HW2 solutions

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7.5m: (m) Veering of the wind with height within the planetary boundary layer is not necessarily an indication of warm advection. Explain.

FRICTIONAL BALANCE BECOMES GEOSTROPHIC ABOVE THE PBL: THAT IS VEERING

Chart

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7.11: Initial conditions are y = -my, decreasing linearly with latitude like the color in Fig. 7.4a.

For a conserved tracer, using subscripts for partial derivatives, yt = -uyx - vyy. In words, *local change is purely due to advection*.

Taking the x partial derivative, using the chain rule, and adding parentheses for clarity:

yxt = -u(yx)x -v(yx)y - uxyx - vxyy

The red terms are *advection of yx*. Taking those terms into the left hand side to form the total d/dt,

d/dt (yx) = - uxyx - vxyy

which for the given initial condition is equal to vxm. (*sign error in book??*)

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which for the given initial condition is equal to vym.

Interpretation: The change in the *initially purely meridional gradient*, moving with the flow (total derivative), is a *twisting into the zonal direction* by uy and a *squeezing or contraction in the meridional direction* by vy. (b) For pure deformation with vy constant, d/dt (yy) = const\*yy clearly has exponential solutions. For shear flow with constant vx, d/dt (yx) = vxm = const is linear growth.

7.19: Geostrophic wind, just formula with units: 20 m/s, everyone got it.

7.21: This is subtle because instead of d/dt being the rate of change *following the wind u,v,* we use the same notation for the rate of change *following a moving ship.* It's just the chain rule, recall: with p(x,y,t), then if xship(t),yship(t) is the position of *any frame of reference we want the result in*, the chain rule gives:

dp/dt = pt + px dx/dt + px dy/dt = pt + px uship + px vship

Everyone plugged the formulas right, given right answers from the Web, but in the old wary undergrad game of writing the least and hoping the grader can't prove whether there is confusion, I wasn't sure if this was clear to all.

7.22: windspeed is 10 m/s and is directed across the isobars from high toward low pressure at an angle 20°. Calculate the magnitude of the frictional drag force and the horizontal pressure gradient force. *Again use geometry on the diagram, and the sohcahtoa rule. Why was it tan() and cot()*? Which lengths are equal on the diagram? This seems like a geometric proof, not a given-formula number-crunch.

A picture containing antenna, stop, traffic, snow

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Balance: 0 = P + C + F

In x direction: 0 = 0 + fv - ku where k is the friction koefficient

In y direction: 0 = P - fu - kv

We are given that u2 + v2 = 100 (m/s)2 and that tan(20o) = v/u. From those 4 equations, can solve for P, k, u, v. Problem asks for P and for the frictional force k{u,v}.

7.24: Seems easy, everyone got it. Gradient wind formula must have a real square root.

7.25: Everyone could write Vg equation and take p or z derivative. The hard part was the latter, which required this key substitution: (Paige)

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7.26: Text

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Advection is the dot product -V·∇T. That involves the *cosine* of the angle between VgB and ∇T, times the strength of each vector. Since VT is perpendicular to ∇T, it is the *sine* of the angle between VgB and VT.

Notice that the thermal wind does not advect temperature (it is parallel to isotherms), so that the advection at levels 1 and 2 are equal: -Vg2·∇T = -(Vg1 + VT)·∇T = -Vg1·∇T

7.42: the concept of propagation is the *tendency leading the anomaly by a quarter-wavelength* (or 90 degrees of phase). With two fields (vectors, contours), propagation involves showing that *both fields progress in the same direction*: vectors change by the PGF (acceleration), and heights change by the divergence or convergence of mass.

Diagram, engineering drawing

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HW3: Write the PE set in various forms (part 0), then plug in numbers for latent-heated air rising for 6h (problems from 1.)

0. Write the PE set (gathered in section 7.3.5):

a. with the 5 prognostic equations in *local change = tendencies* form, with no vector notation, using u and v and derivatives in the (x,y,p,t) coordinate system to express advection.

b. Pull the u and v equations into a single vector horizontal momentum equation, still using u and v and partial derivative notation for the advection.

c. Compress the momentum equation maximally using vector notation.

d. Write the T equation from part a. in flux form, showing with the chain rule and mass continuity that the convergence of {uT, vT, wT} is equal to the advection terms you used in a.

e. Integrate the result of d. over the entire atmosphere from a pressure of 0 to 20000 hPa (the base of the ocean mixed layer) to obtain an equation for global warming. What happens to the transport term (expressed in flux form)? Hint: one term remains.

f. Express the continuity equation in both scalar and vector forms.

g. Express the thermodynamic equation and hydrostatic equation in terms of theta q, the potential temperature, a conversion that uses p (which after all is a coordinate variable, freely invoked without introducing any new unknowns) so that there are still only 5 equations for 5 unknowns.

h. Advection is complicated; another way to treat it is simply to ignore it, and examine weak motions in a resting background state. That is the spirit of the following problem. Write your opening equation in the next problem based on one of your own forms above.