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On large-scale circulations in convecting atmospheres

By KERRY A. EMANUEL[1]*, J. DAVID NEELIN[2] and CHRISTOPHER S. BRETHERTON[3]
[1]*Massachusetts Institute of Technology, USA*
[2]*University of California at Los Angeles, USA*
[3]*University of Washington, USA*

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SUMMARY

The dominant thinking about the interaction between large-scale atmospheric circulations and moist convection holds that convection acts as a heat source for the large-scale circulations, while the latter supply water vapour to the convection. We show that this idea has led to fundamental misconceptions about this interaction, and offer an alternative paradigm, based on the idea that convection is nearly in statistical equilibrium with its environment. According to the alternative paradigm, the vertical temperature profile itself, rather than the heating, is controlled by the convection, which ties the temperature directly to the subcloud-layer entropy. The understanding of large-scale circulations in convecting atmospheres can, therefore, be regarded as a problem of understanding the distribution in space and time of the subcloud-layer entropy. We show that the subcloud-layer entropy is controlled by the sea surface temperature, the surface wind speed, and the large-scale vertical velocity in the convecting layer, and demonstrate how the recognition of this control leads to a simple, physically consistent view of large-scale flows, ranging from the Hadley and Walker circulations to the 30–50-day oscillation. In particular, we argue that the direct effect of convection on large-scale circulations is to reduce by roughly an order of magnitude the effective static stability felt by such circulations, and to damp all of them.

1. INTRODUCTION

The concept of conditional instability has been central to thinking about atmospheric moist convection for more than a century. The roots of this important concept can be traced back at least as far as Espy (1841), who first identified latent-heat release as a crucial factor in the dynamics of convective clouds. The adjective *conditional* is central to this concept, for it implies that the instability is finite amplitude in nature. From the standpoint of parcel dynamics, the state of conditional instability is defined as one in which the adiabatic upward displacement of air samples results in negative buoyancy for small displacements, but positive buoyancy for sufficiently large displacements. The presence of stability to small displacements prevents spontaneous release of instability and permits the accumulation of large amounts of potential energy for convection. The notion that this reservoir of potential energy is tapped when convection is 'triggered' is also central to thinking about the development of convective clouds.

* Corresponding author: Center for Meteorology and Physical Oceanography, Massachusetts Institute of Technology, Cambridge, MA 02139, USA.

The theory of conditional instability received objective confirmation when radiosonde measurements became common in the 1930s and 1940s. Soundings such as the example presented in Fig. 1 leave little doubt that the atmosphere can exist in states of conditional instability, and the observations and numerical simulation of severe convective storms provide definitive evidence that large reservoirs of convective available potential energy (CAPE) are indeed central to the energetics of such phenomena.

In the later 1940s, observations from the Thunderstorm Project in Florida and Ohio (USA) defined the life cycle of ordinary convective showers and seemed to confirm the paradigm of convection as a response to triggering in a conditionally unstable atmosphere. Not long after the completion of this project, Riehl and Malkus (1958) published their analysis of the energetics of the tropical atmosphere. This showed that latent-heat release in tall cumulonimbus clouds is an important energy source for large-scale tropical circulations such as the Hadley cell, and embodied a view of tropical dynamics that persists today; i.e. a view in which convection is regarded as a heat source for an otherwise dry circulation. We shall argue that this 'externalization' of convective heating has had a large and unfortunate effect on thinking about the interaction of moist convection with large-scale flows, and on the formulation of representations of convection for use in numerical models.

Flaws in this 'external' view of moist convection became apparent by the early 1960s, when attempts to simulate the development of hurricanes in a conditionally unstable atmosphere met with failure (see the review by Yanai (1964)). These simulations always showed development at the smallest scales, consistent with the semi-linear theory of 'moist-up, dry-down' convection first presented by Bjerknes (1938) and later quantified by Lilly (1960). The role of surface heat fluxes, earlier emphasized as the driving mechanism of tropical cyclones by Kleinschmidt (1951) and Riehl (1954), was neglected in these analyses as well as in the theory of conditional instability of the second kind (CISK), which attempted to cure their obvious defects. This theory, introduced by Charney and Eliassen (1964), was explicitly motivated by the desire to account for the finite scale of the ascent region of developing tropical cyclones, and adhered to the concept of convection as a heat source in an otherwise unsaturated atmosphere. We shall argue here that the theory of CISK, as well as the representation of convection (Kuo 1965) that was implicitly based on it, has been an influential and lengthy dead-end road in atmospheric science.

The paradigm of convection as a response to triggering in a conditionally unstable atmosphere was ultimately challenged by Arakawa and Schubert (1974) in presenting the quasi-equilibrium closure hypothesis for moist convection. Borrowing from turbulence kinetic-energy closures in turbulence theory, they argued that the kinetic energy of the convective motions themselves responds to changes in large-scale conditions on time-scales very short compared with those characterizing the larger scales, so that the production of available energy for convection by large-scale processes approximately balances its dissipation by convection. Since the available potential energy for convection is approximately invariant under this hypothesis, there cannot be significant conversions of such potential energy to the kinetic energy of large-scale disturbances, as must happen in CISK.

It is our purpose here to compare and contrast the two paradigms of moist convection, to explore the implications of the statistical equilibrium hypothesis fully, and to present a new conceptual framework for thinking about the interaction between cumulus convection and larger-scale flows. In section 2 we review the paradigm of triggered convection and conditional instability as well as CISK and moisture budget closures for convection. The statistical equilibrium paradigm is explored in section 3, starting with the comparatively simple phenomenon of turbulent dry convection and progressing through moist convection. We outline a new conceptual framework for thinking about the interaction between moist convection and large-scale circulations in section 4 and, in section 5, present a few examples of this conceptual framework, applied to tropical circulation systems. Our conclusions are summarized in section 6.

2. Conditional instability

The sounding shown in Fig. 1 portrays a state of conditional instability for the convection of subcloud-layer air. This air, when displaced upward adiabatically, will be negatively buoyant up to its level of free convection (LFC), and positively buoyant from its LFC up to its level of neutral buoyancy. (The thermodynamic path is here assumed to be pseudo-adiabatic without accounting for freezing; other plausible paths will change the magnitude of the buoyancy but, in this case, not the qualitative attributes of the sounding.) The CAPE, which represents the maximum kinetic energy per unit mass achievable by the convection of particular samples of air (in this case the subcloud air), can be shown to be proportional to the area contained between the adiabat characterizing the ascent of subcloud air and the environment, neglecting the direct effect of water substance on density. The available potential energy (APE), defined by Lorenz (1955), and treated for the case of moist convection by Randall and Wang (1992), is the maximum amount of kinetic energy available to the entire closed system. It has been shown by Emanuel (1994) that this can be expressed as an integral of the CAPE over the mass of all the subcloud air (assuming that only it has positive values of CAPE), minus an integral of the static stability of the system. In particular, the existence of CAPE is both a necessary and sufficient condition for the existence of APE in the absence of horizontal

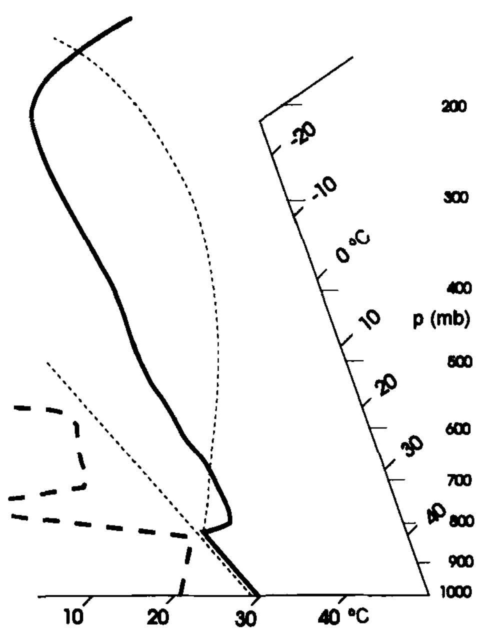

Figure 1. Thermodynamic sounding, 00 GMT 7 May 1986, at Oklahoma City, Oklahoma. Light dashed lines show dry and moist adiabats.

gradients of density, though, if the mass of air with positive CAPE is small, APE will be correspondingly small.

The existence of 'convective inhibition,' i.e. the negative buoyancy of air lifted below its LFC, is essential to the notion of conditional instability. By preventing the spontaneous release of APE, it permits the latter to accumulate under certain conditions. When this happens, the problem of predicting convection and of relating it to large-scale flows is largely one of predicting when and where the convective inhibition will disappear, or when some turbulent or mechanical process will provide enough energy to conditionally unstable parcels to overcome the potential barrier to convection. Once triggered, the convection can reach great intensity and, potentially, can act as an energy source for large scales of motion. The extreme forms of this type of convection have been simulated numerically with some success (e.g. Klemp and Wilhelmson 1978) and it is noteworthy that virtually all extant simulations of convective clouds have proceeded in the 'spin-down' mode, in which convection is triggered by an aberration of the initial condition and thereafter uses the APE of the initial state, which is not resupplied, except, perhaps, advectively. The work of Bretherton and Smolarkiewicz (1989) demonstrates that the APE consumed in this process is used to overcome turbulent dissipation, to excite internal gravity-inertia waves and to generate a quasi-balanced circulation on the scale of the deformation radius based on the depth of the cloud-bearing layer; the proportion being governed by both the details of the convective process and the nature of the large-scale environment.

In essence, the large-scale processes act to accumulate APE and may partially or completely constitute the trigger that determines where and when convection occurs; afterwards, the large-scale flow is, in this view, a passive repository for some of the energy released by the convection.

This simple basic view of convection has been widely disseminated through primary and secondary educational texts and constitutes a generally accepted conceptual framework for understanding some elementary facets of moist convection. We here simply wish to draw attention to two important aspects of this view: first, that the unambiguous existence of conditionally unstable soundings has only been rigorously established in a few continental areas, especially in North America where much of the research on severe convective storms has originated; and, second, that the progression of events described in the preceding paragraph in no sense constitutes a statistical equilibrium between convection and larger-scale flows. (For the present purpose, we consider that a sounding can only be *unambiguously* conditionally unstable if it contains a nontrivial convective inhibition, and if the mixed layer itself (as opposed to the surface layer) is positively buoyant when lifted by a conservative thermodynamic process, such as a reversible adiabatic displacement that includes the weight of condensed water. We shall return to the general problem of assessing conditional instability in section 3.)

At roughly the same time that the theory of moist convection and conditional instability was being developed, there was growing interest in the role of moist convection in large-scale circulations, particularly tropical cyclones and the Hadley circulation. Some of the early theories of tropical cyclogenesis, reviewed by Yanai (1964), attempted to explain the cyclogenesis as a particular mode of release of conditional instability, largely ignoring the earlier work of Kleinschmidt (1951) and Riehl (1954) who had emphasized the importance of the thermodynamic interaction with the ocean. These theories invariably predicted that the ascent region would occupy horizontal scales no greater than the depth of the troposphere, failing to distinguish the cyclone-scale circulation from the cumulus clouds themselves. It was largely this problem that Charney and Eliassen (hereafter CE) addressed in their development of the CISK theory in 1964.

It is instructive to review some of the important features of this enormously influential paper. CE explicitly recognize the problem of scale selection:

> '. . . we should look upon the pre-hurricane depression and the cumulus cell not as competing for the same energy, for in this competition the cumulus cell must win . . .'

and go on to suggest the solution to this problem:

> '. . . rather, we should consider the two as supporting one another—the cumulus cell by supplying the heat energy for driving the depression, and the depression by producing the low-level convergence of moisture into the cumulus cell.'

Note that 'convergence of moisture' is an advective process; by implying that a disturbance could amplify in an initially horizontally homogeneous atmosphere through strictly advective processes, CE implicitly assume that the APE contained in the conditionally unstable atmosphere can be converted to the kinetic energy of the large-scale disturbance. But perhaps the most important statement made by CE is one that contains within it an expression of the dominant mode of thinking about moist convection:

> 'The most striking characteristic of the pre-hurricane depression as well as of the hurricane is the enormous rainfall in the region of low-level convergence; the latent heat energy released is two orders of magnitude greater than the amount needed to maintain the kinetic energy against frictional dissipation.'

Here we have a clear statement of an important misconception in atmospheric science: the implication that heating *per se* leads to production of kinetic energy; the elementary requirement that heating and temperature fluctuations must be positively correlated in order that disturbance energy be produced seems to have been overlooked here. This misconception may be regarded as inherent to the 'external' view of cumulus heating, since an externally applied heat source *will*, in general, produce temperature perturbations that are positively correlated with the heating. But in real atmospheric circulations in which clouds interact with their environment, this correlation can be by no means taken for granted; indeed we shall present some evidence that it is often negative. The last quoted statement by CE is, in an important sense, wrong, as they have failed to consider the thermodynamic efficiency of heat engines.

It would be difficult to overstate the degree to which this misconception pervades atmospheric science. Here are a few examples taken from widely used textbooks:

> Textbook 1: Large-scale motions in the equatorial zone are driven primarily by latent-heat release.
>
> Textbook 2: The hurricane winds are generated and maintained by the condensation of water vapour within a uniform, tropical air mass.

In the tropics, large-scale upward motion is certainly associated with latent-heat release, but the latent heating is nearly balanced by the combination of radiative and adiabatic cooling. Whether or not this heating is positively correlated with temperature is a far more difficult question to answer, and is critical for understanding the dynamics of the convecting atmosphere. In section 4 we shall present a new way of thinking about this interaction, which mostly bypasses the need to consider the heating. For now, we simply wish to point out that small errors in the determination of cumulus heating may lead to large errors in temperature, and thus to large errors in the dynamics of circulations in the convecting atmosphere. This problem occurs prominently in representations of convection based on moisture budgets, which have been used to support the existence of CISK. We here offer a brief critique of such representations.

Kuo-type convective parametrizations (Kuo 1965, 1974) are based on the observation that precipitation is nearly equal to column-moisture convergence in the tropics (Krishnamurti *et al.* 1980). The vertically integrated water-conservation equation may be written

$$\int_0^\infty \frac{\partial \rho q}{\partial t} \mathrm{d}z = -\int_0^\infty \nabla \cdot \rho q \mathbf{V}_\mathrm{H} \, \mathrm{d}z + E_\mathrm{s} - P \tag{1}$$

where q is the specific humidity, ρ is density, $\mathbf{V}_\mathrm{H}$ is the horizontal component of the total velocity vector (the vertical velocity is assumed to vanish at the surface and at the top of the atmosphere), E_s is the surface evaporation, and P is the precipitation reaching the surface. It has been assumed that the storage of condensed water is negligible. The assumption made by Kuo (1974) is that in conditionally unstable air, the left side of (1) is proportional by a small number to the terms on the right; this amounts to a bulk statistical-equilibrium assumption on water vapour and implies, as the observations show, that the difference between surface evaporation and precipitation is nearly balanced by column-moisture convergence. Note that by this assumption, the predictability of water vapour is compromised in convecting columns. This assumption may be stated:

$$P = \beta\left(E_\mathrm{s} - \int_0^\infty \nabla \cdot \rho q \mathbf{V}_\mathrm{H} \, \mathrm{d}z\right) \tag{2}$$

where β is a constant that is, in general, less than but close to unity. (Later formulations (e.g. Anthes 1977) make β a function of certain large-scale conditions such as relative humidity.) From (1), this directly implies that

$$\int_0^\infty \frac{\partial \rho q}{\partial t} \mathrm{d}z = \frac{1-\beta}{\beta} P. \tag{3}$$

Since β is usually close to 1, it is possible for (2) to be well-verified observationally while at the same time there is little observational support for (3). It is immediately obvious from (3) that Kuo schemes fail an important test: There can be no radiative–convective equilibrium unless $\beta = 1$, which in most implementations of Kuo-type schemes requires large-scale saturation through the entire column. This inability to simulate realistic radiative–convective equilibrium implies that even for spatially homogeneous conditions, the Kuo-type schemes must either result in a saturated atmosphere or must produce disturbances in the large-scale flow, rather than producing steady vertical transports by the ensemble average of cumulus-scale motions.

Knowledge of the precipitation from (2) also gives the vertically integrated heating, through the first law of thermodynamics. This is then distributed vertically in proportion to the departure of the actual sounding from a moist adiabat. (Similarly, the vertically integrated moistening given by (3) is distributed according to the degree of subsaturation of the atmosphere.)

The closure on the vertically integrated heating given by (2) emphasizes the importance of the heating itself, and is subject to large errors resulting from the fact that all the dynamics depend on very small percentage differences between convective heating and large-scale adiabatic and radiative cooling. The actual degree of conditional instability has no effect on the magnitude of the heating, though implementations of such schemes usually contain switches that prevent convective heating in the absence of conditional instability. Once the atmosphere becomes infinitesimally unstable, however, the scheme may 'kick in' with finite heating, resulting in a noisy integration. A negative value of moisture convergence (the right-hand side of (2)) also switches the scheme off, though with no discontinuity in the heating.

We wish to emphasize two essential points about Kuo-type schemes. First, by closing on the moisture budget, the predictability of water is compromised severely. Second, the nature of the relationship between heating and moisture convergence permits conditional instability to accumulate under certain conditions in which the right-hand side of (2) is negative or only weakly positive, encouraging an active exchange between disturbance available potential and kinetic energies. This is the fundamental energetic process in CISK. Indeed, the Kuo closure is closely related to that used by CE to demonstrate CISK. It largely ignores the fact that convection is a response to instability.

3. Statistical equilibrium

The notion of statistical equilibrium rests upon the idea that the time-scale of some chaotic, small-scale process is appreciably smaller than that of the large-scale process with which it interacts. Put a slightly different way, statistical equilibrium implies that the large-scale process of interest evolves slowly compared with the small-scale process one wishes to regard in an ensemble-integrated sense. It is instructive to examine the nature of convective statistical equilibrium in a few, relatively simple, cases.

As a first example we use the classical Prandtl convective boundary layer, consisting of a semi-infinite, Boussinesq fluid overlying an infinite horizontal plate. The fluid is cooled at a constant, infinitesimal rate such that the volume-integrated cooling is constant, and the plate is held at a fixed temperature. The lower plate must be considered dynamically rough, so that molecular diffusivities may be neglected, but we will consider statistical aspects of the flow at distances from the plate that are large compared with the roughness scale. On physical grounds, we expect the fluid to convect turbulently.

An important controlling parameter in the problem is the volume-integrated sink of buoyancy:

$$Q_0 \equiv -\int_0^\infty g\gamma\dot{T}\,\mathrm{d}z \tag{4}$$

where g is the acceleration of gravity, γ is the coefficient of thermal expansion, and $\dot{T}$ is the imposed infinitesimal heating rate, which is negative (Q_0 is a positive quantity). In strict statistical equilibrium, this must also equal the surface buoyancy flux per unit area.

The turbulence kinetic-energy (TKE) equation for this system may be written

$$\frac{\partial}{\partial t}(\mathrm{TKE}) = \overline{w'B'} - \mathcal{D} \tag{5}$$

where $\mathcal{D}$ is the dissipation rate, and $\overline{w'B'}$ is the mean correlation of vertical velocity and buoyancy. (There is no mean shear in this problem.) On dimensional grounds, the dissipation rate is generally assumed to be related to the TKE by

$$\mathcal{D} = \frac{c_1(\mathrm{TKE})^{3/2}}{z} \tag{6}$$

where c_1 is some numerical constant. Mixing-length theory relates $\overline{w'B'}$ to a velocity-scale and a length-scale by

$$\overline{w'B'} = -c_2 z(\mathrm{TKE})^{1/2} g\gamma\frac{\mathrm{d}\overline{T}}{\mathrm{d}z} \tag{7}$$

where c_2 is another constant and $\overline{T}$ is the mean temperature. A statistical equilibrium postulate on (5) equates dissipation to buoyant production of TKE, on the grounds that the TKE responds quickly to slowly varying Q_0. Thus if Q_0 varies slowly compared with turbulence time-scales, the left-hand side of (5) may be regarded as small. Then, equating (7) to (6) gives a TKE scale:

$$\mathrm{TKE} \simeq -\frac{c_2}{c_1} z^2 g\gamma \frac{\mathrm{d}\overline{T}}{\mathrm{d}z}. \tag{8}$$

On the other hand, if the mean temperature of the fluid varies slowly in time, then the convergence of the heat flux must be small and $\overline{w'B'} \simeq Q_0$. Using this in (7) and also using (8), we have

$$g\gamma \frac{\mathrm{d}\overline{T}}{\mathrm{d}z} = -c_1^{1/3} c_2^{-1} Q_0^{2/3} z^{-4/3} \tag{9}$$

and

$$\mathrm{TKE} = c_1^{-2/3} Q_0^{2/3} z^{2/3}. \tag{10}$$

Note that both (9) and (10) could have been written from dimensional considerations alone in this one-parameter problem, but by going through this physical argument we can understand how this may be extended to the case of slowly varying Q_0. Given that the time-scale τ of turbulence varies as

$$\tau \sim Q_0^{-1/3} z^{2/3}$$

and the length-scale varies as z, it is clear that the time-scale of Q_0 variability must be much longer than τ, and the space-scale of Q_0 variations much longer than z, for statistical equilibrium to be a good approximation.

Note that this particular physically based derivation of (9) and (10) depends on the twin premises of slowly varying mean temperature and slowly varying surface buoyancy flux, which are of course strongly related in this example.

Now consider a slight variant on this system, illustrated in Fig. 2(a), in which there is an insulated upper boundary, and a horizontal temperature difference, ΔT, is imposed on the lower plate on a scale much larger than the depth of the system. To a first approximation, one might expect that the one-dimensional solution described above applies locally to each column and, on dynamical grounds, one would suppose that a thermally direct circulation would appear, superimposed on the convective motions. Provided that the horizontal heat flux carried by the circulation is weak compared with the vertical convective heat flux, the stabilization effected by the large-scale circulation will be insufficient to counter the destabilization by the interior cooling, and (9) and (10) would still be expected to apply. But if the horizontal heat flux carried by the large-scale circulation is not small, one would imagine that the circumstance illustrated in Fig. 2(b) would result. Here, the thermally direct flow succeeds in establishing a stable thermal stratification, except in a convective boundary layer, and the interior cooling is balanced by subsidence. On the other hand, convection would still occur in the ascent region and in the boundary layer. An interesting question is the extent to which (9) and (10) may still apply in the convective regions of the flow. Perhaps, if the thermally direct circulation is very strong, it may become difficult to distinguish between the 'large-scale' and 'convective' motions.

Very few would describe the large-scale circulations in Fig. 2 as the result of a cooperative interaction between the large-scale flow and the small-scale convection, even though the latter is undeniably important to the dynamics of the system. Fundamentally,

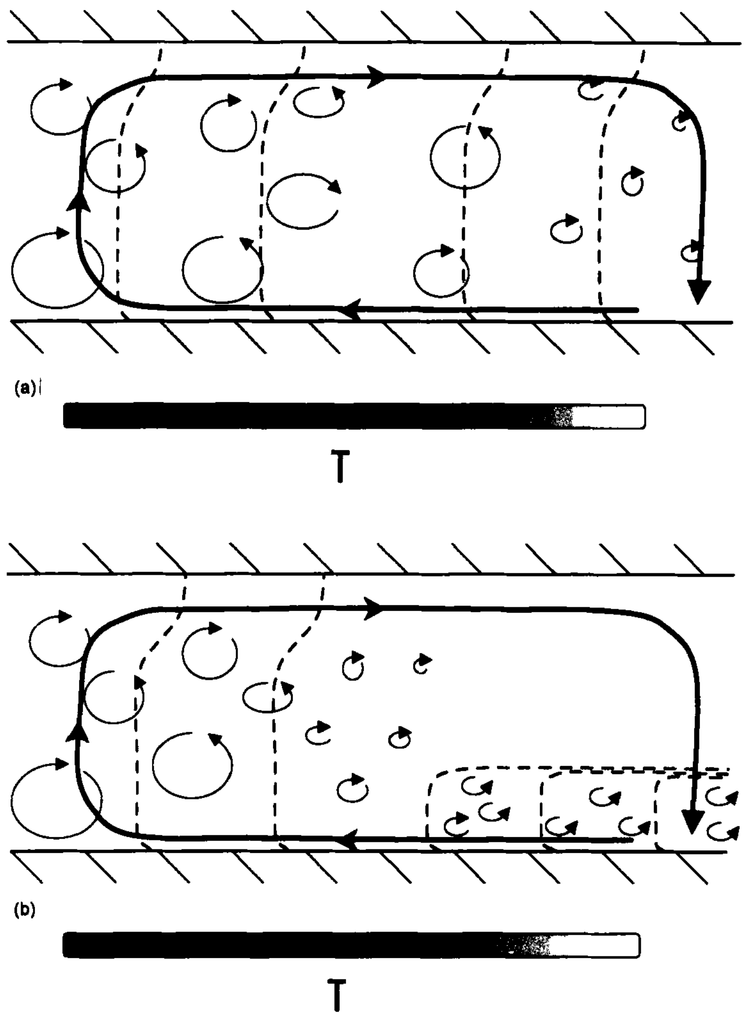

Figure 2. (a) Small-scale convection and large-scale circulation in a parallel-plate apparatus with an imposed horizontal temperature gradient at the boundaries. It is understood that the width of this domain is very much greater than its depth. The dashed lines show contours of temperature, with higher values at the left. In this case the large-scale circulation is not strong enough to suppress convection over the colder side of the apparatus. (b) As (a) but for a case in which the imposed horizontal temperature gradient is strong enough to drive a large-scale circulation that suppresses convection over the colder side of the domain. An inversion-capped boundary layer forms over the colder part of the apparatus in this case.

the large-scale circulation is driven by the imposed horizontal temperature gradient. Provided the time-scales associated with the large-scale flow are large compared with those of the convection, one is entitled to continue to assume that the consumption of APE by convection is equal to its supply by the boundary heating and interior cooling, and by the large-scale circulation.

A natural way to represent the effects of the small-scale convection in this example (neglecting possibly important effects due to convective momentum transport) is simply to assume that the vertical lapse rate of mean temperature, given by (9), applies wherever the large-scale processes are trying to produce an unstable lapse rate. It is worth noting that atmospheric measurements of the constants in (9) (for example see Stull (1988)) give equilibrium lapse rates of potential temperature of only about 0.05 K km^{-1} at an altitude of 100 m, showing that equilibrium lapse rates should be very close to adiabatic, except very near the surface. On a certain level of approximation, the convection determines the vertical lapse rate, which in turn affects the geometry and intensity of the large-scale circulation, but in no sense does it drive it. An easy way to think about the macro-effect of convection is to consider that it maintains a fixed lapse rate of temperature and that the magnitude of the temperature is controlled by both the boundary temperature and the magnitude of the boundary heat flux.

The notion of statistical equilibrium was first applied systematically to the case of moist convection by Arakawa and Schubert (1974). They noted that the fastest energetic processes acting in convective clouds are the conversion of APE to kinetic energy and the dissipation of kinetic energy. Clouds typically develop and decay on time-scales of hours, and it is reasonable to expect that the kinetic energy of ensembles of convective clouds in statistical equilibrium with their large-scale environment responds to changing large-scale conditions on a similar time-scale. Arakawa and Schubert proposed that convective statistical equilibrium can be characterized by a near equality between the rate of production of available energy by the large-scale processes and its consumption and destruction within convective clouds. They called this particular description of statistical equilibrium *quasi-equilibrium.* It is important to recognize that this represents a statistical equilibrium argument applied to a measure of *energy* (and therefore buoyancy) as opposed to water or some other quantity. Its mathematical form can be described as follows. First, use λ to denote a particular subensemble of convective cloud characterized by, say, its characteristic cloud-top altitude. Then the rate at which the subensemble consumes energy is

$$-\left(\frac{\mathrm{d}A_\lambda}{\mathrm{d}t}\right)_\mathrm{c} = \int_{z_{\mathrm{b}_\lambda}}^{z_{\mathrm{t}_\lambda}} M_\lambda B_\lambda \,\mathrm{d}z \tag{11}$$

where A_λ is the 'cloud work function', M_λ is the vertical mass flux by clouds of type λ, B_λ is the buoyancy in clouds of type λ, and z_{b_λ} and z_{t_λ} are the cloud base and cloud top altitudes of clouds of type λ.*

The rate at which the large-scale processes and clouds other than type λ generate potential energy for clouds is given symbolically by

$$\left(\frac{\mathrm{d}A_\lambda}{\mathrm{d}t}\right)_{\mathrm{LS},\lambda'} = F(\lambda) - \sum_{\lambda \neq \lambda'} \int_{z_{\mathrm{b}_{\lambda'}}}^{z_{\mathrm{t}_{\lambda'}}} J(\lambda, \lambda')M(\lambda')\,\mathrm{d}z \tag{12}$$

where $F(\lambda)$ is the rate at which large-scale processes generate potential energy for clouds of type λ, and $J(\lambda, \lambda')$ is the effect of clouds of type λ' on the energy available to clouds of type λ. The quasi-equilibrium postulate simply asserts that the rate of consumption (11) is equal to minus the rate of generation (12):

$$F(\lambda) = \sum_{\lambda} \int_{z_{\mathrm{b}_{\lambda'}}}^{z_{\mathrm{t}_\lambda}} K(\lambda, \lambda')M(\lambda')\,\mathrm{d}z \tag{13}$$

where

$$K(\lambda, \lambda') = J(\lambda, \lambda') + B_\lambda \delta(\lambda, \lambda').$$

* Note that since the integrand of (11) is a quadratic, there is an implicit neglect of variations of M and B within λ-type clouds themselves; this may be a serious limitation, given the observed inhomogeneities of actual clouds. If, on the other hand, λ is identified with individual draughts, (11) is accurate.

Given $F(\lambda)$ for all λ, and a cloud model that determines B_λ, (13) can, in principle, be inverted to find $M(\lambda)$. The kernel $K(\lambda, \lambda')$ will depend strongly on the cloud model used. Arakawa and Schubert assumed that a continuously entraining plume was a good description of a cumulus cloud, though later observations (e.g. Paluch 1979; LaMontagne and Telford 1983; Taylor and Baker 1991) have cast very serious doubts on this. Moreover, the inversion of (13) will in general yield some values of $M(\lambda)$ that are negative, which is unphysical for the plume model. As demonstrated by Lord (1982), this problem can be circumvented in practice by minimizing the difference between the left and right sides of (13), subject to the constraint that $M(\lambda) \geqslant 0$ for all λ. In addition to these problems, more complex cloud models may not yield a unique set of $M(\lambda)$ for a given forcing, especially when downdraughts are included. Since the publication of the Arakawa–Schubert paper, downdraughts have also been found to play an important role in stabilizing the atmosphere (e.g. Cheng 1989), and have been incorporated in the Arakawa–Schubert framework (Arakawa and Cheng 1993).

The basic premise of Arakawa and Schubert, that the rate of change of available energy for convection should be nearly zero, is very well verified in observations (Arakawa and Chen 1987; Arakawa 1993). Figure 3 shows an example of a calculation of the rate of change of available energy by large-scale processes alone, made from an analysis of Marshall Islands sounding data from Yanai *et al.* (1973), versus the observed rate of change of the available energy. It is obvious that the observed tendency of energy is very substantially smaller than that due to large-scale processes acting unopposed. To put things in perspective, the surface fluxes and radiative cooling of the tropical atmosphere alone act to generate about 4000 J kg^{-1} of available energy each day, whereas actual values of CAPE are usually less than 1000 J kg^{-1}.

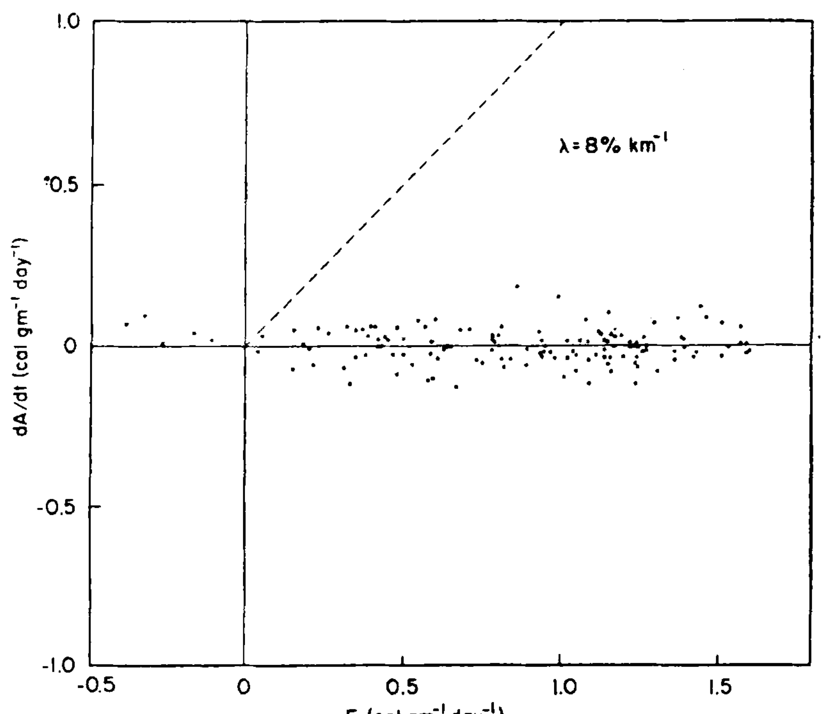

Figure 3. Observed rate of change of cloud work function A (ordinate) plotted against rate of change of A due to large-scale processes only, for a particular value of the entrainment rate. (From Arakawa and Schubert (1974).)

Other evidence of statistical equilibrium comes from observations of the thermodynamic structure of convecting atmospheres (e.g. Betts 1982; Xu and Emanuel 1989). Recall that in the case of dry convection, statistical-equilibrium lapse rates of potential temperature become negligible a short distance above the surface. Unfortunately, an equivalent theory for lapse rates in moist convecting atmospheres has not yet been developed. But the observation of time-invariant energy suggests that some universal thermodynamic profile should characterize moist convecting atmospheres. Betts (1982) first pointed out that the lower tropical troposphere follows a virtual adiabat, defined as the virtual temperature of air lifted reversibly (i.e. carrying the condensed water with the air) from the subcloud layer. Xu and Emanuel (1989) looked at soundings from the tropical western Pacific and found that they could not reject the hypothesis that the tropical atmosphere is exactly neutral to the reversible displacement of air lifted from near the top of the subcloud layer, where and when it is not decidedly stable. This supports the idea that the available energy to convection is nearly invariant in convecting regions. Williams and Rennó (1993) pointed out that this observation should not be interpreted to imply that the tropical atmosphere is literally neutral to convection, since freezing of condensed water will make the soundings unstable. But this in no way changes the conclusion that *the virtual temperature of the convecting atmosphere is uniquely related to the subcloud-layer entropy*. As emphasized by Williams and Rennó, this state does in general contain some potential energy, but the near invariance of this energy content (calculated by any reasonable means) shows that there is little or no conversion of the potential energy to kinetic energy of *large-scale* disturbances. The observed invariance implies, as indicated by Arakawa and Schubert, that the potential energy is consumed by cumulus clouds. But, as of this writing, there is no generally accepted theory for the actual value of CAPE in statistical-equilibrium convecting atmospheres.

4. Statistical-equilibrium thinking

We advocate a mode of thinking about large-scale circulations in convecting atmospheres that rests upon four major points:

1. Textbook examples of conditionally unstable atmospheres, with substantial inhibition to convection, are in reality unusual.
2. In all other cases of moist convecting atmospheres, the amount of CAPE, while not zero, is approximately invariant.
3. This implies that the virtual temperature of convecting atmospheres has approximately a one-to-one relationship with subcloud-layer entropy, θ_e, and that predicting the response of subcloud θ_e to large-scale disturbances would, therefore, account for the major effects of convection on such disturbances.
4. It is nonetheless important to account for the small but nonzero response time of convection to large-scale forcing.

We begin by quantifying the relationship between virtual temperature and subcloud-layer entropy in convecting atmospheres. The convective available potential energy may be defined

$$\mathrm{CAPE} = \int_{p_{\mathrm{LNB}}}^{p_0} (\alpha_\mathrm{p} - \alpha_\mathrm{e})\,\mathrm{d}p \tag{14}$$

where α_p and α_e are the specific volumes of the lifted parcel and its environment, respectively, and p_{LNB} and p_0 are the pressure at the level of neutral buoyancy and at the origin level of the lifted parcel, respectively. In strict quasi-equilibrium (SQE),

$$\frac{\partial \mathrm{CAPE}}{\partial \tau} = \int_{p_{\mathrm{LNB}}}^{p_0} \left(\frac{\partial \alpha_{\mathrm{p}}}{\partial \tau} - \frac{\partial \alpha_{\mathrm{e}}}{\partial \tau}\right) \mathrm{d}p \simeq 0 \tag{15}$$

where the time derivative is understood to be in a reference frame moving with the horizontal velocity of air at the parcel origin level (usually the subcloud layer). (There is no contribution from the time dependence of the limits of integration in (15) because, by definition, the integrand is zero at these points.)

We now relate fluctuations at constant pressure of the parcel specific volume, α_{p}, to fluctuations in saturation entropy, using Maxwell's relations (for example see Emanuel (1994)):

$$(\delta\alpha)_{\mathrm{p}} = \left(\frac{\partial \alpha}{\partial s^*}\right)_{\mathrm{p}} \delta s^* = \left(\frac{\partial T}{\partial p}\right)_{s^*} \delta s^* \tag{16}$$

where s^* is the saturation entropy and we have neglected the small contribution of variable water content to α, in order to streamline this discussion. Thus

$$\left(\frac{\partial \alpha_{\mathrm{p}}}{\partial \tau}\right)_{\mathrm{p}} = \left(\frac{\partial T}{\partial p}\right)_{s^*} \frac{\partial s_{\mathrm{b}}}{\partial \tau} \tag{17}$$

where s_{b} is the actual entropy ($c_p \ln \theta_e$ where c_p is the specific heat at constant pressure) of the parcel to be lifted and we have assumed that the saturation entropy of parcels in the cloud is equal to s_{b} (i.e. entropy is conserved during the lifting and the lifted parcel is saturated).

If the environment of clouds is in hydrostatic balance, then

$$\left(\frac{\partial \alpha_{\mathrm{e}}}{\partial \tau}\right)_{\mathrm{p}} = -\frac{\partial}{\partial \tau}\frac{\partial \varphi}{\partial p} \tag{18}$$

where φ is the geopotential height. Using (17) and (18) in (15) yields an exactly integrable equation, whose result is

$$\frac{\partial}{\partial \tau}(\varphi_{\mathrm{LNB}} - \varphi_0) \simeq (T_0 - T_{\mathrm{LNB}}) \frac{\partial s_{\mathrm{b}}}{\partial \tau}. \tag{19}$$

This shows that fluctuations in the thickness of the SQE convective layer are directly related to fluctuations in subcloud-layer θ_e and is a central relationship in this discussion. As long as the layer is convecting, increases in subcloud-layer θ_e will be associated with increases in the thickness (temperature) of the convecting layer. (Note that had we included the small dependence of α on water content in (16), there would also be a small term in (19) proportional to the time rate of change of subcloud-layer specific humidity.) The relationship (19) was used by Emanuel (1987) and Yano and Emanuel (1991) in models of the Madden–Julian oscillation.

What controls the boundary-layer entropy, s_{b}? The components of the entropy balance of the subcloud layer are illustrated in Fig. 4. The balance is between surface fluxes, entrainment at the top of the subcloud layer, and flux of low-entropy air from the lower and middle troposphere into the boundary layer by convective downdraughts. The budget equation is

$$\frac{\partial s_{\mathrm{b}}}{\partial \tau} = \frac{w_0(s_{\mathrm{s}} - s_{\mathrm{b}}) - w_{\mathrm{e}}(s_{\mathrm{b}} - s_{\mathrm{m}}) - M_{\mathrm{d}}(s_{\mathrm{b}} - s_{\mathrm{d}})}{H_{\mathrm{b}}} + \frac{\dot{Q}_{\mathrm{rad}}}{T} \tag{20}$$

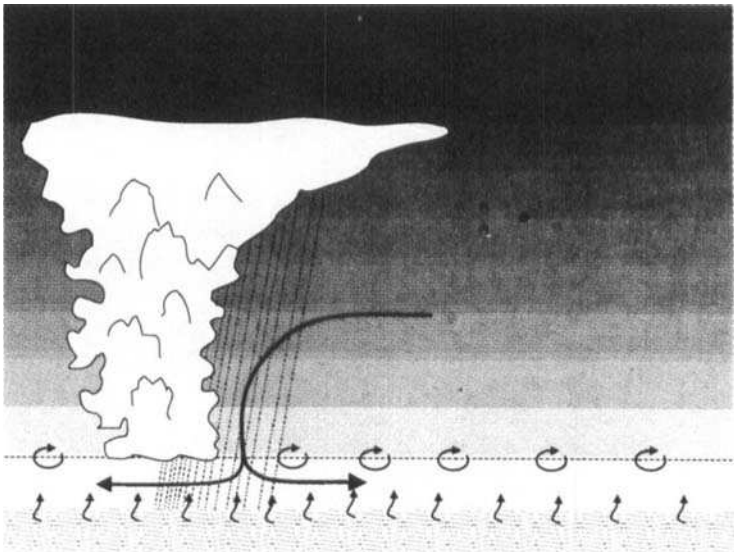

Figure 4. Components of the entropy balance of the subcloud layer. The four major components are radiative cooling (not shown), surface sensible- and latent-heat fluxes, turbulent entrainment at the top of the subcloud layer, and unsaturated downdraughts driven by evaporation of precipitation.

where s_s, s_m, and s_d are the entropies of the sea surface, of air just above the top of the subcloud layer, and of downdraught air, respectively; w_0 is an equivalent surface flux velocity, proportional to the drag coefficient, w_e is the entrainment rate, M_d is the convective downdraught mass flux velocity, H_b is the depth of the subcloud layer, and $\dot{Q}_{rad}$ is the radiative heating (usually negative) of the subcloud layer. In equilibrium,

$$(s_b)_{eq} = \frac{1}{w_0 + w_e + M_d}\left(w_0 s_s + w_e s_m + M_d s_d + \frac{H_b \dot{Q}_{rad}}{T}\right). \tag{21}$$

This shows that the equilibrium subcloud entropy is a weighted average of the entropy associated with the sea surface temperature (SST), the entropy of air just above the top of the subcloud layer, and the entropy of downdraughts, biased downward by radiative cooling of the subcloud layer. The equilibrium entropy thus increases with

(i) increasing SST,
(ii) increasing surface wind speed,
(iii) increasing entropy just above the boundary-layer top, and
(iv) increasing downdraught entropy,

among other conditions. According to (19), higher SSTs should be associated with higher thickness values. Lindzen and Nigam (1987) assumed that SST is the principal quantity affecting (21), and also assumed vanishing geopotential fluctuation at some level, and thereby calculated the climatological surface-pressure distribution from a relation like (19). They obtained good agreement with the climatological surface-pressure distribution, except near the equator, where they argued that a 'back-pressure' term must come into play. We shall demonstrate that this back pressure may be interpreted in terms of convective downdraughts.

Returning to the subcloud-layer entropy equation (20), it is tempting to conclude that an equilibration time-scale for the subcloud layer is

$$\tau_{\mathrm{SCL}} \sim \frac{H_{\mathrm{b}}}{\max(w_0, w_{\mathrm{e}}, M_{\mathrm{d}})}$$

which, for typical values, is of the order of 1/2 day. This is indeed of the order of the observed recovery time of the subcloud layer recently affected by a downdraught. But in a convective ensemble such an estimate would be most misleading, because M_{d}, the convective downdraught mass flux, is a function of convective activity, which is in turn partially related to $\partial s_{\mathrm{b}}/\partial t$. Moreover, quantities like s_{m} respond to variation of convective activity, so that the time-scale of variation of s_{b} has a correspondingly slow component. Because SQE constraints link the rest of the troposphere to the subcloud layer, the effective depth entering the time-scale τ_{SCL} becomes much larger than H_{b}.

(*a*) *Response of convecting atmospheres to large-scale vertical motion*

What happens when an atmosphere in radiative–convective equilibrium is exposed to upward motion associated with a large-scale disturbance? This can be partially understood with the aid of Fig. 5.

One facet of the system that simplifies thinking about this question is that to a good approximation, *the rate of subsidence in clear air is not affected by the presence of mean ascent.* This is so because the actual temperature tendency of the free tropical troposphere is extremely small, so that in the clear air, the relation

$$w_{\mathrm{e}} \frac{\partial \theta}{\partial z} \simeq \frac{\theta}{T} \frac{\dot{Q}_{\mathrm{rad}}}{c_p} \tag{22}$$

is very well satisfied, where w_{e} is the vertical velocity in the clear air. Since $\dot{Q}_{\mathrm{rad}}$ and $\partial\theta/\partial z$ are little affected by mean ascent, w_{e} is nearly invariant. Therefore, *nearly all of the upward motion associated with ensemble-average ascent must appear as increased mass flux in cumulus clouds.**

Thus mean ascent, to first order, just increases the mass flux in the cumulus clouds. What happens to the subcloud-layer entropy? Were it not for convective downdraughts, very little would change (see Fig. 5). Some change in s_{b} might occur if the variations in surface wind speed associated with the wave-mean flow resulted in changing surface fluxes. Also, the changing profile of detrainment from cumuli would slowly alter the entropy content of air entering the subcloud layer from above. The time-scale of this process is the time-scale for clear air to subside through the troposphere, about 30 days. This produces a long (30-day) time-scale component in the response of s_{b} to changing large-scale conditions.

But the most dramatic effect on the subcloud layer is through convective downdraughts. Since the ambient profile of entropy generally shows a minimum in the middle troposphere, the downdraughts decidedly reduce the subcloud-layer entropy, as anyone who has been in a thunderstorm knows. Broadly, the increase in ensemble-mean upward motion and concomitant increase in convective mass flux may be expected to increase convective downdraughts as well, thus producing a negative tendency of s_{b} and, in statistical equilibrium, a negative tendency of temperature in the entire convecting layer (larger at the top of the layer, owing to the divergence of moist adiabats with altitude).

* Observations support changes of $\partial\theta/\partial z$ of no more than about 5%, but changes in the large-scale vertical velocity are of order 100%, thus most of the change in large-scale vertical velocity must be associated with changes in cloud mass flux.

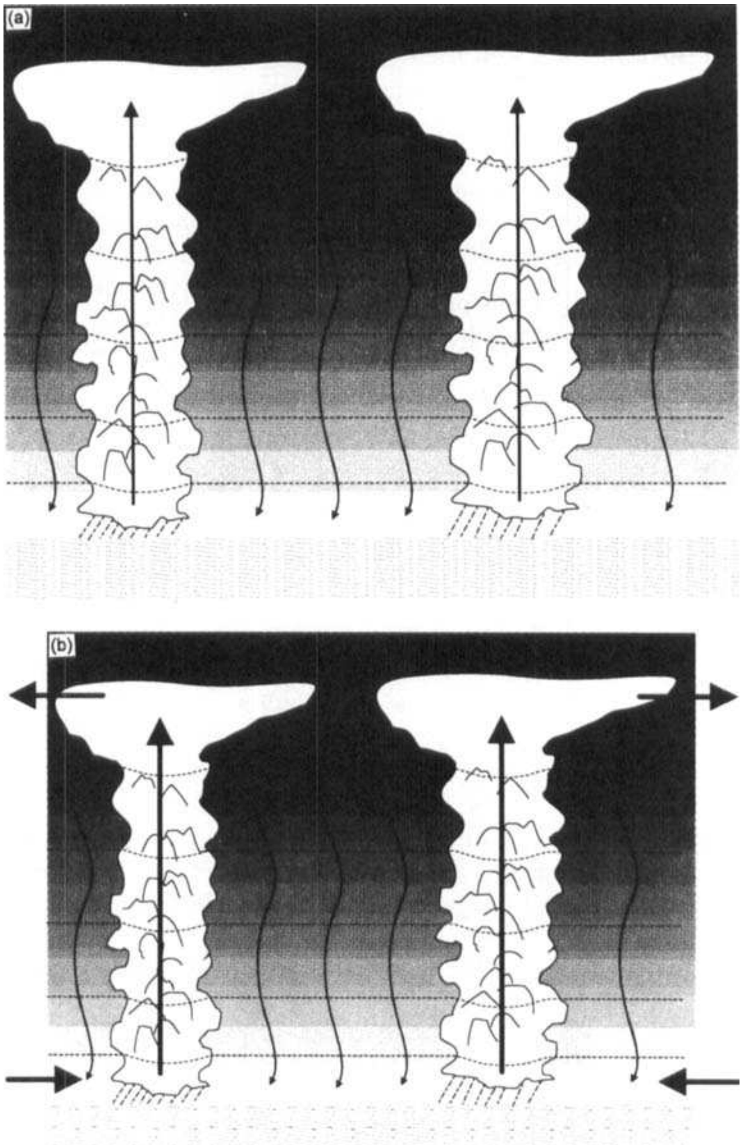

Figure 5. (a) Vertical motions in an atmosphere in radiative–convective equilibrium. Large upward motion in isolated clouds is compensated by weak downward motion in clear air and by cloud-related downdraughts. The dashed lines show surfaces of constant potential temperature. (b) As (a) but with an imposed large-scale upward motion. The upward mass flux in clouds is increased, but the downward motion in clear air remains virtually unchanged owing to the necessity of balancing radiative cooling with subsidence. Cloud-related downdraughts are probably enhanced because of greater precipitation.

Large-scale upward motion in a convecting atmosphere increases convection and thus reduces the boundary-layer entropy, primarily through convective downdraughts. Maintaining moist neutrality, the free-atmosphere temperature is reduced. Thus the large-scale ascent 'feels' an effective, positive static stability. It is evident that the magnitude of this effective static stability depends mostly on the relative magnitude of convective downdraught mass fluxes and on the entropy deficit of the lower and middle troposphere.

The effective static stability to deep, large-scale motions in convecting atmospheres has been quantified in a number of ways. Neelin *et al.* (1987) and Neelin and Held (1987) employed a large-scale budget of moist static energy to deduce an effective stratification. Yano and Emanuel (1991), using a crude two-layer model, expressed the effective static stability as

$$N_{\mathrm{eff}}^2 = (1 - \varepsilon)N^2 \tag{23}$$

where N_{eff} and N are the effective buoyancy frequency and the buoyancy frequency of dry air, respectively, and ε is a bulk precipitation efficiency, which determines the relative magnitude of the convective downdraught mass flux. (If all precipitation falls to the surface without re-evaporation, then there is no evaporation to drive a downdraught, though weak downdraughts may result from precipitation water loading.)

Neelin and Yu (1994) derived an effective static stability to small perturbations to convecting atmospheres using the Betts–Miller convection scheme (Betts 1986; Betts and Miller 1986). We present a slightly reduced approximate treatment of this here, for conceptual simplicity.

We begin with the hydrostatic, inviscid horizontal-momentum equations in inertial Cartesian coordinates, linearized about a resting state:

$$\frac{\partial \mathbf{V}}{\partial t} = -\nabla\phi \tag{24}$$

where $\mathbf{V}$ is the horizontal velocity vector and ϕ is the geopotential. Taking the divergence of this, and making use of mass continuity gives

$$\frac{\partial}{\partial t}\frac{\partial \omega}{\partial p} = \nabla^2\phi \tag{25}$$

where ω is the pressure velocity. Differentiating once in pressure gives

$$\frac{\partial}{\partial t}\frac{\partial^2 \omega}{\partial p^2} = \nabla^2\frac{\partial \phi}{\partial p} = -\nabla^2\alpha \tag{26}$$

where we have made use of the hydrostatic equation. Now we employ Maxwell's relation (16), reflecting fluctuations in α to fluctuations in s^*:

$$\frac{\partial}{\partial t}\frac{\partial^2 \omega}{\partial p^2} = -\left(\frac{\partial T}{\partial p}\right)_{s^*}\nabla^2 s^* \tag{27}$$

where, to be consistent with the linearization, the temperature lapse rate is taken to be that of the basic state.

In SQE, s^* is constant with pressure and equal to s_{b}:

$$\frac{\partial}{\partial t}\frac{\partial^2 \omega}{\partial p^2} = -\left(\frac{\partial T}{\partial p}\right)_{s^*}\nabla^2 s_{\mathrm{b}}. \tag{28}$$

This can be integrated once in pressure to obtain

$$\frac{\partial}{\partial t}\frac{\partial \omega}{\partial p} = C(x, y, t) - T(p)\nabla^2 s_{\mathrm{b}} \tag{29}$$

where C is an integration constant whose value can be obtained by applying boundary conditions on ω. If ω vanishes at $p = p_0$ and $p = p_{\mathrm{t}}$, then integrating (29) between p_{t} and p_0 gives

$$\frac{1}{p_0 - p_t}\frac{\partial}{\partial t}\int_{p_t}^{p_0} \frac{\partial \omega}{\partial p}\,\mathrm{d}p = 0 = C - \overline{T}\nabla^2 s_b \tag{30}$$

where

$$\overline{T} \equiv \frac{1}{p_0 - p_t}\int_{p_t}^{p_0} T(p)\,\mathrm{d}p. \tag{31}$$

(Note that $\overline{T}$ and $T(p)$ are basic-state quantities in this linearized treatment.) Thus (29) becomes

$$\frac{\partial}{\partial t}\frac{\partial \omega}{\partial p} = (\overline{T} - T(p))\nabla^2 s_b. \tag{32}$$

Integrating once in pressure and applying one boundary condition on ω gives

$$\frac{\partial \omega}{\partial t} = \nabla^2 s_b \int_{p}^{p_0} (T(p) - \overline{T})\,\mathrm{d}p. \tag{33}$$

Note that in SQE, *the vertical structure of the vertical velocity of simple linear normal modes is completely determined by the moist adiabatic lapse rate.* Also note that (33) will be a simple wave equation with stable neutral modes if $\partial s_b/\partial t$ is proportional to some weighted integral of ω (or minus the vertical velocity). (Note that if $\partial/\partial t$ is replaced by $\partial^2/\partial t^2 + f^2$ and ∇^2 is replaced by $(\partial/\partial t)\nabla^2 - \beta\partial/\partial x$, then (25)–(33) also hold for motions in a rotating reference frame with Coriolis parameter $f(y)$ and $\beta = \mathrm{d}f/\mathrm{d}y$.)

The only thermodynamic assumption used in deriving (33) is that the atmosphere maintains a moist adiabatic lapse rate at all times. (This can be easily generalized to include reference lapse rates that differ from moist adiabatic.) Betts and Miller make the additional assumption that there exists a *reference specific humidity profile**. This directly implies that the moist static energy, h, also has a reference profile. The actual profiles are relaxed back toward the reference profiles over a specified time constant, τ. In the limit of $\tau \to 0$, the actual temperature, specific humidity, and moist static energy distributions always equal their reference distributions. One implication of this is that

$$\lim_{\tau\to 0} h(p) = \mathscr{F}(h_b) \tag{34}$$

where $\mathscr{F}$ is some function of pressure that can be specified if h_b is known. Here h_b is the moist static energy of the subcloud layer. Integrating (34) in the vertical, we have

$$\lim_{\tau\to 0}\frac{1}{p_0 - p_t}\int_{p_t}^{p_0} h(p)\,\mathrm{d}p = G(h_b) \tag{35}$$

where G is a function of h_b (that turns out to be nearly linear), and p_t and p_0 are the pressures at the top and bottom of the convecting layer, respectively. This can be inverted to get

$$\lim_{\tau\to 0} h_b = G^{-1}\left(\frac{1}{p_0 - p_t}\int_{p_t}^{p_0} h(p)\,\mathrm{d}p\right) \tag{36}$$

where G^{-1} is the inverse function of G. Now differentiate (36) in time to get

* We do not necessarily concur that there is satisfactory theoretical justification for this, but provide the following development to explore fully some of the dynamical consequences that follow from this assumption.

$$\lim_{\tau\to 0}\frac{\partial h_{\mathrm{b}}}{\partial t}\simeq G^{-1}\left(\frac{1}{p_0-p_{\mathrm{t}}}\int_{p_{\mathrm{t}}}^{p_0}\frac{\partial h(p)}{\partial t}\,\mathrm{d}p\right) \tag{37}$$

assuming that G^{-1} is linear and also neglecting variations in p_{t}.

Provided that all of the kinetic energy of convective clouds is converted back into internal and potential energy by dissipation, *convection* cannot change the vertical integral of h. Thus for a simple tropical system in which we neglect horizontal advection and radiative cooling, (37) becomes

$$\lim_{\tau\to 0}\frac{\partial h_{\mathrm{b}}}{\partial t}=-G^{-1}\left(\frac{1}{p_0-p_{\mathrm{t}}}\int_{p_{\mathrm{t}}}^{p_0}\omega\frac{\partial h}{\partial p}\,\mathrm{d}p\right) \tag{38}$$

where ω is the large-scale pressure velocity. The function G^{-1} depends on the nature of the reference profiles of specific humidity and temperature. The slope of G^{-1} is nearly equal to a constant, γ, which is slightly larger than one in the formulation of Betts and Miller (Neelin and Yu 1994). Thus (38) can be approximated as

$$\lim_{\tau\to 0}\frac{\partial h_{\mathrm{b}}}{\partial t}\simeq-\gamma\overline{\omega\frac{\partial h}{\partial p}} \tag{39}$$

where the overbar represents an average over the depth of the convecting layer. The moist static energy, h_{b}, is related to s_{b} by

$$\delta h_{\mathrm{b}}\simeq T_{\mathrm{b}}\delta s_{\mathrm{b}}$$

where T_{b} is a mean subcloud-layer temperature. Using this in (39) and eliminating s_{b} using (33) gives a wave equation:

$$\frac{\partial^2\omega}{\partial t^2}=-\gamma\overline{\nabla^2\omega\frac{\partial h}{\partial p}}\int_{p}^{p_0}\frac{T(p)-\overline{T}}{T_{\mathrm{b}}}\,\mathrm{d}p. \tag{40}$$

Equation (40) implies a unique phase speed associated with linear normal modes. Substituting

$$\omega=\Omega(p)\mathrm{e}^{ik(x-ct)}$$

into (40) gives

$$c^2\Omega(p)=-\gamma\overline{\Omega(p)\frac{\partial h}{\partial p}}\int_{p}^{p_0}\frac{T(p)-\overline{T}}{T_{\mathrm{b}}}\,\mathrm{d}p. \tag{41}$$

Here $\Omega(p)$ is the vertical structure of ω, k is the zonal wave number, and c is the zonal phase speed. Multiplying (41) through by $\partial h/\partial p$ and integrating over the depth of the convecting layer gives

$$\begin{aligned}c^2\frac{1}{p_0-p_{\mathrm{t}}}\int_{p_{\mathrm{t}}}^{p_0}\Omega(p)\frac{\partial h}{\partial p}\,\mathrm{d}p&\equiv c^2\overline{\Omega(p)\frac{\partial h}{\partial p}}\\&=-\gamma\overline{\Omega(p)\frac{\partial h}{\partial p}}\frac{1}{p_0-p_{\mathrm{t}}}\int_{p_{\mathrm{t}}}^{p_0}\frac{\partial h}{\partial p}\left(\int_{p'}^{p_0}\frac{T(p)-\overline{T}}{T_{\mathrm{b}}}\,\mathrm{d}p'\right)\mathrm{d}p.\end{aligned} \tag{42}$$

By integrating the right-hand side of (42) by parts, we obtain

$$c^2 = -\gamma \frac{1}{p_0 - p_t} \int_{p_t}^{p_0} h(p) \left(\frac{T(p) - \overline{T}}{T_b} \right) dp. \tag{43}$$

The above can be further simplified by noting that

$$h = h^* + L_v(q - q^*) \tag{44}$$

where h^* is the saturation moist static energy, q^* is the saturation specific humidity, and L_v is the latent heat of vaporization. Furthermore, in the Betts–Miller formulation, $q = aq^*$, where for simplicity we take a to be constant. Substituting (44) into (43) and assuming that h^* is constant gives

$$c^2 = \gamma(1 - a)L_v \frac{1}{p_0 - p_t} \int_{p_t}^{p_0} q^*(p) \left(\frac{T(p) - \overline{T}}{T_b} \right) dp. \tag{45}$$

This makes it clear that c^2 is positive in Betts–Miller, since q^* decreases rapidly with height. (A more detailed relation similar to (43) has been derived by Neelin and Yu 1994.) Note that for $a = 0.9$, the internal wave speed for a deep mode in a convecting atmosphere, according to (45), is about $10\,\mathrm{m\,s^{-1}}$, compared with $35\,\mathrm{m\,s^{-1}}$ for the corresponding dry mode.

One needs to exercise some caution in interpreting those aspects of (45) that depend on the moisture closure of the Betts–Miller scheme, because although the relaxation of temperature to a universal profile is well supported by observations, there is no corresponding evidence for a universal water-vapour profile. Indeed, the water content of convecting atmospheres varies over a large range, as does its vertical profile. The derivation for the case of the Betts–Miller scheme should be taken as a simple prototype for more general statistical-equilibrium closures.

Brown and Bretherton (1994) used a similar approach to understand very long wavelength evaporation–wind feedback instabilities when the Emanuel parametrization (Emanuel 1991) is used to represent the convection. For modes of period 10 days and longer, the Emanuel parametrization does maintain SQE. However, in the absence of mean lifting, the time-scale for water vapour to adjust to changes in the convection in the Emanuel parametrization is the radiative time-scale (many days). In the free troposphere, water vapour has a phase lag that depends on wave period, so G^{-1} is a complex number that incorporates this phase lag. Since only the temperature perturbation is in phase with h_b, the magnitude of G^{-1} is much smaller than in the Betts–Miller formulation, only about 0.3 for a 40 000 km wavelength (zonal wave number 1) wave. In addition, the radiative–convective equilibrium of the Emanuel parametrization (with the standard parameter values) has a more pronounced and lower altitude minimum in h. A Fourier transformed version of Eq. (38) is still valid if a complex G^{-1} is used. The combination of the different h profile and different G^{-1} cause the effective stratification for the Emanuel scheme to be about three times that of the Betts–Miller formulation, though this is no doubt sensitive to the as yet untuned microphysical parameters in the scheme. This illustrates that the effective stratification (and hence the interaction of convection with large-scale circulations) can be quite sensitive to the convective parametrization, a fact that may help explain why global-circulation models produce a broad range of periods for low-frequency oscillations such as the intraseasonal oscillation.

The inescapable conclusion from all these analyses is that *large-scale ascent in convecting atmospheres is associated with a reduction of temperature*; *i.e. with a positive effective static stability*. Moreover, it is clear from the subcloud-layer equilibrium–entropy relation (21) that, in steady flows, the subcloud-layer entropy (and thus the free tropospheric temperature) will be reduced by ascent, owing to increased convective downdraught flux of low-entropy air. This is equivalent to the 'back-pressure' in Lindzen and Nigam's (1987) terminology: in their steady-state case, positive SST anomalies near the equator lead to positive atmospheric temperature anomalies, giving rise to mean ascent. This excites convection, and the associated downdraughts reduce the boundary-layer entropy, providing a negative feedback on the processes tending to increase the temperature. As an aside, we note that this process will also tend to decrease the SST since the downdraughts increase the surface heat flux. This suggests that over time-scales that are long compared with those associated with the ocean mixed-layer temperature, the *atmospheric* deformation radius will be one of the factors controlling the scale of variation of the SST in deep convecting regions.

(*b*) *WISHE*

Yano and Emanuel (1991) coined the term WISHE (wind-induced surface heat exchange) to denote the source of fluctuations in subcloud-layer entropy arising from fluctuations in surface wind speed. This can be an important source of energy for tropical disturbances (Emanuel 1987; Neelin *et al.* 1987; and Numaguti and Hayashi 1991. The latter two used the term 'evaporation–wind feedback' to characterize the same process). To see the effect of surface fluxes on temperature, return for a moment to relation (37) from the Betts–Miller scheme. Remember that convection cannot change the right-hand side of (37). In addition to vertical motion, surface fluxes can change the right-hand side of (37). Surface fluxes enter the moist static-energy equation in the form

$$\frac{\partial h}{\partial t} = g\frac{\partial}{\partial p}F_h \tag{46}$$

where F_h is the turbulent flux of moist static energy. Substituting (46) into (37) gives

$$\lim_{\tau\to 0}\frac{\partial h_b}{\partial t} = \frac{G^{-1}}{p_0 - p_t}(\rho_s g F'_{h_s}) \tag{47}$$

where F'_{h_s} is the total surface heat flux per unit mass, and ρ_s is the surface air density. Once again approximating G^{-1} by γ, we have

$$\lim_{\tau\to 0}\frac{\partial h_b}{\partial t} = \gamma\frac{p_0}{p_0 - p_t}\frac{F'_{h_s}}{H_s} \tag{48}$$

where H_s is the scale height (RT_s/g) based on the surface temperature and R is the gas constant. It is clear that h_b responds to surface fluxes *as though the flux convergence were spread over the entire depth of the convecting layer.*

An important consideration here is that the surface flux, F'_{h_s}, depends on the surface wind. In the neutral surface layer,

$$F'_{h_s} = C_D|\mathbf{V}_s|(h_0^* - h_b) \tag{49}$$

where C_D is a dimensionless coefficient, $|\mathbf{V}_s|$ is the magnitude of the surface wind, and h_0^* is the saturation static energy of the sea surface. In simple disturbances, fluctuations of $|\mathbf{V}_s|$ are usually out of phase with vertical velocity. Thus in SQE, while the convective heating associated with large-scale ascent is exactly in phase with vertical motion, and thus out of phase with temperature in large-scale waves, the heating associated with fluctuating surface winds can be partially in phase with temperature, leading to wave growth.

(c) *'Soft statistical equilibrium': the effect of finite convective time-scales*

Until now we have treated the interaction of convection with large-scale flow as one characterized by exact statistical equilibrium. It is important to note, however, that the finite time-scale of convection can have a strong effect on large-scale circulations in convecting atmospheres, even though the ratio of convective time-scale to the time-scale of the large-scale circulation is very small. This question has been examined at some length by Emanuel (1993a) and by Neelin and Yu (1994) and Yu and Neelin (1994). The issue is treated quantitatively in these papers; here we illustrate heuristically that finite convective response time actually has a strong *damping* effect on large-scale waves. We refer to this as *moist convective damping*, or MCD.

MCD acts on the circulation in two ways. Consider first an arbitrary perturbation in temperature or moisture introduced into a convecting atmosphere. The first damping effect of convection will be to bring each column into statistical equilibrium rapidly, dissipating the component of the perturbation that is not consistent with the equilibrium being established. The second damping effect is more subtle because it concerns the remaining component of the perturbation that *is* close to the statistical equilibrium profile; in SQE there is no damping effect on this component, so this second effect depends on the equilibrium being 'soft'.

To see this second damping effect, consider a deep equatorial Kelvin wave passing through an atmosphere otherwise in radiative–convective equilibrium, as illustrated in Fig. 6. In exact statistical equilibrium, the perturbation heating associated with the enhanced convection in the ascent phase nearly cancels the adiabatic cooling by the ascent, and the heating is exactly out of phase with the wave temperature perturbation (Fig. 6(a)). (This affects the phase speed of the wave, as we have seen, but not its amplitude.)

Now suppose that the convection lags the forcing (vertical velocity) by a small amount (Fig. 6(b)). The convective heating is slightly displaced into the *cold* phase of the wave, leading to a negative correlation of heating and temperature, and thus to decay of the wave. The magnitude of this effect depends on the ratio of the convective time-scale to the wave period, and is evidently greater for higher-frequency waves. Thus *finite convective response time selectively damps high-frequency oscillations.* In general, waves will experience MCD.

While MCD is explicitly built into the convective parametrizations discussed by Emanuel (1993a) and Neelin and Yu (1994), Brown and Bretherton (1994) find that its effect is also apparent in the Emanuel convective parametrization, which does not incorporate any explicit time delays. At most (though not all) heights, they found that the phase of the temperature perturbation associated with a travelling wave slightly lags the boundary-layer entropy. They also found the expected strong damping of shorter-wavelength modes, in contrast to early studies of wave-CISK (Hayashi 1970; Lindzen 1974) using Kuo-style representations of convection, in which short-wavelength modes amplified most rapidly. As expected, the Arakawa–Schubert (1974) parametrization, in which convection remains perfectly in phase with the forcing, does not produce MCD (Stark 1976; Crum and Stevens 1983).

Note that small lags in the response of convection permit fluctuations in potential energy for convection (CAPE). It is evident from Fig. 6(b) that CAPE will be generated in regions of large-scale ascent. The coincidence of anomalous CAPE and anomalous convection is an indication that convective heating is occurring in anomalously cold air, so that $\overline{Q'T'}$ is negative. It is remarkable that in this case the conversion of potential to

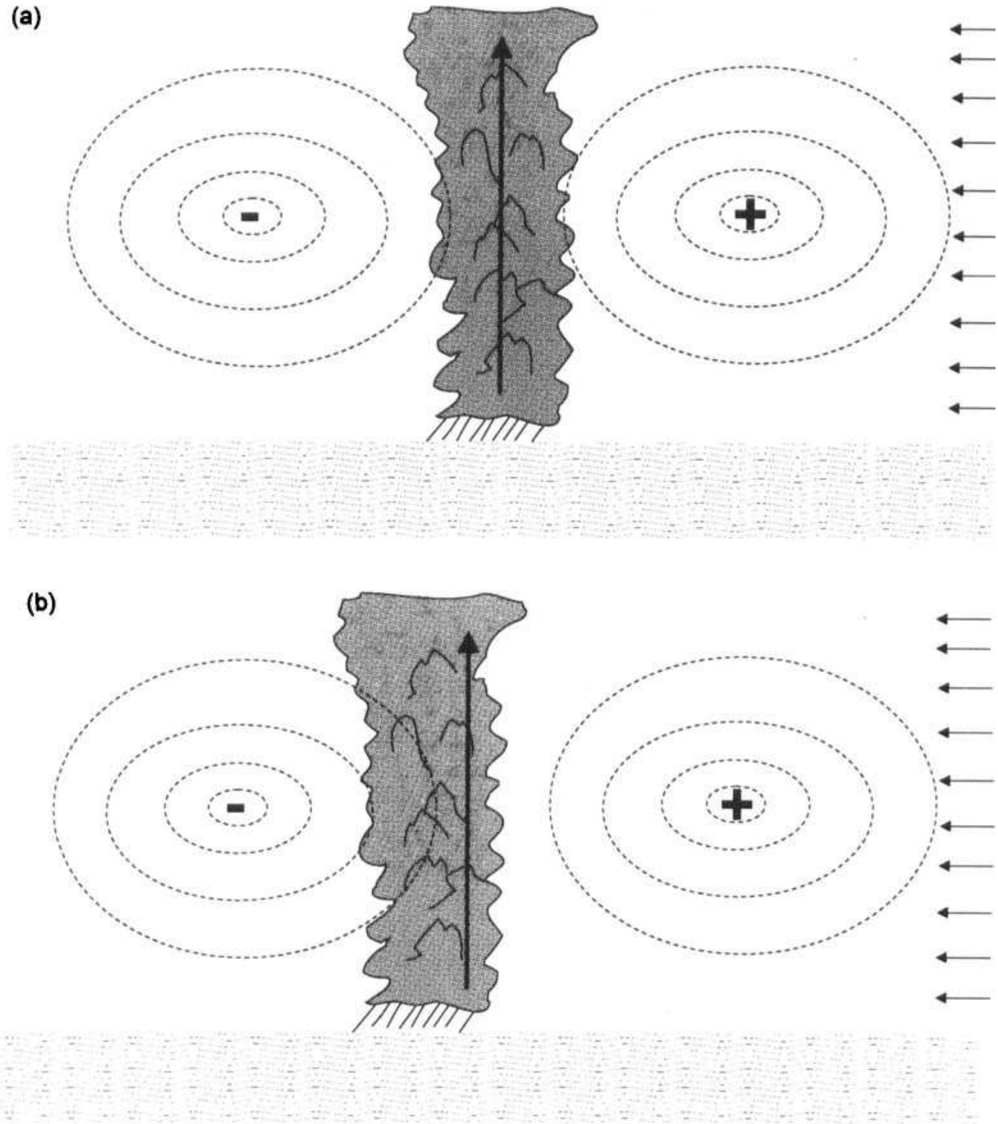

Figure 6. (a) The relationship between convection, temperature perturbations (dashed contours) and vertical motion in an equatorial Kelvin wave passing through a convecting atmosphere in which the convection is in statistical equilibrium with its environment. The arrows at right indicate wave-relative mean flow. The vertical velocity and the convective heating are in quadrature with the wave temperature perturbations; the wave is neutral. (b) As (a) but under the circumstance that the convection responds to forcing over a finite time, causing a phase lag in the convection. (The magnitude of this lag is exaggerated here for illustrative purposes.) Both the convective heating and the vertical velocity are phase shifted toward the cold phase of the wave; the wave is therefore damped.

kinetic energy in clouds is actually associated with the opposite conversion in the large-scale disturbance.

(*d*) *Testing the statistical-equilibrium hypothesis*

Statistical equilibrium is a compelling hypothesis for simplifying our understanding of tropical circulations on a variety of scales (see section 5). Observations already discussed imply that in disturbed parts of the tropics CAPE is nearly constant, with a value of from 0 to 2000 J kg^{-1} depending on how it is defined, what parcel is lifted and, to a lesser extent, when and where it is measured.

A fundamental issue is what fraction of the systematic variations of temperature across a tropical circulation are associated purely with variations in CAPE. Consider for instance the intraseasonal oscillation, which is associated with thickness variations of O (3 dam), corresponding to variations of 1 K in mean tropospheric temperature. If the boundary-layer entropy remained constant through this wave, the CAPE would only have to increase by 300 J kg^{-1} from the warm (low surface pressure) to the cold phase of the wave. Such small systematic variations in CAPE are difficult to detect directly in soundings, and cannot be ruled out by current observations. Conversely, a 1 K change in mean tropospheric temperature could be produced by a 2 K variation in boundary layer θ_e without changes in CAPE. While also not a large signal, this variation may be detectable and could be correlated with thickness or surface pressure perturbations as a test of statistical equilibrium. The phase lags between convection and forcing are probably only 'testable' in the framework of cumulus ensemble modelling of the type performed, for example, by Xu *et al.* (1992).

For the mean tropical circulation, however, systematic variations in CAPE of this magnitude are much less important. Given a mean tropospheric temperature and plausible values of CAPE, the error in boundary-layer θ_e made by assuming constant CAPE would be less than 0.5 K. This is much smaller than the potential variability of boundary-layer θ_e driven by SST variations in the tropics. Current surface and upper-air observations could be used to test how well tropospheric thickness and near-surface θ_e are correlated across the tropics on seasonal time-scales. The diurnal variability of θ_e complicates such a comparison over land.

An attractive feature of statistical-equilibrium thinking is that it simplifies tropical dynamics enough to make indirect tests based on the phase relationships between its predicted flow and thermodynamic fields.

(*e*) *Summary*

We contend that the dynamics of large-scale circulations in convecting atmospheres are best understood through statistical-equilibrium thinking, which is based on the following points:

- The appropriate quantity to consider is the temperature itself, and not the heating.
- In statistical equilibrium, there is a one-to-one correspondence between the mean virtual temperature (thickness) of the convecting layer and the entropy (θ_e) of the subcloud layer.
- Large-scale ascent through a deep layer is associated with enhanced convection, whose downdraughts reduce the subcloud-layer θ_e; this is, according to the previous point, associated with a reduction of the virtual temperature of the convecting layer. Large-scale disturbances therefore 'feel' a positive effective static stability, which is typically about 10% of the dry static stability.
- Fluctuations in the total surface enthalpy (heat) flux change the subcloud-layer entropy as though the flux were converged over the entire depth of the convecting layer. These changes are associated with changes in the mean virtual temperature of the convecting layer. The phase relationship between this temperature change and the wave temperature field can amplify waves under a variety of conditions.
- Small time lags in the response of convection to large-scale ascent displace the convective heating slightly toward the cold phase of the wave, thus damping the wave.

This damping effect increases with wave frequency, and is associated with fluctuations of CAPE. In general, large-scale disturbances experience MCD.

5. Some examples

Here we illustrate the utility of statistical-equilibrium thinking by applying it to several tropical circulation systems.

(*a*) *Tropical cyclones*

Tropical cyclones are quasi-balanced, warm-core systems. They are premier examples of systems arising from WISHE. On the other hand, it is well known that such disturbances do not arise spontaneously from small perturbations, but must be triggered by finite-amplitude perturbations of independent origin.

From the statistical-equilibrium point of view, the wind structure and warm core of developed tropical cyclones must be associated with high subcloud-layer θ_e in the core. This is indeed the case. One small caveat here is that the highly baroclinic core of the hurricane makes it necessary to consider *slantwise* convective adjustment, which occurs along angular-momentum surfaces rather than in the vertical, so that (19) must be modified accordingly (see Emanuel 1986). There is nevertheless a one-to-one correspondence between the distribution of subcloud-layer θ_e and the distribution of temperature (and thus velocity) in the cyclone.

Both the necessity of a finite-amplitude trigger and the intensification and amplitude of the cyclone can be understood through statistical-equilibrium thinking. Consider first a small-amplitude warm-core vortex placed in a convecting atmosphere. An Ekman layer will form and be associated with Ekman pumping near the core. According to the arguments presented in section 4, this Ekman pumping will 'feel' a weak positive stratification, and thus the vortex core will cool. (Accordingly, the vertical penetration depth of the vertical velocity owing to Ekman pumping will scale as fL/N_{eff}, where L is a horizontal length-scale of the vortex. For a length-scale of 500 km, this is virtually the whole depth of the troposphere.) The warm-core vortex will thus decay with time. This has been demonstrated in numerical simulations (e.g. Rotunno and Emanuel 1987; Emanuel 1989). Opposing this tendency are the enhanced surface heat fluxes associated with the vortex surface winds, but these are not quantitatively sufficient to counter the aforementioned reduction of subcloud θ_e produced by the downdraughts associated with the Ekman pumping-induced convection.

How does the tropical cyclone ultimately amplify, given this strong negative feedback? A number of numerical experiments (Emanuel 1989) and a recent field experiment (Emanuel 1993b) suggest that for amplification to occur, the troposphere must become nearly saturated *on the mesoscale* in the core. When this happens, the vertical entropy distribution no longer has a prominent minimum and there is no low-entropy air for downdraughts to import to the subcloud layer. The effective stratification approaches zero in this case, and Ekman pumping becomes inefficient in cooling the core. (In the Betts–Miller formulation, $\alpha \rightarrow 1$ in Eq. (45) and so $c^2 \rightarrow 0$. No cooling would result from lifting.) Now the enhanced surface fluxes associated with strong surface winds near the core can actually increase the subcloud-layer entropy and thus the core temperature. The WISHE process results now in a positive feedback to the warm-core cyclone, and the system amplifies. (It should be remarked here that any effect, such as vertical wind shear, that tends to advect drier air into the core in the lower and middle troposphere will re-establish the Ekman-induced cooling and lead to decay of the vortex; this makes the developing system susceptible to other circulation systems in its environment.)

Statistical-equilibrium thinking also allows one to see why there exists a definite upper bound on the amplitude of tropical cyclones, given certain environmental conditions (Emanuel 1986, 1988). The surface entropy can only increase until the subcloud layer is in thermodynamic equilibrium with the ocean; this will be characterized by saturation of the surface air at ocean temperature. By (19) (integrated in time), this gives an upper bound on the thickness in the core (though in this case the pressure dependence of the saturation entropy must be accounted for). One other dynamical constraint (which is that the absolute vorticity of air at storm top is zero in the steady state; see Emanuel 1986) yields a unique minimum value of the central pressure in a steady axisymmetric tropical cyclone. (One further consideration in the steady-state structure is the radial inward advection of subcloud-layer entropy by the radial inflow in the boundary layer. This influences the radial distributions of subcloud-layer θ_e and thus the structure of the vortex.)

(*b*) *Large-scale steady circulations in the tropics*

The principal insight of Held and Hou (1980) was that a thermally direct circulation must occur if an atmosphere in radiative–convective equilibrium and in thermal-wind balance, with vanishing surface wind, has a physically impossible distribution of momentum. Usually, this 'impossible' momentum distribution is taken to be one with negative absolute vorticity (in the northern hemisphere). Let us consider this problem in a statistical equilibrium, convecting atmosphere. On sufficiently large scales, thermal-wind balance may be assumed to apply. In that case, the thermal-wind equations can be written in pressure coordinates as

$$\frac{\partial u}{\partial p} = \frac{1}{f}\frac{\partial \alpha}{\partial y} \tag{50}$$

$$\frac{\partial v}{\partial p} = -\frac{1}{f}\frac{\partial \alpha}{\partial x} \tag{51}$$

where u and v are the eastward and northward velocities. The specific volume, α, may be regarded as a function of s^* and p, so

$$\left(\frac{\partial \alpha}{\partial x}\right)_p = \left(\frac{\partial \alpha}{\partial s^*}\right)_p \frac{\partial s^*}{\partial x} = \left(\frac{\partial T}{\partial p}\right)_{s^*} \frac{\partial s^*}{\partial x} \tag{52}$$

and

$$\left(\frac{\partial \alpha}{\partial y}\right)_p = \left(\frac{\partial \alpha}{\partial s^*}\right)_p \frac{\partial s^*}{\partial y} = \left(\frac{\partial T}{\partial p}\right)_{s^*} \frac{\partial s^*}{\partial y} \tag{53}$$

where we have used Maxwell's relations as in (16) and again have ignored the direct effect of water on density. In statistical equilibrium, s^* is approximately invariant with p (i.e. the temperature lapse rate is approximately moist adiabatic), so after substituting (52) and (53) into (50) and (51), we can integrate the thermal-wind equations in p, assuming vanishing wind at the surface. The result is

$$u_{\mathrm{T}} = -\frac{1}{f}(T_0 - T_{\mathrm{T}})\frac{\partial s_{\mathrm{b}}}{\partial y} \tag{54}$$

$$v_{\mathrm{T}} = \frac{1}{f}(T_0 - T_{\mathrm{T}})\frac{\partial s_{\mathrm{b}}}{\partial x} \tag{55}$$

where u_{T} and v_{T} are the wind components at the tropopause, T_0 and T_{T} are the temperatures of the surface and tropopause, respectively, and we have used the convective neutrality condition $s^* = s_{\mathrm{b}}$, where s_{b} is the actual subcloud-layer entropy.

The absolute vorticity at the tropopause is therefore given by

$$\eta_{\mathrm{T}} = f + \nabla \cdot \frac{1}{f}(T_0 - T_{\mathrm{T}})\nabla s_{\mathrm{b}}. \tag{56}$$

The requirement that the absolute vorticity at the tropopause have the same sign as f may thus be written

$$1 + \frac{1}{f}\nabla \cdot \frac{1}{f}(T_0 - T_{\mathrm{T}})\nabla s_{\mathrm{b}} \geqslant 0. \tag{57}$$

This may be regarded as a generalization to three dimensions of the requirement for the viability of the radiative–convective equilibrium solution stated in Plumb and Hou (1992). In the zonally symmetric case, (57) reduces to

$$1 + \frac{1}{f^2}\frac{\partial}{\partial y}\left((T_0 - T_{\mathrm{T}})\frac{\partial s_{\mathrm{b}}}{\partial y}\right) - \frac{\beta}{f^3}(T_0 - T_{\mathrm{T}})\frac{\partial s_{\mathrm{b}}}{\partial y} \geqslant 0 \tag{58}$$

which is similar to the expression derived by Plumb and Hou (1992), but differs slightly because the latter authors used exact gradient-wind balance, rather than the geostrophic balance assumed here. It is clear that (58) is always violated if there is a subcloud-layer θ_e gradient at the equator, so that a cross-equatorial Hadley cell must result (Lindzen and Hou 1988). Plumb and Hou (1992) argue that (58) is violated in the southern part of the Tibetan plateau/Indian subcontinent region, giving rise to the monsoon.

The advantage of statistical-equilibrium thinking, in the case of steady circulations in convecting atmospheres, is that criteria like (57) boil down to a determination of the distribution of subcloud-layer θ_e in the radiative–convective equilibrium state, reducing the uncertainty about the vertical profile of equilibrium temperature.

When criteria like (57) are violated, simple scaling arguments suggest that the resulting thermally direct circulation is strong, with the result that the actual temperature distribution should be held close to a critical condition given by setting the right-hand side of (57) to zero. Figure 7 illustrates, for a 'two-column' model, how this might happen. In the case that (57) is weakly violated (Fig. 7(a)), horizontal advection in the subcloud layer and vertical motion over the warm water both act to reduce the subcloud-layer entropy, and drive the subcloud-layer entropy distribution back towards a critical value. In this case there is still convection over the cool water, but its magnitude is reduced. The strength of the meridional circulation can be estimated by calculating the ascent velocity and horizontal advection necessary to keep the subcloud-layer entropy over the warm water close to its critical value from (57). When (57) is strongly violated (Fig. 7(b)), the circulation becomes so strong as to suppress convection altogether over the cool surface. In this case statistical equilibrium fails over the cool surface, and the boundary-layer entropy becomes somewhat uncoupled from the temperature aloft (i.e. a trade cumulus or stratocumulus boundary layer forms). One might still argue that the temperature distribution is driven close to a critical value given by setting the right side of (57) to zero, but substituting the free atmosphere s^* distribution for the s_{b} distribution, since the latter is now uncoupled from the former over the cool surface. The magnitude

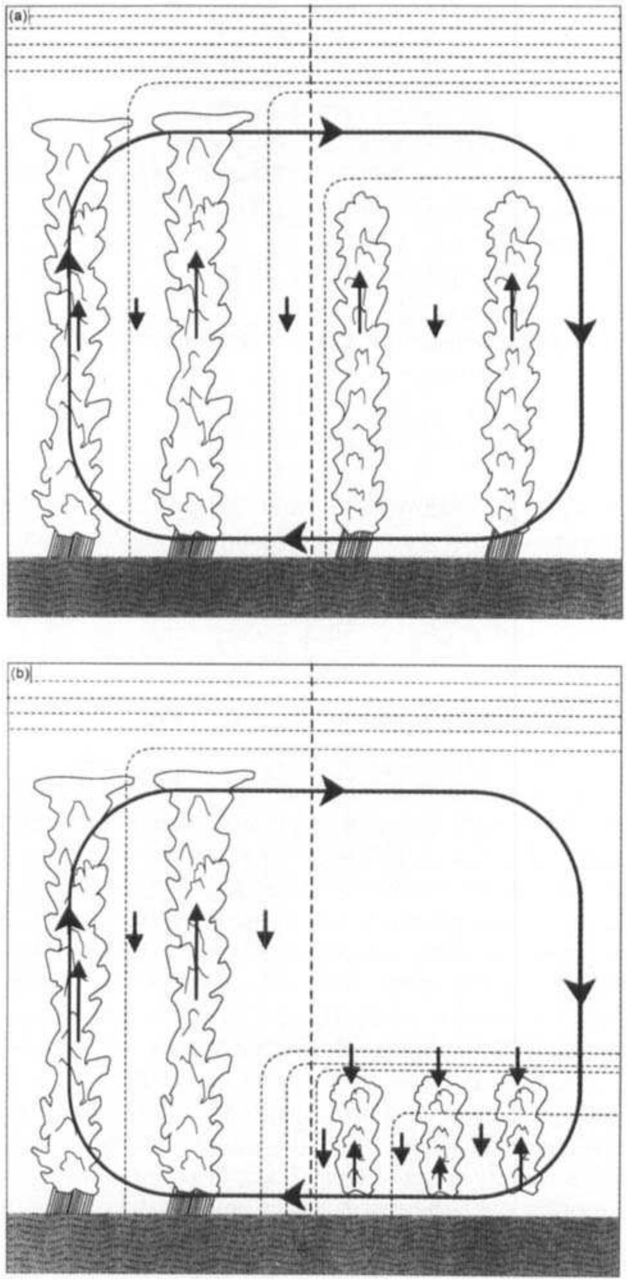

Figure 7. (a) Convection and large-scale circulation in a two-column system in which the sea surface temperature is greater on the left-hand side. The dashed lines are contours of saturated entropy, with higher values on the left. In this case the magnitude of the subsiding motion associated with the large-scale circulation is smaller than the radiatively forced subsiding motion between clouds, and thus the large-scale circulation does not entirely suppress the convection over the colder water. The magnitude of the circulation is such that cold advection into the warm region and convective downdraughts keep the horizontal temperature gradient near the critical value given by Eq. (58). (b) As (a) except that the difference in sea surface temperatures is sufficient to suppress deep convection over the colder water. A trade inversion forms over the cold water, and the temperature profile over the warm water and above the trade inversion is nearly moist adiabatic. The magnitude of the circulation is such that adiabatic warming balances radiative cooling in the subsiding region.

of the thermally direct circulation can be estimated in this case by requiring a balance between radiative cooling and adiabatic warming in the descending air.

(c) *Intraseasonal oscillations*

Statistical-equilibrium thinking also presents a view of the dynamics of wave-like phenomena in convecting atmospheres that is very different from classical thinking, which emphasizes the heating rather than the temperature. We once again remark that the statistical-equilibrium approach outlined in section 4 of this paper, and quantified by Emanuel (1993a) and Neelin and Yu (1994), strongly suggests that the interaction between wave-like perturbations and deep moist convection actually damps the waves; one should think in terms of MCD rather than CISK.

Statistical-equilibrium thinking in this case is illustrated by Fig. 8. The subcloud-layer entropy is reduced under the enhanced convective downdraughts associated with the ascent phase of the wave; this is, in equilibrium, associated with a reduction of the free-atmosphere temperature. A small lag in the response of convection shifts this effect slightly westward (i.e. slightly behind the maximum ascent), thus shifting the convection slightly into the cold phase of the wave. The resulting negative correlation of heating and temperature acts to damp the wave.

At the same time the baroclinic circulation of the wave leads to enhanced surface easterlies to the east of the updraught. The enhanced surface heat fluxes lead to a positive tendency of subcloud-layer θ_e and, therefore, free-atmosphere temperature. This

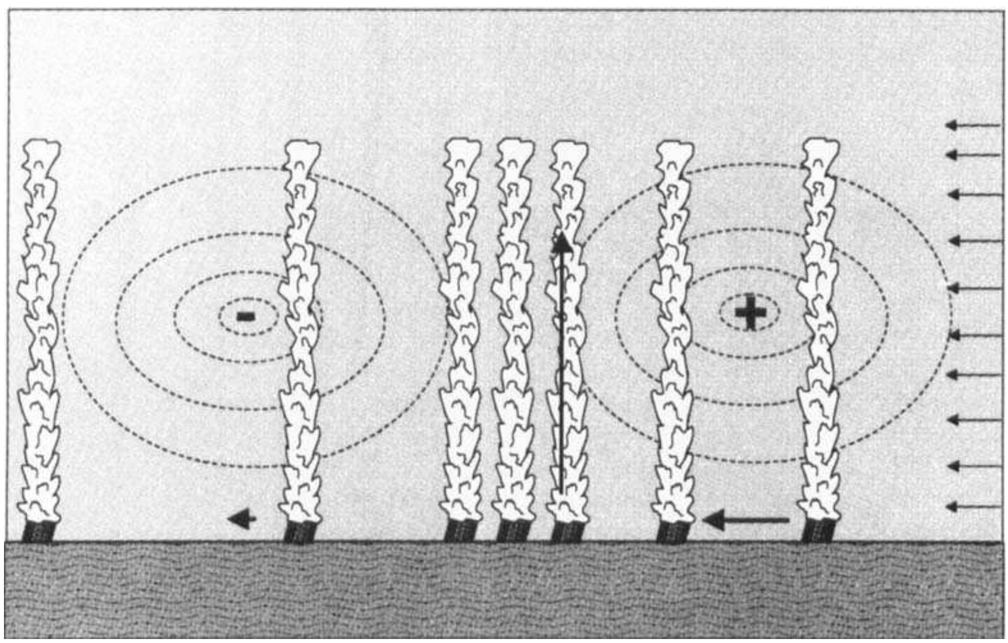

Figure 8. Relationship between convection, temperature perturbations (dashed contours) and vertical motion in an equatorial Kelvin wave passing through a convecting atmosphere and being amplified through its interaction with the ocean. Arrows at right indicate wave-relative mean flow, and it is assumed that the mean surface wind is from the east. Enhanced surface fluxes in the easterly phase of the wave (right) bias the convection toward the warm phase of the wave, in spite of convective lags (Fig. 6(b)). Both the convective heating and the upward motion are phase shifted slightly toward the warm phase of the wave, resulting in wave amplification. Note that the bias is small, so that the bulk of the convective heating still occurs in the ascent phase of the wave.

warming is partially in phase with the wave temperature perturbation, so that the WISHE effect tends to amplify the wave.

Emanuel (1993a) described the behaviour of a model of the equatorial atmosphere linearized about a mean state in radiative–convective equilibrium and with a constant easterly wind. The model assumed statistical-equilibrium convection, but with a slight lag in the response of convection. Such a model has two prominent unstable WISHE modes: an eastward-moving equatorially trapped Kelvin wave with a period around 30 days and low zonal wave number, and a class of synoptic-scale westward-moving disturbances with structures resembling mixed Rossby-gravity waves. These are not always equatorially trapped; f-plane equivalents show poleward and westward propagating synoptic-scale waves. In all cases the high-frequency modes are damped by MCD. Neelin and Yu (1994) and Yu and Neelin (1994) also showed that linear perturbations to model atmospheres with the Betts–Miller scheme are damped by convection, and that WISHE leads to amplifying Kelvin wave-like disturbances of low zonal wave number. (They did not consider nongeostrophic modes.) The damping occurs when the adjustment time-scale in the Betts–Miller scheme is not zero. Neelin and Yu also demonstrate that stochastic, white-noise forcing of convecting atmospheres with WISHE turned off excites a spectrum of 'most slowly decaying' disturbances having maximum power in the zonal wind spectrum at low wave numbers and frequencies, but maximum power in the precipitation rate at higher wave numbers and frequencies.

The advantage of considering the temperature rather than the heating is clear. Most of the convecting *heating* occurs in the ascent phase of the wave, but this heating is slightly *negatively* correlated with temperature and thus acts to damp the wave. The comparatively small amount of heating due to WISHE is more in phase with the wave temperature perturbation, and can amplify the wave.

6. SUMMARY

The understanding of the dynamics of convecting atmospheres has, in our view, been hindered by a tendency to consider the heating associated with convection, rather than directly to consider its effect on temperature. Disturbance growth requires a positive correlation of heating and temperature, and perturbations of the latter are usually very small, so that knowledge of the heating may be of little conceptual or predictive value.

We advocate an alternative mode of thinking based on the observation that most convective systems are nearly in statistical equilibrium with their environment. 'Statistical-equilibrium thinking' hinges on the idea that the virtual temperature of the free atmosphere is directly linked to subcloud-layer entropy, θ_e, in convecting atmospheres; this idea is well supported by observational studies of the tropical atmosphere. The subcloud-layer θ_e is reduced primarily by enhanced convective downdraughts associated with large-scale ascent, and thus the free atmosphere cools as well. For this reason, large-scale vertical motion 'feels' an effective static stability that, under typical conditions, is about an order of magnitude less than the dry static stability.

Very small departures from statistical equilibrium, in the form of small (several-hour) response times of convection, are nonetheless important in the dynamics of convecting atmospheres. They invariably lead to a negative correlation between convective heating and disturbance temperature, and thus to damping of disturbances, in proportion to their frequency. The direct interaction of large-scale disturbances with convection is thus better described as moist convective damping (MCD) than as conditional instability of the second kind (CISK). Other processes, such as wind-induced surface heat exchange (WISHE) can, however, lead to amplifying disturbances in convecting atmospheres.

The recognition of the direct relationship between temperature and subcloud-layer entropy leads to a conceptually clear picture of circulation systems in convecting atmospheres. If, for example, the subcloud-layer entropy distribution (which is controlled mostly by the SST field) is such that the balanced wind field at the tropopause exhibits negative absolute vorticity, a thermally direct circulation is implied. This condition is simply related to the subcloud-layer entropy distribution by an expression like (57) (which has been derived using geostrophic balance). A weak warm-core vortex placed in contact with the sea surface will experience a damping owing to the reduction of subcloud-layer entropy in the core by convective downdraughts excited by Ekman pumping. On the other hand, saturation of the troposphere in the vortex core shuts off the supply of low-entropy air to the subcloud layer (thus reducing the effective static stability to zero) and permits the WISHE mechanism to amplify the vortex. Equatorial oscillations are damped by their interaction with convection, but may be amplified by WISHE.

A number of substantive issues remain unresolved. One such issue is the magnitude of the CAPE in statistical-equilibrium convecting atmospheres. To our knowledge, there exists no theory for this quantity. Given the apparent importance of small time lags in convective response for the dynamics of convecting atmospheres, it may prove desirable to relax the rigid enforcement of statistical equilibrium employed in some convective representations, and also to develop a more physically based way of estimating or calculating such lags.

We believe that the conceptual simplifications inherent in statistical-equilibrium thinking will prove to be a boon to a new generation of tropical dynamicists.

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