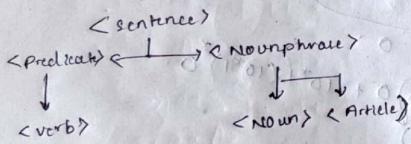


Unit-3 Grammars

Context free language



<> BNF notation.

Ex:-  $x = (ab)^* c$ .

$$\begin{aligned}
 A &\Rightarrow id = E \\
 &\Rightarrow id = E * E \\
 &\Rightarrow id = (E) * E \\
 &\Rightarrow id = (E+E) * E \\
 &\Rightarrow id = (id+E) * E
 \end{aligned}
 \quad
 \begin{aligned}
 &\Rightarrow id = (id+id) * E \\
 &\Rightarrow id = (id+id) * id
 \end{aligned}$$

\* Grammar is a 4 tuple  $(V, T, S, P)$ .

V - set of variables

T - set of terminals

S - set of start symbol

P - set of productions

Conditions: For context free language!

$A \rightarrow x$

$A \in V$

$x \in (V \cup T)^*$

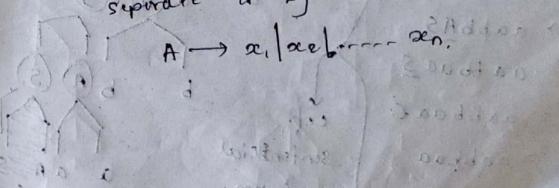
\* Terminals

small letters, operators, digits, punctuation marks,  
Keywords written in triple form.

\* Non Terminals

Upper case letters, letter S, lowercase letters with  
italic font, (Upper case letters (X,Y,Z) can be either termi-  
nal or nonterminal), (lower case (u,v,w,x,y,z) string  
of terminal symbols), (Greek letters (string of gram-  
mar symbols, can be either of them)).

\* More than one body for a head then  
separate it by a vertical line.







Ex:- write a grammar to accept a set of strings with 0 or more combination of an input symbol

$V = \{q_0\}$   
 $T = \{a, b\}$



$$q_0 \rightarrow a q_0 \mid e$$



$$V = \{q_0\} \cup \{q_1\}$$

$$T = \{a, b\}$$

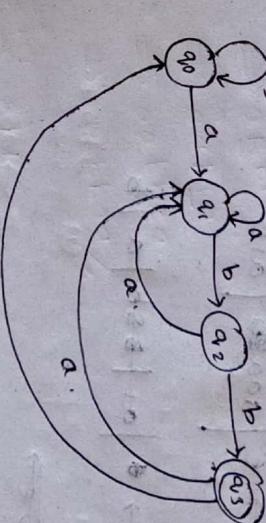
$$S \rightarrow a S \mid b S$$

option

$b^* a^* (a+b)^*$

iii. set of strings ending with abb.

$$(abb)^*$$



$$q_0 \rightarrow b q_0 \mid a q_1$$

$$q_1 \rightarrow a q_1 \mid b q_2$$

$$q_2 \rightarrow a q_1 \mid b q_3$$

$$q_3 \rightarrow a q_1 \mid b q_0$$

write a regular expression in grammar.

\* having substring aa.

$$\begin{array}{l} (\underline{a+b})^* \frac{aa}{\underline{a+b}} (\underline{a+b})^* \\ A \quad B \quad A \\ S \rightarrow ABA \\ A \rightarrow \underline{\underline{aaa}} A \quad e \\ B \rightarrow aa \end{array}$$

$$B \rightarrow aa$$

$$* L = \{ a^n b^n \mid n \geq 0 \}.$$

$$S \rightarrow e \left| \begin{array}{l} aaa \\ aab \end{array} \right| aab$$

$$S \rightarrow \left| \begin{array}{l} aab \\ asb \\ aba \end{array} \right| aab$$

$$* L = \{ a^n b^m \mid m > n \}.$$

$$S \rightarrow as \left| \begin{array}{l} sbb \\ a \\ b \end{array} \right| a \\ b$$

$$S \rightarrow \underline{\underline{as}} \left| \begin{array}{l} \underline{\underline{sbb}} \\ a \\ b \end{array} \right| asb \\ B$$

$$B \rightarrow b \left| \begin{array}{l} \underline{\underline{bB}} \\ bB \end{array} \right| bB$$

$$* L = \{ a^n b^m \mid m < n \}.$$

$$S \rightarrow asb \mid A$$

$$A \rightarrow a \mid aa$$

$$* L = \{ a^n b^m \mid m < n \}.$$

$$S \rightarrow asb \mid A \mid B.$$

$$\begin{array}{l} V = \{ S, A, B \} \\ T = \{ a, b \} \\ S = \# S \\ P = \end{array}$$

$$S \rightarrow a \mid A$$

$$B \rightarrow b \mid bB$$

$$* L = \{ wwr \mid we(a+b)^* \}.$$

$$\Sigma = (a, b)$$

$$w = a^n b^n.$$

$$w^R = b^n a^n$$

$$ww^R = a^n b^n b^n a^n.$$

$$S \rightarrow asa \mid bsb \mid a \mid b \mid e - \text{even aba.}$$

$$S \rightarrow asa \mid bsb \mid e - \text{even abba.}$$

\* signed or unsigned digit.

$$I \rightarrow N \left| \begin{array}{l} s, N \\ \end{array} \right|$$

$$N \rightarrow D \mid ND.$$

$$D \rightarrow 0 \mid 1 \dots \mid 9$$

$$s_1 \rightarrow + \mid -$$

$$T \xrightarrow{1MD} S, N$$

$$\Rightarrow +N$$

$$\Rightarrow +ND$$

$$\Rightarrow +NDD$$

$$\Rightarrow +DDD$$

$$\Rightarrow +1DD$$

$$\Rightarrow +1^2D$$

$$\Rightarrow +1^23/1$$





$$* P_2 \quad S \rightarrow \begin{cases} 1 & | \\ 2 & | \\ 3 & | \\ 4 & | \\ a & | \end{cases}$$

$$X \rightarrow \begin{cases} 1 & | \\ 2 & | \\ 3 & | \\ 4 & | \\ a & | \end{cases}$$

$$V = \{ S, X \}$$

$$T = \{ a \}$$

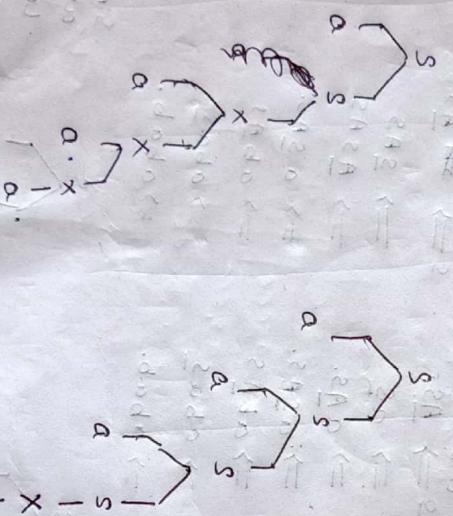
$$S = \{ S \}$$

$$P =$$

$$w = aaaa.$$

$$\begin{aligned} & \text{LMD. } 1, 2, 3, 3, 4 \\ S & \xrightarrow{\text{LMD.}} a \underline{s} \\ & \Rightarrow a \underline{x} \\ & \Rightarrow aax \\ & \Rightarrow aaax \\ & \Rightarrow aaaa. \end{aligned}$$

$$\begin{aligned} & \text{LMD. } 1, 1, 1, 2, 4 \\ S & \xrightarrow{\text{LMD.}} a \underline{s} \\ & \Rightarrow aas \\ & \Rightarrow aas \\ & \Rightarrow aaax \\ & \Rightarrow aaaa. \end{aligned}$$



unit production removing technique.

① writing all non unit production.

$S \rightarrow Aa|B.$

$B \rightarrow A|bb.$

$A \rightarrow a|bc|B.$

② dependency graph for unit production.



As there are two left most derivations, therefore the grammar is ambiguous.

$$\begin{array}{l} S \xrightarrow{*} B \\ S \xrightarrow{*} A \\ B \xrightarrow{*} A \\ A \xrightarrow{*} B \end{array}$$

$$* \quad S \rightarrow ABAC$$

$$A \rightarrow BC$$

$$B \rightarrow b|e$$

$$C \rightarrow D|e$$

$$D \rightarrow d$$

① finding nullable variable  $V_N$

$$V_N = \{ B, C, A \}$$

② exclude e production & rewrite.

$$\begin{aligned} S & \rightarrow ABAc | Bac | AAc | ABA | ac | Ba | Aa \\ & | a. \end{aligned}$$

$$A \rightarrow BC | C | B$$

$$B \rightarrow b$$

$$C \rightarrow D$$

$$D \rightarrow d$$

Removal of useless production.

Useful production: let  $q = \{u, t, s, p\}$  be a context free grammar any variable  $A \in V$  is said to be useful if and only if there is  $x \in T^*$  at least one  $w \in L(G)$  such that  $S \xrightarrow{*} xAy \xrightarrow{*} w$

Ex:

$$S \rightarrow aS \mid A \mid C$$

$$A \rightarrow a$$

$$B \rightarrow aa$$

$$C \rightarrow aCb$$

- ① Remove the variable that doesn't derive any terminal

Skipped

On	NV	Prod
$\phi$	$A, B$	$A \rightarrow a$
		$B \rightarrow aa$
$A, B$	$A, B, S, \epsilon$	$S \rightarrow A$
$A, B, S, \epsilon$	$A, B, S, \epsilon$	$S \rightarrow aS$

- ② Removal of  $\epsilon$  production.

$$V_N = \{A, a, b, c\}$$

$$S \rightarrow a \mid aA \mid b \mid c$$

$$A \rightarrow aB$$

$$B \rightarrow AaA$$

$$C \rightarrow cCD$$

$$D \rightarrow ddd$$

- ③ Removal of unit productions.

but all non-unit productions.

$$S \rightarrow a \mid aA \mid cCD \mid Aa$$

$$A \rightarrow aB$$

$$B \rightarrow AaA$$

$$C \rightarrow cCD$$

$$D \rightarrow ddd$$

dependency graph for unit productions

$$S \xrightarrow{*} B$$

$$S \xrightarrow{*} C$$

useless

$$S \rightarrow A \mid a$$

\* Ex simplify the grammar.

$$S \rightarrow a \mid aA \mid b \mid c$$

$$A \rightarrow aB \mid e$$

$$B \rightarrow Aa$$

$$C \rightarrow cCD$$

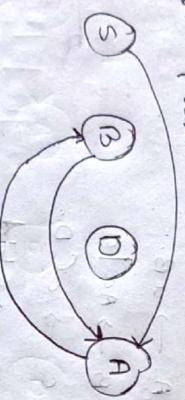
$$D \rightarrow ddd$$

(3) Removal of useless production:

Remove the variable that does not derive any terminal string.

OV	NV	Prod^n.
$\phi$	$S, B, D$	$S \rightarrow a$ $B \rightarrow a$ $D \rightarrow a$
$S, B, D$	$S, B, D, A$	$A \rightarrow aB$ $B \rightarrow aa$
$S, B, D, A$		$S \rightarrow aA$ $B \rightarrow Aa$

Remove the variables that is not taking part in derivation process.

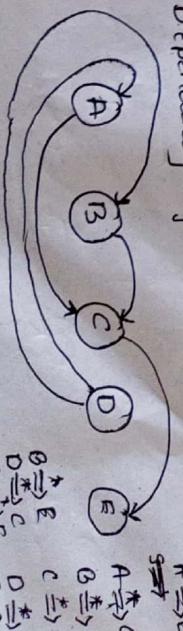


$D$  is not reachable & hence it is useless.

$$\begin{array}{l} S \rightarrow a | aA | Aa \\ A \rightarrow aB \\ B \rightarrow Aa | a \end{array}$$

$D$  is not reachable & hence it is useless.

Dependency graph of unit productions



$$\begin{array}{l} *Ex:- S \rightarrow aAa | bBb | e \\ A \rightarrow C | a \\ B \rightarrow C | b \\ C \rightarrow CDE | e \\ D \rightarrow A | B | b | a | e \end{array}$$

(4) Removal of  $\epsilon$  production:

$$V_N = \{ S, C, A, B, D \}$$

$$S \rightarrow aAa | bBb | aC | bb$$

$$A \rightarrow C | a$$

$$B \rightarrow C | b$$

$$C \rightarrow CDE | DE | CE | E$$

$$D \rightarrow A | B | ab$$

(5) Removal of unit productions.

List of all non-unit production.

~~$$S \rightarrow aAa | bBb | aa | bb$$~~

~~$$A \rightarrow a | CDE | DE | CE$$~~

~~$$B \rightarrow b | CDE | DE | CE$$~~

~~$$C \rightarrow CDE | DE | CE | E$$~~

~~$$D \rightarrow ab | aCDE | DE | CE | b$$~~



Removal of  
Converting to variables

$$\begin{aligned}
 E &\rightarrow E B_+ T \mid T B_* F \mid B_c E B, \quad | id \\
 B_+ &\rightarrow + \\
 B_* &\rightarrow * \\
 B_c &\rightarrow C \\
 B_c &\rightarrow ) \\
 B_c &\rightarrow ) \\
 T &\rightarrow T B_* F \mid B_c E B, \quad | id \\
 F &\rightarrow B_c E B, \\
 F &\rightarrow id, \\
 E &\rightarrow E D_1 \mid T D_2, \quad | B_c D_3 | id, \\
 P_1 &\rightarrow B_+ T \\
 P_2 &\rightarrow B_* F \\
 P_3 &\rightarrow E B, \\
 B_+ &\rightarrow + \\
 B_* &\rightarrow * \\
 B_c &\rightarrow C \\
 B_c &\rightarrow ) \\
 T &\xrightarrow{\text{length 2}} \\
 D_2 &\rightarrow B_* F \\
 D_3 &\rightarrow E B,
 \end{aligned}$$

Ex:  $S \rightarrow AB \xrightarrow{\text{conversion to CNF}} aAb \mid bB$

A context free grammar is said to be in CNF if all productions are of the form  $A \rightarrow \alpha x$  where  $\alpha \in T$  &  $x \in V^*$ .

$S \rightarrow aAb \mid bB$

$A \rightarrow aA \mid bB \mid b$

$B \rightarrow b$ .

\* your specification is used as input to the your tool.

your programming

Using your tool write a parser to validate the syntax of arithmetic expression.

shrp1 :: Grammar

$S \rightarrow e \mid e \cdot e \mid e * e \mid e / e \mid e ^ e \mid ID \mid$

D

Going back NF.

