

Exam format:

word	symbol	nutshell meaning	longer explanation of meaning (concept) in question	Relevant sketch with arrows or little $f(x)$ curve or whatever if appropriate
divergence				
	ρ			xxxxx

...

...

Pure math concepts/words to know

Sphere:

Latitude, longitude, altitude

Zonal, meridional, vertical.

Northward, northerly; eastward, westerly

upward, altitude, pressure level (know Earth's atmosphere layers in p coordinate)

Cartesian: x, y, z i, j, k u, v, w

scalar, vector; scalar field vs. vector field

Scalar functions: domain, coordinates, range, value, curve (1D), surface (2D), field (3D, 4D)

Derivatives: slope, curvature, gradient, Laplacian. Notations for differentiation. Math definition of derivative.

dot product, cross product. For a vector, for a vector *field*.

Dot/cross using del operator (nabla symbol):

vergence (divergence, convergence)

advection $-V \cdot \nabla(T)$

Curl of vector field $\mathbf{V}(x, y, z, t)$

(vorticity if \mathbf{V} is a velocity field)

Curl of gradient vanishes precisely - why?

Nondivergent vs. irrotational decomposition of a vector field

(Sometimes called rotational and divergent)

Integral relationships (opposite of derivative) for gradient, div, curl

Stokes theorem, Gauss' theorem
vanishing of $\text{div}(\text{curl})$, vanishing of loop integral of gradient

Partial derivatives of a field $f(x,y,z,t)$

Local or Eulerian $\partial f / \partial t$

Total or Lagrangian df/dt , following a parcel at position $[x_p(t), y_p(t), z_p(t)]$

Write the relationship to the local derivative $\partial f / \partial t$ (chain rule)

Advection by $\mathbf{V} = [u,v,w] = d/dt [x_p(t), y_p(t), z_p(t)]$

Gradient of a scalar field

Vergence (divergence, convergence) of vector field $\mathbf{V}(x,y,z,t)$

Laplacian (divergence of gradient of a scalar field): *curvature*

ODEs and solutions

exponential solutions to $df/dt = -bf$

sinusoidal solutions to $d^2f/dt^2 = -c^2f$

$\exp()$ with complex numbers combines both

need boundary or initial conditions (constant of integration)

stationary or steady-state solution, equilibrium.

$df/dt = A - B$. Make steady-state assumption. Is it still a diff-eq?

PDEs and solutions: terms and concepts (for our applications)

prognostic vs. diagnostic

boundary conditions, initial conditions

inverse of Laplacian (smoothed, reversed sign)

Physical concepts/words to know

Temperature, pressure, density, wind, humidity (symbols, units)

Basic units (m, K, s, kg) – how defined (originally) from Earth and Water?

Derived units: hPa, mb, J/kg, W/m², ...

Mass, mass fractions (specific ____, mixing ratio of ____, concentration of __)

Conservation of mass (*continuity* of mass flux)

Flux of mass, multiply by specific ____ to get flux of specific ____

Conservation of specific ____

TRANSPORT:

Flux of (anything): what are the units? Flux *convergence*.

Advection: what is the sense of it (and the math)?

how are *advection* vs. *flux convergence* treatments related?

PROGNOSTIC EQUATIONS:

Governing equation, budget, tendency, Eulerian (local), Lagrangian (total)

$$d/dt(\text{something}) = 0 + \text{sources} - \text{sinks}$$

$$\partial/\partial t(\text{something}) = \text{flux convergence} + \text{sources} - \text{sinks}$$

$$\partial/\partial t(\text{something}) = \text{advection} + \text{sources} - \text{sinks}$$

Conserved, tracer

Balance: neglect some time derivative relative to other tendencies

Adjustment ("fast" process leading to restoration of balance)

Kinematics: vorticity, divergence deformation. diffluence/confluence.

recipes: shear = vorticity + deformation

Streamlines, trajectories: know the difference

Diffusion (convergence of a flux that is proportional to a gradient)

Waves

frequency, period, wavelength, wavenumber, amplitude, phase

phase velocity, group velocity

growing, decaying *amplitude* (in space or time)

growing, shrinking *scale* (expressed as wavenumber or wavelength)

Vorticity equations: $d/dt(\text{vorticity}) = 0 + \text{complications}$

Relative vorticity ζ : eliminates PGF from momentum equations

Absolute vorticity $\zeta_a = (f + \zeta)$ which RHS terms are moved to LHS?

Vortex interactions (e.g. for TC steering): 2D reasoning

$1/r$ decay of "induced" wind: $V_{\text{tan}} \propto (1/r) \zeta_{\text{rel}}$

ζ_{rel} is advected by that "induced" flow

Sketch how this plays out for 2 vortices of same/opposite sign

Equation of Motion / Newton's 2nd Law

Pressure gradient force (PGF): Enforcer of continuity

Coriolis force (as 'real' as still air on rotating Earth is 'motionless')

"Inertial forces" (advection of momentum by wind itself)

"Friction" (convergence of momentum flux by small-scale motions)

Scales of variation (m vs. km vs. 1000s of km; hours vs. days vs. months)

Running average (smoothing) isolates *large scales* (filter or grid scale)

Deviations from that are *small scales*: (subfilter, subgrid)

anomaly (in time), *eddy* (in space)

Perturbations: someone/something perturbed something

(an experiment, impact, effect, cause-effect chain)

Beyond exam but current in course

Rossby waves:

explain from $d/dt(\zeta_a)=0$ with $\beta = df/dy$

Phase velocity $c=U - \beta/k^2$: westward relative to U , long waves faster

Group velocity $c_g=U + \beta/k^2$: eastward relative to U , " " "

"downstream development" process

For stationary waves, $c_g = 2U$