CS685: Data Mining
Association Rule Mining

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Mon, Thu 1030-1145 at RM101

Association Rules

- Find which itemsets are associated

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- Called association rule mining or itemset mining or basket mining
- Extremely rare that this will happen always
- Not useful if such itemsets occur rarely

Parameters of Association Rules

- For both A and B to occur, $A \cup B$ must occur
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- Two thresholds or parameters
- Support: A and B should occur in at least s (ratio of) transactions

$$P(A, B) = \frac{|A \cup B|}{|T|} \geq s$$

- Confidence: If A occurs, B should occur in at least c (ratio of) transactions

$$P(B|A) = \frac{|A \cup B|}{|A|} \geq c$$

Example

Transaction Id	Itemsets
1	A, C, D
2	B, C, E
3	A, B, C, E
4	B, E

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Rule	Support	Confidence
$B \implies E$	0.75	1.00
$C \implies E$		

Example

Transaction Id	Itemsets
1	A, C, D
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$B \implies E$	0.75	1.00
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$B, C \implies E$		

Example

Transaction Id	Itemsets
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3	A, B, C, E
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$B, C \implies E$	0.50	1.00
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$E \implies B, C$	0.50	0.67
$A \implies D$	0.25	0.50
$D \implies A$		

Example

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D $\implies$ A	0.25	1.00

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- Strong rule: An association rule whose confidence is more than or equal to the confidence threshold

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- Strong rule: An association rule whose confidence is more than or equal to the confidence threshold
- Weak rule: An association rule whose confidence is less than the confidence threshold

Finding Association Rules

- Mining association rules require two steps
  - Finding frequent itemsets
  - Generating strong association rules

Finding Association Rules

- Mining association rules require two steps
  - Finding frequent itemsets
  - Generating strong association rules
- The first step is more time-consuming

Brute-force Algorithm

- Generate a candidate itemset
- Test its support
- If frequent, accept
- Else, throw away

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- Generate a candidate itemset
- Test its support
- If frequent, accept
- Else, throw away
- Total number of possible itemsets is $2^n - 1$
- Checking each itemset requires scanning the entire transaction database
- Too impractical

Apriori Principle

- Candidate-generation-and-test paradigm
- Apriori principle: If an itemset is frequent, all its subsets must also be frequent
- Conversely, if an itemset X is infrequent, all its supersets are also infrequent
- This is an *anti-monotonic* property: if a set fails, its supersets fail as well

Apriori Algorithm

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- Candidate itemsets of length k is C_k
- Frequent itemsets of length $k-1$ is F_{k-1}
- Join step: $C_k = F_{k-1} \bowtie F_{k-1}$
  - Join two candidates whose $k-2$ items are common
  - Perform subset checking
- Prune step: $F_k = \{I \in C_k : |I| \geq s\}$
  - Retain only frequent itemsets

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  - Retain only frequent itemsets
- Requires k database scans for itemsets up to length k

Apriori Example

Transaction Id	Itemsets
0	1, 2, 5
1	2, 4
2	2, 3
3	1, 2, 4
4	1, 3
5	2, 3
6	1, 3
7	1, 2, 3, 5
8	1, 2, 3
9	6

Support threshold $s = 2$

Apriori example (contd.)

Candidate set C_1

Itemset	Frequency
1	6
2	7
3	6
4	2
5	2
6	1

$\rightarrow$

Apriori example (contd.)

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Itemset	Frequency
1	6
2	7
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Frequent set F_1

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Frequent set F_1

Itemset	Frequency
1	6
2	7
3	6
4	2
5	2

$\rightarrow$

Candidate set C_2

Itemset	Frequency
1, 2	4
1, 3	4
1, 4	1
1, 5	2
2, 3	4
2, 4	2
2, 5	2
3, 4	0
3, 5	1
4, 5	0

$\rightarrow$

Apriori example (contd.)

Candidate set C_1

Itemset	Frequency
1	6
2	7
3	6
4	2
5	2
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→

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5	2

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Candidate set C_2

Itemset	Frequency
1, 2	4
1, 3	4
1, 4	1
1, 5	2
2, 3	4
2, 4	2
2, 5	2
3, 4	0
3, 5	1
4, 5	0

→

Frequent set F_2

Itemset	Frequency
1, 2	4
1, 3	4
1, 5	2
2, 3	4
2, 4	2
2, 5	2

→

Apriori example (contd.)

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Itemset	Frequency
1	6
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Itemset	Frequency
1, 2	4
1, 3	4
1, 4	1
1, 5	2
2, 3	4
2, 4	2
2, 5	2
3, 4	0
3, 5	1
4, 5	0

$\rightarrow$

Frequent set F_2

Itemset	Frequency
1, 2	4
1, 3	4
1, 5	2
2, 3	4
2, 4	2
2, 5	2

$\rightarrow$

Candidate set C_3

Itemset	Frequency
1, 2, 3	2
1, 2, 5	2
(1, 3, 5)	subset
(2, 3, 4)	subset
(2, 3, 5)	subset
(2, 4, 5)	subset

$\rightarrow$

Apriori example (contd.)

Candidate set C_1

Itemset	Frequency
1	6
2	7
3	6
4	2
5	2
6	1

$\rightarrow$

Frequent set F_1

Itemset	Frequency
1	6
2	7
3	6
4	2
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$\rightarrow$

Candidate set C_2

Itemset	Frequency
1, 2	4
1, 3	4
1, 4	1
1, 5	2
2, 3	4
2, 4	2
2, 5	2
3, 4	0
3, 5	1
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Frequent set F_2

Itemset	Frequency
1, 2	4
1, 3	4
1, 5	2
2, 3	4
2, 4	2
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Candidate set C_3

Itemset	Frequency
1, 2, 3	2
1, 2, 5	2
(1, 3, 5)	subset
(2, 3, 4)	subset
(2, 3, 5)	subset
(2, 4, 5)	subset

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Frequent set F_3

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Apriori example (contd.)

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Itemset	Frequency
1	6
2	7
3	6
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5	2
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Frequent set F_1

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1	6
2	7
3	6
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Candidate set C_2

Itemset	Frequency
1, 2	4
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Itemset	Frequency
1, 2	4
1, 3	4
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Itemset	Frequency
1, 2, 3	2
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(1, 3, 5)	subset
(2, 3, 4)	subset
(2, 3, 5)	subset
(2, 4, 5)	subset

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Frequent set F_3

Itemset	Frequency
1, 2, 3	2
1, 2, 5	2

Candidate set C_4

Itemset	Frequency
(1, 2, 3, 5)	subset

Partitioning

- *Transaction-wise partitioning*
  - Partition transactions into different sets
  - Find frequent and infrequent itemsets in each partition with support threshold s' (according to ratio of transactions in each partition)
    - For two equal partitions, $s' = s/2$
  - Report all itemsets that are frequent in all partitions
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  - Report all itemsets that are frequent in all partitions
  - Prune all itemsets that are infrequent in all partitions
- *Item-wise partitioning*
  - Partition items into different sets
  - Find frequent itemsets in each partition
  - Join only these frequent itemsets to form global candidates

FP-Growth

- Frequent pattern (FP)-growth
- Compact representation of entire transaction database as a tree
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- Re-arranges items in every transaction in *flist* order

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- Root is “null”
- Nodes are items with corresponding count
- Each transaction is added as a path in the tree
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- Nodes of same item are linked using node links
- Two database scans

FP-Tree Example

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Support threshold $s = 2$

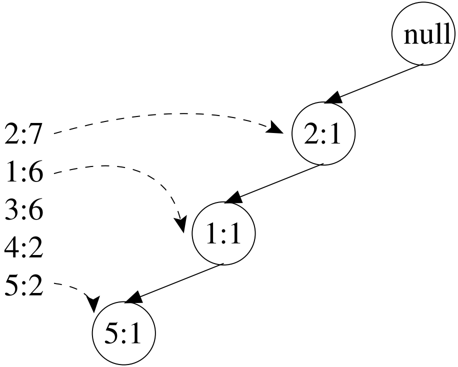
$\rightarrow$

Flist order of items

Item	Frequency
2	7
1	6
3	6
4	2
5	2

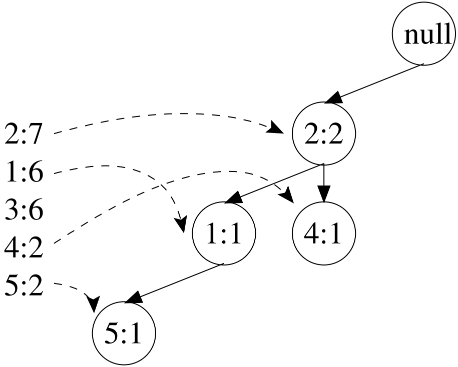
FP-Tree Construction

- Adding transaction 0: 2, 1, 5



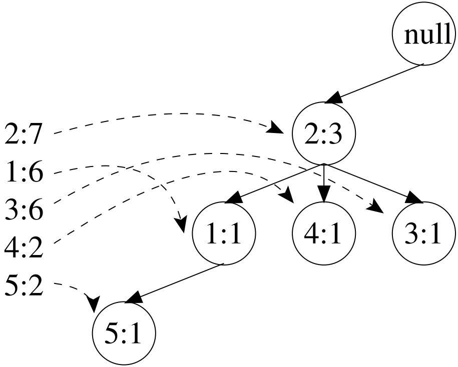
FP-Tree Construction (contd.)

- Adding transaction 1: 2, 4



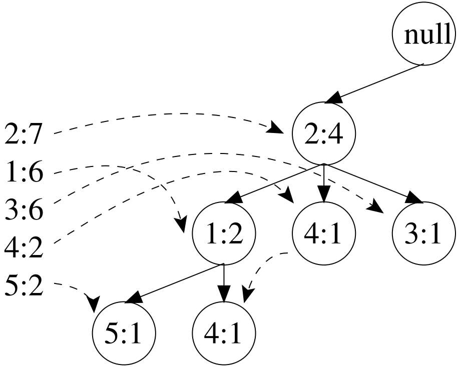
FP-Tree Construction (contd.)

- Adding transaction 2: 2, 3



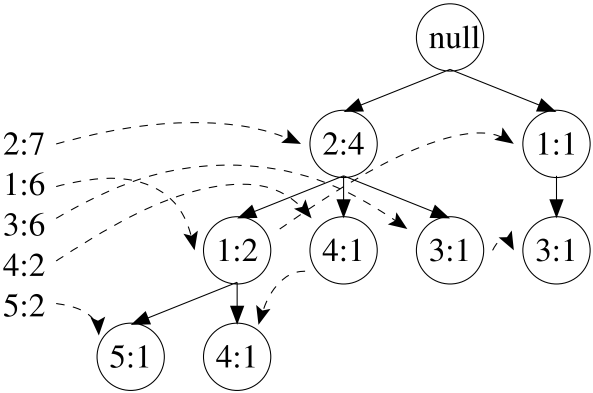
FP-Tree Construction (contd.)

- Adding transaction 3: 2, 1, 4



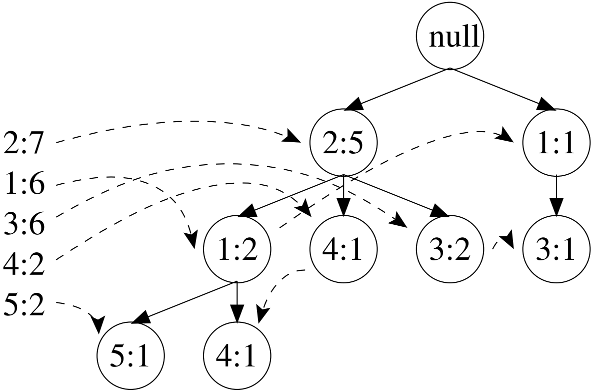
FP-Tree Construction (contd.)

- Adding transaction 4: 1, 3



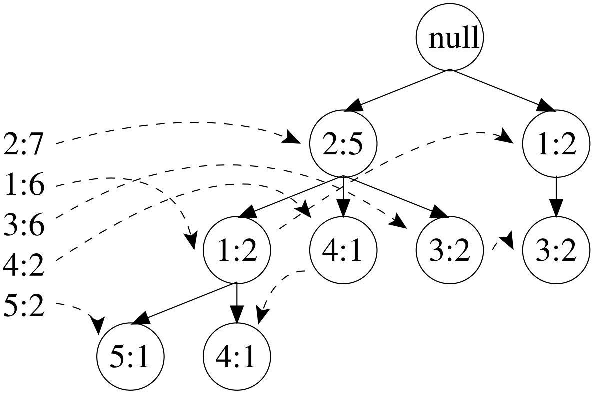
FP-Tree Construction (contd.)

- Adding transaction 5: 2, 3



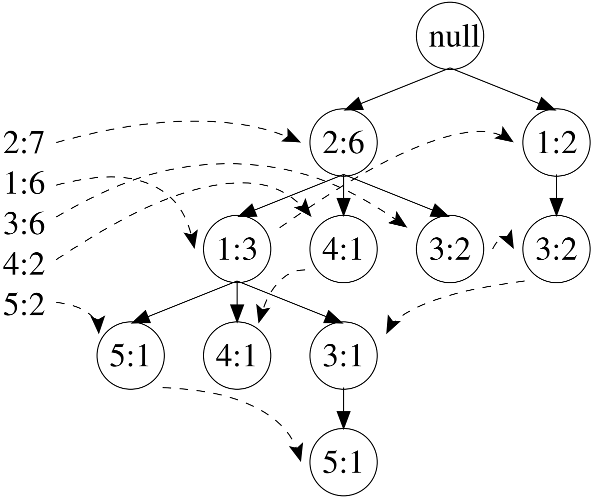
FP-Tree Construction (contd.)

- Adding transaction 6: 1, 3



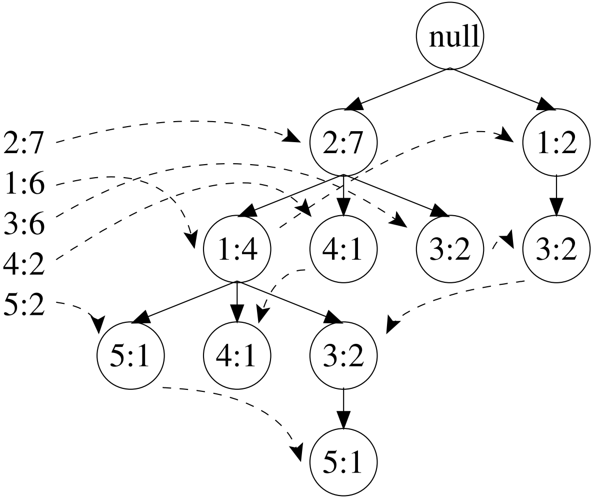
FP-Tree Construction (contd.)

- Adding transaction 7: 2, 1, 3, 5



FP-Tree Construction (contd.)

- Adding transaction 8: 2, 1, 3



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- *Projects* its paths from the base tree
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- A new FP-tree is built with only these paths (equivalently, transactions) with x removed
- This new FP-tree is *recursively* mined to find frequent patterns
- All such frequent patterns are appended with x and returned

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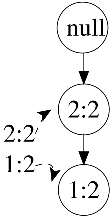
- *Projects* its paths from the base tree
- x is the suffix in all such paths
- A new FP-tree is built with only these paths (equivalently, transactions) with x removed
- This new FP-tree is *recursively* mined to find frequent patterns
- All such frequent patterns are appended with x and returned
- The item with the next lowest count is continued with

FP-Tree Mining Example

- For the least frequent item: 5
- Two prefix paths found by traversing node links are (2, 1): 1 and (2, 1, 3): 1
- This forms the conditional pattern base
- 3 is discarded as its support $(= 1)$ is less than threshold
- From conditional pattern base, conditional FP-tree is then constructed



FP-Tree Mining Example

- For the least frequent item: 5
- Two prefix paths found by traversing node links are (2, 1): 1 and (2, 1, 3): 1
- This forms the conditional pattern base
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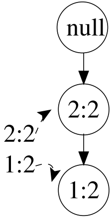

- Frequent patterns found are (1, 5): 2, (2, 1, 5): 2 and (2, 5): 2

FP-Tree Mining Example (contd.)

- For the next least frequent item: 4
- Two prefix paths found by traversing node links are (2, 1): 1 and (2): 1
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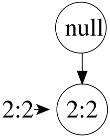

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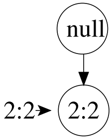

- Frequent patterns found are (2, 4): 2

FP-Tree Mining Example (contd.)

- For the next least frequent item: 3
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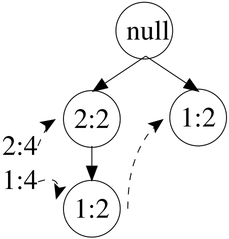

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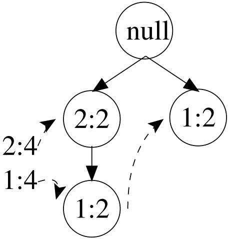

- Frequent patterns found are (1, 3): 4, (2, 1, 3): 2 and (2, 3): 4

FP-Tree Mining Example (contd.)

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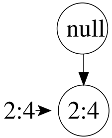

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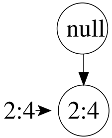

- Frequent patterns found are (2, 1): 4

FP-Tree Mining Example (contd.)

- For the most frequent item: 2
- Nothing needs to be done
- Assumption is that all 1-itemsets are already returned

Projected and Residual Databases

- Consider the item with the largest support, say x
- x partitions transactions into two parts
- Transactions containing x form the projected database P_x of x
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- Consider any (in-)frequent itemset I
  - If $x \in I$, then it will be (in-)frequent in P_x as well
  - If $x \notin I$, then it will be (in-)frequent in R_x as well
    - Frequency of I does not change in R_x

H-Mine

- H-mine is a partitioning-based algorithm
- It first sorts the items in *flist* order
- From each item, a pointer is linked to the first transaction that contain this item as the first in flist order
- All subsequent transactions of the same nature are chained
- Following the chain produces the projected database for that item
- The frequent itemsets are mined recursively then

Mining Closed and Maximally Frequent Itemsets

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- If any subset has same support, remove that subset
- Apriori may be run in reverse direction, starting with all items and then generating subsets as candidates
- A single support threshold across all itemset lengths may not be useful
- Chances of itemsets with larger length occurring are less
- MLMS model: Multiple Length Minimum Support
- Apriori works again
- If support at lesser length is smaller, e.g., $s_k < s_{k+1}$
  - All k-length subsets of frequent itemsets of length $k+1$ are frequent
  - Conversely, if an itemset is pruned at length k, all its supersets of length $k+1$ will be infrequent

Are Strong Association Rules Always Good?

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- Support is 0.4 and confidence is 0.67
- However, support of 2 itself is 0.7
- When there is no influence, 2 occurs more frequently than when 3 is there
- The effect of 3 is thus *negative* on 2
- Just support and confidence thresholds are, therefore, not enough

Lift

- A correlation measure lift
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- Lift of the rule $3 \implies 2$ is $0.67/0.7 = 0.95$
- Thus, 3 and 2 are negatively correlated

Probabilistic Association Rule Mining

- Occurrence of an item in a transaction is not just presence or absence
- It is present with a probability $p \in [0, 1]$
- Applications
  - Medical: a patient may have cancer with 70% chance, hepatitis with 10% chance, etc.

Transaction id	Item A	Item B	Item C	Item D
0	0.9	0.8	0.0	0.2
1	0.7	0.7	1.0	0.3
2	0.2	0.5	0.9	0.5

- Support of 1-itemsets can be found by just adding the columns
- Support of larger itemsets can be found by adding the products of the corresponding probabilities
  - Support of (A) is $0.9 + 0.7 + 0.2 = 1.8$
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