

Semester One Examination, 2019

Question/Answer booklet

MATHEMATICS SPECIALIST UNIT 1

Section Two:

Calculator-assumed

lf	f required by your examination administrator, ple	ase
	place your student identification label in this bo	XC

Student number:	In figures	
	In words	
	Your name	

Time allowed for this section

Reading time before commencing work: ten minutes

Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,

correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper,

and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

- 1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
- Write your answers in this Question/Answer booklet preferably using a blue/black pen.
 Do not use erasable or gel pens.
- 3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.
- 4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
- 5. It is recommended that you do not use pencil, except in diagrams.
- 6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
- 7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Markers use only			
Question	Maximum	Mark	
9	5		
10	8		
11	7		
12	8		
13	8		
14	9		
15	8		
16	7		
17	8		
18	8		
19	7		
20	7		
21	8		
S2 Total	98		
S2 Wt (×0.6633)	65%		

Section Two: Calculator-assumed

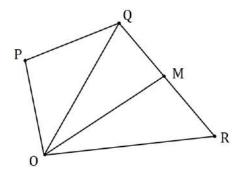
65% (98 Marks)

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 (5 marks)

In the diagram below, M is the midpoint of QR.



If $\overrightarrow{OP} = \mathbf{p}$, $\overrightarrow{OQ} = \mathbf{q}$ and $\overrightarrow{OR} = \mathbf{r}$, express the following in terms of \mathbf{p} , \mathbf{q} and \mathbf{r} .

(a) \overrightarrow{PR} . (1 mark)

(b) \overrightarrow{OM} . (2 marks)

(c) $6\overrightarrow{MP}$. (2 marks)

Question 10 (8 marks)

Points P, Q and R have coordinates (-2, 11), (8, 15) and (17, 3) respectively. Determine

(a) \overrightarrow{PQ} .

(1 mark)

(b) $|\overrightarrow{QR}|$.

(2 marks)

(c) $2\overrightarrow{PQ} - 60\mathbf{u}$, where \mathbf{u} is a unit vector in the direction \overrightarrow{QR} .

(3 marks)

(d) the coordinates of point S, given that $\overrightarrow{RS} = \overrightarrow{QP}$.

(2 marks)

Question 11 (7 marks)

Three vectors are given by $\mathbf{a} = 3\mathbf{i} - 4\mathbf{j}$, $\mathbf{b} = -3\mathbf{i} + 1.5\mathbf{j}$ and $\mathbf{c} = -2\mathbf{i} + y\mathbf{j}$, where y is a constant.

(a) Determine the vector projection of **b** on **a**.

(3 marks)

(b) Determine the value(s) of y if a and c are perpendicular.

(2 marks)

(c) Use your calculator to determine the angle between **a** and **b**, to the nearest degree.

(2 marks)

Question 12 (8 marks)

(a) Show that the vectors (8, -5) and (2.5, 4) are perpendicular.

(2 marks)

(b) Determine, to the nearest degree, the angle between the vectors (3, -2) and (-2, -4). (2 marks)

(c) The vectors (a, 2a + 3) and (a + 3, -2) are perpendicular, where a is a constant. Determine the value(s) of a and the corresponding pair(s) of vectors. (4 marks)

(2 marks)

Question 13 (8 marks)

- (a) Two vectors are $\mathbf{p} = (9.75, -20)$ and $\mathbf{q} = (-3.9, 8)$.
 - (i) State the magnitude of \mathbf{q} and the angle it makes with the positive x-axis. (2 marks)

(ii) Show that the vectors **p** and **q** are parallel.

(b) The points with position vectors (1, a), (4, 1) and (10, b) are collinear, where a and b are constants. Express b in terms of a. (4 marks)

Question 14 (9 marks)

The parts of this question refer to the word AERIFICATION. It has 5 different consonants and 7 vowels, some of which are repeated.

(a) Determine the number of ways that 3 different consonants chosen from the letters of the word can be arranged in a row. (1 mark)

(b) Determine the number of ways that all the letters of the word can be arranged in a row.
(2 marks)

(c) Determine the number of ways that all the letters of the word can be arranged in a row if the vowels must all be adjacent. (3 marks)

(d) Determine how many 3 letter permutations (e.g. TFI, IRI, etc) can be made using the letters of the word. (3 marks)

Question 15 (8 marks)

9

(a) 8-character passwords can be created using both lower or upper case letters as well as digits.

Write an expression that would give the total number of 8-character passwords that start with a digit and end with a consonant, and characters are not repeated.

You must use the notation ${}^n\mathcal{C}_r$ and/or nP_r within your expression. Do not evaluate it.

(3 marks)

(b) Prove that: ${}^{n}C_{r} + {}^{n}C_{r+1} = {}^{n+1}C_{r+1}$ (5 marks)

Question 16 (7 marks)

Three forces a, b and c act on a point in a plane.

The forces are a = -44i + 66j N, b = -12i - 75j N and c = 180i + 102j N.

(a) Determine the magnitude of the resultant force and the direction, to the nearest degree, that the resultant makes with the vector **i**. (3 marks)

When $\lambda \mathbf{a} + \mu \mathbf{b} + \mathbf{c} = 0$, the forces are in equilibrium.

(b) Determine the values of the scalar constants λ and μ for equilibrium to occur. (4 marks)

Question 17		
(a)	A set of cards is numbered from 100 to 999. Determine the minimum numust be selected to ensure that at least 3 cards in the selection have the	
	Justify your answer using the pigeonhole principle.	(3 marks)

11

Eight different books sit on a shelf, one of which has a hardcover and the rest softcovers. (b) A student is told they can take away as many of them as they like but must not leave empty handed. Determine how many different selections can be made of exactly 3 books. (1 mark) (i) (2 marks) (ii) altogether.

(iii) that include the hardcover. (2 marks)

	i on 18 /e to th	te origin, A and B have position vectors $18\mathbf{i}+18\mathbf{j}$ and $21\mathbf{i}-15\mathbf{j}$ respective	(8 marks) ly.
Particl	e P is i	nitially at A and moves with a constant velocity of $8\mathbf{i} - 15\mathbf{j}$ ms ⁻¹ .	
(a) Calculate			
	(i)	the speed of P .	(1 mark)
	(ii)	the position vector of P after 4 seconds.	(1 mark)
	(iii)	the distance of P from B after 4 seconds.	(2 marks)
(b)	Deteri	mine how long after leaving A that P is 157 m from B .	(4 marks)

Question 19 (7 marks)

ABCD is a trapezium with \overrightarrow{AB} parallel and in the same direction to \overrightarrow{DC} .

(a) Sketch a labelled diagram of ABCD.

(1 mark)

(b) Show that $\overrightarrow{AC} + \overrightarrow{DB} = \overrightarrow{AB} + \overrightarrow{DC}$.

(2 marks)

(c) M lies on AC and N lies on BD so that AM:MC=BN:ND=2:1. Use a vector method to prove that ABNM is a trapezium. (4 marks)

Question 20 (7 marks)

Farm A lies 95 km away from farm B on a bearing of 062° . A helicopter leaves farm A at 7:30 am to fly to farm B. The helicopter can maintain a speed of 145 kmh⁻¹ and there is a steady wind of 35 kmh⁻¹ blowing from the north.

Determine the bearing that the helicopter should steer and the time of its arrival at farm B, to the nearest minute.