Assignment 8

Reading Assignment:

1. Chapter 8: Continuous Random Variables.

Problems:

1. Let X be a random variable with probability density function

$$f(x) = \begin{cases} C\left(1 - x^2\right) & -1 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

   (a) What is the value of C?

   (b) What is the cumulative distribution of X?

2. A system consisting of one original unit plus a spare can function for a random amount of time X. If the density of X is given (in units of months) by

$$f(x) = \begin{cases} Cxe^{-x/2} & x > 0 \\ 0 & x \leq 0. \end{cases}$$

   What is the probability that the system functions for at least 5 months?

3. Consider the function

$$f(x) = \begin{cases} C\left(2x - x^3\right) & 0 < x < \frac{5}{2} \\ 0 & \text{otherwise.} \end{cases}$$

   Could f be a probability density function? If so, determine C.

   Repeat if $f(x)$ were given by

$$f(x) = \begin{cases} C\left(2x - x^2\right) & 0 < x < \frac{5}{2} \\ 0 & \text{otherwise.} \end{cases}$$

4. The probability density function of X, the lifetime of a certain type of electronic device (measured in hours), is given by

$$f(x) = \begin{cases} \frac{10}{x^2} & x > 10 \\ 0 & x \leq 10. \end{cases}$$

   (a) Find $\Pr(X > 20)$.

   (b) What is the cumulative distribution function of X?

   (c) What is the probability that, of 6 devices, at least 3 will function for at least 15 hours? What assumptions are you making?

5. (a) A fire station is to be located along a road of length A, $A < \infty$. If fires will occur at points uniformly chosen on $(0, A)$, where should the station be located so as to minimize the expected distance from the fire? That is, choose a so as to

$$\min \mathrm{E}[|X - a|]$$

when X is uniformly distributed over $(0, A)$.

(b) Now suppose that the road is infinite length – stretching from point 0 outward to ∞. If the distance of a fire from point 0 is exponentially distributed with rate λ, where should the fire station now be located? That is, we want to minimize $\mathrm{E}[|X - a|]$, where x is now exponential with rate λ.

6. The time (in hours) required to repair a machine is an exponentially distributed random variable with parameter $\lambda = \frac{1}{2}$. What is

(a) the probability that a repair time exceeds 2 hours;

(b) the conditional probability that a repair takes at least 10 hours, given that its duration exceeds 9 hours?