

Homework 0

Vectors: do the components math, but don't forget to use geometry (sketch graphs and remember parallel, perpendicular, $\cos\theta$, $\sin\theta$)

1. Find the resultant (net) vector \mathbf{V} by adding up its 3 parts:

a. $\mathbf{V}_1 = 3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$, $\mathbf{V}_2 = 4\mathbf{i} - \mathbf{j} - \mathbf{k}$, $\mathbf{V}_3 = -7\mathbf{i} - \mathbf{j}$

b. $\mathbf{V}_1 = \mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{V}_2 = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$, $\mathbf{V}_3 = -2\mathbf{i} + \mathbf{j} + \mathbf{k}$

2. Let $\mathbf{G} = 3\mathbf{j} - 4\mathbf{k}$, $\mathbf{H} = \mathbf{i} - \mathbf{j}$ be two forces. Determine a force \mathbf{F} such that $\mathbf{F}, \mathbf{G}, \mathbf{H}$ are in equilibrium.

Dot Product

3. Find the work W (Joules) done by a force \mathbf{F} (in Newtons) acting on a 1 kg mass while it moves from A to B (measured in meters) when:

a. $\mathbf{F} = 2\mathbf{i} + \mathbf{j}$, $A: (0, 0, 0)$, $B: (0, 1, 0)$

b. $\mathbf{F} = \mathbf{i} + 2\mathbf{j}$, $A: (4, -7, 3)$, $B: (4, -7, 8)$

c. $\mathbf{F} = \mathbf{i} + \mathbf{j} + \mathbf{k}$, $A: (1, -1, 2)$, $B: (2, 1, 3)$

4. Let $\mathbf{b} = 2\mathbf{i} + \mathbf{j}$ and $\mathbf{c} = 2\mathbf{i} + 4\mathbf{j}$. Find all the unit vectors $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ such that the angle between \mathbf{a} and \mathbf{b} is equal to the angle between \mathbf{a} and \mathbf{c} .

Cross Product

5. Concerning the so-called triple product: show that these are equal:
 $(\mathbf{a}\mathbf{b}\mathbf{c}) \equiv \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$

6. Let $\mathbf{a} \times \mathbf{b} = \mathbf{0}$ and $\mathbf{a} \cdot \mathbf{b} = 0$. Does that imply that $\mathbf{a} = \mathbf{0} = \mathbf{b}$?

7. Complete the following identities:

a. $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) =$

b. $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c} =$

8. Show that if $\mathbf{c} = \mathbf{a} \times \mathbf{b}$ and $\mathbf{b} = \mathbf{a} \times \mathbf{c}$ then $\mathbf{b} = \mathbf{0} = \mathbf{c}$.

9. Find the following:

a. $(\mathbf{i} \times \mathbf{j}) \times \mathbf{k} =$

b. $\mathbf{i} \times (\mathbf{j} \times \mathbf{k}) =$

c. $(\mathbf{i} \times \mathbf{j}) \cdot \mathbf{k} =$

d. $\mathbf{i} \cdot (\mathbf{j} \times \mathbf{k}) =$

where $\mathbf{i}, \mathbf{j}, \mathbf{k}$ are the standard unit vectors.

Vector Calculus

10. Find the first partial differential derivatives with respect to (x, y, z)

a. $\mathbf{V}(x, y, z) = x^2\mathbf{i} - z^2\mathbf{j} + y^2\mathbf{k}$

b. $\mathbf{V}(x, y, z) = e^y\mathbf{i} - e^{-z}\mathbf{j} + e^{2x}\mathbf{k}$

11. Show that if $\mathbf{V}(t) = u(t)\mathbf{i} + v(t)\mathbf{j} + w(t)\mathbf{k}$ has length 1, and its derivative $\mathbf{V}'(t) = u'(t)\mathbf{i} + v'(t)\mathbf{j} + w'(t)\mathbf{k}$ is not zero, then \mathbf{V} and \mathbf{V}' are orthogonal.

Velocity and Acceleration

12. Find the centripetal acceleration of the moon $d\mathbf{V}/dt$ towards the earth. The moon orbits the earth at a distance $R = 239,000$ miles. It takes 27.3 earth days for one complete revolution.

The Chain Rule (total derivative):

13. Find $\frac{df}{dt}$ where $f(x, y, z) = x^2 + y^2 + z^2$, and $x(t) = e^t \cos t$, $y(t) = e^t \sin t$, $z(t) = e^t$.

14. Let $f(x, y, z) = \frac{c}{r}$ denote the gravitational potential, where $c = GMm$ is a constant, and $\mathbf{r} = (x - x_0)\mathbf{i} + (y - y_0)\mathbf{j} + (z - z_0)\mathbf{k}$ is the distance vector, with magnitude r . Show that: $\nabla^2 f = 0$.

15. Find the gradient of the following functions.

a. $f(x, y) = e^x \sin y$

- i. Sketch contours of the field f in a in the 2D domain $[-1,1]$ for x and $[-\pi,\pi]$ for y . Sketch arrows showing its gradient.

b. $f(x,y) = \frac{1}{2} \ln(x^2 + y^2)$

16. Find the divergence and curl of the following vector fields \mathbf{V} :

a. $\mathbf{V} = 2x\mathbf{i} + 2y\mathbf{j}$

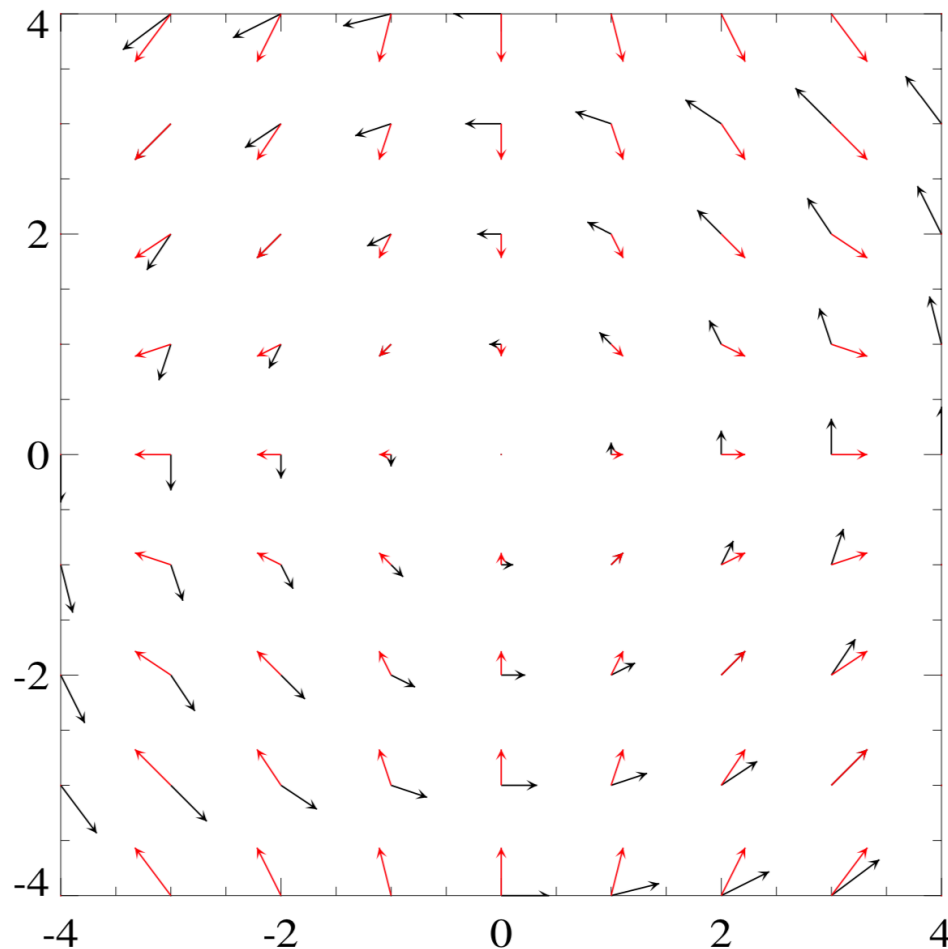
b. $\mathbf{V} = -2y\mathbf{i} + 2x\mathbf{j}$

c. $\mathbf{V} = y^2\mathbf{i}$

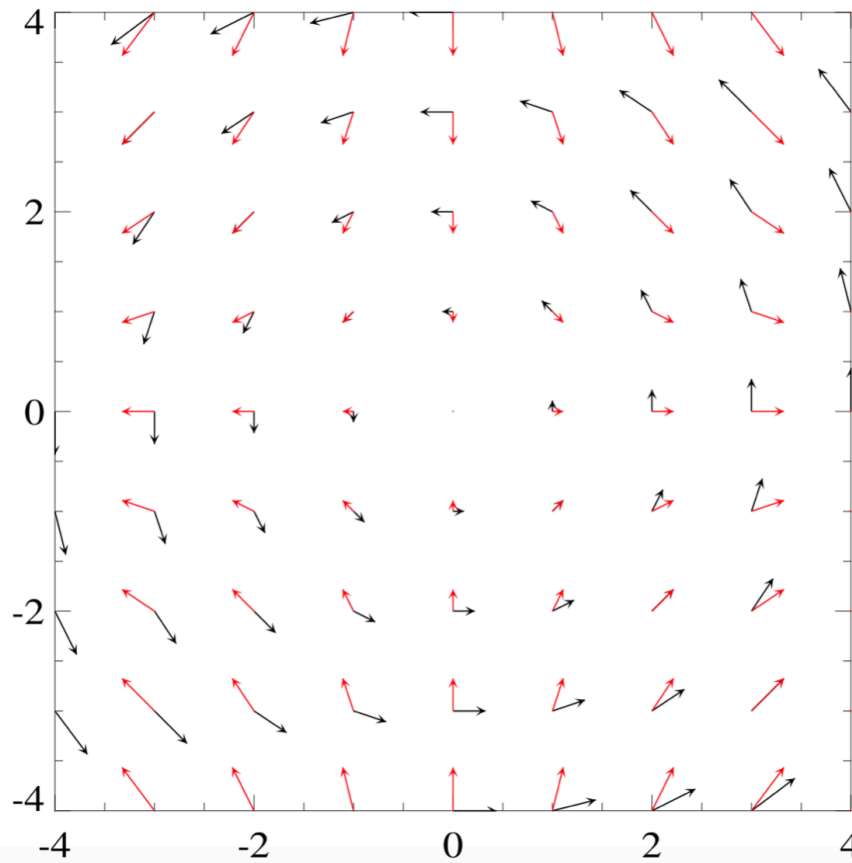
17. Show that $\nabla \times \nabla f = \mathbf{0}$ for any function $f = f(x, y, z)$.

18. Here are two vector fields: pure rotation and pure deformation. Do vector addition at each point to create the summed vector field, fill in some other points, and describe the flow.

rotation and deformation: HOMEWORK #18



19. Sketch contours of the dot (inner) product between these two vector fields.



20.

21. Sketch the curve of a function $f(t)$ whose *slope is positively proportional to its value*. Write a differential equation corresponding to those words. What is its solution mathematically?

22. Sketch the curve of a function $f(t)$ whose *slope is negatively proportional to its value*. Write a differential equation corresponding to those words. What is its solution mathematically?

23. Sketch the curve of a function $f(t)$ whose *curvature is negatively proportional to its value*. Write a differential equation corresponding to those words. What is its solution mathematically?

24. Sketch the curve of a function $f(t)$ whose *curvature is negatively proportional to its value, with that proportionality constant decreasing with time*. Write a differential equation corresponding to those words. What is its solution mathematically?

25. Sketch the curve of a function $f(t)$ whose *curvature is positively proportional to its value*. Write a differential equation corresponding to those words. What is its solution mathematically?