Thermal energy equation for a kg of air.

diabatic heat added = T change + mech. work (Force × displacement)

heating rate $\frac{J}{kg \cdot s}$

$$Q = C_v \frac{dT}{dt} + p \underbrace{\frac{d\alpha}{dt}}_{\text{inconvenient}}$$

grab this sorta

$\alpha = \frac{1}{\rho}$ specific volume

$$= C_v \frac{dT}{dt} + \left[\underbrace{\frac{d}{dt}(p\alpha)}_{\frac{d}{dt}(RT)} - \alpha \frac{dp}{dt} \right]$$

$$Q = \underbrace{(C_v + R)}_{C_p} \frac{dT}{dt} - \alpha \frac{dp}{dt} = \underbrace{0!!!}_{\text{adiabatic; yay}} + \underbrace{L \frac{dq_{sat}}{dt}}_{\text{moist or pseudo adiabatic (where saturate)}} + \underbrace{Q_R + Q_{etc}}_{\text{diabatic}}$$

We need a trick

$$\frac{Q}{T} = c_p \frac{dT}{dt}\Big/T - \alpha \frac{dp}{dt}\Big/T = \frac{d}{dt}(\text{warmth})$$

?

ideal gas

$p\alpha = RT$

1. Divide by T

$$\frac{\alpha}{T} = \frac{R}{p}$$

$$\frac{Q}{T} = c_p \frac{d}{dt}(\ln T) - R\frac{d}{dt}(\ln p)$$

$$= c_p \frac{d}{dt}\ln(\theta) \qquad \theta = T\left(\frac{P_0}{P}\right)^{R/c_p}$$

easier to verify than derive!

"potential T", T if air compressed adiabatically to P_0

$= 1000\ hPa$

$= 10^5\ Pa.$

$S = c_p \ln\theta$

"entropy"

(J/kg K)

(?)

$$\boxed{\underbrace{\frac{d}{dt}(\theta)}_{K} = \underbrace{\frac{Q}{c_p}}_{K/s}\cdot\underbrace{\left(\frac{\theta}{T}\right)}_{\text{no unit}}}$$

$= 0$ in adiabatic case

$= L\left(\frac{dq_{sat}}{dt}\right)$ for cond/evap processes

get θ_e

much clearer conceptually!

Other trick:

$$Q = C_p \frac{dT}{dt} - \alpha \underbrace{\frac{dp}{dt}}$$

$$-\frac{1}{\rho}\frac{dp}{dt}$$ where have we seen this?

$P \approx P_{hyd}$

$$\underbrace{-\frac{1}{\rho}\frac{dP_h}{dz}}_{\text{VPGF}} = \underbrace{g}_{\text{gravity}}$$

$$= \underline{C_p \frac{dT}{dt} - g\frac{dz}{dt}}$$ ← hydrostatic z

$$\underbrace{0!!!}_{\text{(adiabatic)}} = Q = \frac{d}{dt}\underbrace{\left(\overbrace{C_p T}^{\text{enthalpy}} + \overbrace{g z_h}^{\text{grav. potential energy}}\right)}_{\text{"warmth"}}$$

"warmth" — dry static energy acts just like Θ

$$= \underbrace{L\left(\frac{dq_{sat}}{dt}\right)} + Q_{Rad} + Q_{cond,mix}$$

$$\underbrace{Q_{mix}\ Q_R}_{\text{truly diabatic}} + \underbrace{0!!!}_{\text{yay}} = \frac{d}{dt}\underbrace{\left(C_p T + g z + \overbrace{L q_{sv}}^{\text{latent energy of vapor}}\right)}_{\text{moist static energy}}$$