ESc201 : Introduction to Electronics

Transient Analysis of Capacitive and Inductive Circuits

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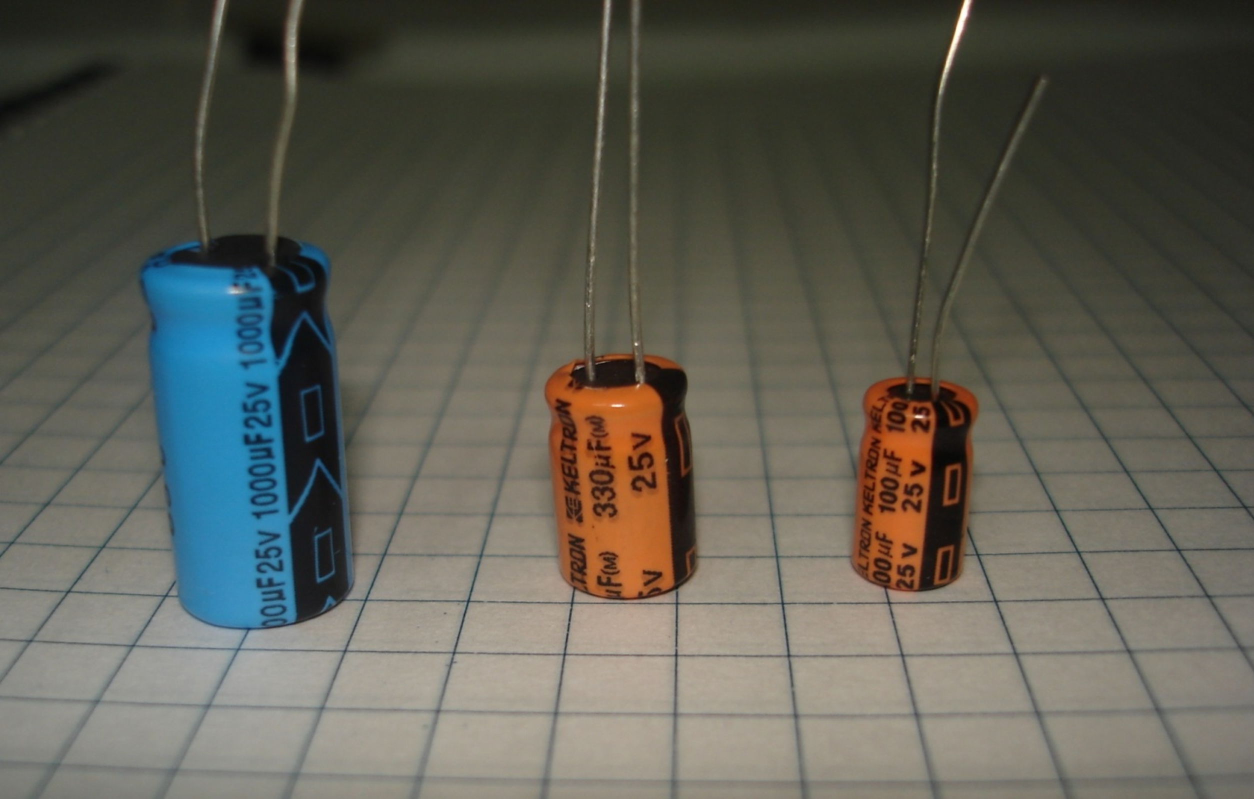
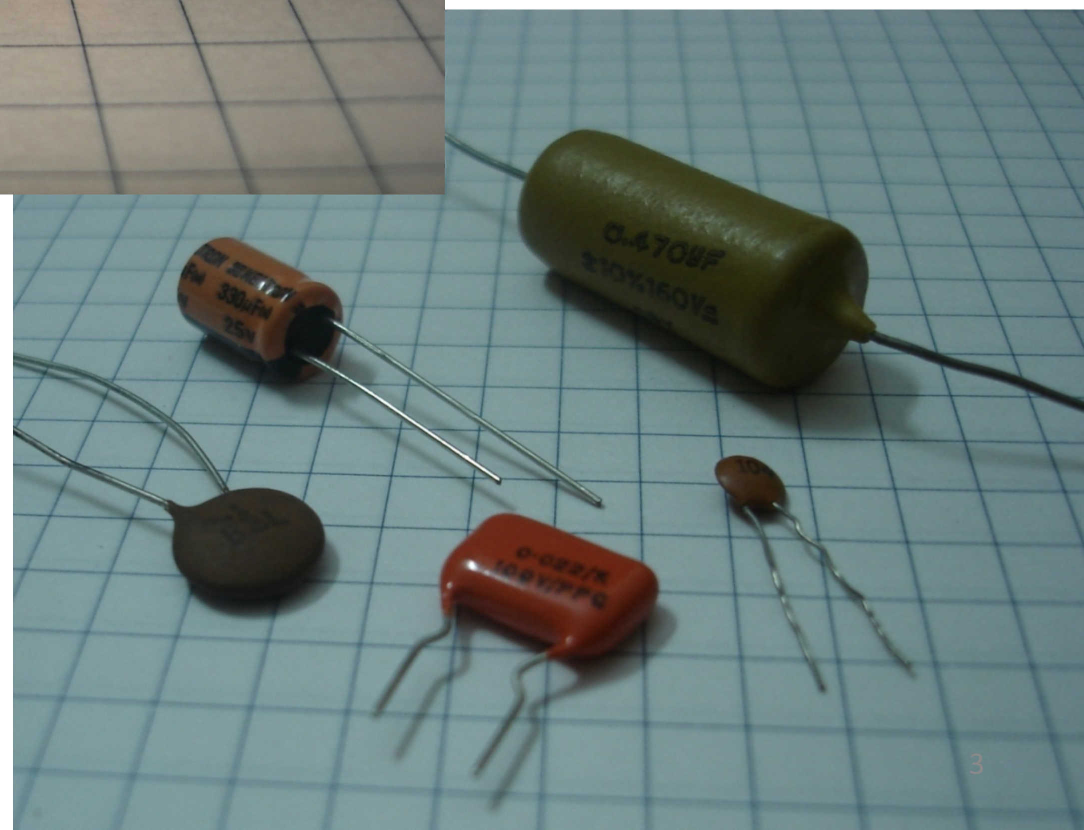
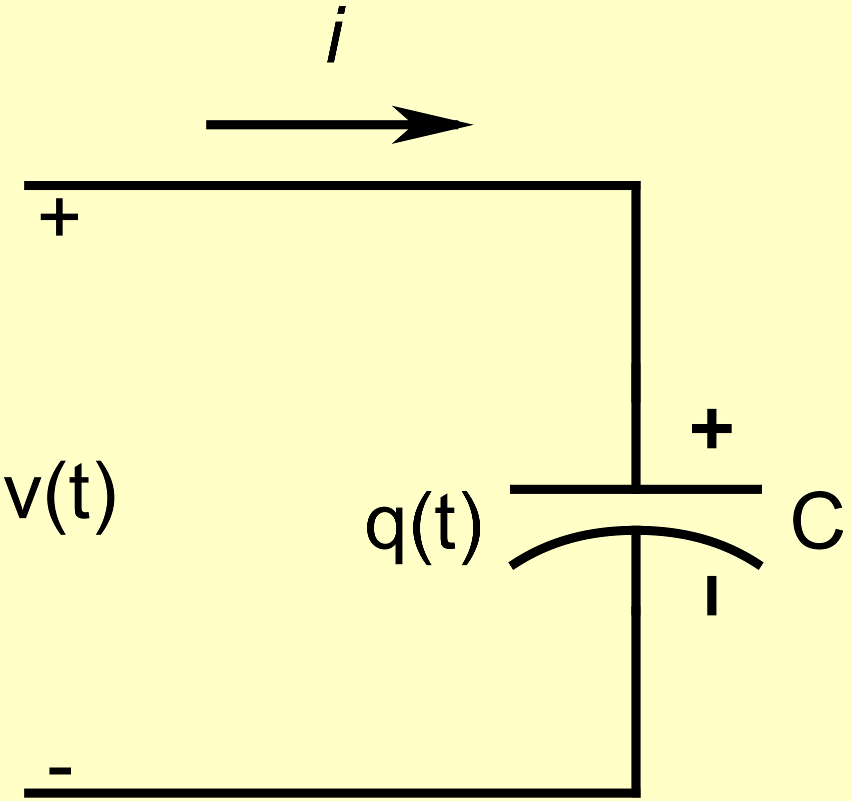
Capacitance

Two sheets of conductors separated by a layer of insulating material

– The insulating material is called dielectric. This could be air, polyester,


1000µF25V 1000µF25V
KELTRON
330µF(M)
25V
100µF
25 v


330µF
25v
0.022/K



$$q = C \times v$$

Coulombs

Farad

Volt

$$i = \frac{dq}{dt}$$

$$i = C\frac{dv}{dt}$$

$$v(t) = \frac{1}{C}\int_{t_o}^{t} i\,dt + v(t_o)$$

$$w_c(t) = \frac{1}{2}C \times v_c^2(t)$$

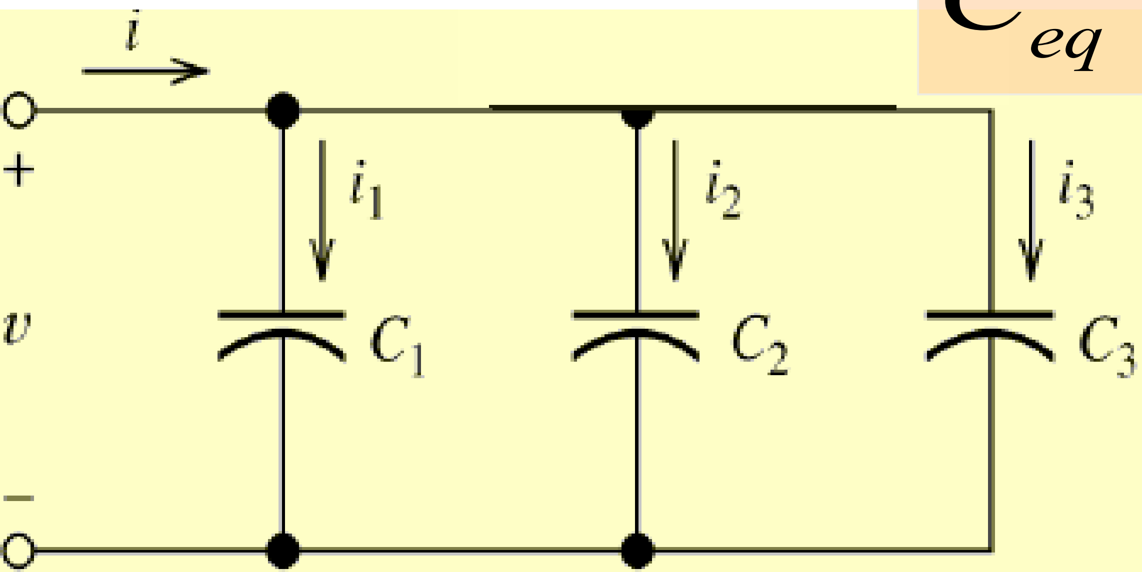
For dc or steady state when the voltage does not vary with time

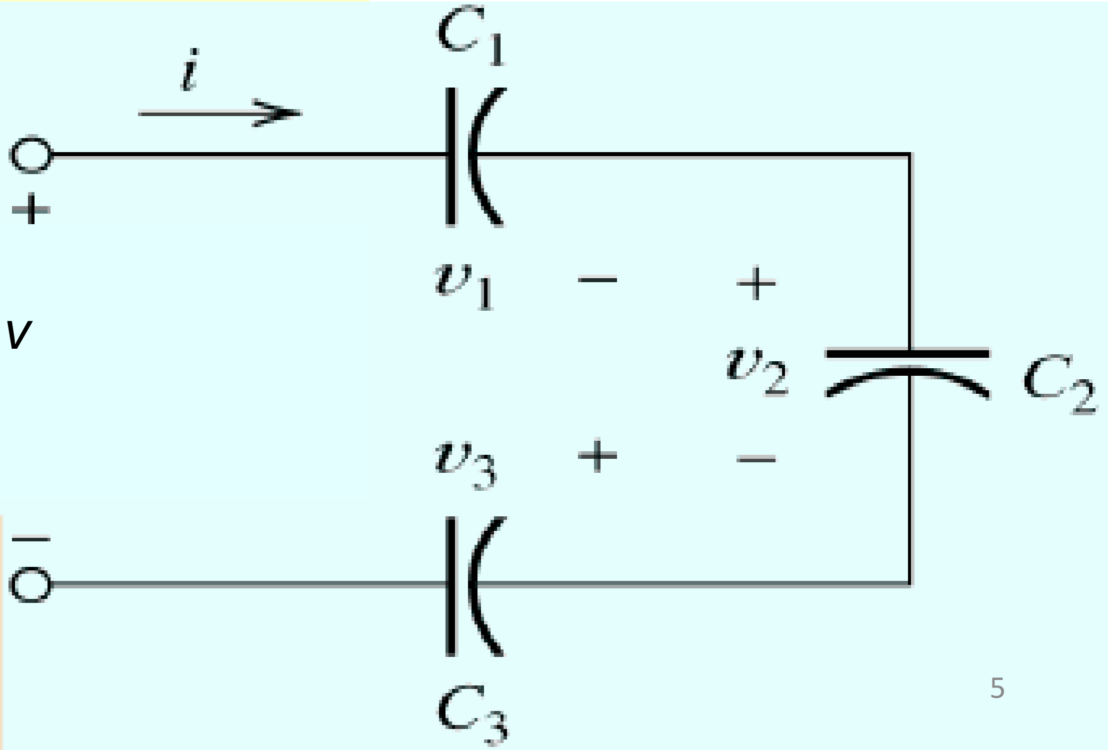
A capacitor under dc or steady state acts like an

open circuit

$$i = 0$$

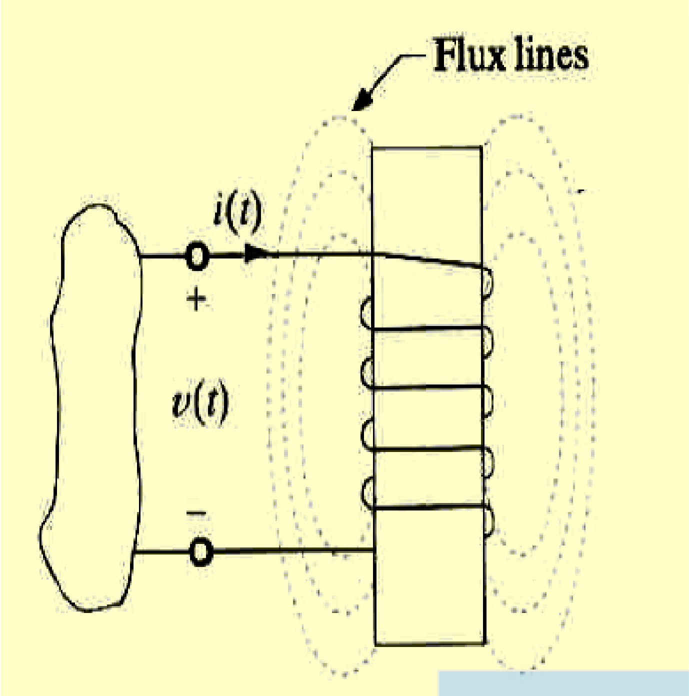
$$C_{eq} = C_1 + C_2 + C_3$$

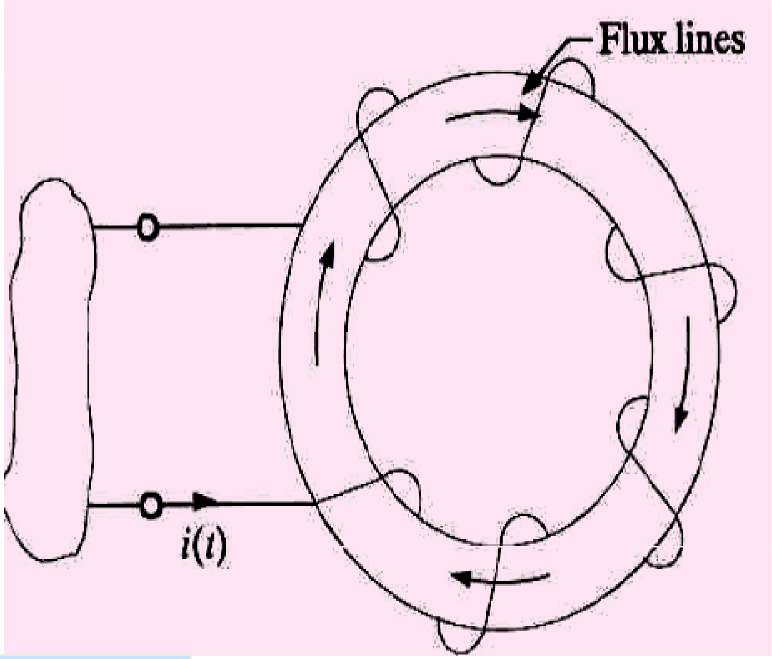




$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Inductance

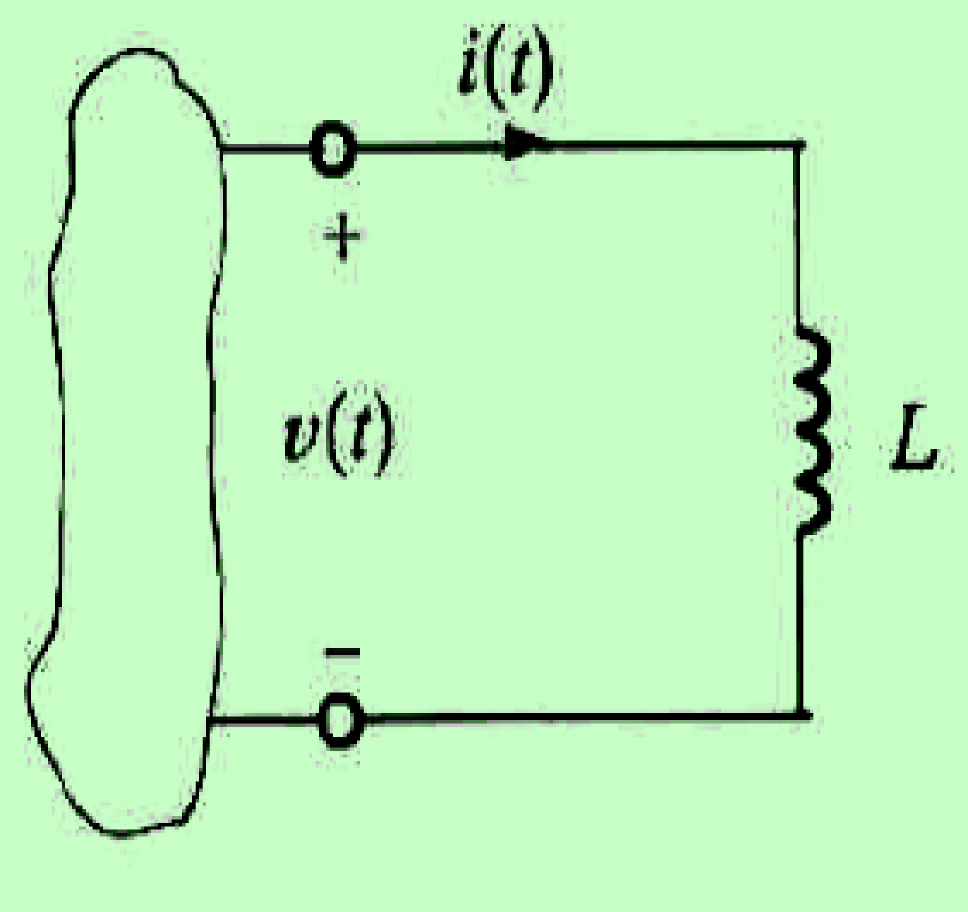




$$\phi = L \times i$$

A time varying flux causes voltage to appear across the device terminals

$$v = \frac{d\phi}{dt} = L \times \frac{di}{dt}$$



$$v = L \times \frac{di}{dt}$$

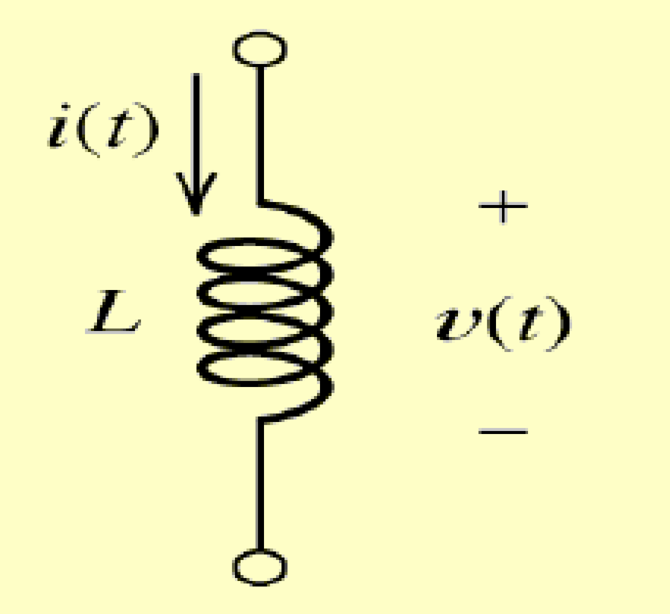
Henry

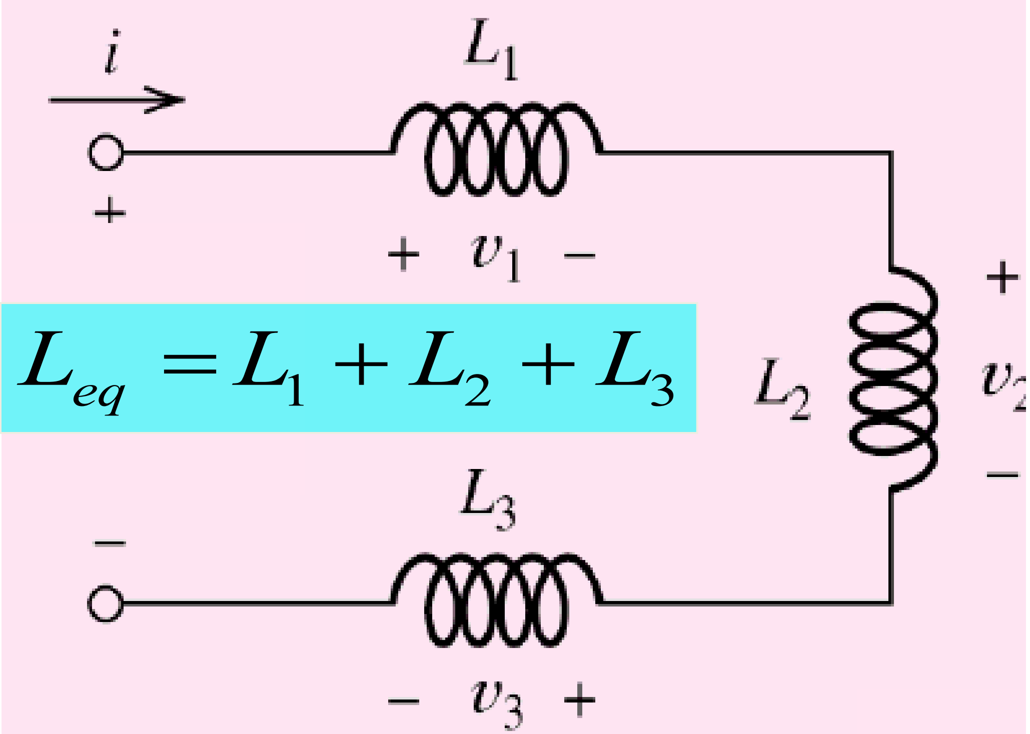
For dc or steady state when the current does not vary with time

$$v = 0$$

An inductor under dc or steady state acts like a short circuit

Typical Inductors

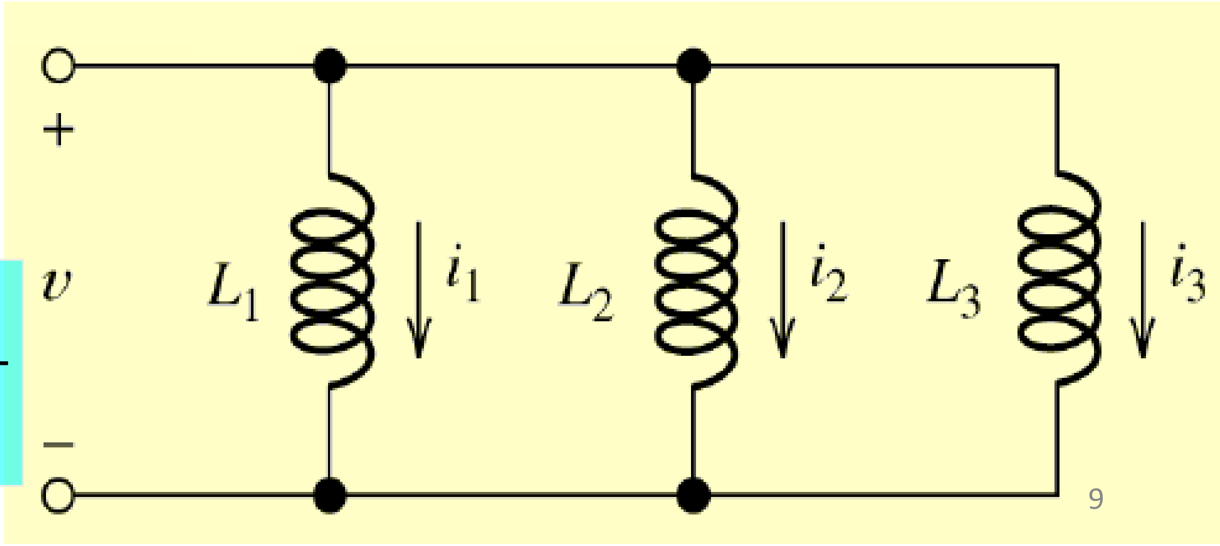




$$L_{eq} = L_1 + L_2 + L_3$$

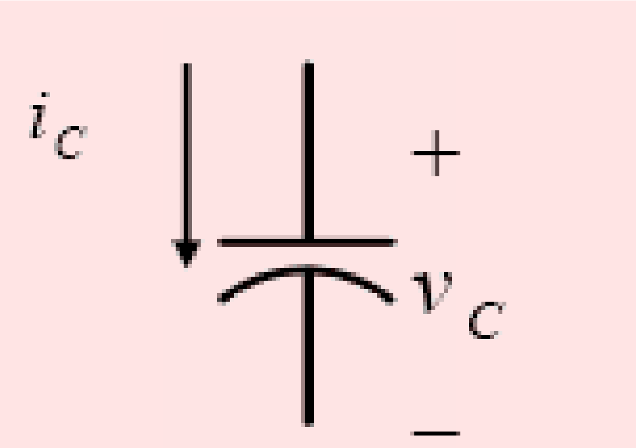
$$w_L(t) = \frac{1}{2} L \times i^2(t)$$

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$



Two important concepts

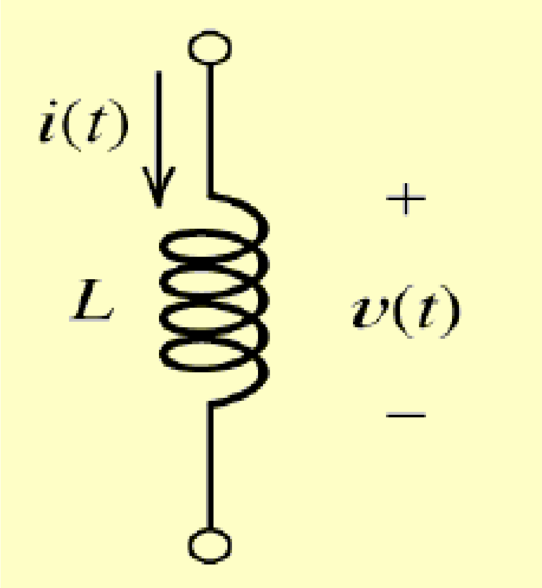
Voltage across a capacitor cannot change instantaneously



$$i_c = C\frac{dv_c}{dt}$$

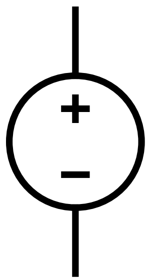
Instant change in voltage implies infinite current!

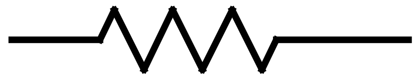
Current through an inductor cannot change instantaneously

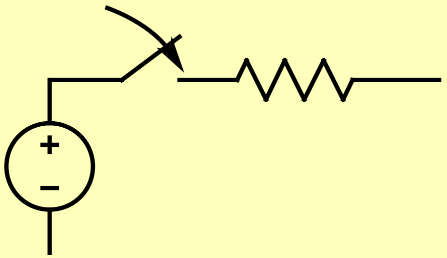


$$v = L\frac{di}{dt}$$

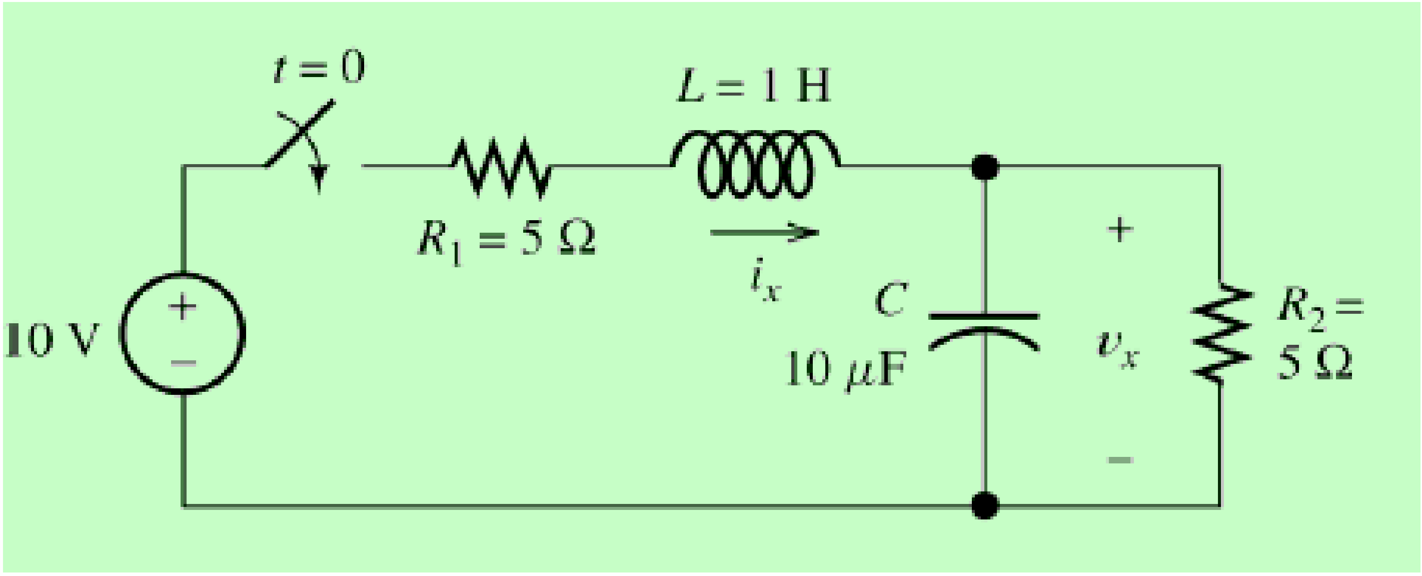
Instant change in current implies infinite voltage!





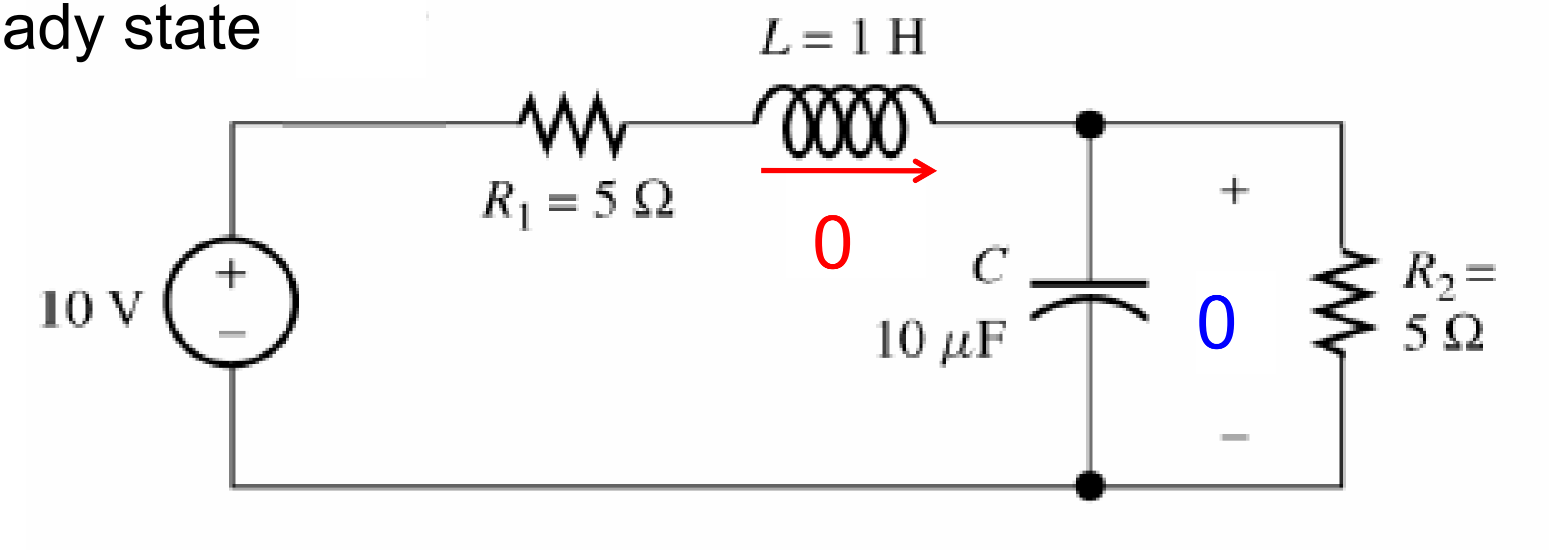


Example



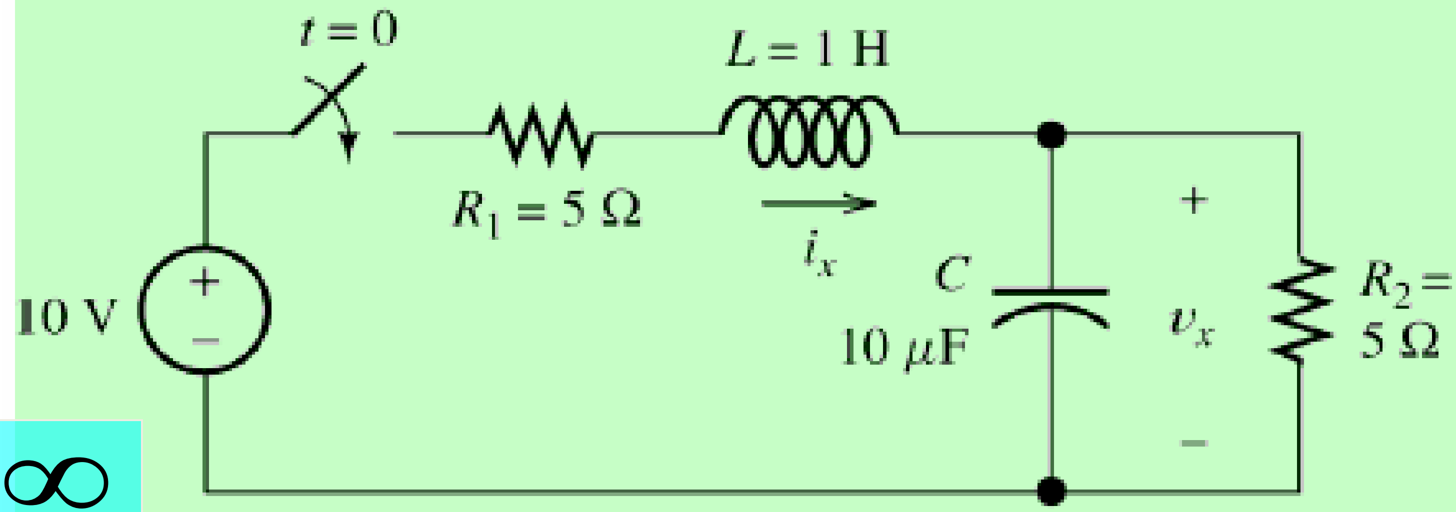
Find voltage and current immediately after closing the switch and in steady state

$t = 0^+$



Current through an inductor cannot change instantaneously

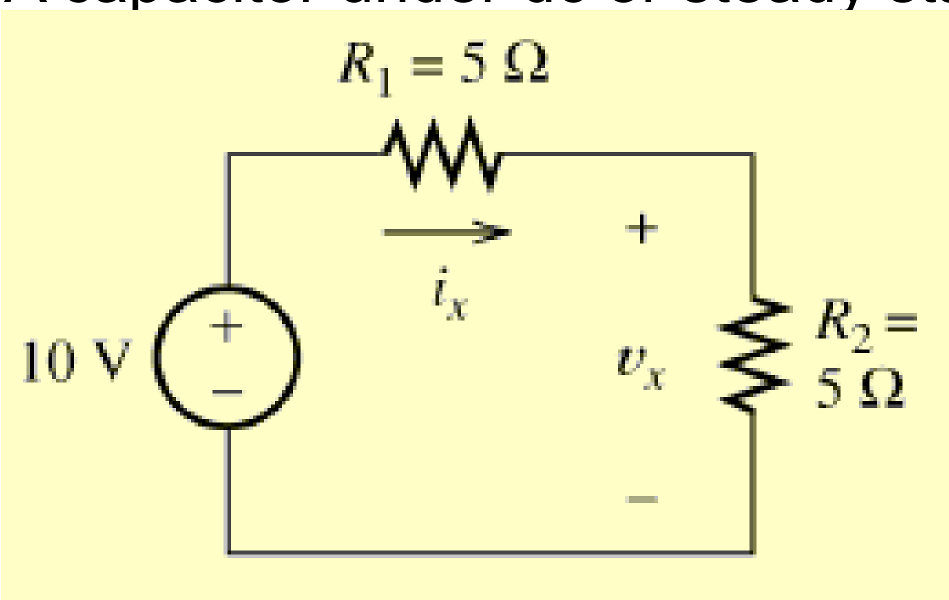
Voltage across a capacitor cannot change instantaneously



$t \to \infty$

An inductor under dc or steady state acts like a short circuit

A capacitor under dc or steady state acts like an open circuit



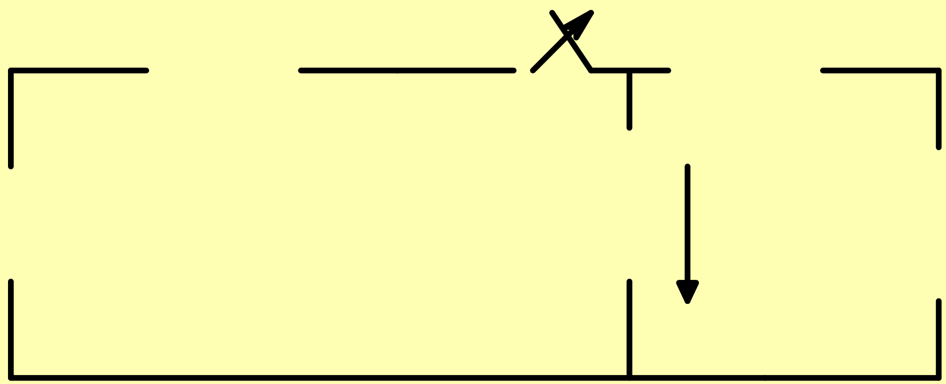
$$i_x = \frac{10}{R_1 + R_2} = 1\ \text{A}$$

$$v_x = R_2 i_x = 5\ \text{V}$$

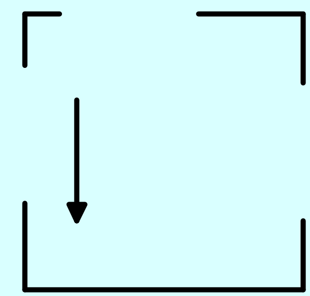
$$i_L(t \to \infty) = 1 \quad \text{A}$$

$$v_c(t \to \infty) = 5 \quad \text{V}$$

Determine the current I_X immediately after switch is opened.



Circuit for t>0

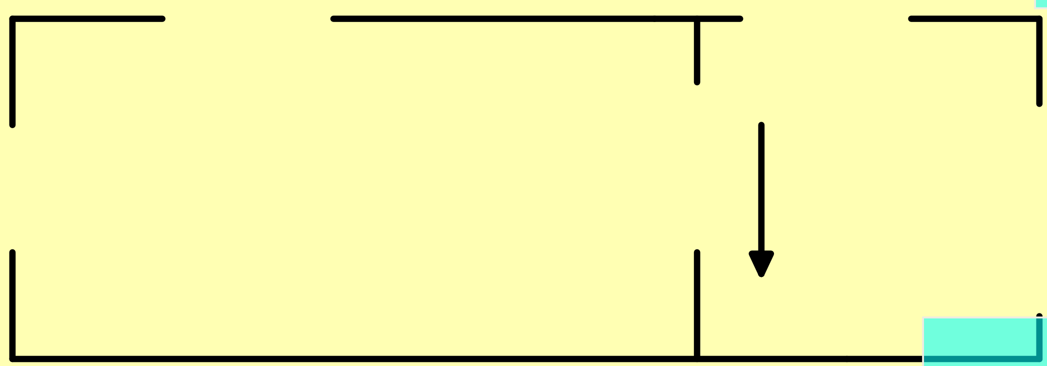


$$i_X(0^+) = \frac{v_C(0^+)}{4K} = 1mA$$

$$v_C(0^+) = v_C(0^-)$$

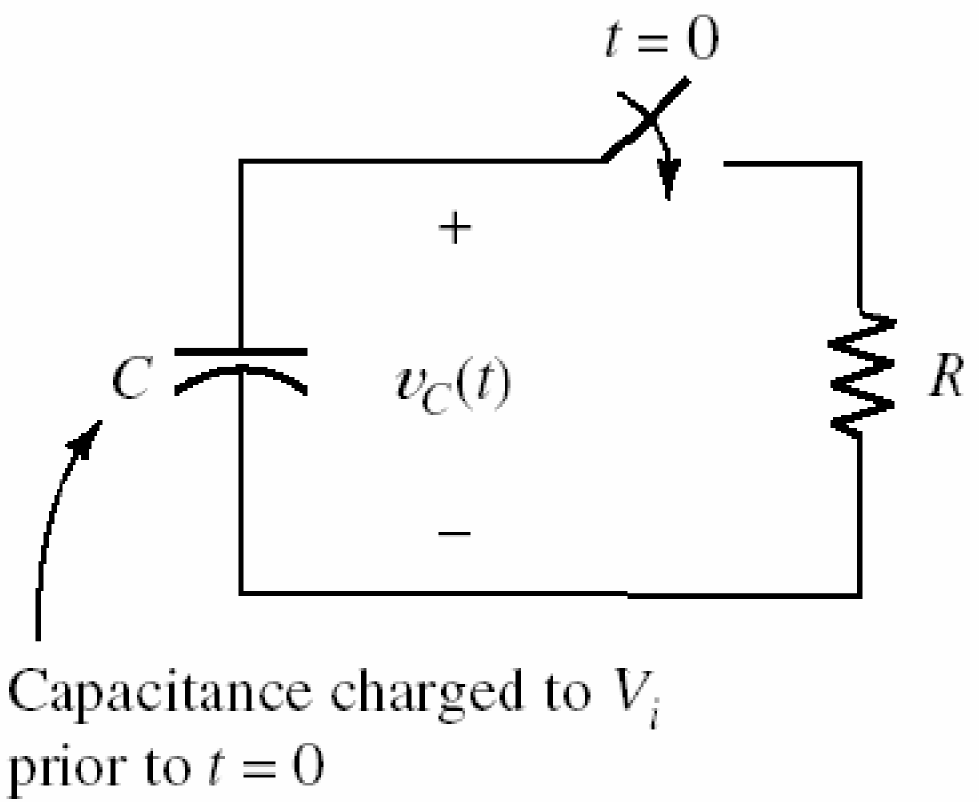
First find voltage $V_C(0^-)$

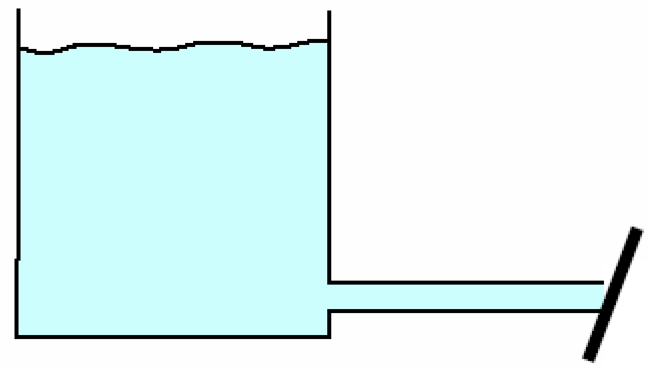
Circuit for t<0



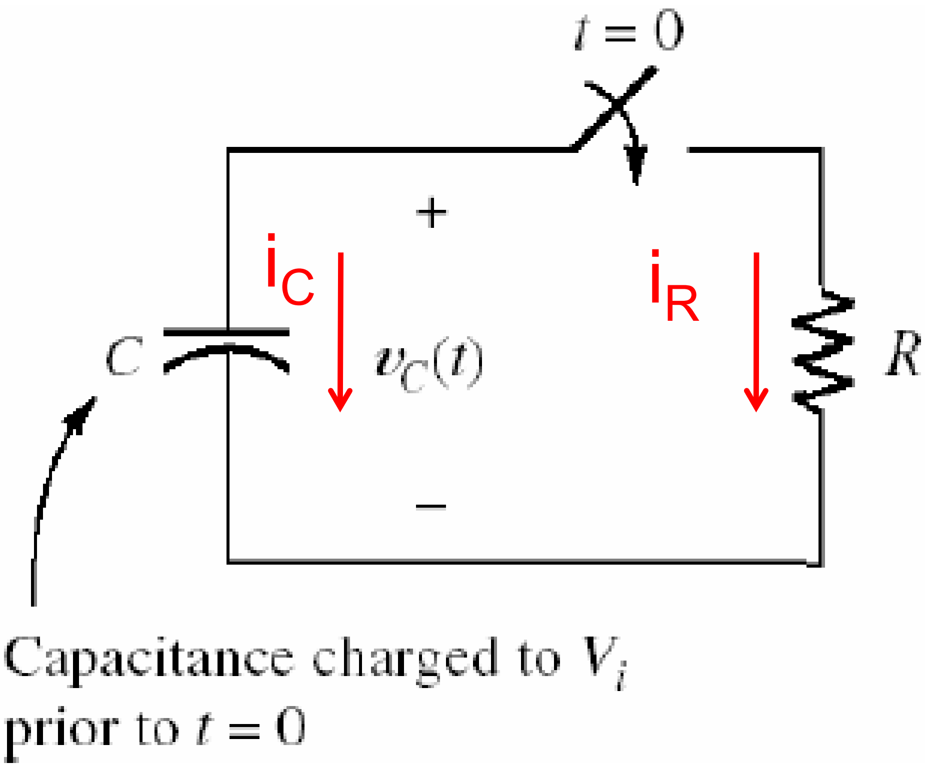
$$v_C(0^-) = \frac{2}{3} \times 6 = 4V$$

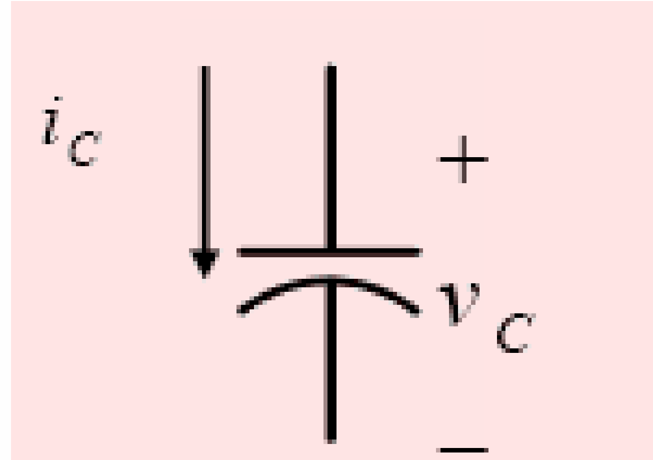
Discharge of a capacitor through a Resistor





How long will it take for capacitor voltage to fall to half its initial value?





$$i_c = C\frac{dv_c}{dt}$$

Write KCL at top node with switch closed:

$$i_c(t) + i_R(t) = 0$$

$$C\frac{dv_C(t)}{dt} + \frac{v_C(t)}{R} = 0$$

$$\frac{dv_C(t)}{dt} = -\frac{1}{RC}v_C(t)$$

First Order Differential Equation

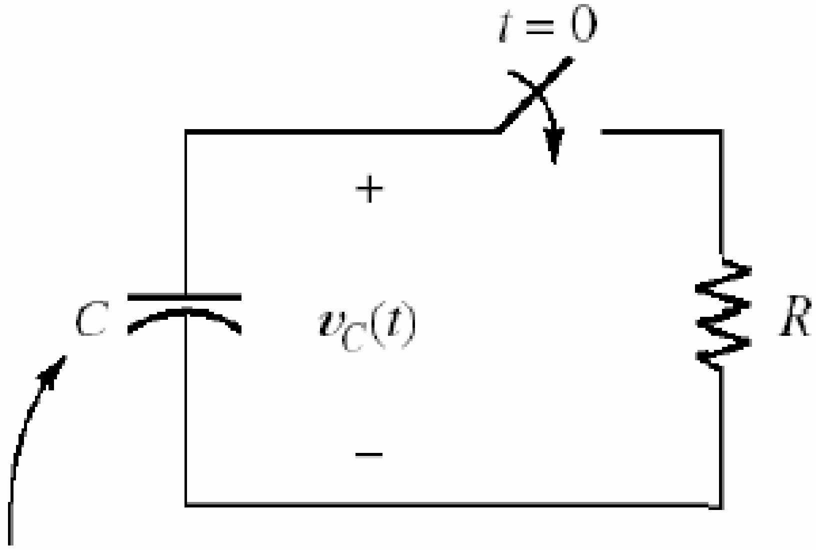
$$\frac{dy}{dt} = -a\,y$$

Solution: $$y(t) = K\,e^{-at}$$

Constant K is often found from the initial condition

$$K = y(0)$$

$$y(t) = y(0)\;e^{-at}$$



Capacitance charged to V_i prior to $t = 0$

$$\frac{dv_C(t)}{dt} = -\frac{1}{RC} v_C(t)$$

$$v_C(t) = v_C(0)\ e^{-\frac{t}{RC}}$$

$$v_C(t) = v_C(0^+)\ e^{-\frac{t}{RC}}$$

$$\frac{dy}{dt} = -a\,y$$

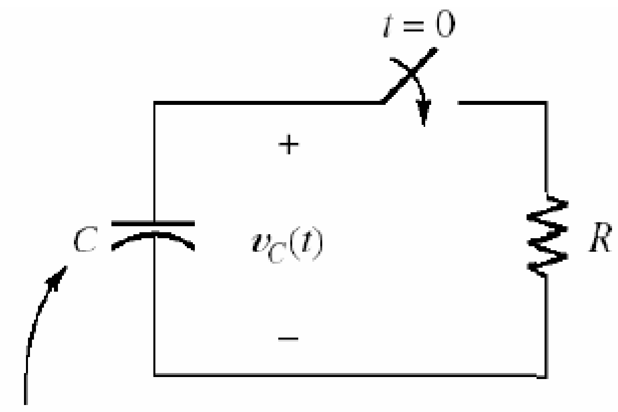
We know: $$v_C(0^-) = V_i$$

$$y(t) = y(0)\ e^{-at}$$

Voltage across a capacitor cannot change instantaneously

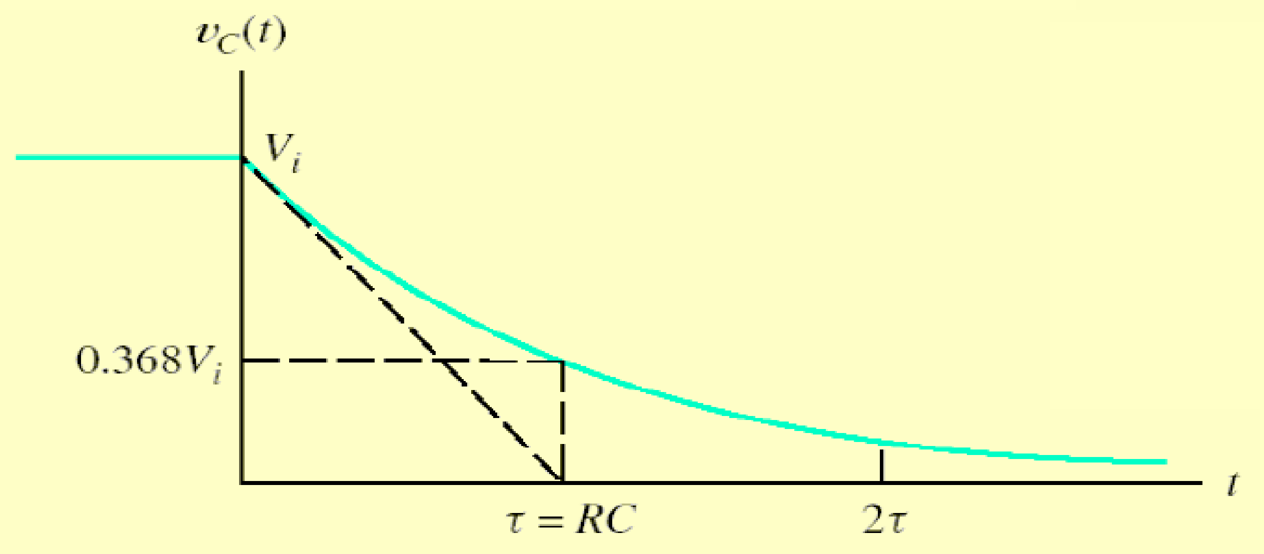
$$v_C(0^+) = v_C(0^-) = V_i$$

$$v_C(t) = V_i\, e^{-\frac{t}{RC}}$$



Capacitance charged to V_i prior to $t = 0$

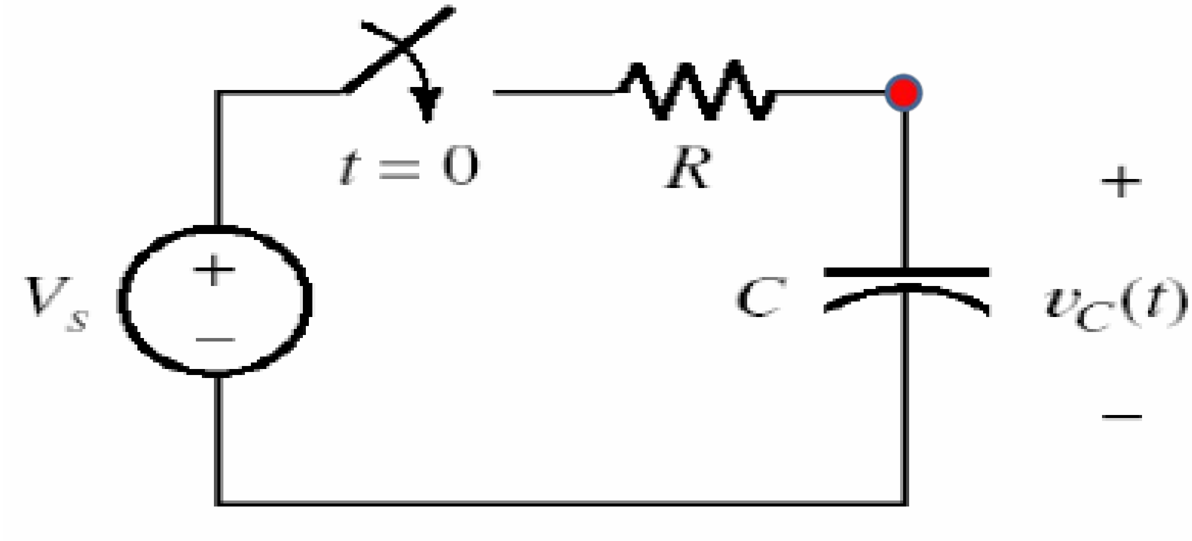
$$v_C(t) = V_i e^{-t/RC}$$



The time interval $\tau = RC$ is called the time constant of the circuit. After about **five time constants**, the voltage remaining on the capacitor will be negligible compared to the initial value

Time	τ	2τ	3τ	4τ	5τ
$V(t)/V_i$	0.368	0.135	.05	0.018	0.0067

Charging a capacitor



$$i_c = C\frac{dv_c}{dt}$$

Application of KCL at the indicated node gives

$$C\frac{dv_c(t)}{dt} + \frac{v_c(t) - V_s}{R} = 0$$

$$RC\frac{dv_c(t)}{dt} + v_c(t) = V_s$$

$$\frac{dx}{dt} = -a_1\, x + a_2$$

Solution: $$x(t) = K_1 + K_2\, e^{-a_1 t}$$

$$RC\frac{dv_c(t)}{dt} + v_c(t) = V_s$$

$$x(\infty) = K_1$$

$$x(t) = x(\infty) + K_2\, e^{-a_1 t}$$

Use initial condition: $$x(0) = x(\infty) + K_2$$

$$x(t) = x(\infty) + \{x(0) - x(\infty)\}\, e^{-a_1 t}$$

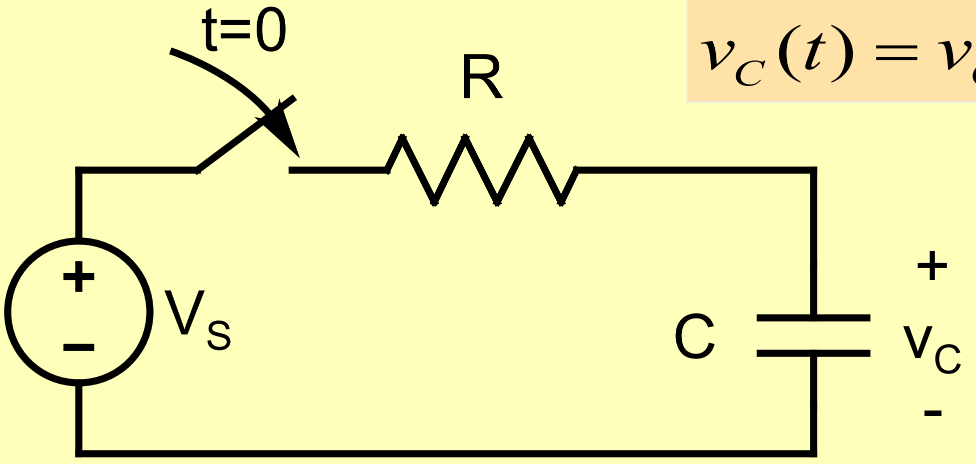
$$\frac{dx}{dt} = -a_1\, x + a_2$$

$$x(t) = x(\infty) + \{x(0) - x(\infty)\}\; e^{-a_1 t}$$

$$RC\,\frac{dv_c(t)}{dt} + v_c(t) = V_s$$

$$a_1 = \frac{1}{RC}$$

$$v_C(t) = v_C(\infty) + \{v_C(0^+) - v_C(\infty)\}\; e^{-\frac{t}{RC}}$$



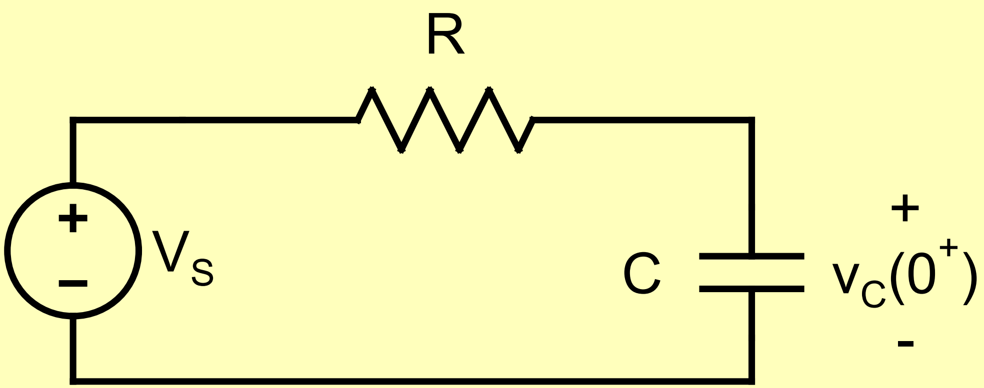
$$v_C(t) = v_C(\infty) + \{v_C(0^+) - v_C(\infty)\}\ e^{-\frac{t}{RC}}$$

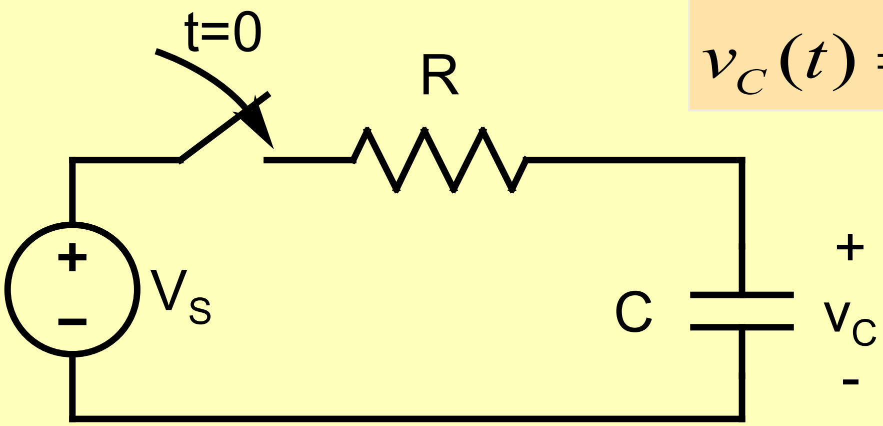
What is $v_C(\infty)$?

A capacitor under dc or steady state acts like an open circuit

$$v_C(\infty) = V_S$$

What is $v_C(0^+)$?





$$v_C(t) = v_C(\infty) + \{v_C(0^+) - v_C(\infty)\}\ e^{-\frac{t}{RC}}$$

$$v_C(0^+) = v_C(0^-)$$

We use the fact that voltage across a capacitor cannot change instantly

If the capacitor does not have any initial charge, then

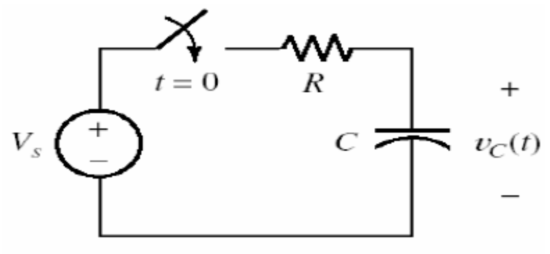
$$v_C(0^+) = v_C(0^-) = 0$$

$$v_C(\infty) = V_S$$

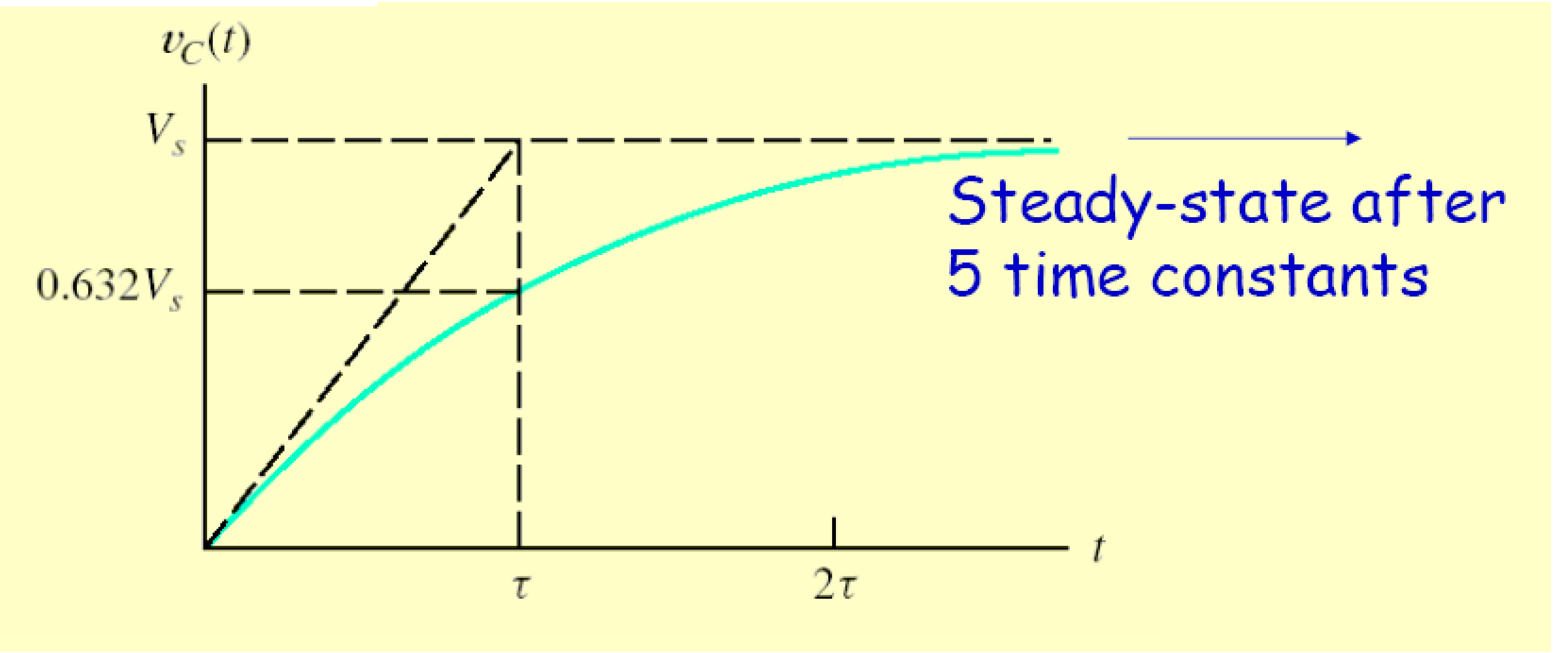
$$\tau = RC$$

$$v_C(t) = V_S(1 - e^{-\frac{t}{RC}})$$

Charging a Capacitor

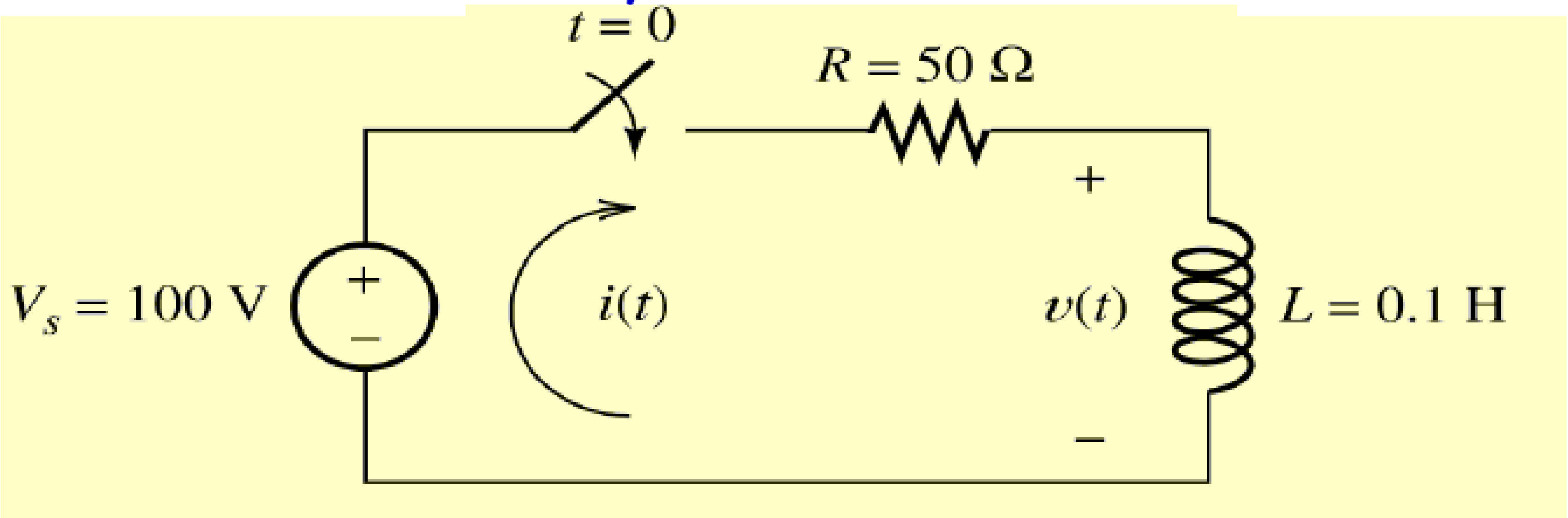


$$v_C(t) = V_s - V_s e^{-t/\tau}$$



Time	τ	2τ	3τ	4τ	5τ
$V(t)/V_i$	0.632	0.865	.95	0.982	0.993

RL Transient Analysis



Write KVL equation:

$$Ri(t) + L\frac{di}{dt} = V_s$$

$$v = L\frac{di}{dt}$$

$$\frac{dx}{dt} = -a_1 x + a_2$$

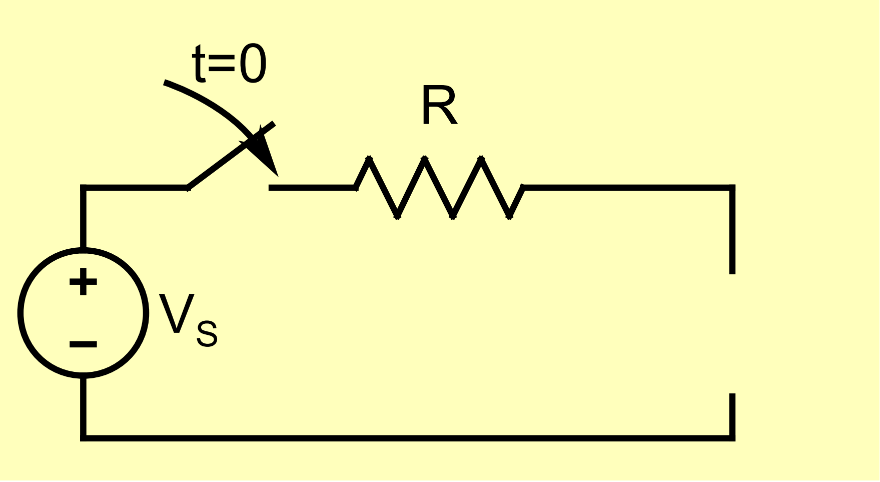
$$x(t) = x(\infty) + \{x(0) - x(\infty)\}\ e^{-a_1 t}$$

$$i(t) = i(\infty) + \{i(0) - i(\infty)\}\ e^{-\frac{R}{L}t}$$

$$e^{-\frac{t}{\tau}}$$

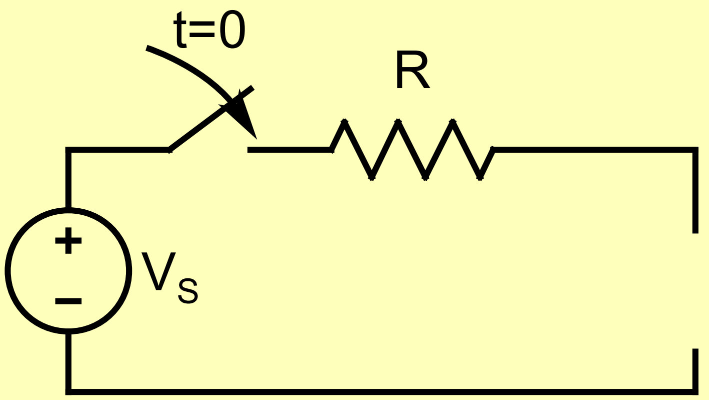
$$\text{Time Constant}: \tau = \frac{L}{R}$$

What is $i(\infty)$?

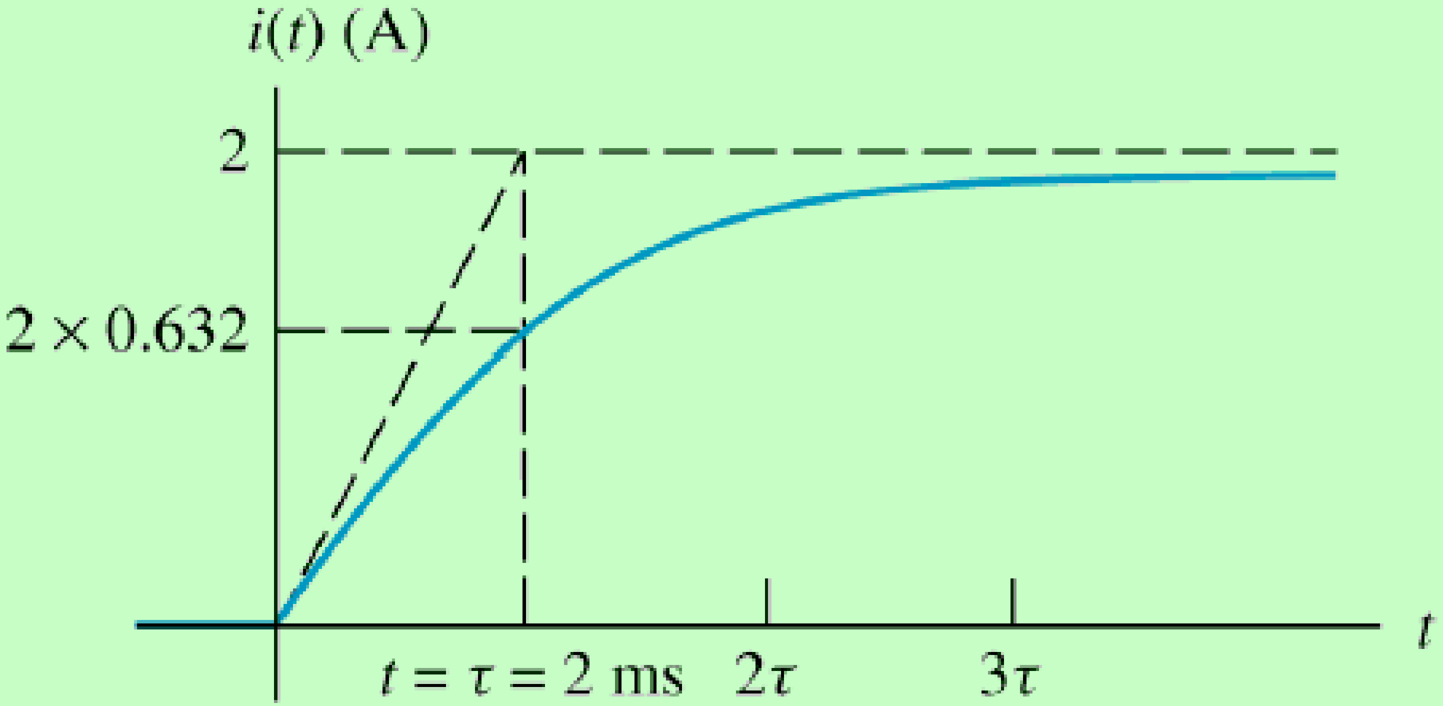


Current through an inductor cannot change instantaneously

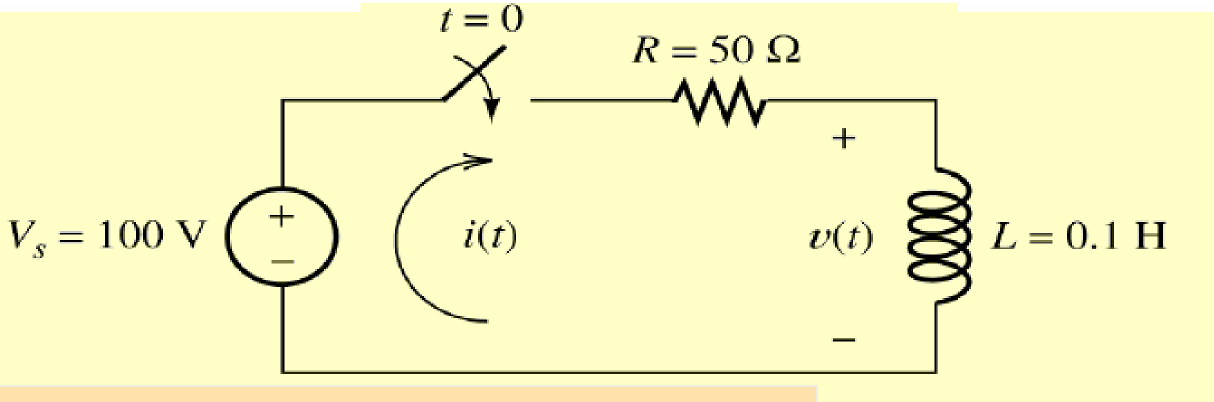
$$i(0^+) = i(0^-)$$



$$i(t) = \frac{V_S}{R} \times (1 - e^{-\frac{t}{\tau}})$$

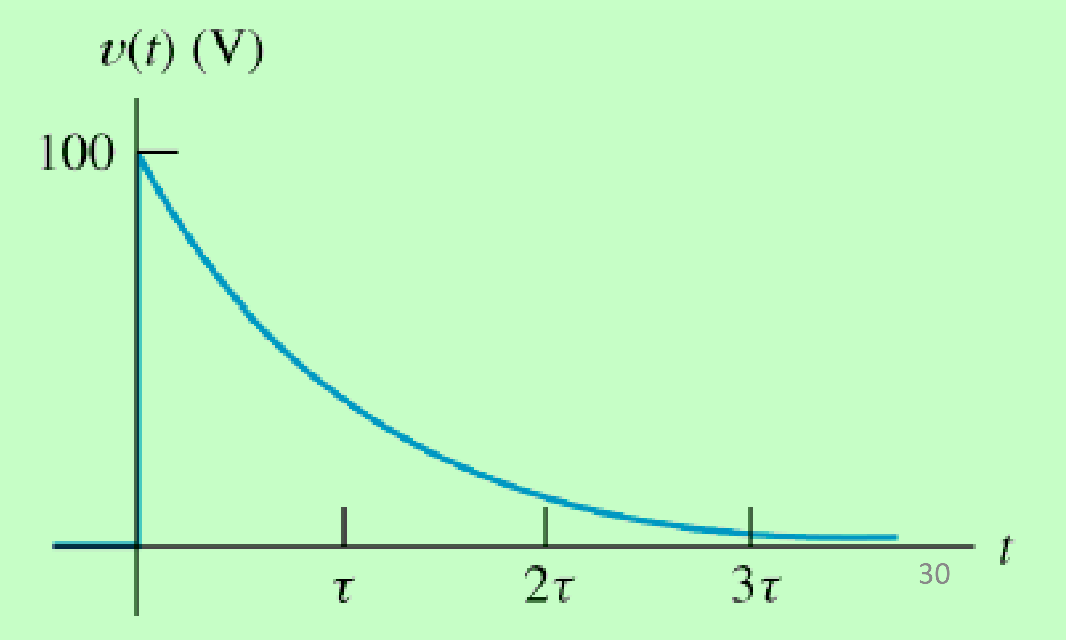


What about voltage across the Inductor?



$$i(t) = \frac{V_S}{R} \times (1 - e^{-\frac{t}{\tau}})$$

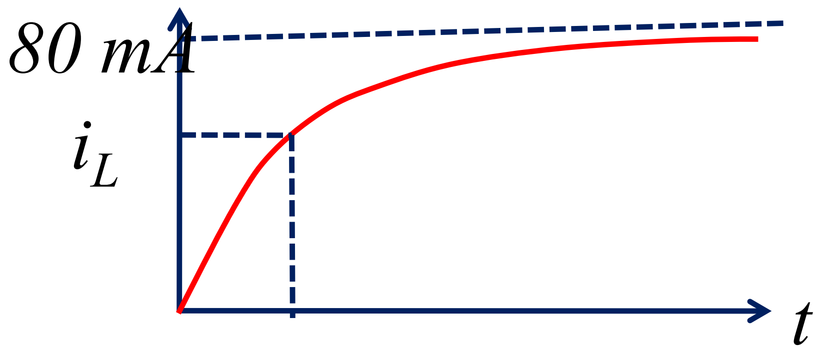
$$v = L\frac{di}{dt} = \frac{L}{R} V_S \times e^{-\frac{t}{\tau}}$$

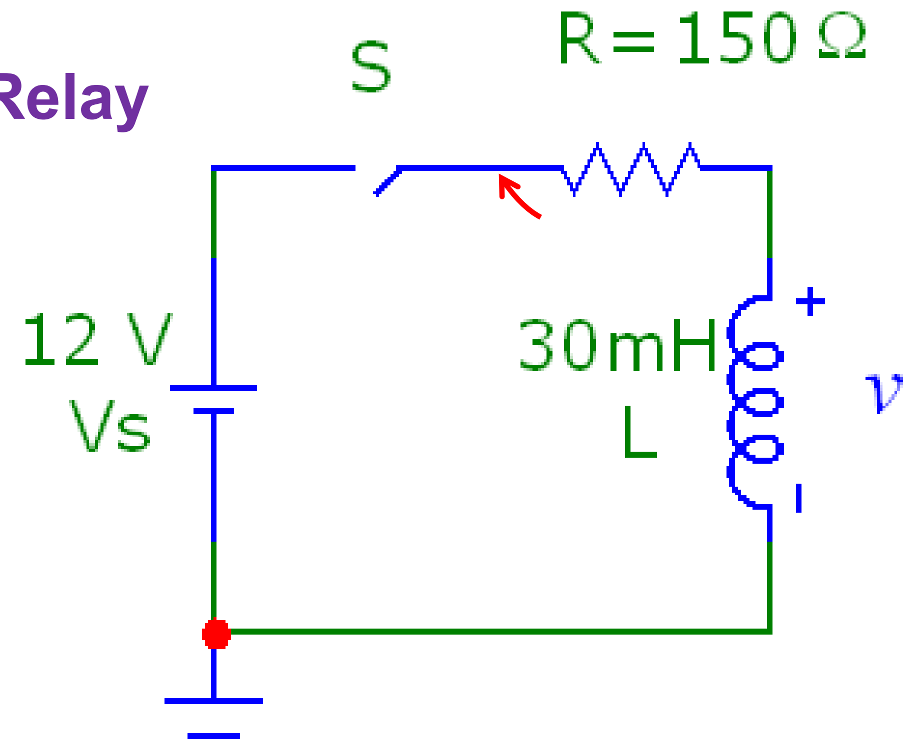


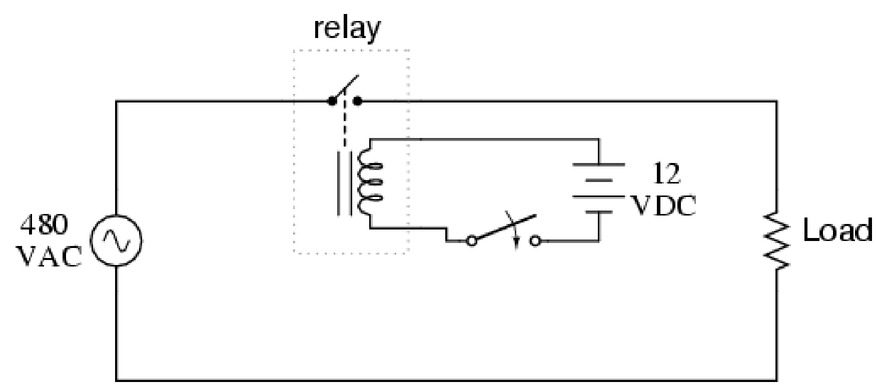
Application

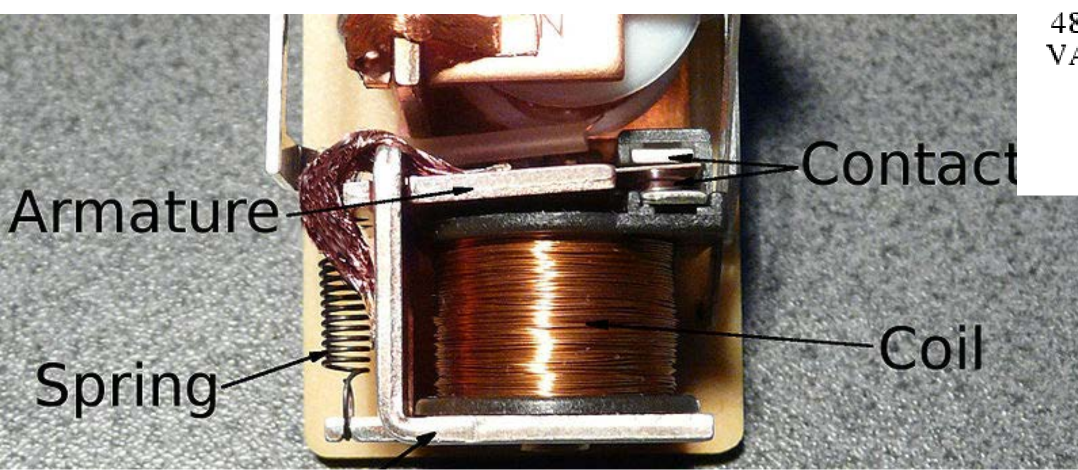
Electromechanical Relay

If the current needed to operate the relay is 50 mA, what is the relay delay time?

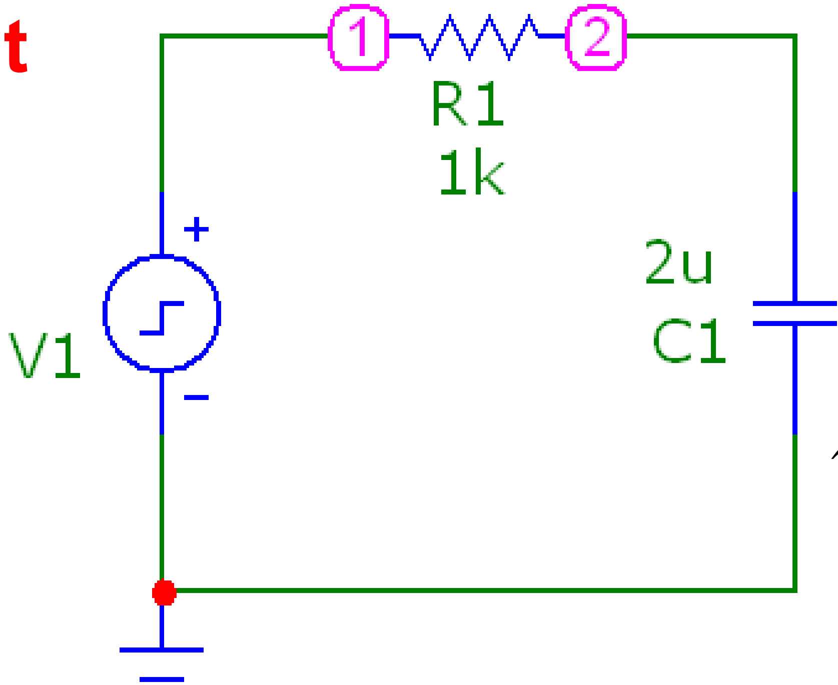






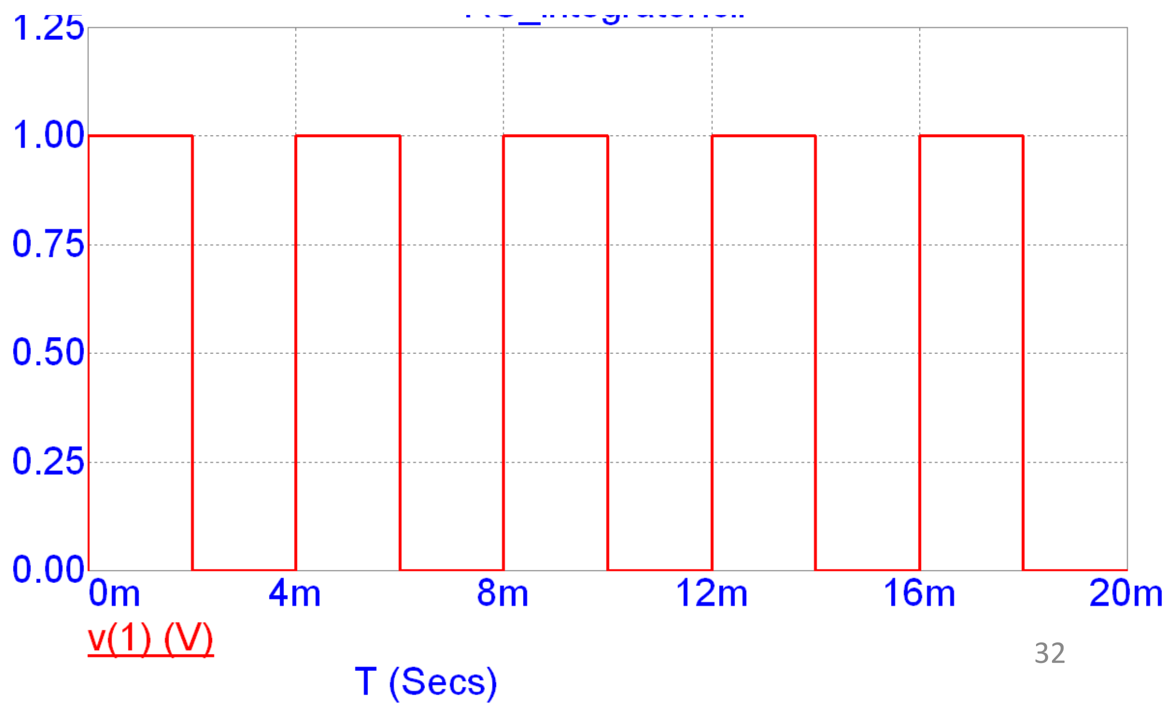


RC Circuit



$$\tau = RC = 2\text{ ms}$$

Input voltage



RC Circuit

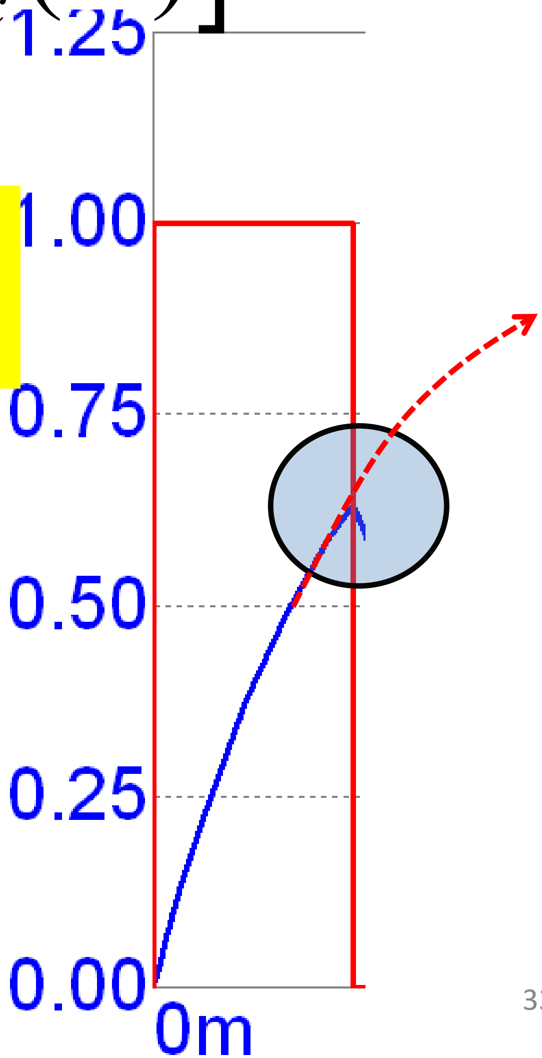
$$\tau = RC = 2\,\text{ms}$$

$$v_c(t) = v_c(\infty) + \left[v_c(0^+) - v_c(\infty)\right]e^{-\frac{t}{\tau}}$$

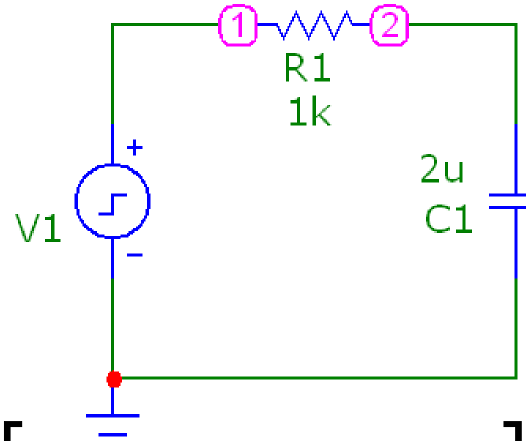
$$v_c(\infty) = 1V; \quad v_c(0^+) = 0;$$

$$v_c(t) = 1 - e^{-\frac{t}{2}}$$

$$v_c(2) = 1 - e^{-\frac{2}{2}} = 0.63V$$



RC Circuit



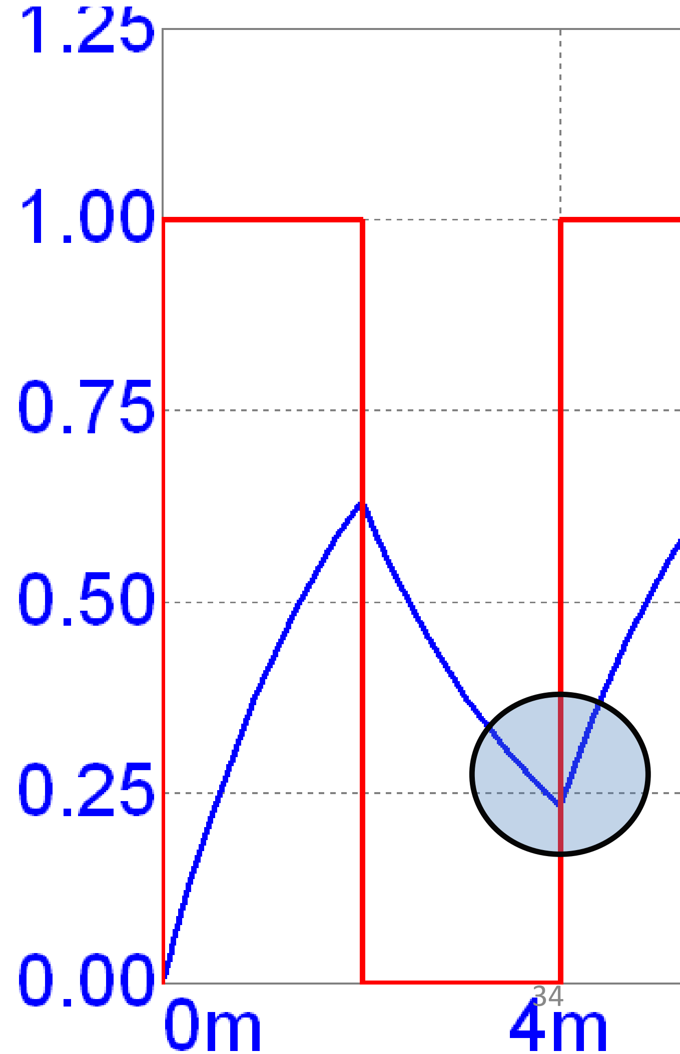
$$\tau = RC = 2\,\text{ms}$$

$$v_c(t) = v_c(\infty) + \left[v_c(0^+) - v_c(\infty)\right]e^{-\frac{t}{\tau}}$$

$$v_c(\infty) = 0;\, v_c(0^+) = 0.63V;$$

$$v_c(t) = 0.63e^{-\frac{(t-2)}{2}};2 \le t \le 4$$

$$v_c(4) = 0.63\,e^{-\frac{2}{2}} = 0.23V$$

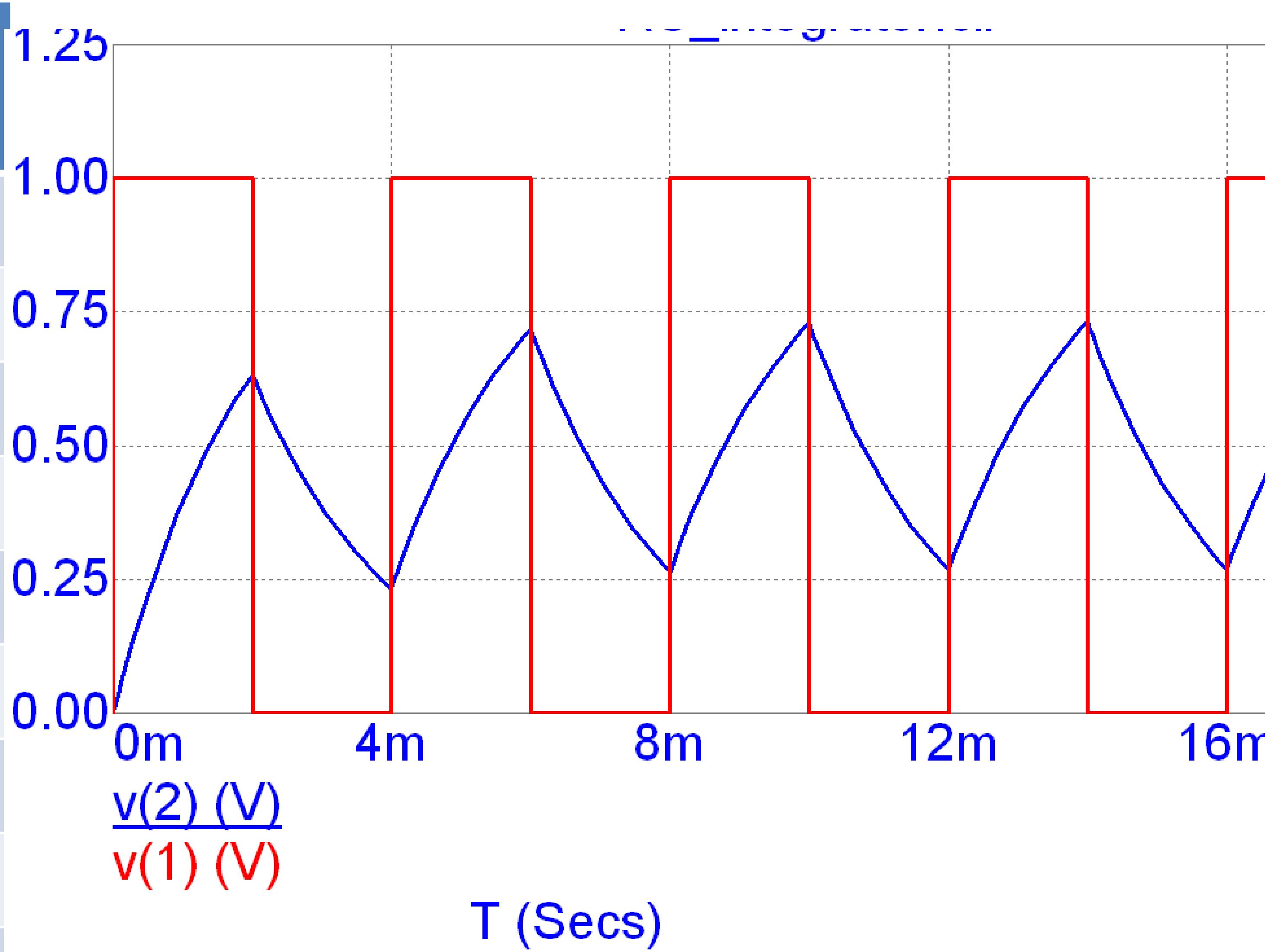


time t is in ms

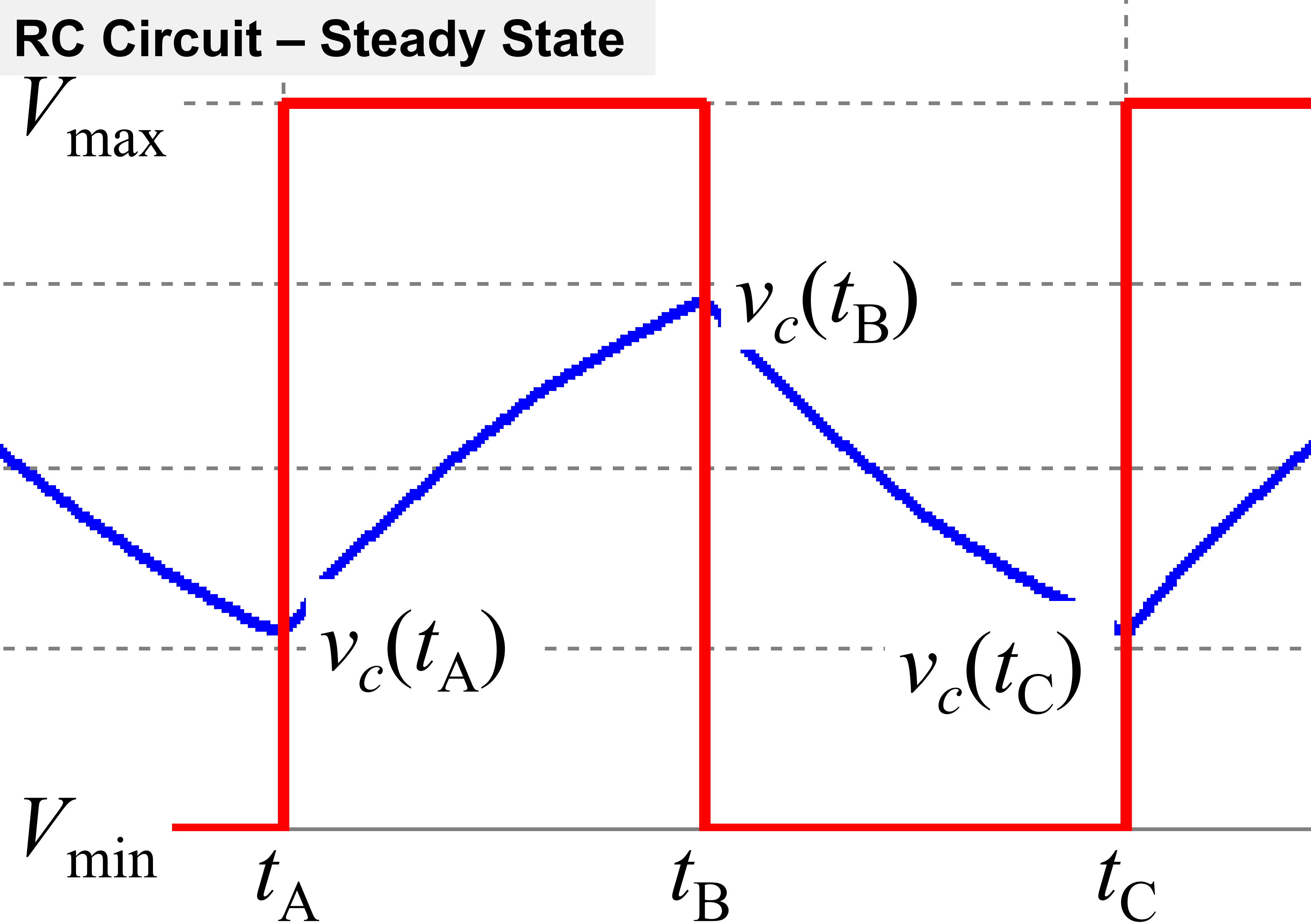
RC Circuit

$$\tau = RC = 2\text{ ms}$$

0	*0*
2	*0.63*
4	*0.23*
6	*0.72*
8	*0.265*
10	*0.73*
12	*0.269*
14	*0.731*
16	*0.269*



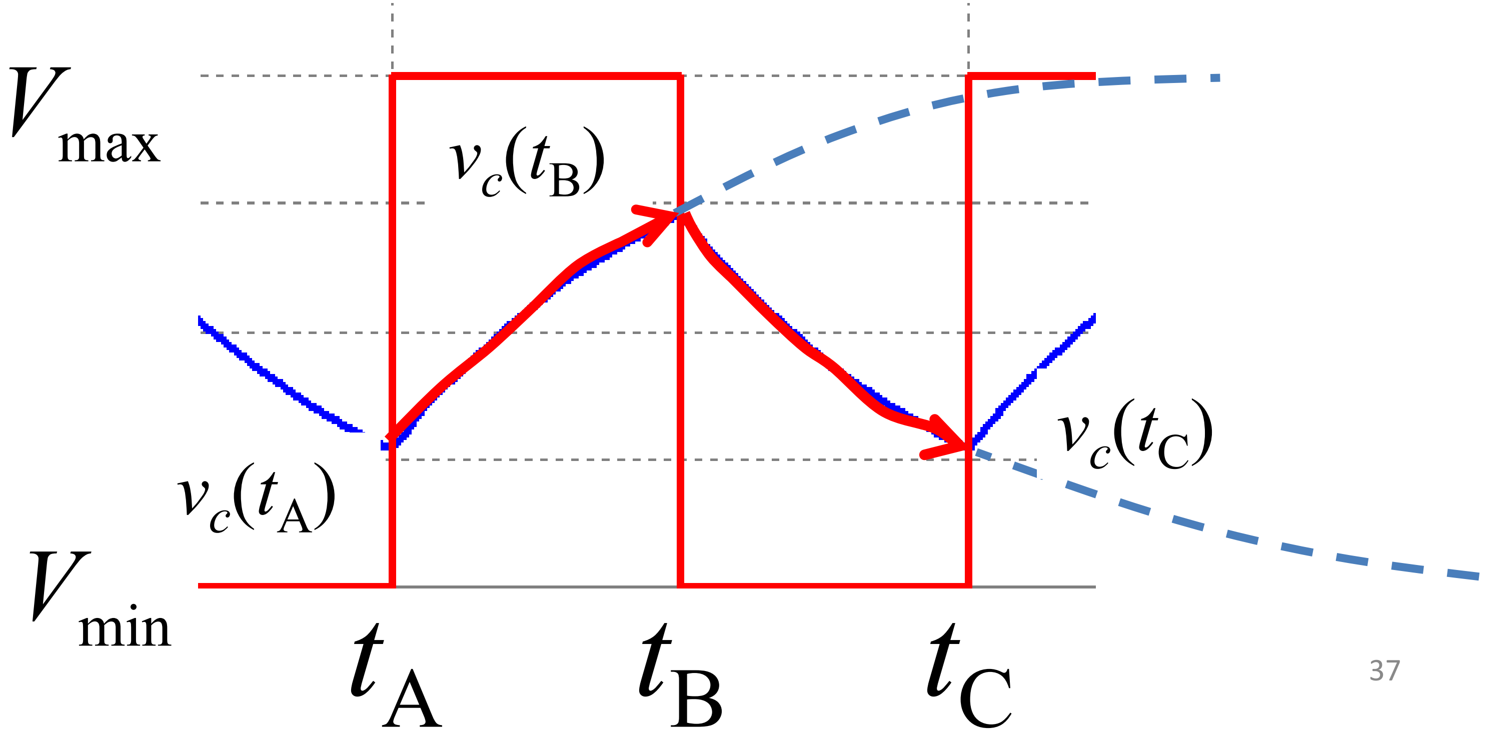
RC Circuit – Steady State



$$v_c(t) = v_c(\infty) + \left[v_c(0^+) - v_c(\infty)\right]e^{-\frac{t}{\tau}}$$

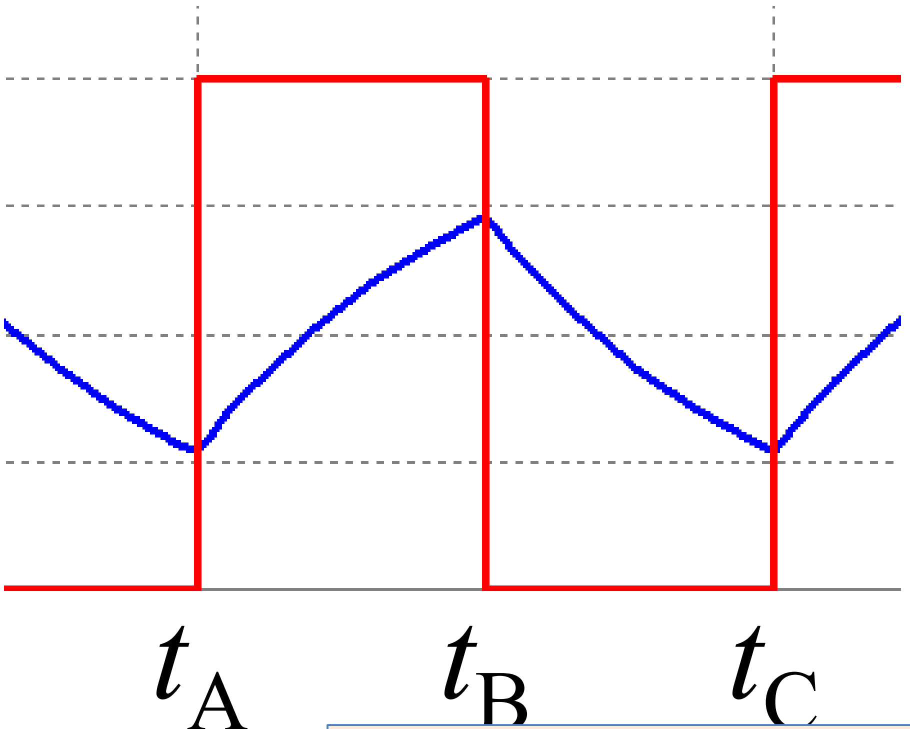
$$v_c(t) = V_{\max} + \left[v_c(t_A) - V_{\max}\right]e^{-\frac{(t-t_A)}{\tau}} \qquad t_A \le t \le t_B$$

$$v_c(t) = V_{\min} + \left[v_c(t_B) - V_{\min}\right]e^{-\frac{(t-t_B)}{\tau}} \qquad t_B \le t \le t_C$$



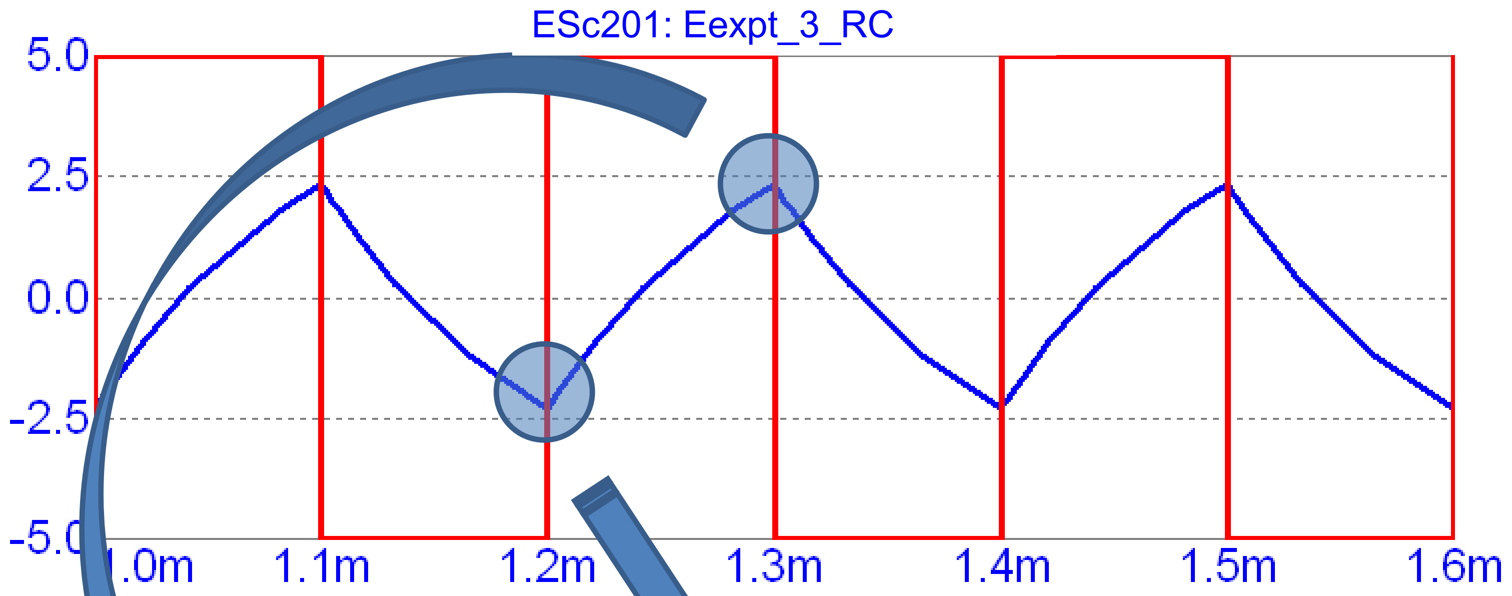
$$v_c(t_B) = V_{\max} + \left[v_c(t_A) - V_{\max}\right] e^{-\frac{(t_B - t_A)}{\tau}}$$

$$v_c(t_C) = V_{\min} + \left[v_c(t_B) - V_{\min}\right] e^{-\frac{(t_C - t_B)}{\tau}} = v_c(t_A)$$



$$\boxed{(t_B - t_A) = (t_C - t_B) = \frac{T}{2}}$$

Determine $v_c(t_A)$ and $v_c(t_B)$ in terms of $V_{\max}$ and $V_{\min}$



$$v_c(t_B) = V_{\max} + \left[v_c(t_A) - V_{\max}\right] e^{-\frac{(t_B - t_A)}{\tau}}$$

$$2.3 = 5 + \left[-2.3 - 5\right] e^{-\frac{0.1\,\text{ms}}{\tau}}$$

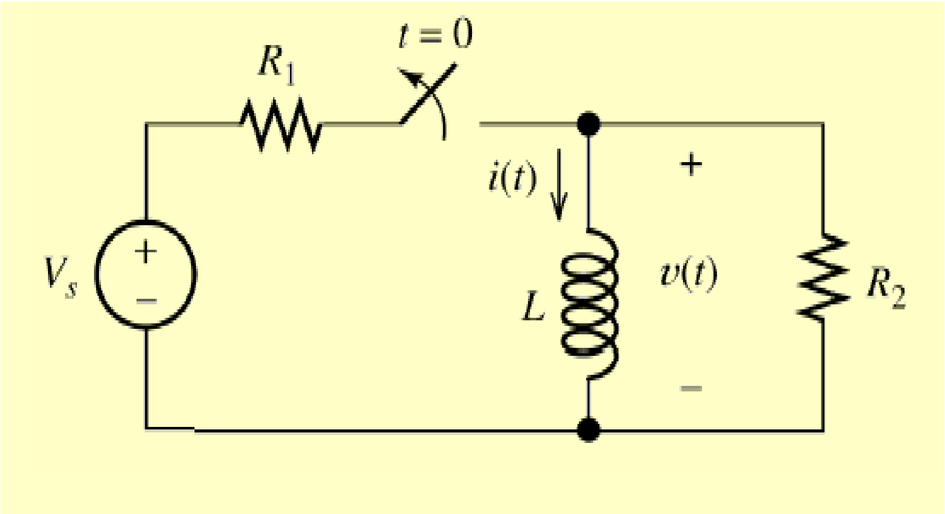
$$\tau = 0.1\,\text{ms}$$

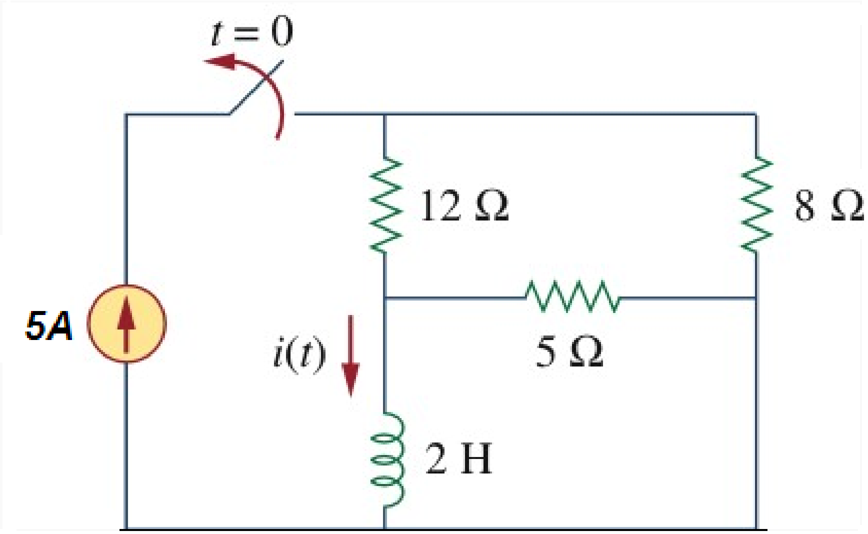
Application

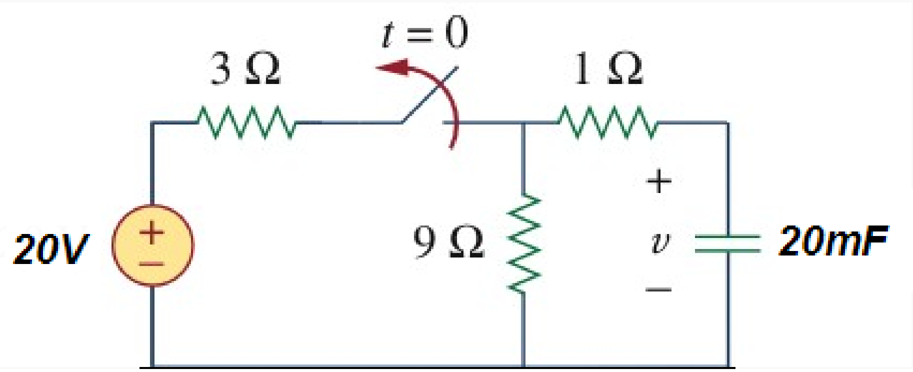
Try in the LAB

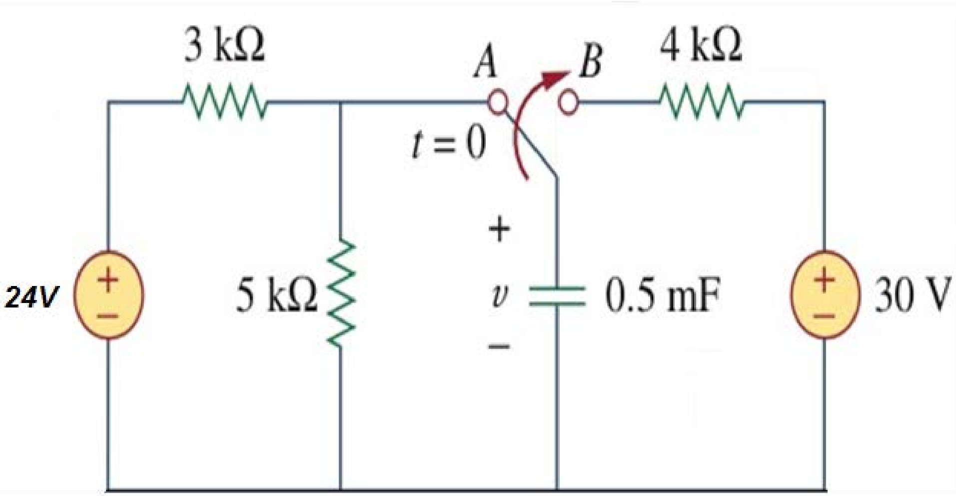
Determining: τ

How do we solve more complex circuits containing a single inductor or a capacitor?

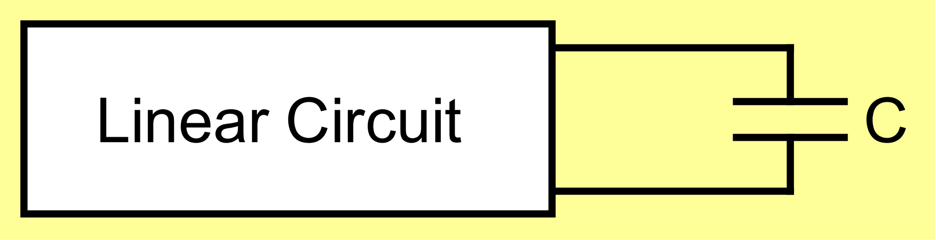




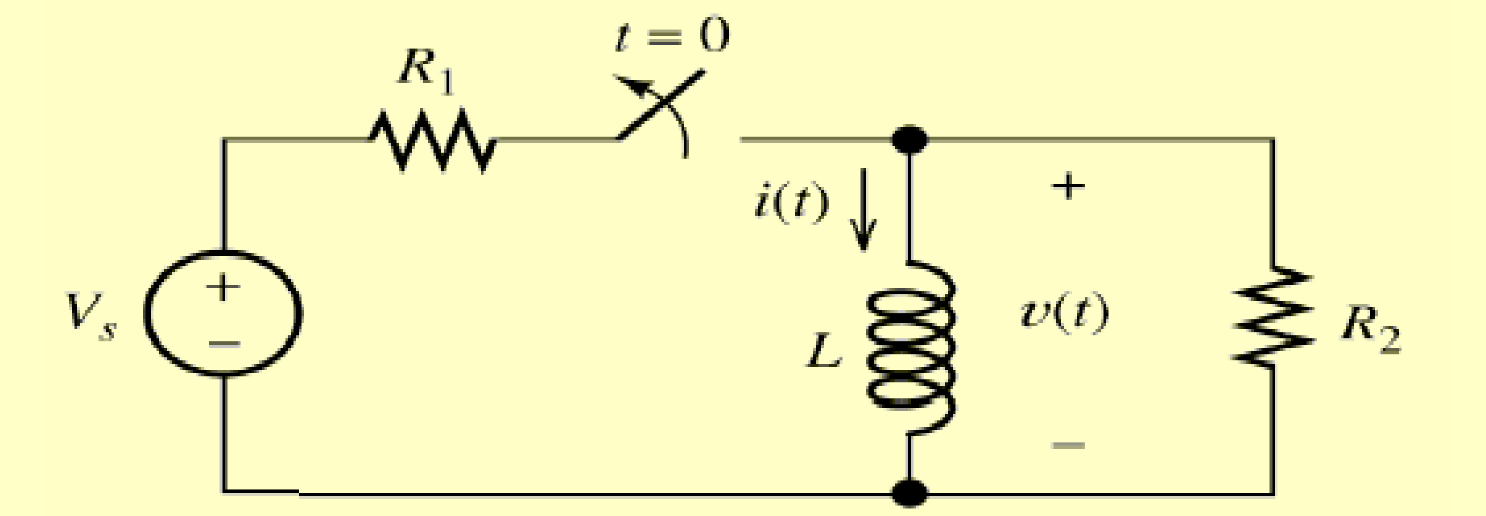




Method for circuits containing a single capacitor or inductor

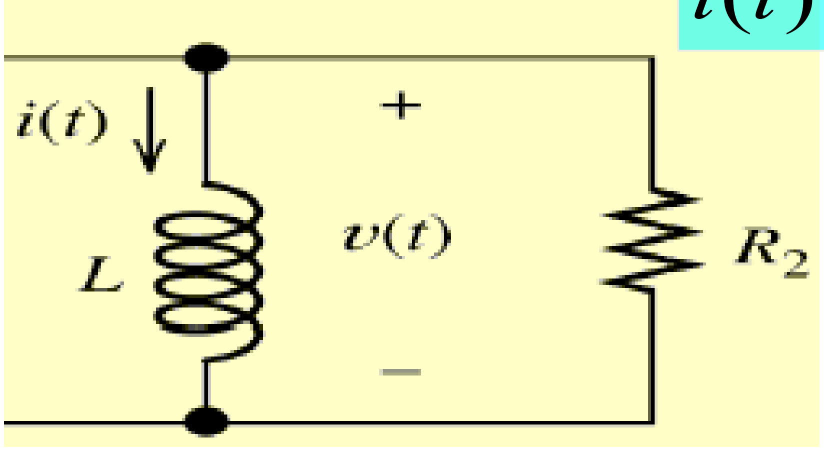


Example-1



$$i(t) = i(\infty) + \{i(0^+) - i(\infty)\} \times e^{-\frac{t}{\tau}}$$

Circuit for t > 0



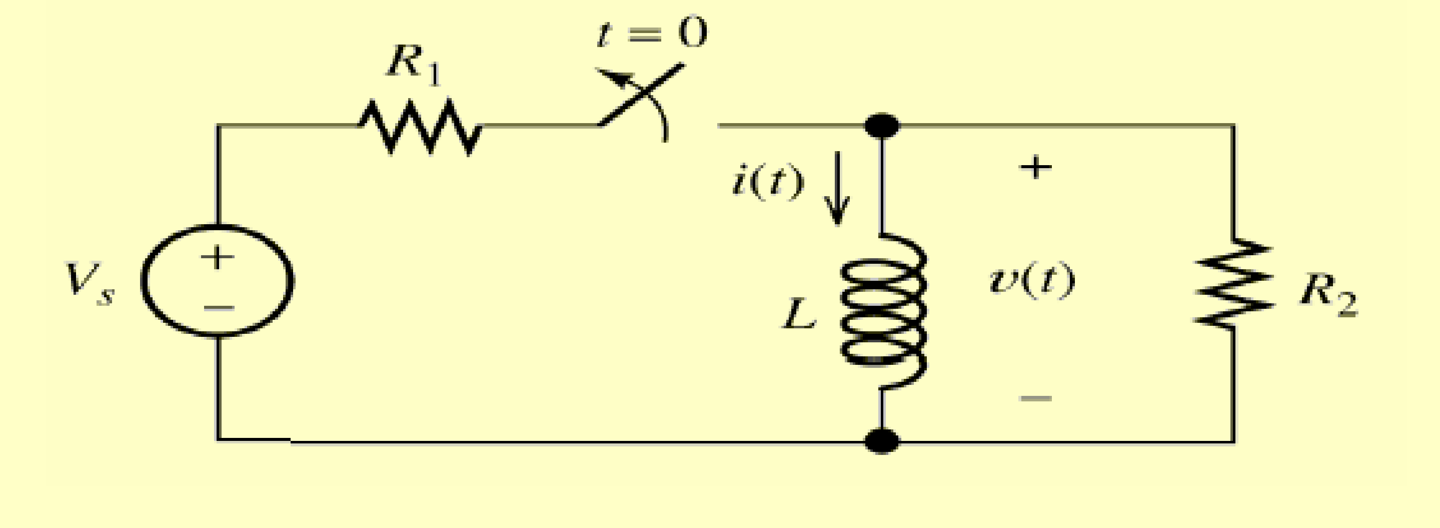
$$\tau = \frac{L}{R_2}$$

Steady state Solution:

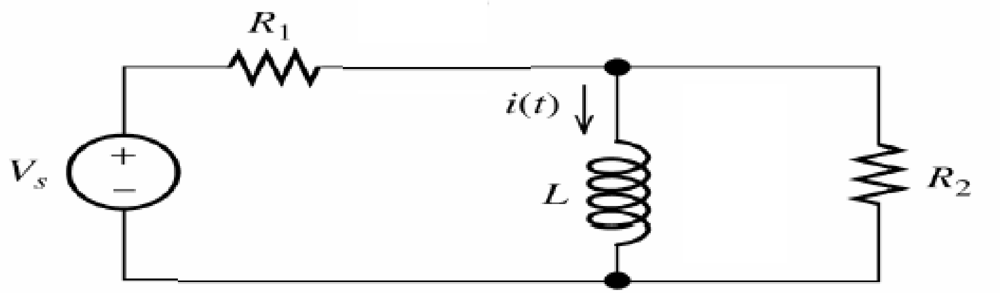
$$i(t \to \infty) = 0$$

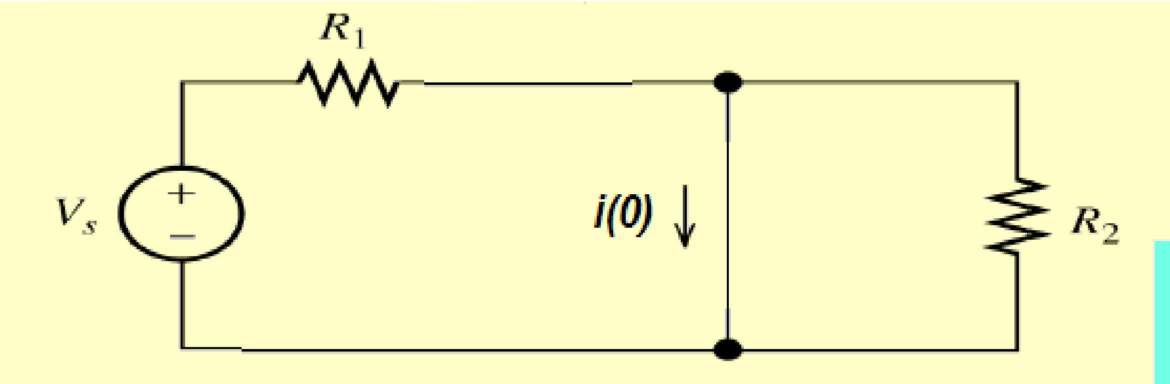
$$i(t) = i(0^+) \times e^{-\frac{t}{\tau}}$$

Initial condition



Circuit for t < 0

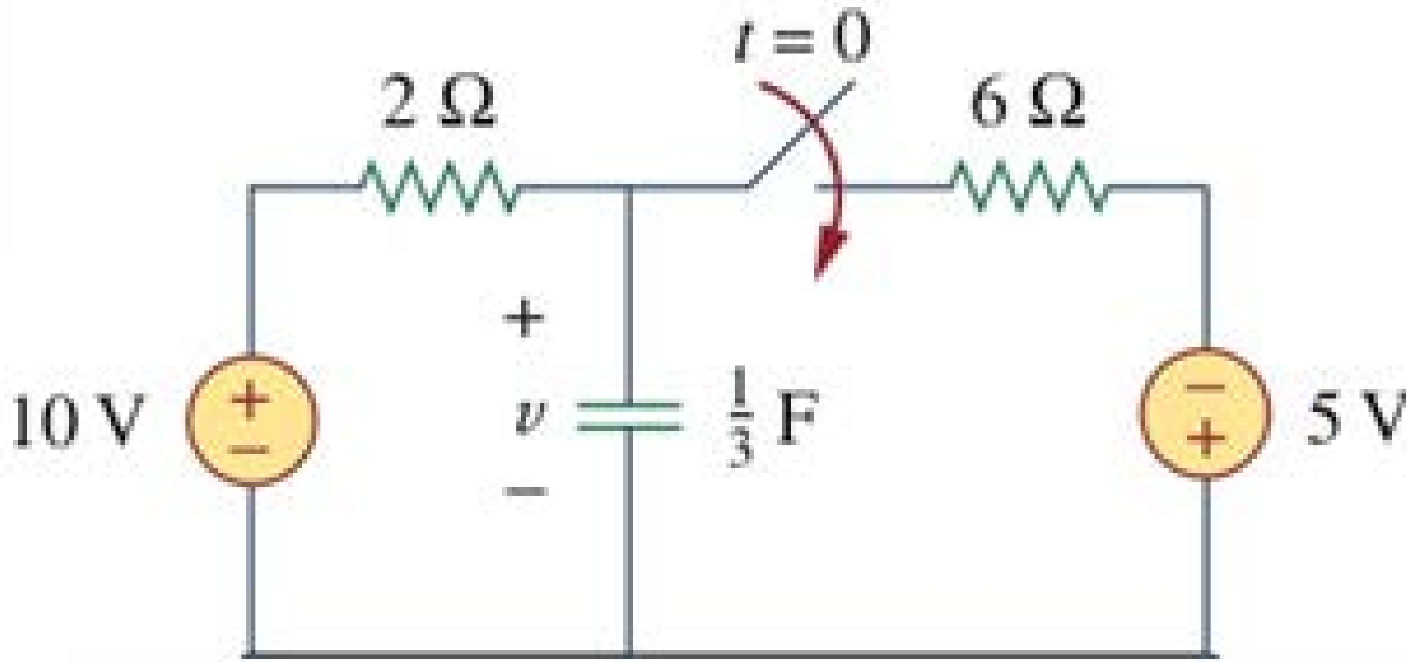




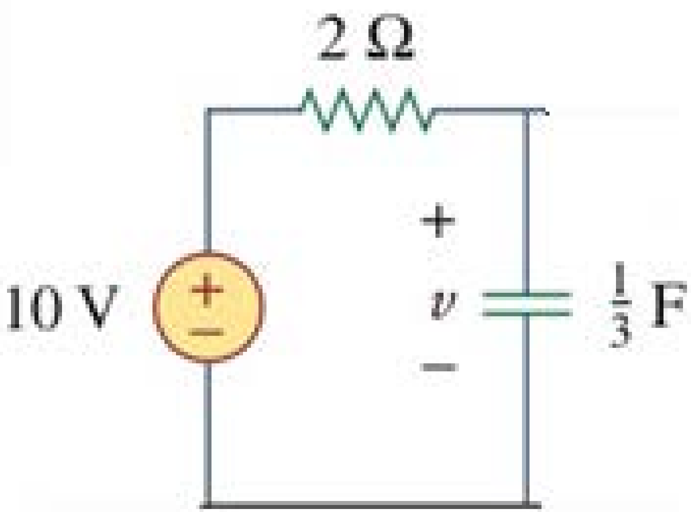
$$i(0^+) = i(0^-) = \frac{V_S}{R_1}$$

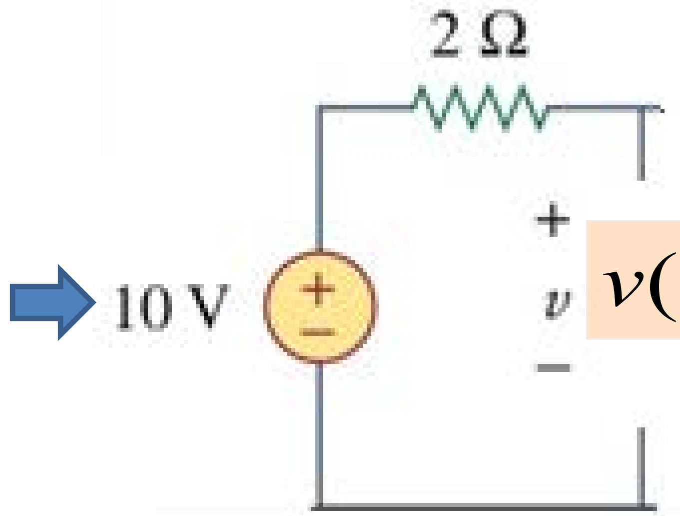
$$i(t) = \frac{V_S}{R_1} e^{-\frac{R_2}{L}t}$$

Determine the capacitor voltage as a function of time.



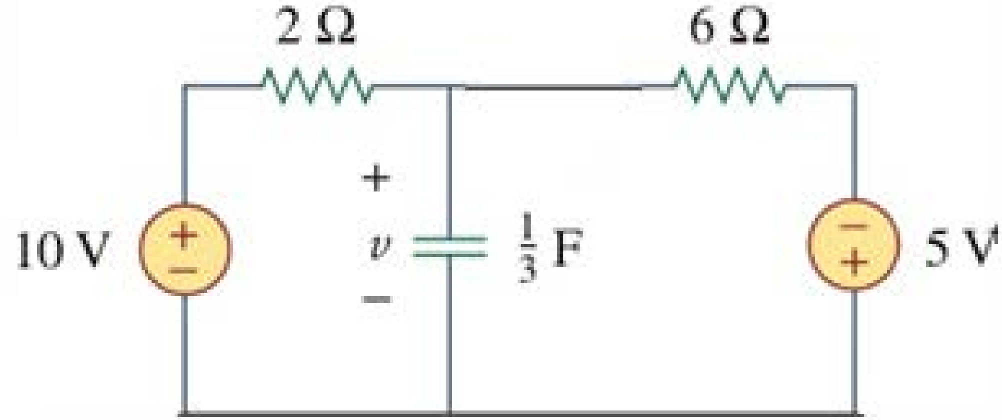
Circuit for t < 0





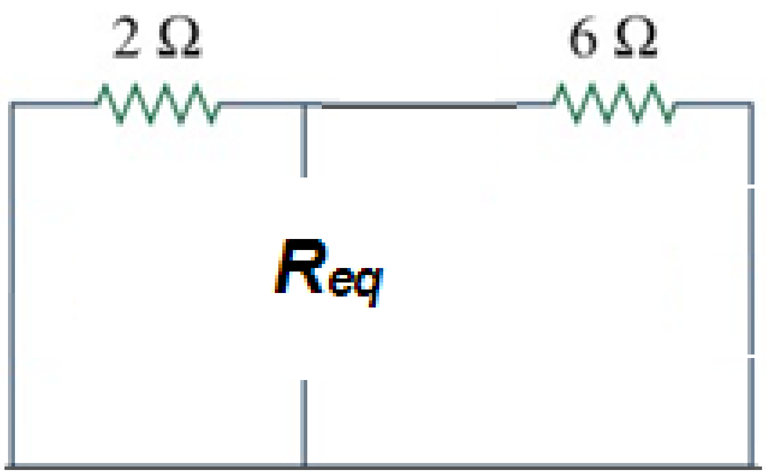
$$v(0^+) = 10 \quad \text{V}$$

Circuit for t > 0



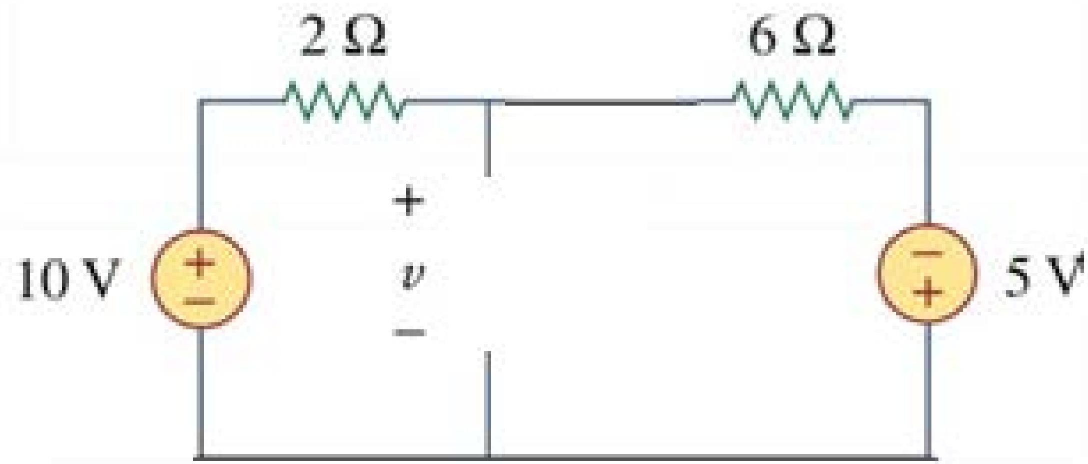
Determine the Thevenin equivalent, as seen by the capacitor:

Equivalent resistance



$$R_{eq} = 2 \| 6 = 1.5\Omega$$

We next find voltage long after closing the switch



$$v(\infty) = \frac{25}{4} \text{ V}$$

Final Solution:

$$v(t) = v(\infty) + \{v(0^+) - v(\infty)\}e^{-\frac{t}{\tau}}$$

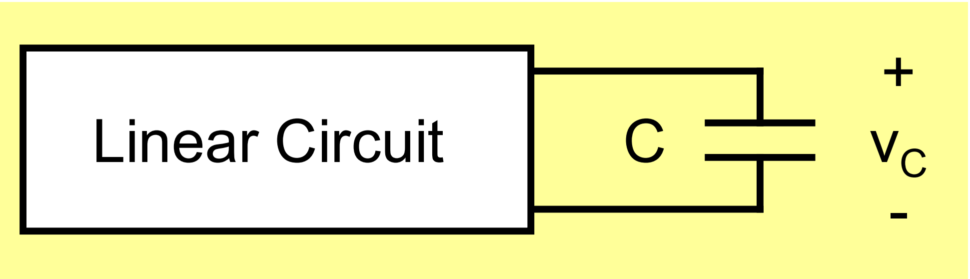
$$v(0^+) = 10 \text{ V}$$

$$v(\infty) = \frac{25}{4} \text{ V}$$

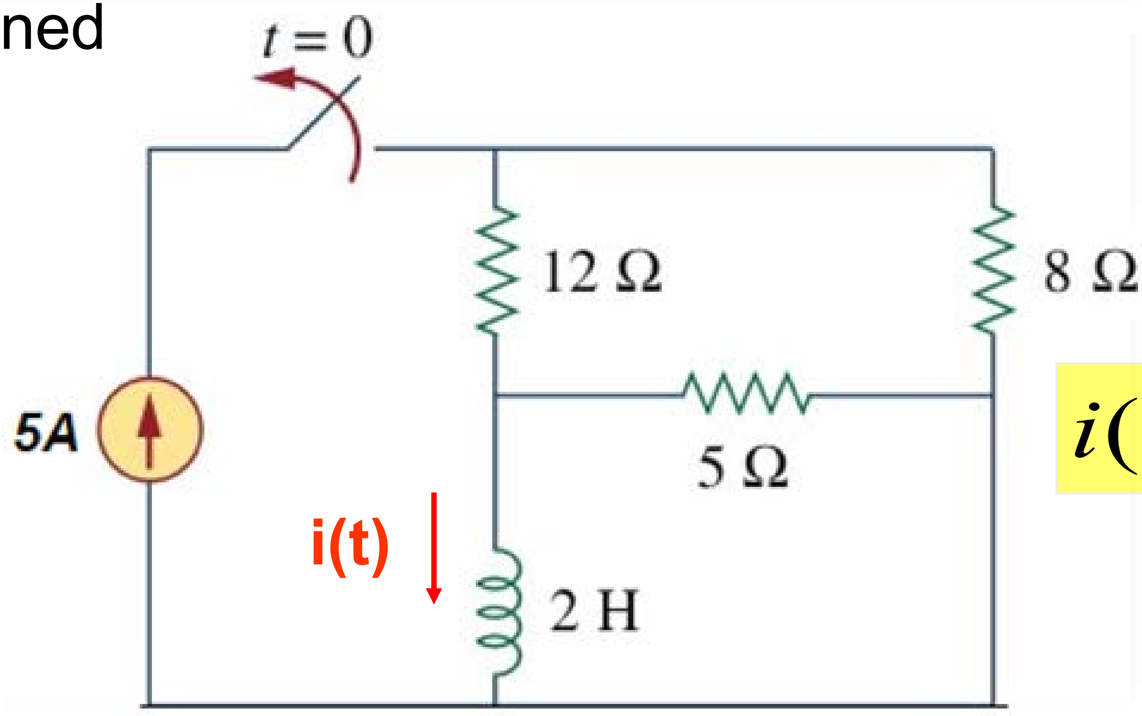
$$\tau = C \times R_{eq} = \frac{1}{3} \times 1.5 = 0.5s$$

$$v(t) = \frac{25}{4} + \frac{15}{4}e^{-2t} \text{ V}$$

How do we find voltages and currents elsewhere in the circuit?



Find current in 8Ω resistor as a function of time after the switch is opened



$$i(t) = 2 \times e^{-2t} \text{ A}$$

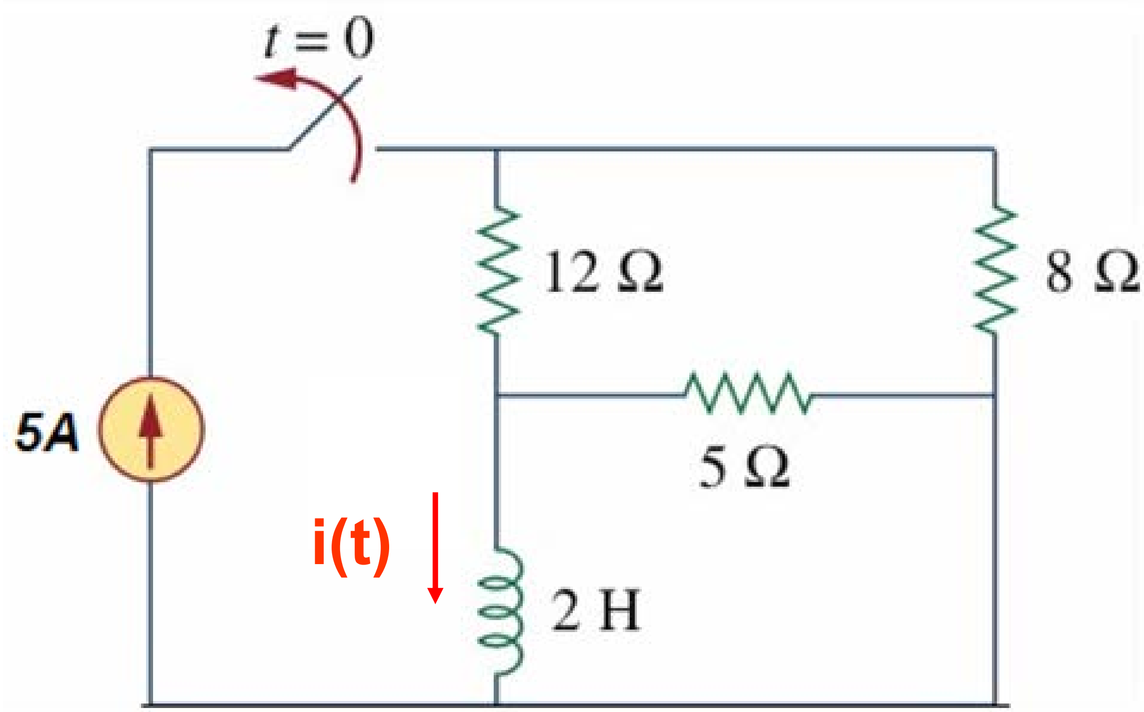
$$i(t = 0^-) = 5A \times \left(\frac{8}{12+8} \right) = 2A$$

No current will flow in 5 Ω resistance for t < 0

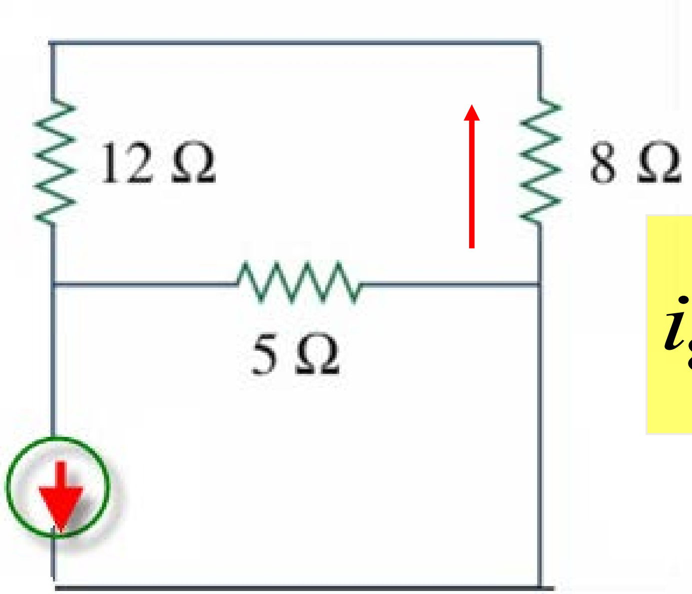
For t > 0

$$R_{eq} = (12+8) \,||\, 5 = 4 \quad \Omega$$

$$\tau = \frac{L}{R_{eq}} = 0.5 \sec$$

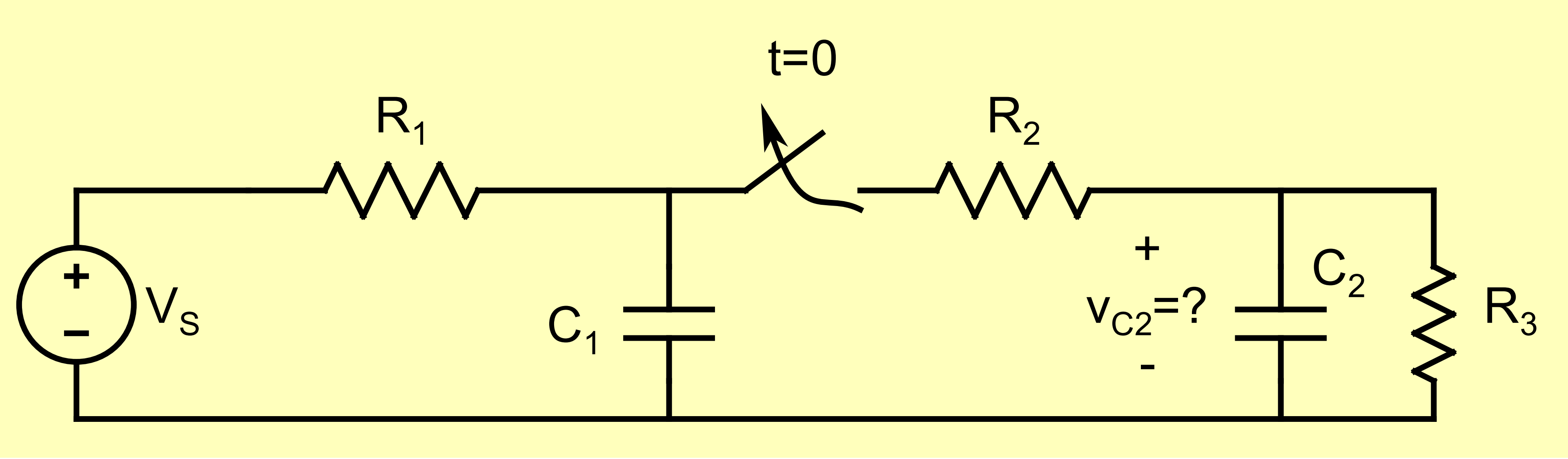


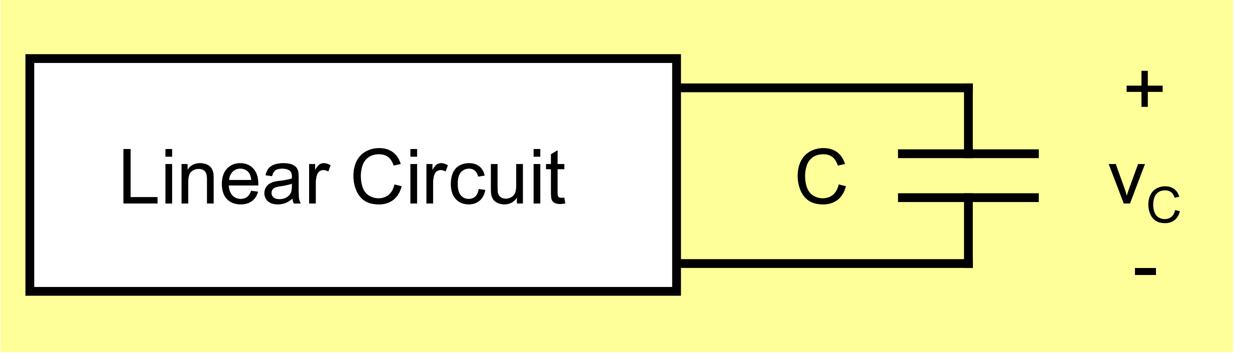
$$i(t) = 2\times e^{-2t}\ \text{A}$$



$$i_8 = i(t)\times\frac{5}{5+20} = 0.4\times e^{-2t}\ \text{A}$$

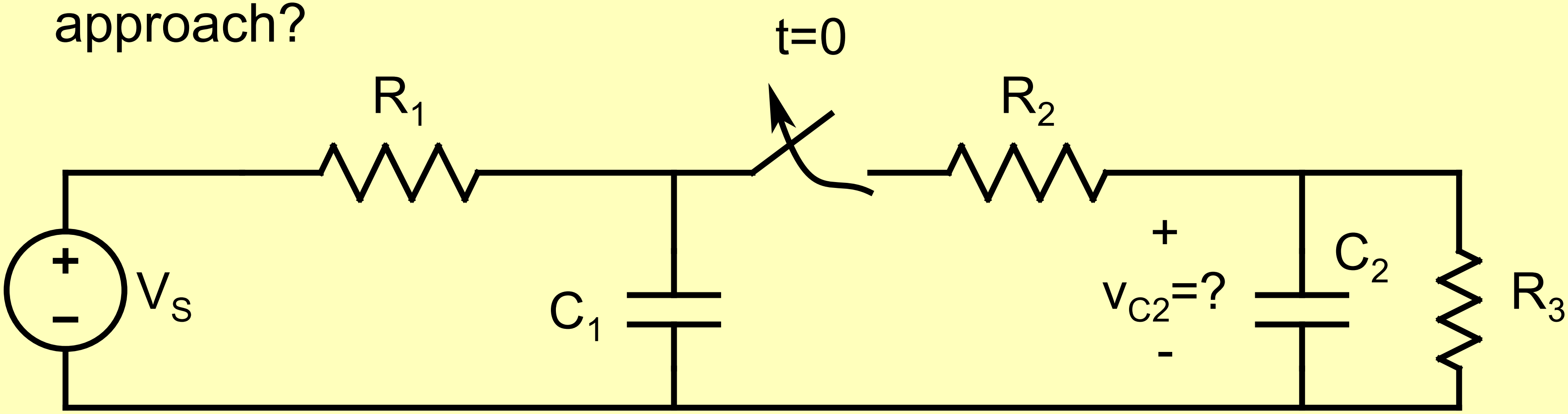
Can we solve this 2 capacitor problem using our present approach?

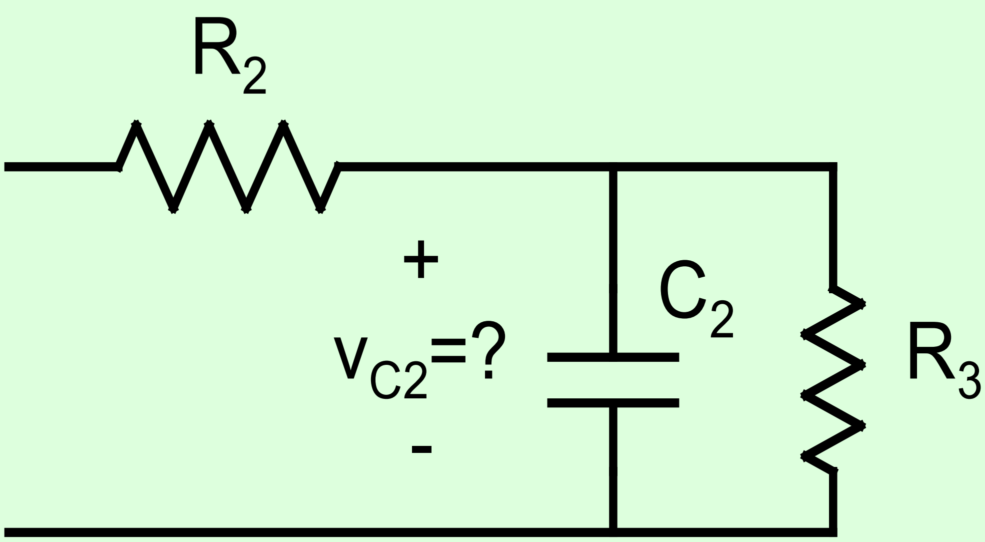




Circuit for t > 0

Can we solve this 2 capacitor problem using our present approach?



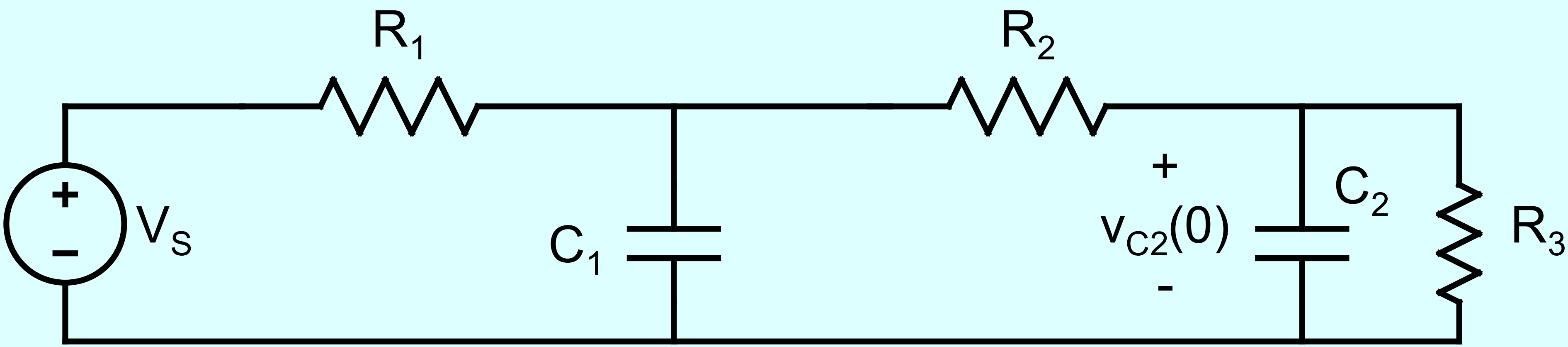


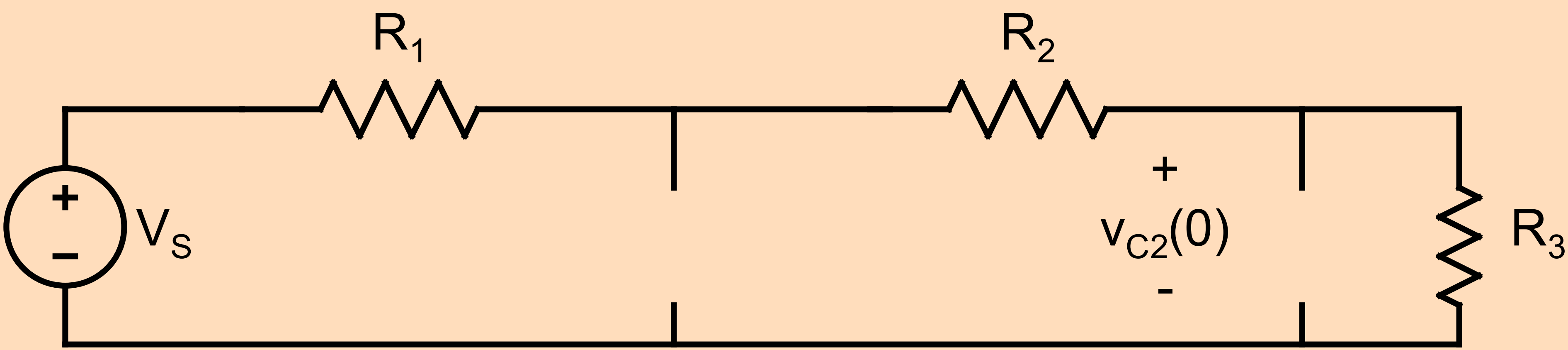
Circuit for t > 0

$$v_{c2}(0^+) = v_{c2}(0^-)$$

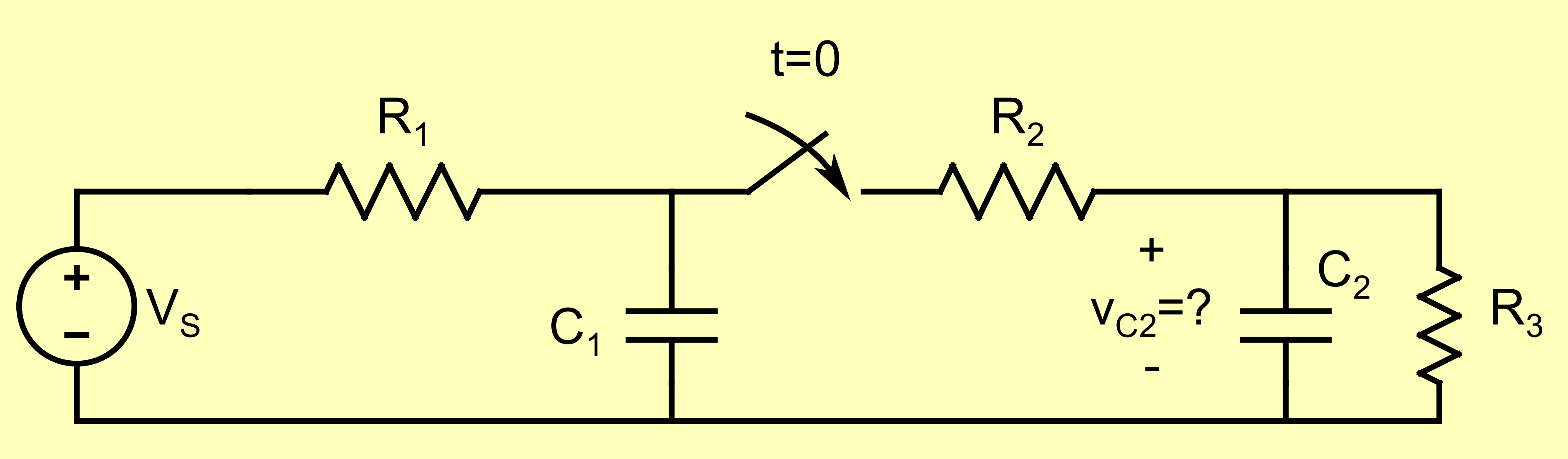
$$v_{c2}(t) = v_{c2}(\infty) + \{v_{c2}(0^+) - v_{c2}(\infty)\}e^{-\frac{t}{\tau}}$$

$$v_{c2}(0^+) = v_{c2}(0^-)$$





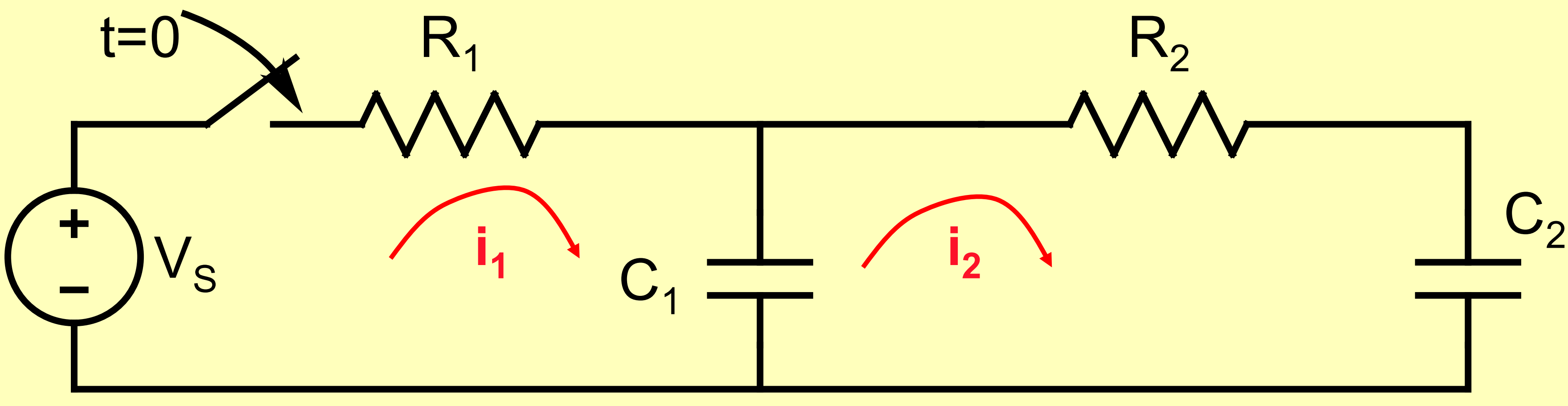
Will our approach work here?



No, because circuit for $t > 0$ has two capacitances

As long as the circuit has single capacitor or inductor for the time interval for which the analysis is being carried out, the stated approach will work fine.

What happens when there is more than one storage element?



$$V_S = i_1 R_1 + v_{C1} \quad (1)$$

$$v_{C1} = i_2 R_2 + v_{c2} \quad (2)$$

$$i_1 - i_2 = C_1 \frac{dv_1}{dt} \quad (3)$$

$$i_2 = C_2 \frac{dv_2}{dt} \quad (4)$$

$$R_1 R_2 C_1 C_2 \frac{d^2 v_{c2}}{dt^2} + (R_1 C_1 + R_1 C_2 + R_2 C_2) \frac{dv_{c2}}{dt} + v_{c2} = V_S$$