第3回 課題　　　　8222124　柳下恭輔

エネルギー保存則

$$\frac{hc}{\lambda} = \frac{hc}{\lambda'} + \frac{1}{2}mv^2 \cdots ①$$

運動量保存則

$$\frac{h}{\lambda} = \frac{h}{\lambda'}\cos\theta + mv\cos\phi \cdots ②$$

$$0 = \frac{h}{\lambda'}\sin\theta - mv\sin\phi \cdots ③$$

$$① \iff m^2v^2 = 2mhc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) \cdots ④$$

$$② \iff h\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\cos\theta\right) = mv\cos\phi \cdots ⑤$$

$$③ \iff \frac{h}{\lambda'}\sin\theta = mv\sin\phi \cdots ⑥$$

⑤^2 + ⑥^2 より

$$m^2v^2(\sin^2\phi + \cos^2\phi) = h^2\left(\frac{1}{\lambda} - \frac{\cos\theta}{\lambda'}\right)^2 + \left(\frac{h}{\lambda'}\sin\theta\right)^2$$

$\sin^2\phi + \cos^2\phi = 1$, ④ より

$$2mhc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = h^2\left(\frac{1}{\lambda^2} - \frac{2\cos\theta}{\lambda\lambda'} + \frac{\cos^2\theta}{\lambda'^2} + \frac{\sin^2\theta}{\lambda'^2}\right)$$

$$2mc\,\frac{\lambda' - \lambda}{\lambda\lambda'} = h \cdot \frac{\lambda'^2 - 2\lambda\lambda'\cos\theta + \lambda^2}{\lambda^2\lambda'^2}$$

$$2mc(\lambda' - \lambda) = \frac{h}{\lambda\lambda'}(\lambda'^2 - 2\lambda\lambda'\cos\theta + \lambda^2)$$

$$\lambda' - \lambda = \frac{h}{2mc}\left(\frac{\lambda'}{\lambda} - 2\cos\theta + \frac{\lambda}{\lambda'}\right) \cdots ⑦$$

ここで $\frac{\lambda}{\lambda'}$ について

$$\frac{\lambda}{\lambda'} = \frac{\lambda}{\lambda + \lambda' - \lambda} = \frac{1}{1 + \frac{\lambda' - \lambda}{\lambda}}$$

$\frac{\lambda' - \lambda}{\lambda} \ll 1$ より $\frac{1}{1+x} \cong 1 - x$ $(x \ll 1)$ を用いると

$$\frac{\lambda}{\lambda'} = 1 - \frac{\lambda' - \lambda}{\lambda} \quad \cdots ⑧$$

⑧を⑦に代入すると

$$\lambda' - \lambda = \frac{h}{2mc}\left(\frac{\lambda'}{\lambda} + 1 - \frac{\lambda'}{\lambda} + 1 - 2\cos\theta\right)$$

$$= \frac{h}{mc}(1 - \cos\theta) \quad //$$