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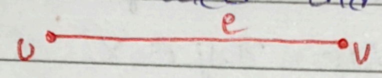
GRAPH UNIT - IV

Graph theory : A graph G consists of a-pair (V, E) where E is the non empty finite set who elements are called vertices and E is another set those elements are called edges. Such that there is a mapping from the set of edges E of the set of ordered or unordered pair of elements of V

- A Graph G X with vertices V and edges E is written as G = (V, E) or G (V, E)
- The edges E is connected the vertices U & V then V, U are called end points of E



- The edge e that joins a node U & V is said to be incident on each of its end points U & V.
- If the two distinct edges E_1, E_2 are incident with a common point, then they are called adjacent edges.
- Any two vertices connected by an edge in a graph ~~then those~~ vertices are called adj vertices.
- In a graph a vertex that is not adj to any other vertex is called an isolated vertex

Finite Graph : A Graph G(V, E) is said to be finite if it has finite no. of vertices and finite no. of edges otherwise it is called infinite graph.

→ The no. of vertices in a finite graph is called order of the graph denoted by V(G)

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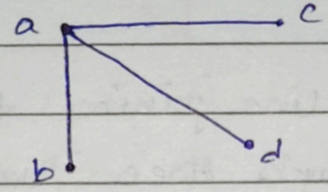
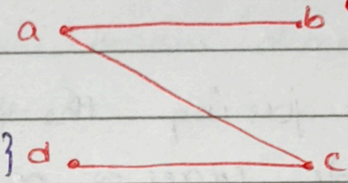
→ The no. of edges in a finite graph G is called size of the graph G denoted by E(G)

Note: A finite graph 'G' consists of N vertices & M edges then it is called as an (N, M) graph

Q. Given V = {a, b, c, d} & E {(a, b), (a, c), (a, d)} from the graph for this information



Q. What can we say about the diagram.

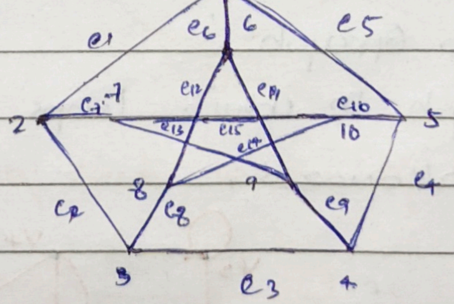


V = {a, b, c, d}
E = {(a, b) (a, c) (c, d)}

order of graph = 4 ; size of graph = 3
(a, b) (a, c) (c, d) are called adj vertices
(a, d) (b, c) (b, d) called non-adj vertices

Q. Consider the graph (10, 15) is what is the information that can be gained from given graph.

(PETERSON'S GRAPH)

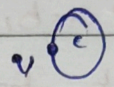


V = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

$E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{12}, e_{13}, e_{14}, e_{15}\}$

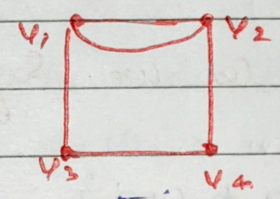
Loop:

An edge of a graph That ~~gives~~ joins its vertex to itself or self loop.



Multi Graph:

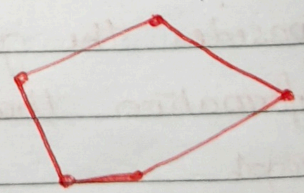
If more than one line joining two vertices are allowed in a graph then the resulting graph is called Multigraph



Lines joining the same vertices are called multiple edges or parallel edges.

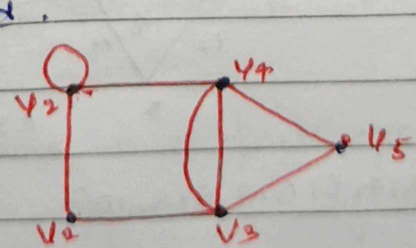
Simple Graph

A graph which has neither loops or multiple edges is called a Simple Graph



Pseudo Graph:

A graph in which loops & multiple edges are allowed.



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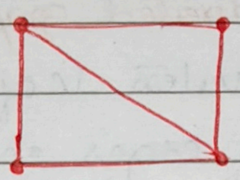
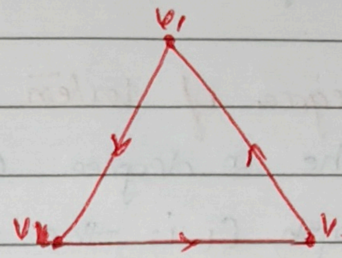
* Directed & Undirected Graph.

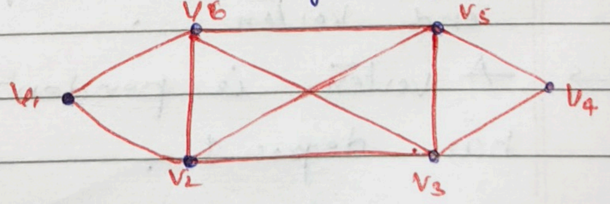
Undirected graph: A undirected graph G has a set of vertices V and a set of edges E such that each edge $e \in E$ is associated with an unordered pair of vertices $(e \in (v_i, v_j)$ and $(v_j, v_i))$



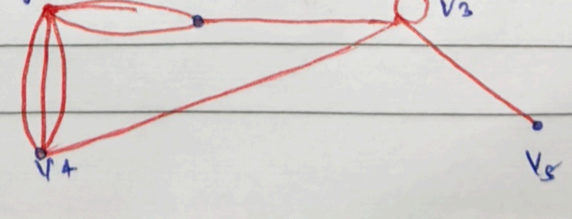


* **Directed Graph**: A directed graph G has a set of vertices V and a set of edges E such that each edges $e \in E$ is associated with an ordered pair of vertices means directions on each edge $(e \in (v_i, v_j))$

* **Degree of a Vertex**: The degree of a vertex v of an undirected graph G is the no. of edges incident with it except that a loop and a vertex contributes twice to the degree of that vertex. It is denoted by deg (v_i)



Q Find the degree of each vertex of graph



$\deg(v_1) = 5$
$\deg(v_2) = 3$
$\deg(v_3) = 5$
$\deg(v_4) = 4$
$\deg(v_5) = 1$

Degree of vertex for directed graph:

→ The in degree of a vertex V of a directional graph G is the no. of edges ending at V and it is denoted by indeg (V) or $\deg^- G(v)$

→ The out degree of a vertex V of a directed graph G is the no. of edges beginning at V it is denoted by outdegree (V) or $\deg^+ G(v)$. The sum of the indegree & outdegree of a vertex is called total degree of a vertex.

A vertex with 0 indegree is called a source & a vertex with 0 outdegree is called sink.

Note: A vertex of degree 0 is called an isolated vertex. A vertex V of degree 1 is called an end vertex.

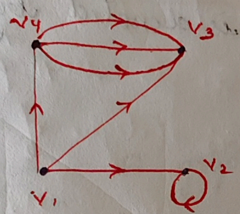
→ A vertex is pendent if and only if it has degree 1.

Find indegree outdegree & total degree for the given directed graph.

→ A vertex is pendant if and only if it has the degree 1.

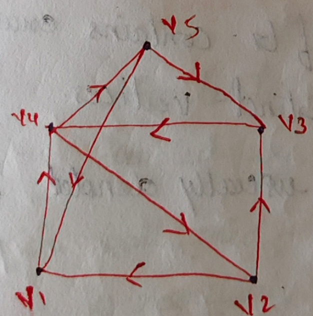
Find indegree, outdegree & total degree for the given directed graph.

(i)



indeg (v_1) = 0	outdeg (v_1) : 3	Total deg (v_1) = 3
indeg (v_2) = 2	outdeg (v_2) = 1	Total deg (v_2) = 3
indeg (v_3) = 4	out deg (v_3) = 0	Total deg (v_3) = 4
indeg $(v_4$ = 1	out deg (v_4) = 3	Total deg (v_4) = 4

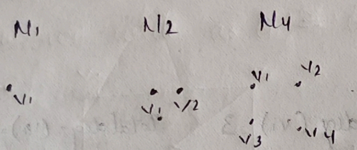
(ii)



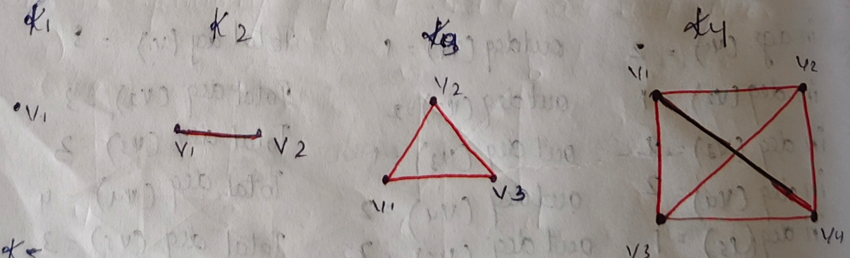
in deg (v_1) = 2	outdeg (v_1) = 1	Total deg (v_1) = 3
in deg (v_2) = 1	out deg (v_2) = 2	Total deg (v_2) = 3
in deg (v_3) = 2	out deg (v_3) = 1	Total ~~deg~~ (v_3) = 3
in deg (v_4) = 2	out deg (v_4) = 2	Total deg (v_4) = 4
in deg (v_5) = 1	out deg (v_5) = 2	Total deg (v_5) = 3

Types of graph

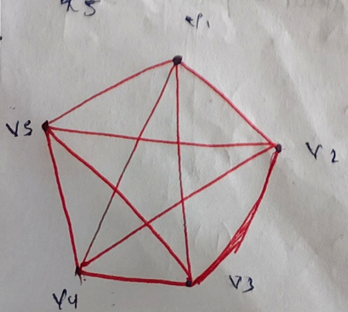
Null graph :- A graph which contains only an isolated vertices is called a null graph. i.e, that set of edges in a null graph is empty.
Null graph with 'n' vertices is denoted by $\boxed{N_n}$



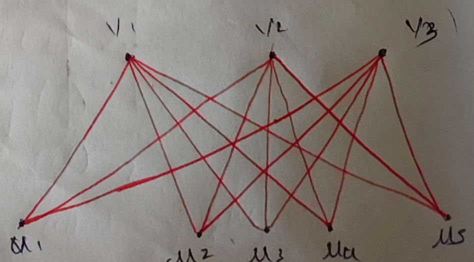
Complete Graph A simple graph G is said to be complete if every vertex in G is connected with every other vertex i.e, if G contains exactly one edge between each pair of distinct vertices.
A complete graph is usually denoted by Kn.
n - no. of vertices



K5



eg: $K_{3,5}$



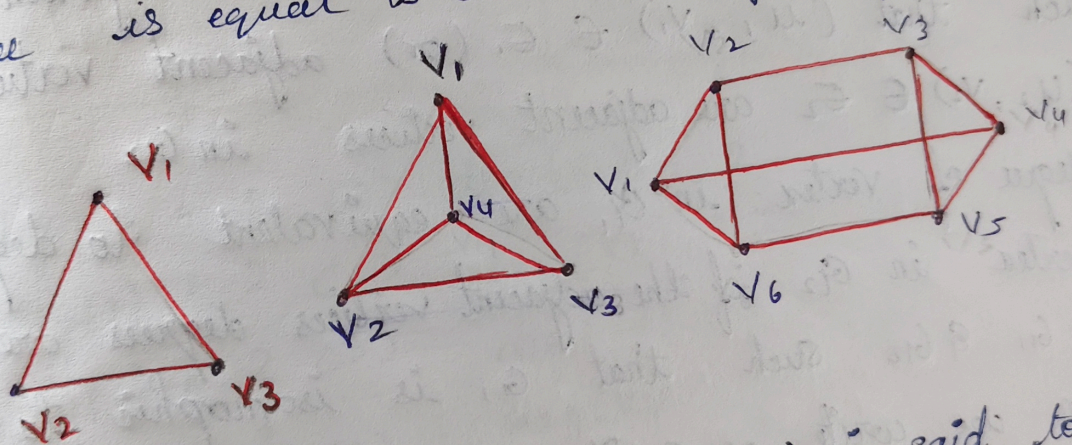
tains only an

aph. i.e. that set

by $\boxed{N_n}$

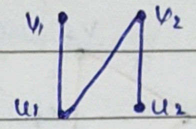
Regular graph:- A graph G has all vertices of the degree is equal is called a Regular graph.

Ex:-



Bipartiate graph:- A graph $G = (V, E)$ is said to be bipartiate graph if the vertex V can be divided ... V_1 & V_2 such that every ... is connects

Bipartiate graph: A graph $G=(V,e)$ is said to be bipartiate graph if the vertex set V can be partioned into two disjoint subsets V_1 & V_2 such that every edge in E connects a vertex in V_1 and a vertex in V_2. So that no edge in G connects either two vertices in V_1 or two vertices in V_2 is called bipartion graph.

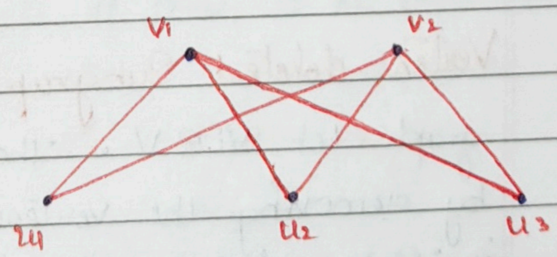


$V_1 = \{v_1, v_2\}$
$V_2 = \{u_1, u_2\}$

Complete Bipartiate graph. A complete bipartiate graph on M and n vertices, denoted by $K_{m,n}$, is a graph whose vertex set is partioned into 2 sets V_1 and V_2 with m vertices and n vertices respectively in which there is an edge between each pair of vertices from the vertex set V_1 to the vertex set V_2.

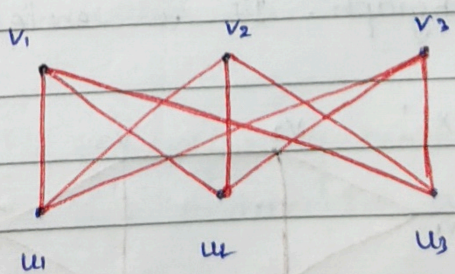
$K_{2,3}$

$V_1 = \{v_1, v_2\}$
$V_2 = \{u_1, u_2, u_3\}$



$K_{3,3}$

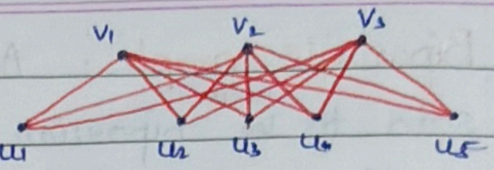
$V_1 = \{v_1, v_2, v_3\}$
$V_2 = \{u_1, u_2, u_3\}$



$K_{3,5}$

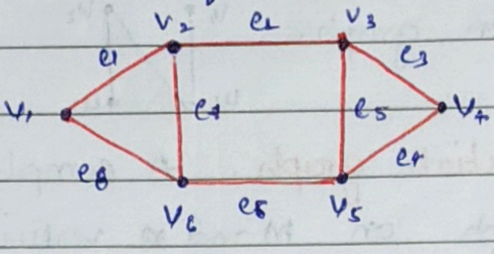
$V_1 = \{v_1, v_2, v_3\}$

$V_2 = \{u_1, u_2, u_3, u_4, u_5\}$



Subgraph :

$H = (V_1, E_1)$ is ~~the~~ called the Subgraph of $G(V,E)$ if $V_1 \subseteq V$ & $E_1 \subseteq E$.

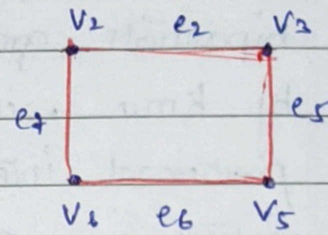


$V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$

$E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$

$V_1 = \{v_2, v_3, v_5, v_6\}$

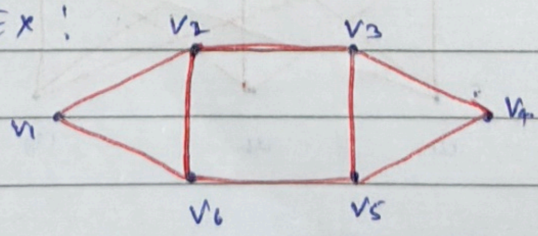
$E_1 = \{e_2, e_5, e_6, e_7\}$



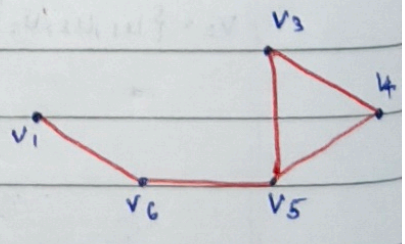
Spanning Subgraph : The subgraph H is called Spanning Subgraph of G if $V_1 = V$.

Vertex deleted Subgraph : Let $G = (V, E)$ be a graph let $v_i \in V$. The subgraph of G obtained by removing the vertex v_i and all the edges incident with ~~E~~ v_i is called Vertex deleted subgraph. It is denoted by $G - v_i$

Ex :



$G - v_2$



Matrix Representation of Graphs.

Any graph can be represented by a matrix. A matrix is very effective and convinient way of representing a graph in a computer for processing.

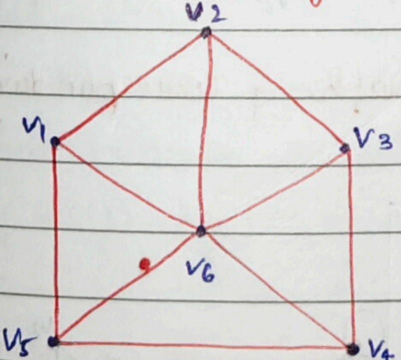
In this section we discuss how to describe a graph using adjacency matrices and incidence matrices, which in computer memory can be stored as 2 dimensional arrays.

Adjacency Matrix for undirected graph:

Let G be a graph with n vertices where $n > 0$ and no parallel edges. The adjacent matrix of G is an $n \times n$ matrix $A = [a_{ij}]$ whose elements are defined as follows.

$a_{ij} = \{1$ if there is an edge between i^{th} and j^{th} vertices, 0 if there is no edge between them$\}$

Find the adjacency matrix of the graph

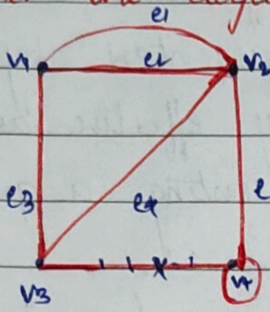


The vertices of the graph are listed as $v_1, v_2, v_3, v_4, v_5, v_6$

The adjacency matrix for this undirected graph is

$$A_G = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

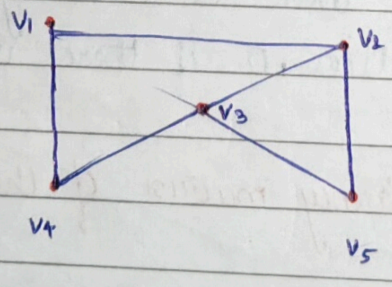
Find the adjacency matrix of the graph



$$AG = \begin{array}{c|cccc} & v_1 & v_2 & v_3 & v_4 \\ \hline v_1 & 0 & 2 & 1 & 0 \\ v_2 & 2 & 0 & 1 & 1 \\ v_3 & 1 & 1 & 0 & 0 \\ v_4 & 0 & 1 & 0 & 1 \end{array}$$

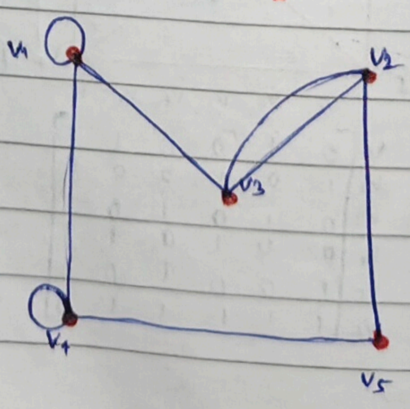
Draw the graph represented by the adjacency matrix AG written as

$$AG = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$



Draw the undirected graph G corresponding to the adjacency matrix

$$AG = \begin{array}{c|ccccc} & v_1 & v_2 & v_3 & v_4 & v_5 \\ \hline v_1 & 1 & 0 & 1 & 1 & 0 \\ v_2 & 0 & 0 & 2 & 0 & 1 \\ v_3 & 1 & 2 & 0 & 0 & 0 \\ v_4 & 1 & 0 & 0 & 1 & 1 \\ v_5 & 0 & 1 & 0 & 1 & 0 \end{array}$$

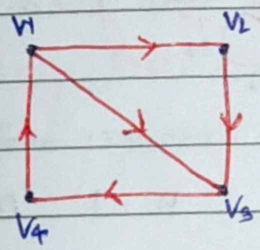


Adjacency Matrix for a directed graph

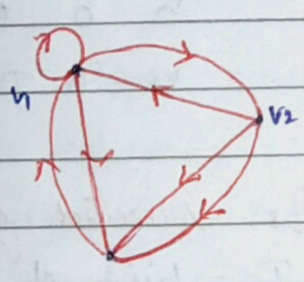
The adjacency ~~matrix~~ for a di-graph with n vertices is the matrix $A_G = n \times n (a_{ij})$

in which $a_{ij} = \begin{cases} 1, & \text{if } (v_i, v_j) \in \text{ie in } D \text{ (Di-graph)} \\ 0, & \text{otherwise} \end{cases}$

Find the adjacency matrix for the graph



$$A_G = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$



$$A_G = \begin{array}{c|ccc} & v_1 & v_2 & v_3 \\ \hline v_1 & 1 & 1 & 1 \\ v_2 & 1 & 0 & 2 \\ v_3 & 1 & 0 & 0 \end{array}$$

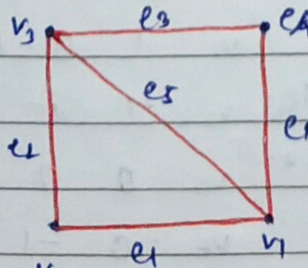
Incidence Matrix for undirected

Let $G = (V, e)$ be an undirected graph with n labelled vertices and m labelled edges.

The incidence matrix $I_G = (b_{ij})_{n \times m}$ where

$b_{ij} = \begin{cases} 1, & \text{if } j^{th} \text{ edge } e_j \text{ incident on the } i^{th} \text{ vertex } v_i \\ 0, & \text{otherwise} \end{cases}$

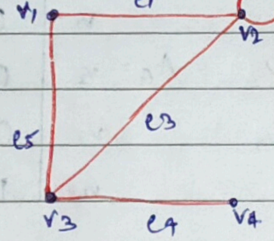
Construct the incidence matrix I_G for the graph G



4 no of Vertex and
5 no of edges

$$I_G = \begin{array}{c|ccccc} & e_1 & e_2 & e_3 & e_4 & e_5 \\ \hline v_1 & 1 & 0 & 0 & 1 & 1 \\ v_2 & 1 & 1 & 0 & 0 & 0 \\ v_3 & 0 & 1 & 1 & 0 & 1 \\ v_4 & 0 & 0 & 1 & 1 & 0 \end{array}$$

Find the incidence matrix for given graph.



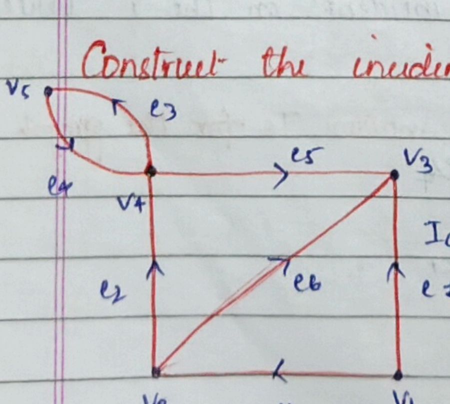
	e_1	e_2	e_3	e_4	e_5
v_1	1	0	0	0	1
v_2	1	2	1	0	0
v_3	0	0	1	1	1
v_4	0	0	0	1	0

Incidence Matrix for Directed graph

The incidence matrix $I_G = b_{ij}$ of a di-graph with n vertices and m edges is the $n \times m$ matrix where,

$$b_{ij} = \begin{cases} 1, & \text{if } j^{th} \text{ edge is incident out of the } i^{th} \text{ vertex } v_i \\ -1, & \text{if } j^{th} \text{ edge } e_j \text{ incident into the } i^{th} \text{ vertex } v_i \\ 0, & \text{otherwise} \end{cases}$$

Construct the incidence matrix I_G for the di-graph



$$I_G = \begin{array}{c|ccccccc} & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 \\ \hline v_1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ v_2 & -1 & 1 & 0 & 0 & 0 & 1 & 0 \\ v_3 & 0 & 0 & 0 & 0 & -1 & -1 & -1 \\ v_4 & 0 & -1 & 1 & -1 & +1 & 0 & 0 \\ v_5 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \end{array}$$

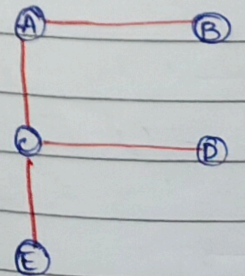
Isomorphic Graph

Two graphs $G_1 = ($
said to be isomo
$f: V_1 \to V_2$ such
in G_1 iff $f(u)$
If G_1 is isomorp
G_1 is isomorphic

Working Rule:

1. The given graphs
vertices
2. The two graphs m
of edges.
3. The two graphs
degree sequence.
4. 1-1 correspondence
graphs must be
i.e. for every ver
be an equivalent
5. Edge preserving s
edge in the gra
edge in graph
6. ~~Edges~~ Adjancent
must be same

Example:



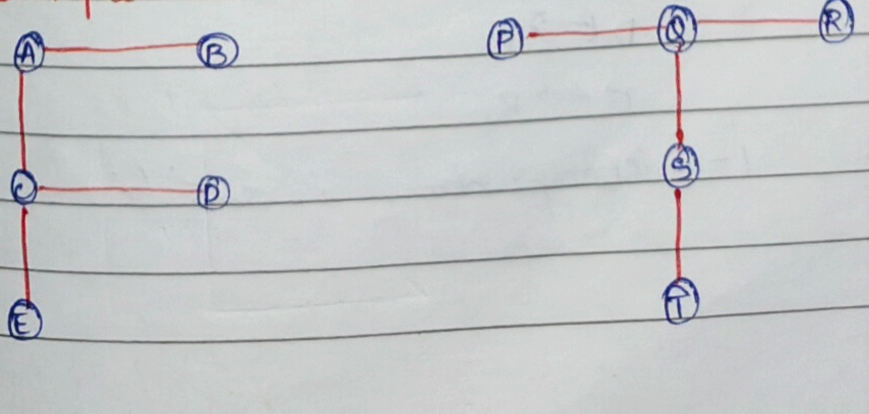
Isomorphic Graph

Two graphs $G_1=(V_1,E_1)$ & $G_2=(V_2,E_2)$ are said to be isomorphic if there exists a bijection $f: V_1 \rightarrow V_2$ such that u and v are adjacent in G_1 iff $f(u)$ & $f(v)$ are adjacent in G_2. If G_1 is isomorphic to G_2, then we write G_1 is isomorphic to G_2 $(G_1 \cong G_2)$

Working Rule:

1. The given graphs must contain same number of vertices
2. The two graphs must contain the same number of edges.
3. The two graphs must contain same number of degree sequence.
4. 1-1 correspondence between the 2 vertices of 2 graphs must be same.
   i.e. for every vertex in the graph G_1 there should be an equivalent vertex in the graph G_2.
5. Edge preserving should be satisfied. i.e each edge in the graph G_1 is equivalent to an edge in graph G_2.
6. ~~Edges~~ Adjacent matrix of both the graphs must be same.

Example:



(1) The no. of vertices in G_1 is 5

The no. of vertices in G_2 is 5

$\therefore$ The no. of vertices in two graphs are same.

(2) The no. of edges in G_1 is 4

The no. of edges in G_2 is 4

$\therefore$ The no. of edges in two graphs are same.

(3) Degree sequence in G_1 (A, B, C, D, E) = (2, 1, 3, 1, 1)

Degree sequence in G_2 (P, Q, R, S, T) = (1, 3, 1, 2, 1)

$\therefore$ The degree sequence in two graphs is same.

(4) 1-1 correspondence.

(2) A – B (1), C (3)

(1) B – A (2)

(2) A, D (1), E (1) – (3) C

(1) D – C (3)

(1) E – C (3)

(1) P – Q (3)

(3) Q – P (1), S (2), R (1)

(1) R – Q (3)

(2) S – Q (3), T (1)

(1) T – S (2)

A $\Leftrightarrow$ S

B $\Leftrightarrow$ T

C $\Leftrightarrow$ Q

D $\Leftrightarrow$ P

E $\Leftrightarrow$ R

1-1 correspondance is satisfied

5) Edge preserving

$A \leftrightarrow B \equiv S \leftrightarrow T$

$A \leftrightarrow C \equiv S \leftrightarrow Q$

$C \leftrightarrow D \equiv Q \leftrightarrow P$

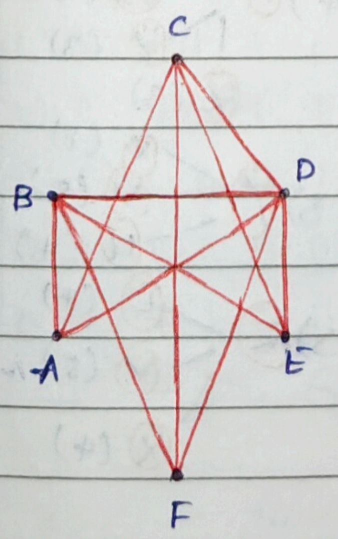
$C \leftrightarrow E \equiv Q \leftrightarrow R$

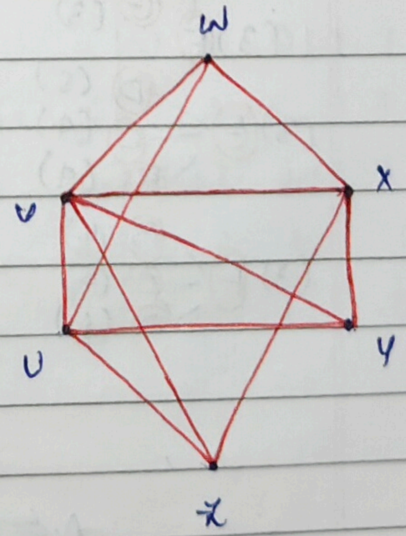
$\therefore$ Edge preserving is satisfied

6) Adjacency matrix

$$A_{G_1} = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}
\qquad
A_{G_2} = \begin{array}{c} \\ S \\ T \\ Q \\ P \\ R \end{array} \begin{array}{c} \begin{array}{ccccc} S & T & Q & P & R \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

$\therefore G_1 \cong G_2$



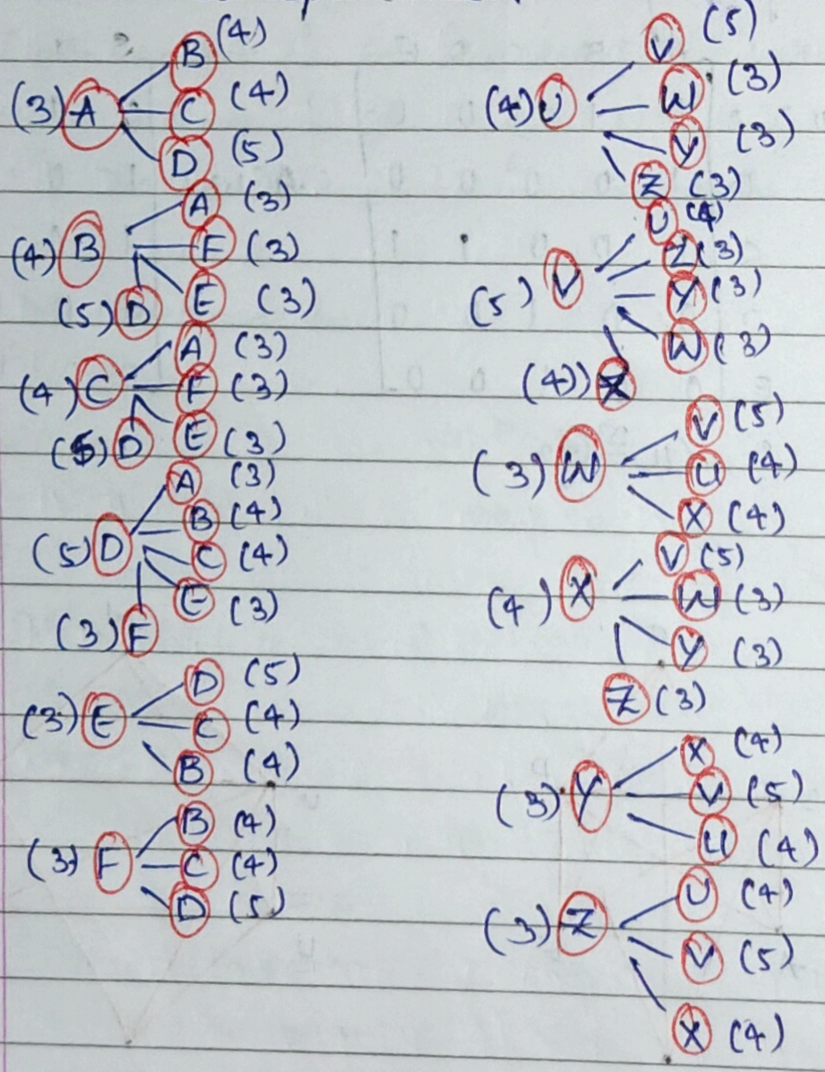


1) The no. of vertices in G_1 is 6
2) The no. of vertices in G_2 is 6

(2) The edges in G_1 is 11
The no. of edges in G_2 is 11
$\therefore$ The no. of edges are same in graphs

(3) The degree sequence in G_1 $(A, B, C, D, E, F) = (3, 4, 4, 5, 3, 3)$
The degree sequence in G_2 $(U, V, W, X, Y, Z) = (4, 5, 3, 4, 3, 3)$
$\therefore$ The degree sequence is same in two graphs

(4) 1-1 correspondence.



A $\Leftrightarrow$ W
B $\Leftrightarrow$ U
C $\Leftrightarrow$ X
D $\Leftrightarrow$ V
E $\Leftrightarrow$ Y
F $\Leftrightarrow$ Z

Edge preserving

$A \leftrightarrow B \Leftrightarrow W \leftrightarrow U$

$A \leftrightarrow C \Leftrightarrow W \leftrightarrow X$

$A \leftrightarrow D \Leftrightarrow W \leftrightarrow V$

$B \leftrightarrow F \Leftrightarrow U \leftrightarrow Z$

$B \leftrightarrow E \Leftrightarrow U \leftrightarrow Y$

$B \leftrightarrow D \Leftrightarrow U \leftrightarrow V$

$C \leftrightarrow F \Leftrightarrow X \leftrightarrow Z$

$C \leftrightarrow E \Leftrightarrow X \leftrightarrow Y$

$C \leftrightarrow D \Leftrightarrow X \leftrightarrow V$

$D \leftrightarrow E \Leftrightarrow V \leftrightarrow Y$

$D \leftrightarrow F \Leftrightarrow V \leftrightarrow Z$

~~E~~ $\therefore$ Edge presсерving is Satisfied

6. Adjacency matrix

$$AG_1 = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array} \begin{array}{c} \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array} \qquad AG_2 = \begin{array}{c} \\ W \\ U \\ X \\ V \\ Y \\ Z \end{array} \begin{array}{c} \begin{array}{cccccc} W & U & X & V & Y & Z \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

$\therefore G_1 \cong G_2$

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Walk: A walk of a graph G is defined as an alternative sequence of vertices and edges $v_0 e_1 v_1 e_2 v_2 e_3 v_3 \ldots\ldots$ beginning and ending with the vertices such that each line e_i is incident with v_{i-1} and v_i.

→ A walk joining v_0 and v_n is called v_0-v_n walk

→ Here v_0 is called initial vertex and v_n is called terminal vertex of the walk.

→ The no. of edges in the walk is known as the length of the walk.

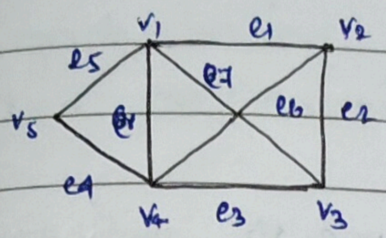
→ If the length of the walk is '0' then the walk has no edges and it contains only a single vertex. such type of walk are called Trivial walks.

TRAIL: A walk is called a trail if all its edges are distinct.

PATH: A walk is called a path if all its vertices are distinct.

CLOSED PATH: A closed path is a path that starts and ends ~~with~~ at the same point

CIRCUIT (or) CYCLE: A circuit is defined as a closed path of non zero length that does not contain a repeated edge.



i) $V_1 e_1 V_2 e_6 V_4 e_3 V_3 e_2 V_2$ - ~~walk~~

ii) $V_1 e_1 V_2 e_2 V_3 e_3 V_4 e_4 V_5$

iii) $V_1 e_8 V_4 e_3 V_3 e_7 V_1 e_8 V_4$

iv) $V_5 e_5 V_1 e_8 V_4 e_3 V_3 e_2 V_2 e_6 V_4 e_4 V_5$

v) $V_2 e_2 V_3 e_3 V_4 e_4 V_5 e_5 V_1 e_1 V_2$

i) $V_1 e_1 V_2 e_6 V_4 e_3 V_3 e_2 V_2$

It is a walk

It is trail and all edges are distinct

It is not a path as the (V_2) is repeated so, closed path of circuit also cannot be considered

ii) $V_1 e_1 V_2 e_2 V_3 e_3 V_4 e_4 V_5$

It is a walk.

It is a trail

It is a path

It is not a path so circuit is also cannot be considered

iii) $V_1 e_8 V_4 e_3 V_3 e_7 V_1 e_8 V_4$

It is a walk

It is not a trail

iv) $V_5 e_5 ~~V_1~~ e_8 V_4 e_3 V_3 e_2 V_2 e_6 V_4 e_4 V_5$

It is a walk

It is a trail
It is not a path.

v) $V_2 e_2 V_3 e_3 e_4 e_4 V_5 e_5 V_1 e_1 V_2$
It is a walk
It is a path
It is a Trail
It is a closed path
It is a circuit.

CONNECTED VERTICES.
Two vertices U & V of a graph G are said to be connected if there exist a path from U to V.

CONNECTED GRAPH: A Graph G is said to be connected if every pair of vertices in G are connected.

DISCONNECTED GRAPH: A Graph which is not connected is said to be a disconnected graph.

** EULERIAN GRAPH: A trail in G is called an eulerian trail if it contains all the edges of G exactly ones and it contains all the vertices of G.

→ A closed eulerian trail is called eulerian circuit

→ A graph having
Called an euleri

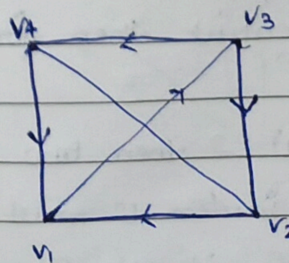
EULER PATH: A path
an euler path
exactly once hen
euler trail.

→ A closed euler pa

NOTE: To determin
an euler circuit
the following

1) List the degree
2) If any value is
connected and hence
or euler circuit.
3) If all the degree
both euler circuit
4) If exactly 2 verte
G has euler pa

Determine whether
or euler circuit



→ A graph having an eulerian circuit is called an eulerian graph.

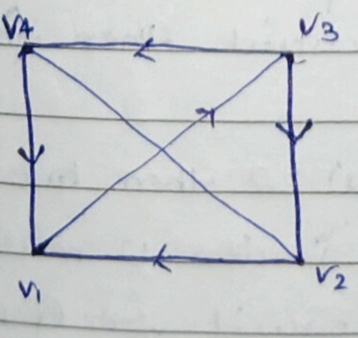
EULER PATH : A path in graph G is called an euler path if it includes every edge exactly once hence euler path is also called euler trail.

→ A closed euler path is called euler circuit

NOTE : To determine whether a graph G has an euler circuit or euler path, we consider the following procedure.

1) List the degree of all the vertices of graph G.
2) If any value is '0', the graph is not connected and hence it cannot have euler path or euler circuit.
3) If all the degrees are even then G has both euler circuit and euler path.
4) If exactly 2 vertices are odd degree or even then G has euler path but no euler circuit

Determine whether the graph G has ~~the~~ euler path or euler circuit



~~i) deg(v2) = 2~~ ~~ii) deg(v)~~.

Here clearly the euler circuit is not possible

Hamiltonian Graph:

A circuit in a graph G is called a hamiltonian circuit if it contains each vertex in G exactly ones except for the starting and ending vertex that appears twice.

→ A graph G is called a hamiltonian graph if it contains hamiltonian circuit.

→ A hamiltonian path is a path that contains all the vertices of G where the end points may be distinct

NOTE: An eulerian circuit traverses any edge exactly once but vertices may be repeat on the other hand

Hamiltonian circuit visits each vertex exactly once. but edges may be repeat

→ To find a hamiltonian circuit in a graph G the following hints prove to be helpful.

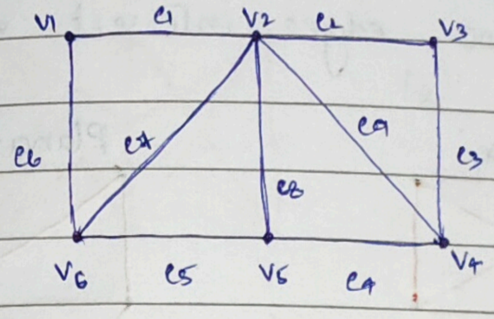
1. If G has hamitonian circuit then for all $\forall v \in V$ $\deg(v) \geq 2$.

2. If $u \in V$ and $\deg(u) = 2$ then two edges are incident with the vertex u must appear in every hamitonian circuit for G.

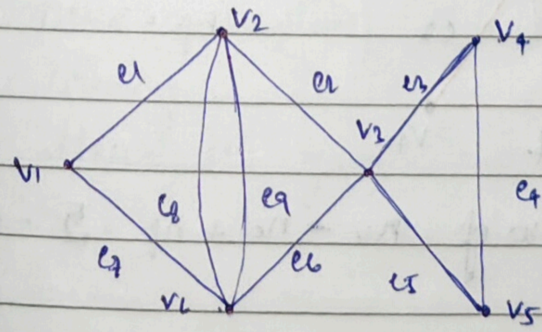
3. If $u \in V$ and $\deg(u) > 2$ then we try to

build a hamiltonian circuit.

Give an example of a graph which is hamiltonian but not eulerian and vice-verse



The given graph is hamiltonian because $v_1 e_1 v_2 e_2 v_3 e_3 v_4 e_4 ~~e_5~~ e_5 v_6 e_6 v_1$
In this circuit all the vertices are distinct except the end vertices.
Hence the given graph contains hamiltonian circuit.
∴ The given graph is hamiltonian graph but it is not eulerian graph.
∵ It is not possible to write eulerian circuit with all distinct edges.



Since all the vertices of a graph have an even degree then it contains euler path and euler circuit

$v_1 e_1 v_2 e_2 v_3 e_3 v_4 e_4 v_5 e_5 v_3 e_6 v_6 e_8 v_2 e_9 v_6 e_7 v_1$

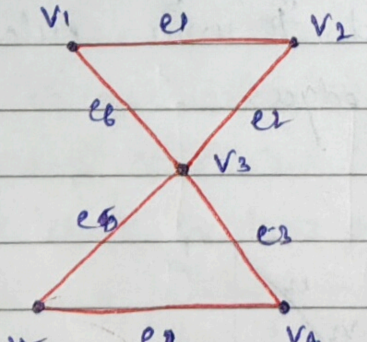
Here all the edges present exactly once.

PLANAR GRAPH : A graph G is called a planar graph if it can be drawn in a plane such that no two edges intersect except at the vertices.

EULER THEOREM

If a connected planar graph G has n_v vertices, n_v edges and n_f regions, then $n_v - n_e + n_f = 2$

Consider the graph given below & discuss the characteristics of the graph in detail



$n_v = 5$

$n_e = 6$

$n_f = 3$

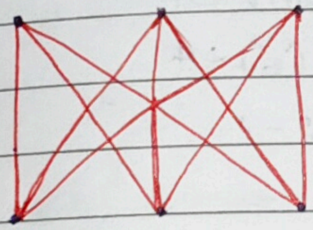
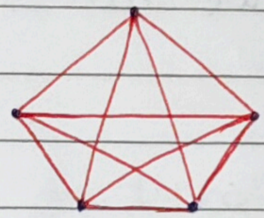
No. of $n_v - n_e + n_f = 5 - 6 + 3 = 2$

IMPORTANT THEOREM

Let G be a simple, connected planar graph with $n_v \geq 3$ and n_e edges. Then $\boxed{n_e \leq 3n_v - 6}$

If G is a connected simple planar graph with $n \geq 3$ vertices and n_e edges and no circuits of length 3 then $n_e \leq 2n_v - 4$

Show that the graph K_5 is not a planar graph

(i)  (ii) 

no. of vertices $n_v \rightarrow 5$
no. of edges $n_e \rightarrow 10$
clearly simple connected graph. There are 3-length circuit (or) cycle.
If suppose K_5 graph is a planar graph then we have

$n_e \leq 3n_v - 6$

$10 \leq 3(5) - 6$

$10 \leq 15 - 6$

$10 \leq 9$

$\therefore$ there is a contradiction

$\therefore$ Our assumption K_5 is planar graph is wrong

Hence K_5 graph a non-planar graph.

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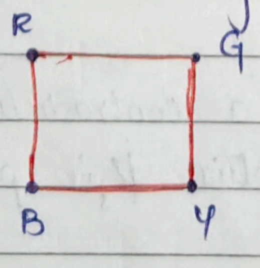
ii) Clearly $K_{3,3}$ graph is simple connected graph and has no circuits of length 3 with the no. of vertices - $n_v = 6$
The no. of edges $n_e = 9$
suppose that $K_{3,3}$ is a planar graph then we have $n_e \leq 2n_v - 4$

$$9 \leq 2(6) - 4$$
$$9 \leq 12 - 4$$
$$9 \leq 8$$

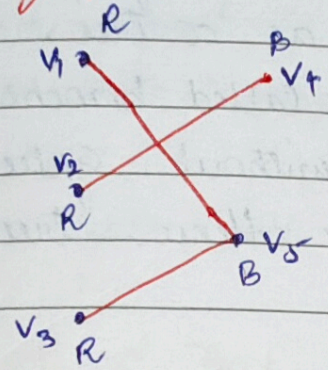
$\therefore$ There is a contradiction
$\therefore$ Our assumption of $K_{3,3}$ is a planar graph is wrong.
Hence $K_{3,3}$ is a non planar graph.

COLOURING OF GRAPH : An assignment of colours to the vertices of a graph so that no two adjacent vertices have the same colour i.e called colouring of graph or vertex colouring



CHROMATIC NUMBER
The chromatic number of the graph G is the minimum number of colours needed to colour the vertices of the graph G and it is denoted by χ_G

Determine the chromatic number of the bipartiate graph.



The no. of colours required for the given bipartiate graph is 2.

$\therefore \chi G = 2$