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p_x. p_y軌道が $\hat{l}_x$、$\hat{l}_y$ の固有状態であることを示す.

定義より.

$\hat{l}_+ = \hat{l}_x + i\hat{l}_y$ … ①

$\hat{l}_- = \hat{l}_x - i\hat{l}_y$ … ②

①+②より.

$\hat{l}_x = \frac{1}{2}(\hat{l}_+ + \hat{l}_-)$ … ③

①-②より.

$\hat{l}_y = \frac{1}{2}i(\hat{l}_+ - \hat{l}_-)$ … ④

また、$Y_{1,-1} = |1 -1\rangle$, $Y_{1,1} = |1.1\rangle$ … ⑤

③.④.⑤より.

$$\hat{l}_x Y_{px} = \frac{1}{2}(\hat{l}_+ + \hat{l}_-)\cdot\frac{1}{\sqrt{2}}(Y_{1,-1} - Y_{1,1})$$

$$= \frac{1}{2\sqrt{2}}(\hat{l}_+ + \hat{l}_-)(|1,-1\rangle - |1.1\rangle)$$

$$= \frac{1}{2\sqrt{2}}(\hat{l}_+|1,-1\rangle - \hat{l}_+|1.1\rangle + \hat{l}_-|1.-1\rangle - \hat{l}_-|1.1\rangle) \quad \cdots ⑥$$

$$\hat{l}_y Y_{py} = -\frac{1}{2}(\hat{l}_+ - \hat{l}_-)\cdot\frac{1}{\sqrt{2}}(Y_{1,-1} + Y_{1,1})$$

$$= \frac{1}{2\sqrt{2}}(\hat{l}_+ - \hat{l}_-)(|1.-1\rangle + |1.1\rangle)$$

$$= \frac{1}{2\sqrt{2}}(\hat{l}_+|1.-1\rangle + \hat{l}_+|1.1\rangle - \hat{l}_-|1.-1\rangle - \hat{l}_-|1.1\rangle) \quad \cdots ⑦$$

$\hat{l}_\pm |lm\rangle = \hbar\sqrt{(l\mp m)(l\pm m+1)}\ |l, m\pm 1\rangle$ より.

$\hat{l}_+ |1.1\rangle = \hbar\sqrt{0\cdot 3}\ |1.2\rangle = 0$

$\hat{l}_+ |1.-1\rangle = \hbar\sqrt{[1-(-1)][1+(-1)+1]}\ |1.-1+1\rangle$

$= \sqrt{2}\hbar |1.0\rangle$

$\hat{l}_- |1.1\rangle = \hbar\sqrt{(1+1)(1-1+1)}\ |1.1-1\rangle$

$= \sqrt{2}\hbar |1.0\rangle$

$\hat{l}_- |1.-1\rangle = \hbar\sqrt{(1+(-1))[1-(-1)+1]}\ |1.-1-1\rangle$

$= \hbar\sqrt{0\cdot 3}\ |1.2| = 0$

これらを⑥.⑦に代入して.

$\hat{l}_x Y_{px} = \frac{1}{2\sqrt{2}}(\sqrt{2}\hbar|1.0\rangle - 0 + 0 - \sqrt{2}\hbar|1.0\rangle) = 0$

$\hat{l}_y Y_{py} = \frac{1}{2\sqrt{2}}(\sqrt{2}\hbar|1.0\rangle + 0 - 0 - \sqrt{2}\hbar|1.0\rangle) = 0$

よって. $\hat{l}_x Y_{px}$, $\hat{l}_y Y_{py}$ 共に0となるため P_x. P_yの軌道は

それぞれ l_x, l_yの固有状態であり. この固有状態は0である。