16/07/24.

UNIT-4 (5Marks)

1) Given that $n=6$; $k=3$; $q=n-k=3$.

$$G = \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right]$$

We know that it is in the form of

$$\boxed{G = [I_{3\times3} : P_{3\times3}].}$$

$$I_{3\times3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad ; \quad P_{3\times3} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Asking

i) Determine the P submatrix from generator matrix

A) From the Generator matrix it is very clear that

$$P_{3\times3} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

ii) Obtain the Equation for check bits using $C = MP$

$$\boxed{C_1 = M\ P}$$

M = message bits

P = Sub matrix.

$$C = \begin{bmatrix} m_1 & m_2 & m_3 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$\therefore$ Equations are

$$c_1 = 0 \times m_1 \oplus 1 \times m_2 \oplus 1 \times m_3$$

$$\boxed{c_1 = m_2 \oplus m_3}$$

$$c_2 = 1 \times m_1 \oplus 0 \times m_2 \oplus 1 \times m_3$$

$$\boxed{c_2 = m_1 \oplus m_3}$$

$$c_3 = 1 \times m_1 \oplus 1 \times m_2 \oplus 0 \times m_3$$

$$\boxed{c_3 = m_1 \oplus m_2}$$

iii) ~~the~~ Determine check bits for every message vector.

The Possible all combinations is 8.

$\because$ We have $n = 6$; $k = 3$

$$q = n - k = 3$$

$$2^q = 2^3 = 8.$$

S.No	m_1	m_2	m_3	c_1	c_2	c_3
1	0	0	0	0	0	0
2	0	0	1	1	1	0
3	0	1	0	1	0	1
4	0	1	1	0	1	1
5	1	0	0	0	1	1
6	1	0	1	1	0	1
7	1	1	0	1	1	0
8	1	1	1	0	0	0

∴ Final ~~codes :-~~ check bits are :-

For 000 :- 000

For 001 :- 110

For 010 :- 101

For 011 :- 011

For 100 :- 011

For 101 :- 101

For 110 :- 110

For 111 :- 000

2) Given Parity check matrix is

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & : & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & : & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & : & 0 & 0 & 1 \end{bmatrix}$$

It is in the form of $H = [P^T ; I]$.

$$P^T = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} ; \quad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

i) Find the Generator matrix (G)?

W.K.T

$$G = [I ; P]$$

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & : & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & : & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & : & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & : & 0 & 1 & 1 \end{bmatrix}$$

$4 \times 4 \qquad 4 \times 3$

ii) List of all code words.

Here $n = 7$; $k = 4$; $2^{q} = 2^{7-4} = 2^{3} = 8$.

For finding the code words we have to know the equations.

$$[C = m \; P]$$

$$C = \begin{bmatrix} m_1 & m_2 & m_3 & m_4 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$c_1 = 1 \times m_1 \oplus 1 \times m_2 \oplus 1 \times m_3 \oplus 0 \times m_4$$

$$\boxed{c_1 = m_1 \oplus m_2 \oplus m_3}$$

$$c_2 = 1 \times m_1 \oplus 1 \times m_2 \oplus 0 \times m_3 \oplus 1 \times m_4$$

$$\boxed{c_2 = m_1 \oplus m_2 \oplus m_4}$$

$$c_3 = 1 \times m_1 \oplus 0 \times m_2 \oplus 1 \times m_3 \oplus 1 \times m_4$$

$$\boxed{c_3 = m_1 \oplus m_3 \oplus m_4}$$

s-0 ; d-1

S.NO	m_1	m_2	m_3	m_4	C_1	C_2	C_3	Weigh of
1	0	0	0	0	0	0	0	0
2	0	0	0	1	0	1	1	3
3	0	0	1	0	1	0	1	3
4	0	0	1	1	1	1	0	4
5	0	1	0	0	1	1	0	3
6	0	1	0	1	1	0	1	4
7	0	1	1	0	0	1	1	4
8	0	1	1	1	0	0	0	3
9	1	0	0	0	1	1	1	4
10	1	0	0	1	1	0	0	3
11	1	0	1	0	0	1	0	3
12	1	0	1	1	0	0	1	4
13	1	1	0	0	0	0	1	3
14	1	1	0	1	0	1	0	3
15	1	1	1	0	1	0	0	4
16	1	1	1	1	1	1	1	7

iii) The minimum distance between code vector

$$\boxed{d_{min} = 3}$$

3)

iv) The errors can be detected :-

$$\boxed{d_{min} \geq s+1}$$ → formula for detection

$$3 \geq s+1$$

$$\boxed{s \leq 2}$$

Two errors can be detected.

$$\boxed{d_{min} \geq 2t+1}$$ → formula for correction

$$3 \geq 2t+1$$

$$2 \geq 2t$$

$$t \leq 1$$

Only one error will corrected.

3) Given $n = 7$; $k = 4$; $q = n - k = 3$.

$$H = \left[\begin{array}{cccc:ccc} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$H = [P^T ; I]$$

We know that Syndrome vector is a q bit vector

Hence "q" value is 3.

$$\boxed{S = E H^T}$$

S.NO	Bit in Error	Bit Error (E)
1	1^{st}	1000000
2	2^{nd}	0100000
3	3^{rd}	0010000
4	4^{th}	0001000
5	5^{th}	0000100
6	6^{th}	0000010
7	7^{th}	0000001

Caluclation Syndrome for each error bit :- 4)

For 1st bit :-

$$S = E H^T$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow 1 \oplus 0 \oplus 0 \oplus 0 \oplus 0 \oplus 0 \oplus 0 = 1 \qquad 0 \oplus 0 \oplus 0 \oplus 0 \oplus 0 \oplus 0 \oplus 0 = 0$$

$$1 \oplus 0 \oplus 0 \oplus 0 \oplus 0 \oplus 0 \oplus 0 = 1$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}$$

Similarly do for remaining all error bits.

4) Given that $n = 6$; $K = 3$; $q = n - K = 3$

$2^q = 2^3 = 8.$

$$H = \left[\begin{array}{ccc:ccc} 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \end{array}\right]$$

$$H^T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Also given that received code word is

Let received code word is X

$X = 100101$

$$S = X H^T = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$\Rightarrow 1 \oplus 0 \oplus 0 \oplus 0 \oplus 0 \oplus 1 = 0$ $\quad 0 \oplus 0 \oplus 0 \oplus 1 \oplus 0 \oplus 1 = 0$

$1 \oplus 0 \oplus 0 \oplus 1 \oplus 0 \oplus 0 = 0$

$\Rightarrow \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$

i) Find the Generator matrix (G)

W.K.T

$$G = [I : P]$$

$$G = \begin{bmatrix} 1 & 0 & 0 & : & 1 & 0 & 1 \\ 0 & 1 & 0 & : & 0 & 1 & 1 \\ 0 & 0 & 1 & : & 1 & 1 & 1 \end{bmatrix}$$

ii) Find the code word for data bit 101

$$X = MG = [101] \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$X \Rightarrow [1\,0\,1\,0\,1\,0]$$

8) Given $n = 7$; $K = 4$; $q = n - K = 7 - 4 = 3$

$2^q = 2^3 = 8$.

$$H = \begin{bmatrix} 1 & 0 & 1 & 1 & : & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & : & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & : & 0 & 0 & 1 \end{bmatrix}$$

$$H = [P^T ; I]$$

$$P^T = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} ; P = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} ; I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

i) construct generator Matrix

$$G = [I ; P]$$

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

ii) The code word begins with 1010

$X = MG$

$$X = \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$X = \begin{bmatrix} 1010011 \end{bmatrix}$$

iii) If received code word $Y = 0111100$; then decode this received codeword.

$$\begin{bmatrix} 0111100 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$

000
001
010
011
100

$\Rightarrow \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}$

$\therefore$ decode received code word is

$$\boxed{Y = 0111110}$$

9)

9) Given that (7, 4)

$n = 7$; $K = 4$; $q = n - K = 4$.

also Given that $G(P) = P^3 + P + 1$

Asking to find all code vectors in non-systematic form.

here $K = 4$ so code vectors from 0000 to 1111.

For 0000 :-

$M(P) = 0000 = 0$ $\quad G(P) = P^3 + P + 1$

$\therefore M(P)\,G(P) = 0\,[P^3 + P + 1]$

$\Rightarrow 0.$

$\therefore$ code vector is 0000000.

For 0001 :-

$M(P) = \overset{P^3\,P^2\,P^1\,P^0}{0001}$ $\quad G(P) = P^3 + P + 1$

$\Rightarrow 1 \times P^0 = 1$

$\therefore M(P)\,G(P) = 1\,[P^3 + P + 1]$

$\Rightarrow P^3 + P + 1$

P^7	P^6	P^5	P^4	P^3	P^2	P^1	P^0
0	0	0	0	1	0	1	1

For 0010 :-

$M(P) = 0010$ $\quad G(P) = P^3 + P + 1$

$\Rightarrow P$

$\therefore M(P)\,G(P) \Rightarrow P(P^3 + P + 1)$

$\Rightarrow P^4 + P^2 + P.$

P^7	P^6	P^5	P^4	P^3	P^2	P^1	P^0
0	0	0	1	0	1	1	0

For 0011 :-

$M(P) = 0011$ $\quad G(P) = P^3 + P + 1$

$= P + 1$

$\therefore M(P)\,G(P) = (P+1)(P^3 + P + 1)$

$\Rightarrow P^4 + P^2 + P + P^3 + P + 1$

$\Rightarrow P^4 + P^3 + P^2 + 1$

P^7	P^6	P^5	P^4	P^3	P^2	P^1	P^0
0	0	0	1	1	1	0	1

Note :- Do ~~for~~ the same for remaining all values.

10) Given that (7,4) $n=7$; $K=4$; $q=n-K=3$

also given that $G(P) = P^3+P+1$

Asking to find all possible code vector in a systemetic order.

here the all possible values are from 0000 to 1111.

For 0000 :- it is always 0000000

For 0001 :-

$M(P) = 1 \times P^0 = 1$

Now, $P^3+P+1) 1 ($

remainder $= 1$

$C(P) = 1$

$\Rightarrow 0001$

$\therefore$ Final code vector $=$ ~~0001~~

$0001 : 001$

$= 1 \ldots$

For 0010 :-

$M(P) = 0010 \Rightarrow P$

Now

$G(P)\,)\,M(P)\,($

$P^3+P+1\,)\ P\ ($

$\underline{P}$

$\underline{P}$

remainder = P'

$\Rightarrow 010$

$0011 \Rightarrow P+1$

$P^3+P+1\,)\ P+1\ (1$

P^3+P+1

P^3

100

$\Rightarrow 0011 : 100$

0100 $\Rightarrow P^2$

$P^3+P+1\,)\ P^2\ (1$

P^3+P+1

P^3+P^2+P+1

11) Write the advantages and disadvantages of cyclic codes?

Sol Advantages:-

i) Error control

ii) Efficient Encoding and decoding

iii) Ease of implementation

iv) They are very fast.

v) This has made cyclic codes a good for many networks.

Disadvantages:-

i) It can be difficult to implement protection mechanisms for global variables.

ii) Bandwidth is more

iii) Extra bit will reducees the bit rate and also its power.

12) Mention the features of BCH codes?

i) It has Error correction capability.

ii) It is in cyclic structure.

iii) It contains the Range of code lengths.

iv) It has minimum distance.

v) They are commonly used in digital communi-cations, storage systems.

vi) These are based on finite fields.

vii) It will provide a solid mathematical foundation.

viii) It is also used in conjunction to acheive even higher levels of error correction.

ix) several efficient algorithms exists for decoding in BCH codes.

1. Define the Parameters

- **n:** Length of the codeword.
- **k:** Length of the message.
- **r:** Number of parity bits, where $r = n - k$.

2. Generate Matrix (G)

The generator matrix G is used to encode the message. It is a $k \times n$ matrix.

Steps:

1. **Identify k message bits.**
2. **Form the Identity Matrix I_k:** This is a $k \times k$ identity matrix.
3. **Form Parity Matrix P:** This is a $k \times r$ matrix.
4. **Combine to form G:** $G = [I_k|P]$, where I_k is the identity matrix and

4. **Combine to Form G:** $G = [I_k|P]$, where I_k is the identity matrix and P is the parity matrix.

3. Parity-Check Matrix (H)

The parity-check matrix H is used for error detection and correction. It is an $r \times n$ matrix.

Steps:

1. **Form the Transpose of Parity Matrix P:** P^T.
2. **Form Identity Matrix I_r:** This is a $r \times r$ identity matrix.
3. **Combine to Form H:** $H = [P^T|I_r]$.

4. Encoding the Message

To encode a message m (a $1 \times k$ row vector):