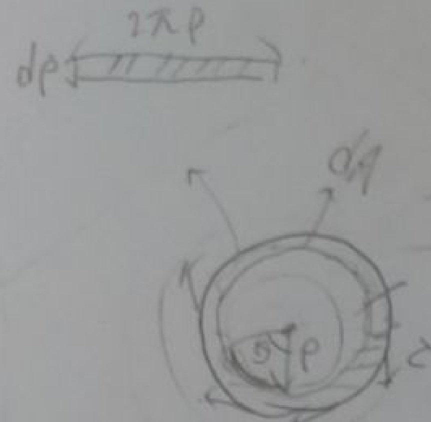
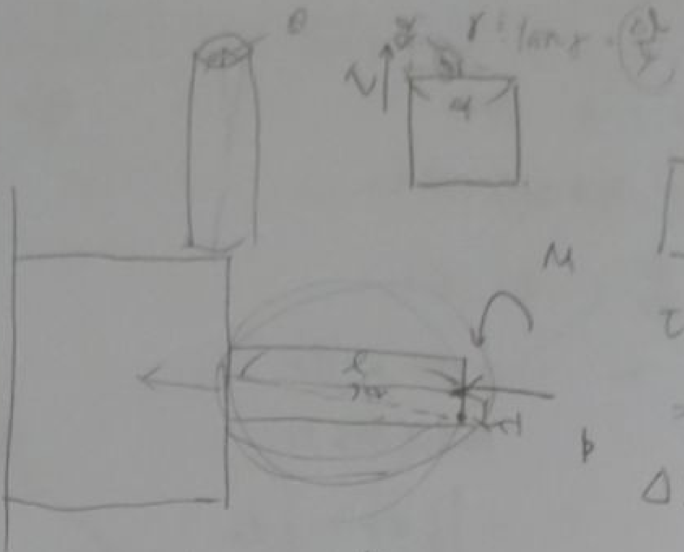
Ⅳ

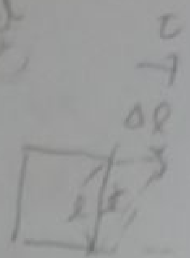
(1)



$$\int_0^r \tau\, dA \times \rho = M$$

$$dA = 2\pi\rho\, d\rho$$





$\tau = G\dfrac{\Delta l}{l}$

$= G\gamma$

$\Delta l = \rho\theta$

$\tau = G\dfrac{\rho\theta}{l}$

BCの材料の変位をΔl_{BC}とおく

$$\frac{P}{A_{BC}} = E\cdot\frac{\Delta l_{BC}}{l_2} \qquad A_{BC} = \left(\frac{d_2}{2}\right)^2\pi = \frac{d_2^2\pi}{4}$$

$$\Delta l_{BC} = \frac{Pl_2}{A_{BC}E} = \frac{4Pl_2}{d_2^2\pi E}$$

ABの材料の変位Δl_{AB}とおく　（ABの材料にも同じように荷重Pがかかる）

ABの断面積　$A_{AB} = \left(\dfrac{d_1}{2}\right)^2\pi = \dfrac{d_1^2\pi}{4}$

$$\frac{P}{A_{AB}} = E\cdot\frac{\Delta l_{AB}}{l_1}$$

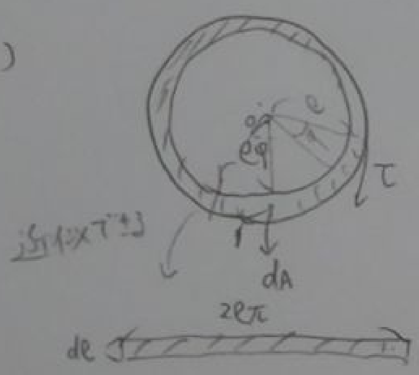
$\dfrac{r\theta}{r} = \theta$

$$\Delta l_{AB} = \frac{Pl_1}{A_{AB}E} = \frac{4Pl_1}{d_1^2E\pi}$$

C部分の軸方向変位 $= \Delta l_{AB} + \Delta l_{BC} = \dfrac{4Pl_1}{d_1^2E\pi} + \dfrac{4Pl_2}{d_2^2\pi E}$

$$= \frac{4P}{\pi E}\left(\frac{l_1}{d_1^2} + \frac{l_2}{d_2^2}\right)$$

(2)



$$M = \int_0^{\frac{d_2}{2}} \underbrace{\tau\, dA}_{力} \times \underbrace{\rho}_{キョリ}$$

$$dA = 2\rho\pi \times d\rho$$

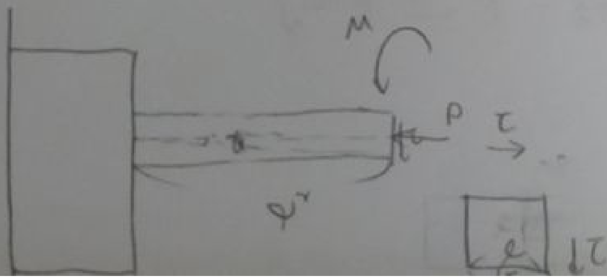
よって

$$M = \int_0^{\frac{d_2}{2}} \underbrace{\tau}_{?}\rho \times 2\rho\pi \times d\rho$$

横弾性係数をGとおくと

$$G = \frac{E}{2(1+\nu)}$$

τをρを使って表す



$$\tau = G\frac{\Delta l}{l} \qquad \Delta l = \rho\varphi$$

$$= G\frac{\rho\varphi}{l_2}$$

$$M = \int_0^{\frac{d_2}{2}} G \cdot \frac{\rho\varphi}{l_2} \cdot \rho \times 2\pi\rho \times d\rho$$

$$= \int_0^{\frac{d_2}{2}} 2G \frac{\pi\rho^3\varphi}{l_2} d\rho$$

$$= \frac{2G\pi\varphi}{l_2} \left[\frac{1}{4}\rho^4 \right]_0^{\frac{d_2}{2}}$$

$$= \frac{2G\pi\varphi}{l_2} \cdot \frac{1}{4} \cdot \frac{d_2^4}{16}$$

$$= \frac{G\pi\varphi d_2^4}{32 l_2}$$

$$\therefore \varphi_{BC} = \frac{32 l_2 M}{G\pi d_2^4} = \frac{64(1+\nu) l_2 M}{E\pi d_2^4}$$ …BCのねじれ角

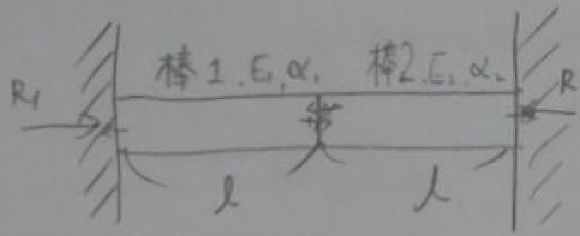
同様にして ABのねじれ角 φ_{AB} は

$$\varphi_{AB} = \frac{64(1+\nu) l_1 M}{E\pi d_1^4}$$ …ABのねじれ角

Cの部分の回転角度 = BCのねじれ角 + ABのねじれ角

$$= \frac{64(1+\nu)M}{E\pi} \left(\frac{l_1}{d_1^4} + \frac{l_2}{d_2^4} \right)$$

7



ΔTを与える
断面積 A

棒1による熱変形(伸び量)

$\Delta l_1 = \alpha_1 \Delta T l$

棒2による熱変形(伸び量)

$\Delta l_2 = \alpha_2 \Delta T l$

壁による圧縮力を受けて ~~縮む~~ 縮み量

棒1

$R_1 = E_1 \cdot \frac{\Delta l_1'}{l}$

$\Delta l_1' = \frac{R_1 l}{E_1}$

棒2

$R_2 = E_2 \cdot \frac{\Delta l_2'}{l}$

$\Delta l_2' = \frac{R_2 l}{E_2}$

$\Delta l_1 + \Delta l_2 - \Delta l_1' - \Delta l_2' = 0$ より

$$\alpha_1 \Delta T l + \alpha_2 \Delta T l - \frac{R_1 l}{E_1} - \frac{R_2 l}{E_2} = 0$$

$R_1 = R_2$ より

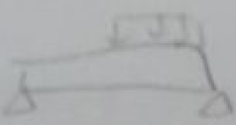
$$\alpha_1 \Delta T l + \alpha_2 \Delta T l - \frac{R_1 l}{E_1} - \frac{R_1 l}{E_2} = 0 \qquad \left(\frac{E_1+E_2}{E_1E_2}\right)$$

$$R_1\left(\frac{l}{E_1} + \frac{l}{E_2}\right) = (\alpha_1 + \alpha_2)\Delta T l$$

$$R_1 = \frac{(\alpha_1+\alpha_2)E_1E_2\Delta T}{E_1+E_2}$$

棒1に発生する熱応力 $= R_1$

$$= \frac{(\alpha_1+\alpha_2)E_1E_2\Delta T}{E_1+E_2}$$



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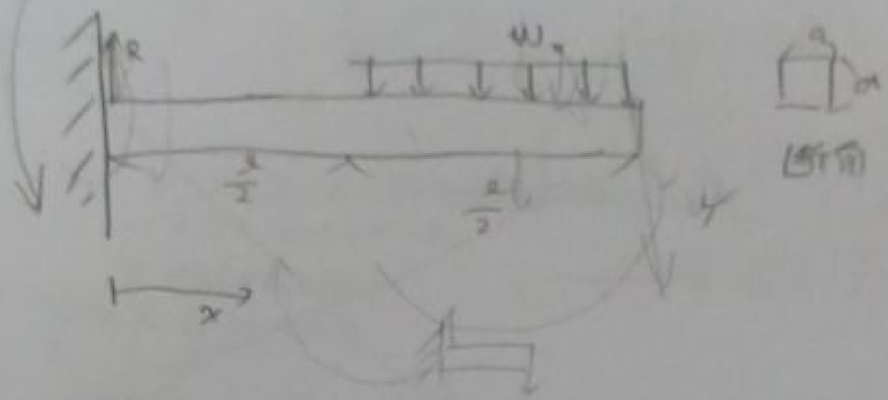


①

$$R = \int_{\frac{1}{2}l}^{l} w_0\,dx = \frac{1}{2}w_0 l$$

$$M = \int_{\frac{1}{2}l}^{l} w_0 x\,dx = \left[\frac{1}{2}w_0 x^2\right]_{\frac{1}{2}l}^{l}$$

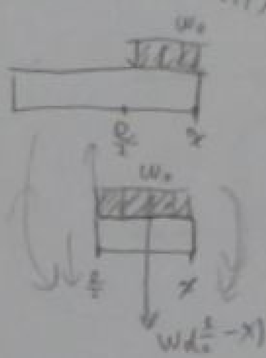
$$= \frac{3}{8}w_0 l^2$$

②

(i) $0 < x < \frac{l}{2}$ のとき

$$M(x) = 0$$

(ii) $\frac{l}{2} < x < l$ のとき



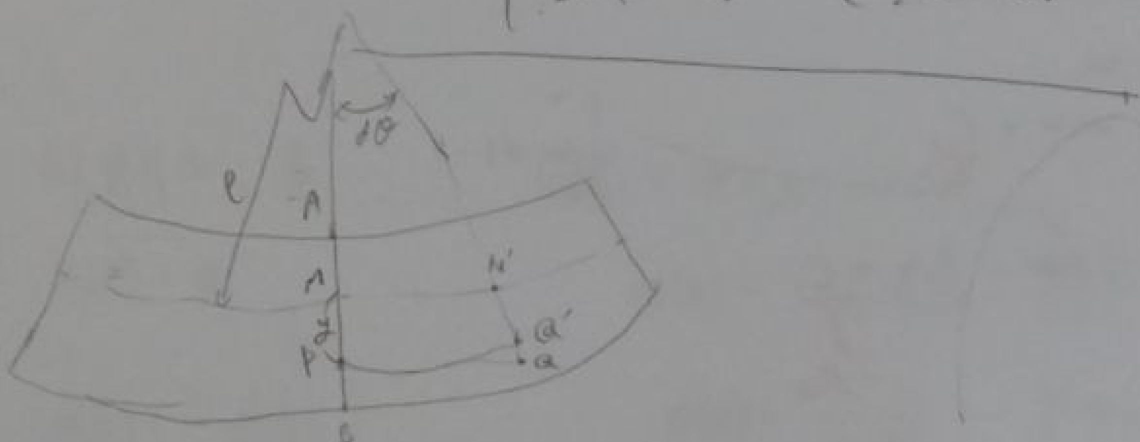
$$w_0\left(x - \frac{l}{2}\right) \times \frac{1}{2}\left(x - \frac{l}{2}\right)$$

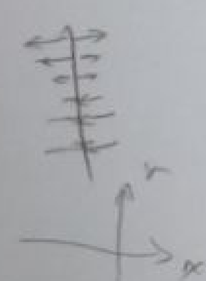
$$= \frac{1}{2}w_0\left(x - \frac{l}{2}\right)^2$$

曲げモーメント $M(x) = -\frac{1}{2}w_0\left(x - \frac{l}{2}\right)^2$

$$M(x) = \begin{cases} 0 & \left(0 < x < \frac{l}{2}\right) \\ -\frac{1}{2}w_0\left(x - \frac{l}{2}\right)^2 & \left(\frac{l}{2} < x < l\right) \end{cases}$$

③



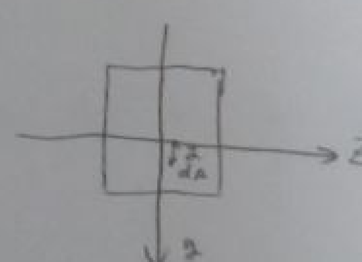


$$PQ' = (\rho + y)\,d\theta$$

$$PQ = MN' = \rho\,d\theta$$

PQ'が伸び

$$\varepsilon = \frac{PQ' - PQ}{PQ} = \frac{y\,d\theta}{\rho\,d\theta} = \frac{y}{\rho}$$



$$\sigma(y) = E\frac{y}{\rho}$$

$$\int_A \sigma\,dA = 0 \Leftrightarrow \int_A E\frac{y}{\rho}\,dA = 0$$

$$dM = y \times \sigma \times dA = \frac{E}{\rho}y^2\,dA$$

全体のモーメント

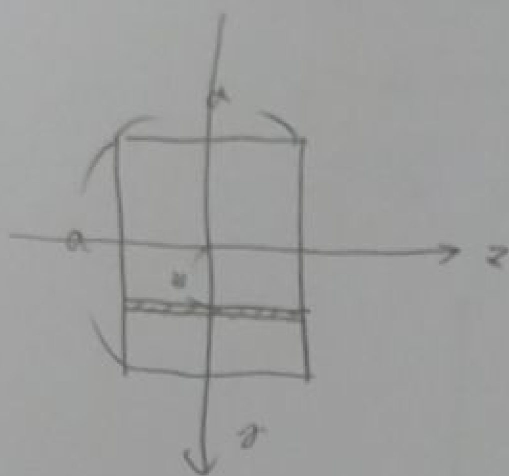
$$M = \int_A \frac{E}{\rho} y^2 dA = \frac{E}{\rho} I \text{ とおく}$$

$$\left(I = \int_A y^2 dA \right)$$

断面2次モーメント

$$\frac{M}{EI} = \frac{1}{\rho}$$

$$\sigma = Ey \times \frac{M}{EI} = M\frac{y}{I}$$



$$I = \int_A y^2 dA = \int_{-\frac{a}{2}}^{\frac{a}{2}} y^2 dy \times a = a\left[\frac{1}{3}y^3\right]_{-\frac{a}{2}}^{\frac{a}{2}} = \frac{a^4}{12}$$

$$\frac{a^3}{8} + \frac{a^3}{8} \quad \frac{2a^3}{8} \quad \frac{a^3}{4}$$

$$-\frac{1}{2}w_0\frac{l^2}{4}$$

$$\sigma_{Max} = M_{Max} \cdot \frac{y}{I} = \frac{w_0 l^2}{8} \times \frac{12 \cdot \frac{a}{2}}{a^4} \quad \frac{3w_0 l^2}{4}$$

④

(i) $0 < x < \frac{l}{2}$, (ii) $\frac{l}{2} < x < l$ のとき.

$$M(x) = 0 \qquad M(x) = -\frac{1}{2}w_0\left(x - \frac{l}{2}\right)^2$$

$$\theta(x) = -\frac{1}{EI}(C) \qquad \theta'(x) = -\frac{1}{EI}\left\{-\frac{1}{6}w_0\left(x-\frac{l}{2}\right)^3 + C'\right\}$$

$$\theta\left(\frac{l}{2}\right) = \theta'\left(\frac{l}{2}\right)$$

$$v(x) = -\frac{1}{EI}(Cx + C_0) \qquad v'(x) = -\frac{1}{EI}\left\{-\frac{1}{24}w_0\left(x-\frac{l}{2}\right)^4 + C'\left(x-\frac{l}{2}\right) + C_0'\right\}$$

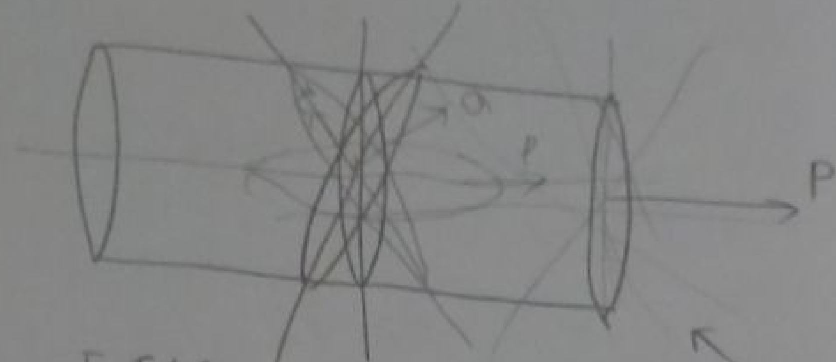
$$v\left(\frac{l}{2}\right) = v'\left(\frac{l}{2}\right)$$

$$\theta(0) = 0 \Rightarrow C = 0 \qquad C' = 0$$

$$v(0) = 0 \Rightarrow C_0 = 0 \qquad C_0' = 0$$

$$v(x) = 0 \qquad v'(x) = \frac{1}{24EI}w_0\left(x-\frac{l}{2}\right)^4 = \frac{1}{2Ea^4}w_0\left(x-\frac{l}{2}\right)^4$$

$\boxed{4}$



長手方向にのみ荷重がかかる

モールの応力円は

$\left(P/A,\ 0\right)$ $(0, 0)$

せん断…ある面だけを
動かすときに生じる

今回は全体を運しているので
せん断力はない