MATHEMATICS METHODS UNITS 3&4

Section Two:
Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

WA student number: In figures ☐☐ ☐☐☐ ☐☐☐

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable): ☐

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.
    Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Markers use only		
Question	Maximum	Mark
9	7	
10	6	
11	7	
12	6	
13	8	
14	8	
15	8	
16	10	
17	8	
18	8	
19	8	
20	6	
21	8	
S2 Total	98	
S2 Wt ($\times 0.6633$)	65%	

Section Two: Calculator-assumed **65% (98 Marks)**

This section has **thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 **(7 marks)**

The percentage distribution of the number of cans of soft drink per order placed with a takeaway food company over a long period of time are shown in the following table.

Number of cans per order	0	1	2	3	4 or more
Percentage of orders	14	24	45	5	12

In the following questions, you may assume that all orders are placed with the company at random and independently.

(a) Determine the probability that the next 10 orders all include at least one can of soft drink.

(2 marks)

(b) During a weekday, a total of 225 orders were placed. Determine the probability that

(i) 40 of these orders included 3 or more cans of soft drink. (3 marks)

(ii) more than 25 of these orders included no cans of soft drink. (2 marks)

See next page
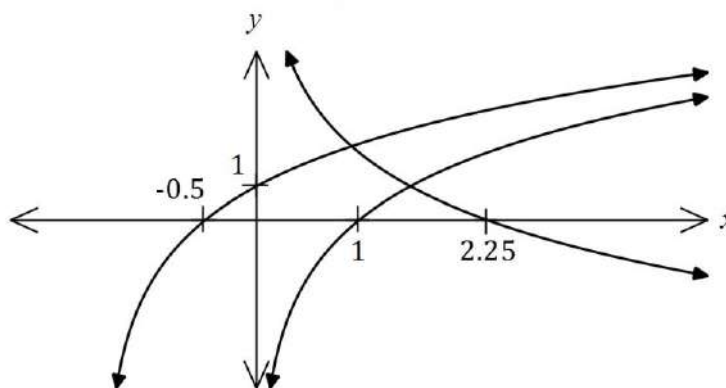
Question 10 **(6 marks)**

(a) Function f is defined by $f(x) = 5\log_3(x + 9) - 4$ over its natural domain. Determine

  (i) the value of the y-intercept of the graph of $y = f(x)$. (1 mark)

  (ii) the equation of the asymptote of the graph of $y = f(x)$. (1 mark)

(b) Function g is defined by $g(x) = \log_n x$ over its natural domain, where n is a constant greater than 1. The graphs shown below have equations $y = g(x)$, $y = a - g(x)$ and $y = g(x + b)$, where a and b are constants. Determine the value of n, a and b. (4 marks)

Question 11 **(7 marks)**

In a sample of 1 325 university students, 64% said that they never look at their phone while driving.

(a) Show how to use the figures from this sample to construct the 95% confidence interval for the proportion of university students who never look at their phone while driving. (3 marks)

(b) According to a newspaper article, "70% of university students never look at their phone while driving". Explain whether the interval from (a) supports this claim. (2 marks)

(c) Another source claims that "the majority of university students never look at their phone while driving". Explain whether the interval from (a) supports this claim. (2 marks)
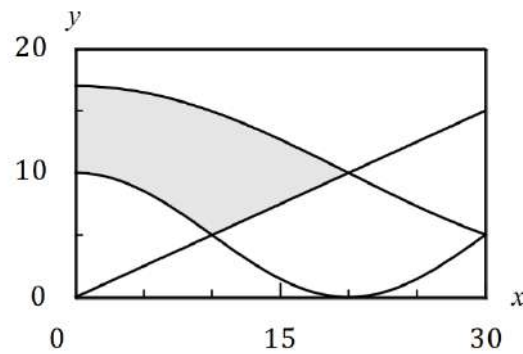
Question 12   **(6 marks)**

The diagram shows a flag design, with dimensions in centimetres.

The shaded region is bounded by the y-axis, $y = f(x)$, $y = g(x)$ and $y = h(x)$ where

$f(x) = 0.5x,$

$g(x) = 5 + 5\cos\left(\dfrac{\pi x}{20}\right)$ and

$h(x) = 10 + 7\sin\left(\dfrac{\pi(x+20)}{40}\right).$

(a)   Let A be the area of another region on the graph, where $A = \displaystyle\int_{20}^{30} [f(x) - h(x)]\, dx.$

     (i)   Clearly mark the region on the diagram with the letter A.   (1 mark)

     (ii)   Determine the value of A, rounded to one decimal place.   (1 mark)

(b)   Determine the exact area of the shaded region.   (4 marks)

Question 13 **(8 marks)**

The weights of boys W in a large study of 5-year-old children are normally distributed with a mean of 18.2 kg and a standard deviation of 2.15 kg.

(a) Determine the probability that a randomly selected boy from the study has a weight

(i) that rounds to 17 kg, to the nearest kilogram. (2 marks)

(ii) no more than 20 kg given that they weigh at least 18.2 kg. (2 marks)

(b) The heaviest 4% of boys were classified as obese. Determine the least weight of a boy to be classified in this manner. (1 mark)

(c) The weights of girls in the study are normally distributed with mean of 17.9 kg and the heaviest 2% of girls, weighing more than 22.5 kg, were classified as obese. Determine the standard deviation of the girls' weights. (3 marks)

Question 14 **(8 marks)**

A cooling system maintains the temperature T of an integrated circuit between $0.5\,°C$ and $1\,°C$. At any instant, T is a continuous random variable defined by the probability density function

$$f(t) = \begin{cases} \dfrac{a}{t} & 0.5 \leq t \leq 1 \\ 0 & \text{elsewhere.} \end{cases}$$

(a) Determine the exact value of the constant a. (2 marks)

(b) Determine a decimal approximation for the probability that a temperature taken at random exceeds $0.85\,°C$. (2 marks)

(c) Determine decimal approximations for the mean and standard deviation of the temperature of the integrated circuit. (4 marks)

Question 15 **(8 marks)**

The voltage generated by a circuit at time t seconds is given by $V(t) = e^{0.1t} \sin(2t)$ for $0 \le t \le 5$.

(a) Show that the voltage is initially increasing. (2 marks)

(b) Using a graphical method, or otherwise, determine the voltage at the instant the rate of change of voltage first starts to decrease. (3 marks)

(c) Use the increments formula to estimate the change in voltage in the one tenth of a second after $t = 3$. (3 marks)

Question 16 **(10 marks)**

Random samples of 165 people are taken from a large population. It is known that 8% of the population have blue eyes.

(a) Use a discrete probability distribution to determine the probability that the proportion of people in one sample who have blue eyes is less than 7%. (3 marks)

(b) Ten consecutive random samples are taken. Determine the probability that the proportion of those with blue eyes is less than 7% in exactly half of these samples. (2 marks)

A large number of random samples of 165 people are taken, the proportion of blue eyed people calculated for each sample and the distribution of these sample proportions analysed.

(c) Describe the continuous probability distribution that these sample proportions approximate, including any parameters. (3 marks)

(d)   Describe how two factors affect the closeness of the approximate distribution in (c) to the true distribution of proportions.   (2 marks)

See next page

Question 17 **(8 marks)**

A charged capacitor discharges through a resistor. Some readings of the deflection D cm of a galvanometer scale in the circuit t seconds after the discharge began are shown on the semilogarithmic graph below.
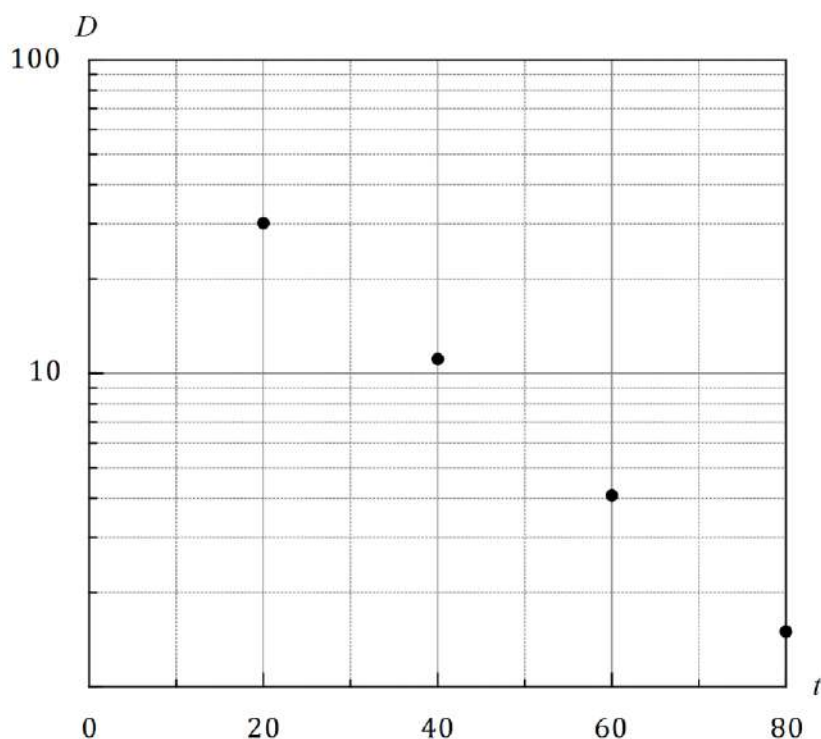
The relationship between the variables is of the form $D = ae^{kt}$, where a and k are constants.

(a) Use the above relationship to obtain an expression for $\ln D$ in terms of a, k and t and hence explain why plotting the data using a logarithmic scale on the vertical axis aligns the points in a straight line. (2 marks)

(b)   Show how to use the relationship and the galvanometer readings at $t = 20$ and $t = 60$ to determine estimates for a and k.   (4 marks)

(c)   Determine

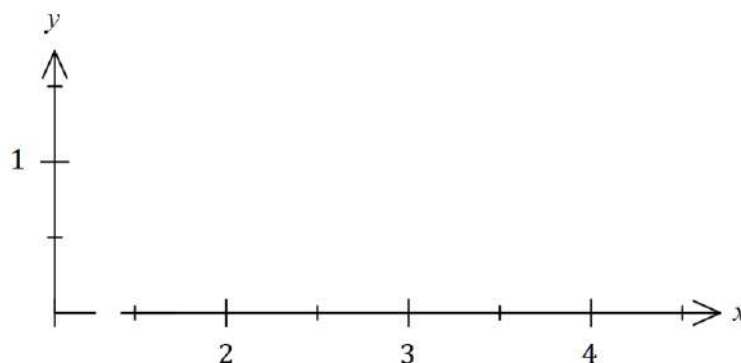   (i)   the deflection after 90 seconds.   (1 mark)

    (ii)   the time for the deflection to reach 1 mm.   (1 mark)

Question 18 **(8 marks)**

Let $f(x) = 1 - (x - 3)^2$ and $g(x) = \ln(x - 1)$.

(a)   Sketch the graphs of $y = f(x)$ and $y = g(x)$ for $x \geq 2$ on the axes below.   (2 marks)

(b)   Show that $\dfrac{d}{dx}((x - 1)\ln(x - 1) - (x - 1)) = \ln(x - 1)$.   (2 marks)

(c)   Show that the area of the region bounded by the graphs of $y = f(x)$ and $y = g(x)$, and the straight line $x = 3$ is exactly $\dfrac{5}{3} - 2\ln 2$ square units.   (4 marks)

Question 19 **(8 marks)**

The cross section of a triangular prism with a volume of 432 cm^3 is an equilateral triangle of side length x cm.

(a) Show that the surface area S cm of the prism is given by $S = \dfrac{\sqrt{3}x^2}{2} + \dfrac{1728\sqrt{3}}{x}$. (4 marks)

(b) Use calculus to determine the minimum surface area of the triangular prism. (4 marks)

See next page

Question 20 **(6 marks)**

A student was set the task of determining the proportion of people in their suburb who use public transport at least once a week.

(a)   Briefly discuss the main source of bias in each of the following sampling methods.

(i)   The student randomly selects staff and students using school records.   (1 mark)

(ii)   The student invites people to respond to their survey using an advert in the suburbs free local paper.   (1 mark)

(b)   The student noted that 36 out of all those sampled said they used public transport at least once a week and went on to construct the confidence interval $(0.32, 0.58)$. Determine the level of confidence of this interval.   (4 marks)

Question 21 **(8 marks)**

When a customer plays an online game of chance, a computer randomly picks one letter from those in the word LUCKY, another from those in the word BOIST, and a third from those in the word GAMER. For example, the computer might pick KSR, YBG, and so on. The customer can see the words but does not know the computer's picks and has to guess the letter it has chosen from each word. The random variable X is the number of letters correctly guessed by a customer in one play of the game.

(a) Complete the table below to show the probability distribution of X. (3 marks)

x	0	1	2	3
$P(X = x)$				

Each game costs a player 25 cents. A player wins a prize of $14 if they guess all three letters correctly, $1.40 if they guess two out of three letters correctly but otherwise wins nothing.

(b) Determine $E(Y)$ and $\mathrm{Var}(Y)$, where the random variable Y is the gain, in cents, made by the customer in one play of the game. (4 marks)

(c) If an average of 250 people from around the world play the game once every 20 seconds, calculate the gross profit expected by the game owners in any 24-hour period. (1 mark)

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____