## **Probabilistic Topic Models**

Piyush Rai

Probabilistic Machine Learning (CS772A)

Oct 26, 2017

## Remaining schedule...

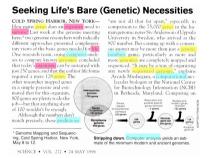
- ullet 6 more classes (+1 make up lecture for Diwali holiday; date TBD)
- Probabilistic Topic models (today)
- Deep Probabilistic Models (Bayesian neural nets, VAE, GAN)
- Models for sequential data (HMM, Linear Dynamical Systems)
- Probabilistic Graphical Models, Message Passing algorithms
- Other misc. topics
  - Semi-supervised Learning
  - Active Learning
  - Other suggestions.. ?

Topic models is a way to model documents: assumes each document to be a mixture of topics

#### Seeking Life's Bare (Genetic) Necessities

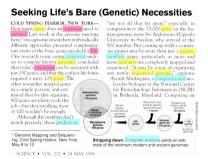


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• The notion of "document" is abstract here (e.g., "document" may represent an internet "user" and the words may represent the textual contents of all the URLs the user visits during a session)

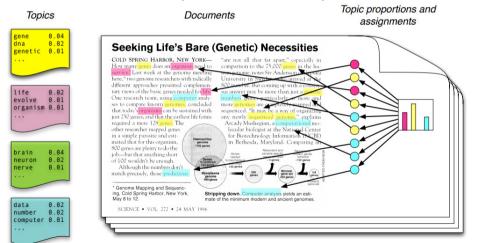
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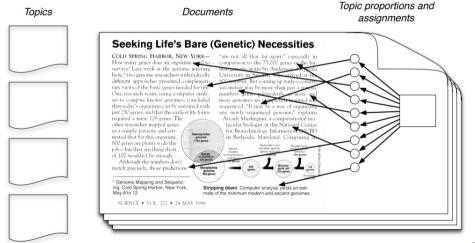
- The notion of "document" is abstract here (e.g., "document" may represent an internet "user" and the words may represent the textual contents of all the URLs the user visits during a session)
- The notion also applies for other type of data, not necessarily text (e.g., images as bag of "visual words" or speech data as phonemes) and a topic model makes sense for such data as well.

Probabilistic Machine Learning - CS772A (Pivush Rai, IITK)

• Goal of topic modeling is to learn the topics that underlie the document collection, and represent each document in terms of these topics (fraction of various topics)



• The input for the topic modeling problem will be the raw contents of the documents and the goal is to infer all the unknowns of the model (topics, topic proportions for each document, etc.)



# **Applications of Topic Modeling**

- Can be used to learn topic-based representation for each document
- These representation are compact (think dimensionality reduction) and semantically meaningful

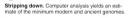
#### Seeking Life's Bare (Genetic) Necessities

COLD SPRING HARBOR, NEW YORK-How many genes does an organism need to survive? Last week at the genome meeting here," two genome researchers with radically different approaches presented complementary views of the basic genes needed for life. One research team, using computer analyses to compare known genomes, concluded that today's organisms can be sustained with just 250 genes, and that the earliest life forms required a mere 128 genes. The other researcher mapped genes in a simple parasite and estimated that for this organism. 800 genes are plenty to do the job-but that anything short of 100 wouldn't be enough Although the numbers don't

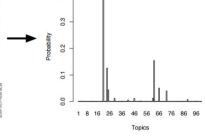
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\* Genome Mapping and Sequencing, Cold Spring Harbor, New York, May 8 to 12.

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SCIENCE • VOL. 272 • 24 MAY 1996



<sup>&</sup>quot;are not all that far apart," especially in comparison to the 75,002 genes in the human genome, notes Siv Andersson of Uppsala University in Sweden, who arrived at the 800 number. But coming up with a consensus answer may be more than just a genetic numbers game, particularly as more and more genomes are completely mapped and sequenced. "It may be a way of organicing any newly sequenced genome," explains

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• Also makes it easy to cluster data naturally by (learned) topics

## Topic Models: The Basic Idea

- Topic models (especially LDA) are essentially based on assigning words in each document to clusters/topics (each cluster can be thought of as representing a "topic")
- Note: Different occurrences of the same word may be assigned to different topics

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Note that word-level topic assignments can be aggregated to get the document's topic proportions

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word	prob.	word	prob.	word	prob.	word	prob.
DRUGS	.069	RED	.202	MIND	.081	DOCTOR	.074
DRUG	.060	BLUE	.099	THOUGHT	.066	DR.	.063
MEDICINE	.027	GREEN	.096	REMEMBER	.064	PATIENT	.061
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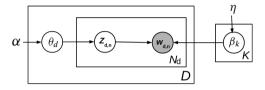
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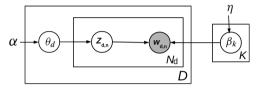
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Models data where each observation (document) consist of a set of discrete-valued tokens (words)

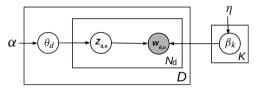


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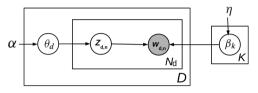
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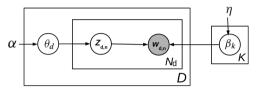
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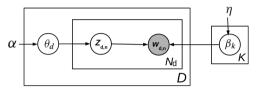


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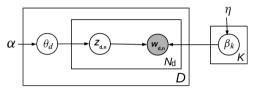


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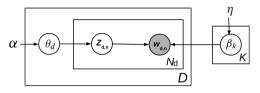
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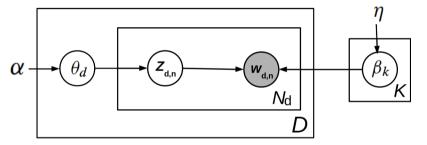
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    - Now generate the word form the chosen topic:  $\mathbf{w}_{d,n} \sim \text{multinoulli}(\beta_{\mathbf{z}_{d,n}})$

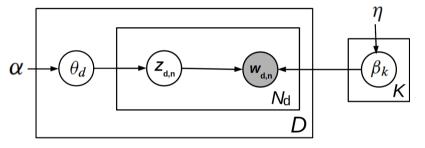
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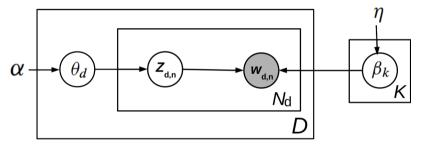
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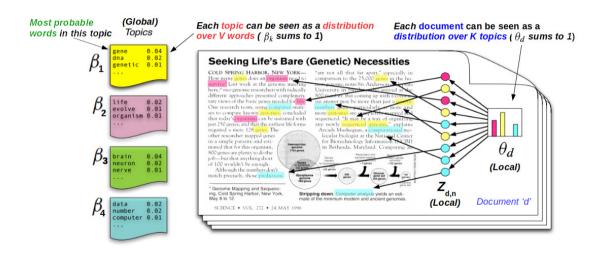
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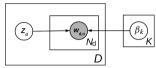
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- Word co-occurrences (within/across document) influences the topics  $\{\beta_k\}_{k=1}^K$  learned by the model

# What LDA Output Looks Like, Qualitatively

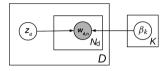


(Figure: Modified from the original by David Blei)

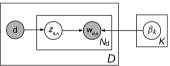
• A naïve mixture model: Each document having a single topic, all words from that topic



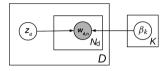
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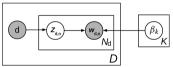
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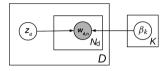


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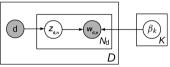


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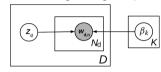


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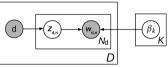


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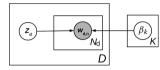


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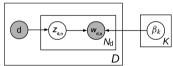


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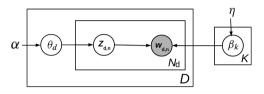


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- Also, unlike LDA, PLSA is not a Bayesian model

### Inference for LDA

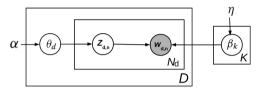


• The goal is to infer the posterior distribution over all the latent variables

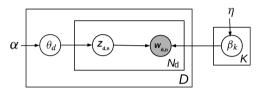
$$p(\mathbf{Z}, \theta, \beta | \mathbf{W}, \alpha, \eta) = \frac{p(\mathbf{W} | \beta, \mathbf{Z}) p(\mathbf{Z} | \theta) p(\beta | \eta) p(\theta | \alpha)}{p(\mathbf{W} | \alpha, \eta)}$$
 (assuming hyperparams  $\alpha, \eta$  are fixed)

- Z: Topic assignments of all words across all documents
- $\theta = \{\theta_1, \dots, \theta_D\}$ : Topic proportion vectors; each  $\theta_d$  is a K-dim vector
- $\beta = \{\beta_1, \dots, \beta_K\}$ : The K topics; each  $\beta_k$  is a V-dim vector
- ullet  ${f W}=\{{m w}_{d,n}\}, d=1,\ldots,D, n=1,\ldots,N_d$ : Collection of all words in all the documents

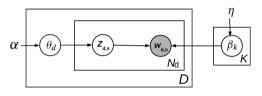




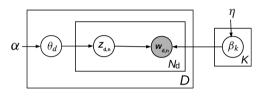
• A wide variety of inference methods exist for LDA (MCMC, VB, EP, batch, online, ...)



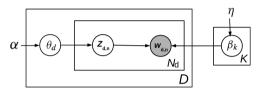
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- Note: Can even collapse some variables and do collapsed Gibbs or collapsed VB

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† Finding Scientific Topics (Griffiths and Stewers 2004)

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- A high average probability (low perplexity) implies a good fit to the data



#### LDA vs Matrix Factorization

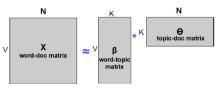
- LDA is sort of equivalent to a non-negative matrix factorization model for discrete data
- Input:  $V \times N$  word x document matrix **X** (of discrete values)

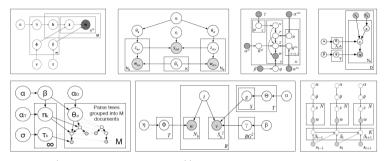
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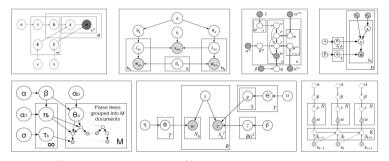
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1																				
Terms	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
data	1	1	0	0	2	0	0	0	0	0	1	2	1	1	1	0	1	0	0	0
examples	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
introduction	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
mining	0	0	0	0	0	0	0	0	0	0	0	1	1	0	1	0	0	0	0	0
network	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	1
package	0	0	0	1	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0

• Output:  $V \times K$  word-topic matrix  $\beta$ ,  $K \times N$  topic-document matrix  $\Theta$ 

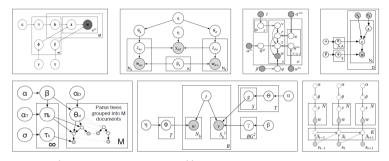




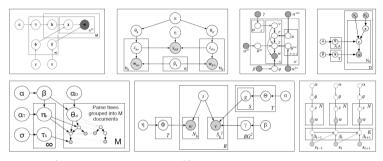


Several extensions of LDA (too many to list here :)). Some include

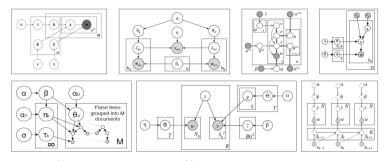
• LDA with document labels (supervised LDA)



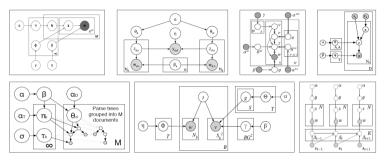
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- LDA with document labels (supervised LDA)
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- LDA with topic hierarchies/correlations
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- LDA with HMM (takes into account word order), and many others..

## **Summary**

- LDA is a simple but powerful model for discrete data (e.g., text documents)
- Easy to extend to more sophisticated models
- A variety of inference algorithms can be developed for doing inference for LDA based models
- Connections to matrix factorization methods, e.g., Poisson likelihood for the document-word count matrix with gamma priors on the latent factors (recall HW3 problem)
- Some recent work on "Deep Topic Models"