Gridpoint vs. spectral models.

Consider the simplest daynamics problem:
the advection-diffusion equation in 1D (x, longitude)

$u(x,t)$

obeys

$$\frac{\partial u}{\partial t} = \overbrace{-u\frac{\partial u}{\partial x}}^{\text{advection}} + \overbrace{K\frac{\partial^2 u}{\partial x^2}}^{\text{diffusion}}$$

How to solve on computer?

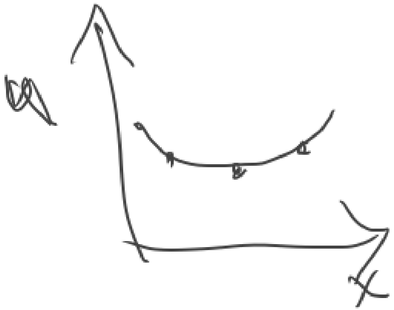
- Time stepping:

$$u(t+1) = u(t) + \Delta t \underbrace{(\text{adv} + \text{diff})}_{\text{tendencies}}$$

- Taking derivatives in space
  - grid
  - Fourier series

Grid:

$$u(x,t+1) = u(x,t) + \Delta t \left(\underbrace{-u(x) \cdot \frac{u(x+\Delta x) - u(x-\Delta x)}{2\Delta x}}_{\text{adv.}} + K \frac{u(x+\Delta x) + u(x-\Delta x) - 2u(x)}{\Delta x^2} \right)$$



Spectral:

Expand $u(x)$ into Fourier components: math says we can!

$$u(x) = a_0 \cos\left(0 \cdot \frac{2\pi}{L} x\right) + a_1 \cos\left(1 \cdot \frac{2\pi}{L} x\right) + b_1 \sin\left(1 \cdot \frac{2\pi}{L} x\right)$$

circumference of Earth

$$+ a_2 \cos(2 \cdot \quad) + b_2 \sin(2 \cdot \quad)$$

$$+ a_m \cos(m \quad) + b_m \sin(m \cdot \quad)$$

"wavenumber" m

$\ldots \rightarrow$ to infinity.

So our adv-diff eq becomes:

vector

$$\begin{bmatrix} a_0 \\ a_1 \\ b_1 \\ a_2 \\ b_2 \\ a_m \\ b_m \\ \vdots \\ \vdots \end{bmatrix}(t+1) = \begin{bmatrix} \\ \\ \\ \\ \vdots \end{bmatrix}(t) + f_{adv}\left(\begin{bmatrix} a_0 \\ a_1 \\ b_1 \\ a_2 \\ b_2 \\ \vdots \end{bmatrix}\right) + f_{dif}\left(\begin{bmatrix} \\ \\ \\ \\ \vdots \end{bmatrix}\right)$$

how many terms?

this will involve summed products like

$\ldots + (a_0 b_5) + \ldots$

advection of wavenumber 5 by mean wind

matrix algebra super power tool

infrastructure exists

?

TRUNCATION is the compromise that cuts small scales