Semester Two Examination, 2019

Question/Answer booklet

MATHEMATICS SPECIALIST UNITS 3 AND 4

Section One: Calculator-free

If required by your examination administrator, please place your student identification label in this box

Student number: In figures | | | | | | | | |

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.
   Do not use erasable or gel pens.
3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Markers use only		
Question	Maximum	Mark
1	6	
2	6	
3	6	
4	6	
5	7	
6	7	
7	7	
8	7	
S1 Total	52	
S1 Wt (×0.6731)	35%	
S2 Wt	65%	
Total	100%	

See next page

Section One: Calculator-free **35% (52 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(6 marks)**

Let $u = 1 + i$, $v = 1 + \sqrt{3}i$ and $z = u^3v^2$.

(a) Determine the modulus and argument of z. (4 marks)

(b) Determine the smallest positive integer k such that z^k is real. (2 marks)

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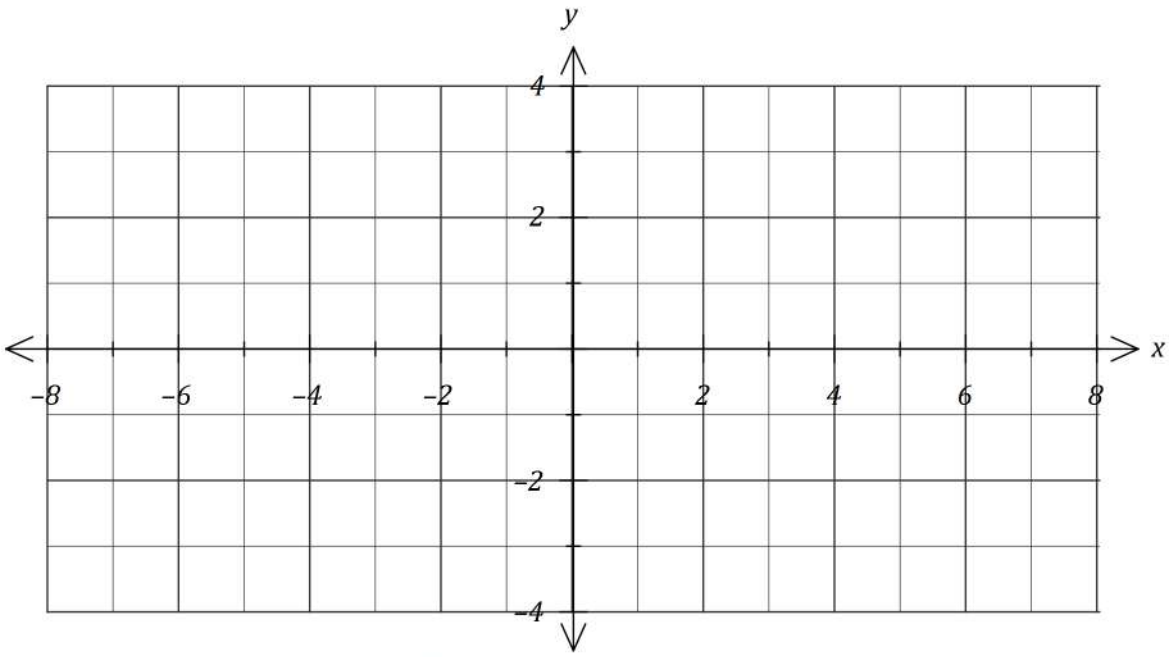
Question 2 **(6 marks)**

Let $f(x) = |x + 1| - 4$.

(a) Sketch the graph of $y = f(x)$ on the axes below. (2 marks)



(b) On the same axes, sketch the graph of $y = \frac{1}{f(x)}$. (3 marks)

(c) Determine all solutions to the equation $\big(f(x)\big)^2 = 1$. (1 mark)

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Question 3 **(6 marks)**

Functions f, g and h are defined as $f(x) = 1 + \sqrt{x}$, $g(x) = 4 - \ln x$ and $h(x) = f \circ g(x)$.

(a) Determine the defining rule for $g^{-1}(x)$ and its domain. (2 marks)

(b) Determine an expression for $h(x)$ and its domain and range. (4 marks)

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Question 4 **(6 marks)**

Let $g(z) = z^4 + 9z^2 + 14$ where $z \in \mathbb{C}$.

(a) Clearly show that $(z - \sqrt{7}i)$ is a factor of $g(z)$. (2 marks)

(b) Solve the equation $g(z) = 0$. (4 marks)

Question 5 **(7 marks)**

A particle leaves the origin at time $t = 0$ with initial velocity $v = 4$ and moves in a straight line with acceleration given by

$$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = x - 4$$

where v is its velocity and x is its displacement from the origin at time t, $t \geq 0$.

(a) Determine an equation for v as a function of x. (4 marks)

(b) Determine an equation for x as a function of time t. (3 marks)

See next page

Question 6 **(7 marks)**

Determine

(a) $\int \frac{5x+7}{x^2+3x+2}dx.$ (3 marks)

(b) $\int \frac{1}{4+x^2}dx$, using the substitution $x = 2\tan\theta$. (4 marks)

See next page

Question 7 **(7 marks)**

The graph of $y = x\sqrt{16 - x^2}$ in the first quadrant is shown below.

(a) Determine the area of the region shown between the curve and the x-axis. (4 marks)

(b) Determine the volume of the solid of revolution formed when the region between the curve, the x-axis, $x = 0$ and $x = 1$ is rotated about the x-axis. (3 marks)

See next page

Question 8 **(7 marks)**

Three planes have equations

$$2x + 2y + 7z = 5$$
$$2x + y - 7z = 1$$
$$4x + ay = 3$$

where a is a constant.

(a) Explain why the planes cannot intersect at a unique point when $a = 3$. (2 marks)

The acute angle between the planes $2x + y - 7z = 1$ and $4x + ay = 3$ is θ, where $\cos\theta = \frac{\sqrt{6}}{18}$.

(b) Determine the value of a. (5 marks)

End of questions

Supplementary page

Question number: _____