Solution: Tutorial 02

1. Solution of non-linear equation

$$f(x) = 600x^4 - 550x^3 + 200x^2 - 20x - 1 = 0$$

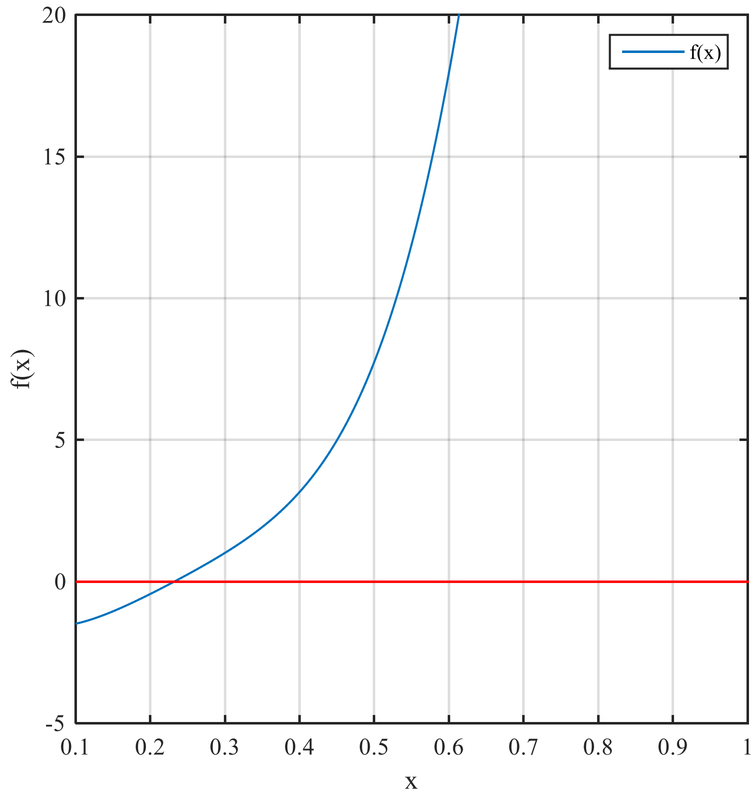

Figure 1: Plot of the non-linear equation.

(a) Bisection method

$$x_l = 0.1 \quad x_u = 1.0 \quad x_r = \frac{x_l + x_u}{2}$$

Iteration	x_l	x_u	x_r	ea (%)
1	0.10000	1.00000	0.55000	-
2	0.10000	0.55000	0.32500	69.231
3	0.10000	0.32500	0.21250	52.941
4	0.21250	0.32500	0.26875	20.930
5	0.21250	0.26875	0.24063	11.688
6	0.21250	0.24063	0.22656	6.207
7	0.22656	0.24063	0.23359	3.010
8	0.22656	0.23359	0.23008	1.528

(b) False position method

$$x_l = 0.1 \quad x_u = 1.0 \quad x_r = x_u - \frac{f(x_u)(x_l - x_u)}{f(x_l) - f(x_u)}$$

Iteration	x_l	x_u	x_r	ea (%)
1	0.10000	1.00000	0.10582	-
2	0.10582	1.00000	0.11146	5.059
3	0.11146	1.00000	0.11691	4.663
4	0.11691	1.00000	0.12217	4.303
5	0.12217	1.00000	0.12722	3.977
6	0.12722	1.00000	0.13209	3.680
7	0.13209	1.00000	0.13675	3.410
8	0.13675	1.00000	0.14122	3.164

(c) Modified false position method

Same as above, except that if one bound is stagnant for two iterations its value is halved.

Iteration	x_l	x_u	x_r	ea (%)
1	0.10000	1.00000	0.10582	-
2	0.10582	1.00000	0.11146	5.059
3	0.11146	1.00000	0.12229	8.860
4	0.12229	1.00000	0.14217	13.980
5	0.14217	1.00000	0.17492	18.724
6	0.17492	1.00000	0.21657	19.231
7	0.21657	1.00000	0.23970	9.649
8	0.21657	0.23970	0.23226	3.203

(d) Newton-Raphson method

$$x_0 = 0.5 \quad x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \quad i = 0,1,2,\ldots,\text{iter}_{\text{max}}$$

Iteration	x	ea (%)
1	0.50000	NaN
2	0.38519	29.808
3	0.28128	36.939
4	0.23485	19.772
5	0.23236	1.072
6	0.23235	0.002

(e) Secant method

$$x_{-1} = 0.1,\ x_0 = 1.0 \quad x_{i+1} = x_i - \frac{f(x_i)\left(x_{i-1} - x_i\right)}{f(x_{i-1}) - f(x_i)} \quad i = 0, 1, 2, \ldots, \text{iter}_{\text{max}}$$

Iteration	x	ea (%)
1	0.10582	-
2	0.11146	5.059
3	0.31178	64.251
4	0.21954	42.013
5	0.23135	5.105
6	0.23236	0.435
7	0.23235	0.004

2. <u>Solution of system of non-linear equation</u>

$$u(x,y) = x^2 - x + y - 0.75 = 0$$
$$v(x,y) = x^2 - 5xy - y = 0$$

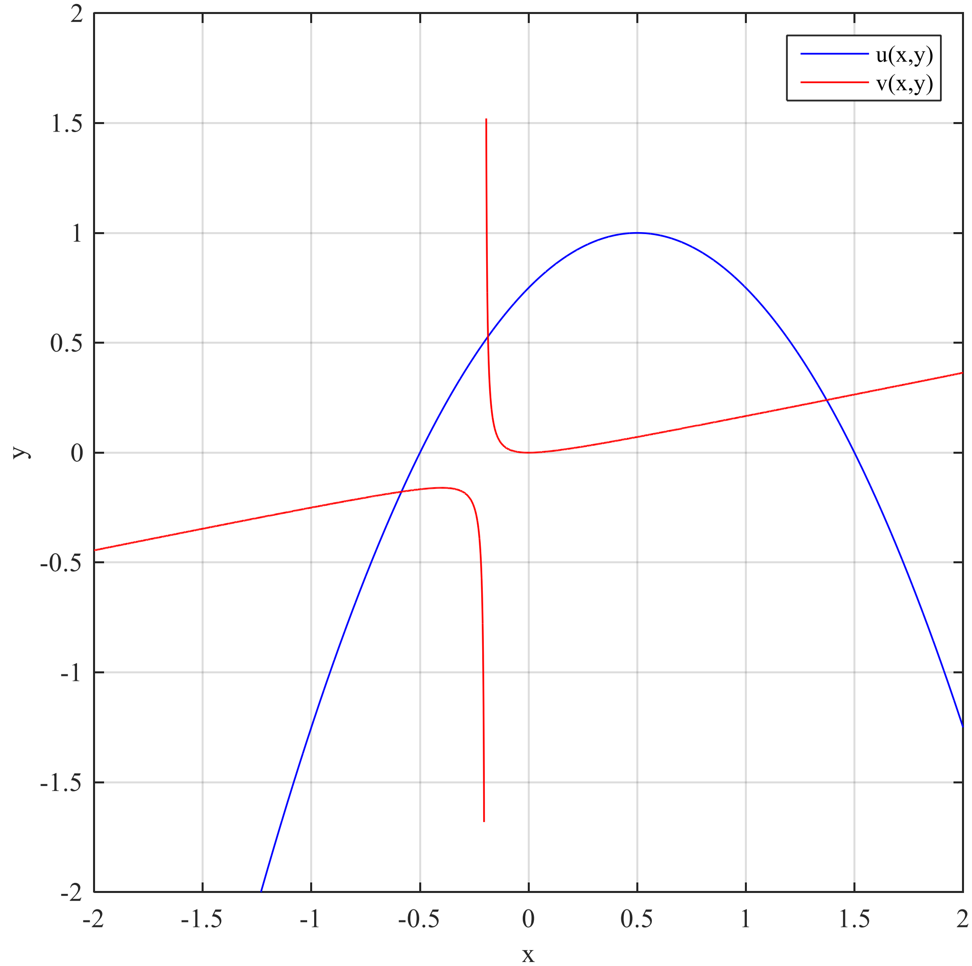

Figure 2: Plot of the non-linear equations. There are three possible solutions for this system of equations

(a) Fixed-point iteration

$$x = \sqrt{x - y + 0.75}$$

$$y = \frac{x^2}{(1+5x)}$$

Table 1: Fixed-point iteration

Iteration	x	y	ea (%) in x	ea (%) in y
0	1.20000	1.20000	-	-
1	0.86603	0.14071	-38.564	-752.820
2	1.21463	0.20858	28.700	32.539
3	1.32516	0.23028	8.341	9.422
4	1.35826	0.23679	2.437	2.749
5	1.36802	0.23871	0.713	0.804
6	1.37088	0.23927	0.209	0.235
7	1.37172	0.23943	0.061	0.069
8	1.37196	0.23948	0.018	0.020
9	1.37204	0.23950	0.005	0.006
10	1.37206	0.23950	0.002	0.002

(b) Newton-Raphson method

$$\frac{\partial u}{\partial x} = 2x - 1; \qquad \frac{\partial u}{\partial y} = 1$$

$$\frac{\partial v}{\partial x} = 2x - 5y; \qquad \frac{\partial v}{\partial y} = -(5x+1)$$

Determinat of Jacobian $D = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} = 5y - 2x - (2x-1)(5x+1)$

Iteration 1

$$x_0 = 1.2 \quad y_0 = 1.2$$
$$u_0 = u(x_0, y_0) = 0.69$$
$$v_0 = v(x_0, y_0) = -6.96$$
$$x_1 = x_0 - \frac{u_0 \frac{\partial v_0}{\partial y} - v_0 \frac{\partial u_0}{\partial y}}{D\left(x_0, y_0\right)} = 1.2 - \frac{2.13}{-6.20} = 1.54355$$
$$y_1 = y_0 - \frac{v_0 \frac{\partial u_0}{\partial x} - u_0 \frac{\partial v_0}{\partial x}}{D\left(x_0, y_0\right)} = 1.2 - \frac{-7.26}{-6.20} = 0.02903$$

Table 2: Newton-Raphson method

Iteration	*x*	*y*	ea (%) in *x*	ea (%) in *y*
0	1.20000	1.20000	-	-
1	1.54355	0.02903	22.257	-4033.333
2	1.39412	0.22287	-10.718	86.974
3	1.37245	0.23929	-1.579	6.862
4	1.37207	0.23950	-0.028	0.087
5	1.37207	0.23950	0.000	0.000