4.1 Graphical analysis for moist thermo and probability

If a kilogram of air lifted to lower pressure can achieve positive buoyancy relative to its environment, convection is possible. There are two challenges to saying any more with the field equations of Part I. First is the unfinished business of predicting buoyancy with moisture folded in as a moist adiabatic process, leaving only the truly diabatic effects in (2.14e) as the necessary externalities of forcing and complications. Section 4.3 said only that $e_s(T)$ is monotonic and upward-curved, not how to reason with that fact, and a complicated formula doesn't actually help: a graphical way to express the curvature is clearest.

A second challenge to an entity-based viewpoint is how to express the vast ambiguous multiplicity of entities, and bracket the complexities that make it silly to take our little elements (parcels and plumes) too literally. Dressing their depictions in probability (a graphical *spaghetti* of *ensembles* of distinct but equally-plausible realizations) helps keep the mind from fixating inappropriately.

Customarily, a one-dimensional *sounding* or profile $T(p)$, $q(p)$ is usually taken as both the environment and the source of a lifted parcel's properties (from the lowest altitude, or the level of greatest h, or some mixture averaged over some convection inflow layer). Data from point sensors on a balloon, released at a particular place and moment and rising slantwise through the sky over an hour, are an imperfect fit to those assumptions, so *representativeness error* sets a useful floor on how fussy to be about precision. The situation-dependent processes of mixing and microphysics also add diversity to outcomes. Even a gridded model's column fails to really be either the environment (for fine grids: parcels don't rise vertically through their own narrow column as sampled at a prior instant) or the parcel (for coarse grids: because gravity naturally selects the *best* air for convection, not the average).

In short, an ensemble of parcels rising and mixing and churning through an environment full of statistical fluctuations must be envisioned. Graphics can help cement this appropriate diversity in the viewer's mind, leaving the necessary assessment of likelihood and weighting of these plausible parcel fates to sensible reasoning. Perhaps better illustrations may leaven a culture of sometimes silly fixation on numerical values of extremely crude indices like undiluted-parcel *convective available potential energy (CAPE)* or *convective inhibition energy (CIN)*. These indices badly underestimate the role of elevated moisture, a factor also wildly distorted in customary *aerological charts* of the pre-computer age, labeled as temperatures vs. pressure and thus expressing moisture as the steeply T-dependent *dewpoint depression*.

4.2 Conserved variables in lifted air

Our only rigorous basis so far for explaining an updraft is Newton's law for vertical acceleration (2.10d, 2.14d), with its b term as the ultimate driver of all fluid motion (problem 2.4.1). We therefore begin with a graphically supported understanding of the thermodynamics of b for rising parcels of air in a quasi-uniform unstable airmass, before turning to the mass-continuity problem of realizable ascending motions.

Air parcel density (or T_v) at any pressure can be easily computed using conserved variables (h and total water mixing ratio q_t), by a quick iterative solution of the mildly nonlinear saturation condition deciding whether $q_t > q_{sat}(T,p)$. These variables are only "conserved" in the absence of surface contact, radiative heating, and precipitation (section 2.1.5), leaving those phenomena to be assessed sensibly by the reasoner. Figure 4.1 shows profiles of the specific static energies s, h, h_s for a Miami summertime sounding[b]. The reader should appreciate all of the following reasoning-relevant aspects of the diagram, as evaluated in the Problems and Exercises of section 4.5.

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[b] Created with the Python (pip) package mseplots-pkg, which rests on thermodynamics in the MetPy package from Unidata. Try the entropy and potential_temperature keywords to see how little these complications of nonlinearity matter on Earth compared to the above ambiguities of the whole 'parcel' conceptualization, as illustrated in Exercises below.

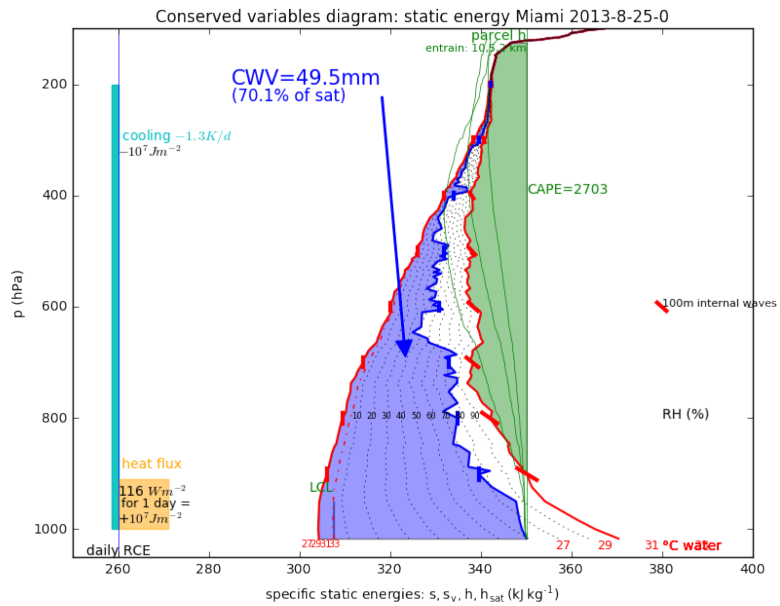

Fig. 4.1. Specific static energy profiles vs. pressure (mass). Static energies from left to right are: dry $s=C_pT+gZ$ (red solid), virtual s_v (red broken), moist $h=s+Lq_v$ (blue), and saturated $h_s=s+Lq_s(T,p)$ (rightmost red solid). Relative humidity RH in 10-90% are dotted (labeled at the 800 hPa level). Blue fill area is proportional to L times column water vapor. Below the lifted condensation level (LCL), buoyancy of a lifted parcel is assessed by the s_v curve. Above the LCL, its sign is indicated by the gap between h_s (red) and h_p for lifted near-surface air parcels (green curves, with entrainment rates of 0 (vertical) and 100% per (10 km,5 km,2 km), which act as the strengths of a linear pull of the rising parcel curve toward the h profile at each level. Thick stubs on *s, h,* h_{sat} profiles every 100 hPa show the effect of hypothetical +/- 100m adiabatic displacements in the air column, comparable to typical internal wave amplitudes, for envisioning their statistical effect on lifted-parcel buoyancy. Red number annotations at bottom show the *s* and h_{sat} of the molecular boundary layer over surface water with temperatures near the sounding's surface temperature, relevant for viewing surface fluxes as another distance-proportional mixing process. Annotations at left centered on 260 kJ/kg indicate typical daily diabatic increments for Earth-like radiative convective equilibrium (RCE), with cyan and orange areas representing 10^7 J m^{-2} of daily static energy changes by radiation (-1.3 K/d, compare Fig. 0.1) and surface flux (orange square, if distributed over a 100 hPa layer).

1. The vertical coordinate is hydrostatic pressure, so its increments are proportional to mass ($\Delta p_{hyd} = g\ \Delta m$).
2. The horizontal coordinate is specific energy (units: kJ/kg).
3. Because of 1 and 2, area on the diagram is proportional to energy.
4. Relative humidity RH (the hygrometer measurement) is proportional to the distance between the two indicators of the thermometer measurement (s *and* h_{sat}), as ruled by the dotted lines labeled 10-90%.
5. The filled blue area is column-integrated water vapor or *precipitable*[c] *water vapor* multiplied by latent heat coefficient L.
6. Unsaturated air conserves s and s_v during vertical displacements, so the stability of unsaturated layers is assessed by the slope of the dashed red s_v curve below the *lifted condensation level* (LCL). A well-mixed layer like the *planetary boundary layer* (PBL) here has vertical s and s_v curves, because absolute instability rapidly mixes a layer if s_v ever decrease with height.
7. An undilute surface parcel lifted through the troposphere conserves h (thick vertical green line) even after condensation commences. The vertically integrated buoyancy of this parcel (assuming instant precipitation of condensate) is labeled with MetPy's pseudo-adiabatic undilute parcel CAPE, which is not quite strictly proportional to the green fill area (nor to any aspect of a realistic convective process; avoid fixating on the number).
8. Lifted buoyancy of any *saturated* parcel (above its LCL) is indicated by the horizontal distance between the green h_p and red h_{sat} curves, because at any given altitude $h_p - h_{sat} = C_p(T_p - T_{env}) + L[\ q_{sat}(T_p) - q_{sat}(T_{env})\ ]$ is monotonically (albeit not linearly) related to $T_p - T_{env}$, as explained in section 2.3.
9. Horizontal mixing with the environment (entrainment) pulls any lifted saturated parcel (green curves) toward the environmental h (blue curve) with a strength linearly related to horizontal diagram distance between the curves.

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[c] This old term for condensed puddle depth in mm or kg m^{-2} (section 1.1) may have stemmed from rough coincidence of its typical values with typical rainstorm totals.

10. Surface flux can be viewed as a mixing process with the microlayer of air that is in thermodynamic equilibrium with the surface. Over water, that microlayer is saturated at the water temperature. If that water temperature is known, latent and sensible heat fluxes (respectively) are proportional to the respective distances between the sounding's lowest-level air values and the water temperature values (red number annotations at the bottom of the diagram are centered on the values).
11. The effect of small adiabatic vertical displacements in the environment (such as by the buoyancy waves of section 2.2.3) is indicated by whiskers on the *s, h,* and h_{sat} curves. These are vertical on the s and h curves (since both are conserved), but are sloped on the h_{sat} curve, expressing the nonlinear saturation relationship $e_{sat}(T)$.
12. Energy added to the mixed-layer PBL by surface fluxes (proportional to area on the diagram, by point 3.) will be spread over a layer whose depth is defined by the area added, and by the condition of verticality of s_v upward from the surface. *Sensible heat flux* SHF increases *s* alone, while the sum of SHF plus *latent heat flux* (LHF) moves the *h* curve to the right (adding to the area encompassed to the left of the h curve). These principles are sufficient to distribute any given sensible and latent surface energy input over a mixed-layer PBL, such as on a morning sounding for anticipating convection on a summer day.

A few other points are worth noting:

- The relative smallness of water's virtual contribution to buoyancy, compared to its latent heating impacts, can be seen graphically by the relative distances s_v-s and h-s.
- There is no fundamental area relation for the distance that a diabatic temperature warming like SHF moves h_s to the right. The h_s curve is the locus of the main nonlinearity, the curvature of $e_s(T)$, and is the reason that a diagram instead of equations is needed for reasoning.
- It is not possible to read absolute *T* off the diagram; annotation along the s curve for the 0C level would be a

useful addition to respect the importance of ice, although the nucleation dependence of freezing complicates its use in reasoning. Qualitatively, the latent heat of freezing adds about 15% to the value of L measured by the h_{sat}-s distance.

4.3 Parcel diversity, dilution, and detrainment profiles

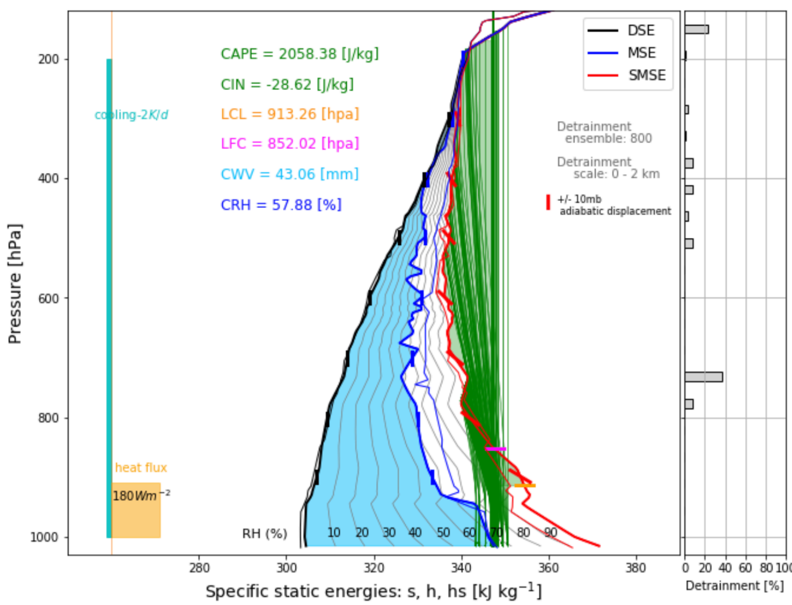

Fig. 4.2. As in Fig. 4.1, but for a different time, and with some statistical features. Surface parcels are sampled randomly from an assumed distribution (loosely based on aircraft data over a tropical ocean) of h values centered on the balloon's data value. Each is then subject to an arbitrary fixed set of different entrainment rates. The bar plot at right shows the resulting histogram of topmost $b>0$ altitudes for members of that parcel ensemble, a crude estimate of cloud "detrainment" (chapter 6) layers that might be produced by natural variability in a sky of which the balloon data is a representative sample. A second balloon sounding one day later is also overlaid, to illustrate the size of day to day variations at this location and season.

Figure 4.2 shows a similar diagram with an ensemble of thermodynamically possible lifted parcels (green curves). These were created with two assumptions (adjustable as keywords in the Python call):

1. Parcel diversity. The balloon is assumed to have sampled the center of a distribution of PBL *h*. Based on aircraft data at low levels over a warm tropical ocean (Kingsmill and Houze 1993), 20 more samples are drawn randomly from a normal distribution with a standard deviation of 2 kJ/kg[d].
2. Mixing diversity. For each parcel, a fixed *ad hoc* distribution of 5 entrainment rate coefficients is considered, ranging from the 1 km^{-1} value typical of shallow convection (chapter 6) to much smaller values that permit nearly-undilute ascent to great heights.

Together these give 100 parcels, whose altitudes of topmost positive buoyancy (summarized in the bar chart at right) are a crude estimate of the profile of likely detrainment altitudes (and hence perhaps of cloud layers one would expect to see in the sky). Sky photographs paired with soundings suggest some relevance to these computations, although feature altitude is hard to estimate visually. A computer exercise invites you to explore this relationship for any place and time, for instance where you can find photographs near a sounding site.

Layers where h_s increases more rapidly tend to cap the ascent of larger numbers of parcels in the distribution. In other words, from a broad distribution of *h* values in buoyant parcels, more will detrain preferentially into stable layers. This is a key mechanism forming a logical basis for a teleological interpretation of convection’s job: adjusting the atmosphere toward a moist adiabat (Betts and Miller 1984, Bretherton and Smolarkiewicz 1989, Bretherton 1993).

Assembling an ensemble of credible parcels into a realizable convective event is not simple. The work done by buoyancy (*b* integrated over height) must be positive for viability, but mass continuity must hold.

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[d] 1 kJ/kg in *h* is about 1K in Kingsmill and Houze's θ_e; this example is narrower than their distributions, and so is a very conservative estimate of PBL diversity in general.