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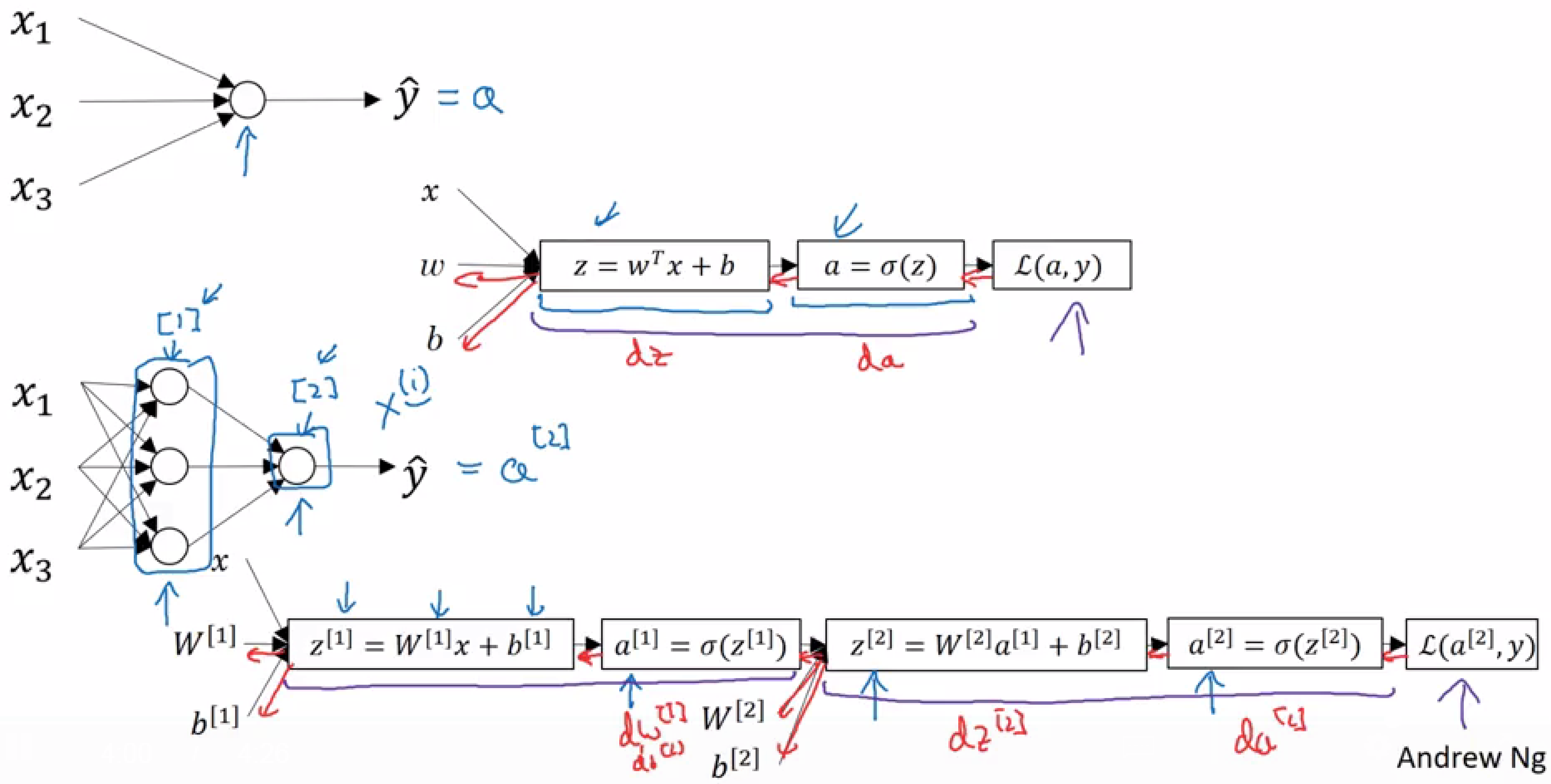
One hidden layer Neural Network

Neural Networks Overview

What is a Neural Network?



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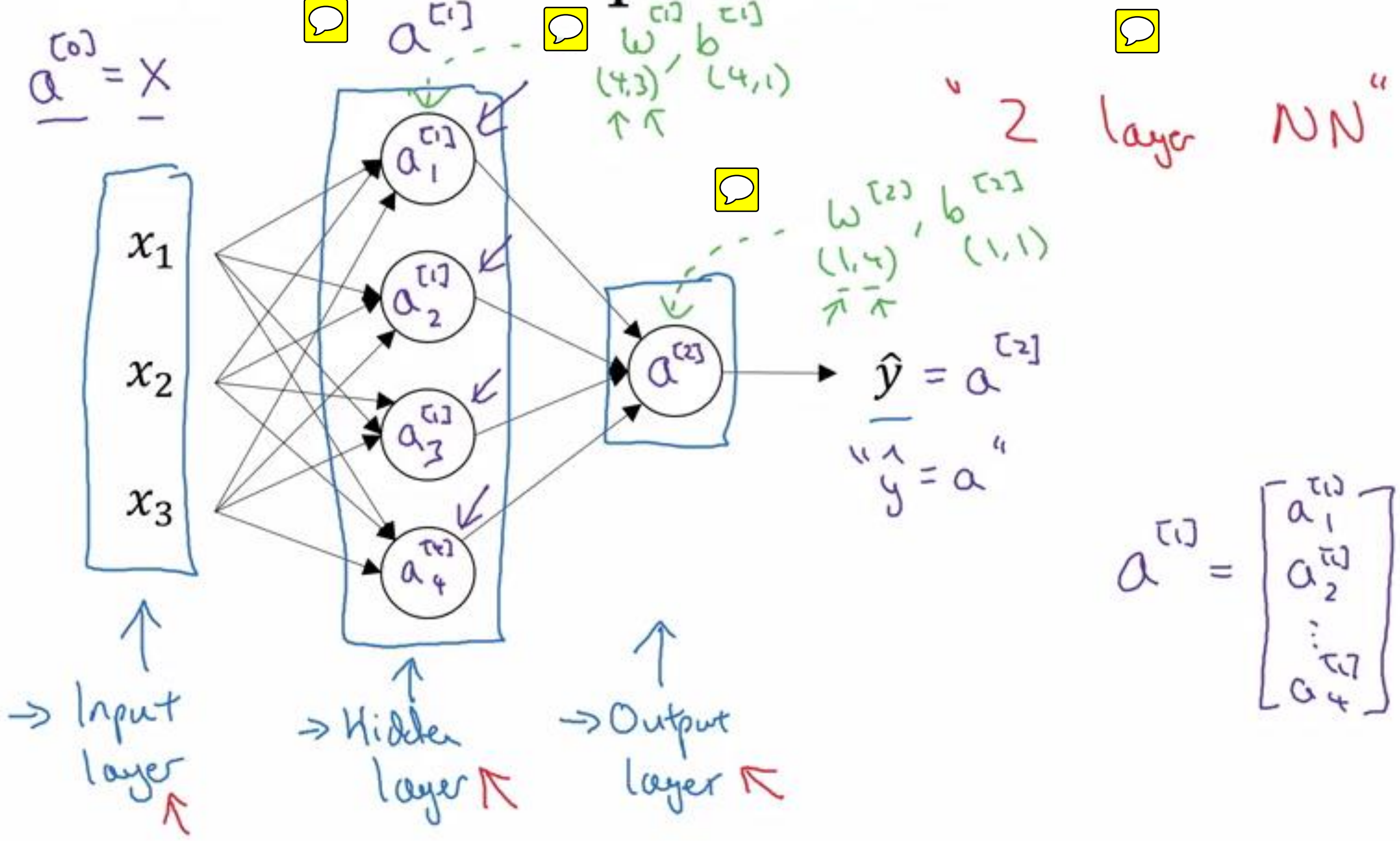
Neural Network Representation

Neural Network Representation



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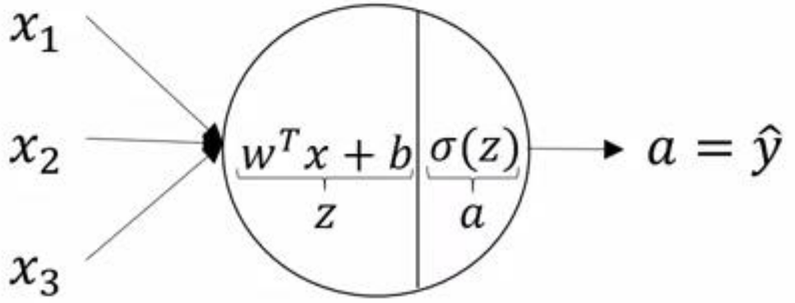
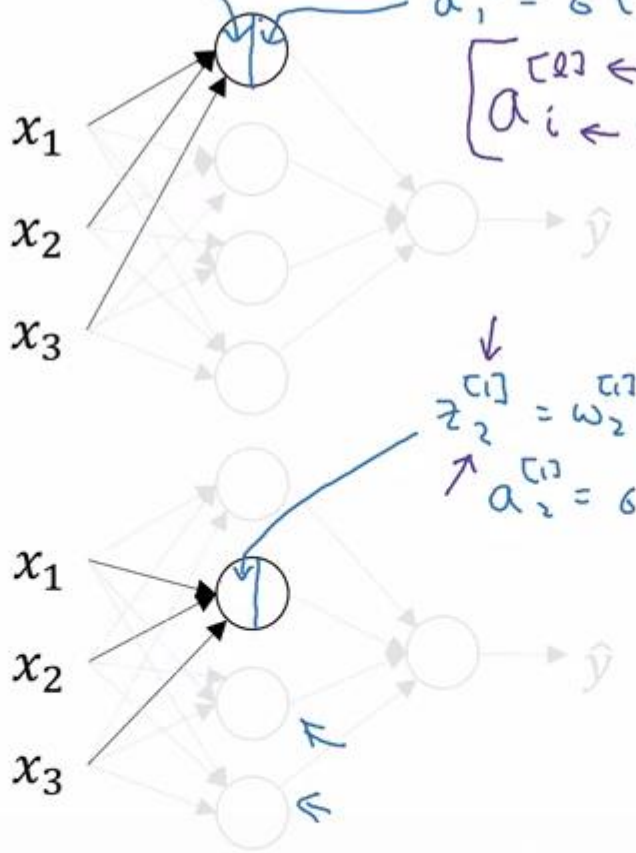
Computing a Neural Network's Output

Neural Network Representation



$$z = w^T x + b$$

$$a = \sigma(z)$$



$$z_1^{[1]} = w_1^{[1]T} x + b_1^{[1]}$$

$$a_1^{[1]} = \sigma(z_1^{[1]})$$

$a_i^{[l]}$ ← layer; i ← node in layer.

$$z_2^{[1]} = w_2^{[1]T} x + b_2^{[1]}$$

$$a_2^{[1]} = \sigma(z_2^{[1]})$$

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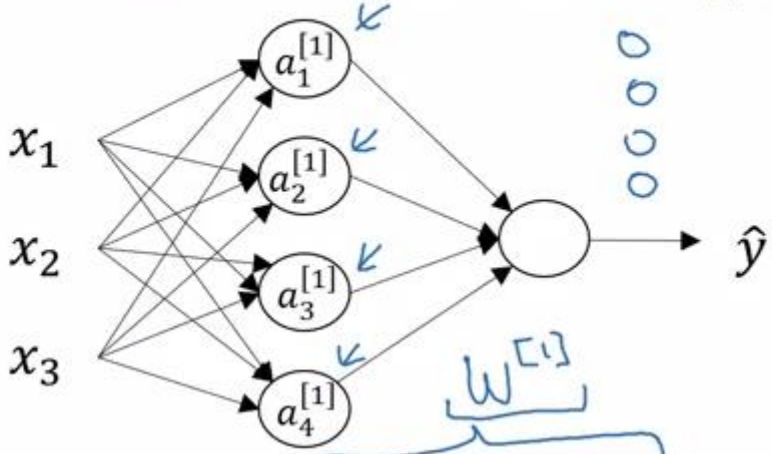
Neural Network Representation



$(w_1^{[1]})^T x$ $a^{[1]}$

$$z_1^{[1]} = w_1^{[1]T} x + b_1^{[1]}, \quad a_1^{[1]} = \sigma(z_1^{[1]})$$
$$z_2^{[1]} = w_2^{[1]T} x + b_2^{[1]}, \quad a_2^{[1]} = \sigma(z_2^{[1]})$$
$$z_3^{[1]} = w_3^{[1]T} x + b_3^{[1]}, \quad a_3^{[1]} = \sigma(z_3^{[1]})$$
$$z_4^{[1]} = w_4^{[1]T} x + b_4^{[1]}, \quad a_4^{[1]} = \sigma(z_4^{[1]})$$

$$z^{[1]} = \overbrace{\begin{bmatrix} — w_1^{[1]T} — \\ — w_2^{[1]T} — \\ — w_3^{[1]T} — \\ — w_4^{[1]T} — \end{bmatrix}}^{W^{[1]}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\begin{bmatrix} b_1^{[1]} \\ b_2^{[1]} \\ b_3^{[1]} \\ b_4^{[1]} \end{bmatrix}}_{b^{[1]}} = \begin{bmatrix} w_1^{[1]T} x + b_1^{[1]} \\ w_2^{[1]T} x + b_2^{[1]} \\ w_3^{[1]T} x + b_3^{[1]} \\ w_4^{[1]T} x + b_4^{[1]} \end{bmatrix} = \begin{bmatrix} z_1^{[1]} \\ z_2^{[1]} \\ z_3^{[1]} \\ z_4^{[1]} \end{bmatrix}$$

$(4,3)$ $(4,1)$

$$a^{[1]} = \begin{bmatrix} a_1^{[1]} \\ \vdots \\ a_4^{[1]} \end{bmatrix} = \sigma(z^{[1]})$$

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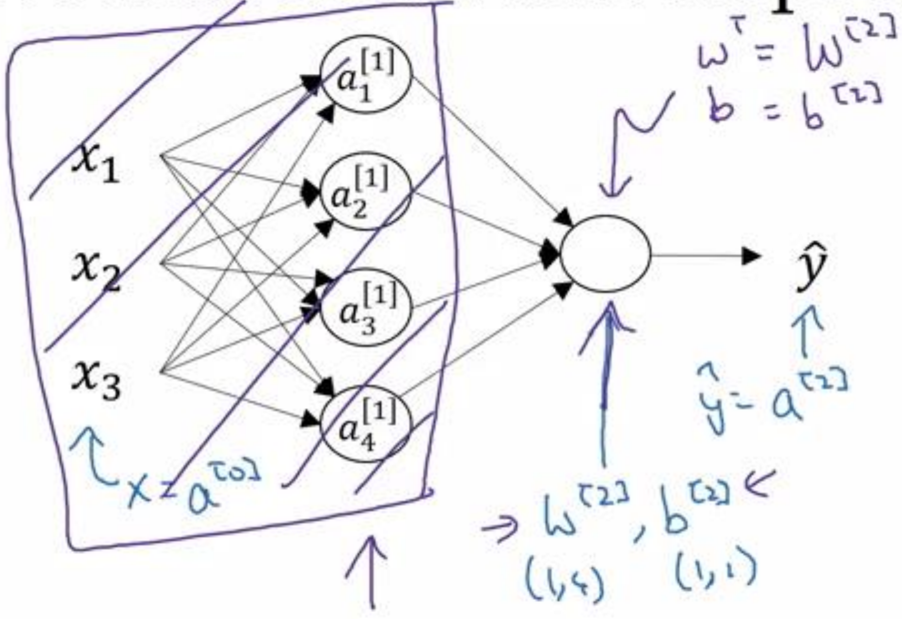
Neural Network Representation learning



$z = w^T x + b$

$\hat{y} = a = \sigma(z)$

Given input x:

$$\underset{(4,1)}{z^{[1]}} = \underset{(4,3)}{W^{[1]}}\underset{(3,1)}{\overset{a^{[0]}}{\cancel{x}}} + \underset{(4,1)}{b^{[1]}}$$

$$\underset{(4,1)}{a^{[1]}} = \sigma(\underset{(4,1)}{z^{[1]}})$$

$$\underset{(1,1)}{z^{[2]}} = \underset{(1,4)}{W^{[2]}}\underset{(4,1)}{a^{[1]}} + \underset{(1,1)}{b^{[2]}}$$

$$\underset{(1,1)}{a^{[2]}} = \sigma(\underset{(1,1)}{z^{[2]}})$$

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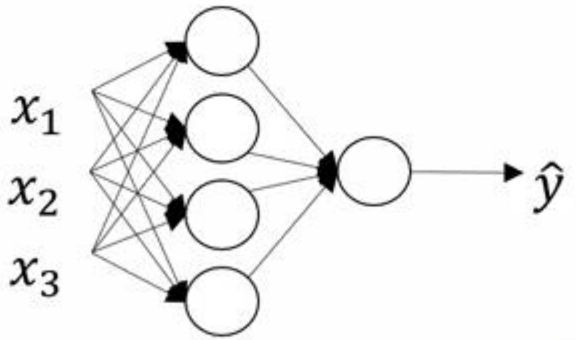
Vectorizing across multiple examples

Vectorizing across multiple examples



$$
\left\{
\begin{aligned}
z^{[1]} &= W^{[1]}x + b^{[1]} \\
a^{[1]} &= \sigma(z^{[1]}) \\
z^{[2]} &= W^{[2]}a^{[1]} + b^{[2]} \\
a^{[2]} &= \sigma(z^{[2]})
\end{aligned}
\right\}
$$

$x \longrightarrow a^{[2]} = \hat{y}$

$x^{(1)} \longrightarrow a^{[2](1)} = \hat{y}^{(1)}$

$x^{(2)} \longrightarrow a^{[2](2)} \quad \hat{y}^{(2)}$

$\vdots$

$x^{(m)} \longrightarrow a^{[2](m)} \quad \hat{y}^{(m)}$

$a^{[2](i)}$ ← example i

← layer 2

$$
\boxed{
\begin{aligned}
&\text{for } i = 1 \text{ to } m, \\
&\quad z^{[1](i)} = W^{[1]} x^{(i)} + b^{[1]} \\
&\quad a^{[1](i)} = \sigma(z^{[1](i)}) \\
&\quad z^{[2](i)} = W^{[2]} a^{[1](i)} + b^{[2]} \\
&\quad a^{[2](i)} = \sigma(z^{[2](i)})
\end{aligned}
}
$$

Vectorizing across multiple examples

```
for i = 1 to m:
```

$$\boxed{z^{[1](i)}} = W^{[1]}x^{(i)} + b^{[1]}$$
$$\boxed{a^{[1](i)}} = \sigma(z^{[1](i)})$$
$$\boxed{z^{[2](i)}} = W^{[2]}a^{[1](i)} + b^{[2]}$$
$$\boxed{a^{[2](i)}} = \sigma(z^{[2](i)})$$

$$\boxed{\begin{aligned} Z^{[1]} &= W^{[1]}X + b^{[1]} \\ A^{[1]} &= \sigma(Z^{[1]}) \\ Z^{[2]} &= W^{[2]}A^{[1]} + b^{[2]} \\ A^{[2]} &= \sigma(Z^{[2]}) \end{aligned}}$$

$$X = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & | \end{bmatrix} \quad (n_x, m)$$

training examples

hidden units.

$$Z^{[1]} = \begin{bmatrix} | & | & & | \\ z^{[1](1)} & z^{[1](2)} & \cdots & z^{[1](m)} \\ | & | & & | \end{bmatrix}$$

$$A^{[1]} = \begin{bmatrix} | & | & & | \\ a^{[1](1)} & a^{[1](2)} & \cdots & a^{[1](m)} \\ | & | & & | \end{bmatrix}$$

hidden units

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Explanation for vectorized implementation

Justification for vectorized implementation

$$z^{[1](1)} = W^{[1]}x^{(1)} + \cancel{b^{[1]}} \rightarrow 0, \quad z^{[1](2)} = W^{[1]}x^{(2)} + \cancel{b^{[1]}} \rightarrow 0, \quad z^{[1](3)} = W^{[1]}x^{(3)} + \cancel{b^{[1]}} \rightarrow 0$$

$$W^{[1]} = \begin{bmatrix} \text{———} \\ \text{———} \\ \text{———} \end{bmatrix} \qquad W^{[1]}x^{(1)} = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} \qquad W^{[1]}x^{(2)} = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} \qquad W^{[1]}x^{(3)} = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix}$$

$$W^{[1]} \underbrace{\begin{bmatrix} | & | & | & \\ x^{(1)} & x^{(2)} & x^{(3)} & \cdots \\ | & | & | & \end{bmatrix}}_{X} = \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix} = \begin{bmatrix} | & | & | & \\ z^{[1](1)} & z^{[1](2)} & z^{[1](3)} & \cdots \\ | & | & | & \\ +b^{[1]} & +b^{[1]} & +b^{[1]} & \end{bmatrix} = Z^{[1]}$$

$$Z^{[1]} = W^{[1]}X + b^{[1]}$$

$$W^{[1]}x^{(i)} = z^{[1](i)}$$

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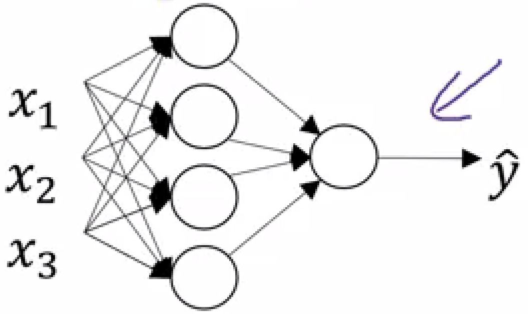
Recap of vectorizing across multiple examples



$$X = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & | \end{bmatrix}$$

$$A^{[1]} = \begin{bmatrix} | & | & & | \\ a^{[1](1)} & a^{[1](2)} & \cdots & a^{[1](m)} \\ | & | & & | \end{bmatrix}$$

for i = 1 to m

$$\boxed{z^{[1](i)} = W^{[1]}x^{(i)} + b^{[1]}}$$
$$a^{[1](i)} = \sigma(z^{[1](i)})$$
$$z^{[2](i)} = W^{[2]}a^{[1](i)} + b^{[2]}$$
$$a^{[2](i)} = \sigma(z^{[2](i)})$$

$A^{[0]}$ $\quad X = a^{[0]}$ $\quad x^{(i)} = a^{[0](i)}$

$$Z^{[1]} = W^{[1]}X + b^{[1]} \leftarrow W^{[1]}A^{[0]} + b^{[1]}$$
$$A^{[1]} = \sigma(Z^{[1]})$$
$$Z^{[2]} = W^{[2]}A^{[1]} + b^{[2]}$$
$$A^{[2]} = \sigma(Z^{[2]})$$

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Activation functions

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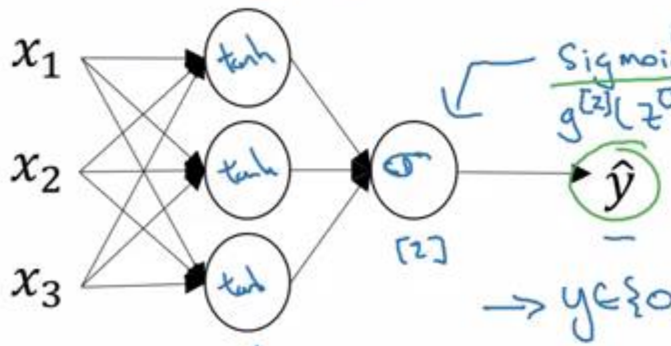
Activation functions

$g^{[1]}(z^{[1]}) = \tanh(z^{[1]})$

Sigmoid

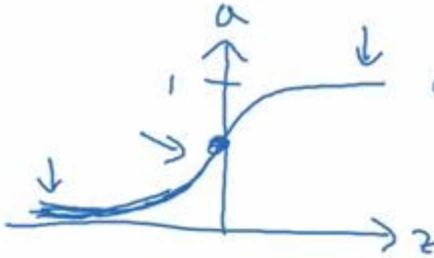
$g^{[2]}(z^{[2]}) = \sigma(z^{[2]})$



$\rightarrow y \in \{0, 1\}$

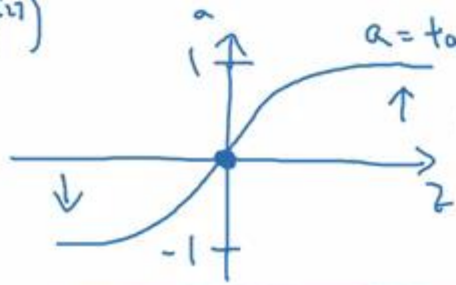
$0 \le \hat{y} \le 1$

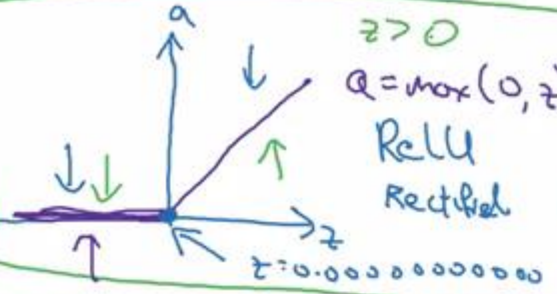
$-1 \qquad 1$



$a = \frac{1}{1+e^{-z}}$

$a = \tanh(z) = \frac{e^{z} - e^{-z}}{e^{z} + e^{-z}}$





$z > 0$

$a = \max(0, z)$

ReLU

Rectified Linear Unit

Leaky ReLU

Given x:

$$z^{[1]} = W^{[1]}x + b^{[1]}$$

$$\rightarrow a^{[1]} = \cancel{\sigma(z^{[1]})}\ g^{[1]}(z^{[1]})$$

$$z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$

$$\rightarrow a^{[2]} = \cancel{\sigma(z^{[2]})}\ g^{[2]}(z^{[2]})$$

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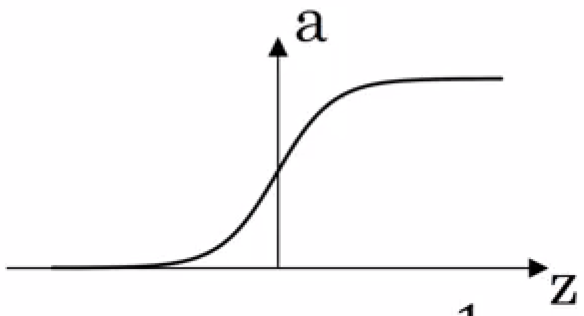
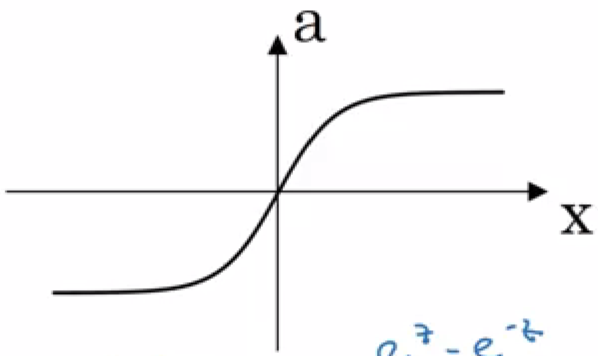
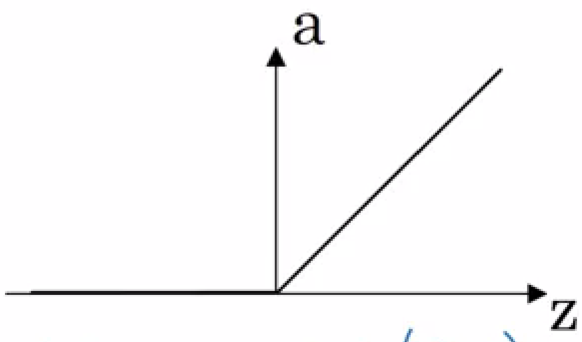
Pros and cons of activation functions



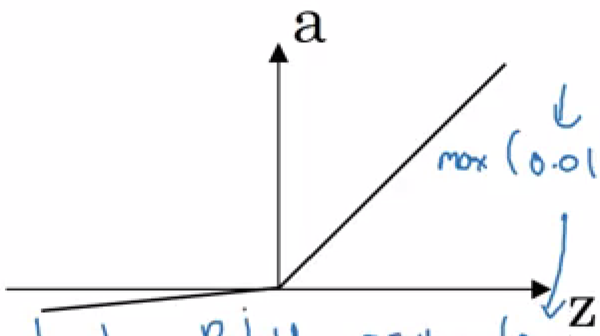
sigmoid: $a = \frac{1}{1 + e^{-z}}$



tanh: $a = \frac{e^{z} - e^{-z}}{e^{z} + e^{-z}}$



ReLU $a = \max(0, z)$



$\max(0.01z, z)$

Leaky ReLU $a = \max(0.01z, z)$

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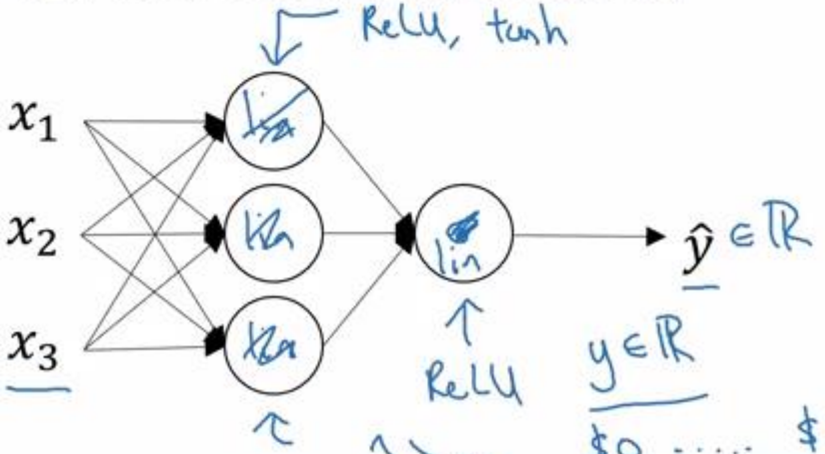
Why do you need non-linear activation functions?

Activation function



$y \in \mathbb{R}$

\$0 ..... \$1,000,000

$\hat{y} \geq 0$..

Given x:

$$\rightarrow z^{[1]} = W^{[1]}x + b^{[1]}$$

$$\rightarrow a^{[1]} = \cancel{g^{[1]}(z^{[1]})}\ z^{[1]}$$

$$\rightarrow z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$

$$\rightarrow a^{[2]} = \cancel{g^{[2]}(z^{[2]})}\ z^{[2]}$$

$g(z) = z$

"linear activation function"

$$a^{[1]} = z^{[1]} = W^{[1]}x + b^{[1]}$$

$$a^{[2]} = z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$

$$a^{[2]} = W^{[2]}\underbrace{(W^{[1]}x + b^{[1]})}_{a^{[1]}} + b^{[2]}$$

$$= \underbrace{(W^{[2]}W^{[1]})}_{W'}x + \underbrace{(W^{[2]}b^{[1]} + b^{[2]})}_{b'}$$

$$= W'x + b'$$

$g(z) = z$

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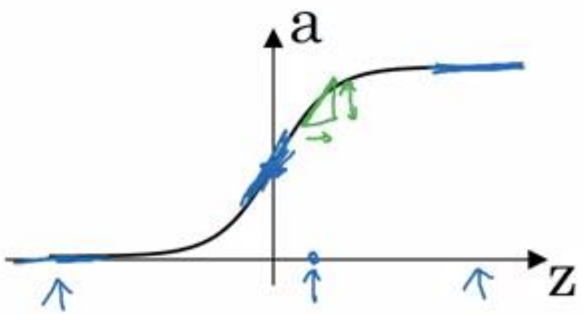
Derivatives of activation functions

Sigmoid activation function



$$g(z) = \frac{1}{1+e^{-z}}$$

$a = g(z) = \frac{1}{1+e^{-z}}$

$\boxed{g'(z)} = \boxed{\frac{d}{dz} g(z)}$ = slope of $g(x)$ at z

$= \frac{1}{1+e^{-z}}\left(1 - \frac{1}{1+e^{-z}}\right)$

$= \underbrace{g(z)\left(1-g(z)\right)}$

$= \boxed{a(1-a)}$

$g'(z) = a(1-a)$

$z = 10.$ $\quad g(z) \approx 1$

$\frac{d}{dz} g(z) \approx 1(1-1) \approx 0$

$z = -10$ $\quad g(z) \approx 0$

$\frac{d}{dz} g(z) \approx 0 \cdot (1-0) \approx 0$

$z = 0$ $\quad g(z) = \frac{1}{2}$

$\frac{d}{dz} g(z) = \frac{1}{2}\left(1 - \frac{1}{2}\right) = \frac{1}{4}$

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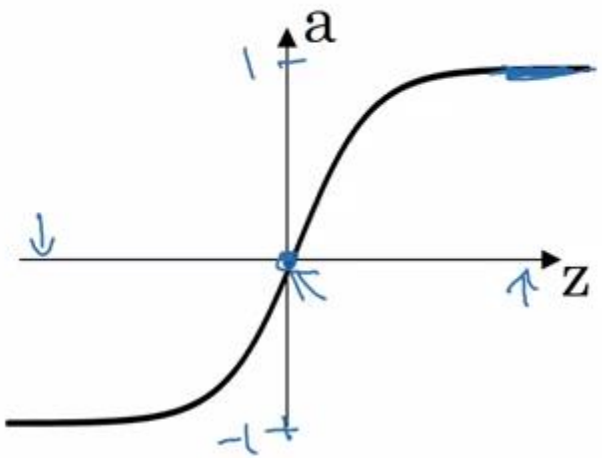
Tanh activation function



$g(z) = \tanh(z)$

$$= \frac{e^z - e^{-z}}{e^z + e^{-z}}$$

$g'(z) = \frac{d}{dz} g(z) =$ slope of $g(z)$ at z

$$= 1 - (\tanh(z))^2$$

$a = g(z), \quad g'(z) = 1 - a^2$

$z = 10 \quad \tanh(z) \approx 1$
$g'(z) \approx 0$

$z = -10 \quad \tanh(z) \approx -1$
$g'(z) \approx 0$

$z = 0 \quad \tanh(z) = 0$
$g'(z) = 1$

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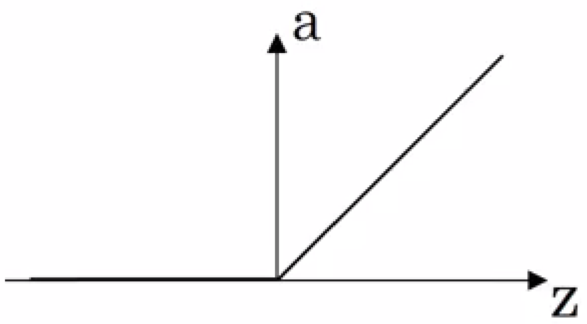
ReLU and Leaky ReLU

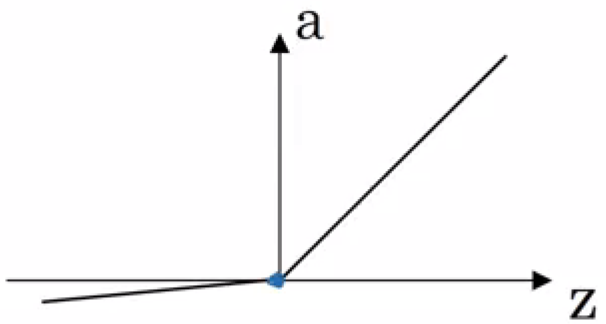


ReLU

$$g(z) = \max(0, z)$$

$$g'(z) = \begin{cases} 0 & \text{if } z < 0 \\ 1 & \text{if } z \geq 0 \\ \cancel{\text{undef} \quad \text{if } z = 0} \end{cases}$$

$z = 0.00000\cdots0$



Leaky ReLU

$$g(z) = \max(0.01z, z)$$

$$g'(z) = \begin{cases} 0.01 & \text{if } z < 0 \\ 1 & \text{if } z \geq 0 \end{cases}$$

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Gradient descent for neural networks

Gradient descent for neural networks

Parameters: $\underset{(n^{[1]},\, n^{[0]})}{W^{[1]}},\ \underset{(n^{[1]},\, 1)}{b^{[1]}},\ \underset{(n^{[2]},\, n^{[1]})}{W^{[2]}},\ \underset{(n^{[2]},\, 1)}{b^{[2]}}$ $\qquad n_x = n^{[0]},\ n^{[1]},\ \underline{n^{[2]} = 1}$

Cost function: $J(W^{[1]}, b^{[1]}, \underline{W^{[2]}}, \underline{b^{[2]}}) = \frac{1}{m} \sum_{i=1}^{n} \mathcal{L}(\underset{\uparrow a^{[2]}}{\hat{y}}, y)$

Gradient descent:

→ Repeat {

→ Compute predicts ($\hat{y}^{(i)}$, $i = 1 \ldots, m$)

$\underline{dW^{[1]}} = \frac{\partial J}{\partial W^{[1]}}, \quad \underline{db^{[1]}} = \frac{\partial J}{\partial b^{[1]}}, \ \ldots$

$W^{[1]} := W^{[1]} - \alpha\, dW^{[1]}$

$b^{[1]} := b^{[1]} - \alpha\, db^{[1]}$

$b^{[2]} := \ldots b^{[2]} -$

Formulas for computing derivatives

Forward propagation:

$Z^{[1]} = W^{[1]} X + b^{[1]}$

$A^{[1]} = g^{[1]}(Z^{[1]})$ ←

$Z^{[2]} = W^{[2]} A^{[1]} + b^{[2]}$

$A^{[2]} = g^{[2]}(Z^{[2]}) = \sigma(Z^{[2]})$

Back propagation:

$dZ^{[2]} = A^{[2]} - Y$ ↙ $\quad Y = [y^{(1)} \ y^{(2)} \ \dots, y^{(m)}]$

$dW^{[2]} = \frac{1}{m} dZ^{[2]} A^{[1]T}$ $\quad (n^{[1]},)$ ←

$\quad (n^{[2]}, 1)$ ←

$db^{[2]} = \frac{1}{m}$ np.sum($dZ^{[2]}$, axis=1, keepdims=True)

$dZ^{[1]} = \underbrace{W^{[2]T} dZ^{[2]}}_{(n^{[1]}, m)} * \underbrace{g^{[1]\prime}(Z^{[1]})}_{(n^{[1]}, m)}$

↑ element-wise product

$dW^{[1]} = \frac{1}{m} dZ^{[1]} X^T$

$db^{[1]} = \frac{1}{m}$ np.sum($dZ^{[1]}$, axis=1, keepdims=True)

$(n^{[1]}, 1)$ $\quad (n^{[1]},)$ $\quad$ reshape ↑

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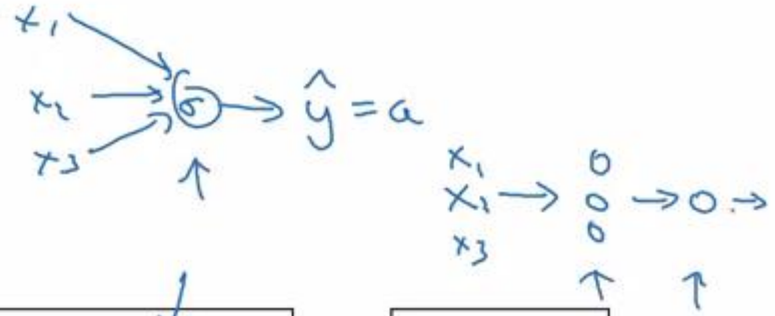
Backpropagation intuition (Optional)

Computing gradients



Logistic regression

$x, w, b \rightarrow \boxed{z = w^T x + b} \rightarrow \boxed{a = \sigma(z)} \rightarrow \boxed{\mathcal{L}(a, y)}$

$dz = a - y$

$dw = dz \cdot x$

$db = dz$

$\boxed{dz = da \cdot g'(z)}$

$g(z) = \sigma(z)$

$$\boxed{\frac{\partial \mathcal{L}}{\partial z} = \frac{\partial \mathcal{L}}{\partial a} \cdot \boxed{\frac{da}{dz}}}$$

"dz" = "da"

$\frac{d}{dz} g(z) = g'(z)$

$da = \frac{d}{da} \mathcal{L}(a, y) = -y \log a - (1-y) \log(1-a)$

$= -\frac{y}{a} + \frac{1-y}{1-a}$

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Neural network gradients

$x_1, x_2, x_3 \rightarrow \circ \rightarrow \hat{y}$

$n_x = n^{[0]}$ $\quad n^{[1]} \quad n^{[2]} = 1$

x, $W^{[1]}$, $b^{[1]}$ $\rightarrow$ $z^{[1]} = W^{[1]}x + b^{[1]}$ $\rightarrow$ $a^{[1]} = \sigma(z^{[1]})$ $\rightarrow$ $z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$ $\rightarrow$ $a^{[2]} = \sigma(z^{[2]})$ $\rightarrow$ $\mathcal{L}(a^{[2]}, y)$

$W^{[2]}$, $b^{[2]}$ $\rightarrow z^{[2]}$

$dW^{[2]}$, $db^{[2]}$

$dW^{[1]}$, $db^{[1]}$

$dz^{[1]} = W^{[2]T} dz^{[2]}$ $\quad da^{[1]} = \cdots$ $\quad dz^{[2]} = a^{[2]} - y$ $\quad da^{[2]}$

$* \, g^{[1]\prime}(z^{[1]})$

element wise product

$dW^{[2]} = dz^{[2]} a^{[1]T}$ $\quad \rightarrow$ "$dw = dz \cdot x$"

$db^{[2]} = dz^{[2]}$

$W^{[1]}: [\;\;\;\;\;\;]$

$W^{[2]}$ $\quad (n^{[2]}, n^{[1]})$

$z^{[2]}, dz^{[2]}$ $\quad (n^{[2]}, 1) \; - (1,1)$

$z^{[1]}, dz^{[1]}$ $\quad (n^{[1]}, 1)$

$dz^{[1]} = \boxed{W^{[2]T} dz^{[2]}} * \boxed{g^{[1]\prime}(z^{[1]})}$

$(n^{[1]}, 1) = (n^{[1]}, n^{[2]}) \; (n^{[2]}, 1) * (n^{[1]}, 1)$

foo $\quad W^{[1]}$

dfoo $\quad dW^{[1]}$

$dW^{[1]} = dz^{[1]} \cdot x^T$ $\quad a^{[0]T}$

$db^{[1]} = dz^{[1]}$

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Summary of gradient descent

$$dz^{[2]} = a^{[2]} - y$$

$$dW^{[2]} = dz^{[2]} a^{[1]^T}$$

$$db^{[2]} = dz^{[2]}$$

$$dz^{[1]} = W^{[2]T} dz^{[2]} * g^{[1]\prime}(z^{[1]})$$

$$dW^{[1]} = dz^{[1]} x^T$$

$$db^{[1]} = dz^{[1]}$$

Vectorized Implementation:

$$z^{[1]} = W^{[1]} x + b^{[1]}$$

$$a^{[1]} = g^{[1]}(z^{[1]})$$

$$Z^{[1]} = \begin{bmatrix} | & | & & | \\ z^{[1](1)} & z^{[1](2)} & \cdots & z^{[1](m)} \\ | & | & & | \end{bmatrix}$$

$$Z^{[1]} = W^{[1]} X + b^{[1]}$$

$$A^{[1]} = g^{[1]}(Z^{[1]})$$

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Summary of gradient descent

$dz^{[2]} = a^{[2]} - y$

$dW^{[2]} = dz^{[2]} a^{[1]^T}$

$db^{[2]} = dz^{[2]}$

$\underset{(n^{[1]},1)}{dz^{[1]}} = W^{[2]T} dz^{[2]} * g^{[1]\prime}(\mathrm{z}^{[1]})$

$dW^{[1]} = dz^{[1]} x^T$

$db^{[1]} = dz^{[1]}$

$dZ^{[2]} = A^{[2]} - Y$

$J(\ldots) = \frac{1}{m} \sum_{i=1}^{n} \mathcal{L}(\hat{y}, y)$

$dW^{[2]} = \frac{1}{m} dZ^{[2]} A^{[1]^T}$

$db^{[2]} = \frac{1}{m} np.sum(dZ^{[2]}, axis = 1, keepdims = True)$

elementwise product

$\underset{(n^{[1]},m)}{dZ^{[1]}} = \underbrace{W^{[2]T} dZ^{[2]}}_{(n^{[1]},m)} * \underbrace{g^{[1]\prime}(\mathrm{Z}^{[1]})}_{(n^{[1]},m)}$

$dW^{[1]} = \frac{1}{m} dZ^{[1]} X^T$

$db^{[1]} = \frac{1}{m} np.sum(dZ^{[1]}, axis = 1, keepdims = True)$

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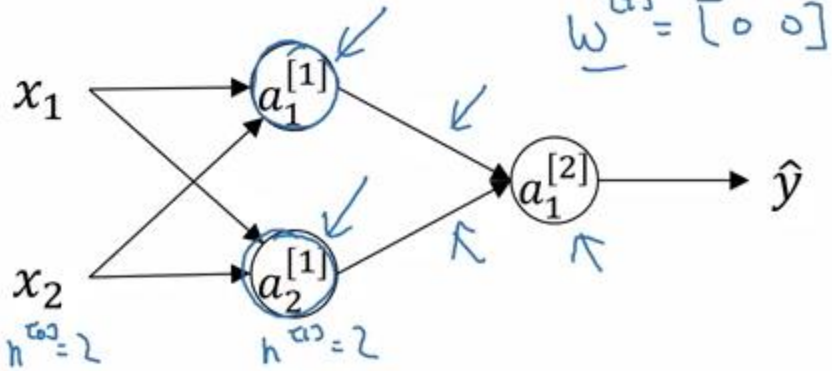
deeplearning.ai

One hidden layer Neural Network

Random Initialization

0:15 / 7:57

What happens if you initialize weights to zero?



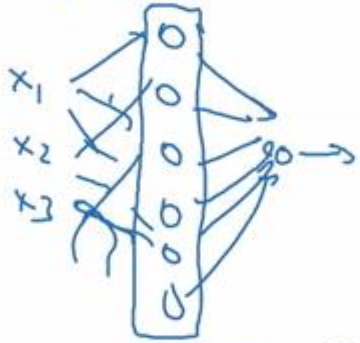
$W^{[2]} = [0 \ 0]$

$W^{[1]} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $\quad b^{[1]} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$a_1^{[1]} = a_2^{[1]}$ $\quad dz_1^{[1]} = dz_2^{[1]}$

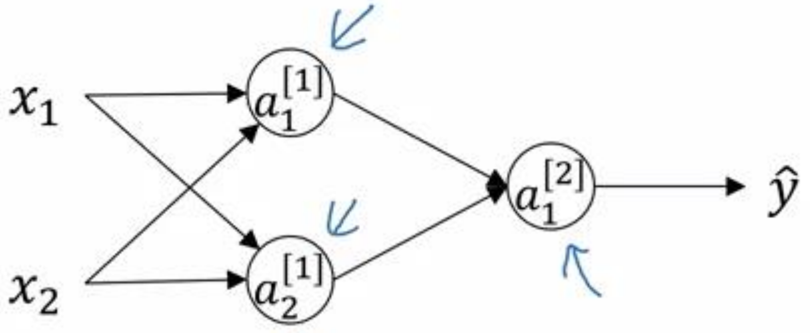
$dw = \begin{bmatrix} u & v \\ u & v \end{bmatrix}$ $\quad W^{[1]} = W^{[1]} - \alpha dW$

Symmetric



$W^{[1]} = \begin{bmatrix} \cdots\cdots \\ \cdots\cdots \end{bmatrix}$

Random initialization



$W^{[1]} = \text{np.random.randn((2,2))} * 0.01$

100?

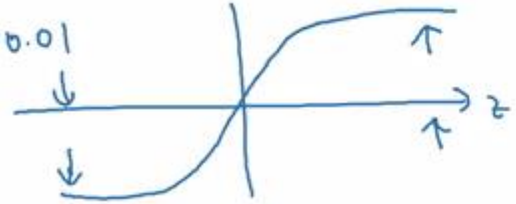
$b^{[1]} = \text{np.zeros((2,1))}$

$W^{[2]} = \text{np.random.randn((1,2))} * 0.01$

$b^{[2]} = 0$

$z^{[1]} = W^{[1]} x + b^{[1]}$

$a^{[1]} = g^{[1]}(z^{[1]})$



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