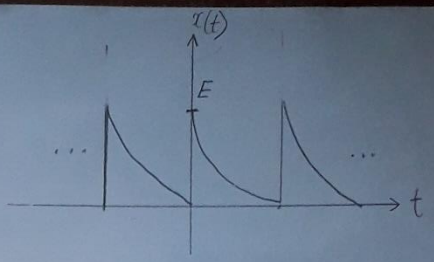
1. Zadatak



$T = 2s$

$$x(t) = \sum_{n=-\infty}^{+\infty} X_n e^{jn\omega_0 t}$$

$$X_n = \frac{1}{T}\int_{-\infty}^{+\infty} x(t) e^{-jn\omega_0 t}\,dt = \frac{1}{T}\int_{0}^{T} E e^{-2t} e^{-jn\omega_0 t}\,dt =$$

$$\frac{E}{T}\int_{0}^{T} e^{-(2+jn\omega_0)t}\,dt = \frac{E}{T}\cdot\frac{1}{-(2+jn\omega_0)} e^{-(2+jn\omega_0)t}\Big|_0^T$$

$$= \frac{-E}{T(2+jn\omega_0)}\left[e^{-(2+jn\omega_0)T} - 1\right]$$

$$= \frac{E}{2T+jn\frac{2\pi}{T}\cdot T}\left(1 - e^{-2T}\cdot e^{-jn\frac{2\pi}{T}T}\right) = \frac{E}{4+jn2\pi}\left(1 - e^{-4}\cdot e^{-jn2\pi}\right)$$

$\omega_0 = \frac{2\pi}{T}$

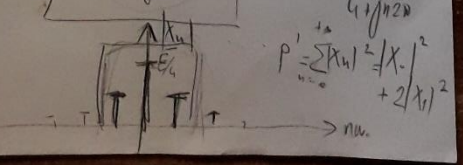
$\cos 2n\pi - j\sin 2n\pi$

$e^{-4}\cdot 1$

$$= \frac{E}{4+jn2\pi} = E\cdot\frac{1}{4+jn2\pi}$$

$$|X_n| = E\cdot\frac{1}{\sqrt{16+4n^2\pi^2}}$$

$P' = \sum_{n=-\infty}^{+\infty}|X_n|^2 = |X_-|^2 + 2|X_1|^2$



$$\omega_0 = \frac{2\pi}{T}$$

$$= \frac{-E}{T(2+jn\omega_0)}\left[e^{-(2+jn\omega_0)T} - 1\right] = \frac{E}{T(2+jn\omega_0)}\left(1 - e^{-2T}\cdot e^{-jn\omega_0 T}\right) =$$

$$= \frac{E}{4+jn2\pi}\left(1 - e^{-4}\left(\cos(n2\pi) - j\sin(2n\pi)\right)\right) = \frac{E}{4+j2n\pi} \quad \checkmark$$

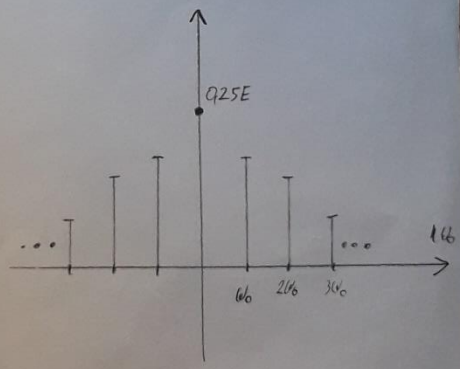
$$|X_n| = E\,\frac{1}{\sqrt{16 + (2n\pi)^2}}$$

$|X_{n=0}| = 0{,}25E$

$|X_{n=1}| = 0{,}134E$

$|X_{n=2}| = 0{,}076E$

$|X_{n=3}| = 0{,}052E$



(b) $$P' = \frac{1}{T}\int_{T} x^2(t)\,dt = \frac{1}{T}\int_{0}^{T} E^2 e^{-4t}\,dt = \frac{E^2}{T}\frac{-1}{4}e^{-4t}\Big|_{0}^{T} = 0{,}125E^2$$

$$P_{out} = |X_0|^2 + 2|X_1|^2 = 0{,}098E^2$$

$$\eta = \frac{P_{out}}{P'} = 0{,}784 = 78{,}4\,[\%]$$

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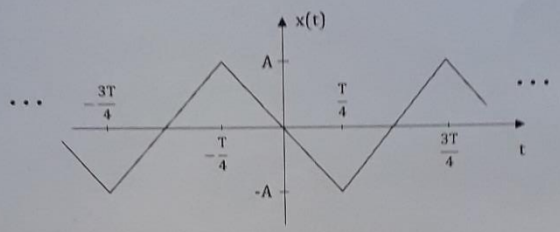
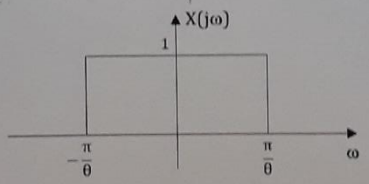
24. 1. 2023.

1. (40%)
   a) (25%) Odrediti (izvesti izraz) i grafički predstaviti amplitudski i fazni spektar signala $x(t)$, prikazanog na slici 1. Odrediti procenat snage koju nose prve četiri komponente u spektru signala $x(t)$.



Slika 1. Signal x(t) u Zadatku 1.a

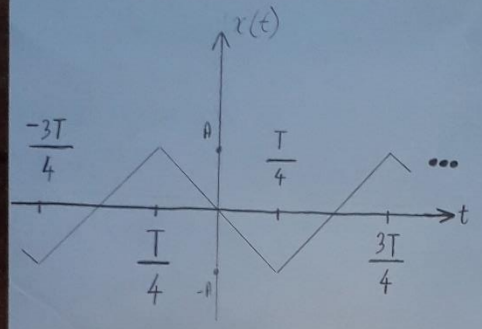
   b) (15%) Odrediti i nacrtati signal čiji je spektar $X(j\omega)$ prikazan na slici 2.

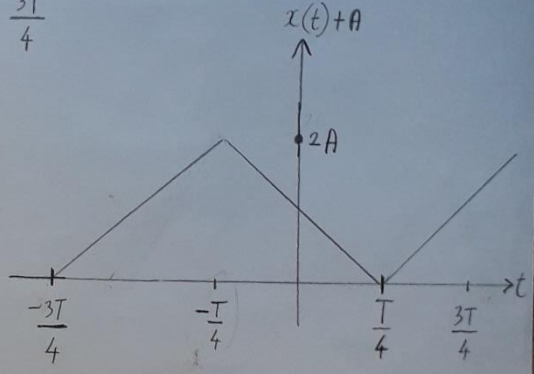


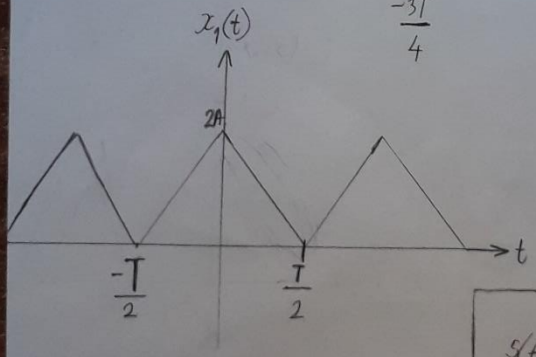
Slika 2. Spektar signala u Zadatku 1.b

2. (20%) Signal $x_1(t) = x(t) + \frac{A}{2}$, gdje je $x(t)$ naponski signal iz Zadatka 1.a, prikazan na slici 1, dovodi se na analogno/digitalni konvertor koji se sastoji od odmjerača, kvantizera i IKM (eng. PCM) kodera. Poznato je da je A=3V, T=8s, perioda odmjeravanja T_s=1s, te da se odmjeravanje vrši u trenucima nT_s, n=0,1,2... Nacrtati vremenski oblik signala na izlazu odmjerača. Kvantovanje se vrši uniformnim kvantizerom sa korakom kvantizacije 0.6V. Nacrtati karakteristiku kvantizera, te vremenski oblik signala na izlazu kvantizera. Nacrtati vremenski oblik signala na izlazu kodera. Smatrati da se vrši kodovanje sa povratkom na nulu (RZ).

1 Задача







$x_1(t)$

$x(t) = \left(\frac{a_0}{2} + \sum a_n \cos(n\omega_1 t)\right) - A$

$s(t) = kt + n = \frac{-4A}{T}t + 2A$

$s(t=0) = 2A \rightarrow n = 2A$

$s\left(t=\frac{T}{2}\right) = 0 \rightarrow k\frac{T}{2} + 2A = 0$

$k\frac{T}{2} = -2A$

$k = \frac{-4A}{T}$

1.

$$x_1(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t) \right)$$

$$a_n = \frac{2}{T} \int_{-T/2}^{T/2} s(t) \cos(n\omega_0 t)\, dt = \frac{2}{T} \cdot 2 \cdot \int_0^{T/2} \left(\frac{-4A}{T} t + 2A \right) \cos(n\omega_0 t)\, dt =$$

$$\frac{4}{T} \left[\frac{-4A}{T} \int_0^{T/2} t \cos(n\omega_0 t)\, dt = \boxed{\begin{array}{c} n\omega_0 t = y \\ t = y/n\omega_0 \\ dt = dy/n\omega_0 \end{array}} + 2A \int_0^{T/2} \cos(n\omega_0 t)\, dt = \boxed{\begin{array}{l} n\omega_0 t = y \\ dt = \frac{dy}{n\omega_0} \end{array}} \right]$$

$$= \frac{4}{T} \left[\frac{-4A}{T} \int_0^{T/2} \frac{1}{(n\omega_0)^2} y \cos(y)\, dy + 2A \int_0^{T/2} \cos(y) \frac{dy}{n\omega_0} \right] =$$

$$\frac{4}{T} \left[\frac{-4A}{T(n\omega_0)^2} \int_0^{T/2} y \cos(y)\, dy + \frac{2A}{n\omega_0} \int_0^{T/2} \cos(y)\, dy \right] =$$

$$\frac{4}{T} \left[\frac{-4A}{T(n\omega_0)^2} \left[y \sin(y) - \int_0^{T/2} \sin(y)\, dy \right] + \frac{2A}{n\omega_0} \sin(y) \Big|_0^{T/2} \right] =$$

$$\frac{4}{T} \left[\frac{-4A}{T(n\omega_0)^2} \left(n\omega_0 t \cdot \sin(n\omega_0 t) + \cos(n\omega_0 t) \Big|_0^{T/2} \right) + \frac{2A}{n\omega_0} \sin(n\omega_0 t) \Big|_0^{T/2} \right]$$

$$\frac{4}{T}\left[\frac{-4A}{T(n\omega)^2}\left(\cos\left(n\omega_0\frac{T}{2}\right)-1\right)\right] = \frac{4}{T}\frac{4A}{T(n\omega)^2}\left(1-\cos\left(n\omega_0\frac{T}{2}\right)\right)$$

$$= \frac{16A}{(Tn\omega)^2}\,2\sin^2\left(n\omega\frac{T}{4}\right) = 2A\,\frac{\sin^2\left(n\omega\frac{T}{4}\right)}{\left(n\omega_0\frac{T}{4}\right)^2}\ \checkmark$$

$$\frac{4^2}{4^2}$$

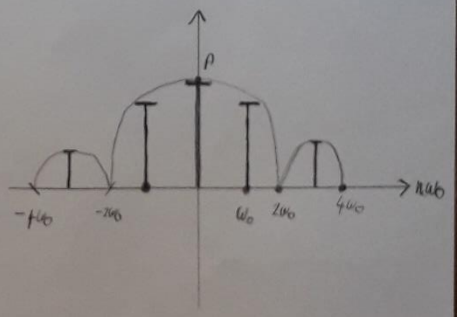
$$a_n = 2A\,\frac{\sin^2\left(n\omega_0\frac{T}{4}\right)}{\left(n\omega_0\frac{T}{4}\right)^2} = 2A\,\frac{\sin^2\left(n\frac{2\pi}{T}\frac{T}{4^2}\right)}{\left(n\frac{2\pi}{T}\frac{T}{4^2}\right)^2} = 2A\cdot\mathrm{sinc}\left(\frac{n}{2}\right)$$

$$\underline{X_{1n}} = \frac{a_n - jb_n}{2} = A\,\mathrm{sinc}\left(\frac{n}{2}\right) = A\,\frac{\sin^2\left(n\omega\frac{T}{4}\right)}{\left(n\omega\frac{T}{4}\right)^2}$$

$n\omega_0\frac{T}{4} = n\pi$, $n = 0, \pm1, \pm2, \ldots$

$n\omega_0 = \frac{4n\pi}{T} = 2\,\frac{2n\pi}{T}$

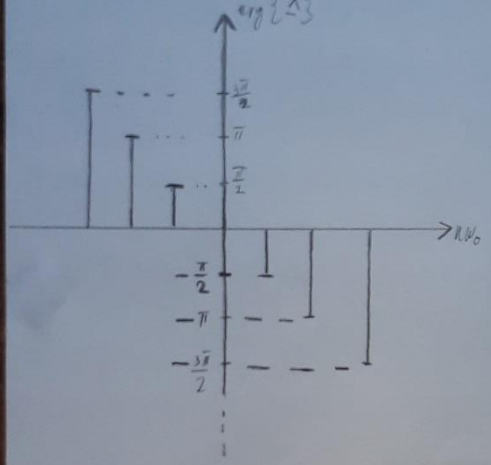
$\omega = 2\omega_0 n$



2.

$$-n\,\omega_0\frac{T}{4} = -n\,\frac{2\pi}{T}\,\frac{T}{4} = -\frac{n\pi}{2}$$

$$x_1(t) = \left|\underline{X}_{n,1}\right| e^{-jn\omega_0\frac{T}{4}}$$



$$P_{ord} = |X_0|^2 + 2\left(|X_1|^2 + |X_2|^2 + |X_3|^2\right)$$

$$P' = \frac{1}{T}\int s^2(t)\,dt$$

$$\eta = \frac{P_{ord}}{P'}$$

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13.9.2022.

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1. (25%) Odrediti i nacrtati spektar signala $x(t)=\cos\omega_0 t$, gdje je f_0=10Hz. Ako se signal $x(t)$ odmjerava idealnom povorkom Dirakovih impulsa minimalne frekvencije odmjeravanja f_s, odrediti i nacrtati spektar odmjerenog signala. Kolika je minimalna frekvencija odmjeravanja f_s?
Nacrtati spektar odmjerenog signala ako je frekvencija odmjeravanja signala $x(t)$ f_s=15Hz. Da li je u tom slučaju iz spektra odmjerenog signala moguća rekonstrukcija originalnog signala $x(t)$? Obrazložiti odgovor.

2. (35%)

a) (25%) Naponski signal $x(t)=A\cdot\cos\omega_0 t$, gdje je A=2V, f_0=1Hz, dovodi se na analogno/digitalni konvertor koji se sastoji od odmjerača, kvantizera i IKM (eng. PCM) kodera. Odmjeravanje se vrši u trenucima nT_s, gdje je T_s=125ms, n=0,1,2... Nacrtati vremenski oblik signala na izlazu odmjerača. Kvantovanje se vrši uniformnim kvantizerom sa korakom kvantizacije 0.8V. Nacrtati karakteristiku kvantizera, te vremenski oblik signala na izlazu kvantizera. Nacrtati vremenski oblik signala na izlazu kodera. Smatrati da se vrši kodovanje sa povratkom na nulu (RZ).

b) (10%) Na kompakt disku (CD) snimljeno je 3h muzičkog sadržaja (stereo zvuk), ukupne količine 3.1104GB. Prilikom snimanja, korisnim podacima dodati su biti za korekciju greške, ekstrakciju takta i kontrolni biti koji ukupno čine zaglavlje od 50%. Izračunati bitsku brzinu potrebnu za prenos korisnih podataka. Ukoliko je korištena frekvencija odmjeravanja od 48kHz, izračunati broj bita kojim je izvršeno kodovanje.

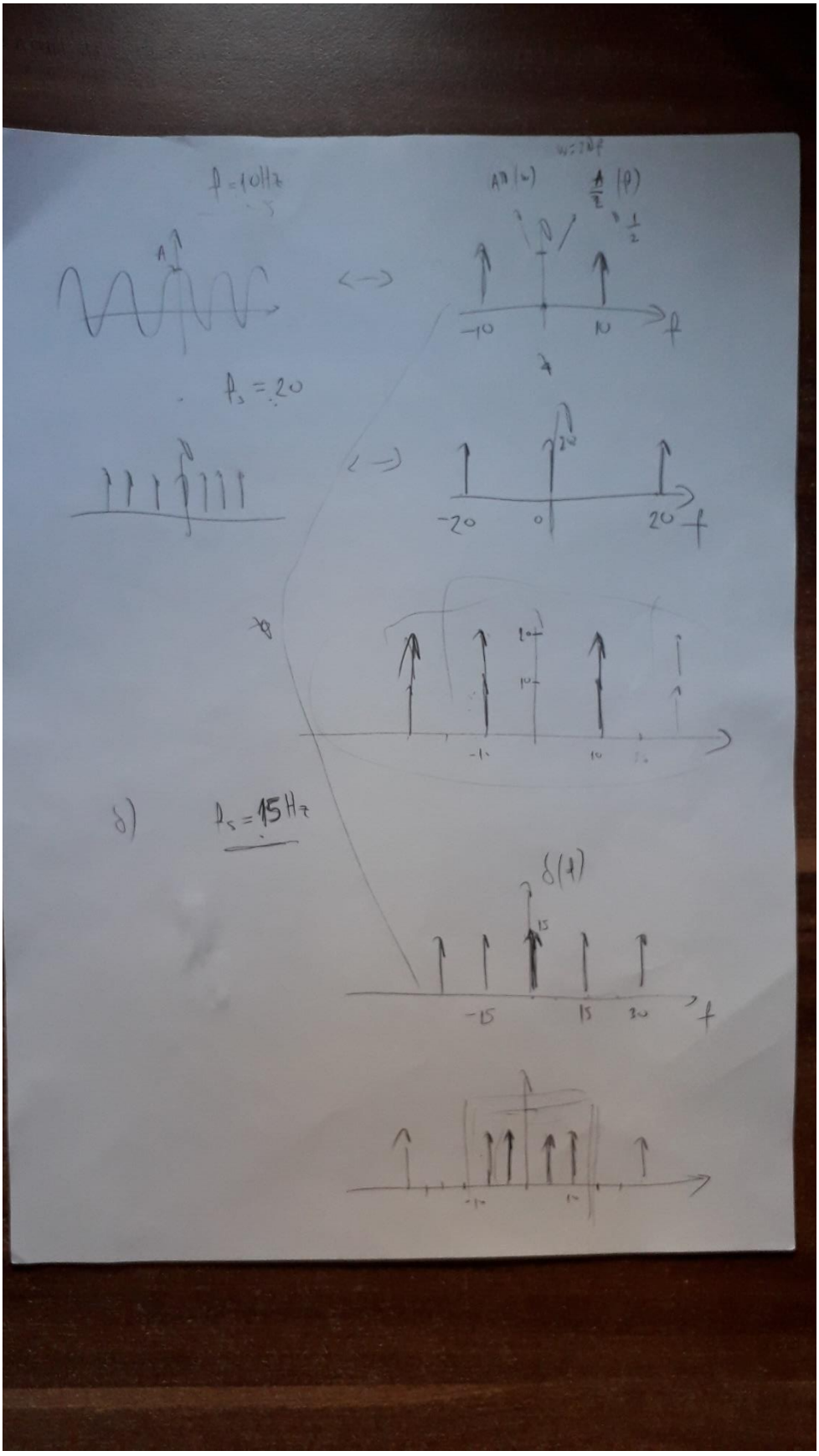
3. (25%) Dat je diskretni izvor bez memorije sa listom simbola S={A, B, C, D, E, F, G, H}. Poznate su vjerovatnoće P(A)=0.03, P(C)=0.19, P(D)=0.14, P(G)=0.23, P(H)=0.11, te je poznato da je P(B) : P(E) : P(F) = 2 : 3 : 1.

a) (4%) Odrediti entropiju i redundansu izvora.
b) (4%) Šenonovim postupkom odrediti kodne riječi. Koliko iznosi prosječna dužina kodnih riječi? Kodovati sekvencu HEFA.
c) (9%) Odrediti kodne riječi primjenom Hafmanovog koda u slučaju kodovanja binarnim simbolima. Koliko iznosi srednja dužina kodne riječi? Da li je dobijena srednja dužina kodne riječi minimalna moguća? Obrazložiti odgovor. Odrediti kodne riječi primjenom Hafmanovog koda u slučaju kodovanja ternarnim (kodna lista {0,1,2}) i kvaternarnim simbolima (kodna lista {0,1,2,3}).
d) (8%) Sekvencu HEFA kodovati aritmetičkim kodovanjem. Koliko bita zauzima poruka?

4. (15%)

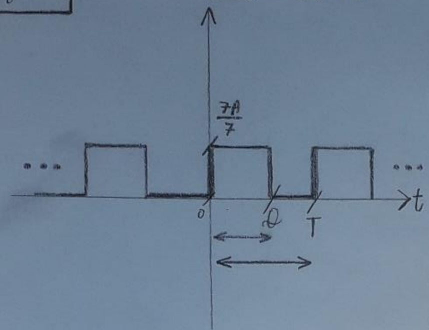
a) (4%) Ispitati da li je kôd {*a, c, ad, abb, bad, deb, bbcde*} jednoznačno dekodiv. Da li je trenutan?
b) (7%) Sekvencu ISPITISPIS PITIISPITI kodovati LZW algoritmom. Odrediti stepen uštede.
c) (4%) Kao zaštitni kôd u telekomunikacionom sistemu se koristi Hemingov kôd (12,8). Kodovati sekvencu 10011001. Kolika je vjerovatnoća greške koja se ne može detektovati ako se na kodovanu riječ doda još jedan bit za opštu provjeru na parnost, tj. ako se koristi Hemingov kôd (13,8)? Vjerovatnoća greške jednog bita je p=0.0001.

Vrijeme izrade: **180 minuta.**


f = 10Hz
A
-10
10
f
$f_s = 20$
-20
0
20
f
$f_s = 15Hz$
δ(f)
15
-15
15
30
f

1. Задатак

$x'(t) = x(t) + 7A$



Значи, приликом интеграла бирати смислено и разумно, тј. логички, а не само да се испоштује шаблон!

Неко стриктно да се шаблонски држи за формулу ће размисли пре него реализује исту...

$$x'(t) = \sum_{-\infty}^{+\infty} X_n' e^{jn\omega_0 t}$$

$$X_n' = \frac{1}{T}\int_{-T/2}^{T/2} x'(t) e^{-jn\omega_0 t}\,dt = \frac{1}{T}\int_0^{\theta} A e^{-jn\omega_0 t}\,dt = \frac{A}{T}\frac{-1}{jn\omega_0} e^{-jn\omega_0 t}\Big|_0^{\theta}$$

$$= \frac{-A}{Tjn\omega_0}\left(e^{-jn\omega_0\theta} - 1\right) = \frac{-A}{Tjn\omega_0} e^{-jn\omega_0\frac{\theta}{2}}\left(e^{jn\omega_0\frac{\theta}{2}} - e^{-jn\omega_0\frac{\theta}{2}}\right)$$

$$= \frac{-A}{Tjn\omega_0} e^{-jn\omega_0\frac{\theta}{2}}\left[-2j\sin\left(n\omega_0\frac{\theta}{2}\right)\right] = \frac{2A}{Tn\omega_0}\sin\left(n\omega_0\frac{\theta}{2}\right)e^{-jn\omega_0\frac{\theta}{2}}$$

$$= 2A\alpha\frac{\sin\left(n\omega_0\frac{\theta}{2}\right)}{n\omega_0\frac{\theta}{2}} e^{-jn\omega_0\frac{\theta}{2}} = 2A\alpha\,\mathrm{sinc}(n\alpha)\,e^{-jn\omega_0\frac{\theta}{2}}$$

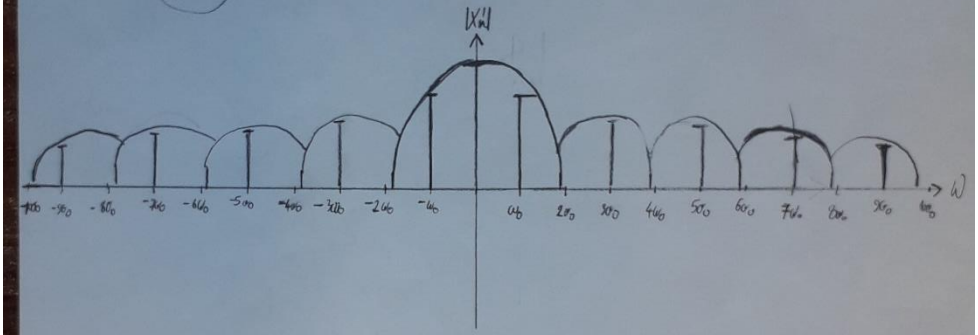
$$n\omega_0\frac{\theta}{2} = n\frac{2\pi}{T}\frac{\theta}{2} = n\pi\alpha$$

* Амплитудни спектър: $|F_n| = \text{нр.}\ |X_n| = \frac{|X_n'|}{2} = A\alpha \left| \frac{\sin\left(n\omega_0 \frac{\theta}{2}\right)}{n\omega_0 \frac{\theta}{2}} \right|$

Нули: $n\omega_0 \frac{\theta}{2} = \pm n\pi$, $n = 0, \pm 1, \pm 2, \pm 3, \pm \ldots$

$\omega \frac{\theta}{2} = n\pi$

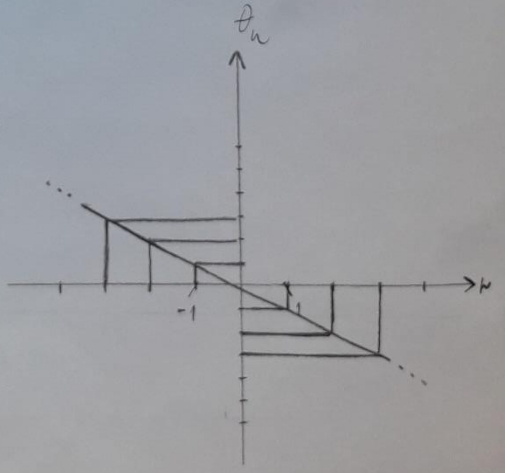
$\omega = \frac{2n\pi}{\theta} \Big/ \frac{T}{T} = \frac{2n\pi}{T} \frac{T}{\theta} = \omega_0 n \frac{1}{\alpha} = \omega_0 n \frac{7}{4} = 1{,}75\omega_0 n$



* Фазов спектър: $\vartheta_n =$

$-n\omega_0 \frac{\theta}{2} = -n\frac{2\pi}{T}\frac{\theta}{2} = -n\pi\alpha$

$|n\pi\alpha| > n$ ↓



Снага: $\downarrow x^2(t)$

$$P' = \frac{1}{T}\int_T x^2(t)\,dt = \frac{1}{T}\left[\int_0^{\vartheta} \frac{9A^2}{49}\,dt + \int_{\vartheta}^{T} \frac{16A^2}{49}\,dt\right] =$$

$$\frac{1}{T}\left[\frac{9A^2}{49}\,t\Big|_0^{\vartheta} + \frac{16A^2}{49}\,t\Big|_{\vartheta}^{T}\right] = \frac{1}{T}\left(\frac{9A^2}{49}\vartheta + \frac{16A^2}{49}T - \frac{16A^2}{49}\vartheta\right)$$

$$= \frac{1}{T}\left(\frac{16A^2}{49}T - \frac{7A^2}{49}\vartheta\right) = \frac{16A^2}{T49}T - \frac{7A^2}{49}\left(\frac{\vartheta}{T}\right)$$

$\alpha = \frac{4}{7}$

$$= 0{,}245\,[A^2]\ \surd$$

$$P_{izl} = |X_0' = 0|^2 + 2\left|\sum_{i=1}^{3} X_i'\right|^2 = 0{,}2148\,[A^2]$$

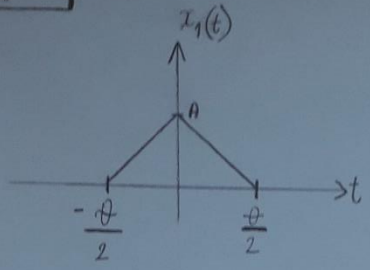
$X_0' = 0$

$$|X_1'| = A\alpha\left|\frac{\sin\left(\frac{2\pi}{T}\frac{\vartheta}{2}\right)}{\frac{2\pi}{T}\frac{\vartheta}{2}}\right| = 931\,[A]$$

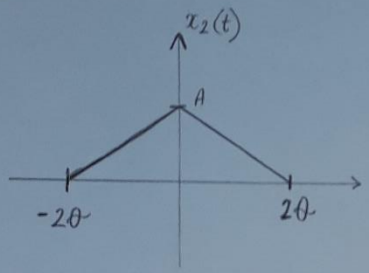
$$|X_2| = A\alpha\left|\frac{\sin\left(2\frac{2\pi}{T}\frac{\vartheta}{2}\right)}{2\frac{2\pi}{T}\frac{\vartheta}{2}}\right| = -907\,[A]$$

$$|X_3| = A\alpha\left|\frac{\sin\left(3\frac{2\pi}{T}\frac{\vartheta}{2}\right)}{3\frac{2\pi}{T}\frac{\vartheta}{2}}\right| = -908\,[A]$$

2 Задача а)



$$\longleftrightarrow \quad X_1(j\omega) = \frac{A\theta}{2} \frac{\sin^2\left(\omega\frac{\theta}{4}\right)}{\left(\omega\frac{\theta}{4}\right)^2}$$



$$\longleftrightarrow \quad X_2(j\omega) = 2A\theta \left(\frac{\sin(2\omega\theta)}{2\omega\theta}\right)^2$$

$$x(t) = x_2(t+2\theta) - x_2(t-2\theta)$$

$$X(j\omega) = 2A\theta \left(\frac{\sin(2\omega\theta)}{2\omega\theta}\right)^2 e^{j\omega 2\theta} - 2A\theta \left(\frac{\sin(2\omega\theta)}{2\omega\theta}\right)^2 e^{-j\omega 2\theta}$$

$$= 2A\theta \left(\frac{\sin(2\omega\theta)}{2\omega\theta}\right)^2 \left(e^{j\omega 2\theta} - e^{-j\omega 2\theta}\right) =$$

$$2A\theta \left(\frac{\sin(2\omega\theta)}{2\omega\theta}\right)^2 \left(2j\sin(2\omega\theta)\right) = \boxed{4A\theta \left(\frac{\sin(2\omega\theta)}{2\omega\theta}\right)^2 \sin(2\omega\theta)\, e^{j\frac{\pi}{2}}}$$

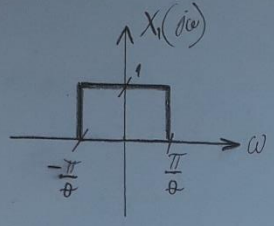
$z = j = e^{j\frac{\pi}{2}}$

$|z| = 1$

$\cos\phi = 0$ $\quad \sin\phi = 1$

3.

b)



$$\begin{cases} x_1(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)\, e^{+j\omega t}\, d\omega \\ X_1(j\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt \end{cases}$$ ima je

$$x_1(t) = \frac{1}{2\pi}\int_{-\pi/\theta}^{\pi/\theta} 1\cdot e^{+j\omega t}\, d\omega = \frac{1}{2\pi}\frac{+1}{jt}\, e^{+j\omega t}\Big|_{-\pi/\theta}^{\pi/\theta} =$$

$$\frac{+1}{2\pi jt}\left(-e^{-jt\frac{\pi}{\theta}} + e^{jt\frac{\pi}{\theta}}\right) = \frac{+1}{2\pi jt}\left(+2j\sin\left(t\frac{\pi}{\theta}\right)\right) =$$

$$\frac{1}{\pi t}\sin\left(t\frac{\pi}{\theta}\right)\ldots/\frac{\theta}{\theta} = \frac{1}{\theta}\frac{\sin\left(t\frac{\pi}{\theta}\right)}{\left(t\frac{\pi}{\theta}\right)}$$

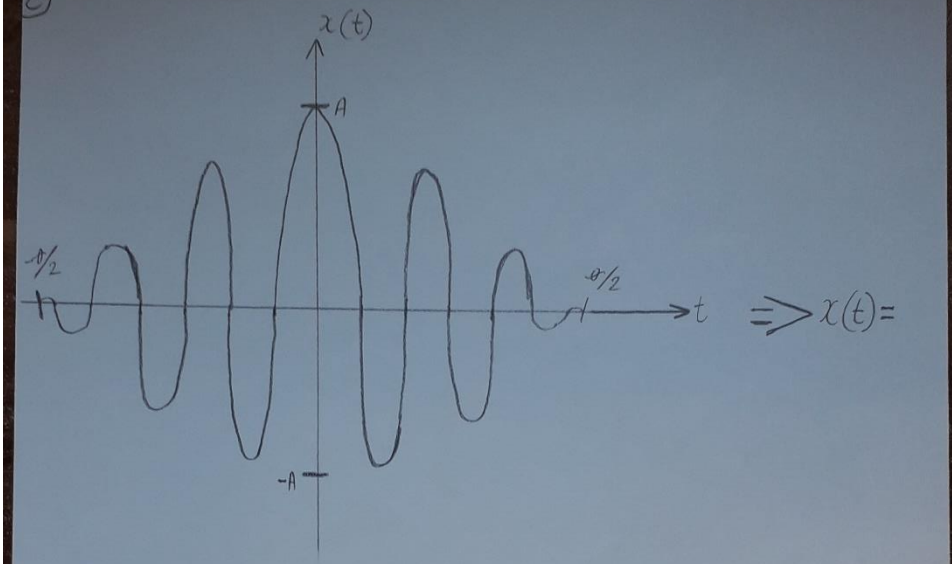
$$X(j\omega) = X_1\left(j(\omega+\omega_0)\right) + X_1\left(j(\omega+0)\right) + X_1\left(j(\omega-\omega_0)\right) \Rightarrow$$

$$x(t) = x_1(t)e^{j\omega_0 t} + x_1(t) + x_1(t)e^{-j\omega_0 t} = x_1(t)\left(e^{j\omega_0 t} + 1 + e^{-j\omega_0 t}\right)$$

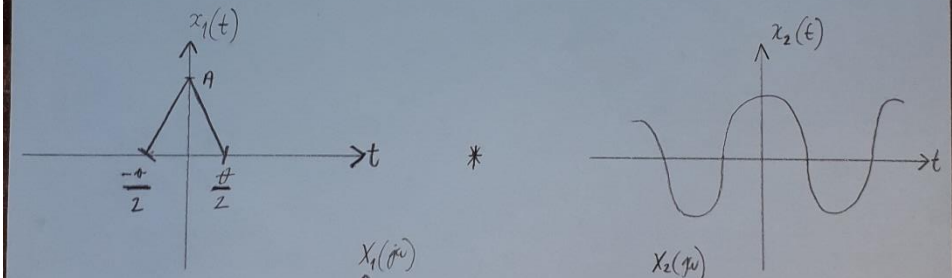
$$= x_1(t)\left(1 + 2\cos(\omega_0 t)\right) = \quad // \operatorname{sinc}(t/\theta)$$

$$\frac{1}{\theta}\frac{\sin\left(t\frac{\pi}{\theta}\right)}{t\frac{\pi}{\theta}}\left(1+2\cos(\omega_0 t)\right)$$

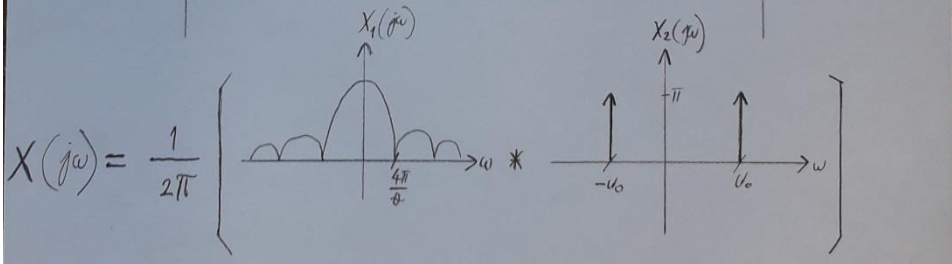
c)



$\Rightarrow x(t) =$



$*$



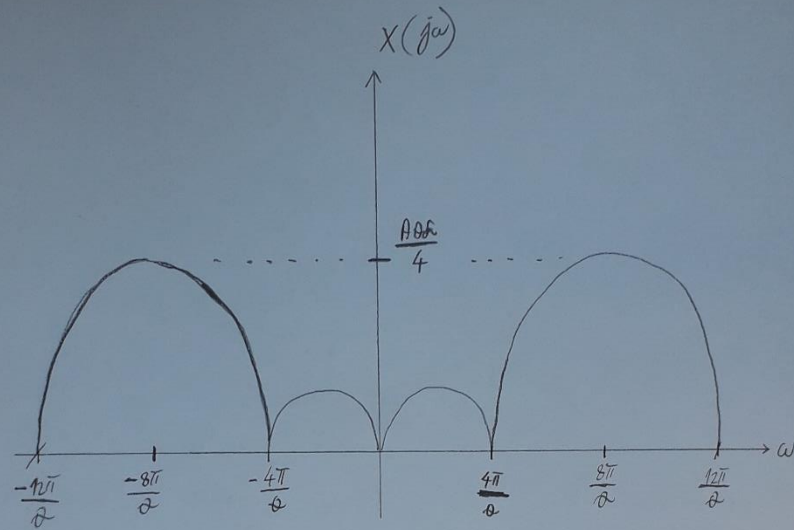
$$X(j\omega) = \frac{1}{2\pi}\left[\quad * \quad\right]$$

$$= \frac{1}{2\pi}\left[\frac{A\theta}{2}\,\frac{\sin^2\left(\omega\frac{\theta}{4}\right)}{\left(\omega\frac{\theta}{4}\right)^2} * \left(\pi\delta(\omega-\omega_0)+\pi\delta(\omega+\omega_0)\right)\right]$$

5.

Zapamti da je FT{Cos()} = Dirac Delta function!!!!

$\omega_0 = \frac{8\pi}{4}$



$$X(j\omega) = \frac{1}{2\pi}\left[\frac{A\theta}{2}\;\frac{\sin^2\left(\omega\frac{\theta}{4}\right)}{\left(\omega\frac{\theta}{4}\right)^2} * \left(\pi\delta(\omega-\omega_0) + \pi\delta(\omega+\omega_0)\right)\right]$$

Због + имамо 2 графика тј. један сигнал али линеарна комбинација од 2 сигнала... Због $\omega-\omega_0$ и $\omega+\omega_0$ графици су померени у односу на координатни почетак...

Нуле:

$$\omega\frac{\theta}{4} = k\pi$$

$$\omega = \frac{4\pi}{\theta}k$$

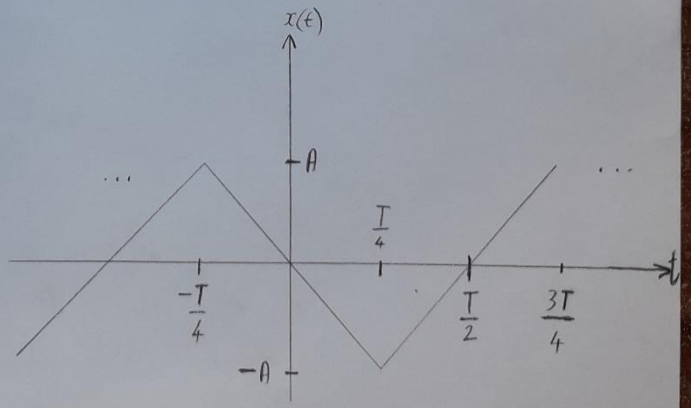
Амплитуда:

$$\frac{1}{2\pi}\cdot\frac{A\theta}{2}\cdot\pi\delta = \frac{A\theta\delta}{4}$$

2. Задатак Сигнал $x_1(t) = x(t) + \frac{A}{2}$, где је $x(t)$ биполарни сигнал из 1. задатка а), приказан на слици 1, доводи се на аналогно-дигитални конвертор који се састоји од одирача, квантизера и КМ кодера.

Познато је да је $A = 3[V]$, $T = 8[s]$, период одабирања $T_s = 1[s]$ тако да се одабирање врши у тренутцима nT_s, $n = 0, 1, 2, \ldots$

Израчунати вредности свих сигнала на излазу одирача. Квантовање се врши униформним квантизером са корацима квантизације $0,6[V]$. Израчунати карактеристике квантизера, те вредности свих сигнала на излазу квантизера. Израчунати вредности свих сигнала на излазу кодера. Скицирати до које врсте долазиће на повратном каналу.



Дато је:

$x_1(t) = x(t) + \frac{A}{2}$

$A = 3[V]$

$T = 8[s]$

$T_s = 1[s]$

$nT_s = 0, T_s, 2T_s, \ldots$

$\Delta n = 0,6$

$$\left.\begin{array}{l} x_1 = 0 \rightarrow y_1 = 0 \\ x_2 = \dfrac{T}{4} \rightarrow y_2 = -A \end{array}\right\} \longrightarrow \frac{y_2 - y_1}{x_2 - x_1} = \frac{-A}{\frac{T}{4}} = \frac{-4A}{T}$$ Или сразу

$$x(t) = \varkappa t + \kappa = \frac{-4A}{T} t$$

$x(t=0) = 0 \rightarrow \kappa = 0$ и $\varkappa$

$x(t = T/4) = -A \rightarrow \varkappa = \frac{-4A}{T}$

$$x_1(t) = \frac{-4A}{T} t - 2A + \frac{A}{2}$$

3. Задатак

$x(t) = A\cos\left(\omega t - \frac{\pi}{4}\right) + 2$

$\frac{T}{T_s} = \frac{4}{0,5} = 8$ одбирака у периоду.

$A = 4\,[V]$

$\Delta = \frac{6-(-2)}{5} = \frac{8}{5} = 1,6\,[V]$

$T = 4\,[s]$

$T_s = 0,5\,[s]$

$Q = 5$

Кратко са одбирањем: T_0?

$x(0) = A\cos\left(-\frac{\pi}{4}\right) + 2 = 4\cos\left(\frac{\pi}{4}\right) + 2 = 4,83\,[V]$

$x(T_s) = 4\cos\left(2\pi\frac{T_s}{T_0} - \frac{\pi}{4}\right) = 4\cos\left(\frac{\pi}{4} - \frac{\pi}{4}\right) + 2 = 6\,[V]$

$x(2T_s) = 4\cos\left(2\pi\frac{2T_s}{T_0} - \frac{\pi}{4}\right) + 2 = 4\cos\left(\frac{\pi}{2} - \frac{\pi}{4}\right) + 2 = 4,83\,[V]$

$x(3T_s) = 4\cos\left(2\pi\frac{3T_s}{T_0} - \frac{\pi}{4}\right) + 2 = 4\cos\left(\frac{3\pi}{4} - \frac{\pi}{4}\right) + 2 = 2\,[V]$

$x(4T_s) = 4\cos\left(\pi - \frac{\pi}{4}\right) + 2 = -0,83\,[V]$

$x(5T_s) = 4\cos\left(\frac{5\pi}{4} - \frac{\pi}{4}\right) + 2 = -2\,[V]$

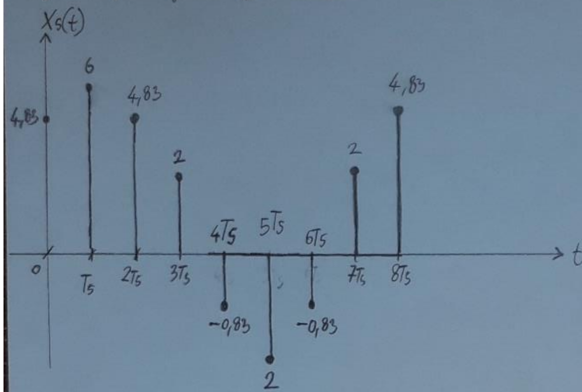
$x(6T_s) = 4\cos\left(\frac{6\pi}{4} - \frac{\pi}{4}\right) + 2 = -0,83\,[V]$

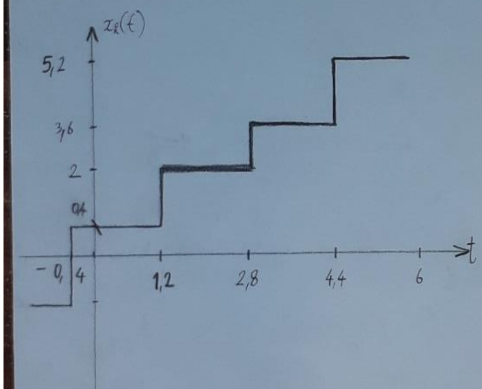
$x(7T_s) = 4\cos\left(\frac{7\pi}{4} - \frac{\pi}{4}\right) + 2 = 2\,[V]$

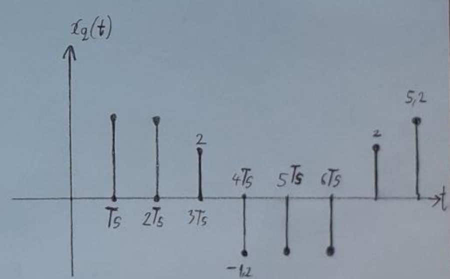
$x(8T_s) = 4,83\,[V]$

6.

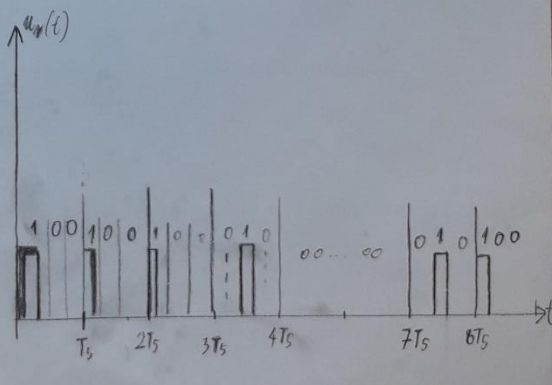
Сљедећи корак јесте да кодирамо одмјерке



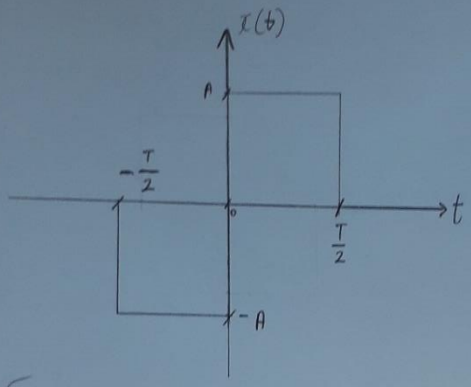


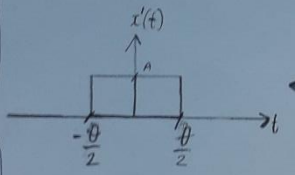


Ниво	0	1	2	3	4
Амплитуда	-1,2	0,4	2	3,6	5,2
Кодна ријеч	000	001	010	011	100



b)





$$\longleftrightarrow \quad A\theta \frac{\sin\left(\omega\frac{\theta}{2}\right)}{\omega\frac{\theta}{2}}$$

$$X'(j\omega) = \frac{AT}{2} \frac{\sin\left(\omega\frac{T}{4}\right)}{\omega\frac{T}{4}}$$

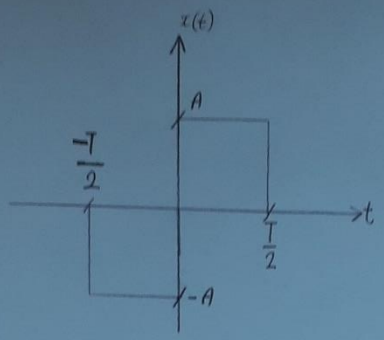
$$x(t) = x'\left(t-\frac{T}{4}\right) - x'\left(t+\frac{T}{4}\right)$$

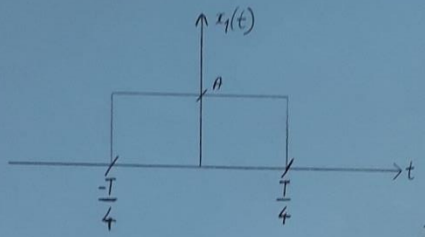
$$X(j\omega) = X'(j\omega)e^{-j\frac{T}{4}\omega} - X'(j\omega)e^{j\frac{T}{4}\omega} = X'(j\omega)\left(e^{-j\omega\frac{T}{4}} - e^{j\omega\frac{T}{4}}\right) =$$

$$X'(j\omega)\left(-2j\sin\left(\omega\frac{T}{4}\right)\right) = -X'(j\omega)\,2j\sin\left(\omega\frac{T}{4}\right) =$$

$$-AT j \frac{\sin^2(\omega T/4)}{\omega T/4} = AT \frac{\sin^2\left(\frac{\omega T}{4}\right)}{\frac{\omega T}{4}} e^{-j\frac{\pi}{2}}$$

b)



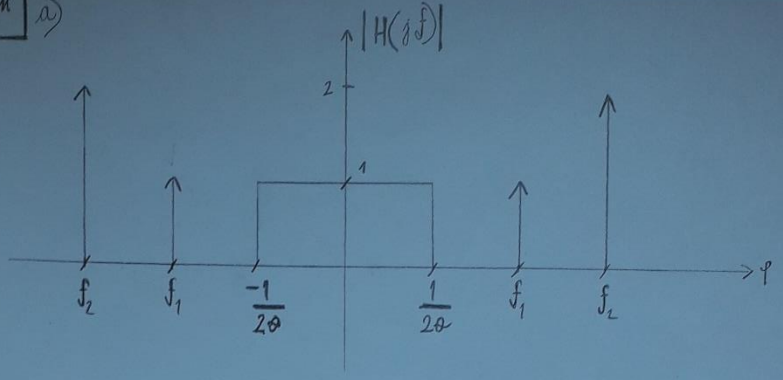


$$X_1(j\omega) = \int_{-T/4}^{T/4} x_1(t) e^{j\omega t}\, dt = \frac{1}{j\omega} e^{j\omega t} \Big|_{-T/4}^{T/4} = \frac{1}{j\omega}\left(e^{j\omega\frac{T}{4}} - e^{-j\omega\frac{T}{4}}\right)$$

$$= \frac{1}{j\omega}\left(+2j\sin\left(\omega\frac{T}{4}\right)\right) \ldots /\tfrac{T}{4}\tfrac{1}{T} =$$

$$\frac{2}{\omega 4T}\,\frac{\sin\left(\omega\frac{T}{4}\right)}{\omega\frac{T}{4}} =$$

1. Задатак а)



$$\mathcal{F}\left\{A\cos(\omega_1 t)\right\} = \mathcal{F}\left\{\frac{A}{2}e^{j\omega_1 t} + \frac{A}{2}e^{j\omega_1 t}\right\} =$$

$$A\pi\delta(\omega-\omega_1) + A\pi\delta(\omega+\omega_1) = \frac{A}{2}\delta(f-f_1) + \frac{A}{2}\delta(f+f_1)$$

$$\Rightarrow \boxed{A=2, \quad x_1(t)=2\cos(\omega_1 t)}$$

$$\mathcal{F}\left\{B\cos(\omega_2 t)\right\} = \mathcal{F}\left\{\frac{B}{2}e^{j\omega_2 t} + \frac{B}{2}e^{-j\omega_2 t}\right\} =$$

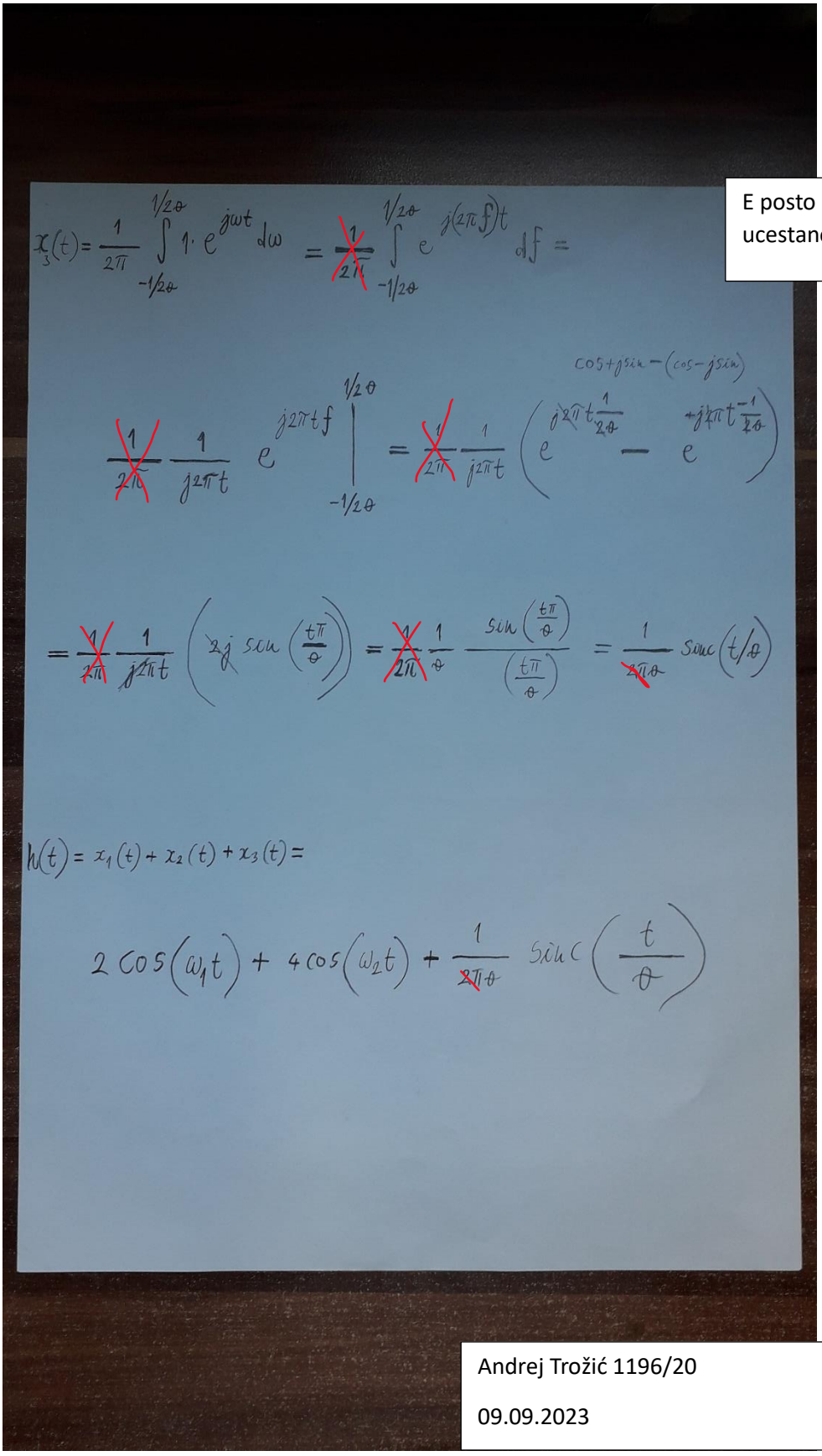
$$B\pi\delta(\omega-\omega_2) + B\pi\delta(\omega+\omega_2) =$$

$$\frac{B}{2}\delta(f-f_2) + \frac{B}{2}\delta(f+f_2) \Rightarrow \boxed{B=4, \quad x_2(t)=4\cos(\omega_2 t)}$$

$$\frac{1}{2\pi}\int_{-1/2\theta}^{1/2\theta} 1\cdot e^{-j\omega t}\,d\omega = \frac{1}{2\pi}\frac{-1}{jt}e^{-j\omega t}\Big|_{-1/2\theta}^{1/2\theta} = \frac{-1}{2\pi jt}\left(e^{-jt\frac{1}{2\theta}} - e^{jt\frac{1}{2\theta}}\right) =$$

$$\frac{+1}{\pi t}\sin\left(\frac{t}{2\theta}\right)$$

6.



$$x_3(t) = \frac{1}{2\pi} \int_{-1/2\theta}^{1/2\theta} 1 \cdot e^{j\omega t} \, d\omega = \frac{1}{2\pi} \int_{-1/2\theta}^{1/2\theta} e^{j(2\pi f)t} \, df =$$

$$\frac{1}{2\pi} \frac{1}{j2\pi t} e^{j2\pi t f} \Big|_{-1/2\theta}^{1/2\theta} = \frac{1}{2\pi} \frac{1}{j2\pi t} \left(e^{j2\pi t \frac{1}{2\theta}} - e^{+j2\pi t \frac{-1}{2\theta}} \right)$$

$\cos + j\sin - (\cos - j\sin)$

$$= \frac{1}{2\pi} \frac{1}{j2\pi t} \left(2j \sin\left(\frac{t\pi}{\theta}\right) \right) = \frac{1}{2\pi} \frac{1}{\theta} \frac{\sin\left(\frac{t\pi}{\theta}\right)}{\left(\frac{t\pi}{\theta}\right)} = \frac{1}{2\pi\theta} \operatorname{sinc}\left(t/\theta\right)$$

$$h(t) = x_1(t) + x_2(t) + x_3(t) =$$

$$2\cos(\omega_1 t) + 4\cos(\omega_2 t) + \frac{1}{2\pi\theta} \operatorname{sinc}\left(\frac{t}{\theta}\right)$$

E posto prelazi ***j*** frekvenciju iz ucestanost..

Andrej Trožić 1196/20

09.09.2023