Have answers ready for class discussion when called upon:

Exercises

7.5 Explain or interpret the following on the basis of the principles discussed in this chapter.

(a) A diffluent flow does not necessarily exhibit divergence.

(b) A flow with horizontal shear does not necessarily exhibit vorticity.

(c) A person of fixed mass weighs slightly less when flying on an eastbound plane than when flying on a westbound plane.

(d) A satellite can be launched in such a way that it remains overhead at a specified longitude directly over the equator in a so-called *geostationary orbit*.

(e) The oblateness of the shape of Jupiter is more apparent than that of the Earth.

(f) The Coriolis force has no discernible effect on the circulation of water going down the drain of a sink.

(g) The vertical component of the Coriolis force is not important in atmospheric dynamics, nor is the Coriolis force induced by vertical motions.

(h) The strong winds encircling hurricanes are highly subgeostrophic.

(i) Cyclones tend to be more intense than anticyclones.

(j) The wind in valleys usually blows up or down the valley from higher toward lower pressure rather than blowing parallel to the isobars.

(k) Surface winds are usually closer to geostrophic balance over oceans than over land.

(l) When high and low cloud layers are observed to be moving in different directions, it can be inferred that horizontal temperature advection is occurring.

(m) Veering of the wind with height within the planetary boundary layer is not necessarily an indication of warm advection.

(n) Areas of precipitation tend to be associated with convergence in the lower troposphere and divergence in the upper troposphere.

(o) The estimation of divergence from wind observations is subject to larger percentage errors than the estimation of vorticity.

(p) The pressure gradient force does not affect the circulation around any closed loop that lies on a pressure surface.

(q) Motions in the middle troposphere tend to be quasi-nondivergent.

(r) The primitive equations assume a simpler form in pressure coordinates than in height coordinates.

(s) The thermodynamic energy equation assumes a particularly simple form in isentropic coordinates.

(t) In middle latitudes, local rates of change of temperature tend to be smaller than the changes attributable to horizontal temperature advection.

(u) Temperatures do not always rise in regions of warm advection.

(v) The release of latent heat in the midtroposphere has the effect of increasing the isentropic potential vorticity of the air in the column below it.

(w) Rising of warm air and sinking of cold air result in the generation of kinetic energy, even in hydrostatic motions.

(x) The Hadley cell does not extend from equator to pole.

(y) A term involving the Coriolis force does not appear in Eq. (7.42).

(z) The easterlies in the lower branch of the Hadley cell are maintained in the presence of friction.

(aa) Baroclinic waves and monsoons tend to be more vigorous in general circulation models that incorporate the effects of moisture than in those that do not.

(bb) Hydroelectric power may be viewed as a by-product of the atmospheric general circulation.

7.6 Describe the vorticity distribution within a flow characterized by counterclockwise circular flow with tangential velocity u inversely proportional to radius r.

Solution: The radial velocity profile is $ur = k$, where k is a constant. Hence the contribution of the shear to the vorticity is $d(k/r)/dr = -k/r^2$, and the contribution of the curvature is

7.7 At a certain location along the ITCZ, the surface wind at 10 °N is blowing from the east–northeast (ENE) from a compass angle of 60° at a speed of 8 m s^{-1} and the wind at 7 °N is blowing from the south–southeast (SSE) (150°) at a speed of 5 m s^{-1}. (a) Assuming that $\partial/\partial y \gg \partial/\partial x$, estimate the divergence and the vorticity averaged over the belt extending from 7 °N to 10 °N. (b) The meridional component of the wind drops off linearly with pressure from sea level (1010 hPa) to zero at the 900-hPa level. The mixing ratio of water vapor within this layer is 20 g kg^{-1}. Estimate the rainfall rate under the assumption that all the water vapor that converges into the ITCZ in the low level flow condenses and falls as rain.

7.8 Consider a velocity field that can be represented as

$$\mathbf{V}_\Psi = \mathbf{k} \times \nabla\Psi$$

or, in Cartesian coordinates,

$$u_\Psi = -\partial\Psi/\partial y; \; v_\Psi = \partial\Psi/\partial x$$

where Ψ is called the *streamfunction*. Prove that $\mathrm{Div}_H\,\mathbf{V}$ is everywhere equal to zero and the vorticity field is given by

$$\zeta = \nabla^2\Psi \tag{7.44}$$

Given the field of vorticity, together with appropriate boundary conditions, the inverse of (7.22); namely

$$\Psi = \nabla^{-2}\zeta$$

may be solved to obtain the corresponding streamfunction field. Because the true wind field at extratropical latitudes tends to be quasi-nondivergent, it follows that $\mathbf{V}$ and $\mathbf{V}_\Psi$ tend to be quite similar.

7.17 A projectile is fired vertically upward with velocity w_0 from a point on Earth. (a) Show that in the absence of friction the projectile will land at a distance

$$\frac{4w_0^3\Omega}{3g^2}\cos\phi$$

to the west of the point from which it was fired. (b) Calculate the displacement for a projectile fired upward on the equator with a velocity of 500 m s^{-1}.

7.18 A locomotive with a mass of 2×10^4 kg is moving along a straight track at 40 m s^{-1} at 43 °N. Calculate the magnitude and direction of the transverse horizontal force on the track.

7.20 (a) Prove that the divergence of the geostrophic wind is given by

$$\nabla \cdot \mathbf{V}_g \equiv \frac{\partial u_g}{\partial x} + \frac{\partial v_g}{\partial y} = \frac{v_g}{f}\frac{\partial f}{\partial y} = -v_g\frac{\cot\phi}{R_E} \tag{7.45}$$

and give a physical interpretation of this result. (b) Calculate the divergence of the geostrophic wind at 45 °N at a point where $v_g = 10$ m s^{-1}.

7.22 At a station located at 43 °N, the surface wind speed is 10 m s^{-1} and is directed across the isobars from high toward low pressure at an angle $\psi = 20°$. Calculate the magnitude of the frictional drag force and the horizontal pressure gradient force (per unit mass).

7.23 Show that if friction is neglected, the horizontal equation of motion can be written in the form

$$\frac{d\mathbf{V}}{dt} = -f\mathbf{k} \times \mathbf{V}_a \qquad (7.46)$$

where $\mathbf{V}_a \equiv \mathbf{V} - \mathbf{V}_g$ is the *ageostrophic* component of the wind.

See problem 7.32 deriving the equation for absolute vorticity, whose solution ends like:

With these substitutions, we obtain

$$\frac{\partial \zeta}{\partial t} = -u\frac{\partial \zeta}{\partial x} - v\frac{\partial \zeta}{\partial y} - (f + \zeta)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) - v\frac{\partial f}{\partial y}$$

Rearranging slightly (noting that $\partial f/\partial t = 0$) and reintroducing vector notation yields the conservation law

$$\frac{\partial}{\partial t}(f + \zeta) = -\mathbf{V} \cdot \nabla(f + \zeta) - (f + \zeta)(\nabla \cdot \mathbf{V})$$

or, in Lagrangian form,

$$\frac{d}{dt}(f + \zeta) = -(f + \zeta)\,(\nabla \cdot \mathrm{V})$$

These equations appear in the text as Eq. (7.21a,b). ■

Show that the final equation can be written as the *horizontal convergence of a flux of absolute vorticity ,* given by (z+f)**V.**

7.33 Consider a sinusoidal wave along latitude ϕ with wavelength L and amplitude v in the meridional wind component. The wave is embedded in a uniform westerly flow with speed U. (a) Show that the amplitude of the geopotential height perturbations associated with the wave is $fvL/2\pi g$ where f is the Coriolis parameter and g is the gravitational acceleration. (b) Show that the amplitude of the associated vorticity perturbations is $(2\pi/L)v$. Show that the maximum values of the advection of planetary and relative vorticity are βv and $(2\pi/L)^2 Uv$, respectively, and that they are coincident and of opposing sign. (c) Show that the advection terms exactly cancel for waves with wavelength

$$L_s = 2\pi\sqrt{\frac{U}{\beta}}.$$

L_S is referred to as the wavelength of a stationary Rossby wave.

7.34 Suppose that the wave in the previous exercise propagating is along 45° latitude and has a wavelength of 4000 km. The amplitude of the meridional wind perturbations associated with the wave is 10 m s^{-1} and the background flow U = 20 m s^{-1}. Assume that the velocity field is independent of latitude. Using the results of the previous exercise, estimate (a) the amplitude of the geopotential height and vorticity perturbations in the waves, (b) the amplitude of the advection of planetary and relative vorticity, and (c) the wavelength of a stationary Rossby wave embedded in a westerly flow with a speed of 20 m s^{-1} at 45° latitude.

7.36 Consider barotropic ocean eddies propagating meridionally along a sloping continental shelf, with depth increasing toward the east, as pictured in Fig. 7.28, conserving barotropic potential vorticity in accordance with (7.27). There is no background flow. In which direction will the eddies propagate? [**Hint**: Consider the vorticity tendency at points A and B.]

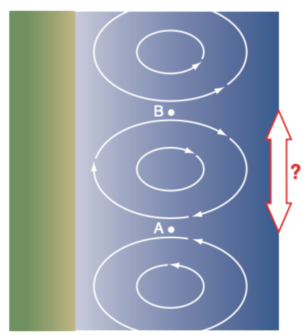

Fig. 7.28 Barotropic ocean eddies propagating along a continental shelf, as envisioned in Exercise 7.36. Lighter blue shading indicates shallower water.

7.39 Figure 7.29 shows an idealized trapezoidal vertical velocity profile in a certain rain area in the tropics. The horizontal convergence into the rain area in the 1000 to 800-hPa layer is $10^{-5}\ s^{-1}$, and the average water vapor content of this converging air is 16 g kg^{-1}. (a) Calculate the divergence in the 200 to 100-hPa layer. (b) Estimate the rainfall rate using the assumption that all the water vapor is condensed out during the ascent of the air.

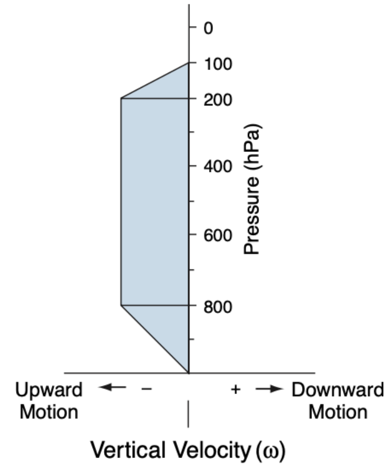

Fig. 7.29 Vertical velocity profile for Exercise 7.39.

(note solution is in book, pleaase go through that derivation as you craft your answer).

7.40 In middle-latitude winter storms, rainfall (or melted snowfall) rates on the order of 20 mm day^{-1} are not uncommon. Most of the convergence into these storms takes place within the lowest 1–2 km of the atmosphere (say, below 850 hPa) where the mixing ratios are on the order of 5 g kg^{-1}. Estimate the magnitude of the convergence into such storms.

7.41 The area of a large cumulonimbus anvil in Fig. 7.30 is observed to increase by 20% over a 10-min period. Assuming that this increase in area is representative of the average divergence within the 300 to 100-hPa layer and that the vertical velocity at 100 hPa is zero, calculate the vertical velocity at the 300-hPa level.

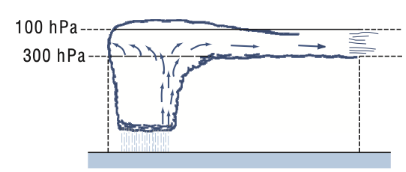

Fig. 7.30 Physical situation in Exercise 7.41.

7.44 Suppose that a parcel of air initially at rest on the equator is carried poleward to 30° latitude in the upper branch of the Hadley cell, conserving angular momentum as it moves. In what direction and at what speed will it be moving when it reaches 30°?

7.45 On average over the globe, kinetic energy is being generated by the cross-isobar flow at a rate of $\sim 2\ \mathrm{W\ m^{-2}}$. At this rate, how long would it take to "spin up" the general circulation, starting from a state of rest?