

Vorticity: the strategic view.

- We have admired & understood eq's like

hungry  
for  
this →

$$\underbrace{\frac{d}{dt}(\text{conserved}) = 0}_{\text{+ transport only!}} + \text{complications} \\ (\text{sources minus sinks})$$

- Reading ch 7 starts in on momentum

$$\frac{d}{dt} \left( \frac{\text{(momentum)}}{\text{per unit mass}} \right) = 0 + \cancel{\rho} + C + F \quad (7.12)$$

terrible  
approximation!

stuck  
with  
this.  
Jg.

$$\left. \begin{aligned} q &= \frac{d\vec{V}}{dt} \end{aligned} \right\}$$

$$= -\vec{\nabla} \Phi + \cancel{F_{\text{ex}}} \times \vec{V} + \cancel{F} \quad (7.14)$$

Hungry mind looks at (7.13b) in  $\varphi$  coordinates

$$\frac{\partial}{\partial y} \left[ \frac{du}{dt} \right] = - \frac{\partial \Phi}{\partial x} + fv + F_x$$

$$-\frac{\partial}{\partial x} \left[ \frac{dv}{dt} \right] = - \frac{\partial \Phi}{\partial y} - fu + F_y$$

Want less  
on right!

How:

1. about  $F_x$  &  $F_y$ : ignore it (above PBL)  
upper level  
fores

2. About  $\Phi$  and PGF:  
take y-derivative of u equation,  
x-derivative of v equation

$$\frac{\partial}{\partial y} \left( \frac{du}{dt} \right) - \frac{\partial}{\partial x} \left( \frac{dv}{dt} \right) = \underline{0!!!} + \left[ \frac{\partial}{\partial y}(fv) - \frac{\partial}{\partial x}(fu) \right] + \cancel{\left( \text{curl of } \frac{\partial \Phi}{\partial z} \right)}$$

Hungry mind clobbered P6F ( $\vec{\nabla} \Phi$ ) term!  
At what price?

$$\frac{\partial}{\partial y} \left( \frac{du}{dt} \right) = \text{how many terms?}$$

$$\frac{\partial}{\partial y} \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right)$$

$$\left( \frac{\partial^2 u}{\partial y \partial t} \right)$$

chain  
rule

↓  
2 terms

↓

↓

2

$$\text{same for } \frac{\partial}{\partial x} \left( \frac{dv}{dt} \right)$$



7

$$7.7 \nabla \cdot \vec{J} = \frac{\partial v}{\partial y} = \frac{-0.05(60) - 0.5 \cos(100\pi t) \cdot 0.05}{3 \cdot 1.1 \times 10^3 \text{ m/deg}} = \frac{-0.9 - 0.33}{3.33 \times 10^3} = -2.5 \times 10^{-5} \text{ s}^{-1}$$

$$RR = -\frac{\nabla \cdot \vec{J}}{g} \frac{\rho f}{2} \omega = -2.5 \times 10^{-5} \text{ s}^{-1} \cdot \frac{1}{2} \cdot \frac{(1010 - 980 \text{ mb})}{9.8 \text{ m/s}^2} \cdot 0.2 \frac{\text{kg}}{\text{m}^3} = 2.81 \times 10^{-4} \text{ kg/s} = 2.42 \text{ kg/d}$$

dp/g units: kg per sq m

$$7.8 \nabla \cdot \vec{J} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \rightarrow \frac{\partial}{\partial x} \left( -\frac{\partial \Psi}{\partial y} \right) + \frac{\partial}{\partial y} \left( \frac{\partial \Psi}{\partial x} \right) = 0 \quad \checkmark$$

$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \rightarrow \left( \frac{\partial}{\partial x} \left( \frac{\partial \Psi}{\partial x} \right) \right) - \left( \frac{\partial}{\partial y} \left( \frac{\partial \Psi}{\partial y} \right) \right) = \frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} \rightarrow \nabla^2 \Psi = \zeta \quad \checkmark$$

$$\frac{\Delta P}{g} = \frac{\rho g}{m/s^2} = \frac{N/m^2}{m/s^2} = \frac{kg/m^2}{m/s^2} = \frac{kg}{m^2 \cdot s^2}$$

of  $\text{kg}/\text{m}^2$

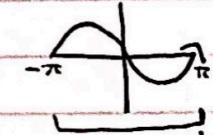
$$7.9 -\Psi = my \quad \begin{matrix} \xrightarrow[m>0]{} & \xrightarrow{} \\ \xrightarrow{} & \xrightarrow[m>0]{} \end{matrix} \quad \Rightarrow \Psi = my + n \cos \frac{2\pi x}{L}, m > 0$$

$\zeta = m(x^2 + y^2)$  2

solid body rotation

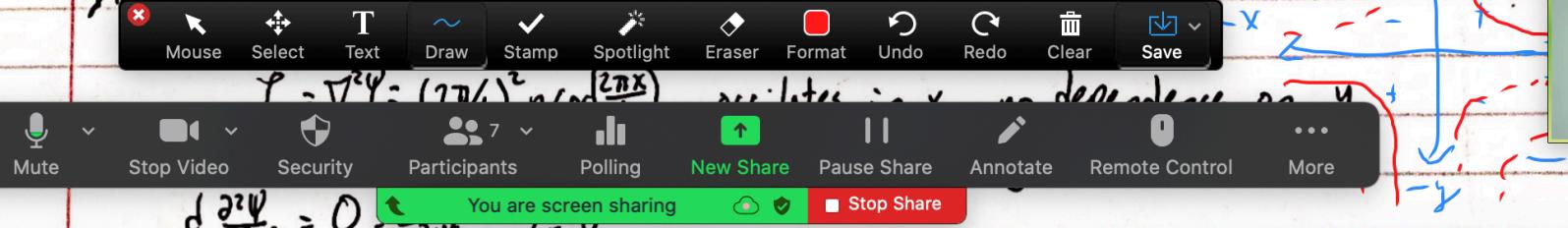
pure deformation

$$\zeta = m(xy) \quad m > 0$$



draw contours, then winds are parallel to those

$$7.10 \frac{\partial^2 \Psi}{\partial x^2} = 0 = \frac{\partial^2 \Psi}{\partial y^2}, \text{ no vorticity in straight flow } \zeta = 0$$



Homework lessons for our vortex interaction  
topic:

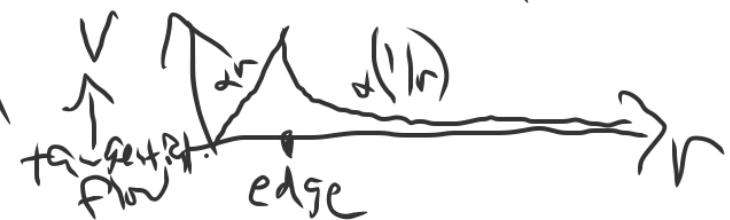
HW prob (7.6): distance in the far field advects tangential flow "induced" by the vortex field advection tracers (like vorticity)

HW prob (7.9):  $\Psi = m(x^2 + y^2) = m(r^2)$   
near-field paradigm: solid body rotation  
 $\zeta = \nabla^2 \Psi = 4m$

Combine these views:  $\zeta = \text{constant to some radius}$

a Rankine vortex

$\zeta = 0$  beyond



Bri | Da | Ma | ATI | GF | NASA | We | Co | h. P7 | AT | Co | UA DES | GF | Ro | Na | AT | AT | RS | da | Na | +

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# WEATHER MAPS

NASA GEOS Forecast  
Initial: 09/14/2020 00Z Hour: 063 Valid: 09/16/2020 15Z

Abs EPV Humidity

Precip & SLP Temperature

Vorticity Vert Velocity

Wind Speed

REGIONS

Atlantic Australia

Global Mid Atlantic

North America N Polar

Pacific Seven Seas

S Polar

LEVELS

10 30

50 100

200 300

500 700

850

Mouse Select Text Draw Stamp Spotlight Eraser Format Undo Redo Clear Save

ights [dam]

V decays is 1/ distance from these "point" vorticity

FO Mute Stop Video Security Participants Polling New Share Pause Share Annotate Remote Control More

14 Sep 2020 00Z You are screen sharing Stop Share