I First Law of Thermo

Energy added Q = Δ internal energy + work done in expanding

$$Q = C_v \frac{dT}{dt} + p\frac{d\alpha}{dt}$$

$\alpha = \frac{1}{\rho}$ = specific volume

$\boxed{p\alpha = RT \text{ ideal gas law}}$

$$p\frac{d\alpha}{dt} + \alpha\frac{dp}{dt} = R\frac{dT}{dt}$$

$$Q = \underbrace{(C_v + R)}_{C_p}\frac{dT}{dt} - \frac{RT}{p}\frac{dp}{dt}$$

We desire one term on the RHS, not two! How?

Divide by T! the entropy route

$\Theta = T\left(\frac{p_0}{p}\right)^{R/c_p}$ potential temperature

$$\frac{Q}{T} = C_p\underbrace{\left(\frac{1}{T}\frac{dT}{dt}\right)}- R\left(\frac{1}{p}\frac{dp}{dt}\right)$$

$$= C_p\frac{d}{dt}(\ln T) - R\frac{d}{dt}(\ln p) = \frac{d}{dt}\left(\underbrace{C_p \ln\Theta}\right) = \frac{d}{dt}(S)$$

entropy

Is there another trick? 

$$Q = C_p \frac{dT}{dt} - \alpha \frac{dp}{dt}$$

want to combine these.

$$\frac{dp}{dz} = -\rho g$$

For a hydrostatic atmosphere.

$$dp = -\rho g \, dz$$

$$Q = C_p \frac{dT}{dt} + g \frac{dz}{dt} = \frac{d}{dt}\left(C_pT + gz\right) \equiv \frac{d}{dt}(s)$$

constant   constant   dry static energy.

- dry static energy is
  - conserved when $Q=0$ (dry adiabatic flow)
    - just like θ!
- At a given height z (related to p), it measures C_pT or T enthalpy
  - and thus parcel bouyancy $b = g\left(\frac{\rho_{parcel} - \rho_{env}}{\rho_{env}}\right)$ fractional density difference
- A layer of constant s has lapse rate:

$$\frac{\partial s}{\partial z} = 0 = C_p \frac{\partial T}{\partial z} + g \Longrightarrow \boxed{\frac{\partial T}{\partial z} = -\frac{g}{C_p} = \frac{-10K}{km}}$$

Suppose Q is condensation only.

if $Q = -L\frac{dq_v}{dt}$ only, then

$$-L\frac{dq_v}{dt} = \frac{d}{dt}\left(c_pT + gz\right)$$ or (Drum Roll: Conservation!)

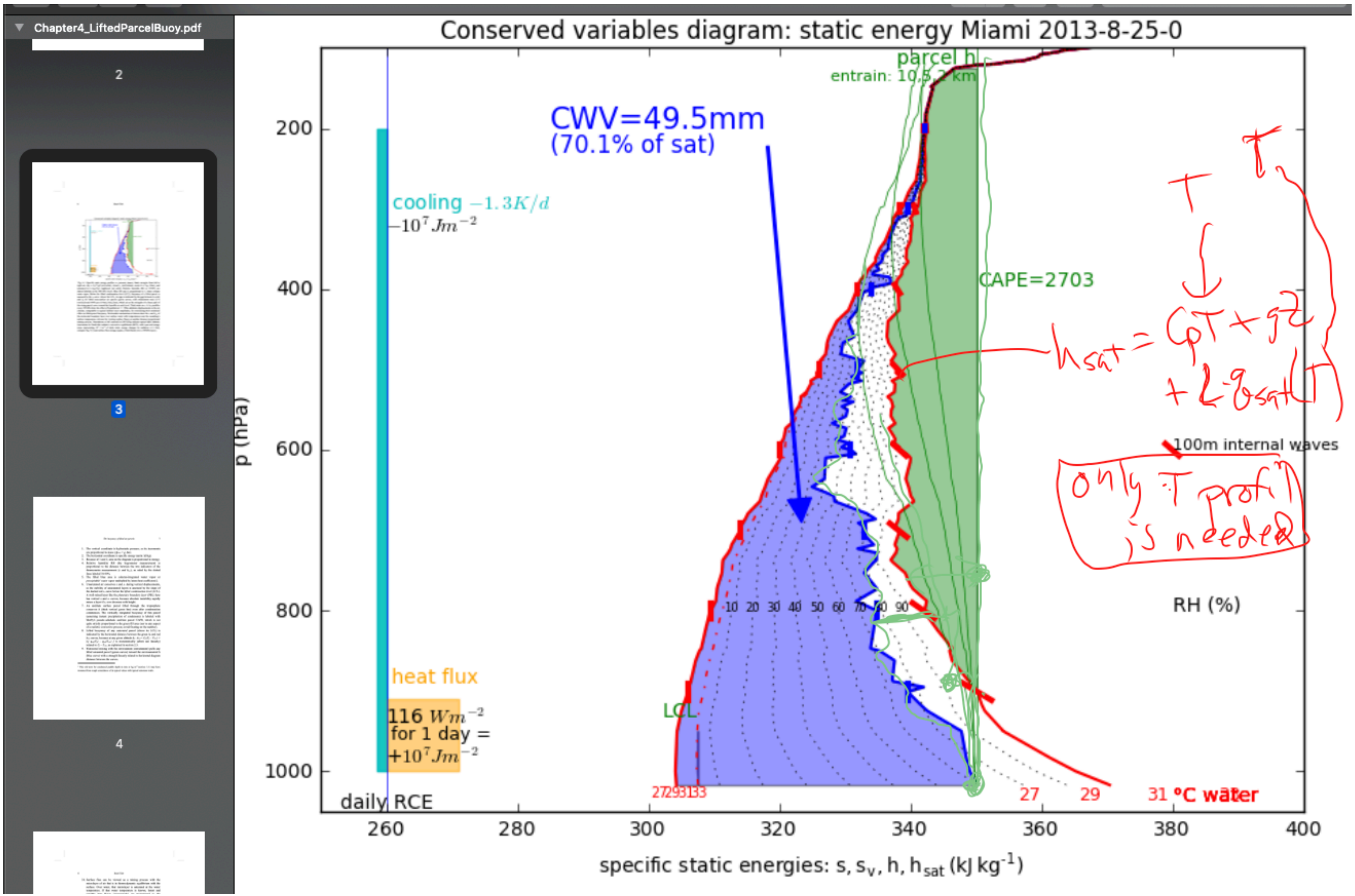
$$\frac{d}{dt}\Big(\underbrace{\underbrace{\underbrace{c_pT}_{\text{enthalpy}} + \underbrace{gz}_{\text{potential energy}}}_{\text{dry static energy}} + \underbrace{Lq_v}_{\text{latent heat}}}_{\text{moist static energy } h}\Big) = 0$$

q_v is $\frac{\text{Kg}_{\text{vapor}}}{\text{Kg}_{\text{air}}}$ (specific humidity)

sometimes just q

h is conserved
- for changes in z (vertical motions)
- even with condensation like θ_e
- conserved when s is (because q is.)

Connection to buoyancy: If $T_{\text{parcel}} > T_{\text{env}}$ and parcel is saturated, then $h_{\text{parcel}} > h_{\text{sat}}(T_{\text{env}})$


Chapter4_LiftedParcelBuoy.pdf
2
3
4
Conserved variables diagram: static energy Miami 2013-8-25-0
parcel h
entrain: 10.5,2 km
CWV=49.5mm
(70.1% of sat)
cooling $-1.3K/d$
$-10^7 Jm^{-2}$
CAPE=2703
100m internal waves
RH (%)
10 20 30 40 50 60 70 80 90
heat flux
116 Wm^{-2}
for 1 day =
$+10^7 Jm^{-2}$
LCL
daily RCE
27 29 31 33
27
29
31 °C water
p (hPa)
200
400
600
800
1000
260
280
300
320
340
360
380
400
specific static energies: s, s_v, h, h_{sat} (kJ kg^{-1})
T
T
$h_{sat} = C_pT + gz + L\,q_{sat}(T)$
only T profile is needed