Overview of Other Topics, Conclusion and Perspectives

Piyush Rai

Probabilistic Machine Learning (CS772A)

Nov 14, 2017

Announcements

- Project presentations from Nov 20-21
- Duration of each presentation is 12+3 mins
- Will send a Google form to pick your preferred slots

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- Important: Final exam venue will be RM-101 (Nov 25, 9am-12pm), not LHC

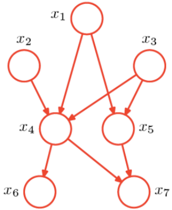
Plan for today

- A quick recap of the previous lecture (graphical models - directed and undirected)
- An overview of other topics
  - Probabilistic models for Active Learning
  - Probabilistic models for Semi-supervised Learning
  - Probabilistic models for Transfer and Multitask Learning
- Conclusion and perspectives

Recap

Directed Graphical Models (DGM)

- Represent the joint distribution of a set of random variables using a directed acyclic graph



Some nodes may be observed (e.g., data or known hyperparams)

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$$p(\boldsymbol{x}) = \prod_{k=1}^{K} p(x_k|\text{pa}_k)$$

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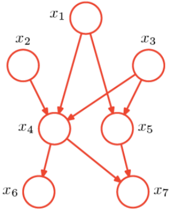

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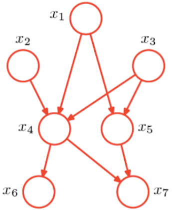

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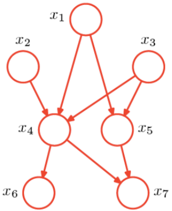

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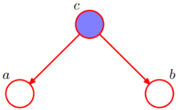
    - For overall $p(\boldsymbol{x})$, will only need $\sum_{k=1}^{K} |x_k| \prod_{i \in \text{pa}_k} |x_i|$ "numbers" to represent $p(\boldsymbol{x})$
    - On the other hand, a naïve representation of $p(\boldsymbol{x})$ would require $\prod_k |x_k|$ numbers (much larger)

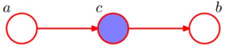
DGM: Tests for Independence

- Some simple tests for conditional independence in a DGM



$a \perp\!\!\!\perp b \mid c$

Tail-to-Tail + Observed: Blocks Path



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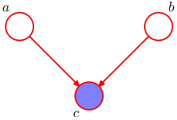

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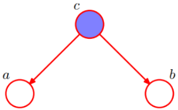

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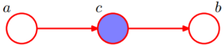

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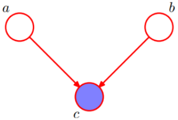

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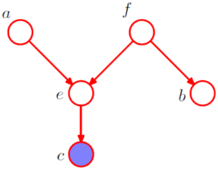
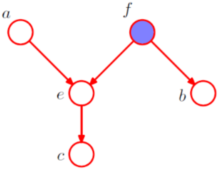
Head-to-Head + Observed: Unblocks Path

- For sophisticated DGM, apply D-separation test (based on a combination of above simple tests)



a to b is unblocked by e

Thus $a \not\!\perp\!\!\!\perp b \mid e$

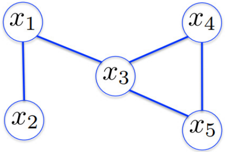


a to b blocked by f

Thus $a \perp\!\!\!\perp b \mid f$

Undirected Graphical Models (UGM) a.k.a. Markov Random Field

- Represent joint distributions as product of non-negative *potentials* $\psi()$ defined over cliques



$$p(x_1, x_2, x_3, x_4, x_5) = \frac{1}{Z}\psi(x_1, x_2)\psi(x_1, x_3)\psi(x_3, x_4, x_5)$$

where $Z = \sum_{x_1,x_2,x_3,x_4,x_5} \psi(x_1, x_2)\psi(x_1, x_3)\psi(x_3, x_4, x_5)$ is a normalizer

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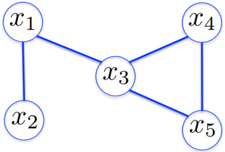

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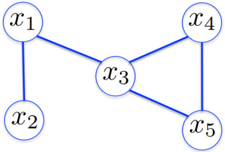

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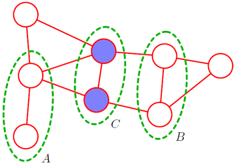
- The joint distribution of a UGM can be then written as

$$p(\boldsymbol{x}) = \frac{1}{Z}\prod_{c\in C}\psi_c(\boldsymbol{x}_c) = \frac{1}{Z}\prod_{c\in C} e^{-E(\boldsymbol{x}_c|\theta_c)} = \frac{1}{Z}e^{-\sum_{c\in C} E(\boldsymbol{x}_c|\theta_c)}$$

UGM: Tests for Independence

- Usually simpler to check than the DGM case (UGMs have no causal relationships)
- $A \perp\!\!\!\perp B | C$ if all paths from A to B pass through one or more nodes in C



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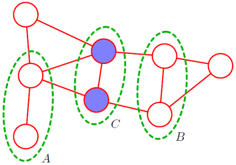

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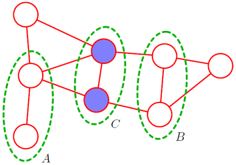

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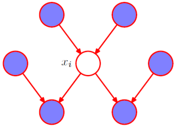
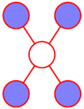
- Also, unlike DGM, Markov blanket of a UGM node only consists only of nodes it is connected to



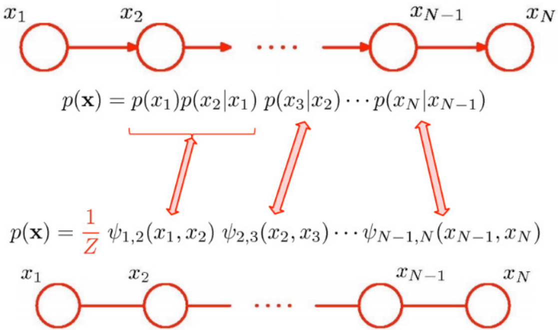
Markov Blanket for a DGM



Markov Blanket for a UGM

DGM and UGM

- In general, any probability distribution $p(\boldsymbol{x})$ can be modeled via both a DGM or a UGM
- One form can be transformed into the other. Straightforward for chain-structured DGM



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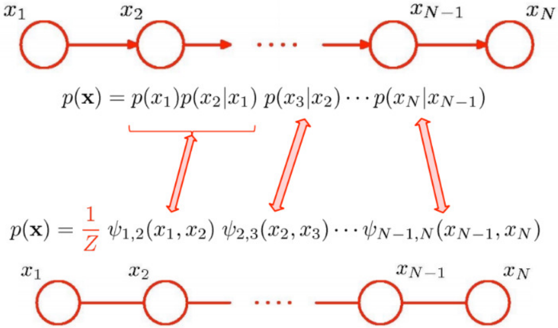

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$$p(\mathbf{x}) = \underbrace{p(x_1)p(x_2|x_1)}\, p(x_3|x_2) \cdots p(x_N|x_{N-1})$$

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$$p(\mathbf{x}) = \frac{1}{Z}\, \psi_{1,2}(x_1, x_2)\, \psi_{2,3}(x_2, x_3) \cdots \psi_{N-1,N}(x_{N-1}, x_N)$$

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- Note: For more general DGMs, some more additional care is required in the conversion

Inference via Message-Passing

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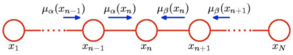
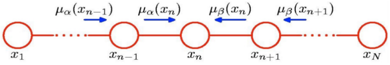
- E.g., for a chain-structured UGM, computing node marginal $p(x_n) = \sum_{x_1, x_2, x_{n-1}, x_{n+1}, \ldots, x_N} p(\boldsymbol{x})$



$$p(x_n) = \frac{1}{Z} \underbrace{\left[\sum_{x_{n-1}} \psi_{n-1,n}(x_{n-1}, x_n) \cdots \left[\sum_{x_2} \psi_{2,3}(x_2, x_3) \left[\sum_{x_1} \psi_{1,2}(x_1, x_2)\right]\right] \cdots\right]}_{\mu_\alpha(x_n)}$$

Cost = O(NK^2)

$$\times \underbrace{\left[\sum_{x_{n+1}} \psi_{n,n+1}(x_n, x_{n+1}) \cdots \left[\sum_{x_N} \psi_{N-1,N}(x_{N-1}, x_N)\right] \cdots\right]}_{\mu_\beta(x_n)}$$



Forward message:
$$\mu_\alpha(x_n) = \sum_{x_{n-1}} \psi_{n-1,n}(x_{n-1}, x_n) \left[\sum_{x_{n-2}} \cdots\right] = \sum_{x_{n-1}} \psi_{n-1,n}(x_{n-1}, x_n) \boxed{\mu_\alpha(x_{n-1})}$$

Backward message:
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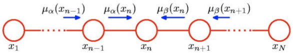

$$p(x_n) = \frac{1}{Z}\underbrace{\left[\sum_{x_{n-1}} \psi_{n-1,n}(x_{n-1},x_n)\cdots\left[\sum_{x_2}\psi_{2,3}(x_2,x_3)\left[\sum_{x_1}\psi_{1,2}(x_1,x_2)\right]\right]\cdots\right]}_{\mu_\alpha(x_n)}$$

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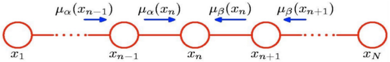

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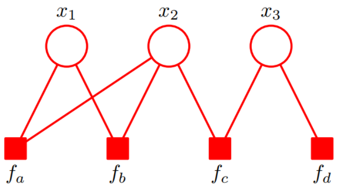
- Significant saving: Assume each node is discrete with K possible values, the cost reduces from $O(K^N)$ to $O(NK^2)$ for computing each node's marginal

Factor Graphs

- A unified representation for general DGM/UGM
- Useful for designing message-passing algos for general DGM/UGM
- A bipartite graph consisting of variable nodes and factor nodes



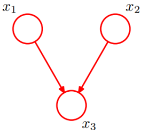
$$p(\mathbf{x}) = f_a(x_1, x_2) f_b(x_1, x_2) f_c(x_2, x_3) f_d(x_3)$$

$$p(\mathbf{x}) = \prod_s f_s(\mathbf{x}_s)$$

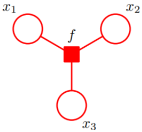
- Basic idea: Original nodes become the variable nodes, conditionals/potentials becomes factor nodes (which represent a computation over the variable nodes connected to the factor node)

Factor Graphs

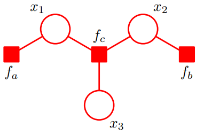
- Both DGM and UGM can be converted into a factor graph representation



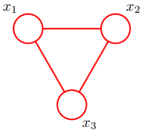
$p(x) = p(x_1)p(x_2)$
$p(x_3|x_1,x_2)$



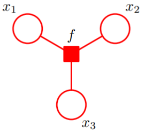
$f(x_1,x_2,x_3) =$
$p(x_1)p(x_2)p(x_3|x_1,x_2)$



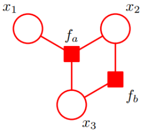
$f_a(x_1) = p(x_1)$
$f_b(x_2) = p(x_2)$
$f_c(x_1,x_2,x_3) = p(x_3|x_2,x_1)$



$\psi(x_1,x_2,x_3)$



$f(x_1,x_2,x_3)$
$= \psi(x_1,x_2,x_3)$



$f_a(x_1,x_2,x_3)f_b(x_2,x_3)$
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- Given the factor graph representation of a distribution $p(\boldsymbol{x})$, can do inference via message passing

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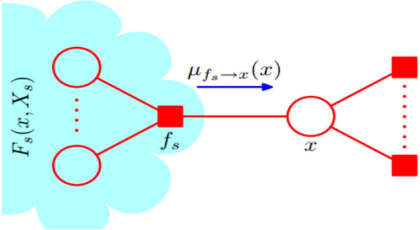
- Given the factor graph representation of a distribution $p(\boldsymbol{x})$, can do inference via message passing
- Example: Computing marginal $p(x) = \sum_{\boldsymbol{x} \setminus x} p(\boldsymbol{x})$ of some node given the factor graph of $p(\boldsymbol{x})$



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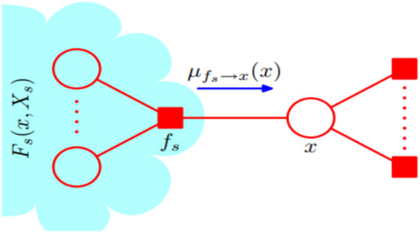

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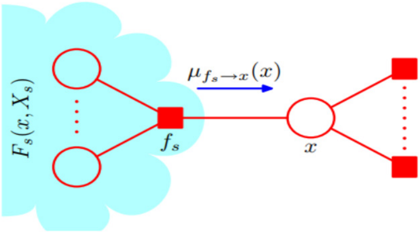

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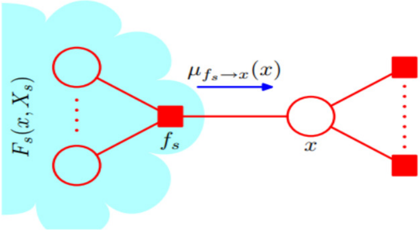

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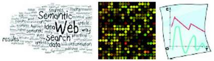
- PRML Chapter 8 contains more details of such algorithms

An Overview of Some Other Topics

Active Learning

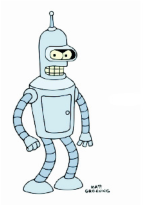
Passive vs Active Learning

Standard supervised learning assumes a "passive" learner



raw unlabeled data

$x_1, x_2, x_3, \ldots$



supervised learner

induces a classifier

expert / oracle

analyzes experiments

to determine labels

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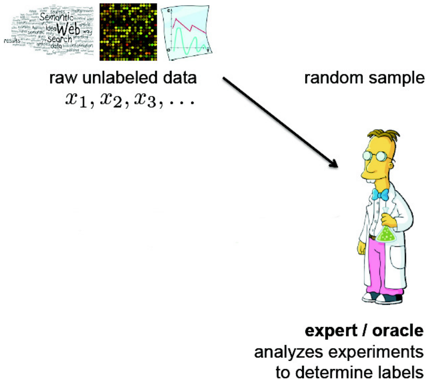

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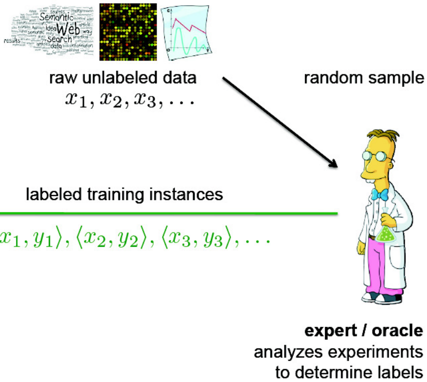

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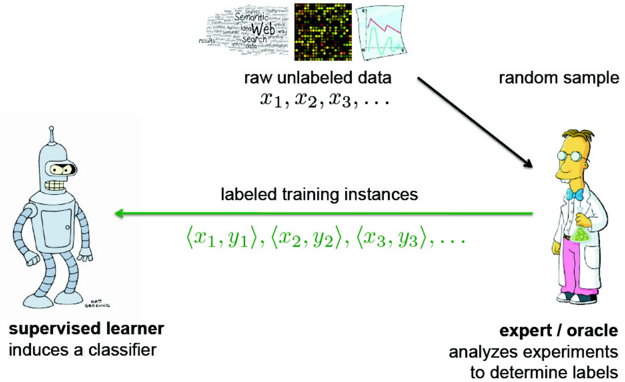
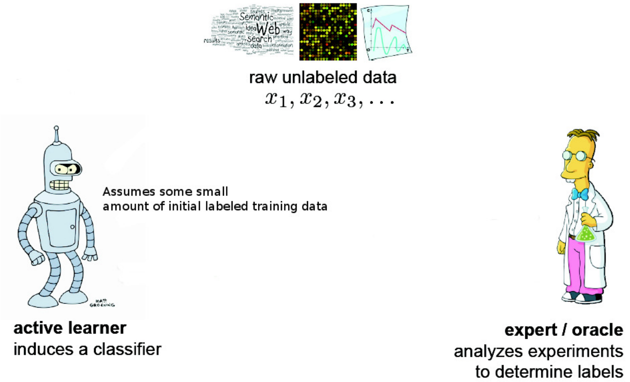


Called "passive" because the learner doesn't have any control over the labeled data it gets to learn

Active Learning

An "active" learner can specifically request labels of "hard" examples that are most useful for learning



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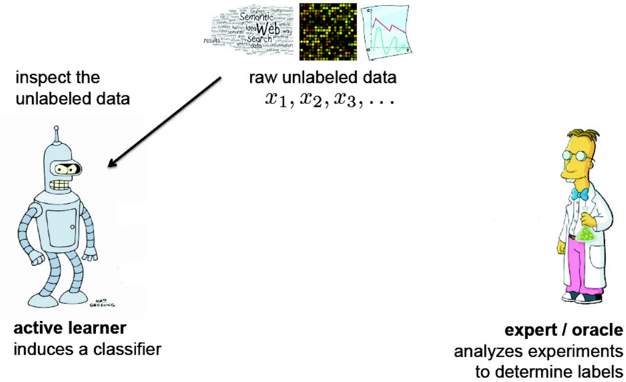

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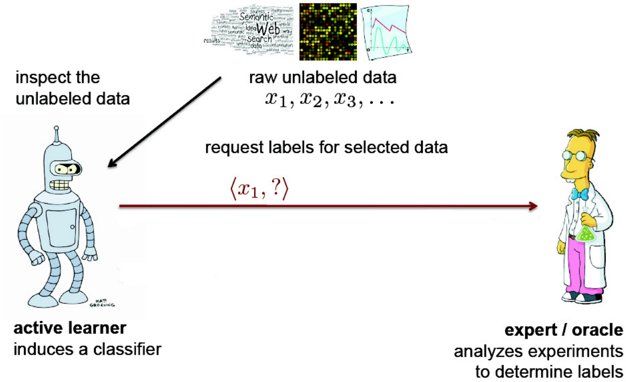

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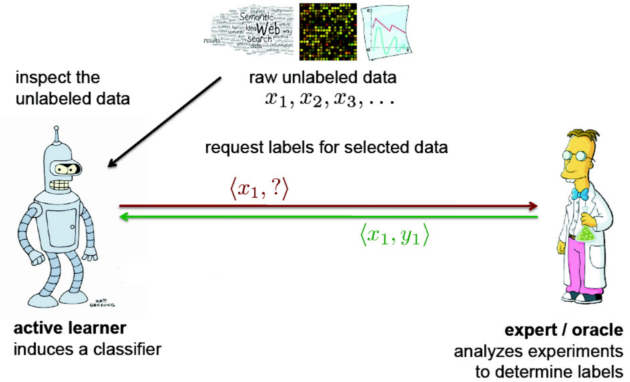

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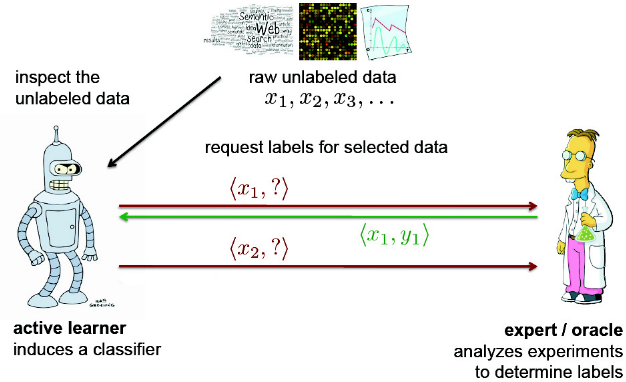

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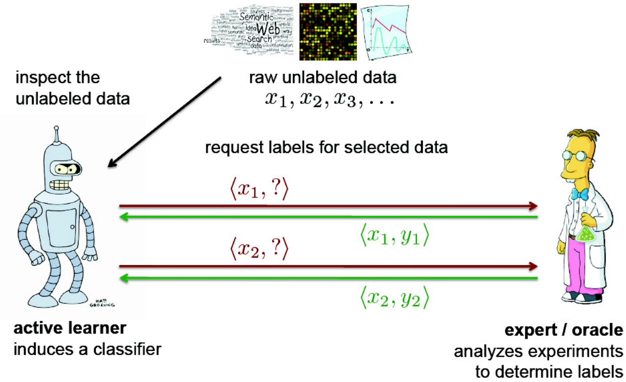

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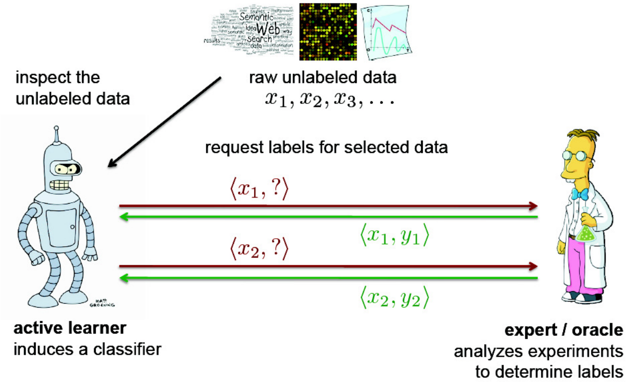

How to quantify the "hardness" or "usefulness" of an unlabeled example?

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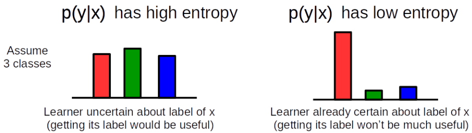

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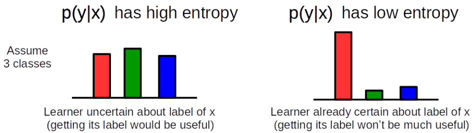

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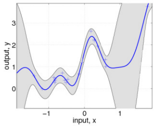
  - Look at the variance of the posterior predictive $p(y|\boldsymbol{x})$. E.g. for regression,



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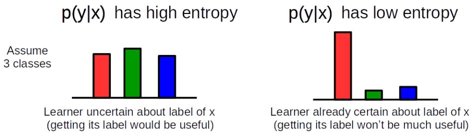

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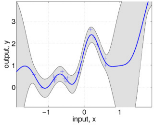

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  - .. many other ways to utilize the information in $p(y|\boldsymbol{x})$

Semi-supervised Learning

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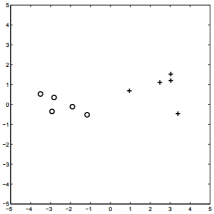

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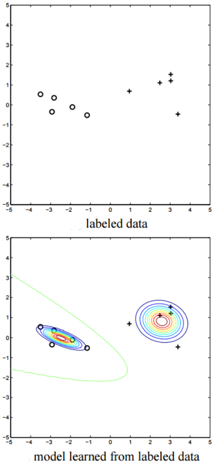

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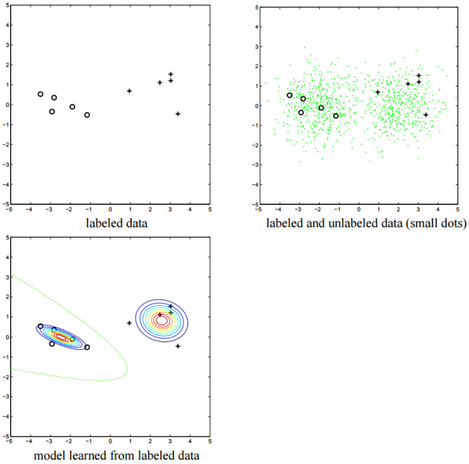

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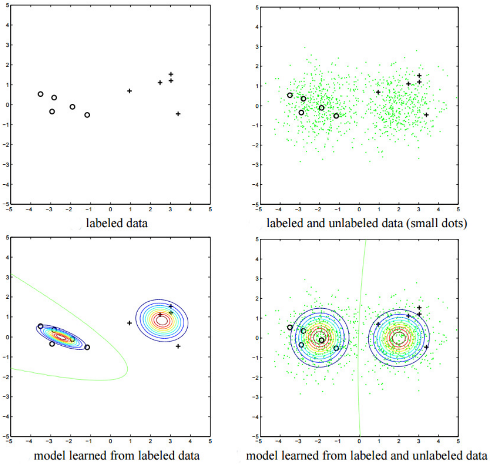

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- A fairly general framework for semi-supervised learning. Can be used for different types of data (by choosing the appropriate $p(\boldsymbol{x}|y)$ distribution)

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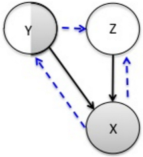

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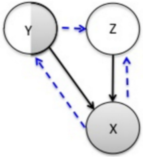

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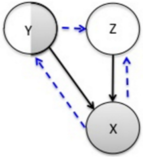

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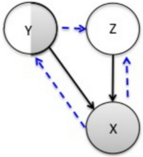

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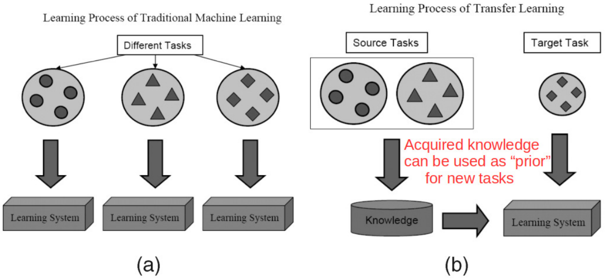
- Many other recent advances in SSL using models such as GAN and hybrids of VAE and GAN (too rapid progress to cover in a course like this; look at follow-up work of Kingman et al (2014))

Semi-Supervised Learning with Deep Generative Models (Kingma et al, 2014)

Transfer and Multi-task Learning

Transfer Learning

- How do we transfer knowledge of what we already know about previous tasks to a new task?



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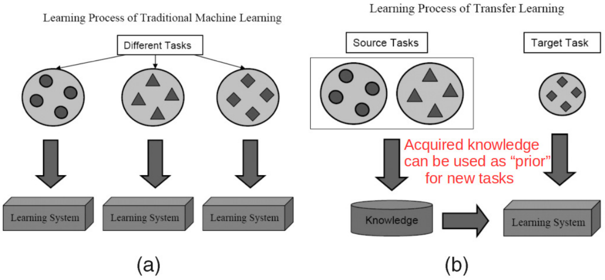

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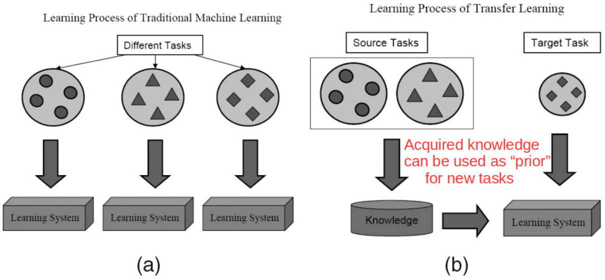

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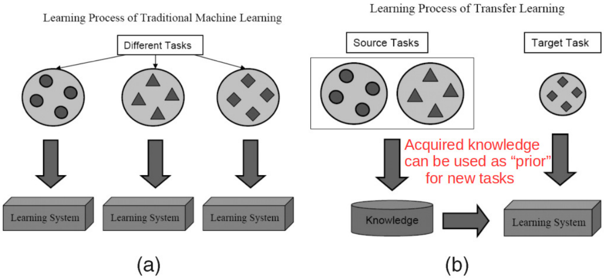

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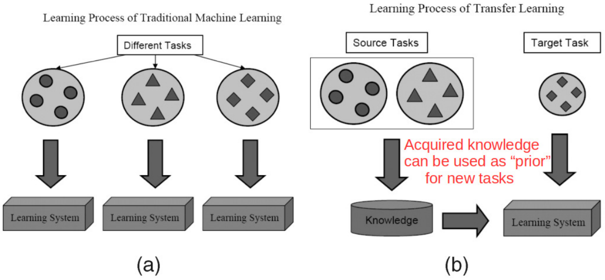
- Probabilistic modeling gives a natural way to transfer knowledge by using past knowledge as prior

Transfer Learning

- How do we transfer knowledge of what we already know about previous tasks to a new task?

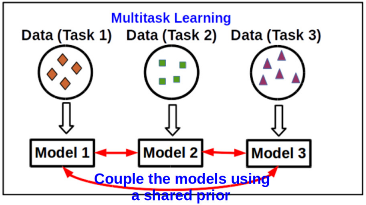


- A very common and popular practice in deep learning (pre-trained networks)
  - However transfer learning (Caruana, 1997) is more general, not limited to pre-training in deep learning
- Probabilistic modeling gives a natural way to transfer knowledge by using past knowledge as prior
  - Train a model for some task, use the learned parameters as hyperparams of prior of the new task

[Multitask Learning (Caruana, 1997)]

Multitask Learning

- Often we may wish to not learn sequentially (learn one task, then the other) but everything jointly



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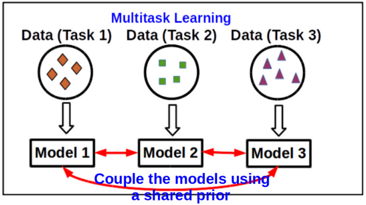
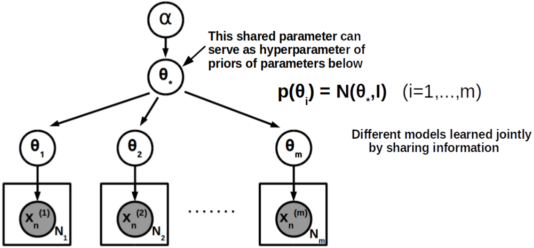


- A probabilistic model is natural to use in such problems



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  - .. and various other benefits as we saw during this course
- Fast-moving field, lots of recent advances on new models and inference methods
  - The course is an attempt to guide you into exploring the area further

We won't be here without..

Thomas Bayes

Pierre-Simon Laplace

Thank You!