"Transport" of conserved stuff

$\left(\text{Eulerian } \partial/\partial t\right)$ vs $\left(\text{Lagrangian } d/dt\right)$ views.

For definiteness, consider water vapor mass concentration (or specific humidity) q $\left(\text{kg}_v/\text{kg}_{air}\right)$

advective transport

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$$\frac{\partial q}{\partial t} = \frac{dq}{dt} \underbrace{- u\frac{\partial q}{\partial x} - v\frac{\partial q}{\partial y} - w\frac{\partial q}{\partial z}}$$

advection: $-\vec{V}\cdot\vec{\nabla} q$ "convergence of"

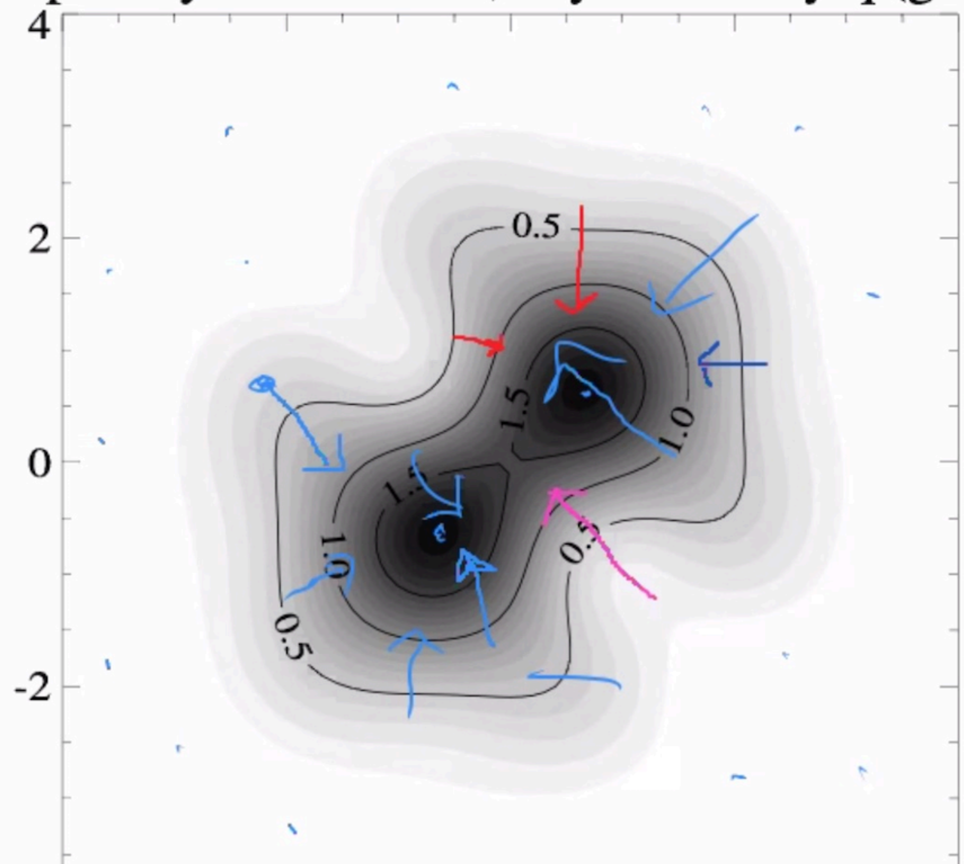
flux and its convergence

- flux of vapor = flux of mass $*\, q$
  $(\text{kg m}^{-2}\text{ s}^{-1})$
  $\boxed{(\rho\vec{V})q}$ vapor flux
- convergence of a flux
  = -divergence " " "
  $= -\vec{\nabla}\cdot(\rho q\vec{V})$ flux

"Advection" $= -\vec{V} \cdot \underbrace{\vec{\nabla} q}_{\text{gradient of } q}$

$\vec{\nabla} q = \hat{i}\boxed{\frac{\partial}{\partial x} q} + \hat{j}\frac{\partial}{\partial y} q$

A patchy scalar field, say humidity q (g/kg)

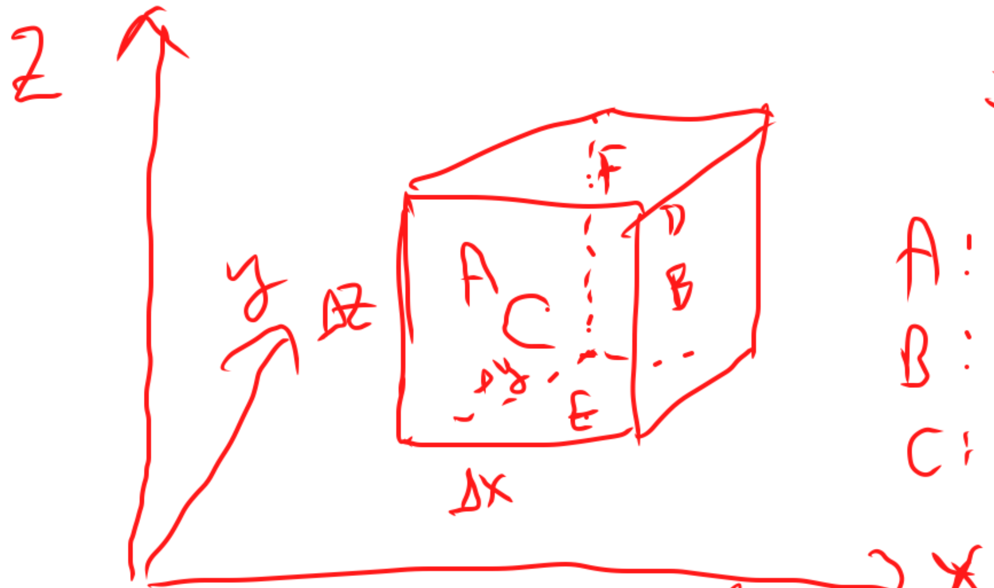


points uphill (toward higher values)

A closer look at flux convergence. Just for mass (totally conserved).

mass flux $= \rho\vec{V}$

$$= \hat{i}(\rho u) + \hat{j}(\rho v) + \hat{k}(\rho w)$$



A: west side D: north side

B: East side E: bottom

C: south side F: top.

mass accumulation in box (per unit volume) is:

$$= \frac{\Big[\underbrace{(\rho u)_A}_{\text{inflow}} - \underbrace{(\rho u)_B}_{\text{outflow}}\Big] + \Big[(\rho v)_C - (\rho v)_D\Big] + \Big[(\rho w)_E - (\rho w)_F\Big]}{\Delta x \, \Delta y \, \Delta z}$$

$\Delta x \cdot \frac{\partial}{\partial x}(\rho u)$

Taylor expansion

$$\boxed{(\rho u)_B = (\rho u)_A + (\Delta x)\cdot\frac{\partial}{\partial x}(\rho u)} + \text{terms involving } \cancel{\Delta x^2}, \cancel{\Delta x^3}, \text{etc.}$$

vanish for small Δx

Mass accumulation in box per unit volume:

$$= \frac{\left[\Delta x \cdot \frac{\partial}{\partial x}(\rho u) + \Delta y \frac{\partial}{\partial y}(\rho v) + \Delta z \frac{\partial}{\partial z}(\rho w)\right]}{\Delta x \Delta y \Delta z}$$

$$= \frac{\frac{\partial}{\partial x}(\rho u)}{\Delta y \Delta z} + \frac{\frac{\partial}{\partial y}(\rho v)}{\Delta x \Delta z} + \frac{\frac{\partial}{\partial z}(\rho w)}{\Delta x \Delta y}$$