Assignment 1

Reading Material:

1. Chapter 1: Mathematical Review;
2. Chapter 2: Combinatorics.

Problems:

1. Consider rolling a six-sided die. Let A be the set of outcomes where the roll is an even number. Let B be the set of outcomes where the roll is greater than 3. Calculate and compare the sets on both sides of De Morgan's laws

$$(A \cup B)^{\mathrm{c}} = A^{\mathrm{c}} \cap B^{\mathrm{c}}, \qquad\qquad (A \cap B)^{\mathrm{c}} = A^{\mathrm{c}} \cup B^{\mathrm{c}}.$$

2. **De Méré's puzzle.** A six-sided die is rolled three times independently. Which is more likely: a sum of 11 or a sum of 12? (This question was posed by the French nobleman de Méré to his friend Pascal in the 17th century.)

3. Ninety students, including Joe and Jane, are to be split into three classes of equal size, and this is to be done at random. What is the probability that Joe and Jane end up in the same class?

4. Eight rooks are placed in distinct squares of an 8×8 chessboard, with all possible placements being equally likely. Find the probability that all the rooks are safe from one another, i.e., that there is no row or column with more than one rook.

5. Consider a group of n persons. A club consists of a special person from the group (the club leader) and a number (possibly zero) of additional club members.

   (a) Explain why the number of possible clubs is $n2^{n-1}$.

   (b) Find an alternative way of counting the number of possible clubs and show the identity

$$\sum_{k=1}^{n} k \binom{n}{k} = n2^{n-1}.$$

6. A parking lot contains 100 cars, k of which happen to be lemons. We select m of these cars at random and take them for a testdrive. Find the probability that n of the cars tested turn out to be lemons.