Probability power: language and topic modelling

Stanislav Protasov

What about the past?

1. How to (mathematically) find harmonics (i.e. wave components) of any sound?
2. How to perform query by humming?
3. What makes our voices different?
4. Why to remove near-duplicate images from search results?
5. How to find the same object on images even if it is rotated?
6. Why texture features are bad at scanned text retrieval?

Agenda

- First, mostly of **language modelling**

- And then, little on **topic modelling**

Language Model: beginning

Language model is used to *generate* texts (e.g. suggest next word) or to compare texts being similar in some case (“*recognize*”)

Prerequisites

What is mathematical **model**?

Why do we need **model**? (in general)

Which **models** did we already consider, and how did we **benefit**:

- Vector Space Model
- Deep ANNs
- Small World Model
- Regular languages for syntax modelling
- ...

What is language

Language is a set. **Language model** serves for two tasks:

- **Guarding predicate** `L = {w|P(w)}    # recognition`
- Set **generation procedure**

And as any other model, it helps to solve tasks stated for its prototype (real language):

- Are these words similar?
- What is the answer for the question?
- Are those texts from the same topics?
- ...
- **propose your own tasks**

Did we already cover any **language models**?

- Vector space and latent space model (*)
- Chomsky hierarchy models…
  - including finite state automata

Very simple model

$$\sum_{s \in \Sigma^*} P(s) = 1$$

$$P_{\text{uni}}(t_1 t_2 t_3) = P(t_1)P(t_2)P(t_3).$$

Unigram model

UM — token (word) frequency as probability estimation

P(s) =

= P("abbcccd") = P("abbcccd[STOP]") =

= [P("a") * P("b")^2 * P("c")^3 * P("d")] *

P(~STOP)^7 * (STOP)

Unigram model

$$P(\text{query}) = \prod_{\text{term in query}} P(\text{term})$$

Consider {a, b} as vocabulary.

$P(\$) = ½$

$P(a\$) = P(b\$) = ½ * ½^2 = ⅛$ | ¼

$P(aa\$) = P(ba\$) = = ½ * ½ * ½^3 = 1/32$ | ⅛

In practice **constant part** is omitted:

1. For same length this is constant
2. In search we compare models. For model comparison constant factor is omitted

Model types

General model

$$P(t_1t_2t_3t_4) = P(t_1)P(t_2|t_1)P(t_3|t_1t_2)P(t_4|t_1t_2t_3)$$

Unigram model

$$P_{\text{uni}}(t_1t_2t_3t_4) = P(t_1)P(t_2)P(t_3)P(t_4)$$

Bigram model

$$P_{\text{bi}}(t_1t_2t_3t_4) = P(t_1)P(t_2|t_1)P(t_3|t_2)P(t_4|t_3)$$

How to find $P(t_1)$, $P(t_2|t_1)$?

Model size for P_{uni}, P_{bi}?

Unigram model under the microscope == multinomial distribution

We don't care about the order, that is why sentence is a bag of words:

```
Professor called Stas ate a dog == Dog called Stas ate a professor
```

Probability of bag ***d*** then is described by a multinomial distribution:

$$P(d) = \frac{L_d!}{\mathrm{tf}_{t_1,d}!\mathrm{tf}_{t_2,d}! \cdots \mathrm{tf}_{t_M,d}!} P(t_1)^{\mathrm{tf}_{t_1,d}} P(t_2)^{\mathrm{tf}_{t_2,d}} \cdots P(t_M)^{\mathrm{tf}_{t_M,d}}$$

Here, $L_d = \sum_{1 \le i \le M} \mathrm{tf}_{t_i,d}$ is the length of document d, M is the size of the term vocabulary, and the products are now over the terms in the vocabulary, not the positions in the document.

Language Model: how to use it in IR

Language model is another way of **comparing texts.**
For this you can use language models of query and text.
And look for such text, which are “better” in generating queries.

Use **KL-divergence** for this.

Ranking, oh yeah!

Likelihood

$$L(\theta \mid x) = p_\theta(x) = P_\theta(X = x)$$

Query Likelihood Model

$$L(d|q) = P(q|d)P(d)/P(q)$$

$$\mathbf{L(d \mid q) \sim P(q \mid d)}$$

$$P(d) = \frac{L_d!}{\mathrm{tf}_{t_1,d}!\mathrm{tf}_{t_2,d}! \cdots \mathrm{tf}_{t_M,d}!} P(t_1)^{\mathrm{tf}_{t_1,d}} P(t_2)^{\mathrm{tf}_{t_2,d}} \cdots P(t_M)^{\mathrm{tf}_{t_M,d}}$$

Each text is a language! ლ(ಠ益ಠლ)

1. Build a model for each text (does it look similar to building TDM?)
2. Compute relevance for each doc (how **likely** this **query** is generated **by the doc language**)

$$P(q|M_d) = K_q \prod_{t \in V} P(t|M_d)^{\mathrm{tf}_{t,d}}$$

$$P(t \mid M_d) \text{ ==?== } P_{Md}(t)$$

But wait, we have ***the*** language model M_c

1. If $\hat{P}(t|M_d) = 0$ (***tf***$_{t,d}$ = 0) $\hat{P}(t|M_d) \leq \mathrm{cf}_t / T$

2. Linear interpolation

$$\hat{P}(t|d) = \lambda \hat{P}_{\mathrm{mle}}(t|M_d) + (1-\lambda)\hat{P}_{\mathrm{mle}}(t|M_c)$$

3. Bayesian smoothing

$$\hat{P}(t|d) = \frac{\mathrm{tf}_{t,d} + \alpha \hat{P}(t|M_c)}{L_d + \alpha}$$

Summary

1. Unigram model $P_{\text{uni}}(t_1 t_2 t_3 t_4) = P(t_1)P(t_2)P(t_3)P(t_4)$

2. Unigram max likelihood query $L(d|q) \sim P(q|d)$

3. $P(t|d)$ **!=** $\text{tf}_{t,d} / D$ as it leads to `0-s`

4. Estimate using **global** language **model**

$$P(d|q) \propto P(d) \prod_{t \in q} ((1-\lambda)P(t|M_c) + \lambda P(t|M_d))$$

Extended LM and motivations

Add **query language model** and use Kullback-Leibler divergence

$$R(d;q) = KL(M_d \| M_q) = \sum_{t \in V} P(t|M_q) \log \frac{P(t|M_q)}{P(t|M_d)}$$

What about **multilingual retrieval**? Just add translation matrix!

$$P(q|M_d) = \prod_{t \in q} \sum_{v \in V} P(v|M_d) T(t|v)$$

Topic modelling

There are (latent) topics, which define behaviour of language models. **Text belongs to a combination of few topics**, which can be discovered using TM.

Topic model

Purpose:

1. Which **topics** a **document** belongs to
2. Which **words** form which **topic**

Which questions answer:

1. How did the topic evolve in media?
2. What are major topics of this author?
3. How many topics are there?
4. ...

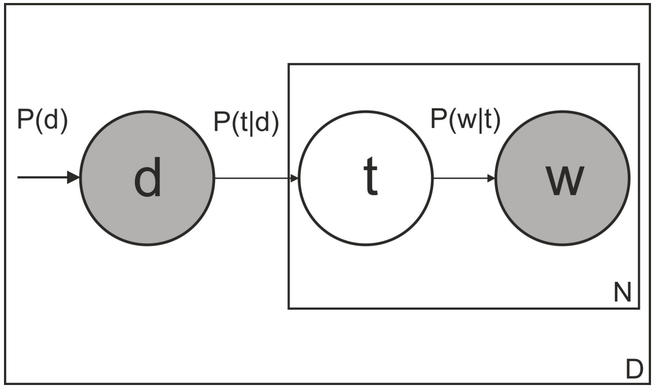
Conditional independence hypothesis

How **words are generated** in the model does not depend on **~~particular document~~**, but **on the topic** of the document.

$$p(w \mid d, t) = p(w \mid t);$$
$$p(d \mid w, t) = p(d \mid t);$$
$$p(d, w \mid t) = p(d \mid t)p(w \mid t).$$

Topic model is (formally)

$$p(d,w) = \sum_{t \in T} p(t)p(w|t)p(d|t) = \sum_{t \in T} p(d)p(w|t)p(t|d) = \sum_{t \in T} p(w)p(t|w)p(d|t)$$



$$p(w \mid d) = \sum_{t \in T} p(t \mid d)\, p(w \mid t).$$

***p*(*w*|*d*)** - how to estimate and model these values?

$$\Phi = ||p(w|t)|| \qquad \Theta = ||p(t|d)||$$

Looking for decomposition F ≈ ΘΦ!

Sparseness hypothesis: document belongs to a very limited topic number. Thus, both Θ and Φ are sparse.

They model **probabilities**, thus, **p(*|*) ∈ [0, 1]** and

$$\sum_w p(w|t) = 1, \sum_t p(t|d) = 1, \sum_t p(t) = 1,$$

SVD and PCA don't work (non-stochastic matrix + bias)

Scientists argue whether **PLSA** (Vorontsov) or **LDA** (Blei, Ng) is better

B(y|e)!