CS685: Data Mining
Basics of Classification

Arnab Bhattacharya
arnabb@cse.iitk.ac.in

Computer Science and Engineering,
Indian Institute of Technology, Kanpur
http://web.cse.iitk.ac.in/~cs685/

1st semester, 2018-19

Mon, Thu 1030-1145 at RM101

Classification

- A dataset of n objects $O_i, i = 1, \ldots, n$
- A total of k classes $C_j, j = 1, \ldots, k$
- Each object belongs to a *single* class
- If object O_i belongs to class C_j, then $C(O_i) = j$
- Given a new object O_q, classification is the problem of determining its class, i.e., $C(O_q)$ out of possible k choices

Classification

- A dataset of n objects $O_i, i = 1, \ldots, n$
- A total of k classes $C_j, j = 1, \ldots, k$
- Each object belongs to a *single* class
- If object O_i belongs to class C_j, then $C(O_i) = j$
- Given a new object O_q, classification is the problem of determining its class, i.e., $C(O_q)$ out of possible k choices
- If, instead of k discrete classes, there is a continuum of values, the problem of determining the value $V(O_q)$ of a new object O_q is called prediction

Sets

- Total available data is divided *randomly* into two parts: training set and testing set
- Classification algorithm or model is built using *only* the training set
- Testing set should not be used *at all*
- Quality of method is measured using testing set
- Sometimes validation set is separated from training set to evalute method

Processes

- Stratified
  - If representation of each class in training set is proportional to the overall ratios

Processes

- Stratified
  - If representation of each class in training set is proportional to the overall ratios
- k-fold cross-validation
  - Training data is divided into k random parts
  - $k - 1$ groups are used as training set and the k^{th} group as validation set
  - Training is repeated k times with a new validation set each time
- Leave-one-out cross-validation (LOOCV): When $k = n$

Processes

- Stratified
  - If representation of each class in training set is proportional to the overall ratios
- k-fold cross-validation
  - Training data is divided into k random parts
  - $k - 1$ groups are used as training set and the k^{th} group as validation set
  - Training is repeated k times with a new validation set each time
- Leave-one-out cross-validation (LOOCV): When $k = n$
- Stratified cross-validation
  - When representation in each of the k random groups is proportional to the overall ratios

Processes

- Stratified
  - If representation of each class in training set is proportional to the overall ratios
- k-fold cross-validation
  - Training data is divided into k random parts
  - $k-1$ groups are used as training set and the k^{th} group as validation set
  - Training is repeated k times with a new validation set each time
- Leave-one-out cross-validation (LOOCV): When $k = n$
- Stratified cross-validation
  - When representation in each of the k random groups is proportional to the overall ratios
- Classification is called supervised learning
  - Algorithm or model is "supervised" by class information

Over-Fitting and Under-Fitting

- Over-fitting
  - Algorithm or model classifies the training set too well
  - It is too complex or uses too many parameters
  - Generally performs poorly with testing set
  - Ends up modeling noise rather than data characteristics

Over-Fitting and Under-Fitting

- Over-fitting
  - Algorithm or model classifies the training set too well
  - It is too complex or uses too many parameters
  - Generally performs poorly with testing set
  - Ends up modeling noise rather than data characteristics
- Under-fitting
  - The opposite problem
  - Algorithm or model does not classify the training set well at all
  - It is too simple or uses too less parameters
  - Generally performs poorly with testing set
  - Ends up modeling overall data characteristics instead of per class

Over-Fitting and Under-Fitting

- Over-fitting
  - Algorithm or model classifies the training set too well
  - It is too complex or uses too many parameters
  - Generally performs poorly with testing set
  - Ends up modeling noise rather than data characteristics
- Under-fitting
  - The opposite problem
  - Algorithm or model does not classify the training set well at all
  - It is too simple or uses too less parameters
  - Generally performs poorly with testing set
  - Ends up modeling overall data characteristics instead of per class
- Bias-variance tradeoff
  - Bias measures errors in the model learnt (under-fitting)
  - Variance measures errors when training set is perturbed (over-fitting)
  - Low bias generally implies higher variance and vice versa

Errors

- Positives (P): Objects that are "true" answers (for a class)
- Negatives (N): Objects that are not answers: $N = D - P$

Errors

- Positives (P): Objects that are “true” answers (for a class)
- Negatives (N): Objects that are not answers: $N = D - P$
- For a particular classification algorithm $\mathcal{A}$,
  - P': Objects returned as positive by $\mathcal{A}$
  - N': Objects not returned as positive by $\mathcal{A}$

Errors

- Positives (P): Objects that are "true" answers (for a class)
- Negatives (N): Objects that are not answers: $N = D - P$
- For a particular classification algorithm $\mathcal{A}$,
  - P': Objects returned as positive by $\mathcal{A}$
  - N': Objects not returned as positive by $\mathcal{A}$
- True Positives (TP):

Errors

- Positives (P): Objects that are "true" answers (for a class)
- Negatives (N): Objects that are not answers: $N = D - P$
- For a particular classification algorithm $\mathcal{A}$,
  - P': Objects returned as positive by $\mathcal{A}$
  - N': Objects not returned as positive by $\mathcal{A}$
- True Positives (TP): Answers returned by $\mathcal{A}$: $P \cap P'$

Errors

- Positives (P): Objects that are "true" answers (for a class)
- Negatives (N): Objects that are not answers: $N = D - P$
- For a particular classification algorithm $\mathcal{A}$,
  - P': Objects returned as positive by $\mathcal{A}$
  - N': Objects not returned as positive by $\mathcal{A}$
- True Positives (TP): Answers returned by $\mathcal{A}$: $P \cap P'$
- True Negatives (TN):

Errors

- Positives (P): Objects that are "true" answers (for a class)
- Negatives (N): Objects that are not answers: $N = D - P$
- For a particular classification algorithm $\mathcal{A}$,
  - P': Objects returned as positive by $\mathcal{A}$
  - N': Objects not returned as positive by $\mathcal{A}$
- True Positives (TP): Answers returned by $\mathcal{A}$: $P \cap P'$
- True Negatives (TN): Non-answers not returned by $\mathcal{A}$: $N \cap N'$

Errors

- Positives (P): Objects that are "true" answers (for a class)
- Negatives (N): Objects that are not answers: $N = D - P$
- For a particular classification algorithm $\mathcal{A}$,
  - P': Objects returned as positive by $\mathcal{A}$
  - N': Objects not returned as positive by $\mathcal{A}$
- True Positives (TP): Answers returned by $\mathcal{A}$: $P \cap P'$
- True Negatives (TN): Non-answers not returned by $\mathcal{A}$: $N \cap N'$
- False Positives (FP):

Errors

- Positives (P): Objects that are "true" answers (for a class)
- Negatives (N): Objects that are not answers: $N = D - P$
- For a particular classification algorithm $\mathcal{A}$,
  - P': Objects returned as positive by $\mathcal{A}$
  - N': Objects not returned as positive by $\mathcal{A}$
- True Positives (TP): Answers returned by $\mathcal{A}$: $P \cap P'$
- True Negatives (TN): Non-answers not returned by $\mathcal{A}$: $N \cap N'$
- False Positives (FP): Non-answers returned by $\mathcal{A}$: $N \cap P'$

Errors

- Positives (P): Objects that are "true" answers (for a class)
- Negatives (N): Objects that are not answers: $N = D - P$
- For a particular classification algorithm $\mathcal{A}$,
  - P': Objects returned as positive by $\mathcal{A}$
  - N': Objects not returned as positive by $\mathcal{A}$
- True Positives (TP): Answers returned by $\mathcal{A}$: $P \cap P'$
- True Negatives (TN): Non-answers not returned by $\mathcal{A}$: $N \cap N'$
- False Positives (FP): Non-answers returned by $\mathcal{A}$: $N \cap P'$
- False Negatives (FN):

Errors

- Positives (P): Objects that are "true" answers (for a class)
- Negatives (N): Objects that are not answers: $N = D - P$
- For a particular classification algorithm $\mathcal{A}$,
  - P': Objects returned as positive by $\mathcal{A}$
  - N': Objects not returned as positive by $\mathcal{A}$
- True Positives (TP): Answers returned by $\mathcal{A}$: $P \cap P'$
- True Negatives (TN): Non-answers not returned by $\mathcal{A}$: $N \cap N'$
- False Positives (FP): Non-answers returned by $\mathcal{A}$: $N \cap P'$
- False Negatives (FN): Answers not returned by $\mathcal{A}$: $P \cap N'$

Errors

- Positives (P): Objects that are "true" answers (for a class)
- Negatives (N): Objects that are not answers: $N = D - P$
- For a particular classification algorithm $\mathcal{A}$,
  - P': Objects returned as positive by $\mathcal{A}$
  - N': Objects not returned as positive by $\mathcal{A}$
- True Positives (TP): Answers returned by $\mathcal{A}$: $P \cap P'$
- True Negatives (TN): Non-answers not returned by $\mathcal{A}$: $N \cap N'$
- False Positives (FP): Non-answers returned by $\mathcal{A}$: $N \cap P'$
- False Negatives (FN): Answers not returned by $\mathcal{A}$: $P \cap N'$

$$P = TP \cup FN \quad N = TN \cup FP \quad P' = TP \cup FP \quad N' = TN \cup FN$$

Errors

- Positives (P): Objects that are "true" answers (for a class)
- Negatives (N): Objects that are not answers: $N = D - P$
- For a particular classification algorithm $\mathcal{A}$,
  - P': Objects returned as positive by $\mathcal{A}$
  - N': Objects not returned as positive by $\mathcal{A}$
- True Positives (TP): Answers returned by $\mathcal{A}$: $P \cap P'$
- True Negatives (TN): Non-answers not returned by $\mathcal{A}$: $N \cap N'$
- False Positives (FP): Non-answers returned by $\mathcal{A}$: $N \cap P'$
- False Negatives (FN): Answers not returned by $\mathcal{A}$: $P \cap N'$

$$P = TP \cup FN \quad N = TN \cup FP \quad P' = TP \cup FP \quad N' = TN \cup FN$$

- In statistics,
  - Type I error: FP
  - Type II error: FN

Confusion Matrix

- Confusion matrix visually represents the information
- Rows indicate "true" answers: P and N
- Columns indicate those returned by $\mathcal{A}$: P' and N'
- Shows which error is more

Sets		Returned by $\mathcal{A}$	
		Positives P'	Negatives N'
True answers	Positives P	TP	FN
	Negatives N	FP	TN

Confusion Matrix for Multiple Classes

- Is more useful when extended for multiple classes
- Shows which classes are confused more against which other classes

Sets		Predicted by $\mathcal{A}$		
		Class C_1'	Class C_2'	Class C_3'
True answers	Class C_1	5	3	0
	Class C_2	2	3	1
	Class C_3	0	2	9

Error Parameters or Performance Metrics

Parameter	*Interpretation*	*Formula*
Precision	Proportion of positives in those returned by $\mathcal{A}$	

Error Parameters or Performance Metrics

Parameter	*Interpretation*	*Formula*
Precision	Proportion of positives in those returned by $\mathcal{A}$	$\frac{\lvert TP\rvert}{\lvert TP \cup FP\rvert} = \frac{\lvert TP\rvert}{\lvert P'\rvert}$
Recall or Sensitivity or True positive rate or Hit rate	Proportion of positives returned by $\mathcal{A}$	

Error Parameters or Performance Metrics

Parameter	*Interpretation*	*Formula*
Precision	Proportion of positives in those returned by $\mathcal{A}$	$\frac{\lvert TP\rvert}{\lvert TP \cup FP\rvert} = \frac{\lvert TP\rvert}{\lvert P'\rvert}$
Recall or Sensitivity or True positive rate or Hit rate	Proportion of positives returned by $\mathcal{A}$	$\frac{\lvert TP\rvert}{\lvert TP \cup FN\rvert} = \frac{\lvert TP\rvert}{\lvert P\rvert}$
Specificity or True negative rate	Proportion of negatives not returned by $\mathcal{A}$	

Error Parameters or Performance Metrics

Parameter	*Interpretation*	*Formula*
Precision	Proportion of positives in those returned by $\mathcal{A}$	$\frac{\lvert TP \rvert}{\lvert TP \cup FP \rvert} = \frac{\lvert TP \rvert}{\lvert P' \rvert}$
Recall or Sensitivity or True positive rate or Hit rate	Proportion of positives returned by $\mathcal{A}$	$\frac{\lvert TP \rvert}{\lvert TP \cup FN \rvert} = \frac{\lvert TP \rvert}{\lvert P \rvert}$
Specificity or True negative rate	Proportion of negatives not returned by $\mathcal{A}$	$\frac{\lvert TN \rvert}{\lvert TN \cup FP \rvert} = \frac{\lvert TN \rvert}{\lvert N \rvert}$
False positive rate	Proportion of negatives returned by $\mathcal{A}$	

Error Parameters or Performance Metrics

Parameter	*Interpretation*	*Formula*
Precision	Proportion of positives in those returned by $\mathcal{A}$	$\frac{\lvert TP \rvert}{\lvert TP \cup FP \rvert} = \frac{\lvert TP \rvert}{\lvert P' \rvert}$
Recall or Sensitivity or True positive rate or Hit rate	Proportion of positives returned by $\mathcal{A}$	$\frac{\lvert TP \rvert}{\lvert TP \cup FN \rvert} = \frac{\lvert TP \rvert}{\lvert P \rvert}$
Specificity or True negative rate	Proportion of negatives not returned by $\mathcal{A}$	$\frac{\lvert TN \rvert}{\lvert TN \cup FP \rvert} = \frac{\lvert TN \rvert}{\lvert N \rvert}$
False positive rate	Proportion of negatives returned by $\mathcal{A}$	$\frac{\lvert FP \rvert}{\lvert TN \cup FP \rvert} = \frac{\lvert FP \rvert}{\lvert N \rvert}$
False negative rate	Proportion of positives not returned by $\mathcal{A}$	

Error Parameters or Performance Metrics

Parameter	*Interpretation*	*Formula*
Precision	Proportion of positives in those returned by $\mathcal{A}$	$\frac{\lvert TP \rvert}{\lvert TP \cup FP \rvert} = \frac{\lvert TP \rvert}{\lvert P' \rvert}$
Recall or Sensitivity or True positive rate or Hit rate	Proportion of positives returned by $\mathcal{A}$	$\frac{\lvert TP \rvert}{\lvert TP \cup FN \rvert} = \frac{\lvert TP \rvert}{\lvert P \rvert}$
Specificity or True negative rate	Proportion of negatives not returned by $\mathcal{A}$	$\frac{\lvert TN \rvert}{\lvert TN \cup FP \rvert} = \frac{\lvert TN \rvert}{\lvert N \rvert}$
False positive rate	Proportion of negatives returned by $\mathcal{A}$	$\frac{\lvert FP \rvert}{\lvert TN \cup FP \rvert} = \frac{\lvert FP \rvert}{\lvert N \rvert}$
False negative rate	Proportion of positives not returned by $\mathcal{A}$	$\frac{\lvert FN \rvert}{\lvert TP \cup FN \rvert} = \frac{\lvert FN \rvert}{\lvert P \rvert}$
Accuracy	Proportion of positives returned and negatives not returned by $\mathcal{A}$	

Error Parameters or Performance Metrics

Parameter	*Interpretation*	*Formula*
Precision	Proportion of positives in those returned by $\mathcal{A}$	$\frac{\lvert TP \rvert}{\lvert TP \cup FP \rvert} = \frac{\lvert TP \rvert}{\lvert P' \rvert}$
Recall or Sensitivity or True positive rate or Hit rate	Proportion of positives returned by $\mathcal{A}$	$\frac{\lvert TP \rvert}{\lvert TP \cup FN \rvert} = \frac{\lvert TP \rvert}{\lvert P \rvert}$
Specificity or True negative rate	Proportion of negatives not returned by $\mathcal{A}$	$\frac{\lvert TN \rvert}{\lvert TN \cup FP \rvert} = \frac{\lvert TN \rvert}{\lvert N \rvert}$
False positive rate	Proportion of negatives returned by $\mathcal{A}$	$\frac{\lvert FP \rvert}{\lvert TN \cup FP \rvert} = \frac{\lvert FP \rvert}{\lvert N \rvert}$
False negative rate	Proportion of positives not returned by $\mathcal{A}$	$\frac{\lvert FN \rvert}{\lvert TP \cup FN \rvert} = \frac{\lvert FN \rvert}{\lvert P \rvert}$
Accuracy	Proportion of positives returned and negatives not returned by $\mathcal{A}$	$\frac{\lvert TP \cup TN \rvert}{\lvert D \rvert}$
Error rate	Proportion of positives not returned and negatives returned by $\mathcal{A}$	

Error Parameters or Performance Metrics

Parameter	*Interpretation*	*Formula*
Precision	Proportion of positives in those returned by $\mathcal{A}$	$\frac{\lvert TP \rvert}{\lvert TP \cup FP \rvert} = \frac{\lvert TP \rvert}{\lvert P' \rvert}$
Recall or Sensitivity or True positive rate or Hit rate	Proportion of positives returned by $\mathcal{A}$	$\frac{\lvert TP \rvert}{\lvert TP \cup FN \rvert} = \frac{\lvert TP \rvert}{\lvert P \rvert}$
Specificity or True negative rate	Proportion of negatives not returned by $\mathcal{A}$	$\frac{\lvert TN \rvert}{\lvert TN \cup FP \rvert} = \frac{\lvert TN \rvert}{\lvert N \rvert}$
False positive rate	Proportion of negatives returned by $\mathcal{A}$	$\frac{\lvert FP \rvert}{\lvert TN \cup FP \rvert} = \frac{\lvert FP \rvert}{\lvert N \rvert}$
False negative rate	Proportion of positives not returned by $\mathcal{A}$	$\frac{\lvert FN \rvert}{\lvert TP \cup FN \rvert} = \frac{\lvert FN \rvert}{\lvert P \rvert}$
Accuracy	Proportion of positives returned and negatives not returned by $\mathcal{A}$	$\frac{\lvert TP \cup TN \rvert}{\lvert D \rvert}$
Error rate	Proportion of positives not returned and negatives returned by $\mathcal{A}$	$\frac{\lvert FP \cup FN \rvert}{\lvert D \rvert}$

Single Measures

- Single measures capturing both precision and recall

Single Measures

- Single measures capturing both precision and recall
- F-score or F-measure or F1-score is the *harmonic mean* of precision and recall

$$Fscore = \frac{2.Precision.Recall}{Precision + Recall}$$

Single Measures

- Single measures capturing both precision and recall
- F-score or F-measure or F1-score is the *harmonic mean* of precision and recall

$$Fscore = \frac{2.Precision.Recall}{Precision + Recall}$$

- In terms of errors

$$Fscore = \frac{2.TP}{2.TP + FN + FP}$$

Single Measures

- Single measures capturing both precision and recall
- F-score or F-measure or F1-score is the *harmonic mean* of precision and recall

$$Fscore = \frac{2.Precision.Recall}{Precision + Recall}$$

- In terms of errors

$$Fscore = \frac{2.TP}{2.TP + FN + FP}$$

- Similarly, G-measure is *geometric mean* of precision and recall

Single Measures

- Single measures capturing both precision and recall
- F-score or F-measure or F1-score is the *harmonic mean* of precision and recall

$$Fscore = \frac{2.Precision.Recall}{Precision + Recall}$$

- In terms of errors

$$Fscore = \frac{2.TP}{2.TP + FN + FP}$$

- Similarly, G-measure is *geometric mean* of precision and recall
- EER or Equal Error Rate is when FP rate is equal to FN rate

Weighting Precision versus Recall

- Suppose recall and precision are weighted at a ratio $\alpha : (1 - \alpha)$
- F-score is the *weighted harmonic mean*

$$\frac{1}{F} = \alpha.\frac{1}{P} + (1 - \alpha).\frac{1}{R}$$

Weighting Precision versus Recall

- Suppose recall and precision are weighted at a ratio $\alpha : (1 - \alpha)$
- F-score is the *weighted harmonic mean*

$$\frac{1}{F} = \alpha . \frac{1}{P} + (1 - \alpha) . \frac{1}{R}$$

- $\beta^2 = \frac{1-\alpha}{\alpha}$ measures the relative importance of precision over recall
  - $\alpha \in [0, 1]$ while $\beta \in [0, \infty]$
  - $\beta > 1$ emphasizes precision, while $\beta < 1$ emphasizes recall

Weighting Precision versus Recall

- Suppose recall and precision are weighted at a ratio $\alpha : (1 - \alpha)$
- F-score is the *weighted harmonic mean*

$$\frac{1}{F} = \alpha . \frac{1}{P} + (1 - \alpha) . \frac{1}{R}$$

- $\beta^2 = \frac{1-\alpha}{\alpha}$ measures the relative importance of precision over recall
  - $\alpha \in [0, 1]$ while $\beta \in [0, \infty]$
  - $\beta > 1$ emphasizes precision, while $\beta < 1$ emphasizes recall
- Using β^2, weighted F-score is

$$F = \frac{(\beta^2 + 1).P.R}{\beta^2.P + R}$$

- When $\beta = 1$, precision and recall are equally weighted ($\alpha = 1/2$)
- F1-score is the harmonic mean

Example

$$
\begin{aligned}
D =& \{O_1, O_2, O_3, O_4, O_5, O_6, O_7, O_8\} \\
\text{Correct answer set} =& \{O_1, O_5, O_7\} \\
\text{Algorithm returns} =& \{O_1, O_3, O_5, O_6\}
\end{aligned}
$$

Example

$$
\begin{aligned}
D =& \{O_1, O_2, O_3, O_4, O_5, O_6, O_7, O_8\} \\
\text{Correct answer set} =& \{O_1, O_5, O_7\} \\
\text{Algorithm returns} =& \{O_1, O_3, O_5, O_6\} \\
\therefore \qquad P =& \\
N =& \\
TP =& \\
TN =& \\
FP =& \\
FN =&
\end{aligned}
$$

Example

$$
\begin{aligned}
D =& \{O_1, O_2, O_3, O_4, O_5, O_6, O_7, O_8\} \\
\text{Correct answer set} =& \{O_1, O_5, O_7\} \\
\text{Algorithm returns} =& \{O_1, O_3, O_5, O_6\} \\
\therefore \qquad P =& \{O_1, O_5, O_7\} \\
N =& \\
TP =& \\
TN =& \\
FP =& \\
FN =&
\end{aligned}
$$

Example

$$
\begin{aligned}
D =& \{O_1, O_2, O_3, O_4, O_5, O_6, O_7, O_8\} \\
\text{Correct answer set} =& \{O_1, O_5, O_7\} \\
\text{Algorithm returns} =& \{O_1, O_3, O_5, O_6\} \\
\therefore \qquad P =& \{O_1, O_5, O_7\} \\
N =& \{O_2, O_3, O_4, O_6, O_8\} \\
TP =& \\
TN =& \\
FP =& \\
FN =&
\end{aligned}
$$

Example

$$
\begin{aligned}
D =& \{O_1, O_2, O_3, O_4, O_5, O_6, O_7, O_8\} \\
\text{Correct answer set} =& \{O_1, O_5, O_7\} \\
\text{Algorithm returns} =& \{O_1, O_3, O_5, O_6\} \\
\therefore \qquad P &= \{O_1, O_5, O_7\} \\
N &= \{O_2, O_3, O_4, O_6, O_8\} \\
TP &= \{O_1, O_5\} \\
TN &= \\
FP &= \\
FN &=
\end{aligned}
$$

Example

$$
\begin{aligned}
D =& \{O_1, O_2, O_3, O_4, O_5, O_6, O_7, O_8\} \\
\text{Correct answer set} =& \{O_1, O_5, O_7\} \\
\text{Algorithm returns} =& \{O_1, O_3, O_5, O_6\} \\
\therefore \qquad P =& \{O_1, O_5, O_7\} \\
N =& \{O_2, O_3, O_4, O_6, O_8\} \\
TP =& \{O_1, O_5\} \\
TN =& \{O_2, O_4, O_8\} \\
FP =& \\
FN =&
\end{aligned}
$$

Example

$$
\begin{aligned}
D =& \{O_1, O_2, O_3, O_4, O_5, O_6, O_7, O_8\} \\
\text{Correct answer set} =& \{O_1, O_5, O_7\} \\
\text{Algorithm returns} =& \{O_1, O_3, O_5, O_6\} \\
\therefore \qquad P &= \{O_1, O_5, O_7\} \\
N &= \{O_2, O_3, O_4, O_6, O_8\} \\
TP &= \{O_1, O_5\} \\
TN &= \{O_2, O_4, O_8\} \\
FP &= \{O_3, O_6\} \\
FN &=
\end{aligned}
$$

Example

$$\begin{aligned} D =& \{O_1, O_2, O_3, O_4, O_5, O_6, O_7, O_8\} \\ \text{Correct answer set} =& \{O_1, O_5, O_7\} \\ \text{Algorithm returns} =& \{O_1, O_3, O_5, O_6\} \\ \therefore \qquad P &= \{O_1, O_5, O_7\} \\ N &= \{O_2, O_3, O_4, O_6, O_8\} \\ TP &= \{O_1, O_5\} \\ TN &= \{O_2, O_4, O_8\} \\ FP &= \{O_3, O_6\} \\ FN &= \{O_7\} \end{aligned}$$

Example

$$
\begin{aligned}
D =& \{O_1, O_2, O_3, O_4, O_5, O_6, O_7, O_8\} \\
\text{Correct answer set} =& \{O_1, O_5, O_7\} \\
\text{Algorithm returns} =& \{O_1, O_3, O_5, O_6\} \\
\therefore \qquad P &= \{O_1, O_5, O_7\} \\
N &= \{O_2, O_3, O_4, O_6, O_8\} \\
TP &= \{O_1, O_5\} \\
TN &= \{O_2, O_4, O_8\} \\
FP &= \{O_3, O_6\} \\
FN &= \{O_7\} \\
\therefore \qquad \text{Recall} = \text{Sensitivity} &= \\
\text{Precision} &= \\
\text{Specificity} &= \\
\text{F-score} &= \\
\text{Accuracy} &= \\
\text{Error rate} &=
\end{aligned}
$$

Example

$$
\begin{aligned}
D =& \{O_1, O_2, O_3, O_4, O_5, O_6, O_7, O_8\} \\
\text{Correct answer set} =& \{O_1, O_5, O_7\} \\
\text{Algorithm returns} =& \{O_1, O_3, O_5, O_6\} \\
\therefore \qquad P =& \{O_1, O_5, O_7\} \\
N =& \{O_2, O_3, O_4, O_6, O_8\} \\
TP =& \{O_1, O_5\} \\
TN =& \{O_2, O_4, O_8\} \\
FP =& \{O_3, O_6\} \\
FN =& \{O_7\}
\end{aligned}
$$

$$
\begin{aligned}
\therefore \qquad \text{Recall} = \text{Sensitivity} =& \ 2/3 = 0.67 \\
\text{Precision} =& \\
\text{Specificity} =& \\
\text{F-score} =& \\
\text{Accuracy} =& \\
\text{Error rate} =&
\end{aligned}
$$

Example

$$
\begin{aligned}
D =& \{O_1, O_2, O_3, O_4, O_5, O_6, O_7, O_8\} \\
\text{Correct answer set} =& \{O_1, O_5, O_7\} \\
\text{Algorithm returns} =& \{O_1, O_3, O_5, O_6\} \\
\therefore \qquad P =& \{O_1, O_5, O_7\} \\
N =& \{O_2, O_3, O_4, O_6, O_8\} \\
TP =& \{O_1, O_5\} \\
TN =& \{O_2, O_4, O_8\} \\
FP =& \{O_3, O_6\} \\
FN =& \{O_7\}
\end{aligned}
$$

$$
\begin{aligned}
\therefore \qquad \text{Recall} = \text{Sensitivity} &= 2/3 = 0.67 \\
\text{Precision} &= 2/4 = 0.5 \\
\text{Specificity} &= \\
\text{F-score} &= \\
\text{Accuracy} &= \\
\text{Error rate} &=
\end{aligned}
$$

Example

$$
\begin{aligned}
D =& \{O_1, O_2, O_3, O_4, O_5, O_6, O_7, O_8\} \\
\text{Correct answer set} =& \{O_1, O_5, O_7\} \\
\text{Algorithm returns} =& \{O_1, O_3, O_5, O_6\} \\
\therefore \qquad P =& \{O_1, O_5, O_7\} \\
N =& \{O_2, O_3, O_4, O_6, O_8\} \\
TP =& \{O_1, O_5\} \\
TN =& \{O_2, O_4, O_8\} \\
FP =& \{O_3, O_6\} \\
FN =& \{O_7\}
\end{aligned}
$$

$$
\begin{aligned}
\therefore \qquad \text{Recall} = \text{Sensitivity} &= 2/3 = 0.67 \\
\text{Precision} &= 2/4 = 0.5 \\
\text{Specificity} &= 3/5 = 0.6 \\
\text{F-score} &= \\
\text{Accuracy} &= \\
\text{Error rate} &=
\end{aligned}
$$

Example

$$
\begin{aligned}
D =& \{O_1, O_2, O_3, O_4, O_5, O_6, O_7, O_8\} \\
\text{Correct answer set} =& \{O_1, O_5, O_7\} \\
\text{Algorithm returns} =& \{O_1, O_3, O_5, O_6\} \\
\therefore \qquad P =& \{O_1, O_5, O_7\} \\
N =& \{O_2, O_3, O_4, O_6, O_8\} \\
TP =& \{O_1, O_5\} \\
TN =& \{O_2, O_4, O_8\} \\
FP =& \{O_3, O_6\} \\
FN =& \{O_7\}
\end{aligned}
$$

$$
\begin{aligned}
\therefore \qquad \text{Recall} = \text{Sensitivity} &= 2/3 = 0.67 \\
\text{Precision} &= 2/4 = 0.5 \\
\text{Specificity} &= 3/5 = 0.6 \\
\text{F-score} &= 4/7 = 0.571 \\
\text{Accuracy} &= \\
\text{Error rate} &=
\end{aligned}
$$

Example

$$
\begin{aligned}
D =& \{O_1, O_2, O_3, O_4, O_5, O_6, O_7, O_8\} \\
\text{Correct answer set} =& \{O_1, O_5, O_7\} \\
\text{Algorithm returns} =& \{O_1, O_3, O_5, O_6\} \\
\therefore \qquad P =& \{O_1, O_5, O_7\} \\
N =& \{O_2, O_3, O_4, O_6, O_8\} \\
TP =& \{O_1, O_5\} \\
TN =& \{O_2, O_4, O_8\} \\
FP =& \{O_3, O_6\} \\
FN =& \{O_7\} \\
\therefore \qquad \text{Recall} = \text{Sensitivity} =& \ 2/3 = 0.67 \\
\text{Precision} =& \ 2/4 = 0.5 \\
\text{Specificity} =& \ 3/5 = 0.6 \\
\text{F-score} =& \ 4/7 = 0.571 \\
\text{Accuracy} =& \ 5/8 = 0.625 \\
\text{Error rate} =&
\end{aligned}
$$

Example

$$
\begin{aligned}
D =& \{O_1, O_2, O_3, O_4, O_5, O_6, O_7, O_8\} \\
\text{Correct answer set} =& \{O_1, O_5, O_7\} \\
\text{Algorithm returns} =& \{O_1, O_3, O_5, O_6\} \\
\therefore \qquad P =& \{O_1, O_5, O_7\} \\
N =& \{O_2, O_3, O_4, O_6, O_8\} \\
TP =& \{O_1, O_5\} \\
TN =& \{O_2, O_4, O_8\} \\
FP =& \{O_3, O_6\} \\
FN =& \{O_7\}
\end{aligned}
$$

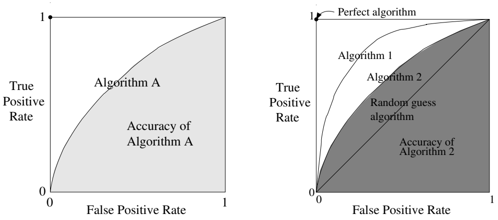
$$
\begin{aligned}
\therefore \qquad \text{Recall} = \text{Sensitivity} &= 2/3 = 0.67 \\
\text{Precision} &= 2/4 = 0.5 \\
\text{Specificity} &= 3/5 = 0.6 \\
\text{F-score} &= 4/7 = 0.571 \\
\text{Accuracy} &= 5/8 = 0.625 \\
\text{Error rate} &= 3/8 = 0.375
\end{aligned}
$$

ROC Curve

- Performance of an algorithm depends on parameters
- To assess over a range of parameters, ROC curve is used
  - 1 - Specificity (x-axis) versus Sensitivity (y-axis)
  - False positive rate (x-axis) versus True positive rate (y-axis)

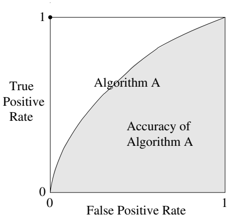
ROC Curve

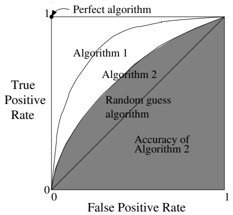
- Performance of an algorithm depends on parameters
- To assess over a range of parameters, ROC curve is used
  - 1 - Specificity (x-axis) versus Sensitivity (y-axis)
  - False positive rate (x-axis) versus True positive rate (y-axis)
- A random guess algorithm is a 45° line



ROC Curve

- Performance of an algorithm depends on parameters
- To assess over a range of parameters, ROC curve is used
  - 1 - Specificity (x-axis) versus Sensitivity (y-axis)
  - False positive rate (x-axis) versus True positive rate (y-axis)
- A random guess algorithm is a 45° line

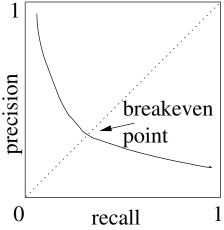




- Area under the ROC curve (AUC or AUROC) measures accuracy (or discrimination)
- What AUC is good?
  - 0.9+: excellent; 0.8+: good; 0.7+: fair; 0.6+: poor; 0.6-: fail
- EER denotes the point in ROC where FP rate is equal to FN rate

Precision-Recall Curve

- Precision versus recall



- Breakeven point where precision is the same as recall