I primitive Equations: 5 unknowns, 5 equations ✓

3 spatial coordinates (x, y, p) ← hydrostatic pressure (weight of air above) (mass · g) " " "

⇒ 3 velocities ("momentums")

$$\left.\begin{aligned} \dot{x} &= \frac{dx}{dt} \equiv u, \\ \dot{y} &= \frac{dy}{dt} \equiv v, \\ \dot{p} &= \frac{dp}{dt} \equiv \omega \end{aligned}\right\} 3$$

$\vec{F} = m\vec{a}$ in 3D

⇒ T which appears in thickness (Φ equation)

$$\frac{dT}{dt} = \ldots$$

$$\Rightarrow 0 = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p}$$ poor cousin King eq.

Momentum Eqs ($F=ma$)

3 1eq ~~1~~ 2 unknowns (u, Φ), ω

$$\frac{du}{dt} = fv - \frac{\partial \Phi}{\partial x} + F_x$$

3 1eq ~~1~~ 2 unks (v, Φ), ω

$$\frac{dv}{dt} = -fu - \frac{\partial \Phi}{\partial y} + F_y$$

1eq : 2 unks (Φ, T)

$$0 = \frac{dw}{dt} = \underbrace{\left(g - \frac{1}{\rho}\frac{\partial p}{\partial z}\right)}_{0} \Rightarrow \text{Force Balance} \quad \boxed{\frac{\partial \Phi}{\partial p}} = \boxed{g\frac{\partial z}{\partial p} = -\frac{RT}{p}}$$

$$p = R\rho T$$

$$1/\rho = \frac{RT}{p}$$

$$\frac{\partial p}{\partial z} = -\rho g$$

$$\frac{\partial z}{\partial p} = -\frac{1}{\rho g}$$

$$g\frac{\partial z}{\partial p} = -\frac{1}{\rho}$$

$$\frac{dT}{dt} = \ldots$$

need a T equation! (First Law of Thermo)

4 eqs, 5 unknowns! Need ω, somehow!

Poor cousin (but really king): mass continuity.
Once was a prognostic eq for density

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\underbrace{\rho \vec{V}}_{\text{mass flux}})$$

When p (mass-related hydrostatic pressure) is the coordinate,

$$\boxed{0 = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p}}$$

rigorous zero!

exact, not a const-density approx!