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応用数学1. 第7講課題

演習

(1)

$$2xy\frac{dy}{dx} = x^2+y^2$$

$$\frac{dy}{dx} = \frac{x^2+y^2}{2xy}$$

$$= \frac{1+\left(\frac{y}{x}\right)^2}{2\left(\frac{y}{x}\right)}$$

$$\frac{y}{x} = u \rightarrow y = ux \rightarrow \frac{dy}{dx} = u + x\frac{du}{dx}$$

$$= \frac{1+u^2}{2u}$$

$$u + x\frac{du}{dx} = \frac{1+u^2}{2u}$$

$$x\frac{du}{dx} = \frac{1-u^2}{2u}$$

$$\frac{2u}{1-u^2}du = \frac{1}{x}dx$$

$$\int \frac{2u}{1-u^2}du = \int \frac{1}{x}dx$$

$$\Leftrightarrow -\log|1-u^2| + C_1 = \log|x| \quad (C_1 \text{は積分定数})$$

$$\Leftrightarrow \log|x(1-u^2)| = C_1$$

$$x(1-u^2) = \pm e^{C_1}$$

$$\downarrow u = \frac{y}{x}$$

$$x\left(1-\frac{y^2}{x^2}\right) = \pm e^{C_1}$$

$$x - \frac{y^2}{x} = C_2 \quad (C_2 = \pm e^{C_1},\ C_2 \neq 0)$$

$$x^2 - y^2 = C_2 x$$

$$\left(x-\frac{C_2}{2}\right)^2 - y^2 = \left(\frac{C_2}{4}\right)^2$$

$$(x-c)^2 - y^2 = c^2 \quad \left(c = \frac{C_2}{2},\ c \neq 0\right)$$

一般解

特異解　$y = mx$ について：

$$\frac{dy}{dx} = m = \frac{1+m^2}{2m}$$

$$2m^2 = 1+m^2$$

$$m^2 = 1$$

$m = \pm 1$ より 実数 $m = \pm 1$ となるので特異解が存在する

特異解 $y = \pm x$

(3)

$$(x+y)\frac{dy}{dx} = x - y$$

$$\frac{dy}{dx} = \frac{x-y}{x+y} = \frac{1-\frac{y}{x}}{1+\frac{y}{x}}$$

(一般解)　$\frac{y}{x} = u$;　$y = ux$　$\frac{dy}{dx} = u + x\frac{du}{dx} = \frac{1-u}{1+u}$

$$u + x\frac{du}{dx} = \frac{1-u}{1+u}$$

$$x\frac{du}{dx} = \frac{1-u}{1+u} - u$$

$$\Leftrightarrow x\frac{du}{dx} = \frac{1-u-u(1+u)}{1+u}$$

$$\Leftrightarrow x\frac{du}{dx} = \frac{1-2u-u^2}{1+u}$$

$$\Leftrightarrow \frac{1+u}{1-2u-u^2}du = \frac{1}{x}\cdot dx$$

$$\Leftrightarrow \int \frac{1+u}{1-2u-u^2}du = \int \frac{1}{x}dx$$

$$\Leftrightarrow -\frac{1}{2}\log|1-2u-u^2| + C_1 = \log|x| \quad (C_1 \text{ は積分定数})$$

$$\Leftrightarrow \log\left|x\sqrt{1-2u-u^2}\right| = C_1$$

$$\Leftrightarrow x\sqrt{1-2u-u^2} = \pm e^{C_1}$$

$$\Leftrightarrow x\sqrt{1-2u-u^2} = C_2 \quad (C_2 = \pm e^{C_1},\ C_2 \neq 0)$$

両辺を2乗すると

$$x^2(1-2u-u^2) = C_2^2$$

$$\downarrow u = \frac{y}{x}$$

$$\Leftrightarrow x^2\left(1-2\left(\frac{y}{x}\right)-\frac{y^2}{x^2}\right) = C_2^2$$

$$x^2 - 2xy - y^2 = C_2^2$$

$$(x-y)^2 = C \quad (C = C_2^2,\ C > 0)$$

一般解

特異解　$y = mx$ について：

$$\frac{dy}{dx} = m = \frac{1-m}{1+m}$$

$$\Leftrightarrow \quad m + m^2 = 1 - m$$

$$m^2 + 2m - 1 = 0$$

$$m = -1 \pm \sqrt{1+1}$$

$$= -1 \pm \sqrt{2} \text{ より}$$

m は実数となるので特異解が存在する

これより特異解　$y = (-1 \pm \sqrt{2})x$

(4)

$$(2x+y) + (x+2y)\frac{dy}{dx} = 0$$

$$(x+2y)\frac{dy}{dx} = -(2x+y)$$

$$\frac{dy}{dx} = -\frac{2x+y}{x+2y}$$

$$= -\frac{2+\frac{y}{x}}{1+2\frac{y}{x}}$$

$$\frac{y}{x} = u, \quad y = ux \quad \frac{dy}{dx} = u + x\frac{du}{dx}$$

$$= -\frac{2+u}{1+2u}$$

$$u + x\frac{du}{dx} = -\frac{2+u}{1+2u}$$

$$x\frac{du}{dx} = -\frac{2+u}{1+2u} - u$$

$$x\frac{du}{dx} = \frac{-2u^2-2u-2}{1+2u}$$

$$\Leftrightarrow \quad \frac{1+2u}{-2u^2-2u-2}\,du = \frac{1}{x}\,dx$$

$$\Leftrightarrow \quad \int \frac{1+2u}{-2u^2-2u-2}\,du = \int \frac{1}{x}\,dx$$

$$-\frac{1}{2}\times\log|-2u^2-2u-2| + C_1 = \log|x| \quad (C_1\text{は積分定数})$$

$$\Leftrightarrow \log\left|x\sqrt{-2u^2-2u-2}\right| = C_1$$

$$\Leftrightarrow x\sqrt{-2u^2-2u-2} = \pm e^{C_1}$$

$$\Leftrightarrow x\sqrt{-2u^2-2u-2} = C_2 \quad (C_2 = \pm e^{C_1},\ C_2 \neq 0)$$

$$\Leftrightarrow x^2(-2u^2-2u-2) = C_2^2$$

$$\downarrow\ u = \frac{y}{x}$$

$$\Leftrightarrow x^2\left(-2\frac{y^2}{x^2} - 2\frac{y}{x} - 2\right) = C_2^2$$

$$\Leftrightarrow -2y^2 - 2\frac{y}{x} - 2x^2 = C_2^2$$

$$\Leftrightarrow y^2 + \frac{y}{x} + x^2 = -\frac{1}{2}C_2^2$$

$$\Leftrightarrow y^2 + \frac{y}{x} + x^2 = C \quad (\cancel{C = -\tfrac{1}{2}C_2^2,\ C<0})$$

$$\Leftrightarrow \underline{xy^2 + y + x^3 = xC \quad (C = -\tfrac{1}{2}C_2^2,\ C<0)}$$

一般解

一般解

$y = mx$ について

$$m = -\frac{2+m}{1+2m}$$

$$m + 2m^2 = -2 - m$$

$$2m^2 + 2m + 2 = 0$$

$$m^2 + m + 1 = 0$$

$$m = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{3}i}{2}$$

実数mは存在しないので

特異解はない