

Q1(a)

(i)

From Left to right direction.

1. $\neg(p \vee q)$ Premise
—————B1 opens—————
2. p Assumption
3. $p \vee q$ $\vee i$ 1, 2
4. \perp $\neg e$ 1, 3
—————B1 closes—————
5. $\neg p$ $\neg i$ 2-4
—————B2 opens—————
6. q Assumption
7. $p \vee q$ $\vee i$ 2, 6
8. \perp $\neg e$ 1, 7
—————B2 closes—————
9. $\neg q$ $\neg i$ 6-8
10. $\neg p \wedge \neg q$ $\wedge i$ 5, 9

From Right to left direction.

1. $\neg p \wedge \neg q$ Premise
—————B1 opens—————
2. $p \vee q$ Assumption
—————B2 opens—————
3. p Assumption
4. $\neg p$ $\wedge e$ 1
5. \perp $\neg e$ 3,4
—————B2 closes—————
—————B3 opens—————
6. q Assumption
7. $\neg q$ $\wedge e$ 1
8. \perp $\neg e$ 6,7
—————B3 closes—————
9. \perp $\vee e$ 2, 3-5, 6-8
—————B1 closes—————
10. $\neg(p \vee q)$ $\neg i$ 2-9

(ii) From Left to right direction.

1. $\neg(p \wedge q)$ premise
 $\text{-----B1 opens-----}$
2. $\neg(\neg p \vee \neg q)$ Assumption
 $\text{-----B2 opens-----}$
3. $\neg p$ Assumption
4. $\neg p \vee \neg q$ $\vee i$ 3
5. \perp $\neg e$ 2,4
 $\text{-----B2 closes-----}$
6. $\neg\neg p$ $\neg i$ 3-5
7. p $\neg\neg e$ 6
 $\text{-----B3 opens-----}$
8. $\neg q$ Assumption
9. $\neg p \vee \neg q$ $\vee i$ 8
10. \perp $\neg e$ 2,9
 $\text{-----B3 closes-----}$
11. $\neg\neg q$ $\neg i$ 8-10
12. q $\neg\neg e$ 11
13. $p \wedge q$ $\wedge i$ 7,12
14. \perp $\neg e$ 1,13
 $\text{-----B1 closes-----}$
15. $\neg\neg(\neg p \vee \neg q)$ $\neg i$ 2-14
16. $\neg p \vee \neg q$ $\neg\neg e$ 15

From Right to left direction.

1. $\neg p \vee \neg q$ premise
 $\text{-----B1 opens-----}$
2. $p \wedge q$ Assumption
 $\text{-----B2 opens-----}$
3. $\neg p$ Assumption
4. p $\wedge e$ 2
5. \perp $\neg e$ 3,4
 $\text{-----B2 closes-----}$
 $\text{-----B3 opens-----}$
6. $\neg q$ Assumption
7. q $\wedge e$ 2
8. \perp $\neg e$ 6,7
 $\text{-----B2 closes-----}$
9. \perp $\vee e$ 1, 3-5, 6-8
 $\text{-----B1 closes-----}$
10. $\neg(p \wedge q)$ $\neg i$ 2-9

(iii)

1. $\neg q \rightarrow \neg p$ premise
—————B1 opens—————
2. p Assumption
—————B2 opens—————
3. $\neg q$ Assumption
4. $\neg p$ MP 1,3
5. \perp $\neg e$ 2,4
—————B2 closes—————
6. $\neg\neg q$ $\neg i$ 3-5
7. q $\neg\neg e$ 6
—————B1 closes—————
8. $p \rightarrow q$ $\rightarrow i$ 2-7

(b)

(i)

1. $\neg(p \rightarrow q)$ premise
—————B1 opens—————
2. q Assumption
—————B2 opens—————
3. p Assumption
4. q Copy 2
—————B2 closes—————
5. $p \rightarrow q \rightarrow i$ 3-4
6. \perp $\neg e$ 1,5
7. p $\perp e$ 6
—————B1 closes—————
8. $q \rightarrow p \rightarrow i$ 2-7

(ii)

1. $(p \rightarrow q) \rightarrow q$ premise
—————B1 opens—————
2. $q \rightarrow p$ Assumption
—————B2 opens—————
3. $\neg p$ Assumption
—————B3 opens—————
4. p Assumption
5. \perp $\neg e$ 3,4
6. q $\perp e$ 5
—————B3 closes—————
7. $p \rightarrow q \rightarrow i$ 4-6
8. q MP 1,7
9. p MP 2,8
10. \perp $\neg e$ 3,9
—————B2 closes—————
11. p PBC 3-10
—————B1 closes—————
12. $(q \rightarrow p) \rightarrow p \rightarrow i$ 2-11