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## 1 Basic

### 1.1 .vimrc

```

1 colo torte
1 syn on
2 se ai ar sm nu rnu is
2 se mouse=a bs=2 ww+=<, >, [, ] so=6 ts=4 sw=4 tt=100
3 se makeprg=g++\ -Wall\ -Wshadow\ -O2\ -std=c++0x\ -o\
3 %\ %
3 au BufNewFile *.cpp 0r ~/default.cpp | :0,22 fo
4 filetype indent on
5
5 map <F7> <ESC>:wa<CR>:make!<CR>
5 imap <F7> <ESC>:wa<CR>:make!<CR>
6 map <C-F7> <ESC>:tabe %<.in<CR>
6 map <F8> :cope <CR>
7 map <S-F8> :ccl <CR>
7 map <F9> :!./%< <CR>
7 map <C-F9> :!./%< < %<.in <CR>

```

### 1.2 IncreaseStackSize

```

10 //stack resize
10 asm( "mov %0,%esp\n" ::"g"(mem+100000000) );
11 //change esp to rsp if 64-bit system
12
12 //stack resize (linux)
12 #include <sys/resource.h>
12 void increase_stack_size() {
13     const rlim_t ks = 64*1024*1024;
13     struct rlimit rl;
13     int res=getrlimit(RLIMIT_STACK, &rl);
14     if(res==0){
14         if(rl.rlim_cur<ks){
15             rl.rlim_cur=ks;
15             res=setrlimit(RLIMIT_STACK, &rl);
16         }
16     }
16 }

```

### 1.3 Default Code

```

16 #include<bits/stdc++.h>
16 #include<unistd.h>
16 using namespace std;
16 #define FZ(n) memset((n),0,sizeof(n))
16 #define FMO(n) memset((n),-1,sizeof(n))
16 #define F first
16 #define S second
16 #define PB push_back
16 #define ALL(x) begin(x),end(x)
16 #define SZ(x) ((int)(x).size())
16 #define IOS ios_base::sync_with_stdio(0); cin.tie(0)
16 template<typename A, typename B>
16 ostream& operator <<(ostream &s, const pair<A,B> &p) {
16     return s<<"("<<p.first<<" "<<p.second<<")";
16 }
16 template<typename T>
16 ostream& operator <<(ostream &s, const vector<T> &c) {
16     s<<"[";
16     for (auto it : c) s << it << " ";
16     s<<"]";
16     return s;
16 }
16 // Let's Fight!
16
16 int main() {
16     return 0;
16 }

```

## 2 Data Structure

### 2.1 Bigint

```

struct Bigint{
    static const int LEN = 60;
    static const int BIGMOD = 100000;

    int s;
    int vl, v[LEN];
    // vector<int> v;
    Bigint() : s(1) { vl = 0; }
    Bigint(long long a) {
        s = 1; vl = 0;
        if (a < 0) { s = -1; a = -a; }
        while (a) {
            push_back(a % BIGMOD);
            a /= BIGMOD;
        }
    }
    Bigint(string str) {
        s = 1; vl = 0;
        int stPos = 0, num = 0;
        if (!str.empty() && str[0] == '-') {
            stPos = 1;
            s = -1;
        }
        for (int i=SZ(str)-1, q=1; i>=stPos; i--) {
            num += (str[i] - '0') * q;
            if ((q * 10) >= BIGMOD) {
                push_back(num);
                num = 0; q = 1;
            }
        }
        if (num) push_back(num);
    }

    int len() const {
        return vl;
        // return SZ(v);
    }
    bool empty() const { return len() == 0; }
    void push_back(int x) {
        v[vl++] = x;
        // v.PB(x);
    }
    void pop_back() {
        vl--;
        // v.pop_back();
    }
    int back() const {
        return v[vl-1];
        // return v.back();
    }
    void n() {
        while (!empty() && !back()) pop_back();
    }
    void resize(int nl) {
        vl = nl;
        fill(v, v+vl, 0);
        // v.resize(nl);
        // fill(ALL(v), 0);
    }

    void print() const {
        if (empty()) { putchar('0'); return; }
        if (s == -1) putchar('-');
        printf("%d", back());
        for (int i=len()-2; i>=0; i--) printf("%.4d", v[i]);
    }
    friend std::ostream& operator << (std::ostream& out,
        const Bigint &a) {
        if (a.empty()) { out << "0"; return out; }
        if (a.s == -1) out << "-";
        out << a.back();
        for (int i=a.len()-2; i>=0; i--) {
            char str[10];
            snprintf(str, 5, "%.4d", a.v[i]);
            out << str;
        }
    }
}

```

```

    return out;
}

int cp3(const Bigint &b) const {
    if (s != b.s) return s - b.s;
    if (s == -1) return -(*this).cp3(-b);
    if (len() != b.len()) return len()-b.len(); //int
    for (int i=len()-1; i>=0; i--)
        if (v[i] != b.v[i]) return v[i]-b.v[i];
    return 0;
}

bool operator < (const Bigint &b) const { return cp3(b) < 0; }
bool operator <= (const Bigint &b) const { return cp3(b) <= 0; }
bool operator == (const Bigint &b) const { return cp3(b) == 0; }
bool operator != (const Bigint &b) const { return cp3(b) != 0; }
bool operator > (const Bigint &b) const { return cp3(b) > 0; }
bool operator >= (const Bigint &b) const { return cp3(b) >= 0; }

Bigint operator - () const {
    Bigint r = (*this);
    r.s = -r.s;
    return r;
}

Bigint operator + (const Bigint &b) const {
    if (s == -1) return -(*this)+(-b);
    if (b.s == -1) return (*this)-(-b);
    Bigint r;
    int nl = max(len(), b.len());
    r.resize(nl + 1);
    for (int i=0; i<nl; i++) {
        if (i < len()) r.v[i] += v[i];
        if (i < b.len()) r.v[i] += b.v[i];
        if (r.v[i] >= BIGMOD) {
            r.v[i+1] += r.v[i] / BIGMOD;
            r.v[i] %= BIGMOD;
        }
    }
    r.n();
    return r;
}

Bigint operator - (const Bigint &b) const {
    if (s == -1) return -(*this)-(-b);
    if (b.s == -1) return (*this)+(-b);
    if ((*this) < b) return -(b-(*this));
    Bigint r;
    r.resize(len());
    for (int i=0; i<len(); i++) {
        r.v[i] += v[i];
        if (i < b.len()) r.v[i] -= b.v[i];
        if (r.v[i] < 0) {
            r.v[i] += BIGMOD;
            r.v[i+1]--;
        }
    }
    r.n();
    return r;
}

Bigint operator * (const Bigint &b) {
    Bigint r;
    r.resize(len() + b.len() + 1);
    r.s = s * b.s;
    for (int i=0; i<len(); i++) {
        for (int j=0; j<b.len(); j++) {
            r.v[i+j] += v[i] * b.v[j];
            if (r.v[i+j] >= BIGMOD) {
                r.v[i+j+1] += r.v[i+j] / BIGMOD;
                r.v[i+j] %= BIGMOD;
            }
        }
    }
    r.n();
    return r;
}

Bigint operator / (const Bigint &b) {
    Bigint r;
}

```

```

r.resize(max(1, len()-b.len()+1));
int oriS = s;
Bigint b2 = b; // b2 = abs(b)
s = b2.s = r.s = 1;
for (int i=r.len()-1; i>=0; i--) {
    int d=0, u=BIGMOD-1;
    while(d<u) {
        int m = (d+u+1)>>1;
        r.v[i] = m;
        if((r*b2) > (*this)) u = m-1;
        else d = m;
    }
    r.v[i] = d;
}
s = oriS;
r.s = s * b.s;
r.n();
return r;
}
Bigint operator % (const Bigint &b) {
    return (*this)-(*this)/b*b;
}
};

```

## 2.2 unordered\_map

```

struct Key {
    int first,second;
    Key () {}
    Key (int _x, int _y) : first(_x), second(_y) {}
    bool operator == (const Key &b) const {
        return tie(F,S) == tie(b.F,b.S);
    }
};
struct KeyHasher {
    size_t operator()(const Key& k) const {
        return k.first + k.second*100000;
    }
};
typedef unordered_map<Key,int,KeyHasher> map_t;

int main(int argc, char** argv){
    map_t mp;
    for (int i=0; i<10; i++)
        mp[Key(i,0)] = i+1;
    for (int i=0; i<10; i++)
        printf("%d\n", mp[Key(i,0)]);

    return 0;
}

```

## 2.3 extc\_balance\_tree

```

#include <bits/extc++.h>
using namespace __gnu_pbds;
typedef tree<int,null_type,less<int>,rb_tree_tag,
    tree_order_statistics_node_update> set_t;

int main()
{
    // Insert some entries into s.
    set_t s;
    s.insert(12);
    s.insert(505);

    // The order of the keys should be: 12, 505.
    assert(*s.find_by_order(0) == 12);
    assert(*s.find_by_order(3) == 505);

    // The order of the keys should be: 12, 505.
    assert(s.order_of_key(12) == 0);
    assert(s.order_of_key(505) == 1);

    // Erase an entry.
    s.erase(12);

    // The order of the keys should be: 505.
}

```

```

assert(*s.find_by_order(0) == 505);

// The order of the keys should be: 505.
assert(s.order_of_key(505) == 0);
}

```

## 2.4 Disjoint Set

```

struct DisjointSet {
    // save() is like recursive
    // undo() is like return
    int n, fa[MXN], sz[MXN];
    vector<pair<int*,int*>> h;
    vector<int> sp;
    void init(int tn) {
        n=tn;
        for (int i=0; i<n; i++) {
            fa[i]=i;
            sz[i]=1;
        }
        sp.clear(); h.clear();
    }
    void assign(int *k, int v) {
        h.PB({k, *k});
        *k=v;
    }
    void save() { sp.PB(SZ(h)) };
    void undo() {
        assert(!sp.empty());
        int last=sp.back(); sp.pop_back();
        while (SZ(h)!=last) {
            auto x=h.back(); h.pop_back();
            *x.F=x.S;
        }
    }
    int f(int x) {
        while (fa[x]!=x) x=fa[x];
        return x;
    }
    void uni(int x, int y) {
        x=f(x); y=f(y);
        if (x==y) return;
        if (sz[x]<sz[y]) swap(x, y);
        assign(&sz[x], sz[x]+sz[y]);
        assign(&fa[y], x);
    }
}djs;

```

## 2.5 Treap

```

const int MEM = 16000004;
struct Treap {
    static Treap nil, mem[MEM], *pmem;
    Treap *l, *r;
    char val;
    int size;
    Treap () : l(&nil), r(&nil), size(0) {}
    Treap (char _val) :
        l(&nil), r(&nil), val(_val), size(1) {}
} Treap::nil, Treap::mem[MEM], *Treap::pmem = Treap::
    mem;

int size(const Treap *t) { return t->size; }
void pull(Treap *t) {
    if (!size(t)) return;
    t->size = size(t->l) + size(t->r) + 1;
}
Treap* merge(Treap *a, Treap *b) {
    if (!size(a)) return b;
    if (!size(b)) return a;
    Treap *t;
    if (rand() % (size(a) + size(b)) < size(a)) {
        t = new (Treap::pmem++) Treap(*a);
        t->r = merge(a->r, b);
    } else {
        t = new (Treap::pmem++) Treap(*b);
        t->l = merge(a, b->l);
    }
}

```

```

    }
    pull(t);
    return t;
}

void split(Treap *t, int k, Treap *&a, Treap *&b) {
    if (!size(t)) a = b = &Treap::nil;
    else if (size(t->l) + 1 <= k) {
        a = new (Treap::pmem++) Treap(*t);
        split(t->r, k - size(t->l) - 1, a->r, b);
        pull(a);
    } else {
        b = new (Treap::pmem++) Treap(*t);
        split(t->l, k, a, b->l);
        pull(b);
    }
}

int nv;
Treap *rt[50005];

void print(const Treap *t) {
    if (!size(t)) return;
    print(t->l);
    cout << t->val;
    print(t->r);
}

int main(int argc, char** argv) {
    IOS;
    rt[nv=0] = &Treap::nil;
    Treap::pmem = Treap::mem;
    int Q, cmd, p, c, v;
    string s;
    cin >> Q;
    while (Q--) {
        cin >> cmd;
        if (cmd == 1) {
            // insert string s after position p
            cin >> p >> s;
            Treap *tl, *tr;
            split(rt[nv], p, tl, tr);
            for (int i=0; i<SZ(s); i++)
                tl = merge(tl, new (Treap::pmem++) Treap(s[i]));
            rt[++nv] = merge(tl, tr);
        } else if (cmd == 2) {
            // remove c characters starting at position
            Treap *tl, *tm, *tr;
            cin >> p >> c;
            split(rt[nv], p-1, tl, tm);
            split(tm, c, tm, tr);
            rt[++nv] = merge(tl, tr);
        } else if (cmd == 3) {
            // print c characters starting at position p, in
            // version v
            Treap *tl, *tm, *tr;
            cin >> v >> p >> c;
            split(rt[v], p-1, tl, tm);
            split(tm, c, tm, tr);
            print(tm);
            cout << "\n";
        }
    }
    return 0;
}

```

## 2.6 Heavy Light Decomposition

```

// only one segment tree / no 0/1 base issue
// getPathSeg return the segment in order u->v
// fa[root] = root
typedef pair<int,int> pii;

int N, fa[MAXN], belong[MAXN], dep[MAXN], sz[MAXN], que[MAXN];
int step, line[MAXN], stPt[MAXN], edPt[MAXN];
vector<int> E[MAXN], chain[MAXN];

void DFS(int u) {
    vector<int> &c = chain[belong[u]];
    for (int i=c.size()-1; i>=0; i--) {

```

```

        int v = c[i];
        stPt[v] = step;
        line[step++] = v;
    }
    for (int i=0; i<(int)c.size(); i++){
        u = c[i];
        for (auto v : E[u]){
            if (fa[u] == v || (i && v == c[i-1])) continue;
            DFS(v);
        }
        edPt[u] = step-1;
    }
}

void build_chain(int st) {
    int fr, bk;
    fr=bk=0; que[bk++] = 1; fa[st]=st; dep[st]=0;
    while (fr < bk) {
        int u=que[fr++];
        for (auto v : E[u]){
            if (v == fa[u]) continue;
            que[bk++] = v;
            dep[v] = dep[u]+1;
            fa[v] = u;
        }
    }
    for (int i=bk-1, u, pos; i>=0; i--){
        u = que[i]; sz[u] = 1; pos = -1;
        for (auto v : E[u]){
            if (v == fa[u]) continue;
            sz[u] += sz[v];
            if (pos == -1 || sz[v]>sz[pos]) pos=v;
        }
        if (pos == -1) belong[u] = u;
        else belong[u] = belong[pos];
        chain[belong[u]].PB(u);
    }
    step = 0;
    DFS(st);
}

int getLCA(int u, int v) {
    while (belong[u] != belong[v]) {
        int a = chain[belong[u]].back();
        int b = chain[belong[v]].back();
        if (dep[a] > dep[b]) u = fa[a];
        else v = fa[b];
    }
    return sz[u] >= sz[v] ? u : v;
}

vector<pii> getPathSeg(int u, int v) {
    vector<pii> ret1, ret2;
    while (belong[u] != belong[v]) {
        int a = chain[belong[u]].back();
        int b = chain[belong[v]].back();
        if (dep[a] > dep[b]) {
            ret1.PB({stPt[a], stPt[u]});
            u = fa[a];
        } else {
            ret2.PB({stPt[b], stPt[v]});
            v = fa[b];
        }
    }
    if (dep[u] > dep[v]) swap(u, v);
    ret1.PB({stPt[u], stPt[v]});
    reverse(ret2.begin(), ret2.end());
    ret1.insert(ret1.end(), ret2.begin(), ret2.end());
    return ret1;
}

// Usage
void build() {
    build_chain(1); //change root
    init(0, step, 0); //init segment tree
}

int get_answer(int u, int v) {
    int ret = -2147483647;
    vector<pii> vec = getPathSeg(u, v);
    for (auto it : vec)
        // check answer with segment [it.F, it.S]
    return ret;
}

```

## 2.7 Link-Cut Tree

```

const int MXN = 100005;
const int MEM = 100005;

struct Splay {
    static Splay nil, mem[MEM], *pmem;
    Splay *ch[2], *f;
    int val, rev, size;
    Splay () : val(-1), rev(0), size(0) {
        f = ch[0] = ch[1] = &nil;
    }
    Splay (int _val) : val(_val), rev(0), size(1) {
        f = ch[0] = ch[1] = &nil;
    }
    bool isr() {
        return f->ch[0] != this && f->ch[1] != this;
    }
    int dir() {
        return f->ch[0] == this ? 0 : 1;
    }
    void setCh(Splay *c, int d) {
        ch[d] = c;
        if (c != &nil) c->f = this;
        pull();
    }
    void push() {
        if (rev) {
            swap(ch[0], ch[1]);
            if (ch[0] != &nil) ch[0]->rev ^= 1;
            if (ch[1] != &nil) ch[1]->rev ^= 1;
            rev=0;
        }
    }
    void pull() {
        size = ch[0]->size + ch[1]->size + 1;
        if (ch[0] != &nil) ch[0]->f = this;
        if (ch[1] != &nil) ch[1]->f = this;
    }
} Splay::nil, Splay::mem[MEM], *Splay::pmem = Splay::mem;
Splay *nil = &Splay::nil;

void rotate(Splay *x) {
    Splay *p = x->f;
    int d = x->dir();
    if (!p->isr()) p->f->setCh(x, p->dir());
    else x->f = p->f;
    p->setCh(x->ch[!d], d);
    x->setCh(p, !d);
    p->pull(); x->pull();
}

vector<Splay*> splayVec;
void splay(Splay *x) {
    splayVec.clear();
    for (Splay *q=x;; q=q->f) {
        splayVec.push_back(q);
        if (q->isr()) break;
    }
    reverse(begin(splayVec), end(splayVec));
    for (auto it : splayVec) it->push();
    while (!x->isr()) {
        if (x->f->isr()) rotate(x);
        else if (x->dir()==x->f->dir()) rotate(x->f), rotate(x);
        else rotate(x), rotate(x);
    }
}

Splay* access(Splay *x) {
    Splay *q = nil;
    for (;x!=nil;x=x->f) {
        splay(x);
        x->setCh(q, 1);
        q = x;
    }
    return q;
}

void evert(Splay *x) {
    access(x);

```

```

    splay(x);
    x->rev ^= 1;
    x->push(); x->pull();
}

void link(Splay *x, Splay *y) {
    // evert(x);
    access(x);
    splay(x);
    evert(y);
    x->setCh(y, 1);
}

void cut(Splay *x, Splay *y) {
    // evert(x);
    access(y);
    splay(y);
    y->push();
    y->ch[0] = y->ch[0]->f = nil;
}

int N, Q;
Splay *vt[MXN];

int ask(Splay *x, Splay *y) {
    access(x);
    access(y);
    splay(x);
    int res = x->f->val;
    if (res == -1) res=x->val;
    return res;
}

int main(int argc, char** argv) {
    scanf("%d%d", &N, &Q);
    for (int i=1; i<=N; i++)
        vt[i] = new (Splay::pmem++) Splay(i);
    while (Q--) {
        char cmd[105];
        int u, v;
        scanf("%s", cmd);
        if (cmd[1] == 'i') {
            scanf("%d%d", &u, &v);
            link(vt[v], vt[u]);
        } else if (cmd[0] == 'c') {
            scanf("%d", &v);
            cut(vt[1], vt[v]);
        } else {
            scanf("%d%d", &u, &v);
            int res=ask(vt[u], vt[v]);
            printf("%d\n", res);
        }
    }

    return 0;
}

```

## 3 Graph

### 3.1 Tarjan

```

const int MAXV = 101000;

int V, E;
vector<int> el[MAXV];
int dfn[MAXV], low[MAXV], did;
bool ins[MAXV];
stack<int> st;
int scc[MAXV], scn;

void tarjan(int u){
    cout << u << endl;
    dfn[u] = low[u] = ++did;
    st.push(u); ins[u] = true;

    for(int i=0; i<(int)el[u].size(); i++){
        int v = el[u][i];
        if(!dfn[v]){
            tarjan(v);
            low[u] = min(low[u], low[v]);
        } else if(ins[v]){

```

```

        low[u] = min(low[u], dfn[v]);
    }
}

if(dfn[u] == low[u]){
    int v;
    do{
        v = st.top();
        st.pop();
        scc[v] = scn;
        ins[v] = false;
    }while(v != u);
    scn ++;
}
}

void calcscc(){
    did = scn = 0;
    for(int i=0; i<V; i++){
        if(!dfn[i]) tarjan(i);
    }
}
}

```

## 3.2 Strongly Connected Components

```

struct Scc{
    int n, nScc, vst[MXN], bln[MXN];
    vector<int> E[MXN], rE[MXN], vec;
    void init(int _n){
        n = _n;
        for (int i=0; i<MXN; i++){
            E[i].clear();
            rE[i].clear();
        }
    }
    void add_edge(int u, int v){
        E[u].PB(v);
        rE[v].PB(u);
    }
    void DFS(int u){
        vst[u]=1;
        for (auto v : E[u])
            if (!vst[v]) DFS(v);
        vec.PB(u);
    }
    void rDFS(int u){
        vst[u] = 1;
        bln[u] = nScc;
        for (auto v : rE[u])
            if (!vst[v]) rDFS(v);
    }
    void solve(){
        nScc = 0;
        vec.clear();
        FZ(vst);
        for (int i=0; i<n; i++)
            if (!vst[i]) DFS(i);
        reverse(vec.begin(),vec.end());
        FZ(vst);
        for (auto v : vec){
            if (!vst[v]){
                rDFS(v);
                nScc++;
            }
        }
    }
};

```

## 3.3 DMST\_with\_sol

```

const int INF = 1029384756;

struct edge_t{
    int u,v,w;
    set< pair<int,int> > add, sub;
    edge_t() : u(-1), v(-1), w(0) {}
    edge_t(int _u, int _v, int _w) {
        u = _u; v = _v; w = _w;
    }
};

```

```

        add.insert({u, v});
    }
    edge_t& operator += (const edge_t& obj) {
        w += obj.w;
        FOR (it, obj.add) {
            if (!sub.count(*it)) add.insert(*it);
            else sub.erase(*it);
        }
        FOR (it, obj.sub) {
            if (!add.count(*it)) sub.insert(*it);
            else add.erase(*it);
        }
        return *this;
    }
    edge_t& operator -= (const edge_t& obj) {
        w -= obj.w;
        FOR (it, obj.sub) {
            if (!sub.count(*it)) add.insert(*it);
            else sub.erase(*it);
        }
        for (auto it : obj.add) {
            if (!add.count(it)) sub.insert(it);
            else add.erase(it);
        }
        return *this;
    }
}eg[MXN*MXN],prv[MXN],EDGE_INF(-1,-1,INF);
int N,M;
int cid,incyc[MXN],contracted[MXN];
vector<int> E[MXN];

edge_t dmst(int rt){
    edge_t cost;
    for (int i=0; i<N; i++){
        contracted[i] = incyc[i] = 0;
        prv[i] = EDGE_INF;
    }
    cid = 0;
    int u,v;
    while (true){
        for (v=0; v<N; v++){
            if (v != rt && !contracted[v] && prv[v].w == INF)
                break;
        }
        if (v >= N) break; // end
        for (int i=0; i<M; i++){
            if (eg[i].v == v && eg[i].w < prv[v].w)
                prv[v] = eg[i];
        }
        if (prv[v].w == INF) // not connected
            return EDGE_INF;
        cost += prv[v];
        for (u=prv[v].u; u!=v && u!=-1; u=prv[u].u);
        if (u == -1) continue;
        incyc[v] = ++cid;
        for (u=prv[v].u; u!=v; u=prv[u].u){
            contracted[u] = 1;
            incyc[u] = cid;
        }
        for (int i=0; i<M; i++){
            if (incyc[eg[i].u] != cid && incyc[eg[i].v] == cid){
                eg[i] -= prv[eg[i].v];
            }
        }
        for (int i=0; i<M; i++){
            if (incyc[eg[i].u] == cid) eg[i].u = v;
            if (incyc[eg[i].v] == cid) eg[i].v = v;
            if (eg[i].u == eg[i].v) eg[i] = eg[--M];
        }
        for (int i=0; i<N; i++){
            if (contracted[i]) continue;
            if (prv[i].u>=0 && incyc[prv[i].u] == cid)
                prv[i].u = v;
        }
        prv[v] = EDGE_INF;
    }
    return cost;
}

void solve(){
    edge_t cost = dmst(0);
}

```

```

for (auto it : cost.add){ // find a solution
    E[it.F].PB(it.S);
    prv[it.S] = edge_t(it.F,it.S,0);
}
}

```

### 3.4 Maximum Clique

```

class MaxClique {
public:
    static const int MV = 210;

    int V;
    int el[MV][MV/30+1];
    int dp[MV];
    int ans;
    int s[MV][MV/30+1];
    vector<int> sol;

    void init(int v) {
        V = v; ans = 0;
        FZ(el); FZ(dp);
    }

    /* Zero Base */
    void addEdge(int u, int v) {
        if(u > v) swap(u, v);
        if(u == v) return;
        el[u][v/32] |= (1<<(v%32));
    }

    bool dfs(int v, int k) {
        int c = 0, d = 0;
        for(int i=0; i<(V+31)/32; i++) {
            s[k][i] = el[v][i];
            if(k != 1) s[k][i] &= s[k-1][i];
            c += __builtin_popcount(s[k][i]);
        }
        if(c == 0) {
            if(k > ans) {
                ans = k;
                sol.clear();
                sol.push_back(v);
                return 1;
            }
            return 0;
        }
        for(int i=0; i<(V+31)/32; i++) {
            for(int a = s[k][i]; a ; d++) {
                if(k + (c-d) <= ans) return 0;
                int lb = a&(-a), lg = 0;
                a ^= lb;
                while(lb!=1) {
                    lb = (unsigned int)(lb) >> 1;
                    lg ++;
                }
                int u = i*32 + lg;
                if(k + dp[u] <= ans) return 0;
                if(dfs(u, k+1)) {
                    sol.push_back(v);
                    return 1;
                }
            }
        }
        return 0;
    }

    int solve() {
        for(int i=V-1; i>=0; i--) {
            dfs(i, 1);
            dp[i] = ans;
        }
        return ans;
    }
};

```

### 3.5 MinimumMeanCycle

```

/* minimum mean cycle */
const int MAXE = 1805;
const int MAXN = 35;
const double inf = 1029384756;
const double eps = 1e-6;
struct Edge {
    int v,u;
    double c;
};
int n,m,prv[MAXN][MAXN], prve[MAXN][MAXN], vst[MAXN];
Edge e[MAXE];
vector<int> edgeID, cycle, rho;
double d[MAXN][MAXN];
inline void bellman_ford() {
    for(int i=0; i<n; i++) d[0][i]=0;
    for(int i=0; i<n; i++) {
        fill(d[i+1], d[i+1]+n, inf);
        for(int j=0; j<m; j++) {
            int v = e[j].v, u = e[j].u;
            if(d[i][v]<inf && d[i+1][u]>d[i][v]+e[j].c) {
                d[i+1][u] = d[i][v]+e[j].c;
                prv[i+1][u] = v;
                prve[i+1][u] = j;
            }
        }
    }
}
double karp_mmc() {
    // returns inf if no cycle, mmc otherwise
    double mmc=inf;
    int st = -1;
    bellman_ford();
    for(int i=0; i<n; i++) {
        double avg=-inf;
        for(int k=0; k<n; k++) {
            if(d[n][i]<inf-eps) avg=max(avg,(d[n][i]-d[k][i])/(n-k));
            else avg=max(avg,inf);
        }
        if (avg < mmc) tie(mmc, st) = tie(avg, i);
    }
    FZ(vst); edgeID.clear(); cycle.clear(); rho.clear();
    for (int i=n; !vst[st]; st=prv[i--][st]) {
        vst[st]++;
        edgeID.PB(prve[i][st]);
        rho.PB(st);
    }
    while (vst[st] != 2) {
        int v = rho.back(); rho.pop_back();
        cycle.PB(v);
        vst[v]++;
    }
    reverse(ALL(edgeID));
    edgeID.resize(SZ(cycle));
    return mmc;
}

```

## 4 Flow

### 4.1 ISAP

```

struct Isap{
    static const int MXN = 10000;
    struct Edge{ int v,f,re; };
    int n,s,t,h[MXN],gap[MXN];
    vector<Edge> E[MXN];
    void init(int _n, int _s, int _t){
        n = _n; s = _s; t = _t;
        for (int i=0; i<n; i++) E[i].clear();
    }
    void add_edge(int u, int v, int f){
        E[u].PB({v,f,SZ(E[v])});
        E[v].PB({u,0,SZ(E[u])-1});
    }
    int DFS(int u, int nf, int res=0){
        if (u == t) return nf;
        for (auto &it : E[u]){
            if (h[u]==h[it.v]+1 && it.f>0){

```



```

    int tf = DFS(it.v,min(nf,it.f));
    res += tf; nf -= tf; it.f -= tf;
    E[it.v][it.re].f += tf;
    if (nf == 0) return res;
}
}
if (nf){
    if (--gap[h[u]] == 0) h[s]=n;
    gap[++h[u]]++;
}
return res;
}
int flow(int res=0){
    FZ(h); FZ(gap);
    gap[0] = n;
    while (h[s] < n) res += DFS(s,2147483647);
    return res;
}
}flow;

```

## 4.2 Dinic

```

struct Dinic{
    static const int MXN = 10000;
    struct Edge{ int v,f,re; };
    int n,s,t,level[MXN];
    vector<Edge> E[MXN];
    void init(int _n, int _s, int _t){
        n = _n; s = _s; t = _t;
        for (int i=0; i<n; i++) E[i].clear();
    }
    void add_edge(int u, int v, int f){
        E[u].PB({v,f,SZ(E[v])});
        E[v].PB({u,0,SZ(E[u])-1});
    }
    bool BFS(){
        FMO(level);
        queue<int> que;
        que.push(s);
        level[s] = 0;
        while (!que.empty()){
            int u = que.front(); que.pop();
            for (auto it : E[u]){
                if (it.f > 0 && level[it.v] == -1){
                    level[it.v] = level[u]+1;
                    que.push(it.v);
                }
            }
        }
        return level[t] != -1;
    }
    int DFS(int u, int nf){
        if (u == t) return nf;
        int res = 0;
        for (auto &it : E[u]){
            if (it.f > 0 && level[it.v] == level[u]+1){
                int tf = DFS(it.v, min(nf,it.f));
                res += tf; nf -= tf; it.f -= tf;
                E[it.v][it.re].f += tf;
                if (nf == 0) return res;
            }
        }
        if (!res) level[u] = -1;
        return res;
    }
    int flow(int res=0){
        while (BFS())
            res += DFS(s,2147483647);
        return res;
    }
}flow;

```

## 4.3 Cost Flow

```

typedef pair<long long, long long> pll;
struct CostFlow {
    static const int MXN = 205;
    static const long long INF = 102938475610293847LL;

```

```

    struct Edge {
        int v, r;
        long long f, c;
    };
    int n, s, t, prv[MXN], prvL[MXN], inq[MXN];
    long long dis[MXN], fl, cost;
    vector<Edge> E[MXN];
    void init(int _n, int _s, int _t) {
        n = _n; s = _s; t = _t;
        for (int i=0; i<n; i++) E[i].clear();
        fl = cost = 0;
    }
    void add_edge(int u, int v, long long f, long long c)
    {
        E[u].PB({v, SZ(E[v]) , f, c});
        E[v].PB({u, SZ(E[u])-1, 0, -c});
    }
    pll flow() {
        while (true) {
            for (int i=0; i<n; i++) {
                dis[i] = INF;
                inq[i] = 0;
            }
            dis[s] = 0;
            queue<int> que;
            que.push(s);
            while (!que.empty()) {
                int u = que.front(); que.pop();
                inq[u] = 0;
                for (int i=0; i<SZ(E[u]); i++) {
                    int v = E[u][i].v;
                    long long w = E[u][i].c;
                    if (E[u][i].f > 0 && dis[v] > dis[u] + w) {
                        prv[v] = u; prvL[v] = i;
                        dis[v] = dis[u] + w;
                        if (!inq[v]) {
                            inq[v] = 1;
                            que.push(v);
                        }
                    }
                }
            }
            if (dis[t] == INF) break;
            long long tf = INF;
            for (int v=t, u, l; v!=s; v=u) {
                u=prv[v]; l=prvL[v];
                tf = min(tf, E[u][l].f);
            }
            for (int v=t, u, l; v!=s; v=u) {
                u=prv[v]; l=prvL[v];
                E[u][l].f -= tf;
                E[v][E[u][l].r].f += tf;
            }
            cost += tf * dis[t];
            fl += tf;
        }
        return {fl, cost};
    }
}flow;

```

## 4.4 Bipartite Matching (Augmenting Path)

```

bool DFS(int u){
    for (auto v : E[u]){
        if (!vst[v]){
            vst[v]=1;
            if (match[v] == -1 || DFS(match[v])){
                match[v] = u; match[u] = v;
                return true;
            }
        }
    }
    return false;
}
int DoMatch(int res=0){
    memset(match,-1,sizeof(match));
    for (int i=1; i<=N; i++){
        if (match[i] == -1){
            memset(vst,0,sizeof(vst));
            DFS(i);

```



```

    }
}
for (int i=1; i<=N; i++)
    if (match[i] != -1) res++;
return res;
}

```

## 4.5 Kuhn Munkres

```

struct KM{
// Maximum Bipartite Weighted Matching (Perfect Match)
static const int MXN = 650;
static const int INF = 2147483647; // long long
int n, match[MXN], vx[MXN], vy[MXN];
int edge[MXN][MXN], lx[MXN], ly[MXN], slack[MXN];
// ^^^^ long long
void init(int _n){
    n = _n;
    for (int i=0; i<n; i++)
        for (int j=0; j<n; j++)
            edge[i][j] = 0;
}
void add_edge(int x, int y, int w){ // long long
    edge[x][y] = w;
}
bool DFS(int x){
    vx[x] = 1;
    for (int y=0; y<n; y++){
        if (vy[y]) continue;
        if (lx[x]+ly[y] > edge[x][y]){
            slack[y] = min(slack[y], lx[x]+ly[y]-edge[x][y]);
        }
        else {
            vy[y] = 1;
            if (match[y] == -1 || DFS(match[y])){
                match[y] = x;
                return true;
            }
        }
    }
    return false;
}
int solve(){
    fill(match, match+n, -1);
    fill(lx, lx+n, -INF);
    fill(ly, ly+n, 0);
    for (int i=0; i<n; i++)
        for (int j=0; j<n; j++)
            lx[i] = max(lx[i], edge[i][j]);
    for (int i=0; i<n; i++){
        fill(slack, slack+n, INF);
        while (true){
            fill(vx, vx+n, 0);
            fill(vy, vy+n, 0);
            if (DFS(i)) break;
            int d = INF; // long long
            for (int j=0; j<n; j++)
                if (!vy[j]) d = min(d, slack[j]);
            for (int j=0; j<n; j++){
                if (vx[j]) lx[j] -= d;
                if (vy[j]) ly[j] += d;
                else slack[j] -= d;
            }
        }
    }
    int res=0;
    for (int i=0; i<n; i++)
        res += edge[match[i]][i];
    return res;
}
}graph;

```

## 4.6 SW-Mincut

```

struct SW{ // O(V^3)
static const int MXN = 514;
int n, vst[MXN], del[MXN];
int edge[MXN][MXN], wei[MXN];

```

```

void init(int _n){
    n = _n;
    FZ(edge);
    FZ(del);
}
void add_edge(int u, int v, int w){
    edge[u][v] += w;
    edge[v][u] += w;
}
void search(int &s, int &t){
    FZ(vst); FZ(wei);
    s = t = -1;
    while (true){
        int mx=-1, cur=0;
        for (int i=0; i<n; i++)
            if (!del[i] && !vst[i] && mx<wei[i])
                cur = i, mx = wei[i];
        if (mx == -1) break;
        vst[cur] = 1;
        s = t;
        t = cur;
        for (int i=0; i<n; i++)
            if (!vst[i] && !del[i]) wei[i] += edge[cur][i];
    }
}
int solve(){
    int res = 2147483647;
    for (int i=0, x, y; i<n-1; i++){
        search(x, y);
        res = min(res, wei[y]);
        del[y] = 1;
        for (int j=0; j<n; j++)
            edge[x][j] = (edge[j][x] += edge[y][j]);
    }
    return res;
}
}graph;

```

## 4.7 Maximum Simple Graph Matching

```

struct GenMatch { // 1-base
static const int MAXN = 250;
int V;
bool el[MAXN][MAXN];
int pr[MAXN];
bool inq[MAXN], inp[MAXN], inb[MAXN];
queue<int> qe;
int st, ed;
int nb;
int bk[MAXN], djs[MAXN];
int ans;
void init(int _V) {
    V = _V;
    FZ(el); FZ(pr);
    FZ(inq); FZ(inp); FZ(inb);
    FZ(bk); FZ(djs);
    ans = 0;
}
void add_edge(int u, int v) {
    el[u][v] = el[v][u] = 1;
}
int lca(int u, int v) {
    memset(inp, 0, sizeof(inp));
    while(1) {
        u = djs[u];
        inp[u] = true;
        if (u == st) break;
        u = bk[pr[u]];
    }
    while(1) {
        v = djs[v];
        if (inp[v]) return v;
        v = bk[pr[v]];
    }
    return v;
}
void upd(int u) {
    int v;
    while (djs[u] != nb) {
        v = pr[u];

```

```

    inb[djs[u]] = inb[djs[v]] = true;
    u = bk[v];
    if(djs[u] != nb) bk[u] = v;
}
}
void blo(int u,int v) {
    nb = lca(u,v);
    memset(inb,0,sizeof(inb));
    upd(u); upd(v);
    if(djs[u] != nb) bk[u] = v;
    if(djs[v] != nb) bk[v] = u;
    for(int tu = 1; tu <= V; tu++)
        if(inb[djs[tu]]) {
            djs[tu] = nb;
            if(!inq[tu]){
                qe.push(tu);
                inq[tu] = 1;
            }
        }
}
void flow() {
    memset(inq,false,sizeof(inq));
    memset(bk,0,sizeof(bk));
    for(int i = 1; i <= V; i++)
        djs[i] = i;

    while(qe.size()) qe.pop();
    qe.push(st);
    inq[st] = 1;
    ed = 0;
    while(qe.size()) {
        int u = qe.front(); qe.pop();
        for(int v = 1; v <= V; v++)
            if(el[u][v] && (djs[u] != djs[v]) && (pr[u] != v)) {
                if((v == st) || ((pr[v] > 0) && bk[pr[v]] > 0))
                    blo(u,v);
                else if(bk[v] == 0) {
                    bk[v] = u;
                    if(pr[v] > 0) {
                        if(!inq[pr[v]]) qe.push(pr[v]);
                    } else {
                        ed = v;
                        return;
                    }
                }
            }
    }
}
void aug() {
    int u,v,w;
    u = ed;
    while(u > 0) {
        v = bk[u];
        w = pr[v];
        pr[v] = u;
        pr[u] = v;
        u = w;
    }
}
int solve() {
    memset(pr,0,sizeof(pr));
    for(int u = 1; u <= V; u++)
        if(pr[u] == 0) {
            st = u;
            flow();
            if(ed > 0) {
                aug();
                ans ++;
            }
        }
    return ans;
}
};

int main() {
    gp.init(V);
    for(int i=0; i<E; i++) {
        int u, v;
        cin >> u >> v;
        gp.edge(u, v);
    }
}

```

```

    }
    cout << gp.solve() << endl;
}

```

## 4.8 Minimum Weight Matching (Clique version)

```

struct Graph {
    // Minimum General Weighted Matching (Perfect Match)
    static const int MXN = 105;

    int n, edge[MXN][MXN];
    int match[MXN],dis[MXN],onstk[MXN];
    vector<int> stk;

    void init(int _n) {
        n = _n;
        FZ(edge);
    }
    void add_edge(int u, int v, int w) {
        edge[u][v] = edge[v][u] = w;
    }
    bool SPFA(int u){
        if (onstk[u]) return true;
        stk.PB(u);
        onstk[u] = 1;
        for (int v=0; v<n; v++){
            if (u != v && match[u] != v && !onstk[v]){
                int m = match[v];
                if (dis[m] > dis[u] - edge[v][m] + edge[u][v]){
                    dis[m] = dis[u] - edge[v][m] + edge[u][v];
                    onstk[v] = 1;
                    stk.PB(v);
                    if (SPFA(m)) return true;
                    stk.pop_back();
                    onstk[v] = 0;
                }
            }
        }
        onstk[u] = 0;
        stk.pop_back();
        return false;
    }

    int solve() {
        // find a match
        for (int i=0; i<n; i+=2){
            match[i] = i+1;
            match[i+1] = i;
        }
        while (true){
            int found = 0;
            FZ(dis); FZ(onstk);
            for (int i=0; i<n; i++){
                stk.clear();
                if (!onstk[i] && SPFA(i)){
                    found = 1;
                    while (SZ(stk)>=2){
                        int u = stk.back(); stk.pop_back();
                        int v = stk.back(); stk.pop_back();
                        match[u] = v;
                        match[v] = u;
                    }
                }
                if (!found) break;
            }
            int ret = 0;
            for (int i=0; i<n; i++)
                ret += edge[i][match[i]];
            ret /= 2;
            return ret;
        }
    }
}graph;

```

## 4.9 2-Commodity Flow

```

const int MAXN = 64;
const int INF = 1029384756;

```

```

int N;
int s1, s2, t1, t2, d1, d2, S, T;
int edge[MAXN][MAXN];
int cap[MAXN][MAXN];

int h[MAXN], gap[MAXN];
bool vis[MAXN];

int isap(int v, int f)
{
    if(v == T) return f;

    if(vis[v]) return 0;
    vis[v] = true;

    for(int i=0; i<N+2; i++)
    {
        if(cap[v][i] <= 0) continue;
        if(h[i] != h[v] - 1) continue;
        int res = isap(i, min(cap[v][i], f));
        if(res > 0)
        {
            cap[v][i] -= res;
            cap[i][v] += res;
            return res;
        }
    }

    gap[h[v]]--;
    if(gap[h[v]] <= 0) h[S] = N + 4;
    h[v]++;
    gap[h[v]]++;

    return 0;
}

int get_flow()
{
    for(int i=0; i<MAXN; i++)
    {
        h[i] = gap[i] = 0;
    }
    gap[0] = N + 2;

    int flow = 0;

    while(h[S] <= N + 3)
    {
        for(int i=0; i<N+2; i++)
        {
            vis[i] = false;
        }

        int df = isap(S, INF);
        flow += df;
    }

    return flow;
}

int main()
{
    ios_base::sync_with_stdio(0);

    int TT;
    cin>>TT;
    while(TT--)
    {
        cin>>N;
        cin>>s1>>t1>>d1>>s2>>t2>>d2;

        for(int i=0; i<MAXN; i++)
        {
            for(int j=0; j<MAXN; j++)
            {
                edge[i][j] = 0;
            }
        }

        for(int i=0; i<N; i++)
    {

```

```

        string s;
        cin>>s;
        for(int j=0; j<N; j++)
        {
            if(s[j] == 'X') edge[i][j] = 0;
            else if(s[j] == 'O') edge[i][j] = 1;
            else if(s[j] == 'N') edge[i][j] = INF;
        }
    }

    int ans = 0;

    S = N;
    T = N + 1;

    //first
    for(int i=0; i<MAXN; i++)
    {
        for(int j=0; j<MAXN; j++)
        {
            cap[i][j] = edge[i][j];
        }
    }

    cap[S][s1] = cap[t1][T] = d1;
    cap[S][s2] = cap[t2][T] = d2;

    ans = get_flow();

    //second
    for(int i=0; i<MAXN; i++)
    {
        for(int j=0; j<MAXN; j++)
        {
            cap[i][j] = edge[i][j];
        }
    }

    cap[S][s1] = cap[t1][T] = d1;
    cap[S][t2] = cap[s2][T] = d2;

    ans = min(ans, get_flow());

    cout<<(ans == d1 + d2 ? "Yes" : "No")<<endl;
}

return 0;
}

```

#### 4.10 (+1) SW-mincut $O(NM)$

```

// {{{ StoerWagner
const int inf=1000000000;
// should be larger than max.possible mincut
class StoerWagner {
public:
    int n,mc; // node id in [0,n-1]
    vector<int> adj[MAXN];
    int cost[MAXN][MAXN];
    int cs[MAXN];
    bool merged[MAXN],sel[MAXN];
    // --8<-- include only if cut is explicitly needed
    DisjointSet djs;
    vector<int> cut;
    //--8<-----
    StoerWagner(int _n:n(_n),mc(inf),djs(_n) {
        for(int i=0;i<n;i++)
            merged[i]=0;
        for(int i=0;i<n;i++)
            for(int j=0;j<n;j++)
                cost[i][j]=cost[j][i]=0;
    }
    void append(int v,int u,int c) {
        if(v==u) return;
        if(!cost[v][u]&&c) {
            adj[v].PB(u);
            adj[u].PB(v);
        }
        cost[v][u]+=c;
        cost[u][v]+=c;
    }

```

```

}
void merge(int v, int u) {
    merged[u]=1;
    for(int i=0; i<n; i++)
        append(v, i, cost[u][i]);
    // --8<-- include only if cut is explicitly
    //      needed
    djs.merge(v, u);
    //      --8<-----
}
void phase() {
    priority_queue<pii> pq;
    for(int v=0; v<n; v++) {
        if(merged[v]) continue;
        cs[v]=0;
        sel[v]=0;
        pq.push({0, v});
    }
    int v, s, pv;
    while(pq.size()) {
        if(cs[pq.top().S]>pq.top().F) {
            pq.pop();
            continue;
        }
        pv=v;
        v=pq.top().S;
        s=pq.top().F;
        pq.pop();
        sel[v]=1;
        for(int i=0; i<adj[v].size(); i++) {
            int u=adj[v][i];
            if(merged[u]||sel[u]) continue;
            cs[u]+=cost[v][u];
            pq.push({cs[u], u});
        }
    }
    if(s<mc) {
        mc=s;
        // --8<-- include only if cut is explicitly
        //      needed -----
        cut.clear();
        for(int i=0; i<n; i++)
            if(djs.getrep(i)==djs.getrep(v)) cut.PB(i);
        // --8<-----
    }
    merge(v, pv);
}
int mincut() {
    if(mc==inf) {
        for(int t=0; t<n-1; t++)
            phase();
        return mc;
    }
    // --8<-- include only if cut is explicitly needed
    // -----
    vector<int> getcut() { // return one side of the
        //      cut
        mincut();
        return cut;
    }
    // --8<-----
};
// }}}

```

## 5 Math

### 5.1 ax+by=gcd

```

typedef pair<int, int> pii;

pii gcd(int a, int b) {
    if(b == 0) return make_pair(1, 0);
    else {
        int p = a / b;
        pii q = gcd(b, a % b);
        return make_pair(q.second, q.first - q.second * p);
    }
}

```

### 5.2 Chinese Remainder

```

int pfn; // number of distinct prime factors
int pf[MAXNUM]; // prime factor powers
int rem[MAXNUM]; // corresponding remainder
int pm[MAXNUM];

inline void generate_primes() {
    int i, j;
    pnum=1;
    prime[0]=2;
    for(i=3; i<MAXVAL; i+=2) {
        if(!nprime[i]) continue;
        prime[pnum++] = i;
        for(j=i*i; j<MAXVAL; j+=i) nprime[j]=1;
    }
}

inline int inverse(int x, int p) {
    int q, tmp, a=x, b=p;
    int a0=1, a1=0, b0=0, b1=1;
    while(b) {
        q=a/b; tmp=b; b=a-b*q; a=tmp;
        tmp=b0; b0=a0-b0*q; a0=tmp;
        tmp=b1; b1=a1-b1*q; a1=tmp;
    }
    return a0;
}

inline void decompose_mod() {
    int i, p, t=mod;
    pfn=0;
    for(i=0; i<pnum&&prime[i]<=t; i++) {
        p=prime[i];
        if(t%p==0) {
            pf[pfn]=1;
            while(t%p==0) {
                t/=p;
                pf[pfn]*=p;
            }
            pfn++;
        }
    }
    if(t>1) pf[pfn++]=t;
}

inline int chinese_remainder() {
    int i, m, s=0;
    for(i=0; i<pfn; i++) {
        m=mod/pf[i];
        pm[i]=(long long)m*inverse(m, pf[i])%mod;
        s=(s+(long long)pm[i]*rem[i])%mod;
    }
    return s;
}

```

### 5.3 Fast Fourier Transform

```
// const int MAXN = 262144;
// (must be 2^k)

typedef long double ld;
typedef complex<ld> cplx;
const ld PI = acos(-1);
const cplx I(0, 1);

cplx omega[MAXN+1];
void pre_fft()
{
    for(int i=0; i<=MAXN; i++)
        omega[i] = exp(i * 2 * PI / MAXN * I);
}

void fft(int n, cplx a[], bool inv=false)
{
    int basic = MAXN / n;
    int theta = basic;
    for (int m = n; m >= 2; m >>= 1) {
        int mh = m >> 1;
        for (int i = 0; i < mh; i++) {
            cplx w = omega[inv ? MAXN-(i*theta%MAXN) : i*
                theta%MAXN];
            for (int j = i; j < n; j += m) {
                int k = j + mh;
                cplx x = a[j] - a[k];
                a[j] += a[k];
                a[k] = w * x;
            }
        }
        theta = (theta * 2) % MAXN;
    }
    int i = 0;
    for (int j = 1; j < n - 1; j++) {
        for (int k = n >> 1; k > (i ^ k); k >>= 1);
        if (j < i) swap(a[i], a[j]);
    }
    if (inv)
        for (i = 0; i < n; i++)
            a[i] /= n;
}
```

### 5.4 (+1) ntt

```
int P=605028353,root=3,MAXNUM=262144;
// Remember coefficient are mod P
/*
p=a*2^n+1
n  2^n      p      a      root
5   32      97      3      5
6   64     193      3      5
7  128     257      2      3
8  256     257      1      3
9  512    7681     15     17
10 1024   12289     12     11
11 2048   12289      6     11
12 4096   12289      3     11
13 8192   40961      5      3
14 16384  65537      4      3
15 32768  65537      2      3
16 65536  65537      1      3
17 131072 786433      6     10
18 262144 786433      3     10 (605028353,
    2308, 3)
19 524288 5767169     11      3
20 1048576 7340033      7      3
21 2097152 23068673     11      3
22 4194304 104857601    25      3
23 8388608 167772161    20      3
24 16777216 167772161   10      3
25 33554432 167772161    5      3 (1107296257, 33,
    10)
26 67108864 469762049    7      3
27 134217728 2013265921 15     31
*/
int bigmod(long long a,int b){
    if(b==0) return 1;
```

```
    return (bigmod((a*a)%P,b/2)*(b%2?a:1))%P;
}
int inv(int a,int b){
    if(a==1) return 1;
    return (((long long)(a-inv(b%a,a))*b+1)/a)%b;
}
std::vector<long long> ps(MAXNUM);
std::vector<int> rev(MAXNUM);
struct poly{
    std::vector<unsigned int> co;
    int n;//polynomial degree = n
    poly(int d){n=d;co.resize(n+1,0);}
    void trans2(int NN){
        int r=0,st,N;
        unsigned int a,b;
        while((1<r)<(NN>>1))++r;
        for(N=2;N<=NN;N<=1,--r){
            for(st=0;st<NN;st+=N){
                int i,ss=st+(N>>1);
                for(i=(N>>1)-1;i>=0;--i){
                    a=co[st+i]; b=(ps[i<r]*co[ss+i])%P;
                    co[st+i]=a+b; if(co[st+i]>=P)co[st+i]-=P;
                    co[ss+i]=a-P-b; if(co[ss+i]>=P)co[ss+i]-=P;
                }
            }
        }
    }
    void trans1(int NN){
        int r=0,st,N;
        unsigned int a,b;
        for(N=NN;N>1;N>=1,++r){
            for(st=0;st<NN;st+=N){
                int i,ss=st+(N>>1);
                for(i=(N>>1)-1;i>=0;--i){
                    a=co[st+i]; b=co[ss+i];
                    co[st+i]=a+b; if(co[st+i]>=P)co[st+i]-=P;
                    co[ss+i]=((a-P-b)*ps[i<r])%P;
                }
            }
        }
    }
    poly operator*(const poly& _b)const{
        poly a=*this,b=_b;
        int k=n+b.n,i,N=1;
        while(N<=k)N*=2;
        a.co.resize(N,0); b.co.resize(N,0);
        int r=bigmod(root,(P-1)/N),Ni=inv(N,P);
        ps[0]=1;
        for(i=1;i<N;++i)ps[i]=(ps[i-1]*r)%P;
        a.trans1(N);b.trans1(N);
        for(i=0;i<N;++i)a.co[i]=((long long)a.co[i]*b.co[i]
            )%P;
        r=inv(r,P);
        for(i=1;i<N/2;++i)std::swap(ps[i],ps[N-i]);
        a.trans2(N);
        for(i=0;i<N;++i)a.co[i]=((long long)a.co[i]*Ni)%P;
        a.n=n+_b.n; return a;
    }
};
```

### 5.5 Mod

```
/// _fd(a,b) floor(a/b).
/// _rd(a,m) a-floor(a/m)*m.
/// _pv(a,m,r) largest x s.t x<=a && x%m == r.
/// _nx(a,m,r) smallest x s.t x>=a && x%m == r.
/// _ct(a,b,m,r) |A| , A = { x : a<=x<=b && x%m == r }.

int _fd(int a,int b){ return a<0?(~a/b-1):a/b; }
int _rd(int a,int m){ return a-_fd(a,m)*m; }
int _pv(int a,int m,int r)
{
    r=(r%m+m)%m;
    return _fd(a-r,m)*m+r;
}
int _nt(int a,int m,int r)
{
    m=abs(m);
    r=(r%m+m)%m;
```

```

    return _fd(a-r-1,m)*m+r+m;
}
int _ct(int a,int b,int m,int r)
{
    m=abs(m);
    a=_nt(a,m,r);
    b=_pv(b,m,r);
    return (a>b)?0:((b-a+m)/m);
}

```

## 5.6 (+1) Miller Rabin

```

// n < 4,759,123,141      3 : 2, 7, 61
// n < 1,122,004,669,633  4 : 2, 13, 23, 1662803
// n < 3,474,749,660,383  6 : pimes <= 13
// n < 2^64              7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
long long power(long long x,long long p,long long mod){
    long long s=1,m=x;
    while(p) {
        if(p&1) s=mult(s,m,mod);
        p>>=1;
        m=mult(m,m,mod);
    }
    return s;
}
bool witness(long long a,long long n,long long u,int t)
{
    long long x=power(a,u,n);
    for(int i=0;i<t;i++) {
        long long nx=mult(x,x,n);
        if(nx==1&&x!=1&&x!=n-1) return 1;
        x=nx;
    }
    return x!=1;
}
bool miller_rabin(long long n,int s=100) {
    // iterate s times of witness on n
    // return 1 if prime, 0 otherwise
    if(n<2) return 0;
    if(!(n&1)) return n==2;
    long long u=n-1;
    int t=0;
    // n-1 = u*2^t
    while(u&1) {
        u>>=1;
        t++;
    }
    while(s--) {
        long long a=randll()%(n-1)+1;
        if(witness(a,n,u,t)) return 0;
    }
    return 1;
}

```

## 5.7 Pollard Rho

```

// does not work when n is prime
long long modit(long long x,long long mod) {
    if(x>=mod) x-=mod;
    //if(x<0) x+=mod;
    return x;
}
long long mult(long long x,long long y,long long mod) {
    long long s=0,m=x%mod;
    while(y) {
        if(y&1) s=modit(s+m,mod);
        y>>=1;
        m=modit(m+m,mod);
    }
    return s;
}
long long f(long long x,long long mod) {
    return modit(mult(x,x,mod)+1,mod);
}
long long pollard_rho(long long n) {
    if(!(n&1)) return 2;
    while (true) {

```

```

        long long y=2, x=rand()%(n-1)+1, res=1;
        for (int sz=2; res==1; sz*=2) {
            for (int i=0; i<sz && res<=1; i++) {
                x = f(x, n);
                res = __gcd(abs(x-y), n);
            }
            y = x;
        }
        if (res!=0 && res!=n) return res;
    }
}

```

## 5.8 Algorithms about Primes

```

/*
 * 12721
 * 13331
 * 14341
 * 75577
 * 123457
 * 222557
 * 556679
 * 999983
 * 1097774749
 * 1076767633
 * 100102021
 * 999997771
 * 1001010013
 * 1000512343
 * 987654361
 * 999991231
 * 999888733
 * 98789101
 * 987777733
 * 999991921
 * 1010101333
 * 1010102101
 * 1000000000039
 * 100000000000037
 * 2305843009213693951
 * 4611686018427387847
 * 9223372036854775783
 * 18446744073709551557
 */

int mu[MX],p_tbl[MX];
vector<int> primes;
void sieve() {
    mu[1] = p_tbl[1] = 1;
    for (int i=2; i<MX; i++) {
        if (!p_tbl[i]) {
            p_tbl[i] = i;
            primes.PB(i);
            mu[i] = -1;
        }
        for (auto p : primes) {
            int x = i*p;
            if (x >= M) break;
            p_tbl[x] = p;
            mu[x] = -mu[i];
            if (i%p==0) {
                mu[x] = 0;
                break;
            }
        }
    }
}

vector<int> factor(int x) {
    vector<int> fac{1};
    while (x > 1) {
        int fn=SZ(fac), p=p_tbl[x], pos=0;
        while (x%p==0) {
            x /= p;
            for (int i=0; i<fn; i++)
                fac.PB(fac[pos++] * p);
        }
    }
    return fac;
}

```

## 5.9 (+1) PolynomialGenerator

```

class PolynomialGenerator {
    /* for a nth-order polynomial f(x), *
    * given f(0), f(1), ..., f(n) *
    * express f(x) as sigma_i{c_i*x^i} */
public:
    int n;
    vector<long long> coef;
    // initialize and calculate f(x), vector _fx should
    // be
    // filled with f(0) to f(n)
    PolynomialGenerator(int _n, vector<long long> _fx)
        : n(_n)
        , coef(_fx) {
        for(int i=0; i<n; i++)
            for(int j=n; j>i; j--)
                coef[j] -= coef[j-1];
    }
    // evaluate f(x), runs in O(n)
    long long eval(int x) {
        long long m=1, ret=0;
        for(int i=0; i<n; i++) {
            ret += coef[i]*m;
            m = m*(x-i)/(i+1);
        }
        return ret;
    }
};

```

## 5.10 Gauss Elimination

```

const int MAX = 300;
const double EPS = 1e-8;

double mat[MAX][MAX];
void Gauss(int n) {
    for(int i=0; i<n; i++) {
        bool ok = 0;
        for(int j=i; j<n; j++) {
            if(fabs(mat[j][i]) > EPS) {
                swap(mat[j], mat[i]);
                ok = 1;
                break;
            }
        }
        if(!ok) continue;

        double fs = mat[i][i];
        for(int j=i+1; j<n; j++) {
            double r = mat[j][i] / fs;
            for(int k=i; k<n; k++) {
                mat[j][k] -= mat[i][k] * r;
            }
        }
    }
}

```

## 5.11 Simplex

```

const int maxn = 111;
const int maxm = 111;
const double eps = 1E-10;

double a[maxn][maxm], b[maxn], c[maxn], d[maxn][maxm];
double x[maxn];
int ix[maxn + maxm]; // !!! array all indexed from 0
// max{cx} subject to {Ax<=b, x>=0}
// n: constraints, m: vars !!!
// x[] is the optimal solution vector
//
// usage :
// value = simplex(a, b, c, N, M);
double simplex(double a[maxn][maxm], double b[maxn],
    double c[maxn], int n, int m) {
    ++m;
    int r = n, s = m - 1;
    memset(d, 0, sizeof(d));
    for (int i = 0; i < n + m; ++i) ix[i] = i;
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < m - 1; ++j) d[i][j] = -a[i][j];
        d[i][m - 1] = 1;
        d[i][m] = b[i];
        if (d[r][m] > d[i][m]) r = i;
    }
    for (int j = 0; j < m - 1; ++j) d[n][j] = c[j];
    d[n + 1][m - 1] = -1;
    for (double dd;; ) {
        if (r < n) {
            int t = ix[s]; ix[s] = ix[r + m]; ix[r + m] = t;
            d[r][s] = 1.0 / d[r][s];
            for (int j = 0; j <= m; ++j) if (j != s) d[r][j] *= -d[r][s];
            for (int i = 0; i <= n + 1; ++i) if (i != r) {
                for (int j = 0; j <= m; ++j) if (j != s)
                    d[i][j] += d[r][j] * d[i][s];
                d[i][s] *= d[r][s];
            }
        }
        r = -1; s = -1;
        for (int j = 0; j < m; ++j) if (s < 0 || ix[s] > ix[j]) {
            if (d[n + 1][j] > eps || (d[n + 1][j] > -eps && d[n][j] > eps)) s = j;
        }
        if (s < 0) break;
        for (int i = 0; i < n; ++i) if (d[i][s] < -eps) {
            if (r < 0 || (dd = d[r][m] / d[r][s] - d[i][m] / d[i][s]) < -eps || (dd < eps && ix[r + m] > ix[i + m])) r = i;
        }
        if (r < 0) return -1; // not bounded
    }
    if (d[n + 1][m] < -eps) return -1; // not executable
    double ans = 0;
    for(int i=0; i<m; i++) x[i] = 0;
    for (int i = m; i < n + m; ++i) { // the missing enumerated x[i] = 0
        if (ix[i] < m - 1) {
            ans += d[i - m][m] * c[ix[i]];
            x[ix[i]] = d[i - m][m];
        }
    }
    return ans;
}

```



## 5.12 Theorem

### 5.12.1 Lucas' Theorem

For non-negative integer  $n, m$  and prime  $p$ ,  $\binom{m}{n} \equiv \prod_{i=0}^k \binom{m_i}{n_i} \pmod{p}$  where  $m_i$  is the  $i$ -th digit of  $m$  in base  $p$ .

### 5.12.2 Sum of Two Squares Thm (Legendre)

For a given positive integer  $n$ , let  
 $D_1 = (\# \text{ of positive integers } d \text{ dividing } N \text{ that } 1 \equiv d \pmod{4})$   
 $D_3 = (\# \text{ of positive integers } d \text{ dividing } N \text{ that } 3 \equiv d \pmod{4})$   
 then  $n$  can be written as a sum of two squares in exactly  
 $R(n) = 4(D_1 - D_3)$  ways.

### 5.12.3 Difference of D1-D3 Thm

let  $n = 2^t \cdot (p_1^{e_1} \cdot \dots \cdot p_r^{e_r}) \cdot \dots \cdot (q_1^{f_1} \cdot \dots \cdot q_s^{f_s})$   
 where  $p_i, q_i$  are primes and  $1 \equiv p_i \pmod{4}, 3 \equiv q_i \pmod{4}$   
 then  $D_1 - D_3 = \begin{cases} (e_1 + 1)(e_2 + 1) \dots (e_r + 1), & \text{if } (f_i)s \text{ all even} \\ 0, & \text{if any } f_i \text{ is odd} \end{cases}$

### 5.12.4 Krush-Kuhn-Tucker Conditions

#### Stationarity

For maximizing  $f(x)$ :  $\nabla f(x^*) = \sum_{i=1}^m \mu_i \nabla g_i(x^*) + \sum_{j=1}^l \lambda_j \nabla h_j(x^*)$   
 For minimizing  $f(x)$ :  $-\nabla f(x^*) = \sum_{i=1}^m \mu_i \nabla g_i(x^*) + \sum_{j=1}^l \lambda_j \nabla h_j(x^*)$

#### Primal feasibility

$g_i(x^*) \leq 0$ , for all  $i = 1, \dots, m$   
 $h_j(x^*) = 0$ , for all  $j = 1, \dots, l$

#### Dual feasibility

$\mu_i \geq 0$ , for all  $i = 1, \dots, m$

#### Complementary slackness

$\mu_i g_i(x^*) = 0$ , for all  $i = 1, \dots, m$

## 6 Geometry

### 6.1 Point operators

```
#include<bits/stdc++.h>
using namespace std;

#define _x first
#define _y second
typedef pair<double, double> pdd;

pdd operator + (const pdd p1, const pdd p2){
    return pdd(p1._x + p2._x, p1._y + p2._y);
}
pdd operator - (const pdd p1, const pdd p2){
    return pdd(p1._x - p2._x, p1._y - p2._y);
}
pdd operator * (const double c, const pdd p){
    return pdd(p._x * c, p._y * c);
}
pdd operator - (const pdd p){
    return (-1.0) * p;
}
double operator * (const pdd p1, const pdd p2){
    return p1._x * p2._x + p1._y * p2._y;
}
double operator % (const pdd p1, const pdd p2){
    return p1._x * p2._y - p2._x * p1._y;
}
```

### 6.2 Intersection of two circles

Let  $O_1 = (x_1, y_1), O_2 = (x_2, y_2)$  be two centers of circles,  $r_1, r_2$  be the radius. If:

$$d = |O_1 - O_2|, u = \frac{1}{2}(O_1 + O_2) + \frac{(r_2^2 - r_1^2)}{2d^2}(O_1 - O_2)$$

$v = \frac{\sqrt{(r_1 + r_2 + d)(r_1 - r_2 + d)(r_1 + r_2 - d)(-r_1 + r_2 + d)}}{2d^2}(y_1 - y_2, -x_1 + x_2)$  then  $u + v, u - v$  are the two intersections of the circles, provided that  $d < r_1 + r_2$ .

```
vector<pdd> interCircle(pdd o1, double r1, pdd o2,
    double r2) {
    ld d2 = (o1 - o2) * (o1 - o2);
    ld d = sqrt(d2);
    if (d > r1+r2) return {};
    pdd u = 0.5*(o1+o2) + ((r2*r2-r1*r1)/(2*d2))*(o1-o2);
    double A = sqrt((r1+r2+d) * (r1-r2+d) * (r1+r2-d) *
        (-r1+r2+d));
    pdd v = A / (2*d2) * pdd(o1.S-o2.S, -o1.F+o2.F);
    return {u+v, u-v};
}
```

### 6.3 Intersection of two lines

```
#include<bits/stdc++.h>

using namespace std;
const double EPS = 1e-9;

pdd interPnt(pdd p1, pdd p2, pdd q1, pdd q2){
    double f1 = (p2 - p1) % (q1 - p1);
    double f2 = (p2 - p1) % (p1 - q2);
    double f = (f1 + f2);

    if(fabs(f) < EPS) return pdd(nan(""), nan(""));

    return (f2 / f) * q1 + (f1 / f) * q2;
}
```

### 6.4 Half Plane Intersection

```
#include<bits/stdc++.h>

using namespace std;

#define PB push_back
#define SZ(x) ((int)(x).size())
#define ALL(x) begin(x), end(x)
#define F first
#define S second
#define x first
#define y second
#define IOS ios::sync_with_stdio(0);cin.tie(0)

const double EPS = 1E-9;

struct point : public pair<double, double> {
    point() {}
    point(double _x, double _y) : pair<double, double>(_x, _y) {}

    point operator + (const point he) const {
        return {x+he.x, y+he.y};
    }
    point operator - (const point he) const {
        return {x-he.x, y-he.y};
    }
    friend point operator * (const double c, const point &p) {
        return {c*p.x, c*p.y};
    }
    point operator * (const double c) const {
        return {c*x, c*y};
    }
    double operator * (const point he) const {
        return x*he.x + y*he.y;
    }
    double cross (const point &he) const {
        return x*he.y - y*he.x;
    }
    friend ostream& operator << (ostream& o, const point &p) {
        cout << "(" << p.x << ", " << p.y << ")";
        return o;
    }
};
```

```

typedef pair<point, point> Line;

point interPnt(Line l1, Line l2, bool &res){
    point p1, p2, q1, q2;
    tie(p1, p2) = l1;
    tie(q1, q2) = l2;
    double f1 = (p2 - p1).cross(q1 - p1);
    double f2 = (p2 - p1).cross(p1 - q2);
    double f = (f1 + f2);

    if(fabs(f) < EPS) {
        res = false;
        return {0, 0};
    }

    res = true;
    return (f2 / f) * q1 + (f1 / f) * q2;
}

bool isin(Line l0, Line l1, Line l2) {
    // Check inter(l1, l2) in l0
    bool res;
    point p = interPnt(l1, l2, res);
    return (l0.S - l0.F).cross(p - l0.F) > 1e-9;
}

/* If no solution, check: 1. ret.size() < 3
 * Or more precisely, 2. interPnt(ret[0], ret[1])
 * in all the lines. (use (l.S - l.F).cross(p - l.F) >
 * 0
 */
vector<Line> halfPlaneInter(vector<Line> lines) {
    int sz = lines.size();
    vector<double> ata(sz), ord(sz);
    for (int i=0; i<sz; i++) {
        ord[i] = i;
        point d = lines[i].S - lines[i].F;
        ata[i] = atan2(d.y, d.x);
    }
    sort(ALL(ord), [&](int i, int j) {
        return ata[i] < ata[j];
    });
    vector<Line> fin;
    for (int i=0; i<sz; i++) {
        if (i and fabs(ata[ord[i]] - ata[ord[i-1]]) <
            EPS) {
            Line li = lines[ord[i]];
            Line lj = lines[ord[i-1]];
            if ((li.S - li.F).cross(lj.S - lj.F) >= 0)
                continue;
            else
                fin.back() = li;
        } else {
            fin.PB(lines[ord[i]]);
        }
    }

    deque<Line> dq;
    for (int i=0; i<SZ(fin); i++) {
        while(SZ(dq) >= 2 and
            not isin(fin[i], dq[SZ(dq)-2], dq[SZ(dq)-1])) {
            dq.pop_back();
        }
        while(SZ(dq) >= 2 and
            not isin(fin[i], dq[0], dq[1])) {
            dq.pop_front();
        }
        dq.push_back(fin[i]);
    }

    while (SZ(dq) >= 3 and
        not isin(dq[0], dq[SZ(dq)-2], dq[SZ(dq)-1])) {
        dq.pop_back();
    }

    while (SZ(dq) >= 3 and
        not isin(dq[SZ(dq)-1], dq[0], dq[1])) {
        dq.pop_front();
    }
}

```

```

vector<Line> res(SZ(dq));
copy(ALL(dq), res.begin());
return res;
}

```

## 6.5 Convex Hull

```

double cross(pdd o, pdd a, pdd b){
    return (a-o) % (b-o);
}

vector<pdd> convex_hull(vector<pdd> pt){
    sort(pt.begin(), pt.end());
    int top=0;
    vector<pdd> stk(2*pt.size());
    for (int i=0; i<(int)pt.size(); i++){
        while (top >= 2 && cross(stk[top-2], stk[top-1], pt[i])
            <= 0)
            top--;
        stk[top++] = pt[i];
    }
    for (int i=pt.size()-2, t=top+1; i>=0; i--){
        while (top >= t && cross(stk[top-2], stk[top-1], pt[i])
            <= 0)
            top--;
        stk[top++] = pt[i];
    }
    stk.resize(top-1);
    return stk;
}

```

## 6.6 Minimum Covering Circle

```

struct Mcc{
    // return pair of center and r^2
    static const int MAXN = 1000100;
    int n;
    pdd p[MAXN], cen;
    double r2;

    void init(int _n, pdd _p[]){
        n = _n;
        memcpy(p, _p, sizeof(pdd)*n);
    }

    double sqr(double a){ return a*a; }
    double abs2(pdd a){ return a.a; }
    pdd center(pdd p0, pdd p1, pdd p2) {
        pdd a = p1-p0;
        pdd b = p2-p0;
        double c1=abs2(a)*0.5;
        double c2=abs2(b)*0.5;
        double d = a % b;
        double x = p0.x + (c1 * b.y - c2 * a.y) / d;
        double y = p0.y + (a.x * c2 - b.x * c1) / d;
        return pdd(x,y);
    }

    pair<pdd, double> solve(){
        random_shuffle(p, p+n);
        r2=0;
        for (int i=0; i<n; i++){
            if (abs2(cen-p[i]) <= r2) continue;
            cen = p[i];
            r2 = 0;
            for (int j=0; j<i; j++){
                if (abs2(cen-p[j]) <= r2) continue;
                cen = 0.5 * (p[i]+p[j]);
                r2 = abs2(cen-p[j]);
                for (int k=0; k<j; k++){
                    if (abs2(cen-p[k]) <= r2) continue;
                    cen = center(p[i], p[j], p[k]);
                    r2 = abs2(cen-p[k]);
                }
            }
        }
        return {cen, r2};
    }
}mcc;

```

## 6.7 KDTree (Nearest Point)

```
const int MXN = 100005;

struct KDTree {
    struct Node {
        int x,y,x1,y1,x2,y2;
        int id,f;
        Node *L, *R;
    }tree[MXN];
    int n;
    Node *root;

    long long dis2(int x1, int y1, int x2, int y2) {
        long long dx = x1-x2;
        long long dy = y1-y2;
        return dx*dx+dy*dy;
    }

    static bool cmpx(Node& a, Node& b){ return a.x<b.x; }
    static bool cmpy(Node& a, Node& b){ return a.y<b.y; }
    void init(vector<pair<int,int>> ip) {
        n = ip.size();
        for (int i=0; i<n; i++) {
            tree[i].id = i;
            tree[i].x = ip[i].first;
            tree[i].y = ip[i].second;
        }
        root = build_tree(0, n-1, 0);
    }

    Node* build_tree(int L, int R, int dep) {
        if (L>R) return nullptr;
        int M = (L+R)/2;
        tree[M].f = dep%2;
        nth_element(tree+L, tree+M, tree+R+1, tree[M].f ?
            cmpx : cmpy);
        tree[M].x1 = tree[M].x2 = tree[M].x;
        tree[M].y1 = tree[M].y2 = tree[M].y;

        tree[M].L = build_tree(L, M-1, dep+1);
        if (tree[M].L) {
            tree[M].x1 = min(tree[M].x1, tree[M].L->x1);
            tree[M].x2 = max(tree[M].x2, tree[M].L->x2);
            tree[M].y1 = min(tree[M].y1, tree[M].L->y1);
            tree[M].y2 = max(tree[M].y2, tree[M].L->y2);
        }

        tree[M].R = build_tree(M+1, R, dep+1);
        if (tree[M].R) {
            tree[M].x1 = min(tree[M].x1, tree[M].R->x1);
            tree[M].x2 = max(tree[M].x2, tree[M].R->x2);
            tree[M].y1 = min(tree[M].y1, tree[M].R->y1);
            tree[M].y2 = max(tree[M].y2, tree[M].R->y2);
        }

        return tree+M;
    }

    int touch(Node* r, int x, int y, long long d2){
        long long dis = sqrt(d2)+1;
        if (x<r->x1-dis || x>r->x2+dis || y<r->y1-dis || y>
            r->y2+dis)
            return 0;
        return 1;
    }

    void nearest(Node* r, int x, int y, int &mID, long
        long &md2) {
        if (!r || !touch(r, x, y, md2)) return;
        long long d2 = dis2(r->x, r->y, x, y);
        if (d2 < md2 || (d2 == md2 && mID < r->id)) {
            mID = r->id;
            md2 = d2;
        }
        // search order depends on split dim
        if ((r->f == 0 && x < r->x) ||
            (r->f == 1 && y < r->y)) {
            nearest(r->L, x, y, mID, md2);
            nearest(r->R, x, y, mID, md2);
        } else {
            nearest(r->R, x, y, mID, md2);
            nearest(r->L, x, y, mID, md2);
        }
    }
}
```

```
int query(int x, int y) {
    int id = 1029384756;
    long long d2 = 102938475612345678LL;
    nearest(root, x, y, id, d2);
    return id;
}

}tree;
```

## 6.8 (+1) MinkowskiSum

```
/* convex hull Minkowski Sum*/
#define INF 1000000000000000LL
class PT{ public:
    long long x,y;
    int POS(){
        if(y==0) return x>0?0:1;
        return y>0?0:1;
    }
};

PT pt[300000],qt[300000],rt[300000];
long long Lx,Rx;
int dn,un;
inline bool cmp(PT a,PT b){
    int pa=a.POS(),pb=b.POS();
    if(pa==pb) return (a^b)>0;
    return pa<pb;
}

int minkowskiSum(int n,int m){
    int i,j,r,p,q,fi,fj;
    for(i=1,p=0;i<n;i++){
        if(pt[i].y<pt[p].y || (pt[i].y==pt[p].y && pt[i].x<
            pt[p].x)) p=i;
    }
    for(i=1,q=0;i<m;i++){
        if(qt[i].y<qt[q].y || (qt[i].y==qt[q].y && qt[i].x<
            qt[q].x)) q=i;
    }
    rt[0]=pt[p]+qt[q];
    r=1; i=p; j=q; fi=fj=0;
    while(1){
        if(((fj&j==q) || ((!fi||i!=p) && cmp(pt[(p+1)%n]-pt
            [p],qt[(q+1)%m]-qt[q]))){
            rt[r]=rt[r-1]+pt[(p+1)%n]-pt[p];
            p=(p+1)%n;
            fi=1;
        }else{
            rt[r]=rt[r-1]+qt[(q+1)%m]-qt[q];
            q=(q+1)%m;
            fj=1;
        }
        if(r<=1 || ((rt[r]-rt[r-1])^(rt[r-1]-rt[r-2]))!=0)
            r
            ++;
        else rt[r-1]=rt[r];
        if(i==p && j==q) break;
    }
    return r-1;
}

void initInConvex(int n){
    int i,p,q;
    long long Ly,Ry;
    Lx=INF; Rx=-INF;
    for(i=0;i<n;i++){
        if(pt[i].x<Lx) Lx=pt[i].x;
        if(pt[i].x>Rx) Rx=pt[i].x;
    }
    Ly=Ry=INF;
    for(i=0;i<n;i++){
        if(pt[i].x==Lx && pt[i].y<Ly){ Ly=pt[i].y; p=i; }
        if(pt[i].x==Rx && pt[i].y<Ry){ Ry=pt[i].y; q=i; }
    }
    for(dn=0,i=p;i!=q;i=(i+1)%n){ qt[dn++]=pt[i]; }
    qt[dn]=pt[q]; Ly=Ry=-INF;
    for(i=0;i<n;i++){
        if(pt[i].x==Lx && pt[i].y>Ly){ Ly=pt[i].y; p=i; }
        if(pt[i].x==Rx && pt[i].y>Ry){ Ry=pt[i].y; q=i; }
    }
    for(un=0,i=p;i!=q;i=(i+n-1)%n){ rt[un++]=pt[i]; }
    rt[un]=pt[q];
}

inline int inConvex(PT p){
```

```

int L,R,M;
if(p.x<Lx || p.x>Rx) return 0;
L=0;R=dn;
while(L<R-1){ M=(L+R)/2;
  if(p.x<qt[M].x) R=M; else L=M; }
if(tri(qt[L],qt[R],p)<0) return 0;
L=0;R=un;
while(L<R-1){ M=(L+R)/2;
  if(p.x<rt[M].x) R=M; else L=M; }
if(tri(rt[L],rt[R],p)>0) return 0;
return 1;
}
int main(){
  int n,m,i;
  PT p;
  scanf("%d",&n);
  for(i=0;i<n;i++) scanf("%I64d %I64d",&pt[i].x,&pt[i].y);
  scanf("%d",&m);
  for(i=0;i<m;i++) scanf("%I64d %I64d",&qt[i].x,&qt[i].y);
  n=minkowskiSum(n,m);
  for(i=0;i<n;i++) pt[i]=rt[i];
  scanf("%d",&m);
  for(i=0;i<m;i++) scanf("%I64d %I64d",&qt[i].x,&qt[i].y);
  n=minkowskiSum(n,m);
  for(i=0;i<n;i++) pt[i]=rt[i];
  initInConvex(n);
  scanf("%d",&m);
  for(i=0;i<m;i++){
    scanf("%I64d %I64d",&p.x,&p.y);
    p.x*=3; p.y*=3;
    puts(inConvex(p)?"YES":"NO");
  }
}

```

## 7 Stringology

### 7.1 Suffix Array

```

const int MAX = 1020304;
int ct[MAX], he[MAX], rk[MAX], sa[MAX], tsa[MAX], tp[
MAX][2];

void suffix_array(char *ip){

  int len = strlen(ip);
  int alp = 256;

  memset(ct, 0, sizeof(ct));
  for(int i=0;i<len;i++) ct[ip[i]+1]++;
  for(int i=1;i<alp;i++) ct[i]+=ct[i-1];
  for(int i=0;i<len;i++) rk[i]=ct[ip[i]];

  for(int i=1;i<len;i*=2){
    for(int j=0;j<len;j++){
      if(j+i>=len) tp[j][1]=0;
      else tp[j][1]=rk[j+i]+1;

      tp[j][0]=rk[j];
    }
    memset(ct, 0, sizeof(ct));
    for(int j=0;j<len;j++) ct[tp[j][1]+1]++;
    for(int j=1;j<len+2;j++) ct[j]+=ct[j-1];
    for(int j=0;j<len;j++) tsa[ct[tp[j][1]]+=j];

    memset(ct, 0, sizeof(ct));
    for(int j=0;j<len;j++) ct[tp[j][0]+1]++;
    for(int j=1;j<len+1;j++) ct[j]+=ct[j-1];
    for(int j=0;j<len;j++) sa[ct[tp[j][0]]+=j]=j;

    rk[sa[0]]=0;
    for(int j=1;j<len;j++){
      if( tp[sa[j]][0] == tp[sa[j-1]][0] &&
         tp[sa[j]][1] == tp[sa[j-1]][1] )
        rk[sa[j]] = rk[sa[j-1]];
      else
        rk[sa[j]] = j;
    }
  }

  for(int i=0,h=0;i<len;i++){
    if(rk[i]==0) h=0;
    else{
      int j=sa[rk[i]-1];
      h=max(0,h-1);
      for(;ip[i+h]==ip[j+h];h++);
    }
    he[rk[i]]=h;
  }
}

```

## 7.2 Suffix Array (SAIS TWT514)

```

struct SA{
#define REP(i,n) for ( int i=0; i<int(n); i++ )
#define REP1(i,a,b) for ( int i=(a); i<=int(b); i++ )
    static const int MXN = 300010;
    bool _t[MXN*2];
    int _s[MXN*2], _sa[MXN*2], _c[MXN*2], x[MXN], _p[MXN], _q[MXN*2], hei[MXN], r[MXN];
    int operator [] (int i){ return _sa[i]; }
    void build(int *s, int n, int m){
        memcpy(_s, s, sizeof(int) * n);
        sais(_s, _sa, _p, _q, _t, _c, n, m);
        mkhei(n);
    }
    void mkhei(int n){
        REP(i,n) r[_sa[i]] = i;
        hei[0] = 0;
        REP(i,n) if(r[i]) {
            int ans = i>0 ? max(hei[r[i-1]] - 1, 0) : 0;
            while(_s[i+ans] == _s[_sa[r[i]-1]+ans]) ans++;
            hei[r[i]] = ans;
        }
    }
    void sais(int *s, int *sa, int *p, int *q, bool *t, int *c, int n, int z){
        bool uniq = t[n-1] = true, neq;
        int nn = 0, nmz = -1, *nsa = sa + n, *ns = s + n, lst = -1;
#define MS0(x,n) memset((x),0,n*sizeof(*(x)))
#define MAGIC(XD) MS0(sa, n); \
        memcpy(x, c, sizeof(int) * z); \
        XD; \
        memcpy(x + 1, c, sizeof(int) * (z - 1)); \
        REP(i,n) if(sa[i] && !t[sa[i]-1]) sa[x[s[sa[i]-1]]++] = sa[i]-1; \
        memcpy(x, c, sizeof(int) * z); \
        for(int i = n - 1; i >= 0; i--) if(sa[i] && t[sa[i]-1]) sa[--x[s[sa[i]-1]]] = sa[i]-1;
        MS0(c, z);
        REP(i,n) uniq &= ++c[s[i]] < 2;
        REP(i,z-1) c[i+1] += c[i];
        if (uniq) { REP(i,n) sa[--c[s[i]]] = i; return; }
        for(int i = n - 2; i >= 0; i--) t[i] = (s[i]==s[i+1] ? t[i+1] : s[i]<s[i+1]);
        MAGIC(REP1(i,1,n-1) if(t[i] && !t[i-1]) sa[--x[s[i]]]=p[q[i]=nn++]=i);
        REP(i, n) if (sa[i] && t[sa[i]] && !t[sa[i]-1]) {
            neq=lst<0||memcmp(s+sa[i],s+lst,(p[q[sa[i]]+1]-sa[i])*sizeof(int));
            ns[q[lst=sa[i]]]=nmz+=neq;
        }
        sais(ns, nsa, p + nn, q + n, t + n, c + z, nn, nmz + 1);
        MAGIC(for(int i = nn - 1; i >= 0; i--) sa[--x[p[nsa[i]]]] = p[nsa[i]]);
    }
}sa;

void suffix_array(int* ip, int len) {
    // should padding a zero in the back
    // s is int array, n is array length
    // s[0..n-1] != 0, and s[n] = 0
    // resulting SA will be length n+1
    ip[len++] = 0;
    sa.build(ip, len, 128);
    // original 1-base
    for (int i=0; i<l; i++) {
        hei[i] = sa.hei[i + 1];
        sa[i] = sa._sa[i + 1];
    }
}

```

## 7.3 Aho-Corasick Algorithm

```

struct AAutomata{
    struct Node{
        int cnt,dp;
        Node *go[26], *fail;
        Node (){
            cnt = 0;
            dp = -1;
            memset(go,0,sizeof(go));
            fail = 0;
        }
    };

    Node *root, pool[1048576];
    int nMem;

    Node* new_Node(){
        pool[nMem] = Node();
        return &pool[nMem++];
    }
    void init(){
        nMem = 0;
        root = new_Node();
    }
    void add(const string &str){
        insert(root,str,0);
    }
    void insert(Node *cur, const string &str, int pos){
        if (pos >= (int)str.size()){
            cur->cnt++;
            return;
        }
        int c = str[pos]-'a';
        if (cur->go[c] == 0){
            cur->go[c] = new_Node();
        }
        insert(cur->go[c],str,pos+1);
    }
    void make_fail(){
        queue<Node*> que;
        que.push(root);
        while (!que.empty()){
            Node* fr=que.front();
            que.pop();
            for (int i=0; i<26; i++){
                if (fr->go[i]){
                    Node *ptr = fr->fail;
                    while (ptr && !ptr->go[i]) ptr = ptr->fail;
                    if (!ptr) fr->go[i]->fail = root;
                    else fr->go[i]->fail = ptr->go[i];
                    que.push(fr->go[i]);
                }
            }
        }
    }
}

```

## 7.4 Z value

```

void Z_value(char *s, int *z, int len) {
    int i,j,left,right;
    left=right=0; z[0]=len;
    for(i=1;i<len;i++) {
        j=max(min(z[i-left],right-i),0);
        for(;i+j<len&&s[i+j]==s[j];j++);
        z[i]=j;
        if(i+z[i]>right) {
            right=i+z[i];
            left=i;
        }
    }
}

```

## 7.5 Z value (palindrome ver.)

```
const int MAX = 1000;
int len;
char ip[MAX];
char op[MAX*2];
int zv[MAX*2];

int main(){
    cin >> ip;
    len = strlen(ip);

    int l2 = len*2 - 1;
    for(int i=0; i<l2; i++){
        if(i&1) op[i] = '@';
        else op[i] = ip[i/2];
    }
    int l=0, r=0;
    zv[0] = 1;

    for(int i=1; i<l2; i++){
        if( i > r ){
            l = r = i;
            while( l>0 && r<l2-1 && op[l-1] == op[r+1] ){
                l--;
                r++;
            }
            zv[i] = (r-l+1);
        }else{
            int md = (l+r)/2;
            int j = md + md - i;
            zv[i] = zv[j];
            int q = zv[i] / 2;
            int nr = i + q;
            if( nr == r ){
                l = i + i - r;

                while( l>0 && r<l2-1 && op[l-1] == op[r+1] ){
                    l--;
                    r++;
                }
                zv[i] = r - l + 1;
            }else if( nr > r ){
                zv[i] = (r - i) * 2 + 1;
            }
        }
    }

    return 0;
}
```

## 7.6 Lexicographically Smallest Rotation

```
string mcp(string s){
    int n = s.length();
    s += s;
    int i=0, j=1, k=0;
    while (j<n && k<n){
        if (s[i+k] == s[j+k]) k++;
        else {
            if (s[i+k] < s[j+k]) {
                j += k + 1;
            } else {
                i = j;
                j = max(j+1, j+k);
            }
            k = 0;
        }
    }
    return s.substr(i, n);
}
```

## 7.7 Suffix Automaton

```
// par : fail link
// val : a topological order ( useful for DP )
```

```
// go[x] : automata edge ( x is integer in [0,26) )
```

```
struct SAM{
    struct State{
        int par, go[26], val;
        State () : par(0), val(0){ FZ(go); }
        State (int _val) : par(0), val(_val){ FZ(go); }
    };
    vector<State> vec;
    int root, tail;

    void init(int arr[], int len){
        vec.resize(2);
        vec[0] = vec[1] = State(0);
        root = tail = 1;
        for (int i=0; i<len; i++)
            extend(arr[i]);
    }
    void extend(int w){
        int p = tail, np = vec.size();
        vec.PB(State(vec[p].val+1));
        for ( ; p && vec[p].go[w]==0; p=vec[p].par)
            vec[p].go[w] = np;
        if (p == 0){
            vec[np].par = root;
        } else {
            if (vec[vec[p].go[w]].val == vec[p].val+1){
                vec[np].par = vec[p].go[w];
            } else {
                int q = vec[p].go[w], r = vec.size();
                vec.PB(vec[q]);
                vec[r].val = vec[p].val+1;
                vec[q].par = vec[np].par = r;
                for ( ; p && vec[p].go[w] == q; p=vec[p].par)
                    vec[p].go[w] = r;
            }
        }
        tail = np;
    }
};
```

## 8 Problems

### 8.1 Find the maximum tangent (x,y is increasing)

```
typedef long long LL;
const int MAXN = 100010;
struct Coord{
    LL x, y;
    Coord operator - (Coord ag) const{
        Coord res;
        res.x = x - ag.x;
        res.y = y - ag.y;
        return res;
    }
}sum[MAXN], pnt[MAXN], ans, calc;

inline bool cross(Coord a, Coord b, Coord c){
    return (c.y - a.y) * (c.x - b.x) > (c.x - a.x) * (c.y - b.y);
}

int main(){
    int n, l, np, st, ed, now;
    scanf("%d %d\n", &n, &l);
    sum[0].x = sum[0].y = np = st = ed = 0;
    for (int i = 1, v; i <= n; i++){
        scanf("%d", &v);
        sum[i].y = sum[i-1].y + v;
        sum[i].x = i;
    }
    ans.x = now = 1;
    ans.y = -1;
    for (int i = 0; i <= n - l; i++){
        while (np > 1 && cross(pnt[np-2], pnt[np-1], sum[i]))
            np--;
        if (np < now && np != 0) now = np;
    }
```

```

    pnt[np++] = sum[i];
    while (now < np && !cross(pnt[now - 1], pnt[now],
        sum[i + 1]))
        now++;
    calc = sum[i + 1] - pnt[now - 1];
    if (ans.y * calc.x < ans.x * calc.y){
        ans = calc;
        st = pnt[now - 1].x;
        ed = i + 1;
    }
}
double res = (sum[ed].y - sum[st].y) / (sum[ed].x - sum[st].x);
printf("%f\n", res);
return 0;
}

```

## 8.2 Orange Protection

```

/*
 * Given a Tree and the power of every node.
 * Each Node can protect the nodes whose distance <=
 * cover[i] with it
 * output the number of each node that it can protect.
 */
const int MXN = 100005;

int cover[MXN], ans[MXN];
int N, ok[MXN];
int fr, bk, que[MXN], vst[MXN], dis[MXN], fa[MXN], sz[MXN];
vector<int> E[MXN];

int bit[MXN];
int lb(int a){ return a & -a; }
void reset_bit(int st){
    for (int i = st+1; i < MXN; i+=lb(i))
        bit[i] = 0;
}
void update(int st){
    for (int i = st+1; i < MXN; i+=lb(i))
        bit[i]++;
}
int query(int st, int ret = 0){
    for (int i = st+1; i > 0; i-=lb(i))
        ret += bit[i];
    return ret;
}

void BFS(int st){
    fr = bk = 0;
    que[bk++] = st;
    vst[st] = 1;
    dis[st] = 0;
    while (fr < bk){
        int u = que[fr++];
        for (auto v : E[u]){
            if (!ok[v] || vst[v]) continue;
            vst[v] = 1;
            dis[v] = dis[u] + 1;
            fa[v] = u;
            que[bk++] = v;
        }
    }
    for (int i=0; i<bk; i++)
        vst[que[i]] = 0;
}

int find_centroid(int st){
    int ret=-1, cnt=MXN+100;
    BFS(st);
    for (int i = bk-1; i>=0; i--){
        int u = que[i], mx = 0;
        sz[u] = 1;
        for (auto v : E[u]){
            if (!ok[v] || v == fa[u]) continue;
            sz[u] += sz[v];
            mx = max(mx, sz[v]);
        }
        mx = max(mx, bk-sz[u]);
        if (mx < cnt){
            ret = u;

```

```

        cnt = mx;
    }
}
return ret;
}

void solve(int u){
    int root = find_centroid(u);
    ok[root] = 0;
    for (auto v : E[root])
        if (ok[v]) solve(v);

    for (auto v : E[root]){
        if (!ok[v]) continue;
        BFS(v);
        for (int i=0; i<bk; i++){
            dis[que[i]]++;
            update(dis[que[i]]);
        }
        for (int i=0; i<bk; i++){
            int it = que[i];
            ans[it] -= query(cover[it] - dis[it]);
        }
        for (int i=0; i<bk; i++)
            reset_bit(dis[que[i]]);
    }
    BFS(root);
    for (int i=0; i<bk; i++) update(dis[que[i]]);
    for (int i=0; i<bk; i++){
        int v = que[i];
        ans[v] += query(cover[v] - dis[v]);
    }
    for (int i=0; i<bk; i++) reset_bit(dis[que[i]]);

    ok[root] = 1;
}

int main(int argc, char** argv){
    scanf("%d", &N);
    for (int i=0; i<N; i++){
        scanf("%d", &cover[i]);
        cover[i] = min(cover[i], N);
    }
    for (int i=0, u, v; i<N-1; i++){
        scanf("%d%d", &u, &v);
        u--; v--;
        E[u].PB(v);
        E[v].PB(u);
    }
    fill(ok, ok+N, 1);
    FZ(vst); FZ(ans); FZ(bit);
    solve(0);
    for (int i=0; i<N; i++)
        printf("%d\n", ans[i]);
    return 0;
}

```