① $-\omega \frac{\partial \theta}{\partial p} \frac{T}{\theta} = -\omega \frac{\partial}{\partial p}\left[T\left(\frac{R}{p}\right)^{R/c_p}\right]\frac{T}{\theta}$

$= -\omega\left[T p_0^{R/c_p}\frac{\partial}{\partial p}\left(\frac{1}{p}\right)^{R/c_p} + \left(\frac{p_0}{p}\right)^{R/c_p}\frac{\partial T}{\partial p}\right]\frac{T}{T\left(\frac{p_0}{p}\right)^{R/c_p}}$

$= -\omega\left[T p^{R/c_p}\frac{\partial}{\partial p}\left(\frac{1}{p}\right)^{R/c_p} + \frac{\partial T}{\partial p}\right]$

$= -\omega\left[T p^{R/c_p}\left(-\frac{R}{c_p}p^{-\frac{R}{c_p}-1}\right) + \frac{\partial T}{\partial p}\right]$

$= -\omega\left[T\left(-\frac{R}{c_p}\right)\frac{1}{p} + \frac{\partial T}{\partial p}\right] = -\omega\left[T\left(\frac{-p}{\rho T}\right)\frac{1}{pc_p} + \frac{\partial T}{\partial p}\right]$

$= -\omega\left[-\frac{1}{\rho c_p} + \frac{\partial T}{-g\rho\partial z}\right] = -\omega\left[-\frac{1}{\rho c_p} - \frac{1}{g\rho}\frac{\partial T}{\partial z}\right]$

$= -\omega\left[-\frac{g}{c_p} - \frac{\partial T}{\partial z}\right](\rho g)$

$= -w\left[\Gamma_d - \Gamma\right]$ ✓

$T = \frac{p}{R\rho}$

Advection of θ in z- coordinates:

$$-w\frac{\partial\theta}{\partial z}\left(\frac{T}{\theta}\right)$$

$$= -w\left(\frac{T}{\theta}\right)\frac{\partial}{\partial z}\left(T\left(\frac{p_0}{p}\right)^{R/c_p}\right)$$

$$= -w\left(\frac{T}{\theta}\right)\left[\left(\frac{p_0}{p}\right)^{R/c_p}\cdot\frac{\partial T}{\partial z} + T\cdot\frac{\partial}{\partial z}\left(\frac{p_0}{p}\right)^{R/c_p}\right]$$

$$= -w\left(\frac{T}{\theta}\right)\left[\left(\frac{p_0}{p}\right)^{\kappa}\cdot\frac{\partial T}{\partial z} + T\,p_0^{\kappa}\,\frac{\partial}{\partial z}p^{-\kappa}\right]$$

$$= -w\left(\frac{T}{\theta}\right)\left[\left(\frac{p_0}{p}\right)^{\kappa}\cdot\frac{\partial T}{\partial z} + Tp_0^{\kappa}(-\kappa)p^{-\kappa-1}\cdot\frac{\partial p}{\partial z}\right]$$

$$= -w\left(\frac{T}{\theta}\right)\left[\left(\frac{p_0}{p}\right)^{\kappa}\cdot\frac{\partial T}{\partial z} - \frac{\kappa T}{p}\left(\frac{p_0}{p}\right)^{\kappa}\cdot(-\rho g)\right]$$

$$= -w\left[\frac{\partial T}{\partial z} + \frac{\kappa T}{p}\rho g\right]$$

$$= -w\left[\frac{\partial T}{\partial p}\frac{\partial p}{\partial z} + \frac{\kappa T}{p}\rho g\right]$$

$$= -w\left[\frac{\partial T}{\partial p}(-\rho g) + \frac{\kappa T}{p}\rho g\right]$$

$$= -w\rho g\left[\frac{\kappa T}{p} - \frac{\partial T}{\partial p}\right]$$

$$= \left(\frac{\kappa T}{p} - \frac{\partial T}{\partial p}\right)\omega_{//}$$ ✓

Shrestha

$\frac{\theta}{T} = \left(\frac{p_0}{p}\right)^{\kappa}$

$\frac{T}{\theta} = \left(\frac{p}{p_0}\right)^{\kappa}$