

HW: use The Primitive Equations to compute how a local heating J drives flow in an initially motionless atmosphere

$$\frac{D_h T}{Dt} \boxed{} = J/C_p$$

1. J causes T to increase
 net change of T = amount of heat added/ C_p

$$\frac{\partial \Phi}{\partial p} = -\frac{RT}{p}$$

2. Warmer T causes increased thickness of the heated column

$$\frac{D}{Dt} \vec{V}_h = \boxed{} - \vec{\nabla}_p \Phi$$

3. High Φ over hot column pushes wind outward

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} = 0$$

4. Surface pressure drops
 (remember, $\omega = Dp/Dt$; Holton eq. 3.44)

HW: use The Primitive Equations to compute how a local heating J drives flow in an initially motionless atmosphere

$$\frac{D}{Dt} \vec{V}_h = \boxed{} - \vec{\nabla}_p \Phi$$

5. Low Φ under hot column pulls wind inward

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} = 0$$

6. Hot air rises (finally!)
 $\omega \approx -\rho g w$

$$\frac{D}{Dt} \vec{V}_h = -f \hat{k} \times \vec{V}_h \boxed{}$$

7. Coriolis force turns inflowing and outflowing air to make round-and-round flow

Homework 6, MSC 405 Spring 2010 – Due Thursday April 13

This problem set walks through the primitive equations for the hot-air-rises process in the class powerpoint, for a 100 km patch of latent heating in an initially motionless atmosphere.

ALL ANSWERS SHOULD CONFIRM/SHOW UNITS. BUT NUMBERS SHOULD BE ROUGH: 1-2 SIGNIFICANT DIGITS. THIS IS A CRUDE ESTIMATION EXERCISE, DON'T COMPLICATE IT BY BEING FUSSY WITH THE DIGITS BEYOND THE DECIMAL PLACE.

1. Suppose the heating J corresponds to the latent heat released during 1cm of rain over a 100km x 100km area. Let's figure out how much ΔT that causes.

- What is the volume of water condensed (m^3)?
- What is the mass of water condensed (kg)?
- How much latent heat is released (Joules)? ($L = 2.5 \times 10^6 \text{ J / kg of water}$).
- How many kg of air are in a 1 square meter column of atmosphere, if surface pressure p is 1000mb? (pressure is weight, so $\text{mass} = p/g$)
- How many kg of air are therefore in a 100km x 100km column?
- How many K can the heat from c. warm the air mass from e.? $C_p \sim 1000 \text{ J/(kg K)}$

2. Moving to the hydrostatic equation now: If the ΔT result from 1f applies to the 1000-200mb layer, roughly what $\Delta\Phi$ is generated at 200mb if the height (geopotential) of the 1000mb surface is unchanged. That is, surface pressure is still constant in this initially motionless atmosphere - the heated column just gets thicker (taller). You can use a crude form of the powerpoint's hydrostatic equation with a layer-mean p on the right: $\Delta\Phi/(-800\text{mb}) = -RT/(600\text{mb})$

3. On to the momentum equation.

- If the upper-level High from 2 has horizontal gradients on a scale of $\Delta x \sim 100 \text{ km}$, estimate the magnitude of the outward pressure gradient force ($\nabla\Phi = \Delta\Phi/\Delta x$) at 200mb.
- Express result 3a in m/s per hour.

4. Computing the surface pressure drop as mass evacuates the column

- If the PGF $-\Delta\Phi/\Delta x$ from 3 acts for 100s, how great a wind speed away from the High will develop? ($\Delta u/\Delta t = -\Delta\Phi/\Delta x$)
- If this horizontal wind speed is directed away from the 200mb High in all directions, estimate the horizontal wind divergence ($\Delta u/\Delta x + \Delta v/\Delta y$).
- If this horizontal divergence from 4b. prevails over the upper half of the troposphere ($\Delta p = 400\text{mb} = \text{the } 600\text{-}200\text{mb layer}$), estimate the rate of change of surface pressure ($\Delta p/\Delta t = \omega$), using the mass continuity equation. Put it into 'weather' pressure drop units: mb per hour.

5. Low-level inflow driven by surface pressure drop, and midlevel rising motion

- How long must the rate of surface pressure decrease in 4c. act before the depression of the 1000mb surface $\Delta\Phi_{1000}$ forms a Low that equal in strength to the upper level High the upward bulge in the 200mb surface $\Delta\Phi_{200}$ from 2)? Note that 1mb in surface pressure is about equal to 10m in Z_{1000} .
- After the time scale computed in (a), the upper-level divergence will be matched by low-level convergence, with upward motion in the middle troposphere. Estimate the vertical *velocity* (in m/s) in the middle troposphere using the mass continuity equation. (Hint: ω at 600mb is the same as the answer to 4c, can you see and explain why? Now you just need to convert that to m/s using the hydrostatic relation, answer is $\sim 20\text{m} = 1\text{mb}$ in the middle troposphere.)

6. Spinup of low-level inflow by Coriolis

Suppose the low-level inflowing velocity u is about equal to the upper-level outflowing velocity computed in 4a. Suppose the Coriolis force act on it for 20000s (a few hours, a typical lifetime of such a 100km sized rain storm) How big a tangential velocity v is generated at the end of the 6h?

