Mean, Variance and S.D of Binomial Distribution:

- Mean (μ):

$$\mu = \sum_{x=0}^{n} x\,p(x)$$

$$= \sum_{x=0}^{n} x\left[{}^nC_x\, p^x q^{n-x}\right]$$

$$= 0 + 1\left[{}^nC_1 p^1 q^{n-1}\right] + 2\left[{}^nC_2 p^2 q^{n-2}\right] + 3\left[{}^nC_3 p^3 q^{n-3}\right] + \dots$$

$$+ \dots n\left({}^nC_n p^n q^0\right)$$

$$= n \cdot p q^{n-1} + 2\left[\frac{n!}{(n-2)!\,2!} p^2 q^{n-2}\right] + 3\left(\frac{n!}{(n-3)!\,3!} p^3 q^{n-3}\right) + \dots$$

$$+ \dots n\left[p^n\right]$$

$$= npq^{n-1} + 2\left[\frac{n(n-1)(n-2)!}{(n-2)!\,2} p^2 q^{n-2}\right] + 3\left(\frac{n(n-1)(n-2)(n-3)!}{(n-3)!\,6} p^3 q^{n-3}\right) + \dots$$

$$+ \dots np^n$$

$$= npq^{n-1} + n(n-1)p^2 q^{n-2} + \frac{n(n-1)(n-2)}{2!} p^3 q^{n-3} + \dots + np^n$$

$$= np\left[q^{n-1} + (n-1)pq^{n-2} + \frac{(n-1)(n-2)}{2!} p^2 q^{n-3} + \dots + p^{n-1}\right]$$

$$= np\,[q+p]^{n-1}$$

$$= np\,(1) = np$$

$$\therefore \boxed{\mu = np}$$

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* **Variance (v):**

$$V = \sum_{x=0}^{n} x^2 p(x) - \mu^2$$

Consider, $\sum_{x=0}^{n} x^2 p(x) = \sum_{x=0}^{n} [x(x-1) + x] p(x)$

$$= \sum_{x=0}^{n} x(x-1)p(x) + \sum_{x=0}^{n} x p(x) = \sum_{x=0}^{n} x(x-1)[{}^nC_x p^x q^{n-x}] + np$$

$$= 0 + 0 + 2\cdot1 [{}^nC_2 p^2 q^{n-2}] + 3\cdot2 [{}^nC_3 p^3 q^{n-3}] + \dots\dots$$

$$+ n(n-1) [{}^nC_n p^n q^0] + np$$

$$= 2\left[\frac{n!}{(n-2)!\,2!} p^2 q^{n-2}\right] + 3\cdot2 \left[\frac{n!}{(n-3)!\,3!} p^3 q^{n-3}\right] + \dots\dots$$

$$+ n(n-1) [p^n] + np$$

$$= 2\left[\frac{n(n-1)(n-2)!}{(n-2)!\,2} p^2 q^{n-2}\right] + 3\cdot2\left[\frac{n(n-1)(n-2)(n-3)!}{(n-3)!\,(3\cdot2)} p^3 q^{n-3}\right] + \dots$$

$$+ n(n-1)(p^n) + np$$

$$= n(n-1)p^2 q^{n-2} + n(n-1)(n-2)p^3 q^{n-3} + \dots + n(n-1)p^n + np$$

$$= n(n-1)p^2 [q^{n-2} + (n-2)pq^{n-3} + \dots p^{n-2}] + np$$

$$= n(n-1)p^2 (q+p)^{n-2} + np$$

$$= n(n-1)p^2(1) + np$$

$$= n^2p^2 - np^2 + np$$

$$= n^2p^2 - np(-1+p)$$

$$= n^2p^2 + npq \qquad \{\because p+q=1\}$$

Now, $V = \sum_{x=0}^{n} x^2 p(x) - \mu^2 = n^2p^2 + npq - n^2p^2 \quad (\because \mu = np)$

$$\boxed{V = npq}$$

$\therefore$ S.D $= \sqrt{V} = \sqrt{npq}$ //

Mean, Variance and S.D of Poission Distribution

i) Mean (μ):

$$\mu = \sum_{x=0}^{\infty} x\,p(x)$$

$$= \sum_{x=0}^{\infty} x\left[\frac{m^x e^{-m}}{x!}\right]$$

$$= 0 + \left[\frac{me^{-m}}{1!} + 2.\frac{m^2e^{-m}}{2!} + 3.\frac{m^3e^{-m}}{3!} + \dots\right]$$

$$= me^{-m}\left[1 + \frac{m}{1!} + \frac{m^2}{2!} + \dots\right]$$

$$= me^{-m}(e^{m})$$

$$\boxed{\mu = m}$$

Variance(V):

$$V = \sum_{x=0}^{\infty} x^2 p(x) - \mu^2$$

Consider, $\sum_{x=0}^{\infty} x^2 p(x) = \sum_{x=0}^{\infty} [x(x-1) + x]\,p(x)$

$$= \sum_{x=0}^{\infty} x(x-1)\,p(x) + \sum_{x=0}^{\infty} x\,p(x)$$

$$= \sum_{x=0}^{\infty} x(x-1)\left\{\frac{m^x e^{-m}}{x!}\right\} + m$$

$$= 0 + 0 + 2.1\left[\frac{m^2e^{-m}}{2!}\right] + 3.2\left[\frac{m^3e^{-m}}{3!}\right] + 4.3\left[\frac{m^4e^{-m}}{4!}\right] + \dots$$

$$+ \dots + m$$

$$= m^2e^{-m} + m^3e^{-m} + \frac{m^4e^{-m}}{2!} + \dots + m$$

$$= m^2e^{-m}\left\{1 + m + \frac{m^2}{2!} + \dots\right\} + \dots + m$$

$$= m^2e^{-m}(e^{m}) = m^2 + m$$

$$V = \sum_{x=0}^{\infty} x^2 p(x) - \mu^2 = (m^2 + m) - m^2$$

$$\boxed{V = m}$$

$$S.D = \sqrt{V} = \sqrt{m}$$