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量子力学 レポート課題

1. 長さLの無限に深い井戸型ポテンシャル中の粒子

$\Psi_n(x) = C\sin\left(\frac{n\pi x}{L}\right) \quad (n = 1, 2, 3, \cdots)$

$2\sin^2\theta = 1 - \cos 2\theta$

$\sin^2\theta =$

存在確率 $P(x) = |\Psi_n(x)|^2 = C^2\sin^2\left(\frac{n\pi x}{L}\right)$

$\cos 2\theta = 1 - 2\sin^2\theta$

$\int_0^L P(x)\,dx = 1$ となる

$$\int_0^L P(x)\,dx = \int_0^L C^2\sin^2\left(\frac{n\pi x}{L}\right)dx = \int_0^L C^2 \cdot \frac{1-\cos\left(\frac{2n\pi x}{L}\right)}{2}dx$$

$$= \int_0^L \left(\frac{1}{2}C^2 - \frac{1}{2}C^2\cos\left(\frac{2n\pi x}{L}\right)\right)dx$$

$$= \left[\frac{1}{2}C^2 x - \frac{1}{2}\cdot C^2\cdot\frac{L}{2n\pi}\cdot\sin\left(\frac{2n\pi x}{L}\right)\right]_0^L$$

$$= \frac{1}{2}C^2 L - \frac{C^2 L}{4n\pi}\sin(2n\pi)$$

$$= \frac{1}{2}C^2 L$$

$\frac{1}{2}C^2 L = 1$

$C^2 = \frac{2}{L}$

$C = \pm\sqrt{\frac{2}{L}}$

$\cos\left(\frac{2n\pi}{L}x\right) + i\sin\left(\frac{2n\pi}{L}x\right)$

2. 長さLのリング上の粒子

$\Psi_n(x) = Ce^{i\frac{2n\pi}{L}x} \quad (n = 0, \pm 1, \pm 2, \pm 3, \cdots)$

存在確率 $P(x) = |\Psi_n(x)|^2 = C^2\left\{\cos\left(\frac{2n\pi}{L}x\right) + i\sin\left(\frac{2n\pi}{L}x\right)\right\}\left\{\cos\left(\frac{2n\pi}{L}x\right) - i\sin\left(\frac{2n\pi}{L}x\right)\right\}$

$$= C^2\left\{\cos^2\left(\frac{2n\pi}{L}x\right) + \sin^2\left(\frac{2n\pi}{L}x\right)\right\}$$

$$= C^2$$

$\int_0^L P(x)\,dx = 1$ となる.

$$\int_0^L P(x)\,dx = \int_0^L C^2\,dx = \left[C^2 x\right]_0^L = C^2 L$$

$C^2 L = 1$

$C = \pm\sqrt{\frac{1}{L}}$