

Assignment 7

1. Consider four independent rolls of a 6-sided die. Let X be the number of 1's and let Y be the number of 2's obtained. What is the joint PMF of X and Y ?
2. Alice passes through four traffic lights on her way to work, and each light is equally likely to be green or red, independently of the others.
 - (a) What is the PMF, the mean, and the variance of the number of red lights that Alice encounters?
 - (b) Suppose that each red light delays Alice by exactly two minutes. What is the variance of Alice's commuting time?
3. You drive to work 5 days a week for a full year (50 weeks), and with probability $p = 0.02$ you get a traffic ticket on any given day, independently of other days. Let X be the total number of tickets you get in the year.
 - (a) What is the probability that the number of tickets you get is exactly equal to the expected value of X ?
 - (b) Calculate approximately the probability in (a) using a Poisson approximation.
 - (c) Any one ticket is \$10 or \$20 or \$50 with respective probabilities 0.5, 0.3, and 0.2, and independently of other tickets. Find the mean and variance of the amount of money you pay in traffic tickets during the year.
 - (d) Suppose you don't know the probability p of getting a ticket, but you got 5 tickets during the year, and you estimate p by the sample mean

$$\hat{p} = \frac{5}{250} = 0.02.$$

What is the range of possible values of p assuming that the difference between p and the sample mean \hat{p} is within 5 times the standard deviation of the sample mean?

4. A fair die is rolled 10 times. Calculate the expected sum of the 10 rolls.
5. Suppose that A and B each randomly, and independently, choose 3 of 10 objects. Find the expected number of objects
 - (a) chosen by both A and B ;
 - (b) not chosen by either A or B ;
 - (c) chosen by exactly one of A and B .
6. A total of n balls, numbered 1 through n , are put into n urns, also numbered 1 through n in such a way that ball i is equally likely to go into any of the urns $1, 2, \dots, i$. Find
 - (a) the expected number of urns that are empty;
 - (b) the probability that none of the urns is empty.