Kennedy Baptist College

Semester Two Examination, 2017

Question/Answer booklet

MATHEMATICS METHODS UNITS 1 AND 2

Section Two: Calculator-assumed

SOLUTIONS

Student Number: In figures | | | | | | | | |

In words ______________________________

Your name ______________________________

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Additional working space pages at the end of this Question/Answer booklet are for planning or continuing an answer. If you use these pages, indicate at the original answer, the page number it is planned/continued on and write the question number being planned/continued on the additional working space page.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed **65% (98 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 **(6 marks)**

(a) The tangent to the curve $y = 16 + 2x - 2x^2$ at $(2, 12)$ intersects the x-axis at $(a, 0)$. Determine the value of a. (3 marks)

Solution
$\frac{dy}{dx} = 2 - 4x = 2 - 4(2) = -6$ $y - 12 = -6(x - 2)$ or $y = -6x + 24$ $y = 0 \Rightarrow x = a = 4$
Specific behaviours
✓ determines gradient of tangent ✓ writes equation of tangent ✓ states value of a

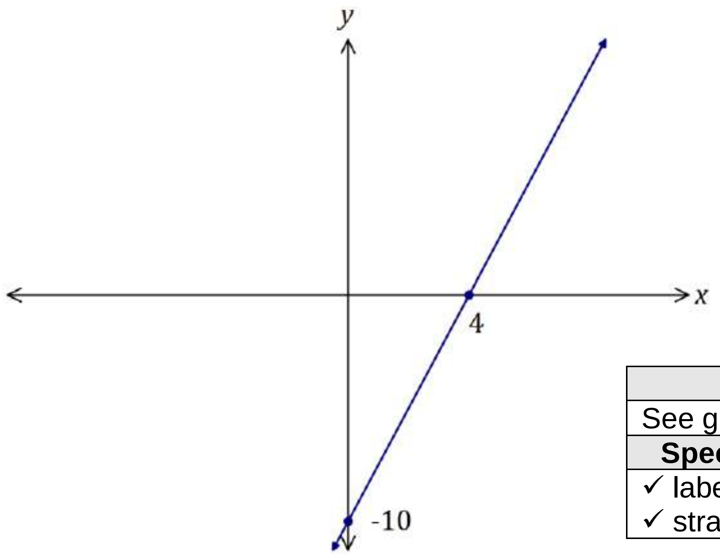
(b) If $f'(x) = 3x - x^3 - 1$ and $f(3) = 0$, determine $f(1)$. (3 marks)

Solution
$f(x) = \frac{3x^2}{2} - \frac{x^4}{4} - x + c$ $f(3) = 0 \Rightarrow c = \frac{39}{4}$ $f(1) = 10$
Specific behaviours
✓ antidifferentiates ✓ determines constant ✓ states value

Question 10 **(6 marks)**

Line L_1 has equation $5x - 2y = 20$.

(a) Sketch the graph of L_1. (2 marks)



Solution
See graph
Specific behaviours
✓ labelled intercepts ✓ straight line

(b) Determine the equation of line L_2 that is parallel to L_1 and passes through the point with coordinates $(-3, -11)$. (2 marks)

Solution
$5(-3) - 2(-11) = -15 + 22 = 7$ $5x - 2y = 5 \quad \left(\text{or } y = \frac{5}{2}x - \frac{7}{2}\right)$
Specific behaviours
✓ uses correct gradient ✓ states equation

(c) Determine the equation of line L_3 that is perpendicular to L_1 and has the same y-intercept as L_1. (2 marks)

Solution
$2x + 5y = k$ $2(0) + 5(-10) = -50$ $2x + 5y = -50 \quad \left(\text{or } y = -\frac{2}{5}x - 10\right)$
Specific behaviours
✓ uses correct gradient ✓ states equation

Question 11 **(8 marks)**

A group of 240 students were asked whether they had bought a drink or a snack from the school canteen. 72 had bought neither, 128 had bought a snack and 48 had bought both.

(a) Determine the number of students who only bought a drink. (2 marks)

Solution
$n(D \cup S) = 240 - 72 = 168$ $n(D \cap \bar{S}) = 168 - 128$ $= 40$
Specific behaviours
✓ Venn diagram or other method   ✓ correct number

(b) Determine the probability that a randomly chosen student from the group had bought

(i) a snack or a drink. (1 mark)

Solution
$P = \frac{168}{240} = \frac{7}{10}$
Specific behaviours
✓ correct probability

(ii) only a snack. (1 mark)

Solution
$P = \frac{128 - 48}{180} = \frac{80}{240} = \frac{1}{3}$
Specific behaviours
✓ correct probability

(iii) a snack given that they had bought a drink. (2 marks)

Solution
$P = \frac{48}{48 + 40} = \frac{48}{88} = \frac{6}{11}$
Specific behaviours
✓ uses conditional probability   ✓ correct probability

(c) For this group of students, are the events buying a snack and buying a drink independent? Justify your answer. (2 marks)

Solution
$P(S) = \frac{128}{240} = \frac{8}{15}$ and $P(S\|D) = \frac{6}{11}$ and so $P(S) \neq P(S\|D)$ and so events are NOT independent.
Specific behaviours
✓ uses probabilities   ✓ correct conclusion

Question 12 **(6 marks)**

The quadratic function $f(x) = ax^2 + bx + c$ passes through $P(5, 9)$ and has roots at $x = -4$ and $x = 7$.

(a) Determine the values of the constants a, b and c. (3 marks)

Solution
$f(x) = a(x+4)(x-7)$ $f(5) = 9 = a(9)(-2)$ $a = -\frac{1}{2}$ $f(x) = -\frac{1}{2}(x+4)(x-7)$ $= -\frac{1}{2}x^2 + \frac{3x}{2} + 14$ $a = -\frac{1}{2} = -0.5, \quad b = \frac{3}{2} = 1.5, \quad c = 14$
Specific behaviours
✓ writes in factored form ✓ uses given point to determine a ✓ expands and clearly states all values

(b) State the location of the y-intercept of the graph $y = -3f(x)$. (1 mark)

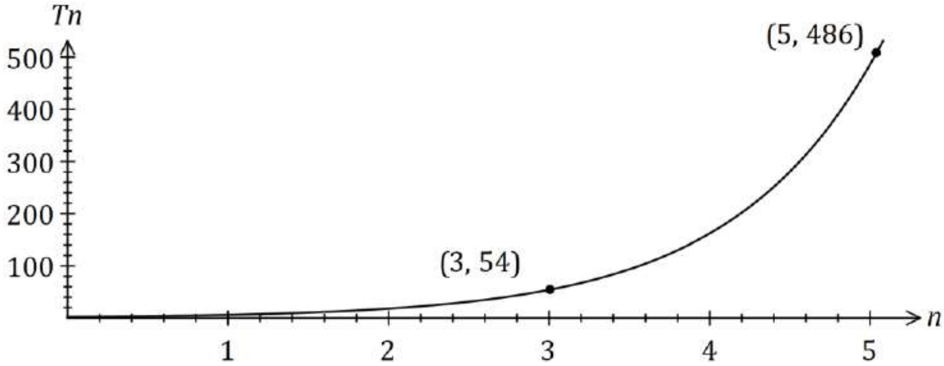
Solution
$y = -3(14) = -42$
Specific behaviours
✓ correct y-value

(c) State the location of the roots of the graph $y = f(4x)$. (2 marks)

Solution
$x = \frac{1}{4}(-4) = -1$ $x = \frac{1}{4}(7) = \frac{7}{4} = 1.75$
Specific behaviours
✓ uses correct horizontal dilation ✓ correct x-values

Question 13 **(8 marks)**

The number of followers of a social media influencer, counted at the start of five successive months, is shown in the exponential graph below.



The number of followers (T_n) at the start of month n can be modelled by the recursive equation $T_{n+1} = rT_n$, $T_1 = a$.

(a) Use the graph to determine the values of r and a. (3 marks)

Solution
$r^2 = 486 \div 54 = 9 \Rightarrow r = 3$ $a = T_3 \div 3^2 = 6$
Specific behaviours
✓ equation using r^2 ✓ states r ✓ states a

(b) Assuming the growth rate continues,

(i) how many followers are expected at the start of month 8? (1 mark)

Solution
$T_8 = 13122$
Specific behaviours
✓ states number

(ii) at the start of which month will the number of followers first exceed 1 million? (1 mark)

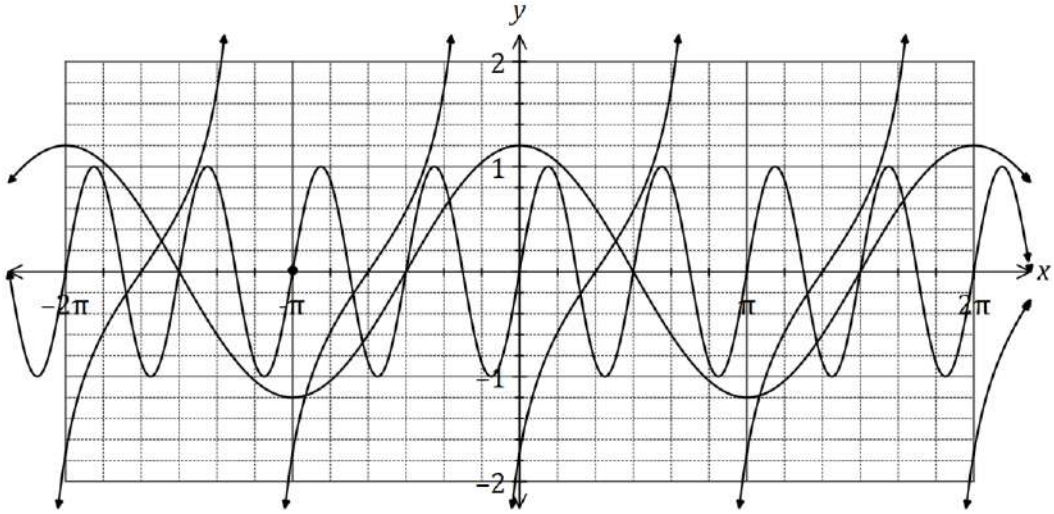
Solution
$T_n = 10^6 \Rightarrow n = 11.9 \Rightarrow \text{Start of month 12}$
Specific behaviours
✓ states month

(c) When the number of followers reached 2 million, the influencer fell out of favour and started to lose 40% of their followers each month. After how many months from this time will they have less than 1 000 followers? (3 marks)

Solution
$T_n = 2000000(0.6)^n$ $1000 = 2000000(0.6)^n$ $n = 14.88$ After 15 months
Specific behaviours
✓ forms equation for nth term ✓ solves equation ✓ rounds correctly

Question 14 **(7 marks)**

(a) The graphs of $y = a\cos x$, $y = \sin(bx)$ and $y = \tan(x + c)$ are shown below.



Determine the values of the constants a and b and the smallest positive value of the constant c. (3 marks)

Solution
$a = 1.2, \qquad b = 4, \qquad c = \dfrac{2\pi}{3}$
Specific behaviours
✓ value of a ✓ value of b ✓ value of c

(b) One day, the depth of water in a tidal basin was modelled by $d = 5.5 + 3.7\sin(0.5t - 0.6)$, where d was the depth in metres and t was the time, in hours, after midnight. For this day, determine

(i) the depth of water at 9.30 am. (2 marks)

Solution
$9.30 \text{ am} \Rightarrow t = 9.5$ $d(9.5) = 2.37 \text{ m}$
Specific behaviours
✓ converts time ✓ determines depth

(ii) the last time that the depth of water was 8.3 m. (2 marks)

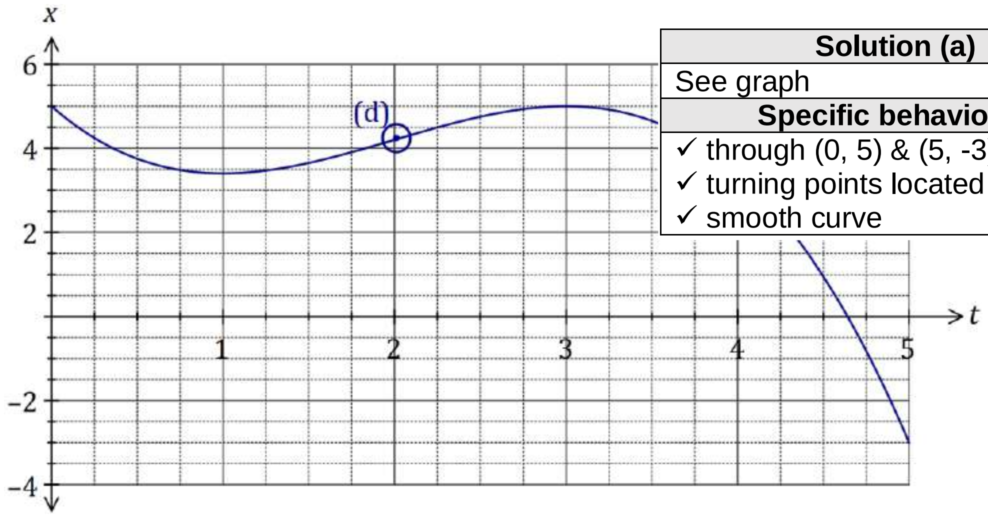
Solution
$5.5 + 3.7\sin(0.5t - 0.6) = 8.3$ Using CAS: $t = 18.33$ h, or 6.20 pm
Specific behaviours
✓ writes equation ✓ solves equation over stated domain

Question 15 **(8 marks)**

A particle is moving along a straight line so that its displacement, x metres, from a fixed point O after t seconds is given by

$$x = 5 - \frac{18t}{5} + \frac{12t^2}{5} - \frac{2t^3}{5}.$$

(a) Sketch the displacement of the particle on the axes below for $0 \le t \le 5$. (3 marks)



Solution (a)
See graph
Specific behaviours
✓ through (0, 5) & (5, -3) ✓ turning points located correctly ✓ smooth curve

(b) Determine the velocity of the particle when $t = 0.5$. (2 marks)

Solution
$v = \dfrac{-(6t^2 - 24t + 18)}{5}$ $v(0.5) = -\dfrac{3}{2} = -1.5\ \text{m/s}$
Specific behaviours
✓ differentiates displacement ✓ states velocity

(c) For how long during the first five seconds is the particle is moving towards O? (1 mark)

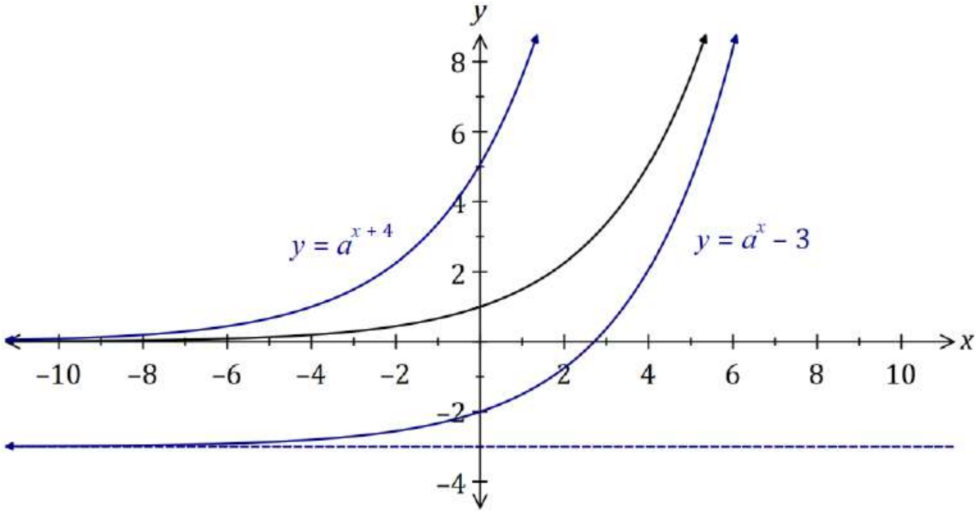
Solution
3.6 seconds (for $0 < t < 1$ and $3 < t < 4.6$).
Specific behaviours

(d) Circle the point on the graph where the particle is moving with the maximum velocity and explain what feature of the graph you used to choose this point. (2 marks)

Solution
See graph. Velocity is maximum at point where curve has greatest positive slope.
Specific behaviours
✓ indicates point of inflection ✓ explanation

Question 16 **(8 marks)**

The graph of $y = a^x$ is shown below, where a is a positive constant.



(a) On the same axes, sketch and label the graphs of

(i) $y = a^{x+4}$. (2 marks)

(ii) $y = a^x - 3$. (2 marks)

Solution
See graph
Specific behaviours
(i) ✓ y-int close to (0, 5); ✓ touches x-axis close to (-10, 0) (ii) ✓ y-int at (0, -2); ✓ clear asymptote at $y = -3$

(b) The graph of $y = a^{x+5}$ intersects the graph of $y = 0.6^x$ when $x = -2.2$.

Determine, giving your answers to 3 significant figures,

(i) the y-coordinate of the point of intersection. (1 mark)

Solution
$y = 0.6^{-2.2}$ $= 3.07657 \ldots$ $\approx 3.08 \ \text{(3sf)}$
Specific behaviours
✓ value that rounds to 3.08

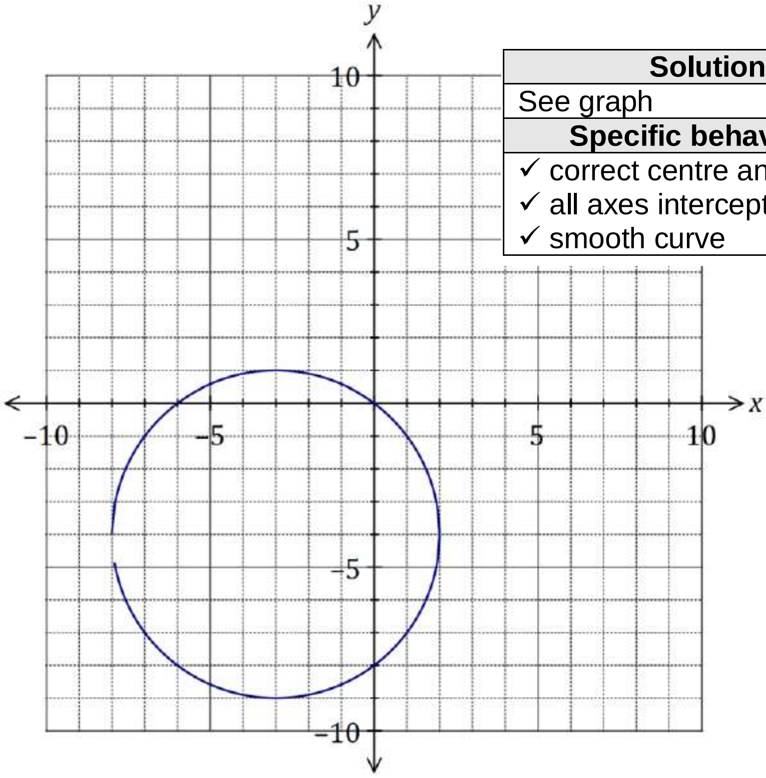
(ii) the value of the constant a. (3 marks)

Solution
$a^{-2.2+5} = 0.6^{-2.2}$ $a = 1.493859 \ldots$ $a \approx 1.49 \ \text{(3sf)}$
Specific behaviours
✓ writes equation ✓ writes solution to equation ✓ rounds answers to (b)(i) & (ii) correctly

Question 17 **(6 marks)**

(a) The variables x and y are related by $(x+3)^2 + (y+4)^2 = 25$.

(i) Sketch the graph of this relationship. (3 marks)



Solution
See graph
Specific behaviours
✓ correct centre and radius ✓ all axes intercepts ✓ smooth curve

(ii) How does the vertical line test indicate that y is not a function of x? (1 mark)

Solution
Any vertical line drawn between $-8 < x < 2$ will cut the graph more than once.
Specific behaviours
✓ explanation

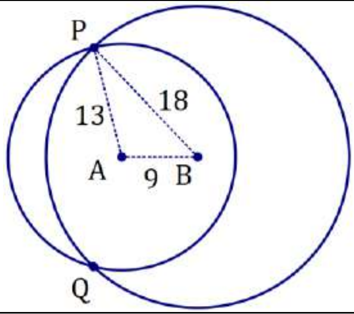
(b) State the domain and range of the function $f(x) = 3 - \sqrt{x-6}$. (2 marks)

Solution
$D_f = \{x : x \in \mathbb{R}, x \geq 6\}$ $R_f = \{y : y \in \mathbb{R}, y \leq 3\}$
Specific behaviours
✓ indicates restriction on x values ✓ indicates restriction on y values

Question 18 **(11 marks)**

Two circles of radii 13 cm and 18 cm have centres at A and B respectively. The centres are 9 cm apart and the circles intersect at P and Q.

(a) Sketch a diagram of the two circles and clearly show triangle ABP. (2 marks)

Solution

Specific behaviours
✓ sketch showing A, B, P & Q ✓ shows triangle ABP

(b) Show that $\angle PBA = 43.2°$, when rounded to one decimal place. (2 marks)

Solution
$\cos B = \dfrac{9^2 + 18^2 - 13^2}{2(9)(18)}$ $\angle PBA = 43.248$ $= 43.2°\ (1\text{dp})$
Specific behaviours
✓ substitutes correctly into cosine rule ✓ states angle to 2 or more dp then rounds

(c) Determine the length of the chord PQ to the nearest millimetre. (2 marks)

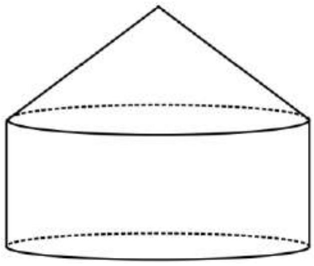
Solution
$\frac{1}{2}PQ = 18\sin 43.2°$ $PQ = 24.6\text{ cm}$
Specific behaviours
✓ uses right-angled trig ✓ states length, rounded to nearest mm

(d) Determine the area common to both circles. (5 marks)

Solution
Segment, centre B: $$\text{Angle: } 2\times 43.2\times\frac{\pi}{180}=1.509$$ $$A=\frac{1}{2}(18)^2(1.509-\sin(1.509))=82.86$$ Segment, centre A: $$\angle PAB=108.44°$$ $$\text{Angle: } 2\times 108.44\times\frac{\pi}{180}=3.785$$ $$A=\frac{1}{2}(13)^2(3.785-\sin(3.785))=370.54$$ Total: $$A=82.86+370.54=453.4\text{ cm}^2$$
Specific behaviours
✓ uses segment formula with angles in radians   ✓ states area of segment, centre B ✓ shows $\angle PAB$ ✓ states area of segment, centre A ✓ correct total area

Question 19 **(8 marks)**

A composite solid is made from a cone and a cylinder, both of height h cm and radius r cm, as shown below.



The dimensions are such that the sum of h and $3r$ is 36 cm.

(a) Show that the volume of the solid is given by $V = 48\pi r^2 - 4\pi r^3$. (3 marks)

Solution
$$h + 3r = 36 \Rightarrow h = 36 - 3r$$ $$V = \frac{1}{3}\pi r^2 h + \pi r^2 h$$ $$= \frac{4}{3}\pi r^2 h$$ $$= \frac{4}{3}\pi r^2 (36 - 3r)$$ $$= 48\pi r^2 - 4\pi r^3$$
Specific behaviours
✓ writes h in terms of r ✓ substitutes into sum of cone and cylinder volumes ✓ simplifies

(b) Use differentiation to determine the values of r and h that will maximise the volume of the solid, and state this maximum volume. (5 marks)

Solution
$$\frac{dV}{dr} = 96\pi r - 12\pi r^2$$ $$= 0 \text{ when } r = 0, 8$$ $$\text{Optimum value of } r = 8 \text{ cm}$$ $$h = 36 - 3(8) = 12 \text{ cm}$$ $$V(8) = 1024\pi \text{ cm}^3 \ (\approx 3217)$$
Specific behaviours
✓ differentiates ✓ determines root of derivative ✓ states optimum value of r ✓ calculates height h ✓ calculates volume V

Question 20 **(8 marks)**

Events A and B occur at random and it is known that $P(B) = 0.6$ and $P(A \cup B) = 0.72$.

(a) Determine $P(A)$ when

(i) A and B are mutually exclusive. (1 mark)

Solution
$P(A) = 0.72 - 0.6 = 0.12$
Specific behaviours
✓ states probability

(ii) $P(\bar{A} \cap B) = 0.55$. (1 mark)

Solution
$P(A) = P(A \cup B) - P(\bar{A} \cap B)$ $= 0.72 - 0.55$ $= 0.17$
Specific behaviours
✓ uses probability laws ✓ states probability

(iii) A and B are independent. (3 marks)

Solution
Let $P(A \cap B) = x$ and $P(A) \times P(B) = P(A \cap B)$ $(0.12 + x) \times 0.6 = x$ $x = 0.18$ $P(A) = 0.12 + 0.18 = 0.3$
Specific behaviours
✓ uses probability law for independence ✓ determines $P(A \cap B)$ ✓ states probability

(b) Determine $P(B|A)$ if $P(B|\bar{A}) = 0.65$. (3 marks)

Solution
Let $P(\bar{A} \cap B) = x$ and $P(\overline{A \cup B}) = 0.28$ $\frac{x}{x + 0.28} = 0.65$ $x = 0.52$ $P(A \cap B) = 0.6 - 0.52 = 0.08$ $P(B|A) = \frac{0.08}{0.12 + 0.08} = \frac{0.08}{0.2} = 0.4$
Specific behaviours
✓ determines $P(\bar{A} \cap B)$ ✓ determines $P(A \cap B)$ ✓ states probability

Question 21 **(8 marks)**

Seven different letters are selected from the thirteen in the word TROUBLEMAKING. The order in which the letters are selected is not important, so that the selection TROUBLE is the same as the selection OUBLERT, and so on.

(a) Determine the number of different selections

(i) of seven letters. (2 marks)

Solution
$n = \binom{13}{7} = 1716$
Specific behaviours
✓ uses combination ✓ correct number

(ii) of seven letters that contain one vowel and six consonants. (2 marks)

Solution
$n = \binom{5}{1} \times \binom{8}{6} = 5 \times 28 = 140$
Specific behaviours
✓ splits selections ✓ multiplies selections

(b) Determine the probability that a random selection of seven different letters

(i) includes the letters T, R and O. (2 marks)

Solution
$P = \dfrac{\binom{3}{3} \times \binom{10}{4}}{1716} = \dfrac{210}{1716} = \dfrac{35}{286} \approx 0.1224$
Specific behaviours
✓ selects (i) M & A (ii) other four ✓ states probability

(ii) includes at least one vowel. (2 marks)

Solution
$P = 1 - \dfrac{\binom{5}{0} \times \binom{8}{7}}{1716} = 1 - \dfrac{8}{1716} = \dfrac{1708}{1716} = \dfrac{427}{429} \approx 0.9953$
Specific behaviours
✓ selects no vowels ✓ states probability

End of questions

Additional working space

Question number: _________

Additional working space

Question number: _________

Additional working space

Question number: _________