演習レポート課題　解答

(2) $y' + y = \sin x$

① 同次方程式

$$\frac{dy}{dx} + y = 0$$

$$\int \frac{1}{y}\,dy = -\int dx$$

$$\log|y| = -x + c_1 \quad (c_1 : \text{積分定数})$$

$$y = \pm e^{-x+c_1}$$

$$= c e^{-x} \quad (c : \text{任意定数})$$

② 定数変化法：$c \to c(x)$

$$y = c(x) e^{-x}$$

与式は、

② 定数変化法：$c \to c(x)$

$$y = c(x)e^{-x}$$

与式は、 $$\frac{d}{dx}\{c(x)e^{-x}\} + c(x)e^{-x} = \sin x$$

$$\left\{e^{-x}\frac{d}{dx}c(x) \underbrace{- c(x)e^{-x}\} + c(x)e^{-x}}_{0}\right. = \sin x$$

$$\frac{d}{dx}c(x) = e^{x}\sin x$$

$$\int dc(x) = \int e^{x}\sin x\,dx$$

$$\frac{d}{dx}(e^{x}\sin x) = e^{x}\sin x + e^{x}\cos x$$

$$-)\ \frac{d}{dx}(e^{x}\cos x) = e^{x}\cos x - e^{x}\sin x$$

$$= 2e^{x}\sin x$$

$$-)\ \frac{d}{dx}(e^x\cos x) = e^x\cos x - e^x\sin x$$

$$= 2e^x\sin x$$

$$\Downarrow$$

$$\frac{1}{2}(e^x\sin x - e^x\cos x) = \int e^x\sin x$$

$$\downarrow$$

$$C(x) = \frac{1}{2}e^x(\sin x - \cos x) + C$$

よって、

$$y = C(x)e^{-x}$$

$$= \frac{1}{2}(\sin x - \cos x) + Ce^{-x} \quad (C\text{：任意定数})$$

(3) $xy' + y = \sin x$

$$\frac{dy}{dx} + \frac{y}{x} = \frac{\sin x}{x}$$

① $\dfrac{dy}{dx} + \dfrac{y}{x} = 0$

$$\int \frac{1}{y}\,dy = -\int \frac{1}{x}\,dx$$

$\log|y| = -\log|x| + c_1$ (c_1：積分定数)

$$\log|yx| = c_1$$

$$yx = \pm e^{c_1}$$

$y = \dfrac{c}{x}$ (c：任意定数)

② $c \to c(x)$

$$y = \frac{c(x)}{x}$$

与式は

② $c \to c(x)$

$$y = \frac{c(x)}{x}$$

与式は、

$$\left\{\frac{1}{\cancel{x}}\frac{d}{dx}c(x) \underbrace{- \frac{c(x)}{x^2}\right\} + \frac{\frac{c(x)}{x}}{x}}_{0} = \frac{\sin x}{\cancel{x}}$$

$$\frac{d}{dx}c(x) = \sin x$$

$$\int dc(x) = \int \sin x \, dx$$

$$c(x) = -\cos x + C \quad (C:\text{積分定数})$$

ゆえに、$\underline{y = \dfrac{c(x)}{x} = \dfrac{-\cos x}{x} + \dfrac{C}{x}}$ //

(4) $xy' + y = e^x$

$$\frac{dy}{dx} + \frac{y}{x} = \frac{e^x}{x}$$

① $\dfrac{dy}{dx} + \dfrac{y}{x} = 0$

(3)より、

$$y = \frac{c}{x} \quad (c : 任意定数)$$

② $c \to c(x)$

$$y = \frac{c(x)}{x}$$

(3)より、

$$\frac{1}{x}\frac{d}{dx}c(x) = \frac{e^x}{x}$$

$$y = \frac{c(x)}{x}$$

(3) より、

$$\frac{1}{x}\frac{d}{dx}c(x) = \frac{e^x}{x}$$

$$\frac{d}{dx}c(x) = e^x$$

$$\int dc(x) = \int e^x dx$$

$$c(x) = e^x + c$$

ゆえに、

$$y = \frac{c(x)}{x}$$

$$= \frac{e^x}{x} + \frac{c}{x} \quad (c:\text{任意定数})$$

3. 2階微分方程式

3.1 2階線形

。2階以上の線形微分方程式

$\longrightarrow$ 1階線形と同様

2階：$\underbrace{\dfrac{d^2y}{dx^2} + P(x)\dfrac{dy}{dx} + Q(x)y}_{y\text{の}1\text{次}} = \underbrace{R(x)}_{y\text{の}0\text{次}}$

$\Downarrow$

同次方程式：$\dfrac{d^2y}{dx^2} + P(x)\dfrac{dy}{dx} + Q(x)y = 0$

の一般解

$+$

非同次方程式の特解

3.2 線形同次方程式

○ e^{px} の微分 ($p = \alpha + i\beta$：複素数)

$$(e^{px})' = \frac{d}{dx} e^{px}$$

$$= \frac{d}{dx} e^{(\alpha + i\beta)x}$$

$$= \frac{d}{dx}(e^{\alpha x} e^{i\beta x})$$

オイラーの関係

$$= \frac{d}{dx}\left\{ e^{\alpha x}(\cos\beta x + i\sin\beta x)\right\}$$

$$= \alpha e^{\alpha x}(\cos\beta x + i\sin\beta x)$$

$$+ e^{\alpha x}(-\beta\sin\beta x + i\beta\cos\beta x)$$

i^2

$$= \alpha e^{\alpha x} e^{i\beta x} + i\beta e^{\alpha x}(\cos\beta x + i\sin\beta x)$$

$$= \alpha e^{\alpha x} e^{i\beta x} + i\beta e^{\alpha x} e^{i\beta x}$$

$$\frac{d}{dx}\left\{e^{\alpha x}(\cos\beta x + i\sin\beta x)\right\}$$

$$= \alpha e^{\alpha x}(\cos\beta x + i\sin\beta x) + e^{\alpha x}(\underbrace{-}_{i^2}\beta\sin\beta x + i\beta\cos\beta x)$$

$$= \alpha e^{\alpha x} e^{i\beta x} + i\beta e^{\alpha x}(\cos\beta x + i\sin\beta x)$$

$$= \alpha e^{\alpha x} e^{i\beta x} + i\beta e^{\alpha x} e^{i\beta x}$$

$$= (\alpha + i\beta) e^{\alpha x} e^{i\beta x}$$

$$= (\alpha + i\beta) e^{(\alpha + i\beta)x}$$

$$= \rho e^{\rho x}$$

よって、 $\dfrac{d}{dx} e^{\rho x} = \rho e^{\rho x}$