

8223036 栗山淳 応用数学 第3回課題

例

4p

$$\begin{aligned} \cos \frac{1}{2}(z_1 + z_2) &= \cos \left(\frac{1}{2}z_1 + \frac{1}{2}z_2 \right) = \frac{e^{i(\frac{1}{2}z_1 + \frac{1}{2}z_2)} + e^{-i(\frac{1}{2}z_1 + \frac{1}{2}z_2)}}{2} \\ &= \frac{e^{i\frac{1}{2}z_1} \cdot e^{i\frac{1}{2}z_2} + e^{-i\frac{1}{2}z_1} \cdot e^{-i\frac{1}{2}z_2}}{2} \\ \cosh z_1 \cosh z_2 &= \frac{e^{z_1} + e^{-z_1}}{2} \cdot \frac{e^{z_2} + e^{-z_2}}{2} = \frac{e^{z_1+z_2} + e^{z_1-z_2} + e^{-z_1+z_2} + e^{-z_1-z_2}}{4} \\ &= \frac{e^{z_1+z_2} + e^{-z_1-z_2}}{4} + \frac{e^{z_1-z_2} + e^{-z_1+z_2}}{4} \\ &= \frac{e^{z_1+z_2} + e^{-z_1-z_2}}{2} \\ &= \cos h(z_1 + z_2) \\ &\text{よ? 証明する?} \end{aligned}$$

(2) $\sinh(z_1 + z_2) = \sinh z_1 \cosh z_2 + \cosh z_1 \sinh z_2$

$$\begin{aligned} (\text{証明}) \quad \sinh(z_1 + z_2) &= \frac{e^{i(z_1 + z_2)} - e^{-i(z_1 + z_2)}}{2i} \\ &= \frac{e^{iz_1} \cdot e^{iz_2} - e^{-iz_1} \cdot e^{-iz_2}}{2i} \end{aligned}$$

$$\begin{aligned} (\text{右辺}) \quad \sinh z_1 \cosh z_2 + \cosh z_1 \sinh z_2 &= \frac{e^{iz_1} - e^{-iz_1}}{2i} \cdot \frac{e^{iz_2} + e^{-iz_2}}{2} + \frac{e^{iz_1} + e^{-iz_1}}{2} \cdot \frac{e^{iz_2} - e^{-iz_2}}{2i} \\ &= \frac{e^{iz_1} \cdot e^{iz_2} + e^{iz_1} \cdot e^{-iz_2} - e^{-iz_1} \cdot e^{iz_2} - e^{-iz_1} \cdot e^{-iz_2}}{4i} + \frac{e^{iz_1} \cdot e^{iz_2} - e^{iz_1} \cdot e^{-iz_2} + e^{-iz_1} \cdot e^{iz_2} - e^{-iz_1} \cdot e^{-iz_2}}{4i} \\ &= \frac{e^{iz_1} \cdot e^{iz_2} - e^{-iz_1} \cdot e^{-iz_2}}{2i} \end{aligned}$$

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$$\begin{aligned} (\text{証明}) \quad \sinh(z_1 + z_2) &= \frac{e^{i(z_1 + z_2)} - e^{-i(z_1 + z_2)}}{2i} \\ &= \frac{e^{iz_1} \cdot e^{iz_2} - e^{-iz_1} \cdot e^{-iz_2}}{2i} \end{aligned}$$

$$\begin{aligned} (\text{右辺}) \quad \sinh z_1 \cosh z_2 + \cosh z_1 \sinh z_2 &= \frac{e^{iz_1} - e^{-iz_1}}{2i} \cdot \frac{e^{iz_2} + e^{-iz_2}}{2} + \frac{e^{iz_1} + e^{-iz_1}}{2} \cdot \frac{e^{iz_2} - e^{-iz_2}}{2i} \\ &= \frac{e^{iz_1} \cdot e^{iz_2} + e^{iz_1} \cdot e^{-iz_2} - e^{-iz_1} \cdot e^{iz_2} - e^{-iz_1} \cdot e^{-iz_2}}{4i} + \frac{e^{iz_1} \cdot e^{iz_2} - e^{iz_1} \cdot e^{-iz_2} + e^{-iz_1} \cdot e^{iz_2} - e^{-iz_1} \cdot e^{-iz_2}}{4i} \\ &= \frac{e^{iz_1} \cdot e^{iz_2} - e^{-iz_1} \cdot e^{-iz_2}}{2i} \end{aligned}$$

(証明)

よ? 証明する?

$$\left(\begin{array}{l} \cos x = \cosh ix \\ \sin x = -i \sinh ix \\ \cos ix = \cosh x \\ \sinh ix = i \sin x \end{array} \right)$$

例

$$\begin{aligned} (1) \quad \cos(z_1) &= \cos z_1 \cos z_2 - \sin z_1 \sin z_2 \\ &= \cos z_1 \cos z_2 - \sin z_1 \sin z_2 \\ &= \frac{e^{iz_1} + e^{-iz_1}}{2} \cdot \frac{e^{iz_2} + e^{-iz_2}}{2} - \frac{e^{iz_1} - e^{-iz_1}}{2i} \cdot \frac{e^{iz_2} - e^{-iz_2}}{2i} \\ &= \frac{e^{iz_1} + e^{-iz_1}}{2} \cdot \frac{e^{iz_2} + e^{-iz_2}}{2} + \frac{e^{iz_1} - e^{-iz_1}}{2} \cdot \frac{e^{iz_2} - e^{-iz_2}}{2} \end{aligned}$$

(2)

$$\begin{aligned} \sinh(1+2i) &= \sinh(1) \cosh(2i) + \cosh(1) \sinh(2i) \\ &= \sinh(1) \cos(2) + i \cosh(1) \sin(2) \end{aligned}$$

(2)

$$\begin{aligned}
 \sinh(1+2i) &= \sinh(i(2-i)) \\
 &= i \sin(2-i) \\
 &= i (\sin 2 \cosh 1 - \cos 2 \sinh 1) \\
 &= i \left(\sin 2 \cdot \cosh 1 - \cos 2 \cdot i \sinh 1 \right) \\
 &= i \left(\frac{e^2+1}{2e} \cdot \sin 2 - i \left(\frac{e^2-1}{2e} \right) \cos 2 \right) \\
 &= \left(\frac{e^2-1}{2e} \right) \cos 2 + i \left(\frac{e^2+1}{2e} \right) \sin 2
 \end{aligned}$$

$$(3) \sin\left(\frac{\pi}{4} + 2i\right) = \sin \frac{\pi}{4} \cosh 2 + \cos \frac{\pi}{4} \sinh 2i$$

$$\begin{aligned}
 &= \frac{1}{\sqrt{2}} \cosh 2 + \frac{1}{\sqrt{2}} \sinh 2i \\
 &= \frac{1}{\sqrt{2}} \cdot \cosh 2 + \frac{1}{\sqrt{2}} \cdot i \sinh 2 \\
 &= \frac{\cosh 2}{\sqrt{2}} + i \cdot \frac{\sinh 2}{\sqrt{2}}
 \end{aligned}$$

$$\begin{aligned}
 (4) \cosh\left(2 + \frac{\pi}{4}i\right) &= \cosh i \left(-2i + \frac{\pi}{4}\right) \\
 &= \cosh i \left(\frac{\pi}{4} - 2i\right) \\
 &= \cos\left(\frac{\pi}{4} - 2i\right) \\
 &= \cos \frac{\pi}{4} \cosh 2i + \sin \frac{\pi}{4} \sinh 2i \\
 &= \frac{1}{\sqrt{2}} \cosh 2i + \frac{1}{\sqrt{2}} \sinh 2i
 \end{aligned}$$

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 &= \cos\left(\frac{\pi}{4} - 2i\right) \\
 &= \cos \frac{\pi}{4} \cosh 2i + \sin \frac{\pi}{4} \sinh 2i \\
 &= \frac{1}{\sqrt{2}} \cosh 2i + \frac{1}{\sqrt{2}} \sinh 2i \\
 &= \frac{1}{\sqrt{2}} \cosh 2 + i \frac{1}{\sqrt{2}} \sinh 2
 \end{aligned}$$