2023年度 線形代数学1 課題 13

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問 1. 次の行列の行列式と余因子行列を求め, 正則ならば逆行列を求めよ.

$$A = \begin{bmatrix} 7 & 3 & 5 \\ 5 & 7 & 3 \\ 3 & 5 & 7 \end{bmatrix}$$

$$|A| = 7\cdot 7\cdot 7 + 3\cdot 3\cdot 3 + 5\cdot 5\cdot 5 - (3\cdot 7\cdot 5 + 3\cdot 5\cdot 7 + 7\cdot 3\cdot 5)$$

$$= 495 - 315$$

$$= \underline{180} \neq 0$$

∴ Aは正則行列

$$a_{11}^* = (-1)^{1+1}|A_{11}| = \begin{vmatrix} 7 & 3 \\ 5 & 7 \end{vmatrix} = 49 - 15 = 34$$

$$a_{12}^* = (-1)^{1+2}|A_{21}| = -\begin{vmatrix} 3 & 5 \\ 5 & 7 \end{vmatrix} = -(21 - 25) = 4$$

$$a_{13}^* = (-1)^{1+3}|A_{31}| = \begin{vmatrix} 3 & 5 \\ 7 & 3 \end{vmatrix} = 9 - 35 = -26$$

$$a_{21}^* = (-1)^{2+1}|A_{12}| = -\begin{vmatrix} 5 & 3 \\ 3 & 7 \end{vmatrix} = -(35 - 9) = -26$$

$$a_{22}^* = (-1)^{2+2}|A_{22}| = \begin{vmatrix} 7 & 5 \\ 3 & 7 \end{vmatrix} = 49 - 15 = 34$$

$$a_{23}^* = (-1)^{2+3}|A_{32}| = -\begin{vmatrix} 7 & 5 \\ 5 & 3 \end{vmatrix} = -(21 - 25) = 4$$

$$a_{31}^* = (-1)^{3+1}|A_{13}| = \begin{vmatrix} 5 & 7 \\ 3 & 5 \end{vmatrix} = 25 - 21 = 4$$

$$a_{32}^* = (-1)^{3+2}|A_{23}| = -\begin{vmatrix} 7 & 3 \\ 3 & 5 \end{vmatrix} = -(35 - 9) = -26$$

$$a_{33}^* = (-1)^{3+3}|A_{33}| = \begin{vmatrix} 7 & 3 \\ 5 & 7 \end{vmatrix} = 49 - 15 = 34$$

$$\widetilde{A} = \begin{bmatrix} 34 & 4 & -26 \\ -26 & 34 & 4 \\ 4 & -26 & 34 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det A}\widetilde{A} = \frac{1}{|A|}\widetilde{A} = \frac{1}{180}\begin{bmatrix} 34 & 4 & -26 \\ -26 & 34 & 4 \\ 4 & -26 & 34 \end{bmatrix}$$