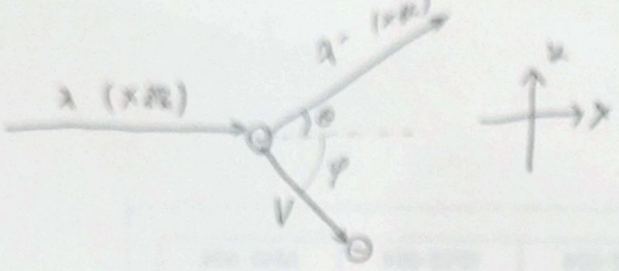
$\Delta\lambda = \lambda' - \lambda = \frac{h}{mc}(1-\cos\theta)$ を示す

$\left(\frac{1}{1+x} \cong 1-x \ (x \ll 1)\right)$



フォトンの運動量

$P = \frac{h}{\lambda}$

エネルギー保存則 $\frac{hc}{\lambda} = \frac{hc}{\lambda'} + \frac{1}{2}mV^2$ … ①

運動量保存則 x成分: $\frac{h}{\lambda} = \frac{h}{\lambda'}\cos\theta + mV\cos\varphi$ … ②

y成分: $0 = \frac{h}{\lambda'}\sin\theta - mV\sin\varphi$ … ③

③より

$mV\sin\varphi = \frac{h}{\lambda'}\sin\theta$ … ④

②より

$mV\cos\varphi = \frac{h}{\lambda} - \frac{h}{\lambda'}\cos\theta$ … ⑤

④^2 + ⑤^2 より

$m^2V^2 = \frac{h^2}{\lambda'^2}\sin^2\theta + \left(\frac{h}{\lambda} - \frac{h}{\lambda'}\cos\theta\right)^2$

$= \frac{h^2}{\lambda'^2}\sin^2\theta + \frac{h^2}{\lambda^2} - \frac{2h^2}{\lambda\lambda'}\cos\theta + \frac{h^2}{\lambda'^2}\cos^2\theta$

$= \frac{h^2}{\lambda'^2} + \frac{h^2}{\lambda^2} - \frac{2h^2}{\lambda\lambda'}\cos\theta$ … ⑥

①と⑥より

$\frac{hc}{\lambda} = \frac{hc}{\lambda'} + \frac{1}{2m}\left(\frac{h^2}{\lambda'^2} + \frac{h^2}{\lambda^2} - \frac{2h^2}{\lambda\lambda'}\cos\theta\right)$

$\Leftrightarrow hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = \frac{1}{2m}\left(\frac{h^2}{\lambda'^2} + \frac{h^2}{\lambda^2} - \frac{2h^2}{\lambda\lambda'}\cos\theta\right)$

$\Leftrightarrow hc\left(\frac{\lambda'-\lambda}{\lambda\lambda'}\right) = \frac{1}{2m}\left(\frac{h^2\lambda^2 + h^2\lambda'^2 - 2h^2\lambda\lambda'\cos\theta}{\lambda^2\lambda'^2}\right)$

$\Leftrightarrow hc(\lambda'-\lambda) = \frac{1}{2m}\left(\frac{h^2\lambda^2 + h^2\lambda'^2 - 2h^2\lambda\lambda'\cos\theta}{\lambda\lambda'}\right)$

$\Leftrightarrow hc(\lambda'-\lambda) = \frac{1}{2m}\left(h^2\frac{\lambda}{\lambda'} + h^2\frac{\lambda'}{\lambda} - 2h^2\cos\theta\right)$

$\Leftrightarrow \lambda'-\lambda = \frac{1}{2mhc}\left\{h^2\left(\frac{\lambda^2+\lambda'^2}{\lambda\lambda'}\right) - 2h^2\cos\theta\right)$

$\Leftrightarrow \lambda'-\lambda = \frac{h}{2mc}\left(\frac{1}{1+\frac{\lambda'-\lambda}{\lambda}} + \frac{1}{1+\frac{\lambda-\lambda'}{\lambda}} - 2\cos\theta\right)$

$\fallingdotseq \frac{h}{2mc}\left\{\left(1-\frac{\lambda'-\lambda}{\lambda}\right) + \left(1-\frac{\lambda-\lambda'}{\lambda'}\right) - 2\cos\theta\right\} \ \left(\because \frac{\Delta\lambda}{\lambda} \ll 1\right)$

$\fallingdotseq \frac{h}{2mc}(2 - 2\cos\theta)$

$= \frac{h}{mc}(1-\cos\theta)$