

Causal Machine Learning

Debiased/Double Machine Learning

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Reference

Chernozhukov, Chetverikov, Demirer, Duflo, Hansen, and Newey (2017):
"Double/Debiased/Neyman Machine Learning of Treatment Effects",
American Economic Review, 107 (5), pp. 261-265, [download](#).

Overview

Inverse Probability Weighting

T-Learner

Double/Debiased Machine Learning

Potential Outcome Framework

Notation:

- ▶ D_i binary treatment dummy (e.g., assignment to training program)
- ▶ $Y_i(1)$ potential outcome under treatment (e.g., earnings under participation in training)
- ▶ $Y_i(0)$ potential outcome under non-treatment (e.g., earnings under non-participation in training)

Infeasible parameter:

- ▶ Individual causal effect: $\delta_i = Y_i(1) - Y_i(0)$

Feasible parameters:

- ▶ Average Treatment Effect (ATE): $\delta = E[Y_i(1) - Y_i(0)] = E[\delta_i]$
- ▶ Average Treatment Effect on the Treated (ATET): $\rho = E[\delta_i | D_i = 1]$

Identifying Assumptions for ATE

- ▶ **Stable Unit Treatment Value Assumption (SUTVA):**

$$Y_i = Y_i(1)D_i + Y_i(0)(1 - D_i)$$

- ▶ **No Support Problems:**

$$\epsilon < \Pr(D_i = 1 | X_i = x) = p(x) < 1 - \epsilon$$

for some small $\epsilon > 0$ and all x in the support of X_i

- ▶ **Conditional Independence Assumption (CIA):**

$$Y_i(1), Y_i(0) \perp\!\!\!\perp D_i | X_i = x$$

for all x in the support of X_i

Modified Outcome Method for ATE

Inverse Probability Weighting:

$$Y_{i,IPW}^* = \frac{D_i}{p(X_i)} Y_i - \frac{1 - D_i}{1 - p(X_i)} Y_i = \frac{D_i - p(X_i)}{p(X_i)(1 - p(X_i))} Y_i$$

with the propensity score $p(x) = Pr(D_i = 1|X_i = x)$.

$$\text{ATE: } \delta = E[Y_{i,IPW}^*] \text{ and } \hat{\delta} = \frac{1}{N} \sum_{i=1}^N \hat{Y}_{i,IPW}^*$$

Proof of Identification

$$\begin{aligned}\delta &= E[Y_i(1)] - E[Y_i(0)] \stackrel{LIE}{=} \int E[Y_i(1)|X_i = x] - E[Y_i(0)|X_i = x] f_X(x) dx \\ &\stackrel{CIA}{=} \int E[Y_i(1)|D_i = 1, X_i = x] - E[Y_i(0)|D_i = 0, X_i = x] f_X(x) dx \\ &= \int E[Y_i|D_i = 1, X_i = x] - E[Y_i|D_i = 0, X_i = x] f_X(x) dx \\ &= \int E[D_i Y_i|D_i = 1, X_i = x] - E[(1 - D_i) Y_i|D_i = 0, X_i = x] f_X(x) dx \\ &\stackrel{LIE}{=} \int E\left[\frac{D_i Y_i}{p(X_i)} \middle| X_i = x\right] - E\left[\frac{(1 - D_i) Y_i}{1 - p(X_i)} \middle| X_i = x\right] f_X(x) dx \\ &= \int E\left[\frac{D_i Y_i}{p(X_i)} - \frac{(1 - D_i) Y_i}{1 - p(X_i)} \middle| X_i = x\right] f_X(x) dx \\ &= \int E\left[\frac{D_i - p(X_i)}{p(X_i)(1 - p(X_i))} Y_i \middle| X_i = x\right] f_X(x) dx \stackrel{LIE}{=} E\left[\frac{D_i - p(X_i)}{p(X_i)(1 - p(X_i))} Y_i\right]\end{aligned}$$

Reference: [Horvitz and Thompson \(1952\)](#)

Modified Outcome Method with IPW

Advantages:

- ▶ Generic approach
- ▶ Sparsity assumptions can be avoided by appropriate choice of estimator for propensity score
- ▶ Heterogeneous treatment effects

Disadvantages:

- ▶ Potentially omitting “weak outcome confounders”
- ▶ Shows weak performance in simulations
([Knaus, Lechner, and Strittmatter, 2018](#))
- ▶ Not \sqrt{N} -consistent in high-dimensional setting

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Conditional Mean Differences

Identification:

$$\begin{aligned}\delta &= E[Y_i(1)] - E[Y_i(0)] \\ &\stackrel{LIE}{=} \int E[Y_i(1)|X_i = x] - E[Y_i(0)|X_i = x] f_X(x) dx \\ &\stackrel{CIA}{=} \int E[Y_i(1)|D_i = 1, X_i = x] - E[Y_i(0)|D_i = 0, X_i = x] f_X(x) dx \\ &= \int \underbrace{E[Y_i|D_i = 1, X_i = x]}_{=\mu_1(x)} - \underbrace{E[Y_i|D_i = 0, X_i = x]}_{=\mu_0(x)} f_X(x) dx\end{aligned}$$

Estimator:

$$\hat{\delta} = \frac{1}{N} \sum_{i=1}^N (\hat{\mu}_1(X_i) - \hat{\mu}_0(X_i))$$

with $\hat{\mu}_1(x) = \hat{E}[Y_i|D_i = 1, X_i = x]$ and $\hat{\mu}_0(x) = \hat{E}[Y_i(0)|D_i = 0, X_i = x]$ being the estimated conditional expectations of the potential outcomes.

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Inverse Probability Weighting

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Double/Debiased Machine Learning (DML)

Efficient Score:

$$\begin{aligned} Y_{i,DML}^* &= \mu_1(X_i) - \mu_0(X_i) + \frac{D_i - p(X_i)}{p(X_i)(1 - p(X_i))} Y_i - \frac{D_i}{p(X_i)} \mu_1(X_i) + \frac{1 - D_i}{1 - p(X_i)} \mu_0(X_i) \\ &= \mu_1(X_i) - \mu_0(X_i) + \frac{D_i(Y_i - \mu_1(X_i))}{p(X_i)} - \frac{(1 - D_i)(Y_i - \mu_0(X_i))}{1 - p(X_i)} \end{aligned}$$

$$\text{ATE: } \delta = E[Y_{i,DML}^*] \text{ and } \hat{\delta} = \frac{1}{N} \sum_{i=1}^N \hat{Y}_{i,DML}^*$$

We can use standard ML methods to estimate $\hat{\mu}_1(x)$, $\hat{\mu}_0(x)$, and $\hat{p}(x)$.

Reference: [Chernozhukov et al., 2017](#)

Proof of Identification

$$\begin{aligned}\delta &= E \left[\mu_1(X_i) - \mu_0(X_i) + \frac{D_i(Y_i - \mu_1(X_i))}{p(X_i)} - \frac{(1 - D_i)(Y_i - \mu_0(X_i))}{1 - p(X_i)} \right] \\&= E \left[\frac{D_i - p(X_i)}{p(X_i)(1 - p(X_i))} Y_i + \frac{(p(X_i) - D_i)\mu_1(X_i)}{p(X_i)} - \frac{(D_i - p(X_i))\mu_0(X_i)}{1 - p(X_i)} \right] \\&= \int E \left[\frac{D_i - p(X_i)}{p(X_i)(1 - p(X_i))} Y_i + \frac{(p(X_i) - D_i)\mu_1(X_i)}{p(X_i)} \right. \\&\quad \left. - \frac{(D_i - p(X_i))\mu_0(X_i)}{1 - p(X_i)} \middle| X_i = x \right] f_X(x) dx \\&= \int \left(E \left[\frac{D_i - p(X_i)}{p(X_i)(1 - p(X_i))} Y_i \middle| X_i = x \right] + \frac{E[p(X_i) - D_i | X_i = x]}{p(x)} \mu_1(x) \right. \\&\quad \left. - \frac{E[D_i - p(X_i) | X_i = x]}{1 - p(x)} \mu_0(x) \right) f_X(x) dx \\&= \int E \left[\frac{D_i - p(X_i)}{p(X_i)(1 - p(X_i))} Y_i \middle| X_i = x \right] f_X(x) dx = E[Y_i(1) - Y_i(0)]\end{aligned}$$

Reference: [Robins and Rotnitzki \(1995\)](#)

DML Cross-Fitting Algorithm

1. Partition the data randomly in samples S^A and S^B
2. Estimate the nuisance parameters $\hat{\mu}_1^A(x)$, $\hat{\mu}_0^A(x)$, and $\hat{p}^A(x)$ in S^A ; and $\hat{\mu}_1^B(x)$, $\hat{\mu}_0^B(x)$, and $\hat{p}^B(x)$ in S^B with ML
3. Calculate the efficient scores in samples S^A and S^B , respectively:

$$\hat{Y}_{i,DML}^{A*} = \hat{\mu}_1^B(X_i^A) - \hat{\mu}_0^B(X_i^A) + \frac{D_i^A(Y_i^A - \hat{\mu}_1^B(X_i^A))}{\hat{p}^B(X_i^A)} - \frac{(1 - D_i^A)(Y_i^A - \hat{\mu}_0^B(X_i^A))}{1 - \hat{p}^B(X_i^A)}$$
$$\hat{Y}_{i,DML}^{B*} = \hat{\mu}_1^A(X_i^B) - \hat{\mu}_0^A(X_i^B) + \frac{D_i^B(Y_i^B - \hat{\mu}_1^A(X_i^B))}{\hat{p}^A(X_i^B)} - \frac{(1 - D_i^B)(Y_i^B - \hat{\mu}_0^A(X_i^B))}{1 - \hat{p}^A(X_i^B)}$$

4. Calculate ATE,

$$\hat{\delta} = \frac{1}{2} \left(\underbrace{\hat{E}[\hat{Y}_{i,DML}^{A*} | S^A]}_{=\hat{\delta}_A} + \underbrace{\hat{E}[\hat{Y}_{i,DML}^{B*} | S^B]}_{=\hat{\delta}_B} \right),$$

Asymptotic Results for ATE

- ▶ Main Regularity Condition: Convergence rate of nuisance parameters is at least $\sqrt[4]{N}$.
- ▶ ATE can be estimated \sqrt{N} -consistently

$$\sqrt{N}(\hat{\delta} - \delta) \xrightarrow{d} N(0, \sigma^2)$$

with $\sigma^2 = \text{Var}(Y_{i,DML}^*)$ and $\text{Var}(\hat{\delta}) = \sigma^2/N$

- ▶ Split sample estimator of σ^2

$$\hat{\sigma}^2 = \frac{1}{2} \left(\hat{\sigma}_A^2 + (\hat{\delta}_A - \hat{\delta})^2 \right) + \frac{1}{2} \left(\hat{\sigma}_B^2 + (\hat{\delta}_B - \hat{\delta})^2 \right)$$

for $\hat{\delta} = 1/2(\hat{\delta}_A + \hat{\delta}_B)$

Advantages of DML

Advantages compared to IPW and Conditional Mean Differences:

- ▶ Treatment and outcome equations are modelled explicitly
- ▶ Double robustness property
- ▶ \sqrt{N} -consistent and asymptotically normal even under high-dimensional confounding

Efficient Score for ATET

$$Y_{i,ATET}^* = \frac{D_i(Y_i - \mu_0(X_i))}{p} - \frac{p(X_i)(1 - D_i)(Y_i - \mu_0(X_i))}{p(1 - p(X_i))}$$

with $p = Pr(D_i = 1)$.

$$\text{ATET: } \rho = E[Y_{i,ATET}^*] \text{ and } \hat{\rho} = \frac{1}{N} \sum_{i=1}^N \hat{Y}_{i,ATET}^*$$

Estimator of Asymptotic Variance:

$$\hat{\sigma}^2 = \frac{1}{N} \sum_{i=1}^N \left(\hat{Y}_{i,ATET}^* - \hat{\rho} \right)^2$$

and $\hat{Var}(\hat{\rho}) = \hat{\sigma}^2/N$

References: [Chernozhukov et al., 2017](#), [Farrell, 2015](#)

Proof of Identification for ATET

$$\begin{aligned}\rho &= E \left[\frac{D_i(Y_i - \mu_0(X_i))}{p} - \frac{p(X_i)(1 - D_i)(Y_i - \mu_0(X_i))}{p(1 - p(X_i))} \right] \\&= \int E \left[\frac{D_i Y_i}{p} - \frac{p(X_i)(1 - D_i) Y_i}{p(1 - p(X_i))} - \frac{(D_i - p(X_i))\mu_0(X_i)}{p(1 - p(X_i))} \middle| X_i = x \right] f_X(x) dx \\&= \int \left(\frac{E[D_i Y_i | X_i = x]}{p} - \frac{p(x)E[(1 - D_i) Y_i | X_i = x]}{p(1 - p(x))} \right. \\&\quad \left. - \frac{E[D_i - p(X_i) | X_i = x]}{p(1 - p(x))} \mu_0(x) \right) f_X(x) dx \\&= \int \left(\frac{E[D_i Y_i | X_i = x]}{p} - \frac{p(x)E[(1 - D_i) Y_i | X_i = x]}{p(1 - p(x))} \right) f_X(x) dx \\&= \int \frac{p(x)}{p} (E[D_i Y_i | D_i = 1, X_i = x] - E[(1 - D_i) Y_i | D_i = 0, X_i = x]) f_X(x) dx \\&= \int (E[Y_i(1) | D_i = 1, X_i = x] - E[Y_i(0) | D_i = 0, X_i = x]) f_{X|D=1}(x) dx \\&= \int (E[Y_i(1) | D_i = 1, X_i = x] - E[Y_i(0) | D_i = 1, X_i = x]) f_{X|D=1}(x) dx \\&= E[Y_i(1) - Y_i(0) | D_i = 1]\end{aligned}$$