

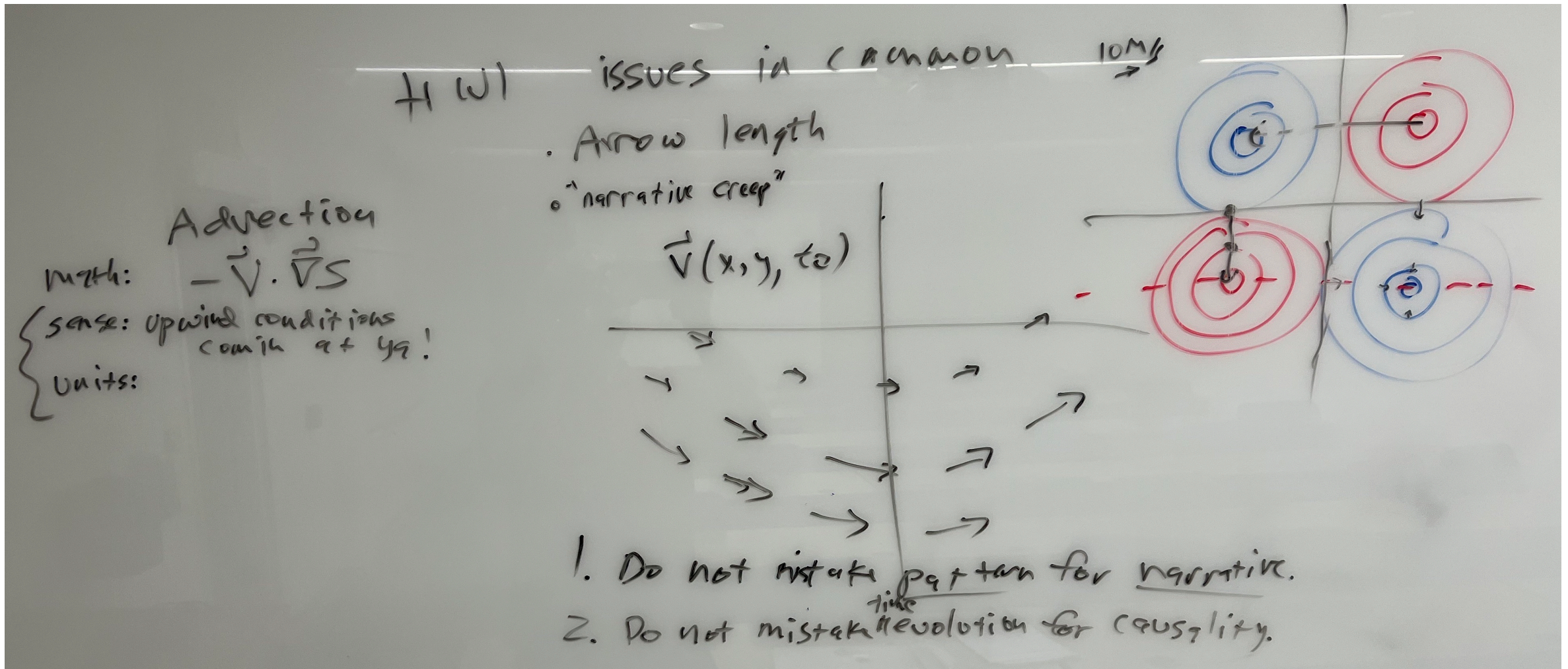
Class lecture

Monday after Labor Day

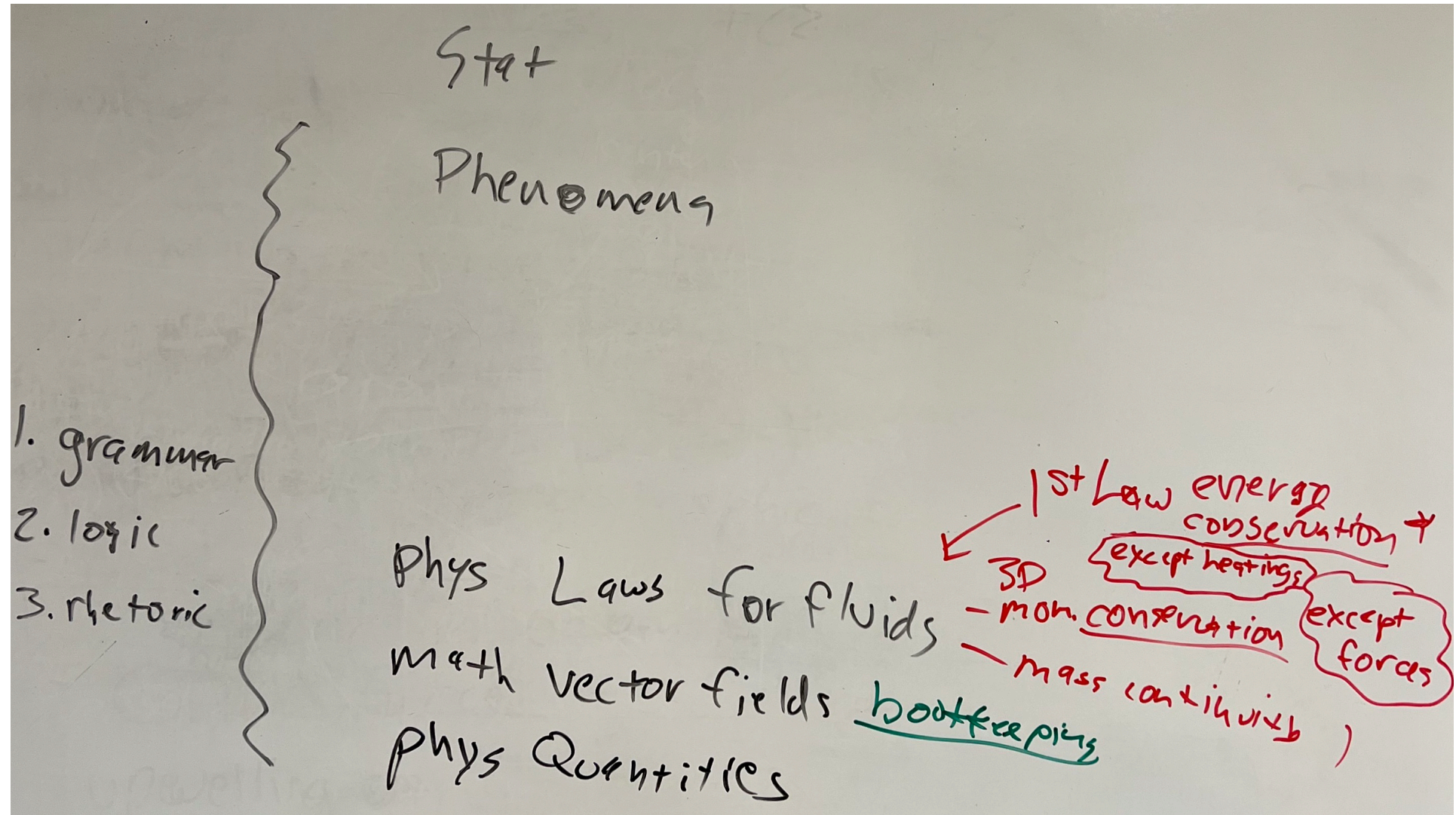
ATM 651 Fall 2022

Brian Mapes

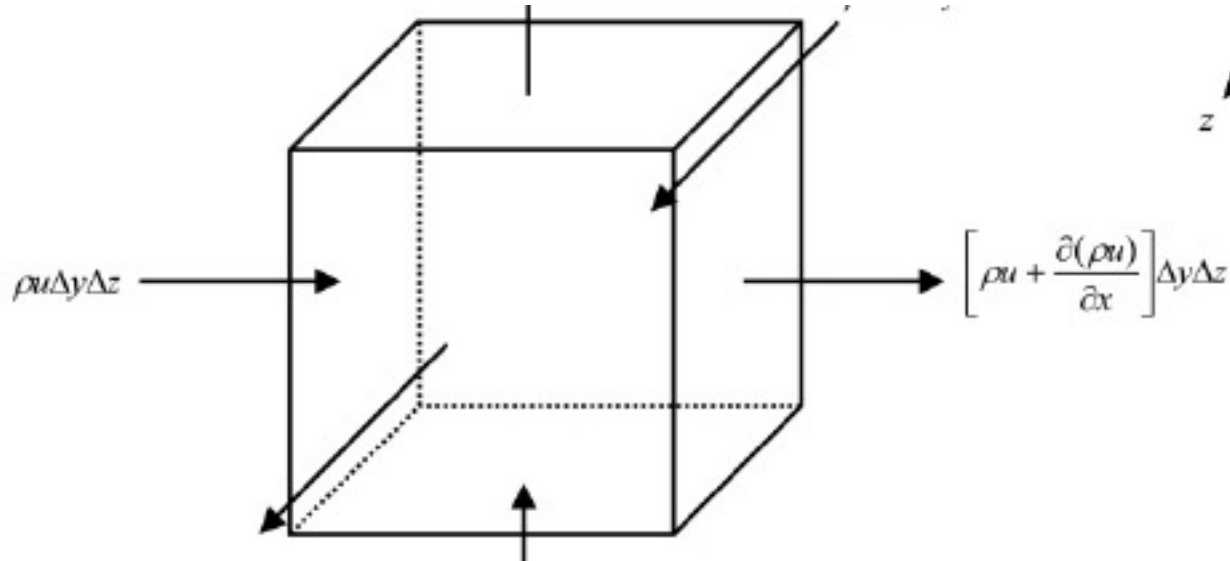
HW1 : some common errors



Where we are working today in course stack



That Cube Thingy for mass continuity



$m = \text{mass in a } \begin{cases} \text{lake} \\ \text{box} \end{cases}$

$\frac{dm}{dt} = (\text{mass flux in}) - (\text{mass flux out})$

$\frac{dm}{dt} = (\rho u)(\Delta y \Delta z) - \left(\rho u + \frac{\partial(\rho u)}{\partial x} \Delta x \right) (\Delta y \Delta z)$

$\frac{dm}{dt} = -\frac{\partial(\rho u)}{\partial x} \Delta x \Delta y \Delta z$

Divide by volume to get density

$\frac{\partial \rho}{\partial t} = -(\rho u)_x - (\rho v)_y - (\rho w)_z$

likewise!

Our First Closed Set!

mass **continuity** (**absolute conservation everywhere**)

momentum **conservation except for forces**

It becomes an equation for pressure →

$\rho = \text{constant } (\rho_0)$ incompressible
(1.4) becomes

cont. ①

$$0 = u_x + v_y + w_z$$

closed set! ②

$$\frac{\partial}{\partial x} \left[u_t = -(u u)_x - (u v)_y - (u w)_z - \pi_x + f v \right]$$

4 unks + ③

$$\frac{\partial}{\partial y} \left[v_t = -(v u)_x - (v v)_y - (v w)_z - \pi_y - f u \right]$$

4 eqs + ④

$$\frac{\partial}{\partial z} \left[w_t = -(w u)_x - (w v)_y - (w w)_z - \pi_z - g \right]$$

② take $\frac{\partial}{\partial x} (2) + \frac{\partial}{\partial y} (3) + \frac{\partial}{\partial z} (4) = 0!$ by ①

Simplify: define $\frac{p}{\rho_0} = \pi$

$$\nabla^2 \pi = \pi_{xx} + \pi_{yy} + \pi_{zz}$$

$$\pi = \nabla^2 \left(\text{div} \left(\begin{array}{l} \text{Coriolis force} \\ \text{per unit mass} \end{array} \right) + \text{div} \left(\begin{array}{l} \text{gravity} \\ \text{force} \\ \text{per unit mass} \end{array} \right) + \text{div} \left(\begin{array}{l} \text{momentum transport} \\ \text{(acceleration)} \\ \text{"force"} \end{array} \right) \right)$$

pressure holds up the atm ocean against gravity

"inertia" or "inertial force"

What is this inverse Laplacian operation?

It *smooths* the complicated fields that are its inputs (divergence of forces).



4,000 x 3,000

International Cloud Atlas - World Meteorological Organization |

Pileus | International Cloud Atlas

Visit

What is Laplacian?

- Curvature

- emphasizes small scales

$$f = \sin(x) + \sin(3x)$$

$$f_{xx} = -\left[\sin(x) + 9\sin(3x)\right]$$

↑
Small scales
Pop out

- used in edge detection

Inverse of Laplacian:

- hides small scales

- a smoothing operation

Tips for:

deriving advective from flux form

Treat $(\rho \cdot V)$ as one thing, don't sub-differentiate it.

The image shows a handwritten derivation on a whiteboard. It starts with the flux form of the continuity equation: $\rho_t = (\rho u)_x + (\rho v)_y + (\rho w)_z$. This equation is enclosed in a blue box. To the left of this box is the label "flux form." with a bracket. Above the box, the text "these 4" is written in blue, with arrows pointing to the four terms in the equation. To the right of the box, the text "cont." is written, with an arrow pointing down to the next equation. The next equation is $(\rho v)_t = -(\rho v u)_x - (\rho v v)_y - (\rho v w)_z$. To the right of this equation is the text "(4/8) of these" in blue. Below this equation is the advective form: $v_t = -u v_x - v v_y - w v_z$. This equation is enclosed in a black box. To the right of this box is the label "advective form".

flux form.

$$\rho_t = (\rho u)_x + (\rho v)_y + (\rho w)_z$$

these 4

cont.

$$(\rho v)_t = -(\rho v u)_x - (\rho v v)_y - (\rho v w)_z$$

(4/8) of these

$$v_t = -u v_x - v v_y - w v_z$$

advective form