HW3: Write the PE set in various forms (part 0), then plug in numbers for latent-heated air rising for 6h (problems from 1.)

0. Write the PE set (gathered in section 7.3.5):

a. with the 5 prognostic equations in *local change = tendencies* form, with no vector notation, using u and v and derivatives in the (x,y,p,t) coordinate system to express advection.

b. Pull the u and v equations into a single vector horizontal momentum equation, still using u and v and partial derivative notation for the advection.

c. Compress the momentum equation maximally using vector notation.

d. Write the T equation from part a. in flux form, showing with the chain rule and mass continuity that the convergence of {uT, vT, ωT} is equal to the advection terms you used in a.

e. Integrate the result of d. over the entire atmosphere from a pressure of 0 to 20000 hPa (the base of the ocean mixed layer) to obtain an equation for global warming. What happens to the transport term (expressed in flux form)? Hint: one term remains.

f. Express the continuity equation in both scalar and vector forms.

g. Express the thermodynamic equation and hydrostatic equation in terms of theta θ, the potential temperature, a conversion that uses p (which after all is a coordinate variable, freely invoked without introducing any new unknowns) so that there are still only 5 equations for 5 unknowns.

h. Advection is complicated; another way to treat it is simply to ignore it, and examine weak motions in a resting background state. That is the spirit of the following problem. Write your opening equation in the next problem based on one of your own forms above.

HW: use The Primitive Equations to compute how a local heating J drives flow in an initially motionless atmosphere

$$\frac{D_h T}{Dt} \boxed{} = J/C_p$$

1. J causes T to increase
net change of T = amount of heat added/Cp

$$\frac{\partial \Phi}{\partial p} = -\frac{RT}{p}$$

2. Warmer **T causes increased thickness** of the heated column

$$\frac{D}{Dt}\vec{V}_h = \boxed{} - \vec{\nabla}_p \Phi$$

3. High Φ over hot column pushes wind outward

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} = 0$$

4. Surface pressure drops
(remember, omega = Dp/Dt; Holton eq. 3.44)

HW: use The Primitive Equations to compute how a local heating J drives flow in an initially motionless atmosphere

$$\frac{D}{Dt}\vec{V}_h = \boxed{} - \vec{\nabla}_p \Phi$$

5. Low Φ under hot column pulls wind inward

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} = 0$$

6. Hot air rises (finally!)
ω =~ $\rho g w$

$$\frac{D}{Dt}\vec{V}_h = -f\hat{k} \times \vec{V}_h \boxed{}$$

7. Coriolis force turns inflowing and outflowing air to make round-and-round flow

Homework 6, MSC 405 Spring 2010 – Due Thursday April 13

This problem set walks through the primitive equations for the hot-air-rises process in the class powerpoint , for a 100 km patch of latent heating in an initially motionless atmosphere.

ALL ANSWERS SHOULD CONFIRM/SHOW UNITS. BUT NUMBERS SHOULD BE ROUGH: 1-2 SIGNIFICANT DIGITS. THIS IS A CRUDE ESTIMATION EXERCISE, DON'T COMPLICATE IT BY BEING FUSSY WITH THE DIGITS BEYOND THE DECIMAL PLACE.

1. Suppose the heating J corresponds to the latent heat released during 1cm of rain over a 100km x 100km area. Let's figure out how much ΔT that causes.

> a. What is the volume of water condensed (m^3)?
>
> b. What is the mass of water condensed (kg)?
>
> c. How much latent heat is released (Joules)? ($L = 2.5 \times 10^6$ J / kg of water).
>
> d. How many kg of air are in a 1 square meter column of atmosphere, if surface pressure p is 1000mb? (pressure is weight, so mass=p/g)
>
> e. How many kg of air are therefore in a 100km x 100km column?
>
> f. How many K can the heat from c. warm the air mass from e.? Cp ~ 1000 J/(kg K)

2. Moving to the hydrostatic equation now: If the ΔT result from 1f applies to the 1000-200mb layer, roughly what $\Delta\Phi$ is generated at 200mb if the height (geopotential) of the 1000mb surface is unchanged. That is, surface pressure is still constant in this initially motionless atmosphere - the heated column just gets thicker (taller). You can use a crude form of the powerpoint's hydrostatic equation with a layer-mean p on the right: $\Delta\Phi/(-800\text{mb}) = -RT/(600\text{mb})$

3. On to the momentum equation.

> a. If the upper-level High from 2 has horizontal gradients on a scale of Δx ~100 km, estimate the magnitude of the outward pressure gradient force ($\nabla\Phi = \Delta\Phi/\Delta x$) at 200mb.
>
> b. Express result 3a in m/s per hour.

4. Computing the surface pressure drop as mass evacuates the column

a. If the PGF $-\Delta\Phi/\Delta x$ from 3 acts for 100s , how great a wind speed away from the High will develop? ($\Delta u/\Delta t = -\Delta\Phi/\Delta x$)

b. If this horizontal wind speed is directed away from the 200mb High in all directions, estimate the horizontal wind divergence ($\Delta u/\Delta x + \Delta v /\Delta y$).

c. If this horizontal divergence from 4b. prevails over the upper half of the troposphere (Δp = 400mb = the 600-200mb layer), estimate the rate of change of surface pressure ($\Delta p/\Delta t = \omega$), using the mass continuity equation. Put it into 'weather' pressure drop units: mb per hour.

5. Low-level inflow driven by surface pressure drop, and midlevel rising motion

a. How long must the rate of surface pressure decrease in 4c. act before the depression of the 1000mb surface $\Delta\Phi_{1000}$ forms a Low that equal in strength to the upper level High the upward bulge in the 200mb surface $\Delta\Phi_{200}$ from 2)? Note that 1mb in surface pressure is about equal to 10m in Z_{1000}.

b. After the time scale computed in (a), the upper-level divergence will be matched by low-level convergence, with upward motion in the middle troposphere. Estimate the vertical *velocity* (in m/s) in the middle troposphere using the mass continuity equation. (Hint: ω at 600mb is the same as the answer to 4c, can you see and explain why? Now you just need to convert that to m/s using the hydrostatic relation, answer is ~20m = 1mb in the middle troposphere.)

6. Spinup of low-level inflow by Coriolis

Suppose the low-level inflowing velocity u is about equal to the upper-level outflowing velocity computed in 4a. Suppose the Coriolis force act on it for 20000s (a few hours, a typical lifetime of such a 100km sized rain storm) How big a tangential velocity v is generated at the end of the 6h?