Kennedy
Baptist College

Semester One Examination, 2019

Question/Answer booklet

MATHEMATICS SPECIALIST UNIT 1

Section One: Calculator-free

If required by your examination administrator, please place your student identification label in this box

Student number: In figures | | | | | | | | |

In words _____

Your name Solutions

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

52

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

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2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.
   Do not use erasable or gel pens.
3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Markers use only		
Question	Maximum	Mark
1	4	
2	8	
3	6	
4	7	
5	6	
6	7	
7	6	
8	8	
S1 Total	52	
S1 Wt (×0.6731)	35%	
S2 Wt	65%	
Total	100%	

Section One: Calculator-free **35% (52 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(4 marks)**

Determine the truth of the following statements, using an example or counter-example to support each answer.

(i) If $z \in \mathbb{R}$ and z^3 is an even number then z is an even number. (2 marks)

$2^3 = 8$ is even. and 2 is even ✓

Statement is true. ✓

(ii) If $x, y \in \mathbb{Z}$ and $x > y$ then $x^2 > y^2$. (2 marks)

False. ✓

If $x = 3$ and $y = -4$.

Then $x > y$

However $x^2 > y^2$ is false as. $9 \not> 16$. ✓

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See next page

Question 2 **(8 marks)**

Let $\mathbf{a} = 4\mathbf{i} - 8\mathbf{j}$, $\mathbf{b} = -3\mathbf{i} + 6\mathbf{j}$ and $\mathbf{c} = 2\mathbf{i} + 3\mathbf{j}$.

(a) Determine

(i) $\mathbf{b} - \mathbf{c}$. (1 mark)

$\langle -3, 6 \rangle - \langle 2, 3 \rangle$

$= \langle -5, 3 \rangle$ ✓

(ii) $3\mathbf{b} + 4\mathbf{a}$. (2 marks)

$3\langle -3, 6 \rangle + 4\langle 4, -8 \rangle$

$= \langle -9, 18 \rangle + \langle 16, -32 \rangle$ ✓ ✓

$= \langle 7, -14 \rangle$ ✓

(iii) $|\mathbf{a} + \mathbf{c}|$. (2 marks)

$= |\langle 4, -8 \rangle + \langle 2, 3 \rangle|$

$= |\langle 6, -5 \rangle|$ ✓

$= \sqrt{36 + 25}$

$= \sqrt{61}$ ✓

(b) Determine a unit vector that is parallel to $\mathbf{a} + \mathbf{b}$ but in the opposite direction. (3 marks)

$\underset{\sim}{a} + \underset{\sim}{b} = \langle 4, -8 \rangle + \langle -3, 6 \rangle$

$= \langle 1, -2 \rangle$

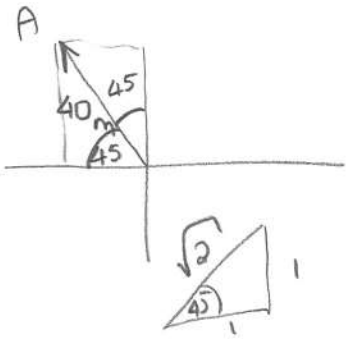
$|\underset{\sim}{a} + \underset{\sim}{b}| = \sqrt{1^2 + 2^2} = \sqrt{5}$ ✓

$\widehat{\underset{\sim}{a} + \underset{\sim}{b}} = \frac{1}{\sqrt{5}} \langle 1, -2 \rangle$ ✓

Opp. direction $= \frac{1}{\sqrt{5}} \langle -1, 2 \rangle$ ✓ opp. direction → sign change.

Question 3 **(6 marks)**

(a) Body A moves 40 m on a bearing of 315°. Express this displacement in component form using unit vectors **i** and **j**. (3 marks)



$$A = -40\cos 45^\circ \mathbf{i} + 40\sin 45^\circ \mathbf{j}$$

$$= \frac{-40}{\sqrt{2}}\mathbf{i} + \frac{40}{\sqrt{2}}\mathbf{j}$$

$$= \frac{-40\sqrt{2}}{2}\mathbf{i} + \frac{40\sqrt{2}}{2}\mathbf{j}$$

$$= -20\sqrt{2}\,\mathbf{i} + 20\sqrt{2}\,\mathbf{j}$$

(b) Body B moves with a velocity of $4\sqrt{3}\mathbf{i} - 4\mathbf{j}$ ms^{-1}. Determine the speed of this body and the bearing it is travelling in. (3 marks)

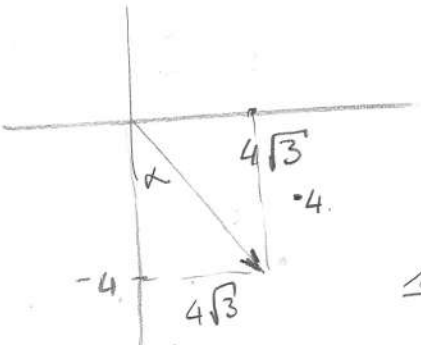
$$\underset{\sim}{v}_B = \langle 4\sqrt{3}, -4 \rangle$$

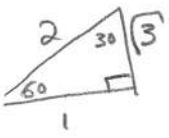
$$|\underset{\sim}{v}_B| = \sqrt{(4\sqrt{3})^2 + 16}$$

$$= \sqrt{16(3) + 16}$$

$$= \sqrt{64}$$

$$= 8 \text{ ms}^{-1}$$





$$\tan\alpha = \frac{4\sqrt{3}}{4} = \frac{\sqrt{3}}{1}$$

$$\alpha = 60^\circ \quad \angle(x\text{-axis}) = -30^\circ$$

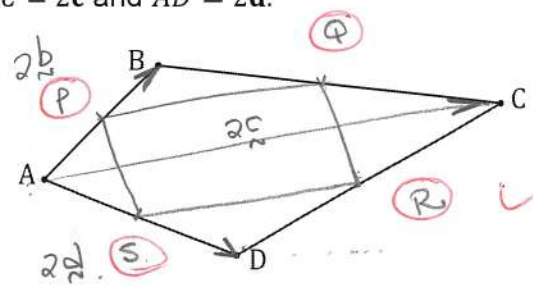
$$\text{Bearing} = 90^\circ + 30^\circ$$

$$= 120^\circ$$

6

Question 4 **(7 marks)**

Quadrilateral $ABCD$ is shown below. The midpoints of sides AB, BC, CD and DA are P, Q, R and S respectively. Let $\overrightarrow{AB} = 2\mathbf{b}, \overrightarrow{AC} = 2\mathbf{c}$ and $\overrightarrow{AD} = 2\mathbf{d}$.



correct quadrilateral. ✓

(a) Sketch quadrilateral $PQRS$ on the diagram above. (1 mark)

(b) Determine expressions for $\overrightarrow{AQ}, \overrightarrow{AR}$ and $\overrightarrow{QR}$ in terms of $\mathbf{b}, \mathbf{c}$ and $\mathbf{d}$. (3 marks)

$\overrightarrow{AQ} = 2\underset{\sim}{b} + \frac{1}{2}\overrightarrow{BC}$

$\overrightarrow{BC} = -2\underset{\sim}{b} + 2\underset{\sim}{c}$

$\therefore \overrightarrow{AQ} = 2\underset{\sim}{b} + \frac{1}{2}(-2\underset{\sim}{b} + 2\underset{\sim}{c})$

$\overrightarrow{AQ} = \underset{\sim}{b} + \underset{\sim}{c}$ ✓

$\overrightarrow{AR} = 2\underset{\sim}{d} + \frac{1}{2}\overrightarrow{DC}$

$\overrightarrow{DC} = -2\underset{\sim}{d} + 2\underset{\sim}{b} + (-2\underset{\sim}{b} + 2\underset{\sim}{c})$

$= -2\underset{\sim}{d} + 2\underset{\sim}{c}$.

$\therefore \overrightarrow{AR} = 2\underset{\sim}{d} + \frac{1}{2}(-2\underset{\sim}{d} + 2\underset{\sim}{c})$

$= 2\underset{\sim}{d} - \underset{\sim}{d} + \underset{\sim}{c}$

$= \underset{\sim}{d} + \underset{\sim}{c}$ ✓

$\overrightarrow{QR} = \overrightarrow{QC} + \overrightarrow{CR}$

$= \frac{1}{2}\overrightarrow{BC} + \frac{1}{2}\overrightarrow{CD}$

$= -\underset{\sim}{b} + \underset{\sim}{c} + \underset{\sim}{d} - \underset{\sim}{c}$

$= -\underset{\sim}{b} + \underset{\sim}{d}$ ✓

(c) Prove that $\overrightarrow{PQ} = \overrightarrow{SR}$ and $\overrightarrow{PS} = \overrightarrow{QR}$. (3 marks)

To Prove: $\overrightarrow{PQ} = \overrightarrow{SR}$

LH $\overrightarrow{PQ} = \overrightarrow{PB} + \overrightarrow{BQ}$

$= \frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC}$

$= \underset{\sim}{b} - \underset{\sim}{b} + \underset{\sim}{c}$

$= \underset{\sim}{c}$ ✓

RH $\overrightarrow{SR} = \overrightarrow{SD} + \overrightarrow{DR}$

$= \frac{1}{2}\overrightarrow{AD} + \frac{1}{2}\overrightarrow{DC}$

$= \underset{\sim}{d} + -\underset{\sim}{d} + \underset{\sim}{c}$

$= \underset{\sim}{c}$ ✓

$\therefore \overrightarrow{PQ} = \overrightarrow{SR}$ QED

To Prove: $\overrightarrow{PS} = \overrightarrow{QR}$

LH $\overrightarrow{PS} = -\frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{AD}$

$= -\underset{\sim}{b} + \underset{\sim}{d}$ ✓

RH $\overrightarrow{QR} = \frac{1}{2}\overrightarrow{BC} + \frac{1}{2}\overrightarrow{CD}$

$= -\underset{\sim}{b} + \underset{\sim}{c} + \underset{\sim}{d} - \underset{\sim}{c}$

$= -\underset{\sim}{b} + \underset{\sim}{d}$ ✓

$\therefore \overrightarrow{PS} = \overrightarrow{QR}$ QED

See next page

Question 5 **(6 marks)**

Consider the following statement that refers to two **isosceles** triangles.

If the triangles have the same area, then the triangles are congruent.

(a) Write the inverse statement and state whether it is true or false. (2 marks)

If two triangles do not have the same area, then they are not congruent. ✓

TRUE ✓

(b) Write the converse statement and state whether it is true or false. (2 marks)

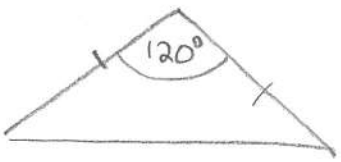
(reverse).

If the triangles are congruent, then they have the same area. ✓

True. ✓

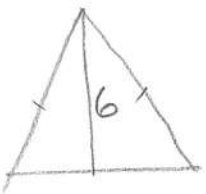
(c) Write the contrapositive statement and use a counter-example to explain why it is false. (2 marks)

If the triangles are not congruent, then they will not have the same area. ✓



Not congruent, but same area.

OR ✓



4

6

Same area, Not congruent.

6

See next page

Question 6 (7 marks)

$W = F \cdot S$

(a) The work done, in joules, by a force of $\mathbf{F}$ Newtons in changing the displacement of an object by $\mathbf{s}$ metres, is given by the scalar product of $\mathbf{F}$ and $\mathbf{s}$. Determine the work done by

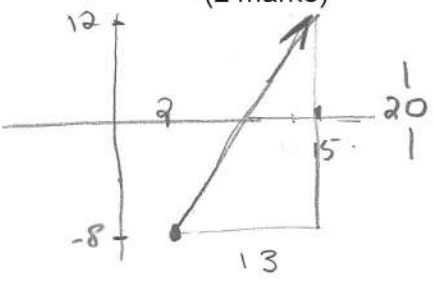
(i) force $\mathbf{F} = (10\mathbf{i} + 8\mathbf{j})$ N that moves a small body from $(2\mathbf{i} - 8\mathbf{j})$ m to $(15\mathbf{i} + 12\mathbf{j})$ m. (2 marks)

$W = F \cdot S$

$= \langle 10, 8 \rangle \cdot \langle 13, 20 \rangle$

$= 130 + 160$

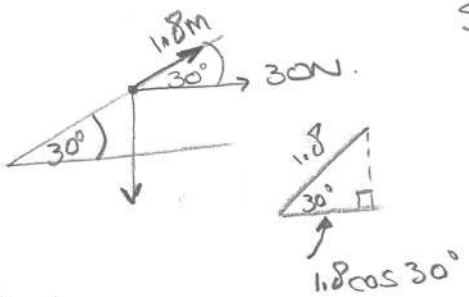
$= 290$ J



$S = \langle 13, 20 \rangle$

(ii) a horizontal force of 30 N that pushes a small body 1.8 m up a slope inclined at 30° to the horizontal. (2 marks)

$S = 1.8$ m



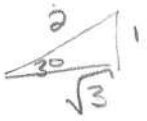
horizontal component.

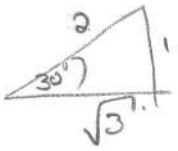
$W = 30 \times 1.8 \cos 30°$

$= 30 \times 1.8 \frac{\sqrt{3}}{2}$

$= 30 \times 0.9 \sqrt{3}$

$= 27\sqrt{3}$ J





30
9
27.0

(b) Determine the vector projection of $(2\mathbf{i} + 4\mathbf{j})$ on $(-3\mathbf{i} + 4\mathbf{j})$. (3 marks)

$\text{Proj}_{\underset{\sim}{S}} \underset{\sim}{U} = (\underset{\sim}{U} \cdot \hat{\underset{\sim}{S}})\hat{\underset{\sim}{S}}$

$= (\langle 2, 4 \rangle \cdot \frac{1}{5}\langle -3, 4 \rangle)\frac{1}{5}\langle -3, 4 \rangle$

$|\underset{\sim}{S}| = 5$

$\hat{\underset{\sim}{S}} = \frac{1}{5}\langle -3, 4 \rangle$

$= \left(-\frac{6}{5} + \frac{16}{5}\right)\frac{1}{5}\langle -3, 4 \rangle$

$= \left(\frac{10}{5}\right)\left(\frac{1}{5}\right)\langle -3, 4 \rangle$

$= \frac{2}{5}\langle -3, 4 \rangle$

Question 7 **(6 marks)**

The position vectors of points P and Q are $\mathbf{p} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ and $\mathbf{q} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ respectively.

(a) Determine the magnitude of the displacement vector $\overrightarrow{PQ}$. (2 marks)

$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$

$= \langle 1, -1 \rangle - \langle 3, 1 \rangle$

$= \langle -2, -2 \rangle$ ✓

$|\overrightarrow{PQ}| = \sqrt{4 + 4}$

$= \sqrt{8}$ ✓

$(2\sqrt{2})$

(b) Determine the values of λ so that $|\mathbf{p} - \lambda\mathbf{q}| = 4$. (4 marks)

$p - \lambda q = \langle 3, 1 \rangle - \lambda \langle 1, -1 \rangle$

$= \langle 3 - \lambda, 1 + \lambda \rangle$ ✓

$|p - \lambda q| = \sqrt{(3-\lambda)^2 + (1+\lambda)^2} = 4$

$\sqrt{9 - 6\lambda + \lambda^2 + 1 + 2\lambda + \lambda^2} = 4$

$\sqrt{10 - 4\lambda + 2\lambda^2}$ ✓ $= 4$

$\sqrt{2\lambda^2 - 4\lambda + 10} = 4$

$2(\lambda^2 - 2\lambda + 5) = 16$

$\lambda^2 - 2\lambda + 5 = 8$ ✓

$\lambda^2 - 2\lambda - 3 = 0$

$(\lambda - 3)(\lambda + 1) = 0$

$\lambda = 3$ ✓ $\lambda = -1$ ✓

6

Question 8 **(8 marks)**

(a) Evaluate $\dfrac{^{2020}P_2}{101 \times {}^{20}P_2}$. (3 marks)

$= \dfrac{2020!}{2018!} \div \left(101 \times \dfrac{20!}{19!}\right)$

$= \dfrac{2020 \times 2019}{101 \times 20}$

$= \dfrac{2020 \times 2019}{2020}$

$= 2019$

$101 \times 20 = 2020$

(b) Given that $^{n+1}P_r = k \times {}^{n}P_r$, determine the constant k in terms of n and/or r. (3 marks)

$\dfrac{(n+1)!}{(n+1-r)!} = \dfrac{k \times n!}{(n-r)!}$

$\dfrac{(n+1)n!}{(n+1-r)(n-r)!} = \dfrac{k \times n!}{(n-r)!}$

$\dfrac{n+1}{n+1-r} = k$

$k = \dfrac{n+1}{n+1-r}$

(c) Given that $^{14}P_{12} = 43\,589\,145\,600$, determine $^{16}P_{12}$. (2 marks)

$\dfrac{14!}{2!} = 43\,589\,145\,600$

$43\,589\,145\,600 \times 20 = 871\,782\,912\,000$

$^{16}P_{12} = \dfrac{16!}{4!}$

$= \dfrac{16 \times 15 \times 14!}{4 \times 3 \times 2}$

$= 20 \times \dfrac{14!}{2!}$

End of questions

8

Supplementary page

Question number: _____

Kennedy
Baptist College

Semester One Examination, 2019

Question/Answer booklet

MATHEMATICS SPECIALIST UNIT 1

Section Two: Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

Student number: In figures | | | | | | | | |

In words _____

Your name Solutions

98

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

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Markers use only		
Question	Maximum	Mark
9	5	
10	8	
11	7	
12	8	
13	8	
14	9	
15	8	
16	7	
17	8	
18	8	
19	7	
20	7	
21	8	
S2 Total	98	
S2 Wt (×0.6633)	65%	

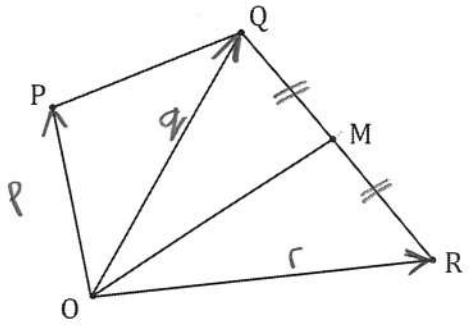
Section Two: Calculator-assumed **65% (98 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 **(5 marks)**

In the diagram below, M is the midpoint of QR.



If $\overrightarrow{OP} = \mathbf{p}, \overrightarrow{OQ} = \mathbf{q}$ and $\overrightarrow{OR} = \mathbf{r}$, express the following in terms of $\mathbf{p}, \mathbf{q}$ and $\mathbf{r}$.

(a) $\overrightarrow{PR}$. (1 mark)

$= -\underset{\sim}{p} + \underset{\sim}{r}$ ✓ $(\underset{\sim}{r} - \underset{\sim}{p})$

(b) $\overrightarrow{OM}$. $= \underset{\sim}{q} + \frac{1}{2}\overrightarrow{QR}$ (2 marks)

$= \underset{\sim}{q} + \frac{1}{2}(-\underset{\sim}{q} + \underset{\sim}{r})$ ✓

$= \frac{1}{2}\underset{\sim}{r} + \frac{1}{2}\underset{\sim}{q}$ ✓

(c) $6\overrightarrow{MP}$. $= 6\left[-\frac{1}{2}\underset{\sim}{r} - \frac{1}{2}\underset{\sim}{q} + \underset{\sim}{p}\right]$ ✓ (2 marks)

$= 6\underset{\sim}{p} - 3\underset{\sim}{r} - 3\underset{\sim}{q}$ ✓

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See next page

Question 10 **(8 marks)**

Points P, Q and R have coordinates $(-2, 11), (8, 15)$ and $(17, 3)$ respectively. Determine

(a) $\overrightarrow{PQ}$. (1 mark)

$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$

$= \langle 8, 15 \rangle - \langle -2, 11 \rangle$

$= \langle 10, 4 \rangle$ ✓

(b) $|\overrightarrow{QR}|$. (2 marks)

$\overrightarrow{QR} = \overrightarrow{OR} - \overrightarrow{OQ}$

$= \langle 17, 3 \rangle - \langle 8, 15 \rangle$

$= \langle 9, -12 \rangle$ ✓

$|\overrightarrow{QR}| = 15$ ✓

(c) $2\overrightarrow{PQ} - 60\mathbf{u}$, where $\mathbf{u}$ is a unit vector in the direction $\overrightarrow{QR}$. (3 marks)

$2\langle 10, 4 \rangle - 60\underset{\sim}{u}$ $\quad \underset{\sim}{u} = \frac{1}{15}\langle 9, -12 \rangle$ ✓

$2\langle 10, 4 \rangle - 60\left(\frac{1}{15}\right)\langle 9, -12 \rangle$ ✓

$= \langle 20, 8 \rangle - \langle 36, -48 \rangle$

$= \langle -16, 56 \rangle$ ✓

(d) the coordinates of point S, given that $\overrightarrow{RS} = \overrightarrow{QP}$. (2 marks)

$\overrightarrow{OS} = \langle x, y \rangle$ $\quad \overrightarrow{RS} = \overrightarrow{QP}$

$\overrightarrow{OS} - \overrightarrow{OR} = \overrightarrow{OP} - \overrightarrow{OQ}$ ✓

$\langle x, y \rangle - \langle 17, 3 \rangle = \langle -2, 11 \rangle - \langle 8, 15 \rangle$

$x - 17 = -2 - 8$ $\quad y - 3 = 11 - 15$

$x = 7$ $\quad y = -1$

$S = (7, -1)$ ✓

8

Question 11 (7 marks)

Three vectors are given by $\mathbf{a} = 3\mathbf{i} - 4\mathbf{j}$, $\mathbf{b} = -3\mathbf{i} + 1.5\mathbf{j}$ and $\mathbf{c} = -2\mathbf{i} + y\mathbf{j}$, where y is a constant.

(a) Determine the vector projection of $\mathbf{b}$ on $\mathbf{a}$. (3 marks)

$\underset{\sim}{a} = \langle 3, -4 \rangle$ $\underset{\sim}{b} = \langle -3, 1.5 \rangle$ $\underset{\sim}{c} = \langle -2, y \rangle$

$\hat{\underset{\sim}{a}} = \frac{1}{5}\langle 3, -4 \rangle$

$\text{Proj}_{\underset{\sim}{a}} \underset{\sim}{b} = (\underset{\sim}{b} \cdot \hat{\underset{\sim}{a}})\hat{\underset{\sim}{a}}$

$= \left(\langle -3, 1.5 \rangle \cdot \frac{1}{5}\langle 3, -4 \rangle\right) \frac{1}{5}\langle 3, -4 \rangle$

$= \frac{-3}{5}\langle 3, -4 \rangle$

$= \left\langle -\frac{9}{5}, \frac{12}{5} \right\rangle$

$\langle -1.8, 2.4 \rangle$

(b) Determine the value(s) of y if $\mathbf{a}$ and $\mathbf{c}$ are perpendicular. (2 marks)

$\underset{\sim}{a} \cdot \underset{\sim}{c} = 0$

$\langle 3, -4 \rangle \cdot \langle -2, y \rangle = 0$

$-6 - 4y = 0$

$y = -\frac{3}{2}$

(c) Use your calculator to determine the angle between $\mathbf{a}$ and $\mathbf{b}$, to the nearest degree. (2 marks)

153°

7

Question 12 **(8 marks)**

(a) Show that the vectors $(8, -5)$ and $(2.5, 4)$ are perpendicular. (2 marks)

$\langle 8, -5 \rangle \cdot \langle 2.5, 4 \rangle$ ✓

$= 20 - 20$

$= 0$ ✓

Therefore $\perp$

(b) Determine, to the nearest degree, the angle between the vectors $(3, -2)$ and $(-2, -4)$. (2 marks)

$\langle 3, -2 \rangle$ and $\langle -2, -4 \rangle$

angle $\approx 83^\circ$ ✓✓

(c) The vectors $(a, 2a + 3)$ and $(a + 3, -2)$ are perpendicular, where a is a constant. Determine the value(s) of a and the corresponding pair(s) of vectors. (4 marks)

$\langle a, 2a+3 \rangle \cdot \langle a+3, -2 \rangle = 0$

$a^2 + 3a - 4a - 6 = 0$ ✓

$a^2 - a - 6 = 0$

$(a - 3)(a + 2) = 0$ ✓

$a = 3$ or $a = -2$

$\langle 3, 9 \rangle$ and $\langle 6, -2 \rangle$ ✓

$\langle -2, -1 \rangle$ and $\langle 1, -2 \rangle$ ✓

Question 13 **(8 marks)**

(a) Two vectors are $\mathbf{p} = (9.75, -20)$ and $\mathbf{q} = (-3.9, 8)$.

(i) State the magnitude of $\mathbf{q}$ and the angle it makes with the positive x-axis. (2 marks)

total $[-3.9, 8]$

$|\underset{\sim}{q}| = 8.9$ ✓ angle $= 116°$ ✓

(ii) Show that the vectors $\mathbf{p}$ and $\mathbf{q}$ are parallel. (2 marks)

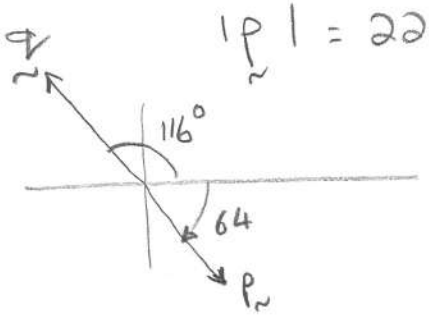
OR $\begin{pmatrix} 9.75 \\ -20 \end{pmatrix} = \lambda \begin{pmatrix} 3.9 \\ 8 \end{pmatrix}$

$9.75 = \lambda(-3.9) \Rightarrow \lambda = -2.5$

$-20 = 8\lambda \Rightarrow \lambda = 2.5$

total $[9.75, -20]$ ✓

$|\underset{\sim}{p}| = 22.25$ angle $= -64°$ ✓



Angle between $\underset{\sim}{p}$ and $\underset{\sim}{q}$ $= 116° + 64°$
$= 180°$

$\underset{\sim}{p}$ and $\underset{\sim}{q}$ are parallel.

A B C

(b) The points with position vectors $(1, a)$, $(4, 1)$ and $(10, b)$ are collinear, where a and b are constants. Express b in terms of a. (4 marks)

on a straight line.

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= \begin{pmatrix} 4 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ a \end{pmatrix}$

$= \begin{pmatrix} 3 \\ 1-a \end{pmatrix}$ ✓

$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$

$= \begin{pmatrix} 10 \\ b \end{pmatrix} - \begin{pmatrix} 4 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 6 \\ b-1 \end{pmatrix}$ ✓

NOW $\overrightarrow{AB} = \lambda \overrightarrow{BC}$ (Parallel too).

$\begin{pmatrix} 3 \\ 1-a \end{pmatrix} = \lambda \begin{pmatrix} 6 \\ b-1 \end{pmatrix}$ ✓

$3 = 6\lambda$, $\lambda = \frac{1}{2}$ → $1 - a = \lambda(b-1)$

$1 - a = \frac{1}{2}(b-1)$

$1 - a = \frac{1}{2}b - \frac{1}{2}$

$b = 2\left(\frac{3}{2} - a\right)$

$b = 3 - 2a$ ✓ (8)

See next page

7 vowels
5 consonants
12

Question 14 **(9 marks)**

The parts of this question refer to the word AERIFICATION. It has 5 different consonants and 7 vowels, some of which are repeated.

(a) Determine the number of ways that 3 different consonants chosen from the letters of the word can be arranged in a row. (1 mark)

C

$^5C_3 = 60$ ✓

(b) Determine the number of ways that all the letters of the word can be arranged in a row. (2 marks)

12 ! — 12 letters

$$\frac{12!}{3! \times 2!} = 39\,916\,800$$ ✓

3! for the 3 I's.

2! for the 2 A's

(c) Determine the number of ways that all the letters of the word can be arranged in a row if the vowels must all be adjacent. (3 marks)

(5 + 1) 5 for the 5 remaining consonants + 1 more for the travelling block of the 7 vowels along the blanks.

$$\frac{7! \times 6!}{3! \times 2!} = 302\,400$$ ✓

vowels together

7 _ _ _ _ _

(d) Determine how many 3 letter permutations (e.g. TFI, IRI, etc) can be made using the letters of the word. (3 marks)

9 8 7 All diffr. letters. = 504 words

×3 for travelling.

(1 1 8) ×3 2 A's + one more = 24 words

3 × (1 1 8) Remaining letters diffr. 2 I's + one more = 24 words.

3 I's = 1 word

Separate cases ✓

553 words ✓

9

Total = 62 vowel = 10 × consonant = 42 ×
26 26

Question 15 **(8 marks)**

(a) 8-character passwords can be created using both lower or upper case letters as well as digits. 10
Write an expression that would give the total number of 8-character passwords that start with a digit and end with a consonant, and characters are not repeated.
You must use the notation ${}^nC_r$ and/or ${}^nP_r$ within your expression. Do not evaluate it.
(3 marks)

10 60 59 58 57 56 55 42.

${}^{10}C_1 \times {}^{60}P_6 \times {}^{42}C_1$ ✓ ✓ ✓

${}^{25}P_5 = \frac{25!}{20!}$

(b) Prove that: ${}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}$ (5 marks)

LHS ${}^nC_r + {}^nC_{r+1}$

$= \frac{n!}{(n-r)!\,r!} + \frac{n!}{(n-r-1)!\,(r+1)!}$ ✓

$= \frac{(n-r-1)(r+1)!\,n! + (n-r)!\,r!\,n!}{(n-r)!\,r!\,(n-r-1)!\,(r+1)!}$

$= \frac{(n-r-1)!\,(r+1)r!\,n! + (n-r)(n-r-1)!\,r!\,n!}{(n-r)!\,r!\,(n-r-1)!\,(r+1)!}$ ✓

$= \frac{(n-r-1)!\,n!\,r!\,((r+1)+(n-r))}{(n-r-1)!\,r!\,(n-r)!\,(r+1)!}$

$= \frac{n!\,(n+1)}{(n-r)!\,(r+1)!}$ ✓

$= \frac{(n+1)!}{(n-r)!\,(r+1)!}$ ✓

RHS ${}^{n+1}C_{r+1}$

$= \frac{(n+1)!}{(n+1-r-1)!\,(r+1)!}$

$= \frac{(n+1)!}{(n-r)!\,(r+1)!}$ ✓

$\therefore$ LHS = RHS.

QED

8

Question 16 **(7 marks)**

Three forces $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ act on a point in a plane.

The forces are $\mathbf{a} = -44\mathbf{i} + 66\mathbf{j}$ N, $\mathbf{b} = -12\mathbf{i} - 75\mathbf{j}$ N and $\mathbf{c} = 180\mathbf{i} + 102\mathbf{j}$ N.

(a) Determine the magnitude of the resultant force and the direction, to the nearest degree, that the resultant makes with the vector $\mathbf{i}$. (3 marks)

→ x-axis

$\langle -44, 66 \rangle + \langle -12, -75 \rangle + \langle 180, 102 \rangle$

$= \langle 124, 93 \rangle$ ✓

toPol $\langle 124, 93 \rangle = 155, \angle 36.87$

$|\underset{\sim}{r}| = 155$ N ✓

angle $\approx 37°$ ✓

When $\lambda\mathbf{a} + \mu\mathbf{b} + \mathbf{c} = 0$, the forces are in equilibrium.

(b) Determine the values of the scalar constants λ and μ for equilibrium to occur. (4 marks)

$\langle -44\lambda, 66\lambda \rangle + \langle -12\mu, -75\mu \rangle + \langle 180, 102 \rangle = \langle 0, 0 \rangle$

$-44\lambda - 12\mu + 180 = 0$ ✓

$66\lambda - 75\mu + 102 = 0$ ✓ } simult.

$\lambda = 3$ ✓ $\mu = 4$ ✓

7

Question 17 **(8 marks)**

(a) A set of cards is numbered from 100 to 999. Determine the minimum number of cards that must be selected to ensure that at least 3 cards in the selection have the same last digit. Justify your answer using the pigeonhole principle. (3 marks)

Pigeonholes: 0, 1, 2, 3 .... , 9. ✓

Pigeons : 2 + 2 + 2 + 2 .... + 2 = 20 pigeons. ✓

One more card = 21 cards ✓

1 Hardcover 7 softcovers.

(b) Eight different books sit on a shelf, one of which has a hardcover and the rest softcovers. A student is told they can take away as many of them as they like but must not leave empty handed. Determine how many different selections can be made

(i) of exactly 3 books. (1 mark)

$^8C_3 = 56$. ✓

(ii) altogether. (2 marks)

choose; 1, 2, 3 .... 8 books.

✓ $2^8 - 1 = 255$ ✓

May not leave empty handed. Last one would have been empty handed thus subtract it

(iii) that include the hardcover. (2 marks)

one hardcover, and choice of 7 others.

$1 \times 2^7 = 128$ ✓ ✓

8

Question 18 (8 marks)

Relative to the origin, A and B have position vectors $18\mathbf{i} + 18\mathbf{j}$ and $21\mathbf{i} - 15\mathbf{j}$ respectively.

Particle P is initially at A and moves with a constant velocity of $8\mathbf{i} - 15\mathbf{j}$ ms^{-1}.

(a) Calculate

(i) the speed of P. (1 mark)

$|P| = \sqrt{8^2 + 15^2}$

$= 17$ m/s ✓

(ii) the position vector of P after 4 seconds. (1 mark)

$\begin{pmatrix} 18 \\ 18 \end{pmatrix} + 4\begin{pmatrix} 8 \\ -15 \end{pmatrix} = \begin{pmatrix} 50 \\ -42 \end{pmatrix}$ ✓

(iii) the distance of P from B after 4 seconds. (2 marks)

ft

$\overrightarrow{PB} = \begin{pmatrix} 21 \\ -15 \end{pmatrix} - \begin{pmatrix} 50 \\ -42 \end{pmatrix} = \begin{pmatrix} -29 \\ 27 \end{pmatrix}$ ✓

$|\overrightarrow{PB}| = \sqrt{29^2 + 27^2}$

$= 1570$

≈ 39.6 m ✓

(b) Determine how long after leaving A that P is 157 m from B. (4 marks)

$\overrightarrow{OP} = \begin{pmatrix} 18 \\ 18 \end{pmatrix} + t\begin{pmatrix} 8 \\ -15 \end{pmatrix}$ ✓

$\overrightarrow{PB} = \begin{pmatrix} 21 \\ -15 \end{pmatrix} - \begin{pmatrix} 18 + 8t \\ 18 - 15t \end{pmatrix}$ ✓

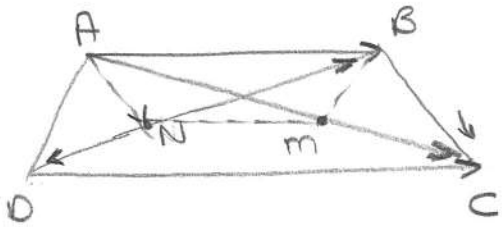
$|\overrightarrow{PB}| = \sqrt{(3 - 8t)^2 + (-33 + 15t)^2} = 157$m ✓

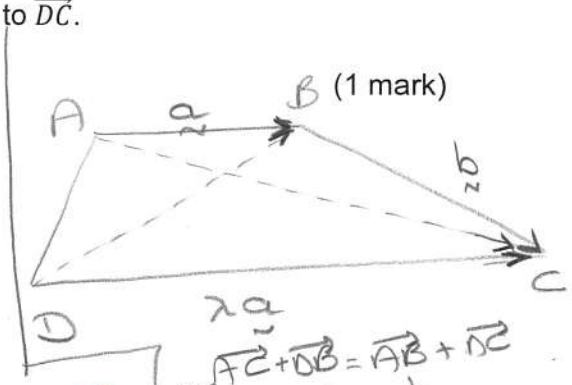
$t = 11$ ✓

Question 19 **(7 marks)**

$ABCD$ is a trapezium with $\overrightarrow{AB}$ parallel and in the same direction to $\overrightarrow{DC}$.

(a) Sketch a labelled diagram of $ABCD$. (1 mark)





(b) Show that $\overrightarrow{AC} + \overrightarrow{DB} = \overrightarrow{AB} + \overrightarrow{DC}$. (2 marks)

$\overrightarrow{AC} + \overrightarrow{DB} = (\overrightarrow{AB} + \overrightarrow{BC}) + (\overrightarrow{DA} + \overrightarrow{AB})$

$= \overrightarrow{AB} + (\overrightarrow{DA} + \overrightarrow{AB} + \overrightarrow{BC})$

$= \overrightarrow{AB} + \overrightarrow{DC}$

QED

$\overrightarrow{AC} + \overrightarrow{DB} = \overrightarrow{AB} + \overrightarrow{DC}$

LHS $\underset{\sim}{a} + \underset{\sim}{b} + \lambda\underset{\sim}{a} - \underset{\sim}{b}$

$a + \lambda a$

RHS $\underset{\sim}{a} + \lambda\underset{\sim}{a}$

LHS = RHS

(c) M lies on AC and N lies on BD so that $AM:MC = BN:ND = 2:1$. Use a vector method to prove that $ABNM$ is a trapezium. (4 marks)

$\overrightarrow{AM} = 2$ $\overrightarrow{AC} = 3$

$\overrightarrow{MC} = 1$

$\overrightarrow{BN} = 2$ $\overrightarrow{BD} = 3$

$\overrightarrow{ND} = 1$

$\overrightarrow{AM} = \frac{2}{3}\overrightarrow{AC}$ $\overrightarrow{AN} = \overrightarrow{AB} + \frac{2}{3}\overrightarrow{BD}$

$\overrightarrow{NM} = \overrightarrow{AM} - \overrightarrow{AN}$

$= \frac{2}{3}\overrightarrow{AC} - (\overrightarrow{AB} + \frac{2}{3}\overrightarrow{BD})$ (without M, N)

$= \frac{2}{3}(\overrightarrow{AC} - \overrightarrow{BD}) - \overrightarrow{AB}$

$= \frac{2}{3}(\underbrace{\overrightarrow{AC} + \overrightarrow{DB}}_{\text{from above}}) - \overrightarrow{AB}$

$= \frac{2}{3}(\overrightarrow{AB} + \overrightarrow{DC}) - \overrightarrow{AB}$ (in terms of $\overrightarrow{AB}, \overrightarrow{DC}$)

But $\overrightarrow{DC} = k\overrightarrow{AB}$ (Parallel as in given).

$\overrightarrow{NM} = \frac{2}{3}(\overrightarrow{AB} + k\overrightarrow{AB}) - \overrightarrow{AB}$

$= \left(\frac{2k-1}{3}\right)\overrightarrow{AB}$ $\therefore$ ABNM is a trapezium

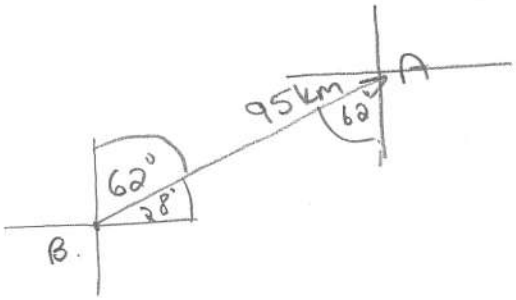
(7)

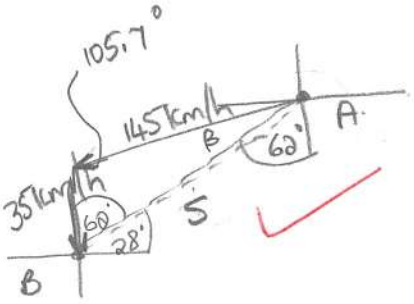
See next page

Question 20 **(7 marks)**

Farm A lies 95 km away from farm B on a bearing of 062°. A helicopter leaves farm A at 7:30 am to fly to farm B. The helicopter can maintain a speed of 145 kmh^{-1} and there is a steady wind of 35 kmh^{-1} blowing from the north.

Determine the bearing that the helicopter should steer and the time of its arrival at farm B, to the nearest minute.



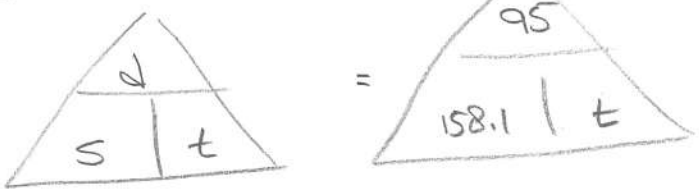


$$\frac{\sin\beta}{35} = \frac{\sin 62^\circ}{145}$$

$$\beta = 12.3^\circ$$

$$s^2 = 145^2 + 35^2 - 2(145)(35)\cos 105.7$$

$$s = 158.1 \text{ km/h}$$



$t = 0.601$ h.

$t = 36$ min

Arrive at 7:30 am + 36 Mh

= 8:06 am

Bearing : $180^\circ + 62^\circ + 12.3^\circ = 254.3^\circ$

4

Question 21 340 integers. **(8 marks)**

Determine how many of the integers between 1 and 340 inclusive are

(a) divisible by 6. (1 mark)

56 ✓

(b) divisible by 6 or 7. (3 marks)

÷ by 6 : 56 ✓
÷ by 7. : 48
÷ by 6 & 7 : 8 ✓

÷ by 6 & 7 : 56 + 48 − 8 = 96. ✓

(c) divisible by 6 ∪ or 7 but not ∩ both. (1 mark)

96 − 8 = 88 ✓

96 ↗ ÷ 6 & 7

8 ↖ 6 ∩ 7 Not

(d) divisible by 6 ∪ or 7 but not 4. (3 marks)

÷ 6 ∩ 4 (12) : 340 ÷ 12. = 28 ✓
÷ 7 ∩ 4 (28) : 340 ÷ 28. = 12 ✓
÷ 4 ∩ 6 ∩ 7 (84) : 340 ÷ 84 = 4. ✓

96 − 28 − 12 + 4. = 60 ✓

8

End of questions

Supplementary page

Question number: _____

Supplementary page

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