**Exponential Functions**

**Introduction**

The exponential function f(x)= a(bx) is a mathematical function that grows or decays exponentially with respect to x. More sophisticated decay or growth functions can have additional constants and constraints. This is widely used in fields such as physics, economics, and engineering, where exponential growth and decay phenomenon are common.

**Motivation**

The exponential function is motivated by the need to model growth and decay in instances where such phenomenon cannot be modelled by linear functions. For example, the spread of a virus, the decay of a radioactive substance, and the growth of a population do not scale linearly with respect to time. The constant a usually represent an initial state, b represents the rate of growth and is called the growth factor, and x denotes time.

**Formal Definition**

f(x)=a(bx) where the domain is {x |x ∈ ℝ}, a ≠ 0, b ≠ 1, b > 0 and a, b ∈ ℝ.   
If a > 0 then the range is (0, ∞).   
If a < 0, then the range is (-∞,0)  
  
**Graphical Representation**

Chart, line chart

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a > 0 and 0 < b < 1 a < 0 and 0 < b < 1

**Chart, line chart

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a > 0 and b > 1 a < 0 and b > 1

**Application**

The exponential function has many applications in various fields. Some examples include:

Biology: Modelling the growth of populations of bacteria, viruses, and other organisms.

Economics: Used in compound interest calculations to model the growth of investments and savings accounts.

Physics: Modelling the decay of radioactive substances and the growth and decay of electromagnetic waves.

Engineering: Used in signal processing to model the growth and decay of signals over time.

A concrete example of an exponential function is as follows: Suppose a given bacteria population doubles every hour, and the initial number of bacteria is 200, then the number of bacteria after x hours is given by the function f(x)=200(2x).

**References**

<https://hal.science/hal-03845390/document>