

# Assignment 5

## Reading Assignment:

1. Chapter 6: Meeting Expectations.

## Problems:

1. A prize is randomly placed in one of ten boxes, numbered from 1 to 10. You search for the prize by asking yes-no questions. Find the expected number of questions until you are sure about the location of the prize, under each of the following strategies.
  - (a) An enumeration strategy: you ask questions of the form “is it in box  $k$ ?”
  - (b) A bisection strategy: you eliminate as close to half of the remaining boxes as possible by asking questions of the form “is it a box numbered less than or equal to  $k$ ?”
2. As an advertising campaign, a chocolate factory places golden tickets in some of its candy bars, with the promise that a golden ticket is worth a trip through the chocolate factory, and all the chocolate you can eat for life. If the probability of finding a golden ticket is  $p$ , find the mean and the variance of the number of candy bars you need to eat to find a ticket.
3. **St. Petersburg paradox.** You toss independently a fair coin and you count the number of tosses until the first tail appears. If this number is  $n$ , you receive  $2^n$  dollars. What is the expected amount that you will receive? How much would you be willing to pay to play this game?
4. A total of 4 buses carrying 148 students from the same school arrives at a football stadium. The buses carry, respectively, 40, 33, 25, and 50 students. One of the students is randomly selected. Let  $X$  denote the number of students that were on the bus carrying this randomly selected student. One of the 4 bus drivers is also randomly selected. Let  $Y$  denote the number of students on her bus.
  - (a) Which of  $E[X]$  or  $E[Y]$  do you think is larger? Why?
  - (b) Compute  $E[X]$  and  $E[Y]$ .
5. An insurance company writes a policy to the effect that an amount of money  $A$  must be paid if some event  $E$  occurs within a year. If the company estimates that  $E$  will occur within a year with probability  $p$ , what should it charge the customer in order that its expected profit will be 10 percent of  $A$ .
6. A sample of 3 items is selected at random from a box containing 20 items of which 4 are defective. Find the expected number of defective items in the sample.