DATE / /
PAGE

UNIT-1

Mathematical Logic

* Statement (or) Preposition: All declarative sentences which can be assigned one and only one of the two possible truth values are called statements (or) prepositions.

→ statements (or) prepositions are denoted by P, Q, R - - - (or) p, q, r - - -

Note:

→ Two possible truth values are true and False
→ True is represented by T (or) 1
→ False is represented by F (or) 0

Example:

1. Tajmahal is in India. The truth value of this sentence is T.
Hence it is a statement.

2. close the door.
It is not a declarative sentence.

Compound Statements : The statements that contains one or more Primary statements and some connections are called compound statements

Connecting statements

1. Nagation : The negation of the statement is generally formed by introducing the word 'not' at a proper place in a statement

→ If p denotes a statement then the negation p is written as $\neg P$ read it as "not p".

→ If the statement p has the truth value T then $\neg P$ has the truth value F

→ If P is false $\neg P$ is true

Truth table for 'not'

P	$\neg P$
T	F
F	T

Example : Consider the preposition P : Chennai is a city

The negation of the statement is Chennai is not a city

2. Conjunction : If the two statements P and q denoted by '$p \wedge q$' read it as 'p and q' is called as conjuction of p and q.

The statement ~~p q~~ $p \wedge q$ has the truth value T whenever both p and q have the truth value T Otherwise it has the truth value F.

Truth table for $p \wedge q$

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Example:

P: Jack went up the hill

Q: Jill went up the hill

$p \wedge q$: Jack and Jill went up the hill

3. Disjunction: The disjunction of two statements p and q is denoted by '$p \vee q$' read it as 'p or q'

$p \vee q$: The statement $p \vee q$ has the truth value F whenever both p and q have the truth value F. Otherwise it has the truth value T

Truth table for $p \vee q$

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Example:

$p \vee q$: Jack or Jill went up the hill

4. **Conditional**: If p and q are any two statements then the statement '$p \rightarrow q$' read as 'if p then q' is called conditional statement

→ The statement $p \rightarrow q$ has the truth value F whenever if p is T and q is F else T

Truth table for $p \rightarrow q$

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Construct the truth table for $p \rightarrow q$ and $q \rightarrow p$ $((p \rightarrow q) \wedge (q \rightarrow p))$

p	q	$p \rightarrow q$	$q \rightarrow p$	Ans
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

5. **Biconditional**: If p and q are any two statements, the statement '$p \leftrightarrow q$' they read it as 'p if and only if q' is called biconditional statement.

→ The statement '$p \leftrightarrow q$' has the truth value T whenever p and q have the identical truth value in all other cases it has the truth value F.

Truth table for $p \leftrightarrow q$

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

$2^3 = 8$ possible truth values are getting

Construct the truth table for following

i $[q \wedge (p \rightarrow q)] \rightarrow p$

ii $\neg [p \vee (q \wedge r)] \leftrightarrow [(p \vee q) \wedge (p \vee r)]$

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$\neg[p \vee (q \wedge r)]$	$(p \vee q)$
T	T	T	T	T	F	T
T	T	F	F	T	F	T
T	F	T	F	T	F	T
T	F	F	F	T	F	T
F	T	T	T	T	F	T
F	T	F	F	F	T	T
F	F	T	F	F	T	F
F	F	F	F	F	T	F

$p \vee r$	$(p \vee q) \wedge (p \vee r)$	$A \leftrightarrow B$
T	T	F
T	T	F
T	T	F
T	T	F
T	T	F
F	F	F
T	F	F
F	F	F

i $[q \wedge (p \to q)] \to p$

p	q	$p \to q$	$q \wedge (p \to q)$	$[q \wedge (p \to q)] \to p$
T	T	T	T	T
T	F	F	F	T
F	T	T	~~F~~ T	F
F	F	T	F	T

Construct the truth tables for the following

i $[p \wedge (p \to q)] \to q$

ii $(p \to q) \leftrightarrow (\neg p \vee q)$

iii $[(p \to q) \wedge (q \to r)] \to (p \vee r)$

iv $(p \leftrightarrow q) \leftrightarrow [(p \wedge q) \vee (\neg p \wedge \neg q)]$

v $(p \leftrightarrow r) \wedge (\neg q \to s)$

vi $[\neg(p \wedge q) \vee \neg r] \vee [(q \leftrightarrow \neg p) \to (r \vee \neg s)]$

vi Sol

p	q	r	s	$p \wedge q$	$\neg(p \wedge q)$	$[\neg(p \wedge q) \vee \neg r]$	$\neg p$	$q \leftrightarrow \neg p$	$\neg s$	$r \vee \neg s$	$(q \leftrightarrow \neg p) \to (r \vee \neg s)$	A∨B
1	1	1	1	1	0	0	0	0	0	1	1	1
1	1	1	0	1	0	0	0	0	1	1	1	1
1	1	0	1	1	0	1	0	0	0	0	1	1
1	1	0	0	1	0	1	0	0	1	1	1	1
1	0	1	1	0	1	1	0	1	0	1	1	1
1	0	1	0	0	1	1	0	1	1	1	1	1
1	0	0	1	0	1	1	0	1	0	0	0	1
1	0	0	0	0	1	1	0	1	1	1	1	1
0	1	1	1	0	1	1	1	1	0	1	1	1
0	1	1	0	0	1	1	1	1	1	1	1	1
0	1	0	1	0	1	1	1	1	0	0	0	1
0	1	0	0	0	1	1	1	1	1	1	1	1
0	0	1	1	0	1	1	1	0	0	1	1	1
0	0	1	0	0	1	1	1	0	1	1	1	1
0	0	0	1	0	1	1	1	0	0	0	1	1
0	0	0	0	0	1	1	1	0	1	1	1	1

v)

p	q	r	s	$p \leftrightarrow r$	$\neg q$	$\neg q \to s$	$(p \leftrightarrow r) \wedge (\neg q \to s)$
1	1	1	1	1	0	1	1
1	1	1	0	1	0	1	1
1	1	0	1	0	0	1	0
1	1	0	0	0	0	1	0
1	0	1	1	1	1	1	1
1	0	1	0	1	1	0	0
1	0	0	1	0	1	1	0
1	0	0	0	0	1	0	0
0	1	1	1	0	0	1	0
0	1	1	0	0	0	1	0
0	1	0	1	1	0	1	1
0	1	0	0	1	0	1	1
0	0	1	1	0	1	1	0
0	0	1	0	0	1	0	0
0	0	0	1	1	1	1	1
0	0	0	0	1	1	0	0

iv)

p	q	$(p \leftrightarrow q)$	$(p \wedge q)$	$\neg p$	$\neg q$	$(\neg p \wedge \neg q)$	$(p \wedge q) \vee (\neg p \wedge \neg q)$	$A \leftrightarrow B$
1	1	1	1	0	0	0	1	1
1	0	0	0	0	1	0	0	1
0	1	0	0	1	0	0	0	1
0	0	1	0	1	1	1	1	1

i)

p	q	$p \to q$	$p \wedge (p \to q)$	$[p \wedge (p \to q)] \to q$
1	1	1	1	1
1	0	0	0	1
0	1	1	0	1
0	0	1	0	1

ii

p	q	$p \to q$	$\neg p$	$\neg p \vee q$	$(p \to q) \leftrightarrow (\neg p \vee q)$
1	1	1	0	1	1
1	0	0	0	0	~~0~~ 1
0	1	1	1	1	~~0~~ 1
0	0	1	1	1	~~0~~ 1

iii $[(p \to q) \wedge (q \to r)] \longrightarrow (p \vee r)$

p	q	r	$p \to q$	$q \to r$	A $(p \to q) \wedge (q \to r)$
1	1	1	1	1	1
1	1	0	1	0	0
1	0	1	0	1	0
1	0	0	0	1	0
0	1	1	1	1	1
0	1	0	1	0	0
0	0	1	1	1	1
0	0	0	1	1	1

B $p \vee r$	$A \to B$
1	1
1	~~0~~ 1
1	~~0~~ 1
1	~~0~~ 1
1	1
0	1
1	1
0	0

How can the following sentence be translated into a logical expression

E: You cannot write the coaster if you are under 5ft tall unless you are older than 18 years

Let P, Q, R represent the prepositions as given below

P: You can ride the coaster

Q: You are under 5ft tall

R: You are older than 18 years

The logical expression for the given statement is

$(q \wedge \neg r) \longrightarrow \neg p$

Converse, Inverse and Contrapositive

The preposition $q \rightarrow p$ is called the converse of $p \rightarrow q$

The preposition $\neg q \longrightarrow \neg p$ is called contrapositive of $p \rightarrow q$

$\neg p \rightarrow \neg q$ is called inverse of $p \rightarrow q$

Well-formed formulas:

Statement formula :- A statement formula is an expression denoted by a string consisting of variables, parenthesis and connective symbols

→ A well formed formula can be generated by the following rules

- A statement variable standing alone is a well formed formula
- If 'A' is a well formed formula then '$\neg A$' is also a well formed formula.

- If 'A' and 'B' are two well formed formulas then $A \vee B$, $A \wedge B$, $A \rightarrow B$, & $A \leftrightarrow B$ are also well formed formulas
- A string of symbols containing the statement variables, connectives and parenthesis is a well formed formulas if and only if it can be obtained by finite application rules of 1, 2, 3

Tautology : A compound preposition that is always true irrespective of the truth values of the prepositions that occur in it, is called a tautology

Contradiction : A compound preposition that is always false irrespective of the truth values of the prepositions that occur in it, is called a contradiction

Contingency : Neither a tautology nor a contradiction then that preposition is called contingency.

Example: Indicate which of the following formulas are tautology or contradiction or contingency

i $p \rightarrow (p \vee q)$

ii $((p \rightarrow \neg p) \rightarrow \neg p)$

iii $(\neg q \wedge p) \wedge q$

iv $(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$

v $(\neg p \rightarrow q) \rightarrow (q \rightarrow p)$

vi $((p \wedge q) \leftrightarrow p)$

i)

p	q	$p \vee q$	$p \rightarrow (p \vee q)$
1	1	1	1
1	0	1	1
0	1	1	1
0	0	0	1

(tautology)

ii

p	$\neg p$	$p \rightarrow \neg p$	$(p \rightarrow \neg p) \rightarrow \neg p$
1	0	0	1
0	1	1	1

(tautology)

iii

p	q	$\neg q$	$\neg q \wedge p$	$(\neg q \wedge p) \wedge q$
1	1	0	0	0
1	0	1	1	0
0	1	0	0	0
0	0	1	0	0

(contradiction)

iv

p	q	r	$q \rightarrow r$	$p \rightarrow q$	$p \rightarrow r$	$(p \rightarrow (q \rightarrow r))$
1	1	1	1	1	1	1
1	1	0	0	1	0	0
1	0	1	1	0	1	1
1	0	0	1	0	0	1
0	1	1	1	1	1	1
0	1	0	0	1	1	1
0	0	1	1	1	1	1
0	0	0	1	1	1	1

~~$(p \to (q \to r)) \to ($~~

$((p \to q) \to (p \to r))$ $A \to B$

	1
1	1
0	1
1	1
1	1
1	1
1	1
1	1
1	1

(tautology)

v.

~~p~~ p	q	$\neg p$	$\overset{A}{\neg p \to q}$	$\overset{B}{q \to p}$	$A \to B$
1	1	0	1	1	1
1	0	0	1	1	1
0	1	1	1	0	0
0	0	1	0	1	1

(contingency)

vi

p	q	$p \wedge q$	$(p \wedge q) \leftrightarrow p$
1	1	1	1
1	0	0	0
0	1	0	1
0	0	0	1

(contingency)

Logically Equivalent : The prepositions p and q are said to be logially equivalent if $p \leftrightarrow q$ is a tautology and it is denoted by $p \Leftrightarrow q$ (or) $p \equiv q$

Logically Equivalent formulae

1) Idempotent Law
   (i) $p \lor p \equiv p$
   (ii) $p \land p \equiv p$

2) Associative Law
   (i) $p \lor (q \lor r) \equiv (p \lor q) \lor r$
   (ii) $p \land (q \land r) \equiv (p \land q) \land r$

3) Commutative Law
   (i) $p \lor q \equiv q \lor p$
   (ii) $p \land q \equiv q \land p$

4) Distributive Law
   (i) $p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$
   (ii) $p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$

5) Identity Law
   (i) $p \land T \equiv p$
   (ii) $p \lor F \equiv p$

6) Domination Law
   (i) $p \lor T \equiv T$
   (ii) $p \land F \equiv F$

7) Negation Law
   (i) $p \lor \neg p \equiv T$
   (ii) $p \land \neg p \equiv F$

8) Demorgan's Law
   (i) $\neg(p \lor q) \equiv \neg p \land \neg q$
   (ii) $\neg(p \land q) \equiv \neg p \lor \neg q$

9) Absorption Law
   (i) $p \lor (p \land q) \equiv p$
   (ii) $p \land (p \lor q) \equiv p$

10) Double Negation Law
   (i) $\neg(\neg p) \equiv p$

i) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$ (C)	$p \vee q$ (A)	$p \vee r$ (B)	$A \wedge B$ (D)
1	1	1	1	1	1	1	1
1	1	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	0	0	0	1	1	1	1
0	1	1	1	1	1	1	1
0	1	0	0	0	1	0	0
0	0	1	0	0	0	1	0
0	0	0	0	0	0	0	0

$C \leftrightarrow D$ = tautology

ii) Demorgans

$\neg(p \vee q) \equiv \neg p \wedge \neg q$

p	q	$\neg p$	$\neg q$	$p \vee q$	$\neg(p \vee q)$	$\neg p \wedge \neg q$	$A \leftrightarrow B$
1	1	0	0	1	0	0	1
1	0	0	1	1	0	0	1
0	1	1	0	1	0	0	1
0	0	1	1	0	1	1	1

iii Absorption

$p \vee (p \wedge q) \equiv p$

p	q	$p \wedge q$	$p \vee (p \wedge q)$	$A \leftrightarrow B$
1	1	1	1	1
1	0	0	1	1
0	1	0	0	1
0	0	0	0	1

$[p \to (q \to r)] \longleftrightarrow [p \to (\neg q \vee r)] \Longleftrightarrow [(p \wedge q) \to r]$

$[\neg p \wedge (\neg q \wedge r)] \vee [(q \wedge r) \underset{\wedge}{\vee} (p \wedge r)] \Longleftrightarrow r$

ii)

p	q	r	$\neg p$	$\neg q$	$\neg q \wedge r$	$\neg p \wedge (\neg q \wedge r)$	$\overset{B}{q \wedge r}$	$\overset{C}{p \wedge r}$	$\overset{D}{B \vee C}$	$A \wedge D$	$(A \vee D) \Leftrightarrow r$
1	1	1	0	0	0	0	1	1	1	1	1
1	1	0	0	0	0	0	0	0	0	0	1
1	0	1	0	1	1	0	0	1	1	1	1
1	0	0	0	1	0	0	0	0	0	0	1
0	1	1	1	0	0	0	1	0	1	1	1
0	1	0	1	0	0	0	0	0	0	0	1
0	0	1	1	1	1	1	0	0	0	1	1
0	0	0	1	1	0	0	0	0	0	0	1

i)

p	q	r	$\neg q$	$q \to r$	$\overset{A}{p \to (q \to r)}$	$\neg q \vee r$	$\overset{B}{p \to \neg q \vee r}$	$A \leftrightarrow B$	$p \wedge q$	$p \wedge q \to r$
1	1	1	0	1	1	1	1	1	1	1
1	1	0	0	0	0	0	0	1	1	0
1	0	1	1	1	1	1	1	1	0	1
1	0	0	1	1	1	1	1	1	0	1
0	1	1	0	1	1	1	1	1	0	1
0	1	0	0	0	1	0	1	1	0	1
0	0	1	1	1	1	1	1	1	0	1
0	0	0	1	1	1	1	1	1	0	1

$A \Leftrightarrow B \Leftrightarrow C$

T

T

T

T

T

T

Show that $[p \rightarrow (q \rightarrow r)] \Leftrightarrow p \rightarrow (\neg q \vee r) \Leftrightarrow (p \wedge q) \rightarrow r$

Ans Formula $\boxed{\begin{array}{l} p \rightarrow q \Leftrightarrow \neg p \vee q \\ p \leftrightarrow q \Leftrightarrow (p \rightarrow q) \wedge (q \rightarrow p) \end{array}}$

$p \rightarrow (q \rightarrow r)$

$\Leftrightarrow p \rightarrow (\neg q \vee r)$

$\Leftrightarrow \neg p \vee (\neg q \vee r)$

$\Leftrightarrow (\neg p \vee \neg q) \vee r$ (Associative Law)

$\Leftrightarrow \neg(p \wedge q) \vee r$ (Demorgan's Law)

$\Leftrightarrow p \wedge q \rightarrow r$

Show that $(\neg p \wedge (\neg q \wedge r)) \vee [(q \wedge r) \vee (p \wedge r)] \leftrightarrow r$

$[\neg p \wedge (\neg q \wedge r)] \vee [(q \wedge r) \vee (p \wedge r)]$

$\Leftrightarrow (\neg p \wedge (\neg q \wedge r)) \vee ((q \vee p) \wedge r)$ (Distributive Law)

$\Leftrightarrow [(\neg p \wedge \neg q) \wedge r] \vee [(p \vee q) \wedge r]$ (Associative & Commutative Law)

$\Leftrightarrow [\neg(p \vee q) \wedge r] \vee [(p \vee q) \wedge r]$ (Demorgan's Law)

$\Leftrightarrow [\neg(p \vee q) \vee (p \vee r)] \wedge r$ (Distributive Law)

$\Leftrightarrow T \wedge r$ (Negation Law)

$\Leftrightarrow r$ (Identity Law)

Show that $\neg(p \vee (\neg p \wedge q))$ and $(\neg p \wedge \neg q)$ are LE

$\neg(p \vee (\neg p \vee q))$ and $(\neg p \wedge \neg q)$

$\cancel{\Leftrightarrow \neg p \wedge \neg(\neg p \vee q)}$

$\cancel{\Leftrightarrow \neg p\ \wedge} \Leftrightarrow \neg((p \vee \neg p) \wedge (p \vee q))$

$\Leftrightarrow \neg(T \wedge (p \vee q))$

$\Leftrightarrow \neg(p \vee q) \Leftrightarrow (\neg p \wedge \neg q)$

DATE / /
PAGE

$\neg(p \vee (\neg p \wedge q))$

$\Leftrightarrow \neg p \wedge \neg(\neg p \wedge q)$

$\Leftrightarrow \neg p \wedge (\neg(\neg p) \vee \neg q)$

$\Leftrightarrow \neg p \wedge (p \vee \neg q)$

$\Leftrightarrow (\neg p \wedge p) \vee (\neg p \wedge \neg q)$

$\Leftrightarrow p \vee (\neg p \wedge \neg q)$

$\Leftrightarrow \neg p \wedge \neg q$

i) Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology without using truth table

ii) Show that $((p \vee q) \wedge \neg(\neg p \wedge (\neg q \vee \neg r))) \vee ((\neg p \wedge \neg q) \vee (\neg p \wedge \neg r))$

i) $(p \wedge q) \longrightarrow (p \vee q)$

$\Leftrightarrow \neg(p \wedge q) \vee (p \vee q)$ $\quad (p \rightarrow \Leftrightarrow \neg p \vee q)$

$\Leftrightarrow (\neg p \vee \neg q) \vee (p \vee q)$ $\quad$ (Demorgan's)

$\Leftrightarrow (\neg p \vee p) \vee (\neg q \vee q)$ $\quad$ (Associative)

$\Leftrightarrow T \vee T$ $\quad$ (Negation)

$\Leftrightarrow T$

ii) $((p \vee q) \wedge \neg(\neg p \wedge (\neg q \vee \neg r))) \vee ((\neg p \wedge \neg q) \vee (\neg p \wedge \neg r))$

$\Leftrightarrow ((p \vee q) \wedge (\neg(\neg p) \vee \neg(\neg q \vee \neg r))) \vee [\neg(p \vee q) \vee \neg(p \vee r)]$

(Demorgan's Law)

$\Leftrightarrow [(p \vee q) \wedge (p \vee (q \wedge r))] \vee \neg[(p \vee q) \wedge (p \vee r)]$

(Double negation & Demorgan's Law)

$\Leftrightarrow (p \vee q) \wedge [(p \vee q) \wedge (p \vee r)] \vee \neg((p \vee q) \wedge (p \vee r))$

(Distributive Law)

$\Leftrightarrow [((p \vee q) \wedge (p \vee q)] \wedge (p \vee r)] \vee \neg[(p \vee q) \wedge (p \vee r)]$

~~Idem.~~ (Associative Law)

$\Longleftrightarrow [(p \vee q) \wedge (p \vee r)] \vee \neg[(p \vee q) \wedge (p \vee r)]$

(Idempotent Law)

$\Longleftrightarrow T$ (Negation Law $(p \vee \neg p \equiv T)$)

i) Show that $(p \rightarrow q) \wedge (r \rightarrow q) \Longleftrightarrow (p \vee r) \rightarrow q$ are L.E

ii) $\neg(p \leftrightarrow q) \Longleftrightarrow [(p \vee q) \wedge \neg(p \wedge q)]$

Duality Law: Two formulas x and x^* are said to be duals of each other if one can be obtained from the other by replacing and by or ($\wedge$ by $\vee$) and or by and ($\vee$ by $\wedge$) and connective $\wedge, \vee$ are also called duals of each other.

Note: If the formula X contains the special variable T and F then its dual X^* is obtained by replacing T by F and F by T

Tautological Implications: A statement X is said to be tautologically imply a statement Y if and only if $X \rightarrow Y$ is a tautology. We shall denote it by $X \Rightarrow Y$ read it as X implies Y

i.e, $X \Rightarrow Y$ if and only if $X \rightarrow Y$ is a tautology

$$\boxed{X \Rightarrow Y \Longleftrightarrow X \rightarrow Y \text{ is a tautology}}$$

NAND : The word NAND is a combination of not and AND where NOT stands for negation and AND stands for conjunction. The connective NAND is denoted by the symbol ↑

Truth table for NAND

p	q	$p \wedge q$	$\neg(p \wedge q) = p \uparrow q$
1	1	1	0
1	0	0	1
0	1	0	1
0	0	0	1

NOR : The word NOR is a combination of NOT and OR where NOT stands for negation and OR stands for Disjunction. The connective NOR is denoted by the symbol ↓

Truth table for NOR

p	q	$p \vee q$	$\neg(p \vee q) = p \downarrow q$
1	1	1	0
1	0	1	0
0	1	1	0
0	0	0	1

NORMAL FORMS

i) $(p \rightarrow q) \wedge (r \rightarrow q) \Longleftrightarrow (p \vee r) \rightarrow q$

$\Longleftrightarrow (\neg p \vee q) \wedge (\neg r \vee q)$ (Distributive)

$\Longleftrightarrow (\neg p \wedge \neg r) \vee q$ (Demorgan's)

$\Longleftrightarrow \neg(p \vee r) \vee q$ $(p \rightarrow q \Longleftrightarrow \neg p \vee q)$

$(p \vee r) \rightarrow q$

$(p \rightarrow q) \wedge (r \rightarrow q) \Longleftrightarrow (p \vee r) \rightarrow q$

ii) $\neg(p \longleftrightarrow q) \Longleftrightarrow [(p \vee q) \wedge \neg(p \wedge q)]$

$\neg(p \longleftrightarrow q)$

from $p \Longleftrightarrow q = (p \rightarrow q) \vee (q \rightarrow p)$

$\Longleftrightarrow \neg((p \rightarrow q) \vee (q \rightarrow p))$

$[\neg(p \rightarrow q) \wedge \neg(q \rightarrow p)]$

from $(p \rightarrow q) = \neg p \vee q$

$\neg(\neg p \vee q) \wedge \neg(\neg q \vee p)$

$(p \wedge \neg q) \wedge (q \wedge \neg p)$

$(p \wedge q) \wedge (\neg p \wedge \neg q)$

$(p \wedge q) \wedge \neg(p \wedge \neg q)$

DATE / /
PAGE

NORMAL FORMS

Elementary Sum: The sum of the variables and their negations is called elementary sum.
For example: p and q are two variables then the elementary sum are $p \vee q$, $p \vee \neg q$, $\neg p \vee \neg q$, $p \vee \neg p \vee \neg q$, $\neg p \vee q \vee \neg q$, $\neg p \vee \neg q \vee q$ etc

Elementary product: Product of variables and their negations is called elementary product.
For example: p and q are two variables then the elementary products are $p \wedge q$, $p \wedge \neg q$, $\neg p \wedge \neg q$, $p \wedge \neg p \wedge \neg p$, $\neg p \wedge q \wedge \neg q$, $\neg p \wedge \neg q \wedge q$ etc.

Disjunctive Normal form (DNF): A formula that is equivalent to the given formula and consists of sum of elementary products is called as disjunctive normal form.

Conjunctive Normal form (CNF): A formula that is equivalent to the given formula and consists of product of elementary sums is called as conjunctive normal form.

Note: $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

Obtain a disjunctive normal form of
$\neg(p\vee q)\longleftrightarrow(p\wedge q)$

Given $\neg(p\vee q)\longleftrightarrow(p\wedge q)$

$\iff [\neg(p\vee q)\wedge(p\wedge q)]\vee[\neg(\neg(p\vee q))\wedge\neg(p\wedge q)]$

$\iff ((\neg p\wedge\neg q)\wedge(p\wedge q))\vee[(p\vee q)\wedge(\neg p\vee\neg q)]$

(Demorgan's & Double Negation Law)

$\iff (\neg p\wedge\neg q\wedge p\wedge q)\vee[((p\vee q)\wedge\neg p)\vee((p\vee q)\wedge\neg q)]$

$(p\wedge(q\vee r)=(p\wedge q)\vee(p\wedge r))$

$\iff (\neg p\wedge\neg q\wedge p\wedge q)\vee(p\wedge\neg p)\vee(q\wedge\neg p)\vee(p\wedge\neg q)\vee(q\wedge\neg q)$

(Distribution Law)

Obtain disjunction normal form of
$p\to((p\to q)\wedge\neg(\neg q\vee\neg p))$

$\iff p\to((\neg p\vee q)\wedge(q\wedge p))$ $(p\to q\equiv\neg p\vee q$

Demorgans & Double negation)

$\iff \neg p\vee((\neg p\vee q)\wedge(q\wedge p))$ $(p\vee q)\wedge r=(p\wedge r)\vee(q\wedge r)$

$\iff \neg p\vee[(\neg p\wedge(q\wedge p))\vee(q\wedge(q\wedge p))]$ (Distribution)

$\iff \neg p\vee(\neg p\wedge q\wedge p)\vee(q\wedge q\wedge p)$

Obtain conjunctive normal form of
$\neg(p\vee q)\longleftrightarrow(p\wedge q)$

$\neg(p\vee q)\longleftrightarrow(p\wedge q)$

$\iff[\neg(p\vee q)\longrightarrow(p\wedge q)]\wedge[(p\wedge q)\longrightarrow\neg(p\vee q)]$

$(p\leftrightarrow q\equiv(p\to q)\wedge(q\to p))$

$\iff[\neg(\neg(p\vee q))\vee(p\wedge q)]\wedge[\neg(p\wedge q)\vee\neg(p\vee q)]$

$(p\to q\equiv\neg p\vee q)$

$\iff [\underbrace{(p\vee q)}_{p}\underbrace{\vee}_{\vee}\underbrace{(p\wedge q)}_{(q\wedge r)}]\wedge[(\neg p\vee\neg q)\vee(\neg p\wedge\neg q)]$

$(p\vee(q\wedge r)=(p\vee q)\wedge(p\vee r))$

$\iff [((p\vee q)\vee p)\wedge((p\vee q)\vee q)]\wedge[((\neg p\vee\neg q)\vee\neg p)$

$\wedge((\neg p\vee\neg q)\vee\neg q)]$

$r \wedge (\neg p \vee q) \quad = (r \wedge \neg p) \vee (r \wedge q)$

$(q \wedge p) \wedge (\neg p \vee q)$

$\leftrightarrow (p \vee q \vee p) \wedge (p \vee q \vee q) \wedge (\neg p \vee q \vee \neg p) \wedge (\neg p \vee \neg q \vee \neg q)$

Obtain conjunctive normal form of

$p \rightarrow ((p \rightarrow q) \wedge \neg(\neg q \vee \neg p))$

$p \rightarrow ((p \rightarrow q) \wedge \neg(\neg q \vee \neg p))$ $\quad (p \rightarrow q \equiv \neg p \vee q)$

$\neg p \vee ((\overset{P}{\neg p} \vee \overset{Q}{q}) \wedge \boxed{(q \wedge p)})$ $\quad$ (Negation Law)

$\neg p \vee ((\neg p \wedge (q \wedge p)) \vee (\cancel{\neg p}\, q \wedge (q \wedge p))$

$\neg p \vee ((\neg p \wedge (q \wedge p)) \vee (q \wedge (q \wedge p)))$

$\neg p \vee ((\underline{\overset{r}{\neg p \vee q}}) \wedge (\underline{\overset{P\ Q}{q \wedge p}}))$.

$\neg p \vee (((\neg p \vee q) \wedge q)$

$\neg p \vee ((\neg p \wedge (q \wedge p)) \vee (q \wedge (q \wedge p)))$

DATE / /
PAGE

Minterms For a given number of variables, the minterms consists of conjunctions in which each Variable or its negation, but not both appears only one
Eg: Let p and q be two statements variables all positive min terms are

Ex. $p, q \quad 2^2 = 4$

$p \wedge q, \neg p \wedge q, p \wedge \neg q, \neg p \wedge \neg q$

$p, q, r \quad = 2^3 = 8$

$\rightarrow p \wedge q \wedge r, \neg p \wedge q \wedge r, p \wedge \neg q \wedge r, p \wedge q \wedge \neg r,$
$\neg p \wedge \neg q \wedge r, \neg p \wedge q \wedge \neg r, p \wedge \neg q \wedge \neg r, \neg p \wedge \neg q, \neg r$

If the statement formula Consisting 3 variables then we get $2^3 = 8$ number of possible minterms. They are given above.

Principle disjunctive normal form (PDNF):
For a given formula, an equivalent formula consists of disjunction of minterms only is known as its principle disjunction normal form

Maxterm: For a given number of variables the maxterm consists of disjunctions in which each variable or its negation, but not both, and appears only once.
Eg: Two variables p, q the number of maxterms are $2^2 = 4$

i.e $p \vee q, \neg p \vee q, p \vee \neg q, \neg p \vee \neg q$

p, q, r . the no. of maxterms are $2^3 = 8$

$p \lor q \lor r$, $p \lor q \lor \neg r$, $p \lor \neg q \lor r$, $p \lor \neg q \lor \neg r$,
$\neg p \lor q \lor r$, $\neg p \lor q \lor \neg r$, $\neg p \lor \neg q \lor r$, $\neg p \lor \neg q \lor \neg r$

Principle conjunctive normal form (PCNF)

For a given formula, an equivalent formula consisting of conjunctions of the maxterms only is known as its principle conjunctive normal form

Obtain the PDNF of negation $\neg p \lor q$

p	q	$\neg p$	$\neg p \lor q$	minterms
T	T	F	T	$p \land q$
T	F	F	F	
F	T	T	T	$\neg p \land q$
F	F	T	T	$\neg p \land \neg q$

~~$\neg p \land$~~ $\neg p \lor q$

$\Leftrightarrow (\neg p \lor T) \lor (q \land T)$ (Identity Law)

$\Leftrightarrow [\neg p \land (q \lor \neg q)] \lor [q \land (p \lor \neg p)]$ (Negation Law)

$\Leftrightarrow [(\neg p \land q) \lor (\neg p \land \neg q)] \lor [(q \land p) \lor (q \land \neg p)]$

$\Leftrightarrow (\neg p \land q) \lor (p \land q) \lor (\neg p \land \neg q)$ (Distribution Law)

Disjunction of minterms which is the required PDNF

Obtain PDNF of $\underbrace{(p \land q)}_{A} \lor \underbrace{(\neg p \land r)}_{B} \lor \underbrace{(q \land r)}_{C}$

p	q	r	$p \land q$ (A)	$\neg p$	$\neg p \land r$ (B)	$q \land r$ (C)	$A \lor B \lor C$	minterm
T	T	T	T	F	F	T	T	$p \land q \land r$
T	T	F	T	F	F	F	T	$p \land q \land \neg r$
T	F	T	F	F	F	F	F	
T	F	F	F	F	F	F	F	

p	q	r	$\overset{A}{p\wedge q}$	$\neg p$	$\overset{B}{\neg p\wedge r}$	$\overset{C}{q\wedge r}$	$A\vee B\vee C$	minterm
F	T	T	F	T	T	T	T	$\neg p\wedge q\wedge r$
F	T	F	F	T	F	F	F	
F	F	T	F	T	T	F	T	$\neg p\wedge \neg q\wedge r$
F	F	F	F	T	F	F	F	

PDNF : $(p\wedge q\wedge r)\vee(p\wedge q\wedge \neg r)\vee(\neg p\wedge q\wedge r)\vee(\neg p\wedge\neg q\wedge r)$

$(p\wedge q)\vee(\neg p\wedge r)\vee(q\wedge r)$

$\Leftrightarrow [(p\wedge q)\wedge T]\vee[(\neg p\wedge r)\wedge T]\vee[(q\wedge r)\wedge T]$

$(P\wedge T\equiv P)$ (Identity law)

$\Leftrightarrow [(p\wedge q)\wedge(r\vee\neg r)]\vee[(\neg p\wedge r)\wedge(q\vee\neg q)]$

$\vee[(q\wedge r)\wedge(p\vee\neg p)]$ (negation law)

$\Leftrightarrow [(p\wedge q\wedge r)\vee(p\wedge q\wedge\neg r)]\vee[(\neg p\wedge r\wedge q)\vee(\neg p\wedge r\wedge\neg q)]$

$\vee[(q\wedge r\wedge p)\vee(q\wedge r\wedge\neg p)]$ (Distribution law)

$\Leftrightarrow (p\wedge q\wedge r)\vee(p\wedge q\wedge\neg r)\vee(\neg p\wedge q\wedge r)\vee(\neg p\wedge\neg q\wedge r)$

(Idempotent and commutative law)

Disjunctions of minterms which is the required PDNF

1) ~~$p\vee r$ $(\neg p\wedge\neg q\wedge r)$~~

i) Obtain the PDNF of $P\rightarrow[(P\rightarrow q)\wedge\neg(\neg q\vee\neg p)]$

ii) Obtain the PDNF of $[(\neg p\rightarrow r)\wedge(q\leftrightarrow p)]$

P	q	$\neg P$	$\neg q$	$\overset{A}{P\rightarrow q}$	$\neg q\vee\neg P$	$\overset{B}{\neg(\neg q\vee\neg P)}$	$\overset{C}{A\wedge B}$	$P\rightarrow C$
T	T	F	F	T	F	T	T	T
T	F	F	T	F	T	F	F	F
F	T	T	F	T	T	F	F	T
F	F	T	T	T	T	F	F	T

minterms : $(P\wedge q)\vee(\neg P\wedge q)\vee(\neg P\wedge\neg q)$

$P\rightarrow[(P\rightarrow q)\wedge\neg(\neg q\vee\neg P)]$

$\Leftrightarrow \neg P\vee[(\neg P\vee q)\wedge\neg(\neg q\vee\neg P)]$

DATE / /
PAGE

$\Leftrightarrow \neg p \vee [(\neg p \vee q) \wedge \neg(\neg(p \wedge q))]$

$\Leftrightarrow \neg p \vee [(\neg p \vee q) \wedge (p \wedge q)]$

$\Leftrightarrow (\neg p \wedge T) \vee [(\neg p \vee q) \wedge (p \wedge q)]$

$\Leftrightarrow (\neg p \wedge (q \vee \neg q)) \vee [(\neg p \wedge (p \wedge q)) \vee (q \wedge (p \wedge q))]$

$\Leftrightarrow ((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee [(\neg p \wedge (p \wedge q)) \vee (q \wedge (p \wedge q))]$

$\Leftrightarrow ((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee [(\neg p \wedge p \wedge q) \vee (p \wedge q \wedge q)]$

$\Leftrightarrow (\neg p \wedge q) \vee (\neg p \wedge \neg q) \vee [(F \wedge q) \vee (p \wedge q)]$

$\Leftrightarrow (\neg p \wedge q) \vee (\neg p \wedge \neg q) \vee [F \vee (p \wedge q)]$

$\Leftrightarrow (\neg p \wedge q) \vee (\neg p \wedge \neg q) \vee (p \wedge q)$

Obtain the PCNF of $\neg p \vee q$

p	q	$\neg p$	$\neg p \vee q$	maxterm
T	T	F	T	
T	F	F	F	$\neg p \vee q$
F	T	T	T	
F	F	T	T	

$\neg(p \vee q)$

p	q	$p \vee q$	$\neg(p \vee q)$	maxterms
T	T	T	F	$\neg p \vee \neg q$
T	F	T	F	$\neg p \vee q$
F	T	T	F	$p \vee \neg q$
F	F	F	T	

$\neg(p \vee q)$

$\Leftrightarrow \neg p \wedge \neg q$ (Demorgan's Law)

$\Leftrightarrow (\neg p \vee F) \wedge (\neg q \vee F)$ (Identity law)

$\Leftrightarrow [\neg p \vee (q \wedge \neg q)] \wedge [\neg q \vee (p \wedge \neg p)]$ (Negation)

$\Leftrightarrow (\neg p \vee q) \wedge (\neg p \vee \neg q) \wedge (\neg q \vee p) \wedge (\neg q \vee \neg p)$ (Distribution)

$\Leftrightarrow (\neg p \lor q) \land (\neg p \lor \neg q) \land (p \lor \neg q)$

(Commutative & Idempotent)

3. $(p \land q) \lor (\neg p \land r) \lor (q \land r)$

p	q	r	$\neg p$	$\overset{A}{p \land q}$	$\overset{B}{\neg p \land r}$	$\overset{C}{q \land r}$	$A \lor B \lor C$	Maxterms
T	T	T	F	T	F	T	T	
T	T	F	F	T	F	F	T	
T	F	T	F	F	F	F	F	$\neg p \lor q \lor \neg r$
T	F	F	F	F	F	F	F	$\neg p \lor q \lor r$
F	T	T	T	F	T	T	T	
F	T	F	T	F	F	F	F	$p \lor \neg q \lor r$
F	F	T	T	F	T	F	T	
F	F	F	T	F	F	F	F	$p \lor q \lor r$

$(p \land q) \lor (\neg p \land r) \lor (q \land r)$

$\Leftrightarrow [(p \land q) \lor F] \lor [(\neg p \land r) \lor F] \lor [(q \land r) \lor F]$ (Identity law)

$\Leftrightarrow [(p \land q) \lor (r \land \neg r)] \lor [(\neg p \land r) \lor (q \land \neg q)] \lor [(q \land r) \lor (p \land \neg p)]$

(Negation law)

iv) $p \rightarrow [(p \rightarrow q) \land \neg(\neg q \lor \neg p)]$

p	q	$\neg p$	$\neg q$	$\overset{A}{p \rightarrow q}$	$\neg q \lor \neg p$	$\overset{B}{\neg(\neg q \lor \neg p)}$	$\overset{C}{A \land B}$	$p \rightarrow C$
T	T	F	F	T	F	T	T	T
T	F	F	T	F	T	F	F	F
F	T	T	F	T	T	F	F	T
F	F	T	T	T	T	F	F	T

maxterms

$\neg p \lor q$

DATE / /
PAGE

$p \rightarrow [(p \rightarrow q) \wedge \neg(\neg q \vee \neg p)]$

$p \rightarrow [(\neg p \vee q) \wedge \neg(\neg(q \wedge p))]$

$p \rightarrow [(\neg p \vee q) \wedge (q \wedge p)]$ (Demorgan's Law)

$\neg p \vee [(\neg p \vee q) \wedge (q \wedge p)]$

$\neg p \vee [(\neg p \vee q \wedge q) \wedge (\neg p \vee q \wedge p)]$ (Distribution Law)

Inference theory for a statement calculus

Consider the set of statements $p_1, p_2, p_3 - - - - p_n$ and the statement q. The compound preposition $p_1 \wedge p_2 \wedge p_3 - - - \wedge p_n \Rightarrow q$ is called an argument. Here the statements $p_1, p_2, p_3 - - p_n$ are called premises and the statement q is called conclusion

→ The above argument can also be written as

$$\begin{array}{l} P_1 \\ P_2 \\ P_3 \\ \vdots \\ P_n \\ \hline \therefore q \\ \hline \end{array}$$

Valid or Invalid argument

An argument with premises $P_1, P_2, P_3 - - P_n$ and the conclusion q is said to be valid if whenever all the premises $p_1, p_2, p_3 - - - p_n$ are true then the conclusion q is also true otherwise the argument is called invalid.

Method 1 : Truth Table Method

Show that the conclusion $C: \neg p$ follows from the premises $H_1: \neg q$; $H_2: p \to q$

The given argument is

$H_1: \neg q$

$H_2: p \to q$

$\therefore C: \neg p$

p	q	$\neg q$ (H_1)	$p \to q$ (H_2)	$\neg p$
T	T	F	T	F
T	F	T	F	F
F	T	F	T	T
F	F	T	T	T

In the truth table the last row consisting the two premises having the truth value True (T) in that case the conclusion is also true then it is a valid argument.

Determine whether the following statement form is a valid or invalid

$H_1: p \to (q \vee \neg r)$

$H_2: q \to (p \wedge r)$

$C: p \to r$

p	q	r	$\neg r$	$q \vee \neg r$	$p \wedge r$	$p \xrightarrow{H_1} (q \vee \neg r)$	$q \xrightarrow{H_2} (p \wedge r)$	$p \to r$
T	T	T	F	T	T	T	T	T
T	T	F	T	T	F	T	F	F
T	F	T	F	F	T	F	T	T
T	F	F	T	T	F	T	T	F
F	T	T	F	T	F	T	F	T
F	T	F	T	T	F	T	F	T
F	F	T	F	F	F	T	T	T
F	F	F	T	T	F	T	T	T

DATE / /
PAGE

The fourth row of the truth table, the two premises having the truth value true, but the conclusion has the truth value false. Hence the given argument is invalid.

Verify the following arguments are valid or invalid

i) $H_1 : p \rightarrow q$ $C : p \rightarrow p \wedge q$

ii) $H_1 : \neg p \vee q$; $H_2 : \neg(q \wedge \neg r)$; $H_3 : \neg r$; $C : \neg p$

p	q	r	$\neg p$	$\neg p \vee q$ (H_1)	$\neg r$	$q \wedge \neg r$ (H_2)	$\neg(q \wedge \neg r)$ (H_3)	~~r~~	$\neg p$ (C)
T	T	T	F	T	F	F	T	F	F
T	T	F	F	T	T	T	F		F
T	F	T	F	F	F	F	T		F
T	F	F	F	F	T	F	T		F
F	T	T	T	T	F	F	T		T
F	T	F	T	T	T	T	F		T
F	F	T	T	T	F	F	T		T
F	F	F	T	T	T	F	T		T

In the truth table the last row consisting the three premises having the truth value True (T) in that case the conclusion is also true then it is a valid argument

DATE / /
PAGE

2^{nd} method : Rules for inference

Rule P : A primise may be introduced at any point in the derivation

Rule T : A formula S may be introduced in a derivation if S is tautologically emplied by one or more of the preceeding formula of the derivation.

Implication formulas :

$I_1 : p \wedge q \Rightarrow p$

$I_2 : p \wedge q \Rightarrow q$

$I_3 : p \Rightarrow p \vee q$

$I_4 : q \Rightarrow p \vee q$

$I_9 : p, q \Rightarrow p \wedge q$

$I_{10} : \neg p, p \vee q \Rightarrow q$

$I_{11} : p, p \rightarrow q \Rightarrow q$

$I_{12} : \neg q, p \rightarrow q \Rightarrow \neg p$

$I_5 : \neg p \Rightarrow p \rightarrow q$

$I_6 : \neg q \Rightarrow p \rightarrow q$

$I_7 : \neg(p \rightarrow q) \Rightarrow p$

$I_8 : \neg(p \rightarrow q) \Rightarrow \neg q$

$I_{13} : p \rightarrow q, q \rightarrow r \Rightarrow p \rightarrow r$

$I_{14} : p \vee q, p \rightarrow q, q \rightarrow r \Rightarrow r$

Note : $E_{18} : (p \rightarrow q) \Leftrightarrow \neg q \rightarrow \neg p$

1. Demostrate that r is a valid ~~valeof~~ inference from the primises $p \rightarrow q$, $q \rightarrow r$ and p

Sol Given Primises

$$\begin{array}{l} p \rightarrow q, \\ q \rightarrow r \\ \underline{p} \\ \therefore r \end{array}$$

{1} (1) $p \rightarrow q$ Rule P

{2} (2) $q \rightarrow r$ Rule P

DATE / /
PAGE

{1,2} (3) $p \rightarrow r$ Rule T
{4} (4) p Rule P
{1,2,4} (5) $p, p \rightarrow r \Rightarrow r$ Rule T (I_{11})

2) Show that $r \vee s$ follows logically from the premises $c \vee d$, $c \vee d \rightarrow \neg h$, $\neg h \rightarrow (a \wedge \neg b)$ and $(a \wedge \neg b) \rightarrow (r \vee s)$

Sol: Given premises
{1} (1) $c \vee d$ (Rule P)
{2} (2) $c \vee d \rightarrow \neg h$ (Rule P)
{1,2} (3) $\neg h$ Rule T (I_{11})
{4} (4) $\neg h \rightarrow a \wedge \neg b$ (Rule P)
{1,2,4} (5) $a \wedge \neg b$ Rule T (I_{11})
{6} (6) $(a \wedge \neg b) \rightarrow (r \vee s)$ Rule P
{1,2,4,6} (7) $r \vee s$ Rule T (I_{11})

H.W. Show that $s \vee r$ is tautologically implied $(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow s)$

Sol: Given premises
$P_1 \rightarrow p \vee q$
$P_2 \rightarrow p \rightarrow r$
$P_3 \rightarrow q \rightarrow s$
$\therefore s \vee r$

{1} (1) $p \vee q$ (Rule P)
{1} (2) $\neg p \rightarrow q$ (Rule T) $p \rightarrow q \equiv \neg p \vee q$
{3} (3) $q \rightarrow s$ (Rule P) $\neg p \rightarrow q \equiv p \vee q$
{1,3} (4) $\neg p \rightarrow s$ (Rule T) I_{13}: $p \rightarrow q, q \rightarrow r \Rightarrow p \rightarrow r$
{1,3} (5) $\neg s \rightarrow p$ (Rule T) E_{13}: $p \rightarrow q \equiv \neg q \rightarrow \neg p$
{6} (6) $p \rightarrow r$ (Rule P)

$\{1,3,6\}$ (7) $\neg S \rightarrow r$ (Rule T) (I_{13})

$\{1,3,6\}$ (8) $S \vee r$ (Rule T) $P \rightarrow q \equiv \neg P \vee q$

$(\neg(\neg S) \vee r = S \vee r)$

Show that $R \wedge (P \vee Q)$ is a valid conclusion from the premises $P \vee Q, Q \rightarrow R, P \rightarrow M, \neg M$

Sol Given premises

$P \vee Q$

$Q \rightarrow R$

$P \rightarrow M$

$\neg M$

$\therefore R \wedge (P \vee Q)$

$\{1\}$ (1) $P \vee Q$ (Rule P)

$\{1\}$ (2) $\neg P \rightarrow Q$ (Rule T)

$\{3\}$ (3) $Q \rightarrow R$ (Rule P)

$\{1,3\}$ (4) $\neg Q \rightarrow R$ (Rule T)

$\{1,3\}$ (5) $\neg R \rightarrow Q$ (Rule T)

$\{6\}$ (6) $P \rightarrow M$ (Rule P)

WRONG

$\{1\}$ (1) $\neg M$ (Rule P)

$\{2\}$ (2) $P \rightarrow M$ (Rule P)

$\{1,2\}$ (3) $\neg P$ (Rule T) I_{13}

$\{4\}$ (4) $P \vee Q$ (Rule P)

$\{1,2,4\}$ (5) Q (Rule T) I_{10}

$\{6\}$ (6) $Q \rightarrow R$ (Rule P)

$\{1,2,4,6\}$ (7) R (Rule T) I_{11}

$\{1,2,4,6\}$ (8) $R \wedge (P \vee Q)$ (Rule T) I_9

Rules of Conditional Proof (or) Deduction theorem.

Rule CP :- If we can derive S from R and a set of premises, then we can derive $R \to S$ from the set of premises alone.

Note : Rule CP is generally used if the ~~conditi~~ conclusion is of the form $R \to S$ in such cases R is taken as an additional premise and S is derived from the given set of premises and R

Show that $r \to s$ is a valid conclusion from premise $P \to (q \to s)$, $\neg r \vee P$ and q

Instead of proving $r \to s$ we can include r as an additional premise and we can derive s. The Given argument is

$$\begin{array}{l} p \to (q \to s) \\ \neg r \vee p \\ \underline{q} \\ \therefore s \end{array}$$

{1} (1) r (rule CP)

{1} (2) $\neg r \vee p$ (rule P)

{2} (3) $r \to p$ (rule T) $(P \to q = \neg P \vee q)$

{1, 2} (4) P (rule T) I_{11} $P, P \to q \Rightarrow q$

{5} (5) $P \to (q \to s)$ (rule P)

{1, 2, 5} (6) $q \to s$ (rule T) I_{11}

{7} (7) q (rule P)

{1, 2, 5, 7} (8) s (rule T) I_{11}

$\therefore r \to s$

Test the validity of the following argument "Sonia is watching TV. If sonia watching TV, then she is not studying. If she is not studying, then her father will not buy her a scooty. Therefore Sonia's father will not buy a scooty."

Sol P : Sonia is watching TV

q : She is studying

r : Sonia's father buy a scooty

The given argument is of the form:

$p, p \to \neg q, \neg q \to \neg r \Rightarrow \neg r$

{1} (1) p (Rule P)

{2} (2) $p \to \neg q$ (Rule P)

{1,2} (3) $\neg q$ (Rule T) I_{11}

{4} (4) $\neg q \to \neg r$ (Rule P)

{1,2,4} (5) $\neg r$ (Rule T) I_{11}

"If there was a ball game, then travelling was difficult. If they arrived on time, then travelling was not difficult. They arrived on time. Therefore there was a no ball game." Show that these statement consis a valid argument.

Sol Let us indicate the statements as follows

P : There was a ball game

q : Travelling was difficult

r : They arrived on time

$p \to q, r \to \neg q, r \Rightarrow \neg p$ are the given arguments in the form,

$\{1\}$ (1) r (Rule P)

$\{2\}$ (2) $r \to \neg q$ (rule P)

$\{1,2\}$ (3) $\neg q$ (rule T)

$\{4\}$ (4) $p \to q$ (rule P)

$\{1,2,4\}$ (5) $\neg p$ (Rule T) I_{12}

By using the method of derivation show that the following statements constitute a valid argument.

"If A works hard, then either B or C will enjoy. If B enjoys then A will not work hard. If D enjoys, then C will not. Therefore If A works hard, D will not enjoy."

Sol Let us indicate the statements as follows

p : A works hard

q : B will enjoy

r : C will enjoy

s : D will enjoy

The given argument is of the form

$p \to (q \vee r),\ q \to \neg p,\ s \to \neg r \Rightarrow p \to \neg s$

$\{1\}$ (1) ~~$q \to \neg p$~~ p (Rule P)

$\{2\}$ (2) $p \to q \vee r$ (Rule P)

$\{1,2\}$ (3) $q \vee r$ (Rule T) I_{11}

$\{1,2\}$ (4) $\neg q \to r$ (Rule T)

$\{1,2\}$ (5) $\neg r \to q$ (Rule T)

$\{6\}$ (6) $q \to \neg r$ (Rule P)

$\{1,2,6\}$ (7) $\neg r \to \neg p$ (Rule T) I_{13}

$\{1,2,6\}$ (8) $p \to r$ (Rule T)

$\{9\}$ (9) $s \to \neg r$ (Rule P)

$\{9\}$ (10) $r \rightarrow \neg s$ (Rule T) E_{18}
$\{1,2,6,8\}$ (11) $P \rightarrow \neg s$ (Rule T) I_{13}

Indirect method of group

To show a conclusion C follows logically from the premises $n_1, n_2, \ldots n_n$. we assume that C is false and negation C is an additional premise. If the new set of premise is inconsistent (F) then they imply a contradiction ∴ The assumption that $\neg C$ is wrong. Hence the conclusion C is true and follows from the given set of premises $H_1, H_2 - - - H_n$.

Show that $\neg(P \wedge Q)$ follows from $\neg P \wedge \neg Q$ by the indirect method.

Sol By the indirect method let us assume that the conclusion $\neg(P \wedge Q)$ is false and take negation of the conclusion as an additional premise.

The new set of premises are $\neg(\neg(p \wedge q))$, $\neg P \wedge \neg q$

$\{1\}$ (1) $\neg(\neg(p \wedge q))$ (Rule P)
$\{1\}$ (2) $P \wedge q$ (Rule T) Double Negation
$\{1\}$ (3) P (Rule T) I_1
$\{4\}$ (4) $\neg P \wedge \neg q$ (Rule P)
$\{4\}$ (5) $\neg P$ (Rule T) I_1
$\{1,4\}$ (6) $P, \neg P \Rightarrow P \wedge \neg P$ (Rule T) I_9
$\{1,4\}$ (7) F (Rule T) Negation Law

Which is contradiction ∴ Our assumption "$\neg(P \wedge Q)$ is false" is wrong. Hence $\neg(P \wedge Q)$ is follows from

PAGE

$\neg P \wedge \neg Q$.

By the indirect method show that $p \to q, q \to r, p \vee r \Rightarrow r$

Sol By the indirect method of & Let us assume that the conclusion r is false and take negation r as an additional premise.

The new set of premises are $\neg r, p \to q, q \to r, p \vee r$

{1} (1) $\neg r$ (Rule P)

{2} (2) $q \to r$ (Rule P)

{1,2} (3) $\neg q$ (Rule T) I_{12}

{4} (4) $p \to q$ (Rule P)

{1,2,4} (5) $\neg p$ (Rule T) I_{12}

{6} (6) $p \vee r$ (Rule P)

{1,2,4,6} (7) r (Rule T) I_{10}

{1,2,4,6} (8) $\neg r, r \Rightarrow \neg r \wedge r$ (Rule T) I_9

{1,2,4,6} (9) F (Rule T) (Negation law)

which is contradiction.

∴ Our assumption " the conclusion r is false" is wrong. Hence $p \to q, q \to r, p \vee r$

Consistency of premises: A set of formulas $H_1, H_2, H_3 - - - H_m$ is said to be consistent if their conjunction has truth value T for some assignment of truth values to the variables appearing in $H_1, H_2, - - - H_m$

Inconsistency of premises: A set of formulas $H_1, H_2 - - H_m$ is said to be consistent if their conjunction has truth value F for some for some assignment of truth values of to the variables appearing in $H_1, H_2 \dots H_m$.

Show that the premises $a \to (b \to c)$, $d \to (b \wedge \neg c)$, and $a \wedge d$ are inconsistent.

Sol
$a \to (b \to c)$
$d \to (b \wedge \neg c)$
$a \wedge d$

{1}	(1)	$a \wedge d$	(Rule P)
{1}	(2)	a	(Rule T) I_1 $P \wedge q \Rightarrow P$
{1}	(3)	d	(Rule T) I_2 $P \wedge q \Rightarrow q$
{4}	(4)	$a \to (b \to c)$	(Rule P)
{1,4}	(5)	$b \to c$	(Rule T) I_{11} $P, P \to q \Rightarrow q$
{1,4}	(6)	$\neg b \vee c$	(Rule T) $(P \to q \equiv \neg P \vee q)$
{7}	(7)	$d \to (b \wedge \neg c)$	(Rule P)
{7}	(8)	$\neg(b \wedge \neg c) \to \neg d$	(Rule T) E_{18} $P \to q \equiv \neg q \to \neg P$
{7}	(9)	$\neg b \vee c \to \neg d$	(Rule T) (Demorgan's Law)
{1,4,7}	(10)	$\neg d$	(Rule T) I_{11} $P, P \to q \Rightarrow q$
{1,4,7}	(11)	$d, \neg d \Rightarrow d \wedge \neg d$	(Rule T) I_9 $P, q \Rightarrow P \wedge q$
{1,4,7}	(12)	F	(Rule T) (Negation Law)

Show that the following set of premises are inconsistent "If the contract is valid, then john is liable for penality. If john is liable for penality, he will go bankrupt. If the bank will loan him money, he will not go bankrupt. As a matter of fact the contract is valid and the bank will loan him money".

DATE / /
PAGE

Sol p : The contract is valid
q : John is liable for penality
r : He will go bankrupt
s : Bank will loan him money.

$p \to q$, $q \to r$, $s \to \neg r$, $p \wedge s$

{1}	(1)	$p \to q$	(Rule P)
{2}	(2)	$q \to r$	(Rule P)
{1,2}	(3)	$p \to r$	(Rule T) I_{13} $p \to q, q \to r \Rightarrow p \to r$
~~{4}~~	~~(4)~~	~~$s \to \neg r$~~	~~(Rule P)~~
~~{5}~~	~~(5)~~	~~$p \wedge s$~~	~~(Rule P)~~
~~{5}~~	~~(6)~~	~~s~~	~~(Rule T) I_1~~
~~{4,5}~~	~~(7)~~	~~$\neg r$~~	~~(Rule T) I_{11}~~
~~{1,2,3}~~	~~(8)~~	~~r~~	~~(Rule T) I_1~~
{4}	(4)	$p \wedge s$	(Rule P)
{4}	(5)	p	(Rule T) I_1
{4}	(6)	s	(Rule T) I_2
{1,2,4}	(7)	r	(Rule T) I_{11}
{8}	(8)	$s \to \neg r$	(Rule P)
{4,8}	(9)	$\neg r$	(Rule T) I_{11}
{1,2,4,8}	(10)	$r \wedge \neg r$	(Rule T) I_9
{1,2,4,8}	(11)	F	(Rule T) (negation Law)

Show that the following premises are inconsistent (i) If jack misses many classes because of illness then he fails highschool.
(ii) If jack fails highschool then he is uneducated
(iii) If jack reads a lot of books, then he is not uneducated.
(iv) Jack misses many classes because of illness

DATE / /
PAGE

and reads a lot of books.

p : Jack misses many classes because of illness
q : he fails high school
R : he is uneducated
s : He reads alot of books

$p \rightarrow q$, $q \rightarrow r$, $s \rightarrow \neg r$, $p \wedge s$

{1} (1) $p \wedge s$ (Rule P)
{1} (2) p (Rule T) I_1
{1} (3) s (Rule T) I_1
{4} (4) $p \rightarrow q$ (Rule P)
{1, 4} (5) q (Rule T)

Predicate : Let us consider a statement.

(i) Ramu is a good boy
Subject Predicate

Let us consider another statement

(ii) Sita is a beautiful girl
Subject predicate

All predicates are represented by capital letters and subject is represented by small letters

The symbolic form of (i) is $G(r)$
The symbolic form of (ii) is $B(s)$

DATE / /
PAGE

Eg: x is a mean — $M(x)$
Subject Predicate

x is a woman — $W(x)$
Subject Predicate

All these are called statement function

Compound Statement function: Two or more statement functions are connected by logical connectives then it is called compound statement function.

Let us consider two statement functions $M(x), W(x)$
The compound statement functions are
$M(x) \wedge W(x)$, $M(x) \vee W(x)$, $\neg W(x)$

Quantifier: - Mainly we have 2 types of Quantifiers
i) Universal Quantifier
ii) existantial Quantifier

Universal Quantifier: If the sentence contains the phrases for all, for each, for every, any, everything etc. is called universal Quantifier. It is denoted by the symbol $(\forall)$ for all and read it as for all.

Existantial Quantifier If the sentence contains the phrases their exists, some, for some, something etc is called existantial Quantifier. and It is denoted by the symbol $(\exists)$ and read it as their exist.

Example :

I Represent the following sentences in Symbolic form

1. Something is good $(\exists x)\ (G(x))$
2. Everything is good $(\forall x)\ (G(x))$
3. Something is not good $(\exists x)\ (\neg(G(x)))$
4. Nothing is good $(\forall x)\ (\neg(G(x)))$

II Represent the following sentences in symbolic form

(i) All men are mortal $(\forall x)[M(x) \longrightarrow L(x)]$

(ii) Every apple is Red $(\forall x)[A(x) \longrightarrow R(x)]$

(iii) Any integer is either positive (or) negative

$$(\forall x)\ [I(x) \longrightarrow (P(x) \vee N(x))]$$

(i) There exist a man (ii) Some men are clever

(iii) Some real (R_1) numbers are rational (R_2)

$$\exists x\ \cancel{\forall x}\ (M(x))$$

$$\exists x\ \cancel{\forall x}\ (M(x) \wedge C(x))$$

$$\exists x\ (R_1(x) \wedge R_2(x))$$

Inference theory of predicate Calculus.

Rule US : Universal specification

$$(x) A(x) \Longrightarrow A(y)$$

From $(x)A(x)$, one can conclude $A(y)$

Rule ES : Existantial Specification

$$(\exists x)\ A(x) \Longrightarrow A(y)$$

Rule EG : Existantial Generalization

$$A(x) \longrightarrow (\exists y)(A(y))$$

Rule UG : Universal Generalization

$$A(x) \longrightarrow (\forall x)(A(x))$$

Verify the validity of the following argument

(1) "All men are mortal. Socrates is a man. Therefore, Socrates is mortal"

$$(\forall x)\,[M(x) \to L(x)] \wedge M(s) \Rightarrow L(s)$$

$M(x)$: x is a men

$L(x)$: x is a mortal

$M(s)$: Socrates is a man

$L(s)$: Socrates is a mortal

{1}	(1)	$(\forall x)[M(x) \to L(x)]$	(Rule P)
{1}	(2)	$M(s) \to L(s)$	(Rule US)
{3}	(3)	$M(s)$	(Rule P)
{1,3}	(4)	$L(s)$	(Rule T) I_{11}

(ii) "Every living thing is a plant or an animal. Joe's goldfish is alive and it is not a plant. All animals have hearts. Therefore, Joe's goldfish has a heart."

(iii) $(x)(p(x) \to Q(x)) \wedge (x)(Q(x) \to R(x)) \Rightarrow (x)(p(x) \to R(x))$

{1}	(1)	$(x)(p(x) \to Q(x))$	(Rule P)
{2}	(2)	$P(y) \to Q(y)$	(Rule US)
{3}	(3)	$(x)(Q(x) \to R(x))$	(Rule P)
{3}	(4)	$Q(y) \to R(y)$	(Rule US)
{1,3}	(5)	$P(y) \to R(y)$	(Rule T) I_{13}
{1,3}	(6)	$(\forall x)(P(x) \to R(x))$	(Rule UG)