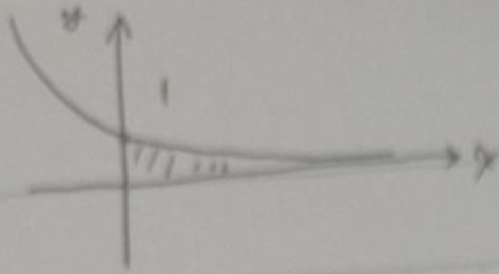

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$$\langle \varepsilon_r \rangle = \sum_{n=0}^{\infty} E_n P(E_n) = \sum_{n=0}^{\infty} (nh\nu) \frac{1}{Z} e^{-nh\nu/k_BT}$$

$$Z = \sum_{n=0}^{\infty} e^{-nh\nu/k_BT} = e^0 + \sum_{n=1}^{\infty} e^{-nh\nu/k_BT}$$

$$= 1 + \frac{e^{-\frac{h\nu}{k_BT}}}{1 - e^{-\frac{h\nu}{k_BT}}} = \frac{1}{1 - e^{-\frac{h\nu}{k_BT}}}$$

初項 $1 \cdot e^{-\frac{h\nu}{k_BT}}$
公比 $(0 <) e^{-\frac{h\nu}{k_BT}} (< 1)$

$$\langle \varepsilon_r \rangle = \frac{h\nu}{e^{\frac{h\nu}{k_BT}} - 1} \fallingdotseq \frac{h\nu}{\left(1 + \frac{h\nu}{k_BT}\right) - 1} = k_BT$$

②

より

$0 < \frac{h\nu}{k_BT}$

$0 < e^{-\frac{h\nu}{k_BT}} < 1$

$$\sum_{n=0}^{\infty} (nh\nu) \frac{1}{Z} e^{-\frac{nh\nu}{k_BT}} = \sum_{n=0}^{\infty} (nh\nu) \times \left(1 - e^{-\frac{h\nu}{k_BT}}\right) e^{-\frac{nh\nu}{k_BT}}$$

$$= h\nu \left(1 - e^{-\frac{h\nu}{k_BT}}\right) \sum_{n=0}^{\infty} n e^{-\frac{nh\nu}{k_BT}} \quad \cdots ①$$

$$S_n = \sum_{n=0}^{\infty} n e^{-\frac{nh\nu}{k_BT}} = \sum_{n=1}^{\infty} n e^{-\frac{nh\nu}{k_BT}}$$

$$S_n = 0 + 1 \cdot e^{-\frac{h\nu}{k_BT}} + 2 \cdot e^{-\frac{2h\nu}{k_BT}} + 3 \cdot e^{-\frac{3h\nu}{k_BT}} + \cdots$$

$$-)\ e^{-\frac{h\nu}{k_BT}} S_n = 1 \cdot e^{-\frac{2h\nu}{k_BT}} + 2 \cdot e^{-\frac{3h\nu}{k_BT}} + \cdots$$

$$\left(1 - e^{-\frac{h\nu}{k_BT}}\right) S_n = 1 \cdot e^{-\frac{h\nu}{k_BT}} + 1 \cdot e^{-\frac{2h\nu}{k_BT}} + 1 \cdot e^{-\frac{3h\nu}{k_BT}}$$

(初項 $e^{-\frac{h\nu}{k_BT}}$
公比 $e^{-\frac{h\nu}{k_BT}}$ $(0 < \ < 1)$)

$$= \frac{e^{-\frac{h\nu}{k_BT}}}{1 - e^{-\frac{h\nu}{k_BT}}}$$

$$S_n = \frac{e^{-\frac{h\nu}{k_BT}}}{\left(1 - e^{-\frac{h\nu}{k_BT}}\right)^2_{(\neq 0)}} \quad \cdots ②$$

① に ② を代入すると

$$① = h\nu\left(1 - e^{-\frac{h\nu}{k_BT}}\right)\sum_{n=0}^{\infty} n e^{-\frac{nh\nu}{k_BT}}$$

$$= h\nu\cancel{\left(1-e^{-\frac{h\nu}{k_BT}}\right)} \cdot \frac{e^{-\frac{h\nu}{k_BT}}}{\left(1-e^{-\frac{h\nu}{k_BT}}\right)^{\cancel{2}}}$$

$$= \frac{\left(h\nu e^{-\frac{h\nu}{k_BT}}\right) \times e^{\frac{h\nu}{k_BT}}}{\left(1-e^{-\frac{h\nu}{k_BT}}\right) \times e^{\frac{h\nu}{k_BT}}}$$

$$= \frac{h\nu}{e^{\frac{h\nu}{k_BT}} - 1}$$

2) $K = \frac{1}{4}cu$, $u = \frac{8\pi^5k^4T^4}{15c^3h^3}$ $\sin 2\theta$ 2°

$$\int_0^{\frac{\pi}{2}}\int_0^{2\pi} c\cos\theta \times \frac{1}{4\pi}u\sin\theta\, d\phi\, d\theta = \frac{uc}{4\pi}\int_0^{\frac{\pi}{2}}\int_0^{2\pi}\cos\theta\sin\theta\, d\phi\, d\theta$$

$$= \frac{uc}{4\pi}\int_0^{\frac{\pi}{2}} 2\pi\cos\theta\sin\theta\, d\theta$$

$$= \frac{uc}{4}\int_0^{\frac{\pi}{2}}\sin 2\theta\, d\theta$$

$\cos\pi$

$$= \frac{uc}{4}\left[-\frac{1}{2}\cos 2\theta\right]_0^{\frac{\pi}{2}}$$ $\frac{1}{2}+\frac{1}{2}$

$$= \frac{uc}{4}\left(-\frac{1}{2}(-1) - \left(-\frac{1}{2}\right)\cdot 1\right)$$

$$= \frac{1}{4}cu$$

$$\left(= \frac{1}{4}c \times \frac{\overset{2}{\cancel{8}}\pi^5k^4T^4}{15c^3h^3}\right)$$

$$\left(= \frac{2\pi^5k^4T^4}{15c^2h^3}\right)$$