Indian Institute of Technology, Kanpur
Department of Electrical Engineering

ESC 201A

Midterm Examination
Monday, 12th September, 2016

Name: ____

Roll Number: ___________________________

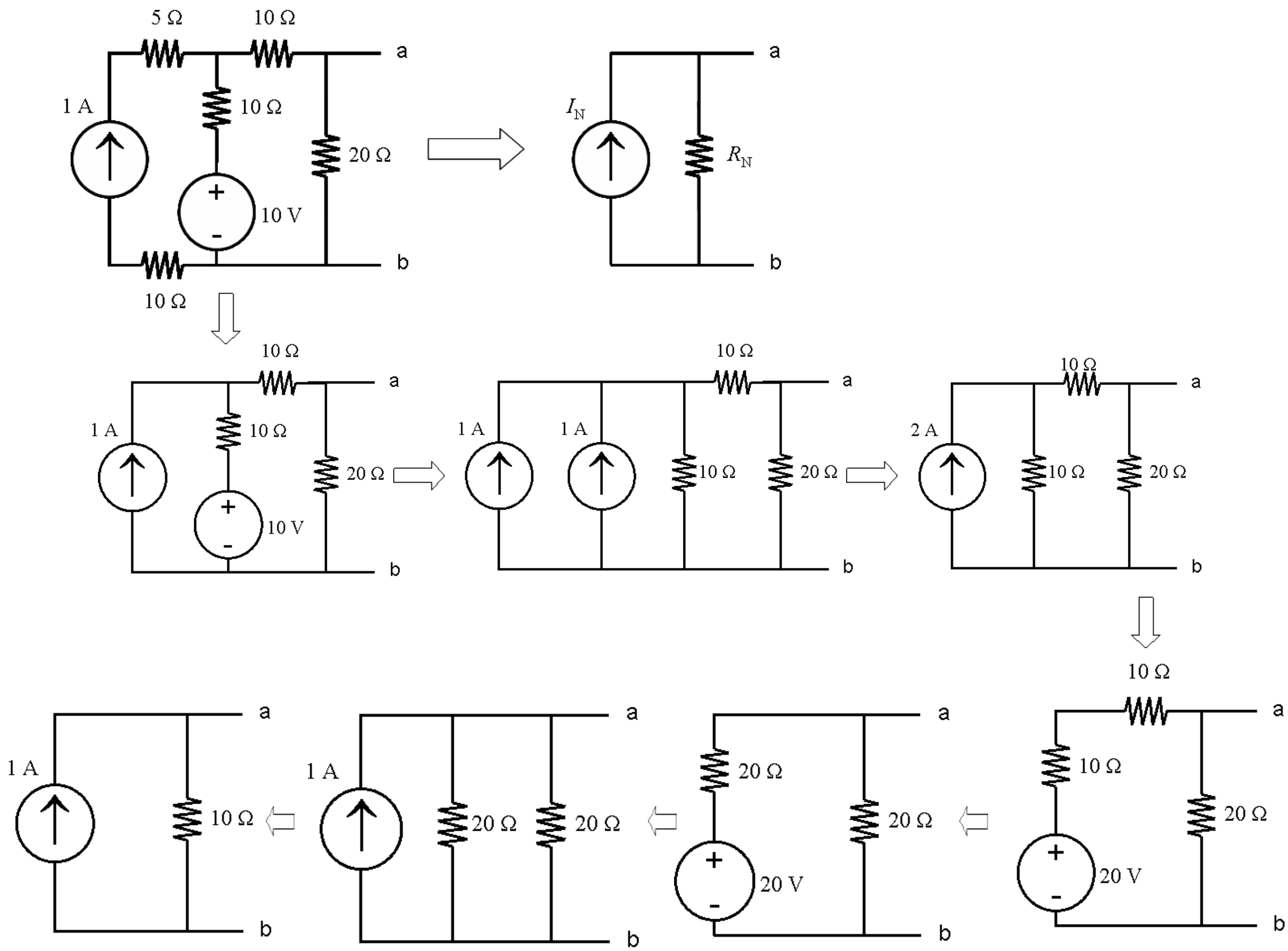
Please confirm there are 14 pages (seven sheets) in this question paper (including the cover pages)

- Maximum time: 120 minutes.
- Answers are to be written in this answer booklet alone to be eligible for grading.
- Feel free to make appropriate approximations in your calculations or assumptions in your analysis. But be sure to mention them clearly in your answer paper.
- Draw the circuit wherever it will help you solve the problem or explain the concept.
- Derive the results analytically as far as possible before plugging in the values.

Question	Score	Maximum Score
1		20
2		20
3		20
4		20
5		20
6		20
Total		**120**

1 (a). (i) Using source transformations show that the circuit below on the left can be transformed into the equivalent circuit shown on the right. Determine I_N and R_N.

(**8 marks**)

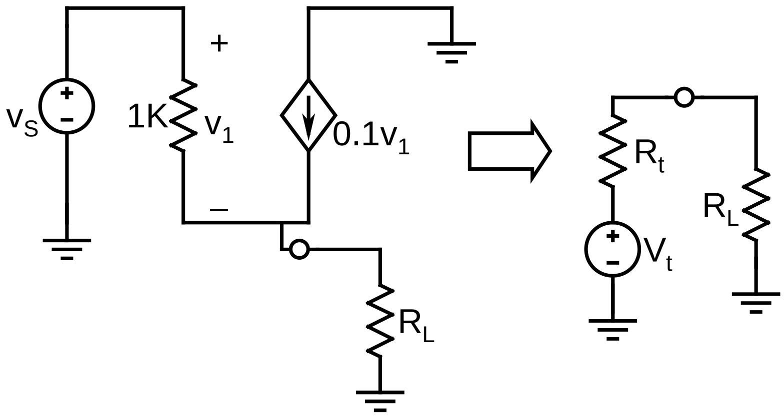


(ii) What should be the load attached across **a** and **b** to have maximum power to be transferred to it?

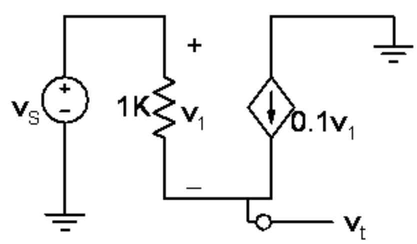
(**2 marks**)

For maximum power transfer, the load should be equal to the Norton equivalent load which is **10 Ω**.

(b). Use Thevenin's theorem to carry out the circuit transformation shown below and determine the value of Thevenin's voltage (V_t) and resistance (R_t). **(10 marks)**



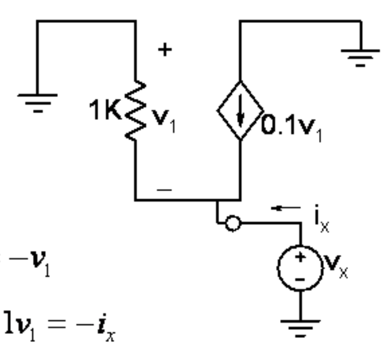
Equivalent circuit for calculation of V_{th}



$$\frac{v_1}{1K} + 0.1v_1 = 0 \Rightarrow v_1 = 0$$

$$\Rightarrow V_t = v_S$$

Equivalent circuit for calculation of R_{th}

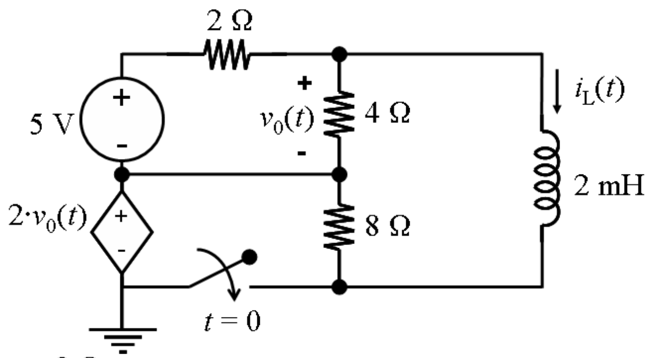


$$v_x = -v_1$$

$$\frac{v_1}{1K} + 0.1v_1 = -i_x$$

$$R_t = \frac{v_x}{i_x} = 9.9\Omega$$

2 (a). The switch in the circuit given below is kept open for a long time, and is closed at $t = 0$.

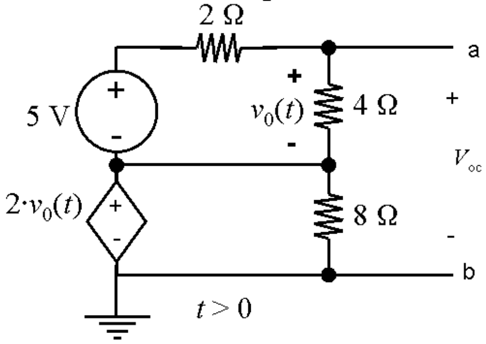


(i) Determine the expressions for the current $i_L(t)$ and the voltage $v_0(t)$ for $t > 0$.

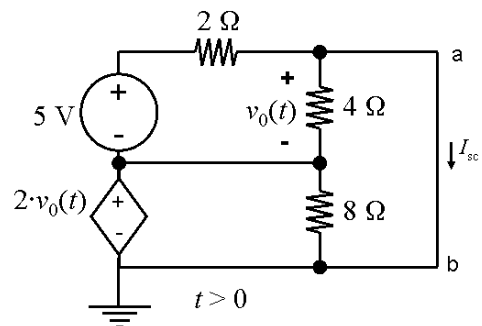
(8 marks)

$i_L(t = 0^-) = \{5/(2 + 4||8)\} \cdot \{4/(4 + 8)\} = 5/14$ A $= i_L(t = 0^+)$

Now for $t > 0$, we could find thevenin equivalent of the circuit to which inductor 2 mH is connected.



V_{th} will be <u>Open circuit voltage across **a** and **b**</u> will be $= v_o(t) + 2 \cdot v_o(t) = 3 \cdot v_o(t) = 3 \cdot \{5 \times 4/(2+4)\} = 10$ V

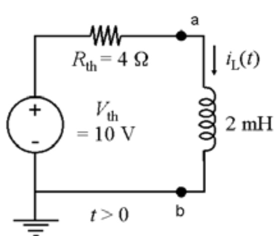


<u>When the terminals **a** and **b** are shorted</u>, we find with KVL

$2 \cdot v_o(t) + v_o(t) = 0 \Rightarrow v_o(t) =$ 0

$\therefore I_{sc} = 5/2 = 2.5$ A

Thus $R_{th} = V_{oc} / I_{sc} = 10/2.5 = 4\ \Omega$.



Thus the $i_L(t \rightarrow \infty) = 10/4 = 2.5$ A

The time constant for the circuit will be $\tau = L/R = 0.002/4 = 1/2000$ s = **0.5 ms**

$\therefore i_L(t) = i_L(t \rightarrow \infty) + [i_L(t = 0^+) - i_L(t \rightarrow \infty)] \cdot \exp(-t/\tau)$

$= 5/2 + (5/14 - 5/2) \cdot \exp(-2000t) =$ $\mathbf{5/2 - (15/7) \cdot \exp(-2000}\boldsymbol{t}\mathbf{)}$ **A**

$\boldsymbol{i}_{\mathbf{L}}\mathbf{(}\boldsymbol{t}\mathbf{) = 2.5 - 2.14 \cdot \exp(-2000}\boldsymbol{t}\mathbf{)}$ **A**

$v_o(t=0^+) = v_o(t=0^-) = 5 \cdot (4||8)/(2 + 4||8) = 20/7$ V. $v_o(t \rightarrow \infty) = 0$;

$\therefore$ $\boldsymbol{v}_{\mathbf{o}}\mathbf{(}\boldsymbol{t}\mathbf{)}$ $= v_o\ (t \rightarrow \infty) + [v_o\ (t = 0^+) - v_o\ (t \rightarrow \infty)] \cdot \exp(-t/\tau) =$ $\mathbf{20/7 \cdot \exp(-2000}\boldsymbol{t}\mathbf{) = 2.86 \cdot \exp(-2000}\boldsymbol{t}\mathbf{)}$ **V**

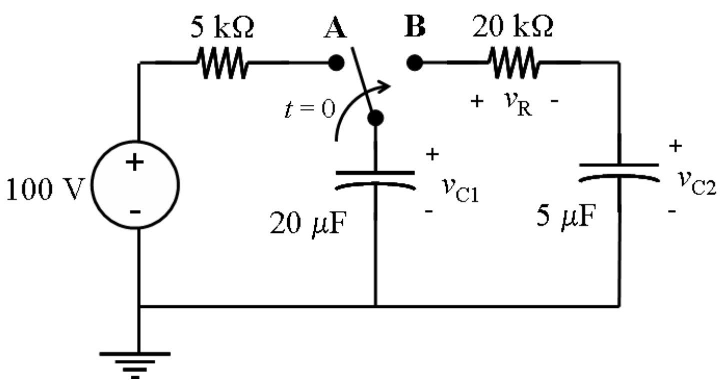
(ii) What is the time constant of variation of voltage across the 2 Ω resistor in the circuit? Why?

(1 + 1 = 2 marks)

The time constant will be the same as above which is **0.5 ms**.

This is because the only reactive element in the circuit is the 2 mH inductor. This, along with the effective resistor parallel to it will determine the time constant for the whole circuit.

(b) A circuit shown below has a switch that can be instantaneously switched from position **A** to position **B**. The switch was in a position **A** for a long time. Assume that the 5 μF capacitor was discharged and has no voltage across it for $t < 0$. The switch position was changed to **B** at $t = 0$.



(i) What are the values of $v_{C1}(0^-)$, $v_{C1}(0^+)$, $v_{C2}(0^+)$, and $v_R(0^+)$? ***(0.5 ×4 = 2 marks)***

$v_{C1}(0^-) = 100$ V = $v_{C1}(0^+)$

$v_{C2}(0^+) = 0$ V; $v_R(0^+) = 100$ V

(iii) What is the time constant of variation of current in the 20 kΩ resistor? ***(1 marks)***

The effective capacitance across 20 kW resistor will be C_1 in series with $C_2 \Rightarrow C_{eff} = [20^{-1} + 5^{-1}]^{-1}$ μF $\Rightarrow C_{eff} = 4$ μF. ∴ time constant of variation of current in 20 kW resistor is $\tau = RC = (20\times10^3)\cdot(4\times10^{-6})$ $\Rightarrow$ **τ = 80 ms**

(v) Determine $v_R(t)$ for $t > 0$. ***(3 marks)***

$v_R(0^+) = 100$ V; $v_R(t \rightarrow \infty) = 0$

$\boldsymbol{v_R(t)} = v_R(t \rightarrow \infty) + [v_R(t = 0^+) - v_R(t \rightarrow \infty)]\cdot\exp(-t/\tau)$

$\boldsymbol{v_o(t)} = 0 + [100 - 0]\cdot\exp(-12.5\cdot t) = 100\cdot\exp(-12.5\cdot t)$ V

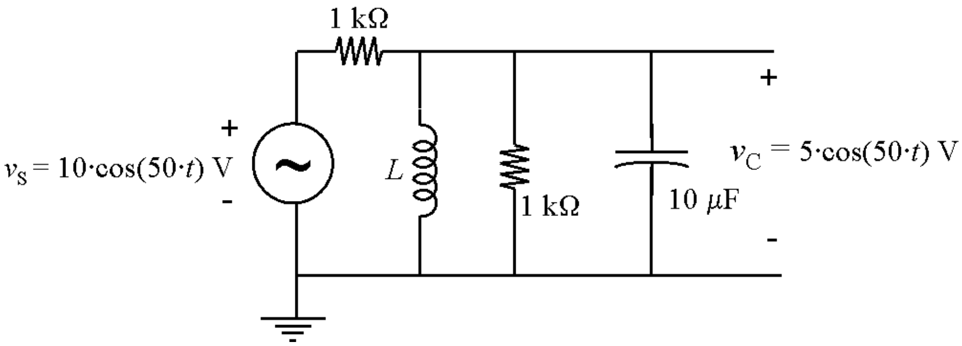
(vii) What is the total energy stored in the 5 μF capacitors as $t \rightarrow \infty$? ***(4 marks)***

$v_{C2}(t \rightarrow \infty) = v_{C1}(0^+)\cdot C_1/(C_1 + C_2) = 100\cdot 20\mu/(20\,\mu + 5\,\mu) = 80$ V

Energy stored in the capacitor C_2 is $\boldsymbol{E_{C2}} = ½\cdot C_2 \cdot v_{C2}^2 = 0.5\cdot5\times10^{-6}\cdot80^2$ **= 16 mJ**

3 (a). (i) What is the frequency of the voltage source in Hz for the circuit given below?

(**2 marks**)



$$f = \omega/2\pi = 50/(2\times3.14) = 7.96 \text{ Hz}$$

(ii) What is the current drawn from the source? (**2 marks**)

Voltage drop across the 1 kΩ resistor is $10\cdot\cos(50\cdot t) - 5\cdot\cos(50\cdot t) = 5\cdot\cos(50\cdot t)$ V

$\therefore$ current is $5\cdot\cos(50\cdot t)/1000 = 5\cdot\cos(50\cdot t)$ mA

(iii) Determine the value of inductor for the voltages indicated in the circuit.

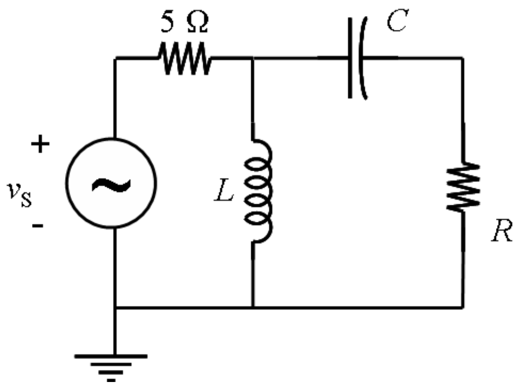
(**6 marks**)

$V_o = 5\angle 0 = [Z_{eq} /(Z_{eq} + 1k)]\times 10\angle 0 \quad \Rightarrow \quad Z_{eq} = 1\ k\Omega$

$Z_{eq} = 1k \parallel j\omega L \parallel -j/\omega C = 1k \quad\quad j\omega L \parallel -j/\omega C \rightarrow \infty$

$L = 1/\omega^2 C = 40$ H

(b) For the circuit shown below $V_s = 20\cdot\cos(5t)$ V, The power dissipated in each of the resistors is 5 W.



(i) What is the v_{rms} of the power supply? ***(1 mark)***

$\boldsymbol{v_{rms}} = V_m / \sqrt{2} = 10\sqrt{2}$ **= 14.14 V**

(i) What is the i_{rms} flowing out of the power supply? ***(2 marks)***

$i_{rms}^2 \cdot R = P$ Power dissipated

Power in 5 Ω is 5W. $\therefore$ $\boldsymbol{i_{rms}}$ **= 1 A**

(ii) What is the power factor for this circuit? ***(3 marks)***

Total power dissipated is $v_{rms}\ i_{rms} \cos(\theta) = 10\sqrt{2}\cdot 1\cdot\cos(\theta) == 5 + 5 = 10$ W

Power factor cos(θ) = 1/√2 = 0.707

(iii) What is the reactive power taken from the power supply? ***(2 marks)***

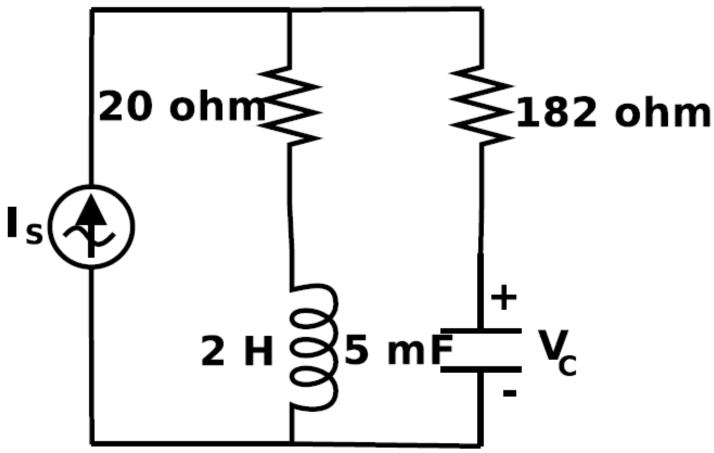
$\theta = \cos^{-1}(0.707) = 45^o$

Reactive power is $\boldsymbol{Q} = v_{rms}\ i_{rms} \sin(\theta) = 10\sqrt{2}\cdot 1\cdot(1/\sqrt{2})$ **= 10 VAR**

(iv) What is the apparent power drawn from the power supply? ***(2 marks)***

Apparent power is $v_{rms}\ i_{rms} = 10\cdot\sqrt{2}$ **= 14.14 VA**

4 (a). Prepare Bode amplitude plot for the transfer function, $H(j\omega) = V_C(j\omega)/I_S(j\omega)$. Show all intermediate steps. **(12 marks)**



Let the current flowing through the capacitor be **I2**, and the current flowing through the inductor be **I1**.

We have $$V_C(j\omega) = \frac{I_2}{j\omega(5X10^{-3})} = \frac{200I_2}{j\omega}$$

We denote the impedance of the inductor in series with 20**Ω** resistor by Z_1, and that of the capacitor in series with 182 **Ω** resistor by Z_2, we have

$$Z_1 = 20 + j2\omega \text{ and } Z_2 = 182 + \frac{200}{j\omega}$$

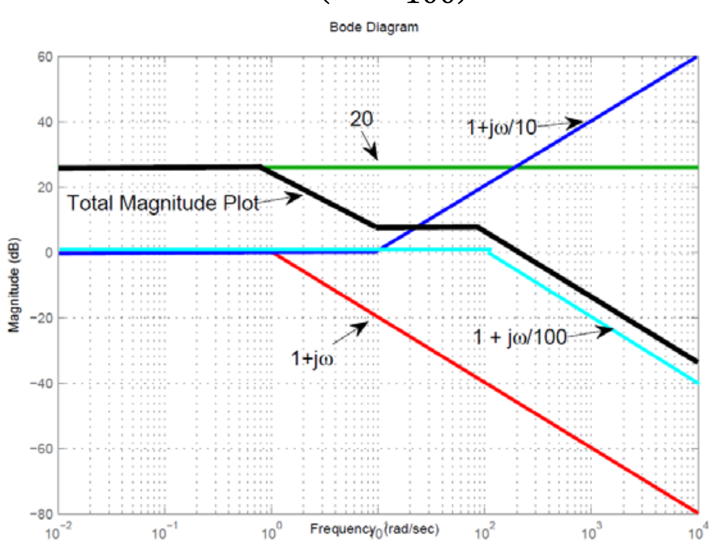
Then, $I_2 = Z_1 * I_S/(Z_1+Z_2)$. Putting this value of I_2 in the expression for V_C, we get

$$V_C(j\omega) = \frac{200}{j\omega}\frac{Z_1}{Z_1+Z_2}I_S$$

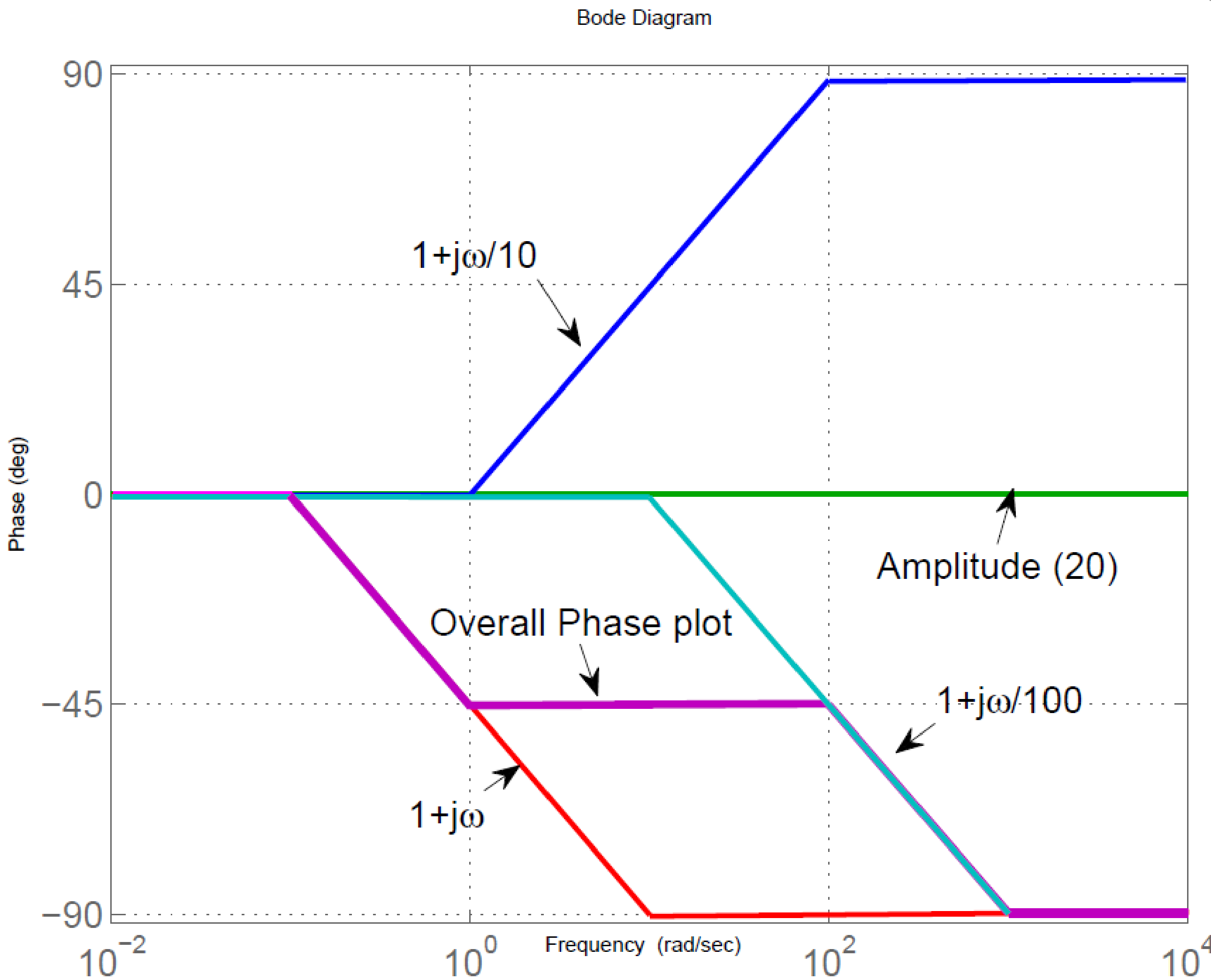
Or we get the transfer function $H(j\omega) = V_C(j\omega)/I_S(j\omega)$ as

$$\begin{aligned}
&= (200/ j\omega) *(20+ j2\omega)/(202+ j2\omega + 200/j\omega) \\
&= 200(20 + 2j\omega)/(200 + 202j\omega + 2(j\omega)^2) \\
&= 100(10 + j\omega)/(100 + 101j\omega + (j\omega)^2) \\
&= 200(10+ j\omega)/((100+ j\omega)(1+ j\omega)) \\
&= (200*10(1+ j\omega/10))/(100*(1+ j\omega/100)*(1+ j\omega))
\end{aligned}$$

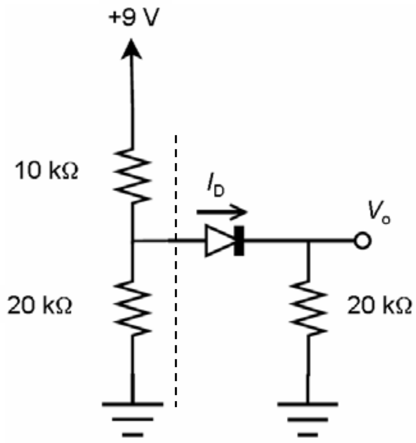
$$H(j\omega) = \frac{20\left(1+j\frac{\omega}{10}\right)}{\left(1+j\frac{\omega}{100}\right)(1+j\omega)}$$

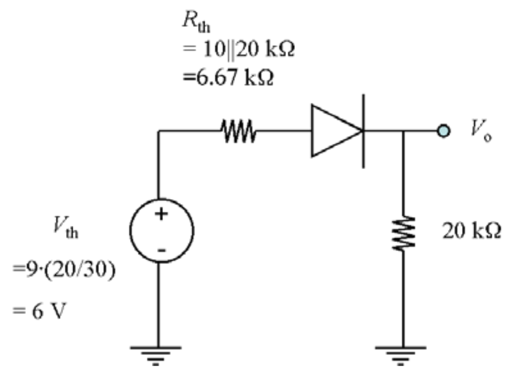


(b). Prepare Bode phase plot for $H(j\omega)$. Show all intermediate steps. **(8 marks)**



5 (a). Consider the diode connected in a circuit as shown alongside. **(i)** Represent the circuit to the left of the diode with its Thévenin equivalent circuit. (**2 marks**)





Evaluate the voltage V_o and current I_D for the following assumption about the diode:
(ii) The diode is ideal. (**1 mark**)

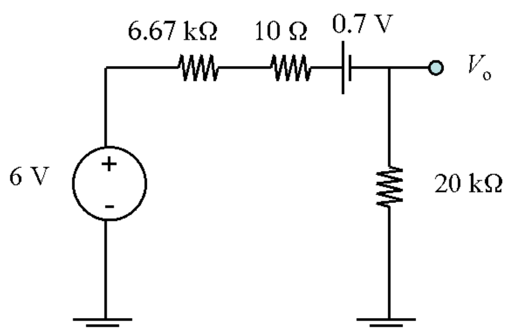
For ideal diode, cut-in voltage is 0 and resistance when on is also zero.

$\therefore$ $V_o = 6 \cdot (20\text{ k}) / (6.67\text{ k} + 20\text{ k}) = \mathbf{4.5\ V}$

$I_D = 6 / (6.67\text{ k} + 20\text{ k}) = \mathbf{225\ \mu A}$

(iii) The diode cut-in voltage $V\gamma = 0.7$ V and on-resistance $r_D = 10\ \Omega$. (**4 marks**)

The diode in forward bias will appear as follows:



$\therefore$ $V_o = (6-0.7) \cdot (20\text{ k}) / (6.67\text{ k} + 20\text{ k} + 10) = \mathbf{3.97\ V}$

and $I_D = (6-0.7) / (6.67\text{ k} + 20\text{ k} + 10) = \mathbf{198.7\ \mu A}$

(iv) The diode reserve saturation current $I_s = 9.5 \times 10^{-15}$ A, ideality factor $n = 1$ and for room temperature, the thermal voltage may be taken to be $V_T = 26$ mV. (**5 marks**)

One may go around the look and from KVL one may write: $-V_{th} + I_D \cdot R_{th} + VD + I_D \cdot 20\text{k} = 0$

Also, from diode equation we know, $I_D = I_s \cdot [\exp(V_D/nV_T) - 1]$

$\therefore$ $(V_{th} - V_D)/(R_{th} + 20\text{ k}) = I_D = I_s \cdot [\exp(V_D/nV_T) - 1] \Rightarrow V_D = nV_T \cdot \ln[(V_{th} - V_D)/\{I_s(R_{th} + 20\text{ k})\} + 1]$

Or $V_D = 0.026 \cdot \ln[(6 - V_D)/\ 9.5\times10^{-15} \cdot 26.67\times10^{-15} + 1]$

First iteration, choose V_D to be 0.7 V and plug it in on the RHS. We get $V_D = 0.6179$ V

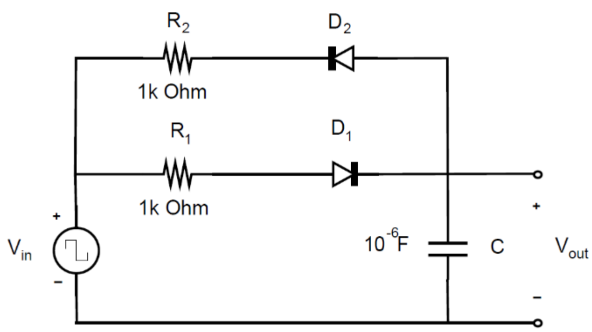
Second iteration, choose V_D to be 0.6179 V and plug it in on the RHS. We get $V_D = 0.6183$ V

Third iteration, choose V_D to be 0.6183 V and plug it in on the RHS. We get $V_D = 0.6183$ V

$\therefore\ V_D \approx 0.62\text{ V} \ \Rightarrow I_D = 9.5\times10^{-15} \cdot [\exp(0.62/0.026) - 1] \approx 215.8\ \mu\text{A}$

(b) For the circuit shown below, sketch the capacitor voltage as a function of time, when the input is a symmetrical square wave with zero average, of frequency 500 Hz and peak-to-peak voltage of 10 V. Find out the positive & negative peak values of the output waveform. Assume the diodes to be ideal.

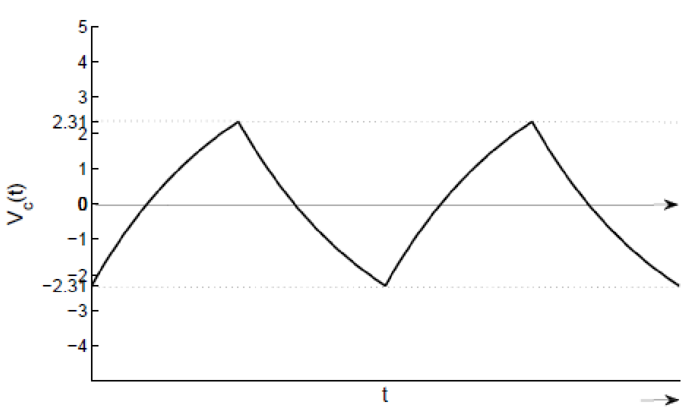
(8 marks)



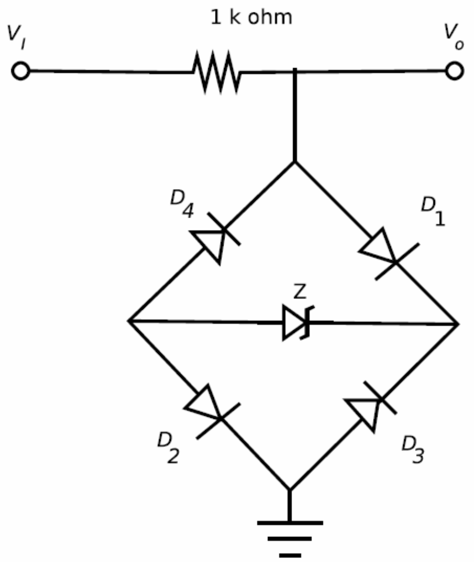
Please note that we are interested in steady state condition. Under steady state condition, the voltage across the capacitor will be symmetrical around x-axis (as charging and discharging time constant are same).

During positive cycle, diode D_1 is on and D_2 is off, and the time constant of the circuit is given by 10^3x10 = 1*ms*. Let the positive peak (under steady state conditions) is given by V', then we have a negative given by -V'. Let the voltage across the capacitor at some time t_0 be given by $v_C(t_0^+) = V'$, and $v_C(\infty) = -5$V, then we have at time $t = t_0 + 1ms$

$$v_C(t) = v_C(\infty) + [v_C(t_0^+) - v_C(\infty)]e^{-(t-t_0)/\tau}$$
$$= -5 + (V' + 5)e^{-1}$$
$$-V' = -5 + (V' + 5)e^{-1}$$
$$\text{or} \quad -V' = (5 - 5e^{-1})/(1 + e^{-1}) = 2.31\text{V}$$



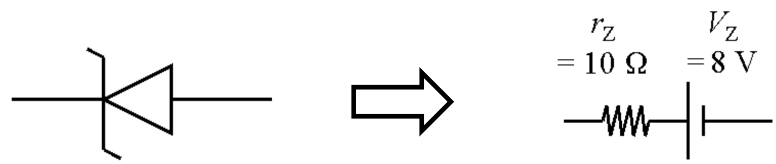
6 (a). The Zener diode in the circuit shown alongside has 8.1 V across it for a current of 10 mA flowing through it. The resistance r_Z in the Zener when it is operating in the breakdown voltage is 10 Ω.



(i) The Zener voltage for D_z is (**2 mark**)

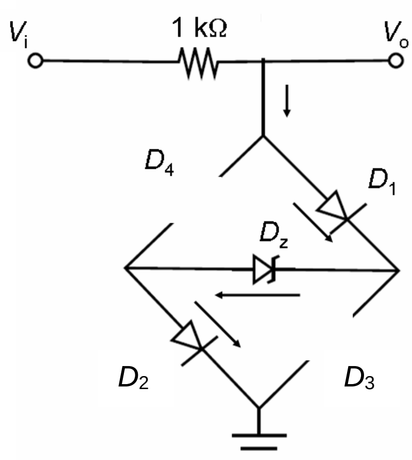
$V_Z + I_Z \cdot r_Z = 8.1$ V where $I_Z = 10$ mA and r_Z is 10 Ω.
$\therefore$ $\mathbf{V_Z} = 8.1 - 10\times10^{-3}\cdot 10 = \mathbf{8\ V}$

(ii) Draw the equivalent circuit of the Zener diode operating in the breakdown regime. (**2 marks**)



(b) The diodes D_1, D_2, D_3 and D_4 have cut-in voltage of $V\gamma = 0.7$ V and a diode resistance of 10 Ω. Ignore the reverse breakdown of these diodes. At what value(s) of V_i will D_z operate in the breakdown regime? Draw the equivalent circuit(s) for these conditions. (**5 + 5 = 10 marks**)

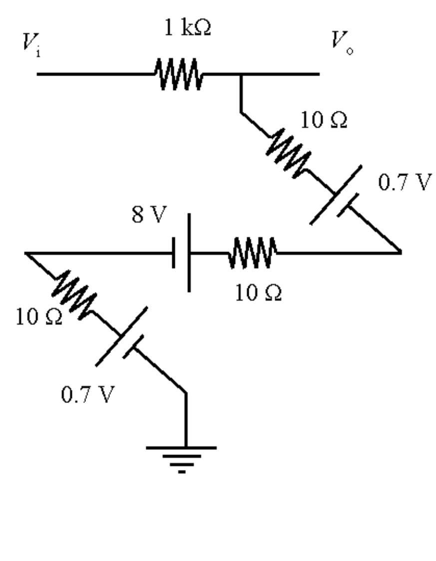
For positive values of V_i when the Zener diode will be in the breakdown regime, the current flow will be as shown below to the left:



When the Zener will be the breakdown regime, $V\gamma$ will drop across D_1 and D_2 and V_Z will drop across D_Z.
Thus $V_o = 2\cdot V\gamma + V_Z = 9.4$ V.
The equivalent circuit is shown to the right.

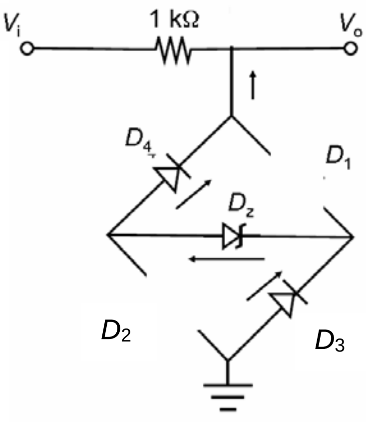
When Zener just starts conducting, the voltage V_i will be 9.4 V and the The series resistance to GND from V_0 will be $2\cdot r_D + r_Z = 30$ Ω. Thus,

$V_o / V_i = (30)/(1000 + 30) = 0.021$



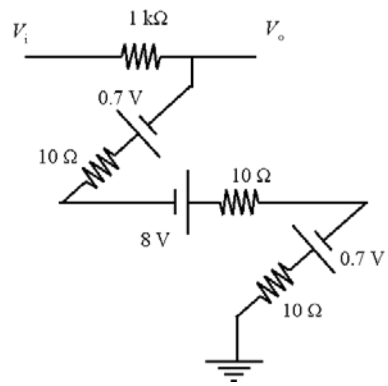
For negative values of V_i when the Zener diode will be in the breakdown regime, the current flow will be as shown in the next page top left:

When the Zener will be the breakdown regime, $V\gamma$ will drop across D_3 and D_4 and V_Z will drop across D_Z.



Thus $V_o = -2 \cdot V\gamma - V_Z = -9.4$ V

The equivalent circuit is shown to the right.



When Zener just starts conducting, the voltage V_i will be 9.4 V and the The series resistance to GND from V_0 will be $2 \cdot r_D + r_Z = 30\ \Omega$. Thus,

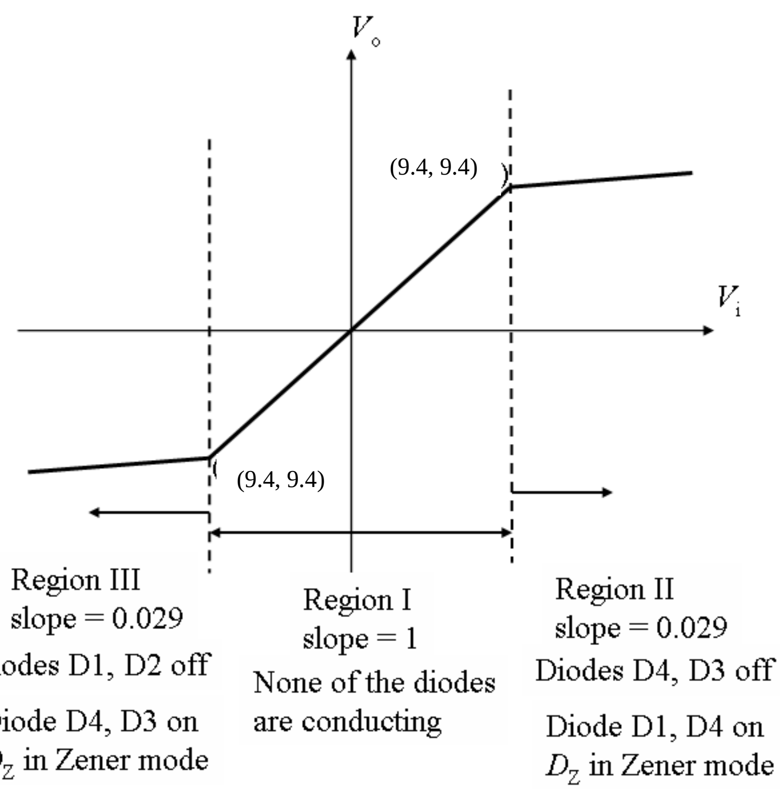
$V_o / V_i = (30)/(1000 + 30) = 0.029$

(c) Sketch and clearly label the transfer characteristic (V_o vs. V_i) of the circuit given in this problem for $-15\text{ V} \leq V_i \leq 15$ V. Clearly indicate voltages at all corner values and the slopes of the characteristics.

(6 marks)

When the Zener diode will not be in the breakdown condition, $V_o = V_i$.

At other times, $V_o / V_i = (30)/(1000 + 30) = 0.029$



Page deliberately left empty for rough work