材料の物理2

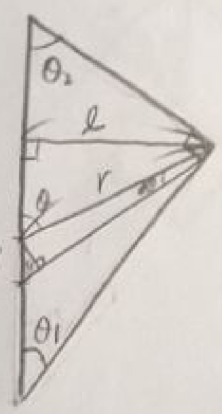
2022年 過去問 ①

ビオ・サバールの磁束密度を求める問題 (2)
+
正方形コイル + 授業でやった例題

図の書き方に注意

[1]

(1)



$$dB = \frac{\mu_0}{4\pi} \cdot \frac{I\,d\boldsymbol{l} \times \boldsymbol{r}}{r^3}$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{I\,dl \times r\sin\theta}{r^3}$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{I\,dl \times \sin\theta}{r^2}$$

$$B(r) = \int_C \frac{\mu_0}{4\pi} \cdot \frac{I\,dl \times \sin\theta}{r^2}$$

$r\sin\theta = l$ および $dl\sin\theta = r\,d\theta$

より

$$dl = \frac{r}{\sin\theta}\,d\theta$$

$$\frac{dl}{d\theta} = r\cos\theta$$

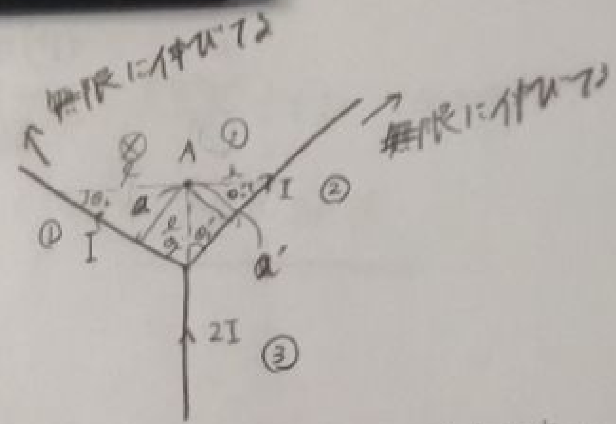
よって

$$B(r) = \frac{\mu_0 I}{4\pi}\int_{\theta_1}^{\pi-\theta_2} \frac{\cancel{\sin\theta}}{r^2} \times \frac{r}{\cancel{\sin\theta}}\,d\theta$$

$$= \frac{\mu_0 I}{4\pi}\int_{\theta_1}^{\pi-\theta_2} \frac{\sin\theta}{l}\,d\theta$$

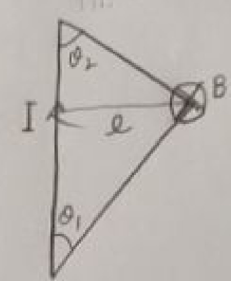
$$= \frac{\mu_0 I}{4\pi}\left[\frac{-\cos\theta}{l}\right]_{\theta_1}^{\pi-\theta_2}$$

$$= \frac{\mu_0 I}{4\pi l}(\cos\theta_2 + \cos\theta_1)$$



(1)より図の電流Iがつくる磁束密度は

$$B = \frac{\mu_0 I}{4\pi a}(\cos\theta_1 + \cos\theta_2)$$



今回の場合、下の電流Iは点Aの磁場に寄与しない

左側の I について

$a = \frac{l}{\sqrt{2}}$, $\theta_1 = 45^\circ$, $\theta_2 = 0^\circ$

$$B_1 = \frac{\mu_0 I}{4\pi\left(\frac{l}{\sqrt{2}}\right)}(\cos 45^\circ + \cos 0^\circ)$$

$$= \frac{\mu_0 I}{4\pi\left(\frac{l}{\sqrt{2}}\right)}\left(\frac{1}{\sqrt{2}} + 1\right)$$

$$= \frac{\sqrt{2}\mu_0 I}{4\pi l}\left(\frac{1}{\sqrt{2}} + \frac{\sqrt{2}}{\sqrt{2}}\right)$$

$$= \frac{\mu_0 I}{4\pi l}(1 + \sqrt{2})$$

右側の I について

$a' = \frac{l}{\sqrt{2}}$, $\theta_1' = 45^\circ$, $\theta_2' = 0^\circ$

$$B_2 = \frac{\mu_0 I}{4\pi\left(\frac{l}{\sqrt{2}}\right)}(\cos 45^\circ + \cos 0^\circ)$$

$$= \frac{\mu_0 I}{4\pi l}(1 + \sqrt{2})$$

合成すると

$B_1 - B_2 = 0$

磁場は 0 になる

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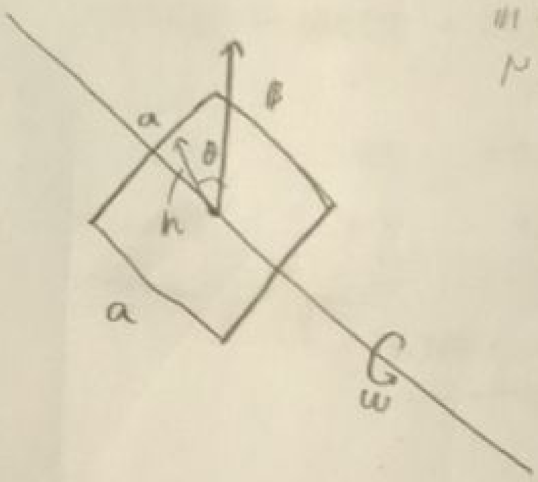
2023年 過去問 ②

2



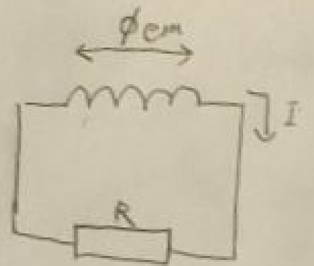
(1)

$\Phi = \mathbb{B}\cdot\mathbb{n} = Ba^2\cos\theta$

(2)

$\phi_{em} = -\frac{d\Phi}{dt} = Ba^2\omega\sin\theta$ $\left(\because \frac{d\theta}{dt} = \omega\right)$

(3), (4)



$I = \frac{\phi_{em}}{R}$

$= \frac{Ba^2\omega\sin\theta}{R}$

$J = \phi_{em} I = \frac{B^2a^4\omega^2\sin^2\theta}{R}$

(5)

$\mathbb{N} = IS\mathbb{n}\times\mathbb{B}$ (モーターの原理)

$= \frac{Ba^2\omega\sin\theta}{R}\times a^2\times |n|\sin\theta\times B$

$= \frac{B^2a^4\omega n\sin^2\theta}{R}$ (トルク)

(6) 仕事 = トルク × 角度

$W = \int_0^\theta N\,d\theta$

$= \frac{B^2a^4\omega}{R}\cdot\frac{1}{2}\left(\theta - \frac{1}{2}\sin 2\theta\right)$

(7)

$\frac{dW}{dt} = \frac{B^2a^4\omega^2}{R}\frac{1}{2}(1-\cos 2\theta)$

$= \frac{B^2a^4\omega^2}{R}\sin^2\theta$