

```

import sys
import math
import sympy as sp
import numpy as np
from tabulate import tabulate

```

*#calculate the nth derivative of fx*  
*#returns a list of [fx, fx', fx'', ... , fx^(n)]*

```

def n_deriv(fx, n):
    x = sp.symbols('x')
    f = fx
    deriv_list = [f]
    for i in range(1, n + 1):
        df_i = deriv_list[-1].diff(x).replace(sp.Derivative, lambda *args: f(x))
        deriv_list.append(df_i)
    return deriv_list

```

*#takes a function, bounds, and step size and finds the x value where the function is maximized*

```

def find_max_error(f_n, bounds, step):
    max_y = None
    max_x = None
    x = sp.symbols('x')
    for i in np.arange(bounds[0], bounds[1] + step, step):
        y = abs(f_n.subs(x, i))
        if (max_y is None and max_x is None) or y > max_y:
            max_y = y
            max_x = i
    return max_x

```

*#takes parameters h, n, f, a, b, and a boolean that checks if we want Simpson's method or not*  
*#returns the calculated numerical integral of f*

```

def numerical_integral(h, n, f, a, b, simpsons):
    x = sp.symbols('x')
    integral = 0

```

*#iterate through all points*

```

for i in range(n):
    #calculate xi by taking the start point + step size * current index
    xi = a + i*h
    xi = round(xi, 10)

```

*#checks to see if we want to use Simpson's method or not*

```

if simpsons:
    #i==0 or i==n-1 corresponds to the two end points
    if i == 0 or i == n - 1:
        integral += f.subs(x, xi)
    #multiply the even terms by 2
    elif i%2 == 0:
        integral += 2 * f.subs(x, xi)
    #multiply the odd terms by 4
    else:
        integral += 4 * f.subs(x, xi)

```

*#if not using Simpson's, we are using Trapezoidal method*

```

else:
    #check for end points
    if i == 0 or i == n - 1:
        integral += f.subs(x, xi)
    #multiply non-end points by 2
    else:
        integral += 2 * f.subs(x, xi)

```

```

if simpsons:
    integral *= h/3
else:
    integral *= h/2

```

**return** integral

"""

*defining our initial variables*

"""

x = sp.symbols('x')

*#define our function 1/(1+e^(-3x))*

f = sp.sympify(1/(1+sp.exp(-3\*x)))

tolerance = float(input("Enter max tolerance: "))

a = float(input("a = "))

b = float(input("b = "))

real = sp.integrate(f, (x, a, b))

"""

*finds where the second and fourth derivatives are maximized*

*used for calculating our h-values*

"""

second\_deriv = n\_deriv(f, 2)[-1]

fourth\_deriv = n\_deriv(f, 4)[-1]

second\_deriv\_max = find\_max\_error(second\_deriv, [a, b], 0.01)

second\_deriv\_max = abs(second\_deriv.subs(x, second\_deriv\_max))

fourth\_deriv\_max = find\_max\_error(fourth\_deriv, [a, b], 0.01)

fourth\_deriv\_max = abs(fourth\_deriv.subs(x, fourth\_deriv\_max))

"""

*calculate h-values, intervals, and points using the Composite error terms*

"""

h\_t = ((tolerance\*12)/((b-a)\*second\_deriv\_max))\*\*(1/2)

h\_s = ((tolerance\*180)/((b-a)\*fourth\_deriv\_max))\*\*(1/4)

interval\_t = math.ceil((b-a)/h\_t)

*#simpsons needs an even interval (resulting in odd # of points)*

interval\_s = math.ceil((b-a)/h\_s)

**if** interval\_s%2 != 0:

interval\_s += 1

points\_t = interval\_t+1

points\_s = interval\_s+1

*#check to make sure the math we did above is actually correct and in tolerance*

**assert** second\_deriv\_max\*((b-a)/points\_t)\*\*2\*((b-a)/12) <= tolerance

**assert** fourth\_deriv\_max\*((b-a)/points\_s)\*\*4\*((b-a)/180) <= tolerance

"""

*setting up our method of calculating each integral*

"""

points = [points\_t, points\_s]

simps = [**False**, **True**]

h\_vals = [h\_t, h\_s]

*#for loop that uses either the trapezoid method or simpsons method because I didn't want to copy/paste*

**for** calc\_type **in** range(len(points)):

print("\n===== \nComposite {0}: \n===== ".format("Trapezoidal" **if** calc\_type == 0 **else** "Simpson's"))

print("Required h = {0}".format(h\_vals[calc\_type]))

print("===== ")

*#sets up our n values*

n\_vals = [11, 41, points[calc\_type]]

err\_vals = [0 **for** \_ **in** range(len(n\_vals))]

```

data = [[] for _ in range(len(n_vals))]

#iterates through n values and calculates integral with each n
for i in range(len(n_vals)):
    n = n_vals[i]

    #calculate our h-value for this specific n
    h = ((b-a)/(n-1))

    integral = numerical_integral(h, n, f, a, b, simp[simps_type])

    #calculates absolute and relative error
    abs_err = abs(real - integral)
    rel_err = abs(abs_err/real)*100

    data_row = [n, h, real, integral, abs_err, rel_err, abs_err <= tolerance]
    data[i] = data_row

#prints out table of information
print(tabulate(data, headers=["n",
                             "h",
                             "Real Value",
                             "Calculated Value",
                             "Absolute Error",
                             "Relative Error",
                             "Within Tolerance"]))

print("\n=====")

```