

⑫

$$(1) E_x = Ax, E_y = Ay, E_z = Az$$

$$\begin{aligned}\phi &= -\int E_x dx - \int E_y dy - \int E_z dz \\ &= -\frac{1}{2}Ax^2 - \frac{1}{2}Ay^2 - \frac{1}{2}Az^2 \\ &= \underline{-\frac{1}{2}A(x^2 + y^2 + z^2)}\end{aligned}$$

$$(2) E_x = 2Ax(y+z), E_y = A(x^2 - z^2), E_z = A(x^2 - z^2)$$

$$\begin{aligned}\phi &= -\int E_x dx - \int E_y dy - \int E_z dz \\ &= -Ax^2(y+z) - A(x^2y - \frac{1}{3}y^3) - A(x^2z - \frac{1}{3}z^3) \\ &= -\cancel{Ax^2y} - \cancel{Ax^2z} - \cancel{Ax^2y} + \frac{1}{3}Ay^3 - \cancel{Ax^2z} + \frac{1}{3}Az^3 \\ &= \underline{-2Ax^2y - 2Ax^2z + \frac{1}{3}Ay^3 + \frac{1}{3}Az^3}\end{aligned}$$

(3)

$$E_x = A(2x^2 - 3y^2 - 3z^2)x,$$

$$E_y = A(2y^2 - 3x^2 - 3z^2)y,$$

$$E_z = A(2z^2 - 3x^2 - 3y^2)z.$$

$$\phi = -\int E_x dx - \int E_y dy - \int E_z dz$$

$$\begin{aligned}&= -\int A(2x^3 - 3xy^2 - 3xz^2) dx - \int A(2y^3 - 3x^2y - 3yz^2) dy - \int A(2z^3 - 3x^2z - 3y^2z) dz \\ &= -A(\frac{1}{2}x^4 - \frac{3}{2}x^2y^2 - \frac{3}{2}x^2z^2) - A(\frac{1}{2}y^4 - \frac{3}{2}x^2y^2 - \frac{3}{2}y^2z^2) - A(\frac{1}{2}z^4 - \frac{3}{2}x^2z^2 - \frac{3}{2}y^2z^2) \\ &= \underline{-\frac{1}{2}A(x^4 + y^4 + z^4) + 3A(x^2y^2 + y^2z^2 + z^2x^2)}\end{aligned}$$