

## &lt; 運動エネルギーの導出 &gt;

$$\begin{aligned} \left\langle \frac{p^2}{2m} \right\rangle &= \int_{-\infty}^{\infty} \frac{p^2}{2m} P(p) dp \\ &= \frac{1}{2m} \int_{-\infty}^{\infty} p^2 \cdot C e^{-\frac{1}{2m k_B T} p^2} dp \quad \dots \textcircled{1} \end{aligned}$$

(ただし  $C$  は比例定数)

$$\text{ここを } \frac{1}{2m k_B T} = a \quad \dots \textcircled{\star} \quad \text{とおく}$$

$$\begin{aligned} \textcircled{1} &= \frac{C}{2m} \int_{-\infty}^{\infty} p^2 e^{-ap^2} dp \\ &= \frac{C}{2m} \frac{1}{2a} \sqrt{\frac{\pi}{a}} \\ &= \frac{C}{4ma} \sqrt{\frac{\pi}{a}} \quad \dots \textcircled{2} \end{aligned}$$

次に  $C$  を求める

$$-\infty \leq p \leq \infty \text{ かつ}$$

$$\int_{-\infty}^{\infty} P(p) dp = 1$$

$$\therefore C \int_{-\infty}^{\infty} e^{-ap^2} dp = 1$$

$$\begin{aligned} \therefore C &= \frac{1}{\int_{-\infty}^{\infty} e^{-ap^2} dp} \\ &= \sqrt{\frac{a}{\pi}} \quad \dots \textcircled{3} \end{aligned}$$

$$\begin{aligned} \textcircled{2} &= \frac{1}{4ma} \sqrt{\frac{\pi}{a}} \cdot \sqrt{\frac{a}{\pi}} \quad (\because \textcircled{3}) \\ &= \frac{k_B T}{2} \quad (\because \textcircled{4}) \\ &\quad \neq \end{aligned}$$

< 位置の導出 >

$$\begin{aligned} \left\langle \frac{1}{2} k x^2 \right\rangle &= \int_{-\infty}^{\infty} \frac{1}{2} k x^2 P(x) dx \\ &= \frac{k}{2} \int_{-\infty}^{\infty} x^2 \cdot D e^{-\frac{E_p}{k_B T}} dx \end{aligned}$$

(ただし  $D$  は比例定数)

$$= \frac{Dk}{2} \int_{-\infty}^{\infty} x^2 \cdot e^{-\frac{k}{2k_B T} x^2} dx \quad \dots \textcircled{4}$$

$$\therefore \frac{k}{2k_B T} = b \quad \dots \textcircled{5} \quad \text{とおく}$$

$$\begin{aligned} \textcircled{4} &= \frac{Dk}{2} \int_{-\infty}^{\infty} x^2 \cdot e^{-bx^2} dx \\ &= \frac{Dk}{2} \cdot \frac{1}{2b} \sqrt{\frac{\pi}{b}} \quad \dots \textcircled{5} \end{aligned}$$

$D$  を  $b$  と同様に求める

$$D = \sqrt{\frac{b}{\pi}} \quad \dots \textcircled{6}$$

$$\begin{aligned} \textcircled{5} &= \frac{k}{4b} \cdot \sqrt{\frac{\pi}{b}} \cdot \sqrt{\frac{b}{\pi}} \quad (\because \textcircled{6}) \\ &= \frac{k}{4b} \\ &= \frac{k_B T}{2} \quad (\because \textcircled{5}) \\ &\quad \neq \end{aligned}$$