熱力学I 第7講 課題 8223026 栗山 淳

$$C_p - C_v = \left(\frac{\partial H}{\partial T}\right)_p - \left(\frac{\partial U}{\partial T}\right)_V$$

$$\left(\frac{\partial H}{\partial T}\right)_p = \left(\frac{\partial (U+PV)}{\partial T}\right)_p = \left(\frac{\partial U}{\partial T}\right)_p + p\left(\frac{\partial V}{\partial T}\right)_p$$

$(\because H = U + PV)$

$$= \left(\frac{\partial U}{\partial T}\right)_p + p\alpha V$$

$$dU = \pi_T dV + C_V dT$$

$$\Rightarrow \left(\frac{\partial U}{\partial T}\right)_p = \pi_T\left(\frac{\partial V}{\partial T}\right)_p + C_V\left(\frac{\partial T}{\partial T}\right)_p$$

$$\Leftrightarrow \left(\frac{\partial U}{\partial T}\right)_p = \pi_T\left(\frac{\partial V}{\partial T}\right)_p + \underbrace{C_V}_{\left(\frac{\partial U}{\partial T}\right)_V}$$

これらより

$$C_p - C_v = \cancel{\left(\frac{\partial U}{\partial T}\right)_p} + p\alpha V + \pi_T\left(\frac{\partial V}{\partial T}\right)_p - \cancel{\left(\frac{\partial U}{\partial T}\right)_p}$$

$$= p\alpha V + \pi_T\left(\frac{\partial V}{\partial T}\right)_p$$

$$= p\alpha V + \pi_T \cdot V\alpha$$

$$= p\alpha V + \left(T\left(\frac{\partial P}{\partial T}\right)_V - p\right)\cdot V\alpha \quad \left(\because \pi_T = T\left(\frac{\partial P}{\partial T}\right)_V - P\right)$$

$$= \cancel{p\alpha V} + T\left(\frac{\partial P}{\partial T}\right)_V \cdot V\alpha - \cancel{PV\alpha}$$

$$= TV\alpha\left(-\left(\frac{\partial P}{\partial V}\right)_T\left(\frac{\partial V}{\partial T}\right)_p\right)$$

$$= TV\alpha\left(-\left(-\frac{1}{V\kappa_T}\right)\cdot V\alpha\right)$$

$$= TV\alpha \cdot \frac{\cancel{V}\alpha}{\cancel{V}\kappa_T}$$

$$= \frac{\alpha^2 TV}{\kappa_T}$$

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