Assignment 4

Reading Assignment:

1. Chapter 5: Discrete Random Variables.

Problems:

1. We are given three coins: one has heads in both faces, the second has tails in both faces, and the third has a head in one face and a tail in the other. We choose a coin at random, toss it, and it comes heads. What is the probability that the opposite face is tails?

2. Two players take turns removing a ball from a jar that initially contains m white and n black balls. The first player to remove a white ball wins. Develop a recursive formula that allows the convenient computation of the probabilty that the starting player wins.

3. We have two jars, each initially containing an equal number of balls. We perform four successive ball exchanges. In each exchange, we pick simultaneously and at random a ball from each jar and move it to the other jar. What is the probability that at the end of the four exchanges all the balls will be in the jar where they started?

4. You just rented a large house and the realtor gave you 5 keys, one for each of the 5 doors of the house. Unfortunately, all keys look identical, so to open the front door, you try them at random.

   (a) Find the PMF of the number of trials you will need to open the door, under the following alternative assumptions: (1) after an unsuccessful trial, you mark the corresponding key, so that you never try it again, and (2) at each trial you are equally likely to choose any key.

   (b) Repeat part (a) for the case where the realtor gave you an extra duplicate key for each of the 5 doors.

5. Let X be a binomial random variable with parameters n and p. Show that its PMF can be computed by starting with $p_X(0) = (1-p)^n$, and by using the recursive formula

$$p_X(k+1) = \frac{p}{1-p} \cdot \frac{n-k}{k+1} \cdot p_X(k),$$

   where $k = 0, 1, ..., n-1$.

6. Two fair dice are rolled. Let X equal the product of the 2 dice. Compute $\Pr\{X = i\}$ for $i = 1, 2, \ldots$