Exam format:

word	symbolic math	nutshell meaning	longer explanation of meaning (concept) in question	Relevant sketch with arrows or little f(x) curve or whatever is appropriate
divergence				
	ρ			xxxxx

...

Essential grammar we have learned: first just math

Latitude, longitude, altitude
Zonal, meridional, vertical
Northward vs. northerly; eastward vs. westerly; poleward vs. equatorward; cyclonic
upward, altitude, pressure level (know Earth's atmosphere layers, z & p depth values)
troposphere, stratosphere, planetary boundary layer (PBL)

Cartesian: x,y,z coords **i,j,k** unit vectors u,v,w components of vector wind **V**
tangent plane to the spherical Earth, locally accurate, *orthogonal*

a scalar vs. a vector
a vector has 2 *properties* (in 2D or 3D): *magnitude* and *direction*
Two kinds of multiplication: dot product, cross product
defined for an individual vector; repeated at every location for a vector *field*
scalar *field* vs. vector *field* -- know your MKS Units on all these quantities

functions apply over a *domain* (coordinates or arguments or inputs)
their *value* (output) spans some *range* of outcomes
in other words, a function is a *mapping from domain to range*
graphically: a function is a *curve* (1D domain), *surface* (2D), or *field* (3D, 4D)
ex: T(x,y,z,t) is *temperature everywhere forever,* $\mathbf{V}_{500}$(x,y) is *hor. wind @500*

Derivatives of a function:
first: *slope* (on 1D domain) is a scalar; *gradient* (on 2D, 3D domain) is **vector field**
second: *curvature* (on any dimension of domain)
second derivative is a *scalar operator*: it flips sign, for *sinuoids* (waves)
equal to *divergence of gradient* on 2D or 3D domain

Del operator: a ***vector operator*** (*nabla* symbol), behaves just like a vector except that things to the left of it are *multiplied*, while things on the right of it are *operated on*.
that is, Del does not *commute* like a regular vector field. $\boldsymbol{V} \cdot \nabla \neq \nabla \cdot \boldsymbol{V}$

* know the *gradient* of a scalar function *of space*, result is a **vector field**
  like *temperature gradient* ∇T where T(x,y,z, maybe t) is a scalar function

* know "del dot" a vector field
  vergence (divergence, convergence) div(**V**) = "del dot **V**" $= \nabla \cdot \boldsymbol{V}$
  convergence of a flux is the tendency due to transport
    advection is another way to express this tendency

* know "del squared": divergence of gradient, second derivative in space
  a measure of *curvature* of a curve, surface, or (abstractly) 3D field
  differentiation emphasizes small scales: *edge finder* in image proc.
  random exchange (molecules) creates a *diffusive flux, down the gradient*
    diffusivity is the constant of proportionality
  convergence of that flux is a *transport tendency* called *diffusive tendency*
    called *viscous force* for diffusive momentum flux

advection **"minus V dot del"** or "minus V dot grad T": *transport from upwind*
  note negative sign $-\boldsymbol{V} \cdot \boldsymbol{\nabla} T$

Curl of vector field **V**, $\nabla \mathrm{x} \boldsymbol{V}$
  Only in 3D! Right hand rule.
    (vector *vorticity,* if **V** is a 3D velocity field)
      we use only its *vertical component,* $\zeta = v_y - u_x$
        (where subscripts indicate partial derivatives)
Curl of gradient vanishes precisely - why?
  round-and-round vs. uphill-downhill are the 2 kinds of motion

Scale of variation (m vs. km vs. 1000s of km; hours vs. days vs. months):
  notice these are logarithmic distinctions, not just "size" (like 10m vs. 5m)
Running average (smoothing) isolates *large scales* (larger than *filter scale*)
  Deviations from that are *small scales*: (*subfilter* scales)
    anomaly (deviation from time average)
    eddy (deviation from space average)
    perturbation: someone/something *perturbed* something
      away from some *control* case
      (to do an *experiment*, capable of isolating *cause and effect*)

Partial derivatives of a field f(x,y,z,t)
  Local or *Eulerian* $\partial f/\partial t$
Total or *Lagrangian* Df/Dt, following a moving parcel at position $<x_p(t), y_p(t), z_p(t)>$
  Know the difference from to the local derivative $\partial f/\partial t$ (chain rule):
    advection $-\boldsymbol{V} \cdot \boldsymbol{\nabla} f$ by wind **V** = <u,v,w> = d/dt $<x_p(t), y_p(t), z_p(t)>$
      because the *rate of change* of *coordinate position* IS *velocity*

Nondivergent vs. irrotational decomposition of a vector field

  equivalently, rotational and divergent, "**components**" *of field decomposition*

    different meaning than *vector **components*** (along the unit vectors)

** streamline, streamfunction, streamwise*: *instantaneous* velocity, *threaded* tip to tail
** trajectory* (different from streamline): motion of a parcel through time

Integral relationships (opposite of derivative) for gradient, div, curl

  Stokes' theorem (circulation), Gauss' theorem (for divergence)

  vanishing of div(curl(**V**(x,y,z))) <--> vanishing of loop integral of gradient

ODEs and solutions

  time tendency is usually put on *left hand side (LHS)*

  exponential solutions to df/dt = -bf

  sinusoidal solutions to $d^2f/dt^2 = -c^2f$

  exp() with complex numbers combines both

  need boundary or initial conditions (constant of integration) to fully solve DEs

  stationary or steady-state solution: equilibrium or "balance"

    df/dt = A - B. Make steady-state assumption. Is it still a diff-eq? NO! A=B

PDEs and solutions: terms and concepts (for our applications)

  prognostic vs. *diagnostic*

  boundary conditions, initial conditions

  inverse of Laplacian (smoothing, the opposite of "edge finding"; reversed sign)

    Streamfunction is inverse-Laplacian of *vorticity*

PROGNOSTIC EQUATIONS:

  Contains a time derivative or *rate of change*, customarily on left-hand side (LHS)

    terms on RHS are then called *partial tendencies*, time *tendencies*

  Governing equation(s), *budgets*, with *partial tendencies* (tendency terms) on RHS

  Eulerian (local) vs. *Lagrangian (total, following-the-flow*) derivatives

  d/dt(something) = 0 + sources - sinks

  $\partial/\partial t$(something) = flux convergence + sources - sinks

  $\partial/\partial t$(something) = advection + sources - sinks

  Conserved tracer an important special case: *sources-sinks negligible*

  Balance special case: *neglect time derivative relative to other tendencies*

    hydrostatic in the vertical (pressure = weight = g*mass of column)

    geostrophic wind balance, gradient wind balance in the horizontal

geostrophic wind $\mathbf{V}_g$, *thermal wind* (upper-level $\mathbf{V}_g$ minus lower-level $\mathbf{V}_g$)
fictional wind fields obeying balance exactly: no divergence, unchanging

jet stream (a momentum feature; sketch forces, p surface slope, thickness)
cool core cyclone: positive PV feature in upper troposphere
warm core cyclone: positive PV feature strongest at low levels
Adjustment (a fast or efficient process of restoration/maintenance of balance)

Dynamics: the physical study of flow (forces, etc.) and its changes (prognostic)
Kinematics: the basis set of 4 2D spatial gradients of 2D velocity field
vorticity, divergence, deformation.
diffluence/confluence may or may not be associated with true divergence
recipes:
shear = vorticity + deformation
line of convergence = convergence + deformation

Waves: terms and concepts
harmonic or sinusoidal functions
frequency, period, wavelength, wavenumber, amplitude, phase
phase velocity, group velocity
growing, decaying *amplitude* (in space or time)
growing, shrinking *scale* (expressed as wavenumber or wavelength)

Physical concepts/words to know

Mass, density, mass fractions (specific __, mixing ratio of __, concentration of _)
Conservation of mass (*continuity* of mass flux)
Flux of mass, multiply by specific __ to get flux of __ (__ = momentum, moisture, etc.)
Conservation of stuff

TRANSPORT:

Flux of (stuff): what are the units? (Stuff) per second per square meter, in 3D
Flux *convergence* is the impact of the flux (*transport's* "drop-off" or "delivery")

Advection: the sense of it (upstream conditions coming at ya) and the math (-**V.del**)
how are *advection* vs. *flux convergence* related?
(Answer: equal, because of mass continuity, as in homework).
Diffusion (convergence of a flux that is negatively proportional to the gradient).

PHYSICAL LAWS

Equation of Motion / Newton's 2nd Law (F=ma)

Gravity force: vertical; it defines vertical
Pressure-gradient force (PGF): Enforcer of continuity, in general
Gradient of *pressure-surface height* or *geopotential height* in p-coords.
Coriolis force (if still air on rotating Earth is 'motionless', this is very real)
f is *Coriolis parameter*, also equal to *planetary vorticity*
"*Inertial forces*" (*advection of momentum* by wind itself)
"*Friction*" (convergence of momentum flux by small-scale motions; *viscosity*)

Vorticity equations: d/dt(vorticity) = 0 + complications
Relative vorticity ζ: eliminates PGF from momentum equations
Absolute vorticity $\zeta_a=(f+\zeta)$ moves v df/dy = vβ term to LHS
mechanism: Coriolis force *torque* on air patch that moves south or north
Potential vorticity PV, eliminates ζ_a div(**V**) "*vortex stretching*" term from RHS

Vortex interactions (e.g. for TC steering): 2D reasoning (not this year)
1/r decay of "induced" rotational wind from vorticity element (point vortex)
$V_{tan} \propto (1/r)$
ζ_{rel} itself is advected by the "induced" flow from all other vortex points
a *point vortex model* of flow and its predictability
Sketch how this plays out for 2 vortices of same/opposite sign

Rossby waves: includes advection of planetary vor (or conservation of absolute vor)
explain from $d/dt(\zeta_a)=0$ with $\beta = df/dy$
Phase velocity $c=U - \beta/k^2$: westward relative to U, long waves faster
Group velocity $c_g=U + \beta/k^2$: eastward relative to U, " " "
"downstream development" process
For stationary waves (c=0), $c_g = 2U$

First Law of Thermodynamics (conservation of microscopic energy)
heat energy added to gas = change in internal energy + work done by gas (p dV/dt)
Per unit mass: $Q = C_v\, dT/dt + p\, d\alpha/dt$
Ideal gas law, an equation of state for air: $p\alpha = RT$
Plug in: $Q = C_p\, dT/dt - \alpha\, dp/dt$
where $C_p = C_v + R$

Mass continuity
Hydrostatic pressure (or mass) coordinate makes it especially clean
$\partial u/\partial x + \partial v/\partial y + \partial \omega/\partial p = 0$
omega = dp/dt is vertical velocity within this coordinate system
notice: *negative for upward motion*
but also "pressure drop with time" on Earth's surface