Compound units

velocity: m/s = momentum per unit mass

acceleration: m/s^2 = force per unit mass

force: $F = ma$! so units: $\frac{kg\,m}{s^2}$ = N ewtons.

pressure or stress: force per unit area $N/m^2 = \frac{kg}{s^2 m}$ = Pascal (Pa)

Energy:

1. kinetic $\frac{1}{2}mv^2$ $kg\frac{m^2}{s^2}$ = J oule
2. work = $\int \vec{F}\cdot\vec{dx}$ $kg\frac{m^2}{s^2}$ = J

Power (rate of work) $\int \vec{F}\cdot\vec{v}$ $\left(kg\frac{m}{s^2}\right)\left(\frac{m}{s}\right) = \frac{J}{s}$ = Watt

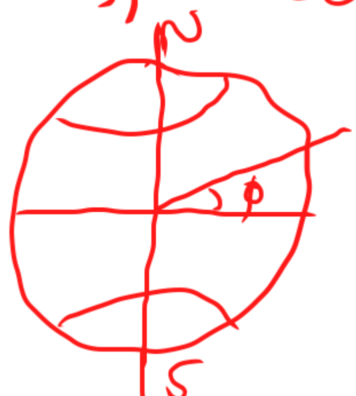
Math book-keeping tools.

Coordinates & functions $f(x)$

- domain or argument, range or value, mapping

$\underbrace{\text{domain or argument}}_{x \text{ input}}$, $\underbrace{\text{range or value}}_{f \text{ output}}$, mapping $f(x)$

For meteorology, coordinates



ϕ = latitude (deg or radians)

λ = longitude (" " ")

z = altitude (m) or height.

"above mean sea level"

For scales smaller than planetary, for convenience, (weather up to continental scales)

use a tangent plane Cartesian domain

(x, y, z): derivatives (velocities): $\{u, v, w\} = \tilde{V}_{3D}$

Use it! $T(x,y,z,t)$. Temperature everywhere, forever.

~~Suppose~~ we have learned the 1st Law of Thermodynamics for a parcel of air (maybe 1kg) trapped in a piston. How will we make that an equation for flowing air?

First law says: (rate of change of T with time) = (energy input).

Derivatives: what is $\dot{f}(t)$ or $f'(t)$ or $\frac{df}{dt}$, for $f(t)$

function of one variable only:

$$\frac{df}{dt} = \lim_{\Delta t \to 0}\left(\frac{f(t+\Delta t) - f(t)}{\Delta t}\right) = \lim_{\Delta t \to 0}\left(\frac{f(t) - f(t-\Delta t)}{\Delta t}\right)$$

for T,

$$\left.\frac{\partial T}{\partial t}\right|_{x,y,z} = \lim_{\Delta t \to 0}\left(\frac{T(x,y,z,t+\Delta t) - T(x,y,z,t)}{\Delta t}\right) \equiv \text{"}\partial_t T = \partial_t(T)\text{"}$$

operator notation

To apply the First Law to free-range parcels of air, we need to define $\frac{dT}{dt}$ for moving parcels. $\boxed{\text{Chain Rule}}$

- Parcel poisition is a function of time

$\{\underline{x_p(t)}, y_p(t), z_p(t)\}$ ← have this set

$T(x_p, y_p, z_p, t)$ is available to us.

$$\underbrace{\frac{dT}{dt}}_{\text{law of physics}} = \underbrace{\frac{\partial T}{\partial t}}_{\text{thing we want}} + \frac{\partial T}{\partial x}\underbrace{\frac{dx_p}{dt}}_{u} + \frac{\partial T}{\partial y}\underbrace{\frac{dy_p}{dt}}_{v} + \frac{\partial T}{\partial z}\underbrace{\frac{dz_p}{dt}}_{w} = \underline{\vec{\nabla}T \cdot \vec{V}} + \frac{\partial T}{\partial t}$$