();
                        int v = stk.back(); stk.pop_back();
                        match[u] = v;
                        match[v] = u;
                    }
                }
            }
            if (!found) break;
        }
    }
}

```

```

    int ret = 0;
    for (int i=0; i<n; i++)
        ret += edge[i][match[i]];
    ret /= 2;
    return ret;
}
}graph;

```

## 4.7 2-Commodity Flow

```

const int MAXN = 64;
const int INF = 1029384756;

int N;
int s1, s2, t1, t2, d1, d2, S, T;
int edge[MAXN][MAXN];
int cap[MAXN][MAXN];

int h[MAXN], gap[MAXN];
bool vis[MAXN];

int isap(int v, int f)
{
    if(v == T) return f;

    if(vis[v]) return 0;
    vis[v] = true;

    for(int i=0; i<N+2; i++)
    {
        if(cap[v][i] <= 0) continue;
        if(h[i] != h[v] - 1) continue;
        int res = isap(i, min(cap[v][i], f));
        if(res > 0)
        {
            cap[v][i] -= res;
            cap[i][v] += res;
            return res;
        }
    }

    gap[h[v]]--;
    if(gap[h[v]] <= 0) h[S] = N + 4;
    h[v]++;
    gap[h[v]]++;

    return 0;
}

int get_flow()
{
    for(int i=0; i<MAXN; i++)
    {
        h[i] = gap[i] = 0;
    }
    gap[0] = N + 2;

    int flow = 0;

    while(h[S] <= N + 3)
    {
        for(int i=0; i<N+2; i++)
        {
            vis[i] = false;
        }

        int df = isap(S, INF);
        flow += df;
    }

    return flow;
}

int main()
{
    ios_base::sync_with_stdio(0);

    int TT;
    cin>>TT;
    while(TT--)

```

```

{
    cin>>N;
    cin>>s1>>t1>>d1>>s2>>t2>>d2;

    for(int i=0; i<MAXN; i++)
    {
        for(int j=0; j<MAXN; j++)
        {
            edge[i][j] = 0;
        }
    }

    for(int i=0; i<N; i++)
    {
        string s;
        cin>>s;
        for(int j=0; j<N; j++)
        {
            if(s[j] == 'X') edge[i][j] = 0;
            else if(s[j] == 'O') edge[i][j] = 1;
            else if(s[j] == 'N') edge[i][j] = INF;
        }
    }

    int ans = 0;

    S = N;
    T = N + 1;

    //first
    for(int i=0; i<MAXN; i++)
    {
        for(int j=0; j<MAXN; j++)
        {
            cap[i][j] = edge[i][j];
        }
    }

    cap[S][s1] = cap[t1][T] = d1;
    cap[S][s2] = cap[t2][T] = d2;

    ans = get_flow();

    //second
    for(int i=0; i<MAXN; i++)
    {
        for(int j=0; j<MAXN; j++)
        {
            cap[i][j] = edge[i][j];
        }
    }

    cap[S][s1] = cap[t1][T] = d1;
    cap[S][t2] = cap[s2][T] = d2;

    ans = min(ans, get_flow());

    cout<<(ans == d1 + d2 ? "Yes" : "No")<<endl;
}

return 0;
}

```

## 4.8 (+1) SW-mincut $O(NM)$

```

// {{{ StoerWagner
const int inf=1000000000;
// should be larger than max.possible mincut
class StoerWagner {
public:
    int n,mc; // node id in [0,n-1]
    vector<int> adj[MAXN];
    int cost[MAXN][MAXN];
    int cs[MAXN];
    bool merged[MAXN],sel[MAXN];
    // --8<-- include only if cut is explicitly needed
    DisjointSet djs;
    vector<int> cut;
    //--8<-----
    StoerWagner(int _n):n(_n),mc(inf),djs(_n) {

```

```

    for(int i=0;i<n;i++)
        merged[i]=0;
    for(int i=0;i<n;i++)
        for(int j=0;j<n;j++)
            cost[i][j]=cost[j][i]=0;
}
void append(int v,int u,int c) {
    if(v==u) return;
    if(!cost[v][u]&&c) {
        adj[v].PB(u);
        adj[u].PB(v);
    }
    cost[v][u]+=c;
    cost[u][v]+=c;
}
void merge(int v,int u) {
    merged[u]=1;
    for(int i=0;i<n;i++)
        append(v,i,cost[u][i]);
    // --8<-- include only if cut is explicitly
    //      needed
    djs.merge(v,u);
    //      -----
}
void phase() {
    priority_queue<pii> pq;
    for(int v=0;v<n;v++) {
        if(merged[v]) continue;
        cs[v]=0;
        sel[v]=0;
        pq.push({0,v});
    }
    int v,s,pv;
    while(pq.size()) {
        if(cs[pq.top().S]>pq.top().F) {
            pq.pop();
            continue;
        }
        pv=v;
        v=pq.top().S;
        s=pq.top().F;
        pq.pop();
        sel[v]=1;
        for(int i=0;i<adj[v].size();i++) {
            int u=adj[v][i];
            if(merged[u]||sel[u]) continue;
            cs[u]+=cost[v][u];
            pq.push({cs[u],u});
        }
    }
    if(s<mc) {
        mc=s;
        // --8<-- include only if cut is explicitly
        //      needed -----
        cut.clear();
        for(int i=0;i<n;i++)
            if(djs.getrep(i)==djs.getrep(v)) cut.PB(i);
        // --8<-----
    }
    merge(v,pv);
}
int mincut() {
    if(mc==inf) {
        for(int t=0;t<n-1;t++)
            phase();
    }
    return mc;
}
// --8<-- include only if cut is explicitly needed
// -----
vector<int> getcut() { // return one side of the
    //      cut
    mincut();
    return cut;
}
// --8<-----
};
// }

```

## 5 Math

### 5.1 $ax+by=gcd$

```

typedef pair<int, int> pii;

pii gcd(int a, int b){
    if(b == 0) return make_pair(1, 0);
    else{
        int p = a / b;
        pii q = gcd(b, a % b);
        return make_pair(q.second, q.first - q.second * p);
    }
}

```

### 5.2 Fast Fourier Transform

```

// const int MAXN = 262144;
// (must be 2^k)

typedef long double ld;
typedef complex<ld> cplx;
const ld PI = acos(-1);
const cplx I(0, 1);

cplx omega[MAXN+1];
void pre_fft()
{
    for(int i=0; i<=MAXN; i++)
        omega[i] = exp(i * 2 * PI / MAXN * I);
}

void fft(int n, cplx a[], bool inv=false)
{
    int basic = MAXN / n;
    int theta = basic;
    for (int m = n; m >= 2; m >>= 1) {
        int mh = m >> 1;
        for (int i = 0; i < mh; i++) {
            cplx w = omega[inv ? MAXN-(i*theta%MAXN) : i*
                theta%MAXN];
            for (int j = i; j < n; j += m) {
                int k = j + mh;
                cplx x = a[j] - a[k];
                a[j] += a[k];
                a[k] = w * x;
            }
        }
        theta = (theta * 2) % MAXN;
    }
    int i = 0;
    for (int j = 1; j < n - 1; j++) {
        for (int k = n >> 1; k > (i ^= k); k >>= 1);
        if (j < i) swap(a[i], a[j]);
    }
    if (inv)
        for (i = 0; i < n; i++)
            a[i] /= n;
}

```

### 5.3 Fast Linear Recurrence

```

ll n,m,dp[N+N];
void pre_dp(){
    dp[0]=1;
    ll bdr = min(m+m,n);
    for(ll i=1; i<=bdr; i++)
        for(ll j=i-1; j>=max(0ll,i-m); j--)
            dp[i]= add(dp[i],dp[j]);
}
vector<ll> Mul(const vector<ll>& v1,const vector<ll>&
    v2){
    int sz1 = (int)v1.size();
    int sz2 = (int)v2.size();
    assert(sz1 == m and sz2 == m);
    vector<ll> _v(m+m);
    for(int i=0; i<m+m; i++) _v[i]= 0;
}

```

```

// expand
for(int i=0; i<sz1; i++)
    for(int j=0; j<sz2; j++)
        _v[i+j+1]= add(_v[i+j+1],mul(v1[i],v2[j]));
// shrink
for(int i=0; i<m; i++)
    for(int j=1; j<=m; j++)
        _v[i+j]= add(_v[i+j],_v[i]);
for(int i=0; i<m; i++)
    _v[i]= _v[i+m];
_v.resize(m);
return _v;
}
vector<ll> I,A;
ll solve(){
    pre_dp();
    if(n <= m+m) return dp[n];
    I.resize(m);
    A.resize(m);
    for(int i=0; i<m; i++) I[i]=A[i]=1;
    // dp[n]= /Sum_{i=0}^{m-1} A_i * dp[n-i-1]
    ll dlt = (n-m) / m;
    ll rdlt = dlt * m;
    while(dlt){
        if(dlt & 1ll) I = Mul(I,A);
        A = Mul(A,A);
        dlt >>= 1;
    }
    ll ans = 0;
    for(int i=0; i<m; i++)
        ans = add(ans,mul(I[i],dp[n-i-1-rdlt]));
    return ans;
}

```

## 5.4 (+1) ntt

```

int P=605028353,root=3,MAXNUM=262144;
// Remember coefficient are mod P
/*
p=a*2^n+1
n    2^n    p    a    root
5    32    97    3    5
6    64    193    3    5
7    128    257    2    3
8    256    257    1    3
9    512    7681    15    17
10   1024    12289    12    11
11   2048    12289    6    11
12   4096    12289    3    11
13   8192    40961    5    3
14   16384    65537    4    3
15   32768    65537    2    3
16   65536    65537    1    3
17   131072    786433    6    10
18   262144    786433    3    10 (605028353,
    2308, 3)
19   524288    5767169    11    3
20   1048576    7340033    7    3
21   2097152    23068673    11    3
22   4194304    104857601    25    3
23   8388608    167772161    20    3
24   16777216    167772161    10    3
25   33554432    167772161    5    3 (1107296257, 33,
    10)
26   67108864    469762049    7    3
27   134217728    2013265921    15    31
*/
int bigmod(long long a,int b){
    if(b==0) return 1;
    return (bigmod((a*a)%P,b/2)*(b%2?a:1ll))%P;
}
int inv(int a,int b){
    if(a==1) return 1;
    return (((long long)(a-inv(b*a,a))*b+1)/a)%b;
}
std::vector<long long> ps(MAXNUM);
std::vector<int> rev(MAXNUM);
struct poly{
    std::vector<unsigned int> co;
    int n;//polynomial degree = n

```

```

poly(int d){n=d;co.resize(n+1,0);}
void trans2(int NN){
    int r=0,st,N;
    unsigned int a,b;
    while((1<<r)<(NN>>1))++r;
    for(N=2;N<=NN;N<=1,--r){
        for(st=0;st<NN;st+=N){
            int i,ss=st+(N>>1);
            for(i=(N>>1)-1;i>=0;--i){
                a=co[st+i]; b=(ps[i<<r]*co[ss+i])%P;
                co[st+i]=a+b; if(co[st+i]>=P)co[st+i]-=P;
                co[ss+i]=a-P-b; if(co[ss+i]>=P)co[ss+i]-=P;
            }
        }
    }
}
void trans1(int NN){
    int r=0,st,N;
    unsigned int a,b;
    for(N=NN;N>1;N>>=1,++r){
        for(st=0;st<NN;st+=N){
            int i,ss=st+(N>>1);
            for(i=(N>>1)-1;i>=0;--i){
                a=co[st+i]; b=co[ss+i];
                co[st+i]=a+b; if(co[st+i]>=P)co[st+i]-=P;
                co[ss+i]=((a-P-b)*ps[i<<r])%P;
            }
        }
    }
}
poly operator*(const poly& _b) const{
    poly a=*this,b=_b;
    int k=n+b.n,i,N=1;
    while(N<=k)N*=2;
    a.co.resize(N,0); b.co.resize(N,0);
    int r=bigmod(root,(P-1)/N),Ni=inv(N,P);
    ps[0]=1;
    for(i=1;i<N;++i)ps[i]=(ps[i-1]*r)%P;
    a.trans1(N);b.trans1(N);
    for(i=0;i<N;++i)a.co[i]=((long long)a.co[i]*b.co[i]
    )%P;
    r=inv(r,P);
    for(i=1;i<N/2;++i)std::swap(ps[i],ps[N-i]);
    a.trans2(N);
    for(i=0;i<N;++i)a.co[i]=((long long)a.co[i]*Ni)%P;
    a.n=n+_b.n; return a;
}
};

```

## 5.5 Mod

```

/// _fd(a,b) floor(a/b).
/// _rd(a,m) a-floor(a/m)*m.
/// _pv(a,m,r) largest x s.t x<=a && x%m == r.
/// _nx(a,m,r) smallest x s.t x>=a && x%m == r.
/// _ct(a,b,m,r) |A| , A = { x : a<=x<=b && x%m == r }.
int _fd(int a,int b){ return a<0?(-a/b-1):a/b; }
int _rd(int a,int m){ return a-_fd(a,m)*m; }
int _pv(int a,int m,int r)
{
    r=(r%m+m)%m;
    return _fd(a-r,m)*m+r;
}
int _nt(int a,int m,int r)
{
    m=abs(m);
    r=(r%m+m)%m;
    return _fd(a-r-1,m)*m+r+m;
}
int _ct(int a,int b,int m,int r)
{
    m=abs(m);
    a=_nt(a,m,r);
    b=_pv(b,m,r);
    return (a>b)?0:((b-a+m)/m);
}

```

## 5.6 (+1) Miller Rabin

```
// n < 4,759,123,141      3 : 2, 7, 61
// n < 1,122,004,669,633  4 : 2, 13, 23, 1662803
// n < 3,474,749,660,383  6 : pimes <= 13
// n < 2^64              7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
// Make sure testing integer is in range [2, n-2] if
// you want to use magic.
long long power(long long x, long long p, long long mod){
    long long s=1, m=x;
    while(p) {
        if(p&1) s=mult(s,m,mod);
        p>>=1;
        m=mult(m,m,mod);
    }
    return s;
}
bool witness(long long a, long long n, long long u, int t)
{
    long long x=power(a,u,n);
    for(int i=0; i<t; i++) {
        long long nx=mult(x,x,n);
        if(nx==1&&x!=1&&x!=n-1) return 1;
        x=nx;
    }
    return x!=1;
}
bool miller_rabin(long long n, int s=100) {
    // iterate s times of witness on n
    // return 1 if prime, 0 otherwise
    if(n<2) return 0;
    if(!(n&1)) return n==2;
    long long u=n-1;
    int t=0;
    // n-1 = u*2^t
    while(!(u&1)) {
        u>>=1;
        t++;
    }
    while(s--) {
        long long a=randll()%(n-1)+1;
        if(witness(a,n,u,t)) return 0;
    }
    return 1;
}
```

## 5.7 Pollard Rho

```
// does not work when n is prime
long long modit(long long x, long long mod) {
    if(x>=mod) x-=mod;
    //if(x<0) x+=mod;
    return x;
}
long long mult(long long x, long long y, long long mod) {
    long long s=0, m=x%mod;
    while(y) {
        if(y&1) s=modit(s+m,mod);
        y>>=1;
        m=modit(m+m,mod);
    }
    return s;
}
long long f(long long x, long long mod) {
    return modit(mult(x,x,mod)+1,mod);
}
long long pollard_rho(long long n) {
    if(!(n&1)) return 2;
    while (true) {
        long long y=2, x=rand()%(n-1)+1, res=1;
        for (int sz=2; res==1; sz*=2) {
            for (int i=0; i<sz && res<=1; i++) {
                x = f(x, n);
                res = __gcd(abs(x-y), n);
            }
            y = x;
        }
        if (res!=0 && res!=n) return res;
    }
}
```

```
}
}
```

## 5.8 Algorithms about Primes

```
/*
 * 12721
 * 13331
 * 14341
 * 75577
 * 123457
 * 222557
 * 556679
 * 999983
 * 1097774749
 * 1076767633
 * 100102021
 * 999997771
 * 1001010013
 * 1000512343
 * 987654361
 * 999991231
 * 999888733
 * 98789101
 * 987777733
 * 999991921
 * 1010101333
 * 1010102101
 * 1000000000039
 * 100000000000037
 * 2305843009213693951
 * 4611686018427387847
 * 9223372036854775783
 * 18446744073709551557
 */
int mu[MX], p_tbl[MX];
vector<int> primes;
void sieve() {
    mu[1] = p_tbl[1] = 1;
    for (int i=2; i<MX; i++) {
        if (!p_tbl[i]) {
            p_tbl[i] = i;
            primes.PB(i);
            mu[i] = -1;
        }
        for (auto p : primes) {
            int x = i*p;
            if (x >= M) break;
            p_tbl[x] = p;
            mu[x] = -mu[i];
            if (i%p==0) {
                mu[x] = 0;
                break;
            }
        }
    }
}
vector<int> factor(int x) {
    vector<int> fac{1};
    while (x > 1) {
        int fn=SZ(fac), p=p_tbl[x], pos=0;
        while (x%p == 0) {
            x /= p;
            for (int i=0; i<fn; i++)
                fac.PB(fac[pos++] * p);
        }
    }
    return fac;
}
```

## 5.9 (+1) PolynomialGenerator

```
class PolynomialGenerator {
    /* for a nth-order polynomial f(x), *
     * given f(0), f(1), ..., f(n) *
     * express f(x) as sigma_i{c_i*C(x,i)} */
}
```



```

public:
    int n;
    vector<long long> coef;
    // initialize and calculate f(x), vector _fx should
    // be
    // filled with f(0) to f(n)
    PolynomialGenerator(int _n,vector<long long> _fx)
        :n(_n)
        ,coef(_fx) {
        for(int i=0;i<n;i++)
            for(int j=n;j>i;j--)
                coef[j]-=coef[j-1];
    }
    // evaluate f(x), runs in O(n)
    long long eval(int x) {
        long long m=1,ret=0;
        for(int i=0;i<n;i++) {
            ret+=coef[i]*m;
            m=m*(x-i)/(i+1);
        }
        return ret;
    }
};

```

## 5.10 Pseudoinverse of Square matrix

```

Mat pinv(Mat m)
{
    Mat res = I;

    FZ(used);
    for(int i=0; i<W; i++)
    {
        int piv = -1;
        for(int j=0; j<W; j++)
        {
            if(used[j]) continue;
            if(abs(m.v[j][i]) > EPS)
            {
                piv = j;
                break;
            }
        }
        if(piv == -1)
            continue;
        used[i] = true;
        swap(m.v[piv], m.v[i]);
        swap(res.v[piv], res.v[i]);

        ld rat = m.v[i][i];
        for(int j=0; j<W; j++)
        {
            m.v[i][j] /= rat;
            res.v[i][j] /= rat;
        }

        for(int j=0; j<W; j++)
        {
            if(j == i) continue;
            rat = m.v[j][i];
            for(int k=0; k<W; k++)
            {
                m.v[j][k] -= rat * m.v[i][k];
                res.v[j][k] -= rat * res.v[i][k];
            }
        }

        for(int i=0; i<W; i++)
        {
            if(used[i]) continue;
            for(int j=0; j<W; j++)
                res.v[i][j] = 0;
        }

        return res;
    }
}

```

## 5.11 Theorem

### 5.11.1 Lucas' Theorem

For non-negative integer  $n$ ,  $m$  and prime  $p$ ,  $\binom{m}{n} \equiv \prod_{i=0}^k \binom{m_i}{n_i} \pmod{p}$  where  $m_i$  is the  $i$ -th digit of  $m$  in base  $p$ .

### 5.11.2 Sum of Two Squares Thm (Legendre)

For a given positive integer  $n$ , let  
 $D_1 = (\# \text{ of positive integers } d \text{ dividing } N \text{ that } 1 \equiv d \pmod{4})$   
 $D_3 = (\# \text{ of positive integers } d \text{ dividing } N \text{ that } 3 \equiv d \pmod{4})$   
 then  $n$  can be written as a sum of two squares in exactly  
 $R(n) = 4(D_1 - D_3)$  ways.

### 5.11.3 Difference of D1-D3 Thm

let  $n = 2^t \cdot (p_1^{e_1} \cdot \dots \cdot p_r^{e_r}) \cdot (q_1^{f_1} \cdot \dots \cdot q_s^{f_s}) \pmod{4}$   
 where  $p_i, q_i$  are primes and  $1 \equiv p_i \pmod{4}, 3 \equiv q_i \pmod{4}$   
 then  $D_1 - D_3 = \begin{cases} (e_1 + 1)(e_2 + 1) \dots (e_r + 1), & \text{if } (f_i)s \text{ all even} \\ 0, & \text{if any } f_i \text{ is odd} \end{cases}$

### 5.11.4 Krush-Kuhn-Tucker Conditions

#### Stationarity

For maximizing  $f(x)$ :  $\nabla f(x^*) = \sum_{i=1}^m \mu_i \nabla g_i(x^*) + \sum_{j=1}^l \lambda_j \nabla h_j(x^*)$   
 For minimizing  $f(x)$ :  $-\nabla f(x^*) = \sum_{i=1}^m \mu_i \nabla g_i(x^*) + \sum_{j=1}^l \lambda_j \nabla h_j(x^*)$

#### Primal feasibility

$g_i(x^*) \leq 0$ , for all  $i = 1, \dots, m$   
 $h_j(x^*) = 0$ , for all  $j = 1, \dots, l$

#### Dual feasibility

$\mu_i \geq 0$ , for all  $i = 1, \dots, m$

#### Complementary slackness

$\mu_i g_i(x^*) = 0$ , for all  $i = 1, \dots, m$

### 5.11.5 Chinese remainder theorem

$x \equiv r_i \pmod{p_i}$   
 $N = \prod p_i$   
 $N_i = N/p_i$   
 $x \equiv \sum r_i N_i (N_i)^{-1} \pmod{N}$

## 5.12 Simplex

```

const int maxn = 111;
const int maxm = 111;
const double eps = 1E-10;

double a[maxn][maxm], b[maxn], c[maxn], d[maxn][maxm];
double x[maxn];
int ix[maxn + maxm]; // !!! array all indexed from 0
// max{cx} subject to {Ax<=b, x>=0}
// n: constraints, m: vars !!!
// x[] is the optimal solution vector
//
// usage :
// value = simplex(a, b, c, N, M);
double simplex(double a[maxn][maxm], double b[maxn],
    double c[maxn], int n, int m) {
    ++m;
    int r = n, s = m - 1;
    memset(d, 0, sizeof(d));
    for (int i = 0; i < n + m; ++i) ix[i] = i;
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < m - 1; ++j) d[i][j] = -a[i][j];
        d[i][m - 1] = 1;
        d[i][m] = b[i];
        if (d[r][m] > d[i][m]) r = i;
    }
    for (int j = 0; j < m - 1; ++j) d[n][j] = c[j];
    d[n + 1][m - 1] = -1;
    for (double dd;; ) {
        if (r < n) {
            int t = ix[s]; ix[s] = ix[r + m]; ix[r + m] = t;
            d[r][s] = 1.0 / d[r][s];

```

```

    for (int j = 0; j <= m; ++j) if (j != s) d[
        r][j] *= -d[r][s];
    for (int i = 0; i <= n + 1; ++i) if (i != r
    ) {
        for (int j = 0; j <= m; ++j) if (j != s
        ) d[i][j] += d[r][j] * d[i][s];
        d[i][s] *= d[r][s];
    }
}
r = -1; s = -1;
for (int j = 0; j < m; ++j) if (s < 0 || ix[s]
> ix[j]) {
    if (d[n + 1][j] > eps || (d[n + 1][j] > -
        eps && d[n][j] > eps)) s = j;
}
if (s < 0) break;
for (int i = 0; i < n; ++i) if (d[i][s] < -eps)
{
    if (r < 0 || (dd = d[r][m] / d[r][s] - d[i
    ][m] / d[i][s]) < -eps || (dd < eps &&
        ix[r + m] > ix[i + m])) r = i;
}
if (r < 0) return -1; // not bounded
}
if (d[n + 1][m] < -eps) return -1; // not
executable
double ans = 0;
for(int i=0; i<m; i++) x[i] = 0;
for (int i = m; i < n + m; ++i) { // the missing
    enumerated x[i] = 0
    if (ix[i] < m - 1)
    {
        ans += d[i - m][m] * c[ix[i]];
        x[ix[i]] = d[i-m][m];
    }
}
return ans;
}
}

```

## 6 Geometry

### 6.1 Point operators

```

#define x first
#define y second

#define cpdd const pdd
struct pdd : pair<double, double> {
    using pair<double, double>::pair;

    pdd operator + (cpdd &p) const {
        return {x+p.x, y+p.y};
    }

    pdd operator - () const {
        return {-x, -y};
    }

    pdd operator - (cpdd &p) const {
        return (*this) + (-p);
    }

    pdd operator * (double f) const {
        return {f*x, f*y};
    }

    double operator * (cpdd &p) const {
        return x*p.x + y*p.y;
    }
};

double abs(cpdd &p) { return hypot(p.x, p.y); }
double arg(cpdd &p) { return atan2(p.y, p.x); }
double cross(cpdd &p, cpdd &q) { return p.x*q.y - p.y*q
    .x; }
double cross(cpdd &p, cpdd &q, cpdd &o) { return cross(
    p-o, q-o); }
pdd operator * (double f, cpdd &p) { return p*f; } //
    !! Not f*p !!

```

### 6.2 Intersection of two circles

Let  $O_1 = (x_1, y_1)$ ,  $O_2 = (x_2, y_2)$  be two centers of circles,  $r_1, r_2$  be the radius. If:  
 $d = |O_1 - O_2|$ ,  $u = \frac{1}{2}(O_1 + O_2) + \frac{(r_2^2 - r_1^2)}{2d^2}(O_1 - O_2)$   
 $v = \frac{\sqrt{(r_1 + r_2 + d)(r_1 - r_2 + d)(r_1 + r_2 - d)(-r_1 + r_2 + d)}}{2d^2}(y_1 - y_2, -x_1 + x_2)$  then  $u + v, u - v$  are the two intersections of the circles, provided that  $d < r_1 + r_2$ .

```

using ld = double;
vector<pdd> interCircle(pdd o1, double r1, pdd o2,
    double r2) {
    ld d2 = (o1 - o2) * (o1 - o2);
    ld d = sqrt(d2);
    if (d > r1 + r2) return {};
    pdd u = 0.5*(o1+o2) + ((r2*r2-r1*r1)/(2*d2))*(o1-o2);
    double A = sqrt((r1+r2+d) * (r1-r2+d) * (r1+r2-d) *
        (-r1+r2+d));
    pdd v = A / (2*d2) * pdd(o1.S-o2.S, -o1.F+o2.F);
    return {u+v, u-v};
}

```

### 6.3 Intersection of two lines

```

const double EPS = 1e-9;

pdd interPnt(pdd p1, pdd p2, pdd q1, pdd q2, bool &res)
{
    double f1 = cross(p2, q1, p1);
    double f2 = -cross(p2, q2, p1);
    double f = (f1 + f2);

    if(fabs(f) < EPS) {
        res = false;
        return {};
    }

    res = true;
    return (f2 / f) * q1 + (f1 / f) * q2;
}

```

### 6.4 Half Plane Intersection

```

const double EPS = 1e-9;

pdd interPnt(Line l1, Line l2, bool &res){
    pdd p1, p2, q1, q2;
    tie(p1, p2) = l1;
    tie(q1, q2) = l2;
    double f1 = cross(p2, q1, p1);
    double f2 = -cross(p2, q2, p1);
    double f = (f1 + f2);

    if(fabs(f) < EPS) {
        res = false;
        return {0, 0};
    }

    res = true;
    return (f2 / f) * q1 + (f1 / f) * q2;
}

bool isin(Line l0, Line l1, Line l2) {
    // Check inter(l1, l2) in l0
    bool res;
    pdd p = interPnt(l1, l2, res);
    return cross(l0.S, p, l0.F) > EPS;
}

/* If no solution, check: 1. ret.size() < 3
 * Or more precisely, 2. interPnt(ret[0], ret[1])
 * in all the lines. (use (l.S - l.F).cross(p - l.F) >
    0
 */
vector<Line> halfPlaneInter(vector<Line> lines) {
    int sz = lines.size();
    vector<double> ata(sz), ord(sz);
    for (int i=0; i<sz; i++) {
        ord[i] = i;
    }
}

```

```

    pdd d = lines[i].S - lines[i].F;
    ata[i] = atan2(d.y, d.x);
}
sort(ALL(ord), [&](int i, int j) {
    if (abs(ata[i] - ata[j]) < EPS) {
        return cross(lines[i].S, lines[j].S, lines[i].F) < 0;
    }
    return ata[i] < ata[j];
});
vector<Line> fin;
for (int i=0; i<sz; i++) {
    if (!i or fabs(ata[ord[i]] - ata[ord[i-1]]) > EPS) {
        fin.PB(lines[ord[i]]);
    }
}

deque<Line> dq;
for (int i=0; i<SZ(fin); i++) {
    while(SZ(dq) >= 2 and
        not isin(fin[i], dq[SZ(dq)-2], dq[SZ(dq)-1])) {
        dq.pop_back();
    }
    while(SZ(dq) >= 2 and
        not isin(fin[i], dq[0], dq[1])) {
        dq.pop_front();
    }
    dq.push_back(fin[i]);
}

while (SZ(dq) >= 3 and
    not isin(dq[0], dq[SZ(dq)-2], dq[SZ(dq)-1])) {
    dq.pop_back();
}

while (SZ(dq) >= 3 and
    not isin(dq[SZ(dq)-1], dq[0], dq[1])) {
    dq.pop_front();
}
vector<Line> res(ALL(dq));
return res;
}

```

## 6.5 Convex Hull

```

vector<pdd> convex_hull(vector<pdd> pt){
    sort(pt.begin(), pt.end());
    int top=0;
    vector<pdd> stk(2*pt.size());
    for (int i=0; i<(int)pt.size(); i++){
        while (top >= 2 && cross(stk[top-2], stk[top-1], pt[i]) <= 0)
            top--;
        stk[top++] = pt[i];
    }
    for (int i=pt.size()-2, t=top+1; i>=0; i--){
        while (top >= t && cross(stk[top-2], stk[top-1], pt[i]) <= 0)
            top--;
        stk[top++] = pt[i];
    }
    stk.resize(top-1);
    return stk;
}

```

## 6.6 Minimum Covering Circle

```

struct Mcc{
    // return pair of center and r^2
    static const int MAXN = 1000100;
    int n;
    pdd p[MAXN], cen;
    double r2;

    void init(int _n, pdd _p[]){

```

```

        n = _n;
        memcpy(p, _p, sizeof(pdd)*n);
    }

    double sqr(double a){ return a*a; }
    double abs2(pdd a){ return a*a; }
    pdd center(pdd p0, pdd p1, pdd p2) {
        pdd a = p1-p0;
        pdd b = p2-p0;
        double c1=abs2(a)*0.5;
        double c2=abs2(b)*0.5;
        double d = a % b;
        double x = p0.x + (c1 * b.y - c2 * a.y) / d;
        double y = p0.y + (a.x * c2 - b.x * c1) / d;
        return pdd(x,y);
    }

    pair<pdd, double> solve(){
        random_shuffle(p, p+n);
        r2=0;
        for (int i=0; i<n; i++){
            if (abs2(cen-p[i]) <= r2) continue;
            cen = p[i];
            r2 = 0;
            for (int j=0; j<i; j++){
                if (abs2(cen-p[j]) <= r2) continue;
                cen = 0.5 * (p[i]+p[j]);
                r2 = abs2(cen-p[j]);
                for (int k=0; k<j; k++){
                    if (abs2(cen-p[k]) <= r2) continue;
                    cen = center(p[i], p[j], p[k]);
                    r2 = abs2(cen-p[k]);
                }
            }
            return {cen, r2};
        }
    }
}mcc;

```

## 6.7 KDTree (Nearest Point)

```

const int MXN = 1000005;

struct KDTree {
    struct Node {
        int x, y, x1, y1, x2, y2;
        int id, f;
        Node *L, *R;
    }tree[MXN];
    int n;
    Node *root;

    long long dis2(int x1, int y1, int x2, int y2) {
        long long dx = x1-x2;
        long long dy = y1-y2;
        return dx*dx+dy*dy;
    }

    static bool cmpx(Node& a, Node& b){ return a.x<b.x; }
    static bool cmpy(Node& a, Node& b){ return a.y<b.y; }
    void init(vector<pair<int, int>> ip) {
        n = ip.size();
        for (int i=0; i<n; i++) {
            tree[i].id = i;
            tree[i].x = ip[i].first;
            tree[i].y = ip[i].second;
        }
        root = build_tree(0, n-1, 0);
    }

    Node* build_tree(int L, int R, int dep) {
        if (L>R) return nullptr;
        int M = (L+R)/2;
        tree[M].f = dep%2;
        nth_element(tree+L, tree+M, tree+R+1, tree[M].f ?
            cmpy : cmpx);
        tree[M].x1 = tree[M].x2 = tree[M].x;
        tree[M].y1 = tree[M].y2 = tree[M].y;

        tree[M].L = build_tree(L, M-1, dep+1);
        if (tree[M].L) {
            tree[M].x1 = min(tree[M].x1, tree[M].L->x1);
            tree[M].x2 = max(tree[M].x2, tree[M].L->x2);

```

```

    tree[M].y1 = min(tree[M].y1, tree[M].L->y1);
    tree[M].y2 = max(tree[M].y2, tree[M].L->y2);
}

tree[M].R = build_tree(M+1, R, dep+1);
if (tree[M].R) {
    tree[M].x1 = min(tree[M].x1, tree[M].R->x1);
    tree[M].x2 = max(tree[M].x2, tree[M].R->x2);
    tree[M].y1 = min(tree[M].y1, tree[M].R->y1);
    tree[M].y2 = max(tree[M].y2, tree[M].R->y2);
}

return tree+M;
}

int touch(Node* r, int x, int y, long long d2){
    long long dis = sqrt(d2)+1;
    if (x<r->x1-dis || x>r->x2+dis || y<r->y1-dis || y>
        r->y2+dis)
        return 0;
    return 1;
}

void nearest(Node* r, int x, int y, int &mID, long
    long &md2) {
    if (!r || !touch(r, x, y, md2)) return;
    long long d2 = dis2(r->x, r->y, x, y);
    if (d2 < md2 || (d2 == md2 && mID < r->id)) {
        mID = r->id;
        md2 = d2;
    }
    // search order depends on split dim
    if ((r->f == 0 && x < r->x) ||
        (r->f == 1 && y < r->y)) {
        nearest(r->L, x, y, mID, md2);
        nearest(r->R, x, y, mID, md2);
    } else {
        nearest(r->R, x, y, mID, md2);
        nearest(r->L, x, y, mID, md2);
    }
}

int query(int x, int y) {
    int id = 1029384756;
    long long d2 = 102938475612345678LL;
    nearest(root, x, y, id, d2);
    return id;
}
}tree;

```

## 6.8 (+1) MinkowskiSum

```

/* convex hull Minkowski Sum*/
#define INF 1000000000000000LL
class PT{ public:
    long long x,y;
    int POS(){
        if(y==0) return x>0?0:1;
        return y>0?0:1;
    }
};
PT pt[300000],qt[300000],rt[300000];
long long Lx,Rx;
int dn,un;
inline bool cmp(PT a,PT b){
    int pa=a.POS(),pb=b.POS();
    if(pa==pb) return (a^b)>0;
    return pa<pb;
}
int minkowskiSum(int n,int m){
    int i,j,r,p,q,fi,fj;
    for(i=1,p=0;i<n;i++){
        if(pt[i].y<pt[p].y || (pt[i].y==pt[p].y && pt[i].x<
            pt[p].x)) p=i; }
    for(i=1,q=0;i<m;i++){
        if(qt[i].y<qt[q].y || (qt[i].y==qt[q].y && qt[i].x<
            qt[q].x)) q=i; }
    rt[0]=pt[p]+qt[q];
    r=1; i=p; j=q; fi=fj=0;
    while(1){
        if((fj&j==q) || ((!fi||i!=p) && cmp(pt[(p+1)%n]-pt
            [p],qt[(q+1)%m]-qt[q]))){

```

```

        rt[r]=rt[r-1]+pt[(p+1)%n]-pt[p];
        p=(p+1)%n;
        fi=1;
    }else{
        rt[r]=rt[r-1]+qt[(q+1)%m]-qt[q];
        q=(q+1)%m;
        fj=1;
    }
    if(r<=1 || ((rt[r]-rt[r-1])^(rt[r-1]-rt[r-2]))!=0)
        r
        ++;
    else rt[r-1]=rt[r];
    if(i==p && j==q) break;
}
return r-1;
}

void initInConvex(int n){
    int i,p,q;
    long long Ly,Ry;
    Lx=INF; Rx=-INF;
    for(i=0;i<n;i++){
        if(pt[i].x<Lx) Lx=pt[i].x;
        if(pt[i].x>Rx) Rx=pt[i].x;
    }
    Ly=Ry=INF;
    for(i=0;i<n;i++){
        if(pt[i].x==Lx && pt[i].y<Ly){ Ly=pt[i].y; p=i; }
        if(pt[i].x==Rx && pt[i].y>Ry){ Ry=pt[i].y; q=i; }
    }
    for(dn=0,i=p;i!=q;i=(i+1)%n){ qt[dn++]=pt[i]; }
    qt[dn]=pt[q]; Ly=Ry=-INF;
    for(i=0;i<n;i++){
        if(pt[i].x==Lx && pt[i].y>Ly){ Ly=pt[i].y; p=i; }
        if(pt[i].x==Rx && pt[i].y<Ry){ Ry=pt[i].y; q=i; }
    }
    for(un=0,i=p;i!=q;i=(i+n-1)%n){ rt[un++]=pt[i]; }
    rt[un]=pt[q];
}

inline int inConvex(PT p){
    int L,R,M;
    if(p.x<Lx || p.x>Rx) return 0;
    L=0;R=dn;
    while(L<R-1){ M=(L+R)/2;
        if(p.x<qt[M].x) R=M; else L=M; }
    if(tri(qt[L],qt[R],p)<0) return 0;
    L=0;R=un;
    while(L<R-1){ M=(L+R)/2;
        if(p.x<rt[M].x) R=M; else L=M; }
    if(tri(rt[L],rt[R],p)>0) return 0;
    return 1;
}

int main(){
    int n,m,i;
    PT p;
    scanf("%d",&n);
    for(i=0;i<n;i++) scanf("%I64d %I64d",&pt[i].x,&pt[i].
        y);
    scanf("%d",&m);
    for(i=0;i<m;i++) scanf("%I64d %I64d",&qt[i].x,&qt[i].
        y);
    n=minkowskiSum(n,m);
    for(i=0;i<n;i++) pt[i]=rt[i];
    scanf("%d",&m);
    for(i=0;i<m;i++) scanf("%I64d %I64d",&qt[i].x,&qt[i].
        y);
    n=minkowskiSum(n,m);
    for(i=0;i<n;i++) pt[i]=rt[i];
    initInConvex(n);
    scanf("%d",&m);
    for(i=0;i<m;i++){
        scanf("%I64d %I64d",&p.x,&p.y);
        p.x*=3; p.y*=3;
        puts(inConvex(p)?"YES":"NO");
    }
}

```

## 7 Stringology

### 7.1 Suffix Array

```
const int MAX = 1020304;
int ct[MAX], he[MAX], rk[MAX], sa[MAX], tsa[MAX], tp[
    MAX][2];

void suffix_array(char *ip){
    int len = strlen(ip);
    int alp = 256;

    memset(ct, 0, sizeof(ct));
    for(int i=0; i<len; i++) ct[ip[i]+1]++;
    for(int i=1; i<alp; i++) ct[i]+=ct[i-1];
    for(int i=0; i<len; i++) rk[i]=ct[ip[i]];

    for(int i=1; i<len; i*=2){
        for(int j=0; j<len; j++){
            if(j+i>len) tp[j][1]=0;
            else tp[j][1]=rk[j+i]+1;

            tp[j][0]=rk[j];
        }
        memset(ct, 0, sizeof(ct));
        for(int j=0; j<len; j++) ct[tp[j][1]+1]++;
        for(int j=1; j<len+1; j++) ct[j]+=ct[j-1];
        for(int j=0; j<len; j++) tsa[ct[tp[j][1]]+j]=j;

        memset(ct, 0, sizeof(ct));
        for(int j=0; j<len; j++) ct[tp[j][0]+1]++;
        for(int j=1; j<len+1; j++) ct[j]+=ct[j-1];
        for(int j=0; j<len; j++) sa[ct[tp[tsa[j]][0]]+j]=tsa[j];

        rk[sa[0]]=0;
        for(int j=1; j<len; j++){
            if( tp[sa[j]][0] == tp[sa[j-1]][0] &&
                tp[sa[j]][1] == tp[sa[j-1]][1] )
                rk[sa[j]] = rk[sa[j-1]];
            else
                rk[sa[j]] = j;
        }

        for(int i=0, h=0; i<len; i++){
            if(rk[i]==0) h=0;
            else{
                int j=sa[rk[i]-1];
                h=max(0, h-1);
                for(; ip[i+h]==ip[j+h]; h++);
            }
            he[rk[i]]=h;
        }
    }
}
```

### 7.2 Suffix Array (SAIS TWT514)

```
struct SA{
#define REP(i,n) for ( int i=0; i<int(n); i++ )
#define REP1(i,a,b) for ( int i=(a); i<int(b); i++ )
    static const int MXN = 300010;
    bool _t[MXN*2];
    int _s[MXN*2], _sa[MXN*2], _c[MXN*2], x[MXN], _p[
        MXN], _q[MXN*2], hei[MXN], r[MXN];
    int operator [] (int i){ return _sa[i]; }
    void build(int *s, int n, int m){
        memcpy(_s, s, sizeof(int) * n);
        sais(_s, _sa, _p, _q, _t, _c, n, m);
        mkhei(n);
    }
    void mkhei(int n){
        REP(i,n) r[_sa[i]] = i;
        hei[0] = 0;
        REP(i,n) if(r[i]) {
            int ans = i>0 ? max(hei[r[i-1]] - 1, 0) :
                0;

```

```
        while(_s[i+ans] == _s[_sa[r[i]-1]+ans]) ans
            ++;
        hei[r[i]] = ans;
    }
}

void sais(int *s, int *sa, int *p, int *q, bool *t,
    int *c, int n, int z){
    bool uniq = t[n-1] = true, neq;
    int nn = 0, nmzx = -1, *nsa = sa + n, *ns = s +
        n, lst = -1;
#define MS0(x,n) memset((x),0,n*sizeof(*(x)))
#define MAGIC(XD) MS0(sa, n); \
    memcpy(x, c, sizeof(int) * z); \
    XD; \
    memcpy(x + 1, c, sizeof(int) * (z - 1)); \
    REP(i,n) if(sa[i] && !t[sa[i]-1]) sa[x[s[sa[i]
        ]-1]]++ = sa[i]-1; \
    memcpy(x, c, sizeof(int) * z); \
    for(int i = n - 1; i >= 0; i--) if(sa[i] && t[
        sa[i]-1]) sa[--x[s[sa[i]-1]]] = sa[i]-1;
    MS0(c, z);
    REP(i,n) uniq &= ++c[s[i]] < 2;
    REP(i,z-1) c[i+1] += c[i];
    if (uniq) { REP(i,n) sa[--c[s[i]]] = i; return;
    }
    for(int i = n - 2; i >= 0; i--) t[i] = (s[i]==s
        [i+1] ? t[i+1] : s[i]<s[i+1]);
    MAGIC(REP1(i,1,n-1) if(t[i] && !t[i-1]) sa[--x[
        s[i]]]=p[q[i]=nn++]=i);
    REP(i, n) if (sa[i] && t[sa[i]] && !t[sa[i]-1])
        {
            neq=lst<0||memcmp(s+sa[i],s+lst,(p[q[sa[i]
                ]+1]-sa[i])*sizeof(int));
            ns[q[lst=sa[i]]]=nmzx+=neq;
        }
    sais(ns, nsa, p + nn, q + n, t + n, c + z, nn,
        nmzx + 1);
    MAGIC(for(int i = nn - 1; i >= 0; i--) sa[--x[s
        [p[nsa[i]]]]] = p[nsa[i]]);
}

}sa;

void suffix_array(int* ip, int len) {
    // should padding a zero in the back
    // s is int array, n is array length
    // s[0..n-1] != 0, and s[n] = 0
    // resulting SA will be length n+1
    ip[len++] = 0;
    sa.build(ip, len, 128);
    // original 1-base
    for (int i=0; i<len; i++) {
        hei[i] = sa.hei[i + 1];
        sa[i] = sa._sa[i + 1];
    }
}
```

### 7.3 Aho-Corasick Algorithm

```
struct ACautomata{
    struct Node{
        int cnt,dp;
        Node *go[26], *fail;
        Node (){
            cnt = 0;
            dp = -1;
            memset(go,0,sizeof(go));
            fail = 0;
        }
    };

    Node *root, pool[1048576];
    int nMem;

    Node* new_Node(){
        pool[nMem] = Node();
        return &pool[nMem++];
    }
    void init(){
        nMem = 0;
        root = new_Node();
    }
}
```

```

}
void add(const string &str){
    insert(root,str,0);
}
void insert(Node *cur, const string &str, int pos){
    if (pos >= (int)str.size()){
        cur->cnt++;
        return;
    }
    int c = str[pos]-'a';
    if (cur->go[c] == 0){
        cur->go[c] = new_Node();
    }
    insert(cur->go[c],str,pos+1);
}
void make_fail(){
    queue<Node*> que;
    que.push(root);
    while (!que.empty()){
        Node* fr=que.front();
        que.pop();
        for (int i=0; i<26; i++){
            if (fr->go[i]){
                Node *ptr = fr->fail;
                while (ptr && !ptr->go[i]) ptr = ptr->fail;
                if (!ptr) fr->go[i]->fail = root;
                else fr->go[i]->fail = ptr->go[i];
                que.push(fr->go[i]);
            }
        }
    }
}
};

```

## 7.4 KMP

```

#include<bits/stdc++.h>
using namespace std;

void build_fail_function(string B, int *fail) {
    int len = B.length(), pos;
    pos = fail[0] = -1;
    for (int i = 1; i < len; i++) {
        while (pos != -1 and B[pos + 1] != B[i])
            pos = fail[pos];
        if (B[pos + 1] == B[i]) pos++;
        fail[i] = pos;
    }
}

void match(string A, string B, int *fail) {
    int lenA = A.length(), lenB = B.length();
    int pos = -1;
    for (int i = 0; i < lenA; i++) {
        while (pos != -1 and B[pos + 1] != A[i])
            pos = fail[pos];

        if (B[pos + 1] == A[i]) pos++;

        if (pos == lenB - 1) {
            // Match ! A[i - lenB + 1, i] = B
            pos = fail[pos];
        }
    }
}

```

## 7.5 Z value

```

void Zval(const char *s, int len, int *z) {
    z[0] = 0;
    for (int b=0, i=1; i<len; i++) {
        z[i] = max(min(z[i-b], z[b] + b - i), 0);
        while (s[i + z[i]] == s[z[i]]) z[i]++;
        if (i+z[i] > b+z[b]) b=i;
    }
}

```

## 7.6 Z value (palindrome ver.)

```

void Zpal(const char *s, int len, int *z) {
    // Only odd palindrome len is considered
    // z[i] means that the longest odd palindrom
    // centered at
    // i is [i-z[i] .. i+z[i]]
    z[0] = 0;
    for (int b=0, i=1; i<len; i++) {
        if (z[b] + b >= i) z[i] = min(z[2*b-i], b+z[b]-i);
        else z[i] = 0;
        while (i+z[i]+1 < len and i-z[i]-1 >= 0 and
            s[i+z[i]+1] == s[i-z[i]-1]) z[i]++;
        if (z[i] + i > z[b] + b) b = i;
    }
}

```

## 7.7 palindromic tree

```

int len[maxn];
int suffLink[maxn];
int to[maxn][2];
int cnt[maxn];
int numV;
char str[maxn];

int v;

void addLetter(int n)
{
    while (str[n - len[v] - 1] != str[n])
        v = suffLink[v];
    int u = suffLink[v];
    while (str[n - len[u] - 1] != str[n])
        u = suffLink[u];
    int u_ = to[u][str[n] - 'a'];
    int v_ = to[v][str[n] - 'a'];
    if (v_ == -1)
    {
        v_ = to[v][str[n] - 'a'] = numV;
        len[numV++] = len[v] + 2;
        suffLink[v_] = u_;
    }
    v = v_;
    cnt[v]++;
}

void init()
{
    memset(to, -1, sizeof to);
    str[0] = '#';
    len[0] = -1;
    len[1] = 0;
    len[2] = len[3] = 1;
    suffLink[1] = 0;
    suffLink[0] = 0;
    suffLink[2] = 1;
    suffLink[3] = 1;
    to[0][0] = 2;
    to[0][1] = 3;
    numV = 4;
}

```

## 7.8 Lexicographically Smallest Rotation

```

string mcp(string s){
    int n = s.length();
    s += s;
    int i=0, j=1;
    while (i<n && j<n){
        int k = 0;
        while (k < n && s[i+k] == s[j+k]) k++;
        if (s[i+k] <= s[j+k]) j += k+1;
        else i += k+1;
        if (i == j) j++;
    }
}

```

```

int ans = i < n ? i : j;
return s.substr(ans, n);
}

```

## 7.9 Suffix Automaton

```

// par : fail link
// val : a topological order ( useful for DP )
// go[x] : automata edge ( x is integer in [0,26) )

struct SAM{
    struct State{
        int par, go[26], val;
        State () : par(0), val(0){ FZ(go); }
        State (int _val) : par(0), val(_val){ FZ(go); }
    };
    vector<State> vec;
    int root, tail;

    void init(int arr[], int len){
        vec.resize(2);
        vec[0] = vec[1] = State(0);
        root = tail = 1;
        for (int i=0; i<len; i++)
            extend(arr[i]);
    }
    void extend(int w){
        int p = tail, np = vec.size();
        vec.PB(State(vec[p].val+1));
        for ( ; p && vec[p].go[w]==0; p=vec[p].par)
            vec[p].go[w] = np;
        if (p == 0){
            vec[np].par = root;
        } else {
            if (vec[vec[p].go[w]].val == vec[p].val+1){
                vec[np].par = vec[p].go[w];
            } else {
                int q = vec[p].go[w], r = vec.size();
                vec.PB(vec[q]);
                vec[r].val = vec[p].val+1;
                vec[q].par = vec[np].par = r;
                for ( ; p && vec[p].go[w] == q; p=vec[p].par)
                    vec[p].go[w] = r;
            }
        }
        tail = np;
    }
};

```

## 8 Problems

### 8.1 Find the maximun tangent (x,y is increasing)

```

typedef long long LL;
const int MAXN = 100010;
struct Coord{
    LL x, y;
    Coord operator - (Coord ag) const{
        Coord res;
        res.x = x - ag.x;
        res.y = y - ag.y;
        return res;
    }
}sum[MAXN], pnt[MAXN], ans, calc;

inline bool cross(Coord a, Coord b, Coord c){
    return (c.y - a.y) * (c.x - b.x) > (c.x - a.x) * (c.y - b.y);
}

int main(){
    int n, l, np, st, ed, now;
    scanf("%d %d\n", &n, &l);
    sum[0].x = sum[0].y = np = st = ed = 0;
    for (int i = 1, v; i <= n; i++){
        scanf("%d", &v);

```

```

        sum[i].y = sum[i - 1].y + v;
        sum[i].x = i;
    }
    ans.x = now = 1;
    ans.y = -1;
    for (int i = 0; i <= n - l; i++){
        while (np > 1 && cross(pnt[np - 2], pnt[np - 1], sum[i]))
            np--;
        if (np < now && np != 0) now = np;
        pnt[np++] = sum[i];
        while (now < np && !cross(pnt[now - 1], pnt[now], sum[i + l]))
            now++;
        calc = sum[i + l] - pnt[now - 1];
        if (ans.y * calc.x < ans.x * calc.y){
            ans = calc;
            st = pnt[now - 1].x;
            ed = i + l;
        }
    }
    double res = (sum[ed].y-sum[st].y)/(sum[ed].x-sum[st].x);
    printf("%f\n", res);
    return 0;
}

```

### 8.2 Painter

```

#include<bits/stdc++.h>
using namespace std;
#define F first
#define S second
#define PB push_back
#define IOS ios_base::sync_with_stdio(0); cin.tie(0);
#define SZ(x) ((int)((x).size()))
#define ALL(x) begin(x),end(x)
#define REP(i,x) for (int i=0; i<(x); i++)
#define REP1(i,a,b) for (int i=(a); i<=(b); i++)

typedef long long ll;
typedef pair<ll,ll> pll;

typedef pll Point;
const int MXN = 100005;

Point operator + (const Point &a, const Point &b) {
    return Point(a.F+b.F, a.S+b.S);
}
Point operator - (const Point &a, const Point &b) {
    return Point(a.F-b.F, a.S-b.S);
}
ll operator * (const Point &a, const Point &b) { return a.F*b.F + a.S*b.S; }
ll operator % (const Point &a, const Point &b) { return a.F*b.S - a.S*b.F; }

struct Segment {
    int v,id;
    Point p,q;
    Segment () {}
    Segment (int _v, int _id, Point _p, Point _q) :
        v(_v), id(_id), p(_p), q(_q) {}
};

bool operator < (const Segment &a, const Segment &b) {
    if (a.p == b.q) return false;
    if (a.q == b.p) return true;
    if (a.p == b.p) return (a.q-a.p) % (b.q-a.p) > 0;
    if (a.q == b.q) return (a.p-a.q) % (b.p-a.q) < 0;
    if (a.p.F == b.p.F) return a.p.S < b.p.S;
    if (a.q.F == b.q.F) return a.q.S < b.q.S;
    if (a.p.F < b.p.F) return (a.q-a.p) % (b.p-a.p) > 0;
    else return (b.q-b.p) % (a.p-b.p) < 0;
}

bool operator == (const Segment &a, const Segment &b) {
    return tie(a.v,a.id,a.p,a.q) == tie(b.v,b.id,b.p,b.q);
}

struct Triangle {
    Point pt[3];
}ip[MXN];

```



```

const int MEM = 350004;
struct Treap {
    static Treap nil, mem[MEM], *pmem;
    Treap *l, *r;
    int sum, presum, size;
    Segment seg;
    Treap () : l(&nil), r(&nil), sum(0), presum(0), size(0), seg() {}
    Treap (Segment _val) : l(&nil), r(&nil), sum(_val.v), presum(max(_val.v, 0)), size(1), seg(_val) {}
} Treap::nil, Treap::mem[MEM], *Treap::pmem = Treap::mem;

int size(const Treap *t) { return t->size; }
void pull(Treap *t) {
    if (!size(t)) return;
    t->size = size(t->l) + size(t->r) + 1;
    t->sum = t->l->sum + t->seg.v + t->r->sum;
    t->presum = max(t->l->presum, t->l->sum + t->seg.v);
    t->presum = max(t->presum, t->l->sum + t->seg.v + t->r->presum);
}

Treap* merge(Treap *a, Treap *b) {
    if (!size(a)) return b;
    if (!size(b)) return a;
    Treap *t;
    if (rand() % (size(a) + size(b)) < size(a)) {
        t = a;
        t->r = merge(a->r, b);
    } else {
        t = b;
        t->l = merge(a, b->l);
    }
    pull(t);
    return t;
}

void split(Treap *t, int k, Treap *&a, Treap *&b) {
    if (!size(t)) a = b = &Treap::nil;
    else if (size(t->l) + 1 <= k) {
        a = t;
        split(t->r, k - size(t->l) - 1, a->r, b);
        pull(a);
    } else {
        b = t;
        split(t->l, k, a, b->l);
        pull(b);
    }
}

int get_rank(Treap *t, Segment x) {
    if (!size(t)) return 0;
    if (x < t->seg) return get_rank(t->l, x);
    return get_rank(t->r, x) + size(t->l) + 1;
}

Treap* find_leftist(Treap *t) {
    while (size(t->l)) t = t->l;
    return t;
}

Treap* find_rightist(Treap *t) {
    while (size(t->r)) t = t->r;
    return t;
}

int N;
vector<int> allx;
vector<Segment> _seg[3*MXN];
#define seg(x) _seg[(x)+100000]

inline void add_seg(Segment s) {
    seg(s.p.F).PB(s);
    if (s.q.F != s.p.F) seg(s.q.F).PB(s);
}

void predo() {
    allx.clear();
    REP(i, N) REP(j, 3) {
        seg(ip[i].pt[j].F).clear();
        allx.PB(ip[i].pt[j].F);
    }
    sort(ALL(allx));
    allx.resize(unique(ALL(allx)) - begin(allx));
    REP(i, N) {
        sort(ip[i].pt, ip[i].pt+3);

```

```

        Point *pt = ip[i].pt;
        Segment seg1 = Segment(1, i, pt[0], pt[1]);
        Segment seg2 = Segment(1, i, pt[0], pt[2]);
        Segment seg3 = Segment(1, i, pt[1], pt[2]);
        if (seg2 < seg1) seg1.v = -1;
        else seg2.v = -1;
        seg3.v = seg1.v;
        add_seg(seg1);
        add_seg(seg2);
        add_seg(seg3);
    }
}

inline int sgn(ll x) { return x < 0 ? -1 : x > 0; }
bool interPnt(Point p1, Point p2, Point q1, Point q2) {
    ll c1 = (p2-p1)%(q1-p1), c2 = (p2-p1)%(q2-p1);
    ll c3 = (q2-q1)%(p1-q1), c4 = (q2-q1)%(p2-q1);
    return sgn(c1) * sgn(c2) <= 0 and sgn(c3) * sgn(c4) <= 0;
}

bool check_error(Segment a, Segment b) {
    if (a.id == b.id) return false;
    return interPnt(a.p, a.q, b.p, b.q);
}

int solve() {
    Treap::pmem = Treap::mem;
    Treap *rt = &Treap::nil;
    int res = 0;
    for (auto i:allx) {
        for (auto l:seg(i)) {
            int k = get_rank(rt, l);
            Treap *t, *tl, *tm, *tr;
            split(rt, k, tl, tr);
            t = find_rightist(tl);
            if (size(t) and check_error(t->seg, l)) return -1;
            t = find_leftist(tr);
            if (size(t) and check_error(t->seg, l)) return -1;
            rt = merge(tl, tr);
            if (l.p.F == i and l.p.F != l.q.F) {
                k = get_rank(rt, l);
                split(rt, k, tl, tr);
                tm = new (Treap::pmem++) Treap(l);
                rt = merge(merge(tl, tm), tr);
            }
        }
        for (auto l:seg(i)) {
            if (l.q.F == i and l.p.F != l.q.F) {
                Treap *tl, *tm, *tr;
                int k = get_rank(rt, l);
                split(rt, k-1, tl, tm);
                split(tm, 1, tm, tr);
                Treap *t1=find_rightist(tl), *t2=find_leftist(tr);
                if (size(t1) and size(t2) and check_error(t1->seg, t2->seg)) return -1;
                rt = merge(tl, tr);
            }
        }
        res = max(res, rt->presum);
    }
    res++;
    return res;
}

int main() {
    IOS;
    int cas = 0;
    while (cin >> N) {
        if (N == -1) break;
        REP(i, N) {
            REP(j, 3) cin >> ip[i].pt[j].F >> ip[i].pt[j].S;
        }
        predo();
        int ans = solve();
        cas++;
        cout << "Case " << cas << ": ";
        if (ans == -1) cout << "ERROR\n";
        else cout << ans << " shades\n";
    }

    return 0;
}

```

## 8.3 Mo-Algorithm on Tree

```
//bcw0x1bd2 {
#include<bits/stdc++.h>
#include<unistd.h>
using namespace std;
#define F first
#define S second
#define MP make_pair
#define PB push_back
#define IOS ios_base::sync_with_stdio(0); cin.tie(0);
#define SZ(x) ((int)((x).size()))
#define ALL(x) begin(x),end(x)
#define REP(i,x) for (int i=0; i<(x); i++)
#define REP1(i,a,b) for (int i=(a); i<=(b); i++)

typedef long long ll;
typedef pair<int,int> pii;
typedef pair<ll,ll> pll;
typedef long double ld;

#ifdef DARKHH
#define FILEIO(name)
#else
#define FILEIO(name) \
    freopen(name".in", "r", stdin); \
    freopen(name".out", "w", stdout);
#endif

#ifdef DARKHH
template<typename Iter>
ostream& _out(ostream &s, Iter b, Iter e) {
    s << "[ ";
    for (auto it=b; it!=e; it++) s << *it << " ";
    s << "]";
    return s;
}

template<typename A, typename B>
ostream& operator << (ostream &s, const pair<A,B> &p) {
    return s<<"("<p.first<<","<p.second<<")";
}

template<typename T>
ostream& operator << (ostream &s, const vector<T> &c) {
    return _out(s,ALL(c));
}

template<typename T, size_t N>
ostream& operator << (ostream &s, const array<T,N> &c)
{ return _out(s,ALL(c)); }

template<typename T>
ostream& operator << (ostream &s, const set<T> &c) {
    return _out(s,ALL(c));
}

template<typename A, typename B>
ostream& operator << (ostream &s, const map<A,B> &c) {
    return _out(s,ALL(c));
}
#endif
// }
// Let's Fight! ~OAO~

const int MX = 500005;
const int SQ = 1400;
const int LOG = 17;

struct BIT {
    int bit[MX];
    int lb(int x) { return x & -x; }
    void add(int p, int v) {
        p++;
        for (int i=p; i<MX; i+=lb(i))
            bit[i] += v;
    }
    int qry() {
        int v = 0;
        for (int i=1<<LOG; i>0; i>>=1) {
            if ((v|i) < MX and bit[v|i]==i) v |= i;
        }
        return v;
    }
} bit;

struct Query {
    int l,r,qid;
} qry[MX];
struct Edge {
```

```
    int v,x;
};

int N,Q,timestamp[MX],ans[MX];
int in[MX],cnt[MX];
vector<Edge> E[MX];
vector<Edge> seq;

void DFS(int u, int f) {
    timestamp[u] = SZ(seq);
    for (auto it:E[u]) {
        if (it.v == f) continue;
        seq.PB(it);
        DFS(it.v,u);
        seq.PB(it);
    }
}

void poke(int id) {
    int v = seq[id].v;
    int x = seq[id].x;
    in[v] ^= 1;
    cnt[x] += in[v] ? 1 : -1;
    if (in[v] and cnt[x] == 1) bit.add(x, 1);
    if (!in[v] and cnt[x] == 0) bit.add(x, -1);
}

int main() {
    IOS;
    cin >> N >> Q;
    REP(_,N-1) {
        int u,v,x;
        cin >> u >> v >> x;
        x = min(x,N);
        E[u].PB({v,x});
        E[v].PB({u,x});
    }
    DFS(1,1);
    REP1(i,1,Q) {
        int u,v;
        cin >> u >> v;
        int l = timestamp[u], r = timestamp[v];
        if (l > r) swap(l,r);
        r--;
        qry[i] = {l,r,i};
    }
    sort(qry+1,qry+1+Q, [](Query a, Query b) {
        return make_pair(a.l/SQ,a.r) < make_pair(b.l/SQ,b.r);
    });

    int curL = 1, curR = 0;
    REP1(i,1,Q) {
        int ql=qry[i].l,qr=qry[i].r;
        while (curL > ql) poke(--curL);
        while (curR < qr) poke(++curR);
        while (curL < ql) poke(curL++);
        while (curR > qr) poke(curR--);
        ans[qry[i].qid] = bit.qry();
    }

    REP1(i,1,Q) {
        cout << ans[i] << "\n";
    }

    return 0;
}
```

## 9.1 Periodic Table

[illegible]