CS 4510: Automata and Complexity 09/06/2022

Lecture 5: NFA To DFA

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Contents

1 Equivalence Of NFAs and DFAs (cont.)

Theorem 1. Every NFA has an equivalent DFA.

Proof. Let $N = (Q, \Sigma, \delta, q_0, F)$ be the NFA recognizing some language A. We construct DFA $M = (Q', \Sigma, \delta', q_0', F')$ recognizing A. Before doing the full construction, first consider the easier case when N has no ε arrows. We will take ε into account later.

1. $Q' = \mathcal{P}(Q)$ (the set of subsets of Q)

   Every state of M is a set of states of N.

2. Σ (the alphabet) doesn't change

3. For $R \in Q'$, and $a \in \Sigma$, let $\delta'(R, a) = \{q \in Q \mid q \in \delta(r, a) \text{ for some } r \in R\}$

   If R is a state of M, it is also a set of states of N. When M reads a symbol a in state R, it shows where A takes each state in R. Because each state may go to a set of states, we take the union of all these sets.

   $$\delta'(R, a) = \bigcup_{r \in R} \delta(r, a)$$

4. $q_0' = \{q_0\}$

   M starts in the state corresponding to the collection containing just the start state of N.

5. $F' = \{R \in Q' \mid R \text{ contains an accepting state of } N\}$

   The machine M accepts if one of the possible states that N could be in at this point is an accepting state.

Now consider the ε arrows. For any state R of M, we define $E(R)$ to be the collection of states that can be reached from members of R by going only along ε arrows, including members of R themselves. Formally, for $R \subseteq Q$, let

$$E(R) = \{q \mid q \text{ can be reached from } R \text{ by traveling along 0 or more } \varepsilon \text{ arrows}\}$$

Then we modify the transition function of M to place additional fingers on all states that can be reached by going along ε arrows after every step. Replacing $\delta(r, a)$ by $E(\delta(r, a))$ achieves this. Finally, we need to modify the start state of M to move the fingers initially to all possible states that can be reached from the start state of N along the ε arrows.

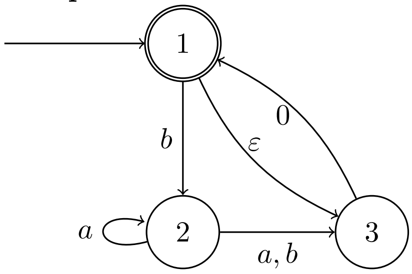
The changes mentioned above to account for ε arrows are shown below:

3. $\delta'(R, a) = \{q \in Q \mid q \in E(\delta(r, a)) \text{ for some } r \in R\}$

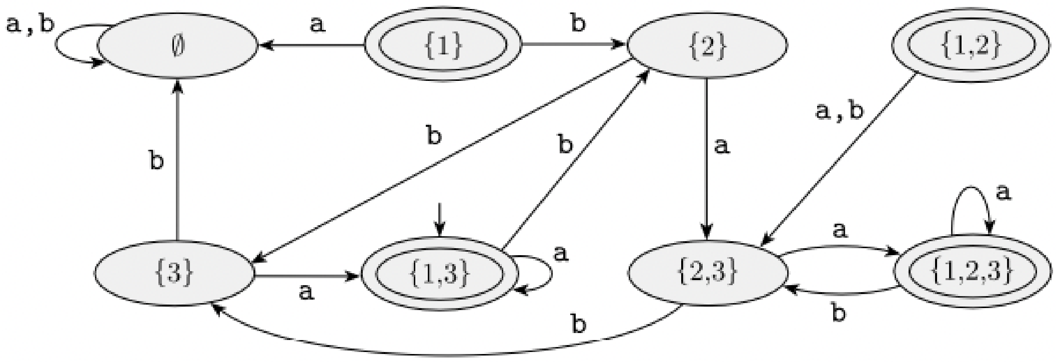
4. $q_0' = E(\{q_0\})$

$\square$

Example 1. Consider the following NFA N:



Note that the DFA will have 8 states, one for each subset of the states of N. The DFA and its transitions are shown below:



The NFA's start state is 1, so the DFA's start state is $E(\{1\}) = \{1,3\}$ (the set of states reachable from 1 by travelling along ε arrows and 1 itself). The NFA's accepting state is 1, so the DFA's accepting states are all sets of states that include 1: $\{\{1\},\{1,2\},\{1,3\},\{1,2,3\}\}$

As for D's transition function, each of D's states goes to one place on input a and one place on input b (by definition of DFA). We will illustrate a few.

- in D, state $\{2\}$ goes to $\{2,3\}$ on input a because in N, state 2 goes to both 2 and 3 on input a.
- in D, state $\{1\}$ goes to $\emptyset$ on input a because no a arrows exit it.
- in D, state $\{1,2\}$ goes to $\{2,3\}$ on input a because in N, state 1 goes nowhere on input a and state 2 goes to both 2 and 3 on input a

NFA with n states $\rightarrow$ DFA with 2^n states.