8223036　栗山淳

材料の物理２　第 2 回課題

①次式を証明せよ

$$rot\,rot\,\boldsymbol{A} = grad\,div\,\boldsymbol{A} - \boldsymbol{\Delta A}$$

(左辺)=$\nabla \times (\nabla \times \boldsymbol{A}) = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \times \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}, \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}, \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$

$$= \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \times \begin{pmatrix} \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \\ \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \\ \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\partial}{\partial y} \cdot \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right) - \frac{\partial}{\partial z} \cdot \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) \\ \frac{\partial}{\partial z} \cdot \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) - \frac{\partial}{\partial x} \cdot \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right) \\ \frac{\partial}{\partial x} \cdot \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) - \frac{\partial}{\partial y} \cdot \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{\partial^2 A_x}{\partial y^2} - \frac{\partial^2 A_x}{\partial z^2} \\ -\frac{\partial^2 A_y}{\partial z^2} - \frac{\partial^2 A_y}{\partial x^2} \\ -\frac{\partial^2 A_z}{\partial x^2} - \frac{\partial^2 A_z}{\partial y^2} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\partial^2 A_x}{\partial x^2} - \frac{\partial^2 A_x}{\partial x^2} - \frac{\partial^2 A_x}{\partial y^2} - \frac{\partial^2 A_x}{\partial z^2} \\ \frac{\partial^2 A_y}{\partial y^2} - \frac{\partial^2 A_y}{\partial y^2} - \frac{\partial^2 A_y}{\partial z^2} - \frac{\partial^2 A_y}{\partial x^2} \\ \frac{\partial^2 A_z}{\partial z^2} - \frac{\partial^2 A_z}{\partial z^2} - \frac{\partial^2 A_z}{\partial x^2} - \frac{\partial^2 A_z}{\partial y^2} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\partial^2 A_x}{\partial x^2} \\ \frac{\partial^2 A_y}{\partial y^2} \\ \frac{\partial^2 A_z}{\partial z^2} \end{pmatrix} - \begin{pmatrix} \frac{\partial^2 A_x}{\partial x^2} + \frac{\partial^2 A_x}{\partial y^2} + \frac{\partial^2 A_x}{\partial z^2} \\ \frac{\partial^2 A_y}{\partial y^2} + \frac{\partial^2 A_y}{\partial z^2} + \frac{\partial^2 A_y}{\partial x^2} \\ \frac{\partial^2 A_z}{\partial z^2} + \frac{\partial^2 A_z}{\partial x^2} + \frac{\partial^2 A_z}{\partial y^2} \end{pmatrix}$$

$$= \ grad\,div\,\boldsymbol{A} - \boldsymbol{\Delta A}$$

$$= (右辺)$$

②ベクトル場の実例を挙げよ

寝坊した人の通勤ルートのベクトル場

寝坊してしまった人が急いで駅まで走るとき，通常の通勤経路ではなく，最短距離をとにかくダッシュする。しかし，駅近くでコンビニに寄ったり，信号に引っかかったりと，毎回想定外の動きをする。そのベクトル場は，直線的な道筋から急にジグザグに動いたり，コンビニの前で一瞬減速するベクトルが見えたする。まさに遅刻防止ベクトル場！！！