Vector model
Distributive semantics
Dimension reduction

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Agenda

- Vector interpretation of boolean query
- Distributive semantics
- Dimension reduction and LSA

Term document matrix			
words\documents	Document1	document2	query term
cat	1	1	0
runs	1	1	0
behind	1	1	0
rat	1	0	1
dog	0	1	0

Term-document matrix

TDM — describes the **frequency of terms** that occur in a collection of documents.

term-document matrix … **documents are the columns** and **terms are the rows** (Wiki)

Term \ Document	information technology	information system	communication technology	software application	telecommunication	computer science
1	0.77	0.55	0.45	0.13	0.14	0.15
2	0	0	0.13	0.53	0.15	0.75

In a *document-term matrix* (*DTM* = *TDM*^{T}), rows correspond to documents in the **collection and columns correspond to terms**

- Column is a description (**vector**, BoW) of a document
- Row is a vector representation of a word
- **Sparse** for short texts

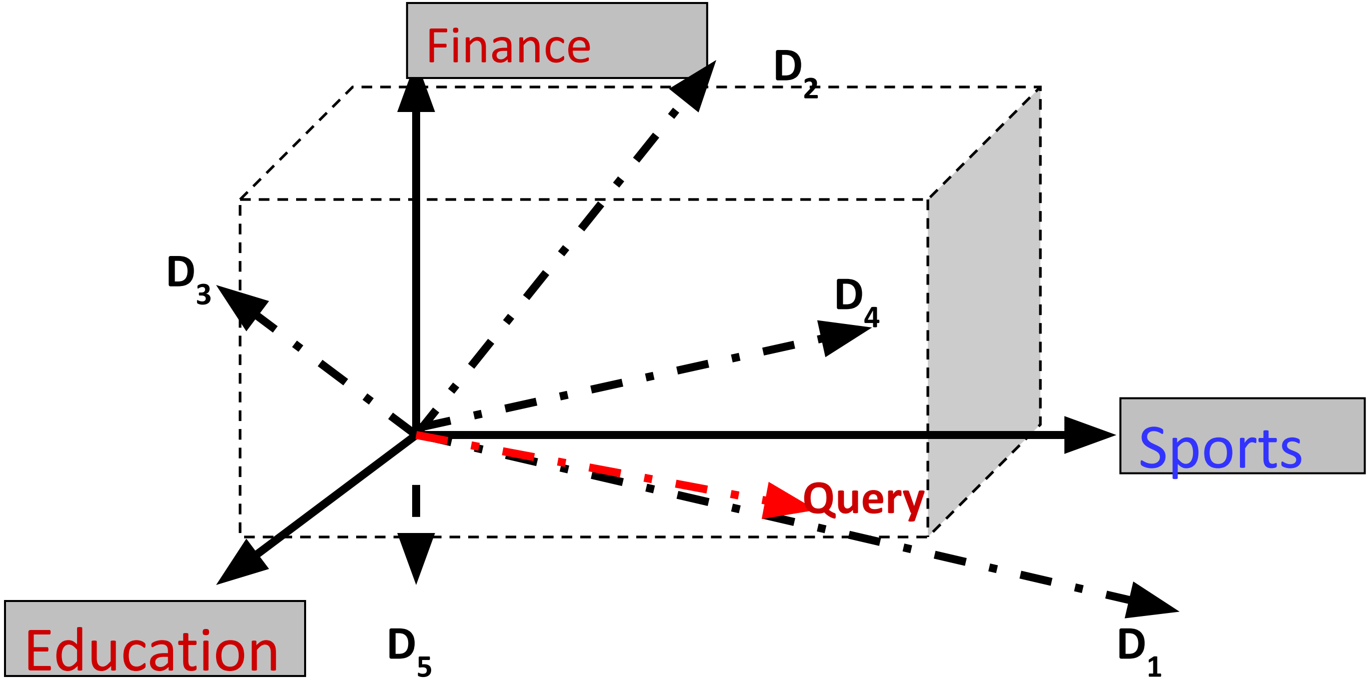
Venue / Keyword	CAMAD	EUNICE	HAISA	HPCC-ICESS	IJESMA	ISCA	KMIS	NMR	SPRINGL	SSV
algorithm	2	8	0	24	0	5	0	2	1	1
cellular	2	1	0	1	0	0	0	0	0	0
game	1	1	0	1	0	0	1	1	0	0
hardwar	1	0	1	4	0	18	0	0	1	0
internet	2	6	2	0	2	0	0	0	0	0
mobil	10	8	0	6	17	5	2	0	2	0
network	58	60	4	38	2	25	12	0	3	0
search	0	1	0	1	2	4	1	0	0	0
secur	4	4	29	5	1	12	3	0	4	0
web	0	2	0	3	3	1	13	0	2	0

Vector space model

- Represents both *doc* and *query* by "<u>concept</u> **vectors**"
  - Each <u>concept</u> defines one **dimension**
  - *K* concepts define a high-dimensional space
  - Element of vector corresponds to concept weight
    - E.g.,for $d=(x_1,\ldots,x_k)^T$, x_i is the "weight" of concept i (e.g. TF-IDF)
- Measures relevance approximation
  - **Distance** between the query vector and document vector in this concept space
  - `relevance ≈ similarity = 1 - distance`
  - How can we define distance?

VS Model: an illustration

- Which document is the closest to the query?

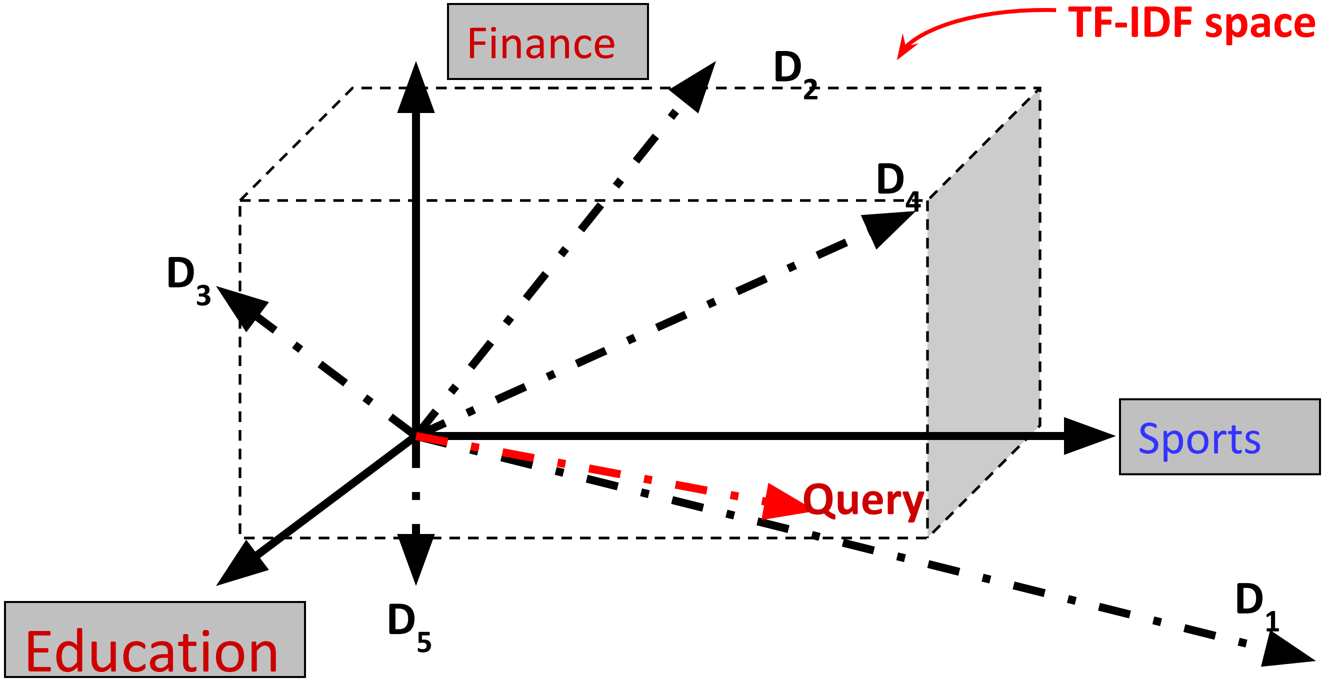


What the VS model doesn't say

- How to define/select the "basic concept"
  - Concepts are assumed to be **orthogonal**
- How to assign **weights**
  - *Weight in a query* indicates importance of the concept for a query
  - *Weight in a doc* indicates how well the concept characterizes the doc
- How to define **distance** measure?

How to define a good similarity measure?

- Euclidean distance?

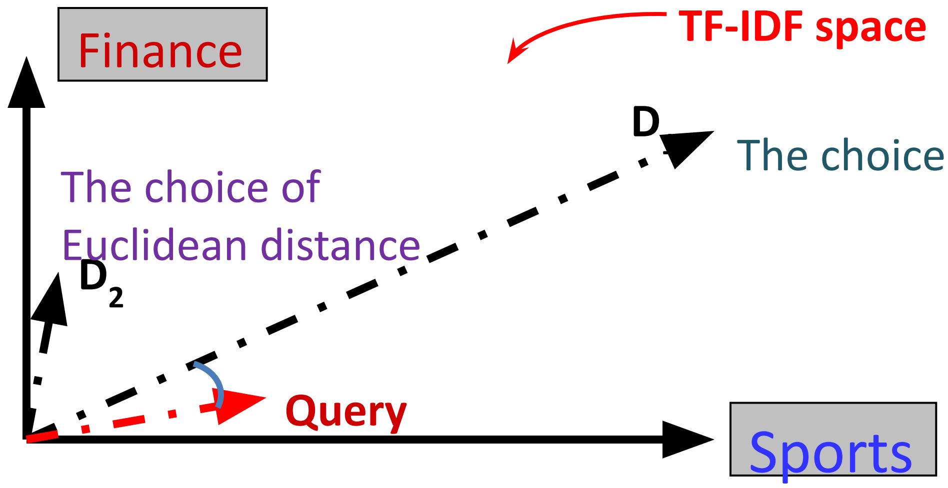


From distance to angle

Cosine similarity – projection of one vector onto another

- ±1 if vectors are collinear
- 0 if vectors are orthogonal

$$similarity = \cos(doc, query) = \frac{\overrightarrow{doc} \cdot \overrightarrow{query}}{\|\overrightarrow{doc}\| * \|\overrightarrow{query}\|}$$



Stop here!

1. We found a way, which allows to **represent any document** (even unseen) **as a vector**.
2. We introduced a **relevance metric** using a simple well-known mathematical concept - **cosine similarity**
3. Still **non-orthogonal** concepts
4. To measure similarity of 2 documents (or doc vs query) we need to do circa **100K arithmetic operations** with floating point numbers

Reduce dimensions!

- Compression approach #1 — works great for **sparse databases**:

```
max_size = N
doc_compressed[i % max_size] += doc[i] (or max, or =)
```

- Compression approach #2:
  - Random projection
  - Or even randomly remove some dimensions!
- Compression approach #3 — latent semantic analysis:
  - LDA, PCA/SVD, GDA, …
  - Embedding using encoder networks (BERT, doc2vec, DSSM, ...)

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Distributional semantics

Recall: **word** is just a **vector** in the vector space model

The **distributional hypothesis**:

- linguistic items with similar distributions have similar meanings
- words that are used and occur in the same contexts tend to purport similar meanings
- You shall know a word by the company it keeps (Firth, J. R. 1957)

Notes on distributional semantics

"*Similar context*" and "*same distribution*" are not well-defined terms. It can go for bag of words model (TDM) or for near-context (as in word2vec).

Distributional hypothesis is a powerful model, which made to happen topic modelling, and all the ML-based NLP.

Consequence:

- If 2 **word**-corresponding **rows** from TDM **correlate**, we fail with orthogonality.
- But we can **infer orthogonal vector** from TDM!

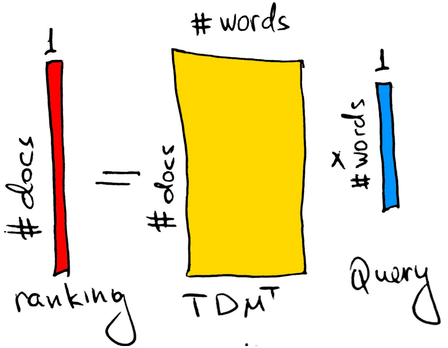
Hypothesis 2: Maybe there is a **latent semantic space** of smaller dimension?

Latent semantic analysis (patent)

Idea: **search for low-rank approximation of TDM!**

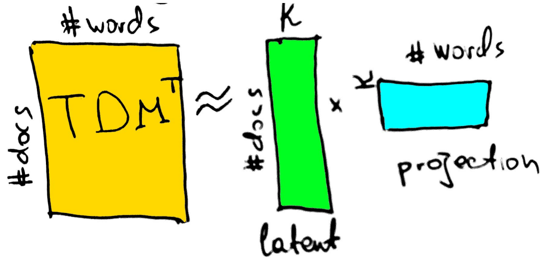
What does it mean? By now, query is (assume vectors are **normed**):

$$\text{Rankings} = \text{TDM}^T * Q_{\text{vector}}$$



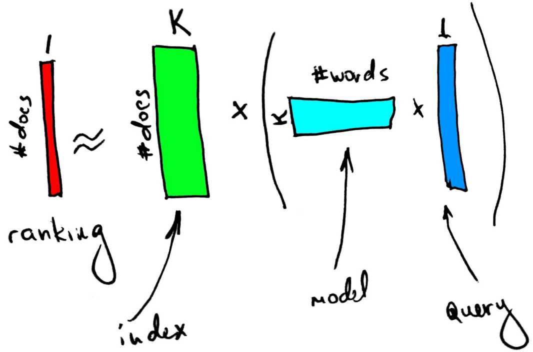
What if

$$\text{TDM}^T = \text{LATENT\_MX} * \text{PROJECTION\_MX}$$

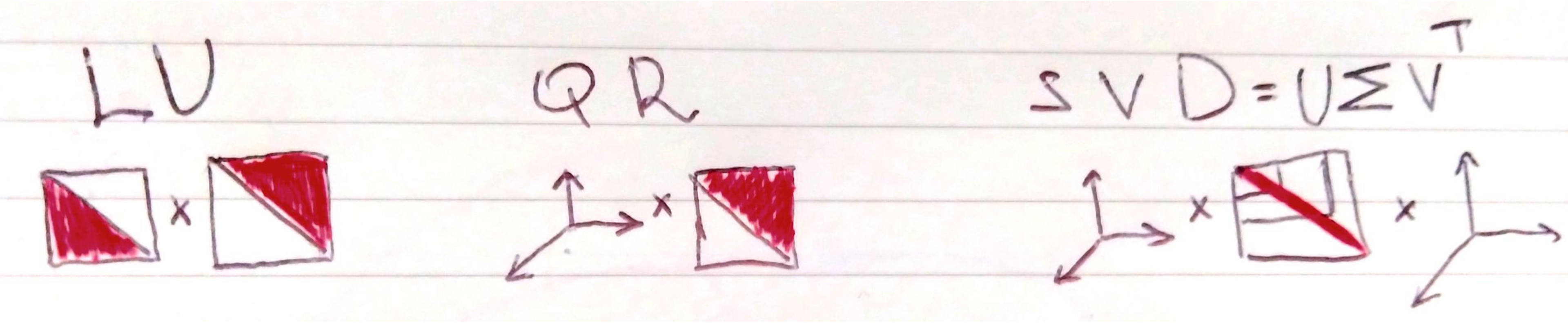


Then

$$\text{Rankings} = \text{LATENT\_MX} * [\text{PROJECTION\_MX} * Q_{\text{vector}}]$$



Decompositions

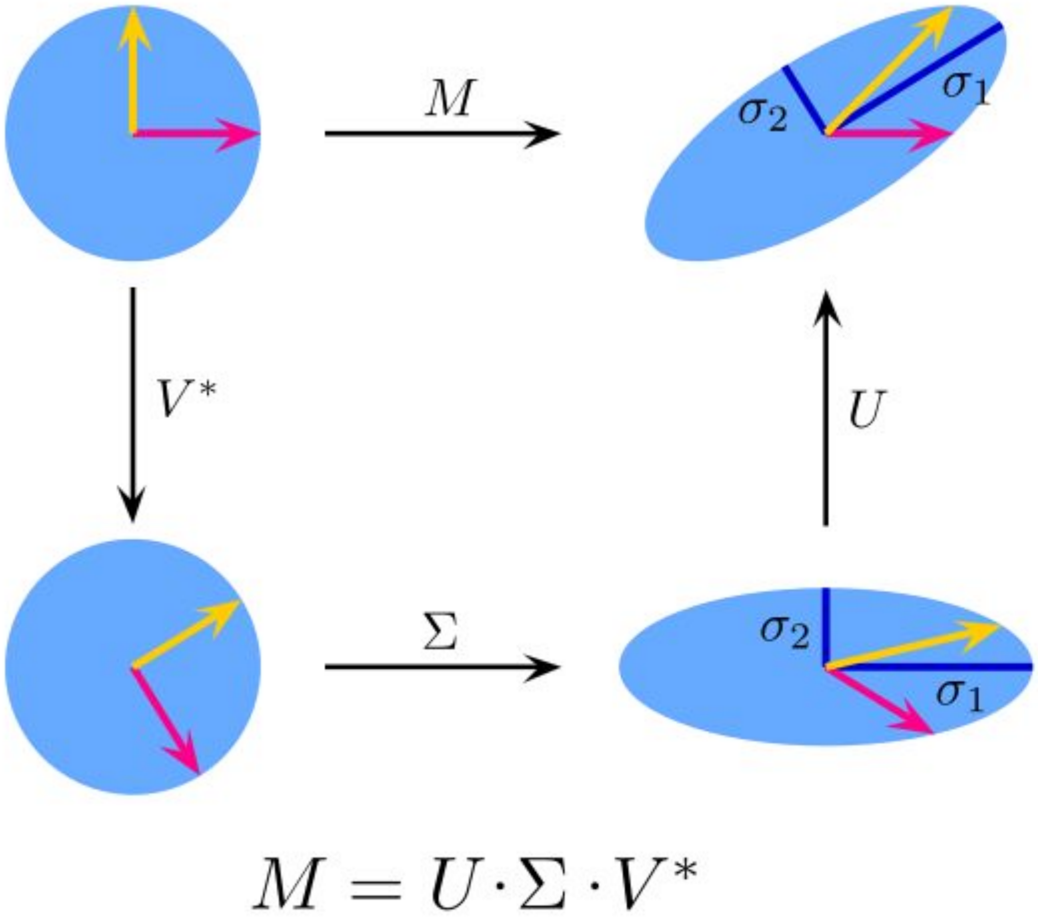


SVD

U is eigenvectors for $\mathbf{MM^T}$

$\mathbf{V^T}$ is eigenvectors for $\mathbf{M^TM}$

Σ is diagonal with square roots of non-negative eigenvalues of $\mathbf{M^TM}$



Matrix approximation

$$M = U_R \Sigma_R V_R^T$$

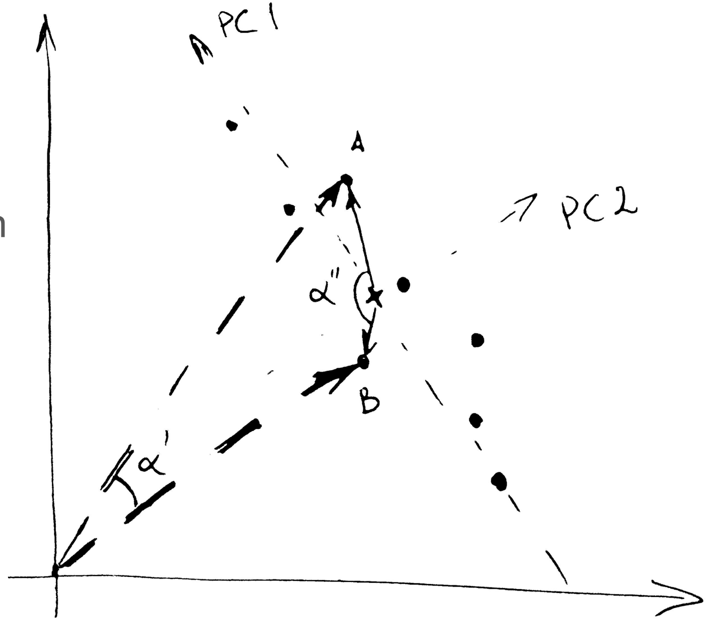
$$TDM^T = [U_R \Sigma_R] * V_R^T$$

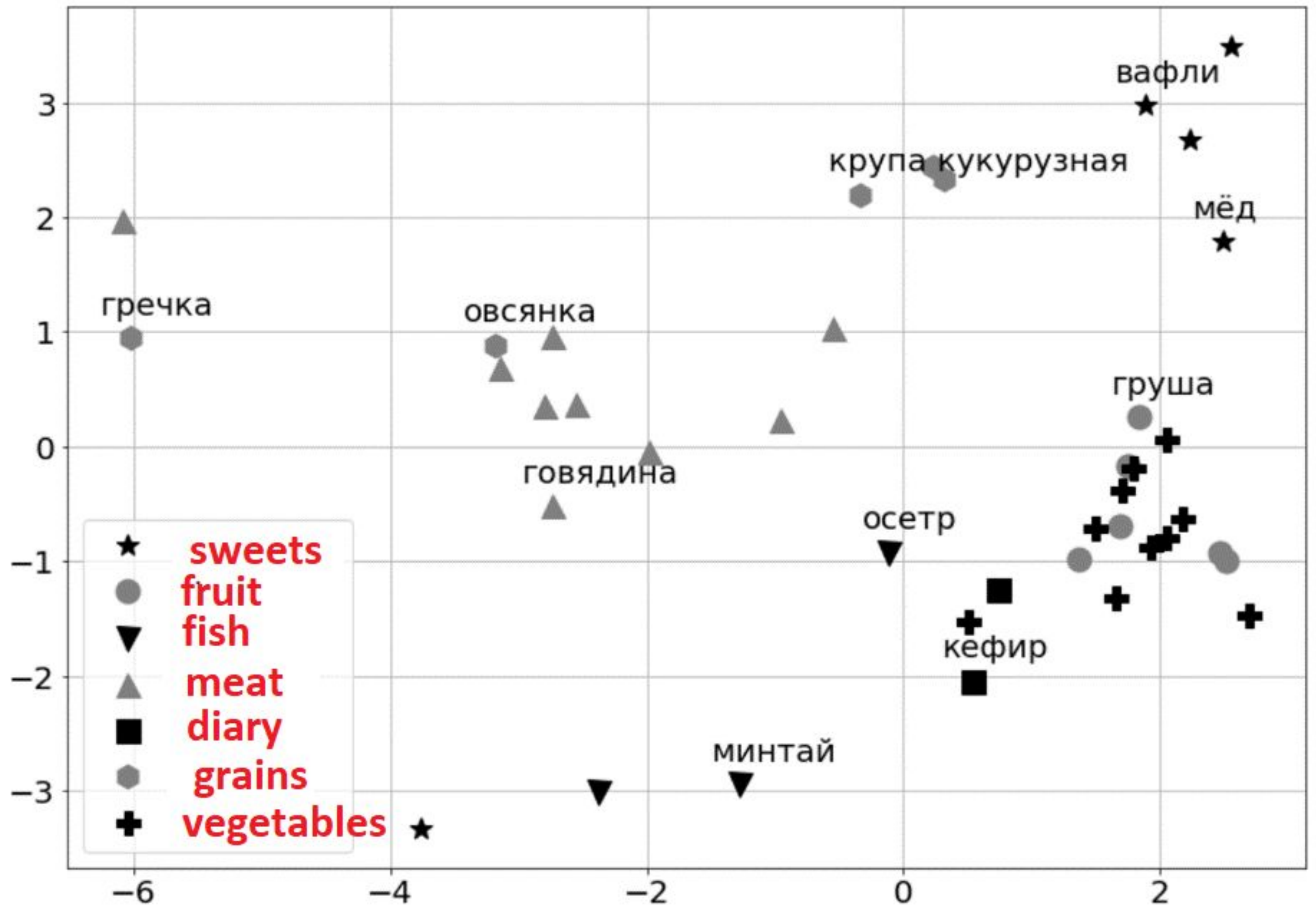
Principal component analysis (similar idea)

… Convert a set of observations of **possibly correlated variables** (entities each of which takes on various numerical values) **into a set of values** of linearly uncorrelated variables … (wiki)

… the **first principal component** has the **largest possible variance ...** and each succeeding component in turn has the highest variance possible under the constraint that it is orthogon to the preceding components (wiki)

NB: Implemented with SVD, PCA requires **data centering** first, which affects the **cosine metric**.





вафли
крупа кукурузная
мёд
гречка
овсянка
груша
говядина
осетр
кефир
минтай
sweets
fruit
fish
meat
diary
grains
vegetables

Stop Here!

1. **Vector space** model is cool, but (1) TDM is sparse (2) concepts are not orthogonal
2. **Distributional hypothesis** gives an insight: words correlate and their distribution defines semantics
3. **Latent semantic analysis** says: yes, and we know that there is a small-dimensional latent space for semantics. TDM is just a **linear projection**
4. **SVD** says: mmmm… We know how this latent space should look like! Orthogonal features + decreasing variance

Reading

- The Book — chapter 6.2-6.5
- All links in this presentation