Homework 6, MSC 405 Spring 2009

This problem set walks through the primitive equations process for rising of heated air (see attached ppt file), in response to a 100 km patch of latent heating in an initially motionless atmosphere.

1. Suppose the heating J corresponds to the latent heat released during 1cm of rain over a 100km x 100km area. Let's figure out how much ΔT that causes.

a. What is the volume of water condensed (m^3)?

100km x 100km x 1cm = 10^8 cubic meters

b. What is the mass of water condensed (kg)?

density of water is 1000kg per cubic meter → 10^11 kg

c. How much latent heat is released (Joules)? ($L = 2.5 \times 10^6$ J / kg of water).

2.5 e17 = 2.5 x 10^17 Joules

d. How many kg of air are in a 1 square meter column of atmosphere, if surface pressure p is 1000mb? (pressure is weight, so mass=p/g)

m = p/g = 10^5 Pa/9.8 = 10^4 kg per square meter

e. How many kg of air are therefore in a 100km x 100km column?

Total heated air mass = 10^4 kg/m^2 x (100km)^2 = 10^14 kg of air

f. How many K can the heat from c. warm the air mass from e.? $C_p \sim 1000$ J/(kg K)

Dividing the available Joules by mass of air and Cp, DT = 2.5 K

2. Moving to the hydrostatic equation now: If the ΔT result from 1f applies to the 1000-200mb layer, roughly what $\Delta\Phi$ is generated at 200mb if the height (geopotential) of the 1000mb surface is unchanged. That is, surface pressure is still constant in this initially motionless atmosphere - the heated column just gets thicker (taller). You can use a crude form of the equation with a layer-mean p on the right: $\Delta\Phi/(-800\text{mb}) = -RT/(600\text{mb})$

Plugging in R=287, the column gets thicker by about 100m.

3. On to the momentum equation.

a. If the upper-level High from 2 has horizontal gradients on a scale of $\Delta x \sim 100$ km, estimate the magnitude of the outward pressure gradient force ($\nabla\Phi = \Delta\Phi/\Delta x$) at 200mb.

About 0.01 m/s/s

b. Express result 3a in m/s per hour.

36 m/s per hour

4. Computing the surface pressure drop as mass evacuates the column

a. If the PGF $-\Delta\Phi/\Delta x$ from 3 acts for 100s, how great a wind speed away from the High will develop? ($\Delta u/\Delta t = -\Delta\Phi/\Delta x$)

about 1m/s

b. If this horizontal wind speed is directed away from the High in all directions, estimate the horizontal wind divergence ($\Delta u/\Delta x + \Delta v/\Delta y$).

DU/DX ~ (2m/s)/(200km), DV/DT is same, so divergence ~ 2 x 10^(-5) /s

c. If this horizontal divergence from 4b. prevails over the upper half of the troposphere (Δp = 400mb = the 600-200mb layer), estimate the rate of change of surface pressure ($\Delta p/\Delta t = \omega$), using the mass continuity equation. Put it into 'weather' units: mb per hour.

Omega = (dp)*(div) = 40000Pa * the above = 0.8 Pa/s = 0.8*36 mb/h ~30mb/h.

5. Low-level inflow driven by surface pressure drop, and midlevel rising motion

a. How long must the rate of surface pressure decrease in 4c. act before the depression of the 1000mb surface $\Delta\Phi_{1000}$ forms a Low that equal in strength to the upper level High the upward bulge in the 200mb surface $\Delta\Phi_{200}$ from 2)?

Have to convert surface pressure changes to Phi1000 or Z1000 changes using the hydrostatic equation. 1mb is about equal to 10m. Think of that next time you ride an elevator! So our 100m thickness change from heating corresponds to about a 10mb SLP depression. That takes **about half an hour** at the pressure fall rate from 4b.

b. After the time scale computed in (a), the upper-level divergence will be matched by low-level convergence, with upward motion in the middle troposphere. Estimate the vertical *velocity* (in m/s) in the middle troposphere using the mass continuity equation. (Hint: ω in the middle troposphere is the same as the answer to 4c, can you see and explain why? Now you just need to convert that to m/s using the hydrostatic relation.)

omega = Dp/Dt under this upper level divergence was already computed (4c). We just convert to Dz/Dt using the z to p conversion above (10m = 1mb = 100Pa near the surface, or more like 20m = 1mb in the middle troposphere). **0.8 Pa/s = 0.008 mb/s = 0.16 m/s.**

6. Spinup of low-level inflow by Coriolis

Suppose the low-level inflowing velocity u is about equal to the upper-level outflowing velocity computed in 4a. Suppose the Coriolis force act on it for 20000s (a few hours, a typical lifetime of such a 100km sized rain storm) How big a tangential velocity v is generated at the end of the 6h?

Dv/Dt = -fu so DV = (20000s) x f x 1m/s **= 2 m/s or so**