# Lec9 - Neural Networks

## CS 783 - Visual Recognition

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# Outline

# How to Improve Representation further

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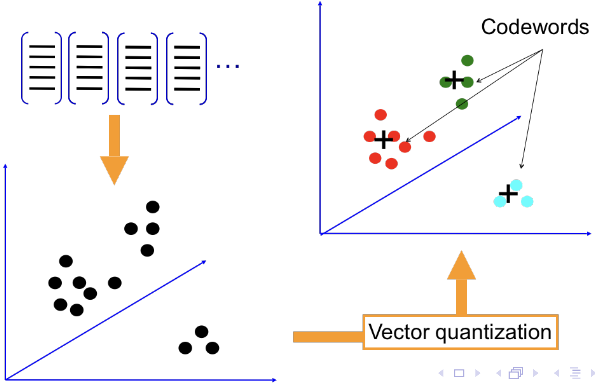

# Kernel Codebook Encoding

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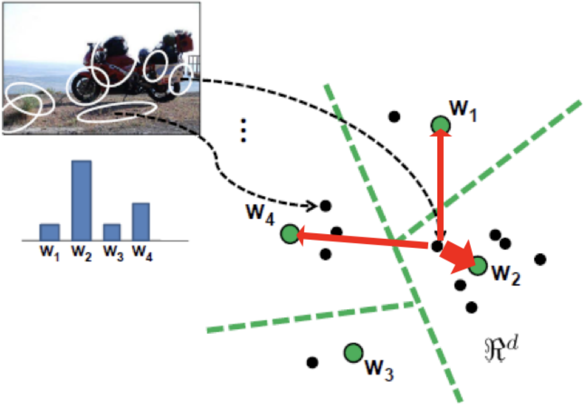

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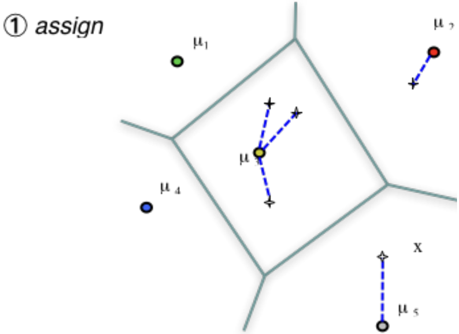

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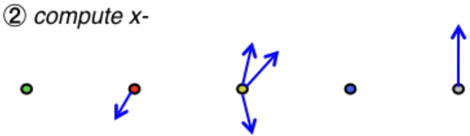

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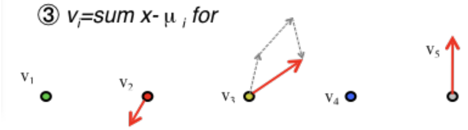


# VLAD

These are the steps for VLAD summarized as follows

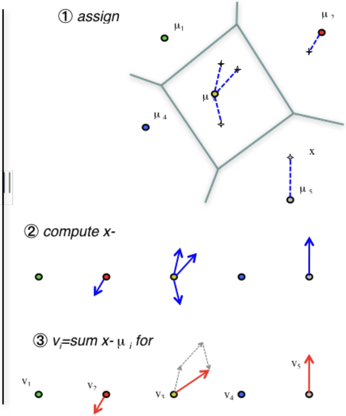
$$\{\mu_i, i = 1 \dots N\}$$

$$X = \{x_t, t = 1 \dots T\}$$

$$\mathrm{NN}(x_t) = \arg\min_{\mu_i} ||x_t - \mu_i||$$

$$v_i = \sum_{x_t : \mathrm{NN}(x_t) = \mu_i} x_t - \mu_i$$

$$\ell_2$$



# VLAD

This can be understood by visualizing the variance for the case where there are 16 centroids and we consider the variance for these 16 centroids. This bag of words representation will have a length of $16 + 16 \times 128$ as its length for the case of 16 centroids

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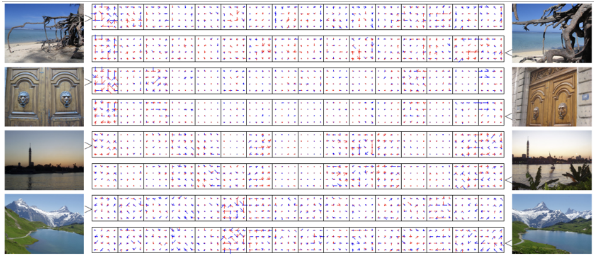

# Locally linear coordinate representation

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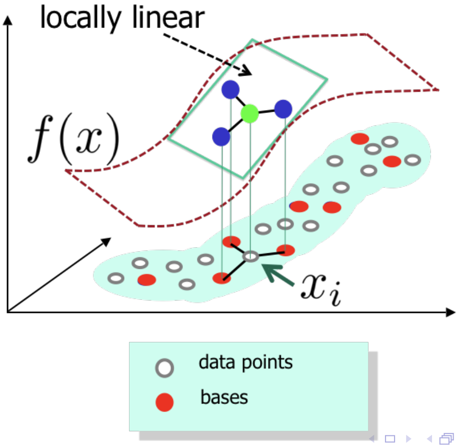

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- Coding for x, to obtain its sparse representation a

  Step 1 – ensure locality: find the K nearest bases

$$[\phi_j]_{j \in J(x)}$$

  Step 2 – ensure low coding error:

$$\min_a \left\| x - \sum_{j \in J(x)} a_{i,j} \phi_j \right\|^2, \quad \text{s.t.} \sum_{j \in J(x)} a_{i,j} = 1$$

# Locally linear coordinate Overview

The pipeline for the whole framework is presented as follows. This appears to be very similar to the conventional deep learning framework that consists of convolution and pooling operations

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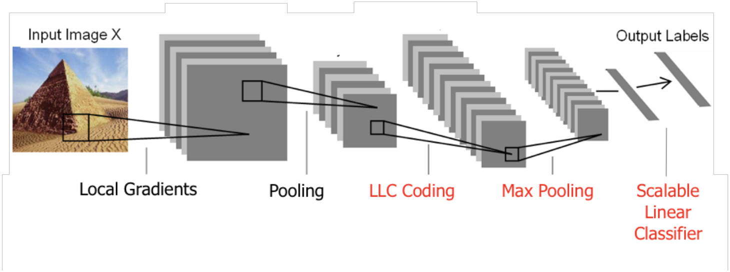

# Outline

1. Earlier Representations
2. Overview: DL for Categorization
3. Learning Perceptrons
4. Multi Layer Perceptron

# AlexNet

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## ImageNet Classification with Deep Convolutional Neural Networks

**Alex Krizhevsky**
University of Toronto
kriz@cs.utoronto.ca

**Ilya Sutskever**
University of Toronto
ilya@cs.utoronto.ca

**Geoffrey E. Hinton**
University of Toronto
hinton@cs.utoronto.ca

# AlexNet Architecture

The architecture adopted by Alex Krizhevsky et al is as shown in the figure below. This architecture consists of various convolution and pooling layers with the parameters being learned. As can be observed these are conceptually similar to the architectures being followed by researchers before deep learning approach was introduced.

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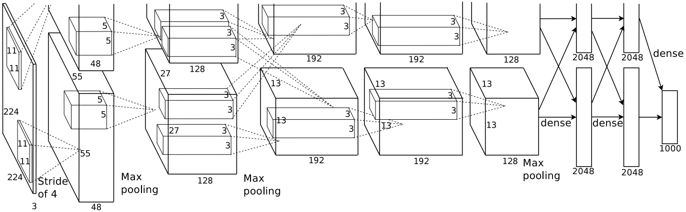

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| **Model** | **Top-1** | **Top-5** |
|---|---|---|
| *Sparse coding [2]* | *47.1%* | *28.2%* |
| *SIFT + FVs [24]* | *45.7%* | *25.7%* |
| CNN | **37.5%** | **17.0%** |

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| **Model** | **Top-1 (val)** | **Top-5 (val)** | **Top-5 (test)** |
|---|---|---|---|
| *SIFT + FVs [7]* | — | — | *26.2%* |
| 1 CNN | 40.7% | 18.2% | — |
| 5 CNNs | 38.1% | 16.4% | **16.4%** |
| 1 CNN* | 39.0% | 16.6% | — |
| 7 CNNs* | 36.7% | 15.4% | **15.3%** |

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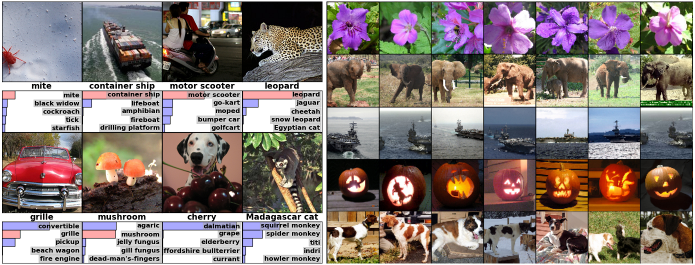


Figure 4: **(Left)** Eight ILSVRC-2010 test images and the five labels considered most probable by our model. The correct label is written under each image, and the probability assigned to the correct label is also shown with a red bar (if it happens to be in the top 5). **(Right)** Five ILSVRC-2010 test images in the first column. The remaining columns show the six training images that produce feature vectors in the last hidden layer with the smallest Euclidean distance from the feature vector for the test image.

# Outline

# A Perceptron

- Consider a model of a linear perceptron that receives an input vector $\mathbf{x}$ of dimension $n$ i.e. $x_1, x_2, \cdots, x_n$. The output for such a model can be obtained through the following equation

$$o = f(\sum_{i=1}^{n} w_i x_i + \theta) \tag{1}$$

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- The bias parameter $\theta$ can be included in the weight vector by assuming a constant input $x_0$ with value 1 allowing the use of $n+1$ weights. In general we therefore consider an input vector $\mathbf{x}$ that is multiplied by a parameter vector $\mathbf{w}$

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- The function $f()$ is a function that assigns for instance 0 for negative argument and 1 for positive argument

# Illustration of a Perceptron

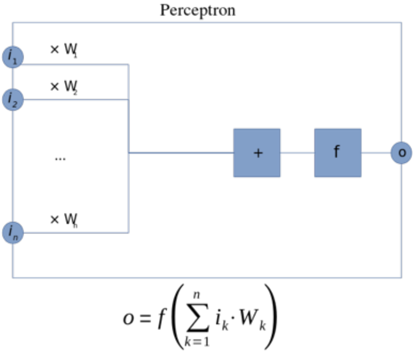


Figure: Perceptron

# The perceptron learning algorithm

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- A naive algorithm would involve generating various random perceptron weight vectors and checking them till we find a weight vector that separates the training samples.

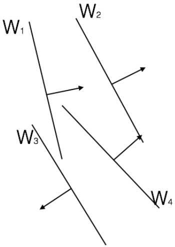


Figure: Naive algorithm for training a perceptron that consists of just sampling random weight vectors

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3. Update the weights according to the following equation:

$$w_j(t+1) = w_j(t) + \eta(d-y)x \tag{2}$$

where
$d$ is the desired output,
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4. Repeat steps 2 and 3 until the iteration error is less than a user-specified error threshold or a predetermined number of iterations have been completed.

# Illustration of the Perceptron learning algorithm

Step 1

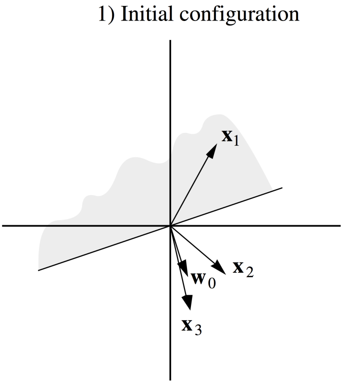


Figure: The different steps for the Perceptron learning algorithm. (fig. credit Rojas 1996)

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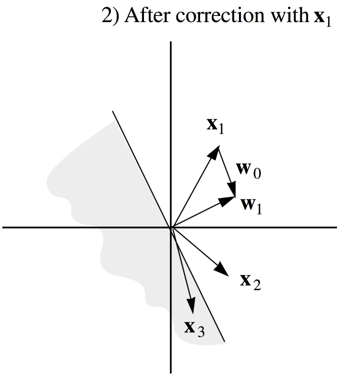
Step 2

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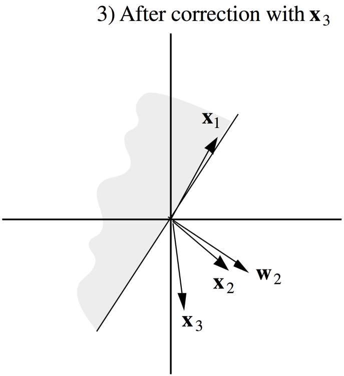


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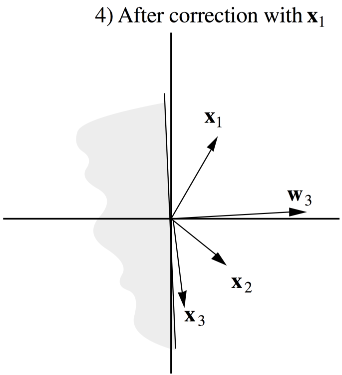
Step 4

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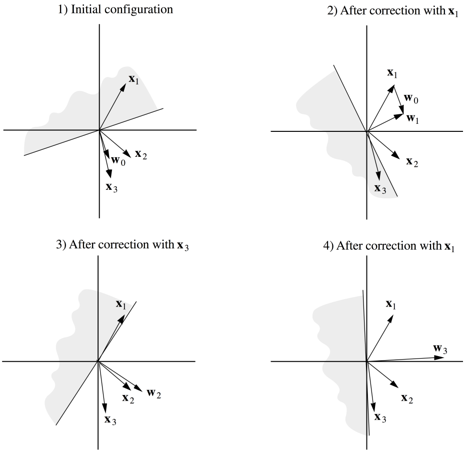


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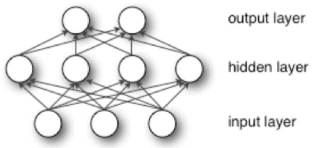
## Next Steps

- The perceptron of optimal stability together with the kernel trick provide the conceptual foundations of the support vector machine. We next consider the model of Multi-layer Perceptrons

# Outline

# Multi-layer Perceptron

- The multi-layer perceptron addressed the drawbacks and limitations of the perceptron.

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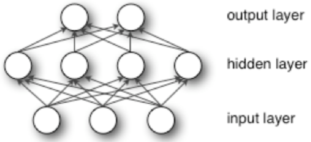


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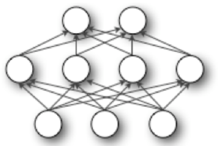
- The multi-layer perceptron addressed the drawbacks and limitations of the perceptron.
- Though it is termed Multi-layer perceptron, it does not use the perceptron algorithm for learning
- For a single perceptron the method used is the delta learning rule and the method used for training the whole network is the backpropagation algorithm

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- An alternative approach is to obtain the error across *all samples* and move the weight vector iteratively to minimize the error
- Suppose we have to guess the price of two items $A$ *and* $B$ from a shopkeeper who just gives the total price
- We do this by buying multiple items of A and B and summing the error from the total price. We then adjust the price based on the total error that we observe. This updation based on error is called the delta learning rule

# Delta learning rule: Example

Consider that we go to a shop and buy some object and observe the error in obtaining the price per sample


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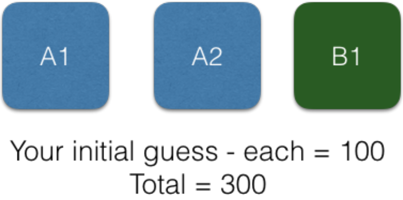


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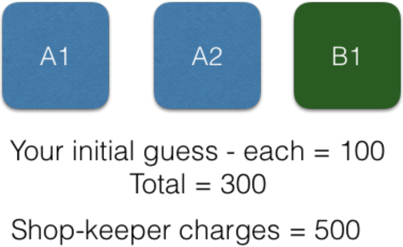


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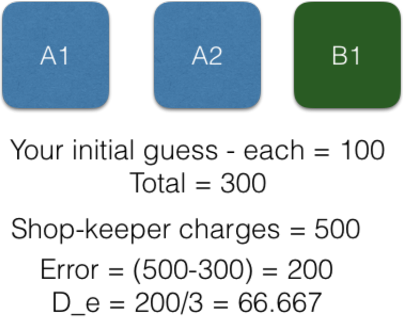


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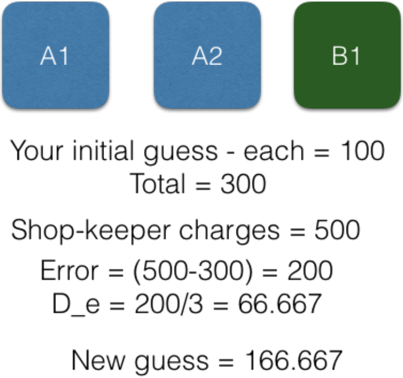


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- Obtain error $E$ given by

$$E = \sum_j \frac{1}{2}(t_j - y_j)^2 \tag{3}$$

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- The contribution of the weight vector $w_{ji}$ to the error can be used to update the weight vector. This is obtained by taking the gradient of the error $E$ with respect to the weight vector.

$$\frac{\partial E}{\partial w_{ji}} = \frac{\partial \left( \sum_j \frac{1}{2}(t_j - y_j)^2 \right)}{\partial w_{ji}} \tag{4}$$

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- The equation 4 can be solved using chain rule as

$$\frac{\partial E}{\partial w_{ji}} = \frac{\partial \left( \sum_j \frac{1}{2}(t_j - y_j)^2 \right)}{\partial y_j} \frac{\partial y_j}{\partial w_{ji}} \tag{5}$$

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- As $\frac{\partial h_j}{\partial w_{ji}}$ is just $x_i$ we obtain the following final expression

$$\frac{\partial E}{\partial w_{ji}} = -(t_j - y_j)g'(h_j)x_i \tag{7}$$

where $g'(h_j)$ is $\frac{\partial y_j}{\partial h_j}$

# The delta learning rule - formulation

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- Based on this we can update the weight vector $w_{ji}$ using the following update rule

$$w_{ji}^{t+1} = w_{ji}^{t} + \eta \triangle w_{ji} \tag{8}$$

  where $\triangle w_{ji}$ is $\frac{\partial E}{\partial w_{ji}}$ and $\eta$ is the learning rate parameter

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- The delta rule is a specialisation of the back-propagation algorithm

# Sigmoidal Activation function

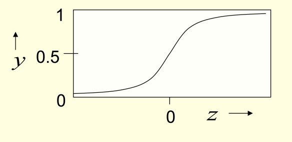


Figure: Sigmoidal Activation function

- So far we have not specified the activation function $g(h_j)$ that is used. We have $y = g(h_j)$ where $h_j = \sum_i x_i w_{ji}$. The learning procedure iteratively approaches the target output and by adapting the learning rate one can approach close to the target output

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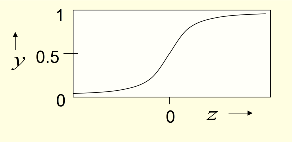


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- One such activation function that can be used is sigmoidal activation function (to approximate the discontinuous Heaviside function used in perceptrons).

# Sigmoidal Activation function

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as $\frac{\partial y_j}{\partial h_j} = \frac{-1}{(1+e^{-h_j})^2}(-e^{-h_j}) = (\frac{1}{1+e^{-h_j}})(\frac{e^{-h_j}}{1+e^{-h_j}}) = y_j(1 - y_j)$

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- The equation 7 can now be obtained by substituting the gradient of the function as

$$\frac{\partial E}{\partial w_{ji}} = -(t_j - y_j)y_j(1 - y_j)x_i \tag{11}$$

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- It relies on the computation of gradient of error $E$ with respect to any weight vector in the network.

# Backpropagation Algorithm

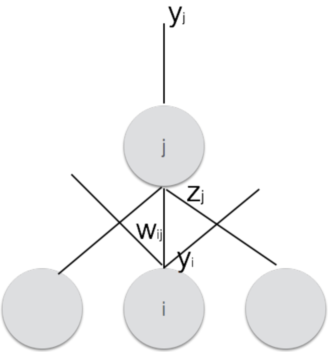


Figure: Backpropagation algorithm

- $E = \frac{1}{2}\sum_j (t_j - y_j)^2$ Therefore we have

$$\frac{\partial E}{\partial y_j} = -(t_j - y_j). \tag{12}$$

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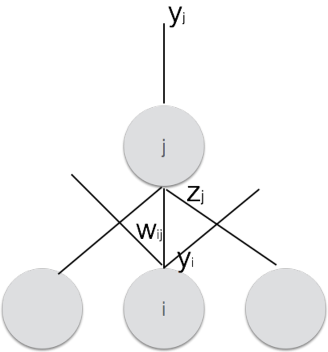


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- Next considering the gradient with respect to $z_j$ we have

$$\frac{\partial E}{\partial z_j} = \frac{\partial y_j}{\partial z_j}\frac{\partial E}{\partial y_j}. \tag{13}$$

which is obtained as

$$\frac{\partial E}{\partial z_j} = y_j(1 - y_j)\frac{\partial E}{\partial y_j}. \tag{14}$$

# Backpropagation Algorithm

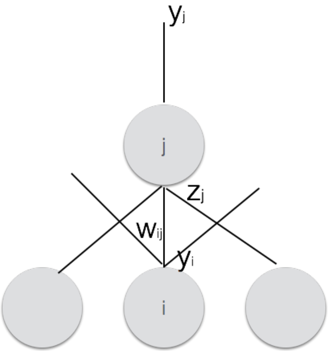


Figure: Backpropagation algorithm

- The expression for error with respect to $y_i$ is obtained as

$$\frac{\partial E}{\partial y_i} = \sum_j \frac{\partial z_j}{\partial y_i} \frac{\partial E}{\partial z_j}. \qquad (15)$$

that is given by

$$\frac{\partial E}{\partial y_i} = \sum_j w_{ij} \frac{\partial E}{\partial z_j}. \qquad (16)$$

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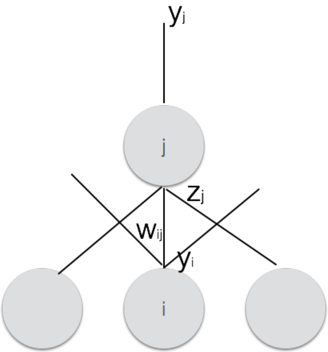


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$$\frac{\partial E}{\partial w_{ij}} = y_i \frac{\partial E}{\partial z_j}. \tag{18}$$

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  - Online: After seeing each training sample
  - Full: After seeing all the training samples
  - mini-batch: After seeing a small set of training samples. This is the most common case.
- Gradient descent with backpropagation is not guaranteed to find the global minimum of the error function, but only a local minimum

# The End