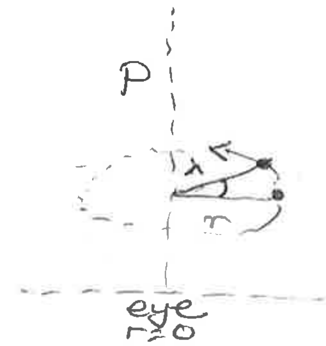
10/2/23 ATM 651 Guest Lecture

Hurricane Diagnostics - Basic Balances



Concepts:

- Isobaric (pressure) vertical coordinates
- Hydrostatic balance
- Gradient wind balance
- Thermal wind balance
- Ideal Gas Law

} applied to a warm-core vortex

Unlike most atmosphere dynamics situations, we use a cylindrical polar coordinate system (r, λ, p).

u = radial wind

v = azimuthal wind ($v>0 \Rightarrow$ cyclonic flow)

w or ω = vertical wind ($w>0$ or $\omega<0 \Rightarrow$ vertical motion). not needed for this class

$v>0$

Here, $\underline{V} = (0, v(r), 0)$ Primary circulation

- A hurricane can be thought of as having an axisymmetric, steady state primary circulation (vortex in thermal wind balance) and an axisymmetric, steady state secondary circulation ("in-up-out" driven by heating or angular momentum sources).
- Plenty that is missing ($\frac{d}{dt}$, asymmetries, air-sea interaction, advection of dry air, wind shear etc.)
- Useful to think about assumptions and approximations every step of the way!
- This class - we make quantitative estimates of the primary circulation and check the characteristics and accuracy of these basic approximations.

[$p = \rho R T$ ideal gas law]

[$\frac{dp}{dz} = -\rho g$ in z coords]

First: hydrostatic balance in p coordinates.

$$\frac{d\Phi}{dz} = g \Rightarrow \frac{d\Phi}{dp} = \underbrace{g}_{\frac{d\Phi}{dz}}\frac{dz}{dp} = g\left(-\frac{1}{\rho g}\right) = -\frac{RT}{p}$$

We use $\boxed{\frac{d\Phi}{dp} = -\frac{RT}{p}}$

Similarly, let's obtain an expression for gradient wind balance in p coordinates.

How are radial pressure gradients (with z constant) related to radial Φ gradients (with p constant - on a p surface)?

$$\left.\frac{\partial p}{\partial r}\right|_p = \left.\frac{\partial p}{\partial r}\right|_z + \left.\frac{\partial z}{\partial r}\right|_p \frac{dp}{dz}$$

$$0 = \left.\frac{\partial p}{\partial r}\right|_z + \frac{1}{g}\frac{\partial \Phi}{\partial r}(-\rho g) \Rightarrow \left.\frac{1}{\rho}\frac{\partial p}{\partial r}\right|_z = \frac{\partial \Phi}{\partial r}.$$

Gradient wind balance

$$\frac{1}{\rho}\frac{\partial p}{\partial r} = \frac{v^2}{r} + fv \Rightarrow \boxed{\frac{\partial \Phi}{\partial r} = \frac{v^2}{r} + fv} \quad (1)$$

Primary circulation $v(r)$ due to the presence of a Φ gradient.

How does this vary with height?

Bring in T, combine ideal gas law + hydrostatic + gradient wind.

Start with $\frac{\partial \Phi}{\partial r} = \frac{v^2}{r} + fv$

Then $\frac{\partial}{\partial p}\left(\frac{\partial \Phi}{\partial r}\right) = \frac{\partial}{\partial p}\left(\frac{v^2}{r} + fv\right) = \left(\frac{2v}{r} + f\right)\frac{\partial v}{\partial p}$

Hydrostatic: $\frac{\partial}{\partial r}\left(\frac{\partial \Phi}{\partial p}\right) = -\frac{R}{p}\frac{\partial T}{\partial r}$

These two are equivalent:

$$\boxed{\underbrace{\left(f + \frac{2v}{r}\right)}_{\text{inertial term}}\underbrace{\frac{\partial v}{\partial p}}_{\text{vertical variation of wind}} = \underbrace{-\frac{R}{p}\frac{\partial T}{\partial r}}_{\text{horizontal T gradient.}}} \quad (2)$$

Thermal wind balance in tropical cyclones!

(1) Calculate Gradient Wind (in eyewall) for Hurricane Lee

950 hPa

Change in r across which gradient is computed : 180 km = 1.8 x 10^5 m

Change in Z = -30m to 420m = 450m. Multiply by g (10) to get change in Φ.

r = radius of eyewall = 60 km
= 6×10^4 m.

$f = 6 \times 10^{-5} s^{-1}$
$= 2\Omega \sin\phi$
24.7°

$$V = -\frac{fr}{2} + \sqrt{\frac{f^2r^2}{4} + r\frac{\partial\Phi}{\partial r}}$$

First : orders of magnitude

$$V \approx \underbrace{\frac{(6\times10^{-5})(6\times10^{4})}{}}_{O(1)} + \sqrt{O(1) + \underbrace{6\times10^4\left(\frac{(450)(10)}{1.8\times10^5}\right)}_{O(10-100)}} \Rightarrow V \approx \sqrt{r\frac{\partial\Phi}{\partial r}}$$

Cyclostrophic Balance
PGF $\approx$ CeF
CoF small.

Then $V \approx \sqrt{(6\times10^4)\frac{450 . 10}{1.8\times10^5}} \approx 50 m/s$
$\approx 100 kt$.

Actual winds on 950 hPa plot :

$\approx$ 100 kt in eyewall.

In this instance, gradient wind is quite a good approximation!

500 hPa

5640 – 5400 = 240m over 150 km.

$V \approx \sqrt{(6\times10^4)\frac{240 . 10}{1.5\times10^5}} \approx 30 m/s$
$\approx 60 kt$

Actual wind in eyewall is $\approx$ 80 kt, stronger than gradient wind
$\Rightarrow$ supergradient.

Notice that gradient wind (and actual wind) decrease in height for a warm core cyclone!

(2) Calculate thermal wind

Recall $\left(f + \frac{2v}{r}\right)\frac{\partial v}{\partial p} = -\frac{R}{p}\frac{\partial T}{\partial r}$.

Warm core: $\frac{\partial T}{\partial r} < 0 \Rightarrow$ sign of RHS > 0

$\Rightarrow$ " " LHS > 0

$\Rightarrow$ " " $\frac{\partial v}{\partial p} > 0$ since $v > 0$

$\Rightarrow$ v increases with p

v decreases with height.

Cold core: reverse signs.

Apply to Lee at 750 hPa.

$$\left(\underbrace{6\times10^{-5}}_{\text{tiny}} + \frac{2\times\overset{ms^{-1}}{\cancel{40}}}{1000000}\right)\frac{\partial v}{\partial p} = -\frac{\overset{J/kg\,K}{287}}{\underset{Pa}{75000}}\left(-\frac{10}{100000}\right)$$

$\frac{\partial v}{\partial p} \approx -2$ m/s or nearly ~~4~~ kt over 50 hPa (by multiplying by 5000 Pa)

If the 750 hPa wind is ~~100~~ 80 kt, then the 700 hPa wind should be ~~96 kt~~ ~~96~~ 76 kt, and the 800 hPa wind should be $\approx$ ~~104 kt~~ 84 kt.

We have estimated how the azimuthal wind changes with height in a tropical cyclone!

Can compare with images.