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The vehicle routing problem in the last decade: variants, taxonomy and metaheuristics

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Abstract

Vehicle routing problem is a NP-hard problem and a combinatorial optimization problem; it appeared first time in 1959 in the paper of the mathematician Dantzig. The goal of VRP is to locate the optimal routes of some vehicles that begin from a depot and serve each customer one time and then return to the depot (i.e., Starting point). From its beginning, the research literature in this area is growing rapidly and causing the extension of VRP to many variants for making it a real-world problem. For solving it, the researchers have tried firstly the exact methods then the heuristics and lastly the metaheuristics.

This paper aims for many targets for instance: (i) discovering the evolution of VRP and its variants over the last decade; (ii) knowing the trends, challenges and opportunities in the next years in this fields by discovering, comparing many recent reviews and papers related either to VRP or to metaheuristics for exploiting these results and building on them in other papers.

Keywords: Vehicle routing problem, Taxonomy, Supply chain and logistics optimization, Combinatorial optimization, metaheuristics, Hybridization of metaheuristics, Machine learning into metaheuristics.

1. Introduction

Supply chain is the collaboration between suppliers, manufacturers, distributors, business partners and consumer.

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So, the transport optimization, the final step of the supply chain, contributes in the optimization of the supply chain and especially the Vehicle routing problem (VRP) that has many variants because the researchers try to adapt it to real life cases. Solving VRP was firstly by using exact methods and then using heuristics and metaheuristics.

The remaining part of this paper is structured as follows: Section 2 presents a mathematical formulation to the classical VRP. Section 3 focuses on VRP and its major variants. In Section 4 explains VRP taxonomy and its evolution, while in section 5, we describe the used metaheuristics for solving VRP. Finally, the last section presents the main conclusions and identifies some research perspectives.

2. VRP mathematical formulation

As we know that the VRP is a combinatorial optimization problem, so to model it we use graph theory. Here in the following Table 1, we will model only the classical VRP as it is in Tan et al., [1].

Table 1. The mathematical formulation of the classical VRP

Mathematical framework	Let G = (V, A) be a graph, where V = {υ_0, υ_1, υ_2, …, υ_N}, where {υ_1, υ_2, …, υ_N} is the node set representing customers to be served and υ_0 is the depot. Each customer is characterized by a demand D_i. A = {(υ_i ,υ_j): υ_i, υ_j Є V} is the arc set (subscript indicates sequence) linking nodes i and j with a distance d_{ij}. Let M_m = {m_1, m_2,…,m_m} denote the vehicle set, where each vehicle has a maximum load capacity cap_m meaning the total load of vehicle m cannot exceed the maximum load capacity cap_m.
The goal of the VRP	Finding the optimal vehicle routes such that each customer is visited exactly once by one vehicle and each vehicle starts and ends its route at the depot.
The Classical VRP assumptions adopted (in this case 5 assumptions)	a. The depot has a demand equal to zero. b. Each customer location is serviced by only one vehicle. c. Each customer's demand is indivisible. d. Each vehicle shall not exceed its maximum load capacity cap_m. e. Each vehicle starts and ends its route at the depot. f. Customer demand, distribution distances between customers, and delivery costs are known.
Fitness or objective function: (the total cost = the fixed cost + variable cost)	Minimize $\sum_{m=1}^{M}\sum_{i=0}^{N}\sum_{j=0}^{N} X_{ij,m} \times fc_m + \sum_{m=1}^{M}\sum_{i=0}^{N}\sum_{j=0}^{N} d_{ij} \times D_{ij,m} \times vc_m$ (1)
Constraint 1 (Routing): Each vehicle should return to the depot, where the subscript is zero.	$\sum_{i=1}^{N} X_{i0,m} = 1 \forall m \in M_m$ (2)
Constraint 2 (Routing): Each node can only be visited once in a route	$\sum_{m=1}^{M}\sum_{i=1}^{N}\sum_{j=1}^{N} X_{ij,m} = 1 \forall (i,j) \in A \forall m \in M_m$ (3)
Constraint 3 (Routing): If a vehicle arrives at a node, it must leave that node, thereby ensuring route continuity.	$\sum_{m=1}^{M}\sum_{i=1}^{N} X_{ip,m} = \sum_{m=1}^{M}\sum_{i=1}^{N} X_{pi,m} \forall p \in V$ (4)

Constraint 4 and 5 (Demand and capacities): They impose restrictions on the amounts of demand and capacity	$\sum_{i=1}^{N}\sum_{j=1}^{N} X_{ij,m} \times D_j = Dm_{ij,m} \forall (i,j) \in A \forall m \in M_m$ (5)
	$\sum_{i=1}^{N} Dm_{0i,m} \le cap_m \forall m \in M_m$ (6)
Constraint 6 (Demand and capacities): It is about the maximum number of available vehicles	$\sum_{m=1}^{M} X_{ij,m} \le veh_m \forall m \in M_m$ (7)
Notations used in this formulation	▪ V: Node set, where v0 is the depot and $\{\upsilon_1, \upsilon_2, \ldots, \upsilon_N\}$ are customers ▪ i,j: Subscripts of the customer nodes, i, j = 1, 2, . . . N ▪ $A = \{(\upsilon_i, \upsilon_j): \upsilon_i, \upsilon_j \in V\}$ is arcs set linking nodes i and j ▪ M_m: The set of vehicles with m types ▪ D_i : Demand of customer i ▪ d_{ij}: Distance between nodes i and j ▪ veh_m : Maximum available number of each vehicle type ▪ cap_m : Maximum load capacity of vehicle type m ▪ fc_m : Fixed cost of vehicle type m ▪ vc_m : Variable cost of vehicle type m ▪ $Dm_{ij,m}$: Amount carried using vehicle type m from i to j ▪ $X_{ij,m}$: Value of one if vehicle type m travels from node i to j. Otherwise, value of zero

Modeling other VRP variant requires some modifications in the constraints or the fitness function. In the case of multi-objective VRP problem, we optimize many variables simultaneously.

After discovering how to formulate VRP, we will explain how the increase in the VRP variants from the Dantzig et al., [2] to many VRP Variants and the characteristics of each one.

3. VRP and its variants

From the VRP of Dantzig, many variants have appeared for corresponding to the complexity of the supply chain. So, the appearance of these variants of VRP was caused by the addition of constraints to the original VRP problem in order to solve a real-world problem. By reading the literature of VRP we notice that the invention of these new variants of VRP was based on different criteria such as: the capacity of the vehicles [3] (CVRP or Capacitated VRP, HFVRP or heterogeneous fleet VRP), need to return to depot [4] (OVRP or Open VRP), dimensions of VRP [5] (2D-VRP or 3D-VRP), Low capacity because Regulation or others [6] (VRPSF or VRP Satellite facilities, 2EVRP or Two-Echelon VRP, MEVRP or Multi-Echelon VRP), Loading capacity [7] (TTVRP or Truck and Trailer VRP), Priority of customers trust [8] (ConsVRP or consistent VRP), Time interval in which the order must be delivered [9] (VRPTW or VRP with time windows, VRP with soft time windows), Time changes in VRP [10] (DVRP or Dynamic VRP), Customers returning product [11] (VRPB or VRP with backhauls, VRPSPD or VRP with simultaneous pickups and deliveries), Environmental and pollution Regulation [12] (GVRP or Green VRP, PRP or Pollution routing problem, EVRP or Electric VRP, HVRP or Hybrid VRP), Travel Time [13] (TDVRP or Time-Dependent VRP), Randomness in VRP [14] (SVRP: Stochastic VRP), depots Number and collaborative distribution [15], [16] (MDVRP or Multi-Depot VRP, Collaborative VRP), Split Delivery [17] (SVRP: Split Delivery VRP), Routes over a planning horizon [18] (PVRP: Periodic VRP), many VRP variants and constraints at the same time [19] (RVRP or rich VRP), and so on. Using the review articles of Konstantakopoulos et al., [20] and Elshaer et al., [21], We have obtained these figures 1 and 2.

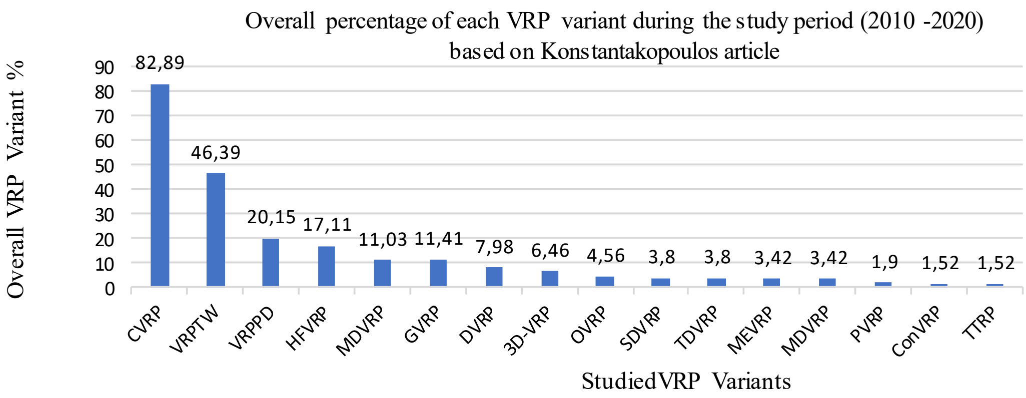

Figure 1. Overall percentage of each VRP variant during the study period (2010 Konstantakopoulos article.

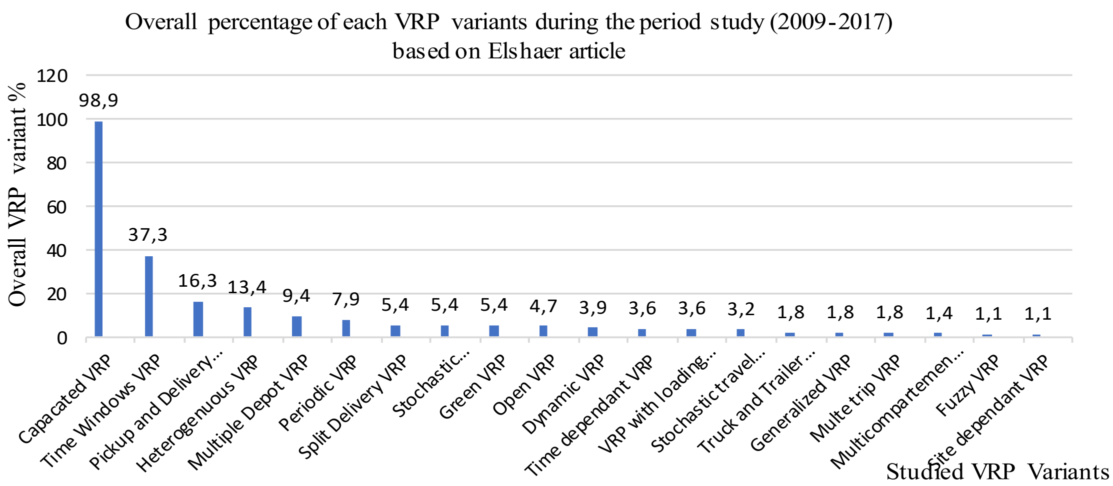

Figure 2. Overall percentage of each VRP variants during the period study (2009-2017) based on Elshaer article.

We notice, based on figure 1 and 2, that: (i) The CVRP is the most studied (82,89% in figure 1 and 98,9 % in figure 2) and the percentage of HFVRP is 17,11% in the figure 1 and it's 13,4 % in the figure 2: We can explain this because CVRP is easier for solving it comparing to HVRP and also for combining it with other variants. We can also justify this result that many companies use the same kind of vehicle for distributing their products. (ii) The percentage of VRPTW is 46.39% in figure 1 and 37,3% in figure 2, so it is almost between half and third of the studied VRP Variants in the two papers: This is probably because it reflects real-life situations. So, the constraint, that the customer must receive the product in a specific time interval, is very important. (iii) The VRPPD attracts the attention of researchers because of its presence in logistics, so it is 20,15 % in the figure 1 and 16,3 % in the figure 2. (iv) The presence of GVRP in research articles will increase thanks to the interest of companies for reducing CO2 emissions recently and the use of electric or hybrid vehicles because of the last negative climatic changes. As we remark that: it is 11,41 % in figure 1 and 5,4 % in figure 2 and this difference maybe because the first study is more recent. (v) There is also a presence, that maybe will increase, of these VRP variants: SVRP, SDVRP, MEVRP. (vi) The less studied VRP variants are ConVRP, TTRP, Fuzzy VRP, Multi-trip VRP, Multicompartment VRP, Generalized VRP and Site dependent VRP. (vii) Both studies have almost the same results about the percentage of presence of each VRP variants in research field. (viii) According to Braekers et al., [22], the VRP variants following: OVRP, DVRP, TDVRP, will become more popular in the next years; especially DVRP because of the evolution of real-time technologies, such as Intelligent Transformation Systems (ITS), Advanced Fleet Management Systems (AFMS) and Global Positioning Systems (GPS). So VRP has many variants that make it a rich research domain with an exponential growth of publications that aggravate this fields for the beginners. How experts will try to organize this area of research?

4. VRP taxonomy and its evolution

Since the first VRP, the research has evolved exponentially because of its importance and its complexity, and also the VRP problem itself has developed into different variants depending on the context of its application. And consequently, the creation of a taxonomy of the VRP has become an obligation for making this subject easy for the understanding, communication, teaching, use, efficient and effective classification of all publications. And also, among its benefits: storing, recalling, sorting, efficient statistical analysis and obtaining classified results that are meaningful and machine-readable, and also allowing the identification of gaps in the literature to exploit them in future research and deepen the understanding of VRP. And even there were some taxonomies before the article of Eksioglu et al., [23], they were insufficient and incomplete.

All these reasons prompted the appearance of Eksioglu's article witch its important contribution was the taxonomy of VRP that will organize this research field. This taxonomy is composed of 5 categories which are: 1. Type of study, 2. Scenario characteristics, 3. Problem physical characteristics, 4. Information characteristics, 5. Data characteristic; and Each category is divided into sub-categories. In 2015, the article Braekers et al., that we have cited before, updates the previous taxonomy of Eksioglu based on VRP articles published between 2009 and 2015 to fill in the gaps of the previous taxonomy. Beside this general VRP taxonomy, the researchers have done some specific taxonomies for the main VRP variants. So next, we will discuss the VRP solving methods especially the metaheuristics and the hybridization either with other metaheuristics or with machine learning.

5. Metaheuristics solving VRP

Firstly, the VRP has been solved using the exact methods based on mathematics. Then the scientists have used heuristic and also metaheuristics [24], [25], [26] based on algorithms and computer sciences that invaded and improved many fields beside optimization for instance [27], [28], [29]. The metaheuristics are divided into local search and population search, and there is an abundance of metaheuristics that still be invented until 2022 by researchers who are inspiring them from many phenomena in life and nature. So, can we have a unified view of these multiple metaheuristics? And even if these metaheuristics have achieved good results in solving combinatorial problems like VRP, researchers continue hybridizing them to benefit from the strengths of each metaheuristic and consequently increase their qualities. Recently, we have started to notice the orientation towards the hybridization of metaheuristics with machine learning in its different levels to create an intelligent and more efficient metaheuristic compared to normal metaheuristics, that it is named machine learning into metaheuristics or learnheuristics: Talbi,

[30], Karimi-Mamaghan et al., [31]. In hybridizing machine learning with metaheuristics, we use a specific method of machine learning in a specific step of designing this metaheuristic.

Based on these papers, that we cited before, and others: Konstantakopoulos, Elshaer, Talbi, Karimi-Mamaghan and also [32], we can conclude that: (i) There is a domination of metaheuristics for solving VRP than heuristics and Exact methods. (ii) Exact methods are not used too much, even they find the optimal solution, because of their limitation in case of VRP with many inputs and also computation complexity. (iii) The local search metaheuristics are most used than the population search metaheuristics, we can explain this result that the local search metaheuristics are faster in finding solutions than the population search. (iv) For local search metaheuristics, the most used are: TS (tabau search) and VNS (Variable neighborhood search); Less often applied are: LNS (large neighborhood search), SA (Simulated annealing method), ILS (Iterated local search) et GRASP (Greedy randomized adaptive search procedure) and rarely used are: GLS (Guided local search). (v) For population search the most used are: GA (Genetic algorithms) and ACO (Ant Colony Optimization) and PSO (Particle Swarm Optimization). (vi) Also, researchers hybridize metaheuristics in order to benefice from the strengths of each one and build more powerful hybrid metaheuristic. Recently, there is a trend toward hybridizing metaheuristics with machine learning for creating an intelligent metaheuristic and import some techniques from machine learning like transfer learning into metaheuristics. (vii) There are a lot of new metaheuristics that still are not used for solving VRP and that we can use them and then compare the obtained results with the previous one.

6. Conclusion and future research

In this paper we have discussed the vehicle routing problem (VRP) and its evolution based on many reviews and other articles done either about it or about metaheuristics solving it. So firstly, we begun by an introduction about the mathematical formulation of VRP. Then, we presented the main VRP variants and the reason of their appearance and the criteria that differentiate between them and also the most and less studied VRP Variants as a guide for the future researchers. Next, we exposed the evolution of the VRP taxonomy and the changes done on it for updating it; we also clarified that this taxonomy give the researchers the tools for organizing and classifying the huge literature related to VRP and it will help them to know the trends of VRP research. Lastly, we demonstrated briefly the relationship between metaheuristics and VRP and we mentioned that the researchers use recently the hybridization in metaheuristics especially the hybridization with machine learning for empowering the metaheuristics, make them smart for broadening our horizon in solving VRP variants even if they are more complex.

For tackling VRP in next years, I suggest that: (i) To try the recent metaheuristics for solving VRP and its variants especially that there is an abundance of metaheuristics, and compare the results. (ii) For searching in VRP, we will focus on VRP variants that are more used VRPTW and VRPPD or new VRP variants that seem to have a brilliant future as Green VRP, EVRP and DVRP and so on. (iii) We can use hybridization of metaheuristics for obtaining a new powerful metaheuristic that can get good solution faster than before. (iv) Also, to hybridize metaheuristics with machine learning by importing some techniques from machine learning such as transfer learning and applying them in many levels in metaheuristics for instance: solution initialization, parameters tuning, problem decomposition, landscape analysis, and so on. (v) Try to apply VRP variants in a context related to Industry 4.0 and Logistics 4.0.

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