

$$6x+21 - 6x + 22$$

例 4.1

$$\begin{aligned} (1) \quad \left( \arctan \sqrt{\frac{3x-11}{2x+7}} \right)' &= \frac{1}{1 + \frac{3x-11}{2x+7}} \cdot \left( \left( \frac{3x-11}{2x+7} \right)^{\frac{1}{2}} \right)' \\ &= \frac{2x+7}{5x-4} \cdot \left( \frac{1}{2} \left( \frac{3x-11}{2x+7} \right)^{-\frac{1}{2}} \cdot \frac{3(2x+7) - (3x-11) \cdot 2}{(2x+7)^2} \right) \\ &= \frac{2x+7}{5x-4} \cdot \left( \frac{1}{2} \sqrt{\frac{2x+7}{3x-11}} \cdot \frac{43}{(2x+7)^2} \right) \\ &= \frac{2x+7}{5x-4} \cdot \frac{1}{2} \sqrt{\frac{2x+7}{3x-11}} \cdot \frac{43}{(2x+7)^2} \\ &= \frac{43}{2(5x-4)\sqrt{3x-11}\sqrt{2x+7}} = \frac{43}{2(5x-4)(2x+7)} \sqrt{\frac{2x+7}{3x-11}} \end{aligned}$$

$$\begin{aligned} (2) \quad \left( \arccos \sqrt{1-9x^2} \right)' &= - \frac{1}{\sqrt{1 - (\sqrt{1-9x^2})^2}} \cdot \left( \sqrt{1-9x^2} \right)' \\ &= - \frac{1}{\sqrt{1 - (1-9x^2)}} \cdot \frac{1}{2\sqrt{1-9x^2}} \cdot (-18x) \\ &= - \frac{1}{2|x|} \cdot \frac{1}{2\sqrt{1-9x^2}} \cdot (-18x) \\ &= \frac{9x}{|x|\sqrt{1-9x^2}} \end{aligned}$$

问题

$$\begin{aligned} (1) \quad \left( \arctan \frac{11x-9}{9x+11} \right)' &= \frac{1}{1 + \left( \frac{11x-9}{9x+11} \right)^2} \cdot \left( \frac{11x-9}{9x+11} \right)' \\ &= \frac{(9x+11)^2}{(9x+11)^2 + (11x-9)^2} \cdot \frac{11(9x+11) - (11x-9) \cdot 9}{(9x+11)^2} \\ &= \frac{202}{(9x+11)^2 + (11x-9)^2} = \frac{1}{x^2+1} \end{aligned}$$

$$\begin{aligned} (2) \quad \left( \arctan \sqrt{\frac{13x-3}{8x+5}} \right)' &= \frac{1}{1 + \frac{13x-3}{8x+5}} \cdot \left( \sqrt{\frac{13x-3}{8x+5}} \right)' \\ &= \frac{8x+5}{8x+5 + 13x-3} \cdot \frac{1}{2\sqrt{\frac{13x-3}{8x+5}}} \cdot \frac{17(8x+5) - (13x-3) \cdot 8}{(8x+5)^2} \\ &= \frac{8x+5}{21x+2} \cdot \frac{1}{2\sqrt{\frac{8x+5}{13x-3}}} \cdot \frac{89}{(8x+5)^2} \\ &= \frac{89}{2(21x+2)(8x+5)} \sqrt{\frac{8x+5}{13x-3}} \end{aligned}$$

$$\begin{aligned} (3) \quad \left( \arcsin \sqrt{1-7x^2} \right)' &= \frac{1}{\sqrt{1 - (1-7x^2)}} \cdot \left( \sqrt{1-7x^2} \right)' \\ &= \frac{1}{\sqrt{7x^2}} \cdot \frac{-14x}{2\sqrt{1-7x^2}} \\ &= \frac{1}{\sqrt{7}|x|} \cdot - \frac{7x}{\sqrt{1-7x^2}} \\ &= - \frac{\sqrt{7}x}{|x|\sqrt{1-7x^2}} \end{aligned}$$

$$\begin{aligned} (4) \quad \left( \arccos \frac{5}{\sqrt{x^2+25}} \right)' &= - \frac{1}{\sqrt{1 - \frac{25}{x^2+25}}} \cdot \left( \frac{5}{\sqrt{x^2+25}} \right)' \\ &= - \frac{1}{\sqrt{\frac{x^2}{x^2+25}}} \cdot \frac{5 \cdot \frac{1}{2\sqrt{x^2+25}} \cdot 2x}{x^2+25} \\ &= - \frac{1}{\sqrt{\frac{x^2}{x^2+25}}} \cdot \frac{5x}{x^2+25\sqrt{x^2+25}} \\ &= \frac{5x}{|x|(x^2+25)} \end{aligned}$$

例

$$\begin{aligned} (3) \quad \left( \arcsin \frac{1}{\sqrt{4x^2+1}} \right)' &= \frac{1}{\sqrt{1 - \frac{1}{4x^2+1}}} \cdot \left( \frac{1}{\sqrt{4x^2+1}} \right)' \\ &= \frac{1}{\sqrt{\frac{4x^2}{4x^2+1}}} \cdot \frac{1}{2\sqrt{4x^2+1}} \cdot (-8x) \\ &= - \frac{1}{\sqrt{\frac{4x^2}{4x^2+1}}} \cdot \frac{8x}{2(4x^2+1)\sqrt{4x^2+1}} \\ &= - \frac{4x}{2|x|(4x^2+1)} \\ &= - \frac{2x}{|x|(4x^2+1)} \end{aligned}$$