8:49 AM Mon Oct 5

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github.com/ATMOcanes/ATM651_2020/blob/master/Exam_and_problems/HW3.PE_hotairris...

0. Write the PE set (gathered in section 7.3.5):

a. with the 5 prognostic equations in *local change = tendencies* form, with no vector notation, using u and v and derivatives in the (x,y,p,t) coordinate system to express advection.

b. Pull the u and v equations into a single vector horizontal momentum equation, still using u and v and partial derivative notation for the advection.

c. Compress the momentum equation maximally using vector notation.

d. Write the T equation from part a. in flux form, showing with the chain rule and mass continuity that the convergence of {uT, vT, ωT} is equal to the advection terms you used in a.

e. Integrate the result of d. over the entire atmosphere from a pressure of 0 to 20000 hPa (the base of the ocean mixed layer) to obtain an equation for global warming. What happens to the transport term (expressed in flux form)? Hint: one term remains.

f. Express the continuity equation in both scalar and vector forms.

g. Express the thermodynamic equation and hydrostatic equation in terms of theta θ, the potential temperature, a conversion that uses p (which after all is a coordinate variable, freely invoked without introducing any new unknowns) so that there are still only 5 equations for 5 unknowns.

h. Advection is complicated; another way to treat it is simply to ignore it, and examine weak motions in a resting background state. That is the spirit of the following problem. Write your opening equation in the next problem based on one of your own forms above.

$$\frac{\partial T}{\partial t} = -u\frac{\partial T}{\partial x} - v\frac{\partial T}{\partial y} + \omega S_p + \frac{Q}{c_p}$$

$$-\omega\frac{\partial T}{\partial p} - \alpha\omega = -\omega\left(\frac{\partial T}{\partial p} + \alpha\right) = S_p$$

$\Downarrow$

$$\frac{\partial T}{\partial t} = -\mathrm{div}(\{uT, vT, \omega T\})$$

Convergence of flux as our ~~transport~~ transport

$$= -\frac{\partial}{\partial x}(uT) - \frac{\partial}{\partial y}(vT) - \frac{\partial}{\partial p}(\omega T)$$

$$= \left\{-u\frac{\partial T}{\partial x} - v\frac{\partial T}{\partial y} - \omega\frac{\partial T}{\partial p}\right\} - \cancel{T\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p}\right)}$$

advection part

$= 0$ by mass continuity!

Advection and convergence of flux are equal in p coords!

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$$\iiint_{\phi,\lambda,p}\left[-\nabla_p\cdot(\vec{V}T)-\frac{\partial}{\partial p}(\omega T)\right]dA\,dp$$

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$$\underbrace{\oint\!\!\oint}_{\text{Area}}\underbrace{-\nabla\cdot(\vec{V}T)}_{\substack{\text{Convergence of flux of heat}\\\text{from one place to another}}}\,dA=-\underbrace{\oint}_{\text{perimeter}}(T\vec{V})\cdot\underbrace{d\hat{n}}_{\text{normal to perimeter}}=0$$

Atmosphere has no perimeter!

adds up to zero globally! (of course; right?)

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4. Computing the surface pressure drop as mass evacuates the column

a. If the PGF $-\Delta\Phi/\Delta x$ from 3 acts for 100s, how great a wind speed away from the High will develop? ($\Delta u/\Delta t = -\Delta\Phi/\Delta x$) $1.2\,m/s$

b. If this horizontal wind speed is directed away from the 200mb High in all directions, estimate the horizontal wind divergence ($\Delta u/\Delta x + \Delta v/\Delta y$).

c. If this horizontal divergence from 4b. prevails over the upper half of the troposphere (Δp = 400mb = the 600-200mb layer), estimate the rate of change of surface pressure ($\Delta p/\Delta t = \omega$), using the mass continuity equation. Put it into 'weather' pressure drop units: mb per hour.

$$\frac{\Delta\Phi}{\Delta x} = 0.012\,m/s^2 \times 100\,s = 1.2\,m/s$$

$$\sim \frac{2\times 1.2\,m/s}{10^5\,m} \left(\frac{\Delta u}{\Delta x}\right) \approx \boxed{\text{a few} \times 10^{-5}\,s^{-1}}$$

3

~1.2 m/s in 100s!

H

5. Low-level inflow driven by surface pressure drop, and midlevel rising motion

a. How long must the rate of surface pressure decrease in 4c. act before the depression of the 1000mb surface $\Delta\Phi_{...}$ forms a Low that equal in strength to the upper level High the upward bulge in the 200mb surface $\Delta\Phi_{...}$ from 2)? Note that 1mb in surface pressure is about equal to 10m in Z....

b. After the time scale computed in (a), the upper-level divergence will be matched by low-level convergence, with upward motion in the middle troposphere. Estimate the vertical *velocity* (in m/s) in the middle troposphere using the mass continuity equation. (Hint: ω at 600mb is the same as the answer to 4c, can you see and explain why? Now you just need to convert that to m/s using the hydrostatic relation, answer is ~20m = 1mb in the middle troposphere.)

$$W_s = \int_0^{P_s} \vec{\nabla}\cdot\vec{v}\,dp$$

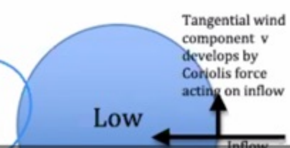
a few × $10^{-5}\,s^{-1}$,

"over a layer depth of 400 mb"

$$W_s = \frac{\partial P_s}{\partial t} = 4\times10^4\,Pa \cdot 3\times10^{-5}\,s^{-1} = 10^5 \cdot 10^{-5} = 1\,\frac{Pa}{s}$$

6. Spinup of low-level inflow by Coriolis

Suppose the low-level inflowing velocity u is about equal to the upper-level outflowing velocity computed in 4a. Suppose the Coriolis force act on it for 20000s (a few hours, a typical lifetime of such a 100km sized rain storm) How big a tangential velocity v is generated at the end of the 6h?



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