# Assignment 6

**Reading Assignment:**

1. Chapter 7: Multiple Discrete Random Variables.

**Problems:**

1. Two fair dice are rolled. Find the joint probability mass function of $X$ and $Y$ when

   (a) $X$ is the largest value obtained on any die and $Y$ is the sum of the values;

   (b) $X$ is the value on the first die and $Y$ is the larger of the two values;

   (c) $X$ is the smallest and $Y$ is the largest value obtained on the dice.

2. Consider a sequence of independent Bernoulli trials, each of which is a success with probability $p$. Let $X_1$ be the number of failures preceding the first success, and let $X_2$ be the number of failures between the first two successes. Find the joint mass function of $X_1$ and $X_2$.

3. Let $X$ and $Y$ be independent random variables, both being equally likely to be any of the values $1, 2, \ldots, m$. Show that

$$E[|X - Y|] = \frac{(m-1)(m+1)}{3m}.$$

4. $N$ people arrive separately to a professional dinner. Upon arrival, each person looks to see if he or she has any friends among those present. That person then either sits at the table of a friend or at an unoccupied table if none of those present is a friend. Assuming that each of the $\binom{N}{2}$ pairs of people are, independently, friends with probability $p$, find the expected number of occupied tables. *Hint*: Let $X_i$ equal 1 or 0 dependent on whether the $i$th arrival sits at a previously unoccupied table.

5. A set of 1000 cards, numbered 1 through 1000, are distributed among 1000 people. Compute the expected number of cards that are given to people whose age matches the number on the card.

6. Ten hunters are waiting for ducks to fly by. When a flock of ducks flies overhead, the hunters fire at the same time, but each chooses his target at random, independently of the others. If each hunter independently hits his target with probability 0.6, compute the expected number of ducks that are hit. Assume that the number of ducks in a flock is a Poisson random variable with mean 6.