Variational Inference with Normalizing Flows

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Introduction

- Calculating the true posterior distribution of inference tasks is in most cases an intractable problem.
- Lots of research on approaches for efficient approximation of the posterior, however the resulting classes prove to be of limited expressiveness.
- The authors in [1] introduce the notion of normalizing flows, sequences of invertible transformations applied to a simple initial density, to efficiently create more expressive families of candidate posteriors to be used for variational inference.
- We compare the performance of different types of normalizing flows on the MNIST dataset.

Our Work

- Reproduced experiment on MNIST using Linear Normalizing Flows
- Reproduced experiment on MNIST using NICE
- Extended the ideas of the paper and experimented with Invertible Convolutional Flows
- Created open-source Github repository with code and results: `github.com/ATML-Group-12/normalising_flows`

Theoretical Background

Normalizing flows are sequences of invertible, smooth mappings $f : \mathbb{R}^d \to \mathbb{R}^d$. We define a flow of length K to be K such transformations as follows:

$$\mathbf{z_K} = f_K \circ \ldots \circ f_2 \circ f_1(\mathbf{z_0})$$

where $\mathbf{z_0}$ is a random variable that has a simple initial distribution, and $\mathbf{z_K}$ is the corresponding random variable after applying the transformations. The properties of normalizing flows allow us to calculate the log-density of $\mathbf{z_K}$ efficiently, using the change of variable theorem:

$$\ln q_K(\mathbf{z_K}) = \ln q_0(\mathbf{z_0}) - \sum_{k=1}^{K} \ln \left| \det \frac{\partial f_k}{\partial \mathbf{z_{k-1}}} \right|$$

The normalizing flow is defined as the path of the successive distributions q_K.

The first class of flows that we use in our experiments is that of linear flows. More specifically, we consider **planar** and **radial** flows, which perform series of contractions and expansions in the direction perpendicular to a fixed hyperplane and around a reference point respectively.

Theoretical Background

TODO We formulate... $\mathcal{S} = \{s_1, s_2, s_3, s_4, s_5, s_6\}$

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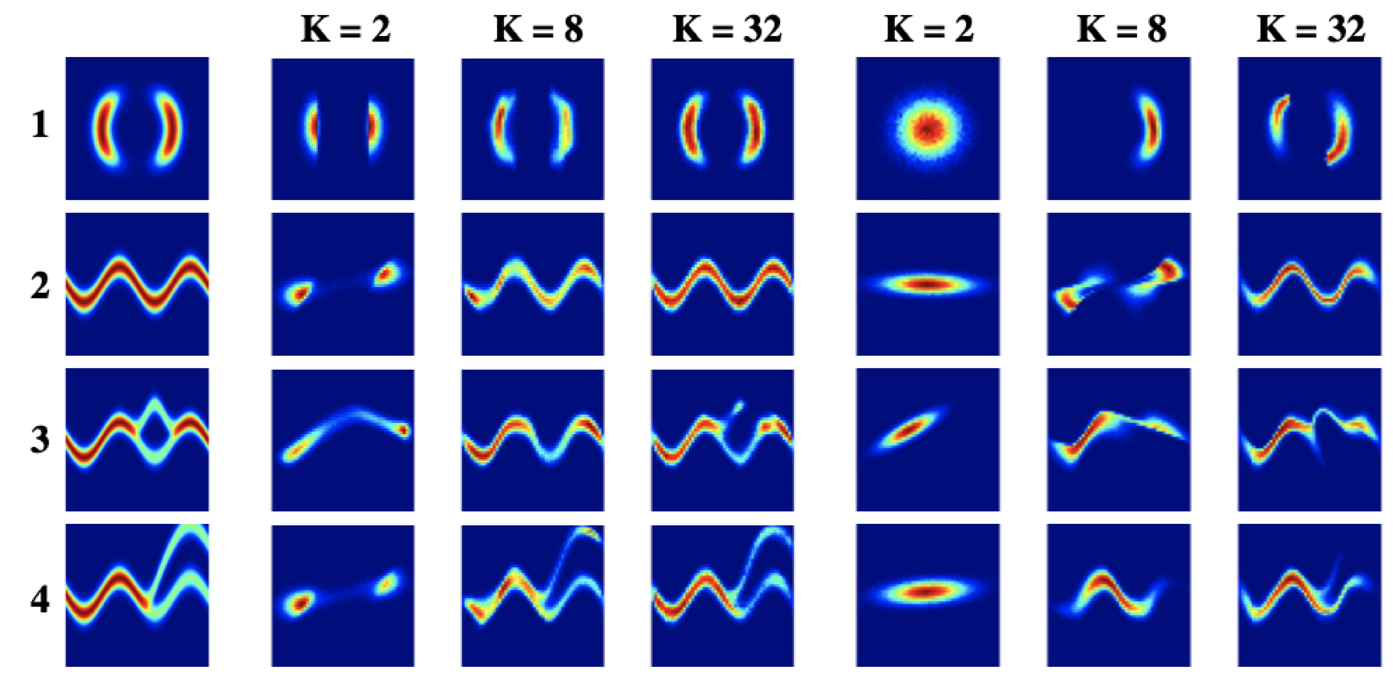
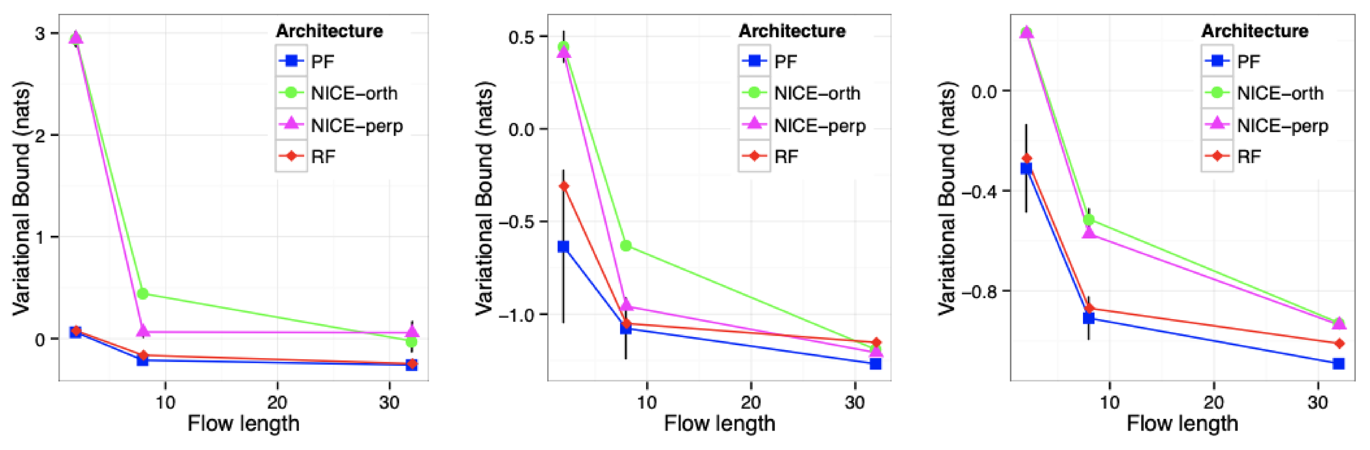
Image example:



(a) (b) Norm. Flow (c) NICE

Experiments

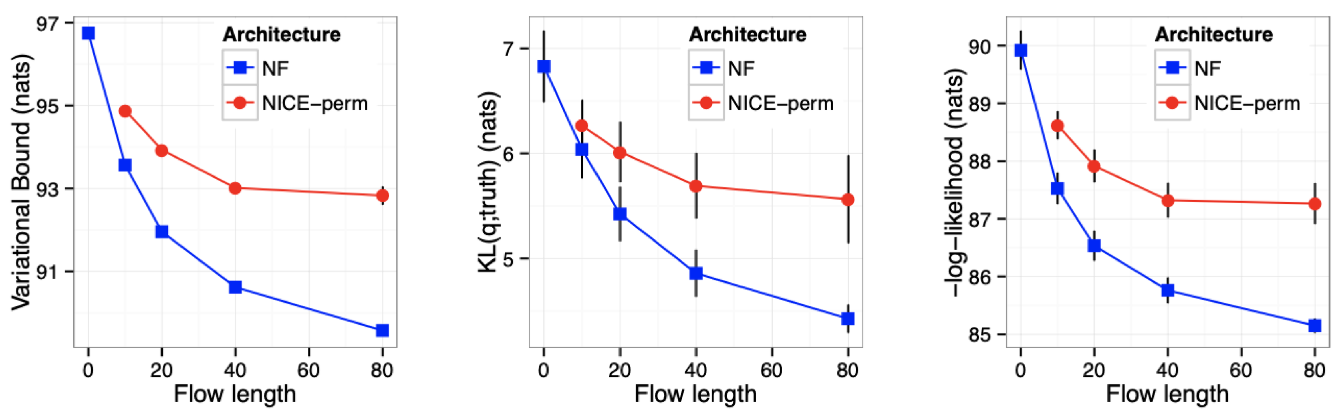
- DLGM + NF
- DLGM + NICE



(d) Comparison of KL-divergences.

Results

TODO!!! Results...



(a) Bound $\mathcal{F}(\mathbf{x})$ (b) $\mathbb{D}_{\mathrm{KL}}(q; p(z|x))$ (c) $-\ln p(\mathbf{x})$

Figure 4. Effect of the flow-length on MNIST.

Our Improvements and Extensions

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References

[1] Danilo Jimenez Rezende and Shakir Mohamed. Variational inference with normalizing flows, 2015.

[2] Laurent Dinh, David Krueger, and Yoshua Bengio. Nice: Non-linear independent components estimation, 2014.