HW2 solutions

Collect (midnight) HW2: Force-balanced wind problems 7.19, 7.21, 7.22, 7.24, 7.42; Thermal wind problems 7.5m, 7.11, 7.25, 7.26

7.5m: (m) Veering of the wind with height within the planetary boundary layer is not necessarily an indication of warm advection. Explain.
FRICTIONAL BALANCE BECOMES GEOSTROPHIC ABOVE THE PBL: THAT IS VEERING

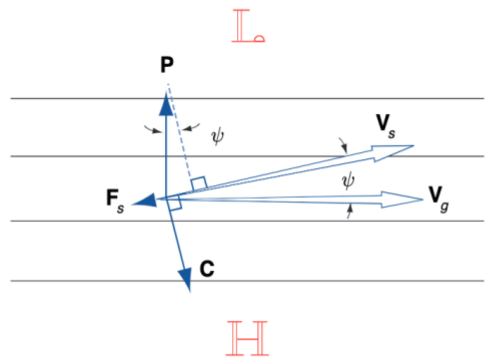


Fig. 7.10 The three-way balance of forces required for

7.11: Initial conditions are ψ = -my, decreasing linearly with latitude like the color in Fig. 7.4a.

For a conserved tracer, using subscripts for partial derivatives, $\psi_t = -u\psi_x - v\psi_y$. In words, *local change is purely due to advection*.

Taking the x partial derivative, using the chain rule, and adding parentheses for clarity:

$$\psi_{xt} = -u(\psi_x)_x - v(\psi_x)_y - u_x\psi_x - v_x\psi_y$$

The red terms are *advection of ψ_x*. Taking those terms into the left hand side to form the total d/dt,

$$d/dt\,(\psi_x) = -u_x\psi_x - v_x\psi_y$$

which for the given initial condition is equal to v_xm. (*sign error in book??*)

Taking the y partial derivative:

$$\psi_{xt} = -u(\psi_y)_x - v(\psi_y)_y - u_y\psi_x - v_y\psi_y$$

The red terms are *advection of ψ_y*. Taking those terms into the left hand side to form the total d/dt,

$$d/dt\,(\psi_y) = -u_y\psi_x - v_y\psi_y$$

which for the given initial condition is equal to v_ym.

Interpretation: The change in the *initially purely meridional gradient*, moving with the flow (total derivative), is a *twisting into the zonal direction* by u_y and a *squeezing or contraction in the meridional direction* by v_y. (b) For pure deformation with v_y constant, d/dt (ψ_y) = const*ψ_y clearly has exponential solutions. For shear flow with constant v_x, d/dt (ψ_x) = v_xm = const is linear growth.

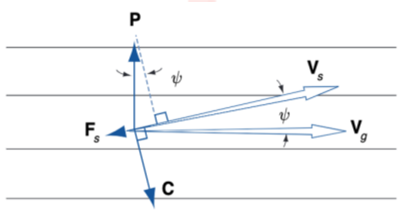
7.19: Geostrophic wind, just formula with units: 20 m/s, everyone got it.

7.21: This is subtle because instead of d/dt being the rate of change *following the wind u,v,* we use the same notation for the rate of change *following a moving ship.* It's just the chain rule, recall: with p(x,y,t), then if $x_{ship}(t),y_{ship}(t)$ is the position of *any frame of reference we want the result in*, the chain rule gives:

$$dp/dt = p_t + p_x\, dx/dt + p_x\, dy/dt = p_t + p_x\, u_{ship} + p_x\, v_{ship}$$

Everyone plugged the formulas right, given right answers from the Web, but in the old wary undergrad game of writing the least and hoping the grader can't prove whether there is confusion, I wasn't sure if this was clear to all.

7.22: windspeed is 10 m/s and is directed across the isobars from high toward low pressure at an angle 20°. Calculate the magnitude of the frictional drag force and the horizontal pressure gradient force. *Again use geometry on the diagram, and the sohcahtoa rule. Why was it tan() and cot()*? Which lengths are equal on the diagram? This seems like a geometric proof, not a given-formula number-crunch.



Balance: 0 = P + C + F
In x direction: 0 = 0 + fv - ku where k is the friction koefficient
In y direction: 0 = P - fu - kv
We are given that $u^2 + v^2 = 100\ (m/s)^2$ and that $\tan(20^o) = v/u$. From those 4 equations, can solve for P, k, u, v. Problem asks for P and for the frictional force k{u,v}.

7.24: Seems easy, everyone got it. Gradient wind formula must have a real square root.

7.25: Everyone could write Vg equation and take p or z derivative. The hard part was the latter, which required this key substitution: (Paige)

hydrostatic

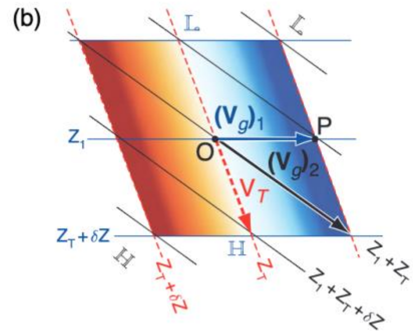
$$\frac{\partial p}{\partial z} = -\rho g = \frac{pg}{RT} \qquad \frac{\partial V_g}{\partial z} = -\frac{g}{f\rho} k \times \nabla p + \frac{1}{f} k \times \nabla p$$

$$\boxed{\frac{\partial \alpha}{\alpha} = \frac{dT}{T} - \frac{dp}{p}} \text{ key} \qquad = -\frac{g}{f\rho} k \times \nabla p + \frac{1}{\alpha}\frac{\partial \alpha}{\partial z}\left(\frac{1}{f} k \times \alpha \nabla p\right)$$

$$\frac{\partial V_g}{\partial z} = \frac{g}{fT} k \times \nabla T + \frac{1}{T}\frac{\partial T}{\partial z} V_g$$

7.26 (a) Prove that the geostrophic temperature advection in a thin layer of the atmosphere (i.e., the rate of change of temperature due to the horizontal advection of temperature) is given by

$$\frac{f}{R \ln (p_B/p_T)} V_{gB} V_{gT} \sin \theta$$



7.26:

Advection is the dot product $-V\cdot\nabla T$. That involves the *cosine* of the angle between V_{gB} and ∇T, times the strength of each vector. Since V_T is perpendicular to ∇T, it is the *sine* of the angle between V_{gB} and V_T.

Notice that the thermal wind does not advect temperature (it is parallel to isotherms), so that the advection at levels 1 and 2 are equal: $-V_{g2}\cdot\nabla T = -(V_{g1} + V_T)\cdot\nabla T = -V_{g1}\cdot\nabla T$

7.42: the concept of propagation is the *tendency leading the anomaly by a quarter-wavelength* (or 90 degrees of phase). With two fields (vectors, contours), propagation involves showing that *both fields progress in the same direction*: vectors change by the PGF (acceleration), and heights change by the divergence or convergence of mass.