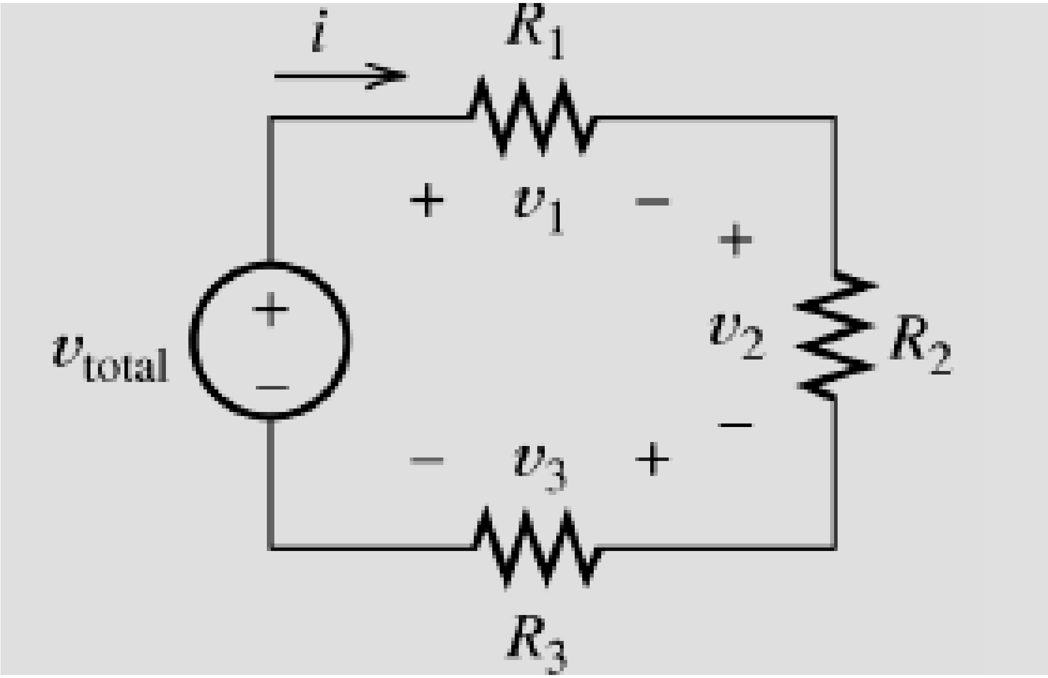
Voltage division

A voltage applied to resistors connected in series will be divided among them

$$i = \frac{v_{total}}{R_1 + R_2 + R_3}$$

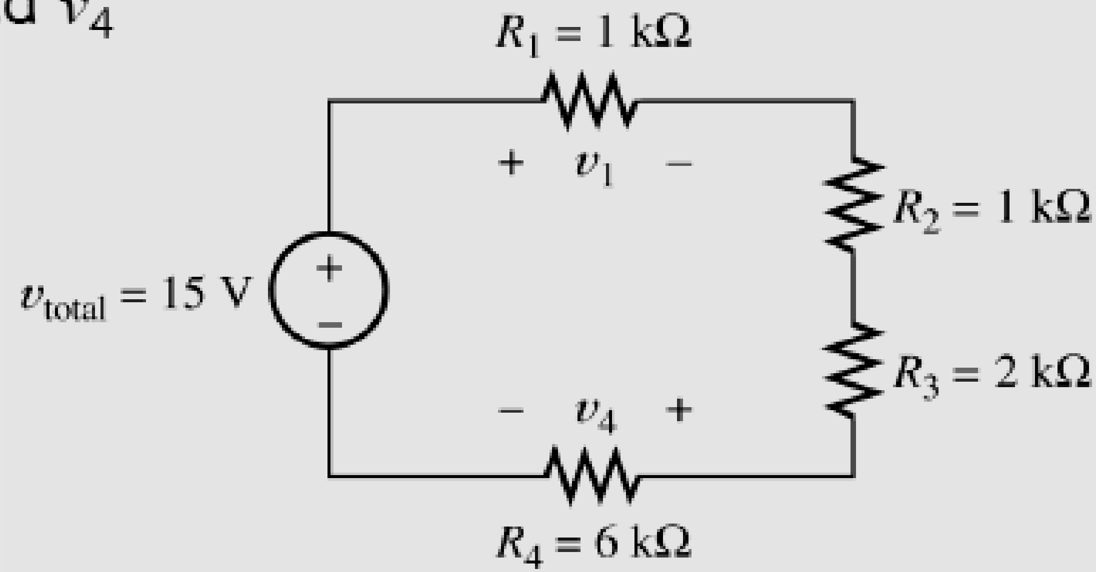
$$v_1 = R_1 i = \frac{R_1}{R_1 + R_2 + R_3} v_{\text{total}}$$

$$v_2 = R_2 i = \frac{R_2}{R_1 + R_2 + R_3} v_{\text{total}}$$



Example

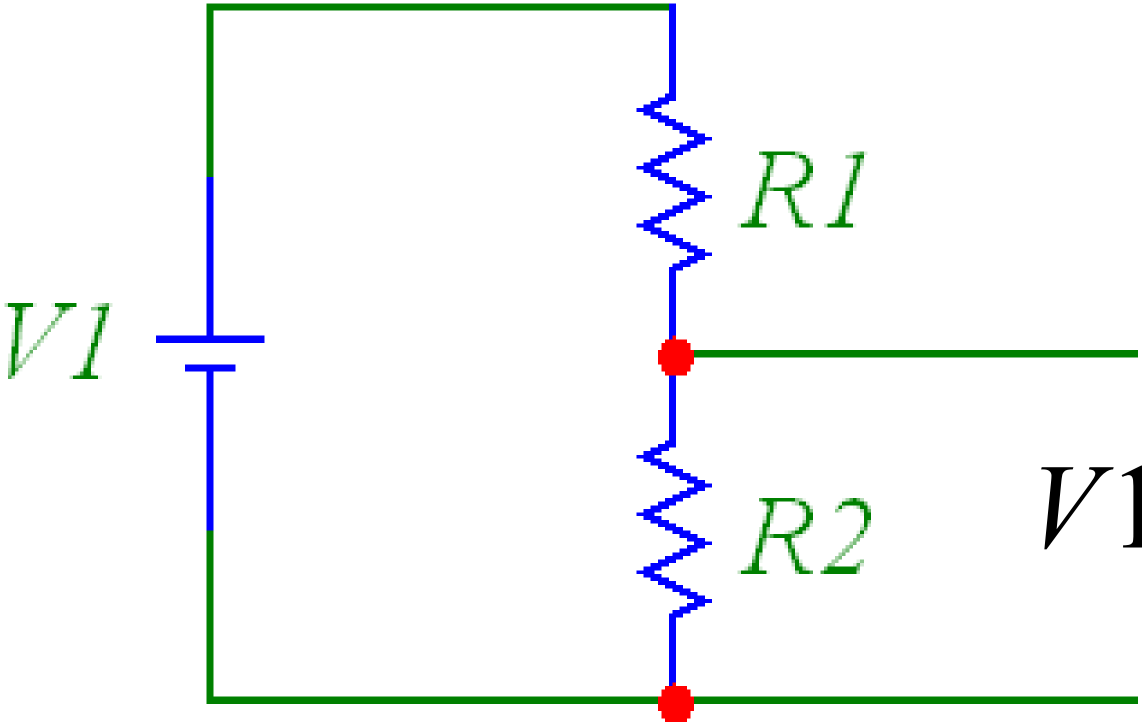
Find v_1 and v_4



$$v_1 = \frac{1000}{1000+1000+2000+6000} \times 15 = 1.5V$$

$$v_4 = \frac{6000}{1000+1000+2000+6000} \times 15 = 9V$$

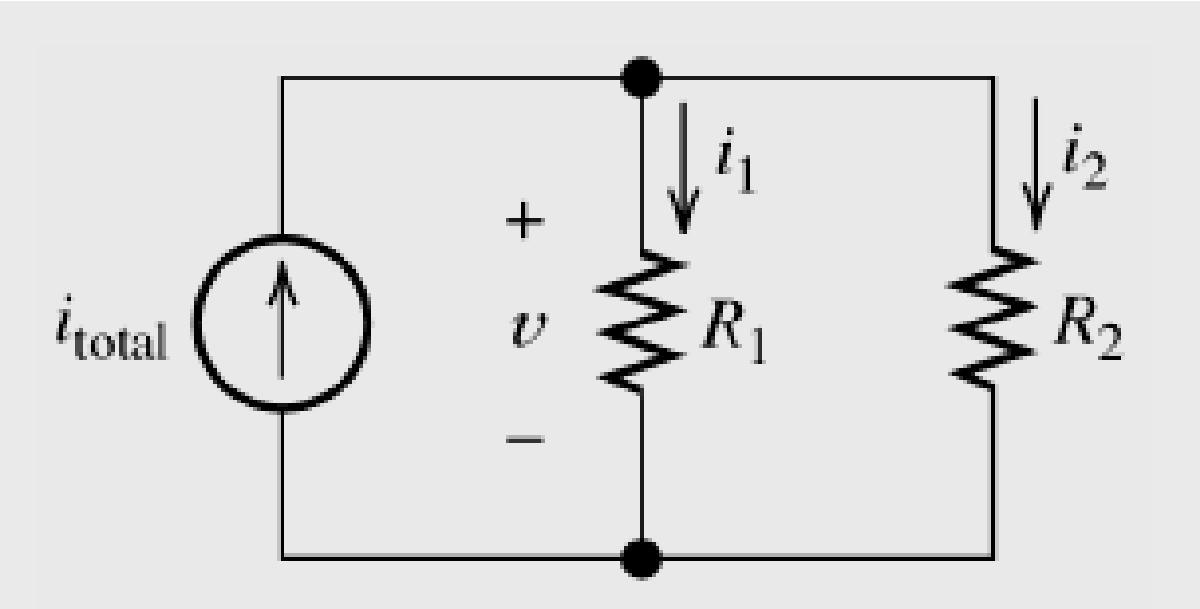
Potential Divider



$$V1\frac{R2}{R1+R2}$$

Current Division

The total current flowing into a parallel combination of resistors will be divided among them



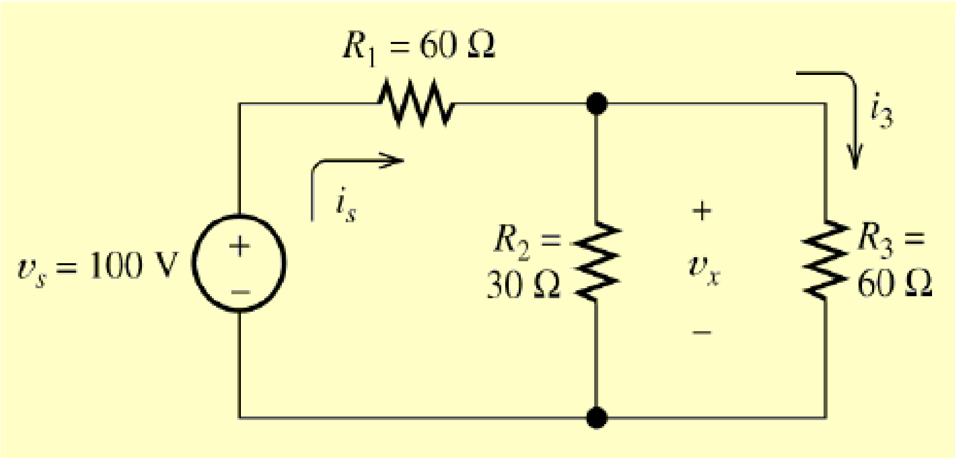
$$v = \frac{R_1 R_2}{R_1 + R_2} i_{total}$$

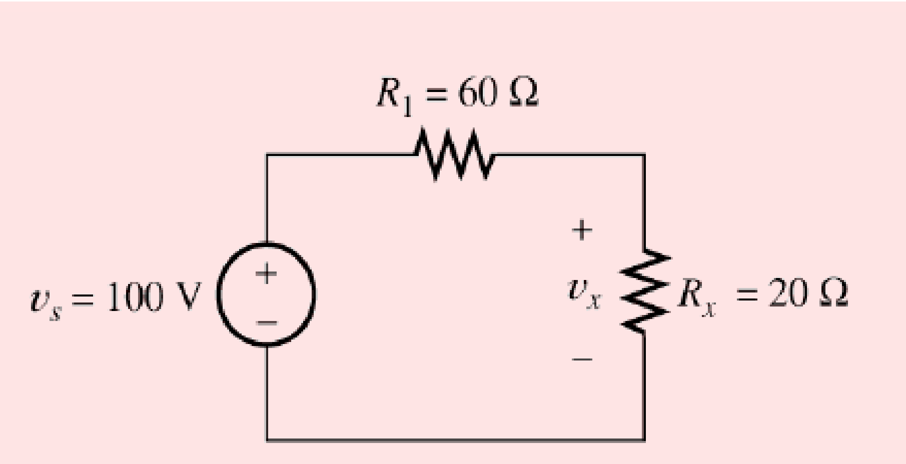
$$i_1 = \frac{v}{R_1} = \frac{R_2}{R_1 + R_2} i_{total}$$

$$i_2 = \frac{v}{R_2} = \frac{R_1}{R_1 + R_2} i_{total}$$

Example

Find v_x *using voltage division and then find* i_s *and use it to find* i_3 *using* current division





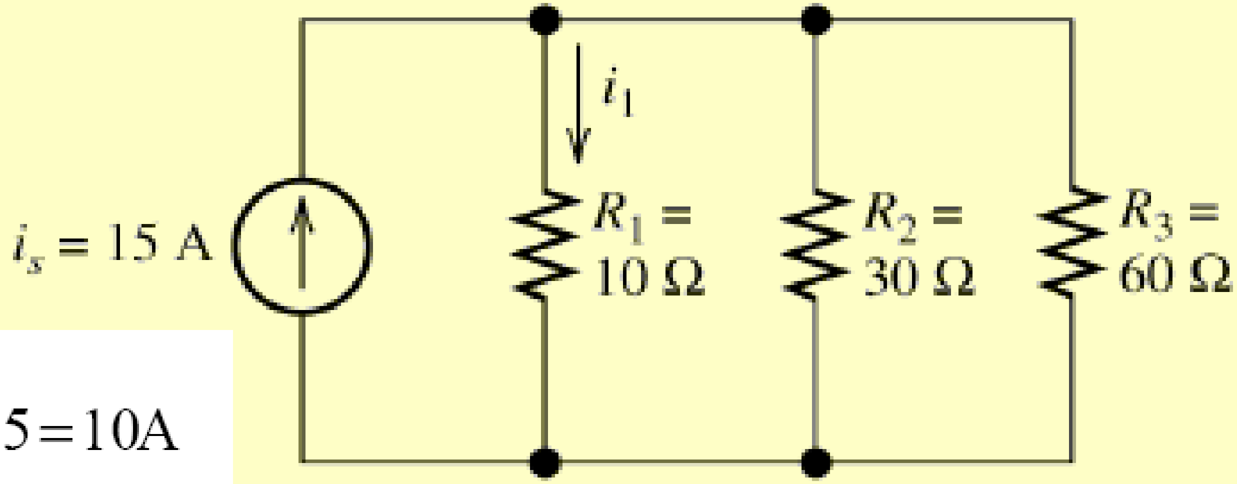
$$R_{\text{eq}} = \frac{R_2 R_3}{R_2 + R_3} = \frac{30 \times 60}{30 + 60} = 20\Omega$$

$$v_x = \frac{R_x}{R_1 + R_x} v_s = \frac{20}{60+20} 100 = 25V$$

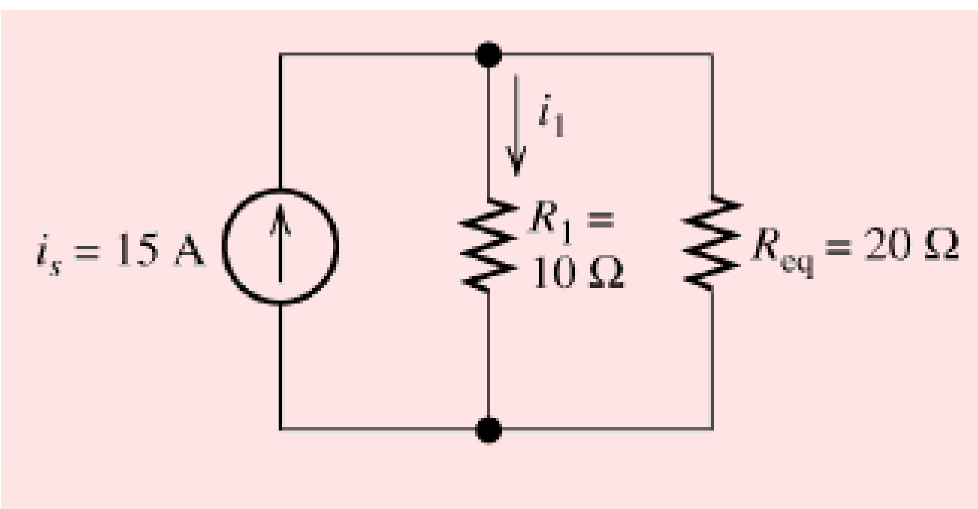
$$i_3 = \frac{R_2}{R_2 + R_3} i_s = \frac{30}{30+60} \times 1.25 = 0.417A$$

$$i_s = \frac{v_s}{R_1 + R_x} = \frac{100}{60+20} = 1.25A$$

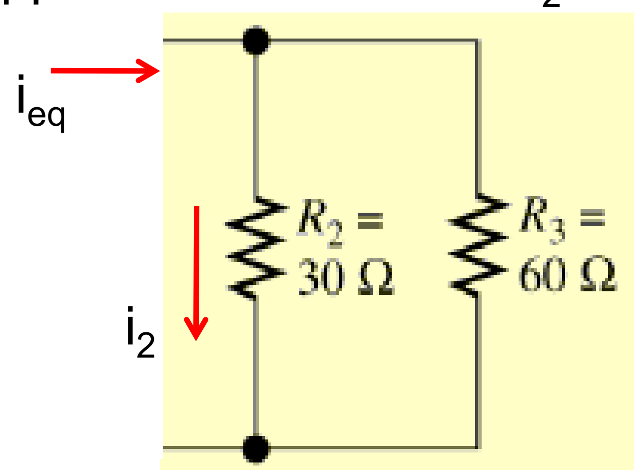
Use current division rule to find i_1



$$i_1 = \frac{R_{eq}}{R_1 + R_{eq}} i_s = \frac{20}{10+20} 15 = 10\text{A}$$



Suppose we want to find i_2 also

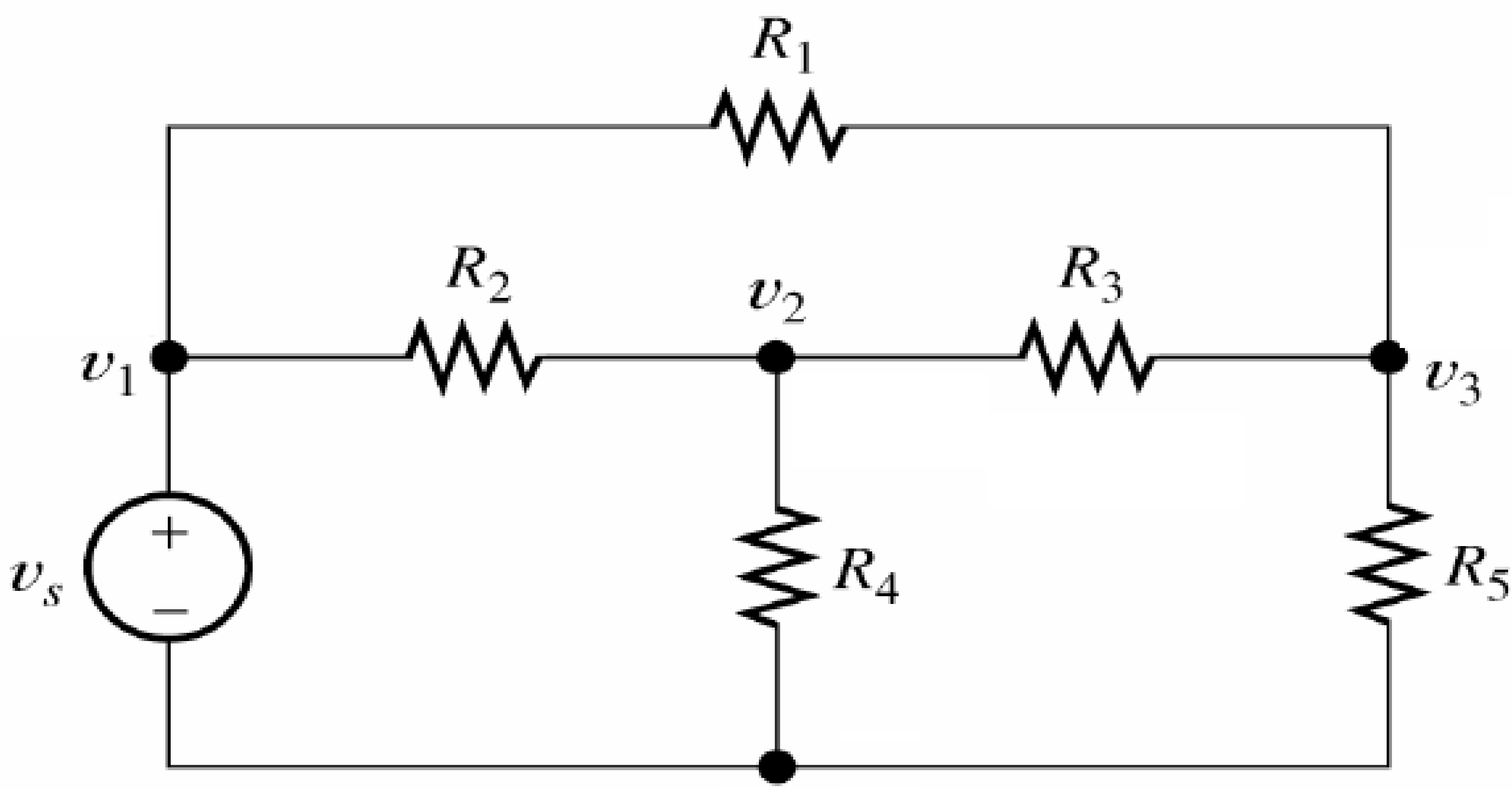


$$i_{eq} = \frac{R_1}{R_1 + R_{eq}} i_s$$

$$i_2 = \frac{R_3}{R_2 + R_3} i_{eq}$$

Limitations

Although series/parallel equivalents and the current/voltage division principles are very important concepts, yet they are not sufficient to solve all circuits !!



Circuit Analysis

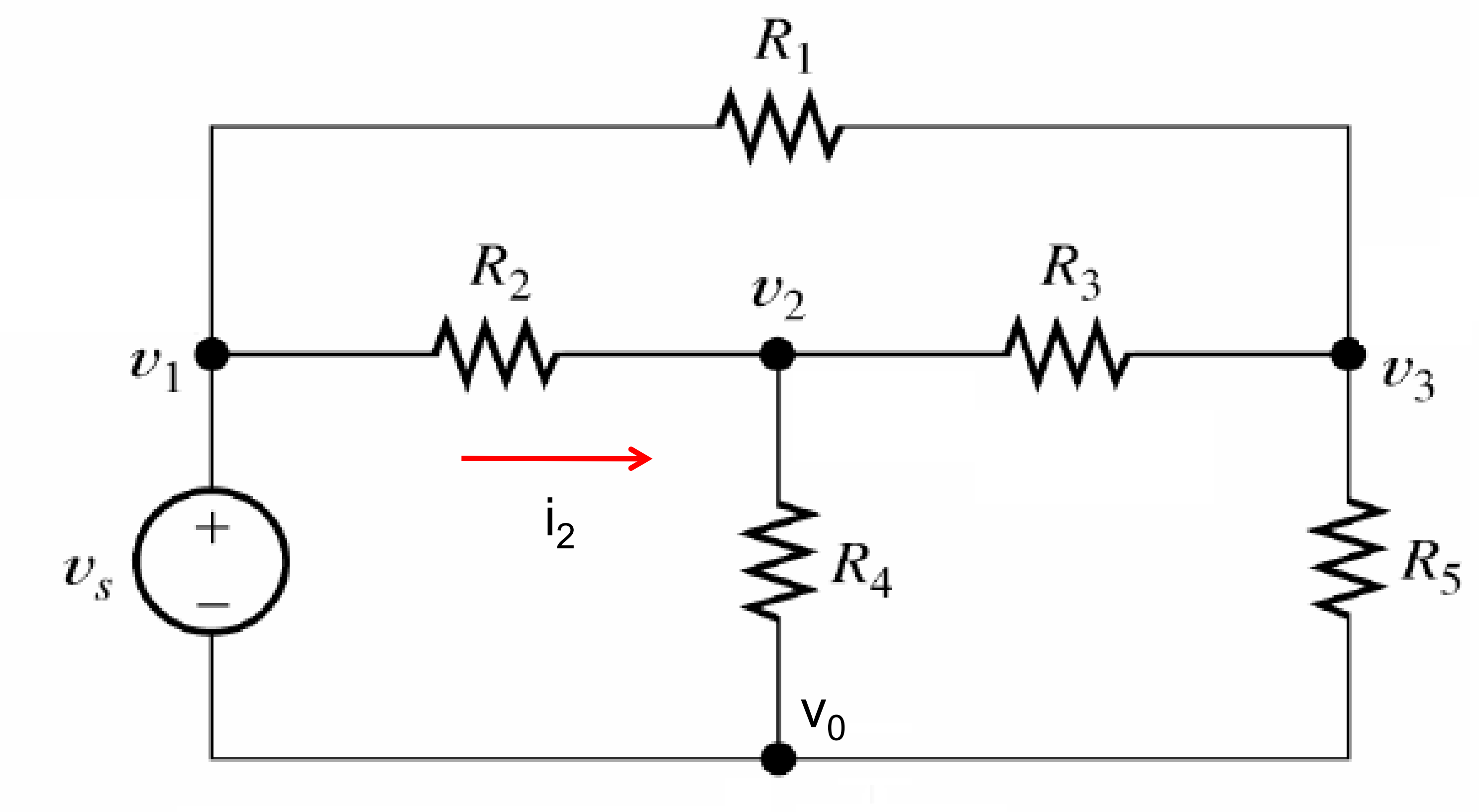
Goal is to find voltages, currents and power in the circuit

If we know voltage and current then power can be easily determined

$$P(t) = v(t) \times i(t)$$

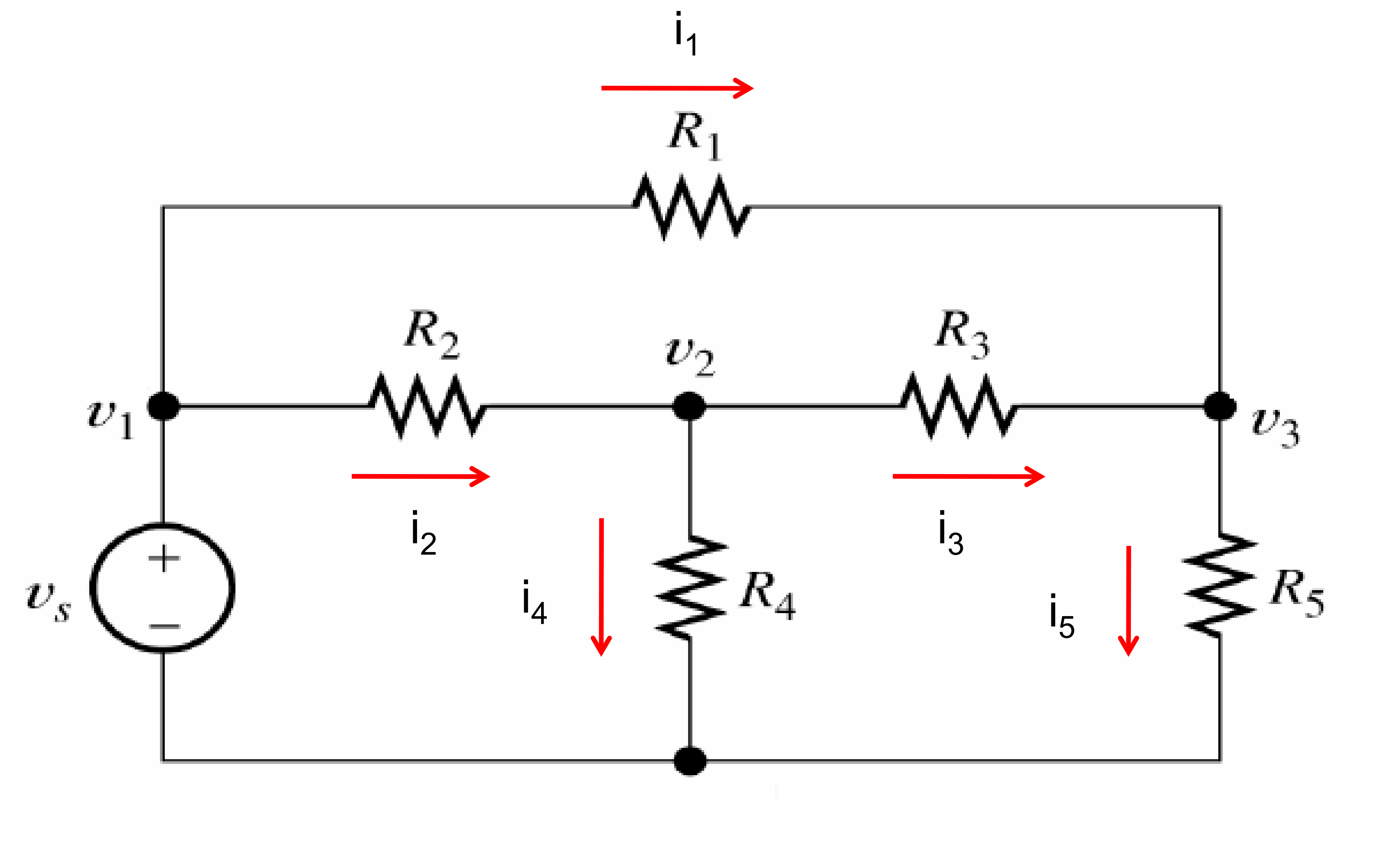
If we determine the voltages, then we can determine the currents using the models of circuit elements

Or if we determine the currents, then we can determine the voltages using the models of circuit elements



If we determine the voltages v_1, v_2, then we can determine the currents as well

$$v_1 - v_2 = i_2 \times R_2$$



If we determine all the currents, then we can determine the voltages as well

$$v_1 - v_2 = i_2 \times R_2$$

General Circuit Analysis Method: Nodal Analysis

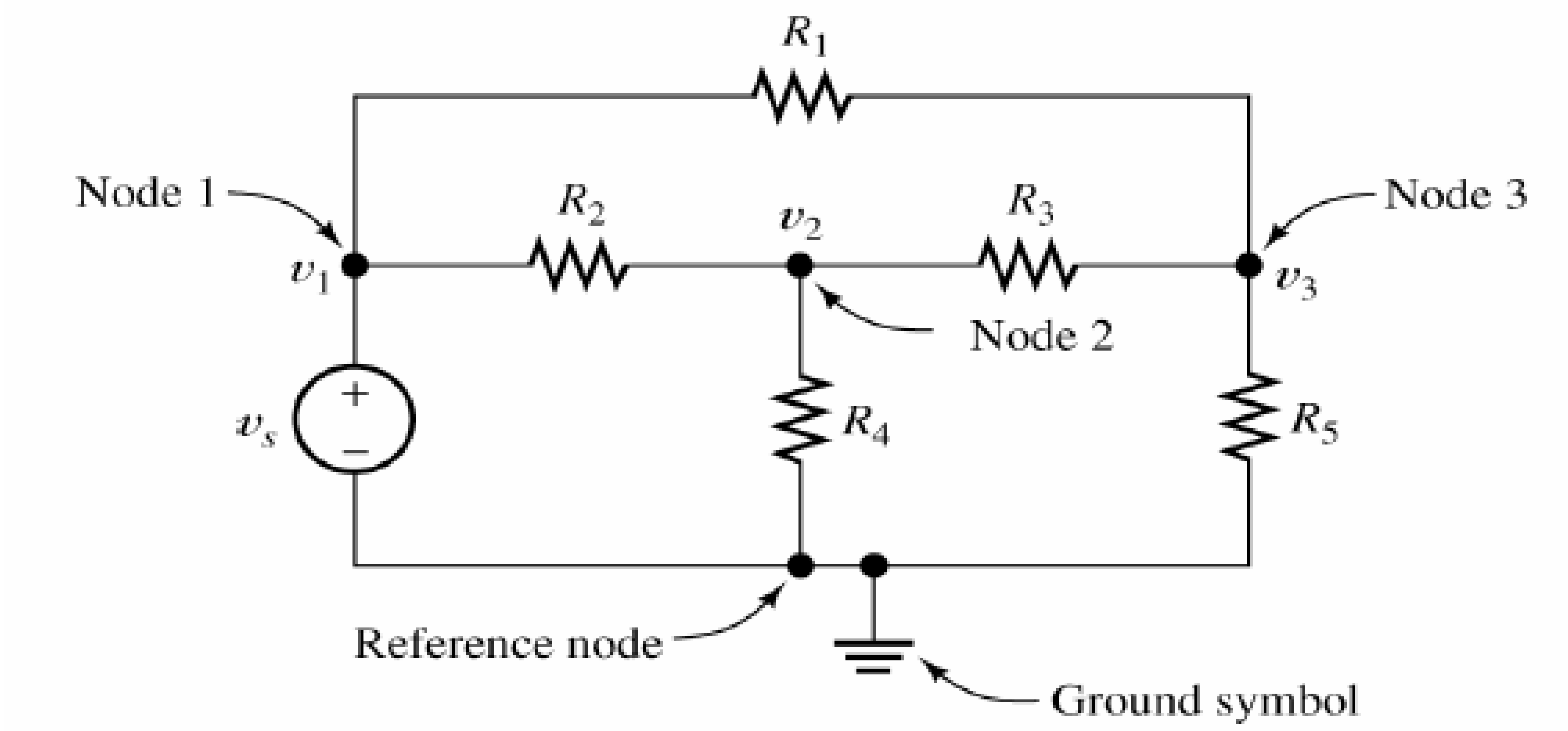
In nodal analysis, the variables used to describe the circuit will be “Node Voltages”

– Nodal voltage are the voltages of each node with respect to a pre-selected reference node

– Usually the reference node has many branches connected to it

– The reference node is also called ground

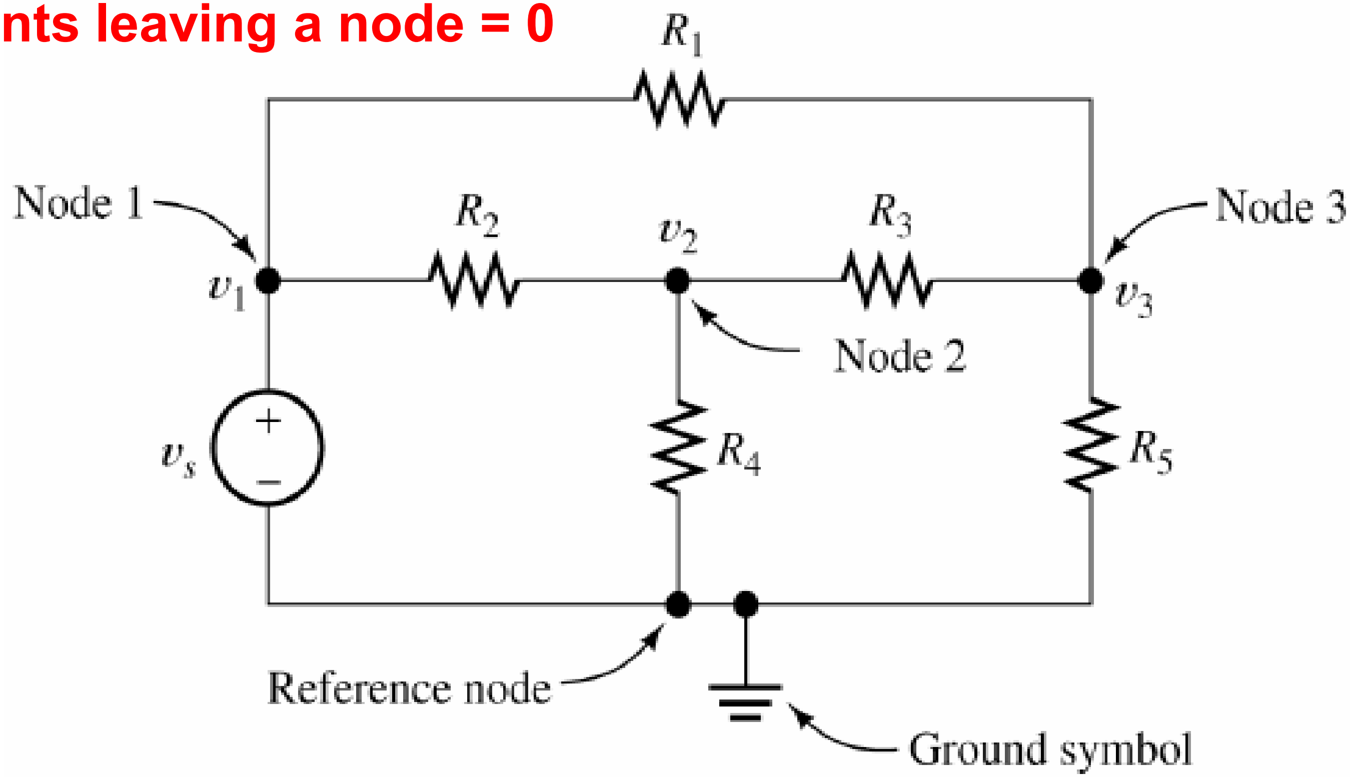
– The node voltages are selected as being positive with respect to the reference node



Nodal Analysis will give values of node voltages v_1, v_2 and v_3 with respect to the reference node

1. Identify and number the nodes
2. Writing KCL Equations in Terms of the Node Voltages

Sum of currents leaving a node = 0

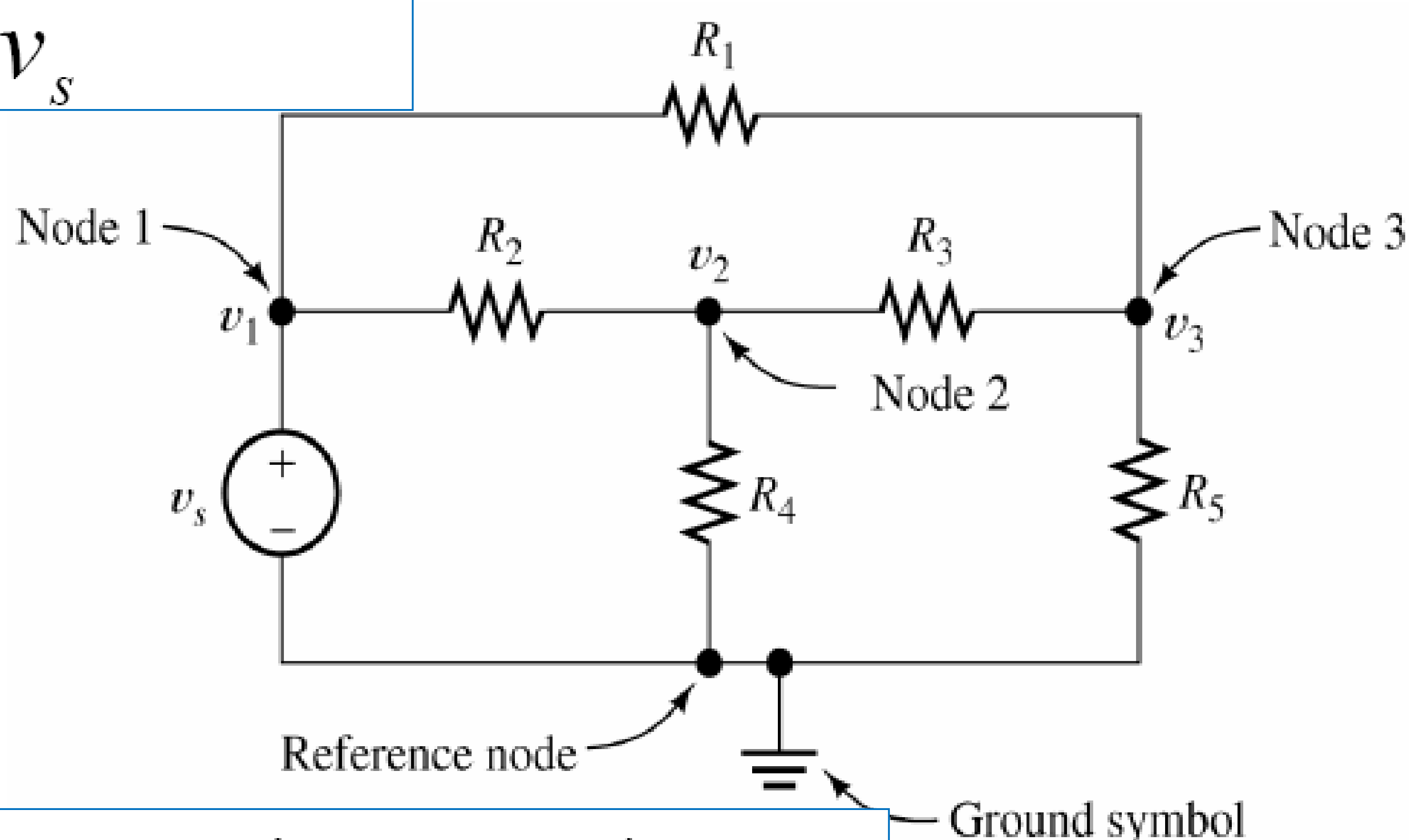


Sum of currents leaving node $2 = 0$

$$\frac{v_2 - v_1}{R_2} + \frac{v_2}{R_4} + \frac{v_2 - v_3}{R_3} = 0$$

Voltage for node 1 is known

$$v_1 = v_s$$

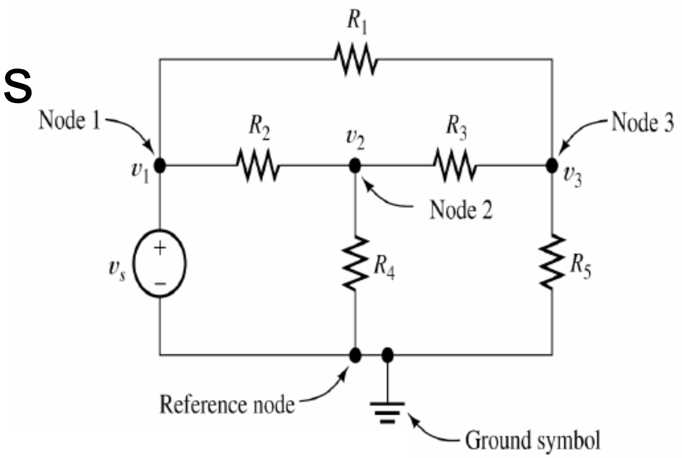


Sum of currents leaving node $3 = 0$

$$\frac{v_3 - v_1}{R_1} + \frac{v_3}{R_5} + \frac{v_3 - v_2}{R_3} = 0$$

Circuit Analysis

- ❑ Transformation of circuit into equations
- ❑ Solution of equations



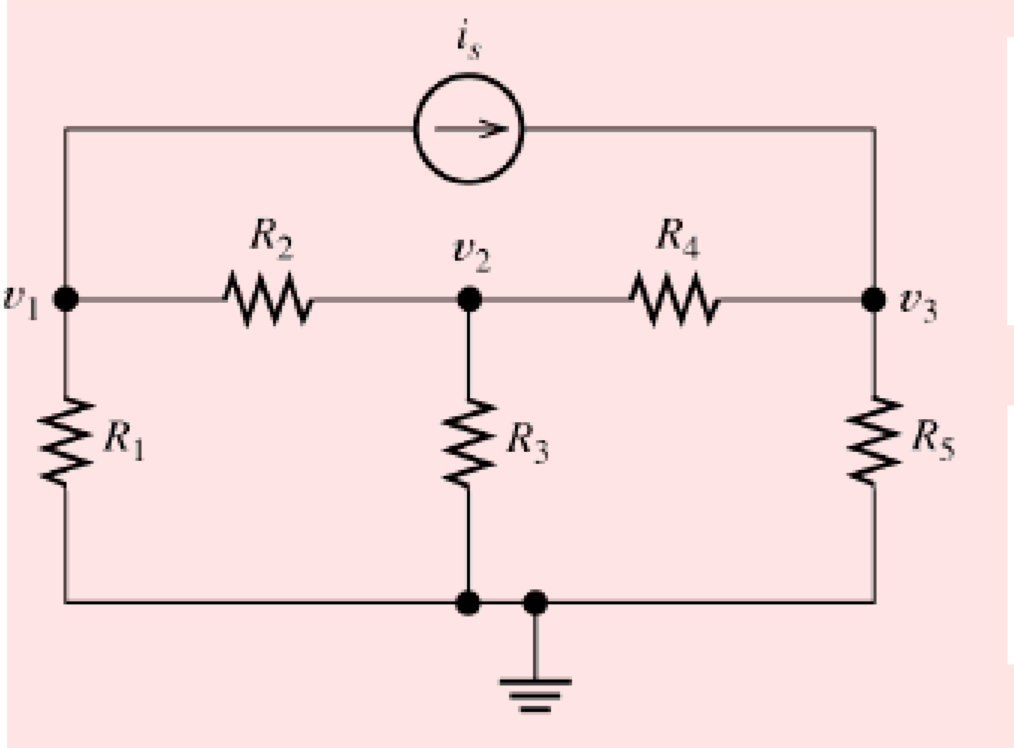
$$\frac{v_2 - v_1}{R_2} + \frac{v_2}{R_4} + \frac{v_2 - v_3}{R_3} = 0$$

$$\frac{v_3 - v_1}{R_1} + \frac{v_3}{R_5} + \frac{v_3 - v_2}{R_3} = 0$$

$$v_1 = v_s$$

Circuits with Independent Current Sources

Sum of currents leaving a node = 0



Node 1:

$$\frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2} + i_s = 0$$

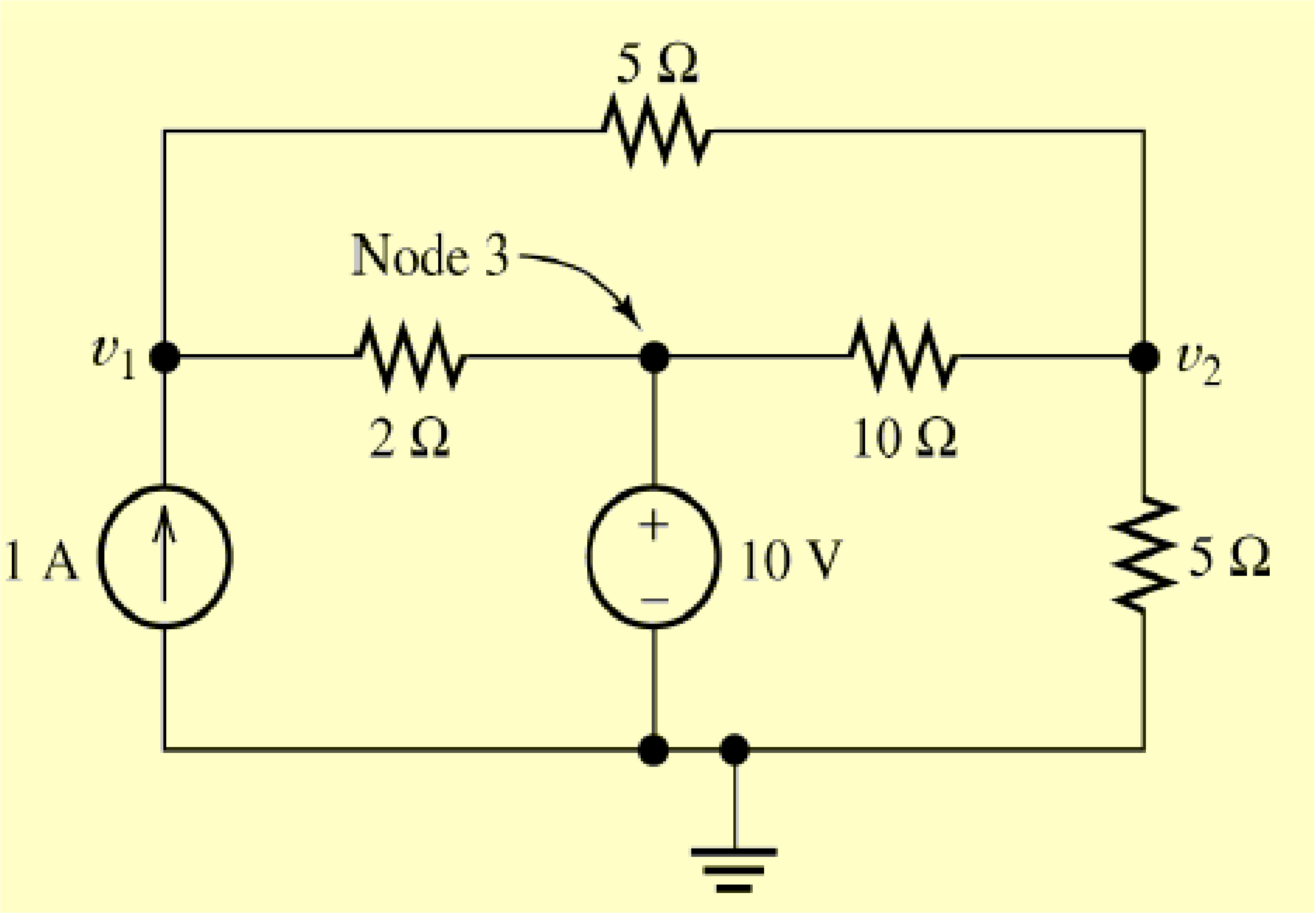
Node 2:

$$\frac{v_2 - v_1}{R_2} + \frac{v_2}{R_3} + \frac{v_2 - v_3}{R_4} = 0$$

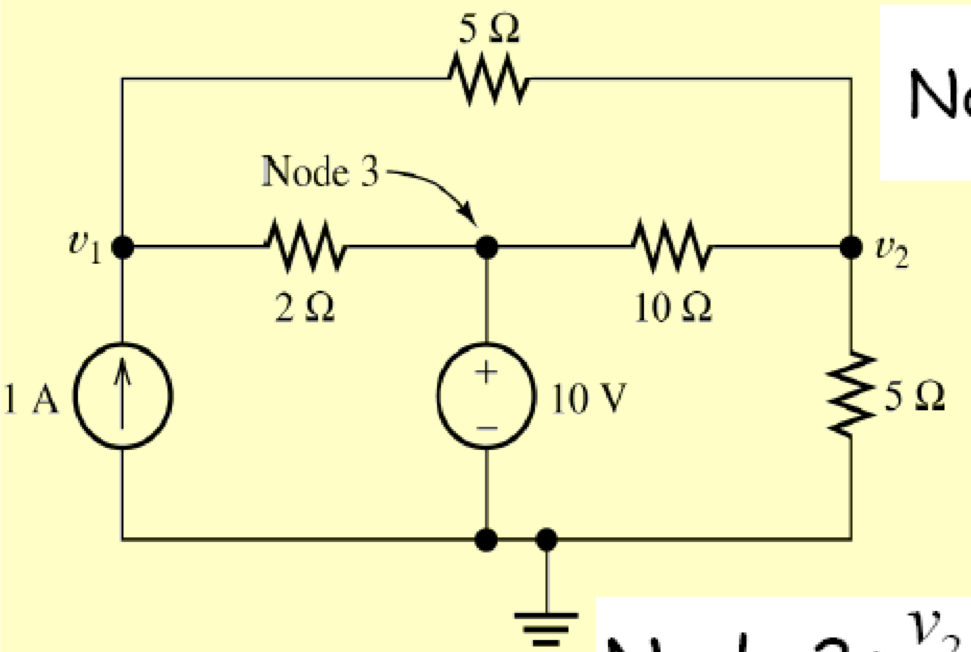
$$Node\,3:$$

$$\frac{v_3}{R_5} + \frac{v_3 - v_2}{R_4} - i_s = 0$$

Find the node voltages v_1 and v_2



Find the node voltages v_1 and v_2



Node1: $\dfrac{v_1 - v_2}{5} + \dfrac{v_1 - 10}{2} = 1 \quad : \text{x}\,10$

$$2v_1 - 2v_2 + 5v_1 - 50 = 10$$

$$0.7v_1 - 0.2v_2 = 6 \qquad (1)$$

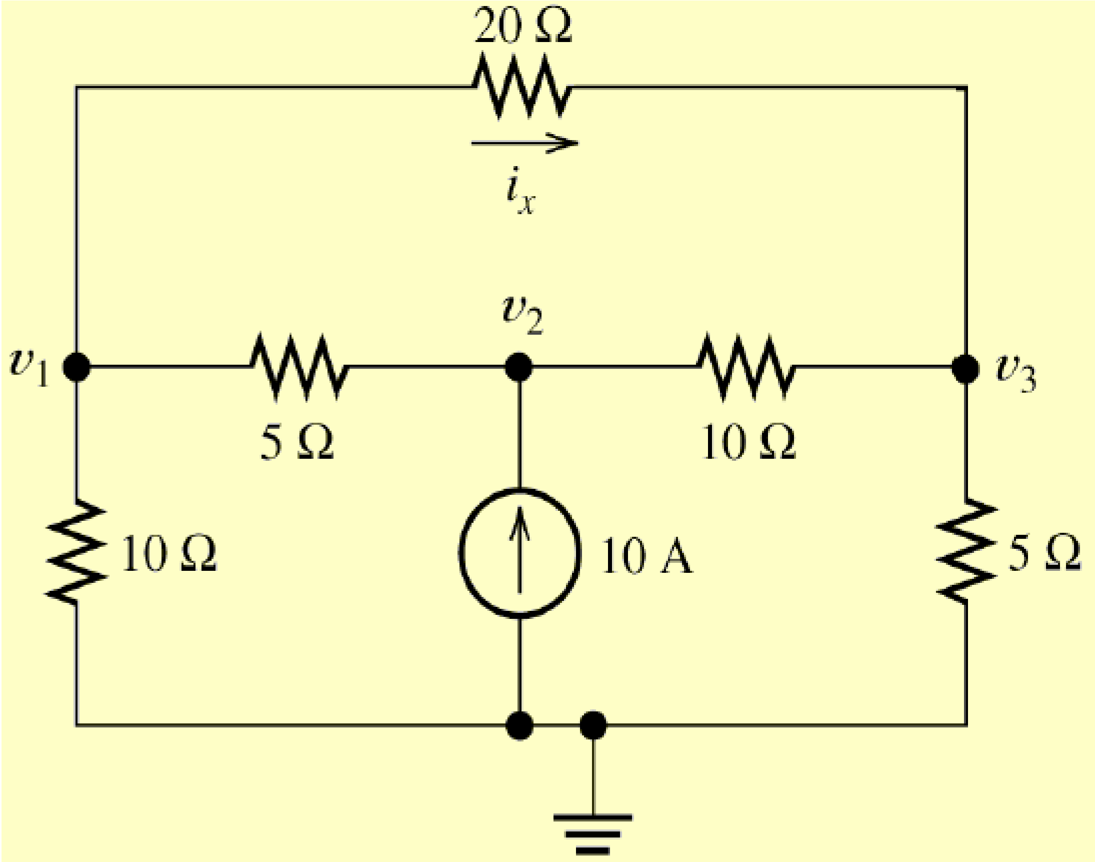
Node 2: $\dfrac{v_2}{5} + \dfrac{v_2 - 10}{10} + \dfrac{v_2 - v_1}{5} = 0 \quad : \text{x}\,10$

$$2v_2 + v_2 - 10 + 2v_2 - 2v_1 = 0$$

$$-0.2v_1 + 0.5v_2 = 1 \qquad (2)$$

Solving (1) and (2) $\quad v_1 = 10.32\,V \qquad v_2 = 6.129\,V$

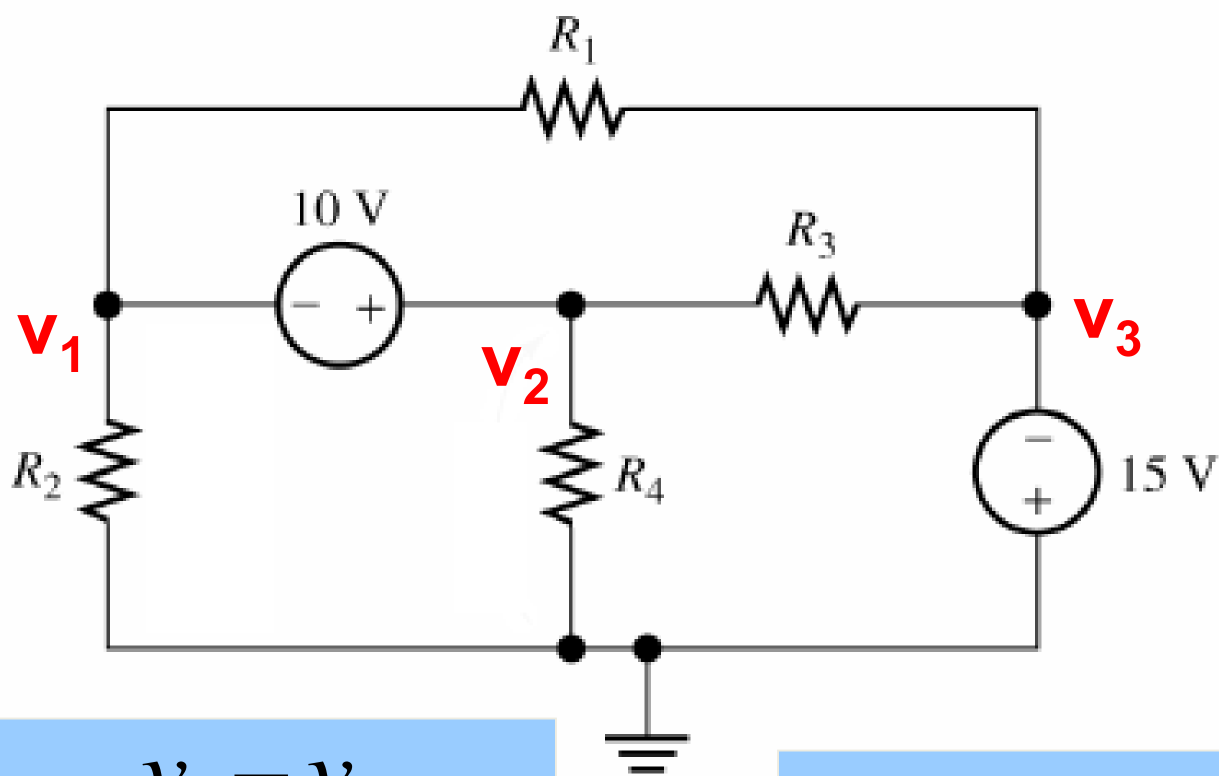
Find i_x using nodal analysis



$$0.35v_1 - 0.2v_2 - 0.05v_3 = 0$$
$$-0.2v_1 + 0.3v_2 - 0.1v_3 = 10$$
$$-0.05v_1 - 0.1v_2 + 0.35v_3 = 0$$

$$v_1 = 45.45\ \mathrm{V} \quad v_2 = 72.73\,V$$
$$v_3 = 27.27\,V$$
$$i_x = \frac{v_1 - v_3}{20} = 0.909\ A$$

Circuit with Voltage Sources



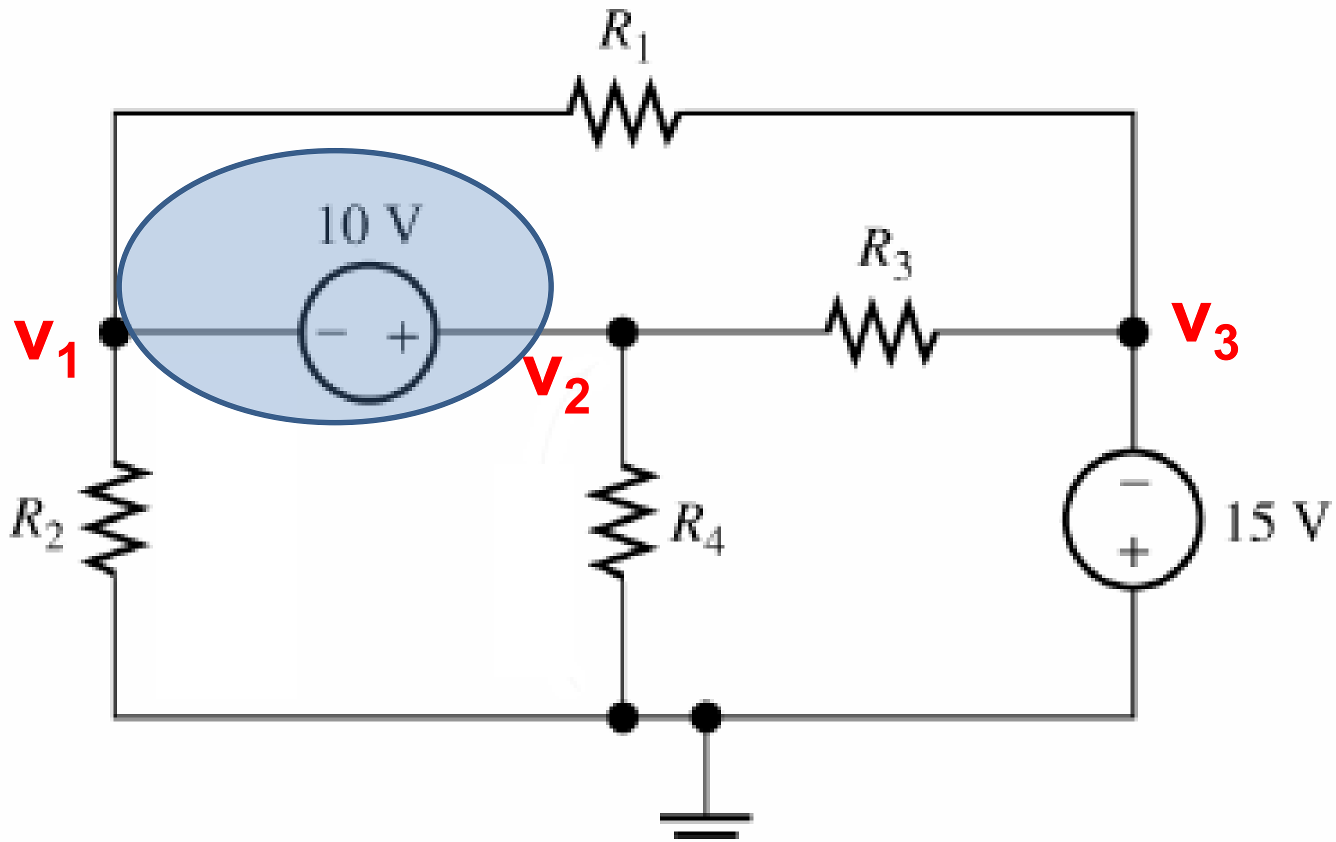
KCL at node 1

$$\frac{v_1}{R_2} + \frac{v_1 - v_3}{R_1} + \frac{v_1 - v_2}{?} = 0$$

$$v_2 - v_1 = 10$$

KCL at node 2

$$\frac{v_2}{R_4} + \frac{v_2 - v_3}{R_3} + \frac{v_2 - v_1}{?} = 0$$



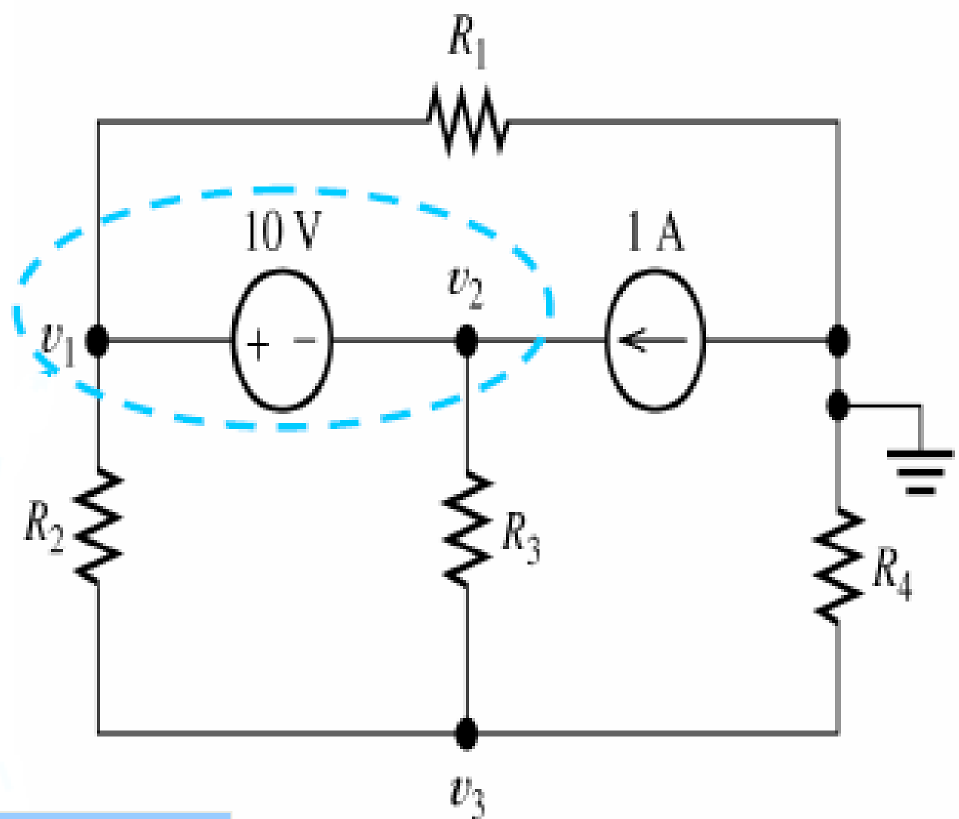
Node 1 and node 2 are merged together into a super node. KCL is applied to the super node

Sum of currents leaving a super node is zero

$$\frac{v_1}{R_2} + \frac{v_1 - v_3}{R_1} + \frac{v_2 - v_3}{R_3} + \frac{v_2}{R_4} = 0$$

Example

$$v_1 - v_2 = 10$$

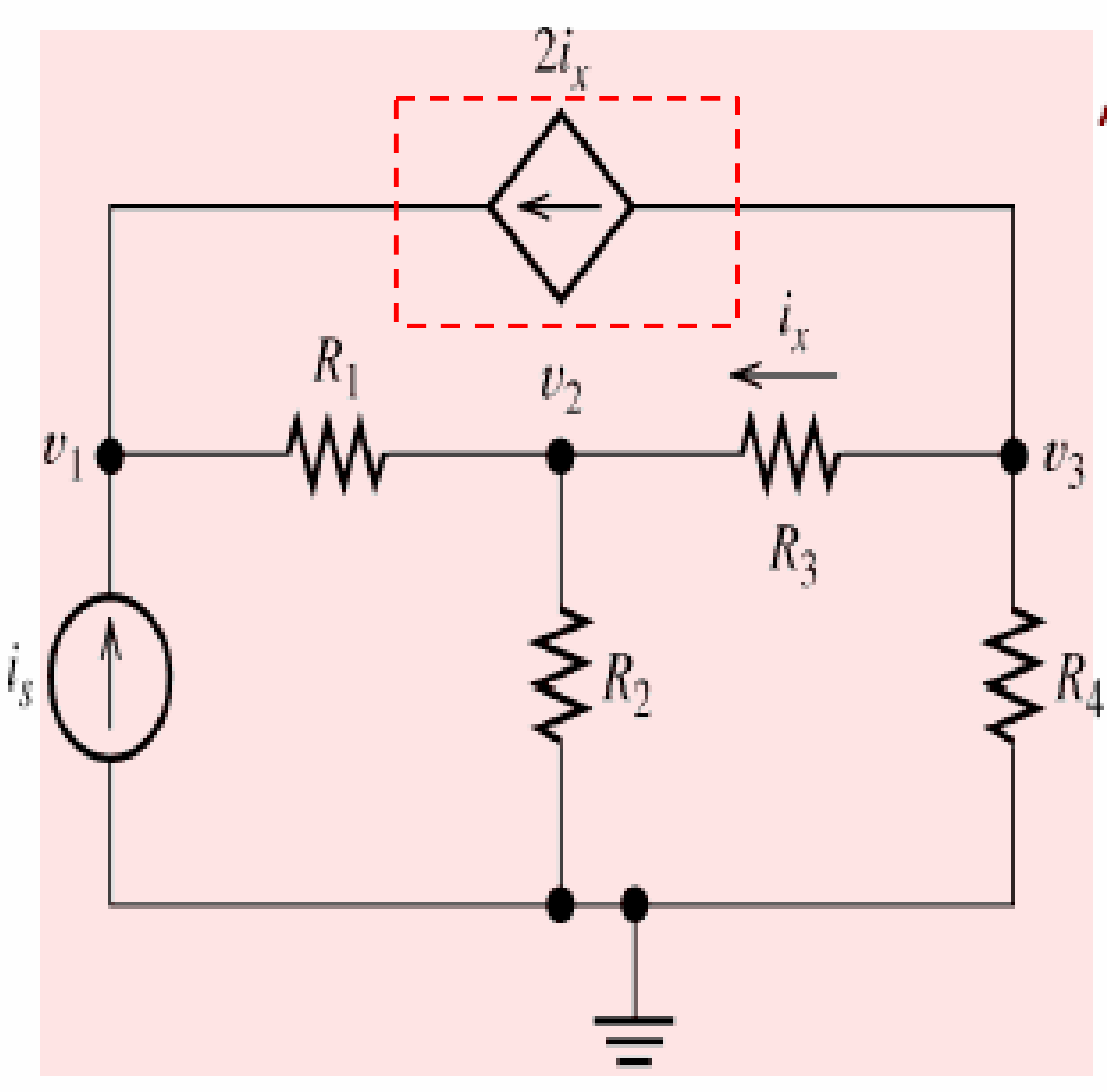


$$\frac{v_1}{R_1} + \frac{v_1 - v_3}{R_2} + \frac{v_2 - v_3}{R_3} - 1 = 0$$

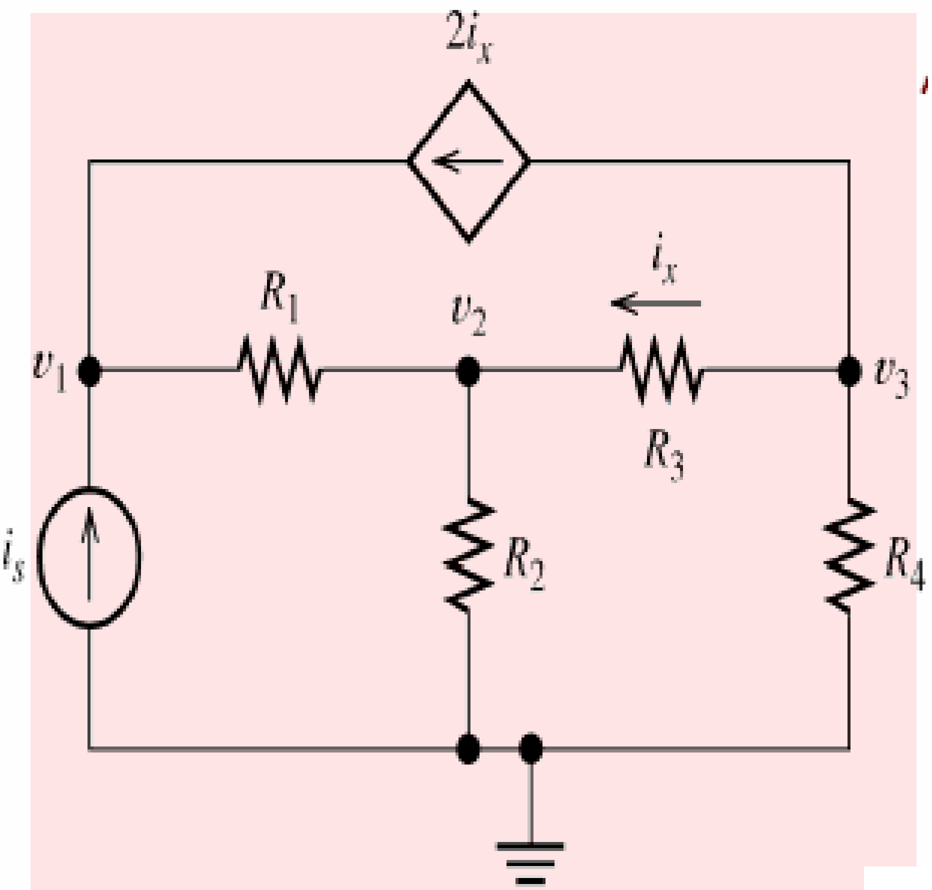
At node 3:

$$\frac{v_3 - v_1}{R_2} + \frac{v_3 - v_2}{R_3} + \frac{v_3}{R_4} = 0$$

Node-Voltage Analysis with a Dependent Source



Node-Voltage Analysis with a Dependent Source



At Node 1

$$\frac{v_1 - v_2}{R_1} - i_s - 2i_x = 0$$

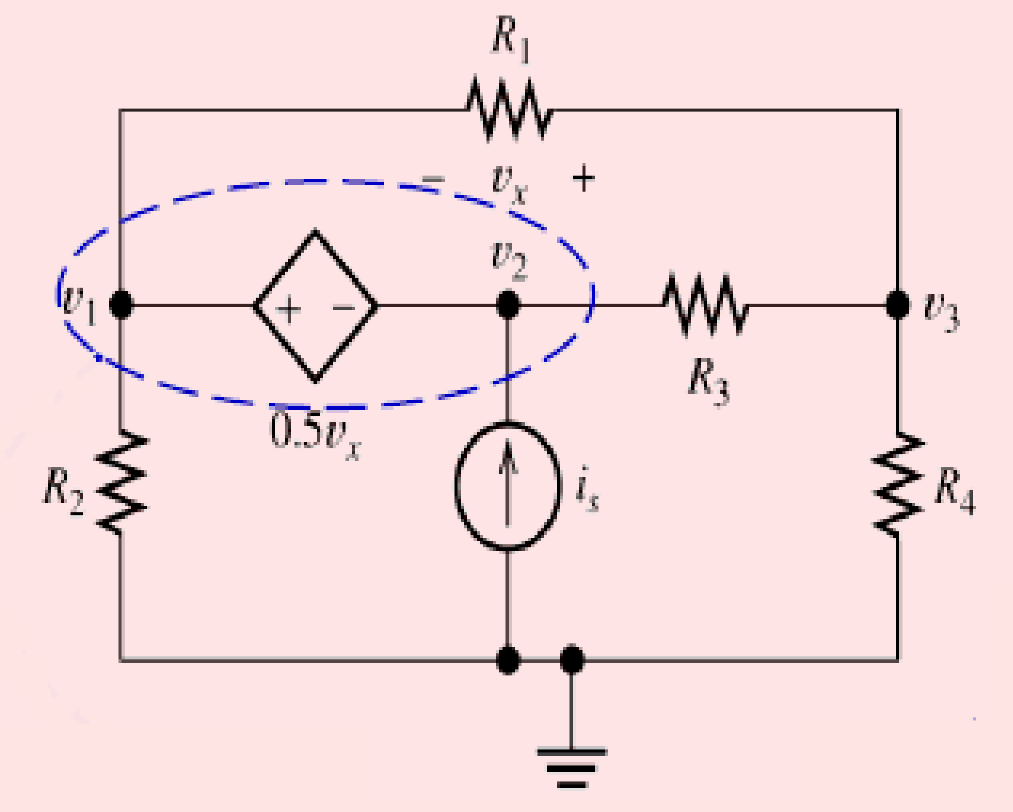
At node 2:

$$\frac{v_2 - v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_2 - v_3}{R_3} = 0$$

$$i_x = \frac{v_3 - v_2}{R_3}$$

At node 3:

$$\frac{v_3 - v_2}{R_3} + \frac{v_3}{R_4} + 2i_x = 0$$



Dependent voltage source :

$$v_1 - v_2 = 0.5v_x$$

Node 3

$$\frac{v_3}{R_4} + \frac{v_3 - v_2}{R_3} + \frac{v_3 - v_1}{R_1} = 0$$

supernode (nodes 1 and 2)

$$\frac{v_1}{R_2} + \frac{v_1 - v_3}{R_1} + \frac{v_2 - v_3}{R_3} = i_s$$

Contolling variable

$$v_x = v_3 - v_1$$

Summary: Node-Voltage Analysis

1. Select a reference node and assign variables for the unknown node voltages

2. Write network equations
- First, use KCL to write current equations for nodes and super nodes
- Then, if you do not have enough equations because of voltage sources connected between nodes, use KVL to write additional equations

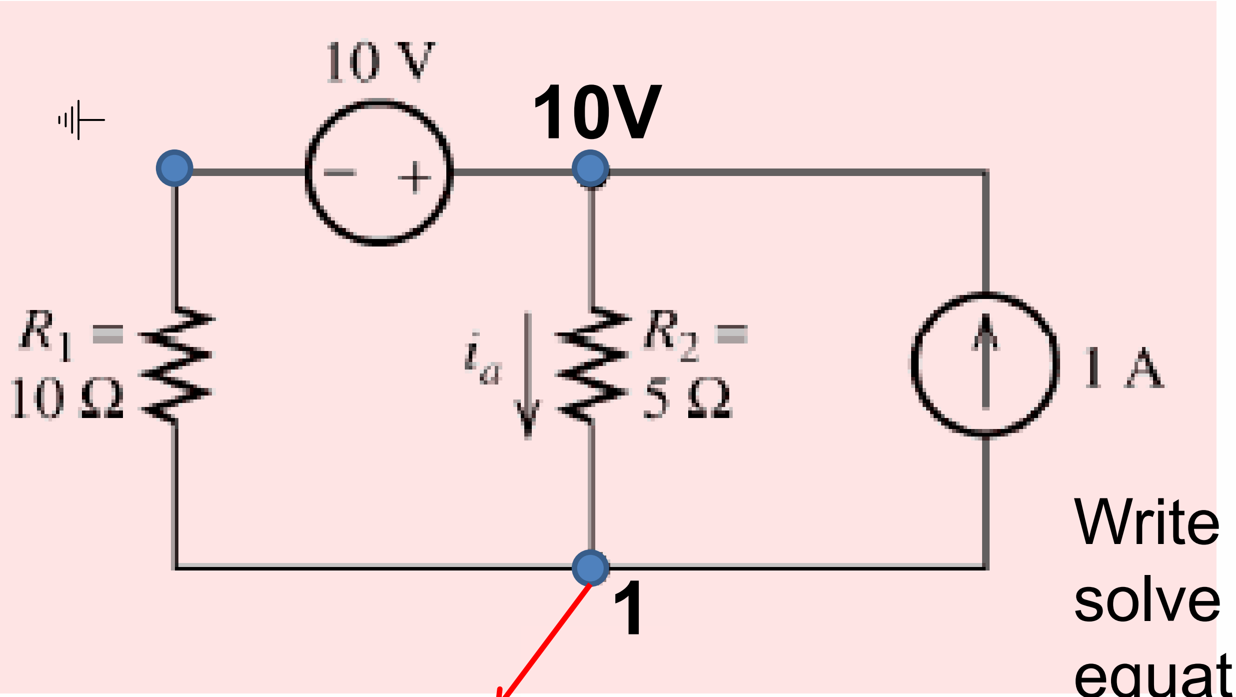
3. If the circuit contains dependent sources
- Find expressions for the controlling variables in terms of the node voltages
- Substitute into the network equations, and obtain equations having only the node voltages as unknowns

4. Put the equations into standard form and solve for the node voltages

5. Use the values found for the node voltages to calculate any other currents or voltages of interest

Find i_a *using nodal analysis*

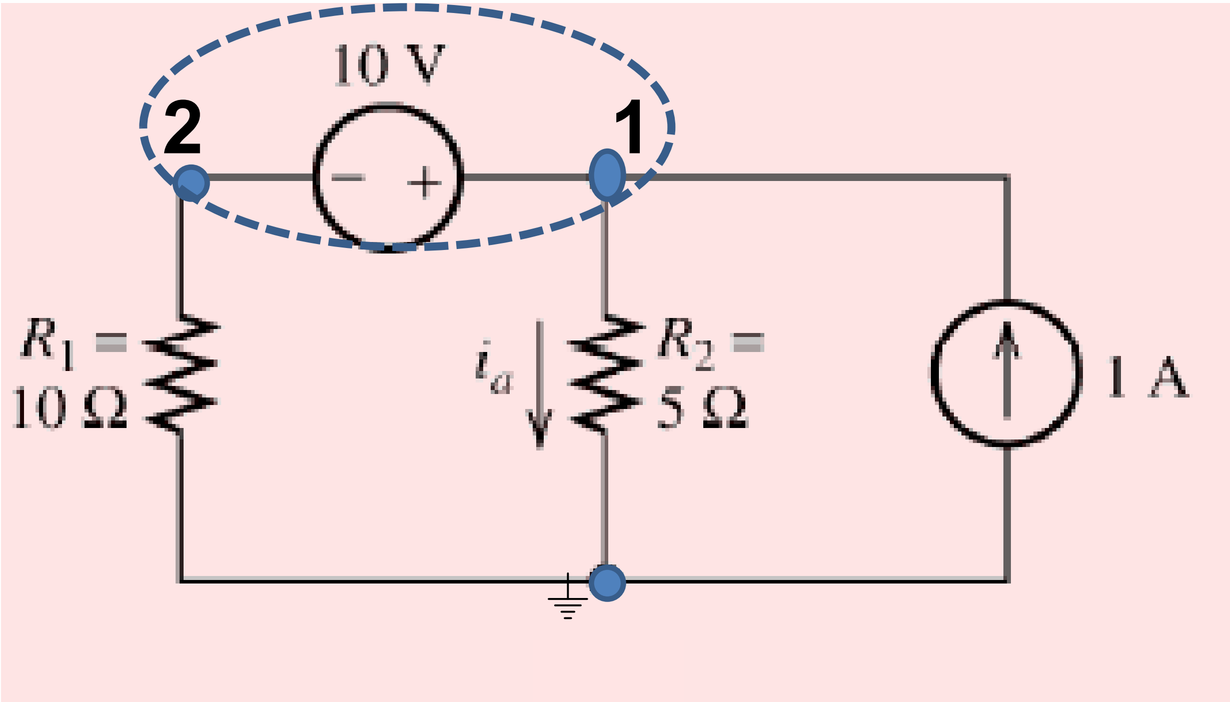
Choose a reference node wisely



Write KCL at nodes and solve the resulting equations

$$\frac{v_1}{10}+\frac{v_1-10}{5}+1=0 \Rightarrow v_1=\frac{10}{3}$$

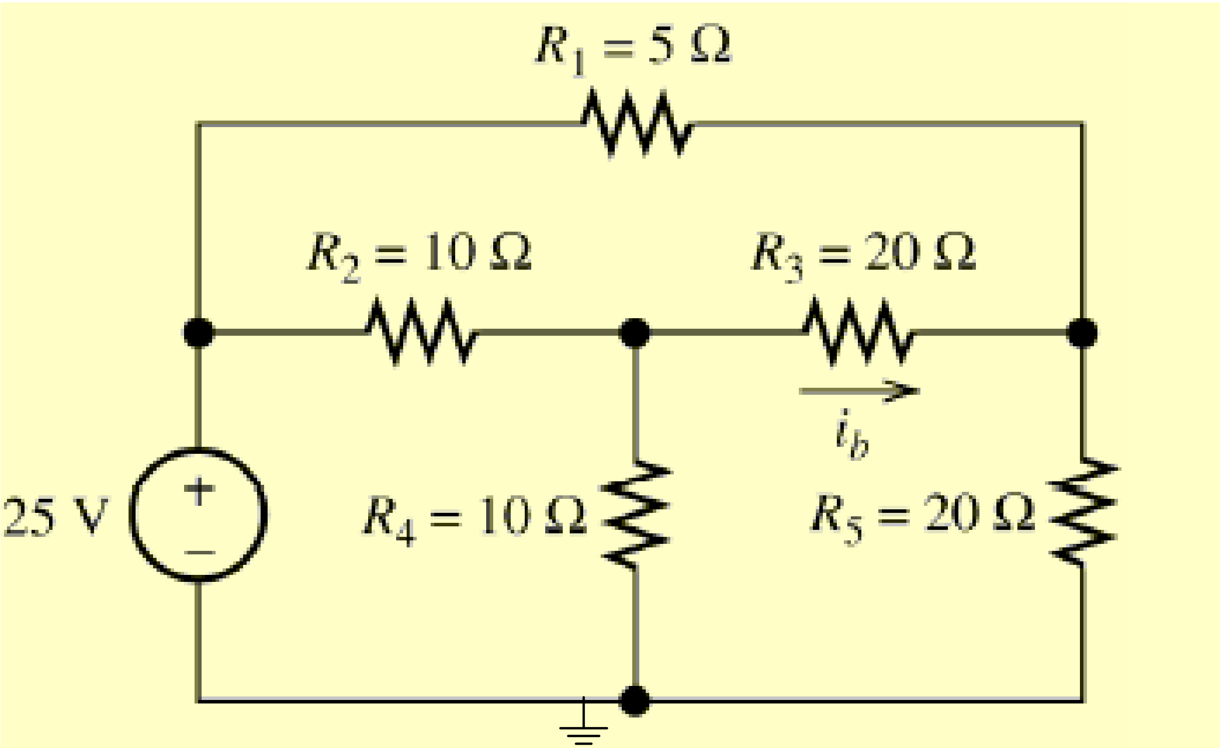
$$i_a=\frac{10-v_1}{5}=1.33A$$



$$v_1 - v_2 = 10V$$

KCL at super node:

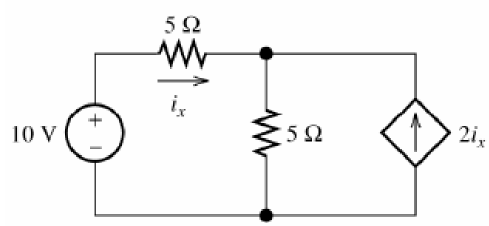
$$\frac{v_1}{5} - 1 + \frac{v_2}{10} = 0$$



Which should be the reference node?

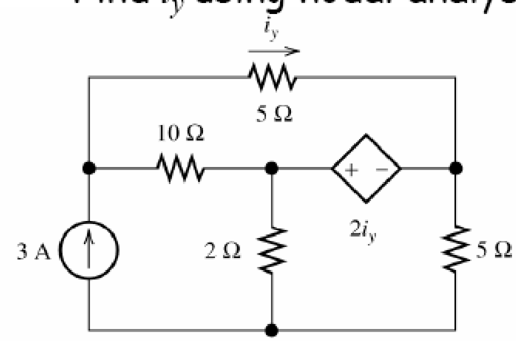
Exercises

Find i_x using nodal analysis



(a)

Find i_y using nodal analysis



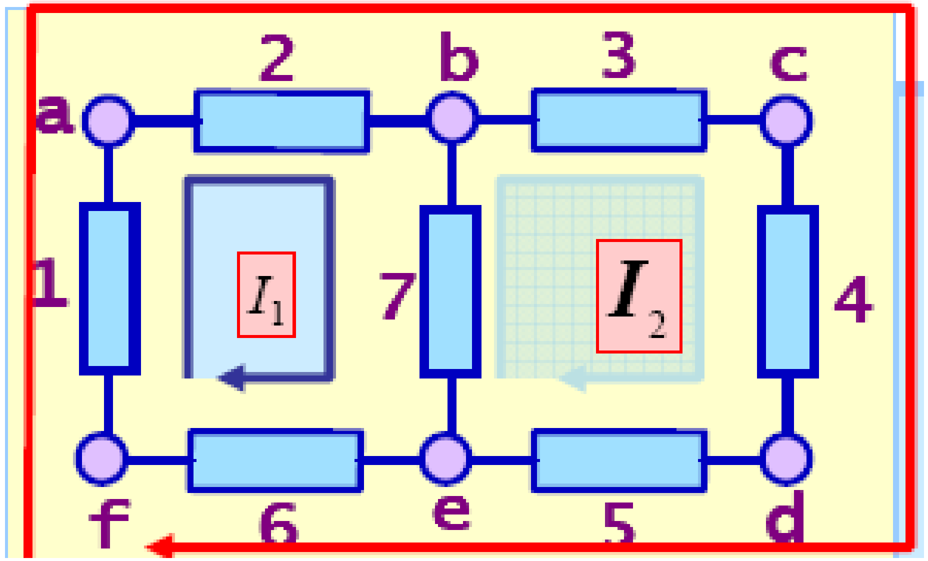
(b)

Mesh Analysis

1. Mesh analysis provides another general procedure for analyzing circuits using mesh currents as the circuit variables.

2. Mesh analysis applies KVL to find unknown currents.

3. A mesh is a loop which does not contain any other loops within it.



A loop is a closed path that does not go twice over any node. This circuit has three loops

A mesh is a loop that does not enclose any other loop. fabef, ebcde are meshes

fabef

ebcde

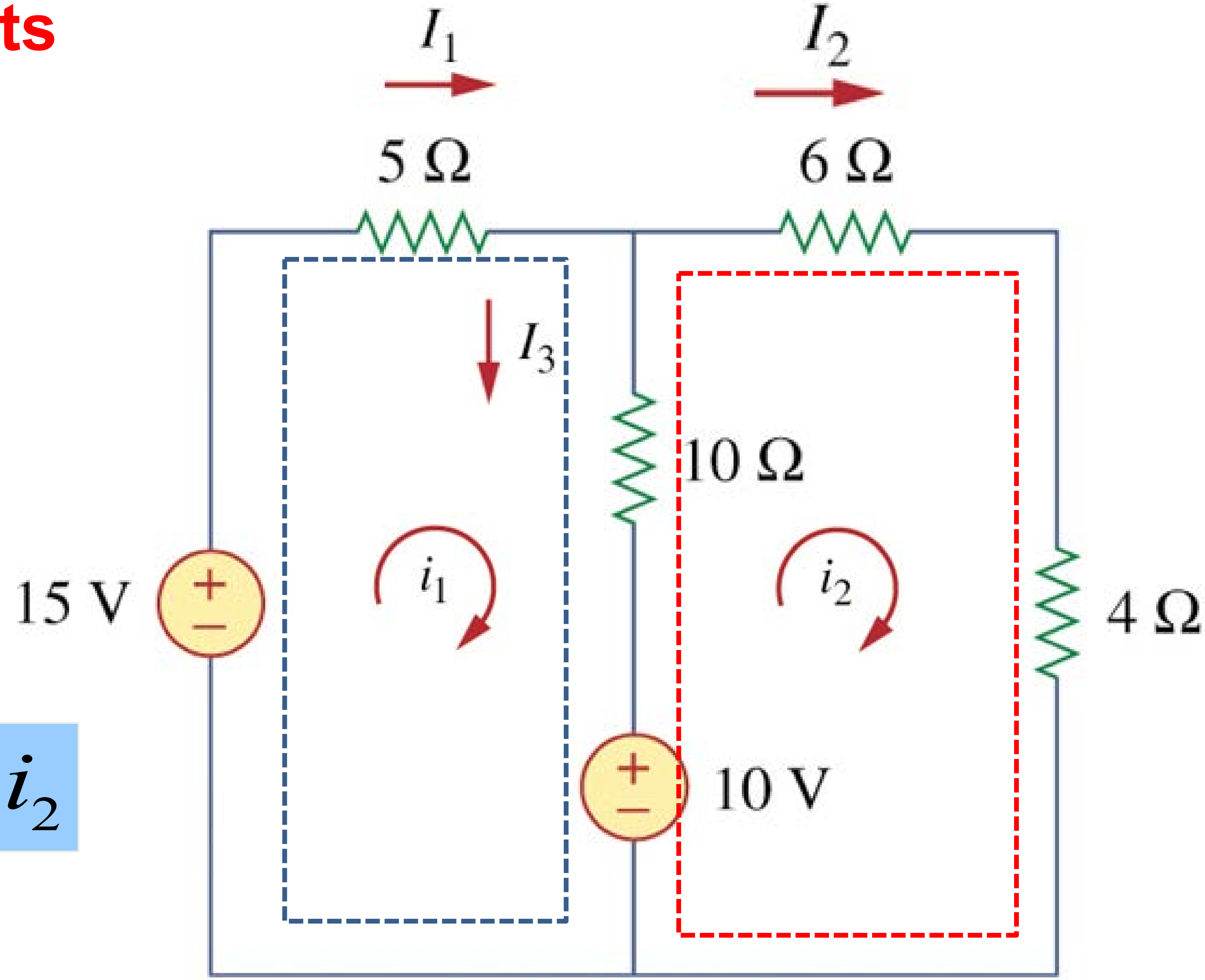
~~fabcdef~~

Mesh Currents

$$I_1 = i_1$$

$$I_2 = i_2$$

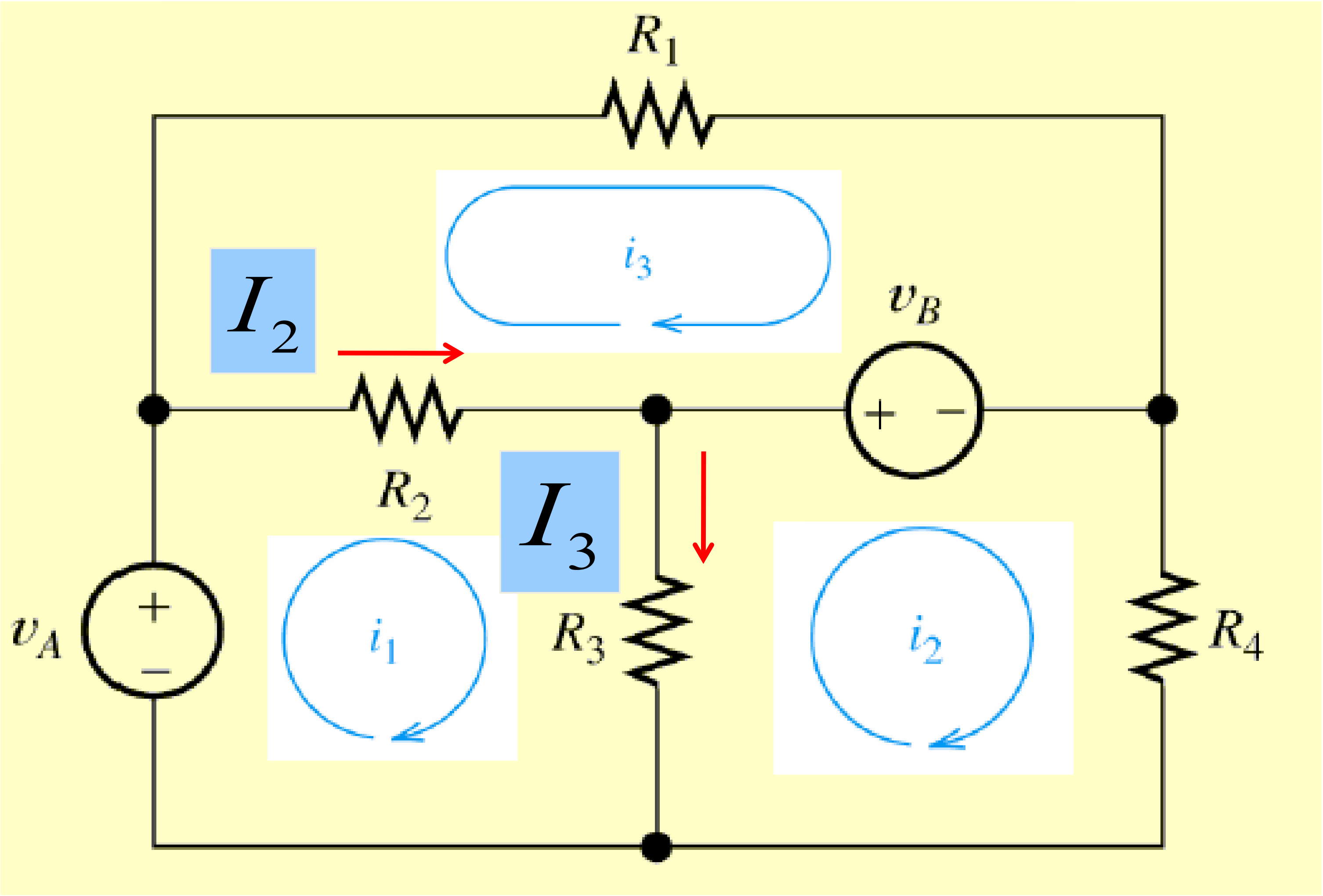
$$I_3 = i_1 - i_2$$



i_1 and i_2 are mesh current (imaginative, not measurable directly)

I_1, I_2 and I_3 are branch current (real, measurable directly)

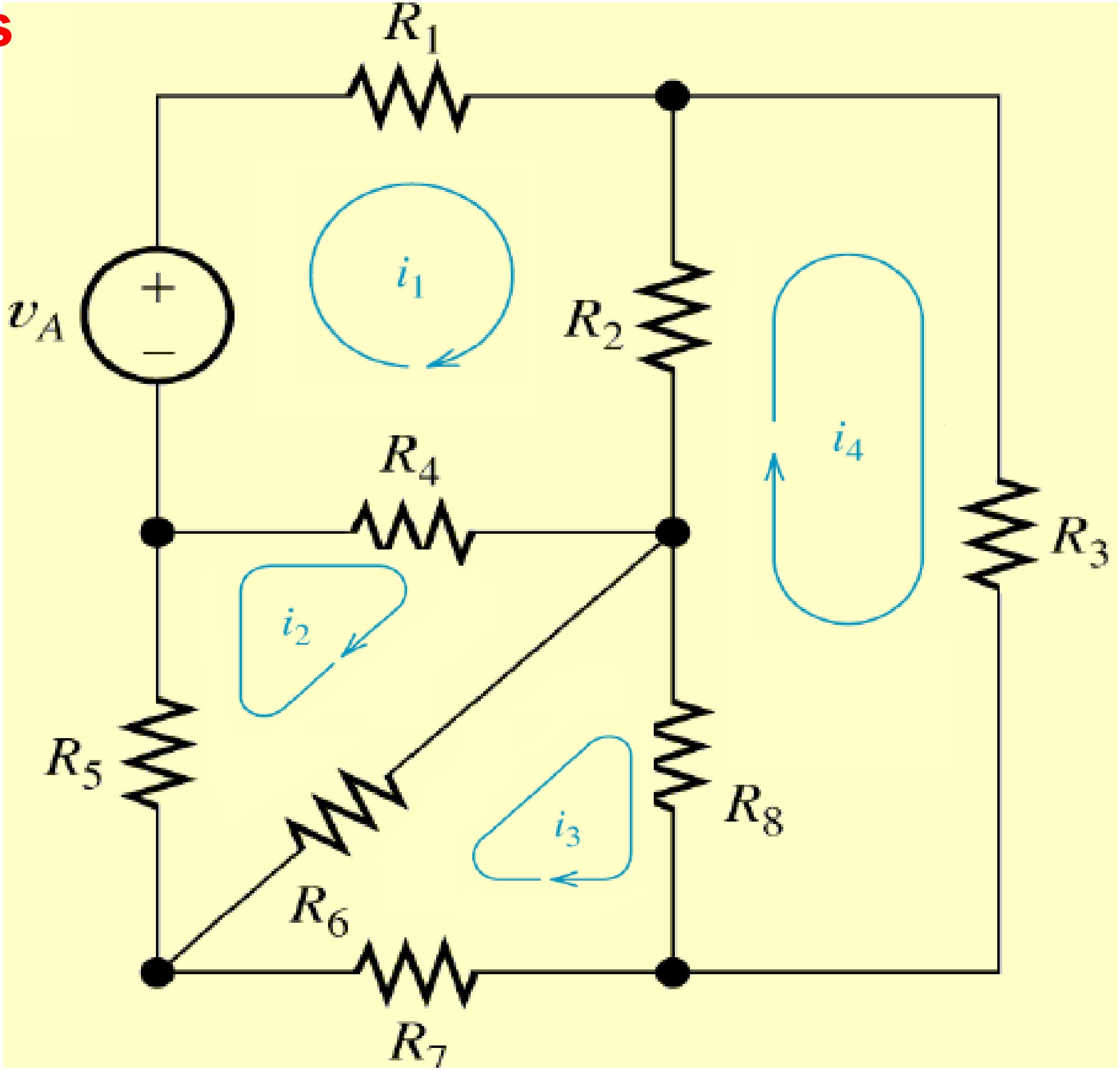
Mesh Currents



$$I_2 = i_1 - i_3$$

$$I_3 = i_1 - i_2$$

Mesh Currents



Mesh Analysis

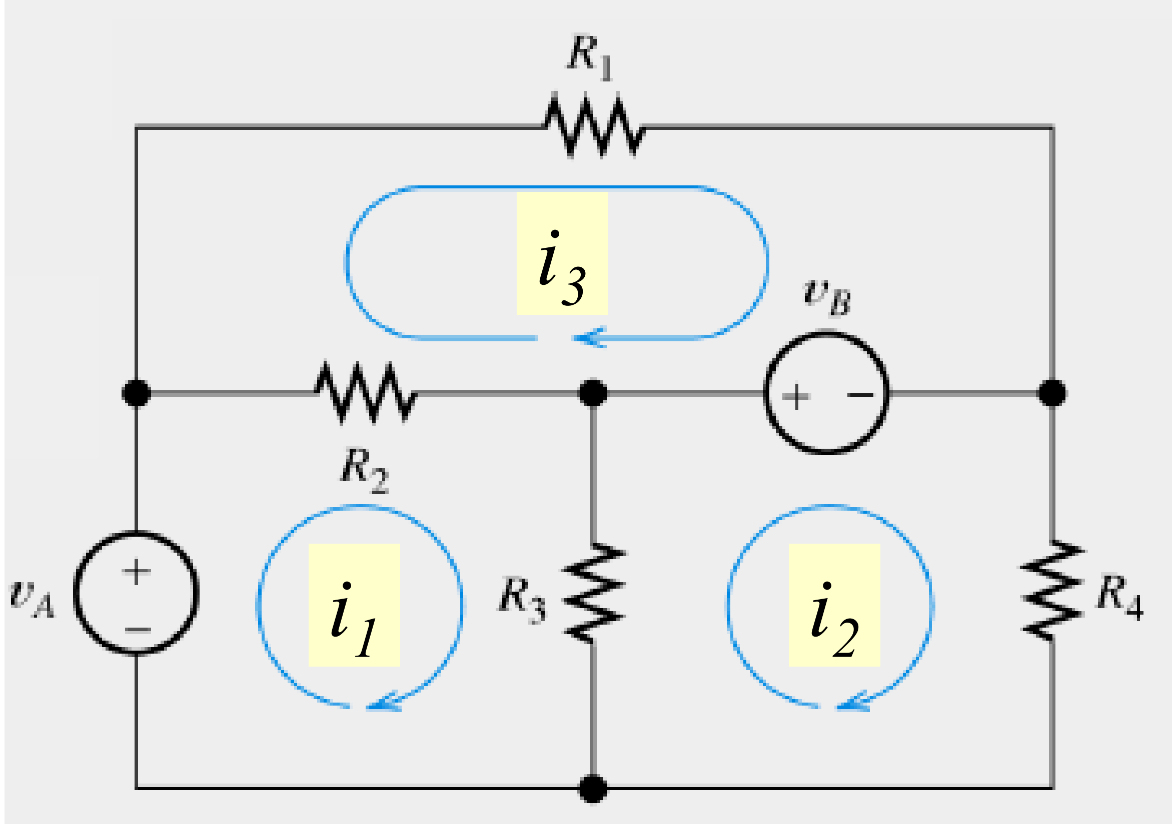
Steps to determine the mesh currents:

1. Assign mesh currents i_1, i_2, , in to the n meshes.

2. Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.

3. Solve the resulting n simultaneous equations to get the mesh currents.

Example



Mesh-1

$$R_2(i_1 - i_3) + R_3(i_1 - i_2) - v_A = 0$$

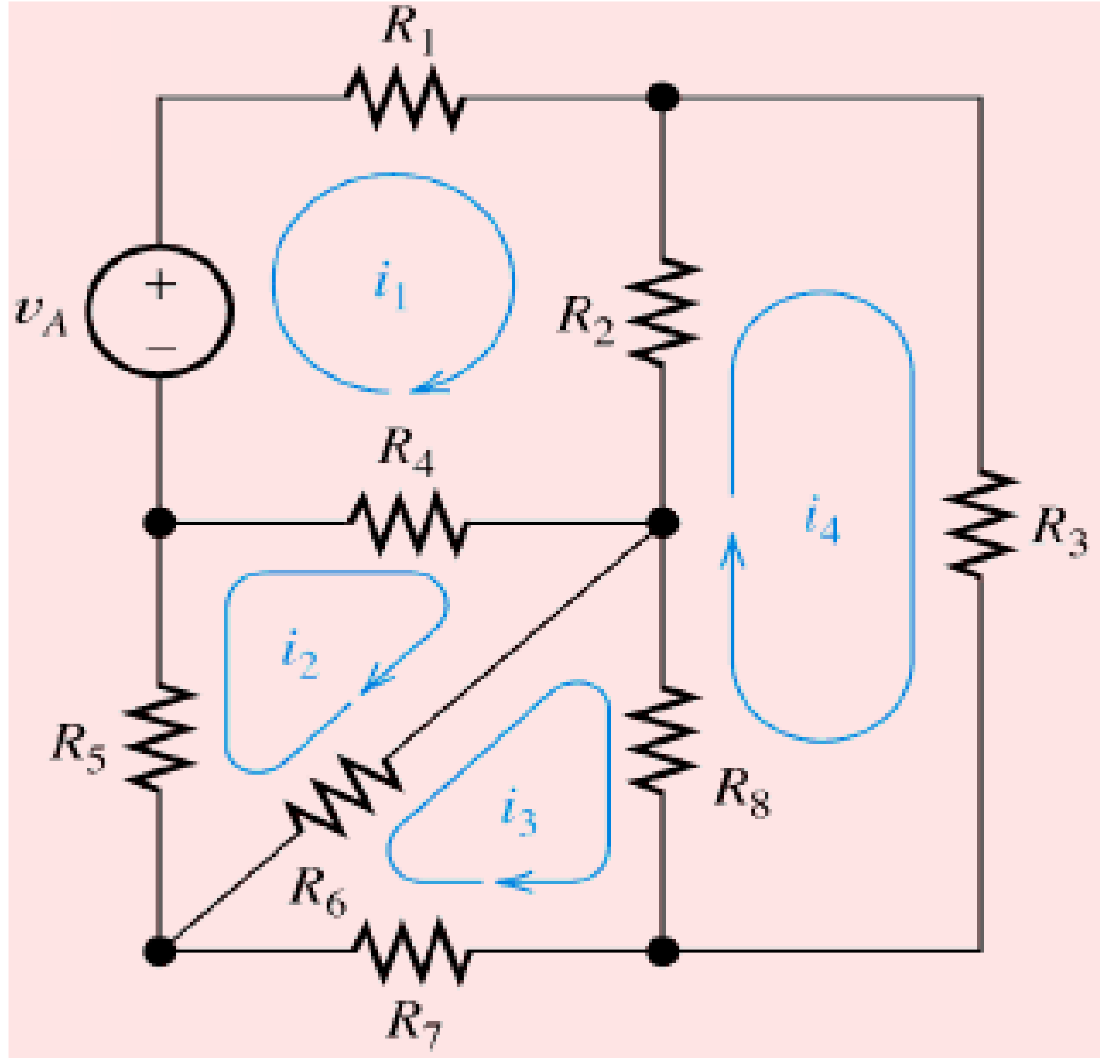
Mesh-2

$$R_3(i_2 - i_1) + v_B + R_4 i_2 = 0$$

Mesh-3

$$R_2(i_3 - i_1) + R_1 i_3 - v_B = 0$$

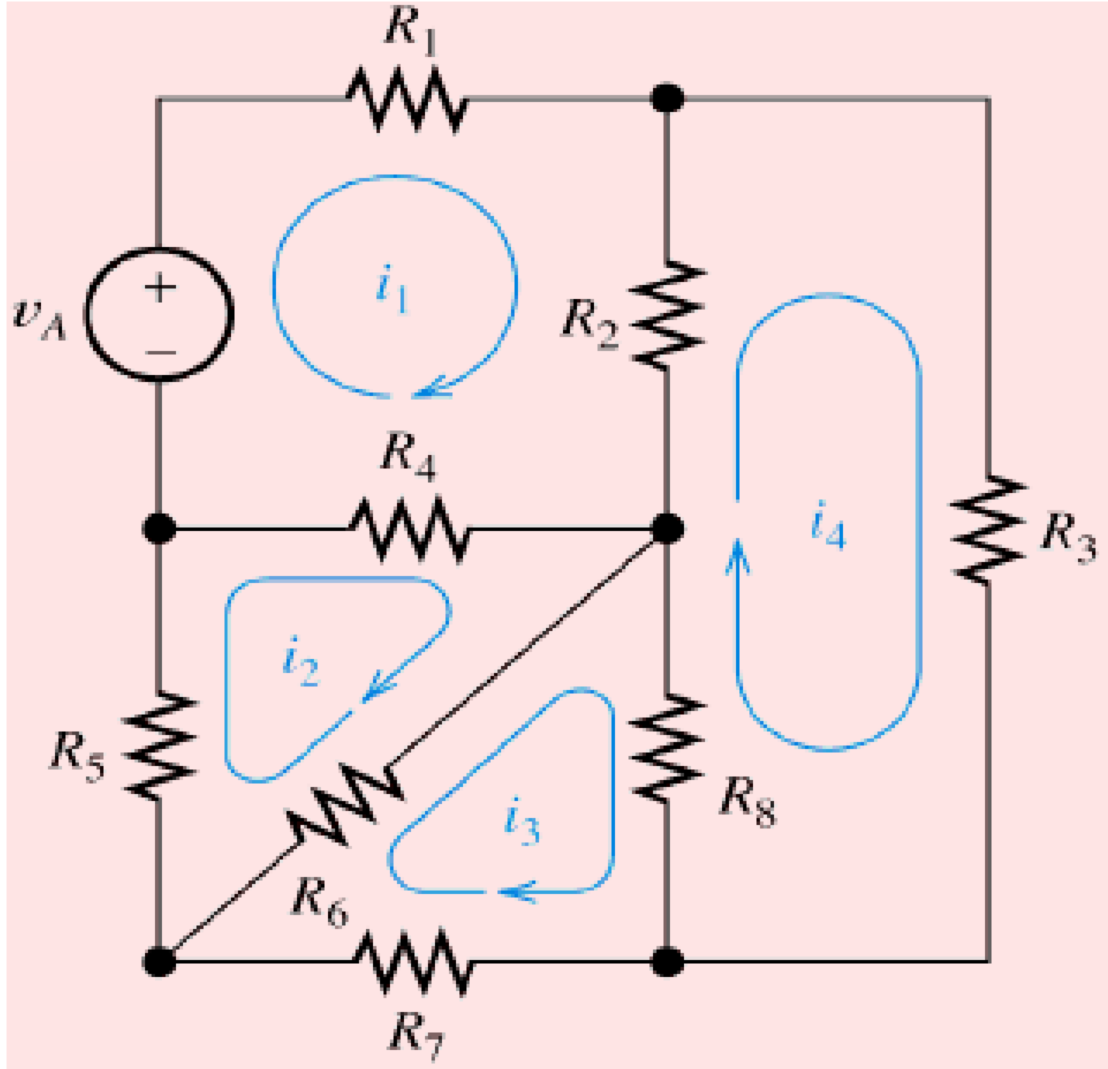
Example



Mesh-1 $$R_1 i_1 + R_2(i_1 - i_4) + R_4(i_1 - i_2) - v_A = 0$$

Mesh-2 $$R_5 i_2 + R_4(i_2 - i_1) + R_6(i_2 - i_3) = 0$$

Example



Mesh-3

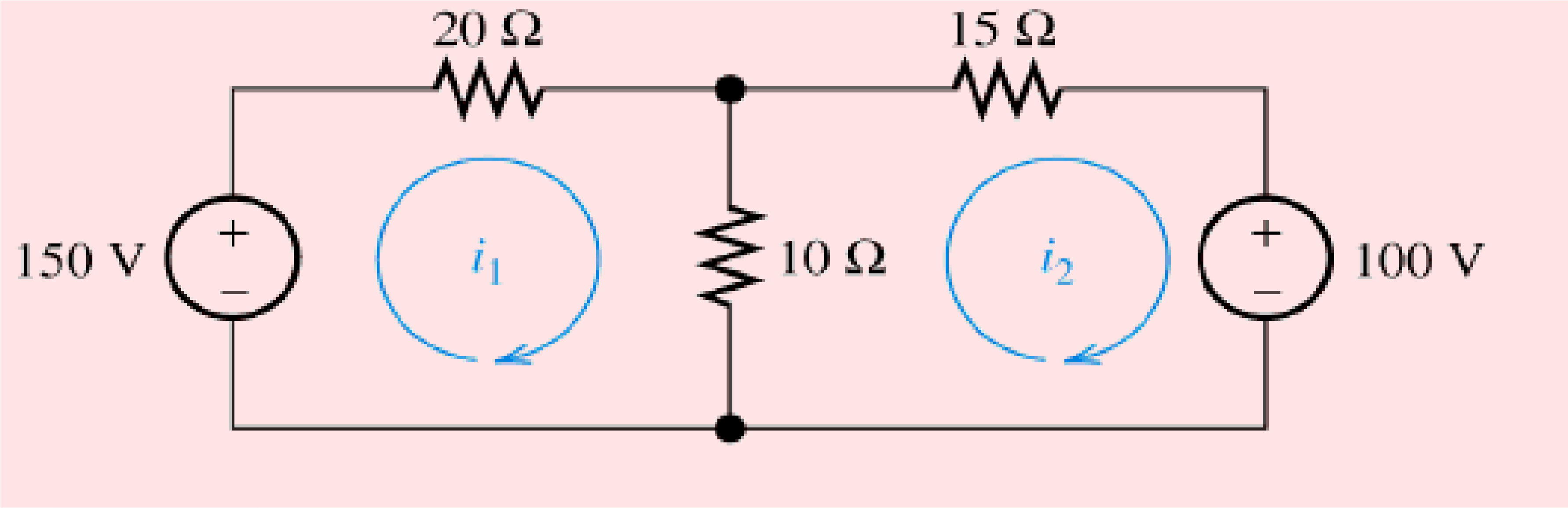
$$R_7 i_3 + R_6(i_3 - i_2) + R_8(i_3 - i_4) = 0$$

Mesh-4

$$R_3 i_4 + R_2(i_4 - i_1) + R_8(i_4 - i_3) = 0$$

Example

Solve for the currents in each element in the circuit



$$\text{mesh}\,1: 20i_1 + 10(i_1 - i_2) - 150 = 0$$

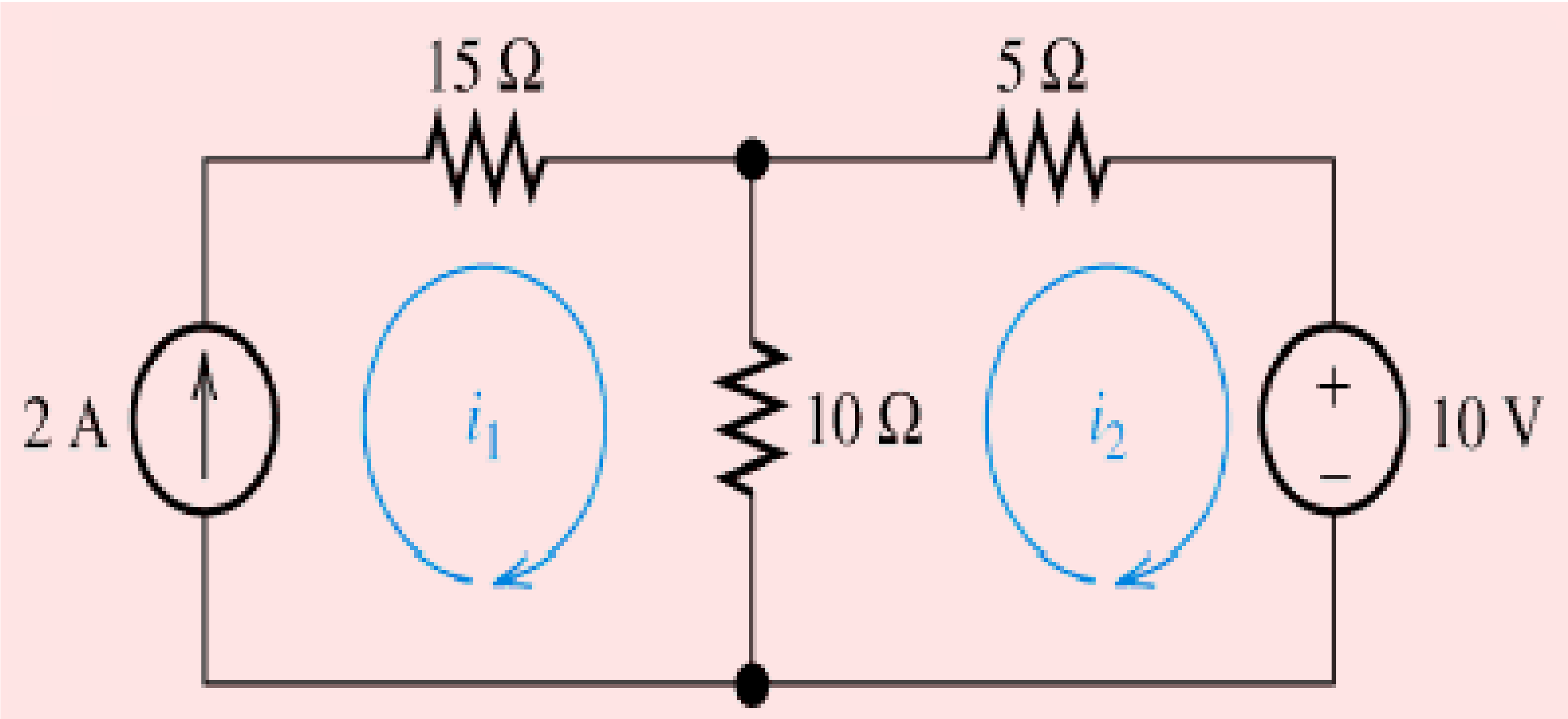
$$\text{mesh}\,2: 10(i_2 - i_1) + 15i_2 + 100 = 0$$

$$30\ i_1 - 10\ i_2 = 150 \qquad i_1 = 4.231\,A$$

$$-10\ i_1 + 25\ i_2 = -100 \qquad i_2 = -2.308\,A$$

The current in the 10 - Ω is i_1-i_2 = 6.539 A

Mesh Currents in Circuits Containing Current Sources



$$15i_1 + 10(i_1 - i_2) + ? = 0$$

$$i_1 = 2\text{A}$$

$$10(i_2 - i_1) + 5i_2 + 10 = 0$$