CHAPTER 2

Exercise 2.1

(a)
$$Y = \overline{AB} + A\overline{B} + AB$$

(b)
$$Y = \overline{ABC} + ABC$$

(c)
$$Y = \overrightarrow{ABC} + \overrightarrow{ABC} + \overrightarrow{ABC} + \overrightarrow{ABC} + \overrightarrow{ABC}$$

(d)

$$Y = \overrightarrow{ABCD} + \overrightarrow{ABCD}$$

(e)

$$Y = \overrightarrow{ABCD} + \overrightarrow{ABCD}$$

Exercise 2.2

(a)
$$Y = \overline{A}B + A\overline{B} + AB$$

(b)
$$Y = \overline{A}\overline{B}C + \overline{A}B\overline{C} + \overline{A}BC + A\overline{B}\overline{C} + AB\overline{C}$$

(c)
$$Y = \overline{ABC} + AB\overline{C} + ABC$$

(d)
$$Y = \overline{A}\overline{B}\overline{C}\overline{D} + \overline{A}\overline{B}C\overline{D} + \overline{A}\overline{B}CD + \overline{A}BCD + \overline{A}BCD + A\overline{B}\overline{C}\overline{D} + A\overline{B}C\overline{D}$$

(e)
$$Y = \overline{ABCD} + \overline{ABCD}$$

(a)
$$Y = (A + \overline{B})$$

(b)
$$Y = (A + B + \overline{C})(A + \overline{B} + C)(A + \overline{B} + \overline{C})(\overline{A} + B + C)(\overline{A} + B + \overline{C})(\overline{A} + \overline{B} + C)$$
(c)
$$Y = (A + B + \overline{C})(A + \overline{B} + \overline{C})(\overline{A} + \overline{B} + C)$$
(d)
$$Y = (A + \overline{B} + C + D)(A + \overline{B} + C + \overline{D})(A + \overline{B} + \overline{C} + D)(A + \overline{B} + \overline{C} + \overline{D})(\overline{A} + B + C + \overline{D})$$
(e)
$$Y = (A + B + C + \overline{D})(A + \overline{B} + C + D)(\overline{A} + \overline{B} + C + \overline{D})(\overline{A} + \overline{B} + \overline{C} + \overline{D})$$
(e)
$$Y = (A + B + C + \overline{D})(A + B + \overline{C} + D)(A + \overline{B} + C + D)(A + \overline{B} + \overline{C} + \overline{D})(\overline{A} + B + C + D)$$

$$(\overline{A} + B + \overline{C} + \overline{D})(\overline{A} + \overline{B} + C + \overline{D})(\overline{A} + \overline{B} + \overline{C} + D)(\overline{A} + \overline{B} + \overline{C} + \overline{D})(\overline{A} + \overline{B} + C + D)$$

Exercise 2.4

(a)
$$Y = A + B$$

(b) $Y = (A + B + C)(\overline{A} + B + \overline{C})(\overline{A} + \overline{B} + \overline{C})$
(c) $Y = (A + B + C)(A + \overline{B} + C)(A + \overline{B} + \overline{C})(\overline{A} + B + C)(\overline{A} + B + \overline{C})$
(d) $Y = (A + B + C + \overline{D})(A + \overline{B} + C + D)(A + \overline{B} + C + \overline{D})(\overline{A} + B + C + \overline{D})$
($\overline{A} + B + \overline{C} + \overline{D}$)($\overline{A} + \overline{B} + C + D$)($\overline{A} + \overline{B} + C + \overline{D}$)($\overline{A} + \overline{B} + \overline{C} + D$)
(e) $Y = (A + B + C + D)(A + B + C + \overline{D})(A + B + \overline{C} + D)(A + \overline{B} + C + D)$
($A + \overline{B} + C + \overline{D}$)($A + \overline{B} + C + D$)($A + \overline{B} + C + D$)($A + \overline{B} + C + D$)
($A + \overline{B} + C + \overline{D}$)($A + \overline{B} + C + D$)($A + \overline{B} + C + D$)

Exercise 2.5

(a)
$$Y = A + B$$

(b) $Y = \overline{ABC} + ABC$
(c) $Y = \overline{AC} + \overline{AB} + AC$
(d) $Y = \overline{AB} + \overline{BD} + AC\overline{D}$
(e) $Y = \overline{ABCD} + \overline$

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(a)
$$Y = A + B$$

(b)
$$Y = A\overline{C} + \overline{A}C + B\overline{C}$$
 or $Y = A\overline{C} + \overline{A}C + \overline{A}B$

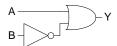
(c)
$$Y = AB + \overline{A}\overline{B}C$$

(d)
$$Y = BC + \overline{B}\overline{D}$$

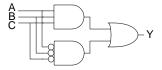
(e)
$$Y = A\overline{B} + \overline{A}BC + \overline{A}CD$$
 or $Y = A\overline{B} + \overline{A}BC + \overline{B}CD$

Exercise 2.7

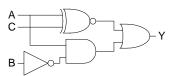
(a)



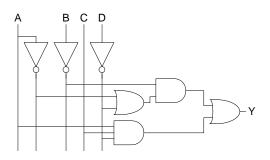
(b)

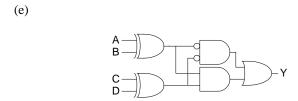


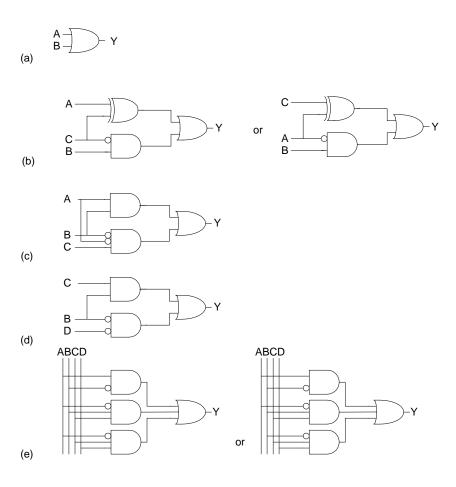
(c)



(d)



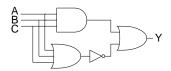




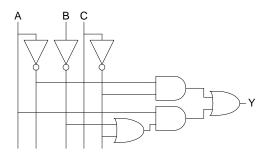
Exercise 2.9

(a) Same as 2.7(a)

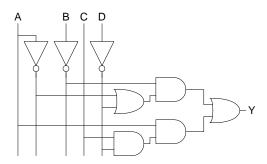
(b)



(c)



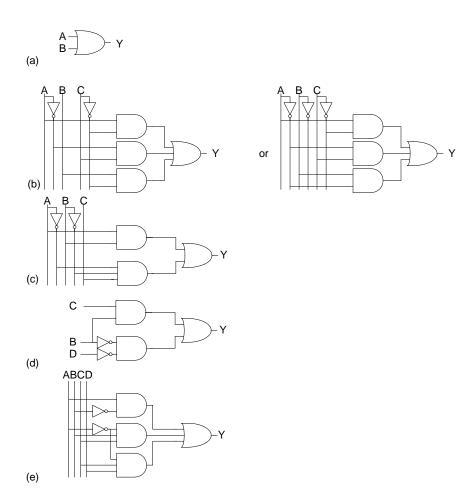
(d)



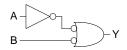
(e)

A B C D Y

Exercise 2.10

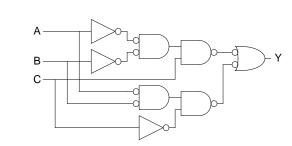


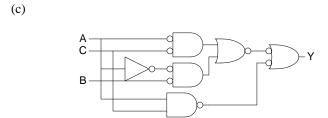
(a)

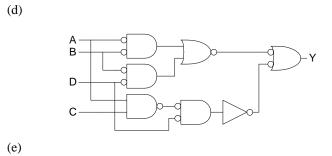


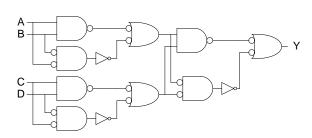
(b)

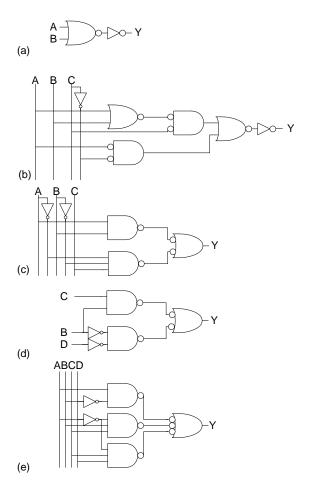
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(a)
$$Y = AC + \overline{B}C$$

(b)
$$Y = \overline{A}$$

(a)
$$Y = AC + \overline{B}C$$

(b) $Y = \overline{A}$
(c) $Y = \overline{A} + \overline{B} \overline{C} + \overline{B} \overline{D} + BD$

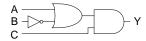
(a)
$$Y = \overline{A}B$$

(b)
$$Y = \overline{A} + \overline{B} + \overline{C} = \overline{ABC}$$

(c)
$$Y = A(\overline{B} + \overline{C} + \overline{D}) + \overline{B}\overline{C}\overline{D} = A\overline{B}\overline{C}\overline{D} + \overline{B}\overline{C}\overline{D}$$

Exercise 2.15

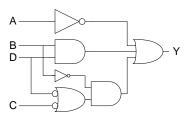
(a)



(b)



(c)



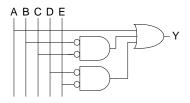
Exercise 2.16

(a)
$$Y = B + \overline{AC}$$



(b)
$$Y = \overline{A}B$$

(c)
$$Y = A + \overline{BC} + \overline{DE}$$



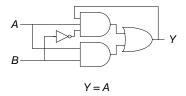
(a)
$$Y = \overline{B} + C$$

(b)
$$Y = (A + \overline{C})D + B$$

(c)
$$Y = B\overline{D}E + BD(\overline{A \oplus C})$$

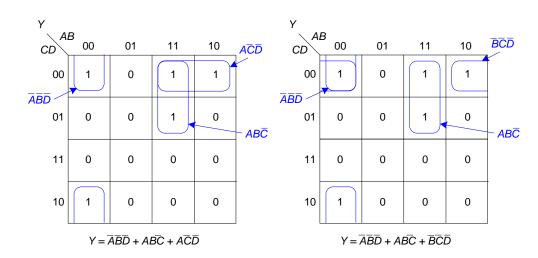
Exercise 2.19

4 gigarows = 4×2^{30} rows = 2^{32} rows, so the truth table has 32 inputs.



Exercise 2.21

Ben is correct. For example, the following function, shown as a K-map, has two possible minimal sum-of-products expressions. Thus, although \overline{ACD} and \overline{BCD} are both prime implicants, the minimal sum-of-products expression does not have both of them.



Exercise 2.22

(a)

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(b)

В	С	D	$(B \bullet C) + (B \bullet D)$	B•(C + D)
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	0	0
1	0	0	0	0
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

(c)

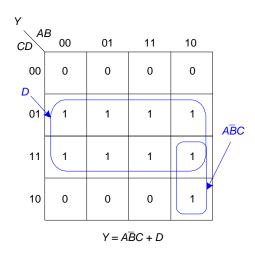
	В	С	$(B \bullet C) + (B \bullet \overline{C})$
•	0	0	0
	0	1	0
	1	0	1
	1	1	1

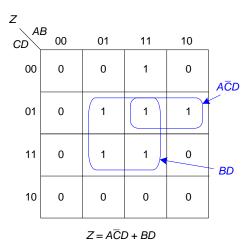
Exercise 2.23

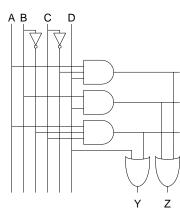
B_2	B_1	B_0	$\overline{B_2 \bullet B_1 \bullet B_0}$	$\overline{B}_2 + \overline{B}_1 + \overline{B}_0$
0	0	0	1	1
0	0	1	1	1
0	1	0	1	1
0	1	1	1	1
1	0	0	1	1
1	0	1	1	1
1	1	0	1	1
1	1	1	0	0

Exercise 2.24

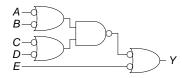
$$Y = \overline{AD} + A\overline{BC} + A\overline{CD} + ABCD$$
$$Z = A\overline{CD} + BD$$



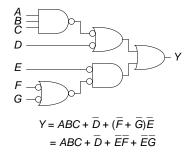




Exercise 2.26

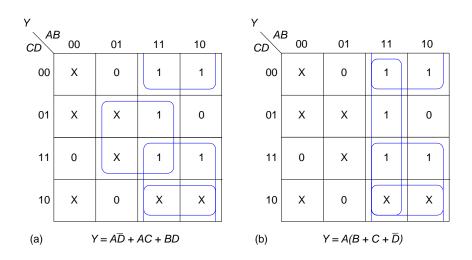


$$Y = (\overline{A} + \overline{B})(\overline{C} + \overline{D}) + \overline{E}$$

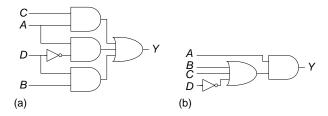


Exercise 2.28

Two possible options are shown below:



Two possible options are shown below:



Exercise 2.30

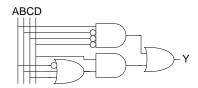
Option (a) could have a glitch when A=1, B=1, C=0, and D transitions from 1 to 0. The glitch could be removed by instead using the circuit in option (b).

Option (b) does not have a glitch. Only one path exists from any given input to the output.

Exercise 2.31

$$Y = \overline{A}D + A\overline{B}\overline{C}\overline{D} + BD + CD = A\overline{B}\overline{C}\overline{D} + D(\overline{A} + B + C)$$

Exercise 2.32



Exercise 2.33

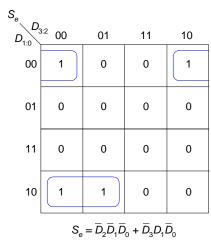
The equation can be written directly from the description:

$$E = S\overline{A} + AL + H$$

(a)

S_c									
S_c $D_{1:0}$	^{3:2} 00	01	11	10					
00	1	1	0	1					
01	1	1	0	1					
11	1	1	0	0					
10	0	1	0	0					
\$ 50.50.55									

$S_c = \overline{D}_3 D_0$	$+ \overline{D}_3 D_2$	$+ \overline{D}_2 \overline{D}_1$
----------------------------	------------------------	-----------------------------------



S_d $D_{1:0}$				
$D_{1:0}$	^{3:2} 00	01	11	10
00	1	0	0	1
01	0	1	0	0
11	1	0	0	0
10	1	1	0	0

$$S_{d} = \overline{D}_{3} D_{1} \overline{D}_{0} + \overline{D}_{3} \overline{D}_{2} D_{1} + \overline{D}_{2} \overline{D}_{1} \overline{D}_{0} + \overline{D}_{3} \overline{D}_{2} \overline{D}_{1} D_{0}$$

$$S_{f}$$

$$D_{3:2} \quad 00 \qquad 01 \qquad 11 \qquad 10$$

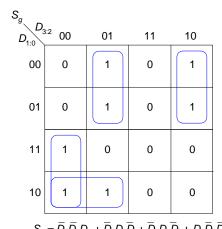
$$00 \qquad 1 \qquad 1 \qquad 0 \qquad 1$$

$$01 \quad 0 \qquad 1 \qquad 0 \qquad 1$$

$$11 \quad 0 \qquad 0 \qquad 0$$

$$10 \quad 0 \qquad 1 \qquad 0$$

$$S_f = \overline{D}_3 \overline{D}_1 \overline{D}_0 + \overline{D}_3 D_2 \overline{D}_1 + \overline{D}_3 D_2 \overline{D}_0 + D_3 \overline{D}_2 \overline{D}_1$$



$$S_g = \overline{D}_3\overline{D}_2D_1 + \overline{D}_3D_1\overline{D}_0 + \overline{D}_3D_2\overline{D}_1 + D_3\overline{D}_2\overline{D}_1$$

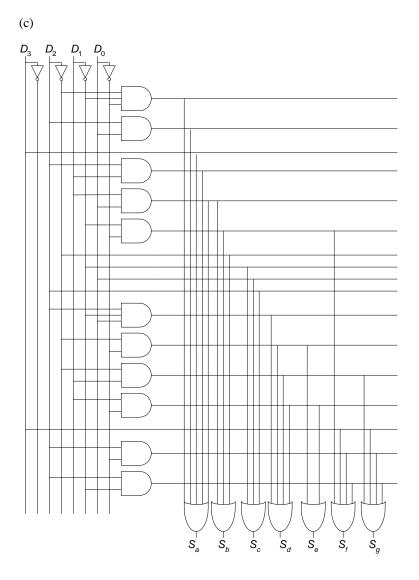
(b)

S _a D ₃					S_b $D_{1:0}$				
$D_{1:0}$	3:2 00	01	11	10	$D_{1:0}$:2 00	01	11	10
00	1	0	X	1	00	1	1	Х	1
01	0	1	х	1	01	1	0	х	1
11	1	1	х	х	11	1	1	Х	х
10	0	1	х	х	10	1	0	x	Х
S _a	$=\overline{D}_2\overline{D}_1\overline{D}_1$	$D_0 + D_2 D_0$	+ D ₃ + D	$D_2D_1 + D_1D_0$) •	S _b =	$\overline{D}_1\overline{D}_0 + D$	$D_1D_0 + \overline{D}_2$	
, D					σ_{λ}				
$D_{1:0}$	^{3:2} 00	$S_a^1 = L$	D ₂ D₁ ¹ Ď₀ +	$D_2 \hat{D}_0^0 + D_3$	$+ D_1 D_{1:0}$	^{:2} 00	01	11	10
S_c $D_{1:0}$ 00	00	$\frac{91}{a} = L$	$D_2D_1^1D_0 + X$	$D_2 \dot{D}_0^0 + D_3$	$S_d + D_1 D_{1:0}$	1	01	11 X	10
D _{1:0}	1								
00	'	1	Х	1	00	1	0	X	1
01	1	1	X	1	00	0	0	X	0

S _e D ₃				
$D_{1:0}$	3:2 00	01	11	10
00	1	0	Х	1
01	0	0	Х	0
11	0	0	Х	Х
10	1	1	Х	X
			_ =	

$S_e = D_2 D_0 + D_1 D_0$ S_a									
S_g $D_{1:0}$	3:2 00	01	11	10					
00	0	1	X	1					
01	0	1	Х	1					
11	1 0		Х	Х					
10	1	1	Х	x					
$S_g = \overline{D}_2 D_1 + D_2 \overline{D}_0 + D_2 \overline{D}_1 + D_3$									

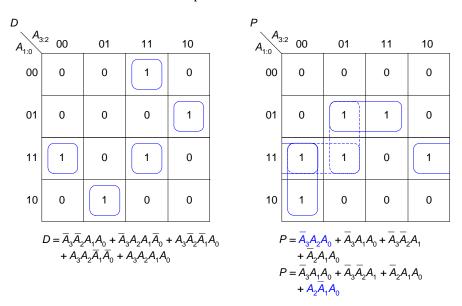
S_f					
S_f $D_{1:0}$	^{3:2} 00	01	11	10	
00	1	1	Х	1	
01	0	1	Х	1	
11	0	0	Х	Х	
10	0	1	Х	х	
	S = T	D + D	<u></u>	+ D	



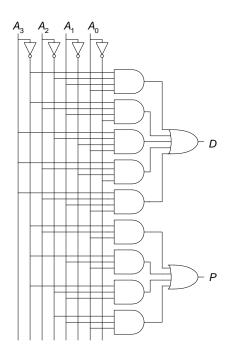
Exercise 2.35

Decimal Value	A_3	A_2	<i>A</i> ₁	A_0	D	Р
0	0	0	0	0	0	0
1	0	0	0	1	0	0
2 3	0	0	1	0	0	1
	0	0	1	1	1	1
4 5	0	1	0	0	0	0
	0	1	0	1	0	1
6	0	1	1	0	1	0
7	0	1	1	1	0	1
8	1	0	0	0	0	0
9	1	0	0	1	1	0
10	1	0	1	0	0	0
11	1	0	1	1	0	1
12	1	1	0	0	1	0
13	1	1	0	1	0	1
14	1	1	1	0	0	0
15	1	1	1	1	1	0

P has two possible minimal solutions:

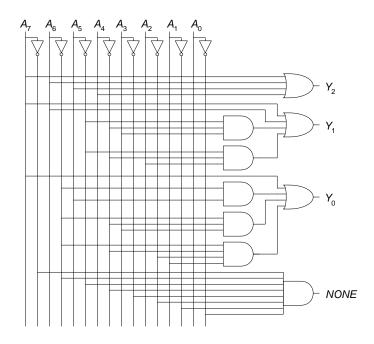


Hardware implementations are below (implementing the first minimal equation given for P).



A_7	A_6	A_5	A_4	A_3	A_2	A_1	A_0	Y ₂	Y_1	Y_0	NONE
0	0	0	0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	1	Х	0	0	1	0
0	0	0	0	0	1	X	Х	0	1	0	0
0	0	0	0	1	X	X	Х	0	1	1	0
0	0	0	1	X	X	X	Х	1	0	0	0
0	0	1	X	X	X	X	Х	1	0	1	0
0	1	X	X	X	X	X	Х	1	1	0	0
1	X	X	Х	X	X	X	Х	1	1	1	0

$$\begin{split} Y_2 &= A_7 + A_6 + A_5 + A_4 \\ Y_1 &= A_7 + A_6 + \overline{A_5} \overline{A_4} A_3 + \overline{A_5} \overline{A_4} A_2 \\ Y_0 &= A_7 + \overline{A_6} A_5 + \overline{A_6} \overline{A_4} A_3 + \overline{A_6} \overline{A_4} \overline{A_2} A_1 \\ NONE &= \overline{A_7} \overline{A_6} \overline{A_5} \overline{A_4} \overline{A_3} \overline{A_2} \overline{A_1} \overline{A_0} \end{split}$$



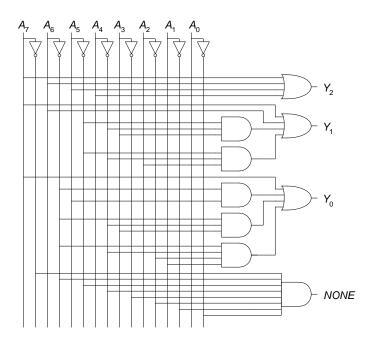
Exercise 2.37

The equations and circuit for $Y_{2:0}$ is the same as in Exercise 2.25, repeated here for convenience.

A_7	A_6	A_5	A_4	A_3	A_2	A_1	A_0	Y ₂	Y ₁	Y_0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	X	0	0	1
0	0	0	0	0	1	X	X	0	1	0
0	0	0	0	1	X	X	X	0	1	1
0	0	0	1	X	X	X	X	1	0	0
0	0	1	X	X	X	X	X	1	0	1
0	1	X	X	X	X	X	X	1	1	0
1	X	X	Х	X	X	X	X	1	1	1

$$\begin{split} Y_2 &= A_7 + A_6 + A_5 + A_4 \\ Y_1 &= A_7 + A_6 + \overline{A_5} \overline{A_4} A_3 + \overline{A_5} \overline{A_4} A_2 \\ Y_0 &= A_7 + \overline{A_6} A_5 + \overline{A_6} \overline{A_4} A_3 + \overline{A_6} \overline{A_4} \overline{A_2} A_1 \end{split}$$

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The truth table, equations, and circuit for $Z_{2:0}$ are as follows.

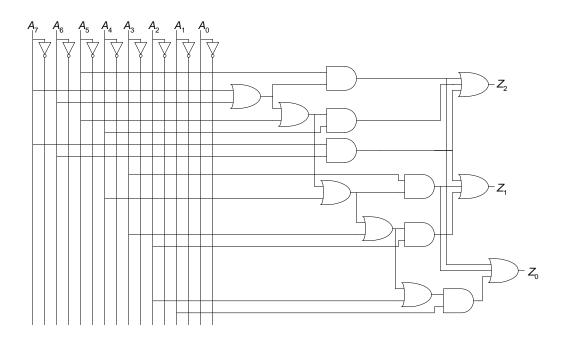
A_7	A_6	A_5	A_4	A_3	A_2	<i>A</i> ₁	A_0	Z_2	Z ₁	Z_0
0	0	0	0	0	0	1	1	0	0	0
0	0	0	0	0	1	0	1	0	0	0
0	0	0	0	1	0	0	1	0	0	0
0	0	0	1	0	0	0	1	0	0	0
0	0	1	0	0	0	0	1	0	0	0
0	1	0	0	0	0	0	1	0	0	0
1	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	1	1	X	0	0	1
0	0	0	0	1	0	1	Х	0	0	1
0	0	0	1	0	0	1	X	0	0	1
0	0	1	0	0	0	1	X	0	0	1
0	1	0	0	0	0	1	X	0	0	1
1	0	0	0	0	0	1	X	0	0	1
0	0	0	0	1	1	X	X	0	1	0
0	0	0	1	0	1	X	X	0	1	0
0	0	1	0	0	1	X	X	0	1	0
0	1	0	0	0	1	X	X	0	1	0
1	0	0	0	0	1	X	X	0	1	0
0	0	0	1	1	X	X	X	0	1	1
0	0	1	0	1	X	Х	X	0	1	1
0	1	0	0	1	X	Х	X	0	1	1
1	0	0	0	1	X	Х	X	0	1	1
0	0	1	1	X	X	X	X	1	0	0
0	1	0	1	X	X	X	X	1	0	0
1	0	0	1	X	X	X	X	1	0	0
0	1	1	Х	Х	Х	Х	Х	1	0	1
1	0	1	Х	Х	Х	Х	Х	1	0	1
1	1	X	Χ	X	X	X	X	1	1	0

$$Z_2 = A_4(A_5 + A_6 + A_7) + A_5(A_6 + A_7) + A_6A_7$$

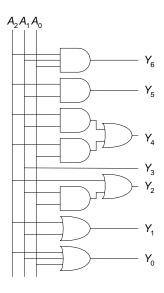
$$\begin{split} Z_1 &= A_2(A_3 + A_4 + A_5 + A_6 + A_7) + \\ A_3(A_4 + A_5 + A_6 + A_7) + A_6A_7 \end{split}$$

$$\begin{split} Z_0 &= A_1(A_2 + A_3 + A_4 + A_5 + A_6 + A_7) + \\ A_3(A_4 + A_5 + A_6 + A_7) + A_5(A_6 + A_7) \end{split}$$

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$$\begin{split} Y_6 &= A_2 A_1 A_0 \\ Y_5 &= A_2 A_1 \\ Y_4 &= A_2 A_1 + A_2 A_0 \\ Y_3 &= A_2 \\ Y_2 &= A_2 + A_1 A_0 \\ Y_1 &= A_2 + A_1 \\ Y_0 &= A_2 + A_1 + A_0 \end{split}$$

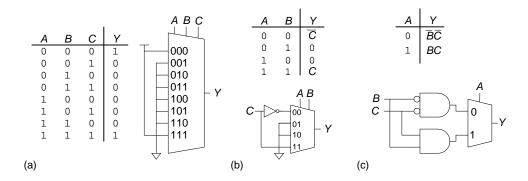


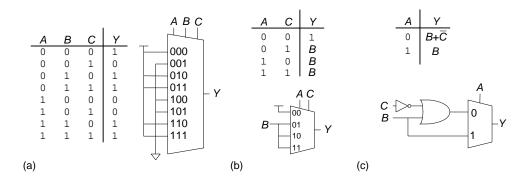
Exercise 2.39

$$Y = A + \overline{C \oplus D} = A + CD + \overline{CD}$$

Exercise 2.40

$$Y = \overline{CD}(A \oplus B) + \overline{AB} = \overline{ACD} + \overline{BCD} + \overline{AB}$$





Exercise 2.43

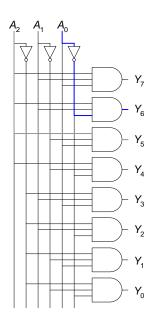
$$t_{pd} = 3t_{pd_NAND2} =$$
60 ps
 $t_{cd} = t_{cd_NAND2} =$ **15 ps**

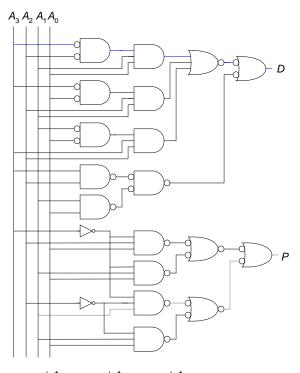
$$\begin{split} t_{pd} &= t_{pd_AND2} + 2t_{pd_NOR2} + t_{pd_NAND2} \\ &= [30 + 2 \ (30) + 20] \text{ ps} \\ &= \textbf{110 ps} \\ t_{cd} &= 2t_{cd_NAND2} + t_{cd_NOR2} \\ &= [2 \ (15) + 25] \text{ ps} \\ &= \textbf{55 ps} \end{split}$$

Exercise 2.45

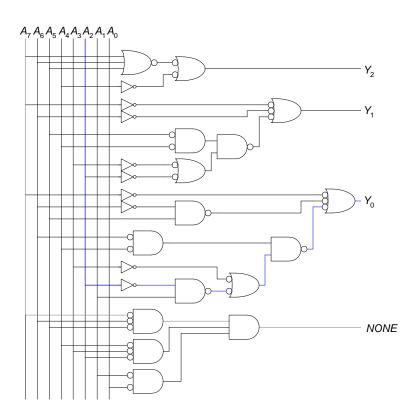
$$t_{pd} = t_{pd_NOT} + t_{pd_AND3}$$

= 15 ps + 40 ps
= **55 ps**
 $t_{cd} = t_{cd_AND3}$
= **30 ps**

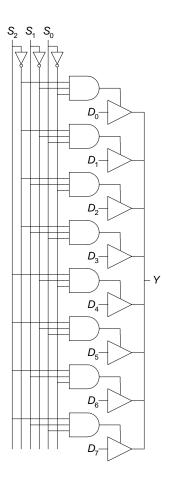




$$\begin{aligned} t_{pd} &= t_{pd_NOR2} + t_{pd_AND3} + t_{pd_NOR3} + t_{pd_NAND2} \\ &= [30 + 40 + 45 + 20] \text{ ps} \\ &= \textbf{135 ps} \\ t_{cd} &= 2t_{cd_NAND2} + t_{cd_OR2} \\ &= [2 \ (15) + 30] \text{ ps} \\ &= \textbf{60 ps} \end{aligned}$$



$$\begin{split} t_{pd} &= t_{pd_INV} + 3t_{pd_NAND2} + t_{pd_NAND3} \\ &= [15 + 3 \ (20) + 30] \text{ ps} \\ &= \textbf{105 ps} \\ t_{cd} &= t_{cd_NOT} + t_{cd_NAND2} \\ &= [10 + 15] \text{ ps} \\ &= \textbf{25 ps} \end{split}$$



$$t_{pd_dy} = t_{pd_TRI_AY}$$
$$= 50 ps$$

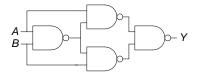
Note: the propagation delay from the control (select) input to the output is the circuit's critical path:

$$t_{pd_sy} = t_{pd_NOT} + t_{pd_AND3} + t_{pd_TRI_SY}$$

= [30 + 80 + 35] ps
= **145 ps**

However, the problem specified to minimize the delay from data inputs to output, t_{pd_dy} .

Question 2.1



Question 2.2

						Υ					
Month	A_3	A_2	A_1	A_0	Ιγ	$A_{1:0}$	3:2 00	01	11	10	1
 Jan	0	0	0	1	1	00	×	0	1	1	
Feb	0	0	1	0	0					'	
Mar	0	0	1	1	1						
Apr	0	1	0	0	0	01	1	1		0	
May	0	1	0	1	1	01	1	1	X	0	$A_3 \rightarrow \nabla$
Jun	0	1	1	0	0						ı "
Jul	0	1	1	1	1						$A_0 \rightarrow L$
Aug	1	0	0	0	1	11	1	1	X	0	
Sep	1	0	0	1	0						
Oct	1	0	1	0	1						
Nov	1	0	1	1	0	10			x	1	
Dec	1	1	0	0	1	10	0	0	X	1	
					•		Y = 2	$\overline{A}_3 A_0 + A_3$	$\overline{A}_{-} = A_{-}$		J

Question 2.3

A tristate buffer has two inputs and three possible outputs: 0, 1, and Z. One of the inputs is the data input and the other input is a control input, often called the *enable* input. When the enable input is 1, the tristate buffer transfers the data input to the output; otherwise, the output is high impedance, Z. Tristate buffers are used when multiple sources drive a single output at different times. One and only one tristate buffer is enabled at any given time.

Question 2.4

- (a) An AND gate is not universal, because it cannot perform inversion (NOT).
- (b) The set {OR, NOT} is universal. It can construct any Boolean function. For example, an OR gate with NOT gates on all of its inputs and output performs the AND operation. Thus, the set {OR, NOT} is equivalent to the set {AND, OR, NOT} and is universal.
- (c) The NAND gate by itself is universal. A NAND gate with its inputs tied together performs the NOT operation. A NAND gate with a NOT gate on its output performs AND. And a NAND gate with NOT gates on its inputs performs OR. Thus, a NAND gate is equivalent to the set {AND, OR, NOT} and is universal.

Question 2.5

A circuit's contamination delay might be less than its propagation delay because the circuit may operate over a range of temperatures and supply voltages, for example, 3-3.6 V for LVCMOS (low voltage CMOS) chips. As temperature increases and voltage decreases, circuit delay increases. Also, the circuit may have different paths (critical and short paths) from the input to the output. A gate itself may have varying delays between different inputs and the output, affecting the gate's critical and short paths. For example, for a two-input NAND gate, a HIGH to LOW transition requires two nMOS transistor delays, whereas a LOW to HIGH transition requires a single pMOS transistor delay.

David Money Harris and Sarah L. Harris, $Digital\ Design\ and\ Computer\ Architecture,\ ©\ 2007$ by Elsevier Inc. Exercise Solutions

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