

# Class lecture

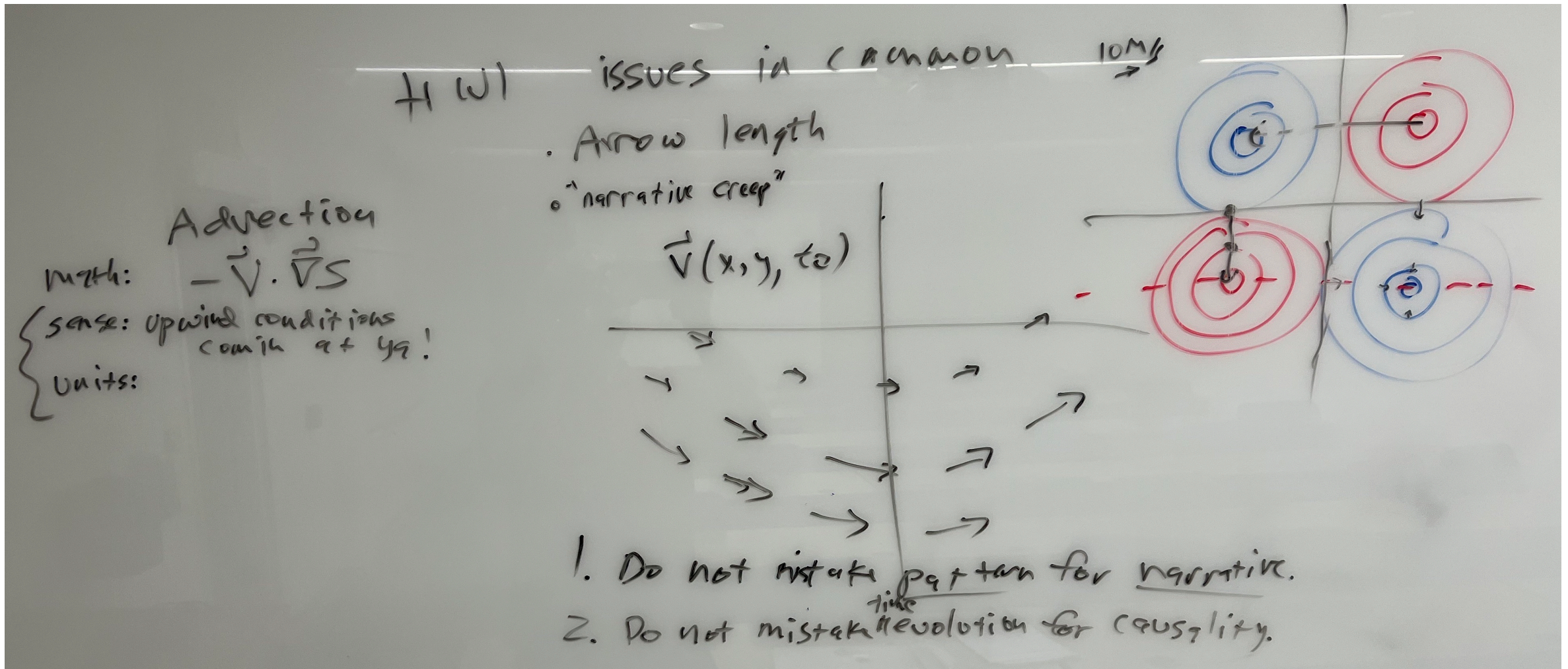
# Monday after Labor Day

ATM 651 Fall 2022

Brian Mapes

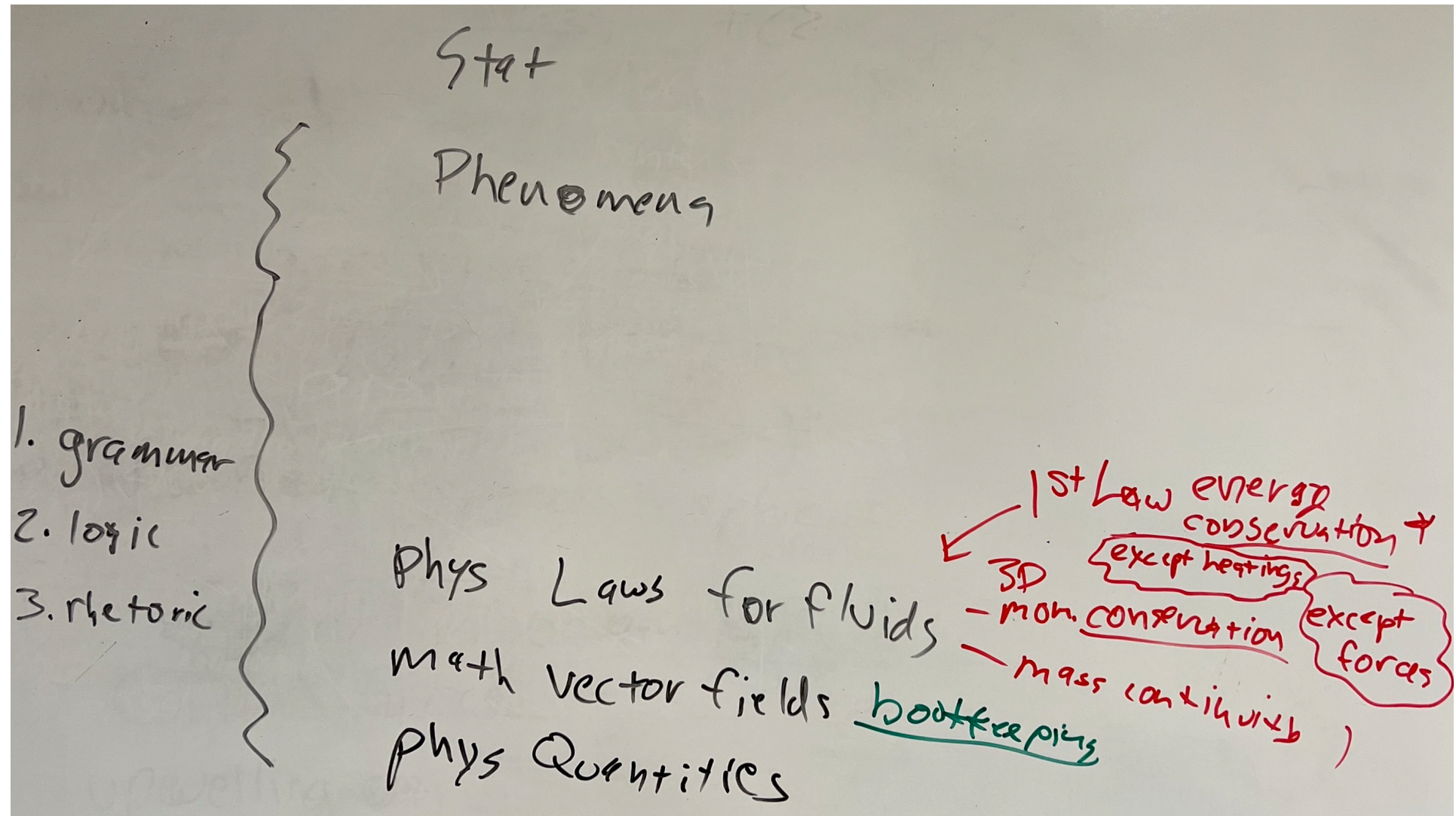
Zoom recording FWIW: [https://miami.zoom.us/rec/share/yfPtyHXWDstOMx2eCYfZ1rzclMImQaCPJ4vH\\_ju8OQl\\_yPo\\_mqYajJi5vtNkB29.QuyNBM0XJQzd4jXq](https://miami.zoom.us/rec/share/yfPtyHXWDstOMx2eCYfZ1rzclMImQaCPJ4vH_ju8OQl_yPo_mqYajJi5vtNkB29.QuyNBM0XJQzd4jXq)  
Passcode: 74e5Z=qi

# HW1 : some common errors



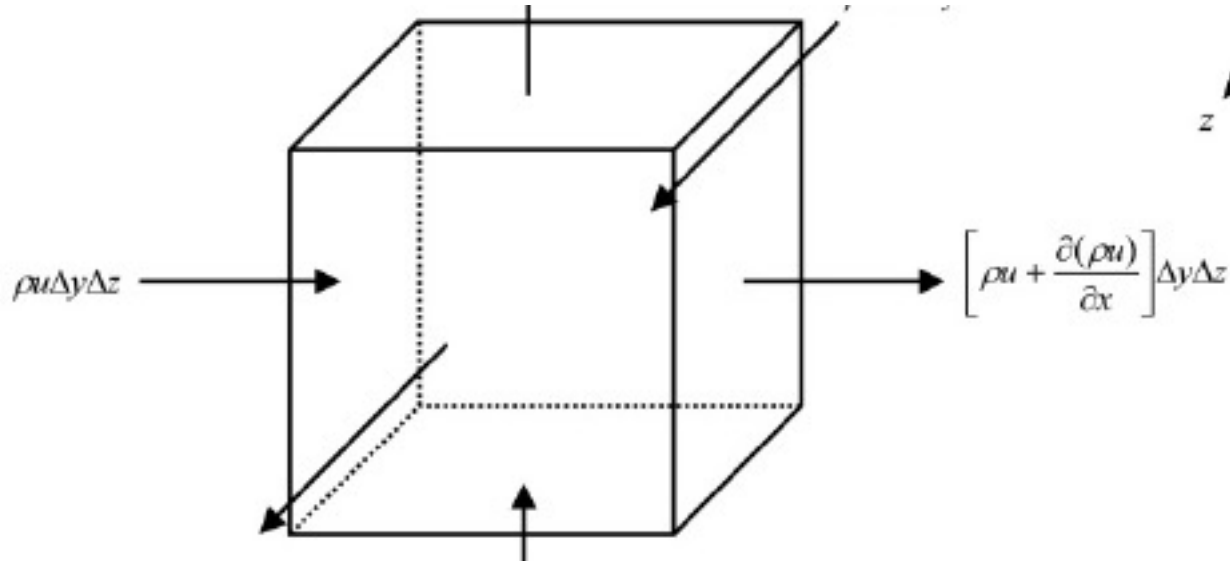


# Where we are working today in course stack





# That Cube Thingy for mass continuity



$m = \text{mass in a } \begin{cases} \text{lake} \\ \text{box} \end{cases}$

$\frac{dm}{dt} = (\text{mass flux in}) - (\text{mass flux out})$

$\frac{dm}{dt} = (\rho u) (\Delta y \Delta z) - \left( \rho u + \frac{\partial(\rho u)}{\partial x} \Delta x \right) (\Delta y \Delta z)$

$\frac{dm}{dt} = - \frac{\partial(\rho u)}{\partial x} \Delta x \Delta y \Delta z$

Divide by volume to get density

$\frac{\partial \rho}{\partial t} = -(\rho u)_x - (\rho v)_y - (\rho w)_z$

likewise!



# Our First Closed Set!

mass **continuity** (**absolute conservation everywhere**)

momentum **conservation except for forces**

It becomes an equation for pressure →

$\rho = \text{constant } (\rho_0)$  incompressible  
(1.4) becomes

cont. ①

$$0 = u_x + v_y + w_z$$

closed set! ②

$$\frac{\partial}{\partial x} \left[ u_t = -(u u)_x - (u v)_y - (u w)_z - \pi_x + f v \right]$$

4 unk's ③

$$+ \frac{\partial}{\partial y} \left[ v_t = -(v u)_x - (v v)_y - (v w)_z - \pi_y - f u \right]$$

4 eq's ④

$$+ \frac{\partial}{\partial z} \left[ w_t = -(w u)_x - (w v)_y - (w w)_z - \pi_z - g \right]$$

⑤ take  $\frac{\partial}{\partial x} (2) + \frac{\partial}{\partial y} (3) + \frac{\partial}{\partial z} (4) = 0!$  by ①

Simplify: define  $\frac{p}{\rho_0} = \pi$

$$\nabla^2 \pi = \pi_{xx} + \pi_{yy} + \pi_{zz}$$

$$\pi = \nabla^2 \left( \text{div} \left( \begin{array}{l} \text{Coriolis force} \\ \text{per unit mass} \end{array} \right) + \text{div} \left( \begin{array}{l} \text{gravity} \\ \text{force} \\ \text{per unit mass} \end{array} \right) + \text{div} \left( \begin{array}{l} \text{momentum transport} \\ \text{(acceleration)} \\ \text{"force"} \end{array} \right) \right)$$

pressure holds up the atm ocean against gravity

"inertia" or "inertial force"

# What is this inverse Laplacian operation?

It *smooths* the complicated fields that are its inputs (divergence of forces).



4,000 x 3,000

International Cloud Atlas - World Meteorological Organization |

Pileus | International Cloud Atlas

Visit

## What is Laplacian?

- Curvature

- emphasizes small scales

$$f = \sin(x) + \sin(3x)$$

$$f_{xx} = -\left[\sin(x) + 9\sin(3x)\right]$$

↑  
Small scales  
pop out

- used in edge detection

## Inverse of Laplacian:

- hides small scales

- a smoothing operation



Tips for:

deriving advective from flux form

Treat  $(\rho \cdot V)$  as one thing, don't sub-differentiate it.

The image shows a handwritten derivation on a whiteboard. It starts with the flux form of the continuity equation:  $\rho_t = (\rho u)_x + (\rho v)_y + (\rho w)_z$ . This equation is enclosed in a blue box, and a bracket to its left is labeled "flux form.". Above the box, the text "these 4" is written in blue, with arrows pointing to the four terms in the equation. To the right of the box, the text "cont." is written, with an arrow pointing down to the next step. The next step is the continuity equation with the product rule applied:  $(\rho V)_t = - (u(\rho V))_x - (v(\rho V))_y - (w(\rho V))_z$ . To the right of this equation, the text "(4/8) of these" is written in blue. Finally, the advective form is derived:  $V_t = -uV_x - vV_y - wV_z$ . This equation is enclosed in a black box, and a bracket to its right is labeled "advective form".

$$\rho_t = (\rho u)_x + (\rho v)_y + (\rho w)_z$$

flux form.

these 4

cont.

eliminate

$$(\rho V)_t = - (u(\rho V))_x - (v(\rho V))_y - (w(\rho V))_z$$

(4/8) of these

$$V_t = -uV_x - vV_y - wV_z$$

advective form