ESO207 Theoretical Assignment 3 Semester II, 2016-17

Due Date: 10th April, 2017 **Maximum Marks: 40**

Instructions

- **Submission without hardcopy (one per team) will be penalized by 20**%
- For questions where you need to design an algorithm, give the pseudocode, proof of correctness and time complexity of the algorithm.
- For this assignment you need to work in groups. The group will the same as earlier. Both members of the group must upload a copy of the assignment.
- You must mention in your assignment the name of your group partner as well.
- You need typeset your assignment on a typesetting system such as LaTeX(recommended) or Word.
- Upload a soft copy of the assignment on moodle by the due date.

---

1. (10 points) An $m \times n$ Young tableau is an $m \times n$ matrix such that the entries of each row are in sorted order(ascending) from left to right and the entries of each column are in sorted order(ascending) from top to bottom. Some entries of a Young tableau may be ∞, which is treated as nonexistent element. Give an $O(m+n)$ time algorithm to implement the following functions on a Young tableau.

   1. EXTRACT-MIN - removes the smallest element from the Young tableau
   2. INSERT - inserts a new element into the Young tableau
   3. SEARCH - determines whether a given number is stored in the Young tableau.

2. (10 points) You are given an n-digit number N and m pairs of indices (i, j) where $1 \leq i, j \leq n$. Each pair (i, j) specifies that you are allowed to swap the i^{th} and j^{th} digit of N. The index of the most significant digit is n and the least significant digit is 1. Design an efficient algorithm to find the largest number that can be obtained from N by applying any number of the allowed swaps in any order.

   Note: Any allowed swap can be applied any number of times (≥ 0)

3. (10 points) For a graph $G = (V, E)$, define

$$s(G) = \max_{u,v} \operatorname{dist}(u, v)$$

   where $\operatorname{dist}(u, v)$ is the distance between u and v in G.

   Given an undirected, connected, acyclic graph $G = (V, E)$ design an $O(|V|)$ algorithm to compute $s(G)$.

4. (10 points) A directed graph $G = (V, E)$ is called a *pseudo-tree* if for all $u \in V$, the subgraph induced by the set of vertices reachable from u, forms a rooted tree. Give an efficient algorithm to determine whether a graph is a pseudo-tree or not.

   Note: This is an open-ended question. Better complexity will fetch more marks.