

Tutorial 12

Elliptic PDE

1. Determine steady state temperature in a square plate of length 3.0 cm for the following conditions.

(a) All the edges are kept at 100°C except the top edge, which is at 0°C.

(b) Same as above, except that the bottom edge is insulated.

Assume that there is no gain or loss of heat from the surface of the plate. Estimate temperature at nodes spaced 1.0 cm apart by applying Liebmann's scheme. Perform iterations till the approximate relative error is less than 1.0 %.

Parabolic PDE

2. Solve the partial differential equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

subject to the following conditions

$$\text{IC: } u(x, 0) = 0$$

$$\text{BCs: } u(0, t) = 0; u(1, t) = t$$

Determine the value of u at $x = 0.5$ and $t = 0.125$ by

- (a) Forward time central space explicit scheme with $\Delta x = 0.5$ and $\Delta t = 0.0625$.
- (b) DuFort-Frankel scheme with $\Delta x = 0.5$ and $\Delta t = 0.0625$.
- (c) Crank-Nicolson scheme with $\Delta x = 0.25$ and $\Delta t = 0.125$.

If the true value of $u(0.5, 0.125)$ is 0.01878, calculate percentage errors in the three estimates.