Reviewing,

$$\left\{\begin{array}{l} \frac{d}{dt}u = -\frac{\partial \Phi}{\partial x}\Big|_p + fv \cancel{+F_x} \\ \frac{d}{dt}v = -\frac{\partial \Phi}{\partial y}\Big|_p - fu \cancel{+F_y} \end{array}\right. \Rightarrow$$

descriptive (no time derivative)

$$0 = -\frac{\partial \Phi}{\partial x}\Big|_p + fv_g$$

$$0 = -\frac{\partial \Phi}{\partial y}\Big|_p - fu_g$$

not assumed exactly.. it defines this

geostrophic part of the total wind field.

"80%" - good approx.

prediction eqs. through conservation

take curl

$\Downarrow$

$$\frac{d\zeta}{dt} = 0 + ..$$

vortical flow advects vorticity

$$v\frac{df}{dy} = v\beta$$

$$\frac{d(\zeta + f)}{dt} = 0$$

same, but also Rossby waves

- westward phase speed
- eastward group speed
- proportional to L^2

$$\frac{dw}{dt} = -\frac{\partial p}{\partial z}\Big/\rho + g \approx 0$$

hydrostatic approx.

Our prediction through cons. eqs. project continues!

$$\frac{d}{dt}(\zeta+f)=0-\underbrace{(\zeta+f)\cdot\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)}_{\substack{=\mathrm{Div}_h V\\ =(\nabla_h\cdot v)\\ \text{small @ 500mb}}}+\text{terms involving } w \text{ or } \omega \text{ (vertical motion)}$$

$$\Downarrow$$

$$\underbrace{\frac{d}{dt}(P.V.)}_{\text{transport only}}=0+C(\zeta+f)\cdot\frac{\partial}{\partial z}(Q)+D\cdot(\text{curl of friction})$$

heating — friction

driver — damper

shapers of the flow

But what is P.V.?

A: It involves θ "potential temperature"

$\frac{d\theta}{dt}=0!$ ← this again!

Potential temperature: the T a parcel of air would have if it were compressed adiabatically to 1000mb $= 10^5$ Pa.

Comes from 1st Law of Thermo: energy is conserved.

internal + external

$$\frac{d}{dt}\left(\begin{matrix}\text{internal}\\ \text{energy}\end{matrix}\right) = \left(\begin{matrix}\text{work done}\\ \text{on env.}\end{matrix}\right) + Q$$

Q ← energy added (zero for adiabatic processes)

$$\frac{d}{dt}\left(C_v T\right) = p\frac{dv}{dt} + Q$$

v: volume per unit mass

C_v: $(J/kg)/K$

ideal gas law (equation of state)

$$\boxed{p\alpha = RT}$$

$\alpha = \frac{1}{\rho}$ "specific volume"

take d/dt.

$$\frac{d}{dt}(p\alpha) = \frac{d}{dt}(RT) = R\frac{dT}{dt}$$

$$p\frac{d\alpha}{dt} + \alpha\frac{dp}{dt} = R\frac{dT}{dt}$$

$$\frac{d}{dt}\left(C_v T\right) = p\frac{d\alpha}{dt} + Q$$

$$\boxed{\frac{d}{dt}\left(c_p T\right) = -\alpha\frac{dp}{dt} + Q}$$

$c_p \equiv (c_v + R)$

Recap of 1^{st} Law in form we like:

$C_p = C_v + R = \underline{\text{const}}$

$$\frac{d}{dt}(c_p T) = \alpha \frac{dp}{dt} + Q \qquad \alpha = \frac{R}{RT}$$

Ideal gas law again

$$c_p \frac{dT}{dt} = \left(\frac{RT}{p}\right)\frac{dp}{dt} + Q$$

Trick: divide by T!

$$\frac{d}{dt}\Big(\underset{\Theta}{\text{something}}\Big) = 0 + Q \ldots$$

how to get here?

$$c_p\left(\frac{1}{T}\frac{dT}{dt}\right) = R\left(\frac{1}{p}\frac{dp}{dt}\right) + \frac{Q}{T}$$

$$c_p \frac{d}{dt}(\ln T) = R\frac{d}{dt}(\ln p) + \frac{Q}{T}$$

$$\boxed{c_p \frac{d}{dt}(\Theta) = 0 + Q\left(\frac{p_0}{p}\right)^{R/c_p}} \qquad \Theta \equiv T\left(\frac{p_0}{p}\right)^{R/c_p}$$

note: $\left(\frac{p_0}{p}\right)^{R/c_p} = \frac{\Theta}{T}$