Using the equation of state $\frac{R_d T}{p} = \frac{1}{\rho}$. Using the hydrostatic equation, $\partial p = -\rho g \partial z$, making these substitutions yield:

$$\left(\frac{\kappa T}{p} - \frac{\partial T}{\partial p}\right)\omega = -\left(\frac{1}{\rho c_p} + \frac{\partial T}{\rho g \partial z}\right)\rho g w$$
$$= -\left(\frac{g}{c_p} - \left(-\frac{\partial T}{\partial z}\right)\right) w$$
$$= -w(\Gamma_d - \Gamma)$$

This came from vert adv. part of $\frac{dT}{dt}$, but $\alpha \frac{dp}{dt}$? non-conserved quantity (like T) is not really "advected" as such.

(θ / T)

$\theta = T \cdot \left(\frac{p_0}{p}\right)^{R/c_p}$

K K unitless

First Law was

Energy in = work done by volume change + $\frac{\Delta}{\delta t}$(internal energy)

$$J = \underbrace{p\frac{d\alpha}{dt}}\ + \underbrace{c_v \frac{dT}{dt}}$$

convert to θ units from T units

$\frac{dT}{dt} = \alpha \frac{dp}{dt} + J/c_p$ (ω)

K/s

$\Rightarrow \frac{d\theta}{dt} = 0 + J/c_p \left(\frac{\theta}{T}\right)$

convert to s units

$\Rightarrow \frac{ds}{dt} = 0 + J/c_p \ (c_p)$

$\frac{\partial s}{\partial t} = -V_h \cdot \nabla s - \omega \frac{\partial s}{\partial p} + J$