Jai Vinayaka

UNIT- III

1) Given that no. of bits = 6

no. of Symbols = 3

They are having same Probabilities

Now

$$P_i \log_2\left(\frac{1}{P_i}\right) \Rightarrow 3 \log_2\left(\frac{1}{P_i}\right).$$

Equate both the Probabilities.

$$2\cancel{6} = \cancel{3}\log_2\left(\frac{1}{P_i}\right)$$

$$2 = \log_2\left(\frac{1}{P_i}\right)$$

$$2^2 = \frac{1}{P_i} \Rightarrow \boxed{P_i = \frac{1}{4}}$$

Jai Vinayaka UNIT- III

2) Given that $x_1 = 0.4$

$x_2 = 0.3$

$x_3 = 0.2$

$x_4 = 0.1$

Asking to find Entropy

W.K.T

$$M = \sum_{i=1}^{n} P_i \log_2\left(\frac{1}{P_i}\right)$$

$$\Rightarrow \sum_{i=1}^{4} P_i \log_2\left(\frac{1}{P_i}\right)$$

$$\Rightarrow 0.4 \log_2\left(\frac{1}{0.4}\right) + 0.3 \log_2\left(\frac{1}{0.3}\right) + 0.2 \log_2\left(\frac{1}{0.2}\right) + 0.1 \log_2\left(\frac{1}{0.1}\right)$$

$$\Rightarrow 0.4 \frac{\log\left(\frac{1}{0.4}\right)}{\log 2} + 0.3 \frac{\log\left(\frac{1}{0.3}\right)}{\log 2} + 0.2 \frac{\log\left(\frac{1}{0.2}\right)}{\log 2} + 0.1 \frac{\log\left(\frac{1}{0.1}\right)}{\log 2}$$

$$\Rightarrow 0.4\left[\frac{0.397}{0.301}\right] + 0.3\left[\frac{0.522}{0.301}\right] + 0.2\left[\frac{0.698}{0.301}\right] + 0.1\left(\frac{1}{0.301}\right)$$

$$\Rightarrow 0.4[1.318] + 0.3[1.734] + 0.2[2.318] + 0.1[3.322]$$

$$\Rightarrow 0.5272 + 0.5202 + 0.463 + 0.3322$$

$$\boxed{M \Rightarrow 1.8426}$$

Jai Vinayaka UNIT- III

3) Given that $x_1 = 0.4$

$x_2 = 0.3$

$x_3 = 0.2$

$x_4 = 0.1$

$$I = \log_2\left(\frac{1}{0.4}\right) + \log_2\left(\frac{1}{0.3}\right) + \log_2\left(\frac{1}{0.2}\right) + \log_2\left(\frac{1}{0.1}\right)$$

$$\Rightarrow \frac{\log\left(\frac{1}{0.4}\right)}{\log_2} + \frac{\log\left(\frac{1}{0.3}\right)}{\log 2} + \frac{\log\left(\frac{1}{0.2}\right)}{\log 2} + \frac{\log\left(\frac{1}{0.1}\right)}{\log 2}$$

$$\Rightarrow 1.318 + 1.734 + 2.318 + 3.222$$

$$\boxed{I \Rightarrow 8.592}$$

REDMI NOTE 10 PRO | VARDAN 16/07/2024 09:18

Jai Vinayaka UNIT - III

5) To Prove that Mutual information is symmetric with Each other

From the formula

$$I(x,y) = \sum_x \sum_y P(x,y) \log_2 \left(\frac{P(x/y)}{x} \right) \Rightarrow I(X;Y).$$

$$\Rightarrow \sum_x \sum_y P(x,y) \log_2 \left(\frac{P(x,y)}{P(y)\,P(x)} \right)$$

$$\Rightarrow \sum_x \sum_y P(x,y) \log_2 \left(\frac{P(y/x)}{y} \right)$$

$$\Rightarrow I(y;x)$$

$\Rightarrow 1\times0.4 + 2\times0.2 + 3\times0.15 + 4\times0.15 + 4\times0.2$

$\Rightarrow$ 2.25 bits/message.

* Shannon - Fano Algorithm :-

Step-1 :- Arrange the given probabilities of all messages in descending order.

Step-2 :- Split the column into 2 partitions such that sum of probabilities in upper ≤ sum of probability in lower partion.

sum of pb in

Step-3 :- Assign 0 & 1 codes for upper & lower partitions respectively.

Step-4 :- The process is repeated till every partition has single message (or) symbol.

The above process generates unique coding for all the messages.

$\frac{1}{8}+\frac{1}{2}+\frac{1}{4}$

$\frac{1+1+2}{8}$

REDMI NOTE 10 PRO | VARDAN 16/07/2024 09:36

Jai Vinayaka **UNIT-III**

6) Given that $m_1 = 0.4$; $m_2 = 0.2$; $m_3 = 0.12$; $m_4 = 0.08$; $m_5 = 0.08$; $m_6 = 0.08$; $m_7 = 0.04$.

Now we have to arrange in descending order

	Step-1 :-	Step-2 :-	Step-3 :-	Step-4.	
0.4	0	0			$n_1 = 2$ bits
0.2	0	1			$n_2 = 2$ bits
0.12	1	0	0		$n_3 = 3$ bits
0.08	1	0	1		$n_4 = 3$ bits
0.08	1	1	0		$n_5 = 3$ bits
0.08	1	1	1	0	$n_6 = 4$ bits
0.04	1	1	1	1	$n_7 = 4$ bits

$$\text{Efficiency}(\eta) = \frac{H}{L}$$

$$L = \sum_{i=1}^{n} P_i n_i$$

$$L \Rightarrow 2\times0.4 + 2\times0.2 + 3\times0.12 + 3\times0.08 + 3\times0.08 + 4\times0.08 + 4\times0.04$$

$$L \Rightarrow 0.8 + 0.4 + 0.36 + 0.24 + 0.24 + 0.32 + 0.16$$

$$\boxed{L \Rightarrow 1.44}$$

6)

$$H = -\sum_{i=1}^{n} P_i \log_2(P_i)$$

$$\Rightarrow -\left[0.4\log_2(0.4) + 0.2\log_2(0.2) + 0.12\log_2(0.12) + 0.08\log_2(0.08) + 0.08\log_2(0.08) + 0.08\log_2(0.08) + 0.04\log_2(0.04)\right]$$

$$\Rightarrow -\left[-0.159 - 0.139 - 0.110 - 0.087 - 0.087 - 0.087 - 0.055\right]$$

$$\Rightarrow -[-0.724] \Rightarrow \underline{\underline{0.724}}$$

$$\therefore \eta = \frac{H}{L} \Rightarrow \frac{0.724}{1.44}$$

$$\boxed{\eta \Rightarrow 0.502}$$

Jai Vinayaka UNIT-III

7) Given that $m_1 = \frac{16}{32}$; $m_2 = \frac{4}{32}$; $m_3 = \frac{4}{32}$; $m_4 = \frac{2}{32}$; $m_5 = \frac{2}{32}$; $m_6 = \frac{2}{32}$

$m_7 = \frac{1}{32}$; $m_8 = \frac{1}{32}$.

Arrange into Descending order.

	Step-1	Step-2	S-3	S-4 :-	S-5	
$\frac{16}{32}$	0					$n_1 = 1$ bit
$\frac{4}{32}$	1	0	0			$n_2 = 3$ bits
$\frac{4}{32}$	1	0	1			$n_3 = 3$ bits
$\frac{2}{32}$	1	1	0	0		$n_4 = 4$ bits
$\frac{2}{32}$	1	1	0	1		$n_5 = 4$ bits
$\frac{2}{32}$	1	1	1	0		$n_6 = 4$ bits
$\frac{1}{32}$	1	1	1	1	0	$n_7 = 5$ bits
$\frac{1}{32}$	1	1	1	1	1	$n_8 = 5$ bits

$$\boxed{\eta = \frac{H}{L}}$$

$$L = \sum_{i=1}^{n} p_i n_i$$

$$\Rightarrow \left[\frac{16}{32}\times 1 + \frac{4}{32}\times 3 + \frac{4}{32}\times 3 + \frac{2}{32}\times 4 + \frac{2}{32}\times 4 + \frac{2}{32}\times 4 + \frac{1}{32}\times 5 + \frac{1}{32}\times 5\right]$$

$$\Rightarrow \left[\frac{16}{32} + \frac{12}{32} + \frac{12}{32} + \frac{24}{32} + \frac{10}{32}\right] \Rightarrow \frac{74}{32} \Rightarrow 2.31$$

Jai Vinayaka UNIT - III

7)

$$H = -\left[\frac{16}{32}\log_2\left(\frac{16}{32}\right) + \frac{4}{32}\log_2\left(\frac{4}{32}\right) + \frac{4}{32}\log_2\left(\frac{4}{32}\right) + \frac{2}{32}\log_2\left(\frac{2}{32}\right)\right.$$

$$\left. + \frac{2}{32}\log_2\left(\frac{2}{32}\right) + \frac{2}{32}\log_2\left(\frac{2}{32}\right) + \frac{1}{32}\log_2\left(\frac{1}{32}\right) + \frac{1}{32}\log_2\left(\frac{1}{32}\right)\right]$$

$$\Rightarrow -\left[-0.150 - 0.112 - 0.112 - 0.075 - 0.075 - 0.075 - 0.047 - 0.047\right]$$

$$\Rightarrow -\left[-0.693\right] \Rightarrow 0.693$$

$$\therefore \eta = \frac{H}{L} \Rightarrow \frac{0.693}{2.3125} \Rightarrow 0.299$$

REDMI NOTE 10 PRO | VARDAN 16/07/2024 10:11

Huffman coding Algorithm :-

i) Arrange the probabilities of symbols in descending order.

ii) Add the least 2 probabilities in the column.

iii) Arrange the remaining probabilities including the above sum in descending order.

iv) Repeat step 2 and 3 until only 2 probabilities remaining.

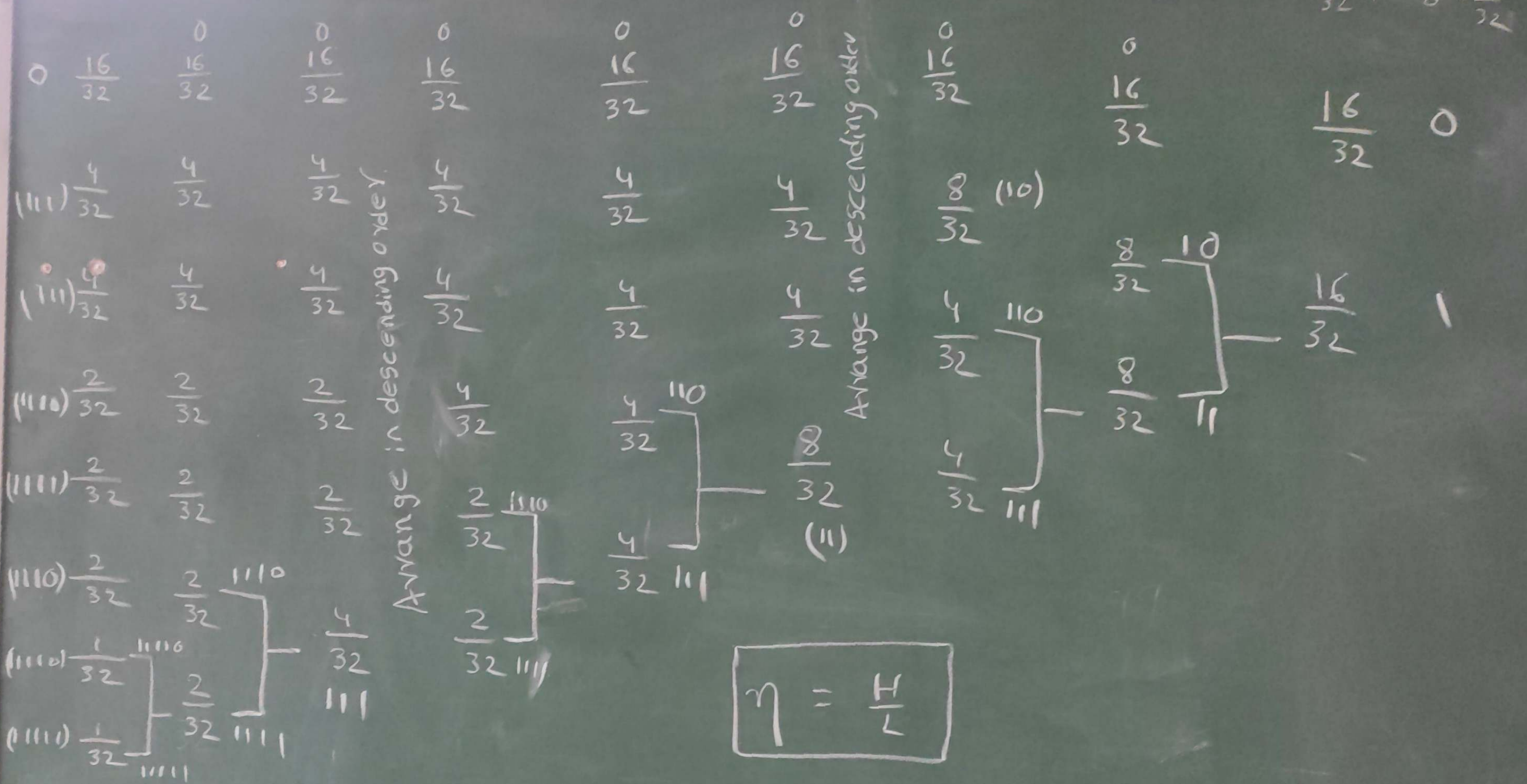
v) we need to Assign ~~codes~~ Binary codes '0' & '1' to the upper and lower probabilities

vi) Repeat step 5 until following the branch of combination from Right to left column

Jai Vinayaka

UNIT-III

3) Given that $m_1 = \frac{16}{32}$; $m_2 = \frac{4}{32}$; $m_3 = \frac{4}{32}$; $m_4 = \frac{2}{32}$; $m_5 = \frac{2}{32}$; $m_6 = \frac{2}{32}$; $m_7 = \frac{1}{32}$; $m_8 = \frac{1}{32}$



$$\boxed{\eta = \frac{H}{L}}$$

$n_1 = \frac{16}{32} = 1$ bit

$n_2 = \frac{4}{32} = 3$ bits

$n_3 = \frac{4}{32} = 3$ bits

$n_4 = \frac{2}{32} = 4$ bits

$n_5 = \frac{2}{32} = 4$ bits

$n_6 = \frac{2}{32} = 4$ bits

$n_7 = \frac{1}{32} = 5$ bits

$n_8 = \frac{1}{32} = 5$ bits

$$L = \frac{16}{32}\times 1 + \frac{4}{32}\times 3 + \frac{4}{32}\times 3 + \frac{2}{32}\times 4 + \frac{2}{32}\times 4 + \frac{2}{32}\times 4 + \frac{1}{32}\times 5 + \frac{1}{32}\times 5$$

$$\Rightarrow \frac{16}{32} + \frac{24}{32} + \frac{24}{32} + \frac{10}{32} \Rightarrow \frac{74}{32} = 2.315$$

$$H = -\left[\frac{16}{32}\log_2\left(\frac{16}{32}\right) + \frac{4}{32}\log_2\left(\frac{4}{32}\right) + \frac{4}{32}\log_2\left(\frac{4}{32}\right) + \frac{2}{32}\log_2\left(\frac{2}{32}\right) + \frac{2}{32}\log_2\left(\frac{2}{32}\right) + \frac{2}{32}\log_2\left(\frac{2}{32}\right) + \frac{1}{32}\log_2\left(\frac{1}{32}\right) + \frac{1}{32}\log_2\left(\frac{1}{32}\right)\right]$$

$$H = -[-0.693] = 0.693$$

$$\therefore \eta = \frac{H}{L} = \frac{0.693}{2.315} \Rightarrow 0.299$$

8)

0 $\frac{16}{32}$, (110) $\frac{4}{32}$, (110) $\frac{4}{32}$, (1110) $\frac{2}{32}$, (1110) $\frac{2}{32}$, (1110) $\frac{2}{32}$, $\frac{1}{32}$ 11110, $\frac{1}{32}$ 11111

0 $\frac{16}{32}$, (110) $\frac{4}{32}$, (110) $\frac{4}{32}$, (1110) $\frac{2}{32}$, (1110) $\frac{2}{32}$, $\frac{2}{32}$ 1110, $\frac{2}{32}$ 1111

0 $\frac{16}{32}$, (110) $\frac{4}{32}$, (110) $\frac{4}{32}$, (1110) $\frac{2}{32}$, (1110) $\frac{2}{32}$, $\frac{4}{32}$ 111

Arrange in desc ord.

0 $\frac{16}{32}$, (110) $\frac{4}{32}$, (110) $\frac{4}{32}$, (110) $\frac{4}{32}$, $\frac{2}{32}$ 1110, $\frac{2}{32}$ 1111

0 $\frac{16}{32}$, (110) $\frac{4}{32}$, (110) $\frac{4}{32}$, $\frac{4}{32}$ 110, $\frac{4}{32}$ 111

0 $\frac{16}{32}$, (110) $\frac{4}{32}$, (110) $\frac{4}{32}$, $\frac{8}{32}$ 11

Arrange in descending order

0 $\frac{16}{32}$, (10) $\frac{8}{32}$, $\frac{4}{32}$ 110, $\frac{4}{32}$ 111

0 $\frac{16}{32}$, $\frac{8}{32}$ 10, $\frac{8}{32}$ 11

$\frac{16}{32}$ 0, $\frac{16}{32}$ 1

$m_1 = \frac{16}{32} = 0 = 1$ bit

$m_2 = \frac{4}{32} = 110 = 3$ bit

$m_3 = \frac{4}{32} = 110 = 8$ bit

$m_4 = \frac{2}{32} = 1110 = 4$ bit

$m_5 = \frac{2}{32} = 1110 - 4$

$m_6 = \frac{2}{32} = 1110 - 4$

$m_7 = \frac{1}{32} = 11110 - 5$

$m_8 = \frac{1}{32} = 11111 - 5$

$$L = \frac{16}{32} + \frac{4}{32}\times 3 + \frac{4}{32}\times 3 + \frac{2}{32}\times 4 + \frac{2}{32}\times 4 + \frac{2}{32}\times 4 + \frac{1}{32}\times 5 + \frac{1}{32}\times$$

$$= \frac{16}{32} + \frac{12}{32} + \frac{12}{32} + \frac{8}{32} + \frac{8}{32} + \frac{8}{32} + \frac{5}{32} + \frac{5}{32}$$

16
24
$\frac{+8}{48}$
24
$\overline{72}$
8

$$\Rightarrow \frac{74}{32} = 3.215$$

$$H = -\left[\frac{16}{32}\log_2\left(\frac{16}{32}\right) + \frac{4}{32}\log_2\left(\frac{4}{32}\right) + \frac{2}{32}\log_2\left(\frac{2}{32}\right) + \frac{2}{32}\log_2\left(\frac{2}{32}\right) + \frac{2}{32}\log_2\left(\frac{2}{32}\right) + \frac{1}{32}\log_2\left(\frac{1}{32}\right) + \frac{1}{32}\log_2\left(\frac{1}{32}\right)\right]$$

$$H = 0.693$$

$$\therefore \eta = \frac{H}{L} = \frac{0.693}{3.215} = 0.299$$

Jai Vinayaka UNIT- III

9) Given $m_1 = 0.4$; $m_2 = 0.2$; $m_3 = 0.2$; $m_4 = 0.1$; $m_5 = 0.1$

0.4	0.4	0.4	0.4	0.4	0.6 0
0.2	0.2	0.2	0.4 00	0.6 0	0.4 1
0.2	0.2 000	0.4 00	0.2 01		
0.1 0010	0.2 001				
0.1 0011					

$\boxed{\begin{array}{l} n_1 = 0.4 = 1 \text{ bit} \\ n_2 = 0.2 = 3 \text{ bits} \\ n_3 = 0.2 = 3 \text{ bits} \\ n_4 = 0.1 = 4 \text{ bits} \\ n_5 = 0.1 = 4 \text{ bits.} \end{array}}$

$$\eta = \frac{H}{L}$$

$$L = 0.4\times1 + 0.2\times3 + 0.2\times3 + 0.1\times4 + 0.1\times4$$

$$\Rightarrow 0.4 + 0.6 + 0.6 + 0.4 + 0.4$$

$$\boxed{L = 2.4}$$

$$H = -\left[0.4\log_2(0.4) + 0.2\log_2(0.2) + 0.2\log_2(0.2) + 0.1\log_2(0.1) + 0.1\log_2(0.1)\right]$$

$$H = -\left[-0.159 - 0.139 - 0.139 - 0.1 - 0.1\right]$$

$$H = -\left[-0.637\right] \Rightarrow 0.637.$$

$$\therefore \eta = \frac{H}{L} = \frac{0.637}{2.4} \Rightarrow \underline{\underline{0.265}}.$$