Lecture - Spatial Surfaces

Miguel Angel Avila Torres -

Ellipse

$$\boxed{\left(\frac{x-h}{a}\right)^2+\left(\frac{y-k}{b}\right)^2=1} \tag{1}$$

with center in $C=(h,k)$

Parabola

$$\boxed{(x-h)^2=4P(y-k)} \tag{2}$$

$$x^2-2hx+h^2=4Py-4Pk \tag{3}$$

again, with center in $C=(h,k)$ and P indicates the positive or negative axis which is taken, being P simultaneously the focus.

Hyperbola

$$\boxed{\left(\frac{x-h}{a}\right)^2-\left(\frac{y-k}{b}\right)^2=1} \tag{4}$$

Elements:

$$F_x(h\pm c,k) \qquad F_y(h,\pm c+k) \tag{5}$$

$$v(h\pm a,k) \qquad C(h,k) \tag{6}$$

$$c=\sqrt{b^2-a^2} \qquad e=\frac{c}{a} \tag{7}$$

Being c the length, from the center C to the focuses on x and y, being e the eccentricity and finally v the vertices.

Ellipsoid

$$\boxed{\left(\frac{x-x_0}{a}\right)^2+\left(\frac{y-y_0}{b}\right)^2+\left(\frac{z-z_0}{c}\right)^2=1} \tag{8}$$

Spherical Equivalence:

$$\begin{cases} x=a\sin\theta\cos\phi \\ y=b\sin\theta\sin\phi \quad 0\le\theta\le\pi\wedge 0\le\phi<2\pi \\ z=c\cos\theta \end{cases} \tag{9}$$

Hyperboloid of a Sheet

$$\boxed{\left(\frac{x-x_0}{a}\right)^2+\left(\frac{y-y_0}{b}\right)^2-\left(\frac{z-z_0}{c}\right)^2=1} \tag{10}$$

Spherical Equivalence:

$$\begin{cases} x=a\cosh\theta\cos\phi \\ y=b\cosh\theta\sin\phi \quad \theta\in\mathbb{R}\wedge 0<\phi\le 2\pi \\ z=c\sinh\phi \end{cases} \tag{11}$$

Hyperboloid of two Sheets

$$\boxed{\left(\frac{x-x_0}{a}\right)^2+\left(\frac{y-y_0}{b}\right)^2-\left(\frac{z-z_0}{c}\right)^2=-1} \tag{12}$$

Spherical Equivalence:

$$\begin{cases} x=a\sinh\theta\cos\phi \\ y=b\sinh\theta\sin\phi \quad \theta\in\mathbb{R}\wedge 0<\phi\le 2\pi \\ z=c\cosh\phi \end{cases} \tag{13}$$

Elliptic Cone

$$\boxed{\left(\frac{x-x_0}{a}\right)^2+\left(\frac{y-y_0}{b}\right)^2-\left(\frac{z-z_0}{c}\right)^2=0} \tag{14}$$

Elliptical Paraboloid

$$\boxed{z=\left(\frac{x-x_0}{a}\right)^2+\left(\frac{y-y_0}{b}\right)^2} \tag{15}$$

Hyperbolic Paraboloid

$$\boxed{z=\left(\frac{y}{b}\right)^2-\left(\frac{x}{a}\right)^2} \tag{16}$$

Cut Planes

For the main planes

$$x=0\Longrightarrow z=\frac{y^2}{b^2} \tag{17}$$

$$y=0\Longrightarrow z=-\frac{x^2}{a^2} \tag{18}$$

For an arbitrary plane

$$z=k\Longrightarrow 1=\left(\frac{y}{\sqrt{k}b}\right)^2-\left(\frac{x}{\sqrt{k}a}\right)^2 \tag{19}$$