Recommendation Systems

CS771: Introduction to Machine Learning

Purushottam Kar

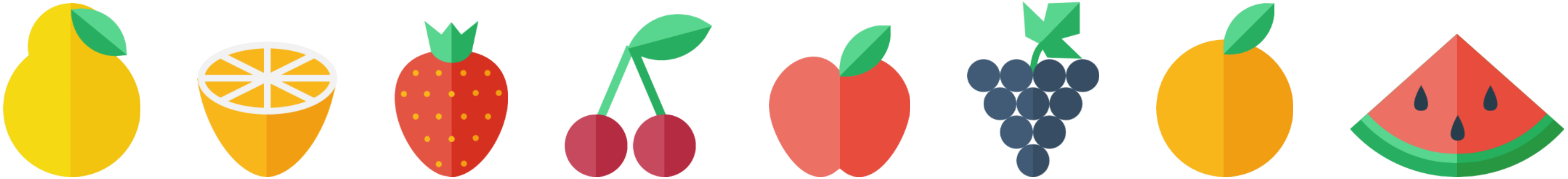
Outline of discussion

- Three major techniques
  - Collaborative Filtering
  - Extreme Classification
  - Multi-armed Bandits
- Barely scratching the surface
- Entire conference RecSys dedicated to recommendation systems
- Look for papers in other conferences too!
  NIPS, ICML, KDD, WWW, WSDM

Problem Statement

- There are users …

- … and there are items …



- … and we need to recommend, to each user, items s/he will “like”
- Users usually come to us one at a time in no particular order

A Powerful Abstraction

User

- Amazon.com user
- Facebook user
- Student
- Patient
- Search query
- …

Items

- Amazon.com products
- Facebook posts
- Study material
- Medicines
- Internet webpages
- …

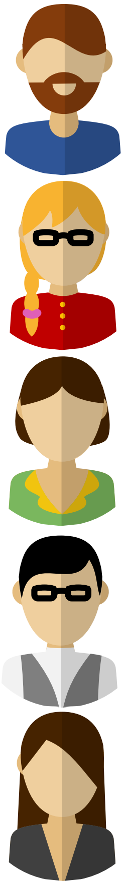
- Users are in millions and thousands come each second
- Items are in millions and each user likes only 5-10 items
- Some items are popular but most items liked by only 5-10 users

Collaborative Filtering

The Matrix Completion Problem

- m users n items
- Rating matrix $X \in \mathbb{R}^{m \times n}$
- X_{ij}: rating indicating how much user i likes item j
- Deeper shade $\Rightarrow$ more like
- Sometimes users rate some items (rarely though)
- So we get to see those entries of X, but only those
- Can we recover the unseen entries i.e. complete X?

The Matrix Completion Problem



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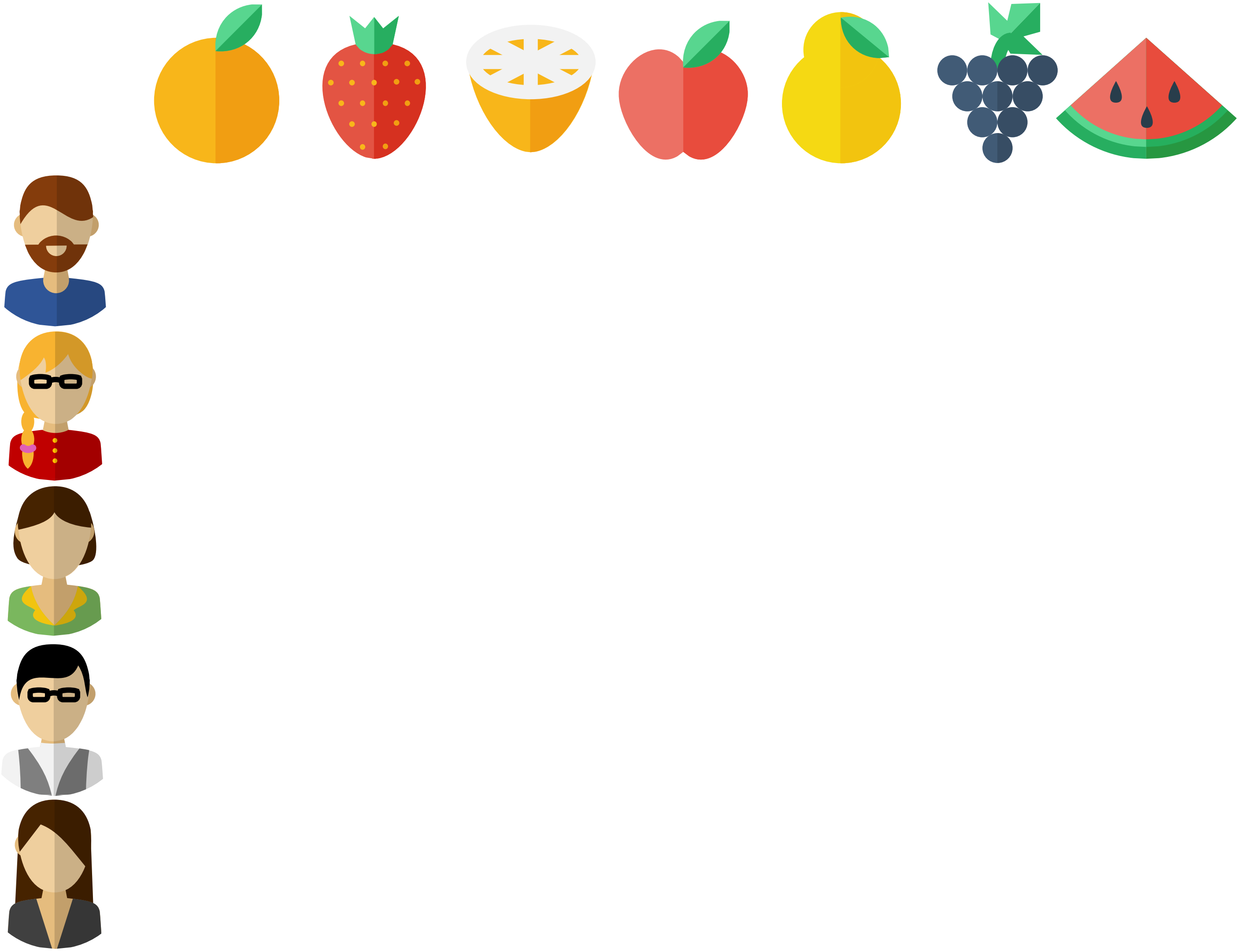

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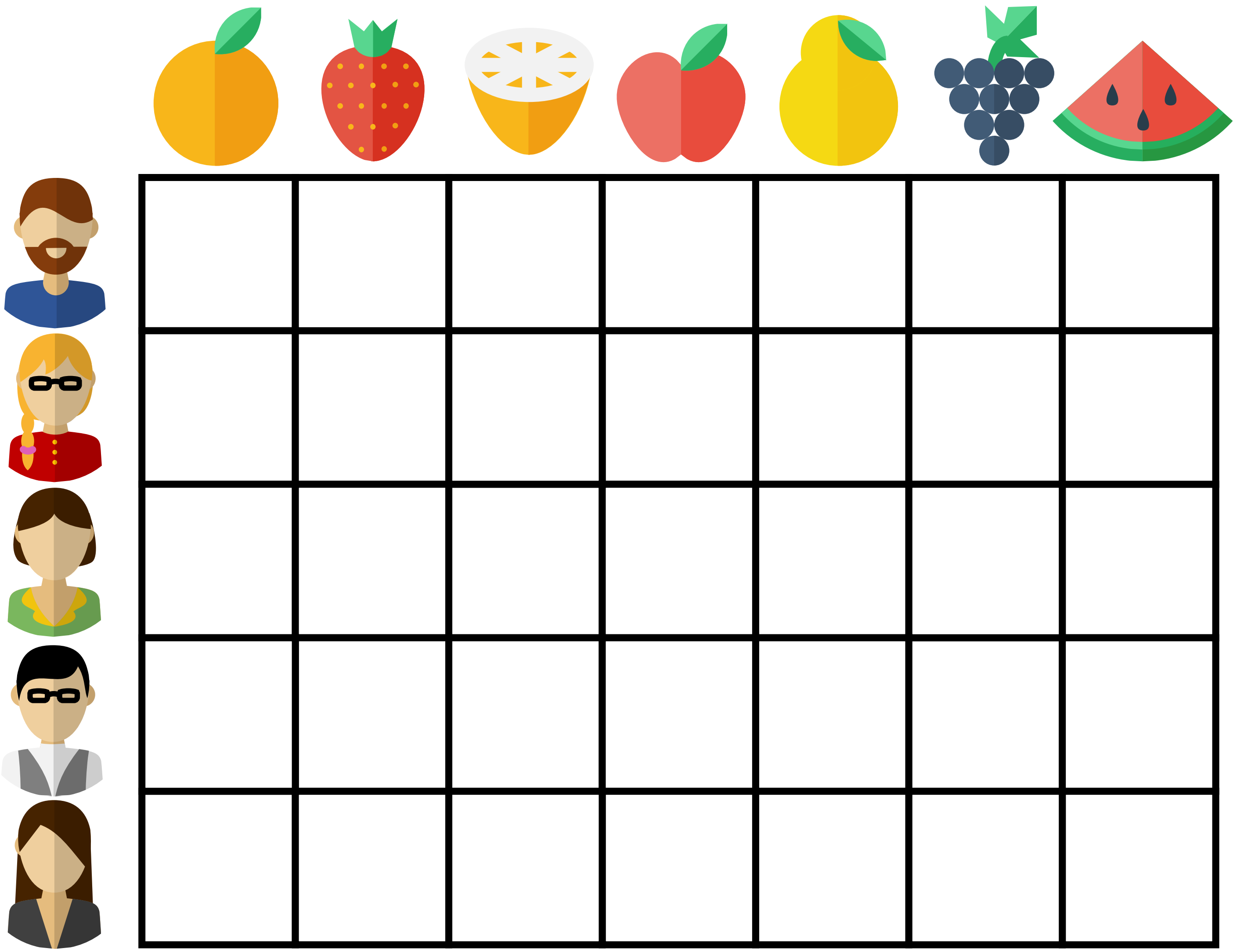

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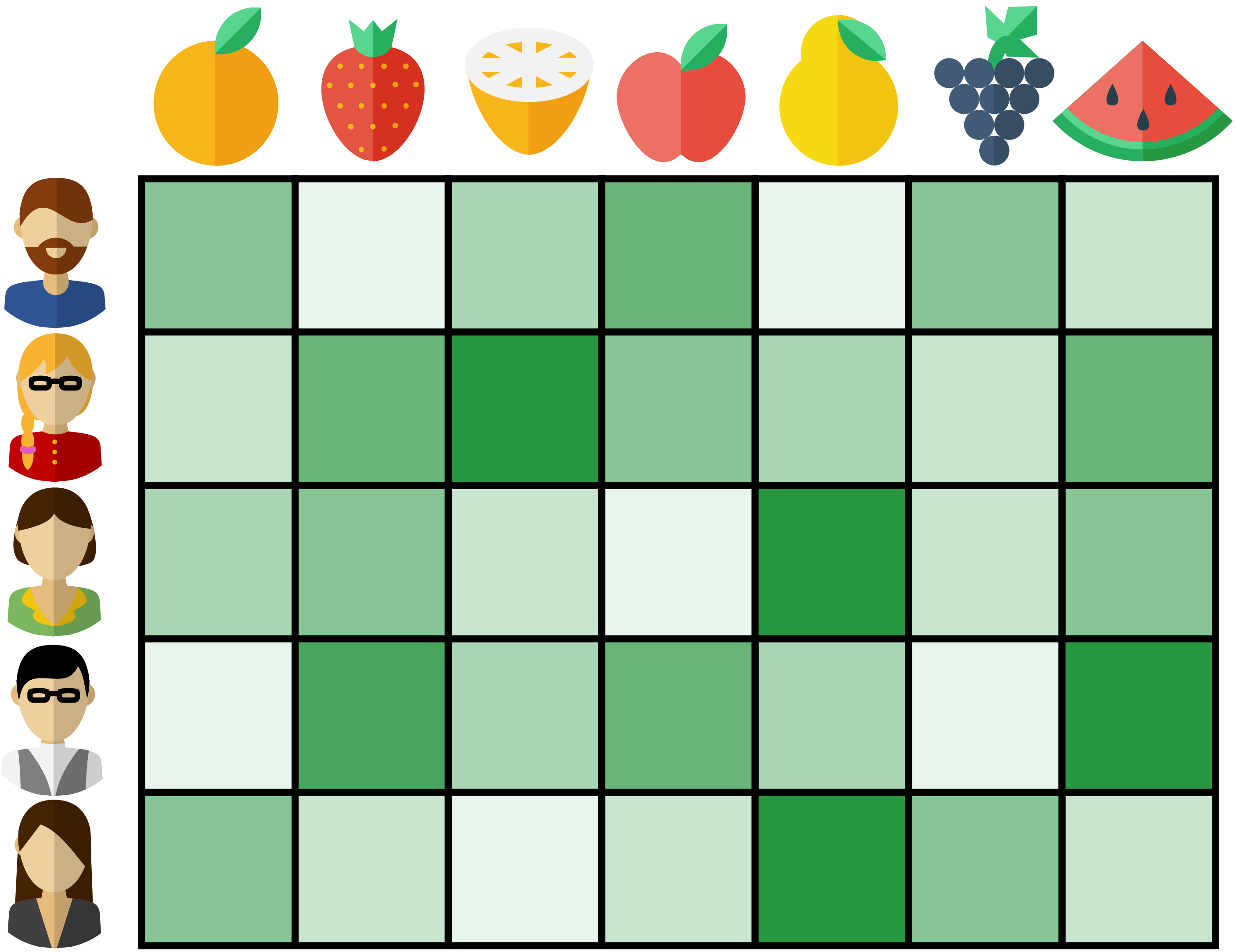

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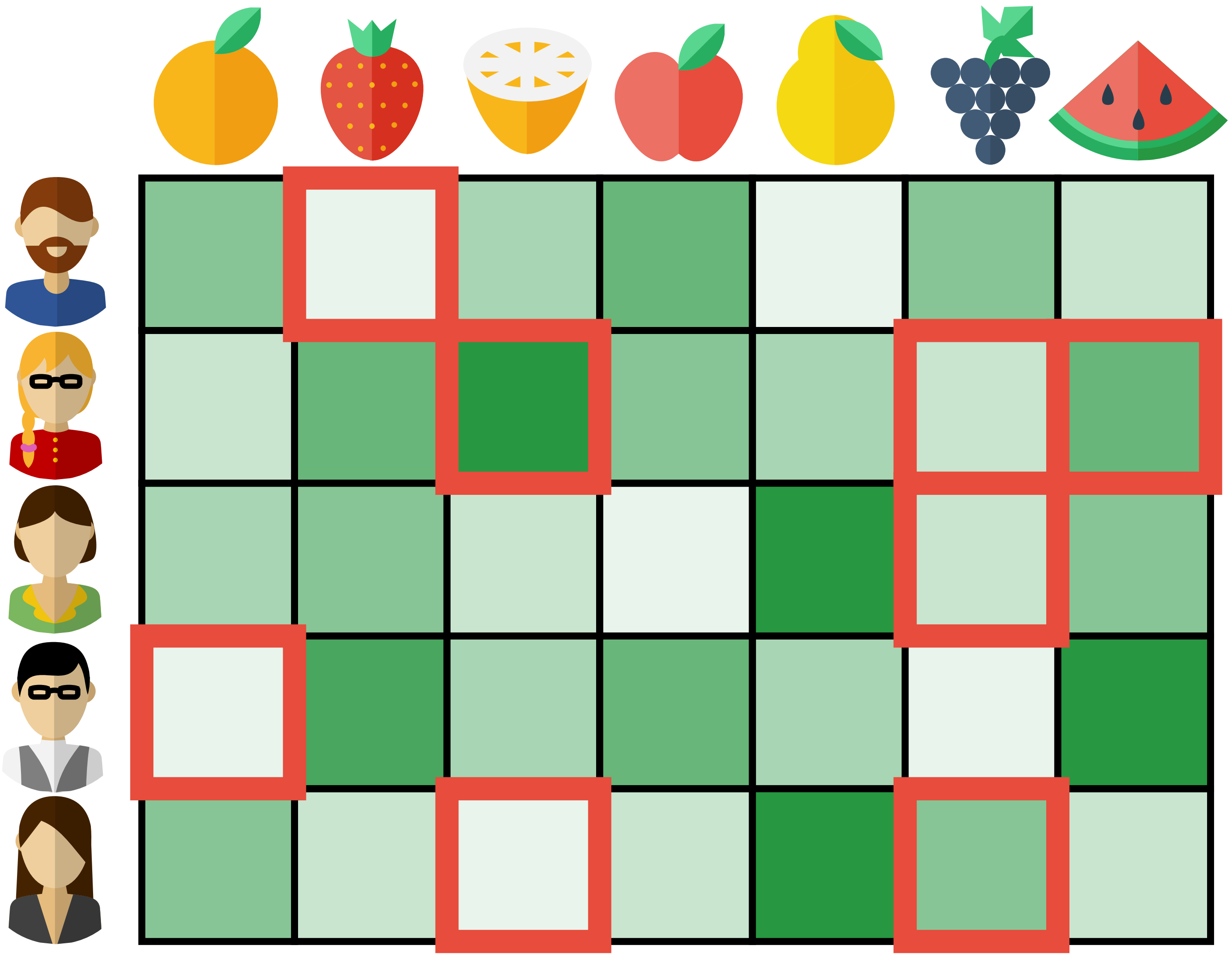

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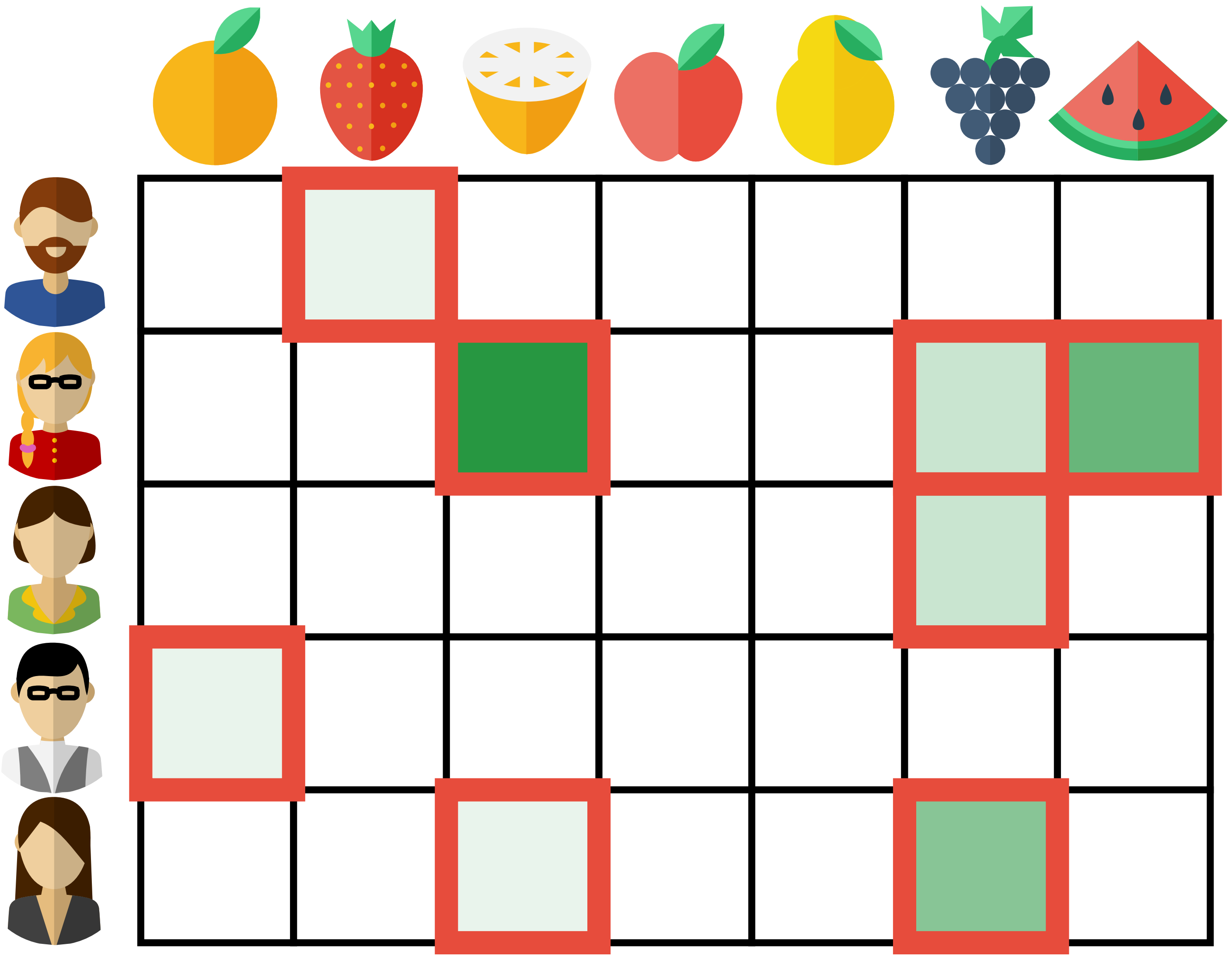

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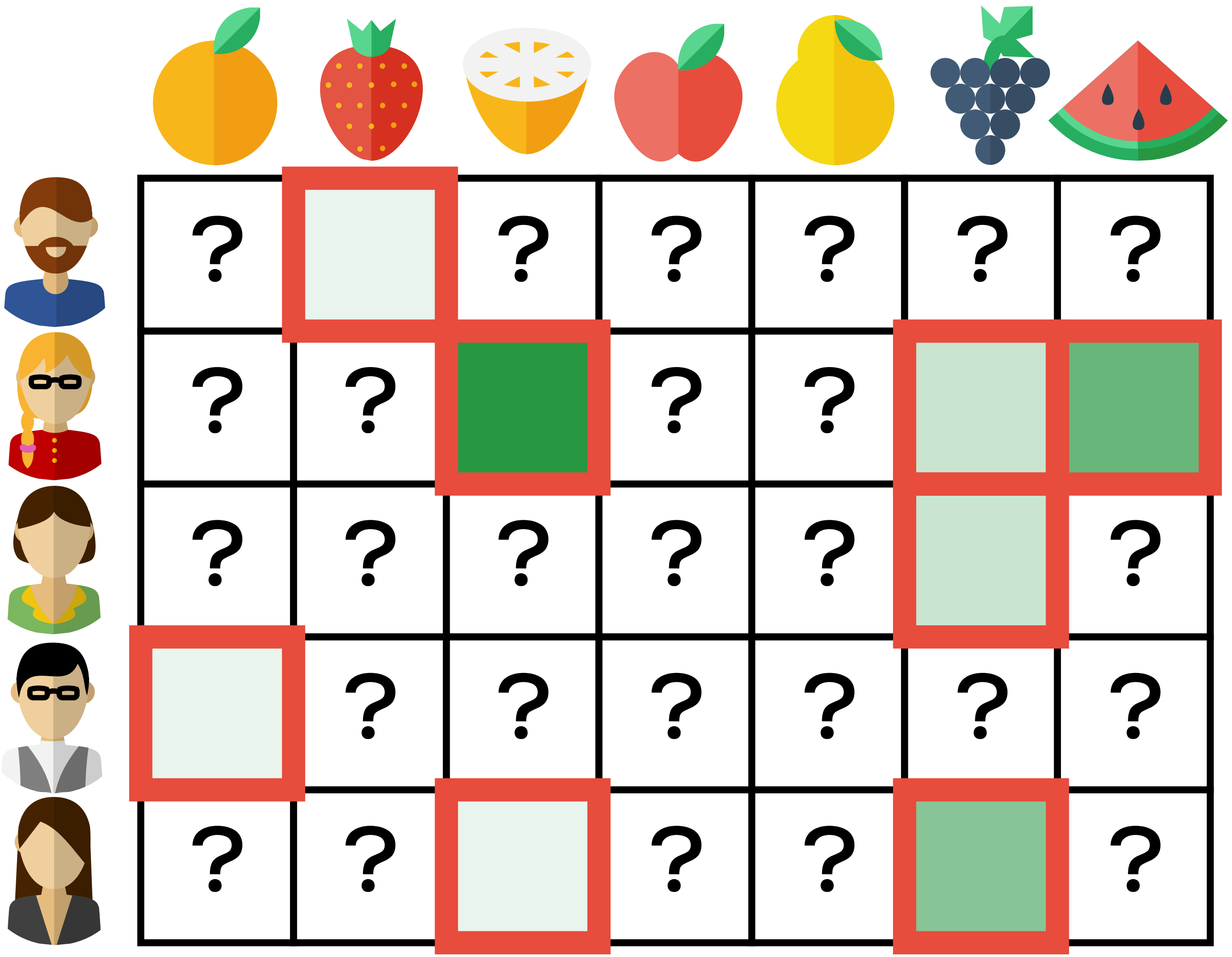

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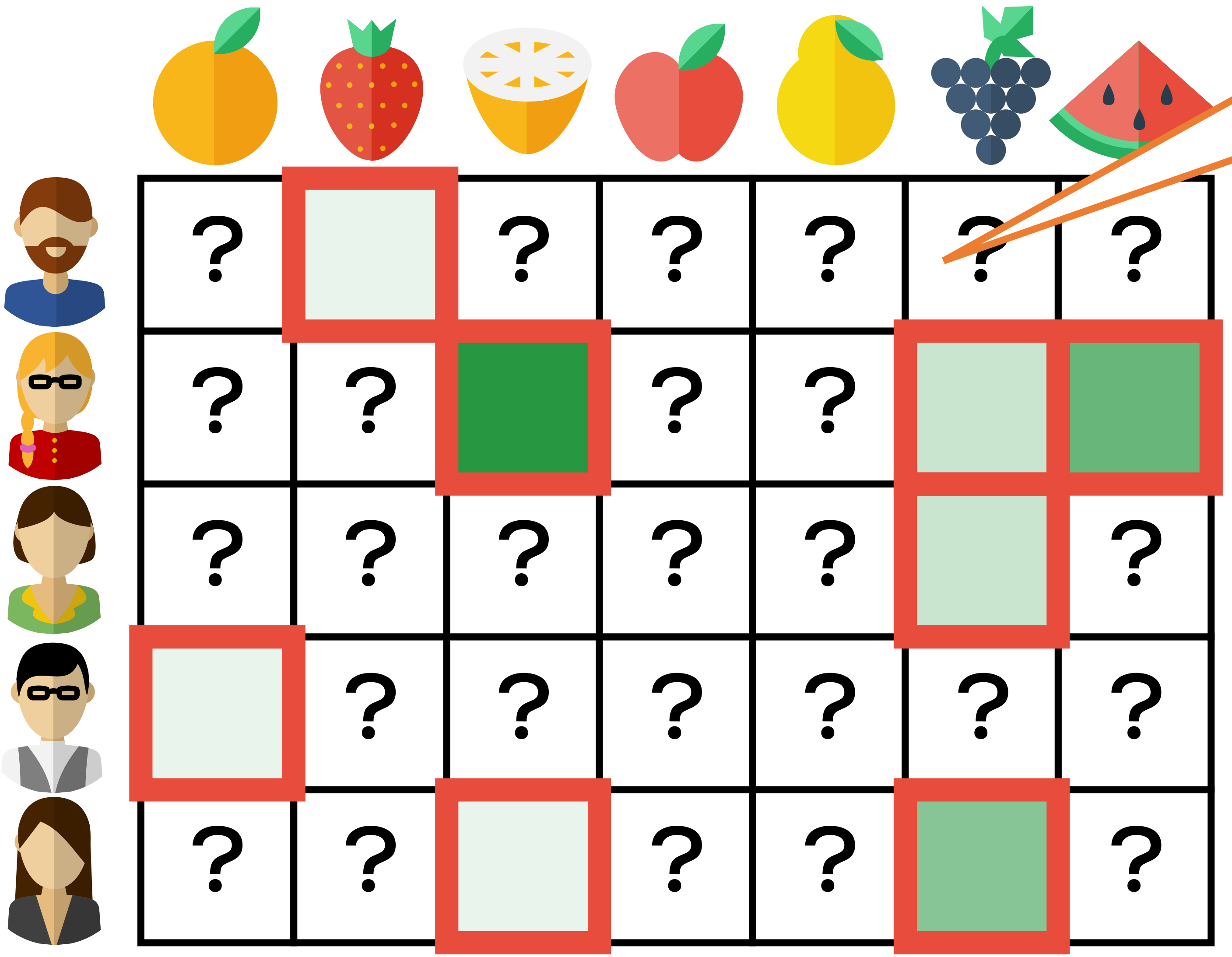

Knowing the entire matrix would reveal the most liked items for every user and allow me to recommend those!

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The Low-rank Matrix Completion Problem

- Unfortunately the matrix completion problem is ill-posed
- No reason why observed entries of the matrix should tell us anything about the unobserved entries
- Absolutely necessary to impose some structure on the matrix
- Popular assumption: the rating matrix X is low rank (say rank k)
- So need to find a low-rank matrix $\hat{X}$ such that X and $\hat{X}$ agree on the observed entries. Let $\Omega \subset [m] \times [n]$ be the locations observed

$$\arg \min_{\mathrm{rank}(\hat{X}) \leq k} \sum_{(i,j) \in \Omega} \left(X_{ij} - \hat{X}_{ij} \right)^2$$

- However, every rank k matrix $\hat{X}$ can be written as $\hat{X} = UV^\top$ where $U \in \mathbb{R}^{m \times k}$ and $V \in \mathbb{R}^{n \times k}$ (show this using singular value decomp!)

The Low-rank Matrix Completion Problem

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- No … entries … tell us any … served
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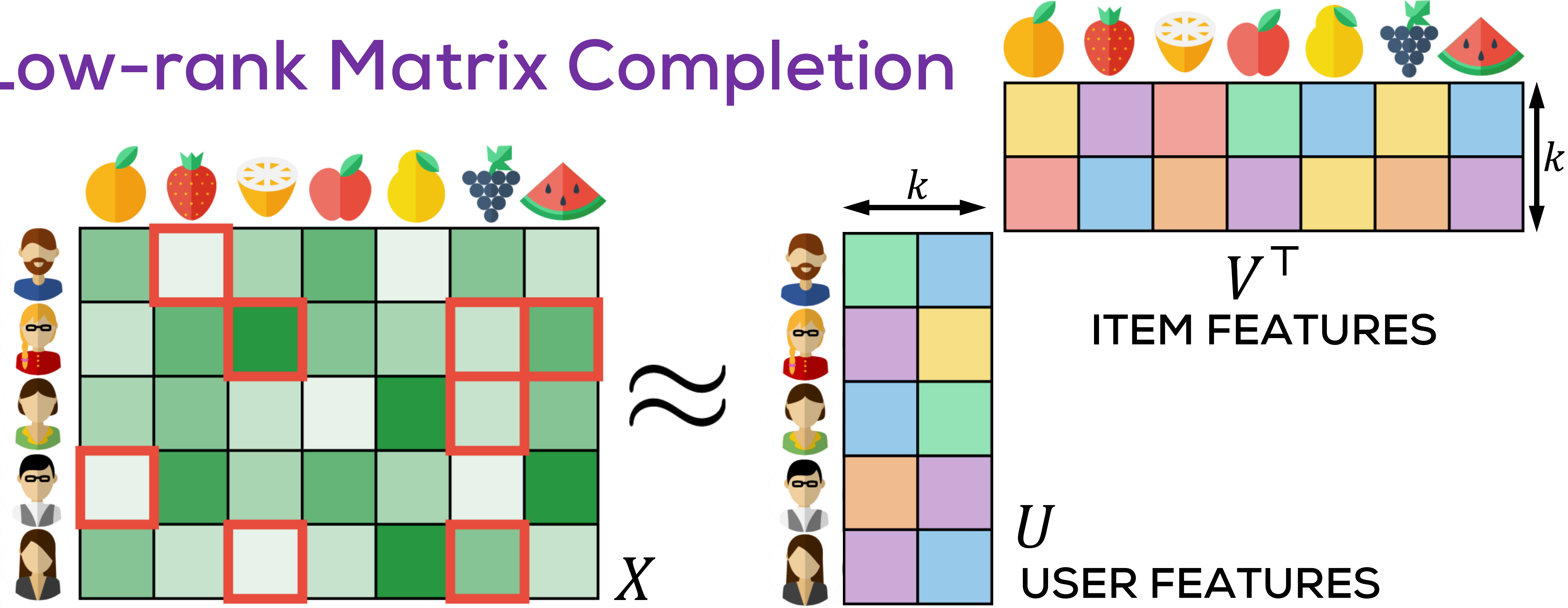
Can encode intuitions like “similar items rated similarly by the same user/similar users”

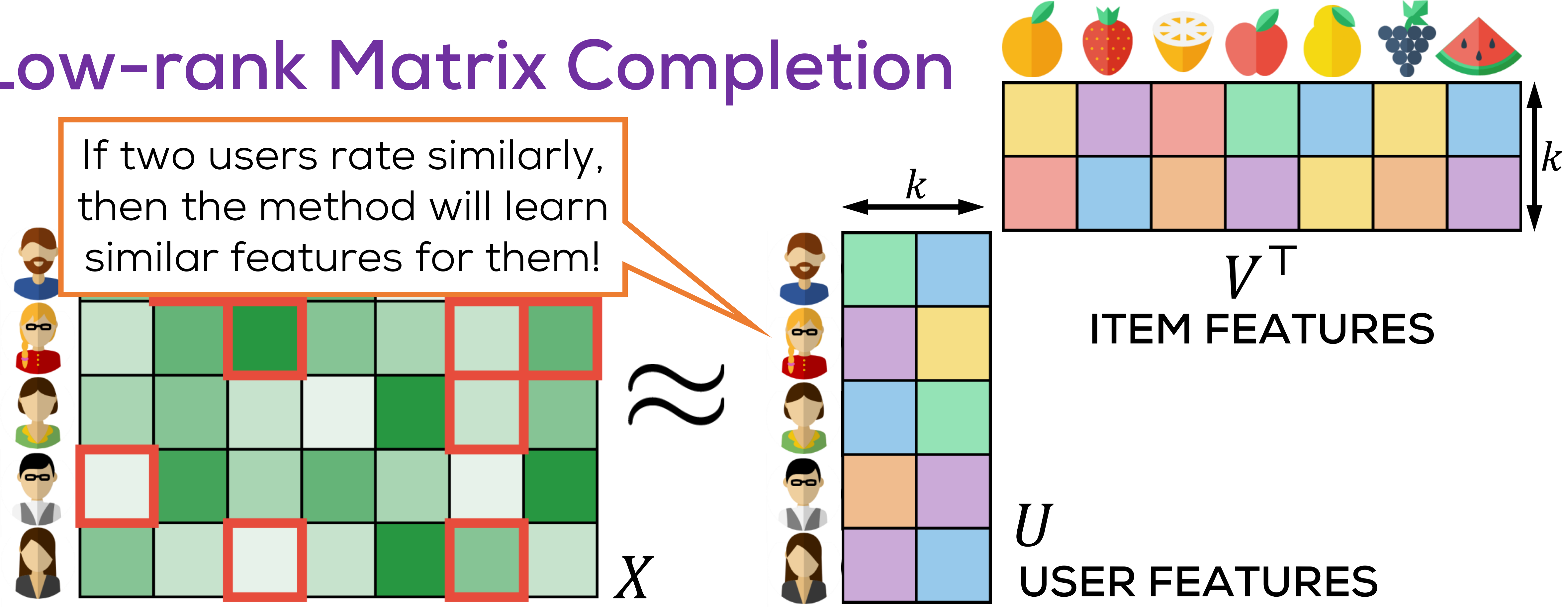
k tuned based on how many entries observed – more entries, larger k

Low-rank Matrix Completion



- So we have $X_{ij} \approx \langle \mathbf{u}^i, \mathbf{v}^j \rangle$ where $U = [\mathbf{u}^1, \dots, \mathbf{u}^m]^\top$ and $V = [\mathbf{v}^1, \dots, \mathbf{v}^n]^\top$
- Nice! As a side effect, vector representations of all users and items
- Such features can be very useful in recommendation (will soon see)
- Can also violate privacy if details users didn't tell are revealed
- E.g. if one of the k features turns out to be correlated with age

Low-rank Matrix Completion



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Solving the LRMC problem

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Solving the LRMC problem

Nice to regularize

$$\arg \min_{\substack{U \in \mathbb{R}^{m \times k} \\ V \in \mathbb{R}^{n \times k}}} \sum_{(i,j) \in \Omega} \left(X_{ij} - \langle \mathbf{u}^i, \mathbf{v}^j \rangle \right)^2 + \lambda \cdot \left(\|U\|_F^2 + \|V\|_F^2 \right)$$

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- Note that loss calculated only over observed entries i.e. $(i,j) \in \Omega$
- How to solve this ? Non convex problem! ☹
- Can use projected gradient descent on $\arg \min_{\mathrm{rank}(\hat{X}) \leq k} \sum_{(i,j) \in \Omega} \left(X_{ij} - \hat{X}_{ij} \right)^2$
- A faster way exists using alternating minimization
- AltMin is very versatile, gives us EM, Lloyd's algo, and now this
- Current state-of-the-art for matrix completion is AltMin

AltMin for LRMC

$$\arg\min_{\substack{U \in \mathbb{R}^{m\times k}\\ V \in \mathbb{R}^{n\times k}}} \sum_{(i,j)\in\Omega} \left(X_{ij} - \langle \mathbf{u}^i, \mathbf{v}^j \rangle\right)^2 + \lambda \cdot \left(\sum_{i=1}^{m} \left\| \mathbf{u}^i \right\|_2^2 + \sum_{j=1}^{n} \left\| \mathbf{v}^j \right\|_2^2 \right)$$

- For now, fix all $\mathbf{v}^j$ and all $\mathbf{u}^i$ except $\mathbf{u}^1$. The problem now becomes

$$\mathbf{u}^1 = \arg\min_{\mathbf{u}\in\mathbb{R}^k} \sum_{(1,j)\in\Omega} \left(X_{1j} - \langle \mathbf{u}, \mathbf{v}^j \rangle\right)^2 + \lambda \cdot \|\mathbf{u}\|_2^2$$

- Wait … this is just a ridge regression problem!
- What happens if I fix all $\mathbf{u}^i$ and all $\mathbf{v}^j$ except $\mathbf{v}^1$

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AltMin for LRMC

Notice that summation is only over items that user 1 has rated – usually very small

$$\ldots - \langle \mathbf{u}^i, \mathbf{v}^j \rangle\big)^2 + \lambda \cdot \Bigg(\ldots$$

$V \in \mathbb{R}^{n \times k}$

$\mathbf{v}^j$ act as the "feature" vectors and X_{1j} as "responses" in this ridge regression problem

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Notice that summation is only over users that rated item 1 – usually small for most items

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This means that these problems can be solved cheaply

- Nice … again just a ridge regression problem!

Alternating Optimization Revisited

ALTERNATING OPTIMIZATION

1. Observed locations Ω and entries $\{X_{ij}: (i,j) \in \Omega\}$
2. Initialize $\mathbf{v}^{1,0}, \mathbf{v}^{2,0}, \ldots, \mathbf{v}^{n,0}$
3. For $t = 1,2, \ldots$
   1. Update the user vectors for $i = 1, \ldots, m$

      $$\mathbf{u}^{i,t} \leftarrow \left(\sum_{(i,j)\in\Omega} \mathbf{v}^{j,t-1} \left(\mathbf{v}^{j,t-1}\right)^\top + \lambda \cdot I \right)^{-1} \left(\sum_{(i,j)\in\Omega} X_{ij} \cdot \mathbf{v}^{j,t-1} \right)$$

   2. Update the item vectors for $j = 1, \ldots, n$

      $$\mathbf{v}^{j,t} \leftarrow \left(\sum_{(i,j)\in\Omega} \mathbf{u}^{j,t} \left(\mathbf{u}^{j,t}\right)^\top + \lambda \cdot I \right)^{-1} \left(\sum_{(i,j)\in\Omega} X_{ij} \cdot \mathbf{u}^{j,t} \right)$$

4. Repeat until convergence

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   2. Update the item vectors for $j = 1, \ldots, n$

      $$\mathbf{v}^{j,t} \leftarrow \Bigg(\sum_{(i,j)\in\Omega} \mathbf{u}^{j,t}\left(\mathbf{u}^{j,t}\right)^{\top} + \lambda \cdot I\Bigg)^{-1}\Bigg(\sum_{(i,j)\in\Omega} X_{ij} \cdot \mathbf{u}^{j,t}\Bigg)$$

4. Repeat until convergence

Inexpensive for most items and users

$\mathcal{O}\left(k^3 + k^2(m+n)\right)$ time per iteration

Don't solve these RR problems optimally, just take an SGD step!

Can speed up even more

Alternating Optimization Revisited

SGD ALTERNATING OPTIMIZATION

1. Observed locations Ω and entries $\{X_{ij} : (i,j) \in \Omega\}$
2. Initialize $\mathbf{u}^1, \mathbf{u}^2, \dots, \mathbf{u}^m$ and $\mathbf{v}^1, \mathbf{v}^2, \dots, \mathbf{v}^n$
3. For $t = 1,2, \dots$
   1. Choose a random observed entry $(i^t, j^t) \in \Omega$
   2. Update the user i^t, and item j^t

      $$\mathbf{u}^{i^t} \leftarrow \mathbf{u}^{i^t} - 2\eta_t \cdot \left(\lambda \cdot \mathbf{u}^{i^t} - \left(X_{i^t j^t} - \left\langle \mathbf{u}^{i^t}, \mathbf{v}^{j^t} \right\rangle\right) \cdot \mathbf{v}^{j^t}\right)$$

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   3. All other user and item vectors stay the same
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Alternating Optimization Revisited

Using simpler notation to avoid clutter

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4. Repeat until convergence

$\mathcal{O}(k)$ time per iteration

Pros and Cons

Pros

- Relies purely on behavioural data
- Does not require users to submit personal information
- Does not require sellers to submit item information
- Collaborative method: lazy users benefit from active users

Cons

- Relies purely on behavioural data
- Cannot utilize user and item information even if present
- Expensive to recompute matrix factorization again and again
- Adding new users and items not simple

Please give your Feedback

http://tinyurl.com/ml17-18afb