Chapter 2 in MSM presents quasigeostrophic theory. Here, we provide an accompanying set of exercises emphasizing the application of QG material in the interpretation and prediction of midlatitude weather systems.

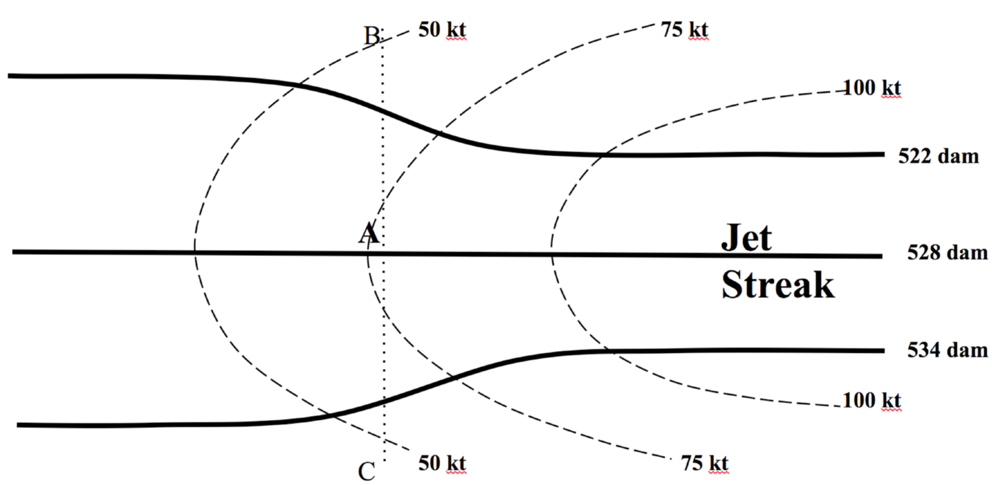
2.1. Conceptual View of QG Omega: Forcing and Response

The QG omega equation is shown below. It states that *forcing for vertical motion* is related to the *differential vorticity advection* and *the Laplacian of the temperature advection*. For both advections, the approximation is made that the geostrophic wind is a good enough approximation for the advecting velocity. Recall that the change in geopotential Φ with pressure in the second term (the thickness dZ per unit mass layer dp) is a measure of layer-averaged temperature. Recall also that the Laplacian (∇^2) of the geopotential in the first term is proportional to geostrophic relative vorticity (with some constants). These ideas may help you to recognize the terms in the omega equation.

$$\left(\nabla^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega = \frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\left[\vec{V}_g \cdot \nabla\left(\frac{1}{f_0}\nabla^2\Phi + f\right)\right] + \frac{1}{\sigma}\nabla^2\left[\vec{V}_g \cdot \nabla\left(-\frac{\partial \Phi}{\partial p}\right)\right] \qquad (2.29)$$

Our objective in this lesson is to understand this diagnostic relationship. In other words, *why* does cyclonic vorticity advection increasing with height, or a horizontal maximum of warm advection, represent forcing for ascent?

This example is inspired by the Durran and Snellman (1987) paper. Consider an idealized jet streak situation. Suppose the 1000-hPa geopotential height surface $Z_{1000} = 0$ everywhere (i.e., the 1000-hPa surface is flat, with no geostrophic wind at that level). On the following diagram, the 500-hPa height contours are shown as bold solid lines, and the 500-hPa isotachs are shown as dashed lines. The cross section B-C is marked for later reference.



a) From the QG momentum equation [Eq. (2.9) in MSM], the local tendency of the geostrophic wind speed is in part *due to geostrophic advection* [evident if one expands the total derivative on the left side of Eq. (2.9)]. Similarly, we could expand the total derivative on the left side of the QG zonal momentum equation [Eq. (2.14)]. **What is the sign of the term at point A?**

$$-u_g \frac{\partial u_g}{\partial x}$$

How would this term tend to change the strength of the vertical wind shear with time (increase or decrease) at point A?

b) Next, consider the *geostrophic advection of temperature* in the lower troposphere. **Is there warm, cold, or neutral advection (+, –, or 0)? Why?**

Explain: **What information do we have about *temperature* on this diagram? What is the thermal wind in the 500–1000-hPa layer? If thickness and height contours are parallel, what does that imply about temperature (thickness) advection?**

c) Recall that thermal wind balance relates the vertical shear of the geostrophic wind to the horizontal gradient of temperature, at every instant in time. Here,

$$u_{g\,U} - u_{g\,L} = -\frac{C}{f}\frac{\partial \overline{T}_v}{\partial y}$$

This is Eq. (1.44) in MSM. Here, the subscripts "U" and "L" correspond to "upper" and "lower" levels, and "C" is a constant for a given pressure layer. **Would thermal wind balance be sustained in locations such as point A as the advective tendencies from a) and b) act together? Why or why not?**

d) If the answer to c) is "no," **explain the sense of the imbalance** that would develop at point A. In other words, would the vertical shear of the westerly flow become too weak for the north–south temperature gradient, or vice versa?

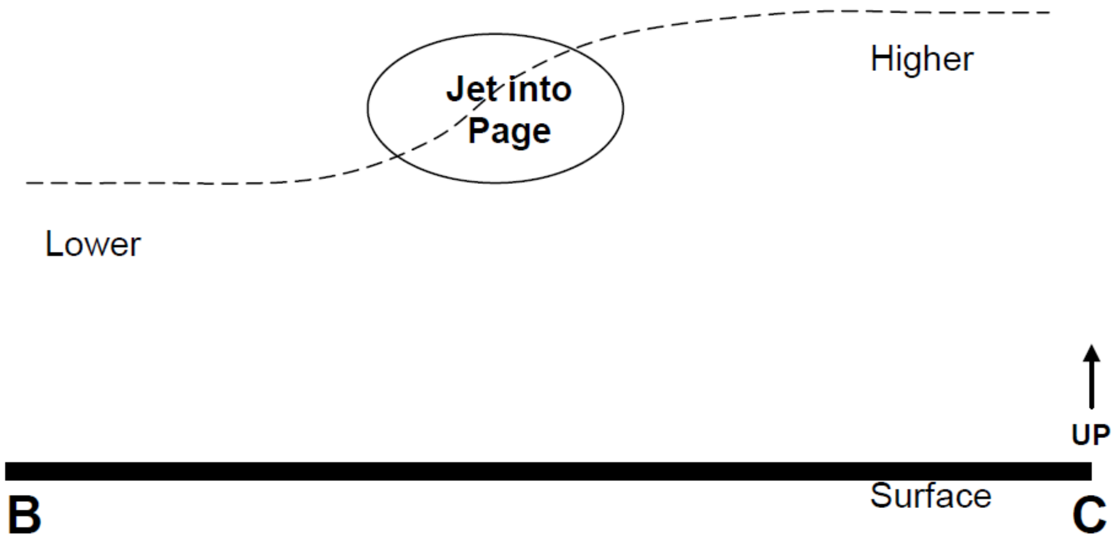
e) **Based on your answer to d), what would need to happen in order to bring the atmosphere back toward thermal wind balance in the vicinity of point A?**

The magnitude of the temperature gradient would need to ______________

AND/OR

The magnitude of vertical wind shear would need to ______________

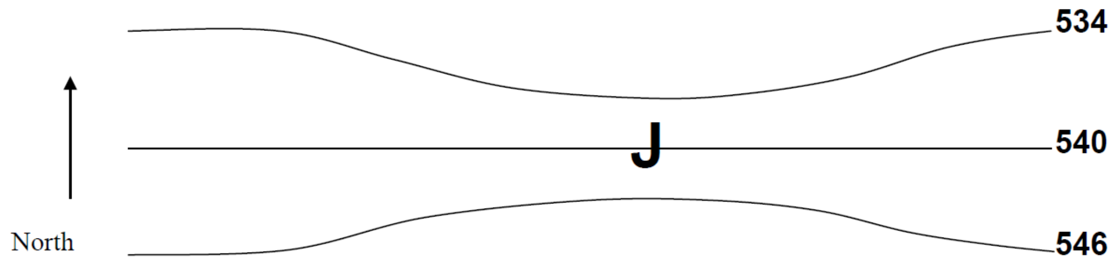
f) What sort of vertical circulation (in this case, in the *y–z* cross-sectional plane) would be required to bring about the needed changes (that is, to restore or maintain thermal wind balance)? Remember, a "circulation" needs to satisfy mass continuity. Consider the vertical cross section below corresponding to the B-C cross section above. The perspective is from the west (i.e., you are standing with your back to the wind, looking eastward).



Sketch the ageostrophic circulation (using vertical and horizontal arrows) that would be needed to bring the atmosphere back towards thermal wind balance after advection has acted for a moment to disrupt that balance. Hint: One useful approach is to think first of how the horizontal wind would need to change with time. Recall from Eq. (2.14) in the text that $du_g/dt_g = fv_{ag}$ (the Coriolis force acting on the ageostrophic wind), so you can deduce what v_{ag} is needed. From v_{ag}, mass continuity then implies a vertical motion to close the circulation.

g) **Does the adiabatic cooling or warming effect of your deduced vertical motion help or hinder the action of fv_{ag} on the momentum field, in the overall two-part effort [as mentioned in e) above] to adjust the situation back to thermal wind balance?**

h) On the jet streak diagram below, **sketch a few vorticity contours, and label regions of cyclonic (use a C) and anticyclonic (use an A) shear vorticity.** Consider the wind orientation, based on the height contours. Combining the wind and vorticity information, **indicate areas of positive and negative vorticity advection.** Given that wind speed is increasing with height in this example, and therefore so is the strength of the advection, **where would you expect rising and sinking motion at the 700-hPa level (based on the first term in the traditional form of the QG omega equation)? Label this area on the diagram**.



i) **Are the areas of ascent and descent you deduced above in a–f for the jet entrance, arising from the *thermal advection* term of Eq. (2.29), consistent with the ascent and descent you deduced in h) from the *vorticity advection* term? In other words, do the two forcing terms work in the same sense, or the opposite sense**, for this case involving self-advection of a geostrophically balanced jet streak?

j) **Optional:** Based on your thinking from the above, **can you imagine a geostrophically balanced flow situation in which the two terms in Eq. (2.29) cancel each other, so that no net QG omega forcing is present?**

2.2. The Traditional QG Omega Equation: Forcing for Ascent during a Winter Storm

The objectives of this exercise are to examine QG forcing mechanisms during a winter storm event and relate these to patterns of vertical motion and cloud cover. For familiarity, we will utilize the same case event from the thermal wind exercise in section 1.3.

Open the LMT_2.2 bundle. You should see displays of `Sea Level Pressure`, `Geopotential height`, and `Wind barbs` at 500 hPa. A magenta square shows the location of a movable `Sounding` probe.

a) Thermal advection at 1200 UTC 18 December 2009

   i. Consider the cyclone in the northern Gulf of Mexico. Based on the angle between the sea-level isobars and 500-hPa height contours, **where do you expect the most pronounced temperature advection at this time? In which areas would the QG omega equation predict forcing for ascent, if we were to base our analysis only on this term? Capture and annotate images to illustrate your answers.** Recall that the equation includes the *Laplacian of* the temperature advection field, so it is not just the *value* of temperature advection itself that is critical, but rather *local maxima* (where the Laplacian is negative) *or minima* (where the Laplacian is positive).

ii. Activate the `QG Temp Adv` display. **Was your intuition correct? Discuss. Use the sounding probe to confirm wind veering vs. backing in the warm vs. cold advection areas, as in exercise 1.3.**

b) Differential vorticity advection

The vorticity advection forcing term is mainly contributed by *upper-level* (500 hPa) vorticity advection, since winds are weaker at low levels.

i. Turn off the `QG Temp Adv` display. Select the `Absolute Vorticity` field in the legend and notice its relationship to the troughs and ridges in geopotential height. **Based on the wind and vorticity fields, where do you expect the most prominent areas of cyclonic vorticity advection? Where are these areas in relation to the locations with significant warm advection?**

ii. Activate the `Geo Avor Advection` display (it is under the `QG diagnostics` category in the legend). **Are the regions of cyclonic vorticity advection consistent with what you expected? Capture images of the comparison between the two QG advection terms, and discuss them.** Since Eq. (2.29) states that the *Laplacian* of omega equals the forcing terms on the RHS, and inverting the Laplacian acts like a smoothing operation, let's smooth the forcing term. Click the blue `Geo Avor Advection` hyperlink in the legend and change the smoothing parameter to a factor of 5. Is this a better fit to the scales of true omega (ascent) suggested by the IR satellite imagery? To answer this, toggle the `IR satellite display` on and off to see the observed cloud tops field. **Capture images to illustrate your assessment. Discuss what aspects of the IR cloudiness display (convective clouds vs. cirrus decks) might be most related to vertical velocity below the 500-hPa level where the vorticity advection term was evaluated.**

c) Vertical profiles at 1200 UTC 18 December 2009

i. Consider the skew-*T* profile in the separate `Sounding_wind` window. Recall that we examined vertical profiles for this case in exercise 1.3a, for geographical reference. Place the skew-*T* marker in the `Map View` window over Louisiana. **Do you expect QG forcing for ascent or descent in this location at this time? Discuss and justify your answer in terms of the two QG forcing terms.**

ii. **Is the midtropospheric dewpoint depression (gap between temperature and dewpoint curves) consistent with the omega implied by the QG forcing terms in (i) over central Louisiana? Discuss.**

iii. Now, move the sounding location to southern Georgia. **Do you expect QG forcing for ascent or descent in this location at this time? Discuss and justify your answer in terms of the two QG forcing terms.**

iv. **Are the midtropospheric temperature and dewpoint profiles (or the dewpoint depression) consistent with the omega implied by the QG forcing terms over southern Georgia? Discuss.**

d) Analyzed omega

The *actual analyzed omega* can differ substantially from the QG omega whose Laplacian equals the forcing terms according to Eq. (2.29) shown at the beginning of section 2.1. Discrepancies can occur because of all the non-QG terms in the equations, including topographic upslope or downslope flow, convection, and other components of the ageostrophic flow field.

In the `Map View` window, toggle on the temperature advection (`QG Temp Adv`), vorticity advection (`Geo Avor Advection`), and the 700-hPa omega field (`Omega - Color-Shaded`) to explore the relationships among the three patterns. **How well does the analyzed region of ascent match the location of QG forcing for ascent diagnosed in a) and b)? Does the analyzed omega better match the vorticity advection term or the temperature advection term? Discuss.**

e) Infrared satellite imagery

Does the `IR satellite imagery` display agree qualitatively with the analyzed omega, QG-predicted omega, the individual QG omega forcing terms, and the sounding's dewpoint depression? Discuss.

f) **Optional: Examine the same set of displays for another weather situation and answer the questions above again.** To do this for a historical case in 1979–2015, use bundle `LMT_2.2_MERRA_1979-2015`. For real time data, use `LMT_2.2_RT`. In either case, to view another area of the globe, zoom out until your desired region is visible and then hold the `Shift` key while rubberbanding a latitude–longitude box with the left mouse button depressed. Then, to adjust the time(s) desired, click the  icon in the animation control area and operate the `Define Animation Times` tab in the popup, reading menu items carefully to avoid excessively large data requests. Unfortunately, the sounding probe will not automatically follow the other displays for IDV versions before 5.3. In that case, you have to zoom out after the data finish loading and move the probe to the area in which your data have been fetched. *If you find a* *case you like, you can save it as a .zidv bundle, and feel free to contact an author of this manual if you wish to share it more broadly.*

g) **Optional:** Explore the excellent widget illustrating all the quantities and terms in the traditional QG omega equation at http://www.meted.ucar.edu/bom/qgoe/qgoe_widget.htm. It is free, but each student will need to register. An instructor's email can be set for quiz score delivery.

We gratefully acknowledge Prof. Jim Steenburgh, University of Utah, for a masterly IDV bundle used in developing aspects of this lesson.

2.3. Q-Vector In-Class Exercise

The objectives of this short lesson are to allow students to compute **Q** vectors for a simple flow, and connect the convergence or divergence of these vectors to forcing terms in the traditional QG omega equation.

The QG omega equation is a tool for diagnosis of the processes that give rise to vertical air motions on the synoptic scale. The "traditional" form of the QG omega equation utilized in the previous exercise is not always well suited for operational weather forecasting. As Durran and Snellman (1987) demonstrated, cancellation can occur between the right-side terms in the traditional omega equation. When the two terms on the right-hand side of the traditional form of the QG omega equation oppose each other, forecasters using that equation as the basis for their interpretation are faced with the difficult challenge of deciding which term is larger! Instead, it is advantageous to combine the right-side terms, as shown by Hoskins and Pedder (1980), to obtain the "**Q** vector" form of the QG omega equation:

$$\left(\nabla^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega = -2\nabla\cdot\vec{Q}, \qquad (2.30)$$

where

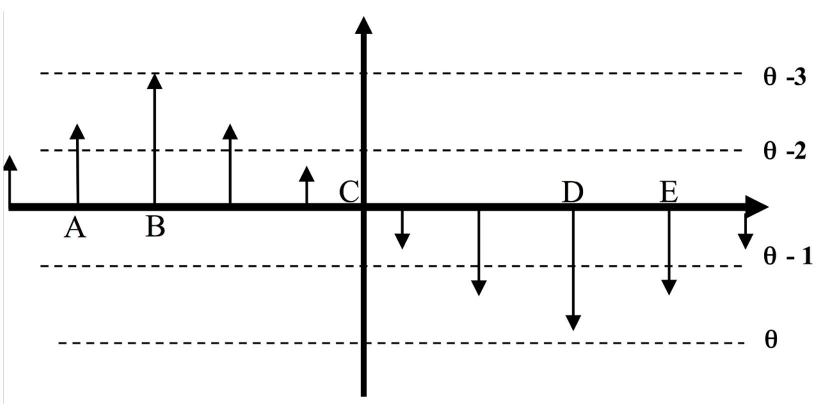
$$\vec{Q} = -\frac{R}{\sigma p}\begin{bmatrix} \dfrac{\partial \vec{V}_g}{\partial x}\cdot\nabla\theta \\ \dfrac{\partial \vec{V}_g}{\partial y}\cdot\nabla\theta \end{bmatrix} = \begin{pmatrix} Q_i \\ Q_j \end{pmatrix} = -\frac{R}{\sigma p}\begin{bmatrix} \dfrac{\partial u_g}{\partial x}\dfrac{\partial\theta}{\partial x} + \dfrac{\partial v_g}{\partial x}\dfrac{\partial\theta}{\partial y} \\ \dfrac{\partial u_g}{\partial y}\dfrac{\partial\theta}{\partial x} + \dfrac{\partial v_g}{\partial y}\dfrac{\partial\theta}{\partial y} \end{bmatrix} \qquad (2.31)$$

Recall that the QG vertical motion is a response to thermal wind balance disruption. It should therefore make sense that the "forcing" for vertical motion on the right side of the omega equation is related to gradients of wind and temperature, evident in the **Q** vector. In fact, the **Q**-vector form of the omega equation is derived by examining the difference between equations that expressed the two different components of the thermal wind relation. This difference reflects advective tendencies that would tend to disrupt balance, and it acts as a "forcing" for circulations that would act to push the atmosphere back towards a state of thermal wind balance.

In Eq. (2.31), the two components correspond to east–west- and north–south-oriented vector components. By evaluating the derivatives and products, we can determine the orientation of **Q** and ultimately locate regions of converging and diverging **Q**, which are clearly of meteorological interest from Eq. (2.30).

In order to illustrate **Q** vectors in a simplified idealized setting, consider the diagram below.

- Define the x axis to be parallel to isentropes, with cold values to the north ($\partial\theta/\partial y < 0$).
- Note that $\partial\theta/\partial x = 0$ in this idealized example (but not in general).
- Assume that v_g at some level varies sinusoidally as indicated in the diagram below.
- For simplicity, ignore the factor $-R/\sigma p$ in the **Q** expression [Eq. (2.31)]—but notice that it is negative.
- **Evaluate Q at each of the indicated points A–E, using "finite difference" techniques.**



$$Q_i = -\frac{\partial u_g}{\partial x}\frac{\partial \theta}{\partial x} - \frac{\partial v_g}{\partial x}\frac{\partial \theta}{\partial y}$$

$$Q_j = -\frac{\partial u_g}{\partial y}\frac{\partial \theta}{\partial x} - \frac{\partial v_g}{\partial y}\frac{\partial \theta}{\partial y}$$

a) **Where would you find ascent? Is this where you would expect it based on the traditional form of the QG omega equation? Explain and discuss.**

b) **Optional: Sketch a different situation, for example involving *westerly* winds and isentropes at some angle,** a more realistic situation than that depicted above. It will help you to simplify the math if you still define your x and y axes so that $\partial\theta/\partial x = 0$, rather than using x as east and y as north. To do this, **sketch a realistic weather situation, then rotate the paper and add the axes** to think about the terms. **Evaluate the Q vectors, and their convergence**.

2.4. Q Vectors Displayed Using Case-Study Data

This exercise examines and interprets **Q** vectors and their divergence at the 700-hPa level, again focusing on the December 2009 case used in exercise 2.2. The objective is to compare and understand how the traditional QG forcing terms relate to the **Q**-vector forcing. Recall from Eq. (2.30) in MSM and from exercise 2.3 above that the right-hand side ("forcing") term in this version of the QG omega equation is proportional to $\nabla \cdot \mathbf{Q}$, the divergence of the **Q**-vector field.

In order to really solve for QG omega, the second derivatives on the left-hand side would have to be inverted via a numerical process such as successive over-relaxation. Qualitatively, such an anti-differentiation process greatly smooths the resulting QG omega field compared to the fine-grained $\nabla \cdot \mathbf{Q}$ field, as the opposite of differentiation (the Laplacian), which acts to enhance small-scale features in a field. As an approximate inverse Laplacian, simple smoothing is used on the $\nabla \cdot \mathbf{Q}$ displays in the bundles below. You can play with smoothing to see the noisy raw $\nabla \cdot \mathbf{Q}$ if you like.

a) Open the LMT_2.2 bundle that we used in section 2.2 above. Activate the displays for `Temperature contours, Hoskins Q-vector, and Wind barbs.` Confirm that all three displays are set to the 700-hPa level. **Examine the Q-vector field in the vicinity of the cyclone in the northern Gulf of Mexico. Capture an image that corresponds to the idealized diagram in problem 2.3 above. In what location are the vectors converging? Where are they diverging? Do these regions match your expectations from QG forcing terms in the traditional form of the omega equation from 2.2? Where are these areas relative to the upper-level trough, and surface cyclone?**

b) Deactivate the `Temperature contours` and `Wind barbs` displays, and activate the `Div(Q vector)` display, which is heavily smoothed as justified above.

   i. Compare the `Div(Q vector)` at 700 hPa to the `Omega` display at 700 hPa. **How well does the Q-vector convergence match up with areas of ascent (purple shading in both cases)? The Q vectors should point towards areas of ascent. Do they?**

   ii. Choose an area of notable **Q**-vector convergence, and **use the other QG diagnostics displays to attribute the forcing for ascent to either differential vorticity advection or temperature advection.**

   iii. Compare the displays of `Div(Q vector), 700-hPa omega, and IR satellite` imagery. **Discuss the degree of correspondence between these three plotted quantities. How well do the areas of Q-vector convergence**

correspond to the convective and/or cirrus cloud decks evident in the satellite image? Notice the discrepancy in areas where convection is prominent. In Eq. (2.31) above, the definition of **Q** involves static stability σ. **What would happen to Q if condensational heating acts to make the *effective static stability* in some region much smaller than the dry static stability σ?**

iv. Change the levels of `Hoskins Q-vector` and `Div(Q vector)` from 700 to 500 hPa and then 850 hPa (to do this, click on these displays' hyperlinks, which will bring up a window where you can choose from a list of isobaric levels). **Are the locations of Q-vector convergence sensitive to the choice of vertical level? What factors should go into determining which level to examine?**

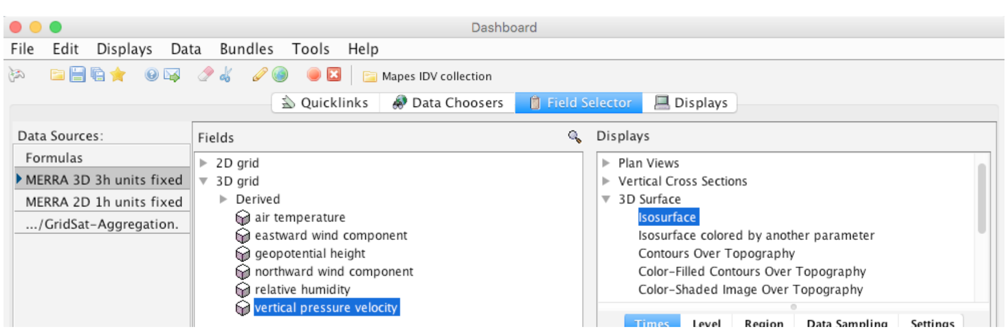
v. Zoom out to view a broader region and again compare the `Div(Q vector)` field to omega. **How good is the correspondence on broader spatial scales? Are there patterns to the discrepancy or agreement?**

c) **Identify an area with the "QG cancellation problem" (e.g., cold advection but cyclonic vorticity advection) based on the sea level pressure and 500-hPa height contours alone.** Use the `Drawing Control` in IDV (the pencil icon in the toolbar) to mark your location of cancellation. If you mismark, you can delete your marks (called glyphs) from the list under the `Shapes` tab in the `Drawing Control`. **Capture an image and indicate the area.**

d) Now, activate the `QG diagnostics` displays needed to check your answer from c) above. **Did you correctly select a location that experiences the QG cancellation problem? Which of the competing QG terms appears to dominate, based on Div(Q vector), the omega field, and IR imagery?**

e) Create a new display, an *isosurface of omega* enclosing the volumes of air with upward and downward motion.

To do this, follow this screen capture:



From the `Field Selector` on the `Dashboard`, highlight the `MERRA 3D 3h` data source under the `Data Sources` panel. Under the `Fields` panel, expand the `3D grid` item and choose `vertical pressure velocity` (the field name will appear as omega). In the `Displays` panel, expand the `3D Surface` item, and select `Isosurface`. Click on `Create Display`.

Now, in the `Legend` of the display window, click on your new `omega-Isosurface` and set the isosurface value to -0.5 Pa s^{-1} in the display control that pops up. The negative value corresponds to a region of *ascent*. Repeating the steps above, create a second isosurface of *descent*, with value $+0.25$ Pa s^{-1}. Adjust the color and transparency of your isosurfaces to taste.

Examine the isosurfaces in three dimensions in this cyclone. Is there a systematic tilt with latitude? Do you see any evidence in the IR imagery for the analyzed omega field being correct or incorrect? Discuss.

2.5. Real-Time Forecast Discussion: Utilize QG Thinking

QG reasoning can be applied to everyday weather situations, as part of a skill set for delivering effective, science-based weather briefings. This exercise works toward those goals by applying QG analysis and forecasting techniques to develop an *Area Forecast Discussion (AFD)* for the current and upcoming weather situation. Specifically, the objective of this lesson is for students to construct a forecast discussion based upon application of QG concepts to a current weather situation.

It is important to recognize that non-QG processes can exert a dominant influence on local weather. Convection and topographically forced processes are two examples. However, it is equally important to understand scale interactions, and knowing how the larger, synoptic-scale atmosphere is evolving is a prerequisite to understanding mesoscale or other non-QG processes. Likewise, recall from chapter 1 that Rossby wave packets are often hemispheric in scale, and when diagnosing QG processes related to a given trough or ridge it is helpful to step back and think about the planetary-scale context for the synoptic systems of interest. Ultimately, students of the atmospheric sciences must integrate information across many scales, and utilize understanding of planetary, synoptic, mesoscale, and perhaps microscale processes in research and forecasting applications. Here, our focus is on the synoptic-scale signals for which the QG equations convey useful understanding.

An important challenge that confronts atmospheric scientists is to boil a large volume of information down into a concise (and actionable) summary and forecast. As you will see