Angel: Interactive Computer Graphics, Fifth Edition

Chapter 5 Solutions

5.1 Eclipses (both solar and lunar) are good examples of the projection of an object (the moon or the earth) onto a nonplanar surface. Any time a shadow is created on curved surface, there is a nonplanar projection. All the maps in an atlas are examples of the use of curved projectors. If the projectors were not curved we could not project the entire surface of a spherical object (the Earth) onto a rectangle.

5.3 Suppose that we want the view of the Earth rotating about the sun. Before we draw the earth, we must rotate the Earth which is a rotation about the y axis. Next we translate the Earth away from the origin. Finally we do another rotation about the y axis to position the Earth in its desired location along its orbit. There are a number of interesting variants of this problem such as the view from the Earth of the rest of the solar system.

5.5 Yes. Any sequence of rotations is equivalent to a single rotation about a suitably chosen axis. One way to compute this rotation matrix is to form the matrix by sequence of simple rotations, such as

$$\mathbf{R} = \mathbf{R_x}\mathbf{R_y}\mathbf{R_z}.$$

The desired axis is an eigenvector of this matrix.

5.7 The result follows from the transformation being affine. We can also take a direct approach. Consider the line determined by the points (x_1, y_1, z_1) and (x_2, y_2, z_2). Any point along can be written parametrically as $(\alpha x_1 + (1-\alpha)x_2, \alpha y_1 + (1-\alpha)y_2, \alpha z_1 + (1-\alpha)z_2)$. Consider the simple projection of this point $\frac{1}{d(\alpha z_1 + (1-\alpha)z_2)}(\alpha x_1 + (1-\alpha)x_2, \alpha y_1 + (1-\alpha)y_2)$ which is of the form $f(\alpha)(\alpha x_1 + (1-\alpha)x_2, \alpha y_1 + (1-\alpha)y_2)$. This form describes a line because the slope is constant. Note that the function $f(\alpha)$ implies that we trace out the line at a nonlinear rate as α increases from 0 to 1.

5.9 The specification used in many graphics text is of the angles the projector makes with x,z and y, z planes, i.e the angles defined by the projection of a projector by a top view and a side view.
Another approach is to specify the foreshortening of one or two sides of a cube aligned with the axes.

5.11 The CORE system used this approach. Retained objects were kept in distorted form. Any transformation to any object that was defined with other than an orthographic view transformed the distorted object and the orthographic projection of the transformed distorted object was incorrect.

5.15 If we use $\theta = \phi = 45$, we obtain the projection matrix

$$\mathbf{P} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

5.17 All the points on the projection of the point $(x.y, z)$ in the direction $d_x, d_y, d_z)$ are of the form $(x + \alpha d_x, y + \alpha d_y, z + \alpha d_z)$. Thus the shadow of the point (x, y, z) is found by determining the α for which the line intersects the plane, that is

$$ax_s + by_s + cz_s = d$$

Substituting and solving, we find

$$\alpha = \frac{d - ax - by - cz}{ad_x + bd_y + cd_z}.$$

However, what we want is a projection matrix, Using this value of α we find

$$x_s = z + \alpha d_x = \frac{x(bd_y + cd_x) - d_x(d - by - cz)}{ad_x + bd_y + cd_z}$$

with similar equations for y_s and z_s. These results can be computed by multiplying the homogeneous coordinate point $(x, y, z, 1)$ by the projection matrix

$$\mathbf{M} = \begin{bmatrix} bd_y + cd_z & -bd_x & -cd_x & -dd_x \\ -ad_y & ad_x + cd_z & -cd_y & -dd_y \\ -ad_z & -bd_z & ad_x + bd_y & -dd_z \\ 0 & 0 & 0 & ad_x + bd_y + cd_z \end{bmatrix}.$$

5.21 Suppose that the average of the two eye positions is at (x, y, z) and the viewer is looking at the origin. We could form the images using the `LookAt` function twice, that is

```
gluLookAt(x-dx/2, y, z, 0, 0, 0, 0, 1, 0);
/* draw scene here */
/* swap buffers and clear */
gluLookAt(x+dx/2, y, z, 0, 0, 0, 0, 1, 0);
/* draw scene again */
/* swap buffers and clear */
```