8223036　栗山淳

応用数学　第 4 回　課題

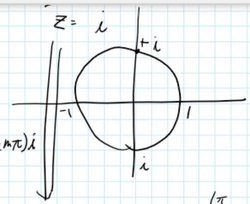
(2) $\sqrt{i} = i^{\frac{1}{2}}$

$\Updownarrow$ 逆関数

$z = w^2$

$z = r \cdot e^{i\theta}$ とすると　$r \cdot e^{i\theta} = 1 \cdot e^{(\frac{\pi}{2}+2m\pi)i}$

$z = i$



$\begin{cases} r = 1 \\ \theta = \frac{\pi}{2} + 2m\pi \\ (m: 整数) \end{cases}$

$z = 1 \cdot e^{(\frac{\pi}{2}+2m\pi)i}$

$\begin{cases} r = 1 \\ \theta = \frac{\pi}{2} + 2m\pi = \left(\frac{1}{2} + 2m\right)\pi \end{cases}$

よって

$W = \sqrt[n]{r} \cdot e^{\frac{i\theta}{n}} = \sqrt[2]{1} \cdot e^{\frac{i(\frac{1}{2}+2m)\pi}{2}}$

$= e^{(\frac{1}{4}+m)\pi i}$

$0 \leq (\frac{1}{4}+m)\pi < 2\pi$ のとき $n=2$ より 2個の Wがあるので

$m = 0, 1$ を代入すると

$W = e^{\frac{\pi}{4}i},\ e^{\frac{5}{4}\pi i}$

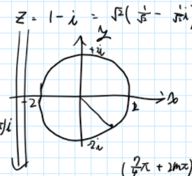
(3) $\sqrt{1-i} = (1-i)^{\frac{1}{2}}$

$\Updownarrow$ 逆関数

$z = W^2$

$z = re^{i\theta}$ とすると　$r \cdot e^{i\theta} = 2 \cdot e^{(\frac{7}{4}\pi+2m\pi)i}$

$z = 1 - i = \sqrt{2}\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right)$



$\begin{cases} r = 2 \\ \theta = \frac{7}{4}\pi + 2m\pi \\ (m: 整数) \end{cases}$

$z = 2 \cdot e^{(\frac{7}{4}\pi+2m\pi)i}$

$\begin{cases} r = 2 \\ \theta = \frac{7}{4}\pi + 2m\pi = \left(\frac{7}{4} + 2m\right)\pi \end{cases}$

よって

$W = \sqrt[n]{r} \cdot e^{\frac{i\theta}{n}} = \sqrt[2]{2} \cdot e^{\frac{(\frac{7}{4}+2m)\pi i}{2}}$

$= \sqrt{2} \cdot e^{(\frac{7}{8}+m)\pi i}$

$0 \leq \frac{7}{8} + m < 2\pi$ のとき $n=2$ より 2個 のWがあるので

$m = 0, 1$ を代入すると

$W = \sqrt{2} \cdot e^{\frac{7}{8}\pi i},\ \sqrt{2}e^{\frac{15}{8}\pi i}$

$\frac{1}{4} + \frac{1}{4}$　$\frac{2}{4}$　$\frac{1}{\sqrt{2}}$

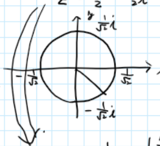
(4) $\frac{1}{\sqrt{1+i}} = (1+i)^{-\frac{1}{2}} = \left(\frac{1}{1+i}\right)^{\frac{1}{2}} = \left(\frac{1}{2} - \frac{1}{2}i\right)^{\frac{1}{2}}$

$\Updownarrow$ 逆関数

$z = W^2$

$z = re^{i\theta}$ とすると　$r \cdot e^{i\theta} = \frac{1}{\sqrt{2}} \cdot e^{(\frac{7}{4}\pi+2m\pi)i}$

$z = \frac{1}{2} - \frac{1}{2}i = \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right)$



$\begin{cases} r = \frac{1}{\sqrt{2}} \\ \theta = \frac{7}{4}\pi + 2m\pi \\ (m: 整数) \end{cases}$

$z = \frac{1}{\sqrt{2}} \cdot e^{(\frac{7}{4}\pi+2m\pi)i}$

$\begin{cases} r = \frac{1}{\sqrt{2}} \\ \theta = \frac{7}{4}\pi + 2m\pi = \left(\frac{7}{4} + 2m\right)\pi \end{cases}$

よって

$W = \sqrt[n]{r} \cdot e^{\frac{i\theta}{n}} = \sqrt[2]{\frac{1}{\sqrt{2}}} \cdot e^{\frac{(\frac{7}{4}+2m)\pi i}{2}}$

$= \sqrt[4]{\frac{1}{2}} \cdot e^{(\frac{7}{8}+m)\pi i}$

$= \frac{1}{\sqrt[4]{2}} \cdot e^{(\frac{7}{8}+m)\pi i}$

$\left(\left(\frac{1}{2}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}$

$\left(\frac{1}{2}\right)^{\frac{1}{4}}$

$0 \leq \frac{7}{8} + m < 2\pi$ のとき $n = 2$ より 2個の Wがあるので

$m = 0.1$ を代入すると

$W = \frac{1}{\sqrt[4]{2}} \cdot e^{\frac{7}{8}\pi i},\ \frac{1}{\sqrt[4]{2}} e^{\frac{15}{8}\pi i}$