量子力学　　　　第12回

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$$Y_{px} = \frac{1}{\sqrt{2}}\left(Y_{1,-1} - Y_{1,1}\right)$$

$$Y_{py} = \frac{i}{\sqrt{2}}\left(Y_{1,-1} + Y_{1,1}\right)$$

絶対値の2乗が方向(θ,ϕ)のどの方向に電子が存在しやすいかを示す

$$\hat{H}\psi = E\psi$$

$|\psi|^2$で粒子の存在確率を示す

$$\hat{l}_\pm = \hat{l}_x \pm i\hat{l}_y \quad \cdots ①$$

$$\hat{l}_\pm |l,m\rangle = \hbar\sqrt{(l\mp m)(l\pm m+1)}\,|l,m\pm 1\rangle \quad \cdots ②$$

①より　昇降演算子

$$\hat{l}_\pm = \hat{l}_x \pm i\hat{l}_y$$

$$\Rightarrow \begin{cases} \hat{l}_+ = \hat{l}_x + i\hat{l}_y \\ \hat{l}_- = \hat{l}_x - i\hat{l}_y \end{cases}$$

$2i\hat{l}_y$

$$\frac{\hat{l}_+ - \hat{l}_-}{2i} = \hat{l}_y$$

$$\hat{l}_+ + \hat{l}_- = 2\hat{l}_x$$

$$\Leftrightarrow \underbrace{\hat{l}_x}_{\text{演算子}} = \frac{\hat{l}_+ + \hat{l}_-}{2} \quad \cdots ③$$

$$\hat{l}_y = \frac{\hat{l}_+ - \hat{l}_-}{2i} \quad \cdots ④$$

1. Y_{px}の場合

$$\hat{l}_x Y_{px} = \frac{\hat{l}_+ + \hat{l}_-}{2}\left\{\frac{1}{\sqrt{2}}\left(Y_{1,-1} - Y_{1,1}\right)\right\} \quad (\because ③)$$

$$= \frac{1}{2\sqrt{2}}\left(\hat{l}_+Y_{1,-1} - \hat{l}_+Y_{1,1} + \hat{l}_-Y_{1,-1} - \hat{l}_-Y_{1,1}\right)$$

ここで②より、

$$\hat{l}_+Y_{1,-1} = \hat{l}_+|1,-1\rangle = \hbar\sqrt{(2)(1)}\,|1,0\rangle = \sqrt{2}\hbar Y_{1,0}$$

$\hat{l}_+Y_{1,1} = \hat{l}_+|1,1\rangle \Rightarrow$ 上昇不可

$\hat{l}_-Y_{1,-1} = \hat{l}_-|1,-1\rangle \Rightarrow$ 降下不可

$\hat{l}_-Y_{1,1} = \hat{l}_-|1,1\rangle \Rightarrow \hbar\sqrt{(2)(1)}\,|1,0\rangle = \sqrt{2}\hbar Y_{1,0}$ となるので

$$\hat{l}_x Y_{px} = \frac{1}{2\sqrt{2}}\left(\sqrt{2}\hbar Y_{1,0} - \sqrt{2}\hbar Y_{1,0}\right)$$

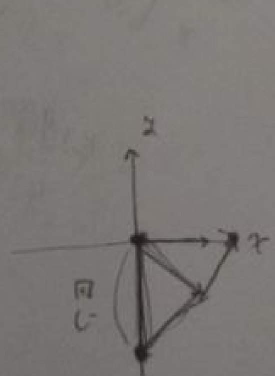
$$= \frac{\hbar}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\left(Y_{1,0} - Y_{1,0}\right)\right)$$

$$= \frac{\hbar}{\sqrt{2}}Y_{px}$$

$\shortparallel$

$Y_{1,-1} - Y_{1,1}$

$$\hat{l}_x = \frac{\hbar}{\sqrt{2}}$$



2. Y_{py} の場合

$$\hat{l}_y Y_{py} = \frac{\hat{l}_+ - \hat{l}_-}{2i}\left\{\frac{i}{\sqrt{2}}\left(Y_{1,-1} + Y_{1,1}\right)\right\} \quad (\because ④)$$

$$= \frac{1}{2\sqrt{2}}\left(\hat{l}_+ Y_{1,-1} + \hat{l}_+ Y_{1,1} - \hat{l}_- Y_{1,-1} - \hat{l}_- Y_{1,1}\right)$$

ここで
②より

$$\hat{l}_+ Y_{1,-1} = \hbar\sqrt{(2)(1)}\,|1,0\rangle = \sqrt{2}\hbar\,|1,0\rangle = \sqrt{2}\hbar Y_{1,0}$$

$\hat{l}_+ Y_{1,1} \Rightarrow$ 上昇不可

$\hat{l}_- Y_{1,-1} = \hat{l}_-|1,-1\rangle \Rightarrow$ 降下不可

$\hat{l}_- Y_{1,1} = \sqrt{2}\hbar Y_{1,0}$ となるので

$$\hat{l}_y Y_{py} = \frac{1}{2\sqrt{2}}\left(\sqrt{2}\hbar Y_{1,0} - \sqrt{2}\hbar Y_{1,0}\right)$$

$$= \frac{\hbar}{\sqrt{2}}\left\{\frac{1}{\sqrt{2}}\underbrace{\left(Y_{1,0} - Y_{1,0}\right)}_{Y_{1,-1}+Y_{1,1}}\right\}$$

$$= \frac{\hbar}{\sqrt{2}} Y_{py}$$

$$\hat{l}_y = \frac{\hbar}{\sqrt{2}}$$