8223036　栗山淳　応用数学　第 3 回課題

問1

(1)

$$\cos h(z_1+z_2) = \cos(hz_1+hz_2) = \frac{e^{i(hz_1+hz_2)}+e^{-i(hz_1+hz_2)}}{2}$$

$$= \frac{e^{ihz_1}\cdot e^{ihz_2}+e^{-ihz_1}\cdot e^{-ihz_2}}{2}$$

$$\cos hz_1 \cos hz_2 - \sin hz_1 \sin hz_2 = \frac{e^{ihz_1}+e^{-ihz_1}}{2}\cdot\frac{e^{ihz_2}+e^{-ihz_2}}{2} - \frac{e^{ihz_1}-e^{-ihz_1}}{2i}\cdot\frac{e^{ihz_2}-e^{-ihz_2}}{2i}$$

$$= \frac{e^{ihz_1}\cdot e^{ihz_2}+\cancel{e^{ihz_1}e^{-ihz_2}}+\cancel{e^{-ihz_1}e^{ihz_2}}+e^{-ihz_1}\cdot e^{-ihz_2}}{4} + \frac{e^{ihz_1}\cdot e^{ihz_2}-\cancel{e^{ihz_1}\cdot e^{-ihz_2}}-\cancel{e^{-ihz_1}\cdot e^{ihz_2}}+e^{-ihz_1}\cdot e^{-ihz_2}}{4}$$

$$= \frac{e^{ihz_1}\cdot e^{ihz_2}+e^{-ihz_1}\cdot e^{-ihz_2}}{2}$$

$$= \cos h(z_1+z_2)$$

よって 証明された

(2) $\sinh(z_1+z_2) = \sinh z_1 \cosh z_2 + \cosh z_1 \sinh z_2$

(左辺) $= \sin h(z_1+z_2) = \sin(hz_1+hz_2)$

$$= \frac{e^{i(hz_1+hz_2)}-e^{-i(hz_1+hz_2)}}{2i}$$

$$= \frac{e^{ihz_1}\cdot e^{ihz_2}-e^{-ihz_1}\cdot e^{-ihz_2}}{2i}$$

(右辺) $= \sin hz_1 \cos hz_2 + \cos hz_1 \sin hz_2$

$$= \frac{e^{ihz_1}-e^{-ihz_1}}{2i}\cdot\frac{e^{ihz_2}+e^{-ihz_2}}{2} + \frac{e^{ihz_1}+e^{-ihz_1}}{2}\cdot\frac{e^{ihz_2}-e^{-ihz_2}}{2i}$$

$$= \frac{e^{ihz_1}\cdot e^{ihz_2}+\cancel{e^{ihz_1}e^{-ihz_2}}-\cancel{e^{-ihz_1}e^{ihz_2}}-e^{-ihz_1}e^{-ihz_2}}{4i} + \frac{e^{ihz_1}\cdot e^{ihz_2}-\cancel{e^{ihz_1}e^{-ihz_2}}+\cancel{e^{-ihz_1}e^{ihz_2}}-e^{-ihz_1}e^{-ihz_2}}{4i}$$

$$= \frac{e^{ihz_1}e^{ihz_2}-e^{-ihz_1}e^{-ihz_2}}{2i}$$

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(右辺) $= \sin hz_1 \cos hz_2 + \cos hz_1 \sin hz_2$

$$= \frac{e^{ihz_1}-e^{-ihz_1}}{2i}\cdot\frac{e^{ihz_2}+e^{-ihz_2}}{2} + \frac{e^{ihz_1}+e^{-ihz_1}}{2}\cdot\frac{e^{ihz_2}-e^{-ihz_2}}{2i}$$

$$= \frac{e^{ihz_1}\cdot e^{ihz_2}+\cancel{e^{ihz_1}e^{-ihz_2}}-\cancel{e^{-ihz_1}e^{ihz_2}}-e^{-ihz_1}e^{-ihz_2}}{4i} + \frac{e^{ihz_1}\cdot e^{ihz_2}-\cancel{e^{ihz_1}e^{-ihz_2}}+\cancel{e^{-ihz_1}e^{ihz_2}}-e^{-ihz_1}e^{-ihz_2}}{4i}$$

$$= \frac{e^{ihz_1}e^{ihz_2}-e^{-ihz_1}e^{-ihz_2}}{2i}$$

$= $ (左辺)

よって 証明された

$$\left(\begin{aligned} \cos x &= \cos ihx \\ \sin x &= -i\sin hix \\ \cos ix &= \cosh x \\ \sin ix &= i\sinh x \end{aligned}\right)$$

問4

(1) $\cos(2+i) = \cos 2\cos i - \sin 2\sin i$

$$= \cos 2\cdot\cos h\cdot 1 - \sin 2\cdot i\sin h\cdot 1$$

$$= \frac{e^2+1}{2e}\cos 2 - i\frac{e^2-1}{2e}\sin 2$$

$$= \frac{e^2+1}{2e}\cos 2 + i\left(\frac{1-e^2}{2e}\right)\sin 2$$

(2)

$$\sin h(1+2i) = \sin h(i(2-i))$$

$$= i\sin(2-i)$$

(2)

$$\sinh(1+2i) = \sinh(i(2-i))$$
$$= i\sin(2-i)$$
$$= i(\sin 2\cos i - \cos 2\sin i)$$
$$= i(\sin 2\cdot\cosh 1 - \cos 2\cdot i\sinh 1)$$
$$= i\left(\frac{e^2+1}{2e}\cdot\sin 2 - i\left(\frac{e^2-1}{2e}\right)\cos 2\right)$$
$$= \left(\frac{e^2-1}{2e}\right)\cos 2 + i\left(\frac{e^2+1}{2e}\right)\sin 2$$

(3) $$\sin\left(\frac{\pi}{4}+2i\right) = \sin\frac{\pi}{4}\cos 2i + \cos\frac{\pi}{4}\sin 2i$$
$$= \frac{1}{\sqrt{2}}\cos 2i + \frac{1}{\sqrt{2}}\sin 2i$$
$$= \frac{1}{\sqrt{2}}\cdot\cosh 2 + \frac{1}{\sqrt{2}}\cdot i\sinh 2$$
$$= \frac{\cosh 2}{\sqrt{2}} + i\cdot\frac{\sinh 2}{\sqrt{2}}$$

(4) $$\cosh\left(2+\frac{\pi}{4}i\right) = \cosh i\left(-2i+\frac{\pi}{4}\right)$$
$$= \cosh i\left(\frac{\pi}{4}-2i\right)$$
$$= \cos\left(\frac{\pi}{4}-2i\right)$$
$$= \cos\frac{\pi}{4}\cos 2i + \sin\frac{\pi}{4}\sin 2i$$
$$= \frac{1}{\sqrt{2}}\cos 2i + \frac{1}{\sqrt{2}}\sin 2i$$

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$$= \frac{1}{\sqrt{2}}\cosh 2 + i\frac{1}{\sqrt{2}}\sinh 2$$