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9.8.22

Homework 1: Review of vector fields for meteorology
See Feynman lectures: Vectors, Differential Calc of Vector Fields, Vector Integral Calculus

In meteorology, we often make use of Cartesian coordinates (x,y,z) for clarity, even though the Earth is round, when discussing weather patterns much smaller than the globe. Strictly speaking, this is a *local tangent plane* assumption, and is good enough for all our purposes this semester. Here we use (x,y) as pseudo-Cartesian distance (east,north *with units of meters*) from some reference point, and we use z as distance above mean sea level. Unit vectors along (x,y,z) axes are $(\hat{\imath}, \hat{\jmath}, \hat{k})$ *while* time t (*with units of seconds or days)* is relative to a convenient t=0 reference time.

Consider the horizontal velocity field for a circular cyclone in a westerly vertical shear S:

$$\vec{V}(x, y, z, t) = \hat{\imath}u + \hat{\jmath}v$$

with

$$u = -Ay \cdot 2^{\mathrm{ct}} + \mathrm{Sz}$$
$$v = \ Ax \cdot 2^{ct}$$

Combining these,

$$\vec{V}(x, y, z, t) = \ \hat{\imath}\,(-Ay \cdot 2^{\mathrm{ct}} + \mathrm{Sz}) + \hat{\jmath}\,(Ax \cdot 2^{ct})$$

Or in the notation where <> indicates vector components,

$$\vec{V}(x, y, z, t) = <(Ay \cdot 2^{ct} + Sz), (Ax \cdot 2^{ct})>$$

Notice we could also write this as two different vector fields:

$$\vec{V}(x, y, z, t) = \ \vec{V_c}(x, y, z, t) + \ \vec{V_s}(x, y, z, t)$$

where one vector field is:

$$\vec{V_c}(x, y, z, t) = <(Ay), (Ax)> \cdot\, 2^{ct} \ = (\ \hat{\imath}\,(-Ay) + \hat{\jmath}\,(Ax)\ ) \cdot 2^{\mathrm{ct}}$$

and the other is:

$$\vec{V_s}(x, y, z, t) = <Sz, 0> = \ \hat{\imath}\,(Sz)$$

For convenience (no calculators needed, *use brain*!), A = 1 m/s $(100\ \mathrm{km})^{-1}$, S = 1m/s km^{-1}, C = 1 day^{-1}, and please consider only the domain $(x, y) \in [-1000km, 1000km]$ since otherwise the values blow up without limit. As a crutch to see the pattern (*but use brain and units and Legend for care in your sketching!*), this site gives a quick view.

$\vec{V} = (-Ay)\hat{i} + (Ax)\hat{j}$; $A = 1$

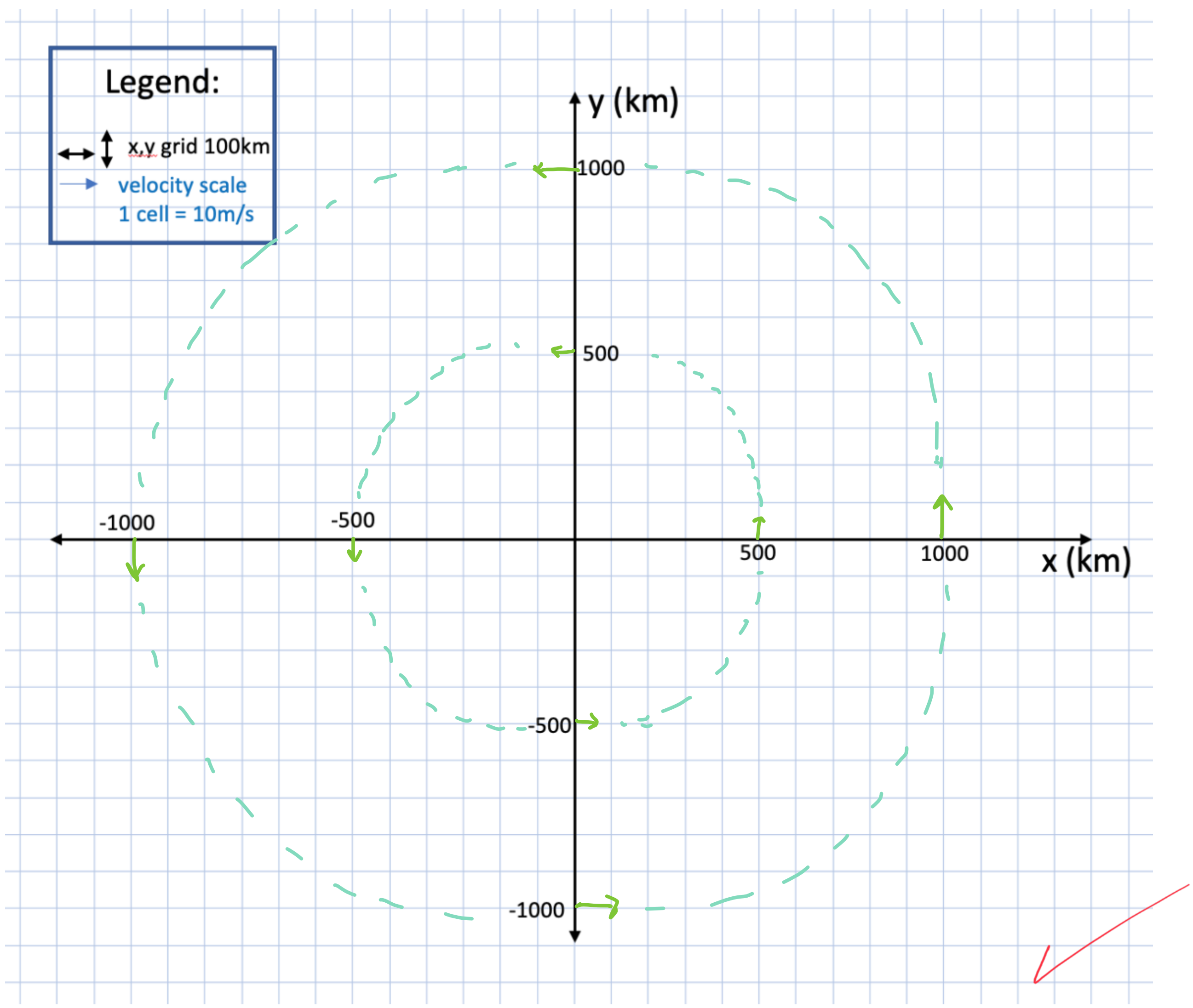
$$\vec{V}(x,y,z,t) = <(-Ay\cdot \cancelto{1}{2^{ct}} + \cancelto{0}{Sz}), (Ax\cdot \cancelto{1}{2^{ct}})>$$
$$= \hat{\imath}\,(-Ay\cdot \cancel{2^{ct}} + \cancel{Sz}) + \hat{\jmath}\,(Ax\cdot \cancel{2^{ct}})$$

A = 1 m/s (100 km)$^{-1}$, S = 1m/s km^{-1}, c = 1 day^{-1}

1. Sketch a few vectors (taking care about their length, using the conversion factor that a 10m/s vector has length 100 km on the distance axes). Draw vectors along the x and y axes, and along the diagonals, by evaluating the formula above *at the surface at the initial time (for z=0 km, t=0 days)*.

$$\vec{V}(x,y,z,t) = \hat{\imath}(-Ay\cdot 2^{ct} + Sz) + \hat{\jmath}(Ax\cdot 2^{ct})$$
for A = 1 m/s (100 km)$^{-1}$, S = 1m/s km^{-1}, C = 1 day^{-1}

Once you see the pattern, lightly sketch a few streamlines (parallel to the vectors everywhere).

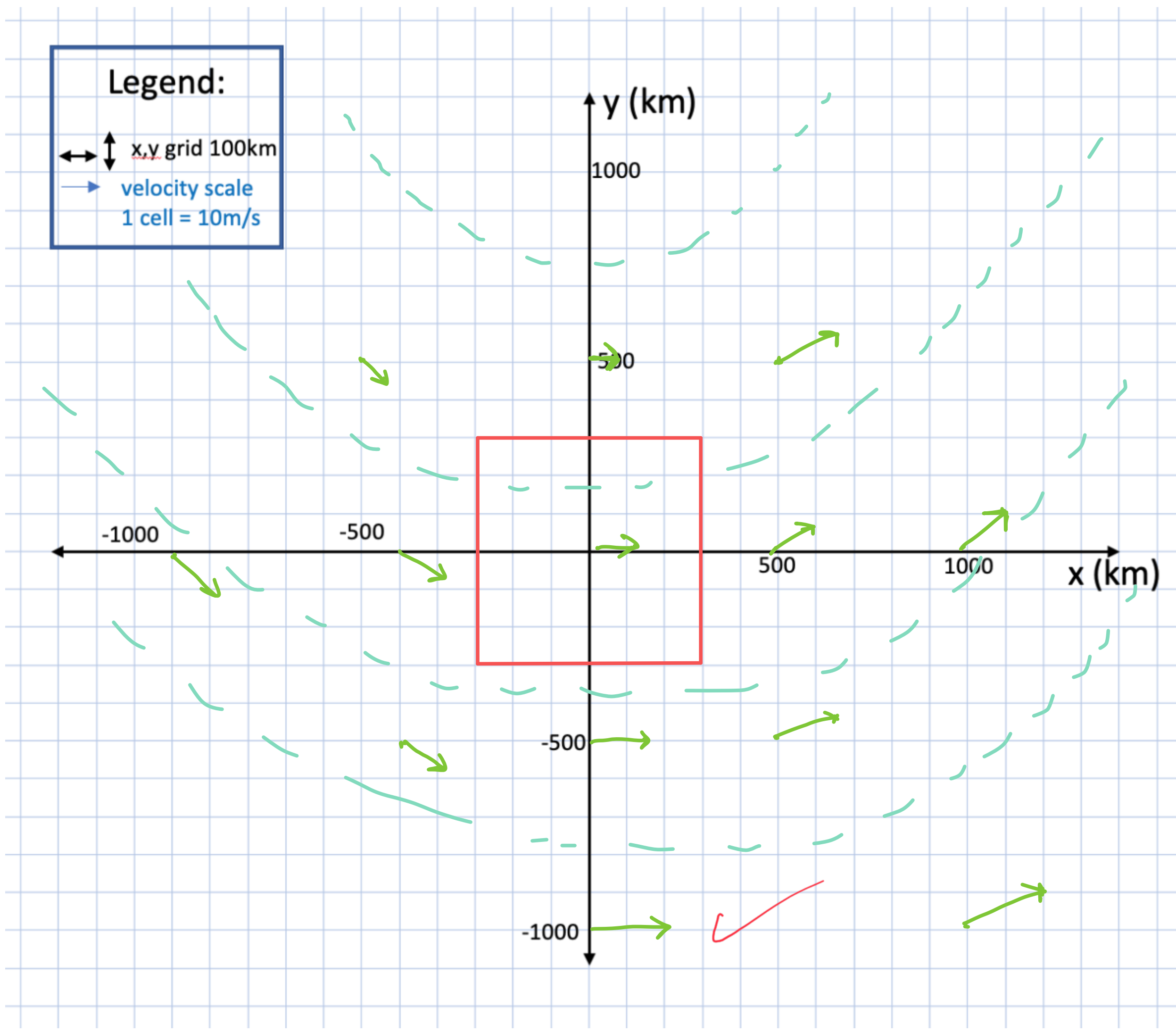


$\vec{V} = (-Ay + 10)\hat{i} + (Ax)\hat{j}$ & $A = 1$

$$\vec{V}(x,y,z,t) = <(-Ay \cdot \cancel{2^{ct}}^{\,1} + Sz), (Ax \cdot \cancel{2^{ct}}^{\,1})>$$
$$= \hat{\imath}\,(-Ay \cdot \cancel{2^{ct}} + Sz) + \hat{\jmath}\,(Ax \cdot \cancel{2^{ct}})$$

A = 1 m/s (100 km)$^{-1}$, S = 1m/s km^{-1}, c = 1 day^{-1}

2. Repeat the exercise for z = 10 km, t = 0 days. Describe the flow in a sentence or two of words. Yes, words for flow are a bit challenging, but this is the point: to learn some. You may wish to use geographical directions (north, south, east west). Where are the winds *southwesterly?* (*from* the southwest)? Where are they *eastward?* (*toward* the east)



Flow is counterclockwise & centered 1000 km north of given origin. Flow from northwesterlies to southwesterlies is indicative of a trough.
Due westerlies (eastward flow) is along y-axis (x=0) and southwesterlies occur in upper & lower right quadrants (east side).

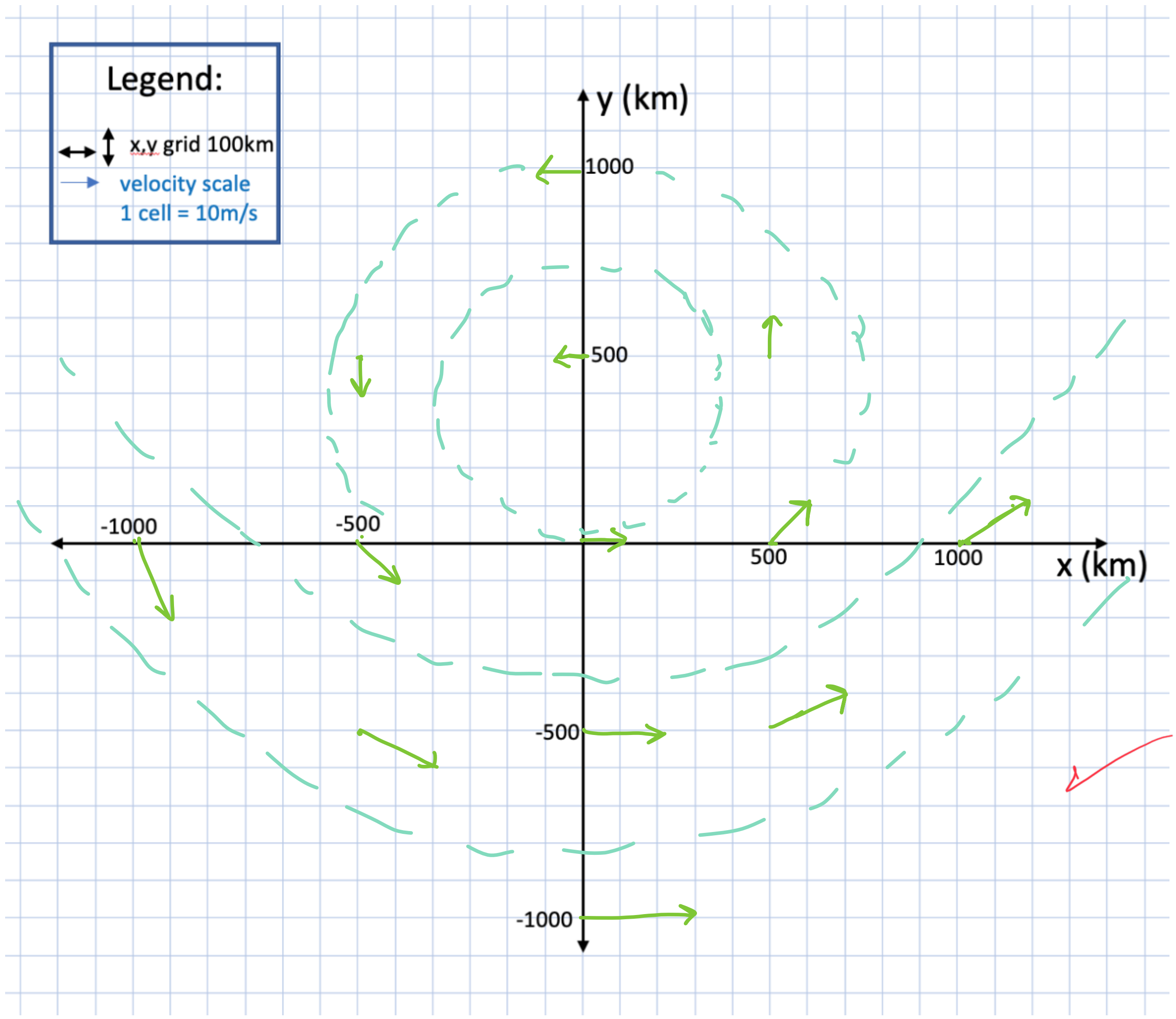
$\vec{V} = (-2Ay + 10)\hat{\imath} + (2Ax)\hat{\jmath}$

$$\vec{V}(x, y, z, t) = \langle (-Ay \cdot 2^{ct} + Sz), (Ax \cdot 2^{ct}) \rangle$$
$$= \hat{\imath}\,(-Ay \cdot 2^{ct} + Sz) + \hat{\jmath}\,(Ax \cdot 2^{ct})$$

A = 1 m/s (100 km)$^{-1}$, S = 1m/s km^{-1}, c = 1 day^{-1}

3. Repeat the exercise for z = 10 km, t = 1 day. Are there any *easterly* (that is, *westward)* winds?

yes. north of y=0 along axis. (0,500) (0,1000)



Flow is again counter clockwise shifting from N. westerlies to S. westerlies but develops a closed circulation centered on ~500 km north.

Easterlies only occur north of ~500 km along y axis (x=0)
(0, 500)
(0, 1000)

$$\vec{V}(x,y,z,t) = < (-Ay \cdot \overset{1}{\cancel{2^{ct}}} + Sz), (Ax \cdot \overset{1}{\cancel{2^{ct}}}) >$$
$$= \hat{\imath}\,(-Ay \cdot \cancel{2^{ct}} + Sz) + \hat{\jmath}\,(Ax \cdot \cancel{2^{ct}})$$

A = 1 m/s (100 km)$^{-1}$, S = 1m/s km^{-1}, c = 1 day^{-1}

4. Consider this difference of vectors *at two specific points*:

$$\Delta\vec{V} = \vec{V}_2 - \vec{V}_1 =$$
$$\vec{V}(500km, 500km, 10km, 0day) - \vec{V}(500km, 500km, 0km, 0day)$$

Because all the function arguments in parentheses (Careful! These are not vector components which some math teachers put in angle brackets <>) are definite locations, these are *individual velocity vectors at that location,* not vector fields. *They have magnitude and direction but not position: you can move them around in order to add and subtract them.*

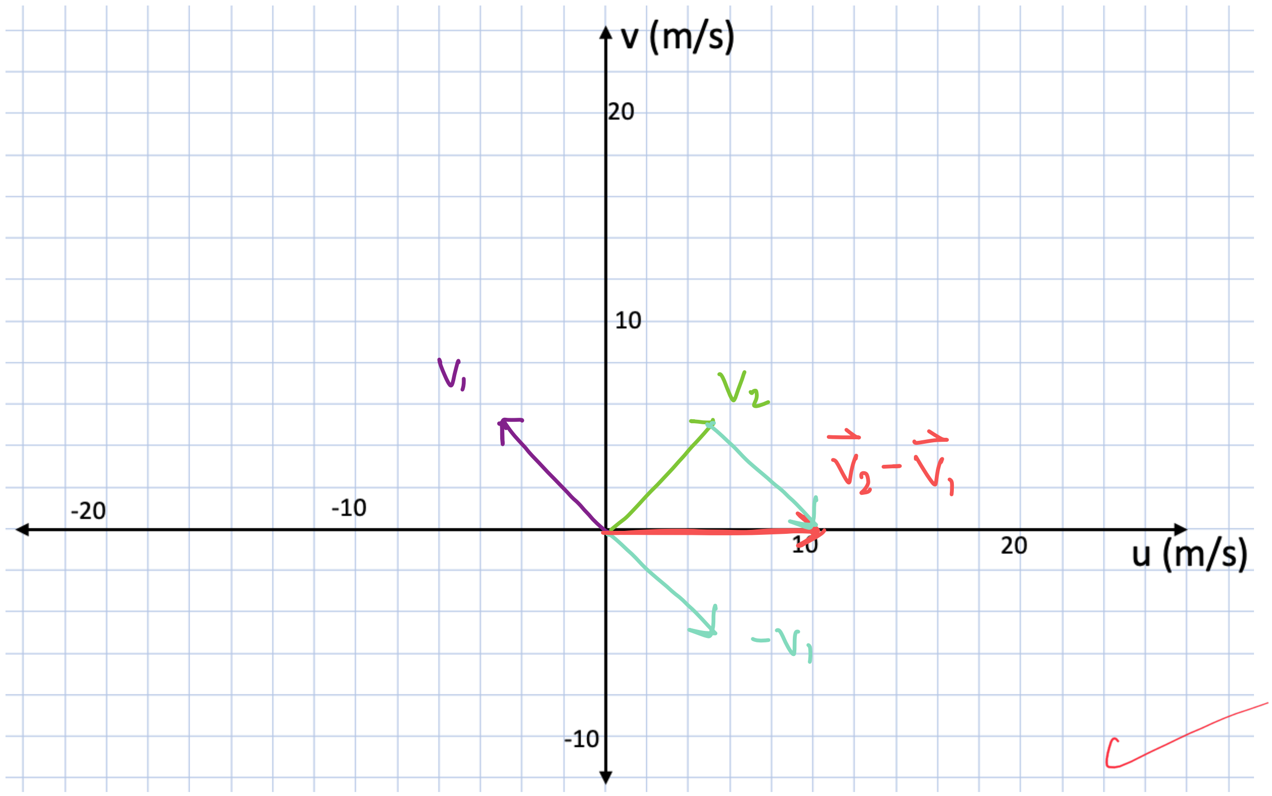
a. **Make a clear sketch of these two vectors and their difference,** showing your graphical subtraction work by your relative positioning of the vectors, labeling, etc. See the caution of Fig. 11-7 here about how individual vectors are repositioned for this purpose. Remember, you can think of subtraction as *adding the negative of the second vector.* I find vector addition (tip to tail) much more intuitive, like a journey, so I use this negation trick constantly.

Notice that the axes of this sketch are u and v components, not x and y (position):

$$\vec{V}_2 = (-5\cancel{00} + 10)\,\hat{\imath} + (5\cancel{00})\,\hat{\jmath}$$
$$5\hat{\imath} + 5\hat{\jmath}$$

$$\vec{V}_1 = (-5\cancel{00})\,\hat{\imath} + (5\cancel{00})\,\hat{\jmath} = -5\hat{\imath} + 5\hat{\jmath}$$

$$\vec{V}_2 - \vec{V}_1 \qquad \begin{array}{r} 5\hat{\imath} + 5\hat{\jmath} \\ -[-5\hat{\imath} - 5\hat{\jmath}] \\ \hline 10\hat{\imath} + 0\hat{\jmath} \end{array}$$



$$\vec{V}(x,y,z,t) = <(-Ay\cdot 2^{ct} + Sz),(Ax\cdot 2^{ct})> \\ = \hat{\imath}\,(-Ay\cdot 2^{ct} + Sz) + \hat{\jmath}\,(Ax\cdot 2^{ct})$$

A = 1 m/s (100 km)$^{-1}$, S = 1m/s km^{-1}, c = 1 day^{-1}

can also use...

$\vec{V_2} = \vec{V_1} + \vec{\Delta V}$

b. Compute the value (with units!) of $\Delta\vec{V}$ from the mathematical formula (function) for $\vec{V}$. What is the name for this quantity? Hint: I chose the coefficient letters carefully.

$\vec{V_2} - \vec{V_1}$

$$\begin{array}{r} 5\hat{\imath} + 5\hat{\jmath} \\ -[-5\hat{\imath} - 5\hat{\jmath}] \\ \hline 10\hat{\imath} + 0\hat{\jmath} \end{array}$$

10 m/s ; ΔV = shear

V = velocity ; ΔV w/ height = shear

c. What is the inner or dot product $\vec{V_1}\cdot\vec{V_2}$? Explain the geometric meaning of this quantity as seen on your diagram (that is, how can you tell its sign just by glancing at the vectors? see section 11-7 here). Also, calculate its value mathematically. Does the sign agree with your graphical reasoning? Discuss (that is, show me your knowledge clearly please).

$(-5\hat{\imath} + 5\hat{\jmath})\cdot(5\hat{\imath} + 5\hat{\jmath}) = -25 + 25 = 0$

(ie: $\perp$)

The dot product of 2 vectors that are orthogonal should be zero. The math agrees as the product is a scalar & can be combined to ∅. $\vec{V_{\hat{\imath}}}\cdot\vec{V_{\hat{\jmath}}} = 0,\ \hat{\imath} \neq \hat{\jmath}$

d. What direction is the outer or cross product product $\overrightarrow{V_1} \times \overrightarrow{V_2}$? Explain how you can tell its direction (up or down) just by glancing at the vectors. Cross product formulas and discussion about the "right hand rule" are here, or easily web-searched, or ask if this is unfamiliar.

down / into page $\otimes$; can't break fingers of RHR

$\therefore$ Index is V_1, so thumb points down (neg. direction)

$$\vec{V}_1 \times \vec{V}_2 = [5 \cdot (-5)] - [5 \cdot 5]$$

$$= -25 - 25 = -50 \underline{\left(\frac{m}{s}\right)^2}$$

Ok but units

$$\vec{V}(x,y,z,t) = < (-Ay \cdot 2^{ct} + Sz), (Ax \cdot 2^{ct}) >$$
$$= \hat{\imath}\,(-Ay \cdot 2^{\mathrm{ct}} + \mathrm{Sz}) + \hat{\jmath}\,(Ax \cdot 2^{ct})$$

A = 1 m/s (100 km)$^{-1}$, S = 1m/s km^{-1}, c = 1 day^{-1}

5. We often use the dot product formula in a straightforward way, but with a special kind of pseudo "vector field": a *vector* ***operator***, denoted by an upside-down triangle called "del". This vector Del notation is "extremely amusing and ingenious" as discussed in section 2-4 here. Study it well, this is close to the heart of our material.

First, you should know what a derivative "operator" is. If you understand that df/dt = d/dt(f) is the slope of the curve f(t), then just consider d/dt or d/dt() as the *operator* that takes the derivative (or "measures the slope") of whatever function of time is put to its right. But be careful! Whatever is put to the left of d/dt is simply multiplied, as in regular algebra. For example, d/dt(5) = 0, but 5d/dt or 5d/dt() is five times the derivative of whatever function is put to the right of d/dt or in the parentheses.

Del is written as $\vec{\nabla}$ or ∇ (the symbol is called a "nabla"). Sometimes it has subscripts, to emphasize whether it is derivative in 3D space, or a 2D derivative along some surface, like a horizontal surface of constant z, or a constant-p surface.

When Del is applied to a ***scalar field***, it creates a vector field called the ***gradient***. Please learn this word well, for instance from diagram 2-1 here.

When Del is combined with vector fields in various ways, it has other names. When Del is dotted into a vector field, it measures the ***vergence or divergence*** *(meaning: arrows-pointing-apart)* of that field. When divergence is negative, or if we are speaking of the negative of vergence, we use the word ***convergence.*** (arrows-pointing-together).

a. Use the dot product definition to derive the mathematical form of the divergence of our vector field, $\nabla \cdot \vec{V}$ from the first page. To do this, write down $\nabla = \hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y}$ and write down $\vec{V}$. Then, just naively apply the dot product formula, like you would for any two vectors. Finally, in a separate step, apply the partial derivative operations in that formula, to evaluate the result.

no con- or di- verg.

$$\nabla \cdot \vec{V} = \left(\frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial y}\hat{\jmath}\right) \cdot \left[(-Ay \cdot 2^{ct} + Sz)\hat{\imath} + (Ax \cdot 2^{ct})\hat{\jmath}\right]$$
$$= \frac{\partial}{\partial x}(-Ay \cdot 2^{ct} + Sz)\hat{\imath} + \frac{\partial}{\partial y}(Ax \cdot 2^{ct})\hat{\jmath} = 0\hat{\imath} + 0\hat{\jmath} = 0$$

b. Does the result make sense, in terms of the image of arrows "verging" together (con-) or apart (di-) in the sketches? Put it in your own words.

yes, sketches do no show any vectors pointing toward (con-) or away (di-) from each other.

$$\vec{V}(x,y,z,t) = < (-Ay \cdot 2^{ct} + Sz), (Ax \cdot 2^{ct}) >$$
$$= \hat{\imath}\,(-Ay \cdot 2^{ct} + Sz) + \hat{\jmath}\,(Ax \cdot 2^{ct})$$

A = 1 m/s (100 km)$^{-1}$, S = 1m/s km^{-1}, c = 1 day^{-1}

c. When Del is crossed into a vector field, it measures the ***curl*** *(arrows-going-around)* of that field. Derive $\nabla \times \vec{V}$ for all times, altitudes, and places by doing this mathematical operation. Finally, in a separate step, apply the partial derivative operations in that formula, to evaluate the result.

$$\nabla \times \vec{V} = \left(\frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial y}\hat{\jmath} + \frac{\partial}{\partial z}\hat{k}\right) \times \left[(-Ay \cdot 2^{ct} + Sz)\hat{\imath} + (Ax \cdot 2^{ct})\hat{\jmath}\right]$$

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (-Ay \cdot 2^{ct} + Sz) & (Ax \cdot 2^{ct}) & \varnothing \end{vmatrix} =$$

$$\left[\frac{\partial}{\partial y}(\varnothing) - \frac{\partial}{\partial z}(Ax \cdot 2^{ct})\right]\hat{\imath} - \left[\varnothing - \frac{\partial}{\partial z}(-Ay \cdot 2^{ct} + Sz)\right]\hat{\jmath} + \left[\frac{\partial}{\partial x}(Ax \cdot 2^{ct}) - \frac{\partial}{\partial y}(-Ay \cdot 2^{ct} + Sz)\right]\hat{k}$$

$$= 0\,\hat{\imath} - S\hat{\jmath} + (2^{ct}A + 2^{ct}A)\hat{k}$$

$$\boxed{= -S\hat{\jmath} + (2A \cdot 2^{ct})\hat{k}}$$

$+$

counter-clockwise

$$\vec{V}(x,y,z,t) = < (-Ay \cdot 2^{ct} + Sz), (Ax \cdot 2^{ct}) >$$
$$= \hat{\imath}\,(-Ay \cdot 2^{ct} + Sz) + \hat{\jmath}\,(Ax \cdot 2^{ct})$$

A = 1 m/s (100 km)$^{-1}$, S = 1m/s km^{-1}, c = 1 day^{-1}

6. In this part, you will compute a *line integral* of the *perimeter-normal component* (that is, the inflow or outflow component) of $\vec{V}$ around the square box $(x, y) \in [-300km, 300km]$ centered on the origin. **Sketch this box on your diagram from problem 2. for reference.**

a. Mathematically, by adding up 4 terms for the 4 sides, construct the line integral:

$$\oint \vec{V}(x,y,z = 10\ km, t = 0\ day) \cdot \widehat{dn}$$

where $\widehat{dn}$ is the distance increment ds (dx or dy for the edges of a square box), times the unit vector pointing directly ***out of*** the box. That is, on the east side $\widehat{dn}$ = $\hat{\imath}\,dy$, on the north side it is $\hat{\jmath}\,dx$, on the west side it is $-\hat{\imath}$ dy, and so on. Notice that this dot product with the unit vector extracts just one component of $\vec{V}$ for each box edge, so it's not too ugly or gory.

$$\oint \vec{V} = \int (-Ay \cdot \cancel{2}^{\overset{1}{\cancel{ct}}} + S\underset{10}{z})\,\hat{\imath} + (Ax \cdot \cancel{2}^{\overset{1}{\cancel{ct}}})\,\hat{\jmath}$$

see next pg

$\boxed{\text{ans: } 0\ km^3/s}$

n

ambiguous: $(Km)^2$?

$K(m^2)$

not

b. What are the units of this integral? $km^2 s^{-1}$

m^2/s

c. Read the essential sense of Gauss's theorem in section 3-3 here. How does your integral relate to the result from part 5. ? Do the values agree and make sense? Explain.

This value is equal to 5 so they agree. #5 showed no con/di-vergence. Therefore flow in & out of the box is the same yielding a flux of Ø.

$$\oint \vec{V} = \int \left(-Ay \cdot \cancel{2^{ct}}^{\,1} + \cancel{5/2}^{\,10}\right)\hat{i} + \left(Ax \cdot \cancel{2^{ct}}^{\,1}\right)\hat{j} = \int (-Ay + 10)\hat{i} + (Ax)\hat{j}$$

EAST: $\displaystyle\int_{-300}^{300} \vec{V} \cdot \hat{i}\, dy = \int_{-300}^{300} (-Ay + 10)\hat{i} + \varnothing$

$$= -\tfrac{1}{2}Ay^2 + 10y \Big|_{-300}^{300} \Rightarrow \left[-\tfrac{1}{2}(.01)(300^2) + 3000\right] - \left[-\tfrac{1}{2}(.01)(-300^2) - 3000\right]$$

$$= 2550 - (-2550)$$

$$= 5100 \, km^2/s$$

NORTH: $\displaystyle\int_{300}^{-300} \vec{V} \cdot \hat{j}\, dx = \int_{300}^{-300} (Ax)\hat{j} = \tfrac{1}{2}Ax^2 \Big|_{300}^{-300}$

$$= \tfrac{1}{2}(.01)(-300)^2 - \tfrac{1}{2}(.01)(300)^2$$

$$= 450 - 450 = 0 \, km^2/s$$

WEST: $\displaystyle\int_{300}^{-300} \vec{V} \cdot \hat{i}\, dy \Rightarrow -\tfrac{1}{2}Ay^2 + 10y \Big|_{300}^{-300}$

$$= -\tfrac{1}{2}(.01)(-300^2) + 10(-300) - \left[-\tfrac{1}{2}(.01)(300)^2 + 10(300)\right]$$

$$= -2550 - 2550 = -5100 \, km^2/s$$

SOUTH: $\displaystyle\int_{-300}^{300} \vec{V} \cdot \hat{j}\, dx \Rightarrow \tfrac{1}{2}Ax \Big|_{-300}^{300}$

$$= \tfrac{1}{2}(.01)(300^2) - \tfrac{1}{2}(.01)(-300)^2 = 0 \, km^2/s$$

$$5100 + 0 - 5100 + 0 = 0 \, km^2/s$$

$$\vec{V}(x,y,z,t) = < (-Ay \cdot 2^{ct} + Sz), (Ax \cdot 2^{ct}) >$$
$$= \hat{\imath}\,(-Ay \cdot 2^{ct} + Sz) + \hat{\jmath}\,(Ax \cdot 2^{ct})$$

A = 1 m/s (100 km)$^{-1}$, S = 1m/s km^{-1}, c = 1 day^{-1}

7. In this part, compute the other line integral, of the *along-perimeter component* for the same box. The convention is to integrate in the ANTI-CLOCKWISE direction.

a. Mathematically, by adding up 4 terms for the 4 sides, construct the line integral:

$$\oint \vec{V}(x, y, z = 10\ km, t = 0\ day) \cdot \widehat{dl}$$

where $\widehat{dl}$ is the distance increment ds (dx or dy, for a square box), times the unit vector pointing DIRECTLY ALONG THE PERIMETER of the box. That is, up the east side it is $\hat{\jmath}\ dy$, on the north side it is -$\hat{\imath}\ dx$, down the west side it is -$\hat{\jmath}$ dy, and so on.

see next pg.

$\boxed{\text{ans: } 12{,}000\ km^2/s}$ ✓

$\left(\frac{m}{s}\right) \cdot km$

or better to say

$\boxed{1.2 \times 10^7\ m^2 s^{-1}}$

b. What are the units of this integral? $km^2 s^{-1}$ $m^2 s^{-1}$ no k

c. Read the essential sense of the Circulation theorem (Stokes's theorem) in section 3-5 and 3-6 of Feynman lectures [here (https://www.feynmanlectures.caltech.edu/II_03.html)](https://www.feynmanlectures.caltech.edu/II_03.html). Do the values and sign of your integral make sense for the swirl of your arrows in your sketch of Part 2, and the curl you calculated in problem 5d. above? Explain.

curl = $\int C \cdot ds = \int (\nabla \times C)_n\, da$; it does agree w/ #2

curl/swirl is (& should be) counterclockwise (positive)

(don't see a 5d... 5c?) Thanks! will fix

$$\oint \vec{v} = \int \left(-Ay \cdot \cancel{2^{ct}}^{\,1} + \cancel{5/2}^{\,10}\right)\hat{i} + \left(Ax \cdot \cancel{2^{ct}}^{\,1}\right)\hat{j} = \int (-Ay + 10)\hat{i} + (Ax)\hat{j}$$

EAST side: $\int_{-300}^{300} \vec{v} \cdot \hat{j}\, dy = \int_{-300}^{300} Ax = Axy \Big|_{-300}^{300} \Rightarrow (.01)(300)(300) - (.01)(300)(-300)$

$= 900 - (-900) = 1800\ km^2/s$

NORTH side: $\int_{300}^{-300} \vec{v} \cdot \hat{i}\, dx = \int_{300}^{-300} -Ay + 10 \Rightarrow -Axy + 10x \Big|_{300}^{-300}$

$\therefore -\hat{i}\, dx$

$[(-.01)(-300)(300) + 10(-300)] - [(-.01)(300)(300) + 10(300)]$

$-\hat{i}\,[(900 - 3000) - (-900 + 3000)]$

$-1[(-2100) - (2100)] = 4200\ km/s$

WEST side: $\int_{300}^{-300} Ax = Axy \Big|_{300}^{-300} \Rightarrow (.01)(300)(-300) - (.01)(300)(300)$

$\therefore -\hat{j}\, dy$

$= [(.01)(-300)(-300) - (.01)(-300)(300)]$

$= -[900 + 900] = \underset{(-\hat{j})}{-1}(1800\ m^2/s) \leftarrow -1800\ km^2/s$

SOUTH side: $\int_{-300}^{300} \vec{v} \cdot \hat{i}\, dx = \int_{-300}^{300} -Ay + 10 \Rightarrow -Axy + 10x \Big|_{-300}^{300}$

$= -.01(300)(-300) + 10(300) - [-.01(-300)(-300) + 10(-300)]$

$= 7800\ km^2/s$

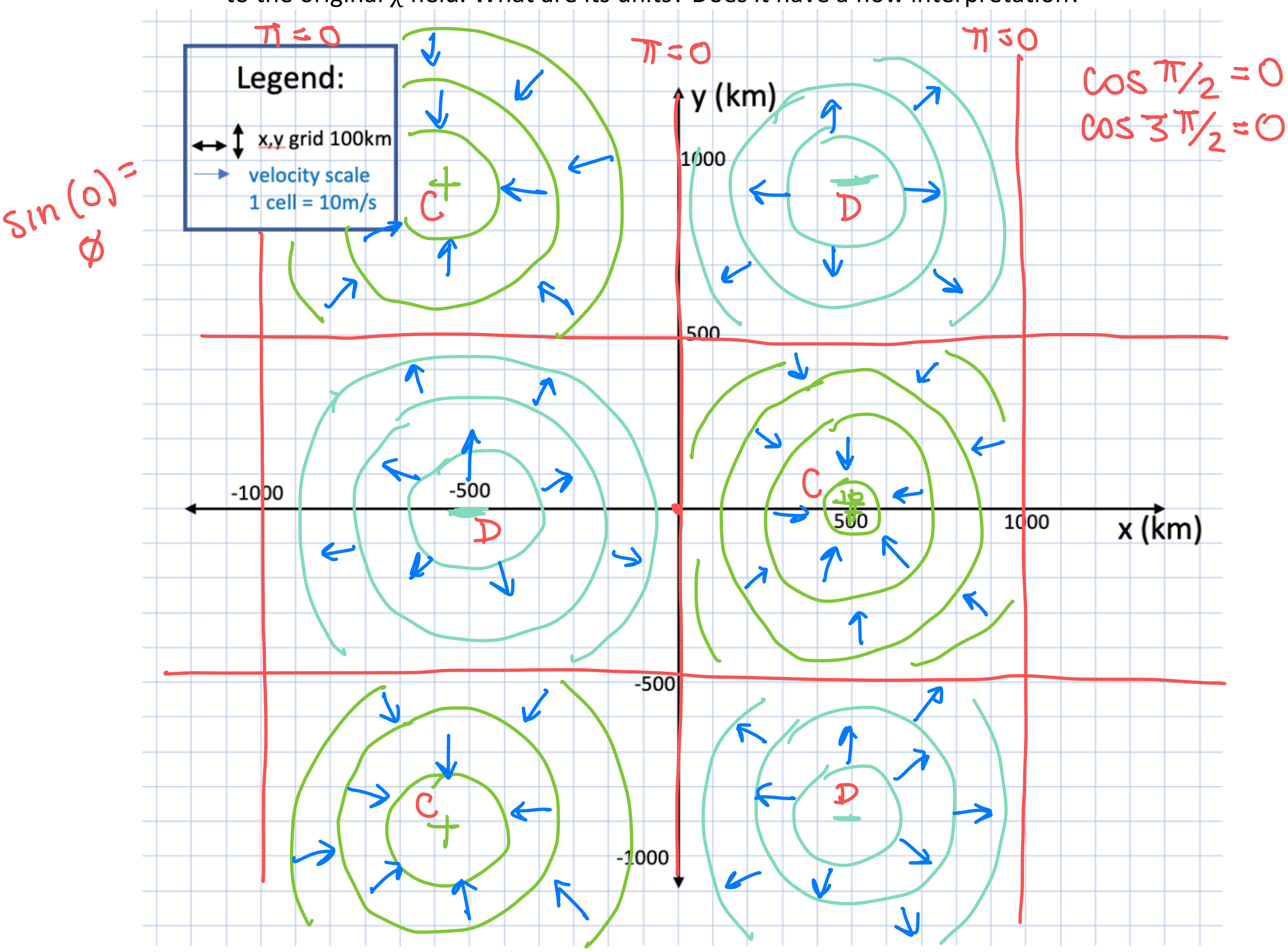
$7800 + 1800 - 1800 + 4200 = 12{,}000\ km^2/s$

use $0.2 \times 10^7 \ m^2/s$

$$\chi(x,y) = \underbrace{(10^7 m^2/s)}\sin\left(\pi \frac{x}{1000km}\right)\cos\left(\pi \frac{y}{1000km}\right)$$

8. Let's get a glimpse of the **Laplacian**: the ***divergence of the gradient*** of a scalar field.
   a. Sketch contours of χ every ~~$20m^2/s$~~, using broken contours or a different color for negative values, clearly labeling the max and min values with H and L symbols. This field is called a *velocity potential.*
   b. Sketch the vector field $\nabla\chi$. What are its units? Use the right arrow size. m/s
   c. Draw a C where the gradient vectors converge and D where they diverge. Is C in the concavities (bowls = dips = troughs), or on the convex places (peaks = crests)?
   d. Calculate the divergence of the gradient, mathematically. Notice that this is just the second derivative wrt x, plus the second derivative wrt y. It is a scalar field (the simple sum of those two terms), not a vector field.
   e. Express in words how the Laplacian = second derivative = curvature = concavity relates to the original χ field. What are its units? Does it have a flow interpretation?

diffusion w/ time?



@ y=0, x=0

$$\vec{\nabla}\chi = \frac{\partial\chi}{\partial x}\hat{i} + \frac{\partial\chi}{\partial y}\hat{j}$$

$$\vec{\nabla}\cdot\vec{\nabla}\chi \rightarrow \underbrace{\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j}\right)}_{\vec{\nabla}}\cdot\underbrace{\left(\frac{\partial\chi}{\partial x}\hat{i} + \frac{\partial\chi}{\partial y}\hat{j}\right)}_{\vec{\nabla}\chi}$$

class notes.

$$\frac{\partial}{\partial x}\left(\frac{\partial \chi}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial \chi}{\partial y}\right)$$

$$\frac{\partial^2}{\partial x^2}\chi + \frac{\partial^2}{\partial y^2}(\chi)$$

divergent component of wind

$\boxed{\text{units m/s}}$

$$-\left(\frac{\pi}{10^6 m}\right)^2 \cdot \left(10^7 \frac{m^2}{s}\right)\sin(\quad)\cos(\quad)$$

$$\approx 10^{-5}\ s^{-1}$$

wind divergence

$\boxed{\text{units } s^{-1}}$

$$\nabla^2 \chi = \frac{d^2}{dx^2}\chi + \frac{d^2}{dy^2}\chi$$

1st 2nd.

$$\frac{d}{dx}\chi = \frac{d}{dx}\left[\underbrace{.2\left(10^7 m^2/s\right)}_{\text{const.}} \sin\left(\pi \frac{x}{1000\text{ km}}\right)\cos\left(\pi\frac{y}{1000\text{ km}}\right)\right]$$

$$= .2(10^7)\frac{\pi}{1000}\cos\left(\pi\frac{x}{1000}\right)\cos\left(\pi\frac{y}{1000}\right)$$

$$\frac{d^2}{dx^2} = -\left[.2(10^7)\frac{\pi^2}{1000^2}\sin\left(\pi\frac{x}{1000}\right)\cos\left(\pi\frac{y}{1000}\right)\right]$$

$$\frac{d}{dy}x = \frac{d}{dy}\left[.2(10^7\, m/s)\sin\left(\pi\frac{x}{1000}\right)\cos\left(\pi\frac{y}{1000}\right)\right]$$

$$= -.2(10^7)\sin\left(\pi\frac{x}{1000}\right)\frac{\pi}{1000}\sin\left(\pi\frac{y}{1000}\right)\Big]$$

$$\frac{d^2}{dy^2} = -.2(10^7)\sin\left(\pi\frac{x}{1000}\right)\frac{\pi^2}{1000^2}\cos\left(\pi\frac{y}{1000}\right)$$

$$= -.2(10^7)\frac{\pi^2}{1000^2}\sin\left(\pi\frac{x}{1000}\right)\cos\left(\pi\frac{y}{1000}\right)$$

add... $\frac{d^2}{dx^2} + \frac{d^2}{dy^2}$

$$-\left[.2(10^7)\frac{\pi^2}{1000^2}\sin\left(\pi\frac{x}{1000}\right)\cos\left(\pi\frac{y}{1000}\right)\right] + -.2(10^7)\frac{\pi^2}{1000^2}\sin\left(\pi\frac{x}{1000}\right)\cos\left(\pi\frac{y}{1000}\right)$$

$$\nabla^2 x = -2\left[.2(10^7)\frac{\pi^2}{1000^2}\sin\left(\pi\frac{x}{1000}\right)\cos\left(\pi\frac{y}{1000}\right)\right]$$

$$= -.4\pi^2 10^{.5}\sin\left(\pi\frac{x}{1000}\right)\cos\left(\pi\frac{y}{1000}\right)$$ ✓

d). units are s^{-1}; ✓

Laplacian is (+) in valleys/convex areas where gradient diverges & (−) at the peak/top/crest where gradient would converge in concave regions.

Words: other way round

valleys are "concave" (positive or upward curvature)