Faster Multiplication

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Introduction

When we are multiplying two 2-digit numbers, we can reduce a few cases into a simple formula, rather than actually calculating the multiplication.

Case Analysis for 2-digit Multiplication

For 2-digit multiplication of two numbers, we have two cases for which our reduction method will work. [?]

Case 1.1: When first digits are same and second digits add up to 10

Case 1.2: When second digits are same and first digits add up to 10

Case 1.1

Claim: *Let the numbers be a = x:y and b = x:z, then multiplication result (c) will be x*(x+1):y*z*

Proof.

$$
\begin{aligned}
&a = x : y = 10x + y \quad \text{and} \quad b = x : z = 10x + z \\
&c = a * b \\
&\quad = (10x + y) * (10x + z) \\
&\quad = 100x^2 + 10x * (y + z) + yz \\
&\quad = 100x^2 + 10x * 10 + yz && (\because y + z = 10) \\
&\quad = 100x * (x + 1) + yz \\
&\quad = x * (x + 1) : yz
\end{aligned}
$$

Note [1] $\square$

Example 1.2.1: Let the two numbers be 66 and 64 (See Figure 1)

$$
\begin{aligned}
6 * (6 + 1) &= 42 \\
6 * 4 &= 24 \\
66 * 64 = 42 : 24 &= 4224
\end{aligned}
$$

```
  66
X 64
 264
396
4224
```

Figure 1: Long multiplication method for example 1

Pseudocode:

Algorithm 1 Vedic Multiplication: Case 1.1

procedure MULTIPLY(a, b) ▷ Where a - first number, b - second number
$x = a/10$ ▷ Where / is integer division
$y = a\%10$
$z = b\%10$
$\alpha = x * (x + 1)$
$\beta = yz$
return $100 * \alpha + \beta$
end procedure

[1]if $yz < 10$, then $c = x * (x + 1) : 0 : yz$

Case 1.2

Claim: *Let the numbers be a = x:y and b = x:z, then multiplication result (c) will be x*y+z:z*z*

Proof.

$$
\begin{aligned}
a &= x : z = 10x + z \quad \text{and} \quad b = y : z = 10y + z \\
c &= a * b \\
&= (10x + z) * (10y + z) \\
&= 100xy + 10z * (x + z) + z^2 \\
&= 100xy + 10z * 10 + z^2 && (\because x + y = 10) \\
&= 100(xy + z) + yz \\
&= xy + z : z^2
\end{aligned}
$$

Note [1] □

Example 1.2.1: Let the two numbers be 34 and 74 (See Figure 2)

$$
\begin{aligned}
3 * 7 + 4 &= 25 \\
4 * 4 &= 16 \\
34 * 74 = 25 : 16 &= 2516
\end{aligned}
$$

```
   34
 X 74
  136
 238
 2516
```

Figure 2: Long multiplication method for example

Pseudocode:

Algorithm 2 Vedic Multiplication: Case 1.2

procedure MULTIPLY(a, b) ▷ Where a - first number, b - second number
  $x = a/10$ ▷ Where / is integer division
  $y = b/10$
  $z = a\%10$
  $\alpha = xy + z$
  $\beta = z^2$
    return $100 * \alpha + \beta$
end procedure

[1] if $z^2 < 10$, then $c = xy + z : 0 : z^2$

Do It Mentally

1. Differentiate the case
2. Compute the two different subparts of the answer, i.e. α, β
3. Generate the answer by appending α and β (add 0 before β if $\beta < 10$)

Generalization

For general case, assume the numbers are $(x_1 x_2 ... x_n)$ and $(y_1 y_2 ... y_n)$. We can divide this into a simpler case (with n-1 digits) using the following two cases: [**?**]

Condition	Case	Answer
$x_1 = y_1$	Case **??**	$x_1 * (x_1 + 1) : (x_2 x_3 ... x_n) * (y_2 y_3 ... y_n)$
$x_n = y_n$	Case **??**	$(x_1 x_2 ... x_{n-1}) * (y_1 y_2 ... y_{n-1}) + x_n : x_n^2$

For more details, visit this link