MSC 405 Final 2011 Name___________

Here is the primitive equation set for hydrostatic flow using pressure p as the vertical coordinate. Vector **V** is horizontal wind and ∇_p is the horizontal gradient operator with its derivatives evaluated at constant p.

	A	B	C	D	E
1	$\frac{\partial Z}{\partial p}$	$= -RT/pg$			
2	$\frac{\partial T}{\partial t}$	$+ (\vec{V}.\vec{\nabla}_p)T$	$+ \sigma_p \omega$	$= Q/C_p$	
3	$\frac{\partial}{\partial t}\vec{V}$	$= -(\vec{V}.\vec{\nabla}_p)\vec{V}$	$- \omega \frac{\partial \vec{V}}{\partial p}$	$- f\hat{k} \times \vec{V}_h$	$- \vec{\nabla}_p \Phi$
4	$\frac{\partial \omega}{\partial p}$	$= -\vec{\nabla} \cdot \vec{V}$			

1. (20) This question just tests general equation-set orientation. You don't actually calculate anything specific, you just show how influences flow between equations in describing fluid flow.

Suppose the atmosphere were initially at rest. Then, the air in some local region, in the middle of the troposphere, gets cooled by radiation. You know that the cooled air will sink downward, then later develop into a cold core vortex as Coriolis effects get involved and geostrophic balance eventually becomes established. But it takes several logical steps through the equation set to express the processes. **Write** equations that express the key process at each logical step, equating **the 2 dominant terms (term1 = term2)**.

Part a has been done for you as an example (on page 2).

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1. Continued: write term1 = term2 equations relevant to each step.

   a) A physical cooling process makes T decrease with time. (Example done for you): 2A = 2D or $\frac{\partial T}{\partial t} = Q/C_p$

   b) Cooler air has smaller thickness, so that a depression of the 200mb surface occurs above the cooled column of air.

      1A = 1B $\quad \Delta z = -\frac{RT}{pg} \Delta p$

   c) Air accelerates horizontally toward the depression.

      3A = 3E $\quad \frac{dV}{dt} = PGF$

   d) Column-integrated horizontal mass convergence causes surface pressure to rise.

      4A = 4B $\quad \omega = \frac{dp}{dt}$ is surface pressure change here

   e) Air at the surface accelerates away from the high pressure.

      3A = 3E $\quad \frac{dV}{dt} = PGF$ again

   f) Middle-tropospheric upward motion is implied by the upper level divergence and low-level convergence.

      4A = 4B $\quad \omega = \frac{dp}{dt}$ is vertical motion here

   g) A tangential component of wind begins to develop, due to the Coriolis force acting on the inflow and outflow winds.

      3A = 3D $\quad \frac{dV}{dt} =$ Coriolis force

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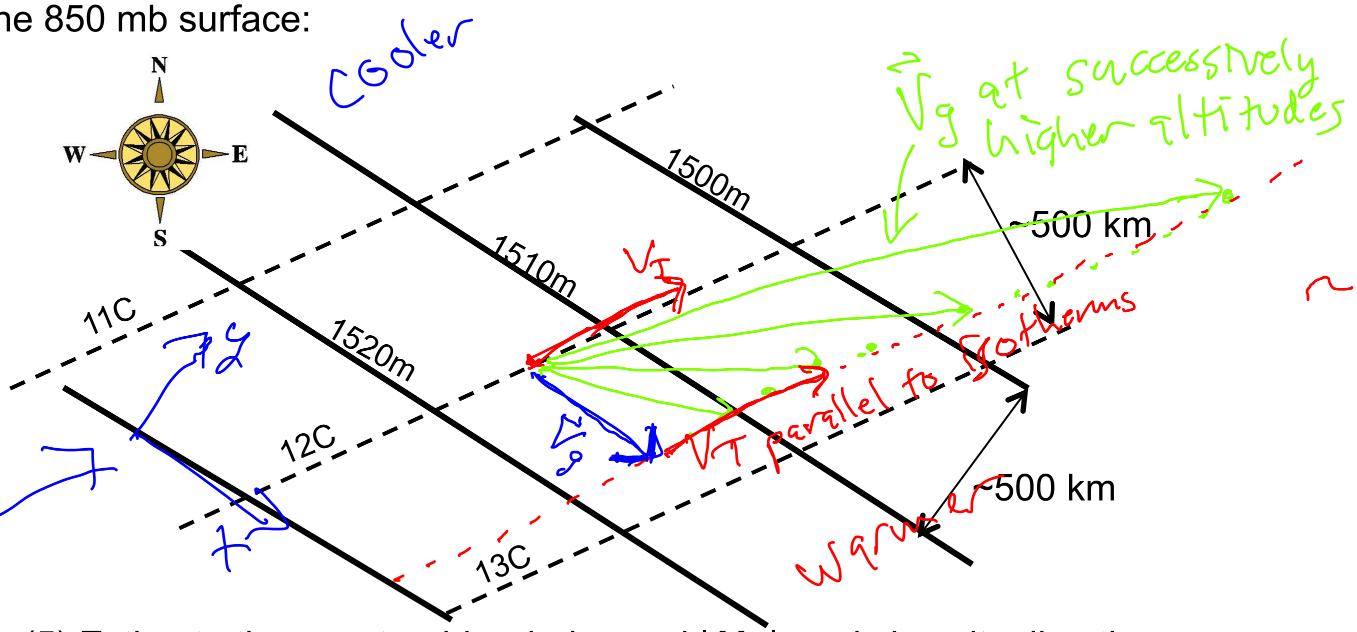
2. (30) Here are some isotherms (dashed) and height contours Z_{850} on the 850 mb surface:



a. (5) Estimate the geostrophic wind speed | $\mathbf{V}_g$ |, and show its direction by an arrow on the diagram. Assume f = 10^{-4} s^{-1}.

$f|V_g| = |\nabla \Phi|$ force balance

$= g \cdot \left(\frac{10m\ \Delta Z}{500\ Km\ \Delta X}\right)$ so $|V_g| = \frac{g}{f} \cdot \frac{20m}{10^6 m} = 2\times 10^6 / 10^6$ $\boxed{= 2\ m/s}$

$g \leftarrow 10$, $f \leftarrow 10^{-4}$

b. (5) Draw an arrow showing the direction of the thermal wind vector ($\mathbf{V}_T$ = $-\partial \mathbf{V}_g/\partial p$). Draw vector sum arrows showing how $\mathbf{V}_g$ at two higher levels (like 700 and 500mb) are related to the $\mathbf{V}_{g850}$ and thermal wind $\mathbf{V}_T$ vectors.

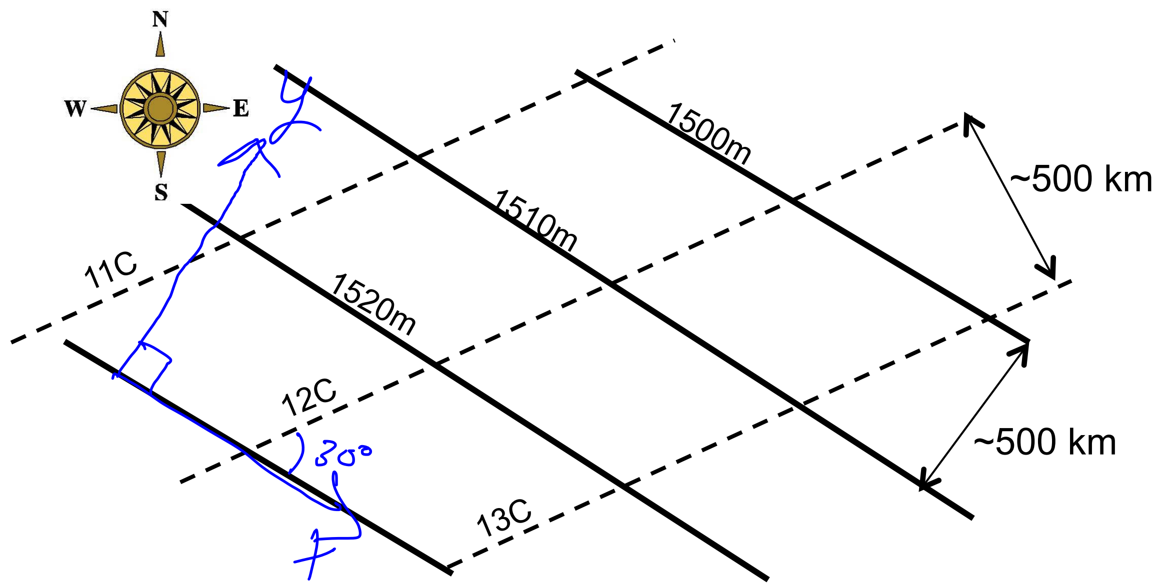
green arrows: $\vec{V}_{upper} = \vec{V}_{lower} + c\vec{V}_T$

↑ various amounts of V_T added

c. (5) Will temperature advection be easier to calculate if you align your x axis with the isobars (Z contours), or with the isotherms, or east-west? Explain your considerations.

Nicer to just have (one component) Ug be the wind
(align x-coordinate along height contours)

2. (continued)



d. (5) Estimate the temperature advection, in C/day (1 day ~ 10^5 s). The angle between isobars and isotherms may be taken as ~30° for simplicity. Get the sign right for sure: check it conceptually.

$U_g = 2$ m/s as in part A.

advection $= U_g \frac{\partial T}{\partial x}$ so need $\frac{\partial T}{\partial x}$. Clearly negative.

$|\vec{\nabla} T| = \frac{2K}{10^6 m}$. x component is $\sin(30°)$ or 0.5 of that

$\boxed{ADV} = (2\,m/s)\left(\frac{2K}{10^6 m}\right)(\cancel{0.5}) = 2\times10^{-6}\,\frac{K}{s} \approx \boxed{2\times10^{-1}\,K/d}$

e. (5) What is the temperature advection at the 500mb level, assuming the T gradient is independent of height? **HINT**: Remember, wind aloft = low-level wind + thermal wind. What is the dot product of the thermal wind vector with the temperature gradient? *You don't need to do any calculations in this part, just think it through and explain the reasoning*.

$(V_{500}\cdot\nabla T) = (V_{850}+V_T)\cdot\nabla T = V_{850}\cdot\nabla T$ = same

$+ \; V_T\cdot\nabla T$

ZERO!

THERMAL WIND IS ∥ to ISOTHERMS!

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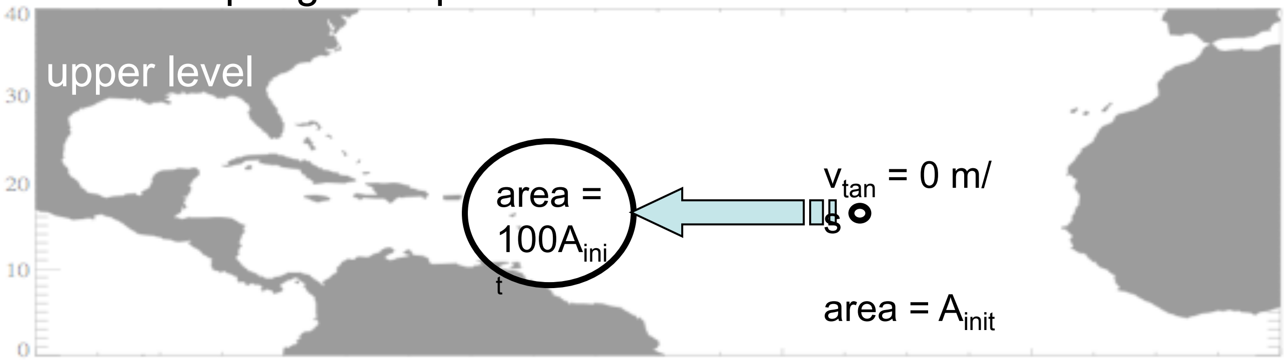


3. Circulation and vorticity (30 points)

a. (5 points) Suppose an *initially motionless* upper level loop of air at 20N (you may take f = 5 x 10^{-5} s^{-1} to be a constant) expands in divergent flow, (like the outflow layer of a hurricane), conserving its absolute circulation. If its enclosed area increases by a factor of 100, **what is the final absolute vorticity** within the expanded loop?

$$\int_{t_1}^{t_2} \frac{d \ln \zeta_a}{dt}\, dt = \left(\frac{\zeta_{a_2}}{\zeta_{a_1}}\right) = \left(\frac{A_1}{A_2}\right) = 1/100$$

abs. vort. is very small (near zero)

so $\zeta_{a_2} = 1/100\, \zeta_{a_1} = 1/100\,(f) = \boxed{5\times 10^{-7}\ s^{-1}}$

b. (5 points) **What is the final relative circulation** around the loop?

$(f+\zeta) = 10^{-2} f$ so $\zeta = (10^{-2}-1) f = -0.99f$

Circulation is

$$C = \oint v \cdot dl = \iint_A (-0.99f)\, dA = \boxed{(-0.99f)\times \pi r^2}$$

c. (5 points) **What tangential wind speed** does this imply? **Clockwise or counter-clockwise**?

To answer, we need the actual final radius r!

$$C = (V_{tan})(2\pi r) = (-0.99f)\cdot(\pi r^2) \quad \text{or} \quad \boxed{V_{tan} = \frac{(-0.99f)\, r}{2}}$$

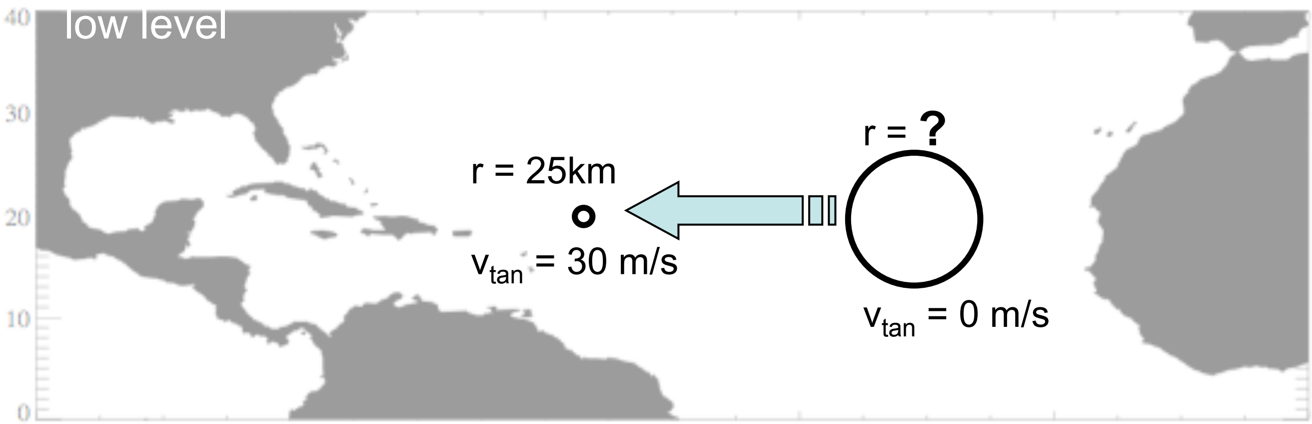
d. (5 points) Answer part c again if the final area= 1000 $A_{init,}$ or 10000 A_{init}. *Words explaining an approximate value (the sense of the answer) are sufficient, I don't really want you to redo the calculations to 3 or 4 decimal places.*

$\zeta = (10^{-3}-1) f$ or $(10^{-4}-1) f$, almost $-f$

and $V_{tan} = \frac{\zeta r}{2}$

3. Circulation and vorticity (continued)

e. (15 points) Suppose a hurricane forms by an *initially motionless* circular loop of air shrinking in low-level convergence at 20N (where $f = 5 \times 10^{-5}\ s^{-1}$ may be taken as constant), conserving its absolute circulation. If the tangential wind speed is 30 m/s when the loop is 25km in radius, **how big was the loop initially**, when it had only planetary circulation?



$V_{tan} = \zeta \frac{r}{2}$ as seen above, or $\zeta = \frac{2V_{tan}}{r} = \frac{60\ m/s}{2.5\times10^4\ m}$

$C = (V_{tan})\cdot(2\pi r) = (30\ \frac{m}{s})\cdot 2\pi\cdot(2.5\times10^4\ m) = (150\times10^4)\pi\ m^2/s$

Q: How big a loop had initial area-avg vort $\iint (f)\, dA = (f)(\pi r^2) = C$?

A: $\longrightarrow$ $r^2 = \frac{C}{f\pi}$, $r = \sqrt{C/f\pi} = \sqrt{\frac{(150\times10^4)\cancel{\pi}\ m^2/s}{5\times10^{-5}\ s^{-1}\cancel{\pi}}} = \sqrt{30\times10^9\ m^2}$

$\boxed{r = \sqrt{3}\times10^5\ m} = 173\ km$

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4. (10) Derive a thermal wind equation, by writing an equation for geostrophic balance (or geostrophic wind), differentiating it in p, and then substituting in the hydrostatic equation. Pick one component (u_g or v_g) for brevity.

$$\frac{du}{dt} = -\frac{\partial \Phi}{\partial x} + f v_g = 0 \Rightarrow v_g = \frac{1}{f}\frac{\partial \Phi}{\partial x}$$

$$\frac{\partial}{\partial p}(v_g) = \frac{1}{f}\frac{\partial}{\partial x}\left(\frac{\partial \Phi}{\partial p}\right) = \frac{1}{f}\frac{\partial}{\partial x}\left(\frac{-RT}{p}\right)$$

hydrostatic

5. (10) Sketches of balanced flow relationships

a. Sketch diagrams with contours (isopleths) of T, p, and v fields in the x-z plane, for an east-west cross section through a warm core cyclone (like a hurricane). You can sketch T' instead of T (anomaly relative to a background T(z) profile), which might make it easier to show the warm core. I will rank these during grading, so you are competing against your fellow students (ties are possible, of course). Practice on scratch paper, and label all lines.

i. Show contours of T and p together on the same diagram, with the thickness (hypsometric) relationship made clear.

ii. Show contours of p and v together on the same diagram, with gradient wind balance relationship made clear.

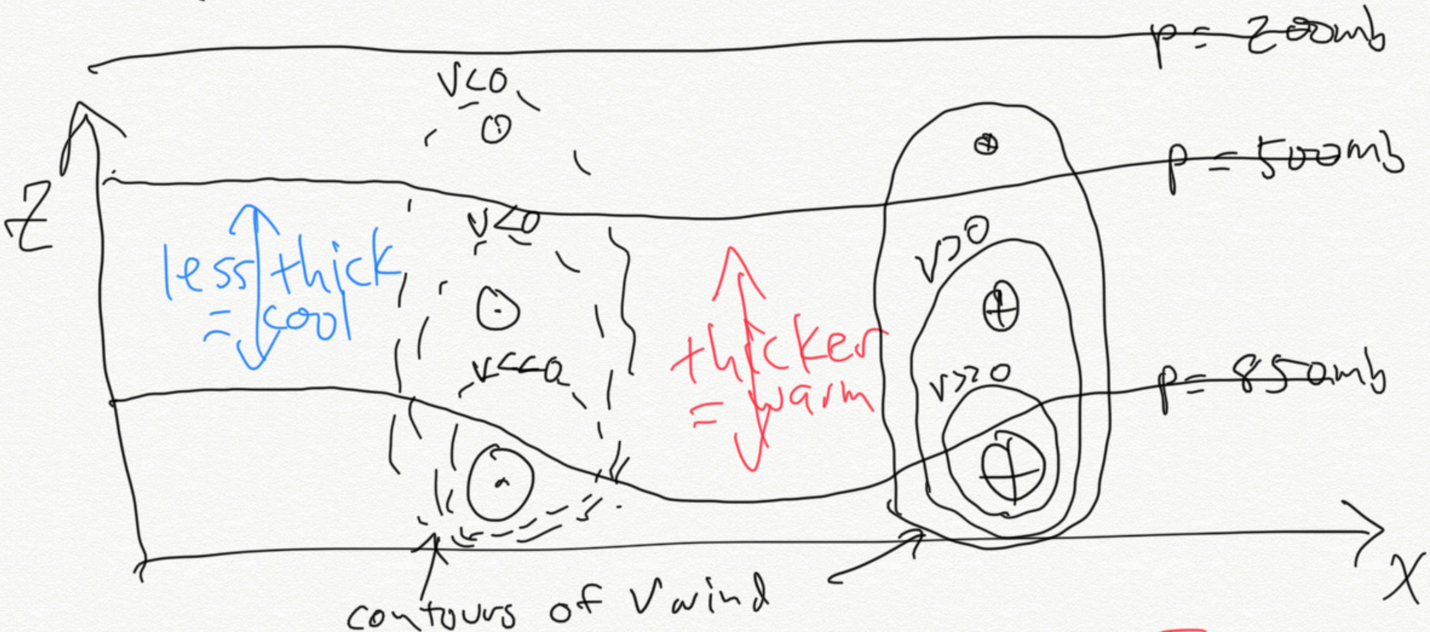
iii. Show contours of T and v together on the same diagram: the thermal wind relationship.

Or of course they could all be on one master diagram.

You may want to use carbon paper to transcribe some lines.
I couldn't find tracing paper.

Thermal wind sketches

for a warm core vortex p surfaces:



Careful! → (downward θ contours) bulge of