ESc201 : Introduction to Electronics

Frequency Domain Response

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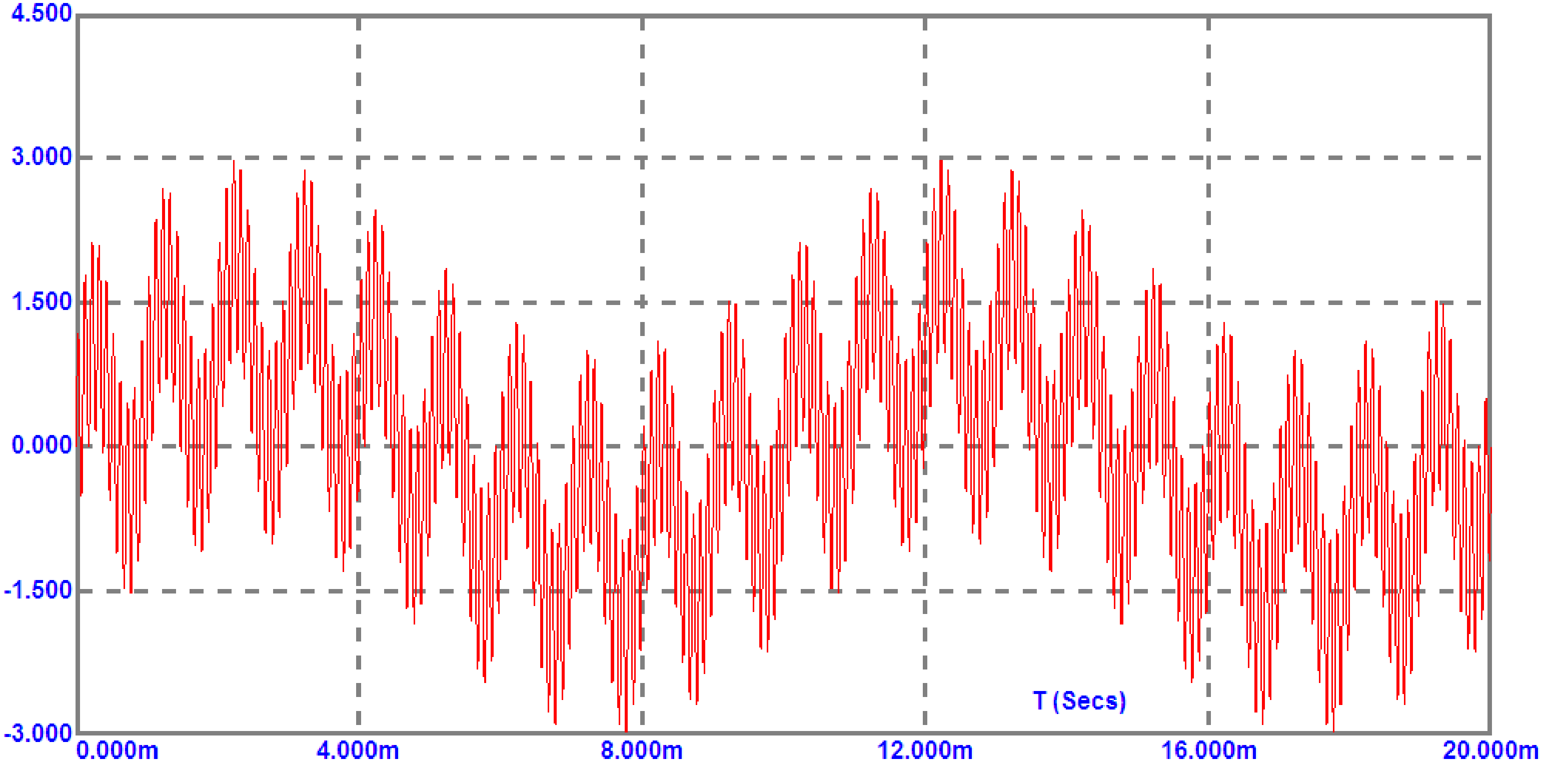
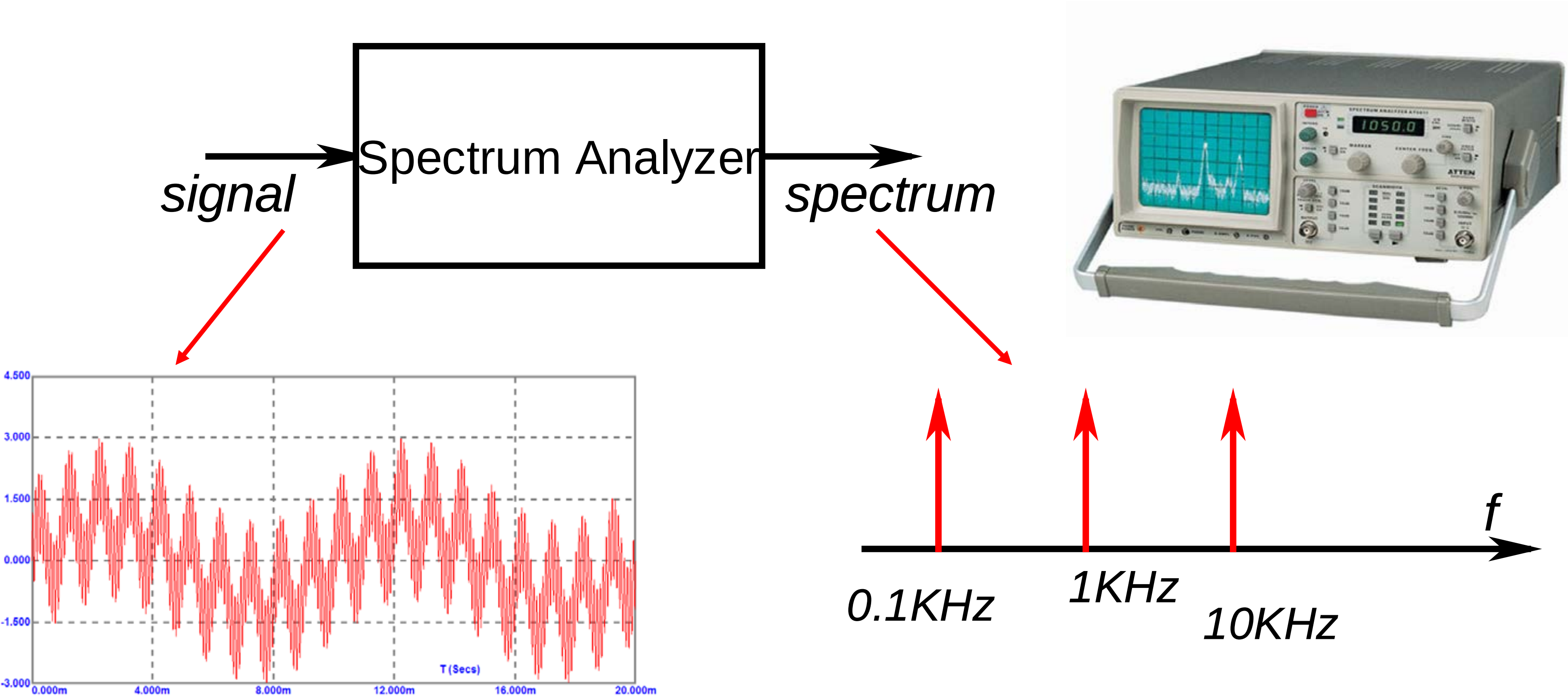
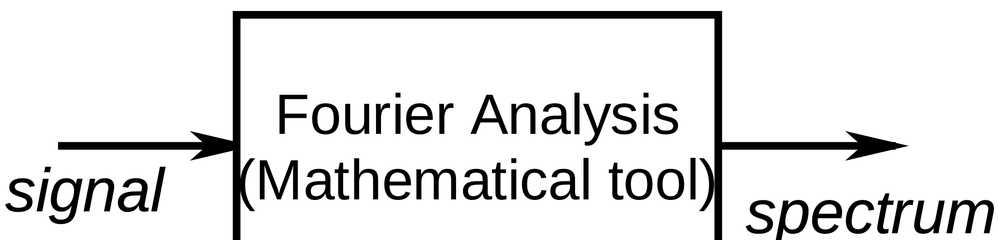
Time domain vs. Frequency domain analysis

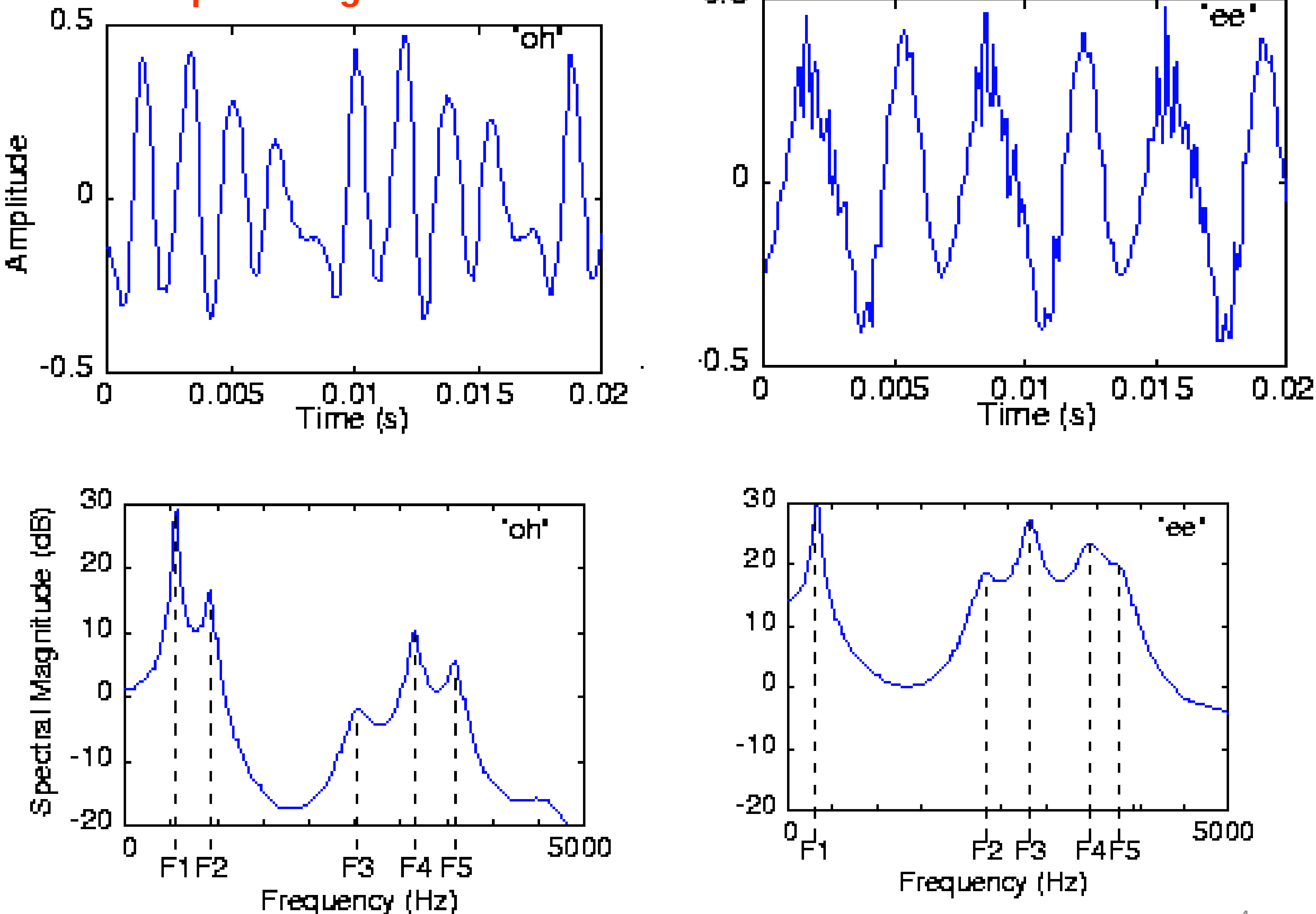
Signal




signal
Spectrum Analyzer
spectrum
1050.0
ATTEN
4.500
3.000
1.500
0.000
-1.500
-3.000
0.000m
4.000m
8.000m
12.000m
16.000m
20.000m
T (Secs)
f
0.1KHz
1KHz
10KHz

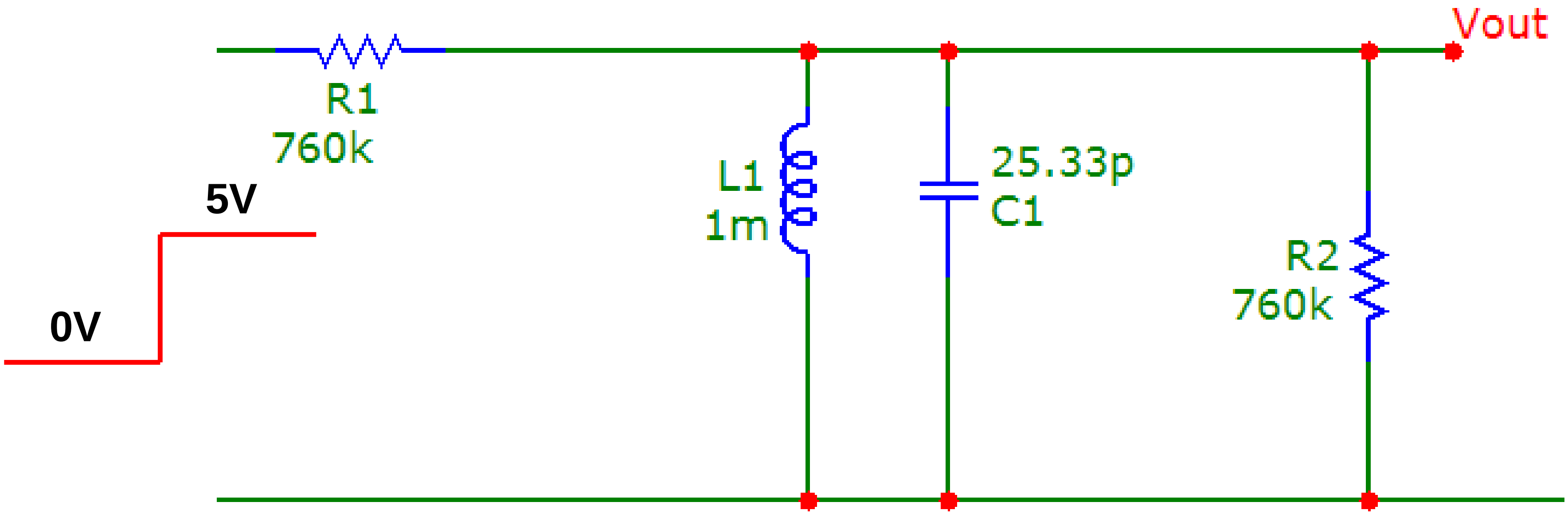

signal
Fourier Analysis
(Mathematical tool)
spectrum

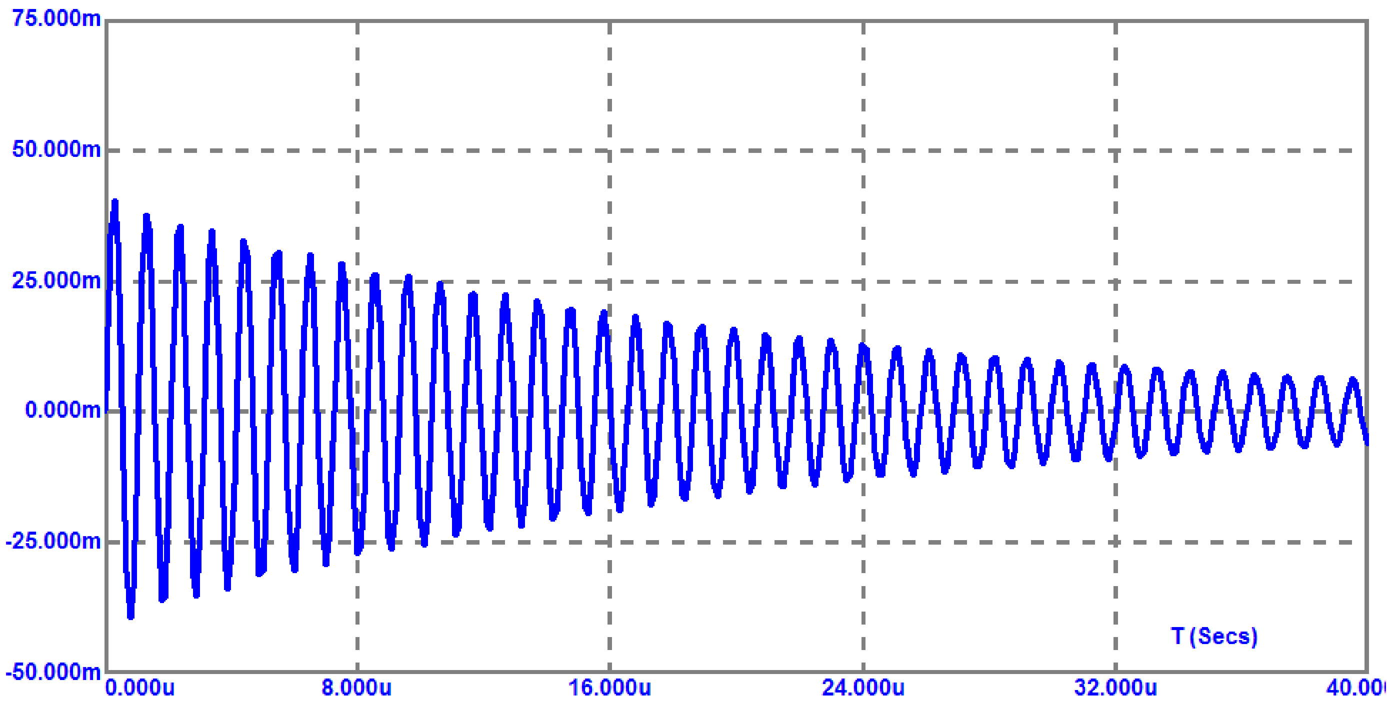
Speech signal



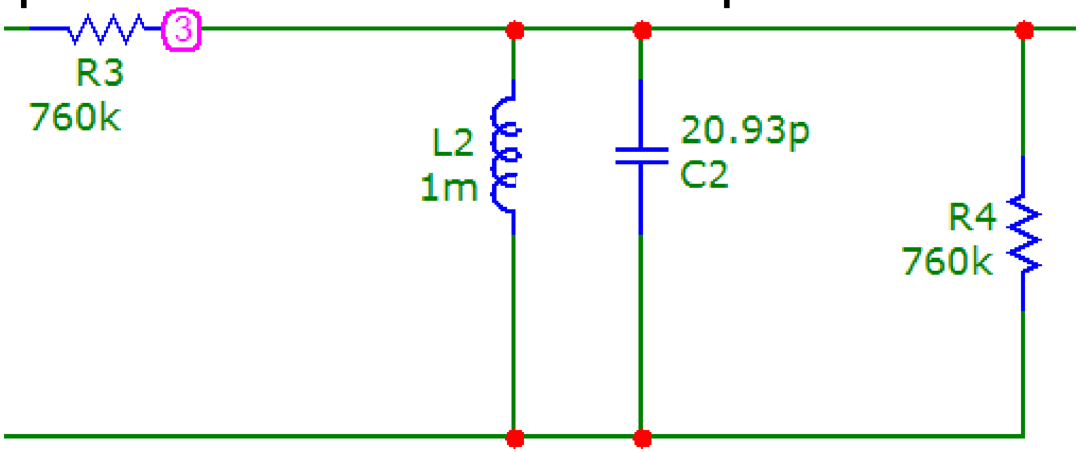
System

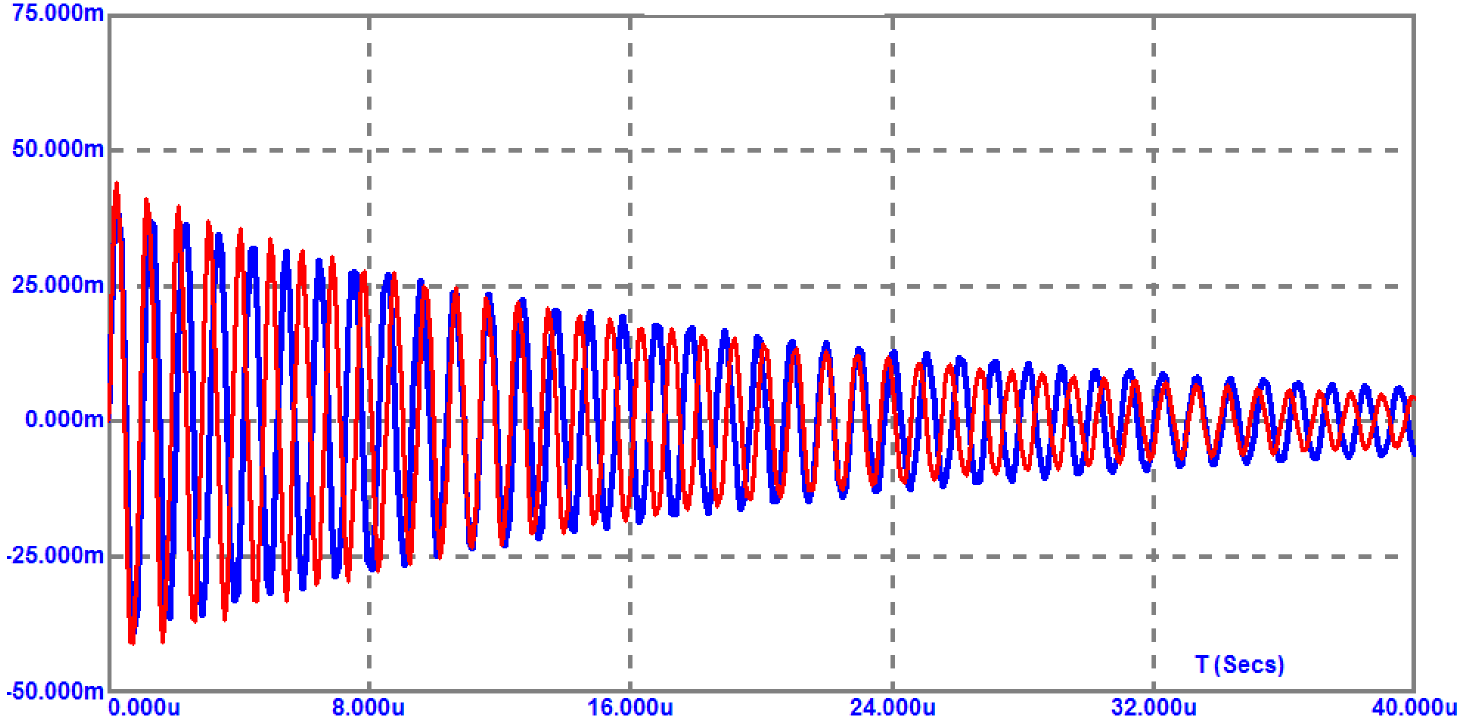
What does this circuit do ?





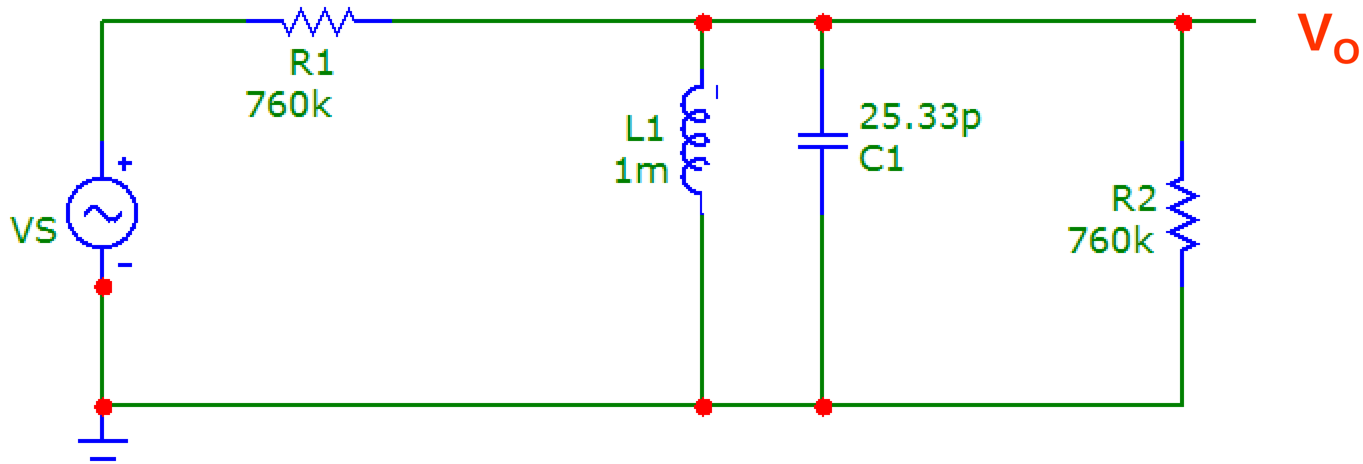
Suppose the capacitor is reduced to ~21pF.

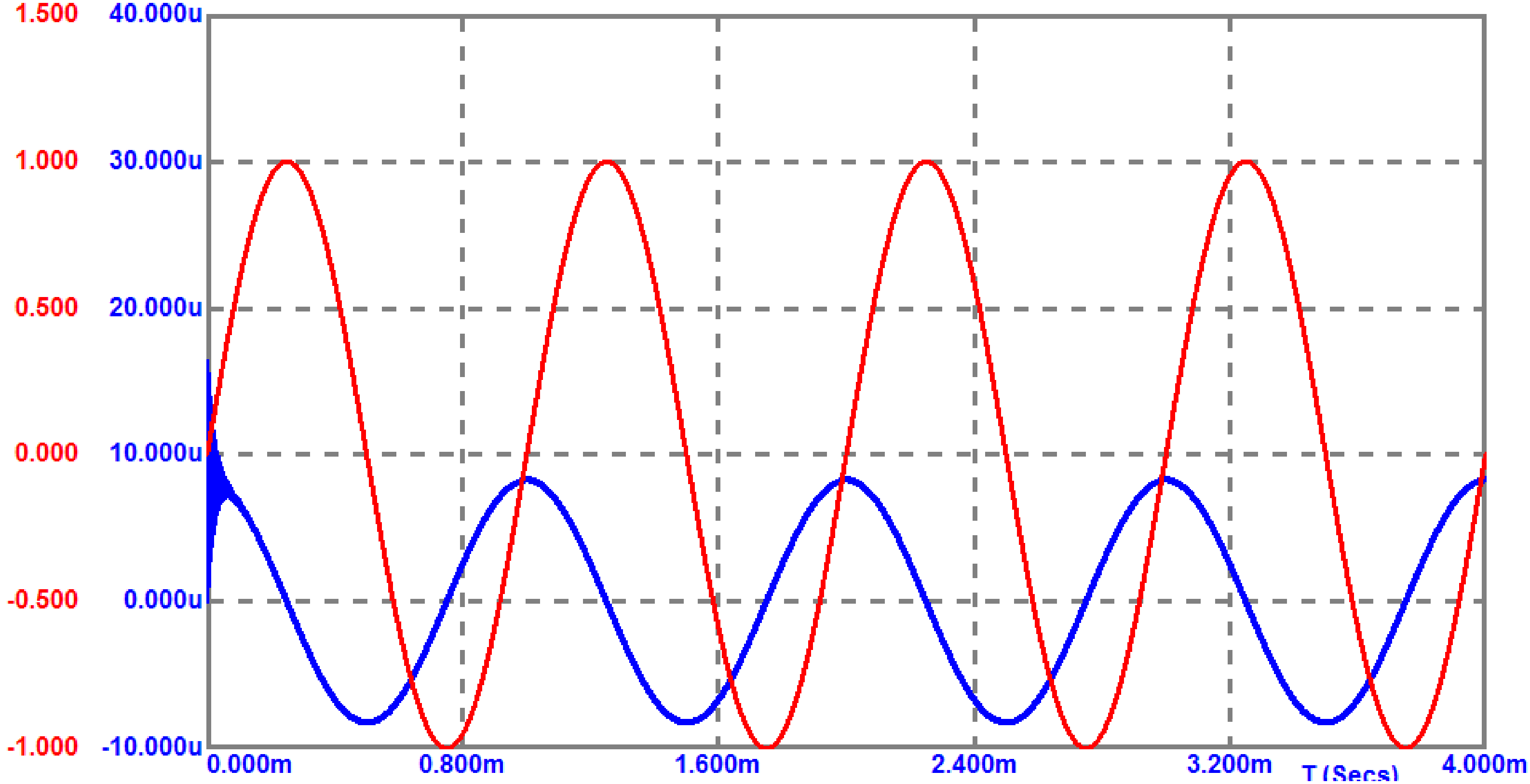




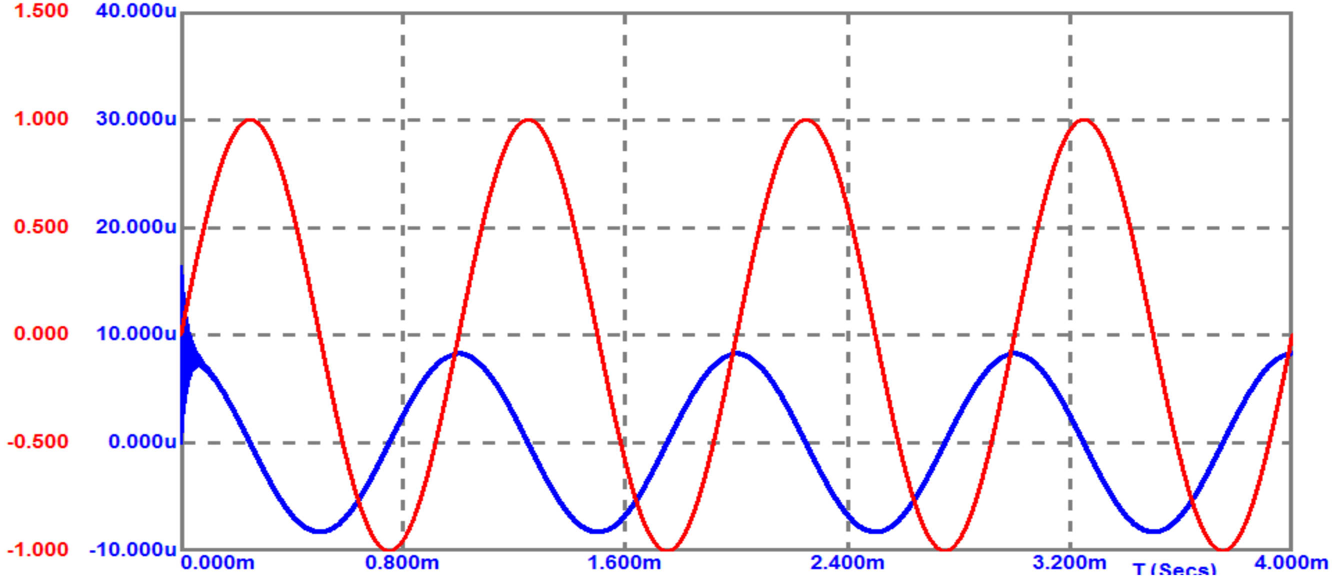
It is hard to find out what impact the change in capacitor has on circuit behavior

Frequency domain analysis

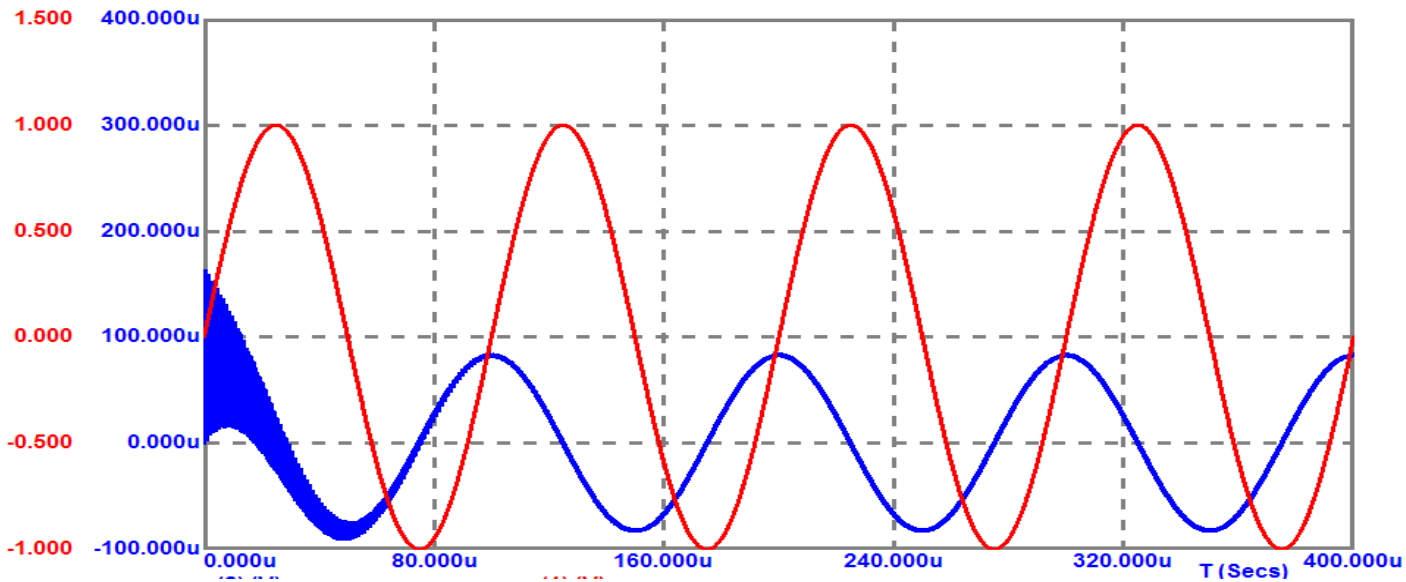




Measure response at many different frequencies for a constant input amplitude

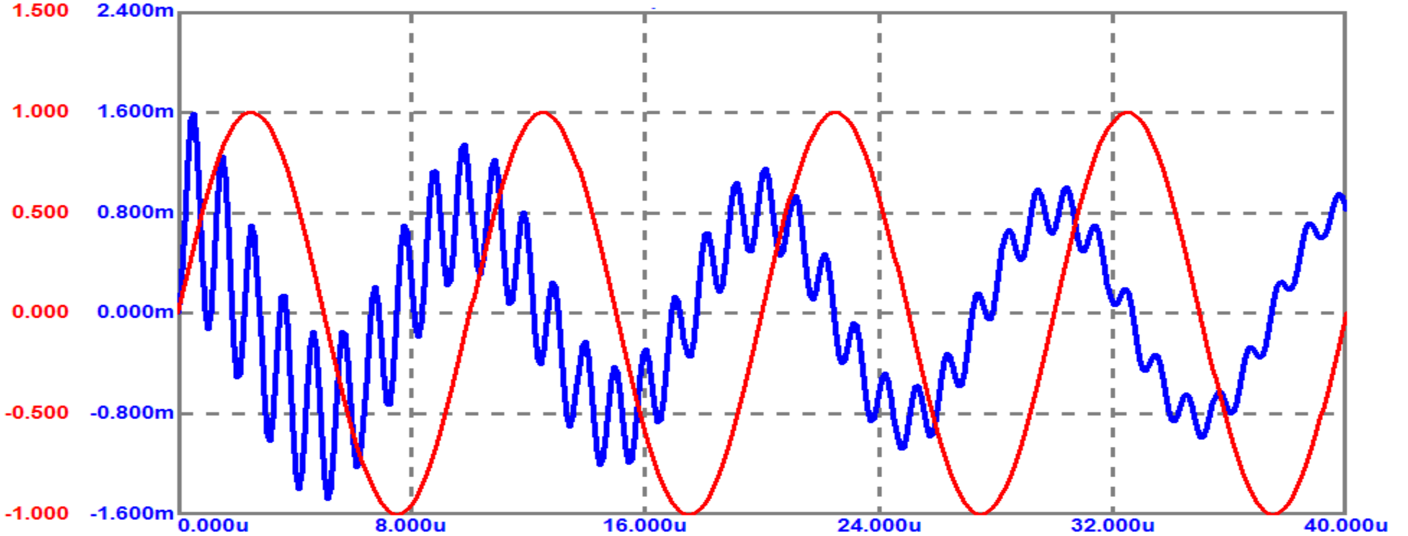


f=1KHz

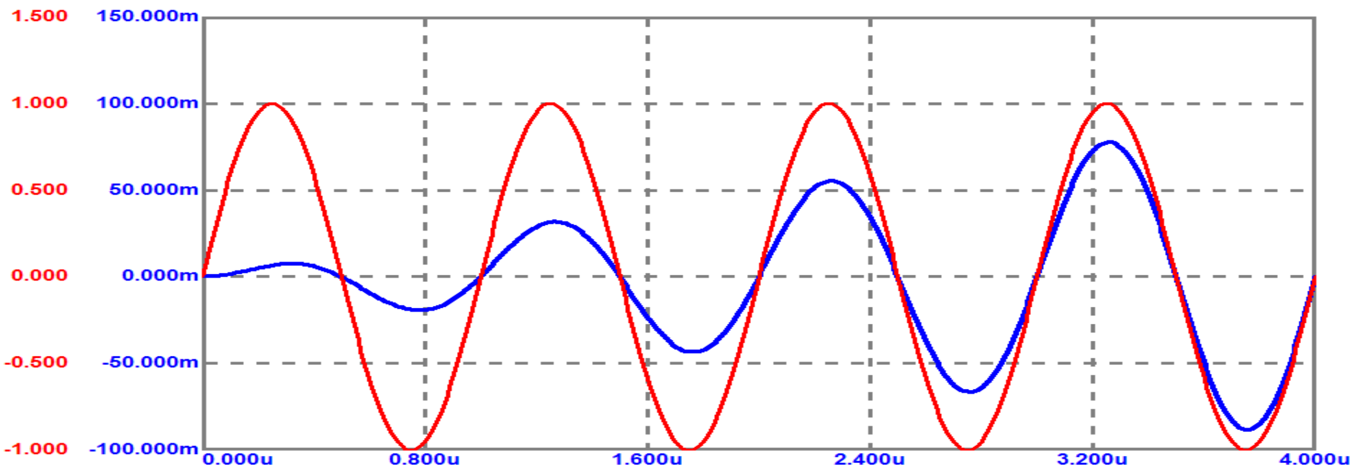


f=10KHz

Measure response at many different frequencies for a constant input amplitude



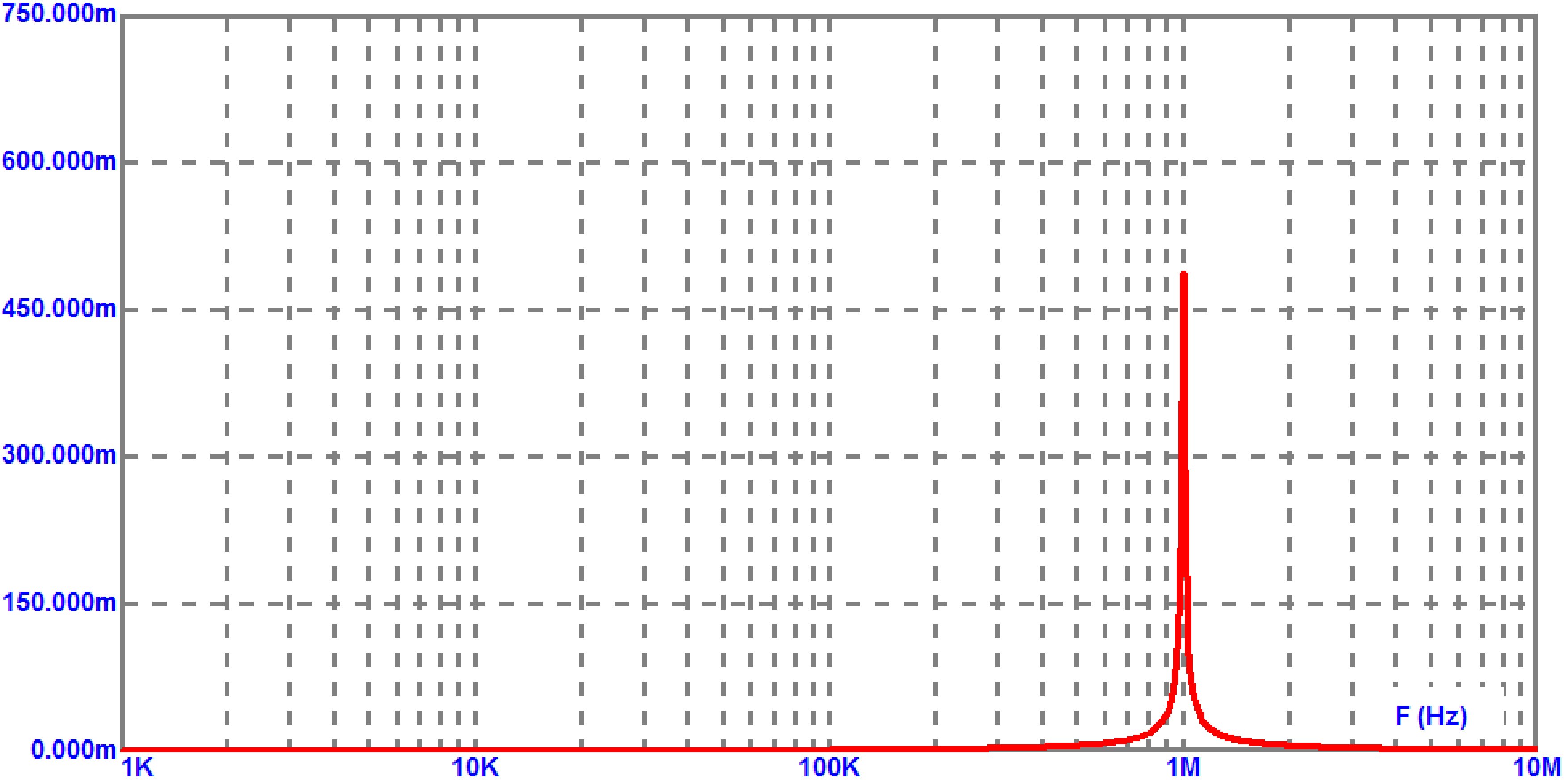
f=100KHz



f=1000KHz

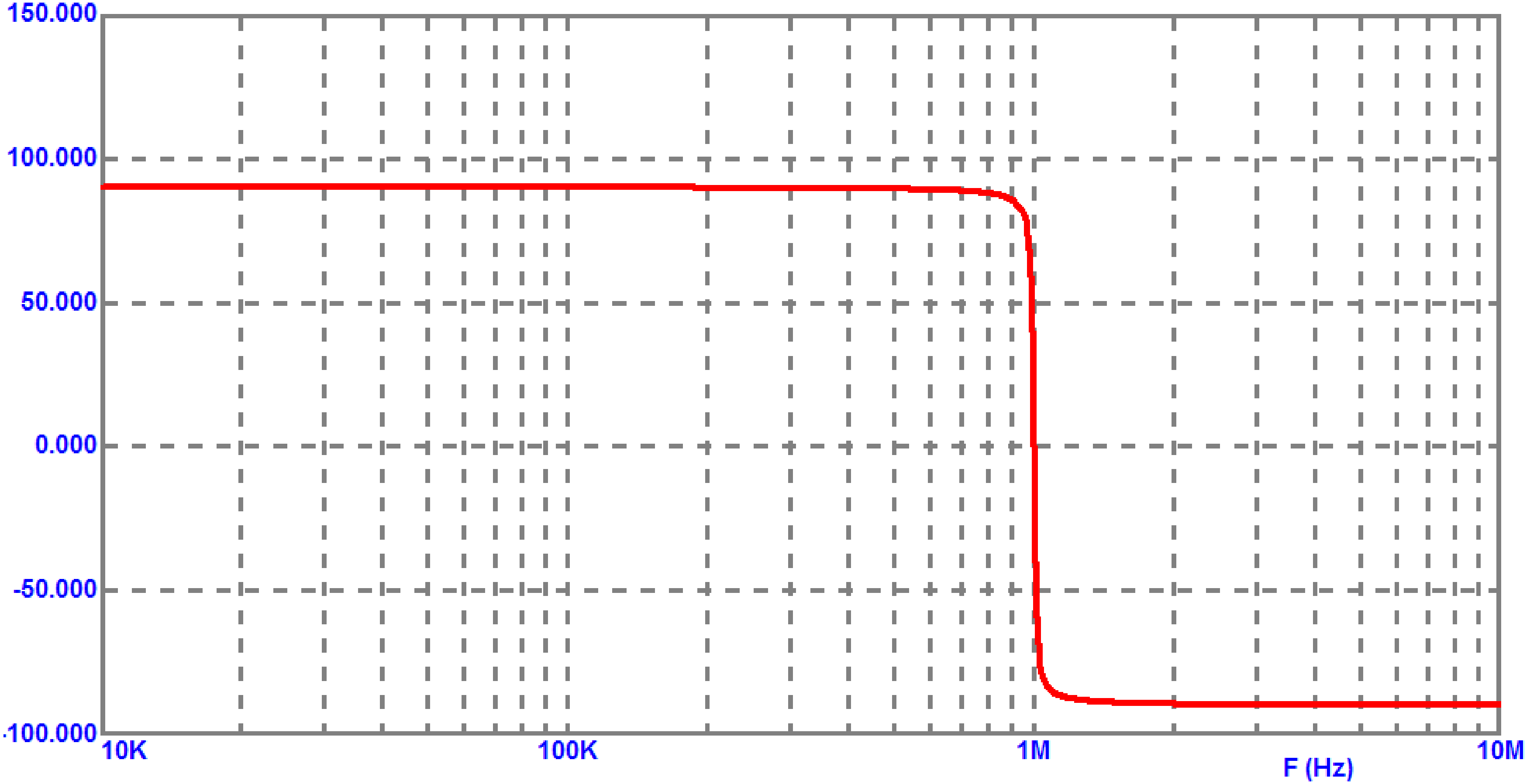
Plot the amplitude and phase as a function of frequency

Amplitude as a function of frequency

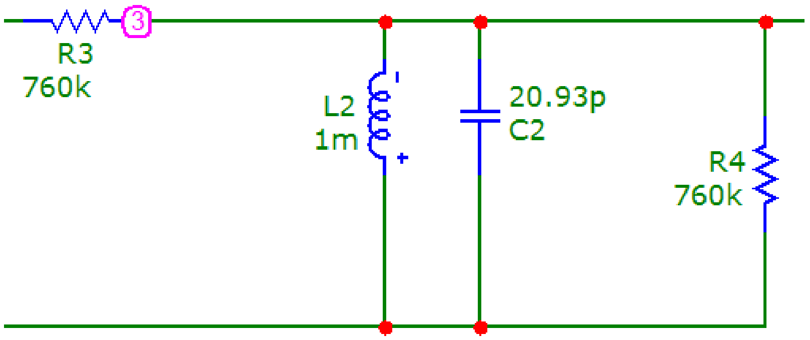


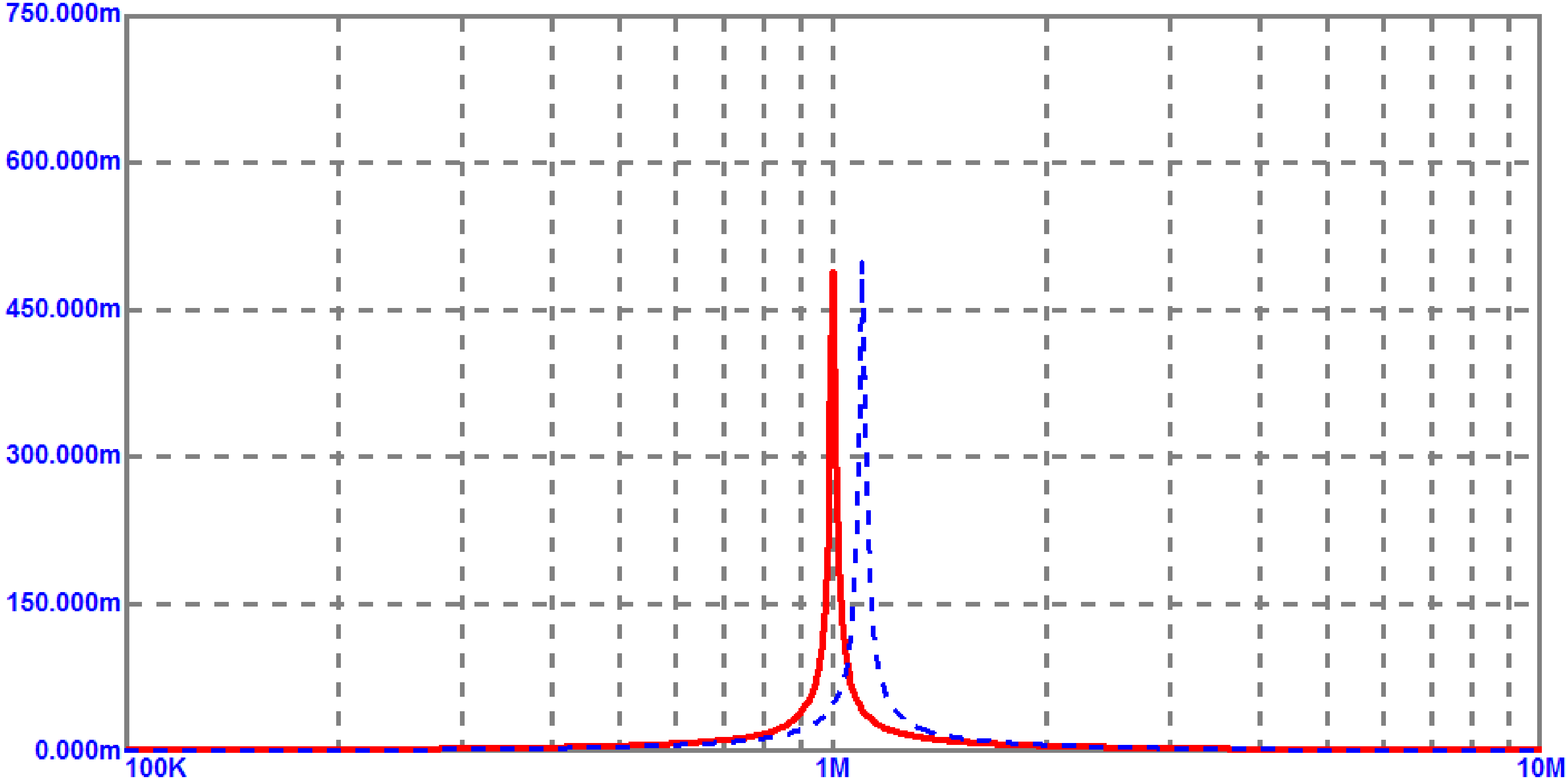
One can clearly see the frequency selective (often called a filter) nature of the circuit

Phase as a function of frequency



Suppose the capacitor is reduced to ~21pF.

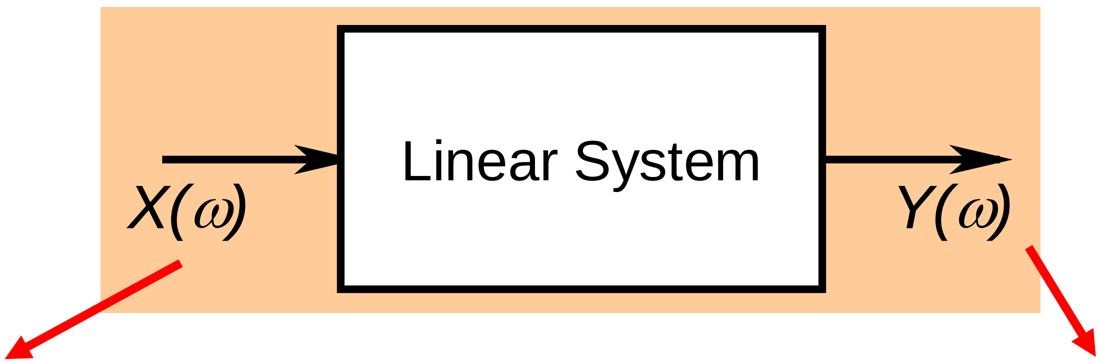




Analysis of signals and systems in frequency domain often provides useful insight into their behavior.

Frequency domain analysis

Transfer function is a useful tool for finding the frequency response of a system



Phasor (V or I) Phasor (V or I)

$$\text{Transfer Function:} \quad H(\omega) = \frac{Y(\omega)}{X(\omega)}$$

Transfer function has a magnitude and a phase

$$H(\omega) = \frac{Y(\omega)}{X(\omega)}$$

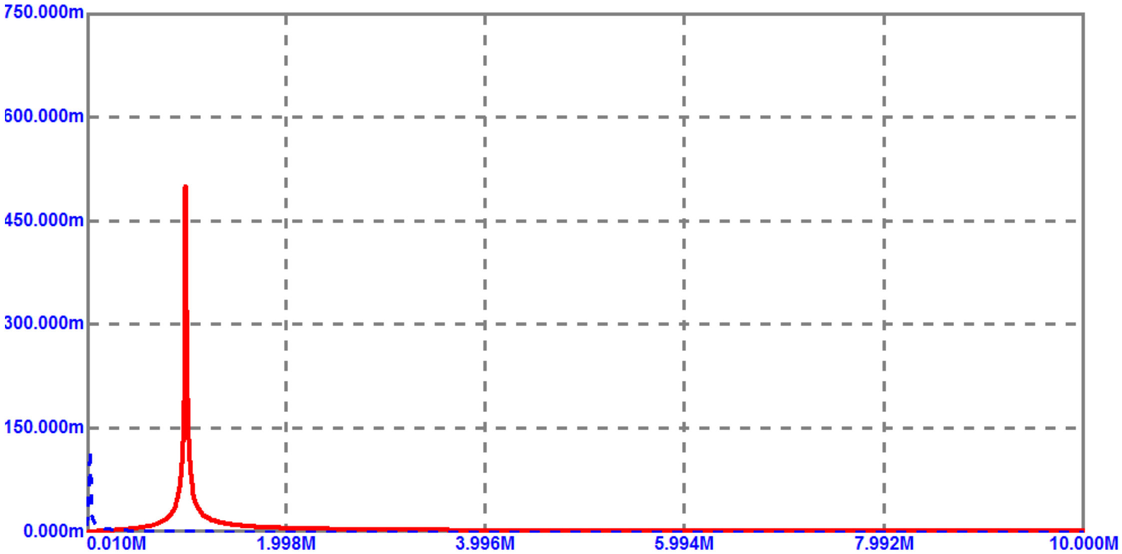
$$\mathbf{H}(\omega) = \text{Voltage gain} = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}$$

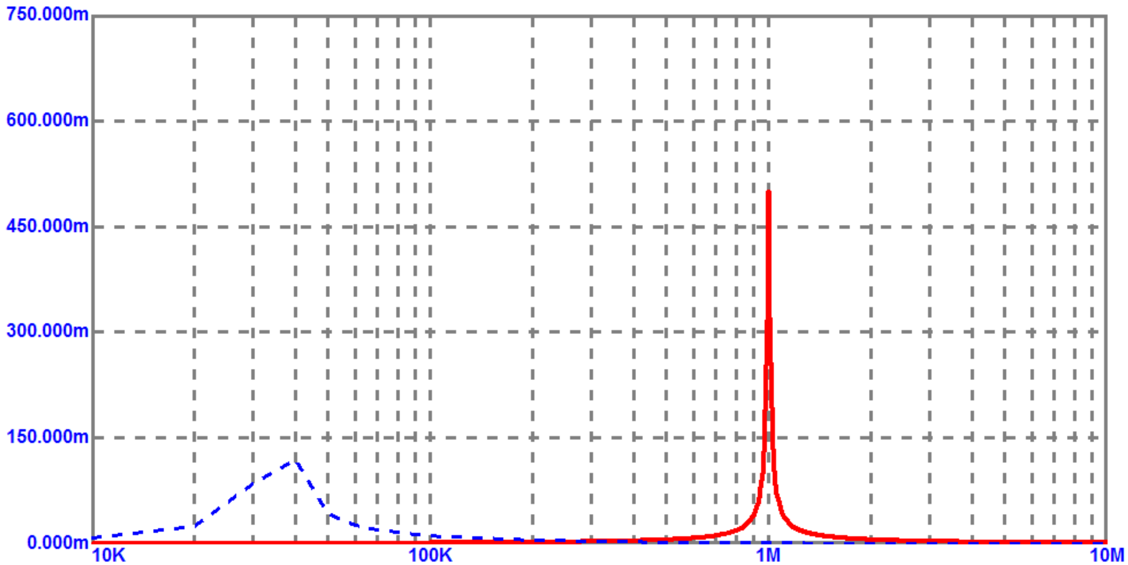
$$\mathbf{H}(\omega) = \text{Current gain} = \frac{\mathbf{I}_o(\omega)}{\mathbf{I}_i(\omega)}$$

$$\mathbf{H}(\omega) = \text{Transfer Impedance} = \frac{\mathbf{V}_o(\omega)}{\mathbf{I}_i(\omega)}$$

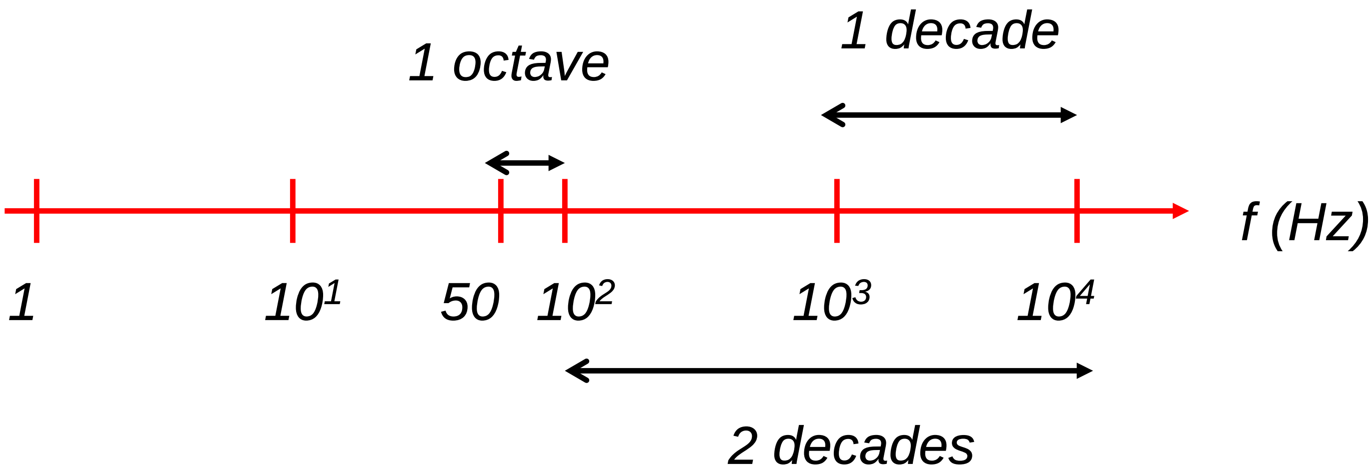
$$\mathbf{H}(\omega) = \text{Transfer Admittance} = \frac{\mathbf{I}_o(\omega)}{\mathbf{V}_i(\omega)}$$

Because of the wide dynamic range of frequency, plotting frequency on log axis is often more revealing !





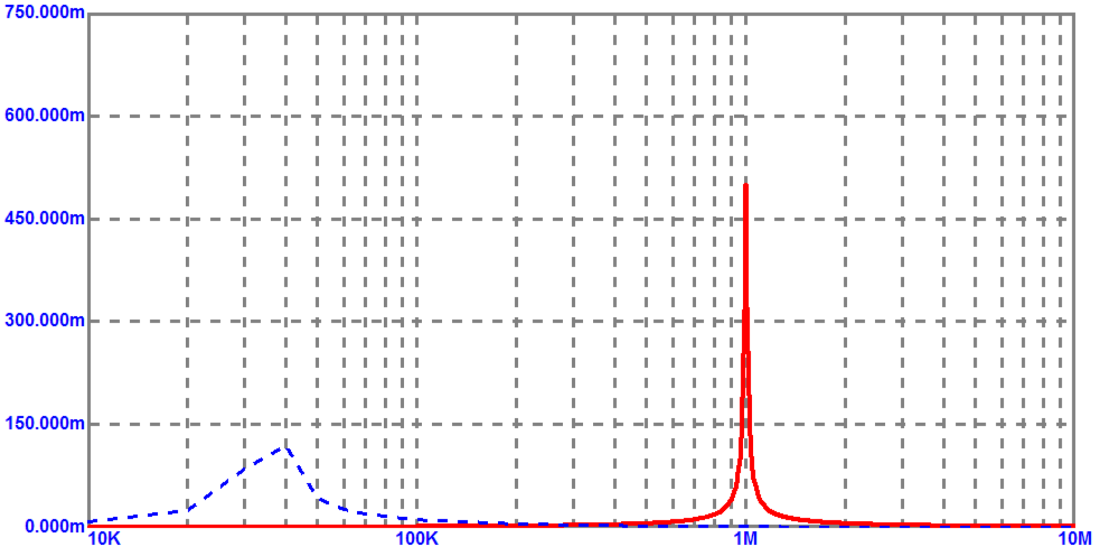
Logarithmic frequency scale

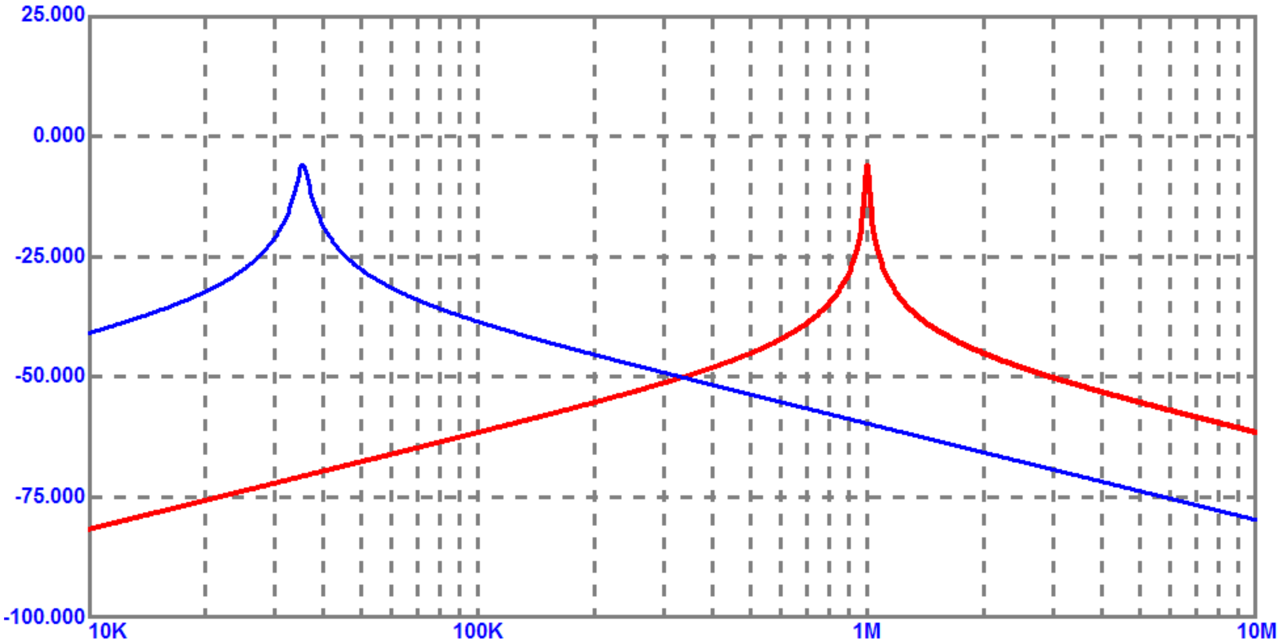


$$\text{No. of decades} = \log_{10}\left(\frac{f_2}{f_1}\right)$$

$$\text{No. of octaves} = \log_{2}\left(\frac{f_2}{f_1}\right) = \frac{\log_{10}\left(\frac{f_2}{f_1}\right)}{\log_{10}(2)}$$

Decibel scale often reveals more information about behavior





The magnitude of transfer function is often specified in **decibels**

$$G_{dB} = 10\log_{10}(\frac{P_2}{P_1})$$

Because power is proportional to V^2 or I^2, voltage gain and current gain in decibels is specified as

$$G_{dB} = 20\log_{10}(\frac{V_2}{V_1})$$

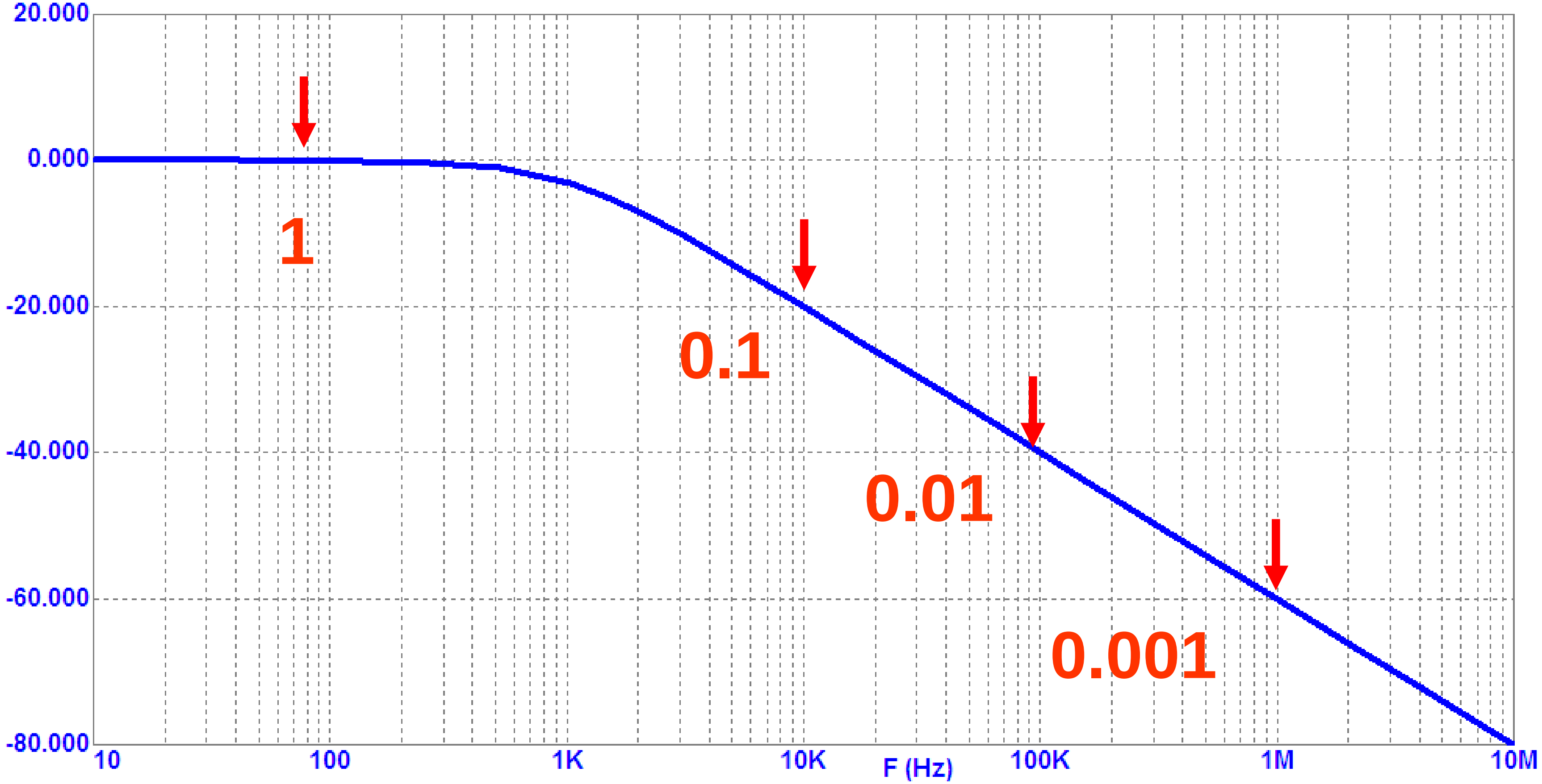
$$G_{dB} = 20\log_{10}(\frac{I_2}{I_1})$$

Decibel scale is more convenient for our perception of hearing

Decibel Scale

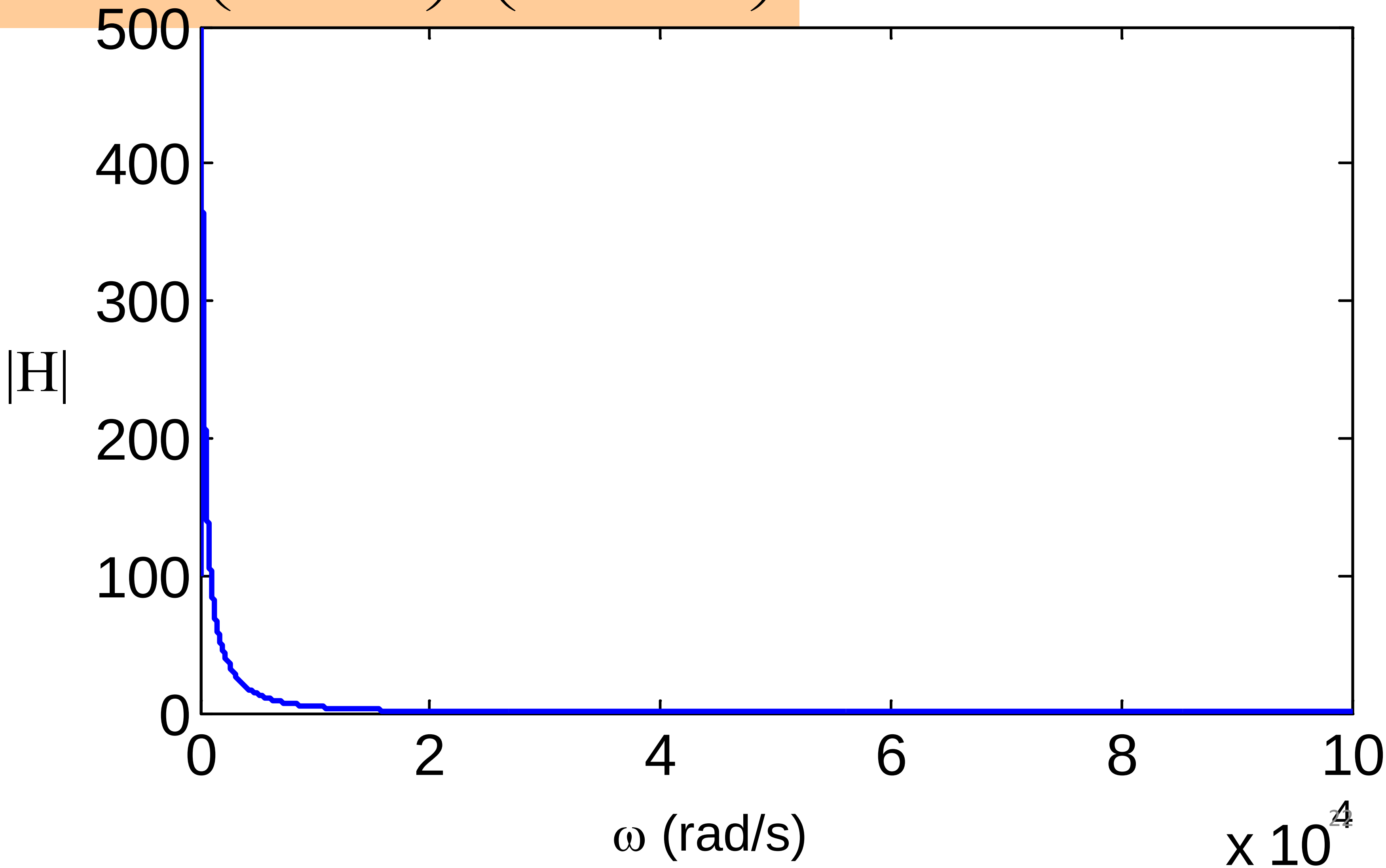
$\|H\|$	$20\text{Log}_{10}(\|H\|)$
1000	60
100	40
10	20
2	6
$\sqrt{2}$	3
1	0
$1/\sqrt{2}$	-3
0.5	-6
0.1	-20
0.01	-40

dB Scale

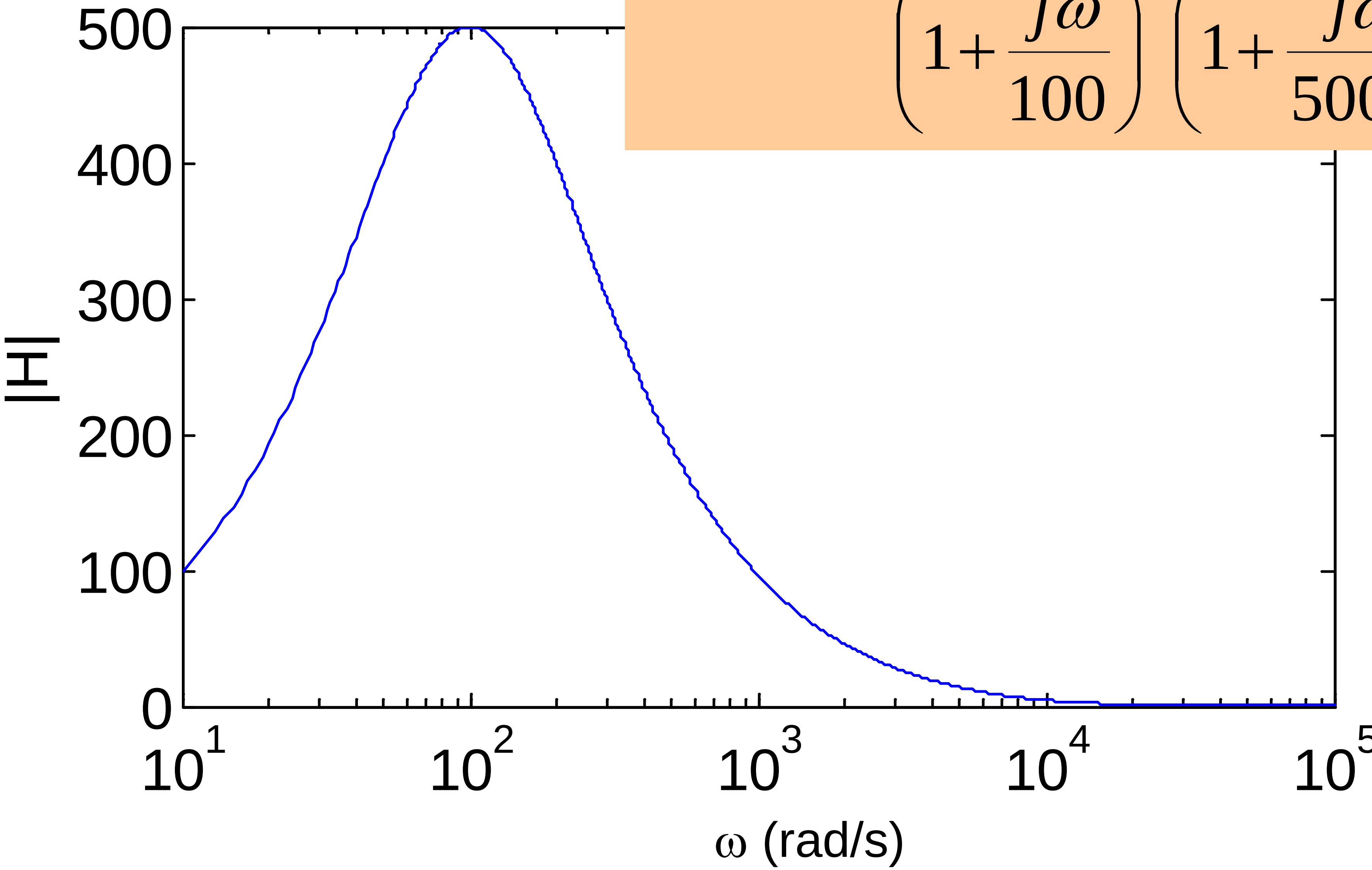


A plot of the decibel magnitude of transfer function versus frequency using a logarihmic scale for frequency is called a **Bode plot**

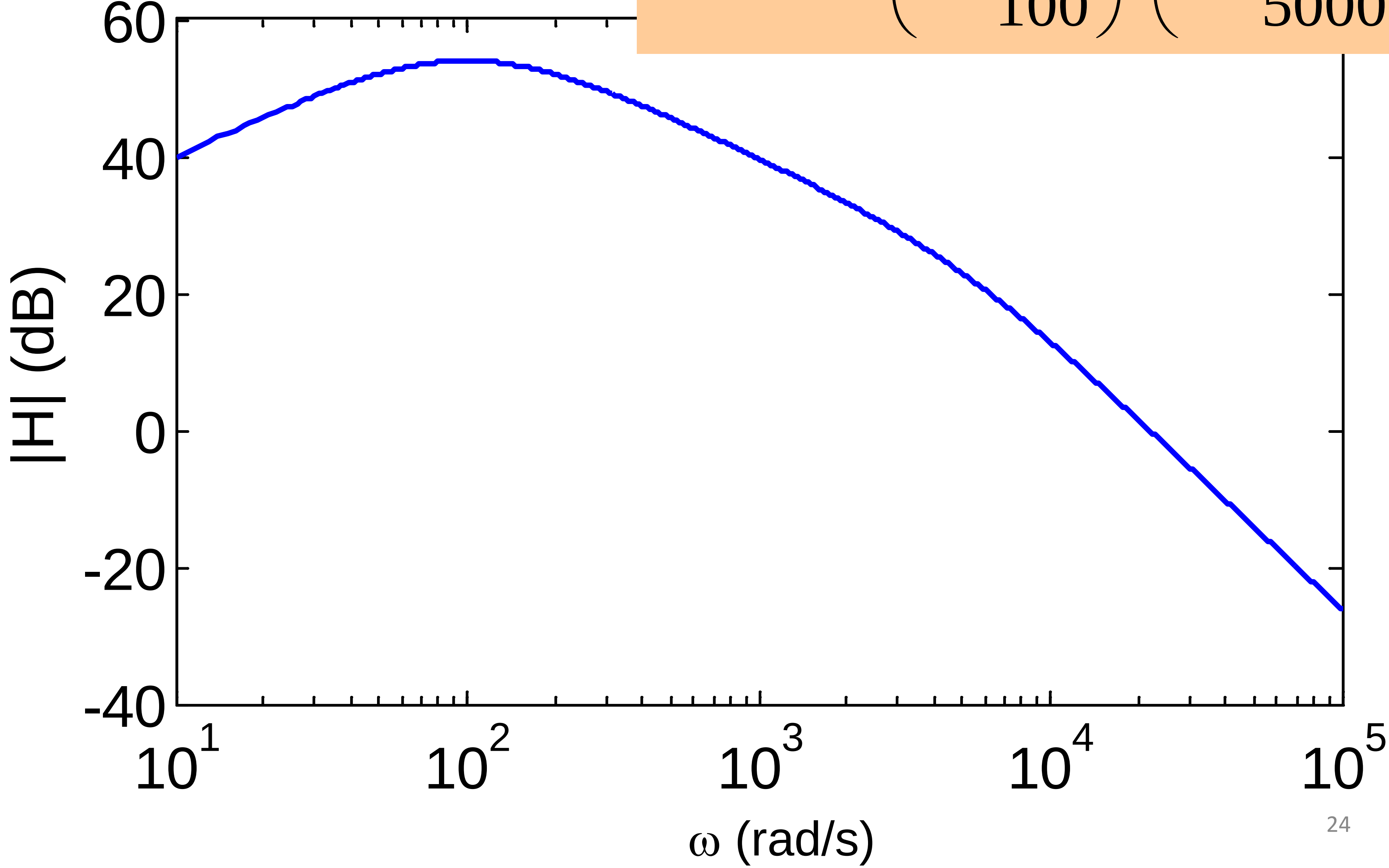
$$H(j\omega)=\frac{10\,j\omega}{\left(1+\dfrac{j\omega}{100}\right)^{2}\left(1+\dfrac{j\omega}{5000}\right)}$$



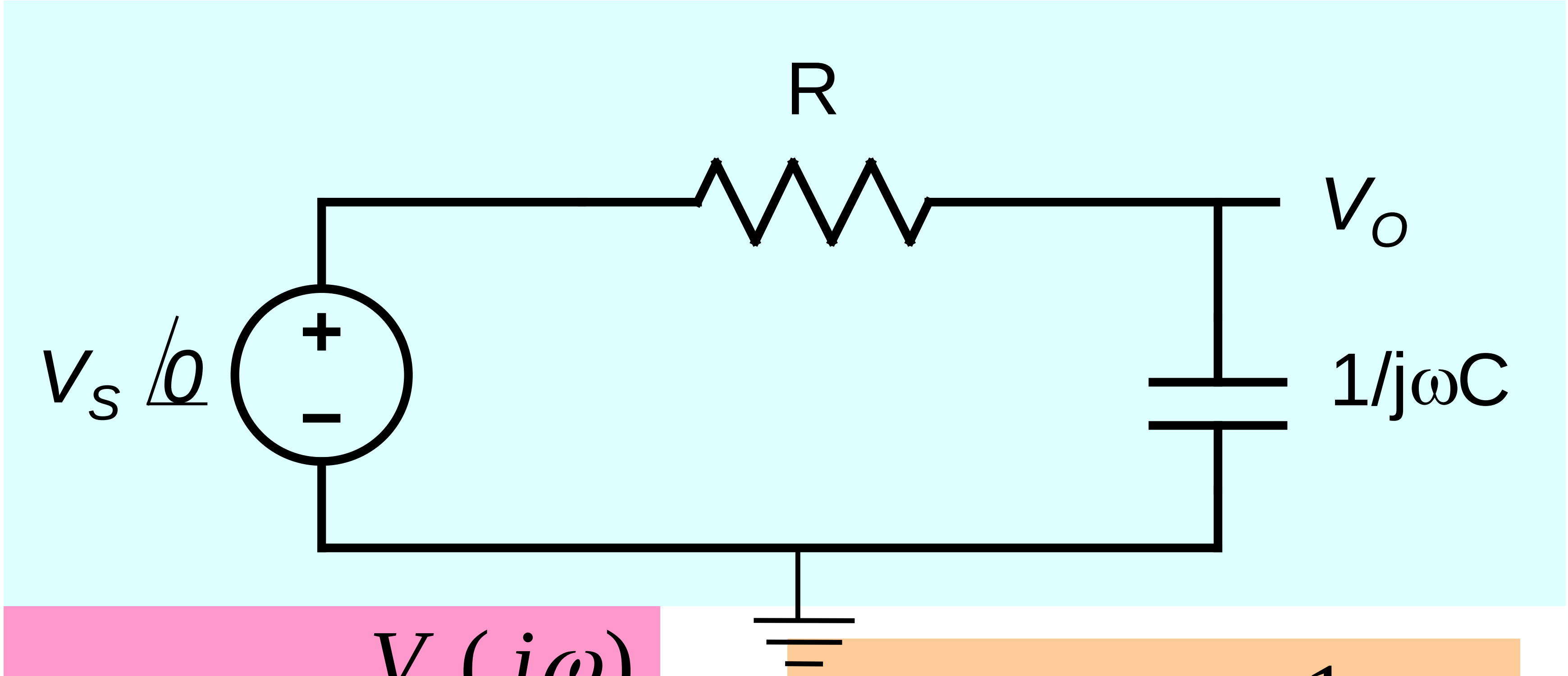
$$H(j\omega) = \frac{10\,j\omega}{\left(1+\dfrac{j\omega}{100}\right)^2\left(1+\dfrac{j\omega}{5000}\right)}$$



$$H(j\omega)=\frac{10\,j\omega}{\left(1+\frac{j\omega}{100}\right)^2\left(1+\frac{j\omega}{5000}\right)}$$



How to determine the transfer function?



$$H(j\omega) = \frac{V_0(j\omega)}{V_S(j\omega)}$$

$$H(j\omega) = \frac{1}{1 + j\omega RC}$$

$$\left|H(j\omega)\right| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

$$\phi(\omega) = -\tan^{-1}(\omega CR)$$

Plot Magnitude

$$\left|H(j\omega)\right| = \frac{1}{\sqrt{1+(\omega RC)^2}}$$

$$H_{dB} = -20\log_{10}\sqrt{1+(\omega RC)^2}$$

$$\omega_{3dB} = \frac{1}{RC}$$

$$H_{dB} = -20\log_{10}\sqrt{1+\left(\frac{\omega}{\omega_{3dB}}\right)^2}$$

$$\omega << \omega_{3dB}$$

$$H_{dB} \approx -20\log_{10}(1) = 0$$

$$\omega >> \omega_{3dB}$$

$$H_{dB} \approx -20\log_{10}\left(\frac{\omega}{\omega_{3dB}}\right)$$

$$\omega >> \omega_{3dB}$$

$$H_{dB} \approx -20\log_{10}\left(\frac{\omega}{\omega_{3dB}}\right)$$

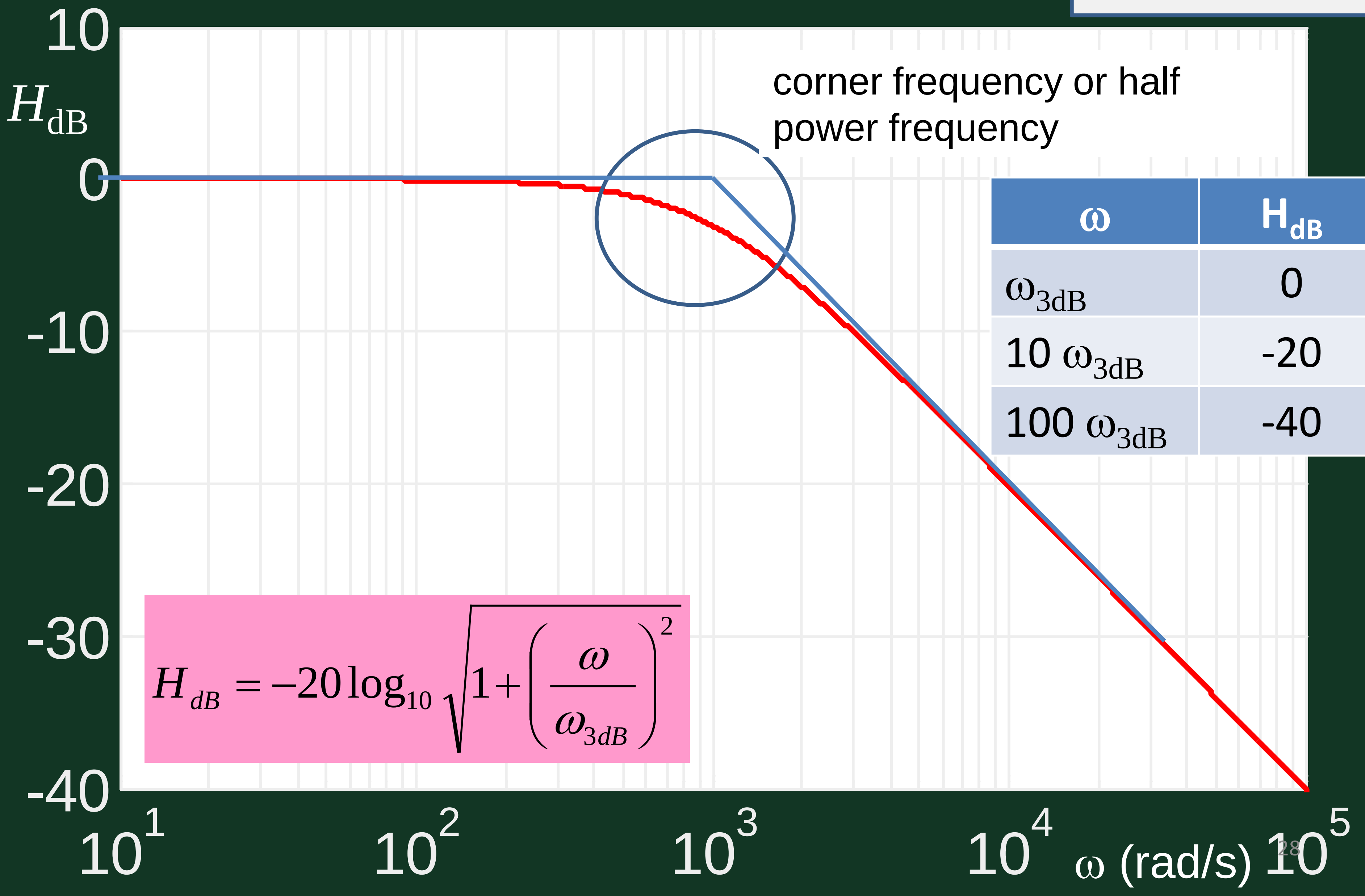
ω	H_{dB}
ω_{3dB}	0
10 ω_{3dB}	-20
100 ω_{3dB}	-40

-20 dB per decade

$$\log_{10} 10 = 1$$

3dB point

$$\omega_{3dB} = \frac{1}{RC}$$



ω	H_{dB}
ω_{3dB}	0
10 ω_{3dB}	-20
100 ω_{3dB}	-40

$$H_{dB} = -20\log_{10}\sqrt{1+\left(\frac{\omega}{\omega_{3dB}}\right)^2}$$

Example

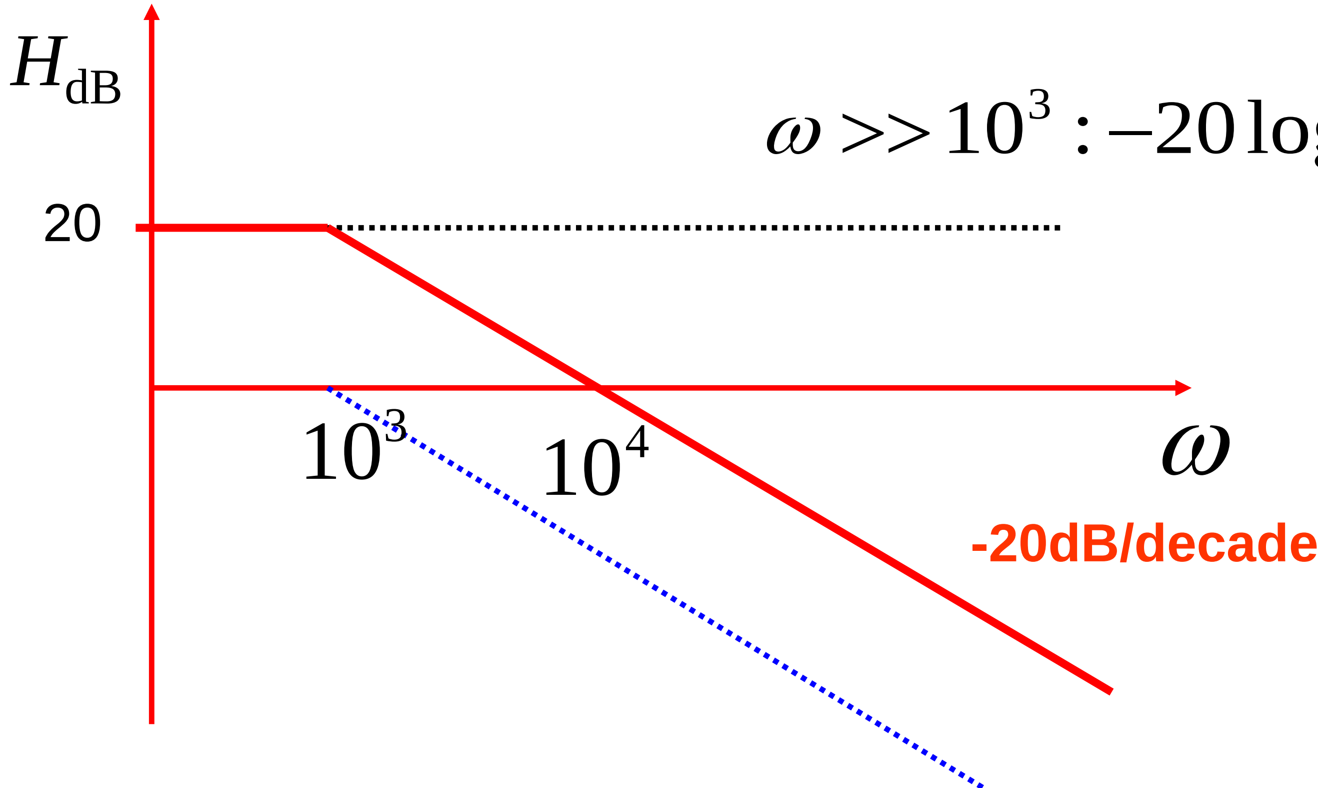
$$H(j\omega) = \frac{10}{1 + j\omega 10^{-3}}$$

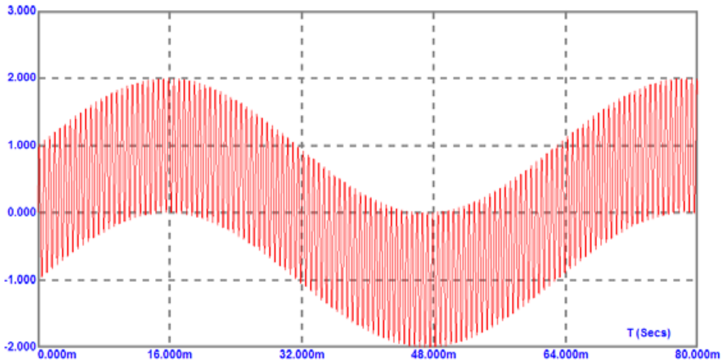
$$\omega_{3dB} = 10^3$$

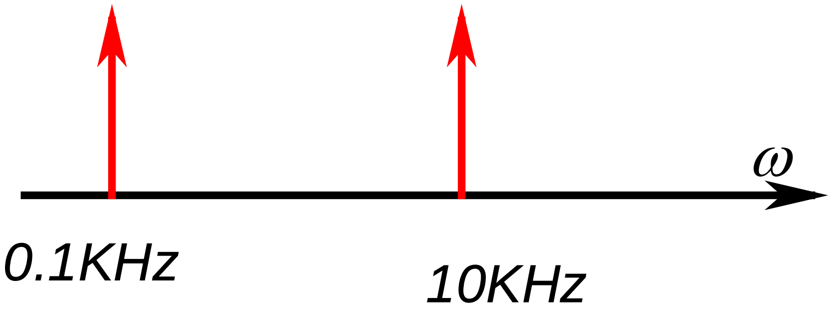
$$H_{dB} = 20 - 20\log_{10}\sqrt{1 + \left(\frac{\omega}{10^3}\right)^2}$$

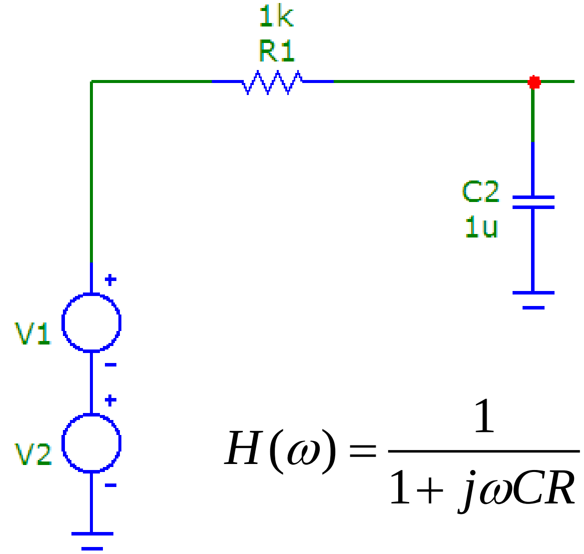
$$\omega << 10^3 : 0dB$$

$$\omega >> 10^3 : -20\log_{10}\frac{\omega}{10^3}$$



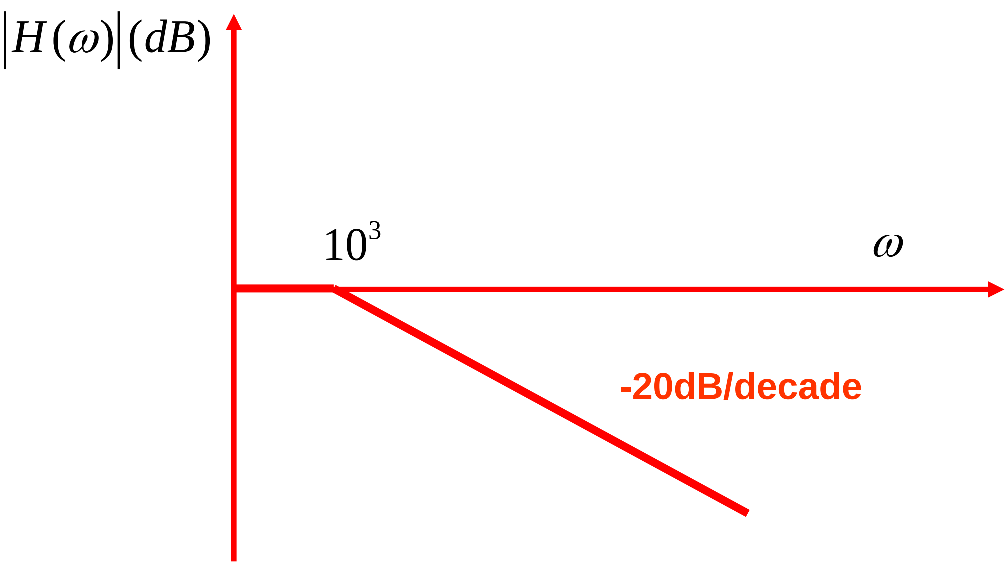




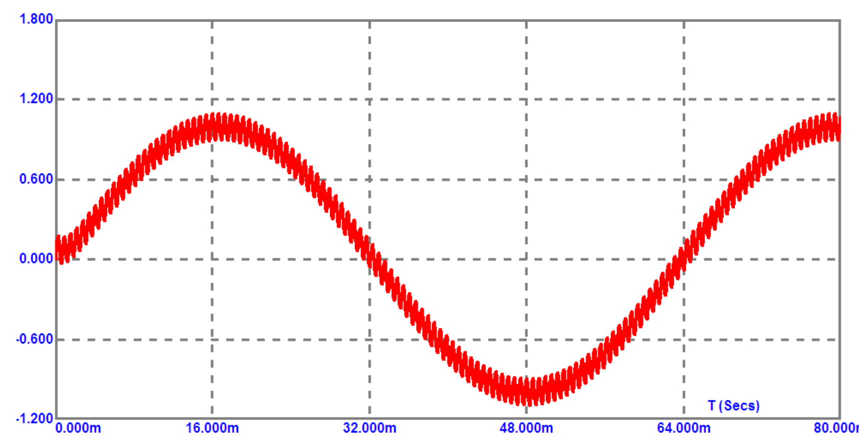


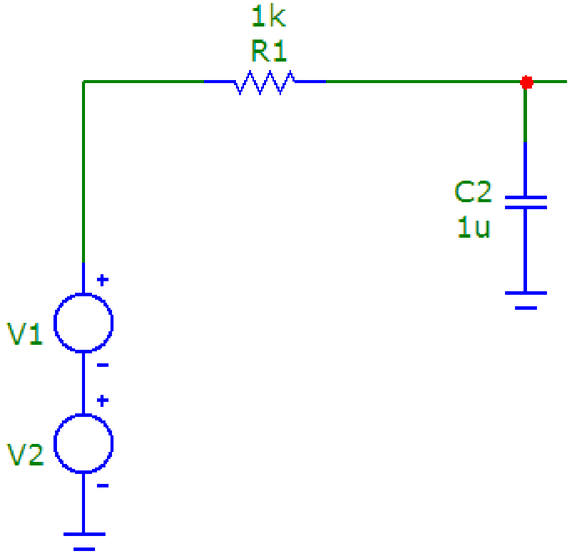
$$H(\omega) = \frac{1}{1 + j\omega CR}$$

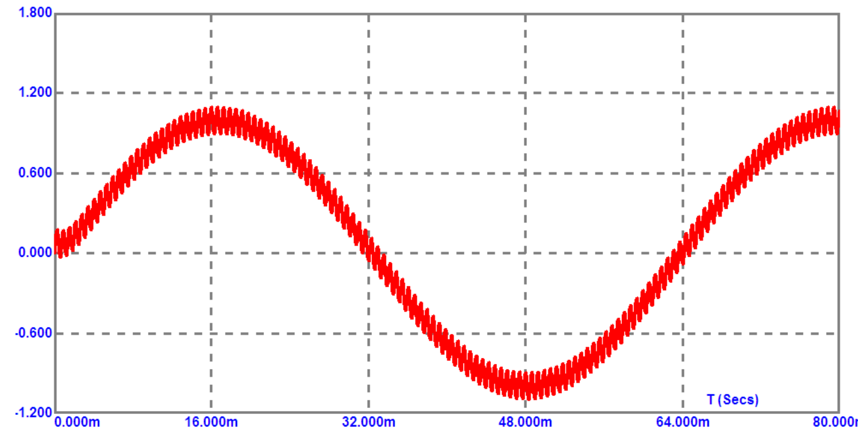
$$H(\omega) = \frac{1}{1 + j\omega 10^{-3}} = \frac{1}{1 + j\dfrac{\omega}{10^{3}}}$$

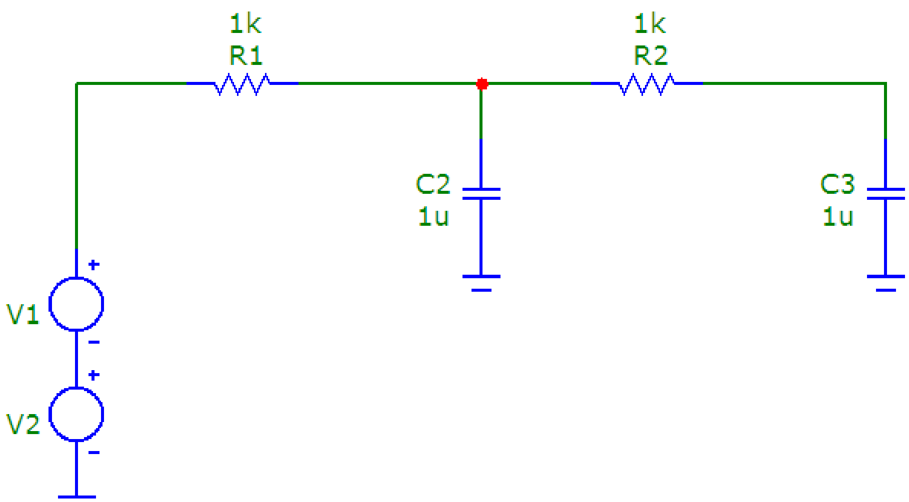


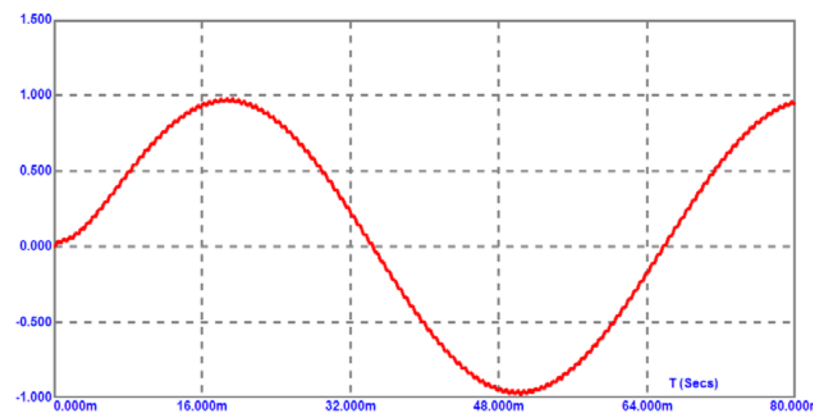
$$V_O(t) = 1Sin(100t) + 0.1Sin(10^4 t)$$

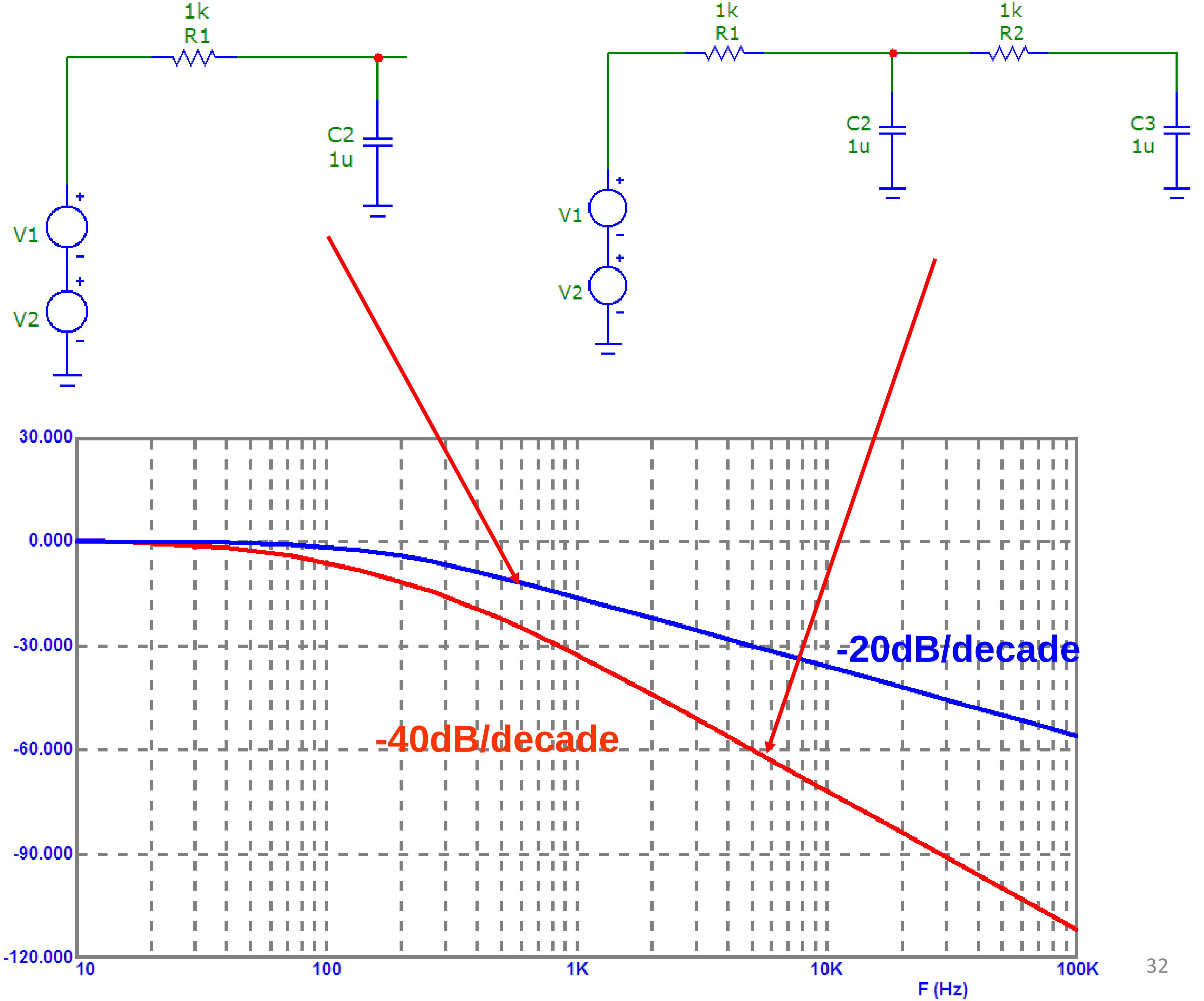



1k
R1
C2
1u
V1
V2

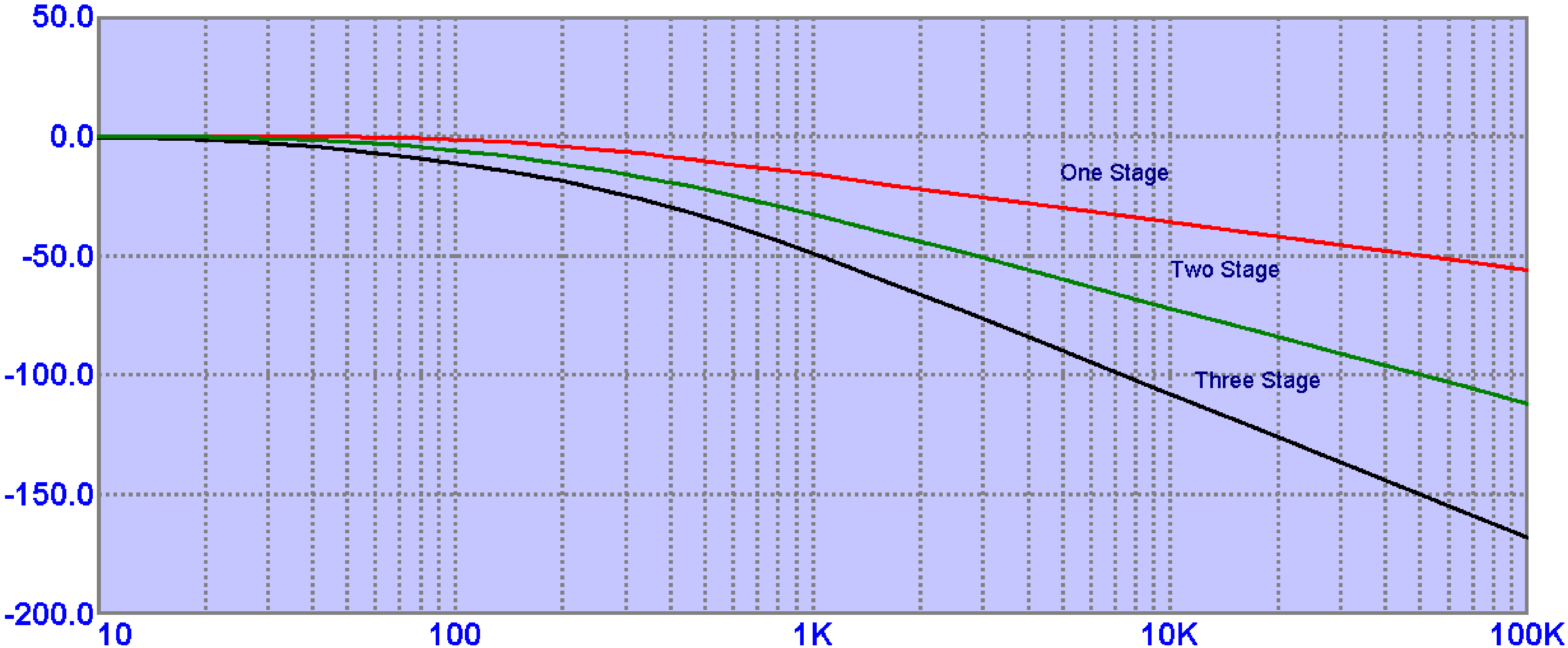

1.800
1.200
0.600
0.000
-0.600
-1.200
0.000m
16.000m
32.000m
48.000m
64.000m
80.000m
T (Secs)


1k
R1
1k
R2
C2
1u
C3
1u
V1
V2


1.500
1.000
0.500
0.000
-0.500
-1.000
0.000m
16.000m
32.000m
48.000m
64.000m
80.000m
T (Secs)


1k
R1
C2
1u
V1
V2
1k
R1
1k
R2
C2
1u
C3
1u
V1
V2
30.000
0.000
-30.000
-60.000
-90.000
-120.000
10
100
1K
10K
100K
F (Hz)
-20dB/decade
-40dB/decade

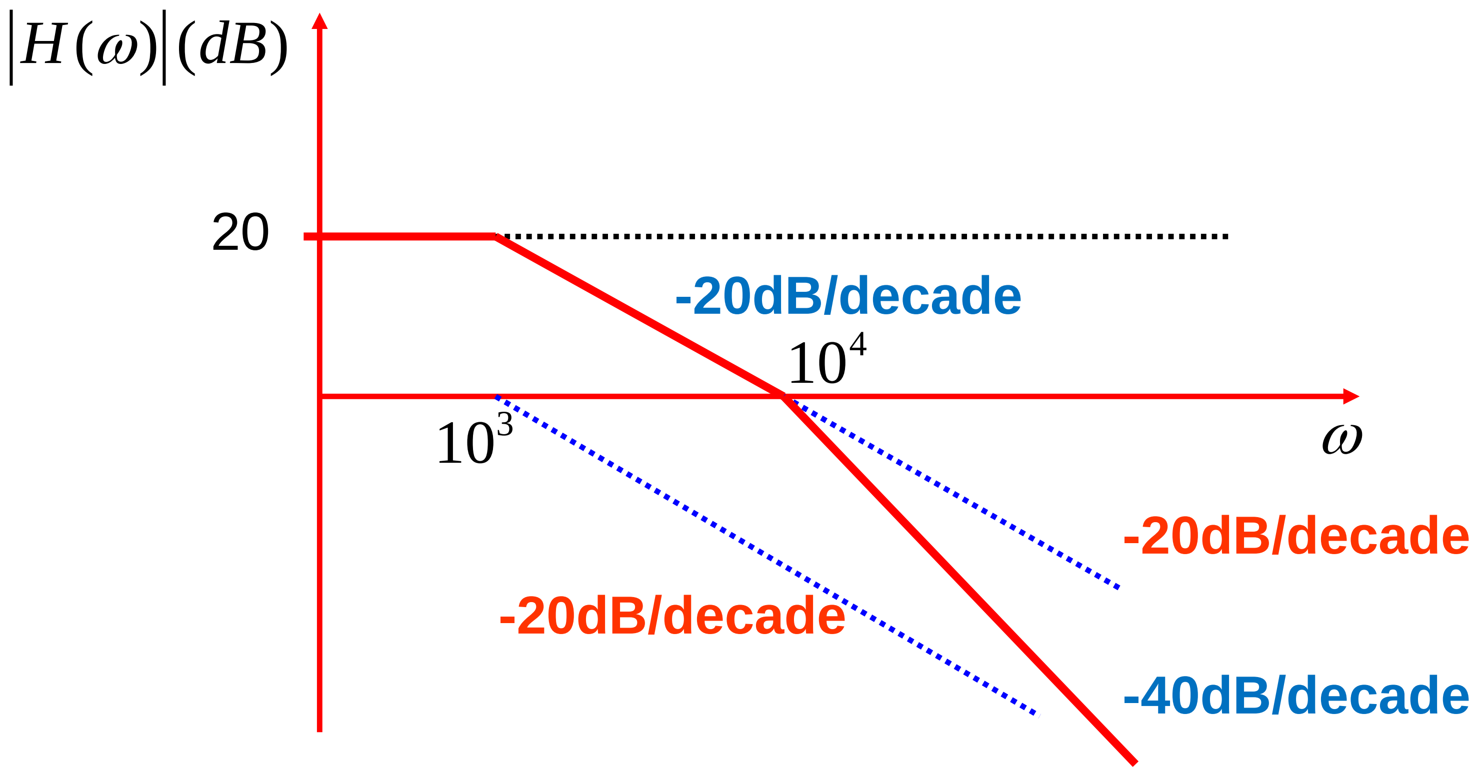
Adding more RC stages, makes the characteristics sharper



Sketching of Transfer function:
Bode Magnitude Plot

$$H(\omega)=\frac{10}{1+j\frac{\omega}{10^3}}\times\frac{1}{1+j\frac{\omega}{10^4}}$$

$$20\mathrm{Log}_{10}(|H(\omega)|)=20-20Log_{10}\sqrt{(1+(\frac{\omega}{10^3})^2)}-20Log_{10}\sqrt{(1+(\frac{\omega}{10^4})^2)}$$

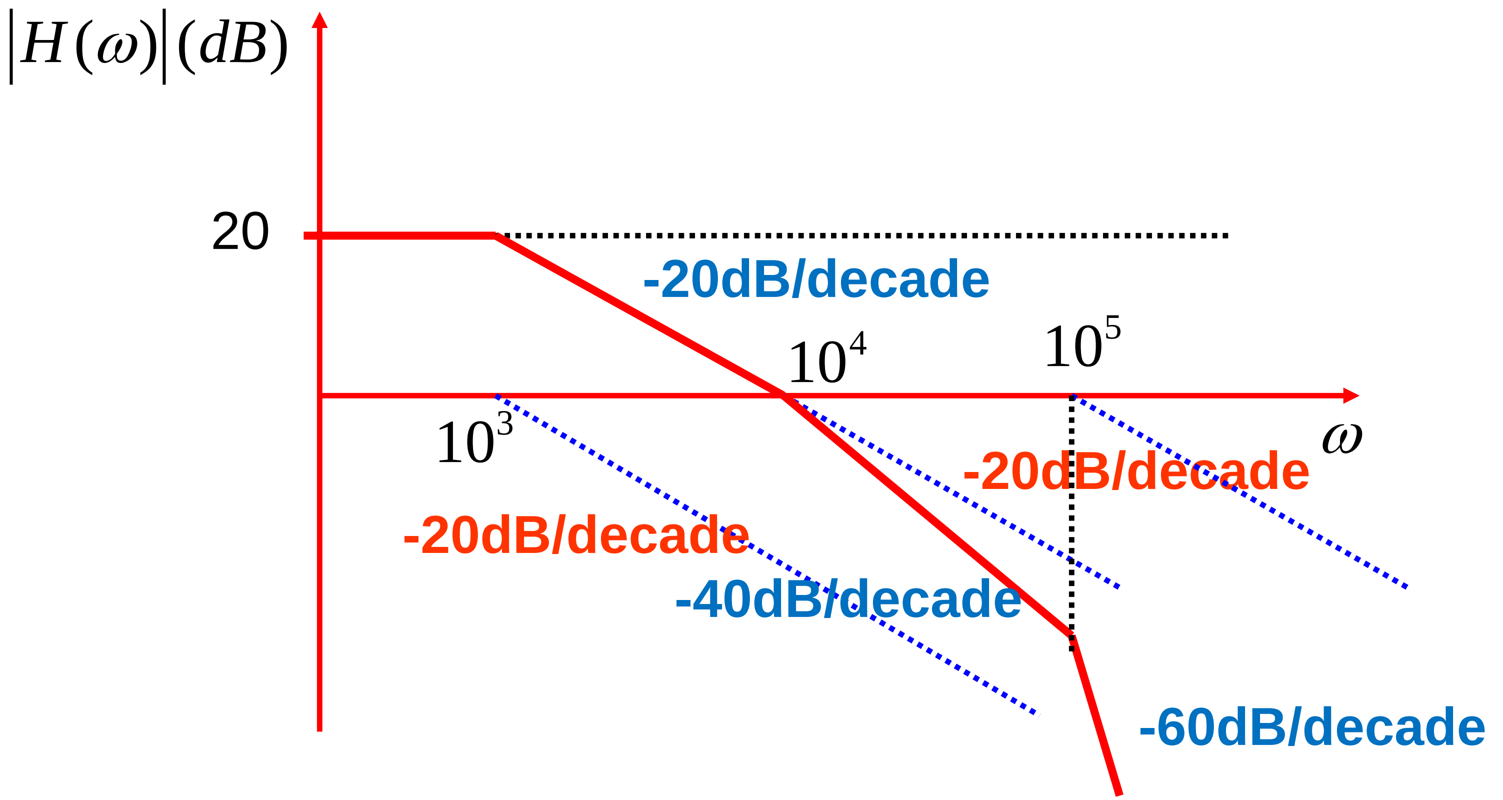


Sketching of Transfer function

Bode Magnitude Plot

$$H(\omega) = \frac{10}{1 + j\frac{\omega}{10^3}} \times \frac{1}{1 + j\frac{\omega}{10^4}} \times \frac{1}{1 + j\frac{\omega}{10^5}}$$

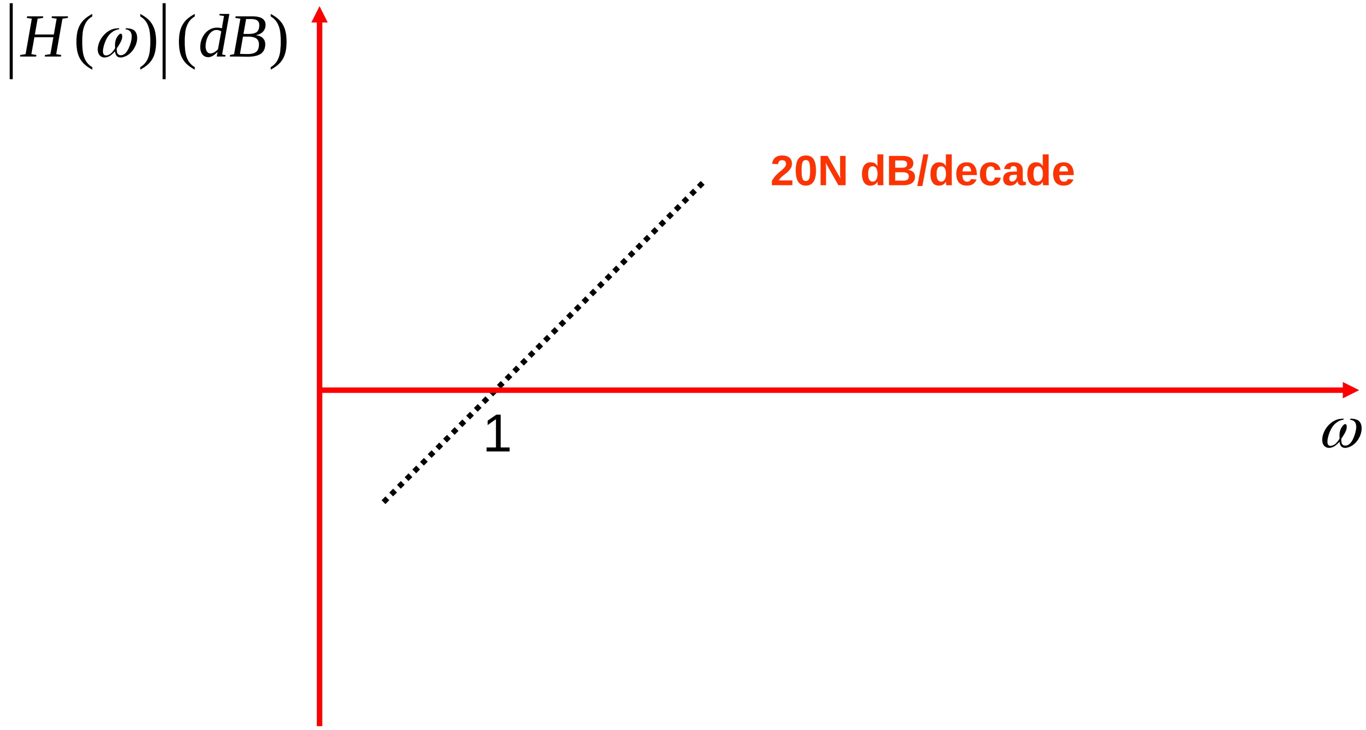
$$20\mathrm{Log}_{10}(|H(\omega)|) = 20 - 20Log_{10}\sqrt{(1+(\frac{\omega}{10^3})^2)} - 20Log_{10}\sqrt{(1+(\frac{\omega}{10^4})^2)} - 20Log_{10}\sqrt{(1+(\frac{\omega}{10^5})^2)}$$



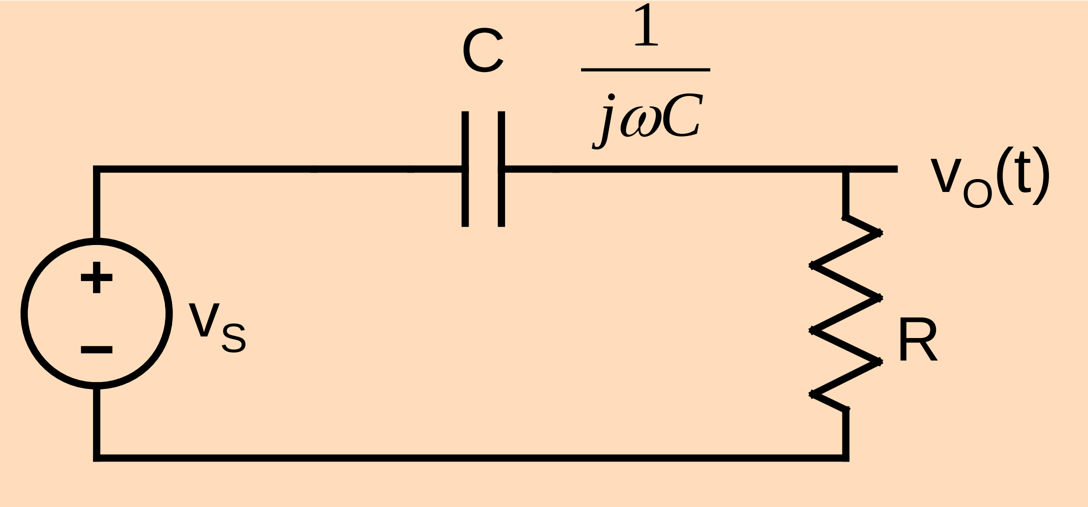
Bode Magnitude Plot

$$H(\omega) = (j\omega)^N$$

$$20\text{Log}_{10}(|H(\omega)|) = 20N \times Log_{10}(\omega)$$



Determine transfer function?



$$H(\omega) = \frac{V_O(\omega)}{V_S(\omega)}$$

$$H(\omega) = \frac{j\omega CR}{1 + j\omega CR}$$

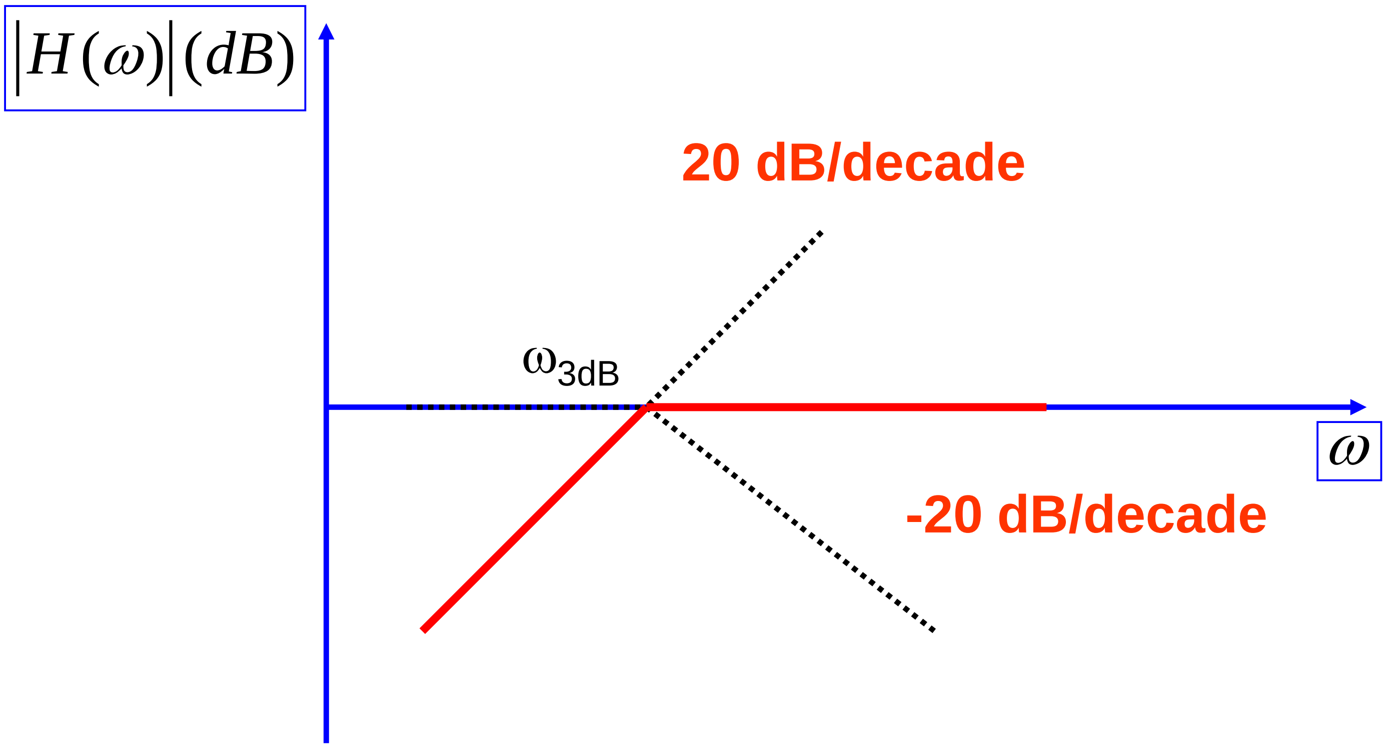
$$H(\omega) = \frac{j(\omega / \omega_{3dB})}{1 + j(\omega / \omega_{3dB})}$$

$$\omega_{3\mathrm{dB}} = \frac{1}{\mathrm{RC}} \quad ; \; \mathrm{f}_{3\mathrm{dB}} = \frac{1}{2\pi \mathrm{RC}}$$

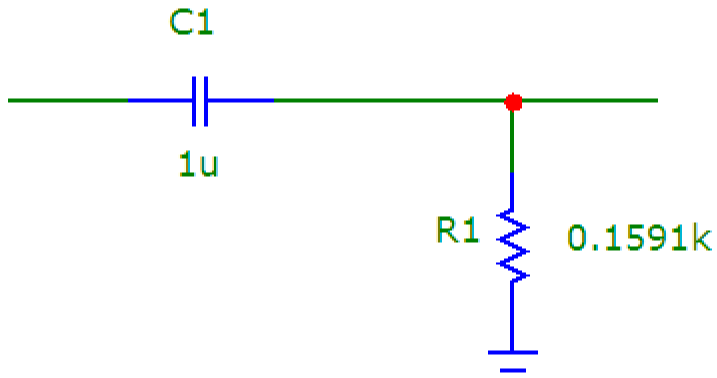
$$20\mathrm{Log}_{10}(\left|H(\omega)\right|) = 20 log_{10}(\frac{\omega}{\omega_{3dB}}) - 20 log_{10}\sqrt{(1 + (\frac{\omega}{\omega_{3dB}})^2)}$$

Bode Magnitude Plot

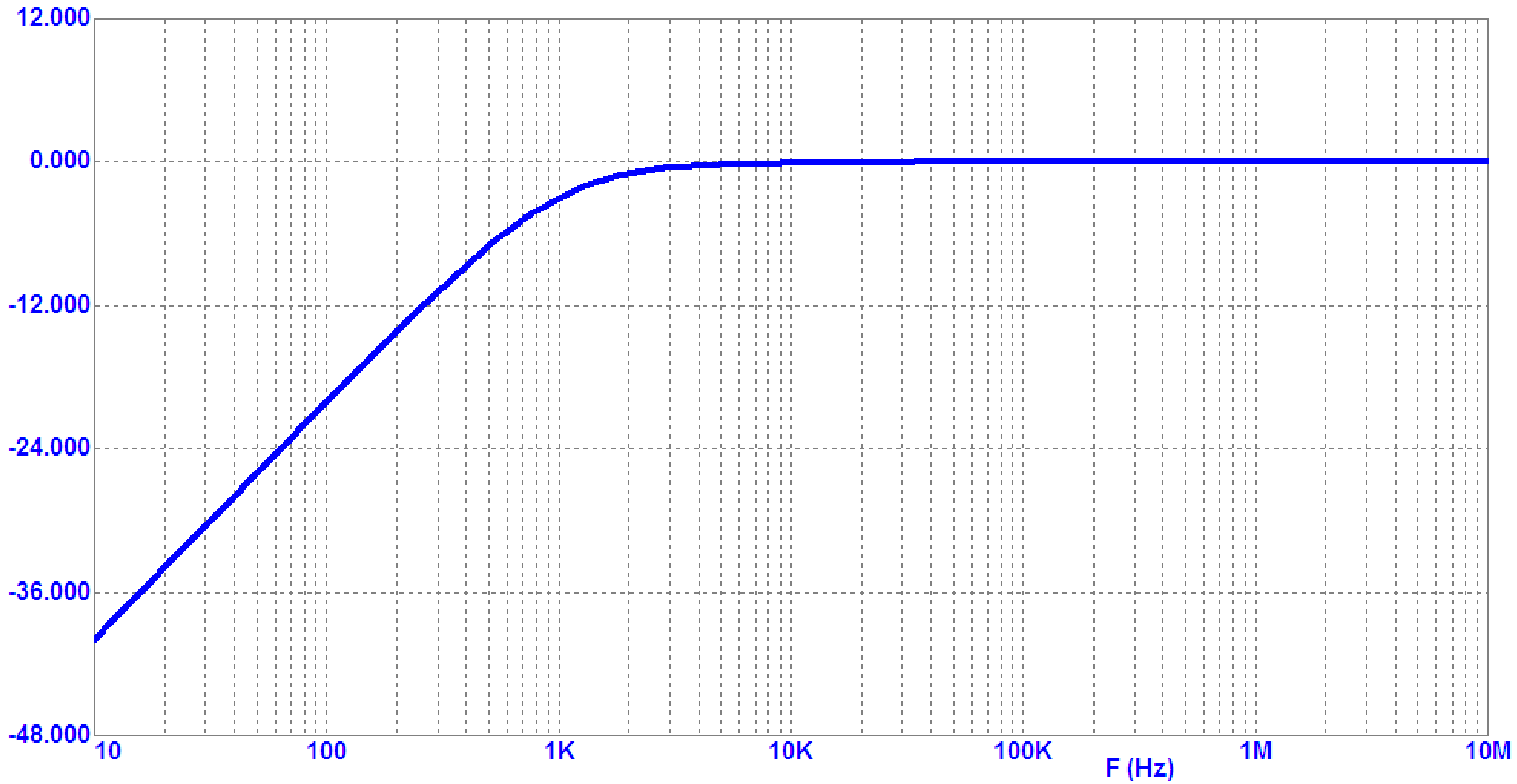
$$20\mathrm{Log}_{10}(|H(\omega)|) = 20log_{10}(\frac{\omega}{\omega_{3dB}}) - 20log_{10}\sqrt{(1+(\frac{\omega}{\omega_{3dB}})^2)}$$



High Pass Filter



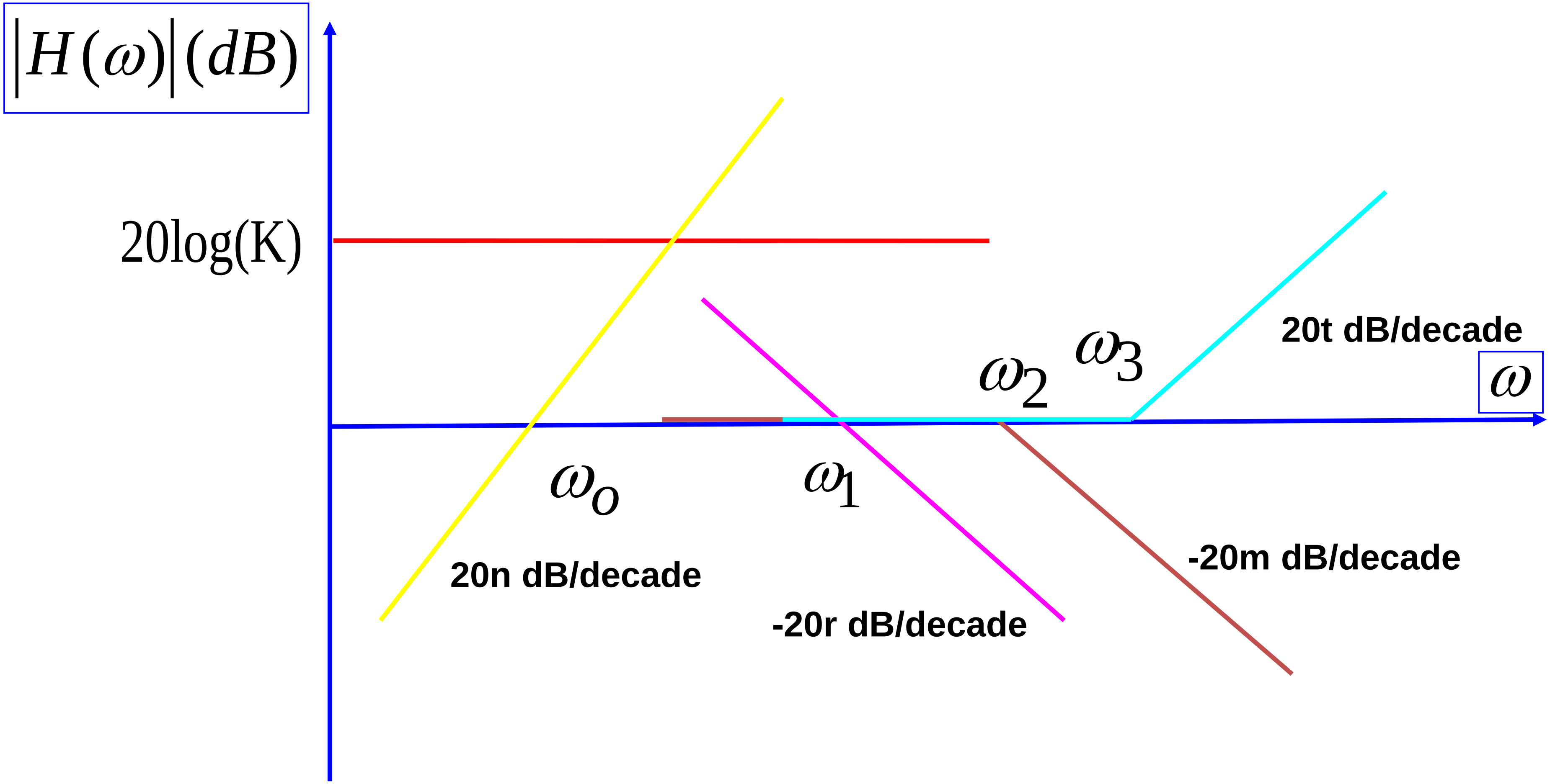
$$f_{3dB} = \frac{1}{2\pi RC} = 10^3\,Hz$$



High Pass Filter

Bode Plot segments

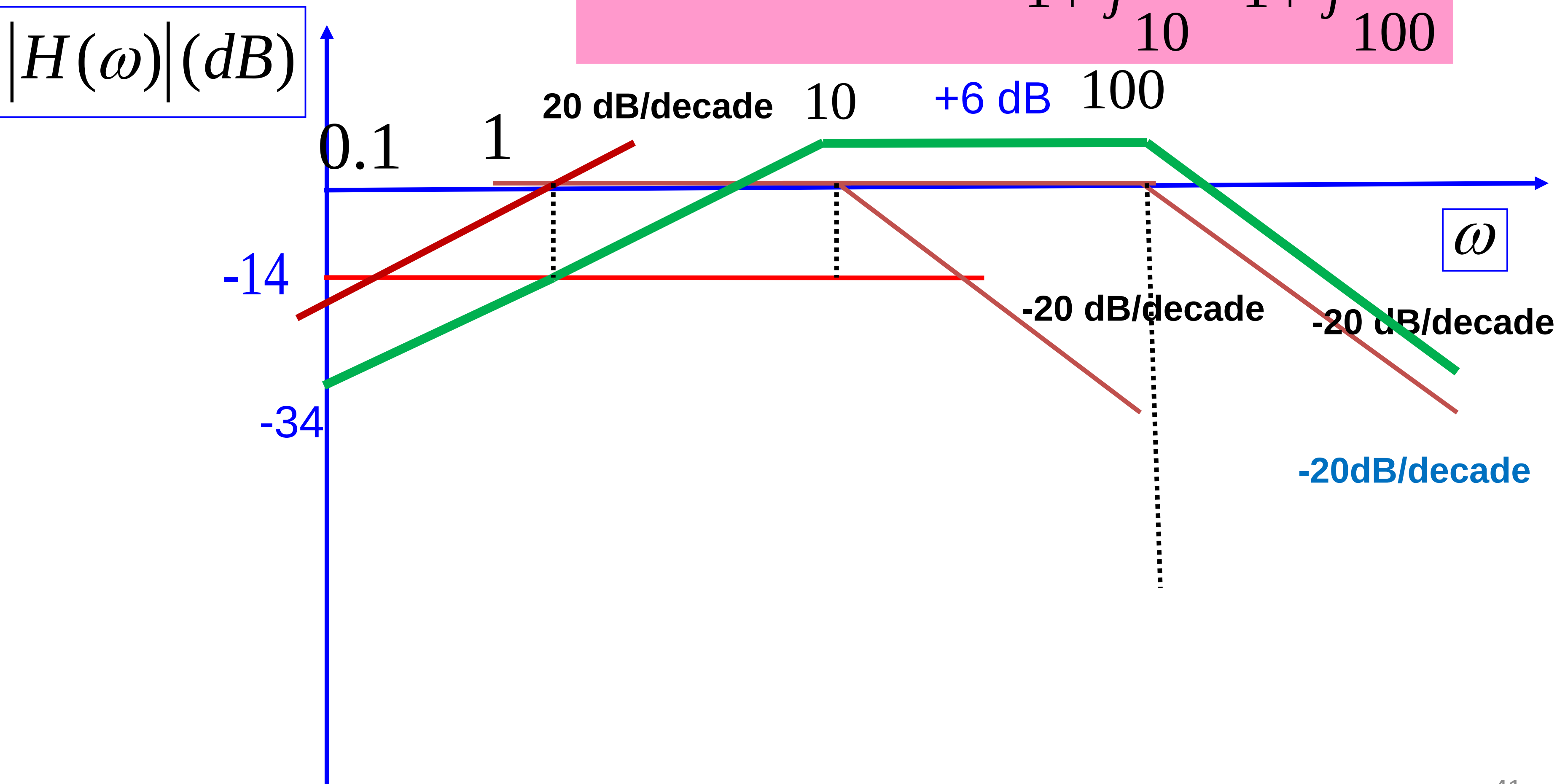
$$H(\omega) = K \times j(\omega/\omega_o)^n \times \frac{1}{j(\omega/\omega_1)^r} \frac{1}{\{1 + j(\omega/\omega_2)\}^m} \times \{1 + j(\omega/\omega_3)\}^t$$



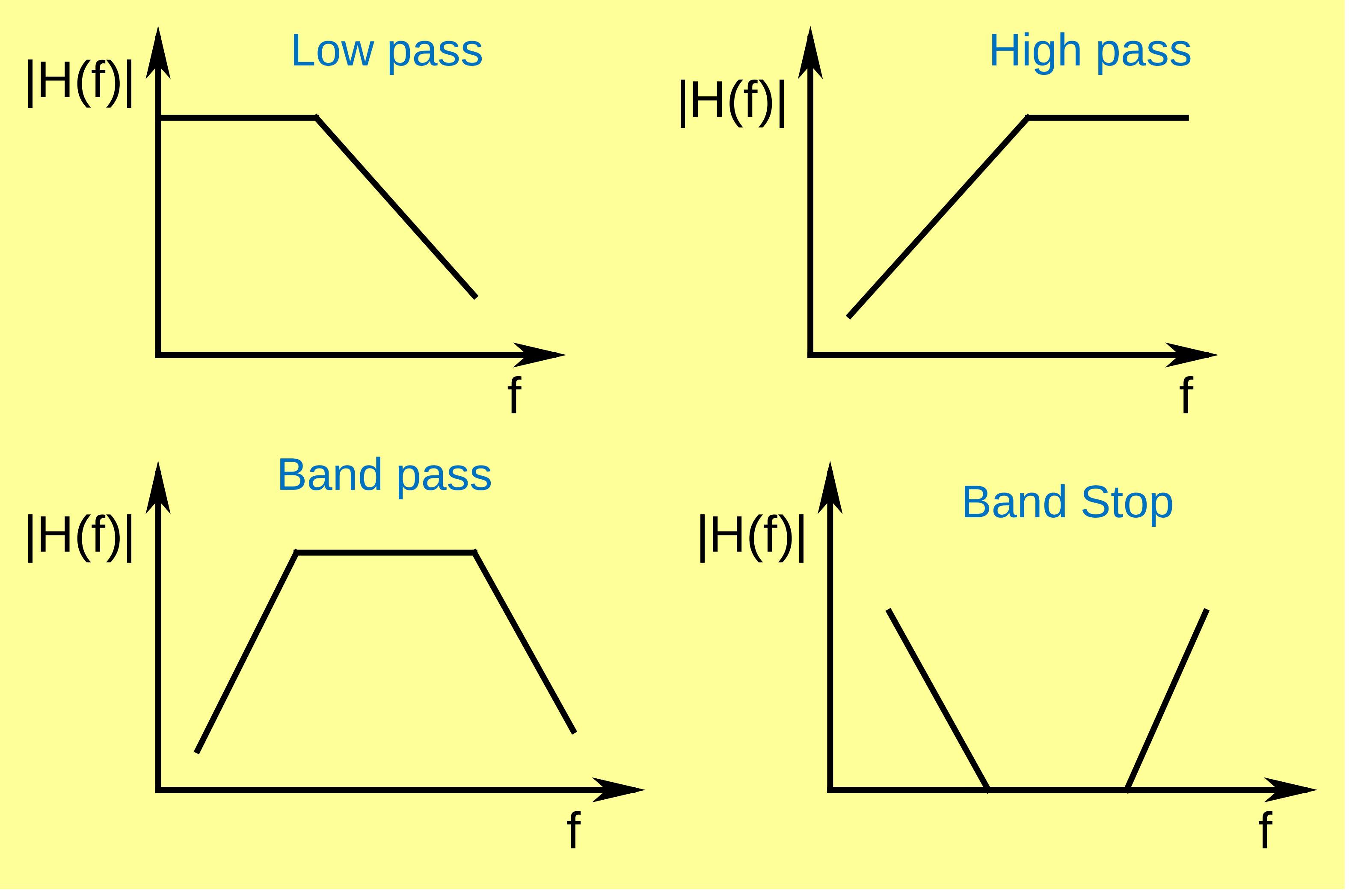
Example:

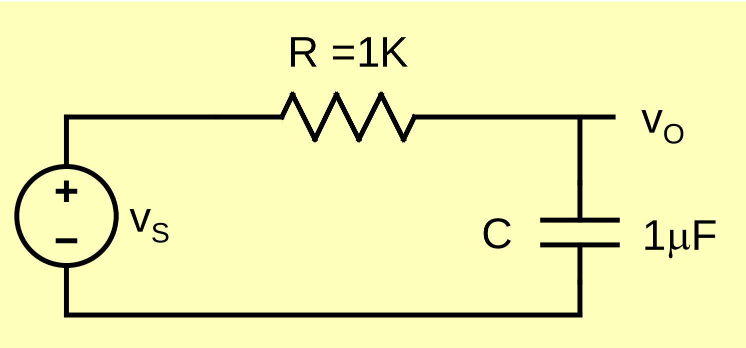
$$H(\omega) = 200 \times j\omega \times \frac{1}{10 + j\omega} \times \frac{1}{100 + j\omega}$$

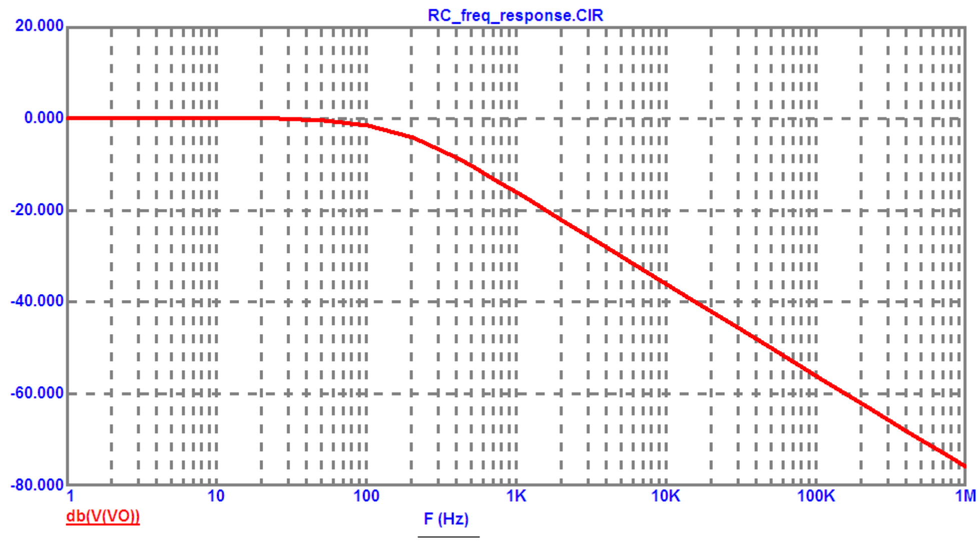
$$H(\omega) = 0.2 \times j\omega \times \frac{1}{1 + j\dfrac{\omega}{10}} \times \frac{1}{1 + j\dfrac{\omega}{100}}$$

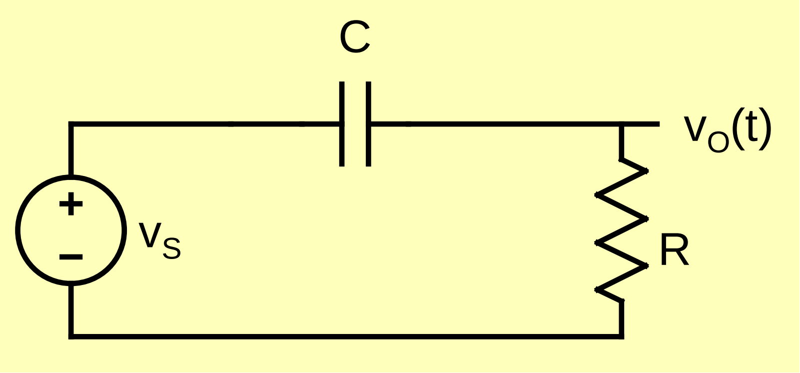


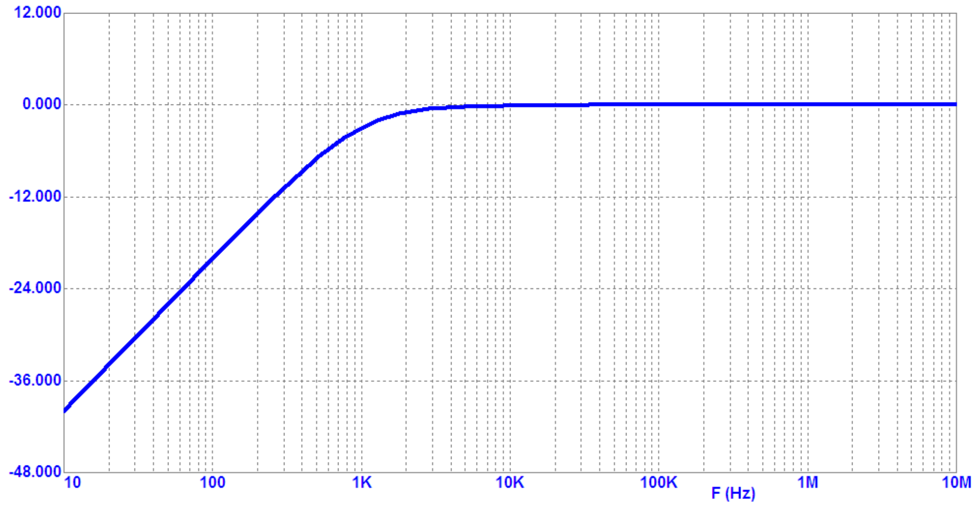
Filter -pass a band of frequency and reject the remaining



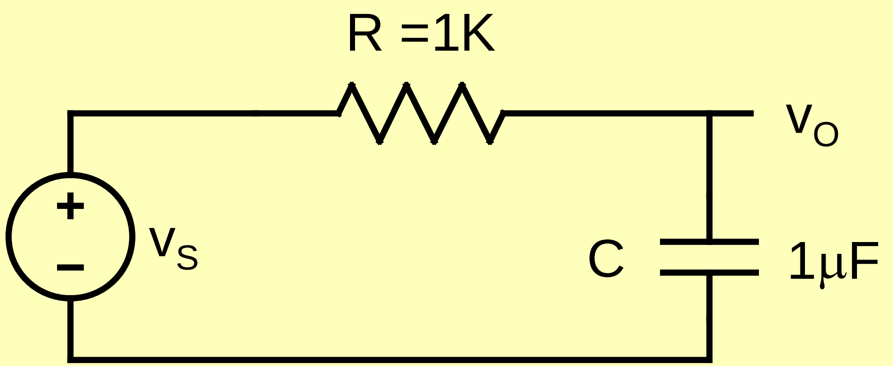

R =1K
v_O
v_S
C
1μF


RC_freq_response.CIR
20.000
0.000
-20.000
-40.000
-60.000
-80.000
1
10
100
1K
10K
100K
1M
db(V(VO))
F (Hz)

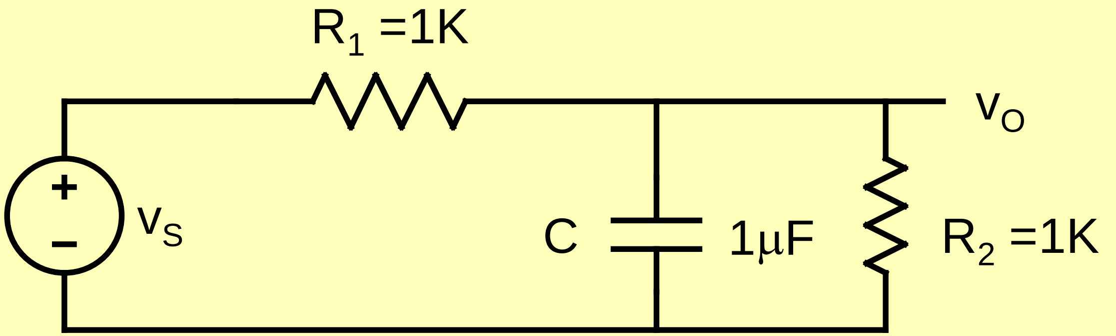

C
$v_O(t)$
v_S
R


12.000
0.000
-12.000
-24.000
-36.000
-48.000
10
100
1K
10K
100K
F (Hz)
1M
10M

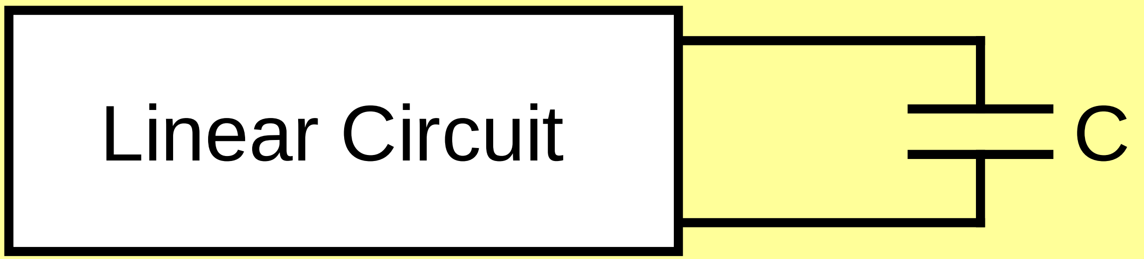
3dB Frequency of single capacitor filters



$$\omega_{3dB} = \frac{1}{RC} = 10^3\, rad/s$$

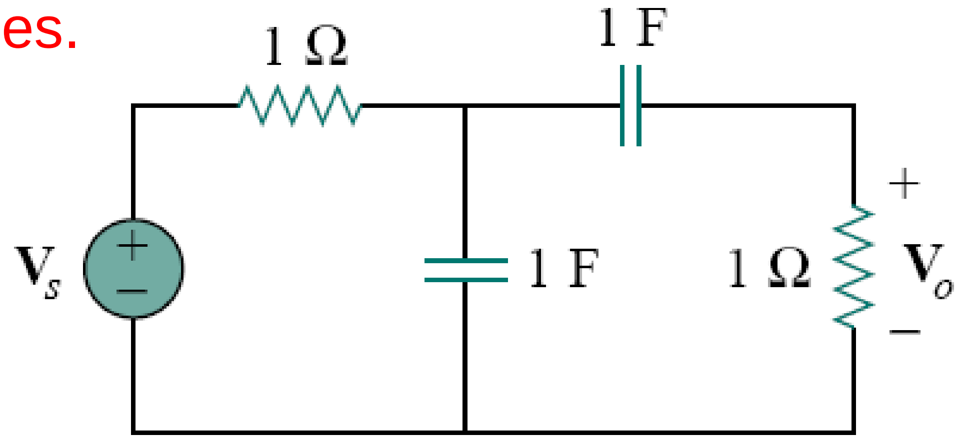


$$\omega_{3dB} = \frac{1}{R_1 \| R_2\, C}$$



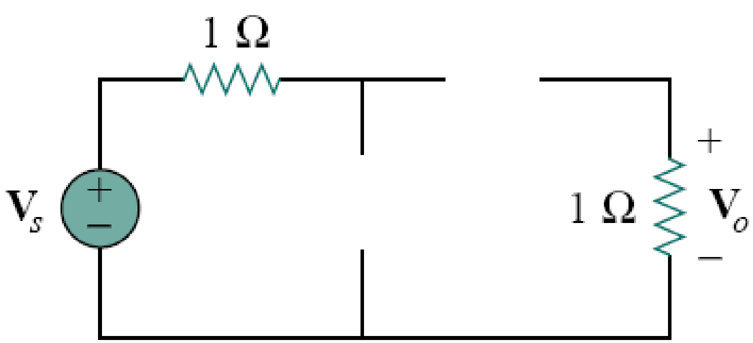
$$\omega_{3dB} = \frac{1}{\tau} = \frac{1}{R_{eq} C}$$

One can often tell the type of filter by looking at behavior at very low and very high frequencies and keeping in mind that capacitor offers very high impedance at low frequencies and very low impedance at high frequencies.

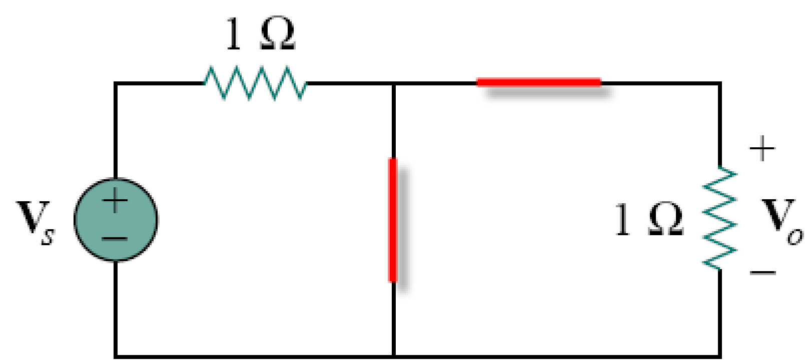


Low *f*

High *f*



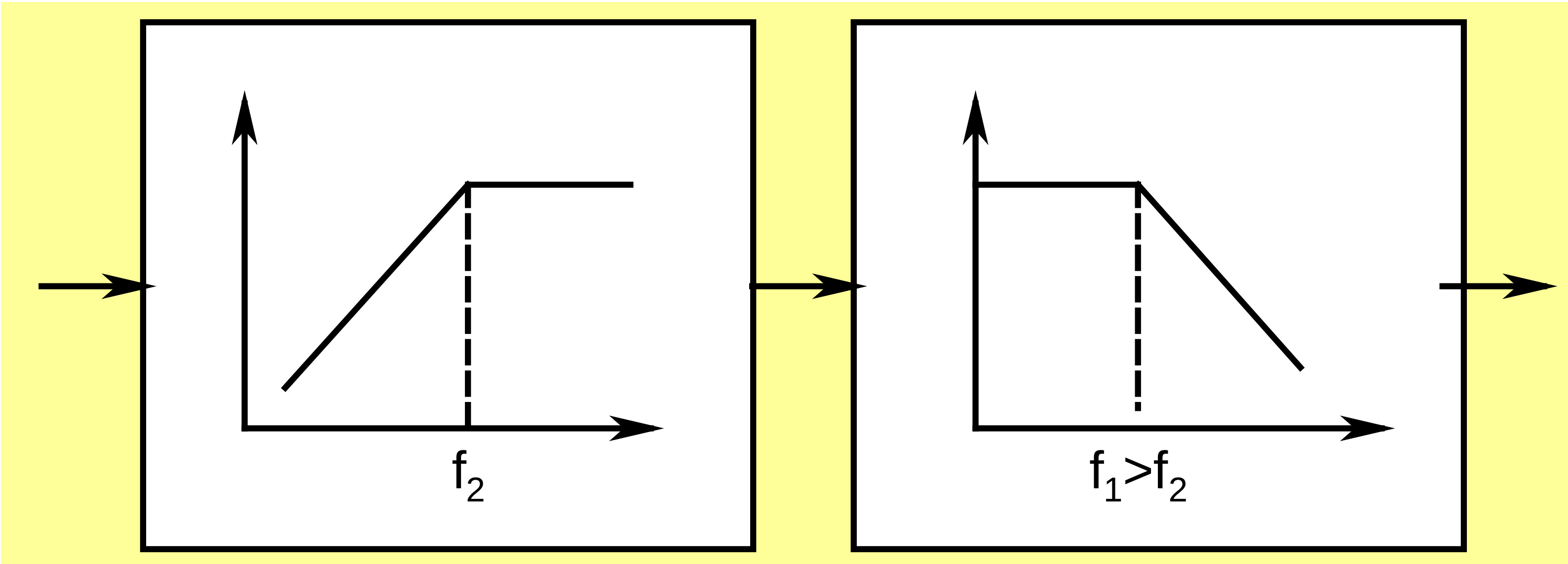
Vo ~0

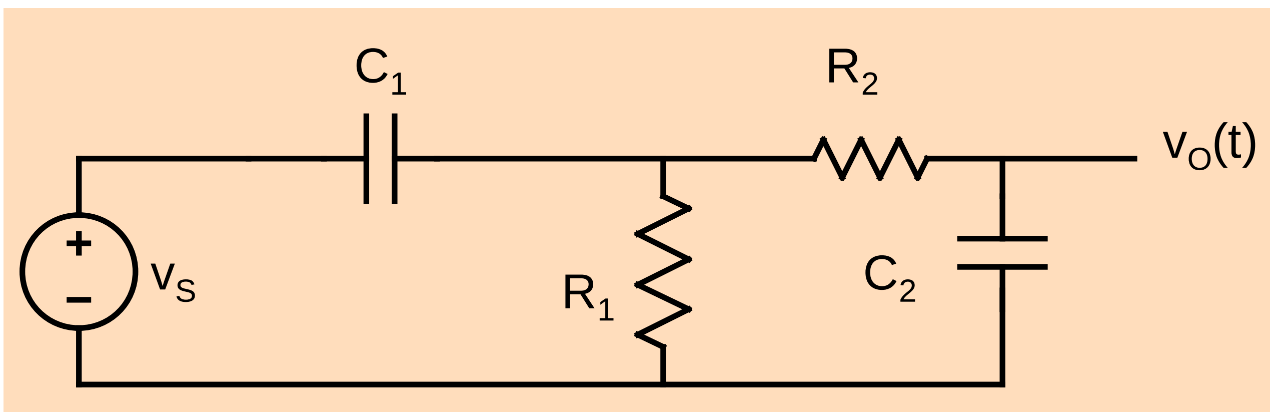


Vo ~0

Bandpass filter

Bandpass Filter





$$f_2 \cong \frac{1}{2\pi R_1 C_1} \; ; \; f_1 \cong \frac{1}{2\pi R_2 C_2}$$

Example: Band Pass filter