

$$\left\langle \frac{p^2}{2m} \right\rangle = \int_{-\infty}^{\infty} \frac{p^2}{2m} P(p) dp$$

$$P(p) dp = \frac{e^{-\frac{E_p}{k_B T}} dp}{\int_{-\infty}^{\infty} e^{-\frac{E_p}{k_B T}} dp}$$

$$\therefore = \int_{-\infty}^{\infty} \frac{p^2}{2m} \cdot \frac{e^{-\frac{E_p}{k_B T}} dp}{\int_{-\infty}^{\infty} e^{-\frac{E_p}{k_B T}} dp}$$

$$\left(\int_{-\infty}^{\infty} e^{-\frac{E_p}{k_B T}} dp = \int_{-\infty}^{\infty} e^{-\frac{p^2}{2mk_B T}} dp = \sqrt{2\pi mk_B T} \right)$$

$$= \int_{-\infty}^{\infty} \frac{p^2}{2m} \cdot \frac{e^{-\frac{E_p}{k_B T}}}{\sqrt{2\pi mk_B T}} dp$$

$$= \frac{1}{2m\sqrt{2\pi mk_B T}} \int_{-\infty}^{\infty} p^2 \cdot e^{-\frac{p^2}{2mk_B T}} dp$$

$$= \frac{1}{2m\sqrt{2\pi mk_B T}} \cdot mk_B T \cdot \sqrt{2\pi mk_B T}$$

$$= \frac{k_B T}{2}$$

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