



PERTH MODERN SCHOOL

Exceptional schooling. Exceptional students.

INDEPENDENT PUBLIC SCHOOL

WAEP Semester One Examination, 2018

Question/Answer booklet

MATHEMATICS SPECIALIST UNIT 1

**Section Two:
Calculator-assumed**

If required by your examination administrator, please
place your student identification label in this box

Student number: In figures

--	--	--	--	--	--	--	--

In words

Your name

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper,
and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	53	35
Section Two: Calculator-assumed	13	13	100	97	65
Total					100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
4. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you do not use pencil, except in diagrams.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed

65% (97 Marks)

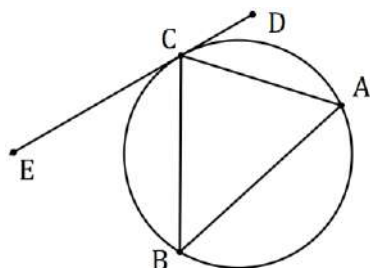
This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

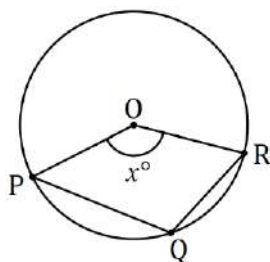
Question 9

(7 marks)

- (a) In the diagram below, points A and B lie on a circle, DE is a tangent to the circle at C , $\angle ACD = 64^\circ$ and $\angle BAC = 49^\circ$. Determine the sizes of $\angle ABC$, $\angle BCE$ and $\angle BCA$. (3 marks)



- (b) In the next diagram, P, Q and R lie on a circle with centre O and $\angle POR = x^\circ$. Determine, with reasons, the size of $\angle PQR$ in terms of x . (4 marks)



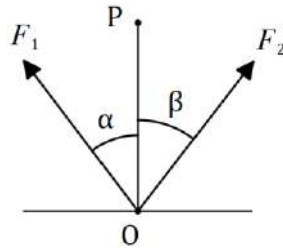
Question 10**(7 marks)**

- (a) A body travels with a velocity $12\mathbf{i} - 5\mathbf{j} \text{ ms}^{-1}$. Determine its speed and the bearing on which it is moving, assuming the positive y -axis to be due north. (3 marks)

- (b) Given that $\lambda(5\mathbf{i} - 2\mathbf{j}) + \mu(-7\mathbf{i} + 4\mathbf{j}) = 25\mathbf{i} - 13\mathbf{j}$, determine the values of λ and μ . (4 marks)

Question 11**(8 marks)**

Two forces, $F_1 = 230 \text{ N}$ and $F_2 = 272 \text{ N}$, act on a body at O , and make angles of $\alpha = 15^\circ$, and $\beta = 20^\circ$ respectively with the vertical OP , as shown in the diagram below.



- (a) Determine the magnitude of the resultant force and the angle it makes with the vertical. (5 marks)
- (b) The magnitude of F_2 is to be adjusted so that the direction of the resultant is vertical. Determine the required magnitude of F_2 . (3 marks)

Question 12**(6 marks)**

The largest Australian family recently met with the largest English family. Between them, these two families had 37 children.

- (a) Three of the children were chosen at random to feature in a TV documentary about the two families. Determine the number of different selections of three children that were possible. (1 mark)

- (b) Prove that at least four of the children were born in the same month of the year. (2 marks)

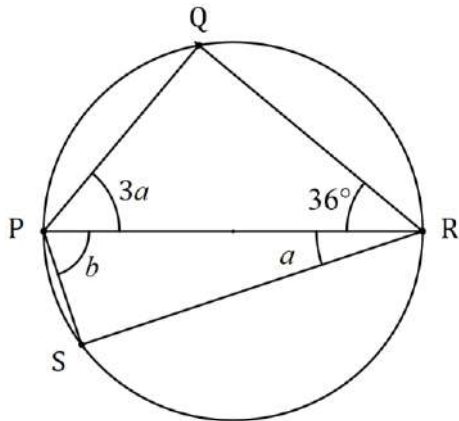
There were more children in the English family than the Australian family and the English children all had blue, brown, hazel or grey coloured eyes.

- (c) Show that at least five English children had the same eye colour. (3 marks)

Question 13

(8 marks)

- (a) Determine the size of angles a and b in the diagram below, where Q and S lie on the circumference of the circle with diameter PR . (3 marks)



- (b) Triangle ABC has sides of length $AB = 5$ cm, $BC = 7$ cm and $AC = 9$ cm. Prove, using the method of contradiction, that if AC is a diameter of a circle then B does not lie on the circumference of the circle. (5 marks)

Question 14

(8 marks)

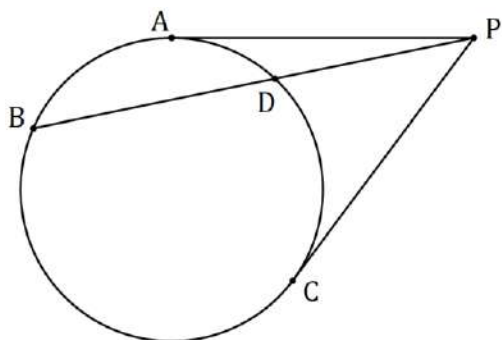
- (a) Simplify $(4\mathbf{a} - 2\mathbf{b}) \cdot (\mathbf{a} - 3\mathbf{b})$ given that $|\mathbf{a}| = 5$, $|\mathbf{b}| = 3$ and vector \mathbf{a} is parallel and in the opposite direction to vector \mathbf{b} . (4 marks)

- (b) Using $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$, demonstrate a vector method to show that if the diagonals \overrightarrow{OB} and \overrightarrow{AC} of parallelogram $OACB$ are perpendicular, then the parallelogram is a rhombus. (4 marks)

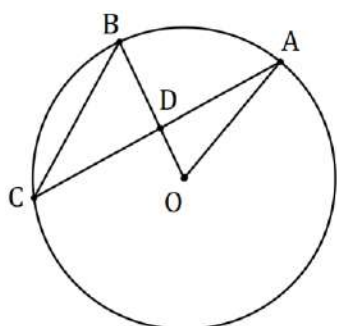
Question 15

(9 marks)

- (a) In the diagram below, PA and PC are tangents to the circle, with $PA = 58$ cm. Secant PB cuts the circle at D , so that $PD = 40$ cm. Determine the lengths of PC and BD . (4 marks)



- (b) In the diagram below, A , B and C lie on the circumference of the circle with centre O , with AC intersecting OB at D . Prove that $\angle DAO = \angle DBC - \angle DCB$. (5 marks)



Question 16**(9 marks)**

- (a) Determine the number of integers between 1 and 370 that are divisible by 4 or 7.

(4 marks)

- (b) A pigeon fancier has 5 Florentine, 6 King and 8 Maltese pigeons and must choose three of them to enter in a local show. Determine the number of different ways the three pigeons can be chosen if

- (i) there are no restrictions. **(1 mark)**

- (ii) the fancier decides to take one of each breed. **(2 marks)**

- (iii) the fancier decides to take at least two Maltese pigeons. **(2 marks)**

Question 17**(6 marks)**

Three vectors are $\mathbf{u} = a\mathbf{i} + b\mathbf{j}$, $\mathbf{v} = -3\mathbf{i} + 5\mathbf{j}$ and $\mathbf{w} = -\mathbf{i} + 4\mathbf{j}$.

(a) Determine the vector projection of \mathbf{v} on \mathbf{w} in exact form.

(2 marks)

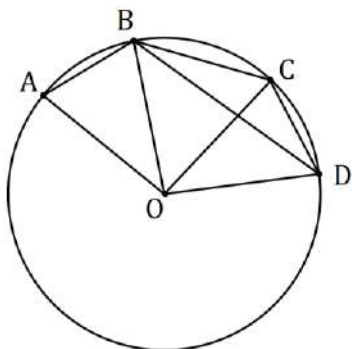
(b) If \mathbf{u} has the same magnitude as \mathbf{v} and is perpendicular to \mathbf{w} , determine the exact values of the coefficients a and b .

(4 marks)

Question 18

(7 marks)

- (a) In the diagram below, points B and C lie on the minor arc AD of the circle with centre O . The lengths of chords AB and CD are congruent, $\angle BOC = 37^\circ$ and $\angle AOD = 163^\circ$. Determine the size of $\angle CBD$. (3 marks)



- (b) Line segment AC intersects line segment BD at N . Given that AC and BD are non-parallel and the lengths AN, AC, BN and BD are 6, 41, 21 and 31 cm respectively, explain whether the points A, B, C and D are concyclic. (4 marks)

Question 19**(8 marks)**

- (a) Triangle ABC has vertices with position vectors $A(2, -6)$, $B(-3, 14)$ and $C(6, 8)$. Point P lies on side BC so that $2\overrightarrow{BP} = \overrightarrow{PC}$. Determine the vector \overrightarrow{AP} . (4 marks)

- (b) $OPQR$ is a parallelogram. Point M is the midpoint of side PQ and point N is on side QR so that $QN = \frac{3}{5}QR$. If $\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OR} = \mathbf{r}$, determine \overrightarrow{MN} in terms of \mathbf{p} and \mathbf{r} . (4 marks)

Question 20**(7 marks)**

A small boat leaves jetty *A* to travel to jetty *B*, 276 m away on a bearing of 240° . A steady current of 2.5 ms^{-1} runs in the river between the jetties on a bearing 130° . If the small boat travels at a constant speed of 6.5 ms^{-1} , determine the bearing it should steer to reach jetty *B* and how long the journey will take.

Question 21**(7 marks)**

A child is playing with thirteen coloured cubes, all the same size. There are six pink cubes, three navy and one each of red, blue, orange and green.

- (a) If the child stacks cubes one on top of another to make a column, determine the number of different coloured columns that can be made using
- (i) all the red, blue and green cubes. (1 mark)
 - (ii) all the pink, red and orange cubes. (2 marks)
 - (iii) all the cubes. (2 marks)
- (b) If all but one of the cubes are used to make a column, determine the number of different coloured columns that can now be made. Justify your answer. (2 marks)

Supplementary page

Question number: _____

Supplementary page

Question number: _____

Supplementary page

Question number: _____

Supplementary page

Question number: _____

