$$\left\langle \frac{1}{2}kx^2 \right\rangle = \int_{-\infty}^{\infty} \frac{1}{2}kx^2 P(x)\,dx$$

$$\left(P(x)\,dx = \frac{e^{-\frac{E_p}{k_BT}}}{\int_{-\infty}^{\infty} e^{-\frac{E_p}{k_BT}}} \right)$$

$$= \int_{-\infty}^{\infty} \frac{1}{2}kx^2 \frac{e^{-\frac{E_p}{k_BT}}\,dx}{\int_{-\infty}^{\infty} e^{-\frac{E_p}{k_BT}}\,dx}$$

$$\left(\int_{-\infty}^{\infty} e^{-\frac{kx^2}{2k_BT}}\,dx = \sqrt{\frac{2\pi k_BT}{k}} \right)$$

$$= \int_{-\infty}^{\infty} \frac{1}{2}kx^2 \frac{e^{-\frac{kx^2}{2k_BT}}}{\sqrt{\frac{2\pi k_BT}{k}}}\,dx$$

$$= \frac{k}{2}\sqrt{\frac{k}{2\pi k_BT}} \int_{-\infty}^{\infty} x^2 e^{-\frac{kx^2}{2k_BT}}\,dx$$

$$= \frac{k}{2}\sqrt{\frac{k}{2\pi k_BT}} \cdot \frac{2k_BT}{2k}\sqrt{\frac{2\pi k_BT}{k}}$$

$$= \frac{k_BT}{2} \quad //$$

8223001 青 怏桂寸