Math 477- Project 8, Numerical Integration Due: May 8, 2022

Project Goal

Your goal is to explore the numerical integration of $g(x) = \frac{1}{1+e^{-3x}}$ on the interval $[-1, 3]$. To achieve this goal you will write one report (submitted as one pdf) that discusses pre-program analysis, provides a pdf of the program, presents post-program results, and provides overall program analysis.

Analysis - 25%

A narrative of the work that you do before you use a computer to optimize the computational program.

1. Calculate the integral $\int_{-1}^{3} g(x)dx$ exactly.
   Hint: Use the substitution $y = e^{-3x}$ and Calculus 'tricks' to integrate rational functions.
2. Analyze the error bound associated with n = 11, n = 41 with the Composite Trapezoid Method. Note: n should be the number of points.
3. Using the formula for the error of the Composite Trapezoidal method, estimate very roughly how small the subinterval h should be to give an absolute error less than 10^{-4}. Then convert that to the number of points you'll need.
4. Analyze the error bound associated with n = 11, n = 41 with the Composite Simpson's Method.
5. Using the formula for the error of the Composite Simpson's method, estimate very roughly how small the subinterval h should be to give an absolute error less than 10^{-4}. Then convert that to the number of points you'll need.

Analyze which of these expressions can be expected to give the best approximation. Explain why.
Also, make a prediction about the approximations based on your analysis above. To receive full credit for this section your discussion comparing and selecting a best method should be at least one paragraph.

Computer Program - 25%

A computational algorithm, setup in Mathematica or any other program.

1. Write a computer code implementing the Composite Trapezoidal method and use it to compute the integral numerically with $n = 11$, $n = 41$, and your h from the Analysis section.
   In this problem n = number of points in your interval, $[-1, 3]$, not the number of sub-intervals in $[-1, 3]$.
2. Write a computer code implementing the Composite Simpson's method and use it to compute the integral numerically with $n = 11$, $n = 41$,and your h from the Analysis section.

To receive credit for this section provide a pdf of your commented program code, appended to the end of this report.

Results- 25%

Narrative that compares the results and errors for these approximations. Use the program that you wrote to approximate the integral of $g(x) = \frac{1}{1+e^{-3x}}$ and associated error. Compare the results and errors for these methods:

1. Discuss results:
   - Make a table highlighting the results at different n's for the different approximations (n=11, n=41, n so that the approximation is guaranteed to be within the tolerance 10^{-4}).
2. Discuss error:
   - Make a table highlighting the the error results at different n's for the approximations (n=11, n=41, n so that the approximation is guaranteed to be within the tolerance 10^{-4}).
   - Discuss how your error: What is the actual error? How does it compare with the analytical estimate?

Discuss your results. In particular, which of the methods give the *best* approximation to the integral? Discuss advantages and flaws of the methods used. Discuss anything that you notice from this set of results.

Style- 10%

Refer to the project format notes and the syllabus to ensure you receive full credit for this assignment.

Post Assignment Review and Reflection- 15%

For the post assignment review and reflection you should first review the comments from the instructor, fix any mathematical errors and review potential corrections in grammar and phrasing. Then you should reflect on what you learned. This reflection should be a minimum of 200 words and include:

- What changes you made when updating your paper based on the comments from the instructor.
- What you learned in making these changes.
- What you learned in doing this project.

The post assignment review and reflection will be due 1 week after the assignment is returned. Please submit the updated project paper and the 200 word reflection.

Notes from previous years

- Be careful with your notation/variable convention. If you use n to be the number of points then don't also use n to the the number of intervals. Be consistent.
- Dr. Brandy always wants references
- Be clear when you're discussing error vs error bound
- ADD TITLES or (*Comments*) to explain graphs.
- Graph errors as individual points (vs continuous graph)