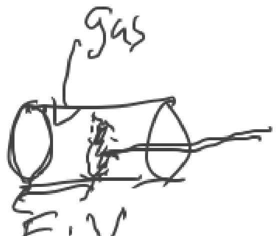
First Law of Thermo: cons of energy

$$\begin{pmatrix}\text{energy added}\\ \text{to gas}\end{pmatrix} = \begin{pmatrix}\text{work on}\\ \text{external}\\ \text{environment}\end{pmatrix} + \begin{pmatrix}\text{Change of}\\ \text{internal}\\ \text{energy}\end{pmatrix}$$

For 1kg:

α (specific volume)

$$J = $$



$$= \boxed{p\frac{dV}{dt}} + C_v\frac{dT}{dt}$$

forbid ⇒ all internal

$\frac{d}{dt}(p\alpha = RT)$ is ideal gas

$$p\frac{d\alpha}{dt} + \alpha\frac{dp}{dt} = R\frac{dT}{dt} \quad \text{or} \quad p\frac{d\alpha}{dt} = R\frac{dT}{dt} - \alpha\frac{dp}{dt}$$

$$J = \underbrace{R\frac{dT}{dt} + C_v\frac{dT}{dt}}_{C_p\frac{dT}{dt}} - \alpha\frac{dp}{dt}$$

where $C_p = C_v + R$

we hate 3-term equations,

$$C_p \frac{dT}{dt} = -\alpha \frac{dp}{dt} + J$$

Can we make a 2-term equation

$$\frac{d}{dt}(\sim) = 0 + \cdots$$

$p\alpha = RT$

$\frac{\alpha}{T} = \frac{R}{p}$

Two tricks:

① Divide both sides by ... T!

$$C_p \frac{1}{T}\frac{dT}{dt} = -\frac{\alpha}{T}\frac{dp}{dt} + J/T$$

$$\underline{C_p \frac{d}{dt}(\ln T)} = \underline{-R \frac{d}{dt}(\ln p)} + J/T$$

can be combined! into entropy eqn. or

entropy $= C_p \ln\theta$

$$\Rightarrow \frac{d\theta}{dt} = 0 + \frac{J}{C_p}\left(\frac{p_0}{p}\right)^{R/C_p}$$

$g\,dz = -\alpha\,dp$

$\frac{\partial p}{\partial z} = -\rho g$

② Other trick for hydrostatic atmosphere

$$-\alpha \frac{dp}{dt} = g\frac{dz}{dt}!$$

Trick 2 becomes:

$$c_p \frac{dT}{dt} = -g \frac{dz}{dt} + J$$

$$\frac{d}{dt}\Big(\underbrace{c_pT + gz}_{\text{"dry static energy"}}\Big) = J$$

Furthermore, if $J = -L \frac{dq_{vapor}}{dt}$ ← water vapor)

h = moist static energy

$$\frac{d}{dt}\left(c_pT + gz\right) = L\frac{dq}{dt} \quad \text{or} \quad \frac{d}{dt}\big(\overbrace{c_pT + gz + Lq}^{h}\big) = 0!$$