ESc201 : Introduction to Electronics

Digital Circuits-1

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Numbers

Every number system is associated with a base or radix

A positional notation is commonly used to express numbers

$$(a_5a_4a_3a_2a_1a_0)_r = a_5r^5 + a_4r^4 + a_3r^3 + a_2r^2 + a_1r^1 + a_0r^0$$

The decimal system has a base of 10 and uses symbols (0,1,2,3,4,5,6,7,8,9) to represent numbers

$$(2009)_{10} = 2\times10^3 + 0\times10^2 + 0\times10^1 + 9\times10^0$$

$$(123.24)_{10} = 1\times10^2 + 2\times10^1 + 3\times10^0 + 2\times10^{-1} + 4\times10^{-2}$$

Numbers

An octal number system has a base 8 and uses symbols (0,1,2,3,4,5,6,7)

$$(2007)_8 = 2\times 8^3 + 0\times 8^2 + 0\times 8^1 + 7\times 8^0$$

What decimal number does it represent?

$$(2007)_8 = 2\times 512 + 0\times 64 + 0\times 8^1 + 7\times 8^0 = 1033$$

$$(2007)_8 = \left(1033\right)_{10}$$

Numbers

A hexadecimal system has a base of 16

$$(2BC9)_{16} = 2\times16^3 + B\times16^2 + C\times16^1 + 9\times16^0$$

How do we convert it into decimal number?

$$(2BC9)_{16} = 2\times4096 + 11\times256 + 12\times16^1$$
$$+9\times16^0 = 11209$$

$$(2BC9)_{16} = \left(11209\right)_{10}$$

Number	Symbol
0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
10	A
11	B
12	C
13	D
14	E
15	F

A Binary system has a base 2 and uses only two symbols 0, 1 to represent all the numbers

$$(1101)_2 = 1\times 2^3 + 1\times 2^2 + 0\times 2^1 + 1\times 2^0$$

Which decimal number does this correspond to ?

$$(1101)_2 = 1\times 8 + 1\times 4 + 0\times 2^1 + 1\times 2^0 = 13$$

1 1 0 1 . 1 0 0 1

2^3 2^2 2^1 2^0 2^{-1} 2^{-2} 2^{-3} 2^{-4}

13. 5625

2^0	1
2^1	2
2^2	4
2^3	8
2^4	16
2^5	32
2^6	64
2^7	128
2^8	256
2^9	512
2^{10}	1024(K)
2^{20}	1048576(M)

2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}	2^{-6}
0.5	0.25	0.125	0.0625	0.03125	0.015625

Developing Fluency with Binary Numbers

1 1 0 0 1 = ?

25

1100001 = ?

64+32+1=97

0.101 = ?

0.5+0.125=0.625

11.001 = ?

3+0.125=3.125

Converting decimal to binary number

Convert 45 to binary number

$$(45)_{10} = b_n b_{n-1} \ldots\ldots b_0$$

$$45 = b_n 2^n + b_{n-1} 2^{n-1} \ldots\ldots + b_1 2^1 + b_0$$

Divide both sides by 2

$$\frac{45}{2} = 22.5 = b_n 2^{n-1} + b_{n-1} 2^{n-2} \ldots\ldots + b_1 2^0 + b_0 \times 0.5$$

$$22 + 0.5 = b_n 2^{n-1} + b_{n-1} 2^{n-2} \ldots\ldots + b_1 2^0 + b_0 \times 0.5$$

$$\Rightarrow \quad b_0 = 1$$

$$22+0.5=b_n 2^{n-1}+b_{n-1}2^{n-2}.......+b_1 2^0+b_0\times 0.5 \quad \Rightarrow \; b_0=1$$

$$22=b_n 2^{n-1}+b_{n-1}2^{n-2}.......+b_2 2^1+b_1 2^0$$

Divide both sides by 2

$$\frac{22}{2}=11=b_n 2^{n-2}+b_{n-1}2^{n-3}.......+b_2 2^0+b_1\times 0.5 \quad \Rightarrow \; b_1=0$$

$$11=b_n 2^{n-2}+b_{n-1}2^{n-3}......+b_3 2^1+b_2 2^0$$

$$5.5=b_n 2^{n-3}+b_{n-1}2^{n-4}......+b_3 2^0+0.5b_2 \quad \Rightarrow \; b_2=1$$

$$5=b_n 2^{n-3}+b_{n-1}2^{n-4}......+b_4 2^1+b_3 2^0$$

$$5 = b_n 2^{n-3} + b_{n-1} 2^{n-4} \ldots\ldots + b_4 2^1 + b_3 2^0$$

$$2.5 = b_n 2^{n-4} + b_{n-1} 2^{n-5} \ldots\ldots + b_4 2^0 + 0.5 b_3$$ $$\Rightarrow \; b_3 = 1$$

$$2 = b_n 2^{n-4} + b_{n-1} 2^{n-5} \ldots\ldots + b_5 2^1 + b_4 2^0$$

$$1 = b_n 2^{n-5} + b_{n-1} 2^{n-6} \ldots\ldots + b_5 2^0 + 0.5 b_4$$ $$\Rightarrow \; b_4 = 0$$

$$\Rightarrow \; b_5 = 1$$

$$(45)_{10} \; = b_5 b_4 b_3 b_2 b_1 b_0 = 101101$$

Converting decimal to binary number

Method of successive division by 2

45	remainder
22	1
11	0
5	1
2	1
1	0
0	1

45 = 1 0 1 1 0 1

44 = 1 0 1 1 0 0

Convert $(153)_{10}$ to octal number system

$$(153)_{10} = (b_n b_{n-1} \ldots\ldots b_0)_8$$

$$(153)_{10} = b_n 8^n + b_{n-1} 8^{n-1} \ldots\ldots b_1 8^1 + b_0$$

Divide both sides by 8

$$\frac{153}{8} = 19.125 = b_n 8^{n-1} + b_{n-1} 8^{n-2} \ldots\ldots b_1 8^0 + \frac{b_0}{8}$$

$$\Rightarrow \frac{b_0}{8} = 0.125$$

$$\Rightarrow b_0 = 1$$

153	remainder
19	1
2	3
0	2

153 = $(231)_8$

Converting decimal to binary number

Convert $(0.35)_{10}$ to binary number

$$(0.35)_{10} = 0.b_{-1}b_{-2}b_{-3}\ldots\ldots b_{-n}$$

$$0.35 = 0 + b_{-1}2^{-1} + b_{-2}2^{-2} + \ldots\ldots b_{-n}2^{-n}$$

How do we find the b_{-1} b_{-2} …coefficients?

Multiply both sides by 2

$$0.7 = b_{-1} + b_{-2}2^{-1} + \ldots\ldots b_{-n}2^{-n+1}$$

$$\Rightarrow b_{-1} = 0$$

$$0.7 = b_{-2}2^{-1} + b_{-3}2^{-2} + \ldots\ldots b_{-n}2^{-n+1}$$

$$0.7 = b_{-2}2^{-1} + b_{-3}2^{-2} + \ldots\ldots b_{-n}2^{-n+1}$$

Multiply both sides by 2

$$1.4 = b_{-2} + b_{-3}2^{-1} + \ldots\ldots b_{-n}2^{-n+2}$$

$$\Rightarrow \; b_{-2} = 1$$

Note that ½+1/4+1/8+……≤1

$$0.4 = b_{-3}2^{-1} + b_{-4}2^{-2} \ldots\ldots b_{-n}2^{-n+2}$$

$$0.8 = b_{-3} + b_{-4}2^{-1} \ldots\ldots b_{-n}2^{-n+3}$$

$$\Rightarrow \; b_{-3} = 0$$

Converting decimal to binary number

0.125 = ?

0 .	125	
		x2
0 .	25	
		x2
0 .	5	
		x2
1.	0	

$0.125 = (.001)_2$

0.8125 = ?

0 .	8125	
		x2
1 .	625	
		x2
1 .	25	
		x2
0.	5	
		x2
1.	0	

$0.8125 = (.1101)_2$

Binary numbers

Most significant bit or MSB

Least significant bit or LSB

1011000111

Binary digit = bit

This is a 10 bit number

N-bit binary number can represent numbers from 0 to $2^N - 1$

decimal	2bit	3bit	4bit	5bit
0	00	000	0000	00000
1	01	001	0001	00001
2	10	010	0010	00010
3	11	011	0011	00011
4		100	0100	00100
5		101	0101	00101
6		110	0110	00110
7		111	0111	00111
8			1000	01000
9			1001	01001
10			1010	01010
11			1011	01011
12			1100	01100
13			1101	01101
14			1110	01110
15			1111	01111

Converting Binary to Hex and Hex to Binary

$$(b_7 b_6 b_5 b_4 b_3 b_2 b_1 b_0)_b = (h_1, h_0)_{Hex}$$

$$b_7 2^7 + b_6 2^6 + b_5 2^5 + b_4 2^4 + b_3 2^3 + b_2 2^2 b_1 2^1 + b_0$$
$$= h_1 16^1 + h_0$$

$$\underbrace{(b_7 2^3 + b_6 2^2 + b_5 2^1 + b_4)}_{h_1} 2^4 + \underbrace{(b_3 2^3 + b_2 2^2 b_1 2^1 + b_0)}_{h_0} = h_1 16^1 + h_0$$

$$(10110011)_b = (1011)(0011) = (B3)_{Hex}$$

$$(110011)_b = (11)(0011) = (33)_{Hex}$$

$$(EC)_{Hex} = (1110)(1100) = (11101100)_b$$

Number	Symbol
0(0000)	0
1(0001)	1
2(0010)	2
3(0011)	3
4(0100)	4
5(0101)	5
6(0110)	6
7(0111)	7
8(1000)	8
9(1001)	9
10(1010)	A
11(1011)	B
12(1100)	C
13(1101)	D
14(1110)	E
15(1111)	F

Binary Addition/Subtraction

$$\begin{array}{r} 0 \\ \underline{0} \\ \underline{0} \end{array}$$

$$\begin{array}{r} 1 \\ \underline{0} \\ \underline{1} \end{array} \qquad \begin{array}{r} 0 \\ \underline{1} \\ \underline{1} \end{array}$$

$$\begin{array}{r} 1 \\ \underline{1} \\ \underline{1\ 0} \end{array}$$

$$\begin{array}{r} 1 \\ 1 \\ \underline{1} \\ \underline{1\ 1} \end{array}$$

$$\begin{array}{r} 1\ 0\ 1 \\ \underline{1\ 1\ 0} \\ \underline{1\ 0\ 1\ 1} \end{array}$$

$$\begin{array}{r} 1\ 1\ 0\ 1 \\ \underline{+\ 1\ 1\ 1\ 0} \\ 1\ \underline{1\ 0\ 1\ 1} \end{array}$$

Complement of a number

Decimal system: → 9's complement

→ 10's complement

9's complement of n-digit number x is $10^n - 1 - x$

10's complement of n-digit number x is $10^n - x$

9's complement of 85 ? $10^2 - 1 - 85$ $99 - 85 = 14$

$$\text{9's complement of 123} = 999 - 123 = 876$$

$$\text{10's complement of 123} = \text{9's complement of 123} + 1 = 877$$

Complement of a binary number

Binary system: → 1's complement
→ 2's complement

1's complement of n-bit number x is $2^n - 1 - x$

2's complement of n-bit number x is $2^n - x$

1's complement of 1011 ? $2^4 - 1 - 1011$ $1111 - 1011 = 0100$

1's complement is simply obtained by flipping a bit (changing 1 to 0 and 0 to 1)

1's complement of 1001101 = ?

0110010

$$2\text{'s complement of } 1010 = 1\text{'s complement of } 1010+1$$
$$= 0101 + 1 = 0110$$

$$2\text{'s complement of } 110010 =$$

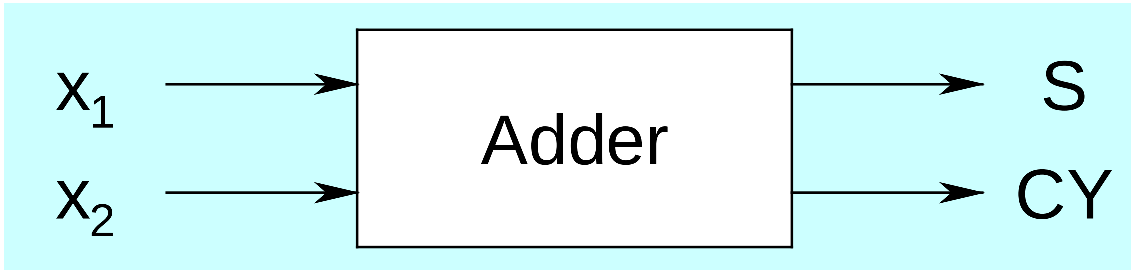
Leave all least significant 0’s as they are, leave first 1 unchanged and then flip all subsequent bits

$$001110$$

$$1011 \rightarrow 0101$$

$$101101100 \rightarrow 010010100$$

Advantages of using 2's complement



10 – 6 = ?

$$10_{10} \Rightarrow (1010)_2$$

$$6_{10} \Rightarrow (0110)_2$$

$$\text{2's complement of } 0110 = 1010$$

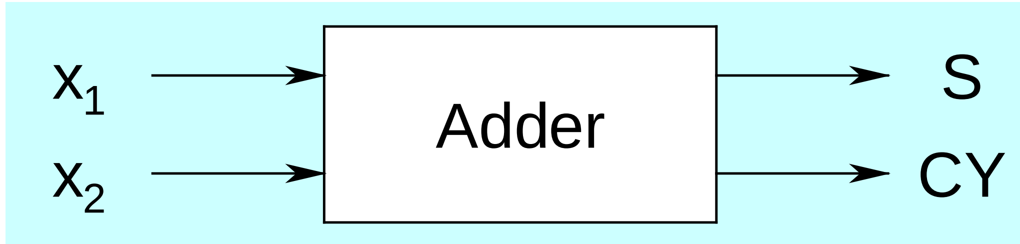
$$\begin{array}{r} 1010 \\ \underline{+1010} \\ 10100 \end{array}$$

If Carry is 1; then number you get is positive

$$(0100)_2 \Rightarrow 4_{10}$$

Answer is +4

Advantages of using 2’s complement



6 – 10 = ?

$$6_{10} \Rightarrow (0110)_2$$

$$10_{10} \Rightarrow (1010)_2$$

$$\text{2's complement of } 1010 = 0110$$

$$\begin{array}{r} 0110 \\ +0110 \\ \hline 01100 \end{array}$$

If Carry is 0; then number you get is negative

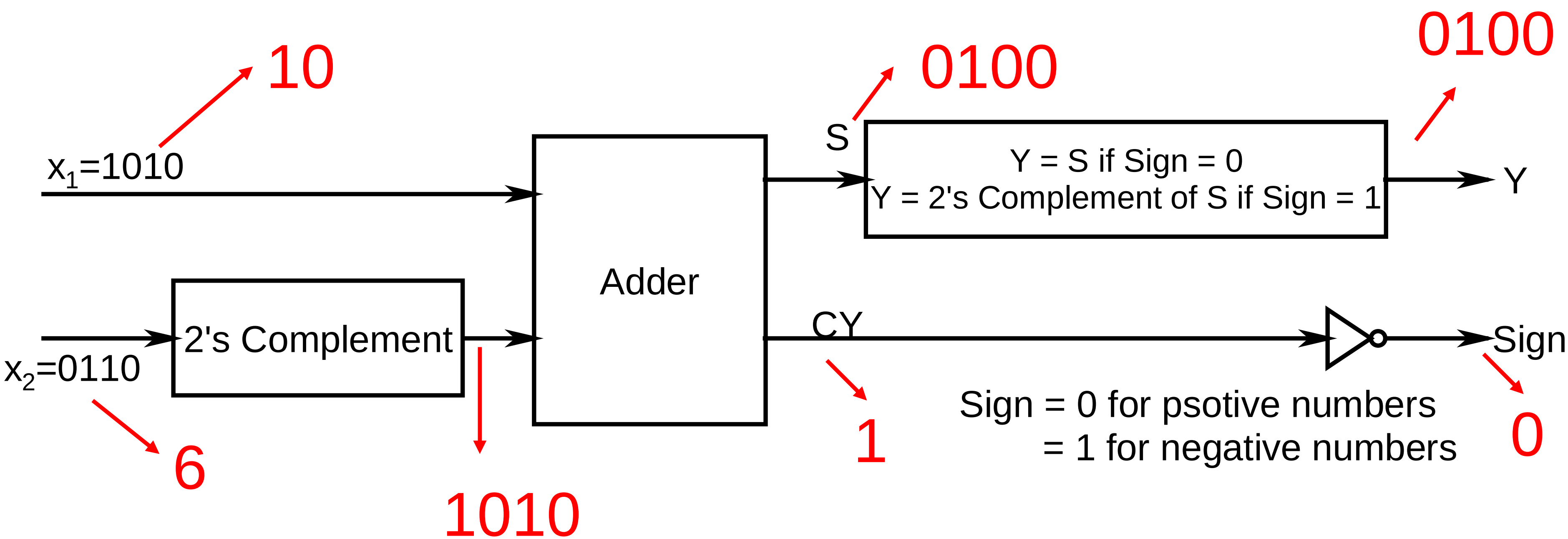
Take the 2’s complement of number

$$\text{2's complement of } 1100 = 0100$$

$$(0100)_2 \Rightarrow 4_{10}$$

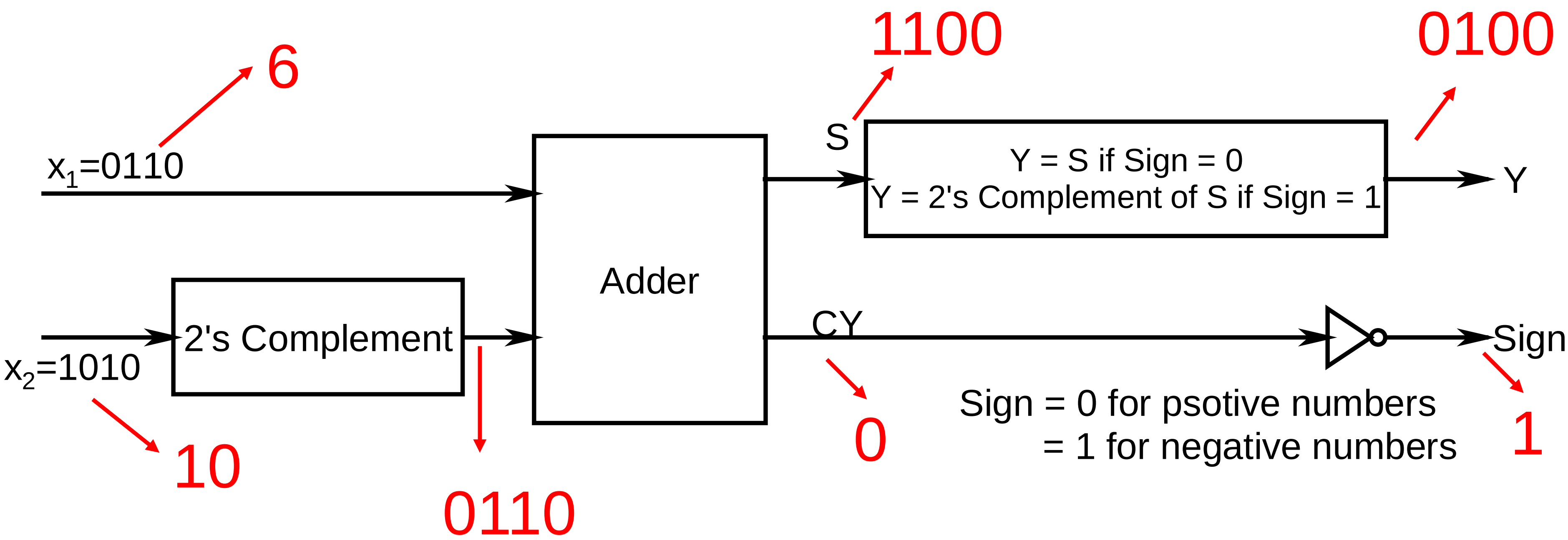
Answer is -4

Example



$$
\begin{array}{r}
1\,0\,1\,0 \\
+\ 1\,0\,1\,0 \\
\hline
1\,0\,1\,0\,0 \\
\hline
\end{array}
$$

Example



$$\begin{array}{r} 0\,1\,1\,0 \\ +\ 0\,1\,1\,0 \\ \hline 1\,1\,0\,0 \\ \hline \end{array}$$

It makes sense to use adder as a subtractor as well provided additional circuit required for carrying out 2’s complement is simple

Representing positive and negative binary numbers

One extra bit is required to carry sign information. Sign bit = 0 represents positive number and Sign bit = 1 represents negative number

decimal	Signed Magnitude
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
-0	1000
-1	1001
-2	1010
-3	1011
-4	1100
-5	1101
-6	1110
-7	1111

decimal	Signed 1’s complement
-1	1110
-2	1101
-3	1100
-4	1011
-5	1010
-6	1001
-7	1000

decimal	Signed 2’s complement
-1	1111
-2	1110
-3	1101
-4	1100
-5	1011
-6	1010
-7	1001

Example

$$\begin{array}{r} +5 \\ \underline{+2} \\ \underline{+7} \end{array}$$	$$\begin{array}{r} 0101 \\ \underline{+0010} \\ \underline{0111} \end{array}$$

$$\begin{array}{r} +5 \\ \underline{-2} \\ \underline{+3} \end{array}$$	$$\begin{array}{r} 0101 \\ \underline{+1110} \\ \underline{0011} \end{array}$$

$$\begin{array}{r} -5 \\ \underline{+2} \\ \underline{-3} \end{array}$$	$$\begin{array}{r} 1011 \\ \underline{+0010} \\ \underline{1101} \end{array}$$

$$\begin{array}{r} -5 \\ \underline{-2} \\ \underline{-7} \end{array}$$	$$\begin{array}{r} 1011 \\ \underline{+1110} \\ \underline{1001} \end{array}$$

2’s complement is 011 = 3

2’s complement is 111 = 7

Example

+ 6	00000110
+13	00001101
+19	00010011

− 6	11111010
+13	00001101
+ 7	00000111

+ 6	00000110
−13	11110011
− 7	11111001

− 6	11111010
−13	11110011
−19	11101101

2's complement is 11111001:

00000111= 7

Boolean Algebra

Algebra on Binary numbers

A variable x can take two values {0,1}

0
- False
- No
- Low voltage

1
- True
- Yes
- High voltage

Basic operations:

AND: $y = x_1 . x_2$

y is 1 if and only if both x_1 and x_2 are 1, otherwise zero

Truth Table

x_1	x_2	y
0	0	0
0	1	0
1	0	0
1	1	1

Basic operations:

$$\text{OR:} \quad y = x_1 + x_2$$

y is 1 if either x_1 or x_2 is 1. y= 0 if and only if both variables are zero

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	1

$$\text{NOT:} \quad y = \overline{x}$$

x	y
0	1
1	0

Boolean Algebra

Basic Postulates

P1.a: $x + 0 = x$	P1.b: $x \,.\, 1 = x$	Identity element
P2.a: $x + y = y + x$	P2.b: $x \,.\, y = y \,.\, x$	Commutative
P3.a: $x.(y+z) = x.y+x.z$	P3.b: $x+y.z = (x+y).(x+z)$	Distributive
P4.a: $x + \overline{x} = 1$	P4.b: $x \,.\, \overline{x} = 0$	Complement

Basic Theorems

T1.a: $x + x = x$	T1.b: $x \,.\, x = x$
T2.a: $x + 1 = 1$	T2.b: $x \,.\, 0 = 0$
T3.a: $\overline{(\overline{x})} = x$	
T4.a: $x + (y+z) = (x+y)+z$	T4.b: $x \,.\, (y.z) = (x.y).z$
T5.a: $\overline{(x+y)} = \overline{x} \,.\, \overline{y}$ (DeMo	T5.b: $\overline{(x.y)} = \overline{x} + \overline{y}$ (DeMorgan
T6.a: $x + x.y = x$	T6.b: $x.(x+y) = x$

Proving Theorems

P1.a: $x + 0 = x$	P1.b: $x . 1 = x$
P2.a: $x + y = y + x$	P2.b: $x . y = y . x$
P3.a: $x.(y+z) = x.y+x.z$	P3.b: $x+y.z = (x+y).(x+z)$
P4.a: $x + \overline{x} = 1$	P4.b: $x . \overline{x} = 0$

Prove T1.a: $x + x = x$

$x + x = (x+x). 1$ (P1.b)

$= (x+x). (x+\overline{x})$ (P4.a)

$= x + x.\overline{x}$ (P3.b)

$= x + 0$ (P4.b)

$= x$ (P1.a)

Prove T1.b: $x . x = x$

$x . x = x.x+ 0$ (P1.a)

$= x.x + x.\overline{x}$ (P4.b)

$= x . (x+\overline{x})$ (P3.a)

$= x . 1$ (P4.a)

$= x$ (P1.b)

Proving Theorems

P1.a: $x + 0 = x$

P2.a: $x + y = y + x$

P3.a: $x.(y+z) = x.y+x.z$

P4.a: $x + \overline{x} = 1$

P1.b: $x . 1 = x$

P2.b: $x . y = y . x$

P3.b: $x+y.z = (x+y).(x+z)$

P4.b: $x . \overline{x} = 0$

Prove : $x + 1 = 1$

$x + 1 = x+(x+ \overline{x})$

$= (x+x)+ \overline{x}$

$= x + \overline{x}$

$= 1$

$x + x .y = x$

$= x . 1 + x. y$

$= x. (1+ y)$

$= x . 1$

$=x$

$x + \overline{x} .y = x+y$

$= (x + \overline{x}). (x+ y)$

$= 1. (x+ y)$

$= x + y$

DeMorgan's Theorem

$$\overline{(x_1 + x_2 + x_3 + ...)} = \overline{x_1} . \overline{x_2} . \overline{x_3} .$$

$$\overline{(x_1 . x_2 . x_3 .....)} = (\overline{x_1} + \overline{x_2} + \overline{x_3} + ...)$$

Simplification of Boolean expressions

$$\overline{(x_1 + x_2 + x_3 + \ldots)} = \overline{x_1} \cdot \overline{x_2} \cdot \overline{x_3} \cdot$$

$$\overline{(x_1 \cdot x_2 \cdot x_3 \ldots)} = (\overline{x_1} + \overline{x_2} + \overline{x_3} + \ldots)$$

$$\overline{(\overline{x_1} . x_2 + \overline{x_2} . x_3)} = ?$$

$$= (\overline{\overline{x_1} . x_2}) . (\overline{\overline{x_2} . x_3})$$

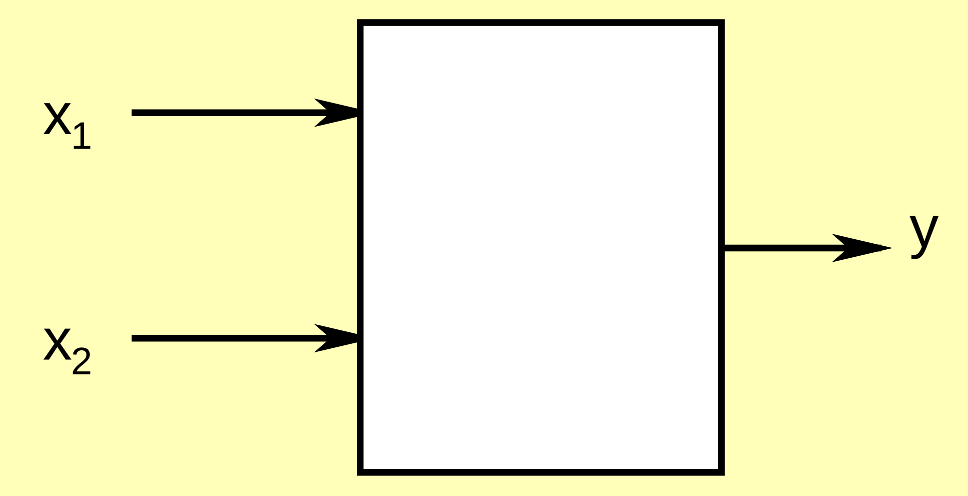
$$= (\overline{\overline{x_1}} + \overline{x_2}) . (\overline{\overline{x_2}} + \overline{x_3})$$

$$= (x_1 + \overline{x_2}) . (x_2 + \overline{x_3})$$

$$= x_1 . x_2 + \overline{x_2} . x_2 + x_1 . \overline{x_3} + \overline{x_2} . \overline{x_3}$$

$$= x_1 . x_2 + x_1 . \overline{x_3} + \overline{x_2} . \overline{x_3}$$

Function of Boolean variables



x_1	x_2	y
0	0	0
0	1	1
1	0	0
1	1	0

y = 1 when x_1 is 0 and x_2 is 1

$$y = \overline{x_1} \cdot x_2$$

Boolean expression

Obtaining Boolean expressions from truth Table

x_1	x_2	y
0	0	1
0	1	0
1	0	0
1	1	0

$$y = \overline{x_1} . \overline{x_2}$$

x_1	x_2	y
0	0	0
0	1	0
1	0	1
1	1	0

$$y = x_1 . \overline{x_2}$$

x_1	x_2	y
0	0	1
0	1	0
1	0	0
1	1	1

$$\overline{x_1} . \overline{x_2}$$

$$x_1 . x_2$$

$$y = \overline{x_1} . \overline{x_2} + x_1 . x_2$$

Obtaining Boolean expressions from truth Table

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	0

$$y = \overline{x_1} . x_2 + x_1 . \overline{x_2}$$

Instead of writing expressions as sum of terms that make y equal to 1, we can also write expressions using terms that make y equal to 0

x_1	x_2	y
0	0	1
0	1	1
1	0	1
1	1	0

$$y = \overline{x_1} . \overline{x_2} + \overline{x_1} . x_2 + x_1 . \overline{x_2}$$

$$y = \overline{x_1} + \overline{x_2}$$

x_1	x_2	y
0	0	1
0	1	0
1	0	1
1	1	1

$$y = x_1 + \overline{x_2}$$

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	1

$$y = x_1 + x_2$$

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	0

$$x_1 + x_2$$

$$y = (x_1 + x_2).(\overline{x_1} + \overline{x_2})$$

$$\overline{x_1} + \overline{x_2}$$

Obtaining Boolean expressions from truth Table

$$y = \overline{x_1}.\overline{x_2}.x_3 + \overline{x_1}.x_2.x_3 + x_1.\overline{x_2}.x_3 + x_1.x_2.x_3$$

x_1	x_2	x_3	y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

Sum of Products (SOP) form

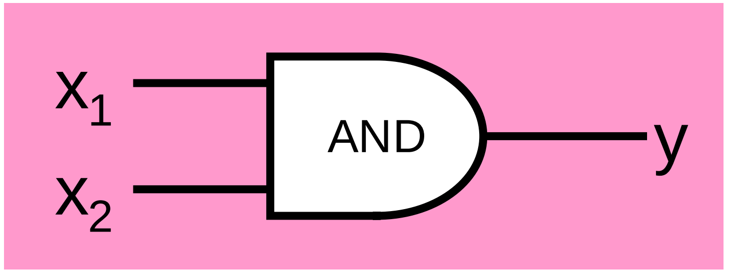
$$y = (x_1 + x_2 + x_3).(x_1 + \overline{x_2} + x_3).(\overline{x_1} + x_2 + x_3).(\overline{x_1} + \overline{x_2} + x_3)$$

Product of Sum (POS) form

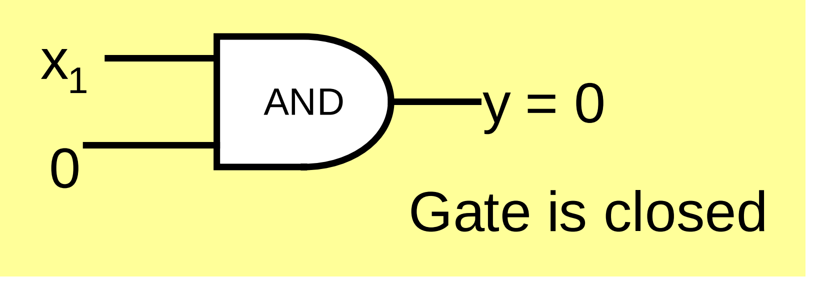
Implementing Boolean expressions

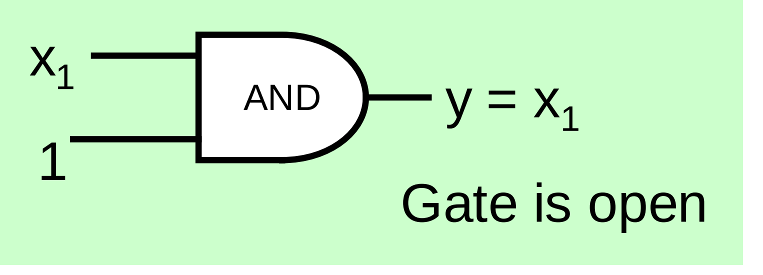
Elementary Gates

$$\text{AND:}\quad y = x_1 . x_2$$

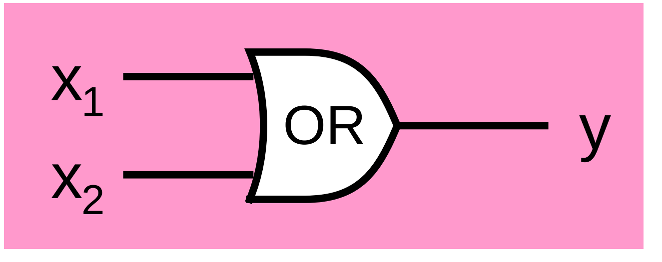


Why call it a gate?

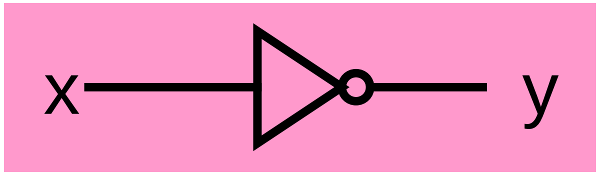




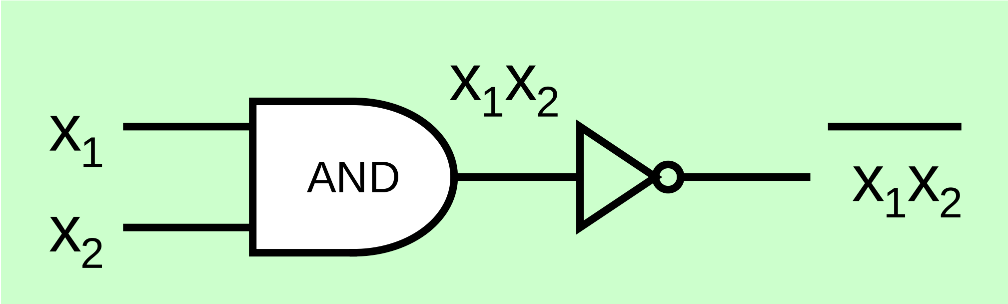
$$\text{OR:}\quad y = x_1 + x_2$$

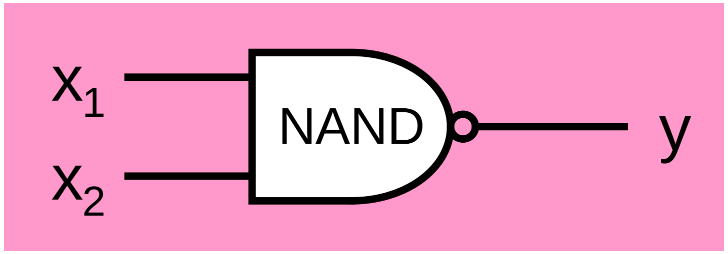


$$\text{NOT:}\quad y = \overline{x}$$

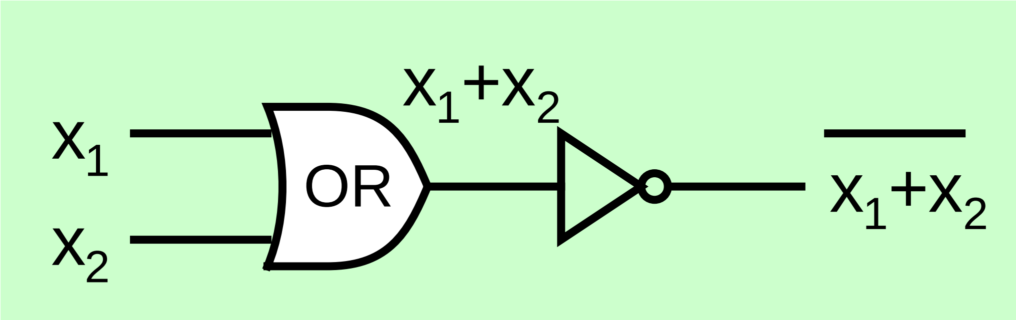


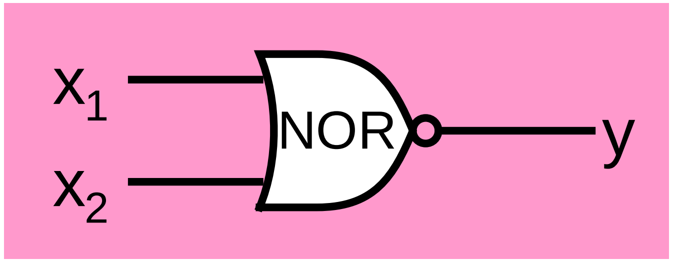
NAND: $y = \overline{x_1 . x_2}$





NOR: $y = \overline{x_1 + x_2}$

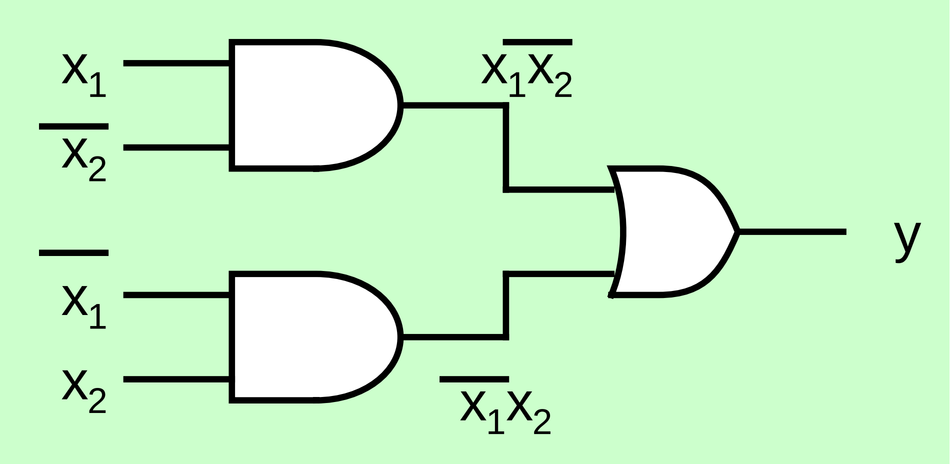


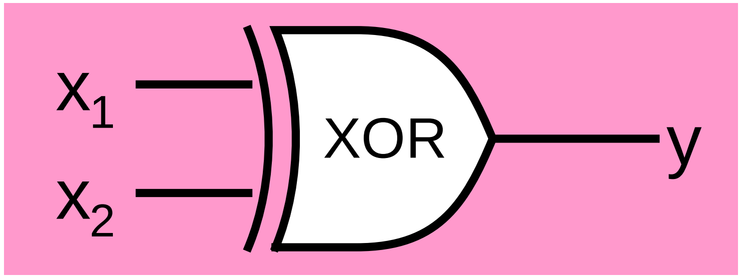


XOR: $y = x_1 \oplus x_2 = x_1 . \overline{x_2} + \overline{x_1} . x_2$

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	0

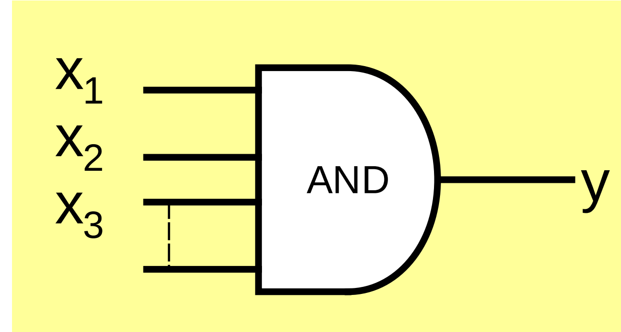
y is 1 if only one variable is 1 and the other is zero



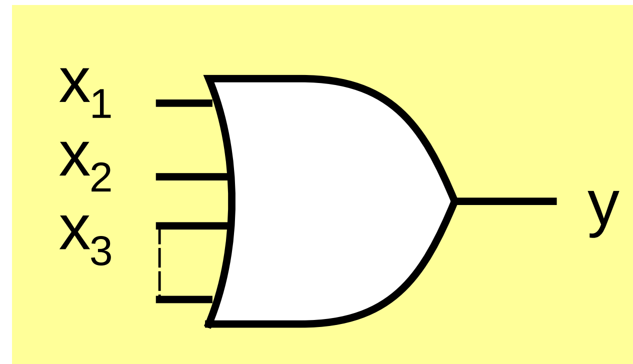


Gates with more than 2 inputs

AND: $y = x_1 . x_2 . x_3 \ldots$



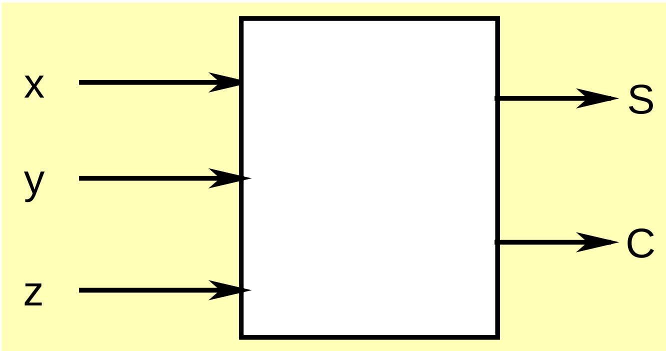
OR: $y = x_1 + x_2 + x_3 + \ldots .$



XOR: $y = x_1 \oplus x_2 \oplus x_3 = x_1 . \overline{x_2}\,\overline{x_3} + \overline{x_1} . x_2 . \overline{x_3} + \overline{x_1} . \overline{x_2} . x_3 + x_1 . x_2 . x_3$

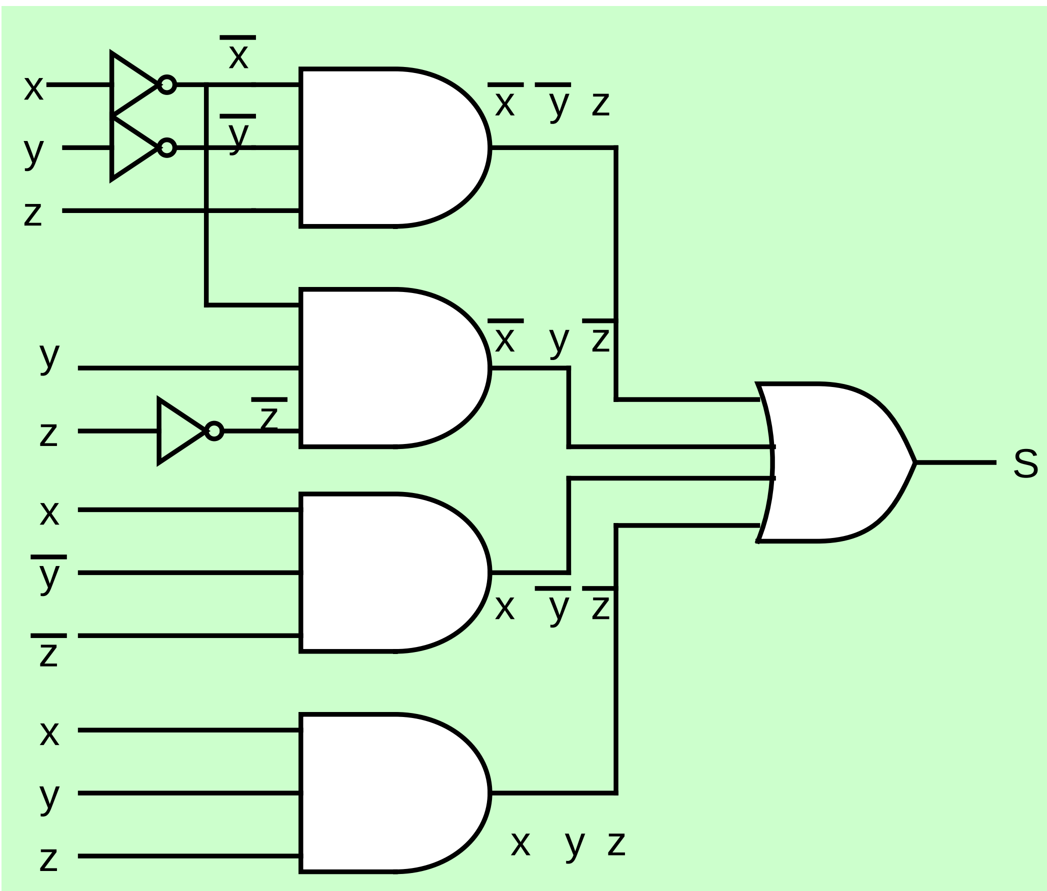
y = 1 only if odd number of inputs is 1

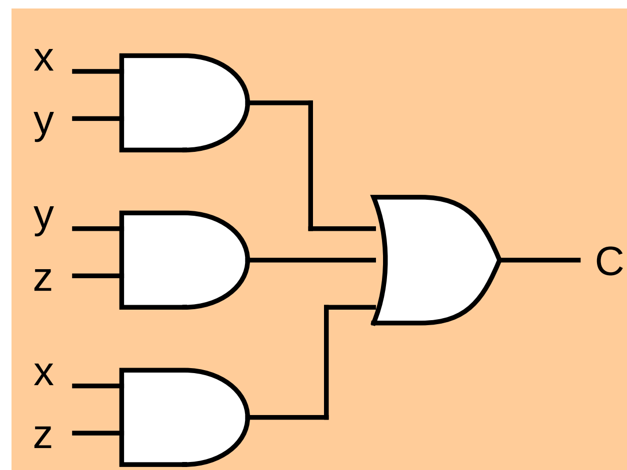
Implementing Boolean expressions using gates



$$S = \overline{x}.\overline{y}.z + \overline{x}.y.\overline{z} + x.\overline{y}.\overline{z} + x.y.z$$

$$C = x.y + x.z + y.z$$





Representation of Boolean Expressions

x	y	f_1
0	0	0
0	1	1
1	0	1
1	1	0

x	y	min term	
0	0	$\bar{x}\,.\,\bar{y}$	m0
0	1	$\bar{x}\,.\,y$	m1
1	0	$x\,.\,\bar{y}$	m2
1	1	$x\,.\,y$	m3

$$f_1 = \bar{x}.\,y + x.\bar{y}$$

$$f_1 = m_1 + m_2$$

$$f_1 = \sum(1,2)$$

$$f_2 = \sum(0,2,3) = ?$$

$$f_2 = \bar{x}.\,\bar{y} + x.\bar{y} + x\,.\,y$$

A minterm is a product that contains all the variables used in a function

Three variable functions

x	y	z	min terms	
0	0	0	$\overline{x}.\overline{y}.\overline{z}$	m0
0	0	1	$\overline{x}.\overline{y}.z$	m1
0	1	0	$\overline{x}.y.\overline{z}$	m2
0	1	1	$\overline{x}.y.z$	m3
1	0	0	$x.\overline{y}.\overline{z}$	m4
1	0	1	$x.\overline{y}.z$	m5
1	1	0	$x.y.\overline{z}$	m6
1	1	1	$x.y.z$	m7

$$f_2 = \sum(1,4,7) = ?$$

$$f_2 = \overline{x}.\overline{y}.z + x.\overline{y}.\overline{z} + x.y.z$$

Product of Sum Terms Representation

x	y	f_1
0	0	0
0	1	1
1	0	1
1	1	0

x	y	Max term	
0	0	$x + y$	M0
0	1	$x + \overline{y}$	M1
1	0	$\overline{x} + y$	M2
1	1	$\overline{x} + \overline{y}$	M3

$$f_1 = (x + y) \cdot (\overline{x} + \overline{y})$$

$$f_1 = M_0 \cdot M_3$$

$$f_1 = \prod (M_0, M_3)$$

x	y	z	Max. terms	
0	0	0	$x + y + z$	M0
0	0	1	$x + y + \overline{z}$	M1
0	1	0	$x + \overline{y} + z$	M2
0	1	1	$x + \overline{y} + \overline{z}$	M3
1	0	0	$\overline{x} + y + z$	M4
1	0	1	$\overline{x} + y + \overline{z}$	M5
1	1	0	$\overline{x} + \overline{y} + z$	M6
1	1	1	$\overline{x} + \overline{y} + \overline{z}$	M7

$$f_1 = \Pi(1,5,7) = ?$$

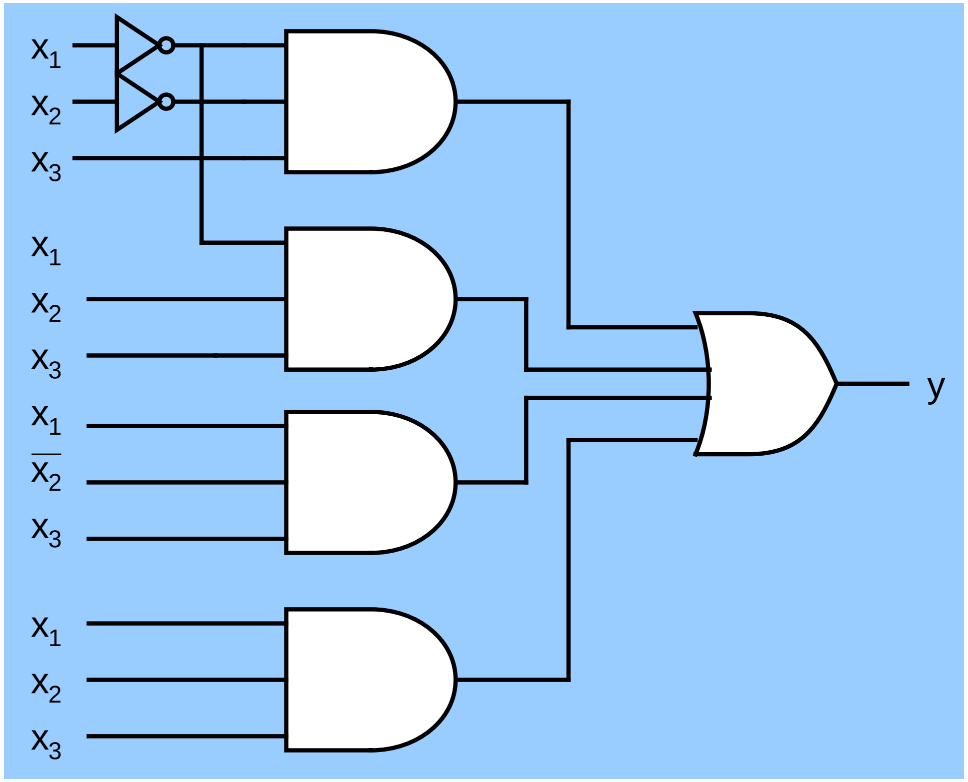
$$f_2 = (x + y + \overline{z}).(\overline{x} + y + \overline{z}).(\overline{x} + \overline{y} + \overline{z})$$

Simplification

x_1	x_2	x_3	y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

$$y = \sum(1,3,5,7)$$

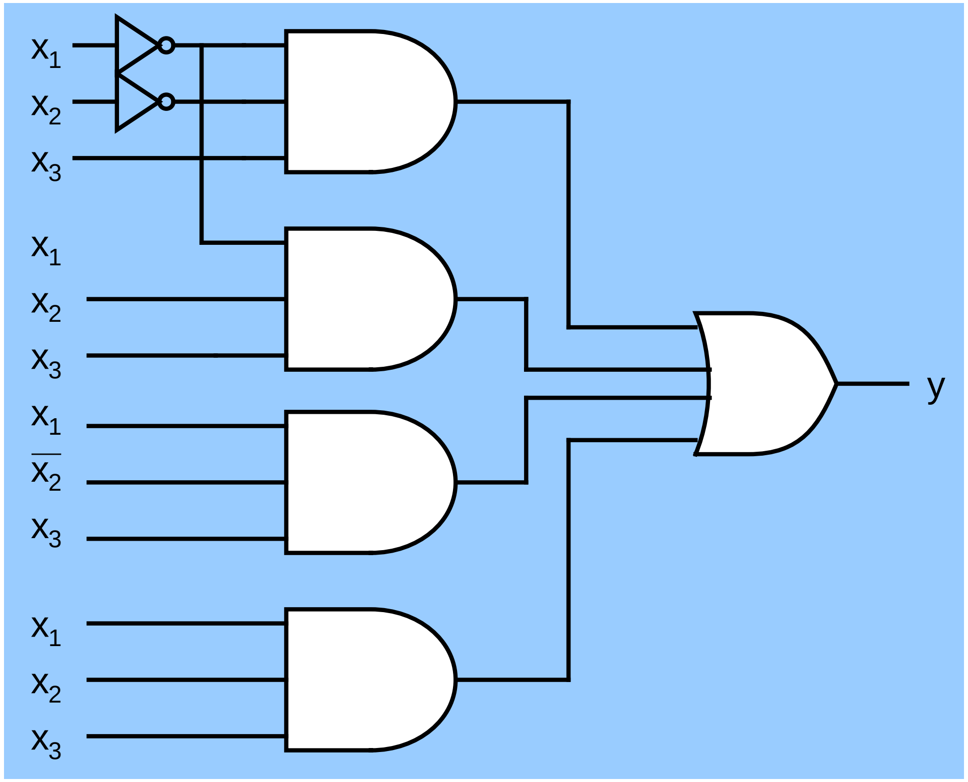
$$y = \overline{x_1}.\overline{x_2}.x_3 + \overline{x_1}.x_2.x_3 + x_1.\overline{x_2}.x_3 + x_1.x_2.x_3$$



Simplification of Boolean expression yields : $y = x_3$!! which does not require any gates at all !

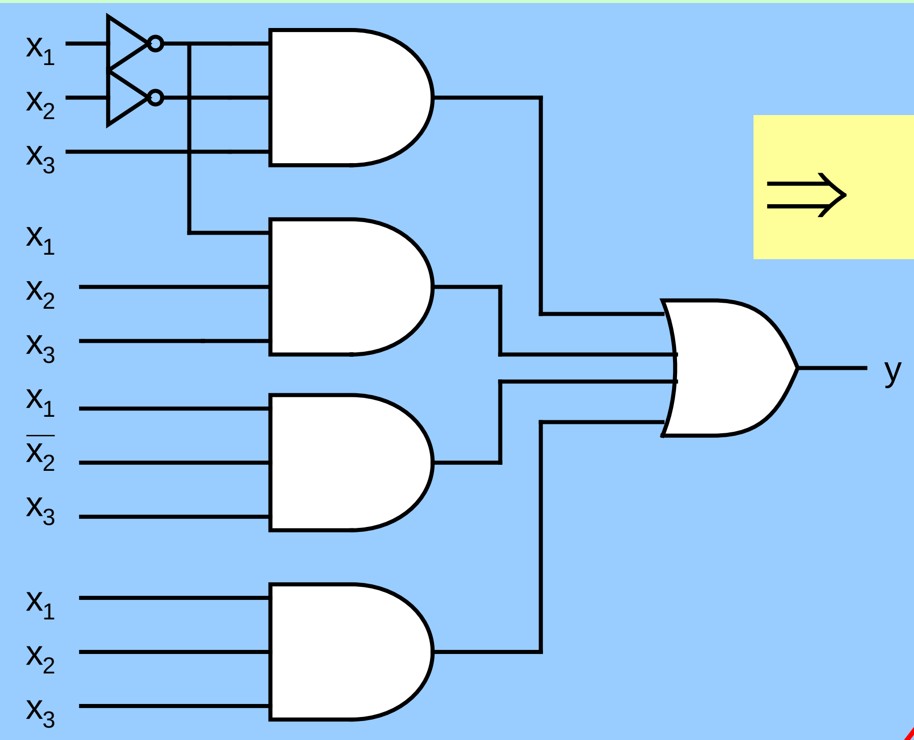
Goal of Simplification

$$y = \overline{x_1} \,.\, \overline{x_2} \,.\, x_3 + \overline{x_1} \,.\, x_2 \,.\, x_3 + x_1 \,.\, \overline{x_2} \,.\, x_3 + x_1 \,.\, x_2 \,.\, x_3$$

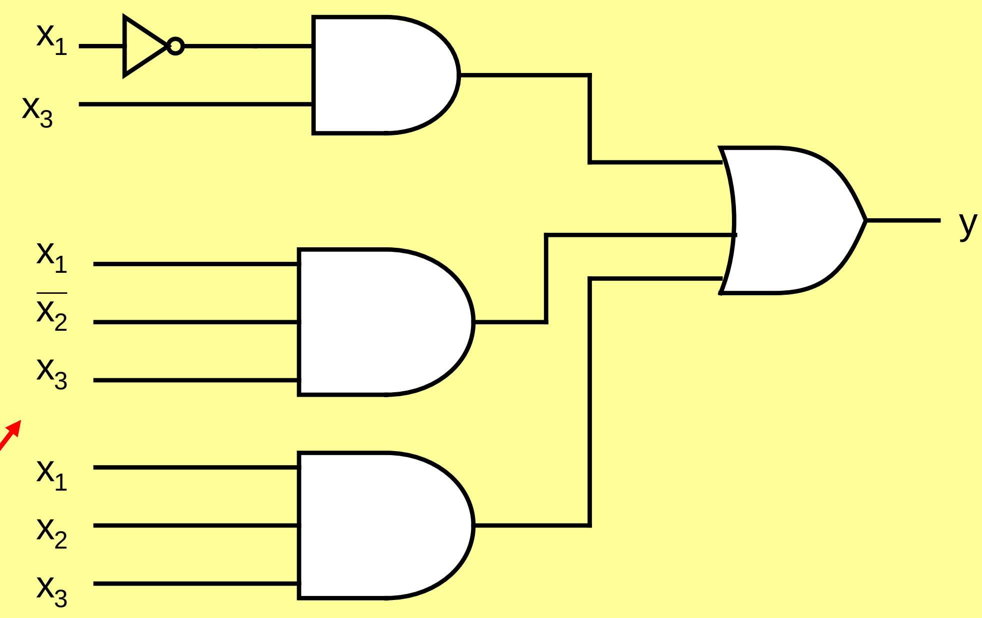


Goal of simplification is to reduce the complexity of gate circuit. This requires that we minimize the number of gates.

$$y = \overline{x_1} \cdot \overline{x_2} \cdot x_3 + \overline{x_1} \cdot x_2 \cdot x_3 + x_1 \cdot \overline{x_2} \cdot x_3 + x_1 \cdot x_2 \cdot x_3$$



$$\Rightarrow \quad y = \overline{x_1} \cdot x_3 + x_1 \cdot \overline{x_2} \cdot x_3 + x_1 \cdot x_2 \cdot x_3$$



This circuit is simpler not just because it uses **4 gates instead of 5** but also because circuit-2 uses **one 2-input and three 3-input gates** as compared **to five 3-input gates used in circuit-1**