Exam format:

word	symbol	nutshell meaning	longer explanation of meaning (concept) in question	Relevant sketch with arrows or little f(x) curve or whatever if appropriate
divergence				
	ρ			xxxxx

...
...

Domains and coordinates and fields as functions of them:

Latitude, longitude, altitude
Zonal, meridional, vertical.
Northward vs. northerly; eastward vs. westerly
upward, altitude, pressure level (know Earth's atmosphere layers, p coordinate)
troposphere, stratosphere, planetary boundary layer (PBL)

Cartesian: x,y,z **i,j,k** u,v,w

scalar, vector; scalar field vs. vector field All Have Units!
vectors have 2 *properties* (in 2D or 3D): magnitude and direction
even though they involve a set of 3 components (numbers) in 3D

dot product, cross product. For a vector; same at every point for a vector *field*.

Scalar functions. Domain (coordinate or argument) vs. value (range)
curve (1D), surface (2D), field (3D, 4D)

Derivatives:
first derivative: slope (1D) or gradient (2D, 3D)
second deriv: curvature or Laplacian (flips sign, for sin/cos)

Del operator (nabla symbol):
gradient (of a scalar function) like temperature gradient ∇T
vergence (divergence, convergence) div(**V**) = "del dot **V**" = $\nabla \cdot \boldsymbol{V}$
advection **"minus V dot del**(T)" or "minus V dot grad T)
note negative sign $-\boldsymbol{V} \cdot \boldsymbol{\nabla} T$
Curl of vector field **V**, $\nabla \mathrm{x} \boldsymbol{V}$
Only in 3D! Right hand rule
(vector vorticity, if **V** is a 3D velocity field)
we use only its *vertical component,* $\zeta = v_y - u_x$
(where subscript indicated partial derivative)

Curl of gradient vanishes precisely - why?

Scale of variation (m vs. km vs. 1000s of km; hours vs. days vs. months): notice these are logarithmic (power of 10) distinctions, not just "size" (like 10m vs. 5m)

- Running average (smoothing) isolates *large scales* (filter scale)
- Deviations from that are *small scales*: (subfilter)
    - *anomaly* (deviation from time average)
    - *eddy* (deviation from space average)
    - *perturbation*: someone/something *perturbed* something
        - away from some control case
        - (an experiment, isolating cause and effect)

Partial derivatives of a field f(x,y,z,t)

- Local or Eulerian $\partial f/\partial t$

Total or Lagrangian df/dt, following a parcel at position $[x_p(t), y_p(t), z_p(t)]$

- Write the relationship to the local derivative $\partial f/\partial t$ (chain rule): it gives
    - advection by $\mathbf{V} = [u,v,w] = d/dt\,[x_p(t), y_p(t), z_p(t)]$

Nondivergent vs. irrotational decomposition of a vector field

- (rotational and divergent) "components"
    - different meaning than vector *components in the axis directions*

streamline, streamfunction, streamwise
trajectory (different from streamline)

Integral relationships (opposite of derivative) for gradient, div, curl

- Stokes theorem (circulation), Gauss' theorem (for divergence)
- vanishing of div(curl), vanishing of loop integral of gradient

ODEs and solutions

- exponential solutions to $df/dt = -bf$
- sinusoidal solutions to $d^2f/dt^2 = -c^2f$
- exp() with complex numbers combines both
- need boundary or initial conditions (constant of integration) to solve

- stationary or steady-state solution; equilibrium or "balance"
    - $df/dt = A - B$. Make steady-state assumption. Is it still a diff-eq? NO!

PDEs and solutions: terms and concepts (for our applications)

- prognostic vs. diagnostic
- boundary conditions, initial conditions
- inverse of Laplacian (smoothed, reversed sign)

<u>PROGNOSTIC EQUATIONS</u>:

Governing equation, budget, tendency

Eulerian (local) vs. Lagrangian (total) derivatives

d/dt(something) = 0 + sources - sinks

∂/∂t(something) = flux convergence + sources - sinks

∂/∂t(something) = advection + sources - sinks

Conserved tracer special case: *sources-sinks negligible*

Balance special case: *neglect time derivative relative to other tendencies*

Adjustment ("fast" process leads to restoration/maintenance of balance)

"Kinematics": spatial gradients of velocity

vorticity, divergence, deformation. diffluence/confluence.

recipes: shear = vorticity + deformation

Streamlines, trajectories: know the difference

Waves: terms and concepts

frequency, period, wavelength, wavenumber, amplitude, phase

phase velocity, group velocity

growing, decaying *amplitude* (in space or time)

growing, shrinking *scale* (expressed as wavenumber or wavelength)

Physical concepts/words to know

Mass, mass fractions (specific ____, mixing ratio of ____, concentration of ___)

Conservation of mass (*continuity* of mass flux)

Flux of mass, multiply by specific ___ to get flux of specific ___

Conservation of specific ___

TRANSPORT:

Flux of (anything): what are the units? Stuff per second per square meter (in 3D)

Flux *convergence* is the effect of the flux (*transport's* "drop-off")

Advection: what is the sense of it (upstream coming at ya) and the math (-**V.del**)?

how are *advection* vs. *flux convergence* treatments related?

(equal, because of mass continuity as in homework 3).

Diffusion (convergence of a flux that is proportional to a gradient).

PHYSICAL LAWS

Equation of Motion / Newton's 2^{nd} Law

Pressure gradient force (PGF): Enforcer of continuity

Coriolis force (as 'real' as still air on rotating Earth is 'motionless')

"Inertial forces" (advection of momentum by wind itself)

"Friction" (convergence of momentum flux by small-scale motions)

Vorticity equations: d/dt(vorticity) = 0 + complications

Relative vorticity ζ: eliminates PGF from momentum equations

Absolute vorticity $\zeta_a=(f+\zeta)$ moves v df/dy = v*beta term to LHS

Potential vorticity PV eliminates $\zeta_a \mathrm{div}(\mathbf{V})$ "stretching" term from RHS

Vortex interactions (e.g. for TC steering): 2D reasoning

1/r decay of "induced" rotational wind from vorticity element

$V_{tan} \alpha (1/r) \zeta_{rel}$

ζ_{rel} is advected by that "induced" flow

Sketch how this plays out for 2 vortices of same/opposite sign

Rossby waves: includes advection of planetary vorticity (or conservation of absolute)

explain from $d/dt(\zeta_a)=0$ with $\beta = df/dy$

Phase velocity $c=U - \beta/k^2$: westward relative to U, long waves faster

Group velocity $c_g=U + \beta/k^2$: eastward relative to U, " " "

"downstream development" process

For stationary waves, $c_g = 2U$

First Law of Thermodynamics (conservation of microscopic energy)

heat energy added to gas = change in internal energy + work done by gas (p dV/dt)

Per unit mass: $Q = C_v\, dT/dt + p\, d\alpha/dt$

Ideal gas law: $p\alpha = RT$

Plug in: $Q = C_p\, dT/dt + \alpha\, dp/dt$

where $C_p = C_v + R$

Mass continuity

Hydrostatic pressure (mass) coordinate makes it especially clean

$$\partial u/\partial x + \partial v/\partial y + \partial \omega/\partial p \partial = 0$$

omega = dp/dt is vertical velocity, but also "pressure drop" at the surface for instance

Primitive Equations: write them like a sonnet (perfectly), as in HW3

see attached exam prep doc