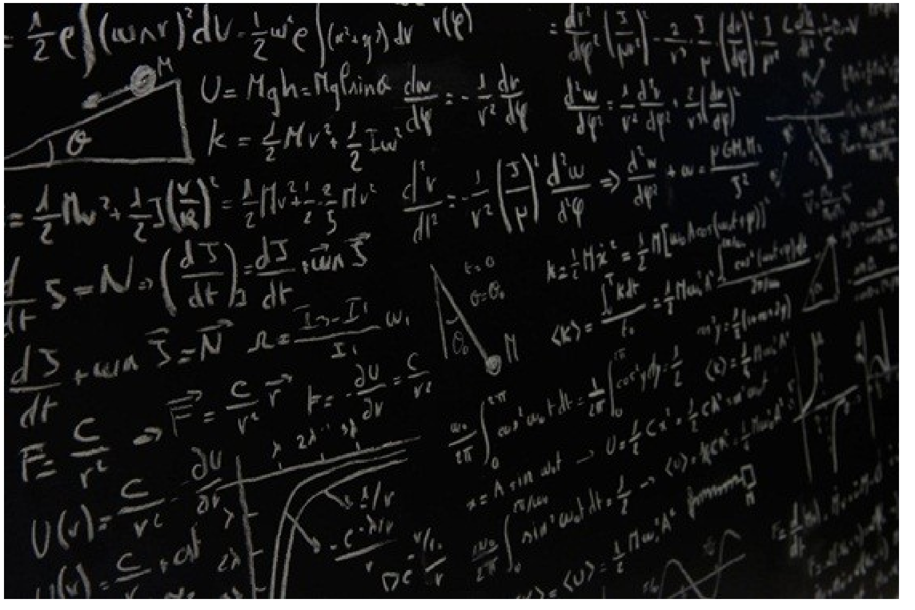
Estimation and Practical Physics Calculations



Sean Hodgman

He* BEC Group

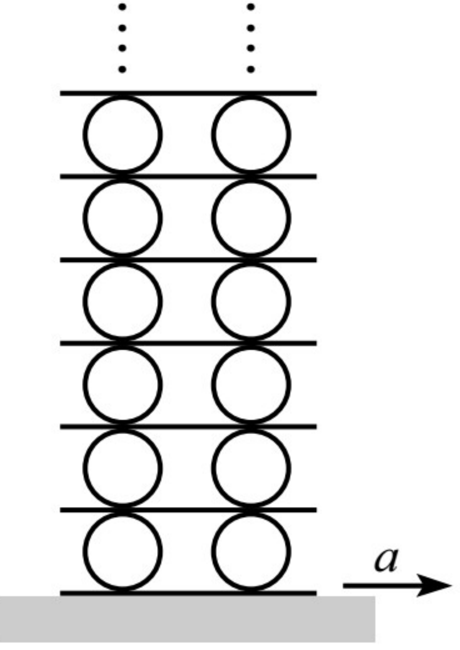
Australian National University

Canberra

sean.hodgman@anu.edu.au

A 'Typical' Physics Problem

Consider the infinitely tall system of identical massive cylinders and massless planks shown below. The moment of inertia of the cylinders is $I = MR^2=2$. There are two cylinders at each level, and the number of levels is infinite. The cylinders do not slip with respect to the planks, but the bottom plank is free to slide on a table. If you pull on the bottom plank so that it accelerates horizontally with acceleration a, what is the horizontal acceleration of the bottom row of cylinders?



'Real' Research Problems

- Actual research problems very different
- Formulation can be imprecise/inexact
- In some respects both simpler and more complicated
- Quick answers often more valuable than long in-depth calculations

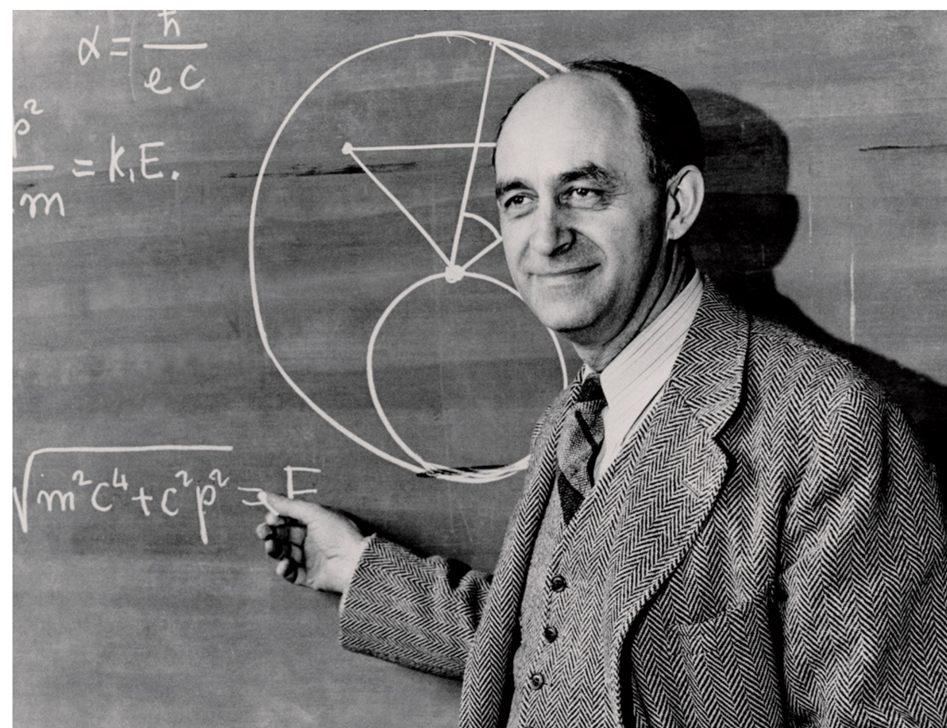
Outline

- Back-of-the-envelope calculations
- Fermi problems

- Research calculations
- Background to experimental ultracold atoms
- Lab problems

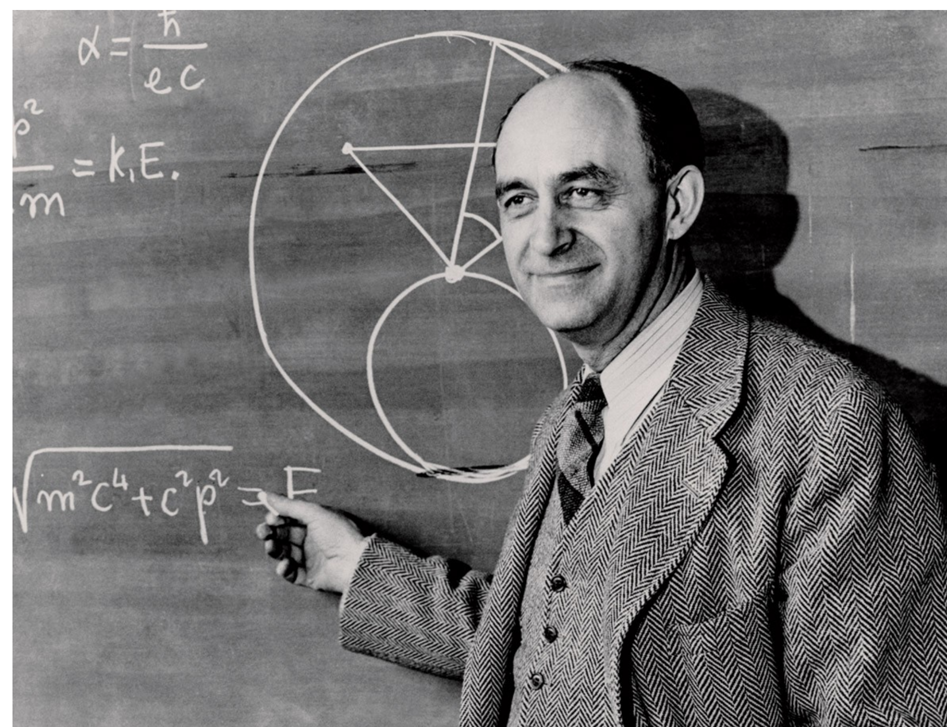
Fermi Estimation

- Often, a rough estimate is more valuable than an in depth calculation
- Simple estimates and rough approximations can quickly yield an order-of-magnitude answer
- Often called 'Fermi estimation'
- No set recipe, but several hints
- Best learnt through practice



Hint 1: Educated Guesses

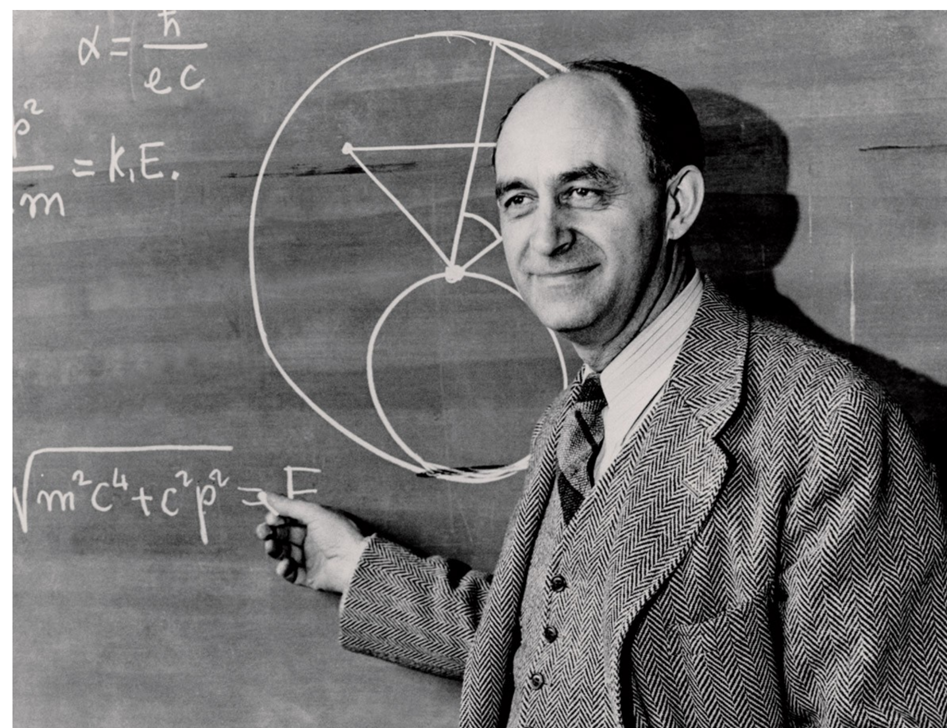
- A key part of Fermi estimation is 'educated guessing' of various quantities
- Can be applied to almost anything – not just physics
  - Hint 1: use upper and lower bounds
  - Hint 2: use known values
- Errors in multiple guesses often cancel



Examples:
How many pet dogs in Australia?
What is the area of Australia?

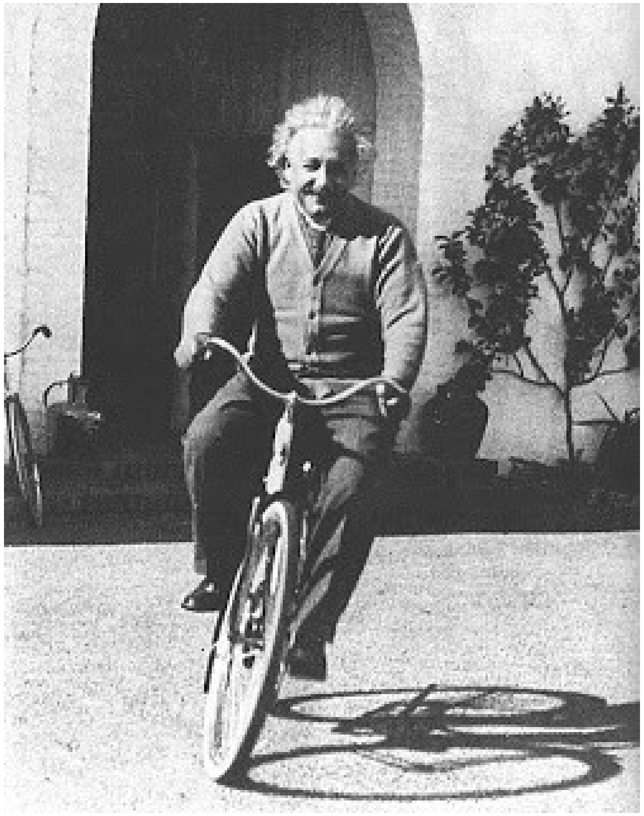
Q1: Fermi's Piano Tuners

- How many piano tuners are in Chicago?



Q1: Fermi's Piano Tuners (updated)

- ~~How many piano tuners in Chicago?~~
- How many bicycle repair shops in Canberra?



Q2: Plane Crash Fatalities

- What is the probability of dying in a passenger plane crash per flight?

Hint 2: Approximations

- Usually ignore factors of 2, π etc.

- A good trick is to round logarithmically:
  - $\leq$ 3 becomes 1
  - $>$ 3 becomes 10
  - e.g. 2 $\rightarrow$ 1, 4 $\rightarrow$ 10 etc

- Use trig approximations for small angles ($\theta <$ 15°):
  - $\sin(\theta) \sim \theta$
  - $\tan(\theta) \sim \theta$

Q3: Photons from a Light Bulb

- How many photons are emitted from a light-bulb in 1 second?

Hint 3: Simplifications

- Simplify the geometry:
  - 2D objects become circles or squares
  - 3D objects become spheres or cubes
  - Density, distribution etc are often uniform

- Use ratios/comparison with known quantities:
  - e.g. gravity between earth and moon, density of lots of objects is ~ water etc

- Discretise smooth functions

Q4: Bird Droppings

- If you went outside and lay down on your back with your mouth open, how long would you have to wait until a bird pooped in it?

- ~ 300 billion birds on earth

K. J. Gaston, T. M. Blackburn *Biodiversity & Conservation*, **6**, 615 (1997)

what if?

what-if.xkcd.com

Hint 4: Dimensional Analysis

- Can often guess form of solution from dimensionality of important parameters
- Can solve a lot of problems just by looking at scaling
- Put numbers in only at the end
- Also a good check of answer

Q6: Animals Jumping

- How does the height an animal can jump scale with its size?

Hint 5: Apply Physics

- Remember that we are (usually) solving a problem involving physics
- Use conservation laws and other basic physics to simplify problem
- Check solution for plausibility
  - Dimensional analysis
  - Violation of laws of physics
  - 'Common sense'

Q7: Fire from Moonlight

- Can you use a magnifying glass and moonlight to light a fire?

  (assume any magnifying glass is possible – ignore practicalities)

what if?

what-if.xkcd.com

Hint 6: Practice!

- Get better and more confident with practice

- Lots of sources online

  http://www.fermiquestions.com/play
  http://www.physics.umd.edu/perg/fermi/fermi.htm
  http://www.physics.uwo.ca/science_olympics/events/puzzles/fermi_questions.html
  http://www.vendian.org/envelope/dir0/fermi_questions.html
  https://what-if.xkcd.com/

- Or formulate your own from real life or films/TV etc

- Have fun!

Outline

- Back-of-the-envelope calculations
- Fermi problems
- Research calculations
- Background to experimental ultracold atoms
- Lab problems

Q8: Armageddon

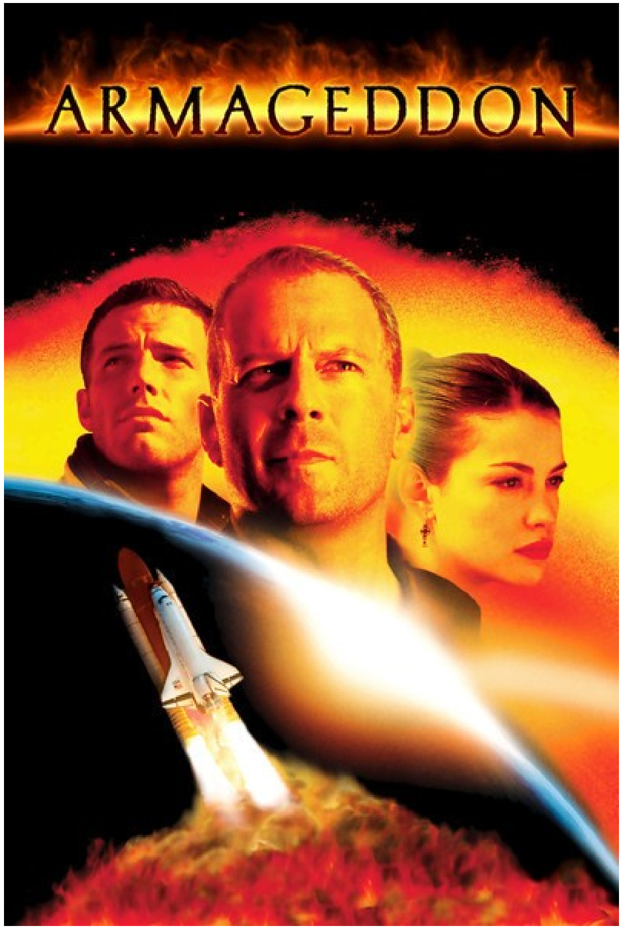
- Bruce Willis and co are sent to split an asteroid in two using a nuclear bomb to deflect it from hitting earth. Will the plan work or is earth doomed?

- Asteroid velocity = 22,500 mi/h

- ‘zero barrier’ = 2 hours

- Size of Texas=691,000km^3

- 1 ton TNT = $4.2 * 10^9$J



https://www.youtube.com/watch?v=kg_jH47u480
https://www.youtube.com/watch?v=DjwTaONIClE

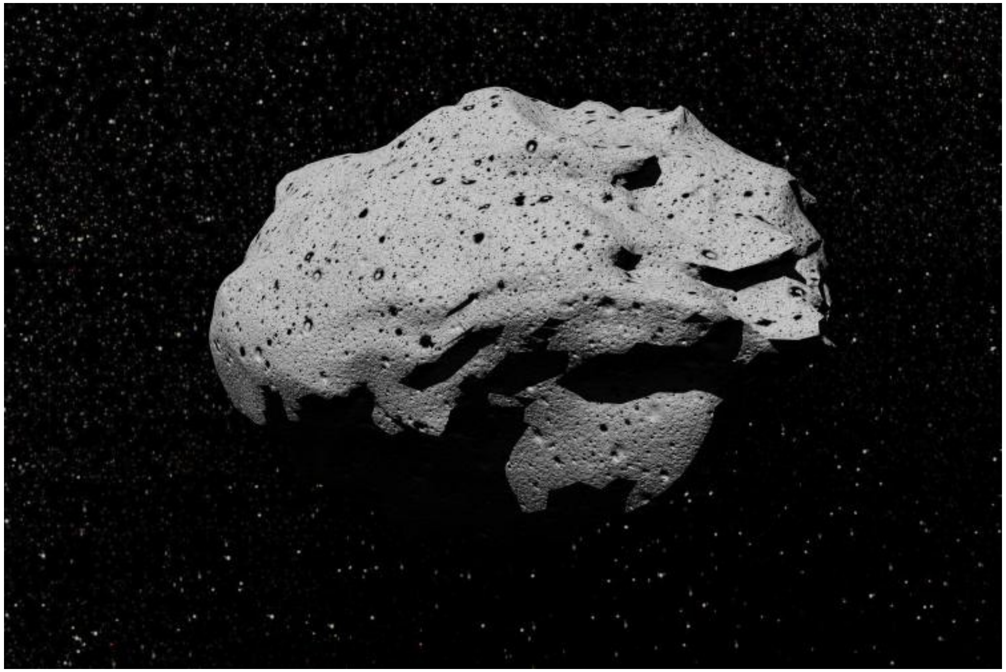
Q9: 2001: A Space Odyssey

- Is the space station in the movie spinning fast enough to generate the gravity shown?

https://www.youtube.com/watch?v=q3oHmVhviO8

Q10: Jumping off an Asteroid

- What is the maximum size of an asteroid from which you can escape by jumping?

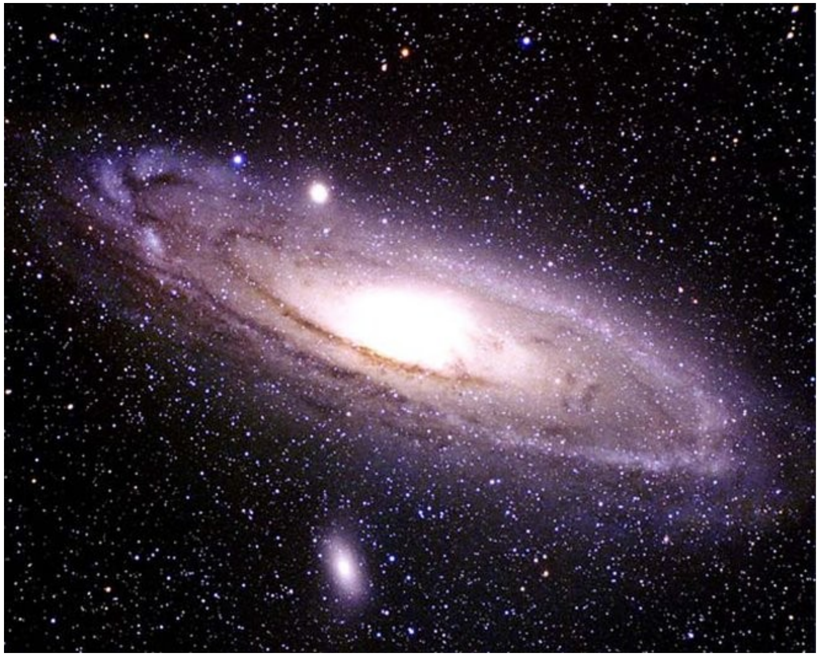


Q11: Grains of Sand

- How many grains of sand are on Kioloa beach?

Q12: Hitting a Star

- If you draw a straight line from earth, what is the probability that it will hit a star in our galaxy?

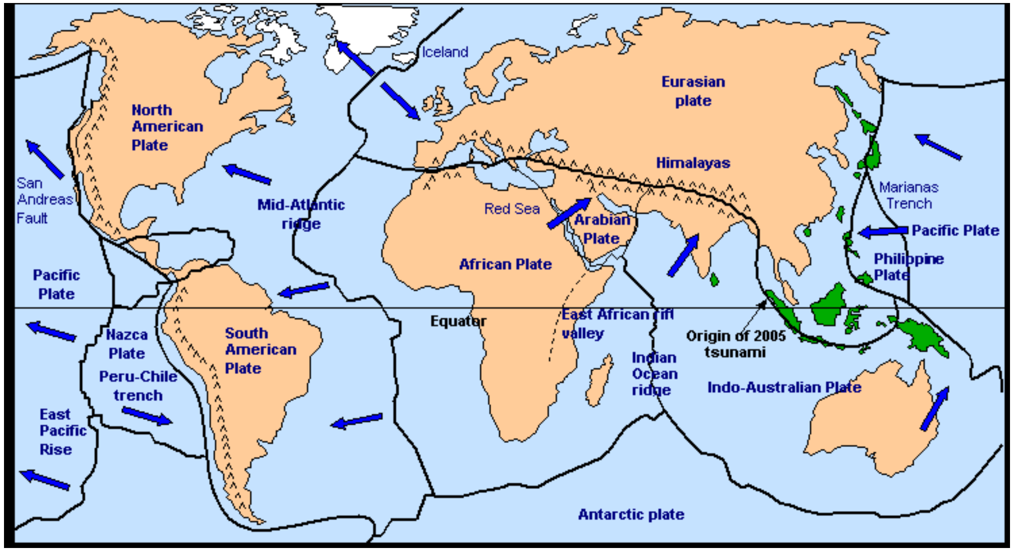


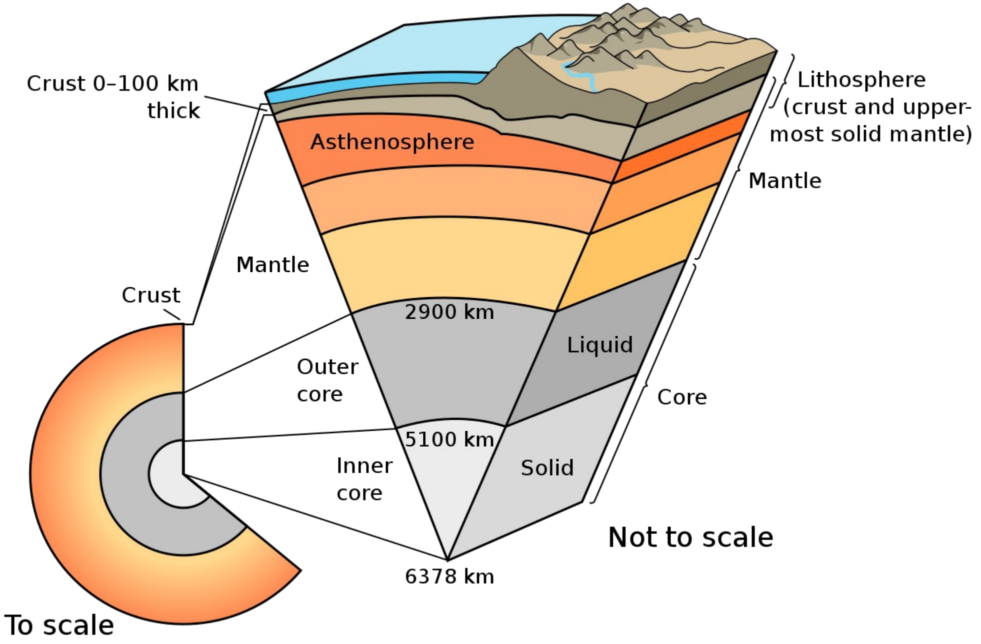
Q13: Waves from Asteroid

- A 1km diameter asteroid travelling 10km/s crashes into the middle of the Pacific Ocean. How high will the wave be that strikes the coast of Australia?

Q14: Kinetic Energy of Continental Drift

- What is the kinetic energy of the Indo-Australian continental plate (as seen from the reference frame of the adjacent plate)?





Q15: Coriolis Force in a Sink

- Is the coriolis force from the rotation of the earth sufficient to cause the water in a sink to spiral in opposite directions in the northern and southern hemispheres?

Q4: Going Bald

- How many hairs on a human head?
- Can you use this to predict how long it will take to go bald?

Are These Methods Still Relevant?

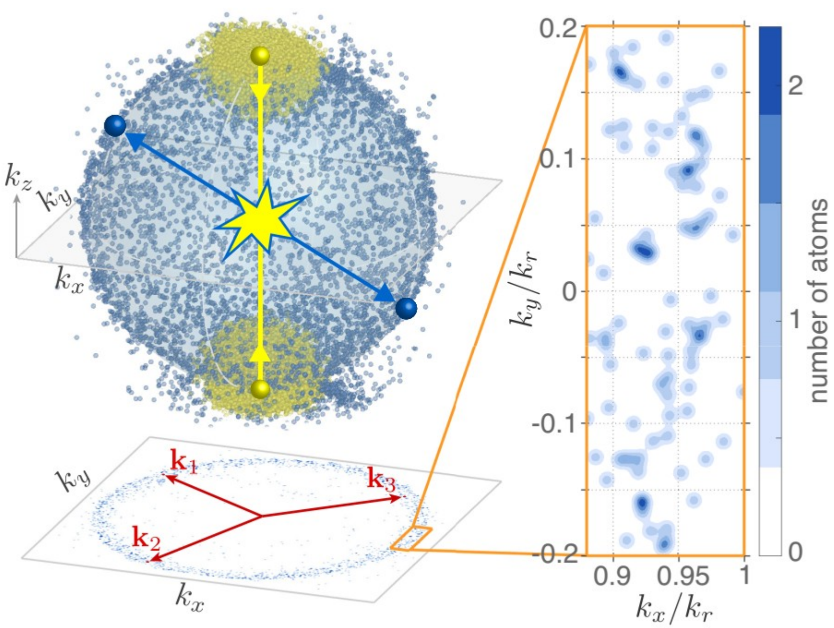
- Fermi didn't have access to calculators, google etc
- Does Fermi estimation still have a place apart from entertainment value?
- Still relevant because:
  - Don't always have access to calculators etc
  - Sometimes quicker
  - Can't look up all answers
  - Good training at formulating research problems!

Outline

- Back-of-the-envelope calculations
- Fermi problems
- Research calculations
- Background to experimental ultracold atoms
- Lab problems

'Real' Research Problems

- A quick answer often more important than a detailed model
- A lot of similarities to Fermi estimation
- Differences as well...



Start by Identifying the Physics

- Identify what physics is contained in the problem
- Formulate quick conceptual model of key aspects of problem
- Diagrams very important
- Work out what you do and don’t know

Appropriate Level of Accuracy

- In Fermi estimation, correct to an order of magnitude is normally acceptable
- Often not the case in actual research
- Within a factor of ~2 is normally a good aim – although may need to be more or less precise

Learn (some) Important Constants

- Every field of research has some constants/parameters that continually arise
- Memorising several of the important ones will save you a lot of time
- Some general ones that most physicists should know:

  g, k_B, m_p, m_e, h (or $\hbar$), q, π, etc

Keep it Simple

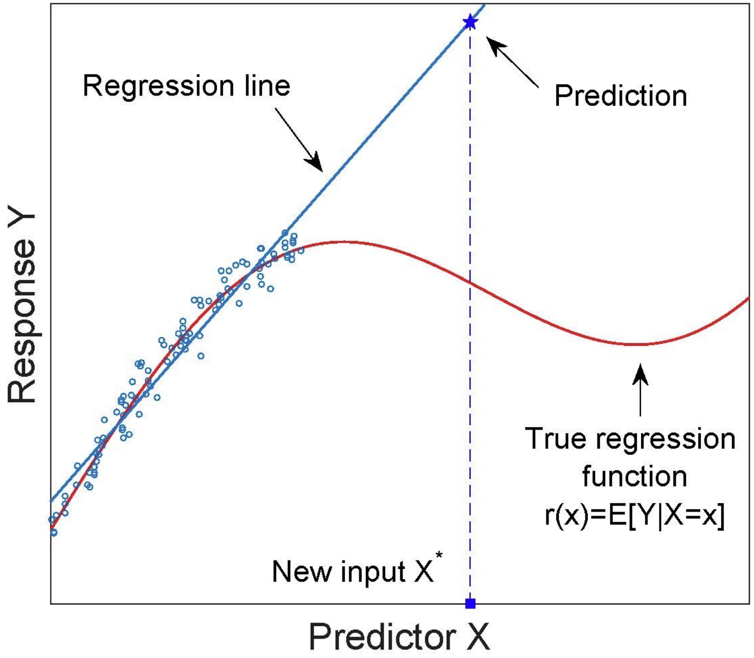
- Don’t over-complicate things
- Try to just look at the important parts of the problem
- First year physics will get you a long way!

Look Things Up & Ask Questions

- You aren’t expected to know everything
- A quick google search can save you a lot of time
- Research is done in a team for a reason!
- Ask supervisor, postdocs, senior PhD students etc about physics of problems and relevant parameters

Possible Pitfalls

- Bad estimation of parameters
- Setting up the problem incorrectly
- Non-linear behaviour/scaling



Outline

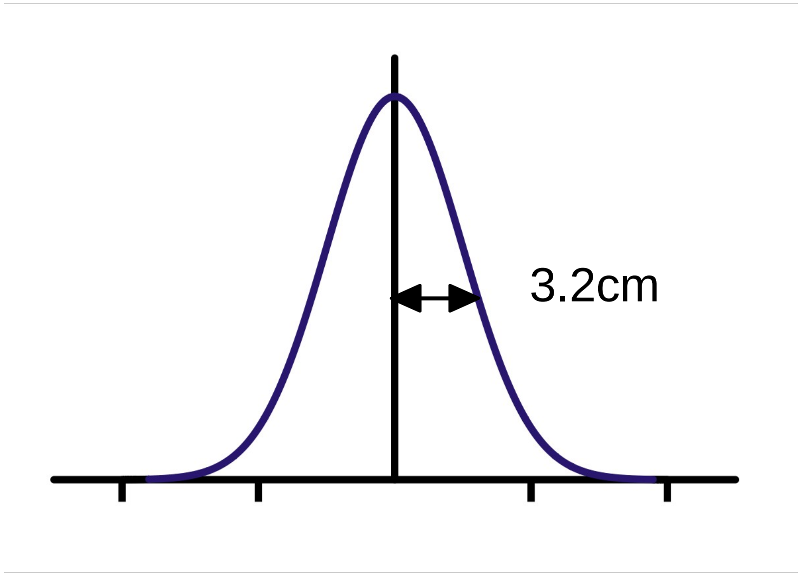
- Back-of-the-envelope calculations
- Fermi problems
- Research calculations
- Background to experimental ultracold atoms
- Lab problems

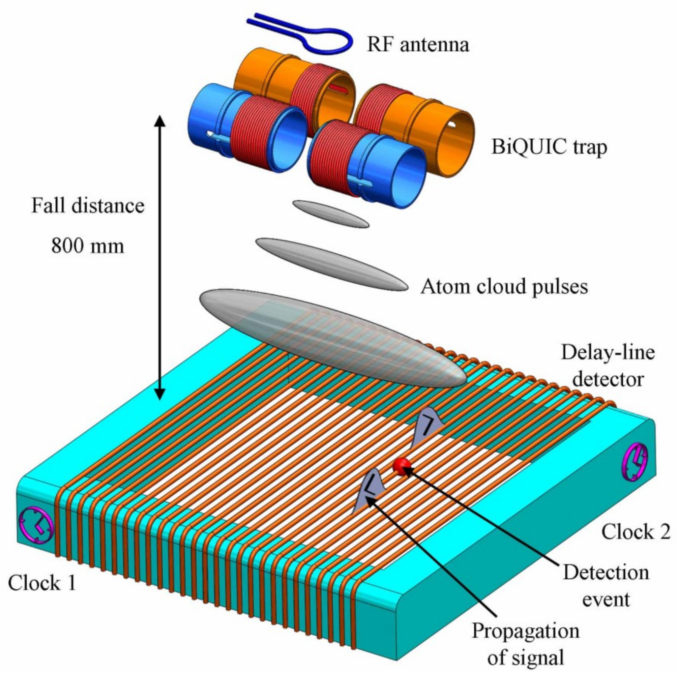
Outline

- Back-of-the-envelope calculations
- Fermi problems
- Research calculations
- Background to experimental ultracold atoms
- Lab problems

Temperature from TOF

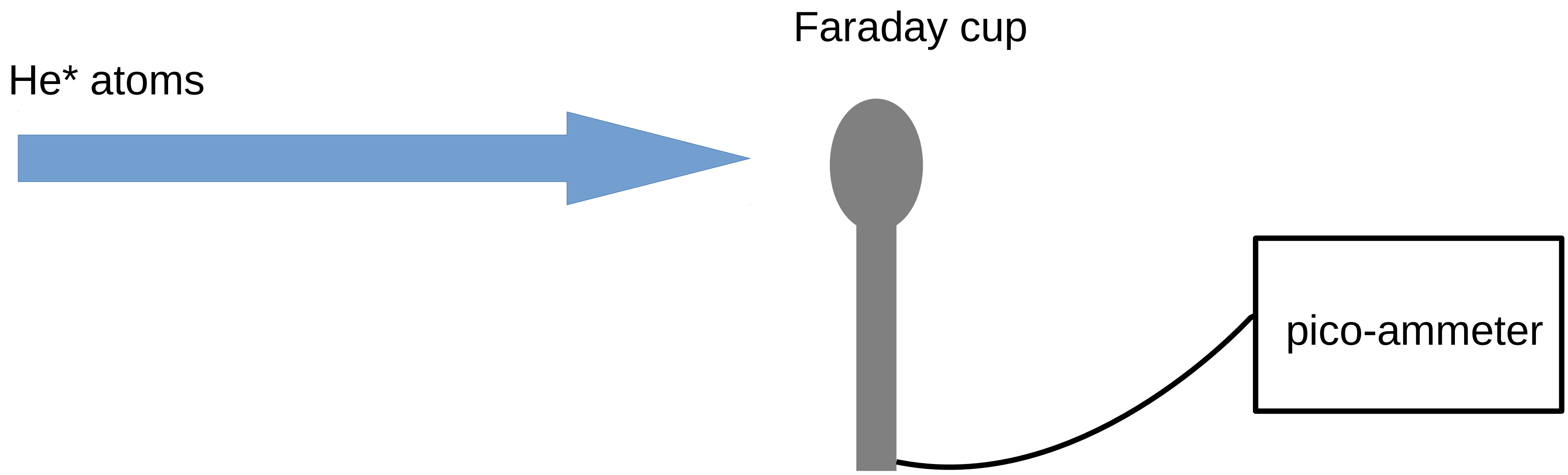
- A cloud of thermal He* atoms is released from a trap and allowed to fall 848mm, where they are detected. The cloud shape is approximately gaussian, with a measured width of 3.2cm. What was the temperature of the atoms in trap?





Faraday Cup Current

- A beam of He* atoms is incident on a ‘Faraday Cup’ (essentially a 10mm diameter piece of stainless steel). A pico-ampmeter connected to the cup measures a current of 4nA. What is the flux of He* atoms?

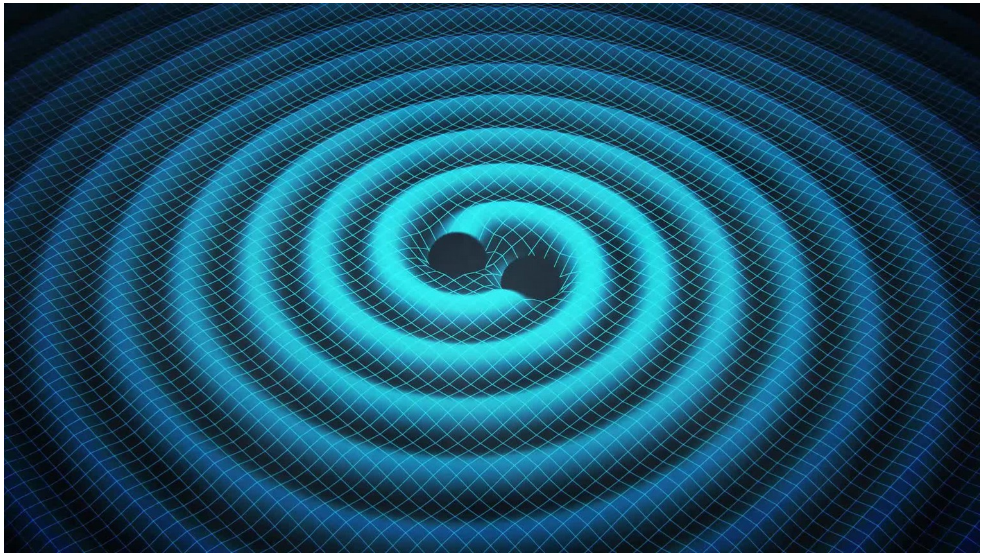


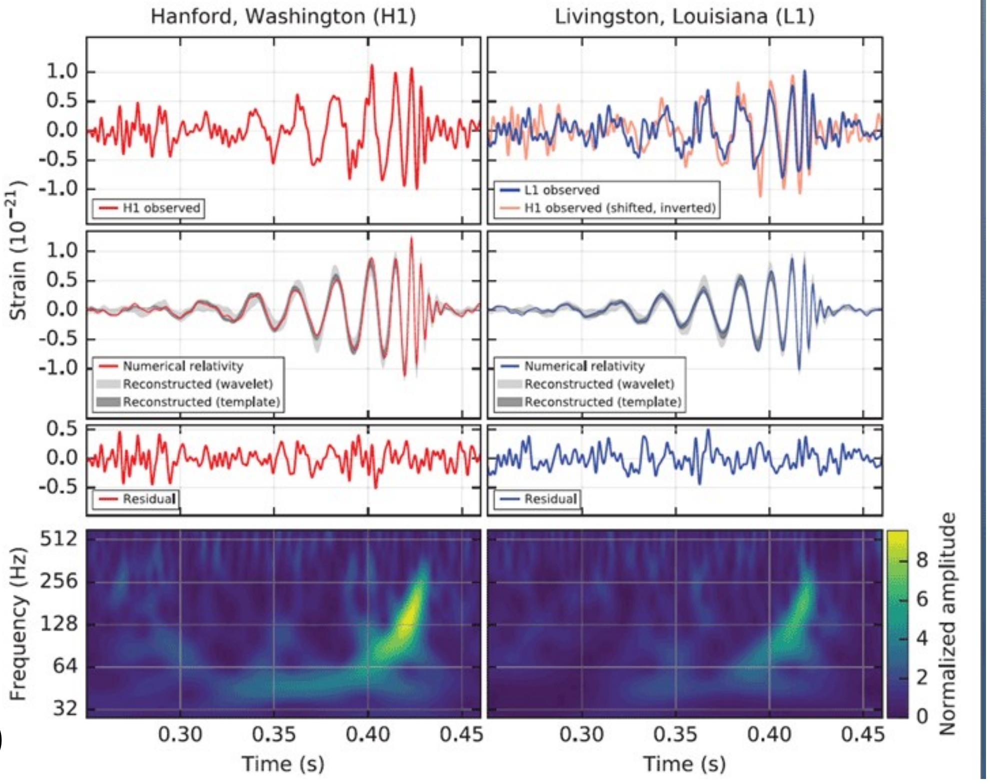
Dipole trap overlap

- A crossed dipole trap consists of overlapping two perpendicular, focused laser beams. The waists of the beams is ~100um. Both can be aligned on atoms in a magnetic trap (~3mm width). To overlap them the steering mirror on one beam has to be scanned from one side of the cloud to the other and images taken with small enough step size to not miss the trap overlap. If 10 shot averages are required for good SNR in each image and the experimental sequence has a 10s duty cycle, how long will it take to thoroughly check the alignment?

Gravity Waves

- The first gravitational wave signal was observed from the inspiral and merger of two black holes, each with ~ 30 solar masses. What frequency signal would you expect to see from the merger event?

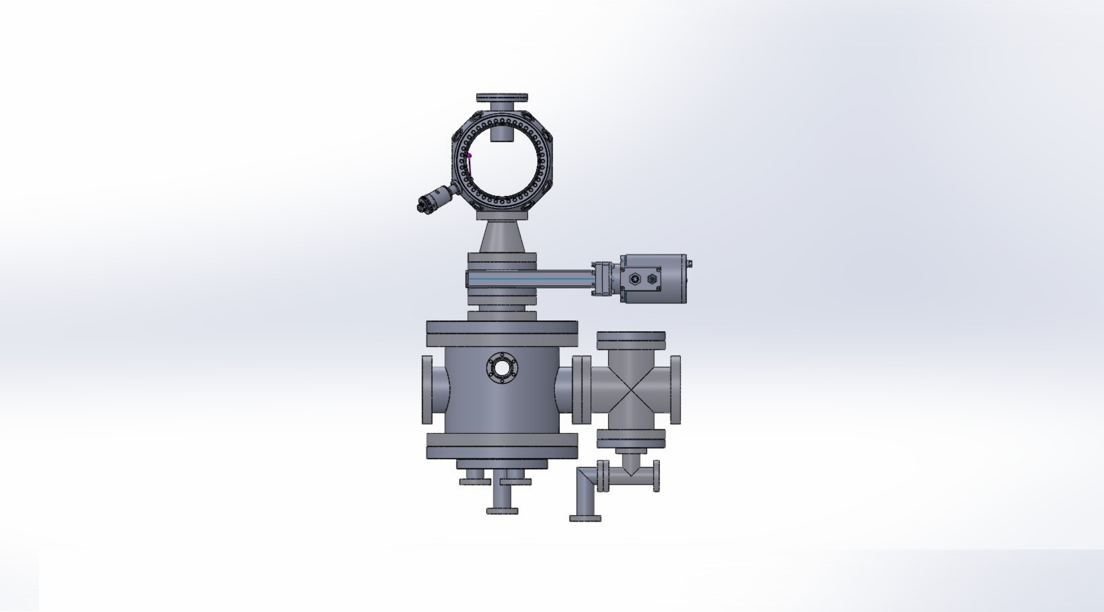


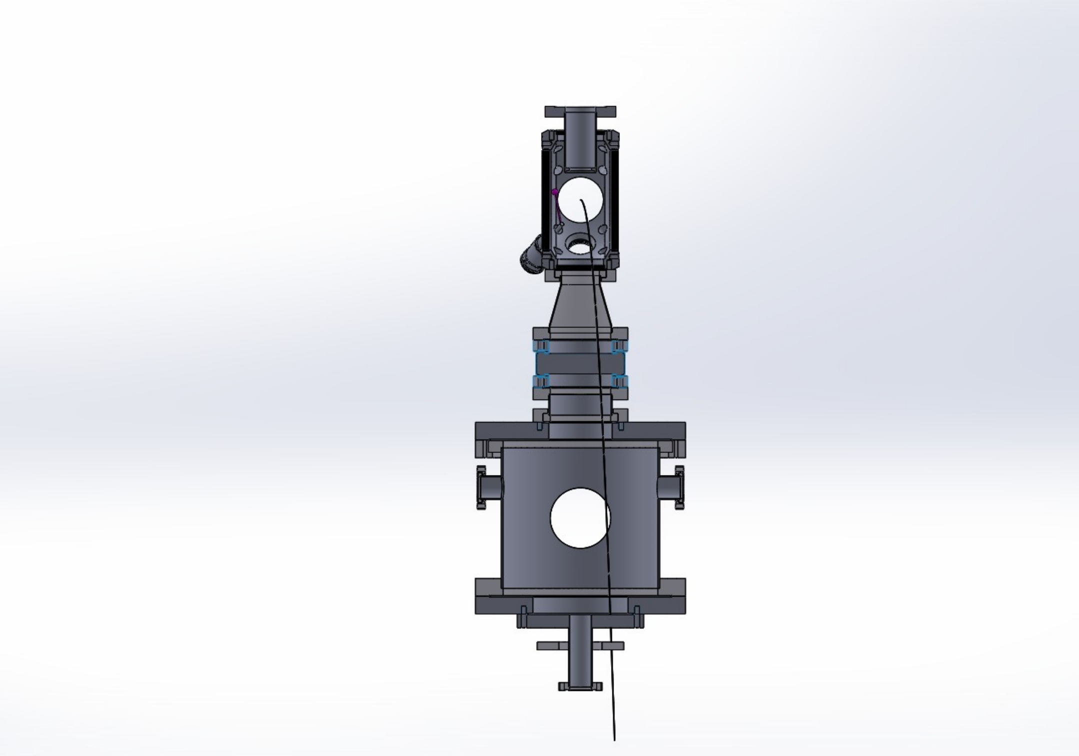


B. P. Abbott et al. *Phys. Rev. Lett.* **116,** 061102 (2016)

Bragg Peaks on Detector

- In an optical lattice experiment, a BEC is diffracted off a grating formed by an optical lattice, which is created by retro-reflecting 1550nm light. In order to be able to detect the first order diffraction peaks, what is the furthest an 80mm diameter detector can be placed below the BEC?





Laser Power on Photodiode

- An InGaAs photodiode (model: Thorlabs PDA10CS) measures a voltage of 5.21V when illuminated with 1083nm light. What is the incident laser power?

Atoms in trap from sat fluoro

- To measure the atom number in a magneto-optical trap, it is illuminated with resonant light with $I \gg I_{sat}$. The scattered 1083nm light is focused with a 1” diameter lens 200mm away onto a PDA10CS photodiode. The PD records 310mV of signal. How many atoms does this correspond to in the trap?

OHS: Asphyxiation from N_2

- If we have a 200L dewar of liquid nitrogen in a lab that is 6*4*3m, is it dangerous if the dewar catastrophically ruptures?

- What about if the dry nitrogen gas line (capable of supplying 180 psi through 1/4” tubing) were to rupture?

Gaussian Beams

- To create an optical dipole trap for a BEC, we need to focus a gaussian beam down to a 100μm waist. The beam is initially collimated with a 2mm waist. Due to space constraints the final lens can be no closer than ~500mm from the chamber, and there is only ~150mm before this for any additional lenses. Find a lens combination that will achieve this from the available lenses of focal lengths:
  50mm, 75mm, 100mm, 125mm, 150mm, 200mm, 300mm, 400mm, 500mm, -100mm, -150mm, -200mm.

Mean Free Path in a Gas

- In our final ultra-high vacuum chamber, we have a pressure of (probably) ~ 10^{-11}Torr. What is the mean free path of an atom in this chamber? How does this compare to air? Can we use this to estimate the collisional lifetime of an atom in the trap?

Conclusion

- Real research problems often very different from textbook undergraduate problems
- Fermi estimation is often useful to provide a quick estimate
- Hopefully learnt some techniques that will help with whatever PhD research project you end up studying