第2回 課題 8222104 桧崎 太輔

(1) $\langle \varepsilon_\nu \rangle = \sum_{n=0}^{\infty} nh\nu \frac{1}{Z} e^{-\frac{nh\nu}{k_B T}}$ $\quad \left(Z = \sum_{n=0}^{\infty} e^{-\frac{nh\nu}{k_B T}} \right)$

$= h\nu \sum_{n=0}^{\infty} n \frac{e^{-\frac{nh\nu}{k_B T}}}{\sum_{n=0}^{\infty} e^{-\frac{nh\nu}{k_B T}}}$ ここで $\frac{nh\nu}{k_B T} = \alpha$ とおくと.

与式 $= h\nu \sum_{n=0}^{\infty} n \frac{e^{-\alpha n}}{\sum_{n=0}^{\infty} e^{-\alpha n}}$

$= h\nu \sum_{n=0}^{\infty} \frac{ne^{-\alpha n}}{\frac{1}{1-e^{-\alpha}}}$

$= h\nu (1-e^{-\alpha}) \sum_{n=0}^{\infty} ne^{-\alpha n}$ ここで $\sum_{n=0}^{m} ne^{-\alpha n}$ を S_m とおき.

$S_m - e^{-\alpha} S_m$ を計算する.

$S_m = 0 + 1 \cdot e^{-\alpha} + 2 \cdot e^{-2\alpha} + \cdots\cdots + m \cdot e^{-m\alpha}$

$e^{-\alpha} S_m = 0 + 1 \cdot e^{-2\alpha} + \cdots\cdots + (m-1) e^{-m\alpha} + me^{-(m+1)\alpha}$

$(1-e^{-\alpha}) S_m = e^{-\alpha} + e^{-2\alpha} + \cdots\cdots + e^{-m\alpha} - me^{-(m+1)\alpha}$

$= e^{-\alpha} \frac{1-(e^{-\alpha})^m}{(1-e^{-\alpha})^2} - \frac{me^{-(m+1)\alpha}}{1-e^{-m}}$

よって. $\lim_{m \to \infty} S_m = \frac{e^{-\alpha}}{(1-e^{-\alpha})^2}$

ゆえに. 与式 $= h\nu (1-e^{-\alpha}) \frac{e^{-\alpha}}{(1-e^{-\alpha})^2} = \frac{h\nu e^{-\alpha}}{1-e^{-\alpha}} = \frac{h\nu}{e^{\alpha}-1}$

$\therefore$ 与式 $= \frac{h\nu}{e^{\frac{h\nu}{k_B T}}-1}$

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(2) $\int_0^{\frac{\pi}{2}}\int_0^{2\pi} c\cos\theta \times \frac{1}{4\pi} U \sin\theta \, d\theta \, d\phi$

$= \frac{UC}{4\pi} \cdot \int_0^{\frac{\pi}{2}} (\sin\theta \cos\theta) 2\pi \, d\theta$

$= \frac{UC}{2} \cdot \int_0^{\frac{\pi}{2}} \frac{1}{2} \sin 2\theta \, d\theta$

$= \frac{1}{4} UC \left[-\frac{1}{2} \cos 2\theta \right]_0^{\frac{\pi}{2}}$

$= \frac{1}{4} CU$