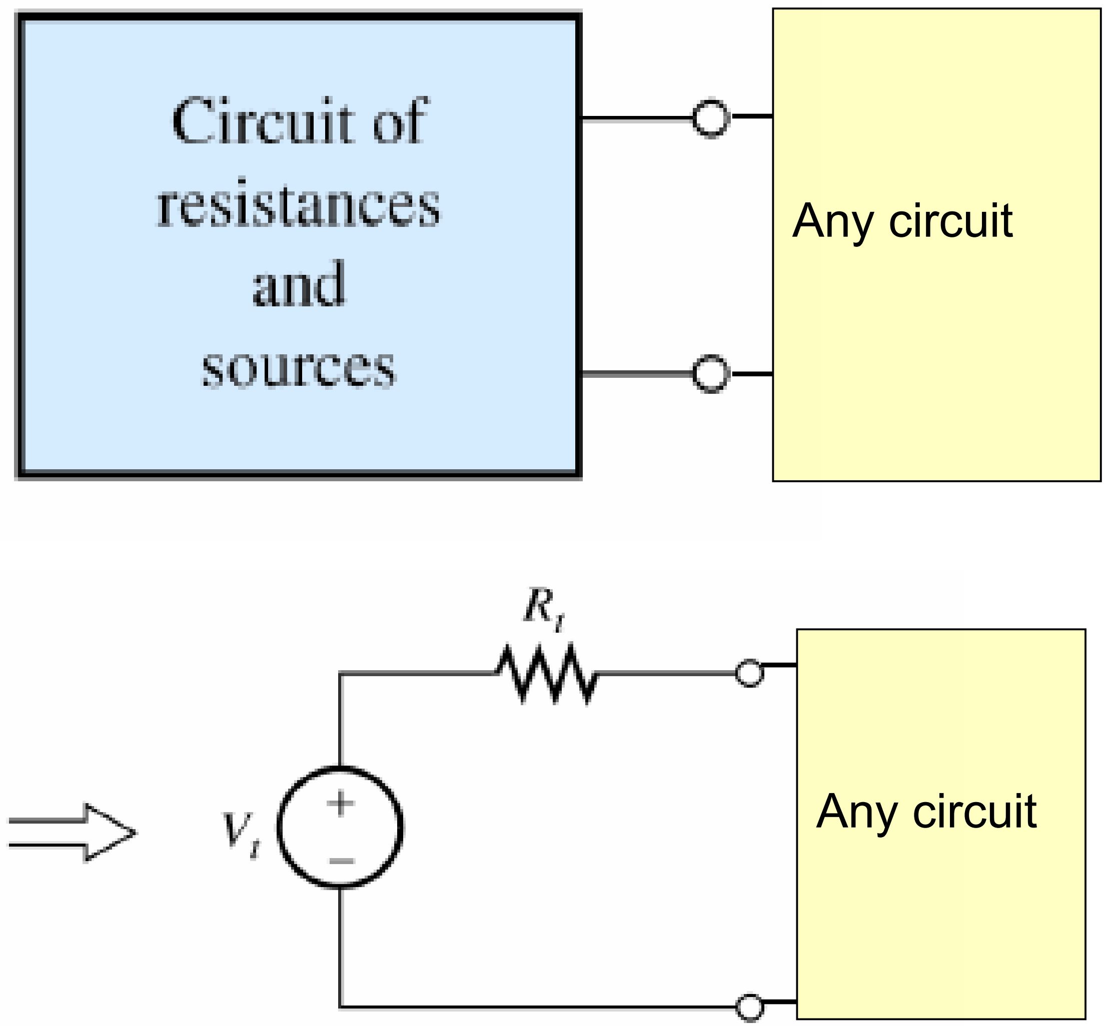
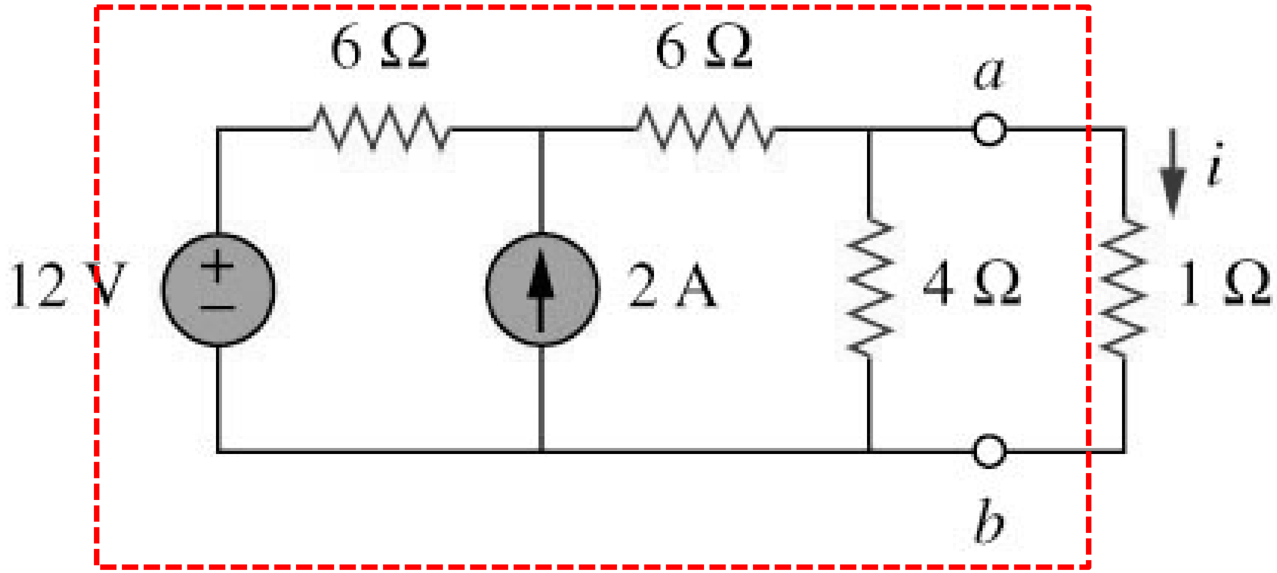
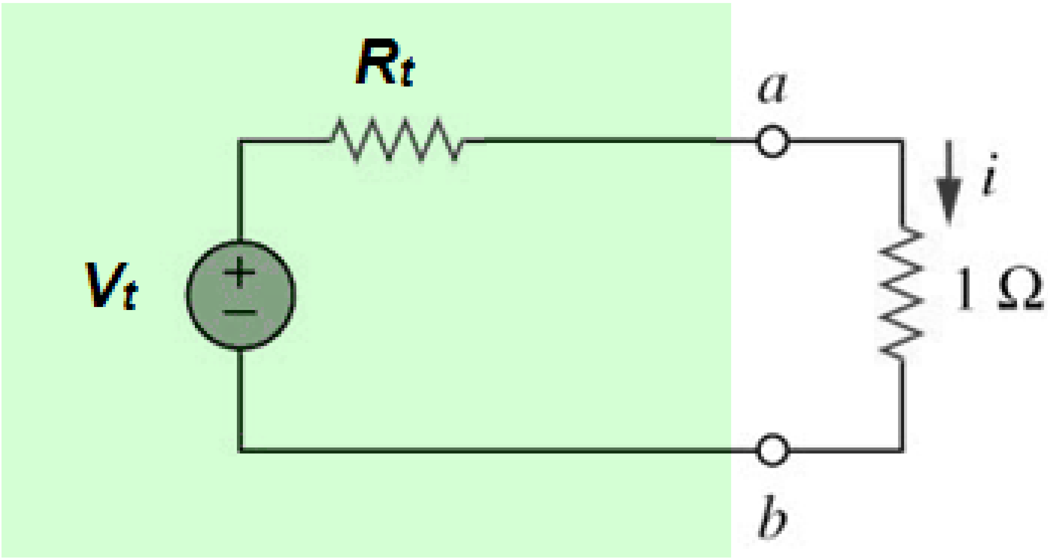
Thévenin Equivalent Circuits



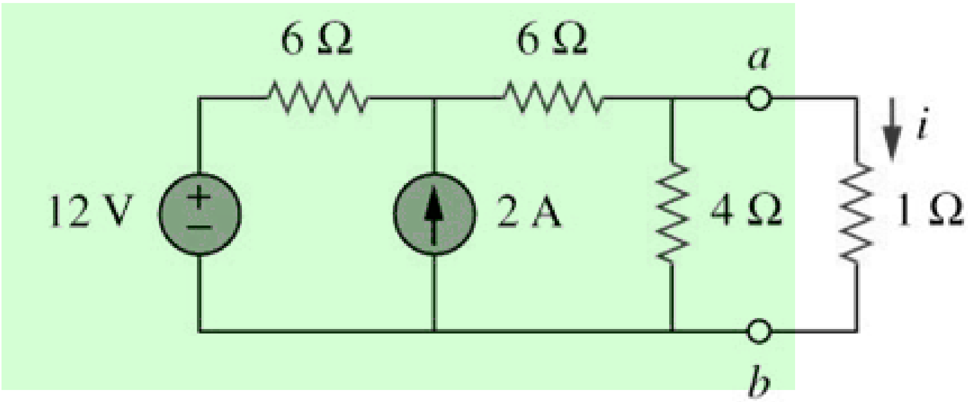
Thévenin equivalent circuit

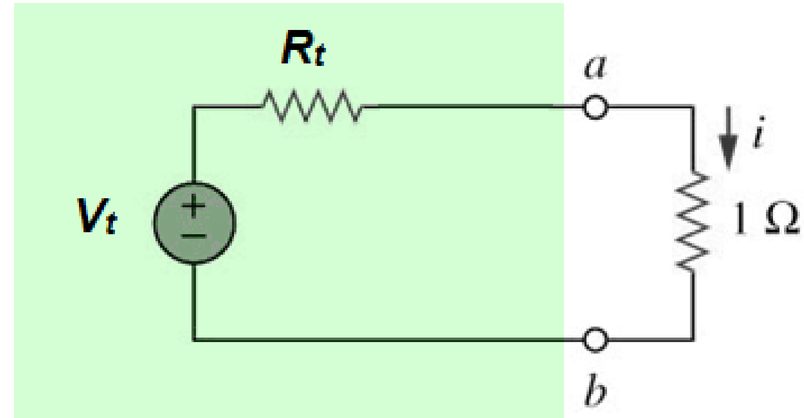
Thévenin Equivalent Circuits

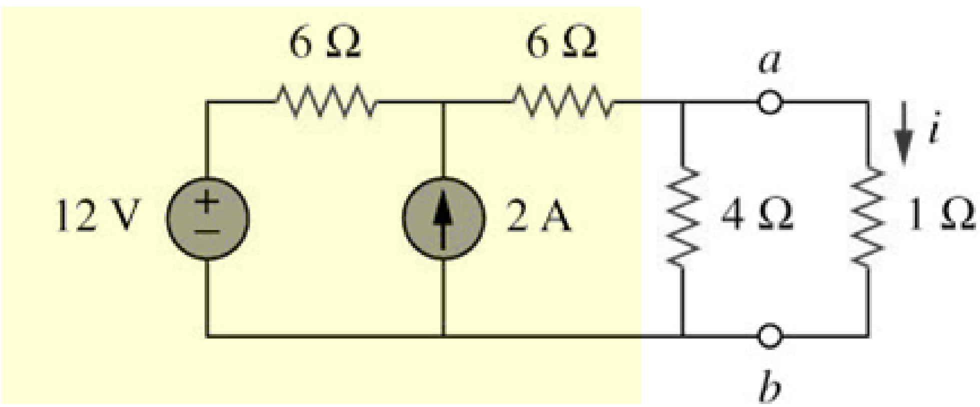


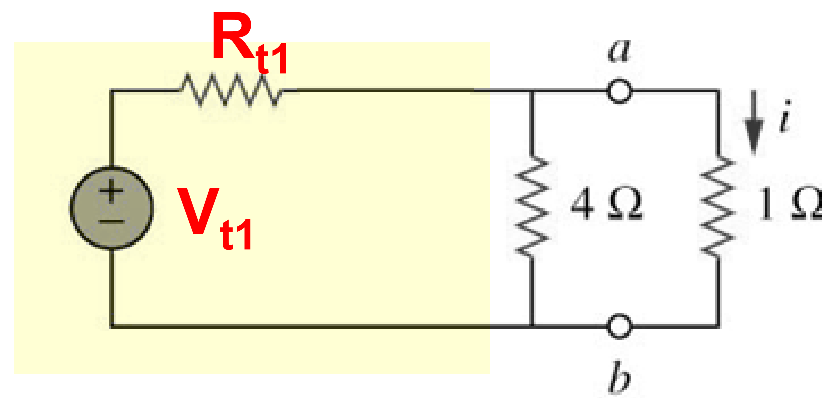


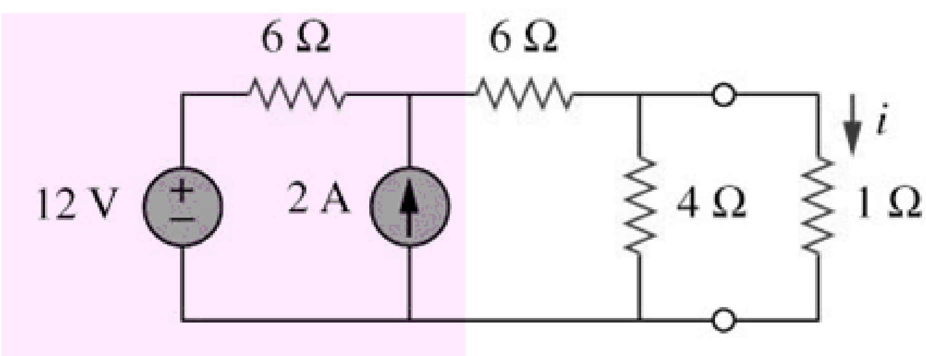
We can apply Thevenin's theorem to any part of the circuit

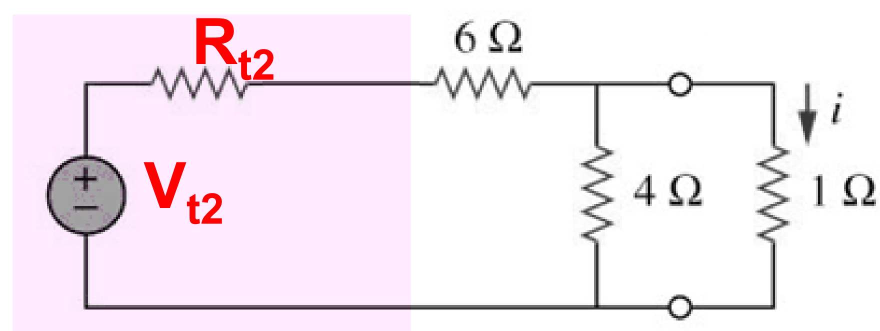




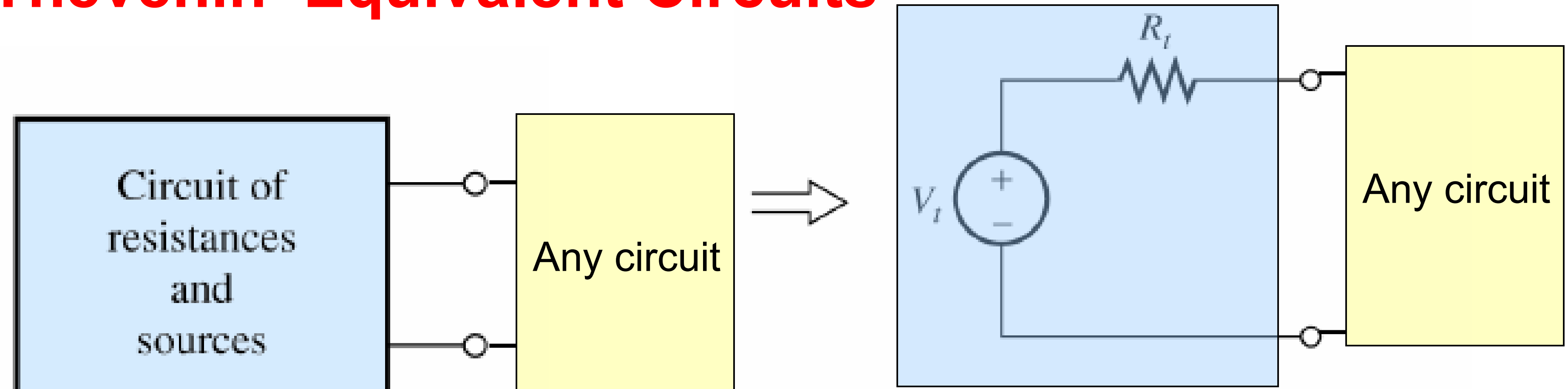






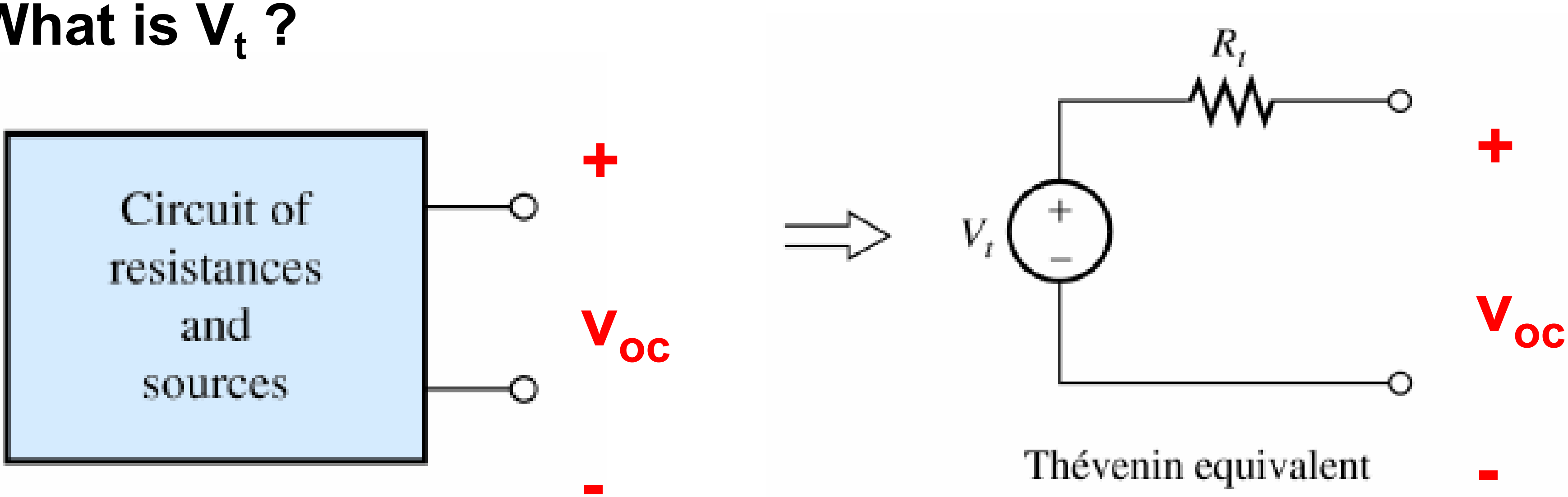


Thévenin Equivalent Circuits



Thévenin equivalent circuit

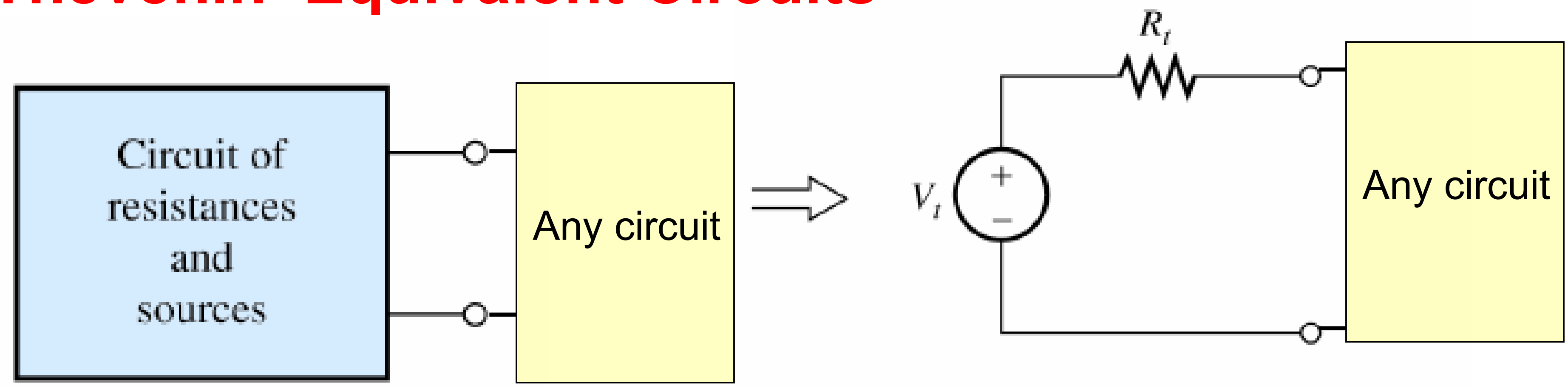
What is V_t ?



Thévenin equivalent circuit

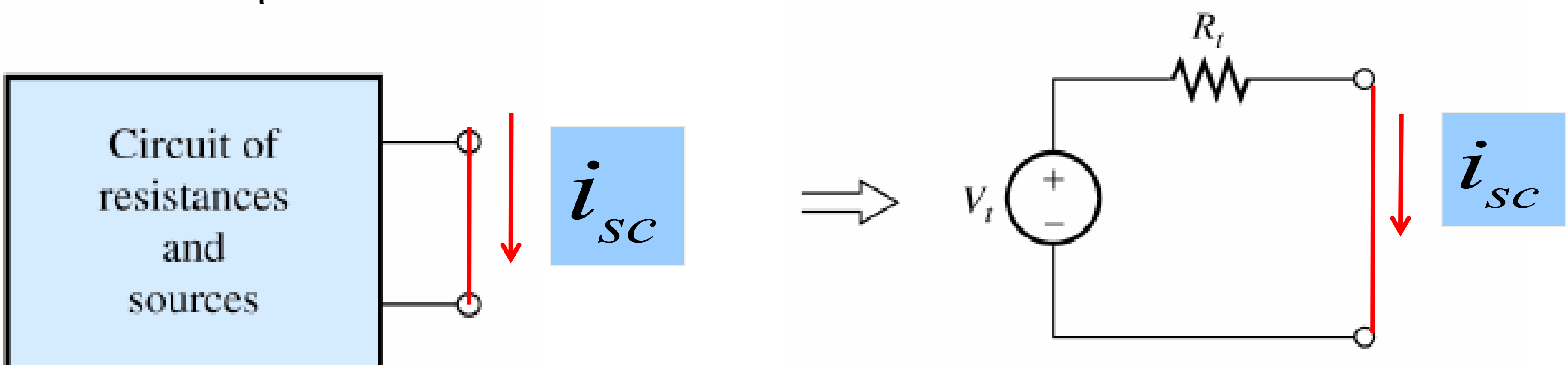
$$V_t = v_{oc}$$

Thévenin Equivalent Circuits



Thévenin equivalent circuit

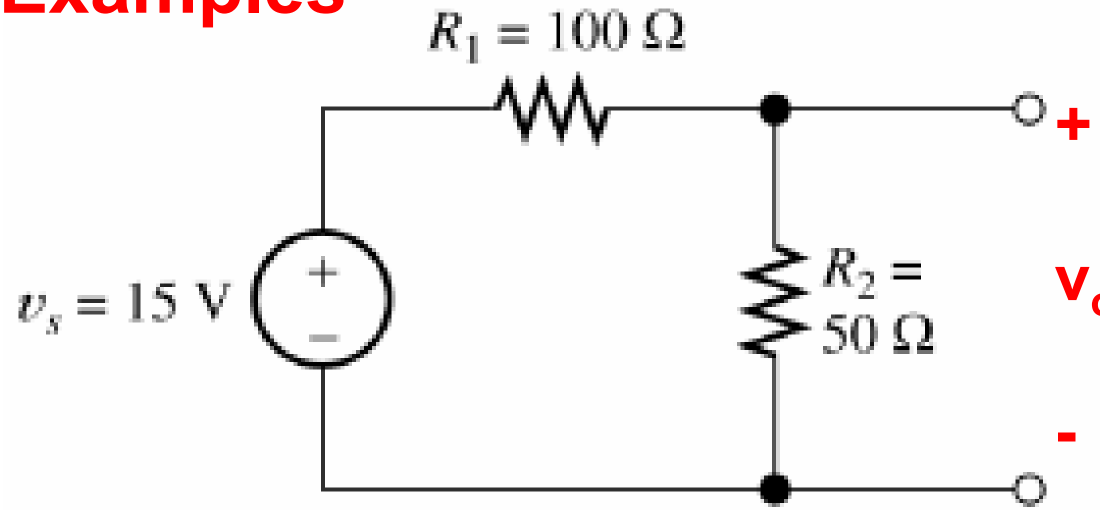
What is R_t ?

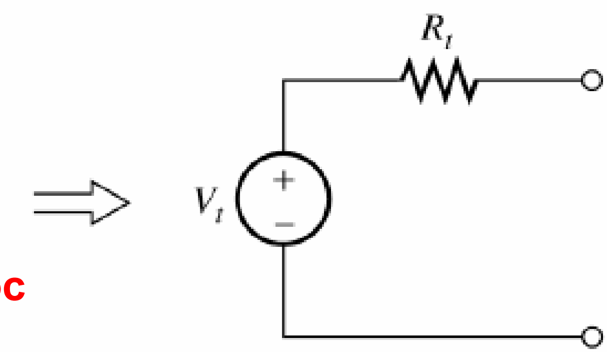


Thévenin equivalent circuit

$$R_t = \frac{V_t}{i_{sc}} = \frac{v_{oc}}{i_{sc}}$$

Examples

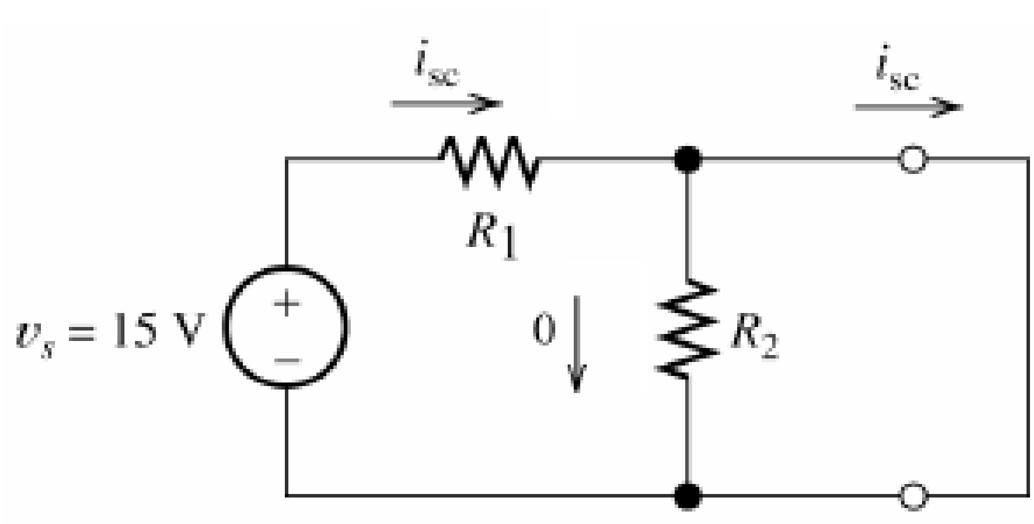




Thévenin equivalent circuit

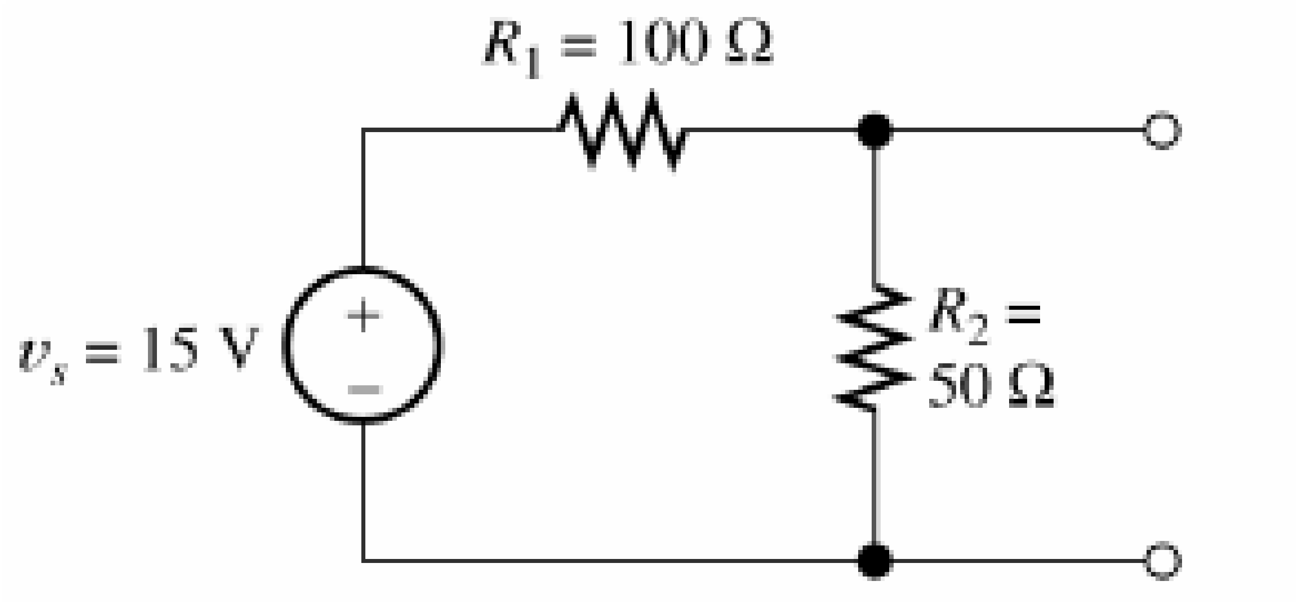
$$V_t = v_{oc}$$

$$V_t = \frac{R_2}{R_2 + R_1} \times 15 = 5$$



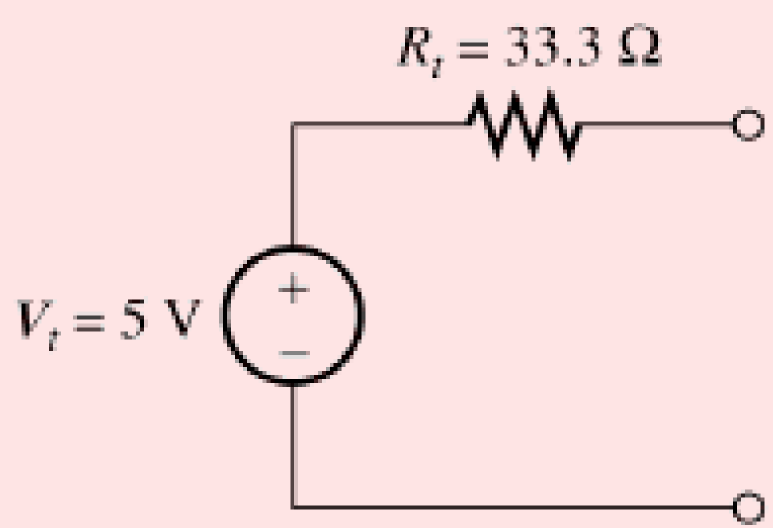
$$i_{sc} = \frac{v_s}{R_1} = 0.15A$$

$$R_t = \frac{v_{oc}}{i_{sc}} = 33.3\Omega$$

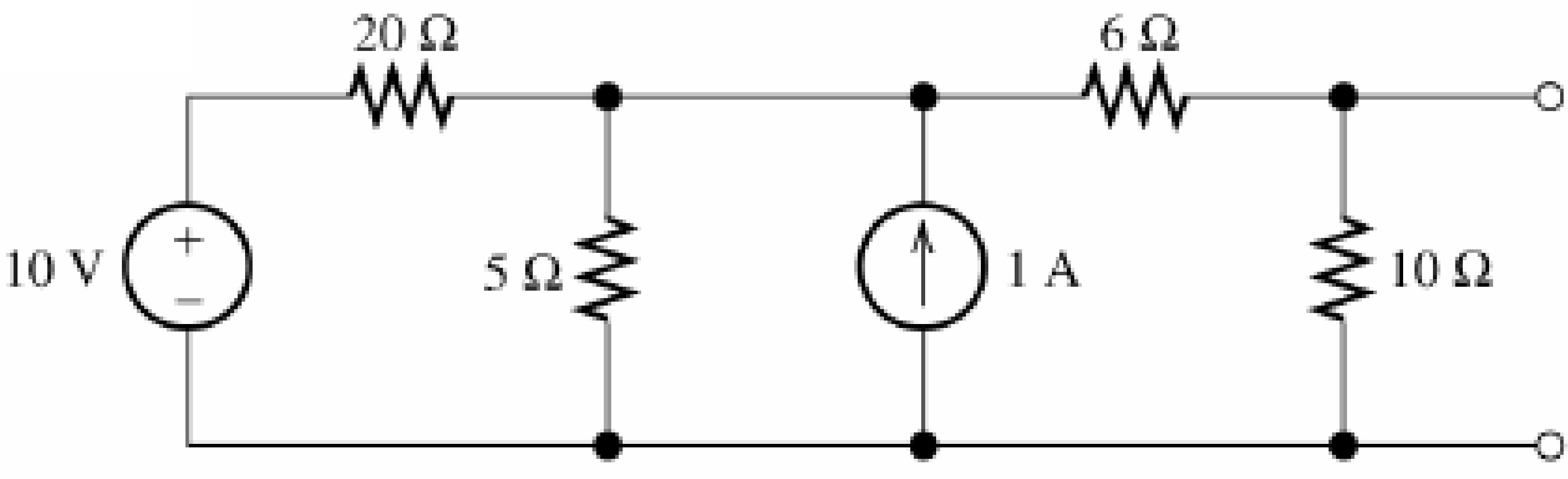


$$V_t = 5$$

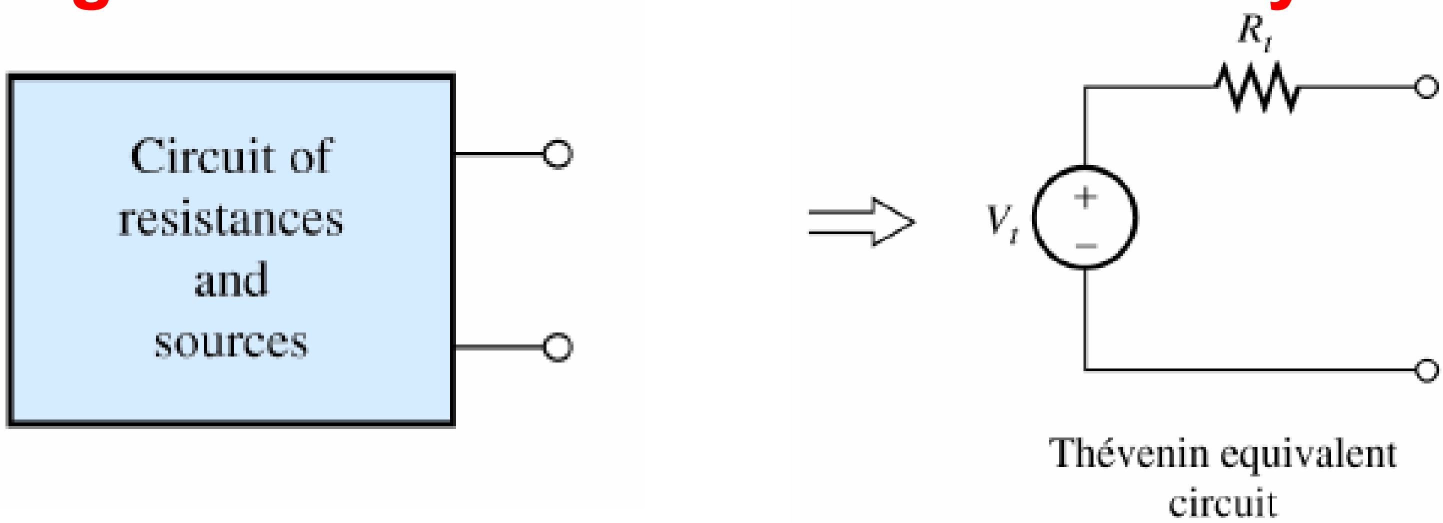
$$R_t = 33.3\Omega$$



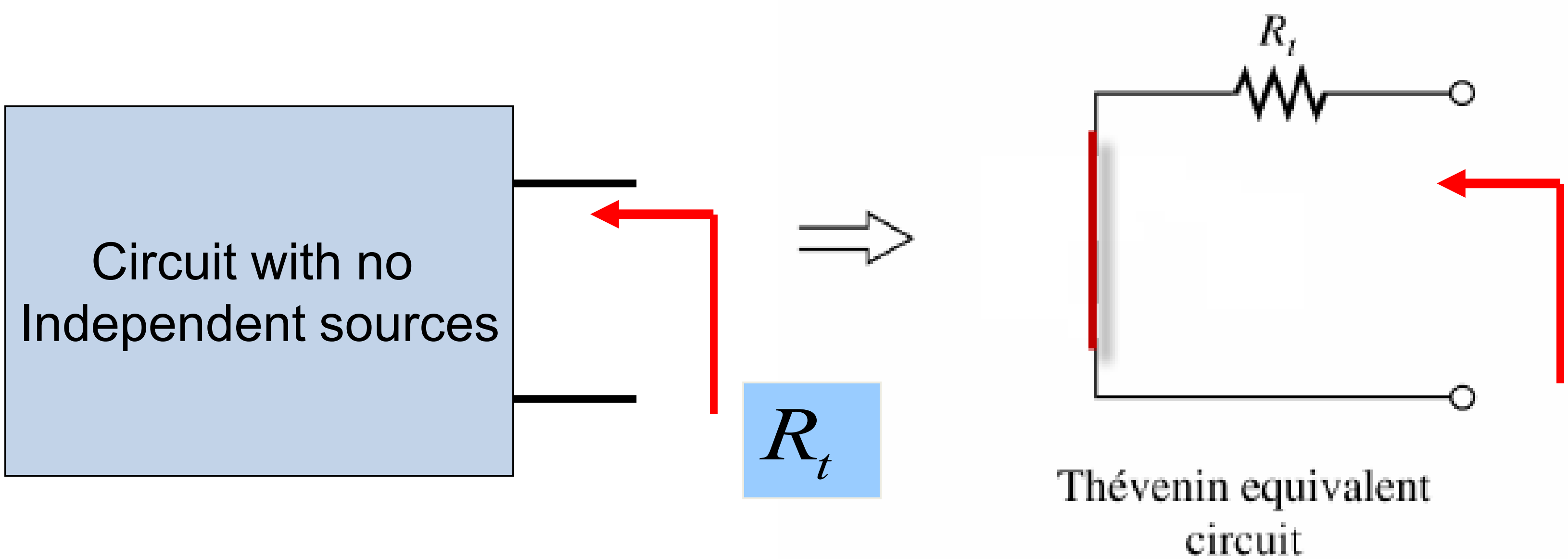
Currents and voltages in the circuit are only due to Independent Sources



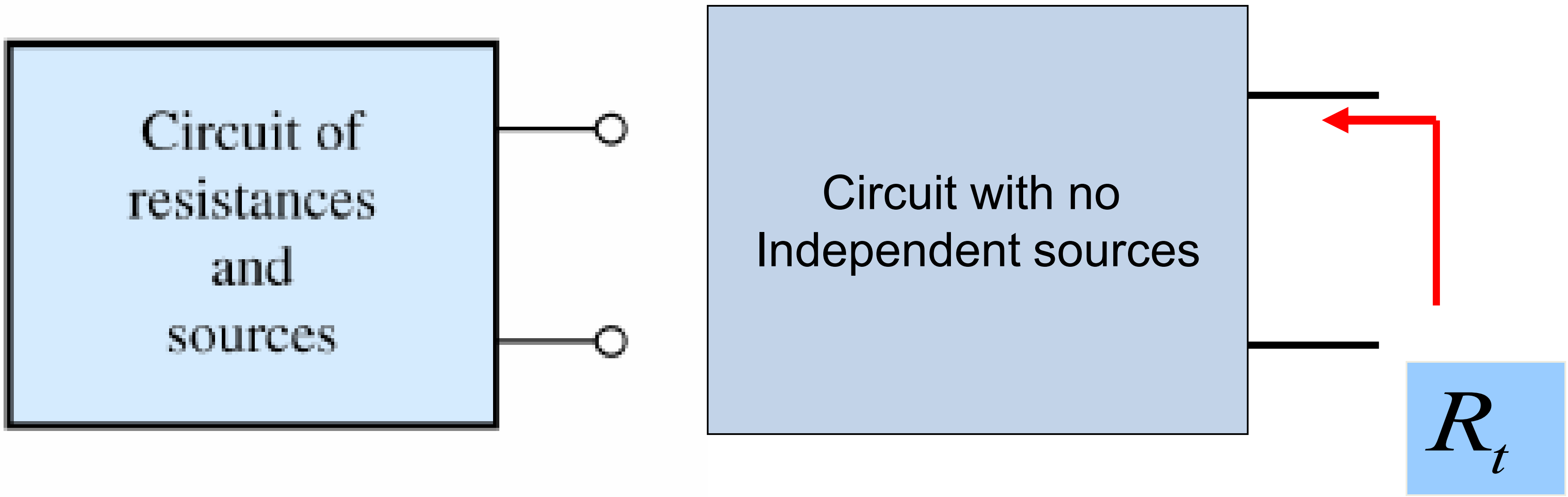
Finding the Thévenin Resistance Directly



Suppose we make all independent sources zero in the circuit



Finding the Thévenin Resistance Directly



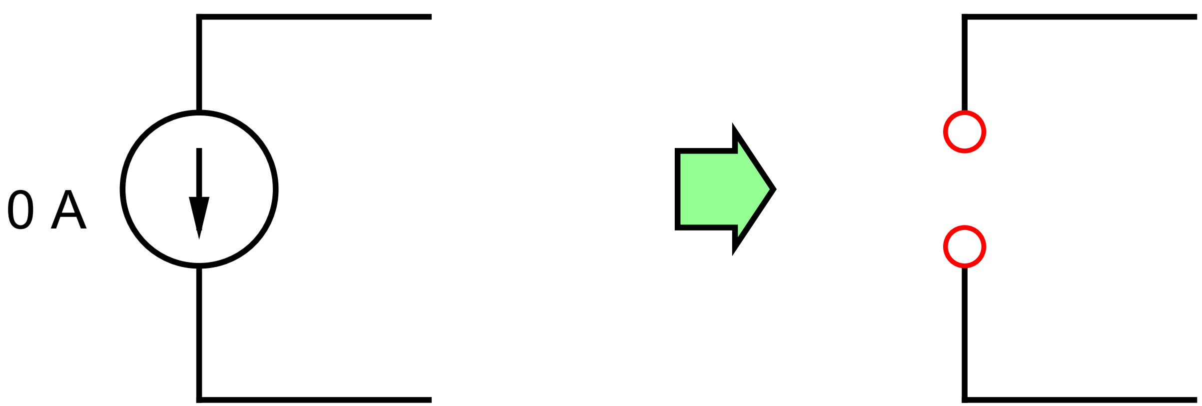
1. Turn off independent sources in the original network:

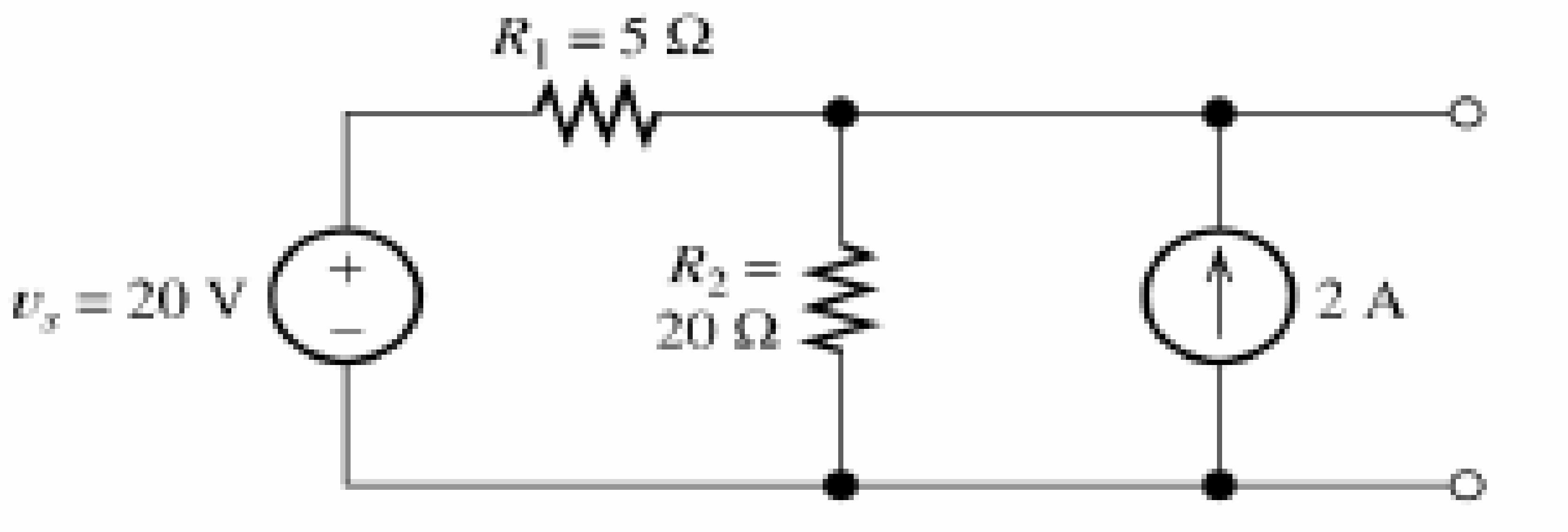
   -A voltage source becomes a short circuit

   -A current source becomes an open circuit

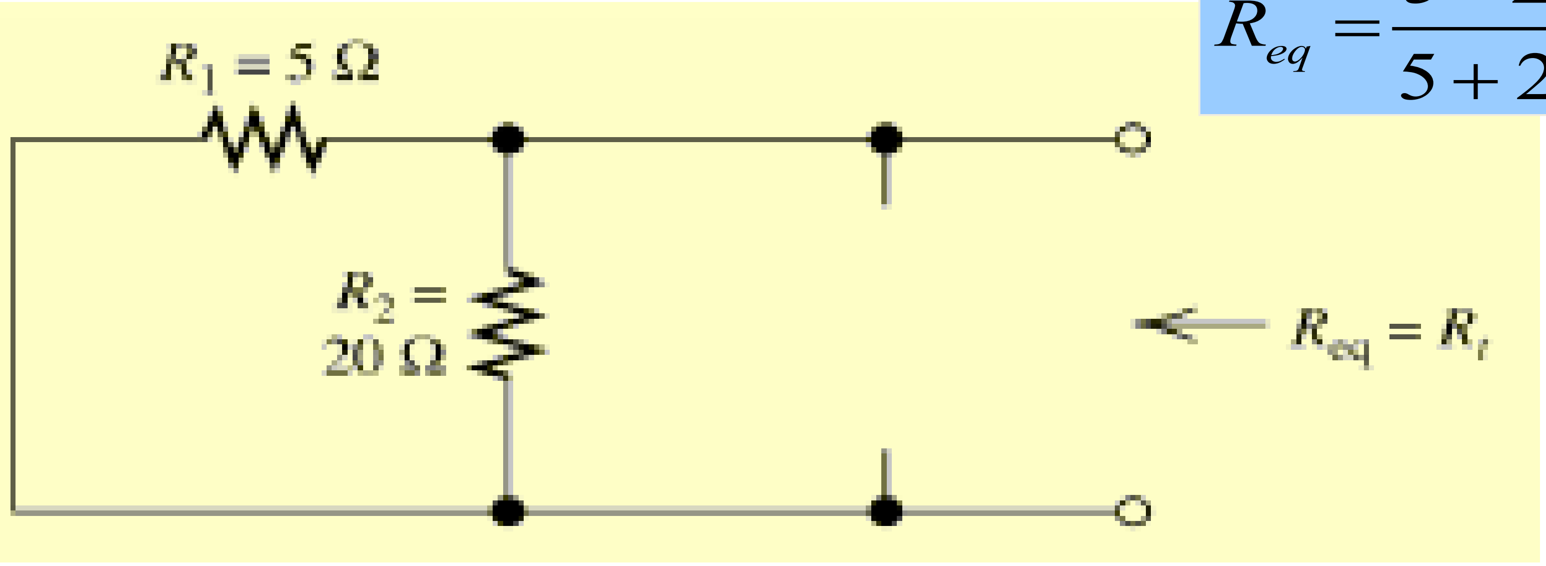
2. Compute the resistance between the terminals


0 V
+
–

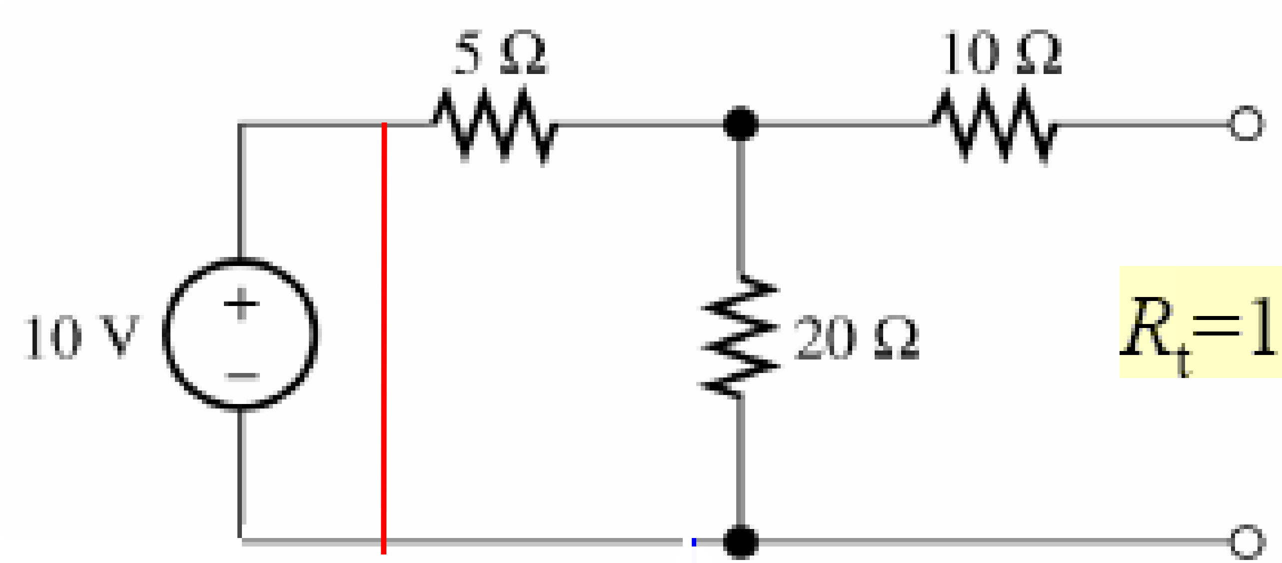

0 A



$$R_{eq} = \frac{5\times 20}{5+20} = 4\Omega$$

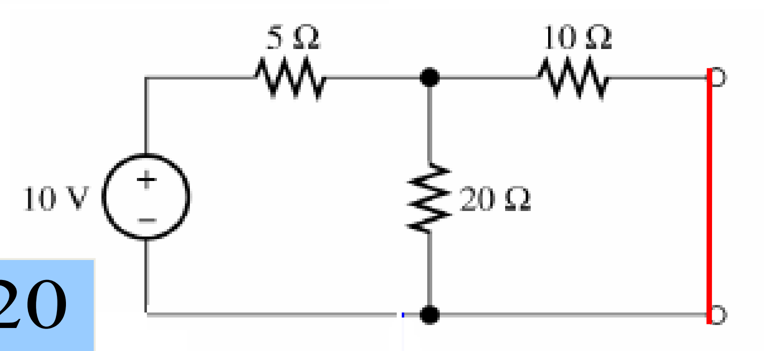


Find Thevenin resistance for each of the circuits shown below



$$R_t = 10 + (5 \| 20) = 14$$

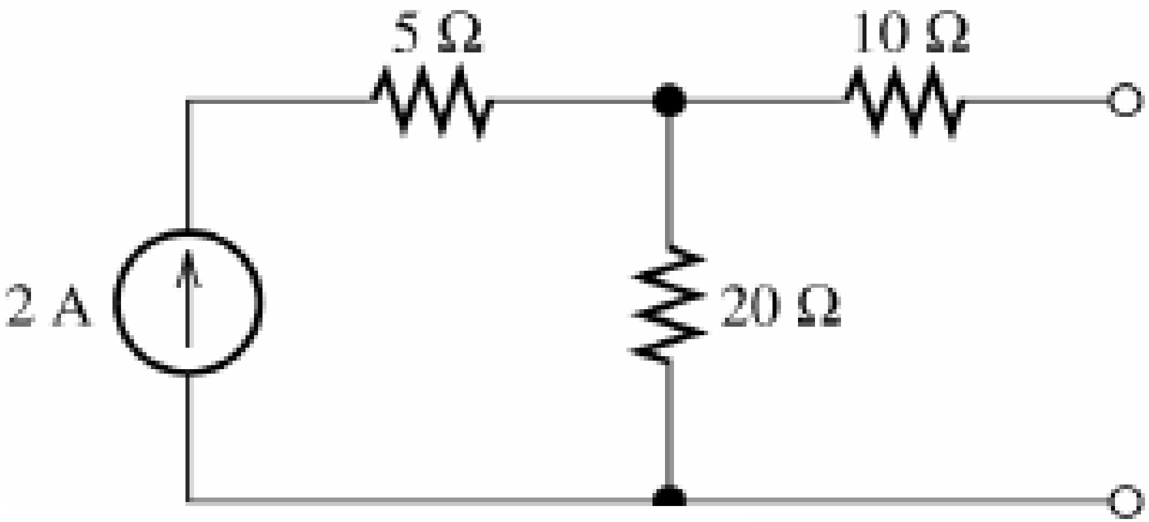
$$V_{OC} = V_t = \frac{20}{20+5} \times 10 = 8$$

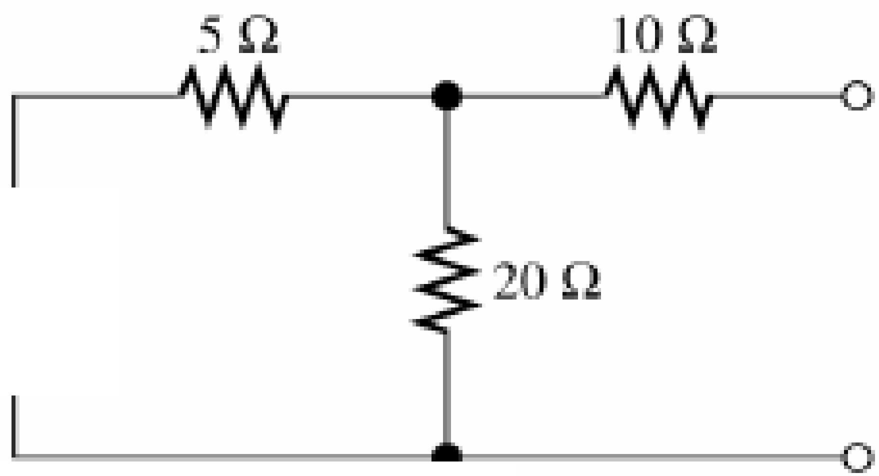


$$i_{sc} = \frac{10}{5 + (10 \| 20)} \cdot \frac{20}{20+10} = \frac{20}{35}$$

$$R_t = \frac{v_{oc}}{i_{sc}} = \frac{8 \times 35}{20} = 14$$

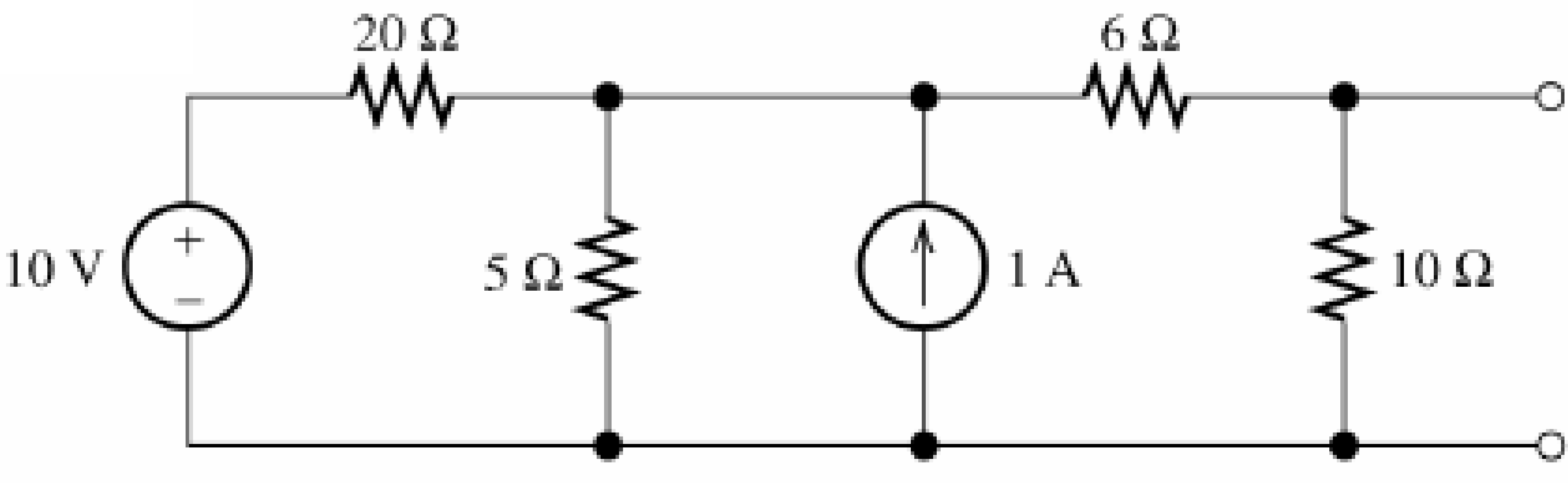
Find Thevenin resistance for each of the circuits shown below





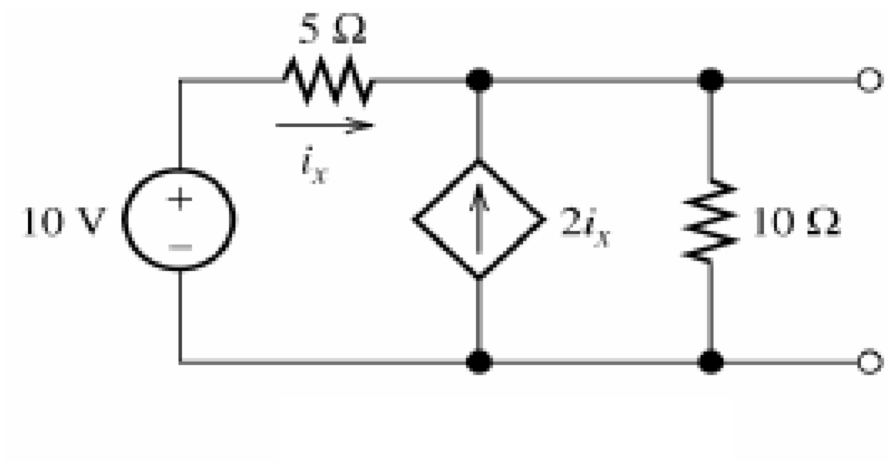
$R_t = 10+20 = 30$

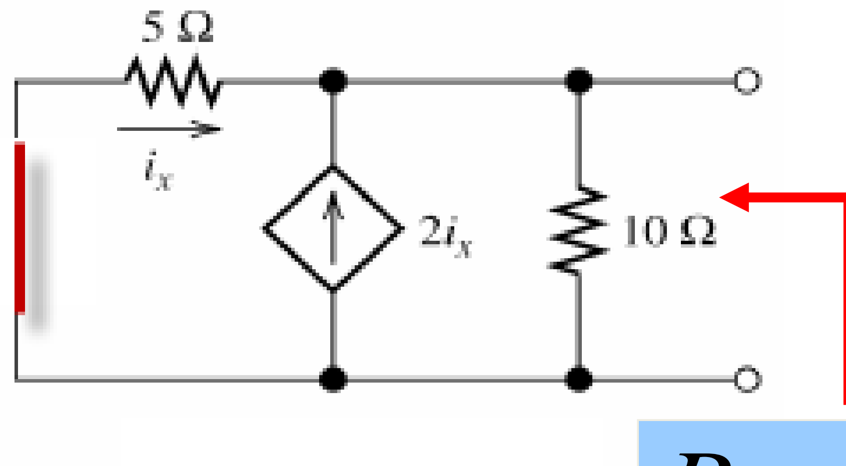
Find Thevenin resistance for each of the circuits shown below



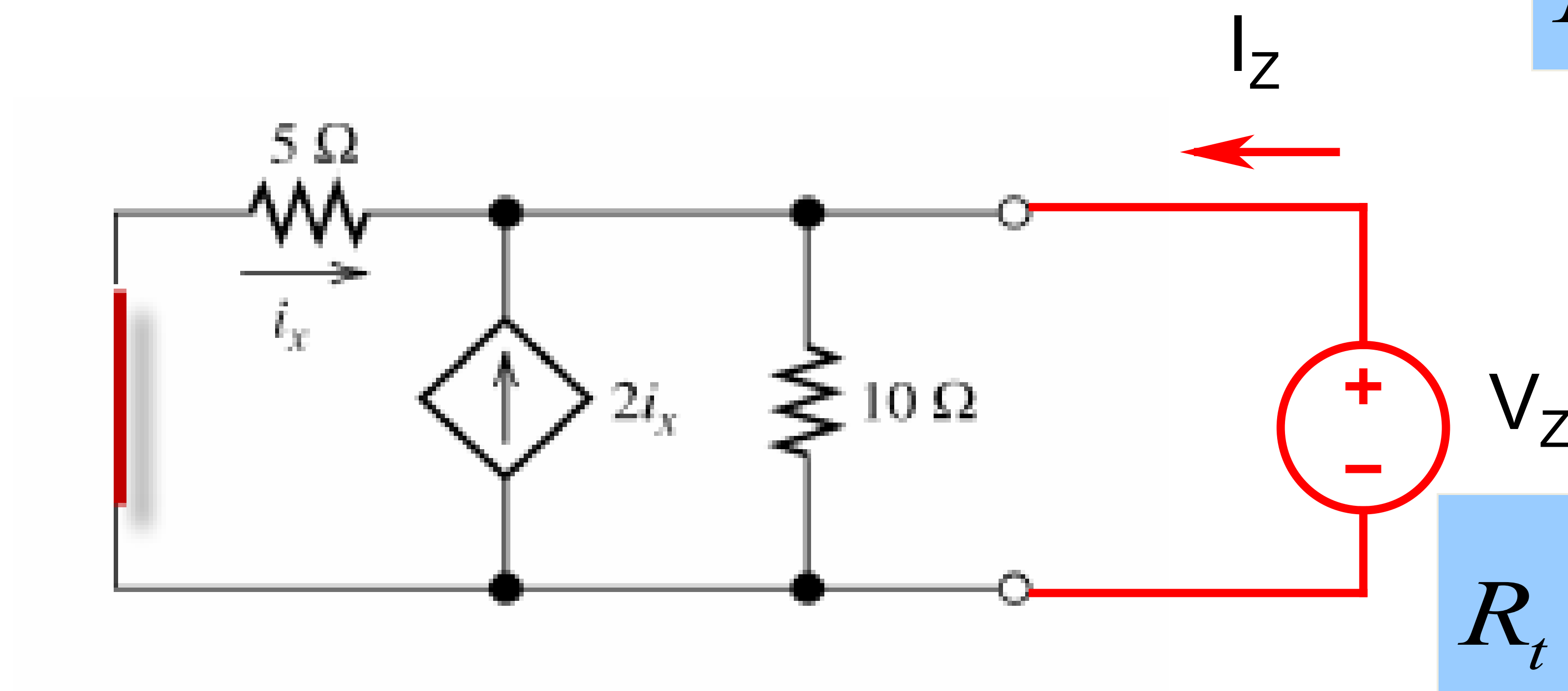
$$R_t = ((20 \| 5) + 6) \| 10) = 5$$

Circuit with dependent Sources



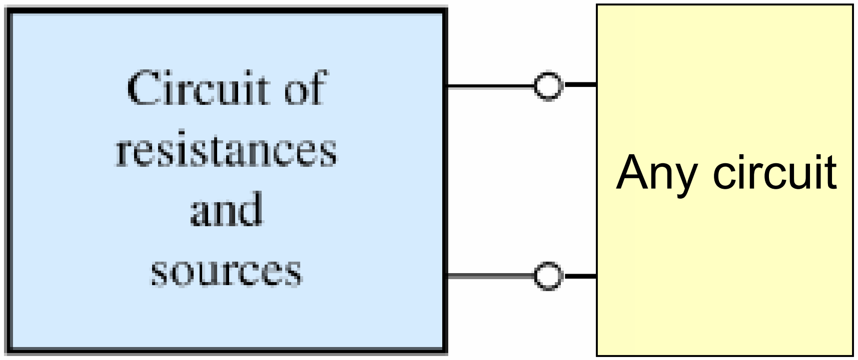


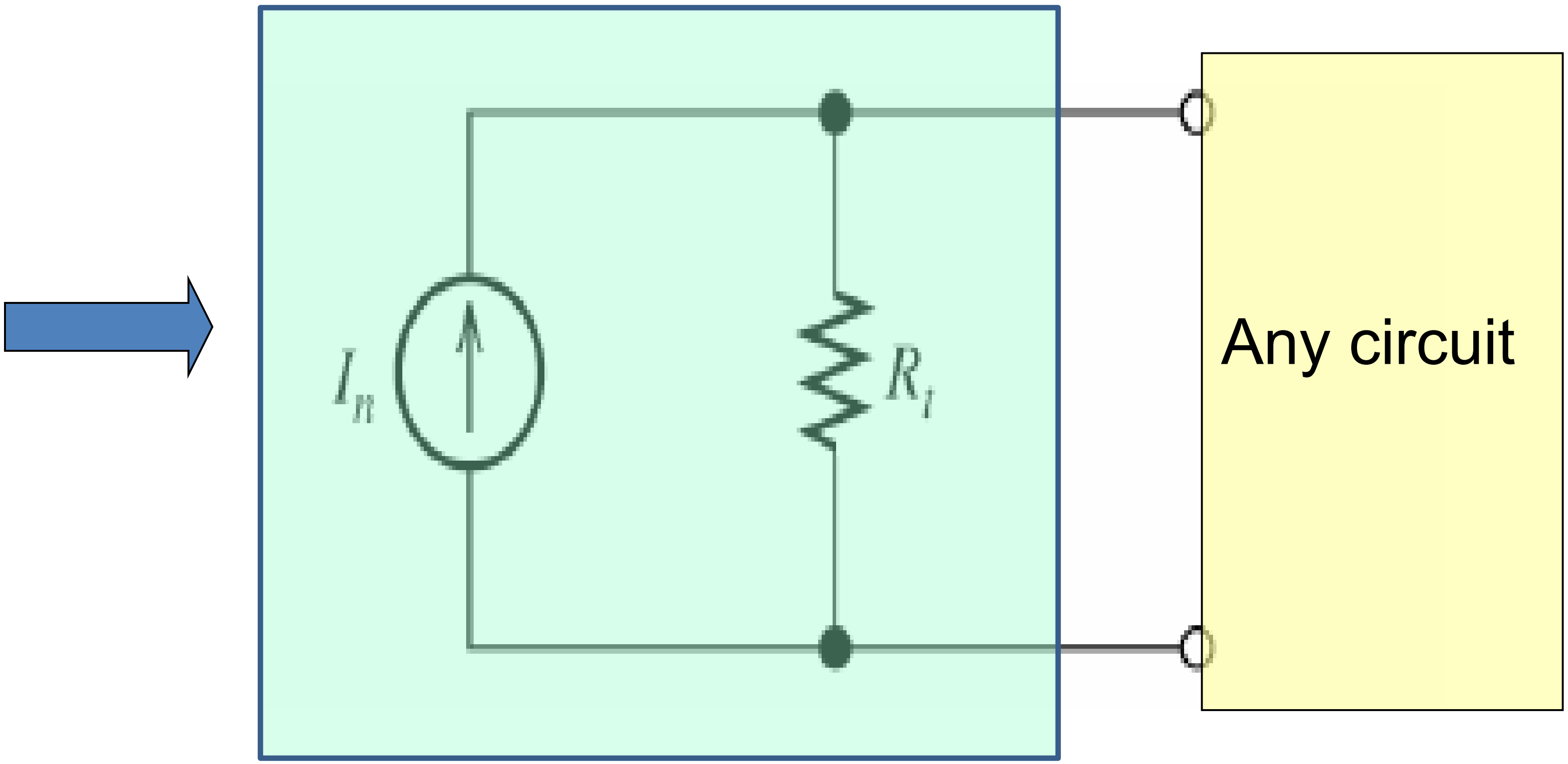
$$R_t = ?$$



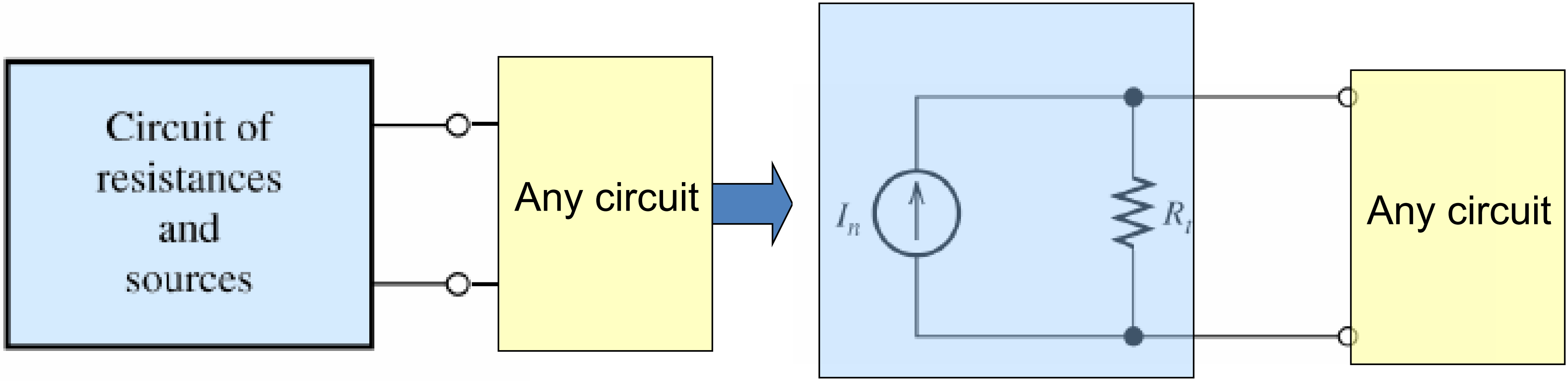
$$R_t = \frac{V_Z}{I_Z}$$

Norton's equivalent

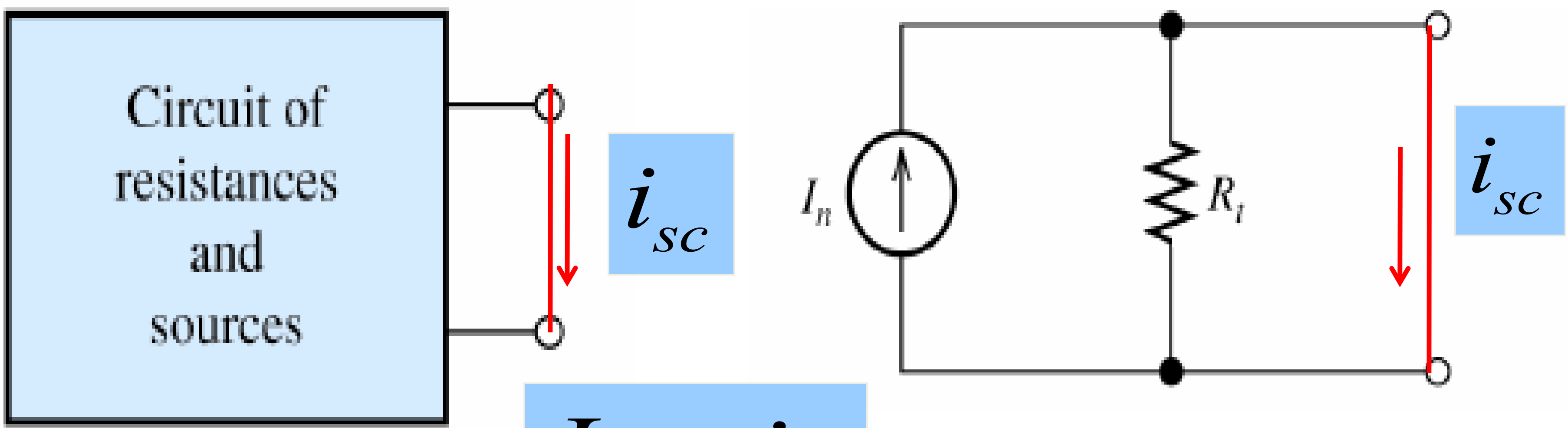




Norton's equivalent

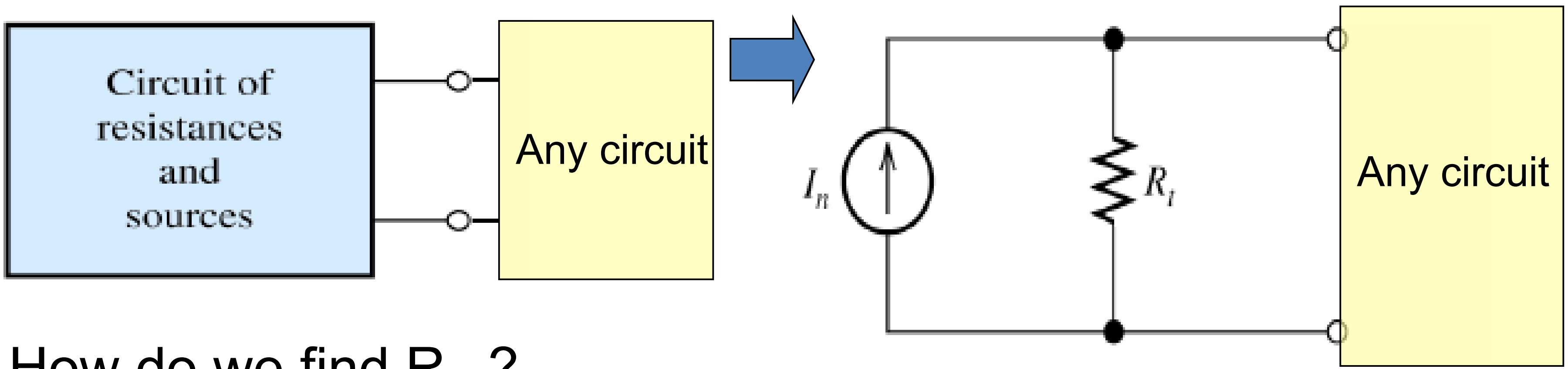


How do we find I_N ?

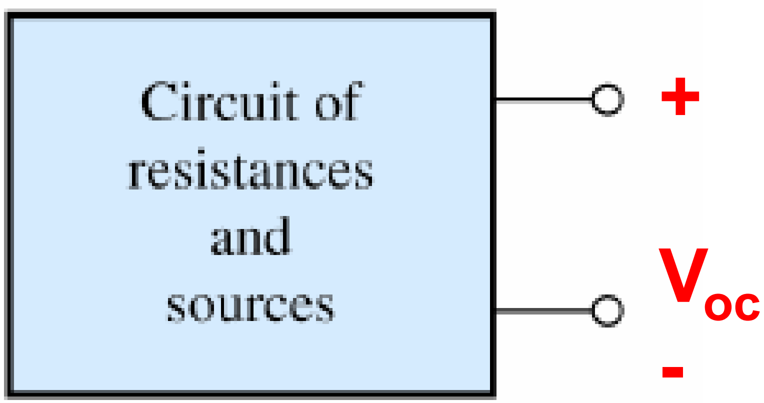


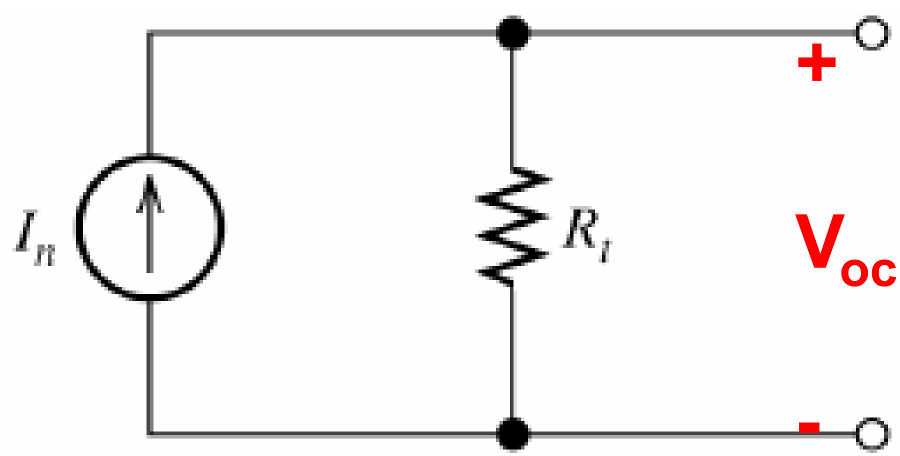
$$I_n = i_{sc}$$

Norton's equivalent



How do we find R_N ?

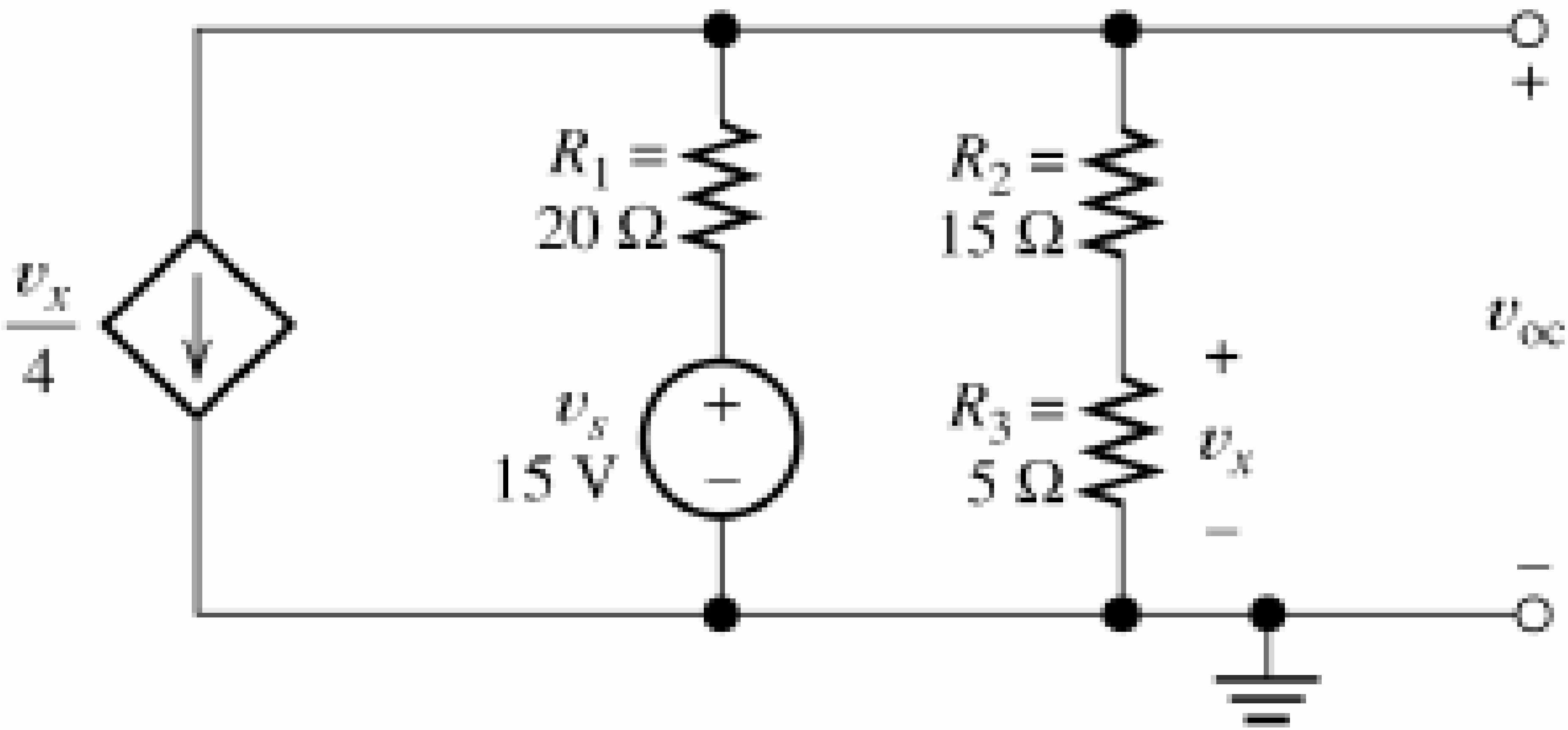


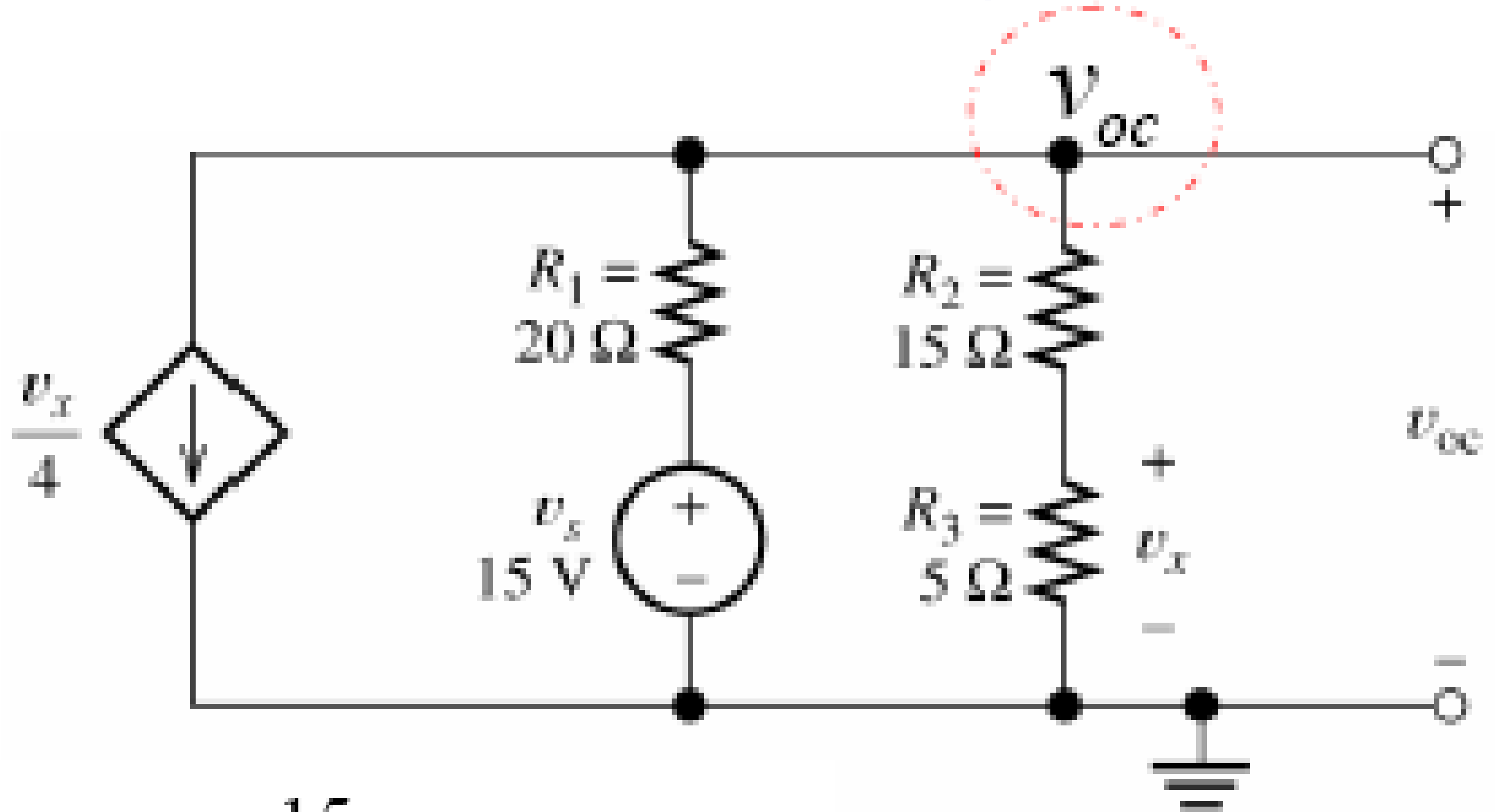


$$R_t = \frac{v_{oc}}{i_{sc}}$$

$$v_{oc} = I_n \times R_t$$

Example: Find the Norton equivalent for the following circuit

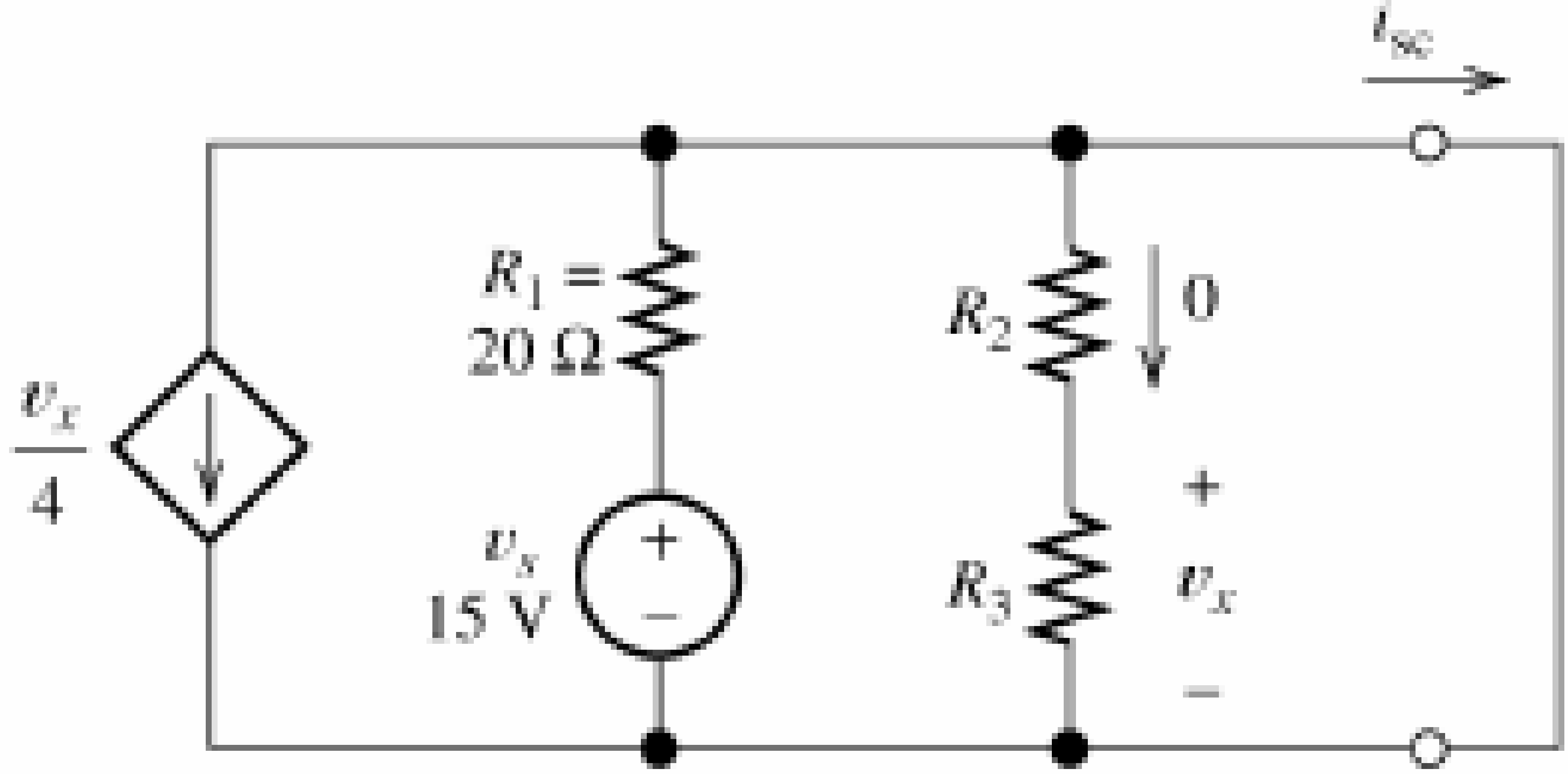




$$\frac{v_x}{4} + \frac{v_{oc} - 15}{R_1} + \frac{v_{oc}}{R_2 + R_3} = 0$$

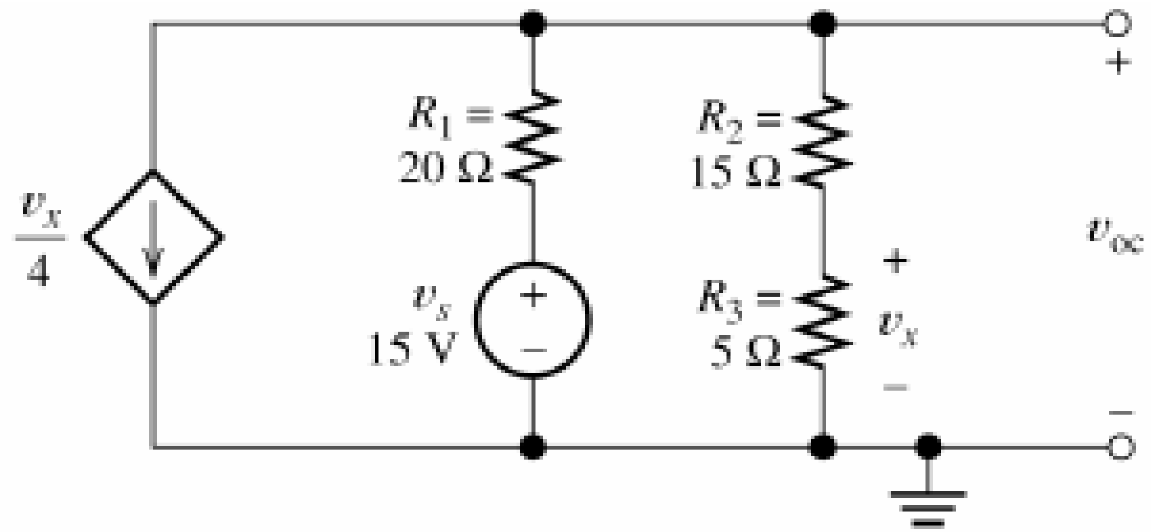
$$v_x = \frac{R_3}{R_2 + R_3} v_{oc} = 0.25 v_{oc}$$

$$\boxed{v_{oc} = 4.62\text{V}}$$



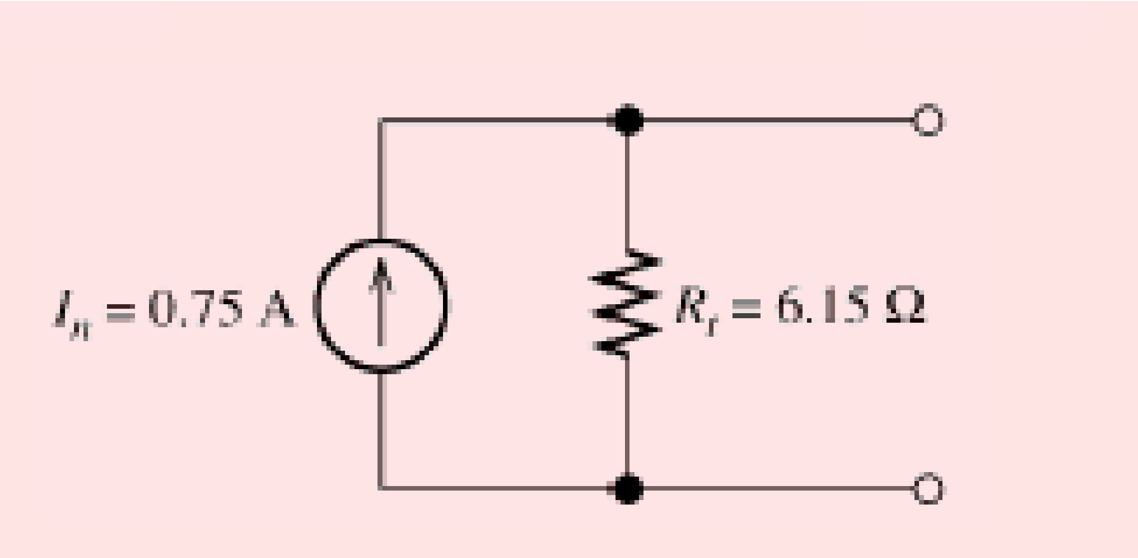
$$i_{sc} = \frac{v_s}{R_1} = \frac{15\text{V}}{20\Omega} = 0.75\text{A}$$

$$R_t = \frac{v_{oc}}{i_{sc}} = \frac{4.62\text{V}}{0.75\text{A}} = 6.15\Omega$$



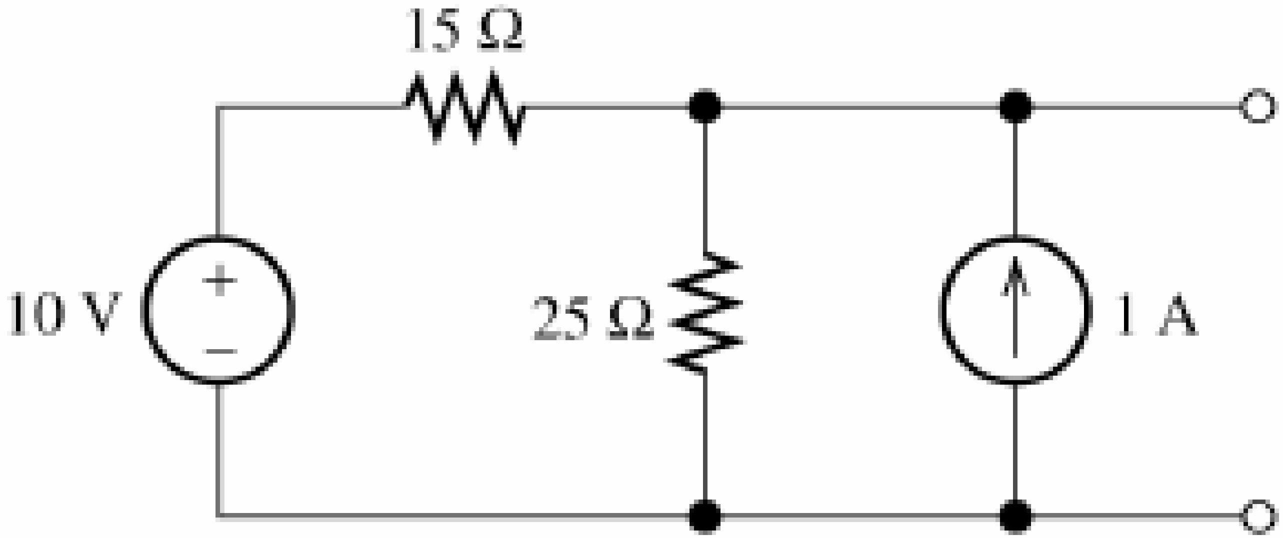
Norton's equivalent

$$\boxed{v_{oc} = 4.62\text{V}}$$

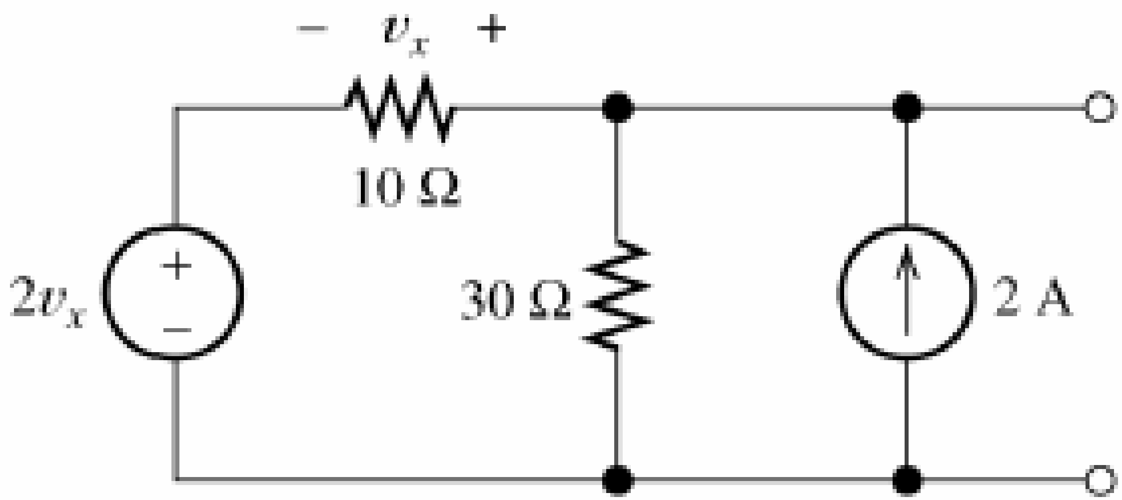


$R_t = 6.15\ \Omega$

Examples



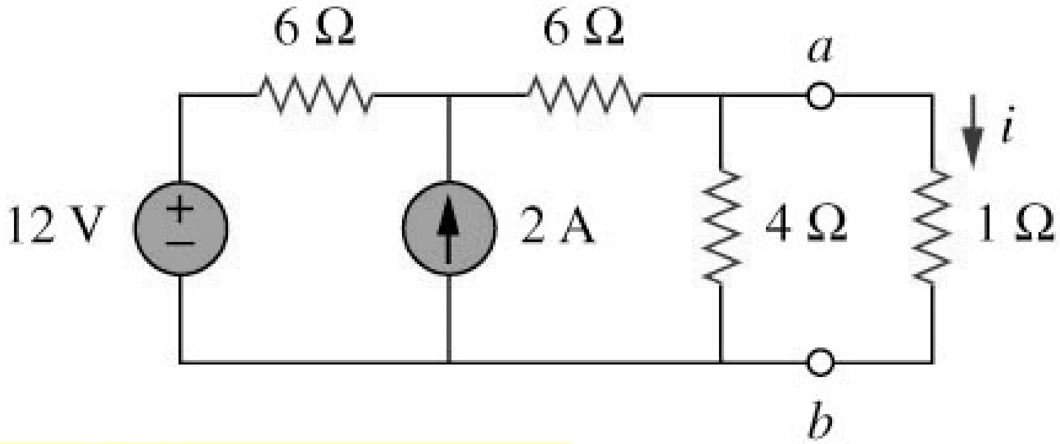
$I_n = 1.67\text{A}, R_t = 9.375\Omega$

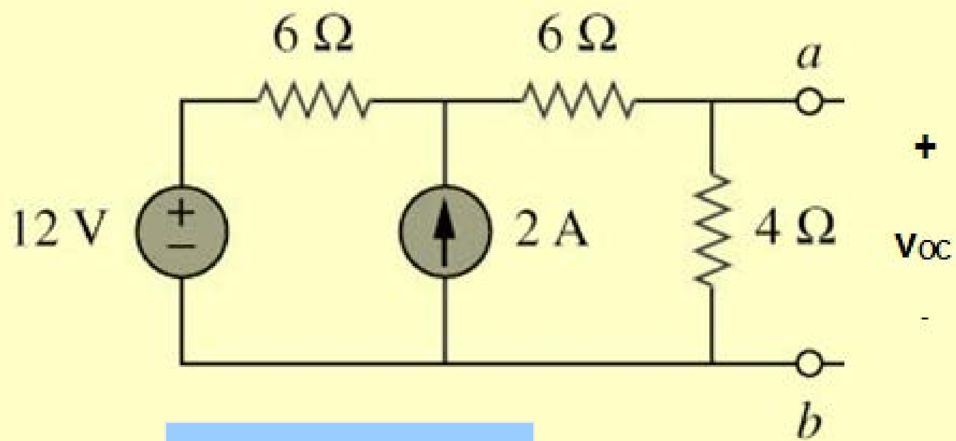


Solve for Rt

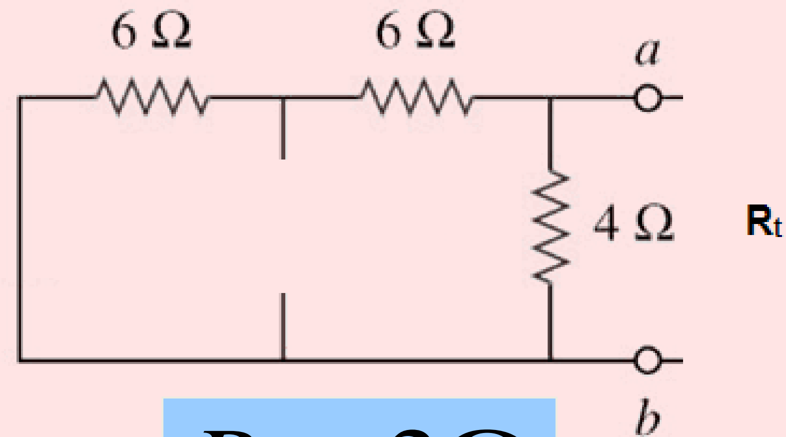
$I_n = 2\text{A}, R_t = 15\Omega$

Using Thevenin's theorem, find the equivalent circuit to the left of the terminals in the circuit shown below. Hence find i.

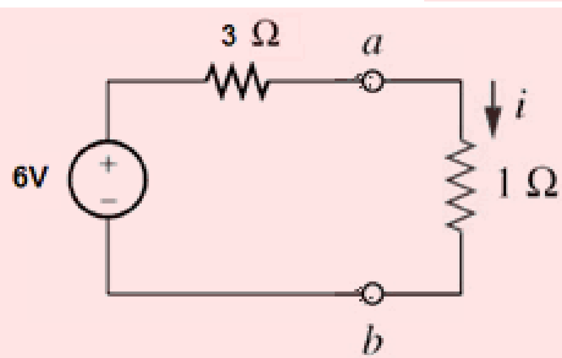




$$v_{oc} = 6V$$



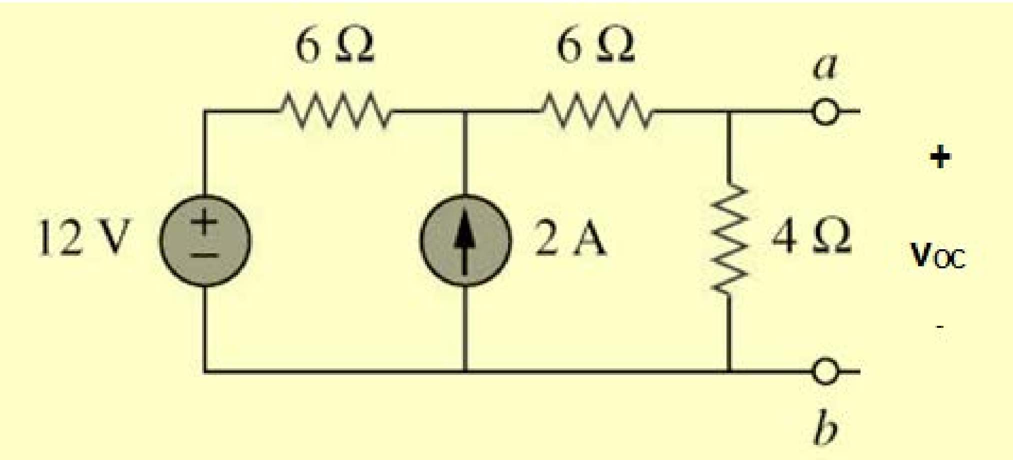
$$R_t = 3\Omega$$



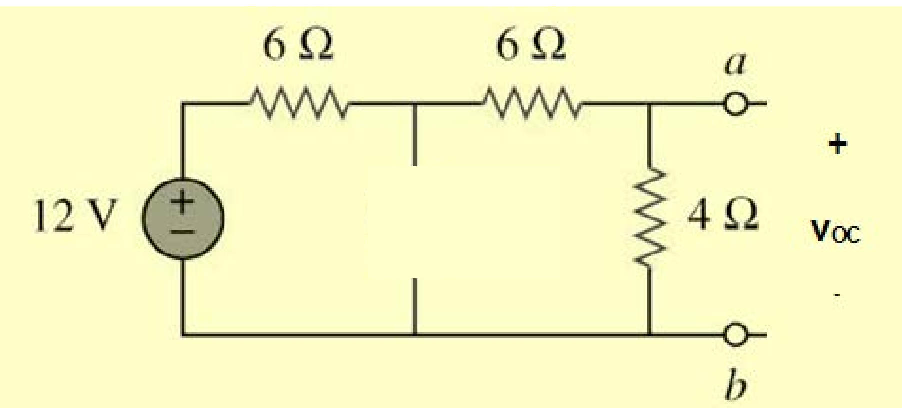
$$i = 1.5\,A$$

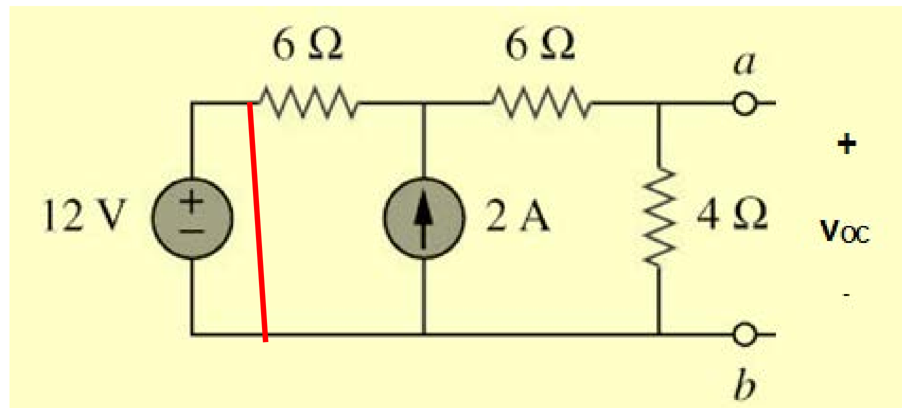
Use Superposition

$$v_{oc} = 6V$$



$$V_{oc} = V_{oc1} + V_{oc2} = 6$$

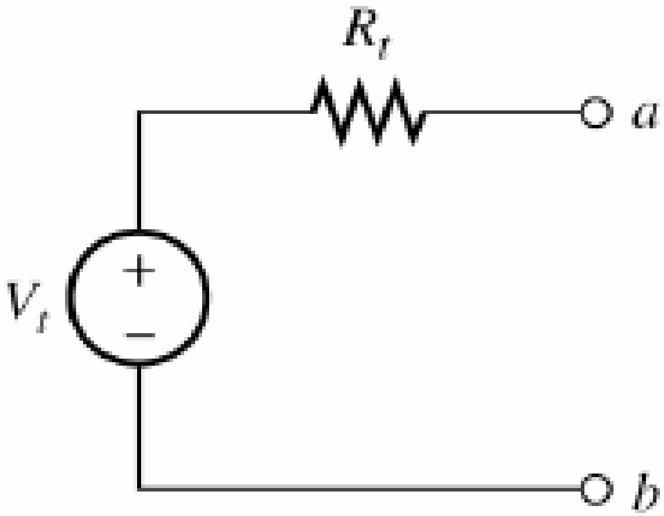


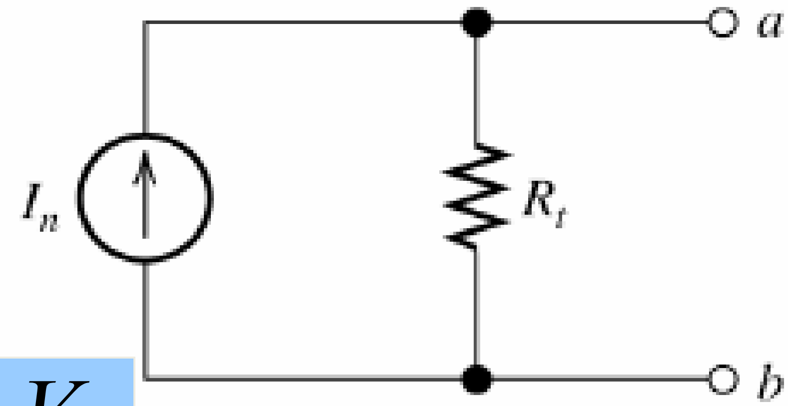


$$V_{oc1} = \frac{4}{4+12} \times 12 = 3$$

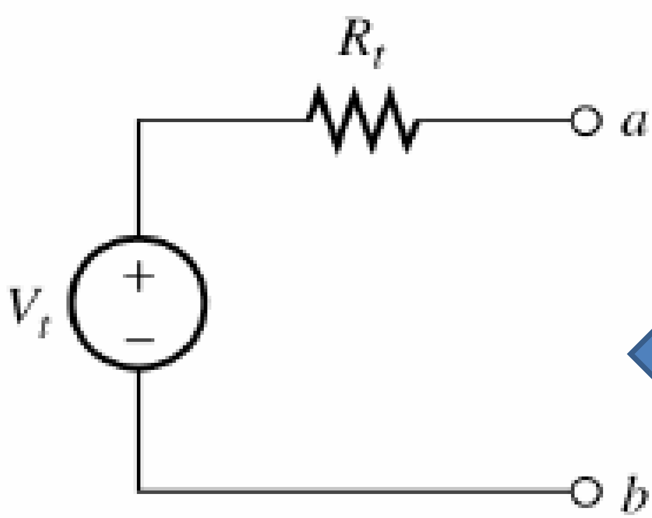
$$V_{oc2} = 4 \times \left(2 \times \frac{6}{6+10} \right) = 3$$

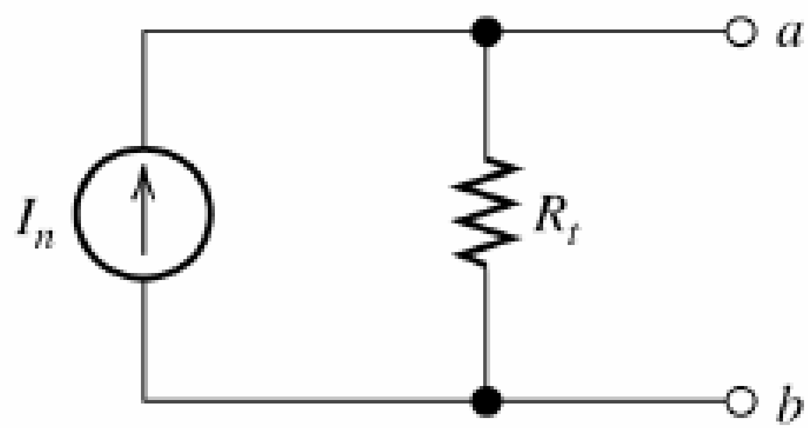
Source Transformation





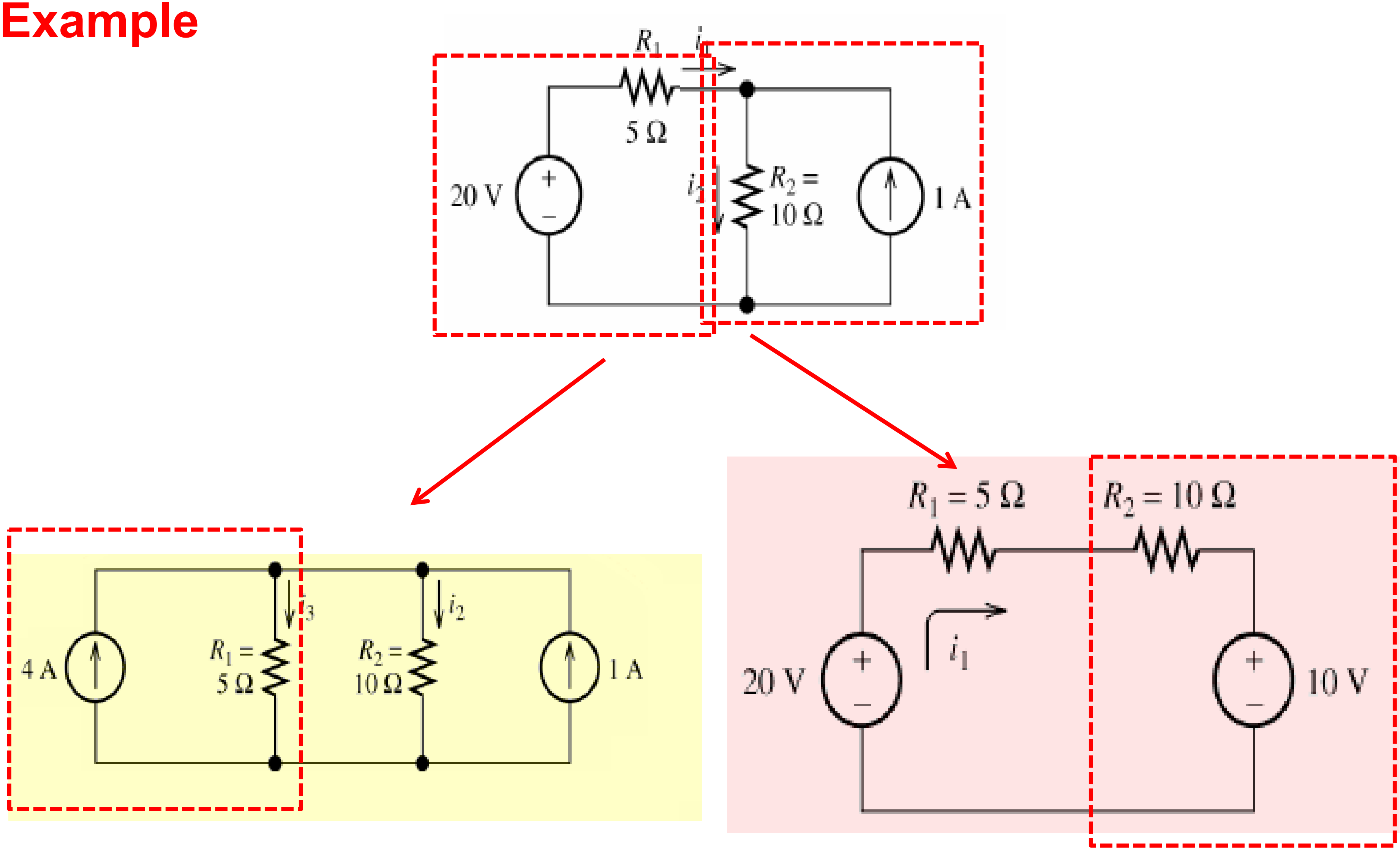
$$I_n = \frac{V_t}{R_t}$$





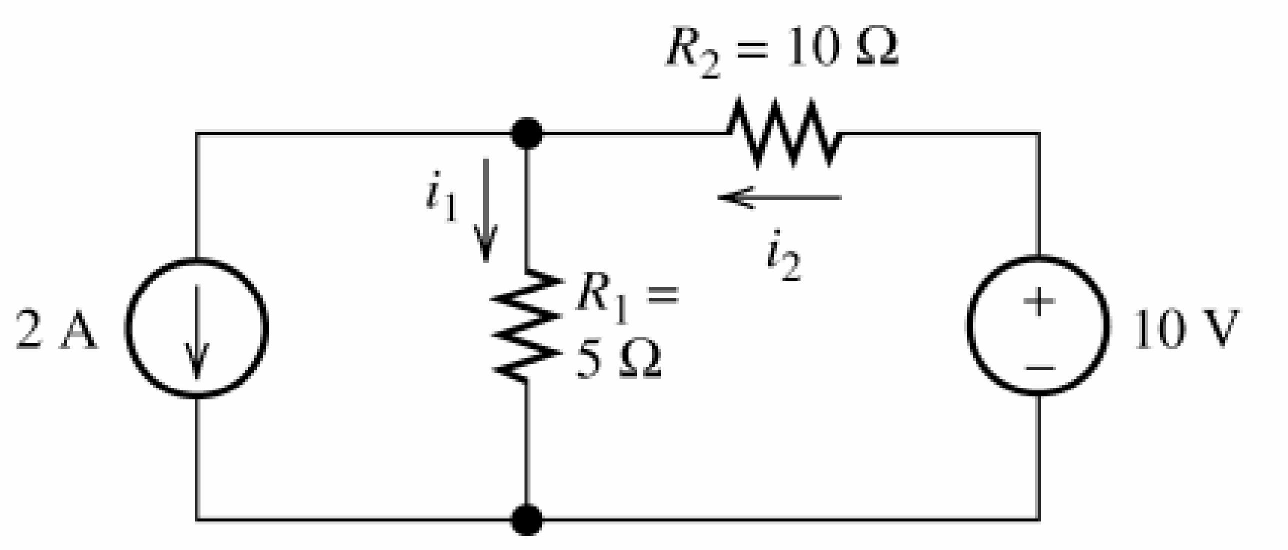
$$V_t = I_n \times R_t$$

Example

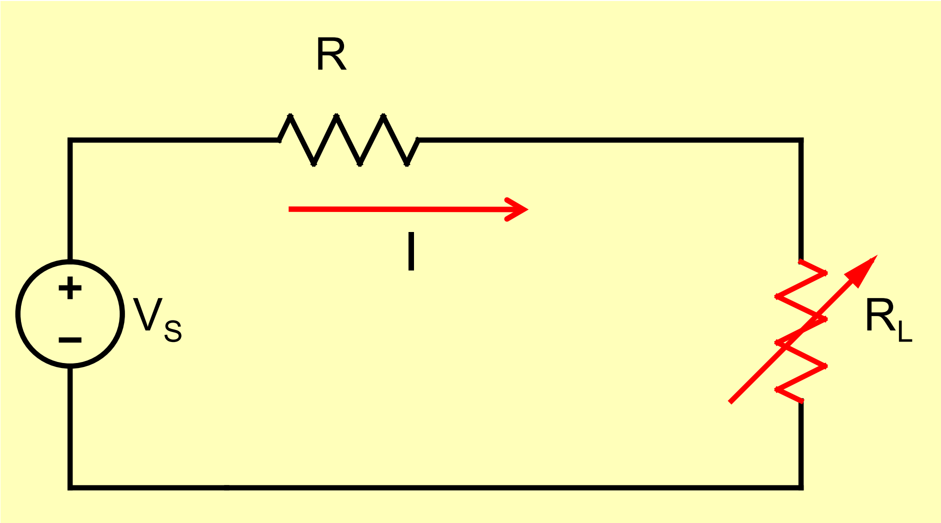


R_1
i_1
$5\ \Omega$
20 V
i_2
$R_2 =$
$10\ \Omega$
1 A
4 A
$R_1 =$
$5\ \Omega$
i_3
i_2
$R_2 =$
$10\ \Omega$
1 A
$R_1 = 5\ \Omega$
$R_2 = 10\ \Omega$
i_1
20 V
10 V

Use source transformation to solve for the indicated currents



Maximum Power Transfer for dc circuits



What value of R_L will give rise to maximum load power ?

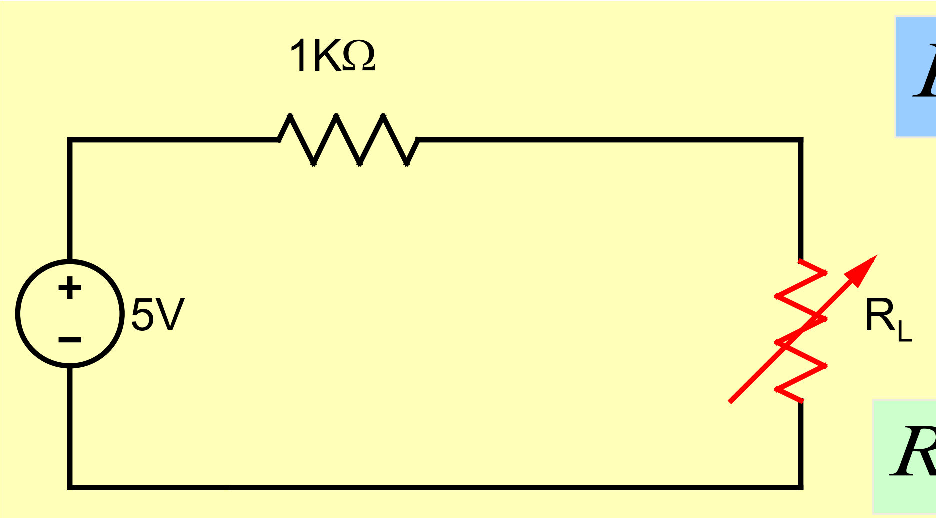
$$I = \frac{V_S}{R + R_L}$$

$$P_L = I^2 R_L = V_S^2 \times \frac{R_L}{(R + R_L)^2}$$

$$\frac{\partial P_L}{\partial R_L} = 0$$

$$R_L = R$$

$$P_{L\max} = \frac{V_S^2}{4R_L}$$

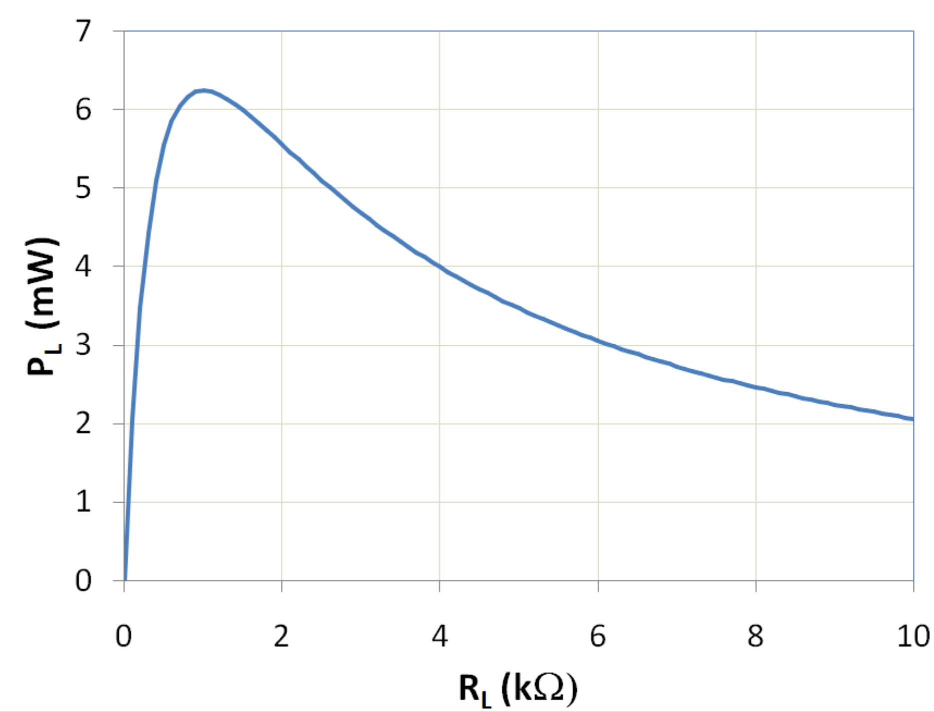


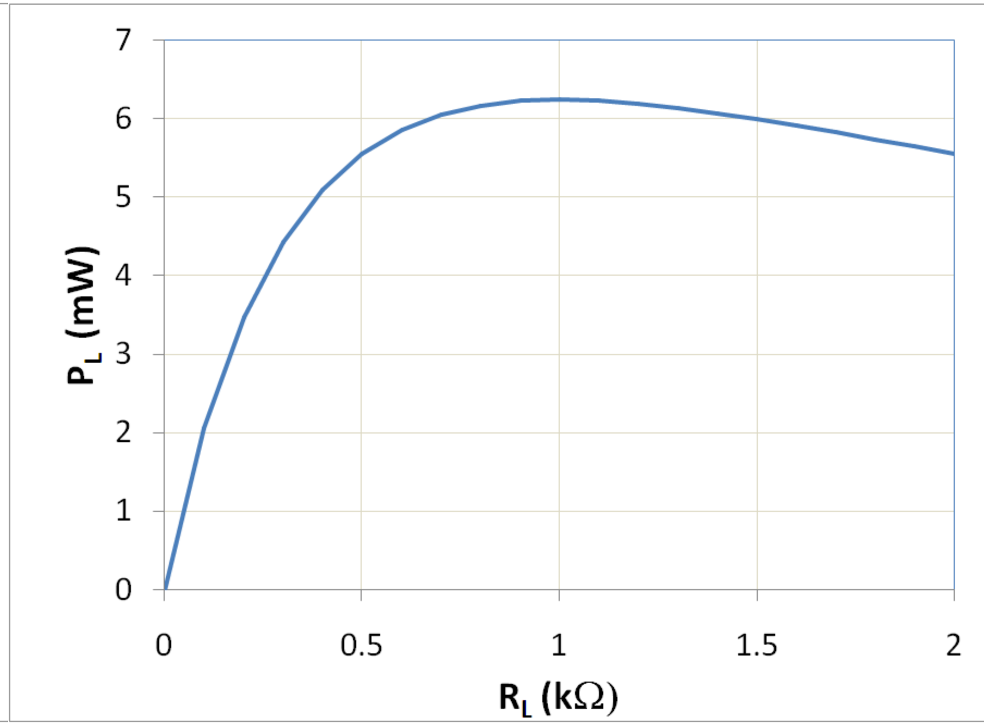
$$R_L = 1K \Rightarrow P_L = 6.25mW$$

$$R_L = 10K \Rightarrow P_L = 2mW$$

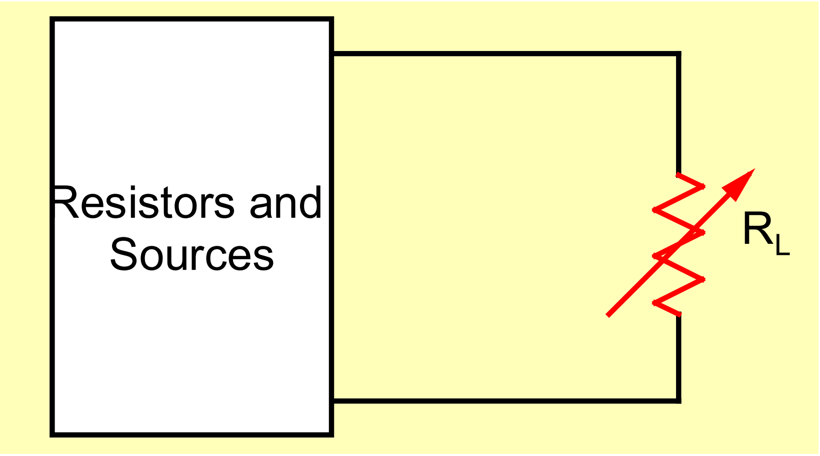
$$R_L = 0.2K \Rightarrow P_L = 3.47mW$$

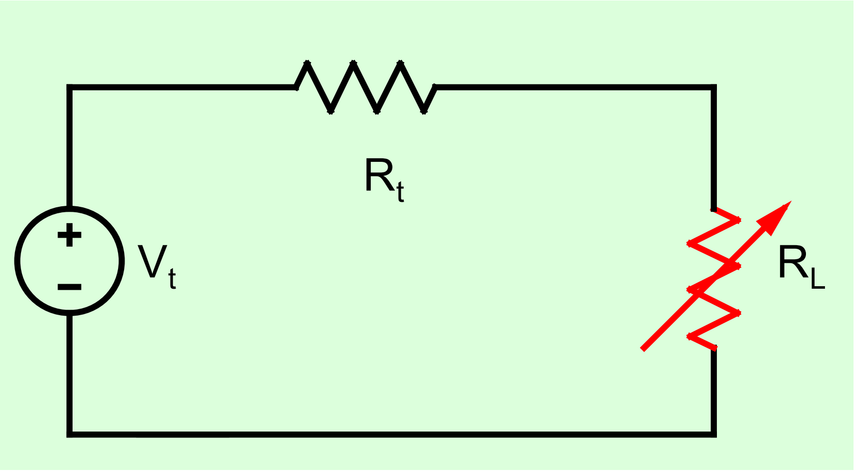
Maximum power is delivered to the load when **R_L = R**





General Case

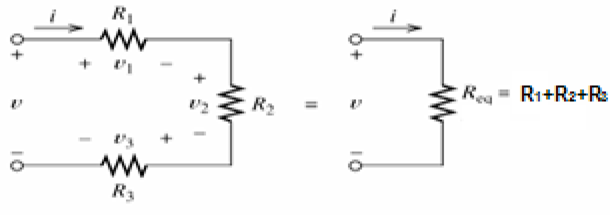




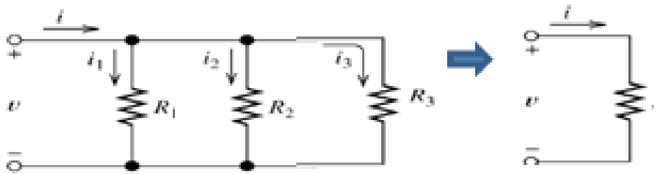
Maximum power is delivered to the load when **$R_L = R_t$**

Summary

Series/Parallel resistances

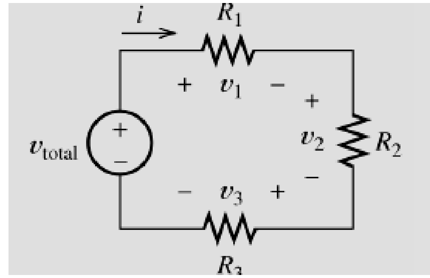


(a) Three resistances (b) Equivalent



$$R_{eq} = \frac{1}{1/R_1 + 1/R_2 + 1/R_3}$$

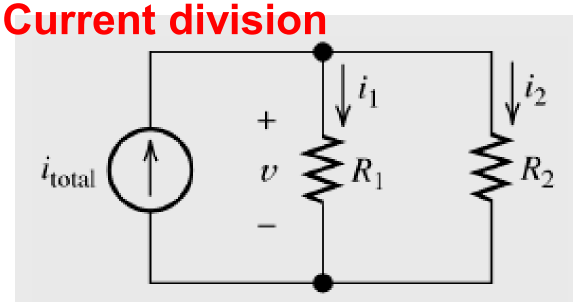
Voltage division



$$v_2 = R_2 i = \frac{R_2}{R_1 + R_2 + R_3} v_{total}$$

$$i_2 = \frac{v}{R_2} = \frac{R_1}{R_1 + R_2} i_{total}$$

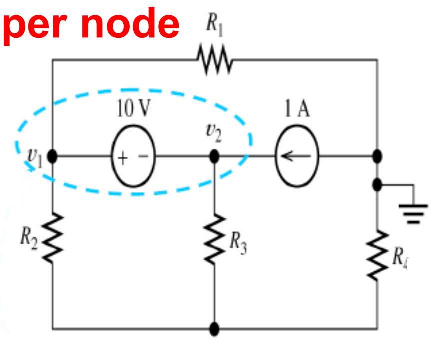
Current division



Mesh Analysis

1. Assign mesh currents i_1, i_2, , in to the n meshes.
2. Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.
3. Solve the resulting n simultaneous equations to get the mesh currents.

Super node

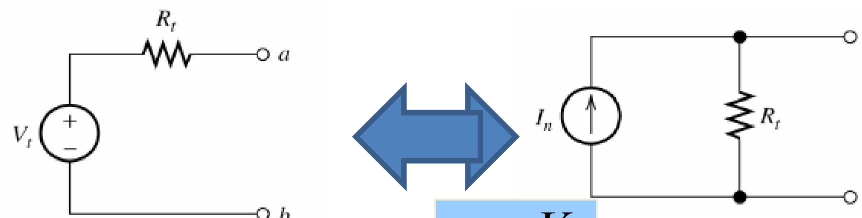


Nodal Analysis:

1. Identify and number the nodes
2. Choose a reference node
3. Write KCL for each node such that

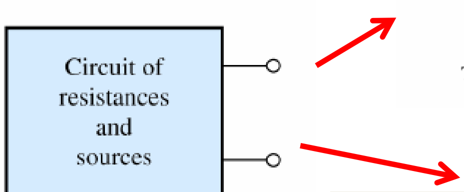
Sum of currents leaving a node is zero.

Source Transformation

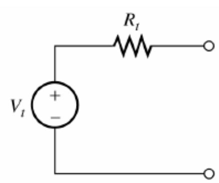


$$V_t = I_n \times R_t$$

$$I_n = \frac{V_t}{R_t}$$



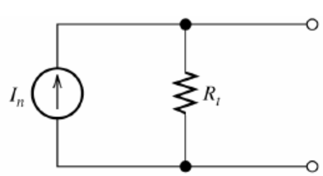
$$V_t = v_{oc}$$



Thévenin equivalent circuit

$$R_t = \frac{v_{oc}}{i_{sc}}$$

$$I_n = i_{sc}$$



Thevenin & Norton

The **superposition principle** states that the total response is the sum of the responses to each of the independent sources acting individually.