Rossmoyne Senior High School

Semester Two Examination, 2017

Question/Answer booklet

MATHEMATICS SPECIALIST UNITS 3 AND 4

Section Two: Calculator-assumed

Your name ____

Your Teacher ____

Time allowed for this section

Reading time before commencing work:	ten minutes
Working time:	one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items:	pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters
Special items:	drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Additional working space pages at the end of this Question/Answer booklet are for planning or continuing an answer. If you use these pages, indicate at the original answer, the page number it is planned/continued on and write the question number being planned/continued on the additional working space page.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Markers use only		
Question	Maximum	Mark
9	6	
10	5	
11	5	
12	8	
13	6	
14	8	
15	6	
16	8	
17	9	
18	9	
19	12	
20	7	
21	9	
S2 Total	98	
S2 Wt (×0.6633)	65%	

Section Two: Calculator-assumed **65% (98 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 **(6 marks)**

Three planes have the following equations, where a and b are constants.

$$\begin{aligned} x + z &= 3 \\ 2x - y + 3z &= 9 \\ 2x + y + az &= b \end{aligned}$$

(a) Determine the coordinates of the point of intersection of the three planes when $a = -3$ and $b = 3$. (2 marks)

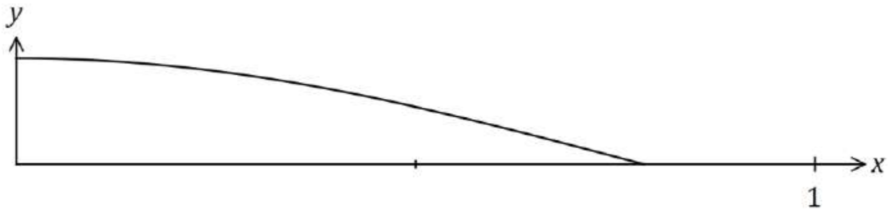
(b) Determine any restrictions on the constants a and b if the planes

(i) intersect in a straight line. (3 marks)

(ii) neither intersect at a point nor in a straight line. (1 mark)

Question 10 **(5 marks)**

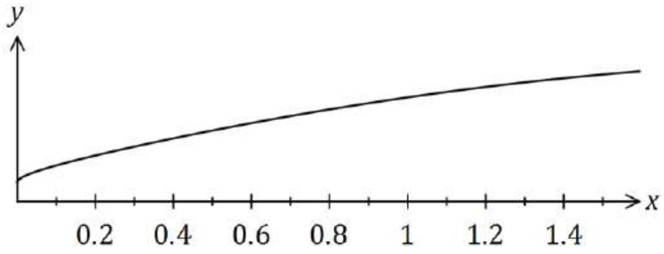
The graph of $y = 8\cos 2x$ is shown below.



Show that when the part of the curve between $x = 0$ and $x = \frac{\pi}{4}$ is rotated about the x axis, the volume of the solid generated is $8\pi^2$. Clearly indicate all trigonometric identities used.

Question 11 **(5 marks)**

Part of the graph of $y = e^{2\sin\sqrt{x}}$ is shown below.



(a) Use numerical integration with three equal width trapeziums to estimate the area between the curve, the x-axis, the y-axis and the line $x = 1.5$. (4 marks)

(b) Briefly explain whether your estimate is too small or large. (1 mark)

Question 12 **(8 marks)**

The position of particle P at any time s seconds is given by $\mathbf{r} = \mathbf{i} - 3\mathbf{j} + 12\mathbf{k} + s(-\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$, where distances are in metres.

(a) Show that when $s = 3$, P is 7 m from the origin. (2 marks)

When $s = 0$, particle Q leaves the point $(9, 13, 0)$ and moves with constant velocity, passing through the point $(6, 11, 1)$ one second later.

(b) Describe the path of Q as a vector function of time t seconds. (2 marks)

(c) Determine where the paths of P and Q cross, and explain whether the particles meet. (4 marks)

Question 13 **(6 marks)**

When used in a torch, the lifetime of a single AAA battery was observed to be normally distributed with a mean of μ hours and a standard deviation of σ hours.

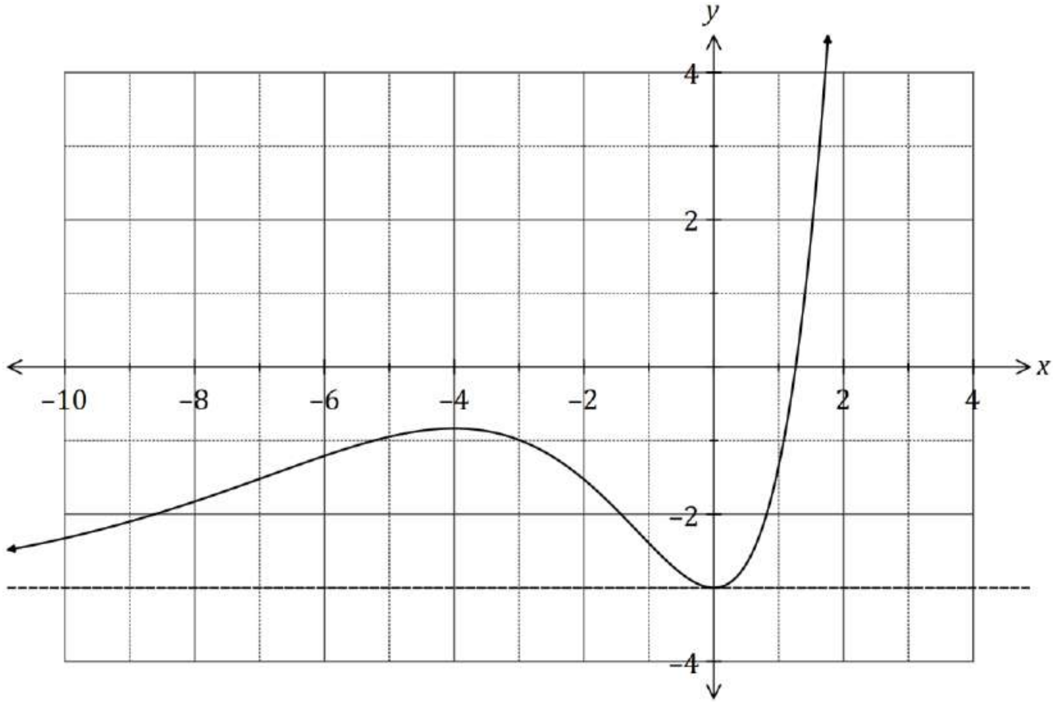
A student bought 40 boxes of these batteries, with 48 batteries in each box, and calculated the average lifetime for the batteries in each box. The mean of the averages was 8.31 hours and the standard deviation of the averages was 0.05 hours.

(a) Use this information to determine estimates for μ and σ. (3 marks)

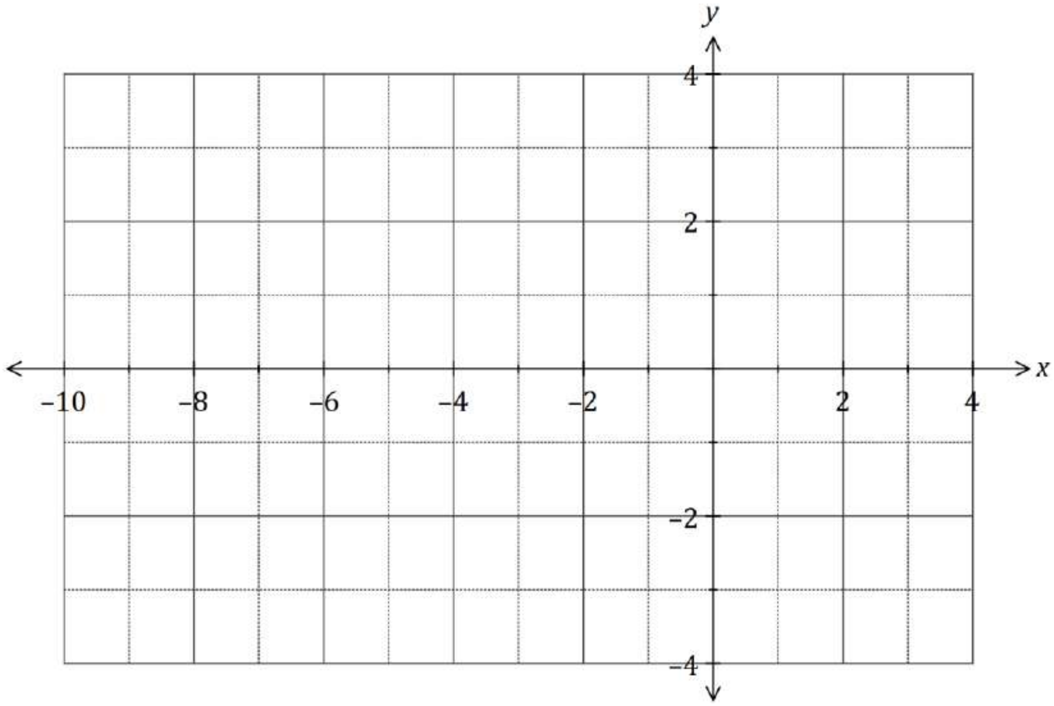
(b) The batteries in one of the boxes lasted for a total of 396 hours. Use this sample of 48 batteries to construct a 95% confidence interval for the lifetime of this type of AAA battery.(3 marks)

Question 14 **(8 marks)**

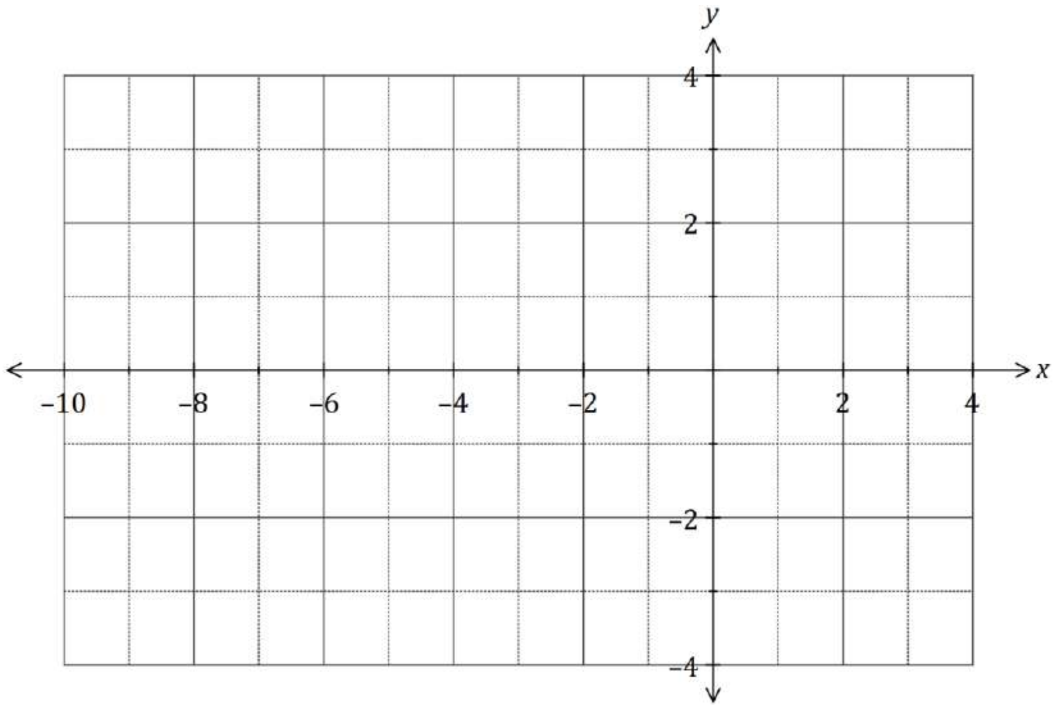
The graph of $y = f(x)$ has asymptote with equation $y = -3$, and is shown below.



(a) Sketch the graph of $y = \dfrac{1}{f(x)}$ on the axes below. (5 marks)



(b) Sketch the graph of $y = |f(|x|)|$ on the axes below. (3 marks)



Question 15 **(6 marks)**

A particle in Simple Harmonic Motion travels from rest to rest, a distance of 30m in 5 seconds.

Find the particle's:

(a) maximum speed. (3 marks)

(b) minimum acceleration and its displacement relative to the mean position at this instant. (3 marks)

Question 16 **(8 marks)**

Consider the function $h(x) = \dfrac{4}{x \ln x}$.

(a) Using your calculator, or otherwise, write down the exact area bounded by $y = h(x)$ and the lines $y = 4, x = e$ and $x = e^4$. (2 marks)

(b) $h(x)$ can be written in the form $f\big(g(x)\big) \cdot g'(x)$. State the functions f and g. (2 marks)

(c) Show how to use integration to obtain the answer to (a) without a CAS calculator. (4 marks)

Question 17 **(9 marks)**

The serving sizes of peanuts dispensed by a machine have been observed to have a mean of 220 g and a standard deviation of 3.4 g.

(a) A random sample of 75 serves of peanuts are taken from the machine and the serving size measured in each case. Determine the probability that

(i) the sample mean will be no more than 220.2 mL. (3 marks)

(ii) the total weight of peanuts dispensed will be between 16.482 kg and 16.509 kg. (3 marks)

(b) After servicing of the machine, an inspector plans to construct a 95% confidence interval for the serving size dispensed by the machine. Determine the sample size they should take so that the width of the interval is no more than 1.5 g, and note any assumptions made. (3 marks)

Question 18 **(9 marks)**

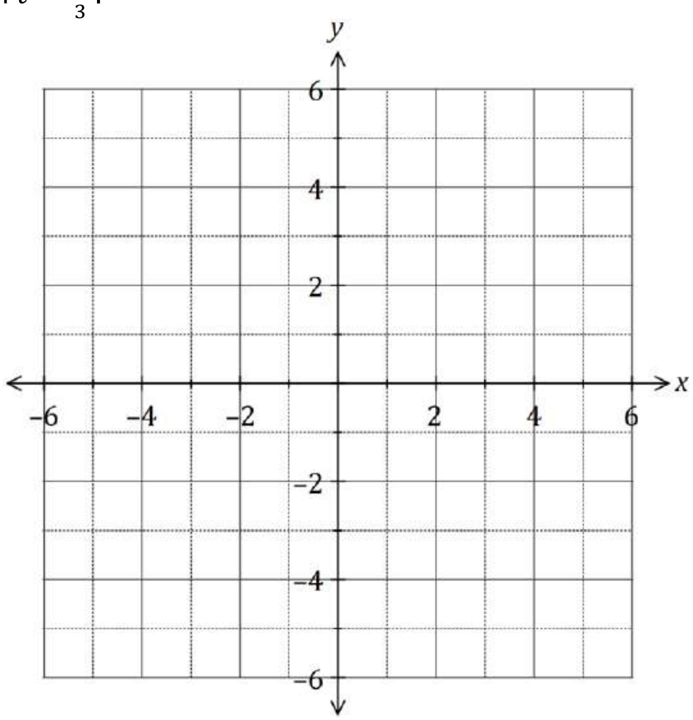
The position vector of a particle at time t seconds, $t \geq 0$, is shown below, with distances in cm.

$$\mathbf{r}(t) = \begin{pmatrix} 4\cos t - 1 \\ 3\sin t - 1 \end{pmatrix}$$

(a) Determine the speed of the particle when $t = \frac{2\pi}{3}$. (3 marks)

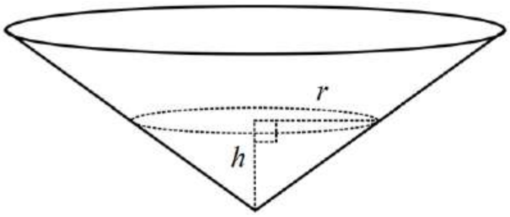
(b) Express the path of the particle as a Cartesian equation. (2 marks)

(c) Sketch the path of the particle on the axes below, indicating its position and the direction it is moving when $t = \frac{2\pi}{3}$. (4 marks)



Question 19 **(12 marks)**

An inverted right cone of diameter 90 cm and height 15 cm is being filled with water at a constant rate of 5π cm^3 per second. Initially the cone contains 56π cm^3 of water. Let r be the radius of the surface of the water and h be the depth of water after t seconds.



(a) Show that the relationship between the volume of water in the cone, V cm^3, and the radius is given by $V = \frac{\pi}{9}r^3$. (2 marks)

(b) Show that $\dfrac{dr}{dt} = \dfrac{15}{r^2}$. (2 marks)

(c) Determine the rate of change of radius r when $t = 5$. (2 marks)

(d) Use the differential equation from (b) to determine a relationship between the radius r and time t. (4 marks)

(e) Calculate the time required to completely fill the cone. (2 marks)

Question 20 **(7 marks)**

(a) Sketch on an Argand diagram the locus L of the complex number z given by $\arg z = \frac{\pi}{3}$.
(1 mark)

(b) A circle C, of radius 9, has its centre lying on L and just touches the line $\text{Im}(z) = 0$.

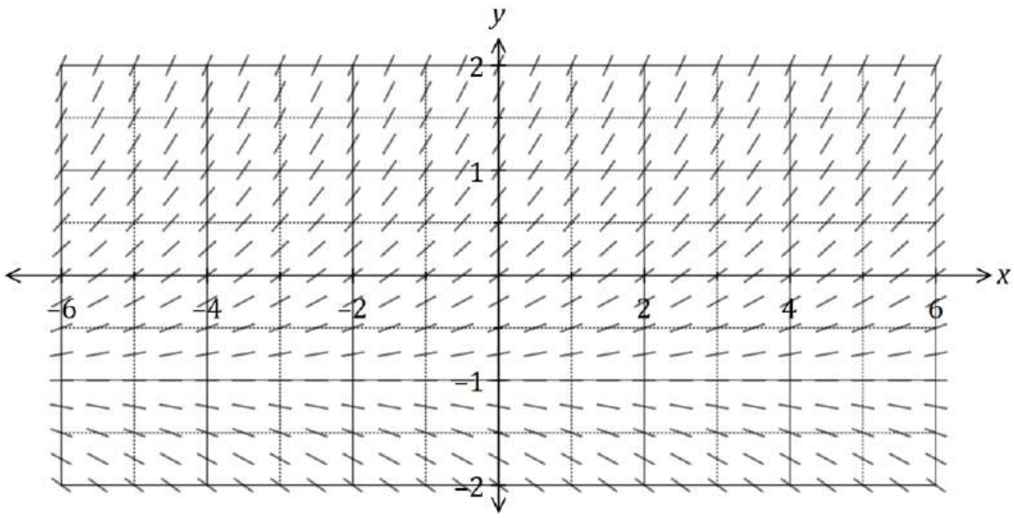
(i) Draw C on your diagram above. (2 marks)

(ii) Determine the equation of C in the form $|z - z_0| = k$. (2 marks)

(iii) The complex number z_1 lies on C. Determine the maximum value of $\arg z_1$, where $-\pi < \arg z_1 \leq \pi$.
(2 marks)

Question 21 **(9 marks)**

A first-order differential equation has a slope field as shown below.



(a) Sketch the solution of the equation that passes through $P(3, 0)$, where the value of the slope is 0.5. (3 marks)

(b) The general differential equation for the slope field is of the form below, where a and b are constants:

$$\frac{dy}{dx} = a(y + b)$$

Determine the solution to this equation that passes through P in the form $y = f(x)$. (6 marks)

End of questions

Additional working space

Question number: _________

Additional working space

Question number: _________