

Tutorial 11

ODE: Boundary Value Problem

Consider the following differential equation

$$\frac{d^2 y}{dx^2} - y = 0; \quad x \in (0, 2)$$

with boundary conditions $y(0) = 0$ and $y(2) = 3.627$.

Determine the solution over the interval $x = 0$ to 2 by the following methods. Estimate the true error in the numerical result by comparing it against the analytical result at $x = 1.0$.

- a. Shooting method with the Midpoint method and step size $h = 0.5$. Assume first two approximations of $y'(0)$ as 1.0 and 2.0 .
- b. Finite difference method by approximating $\frac{d^2 y}{dx^2}$ using a central difference formula of $O(h^2)$. Take the step size $h = 1.0$.
- c. Repeat (b) by taking the step size $h = 0.5$.
- d. Use Richardson extrapolation to improve the estimate of $y(1)$ obtained in (b) and (c).