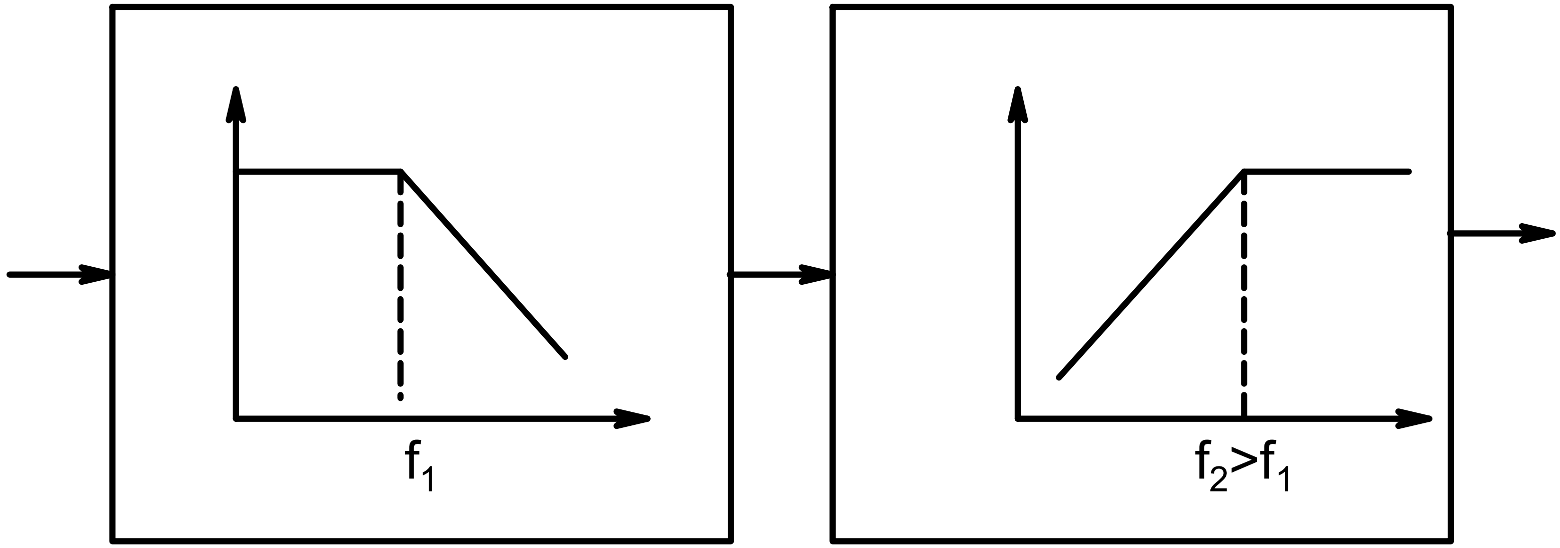
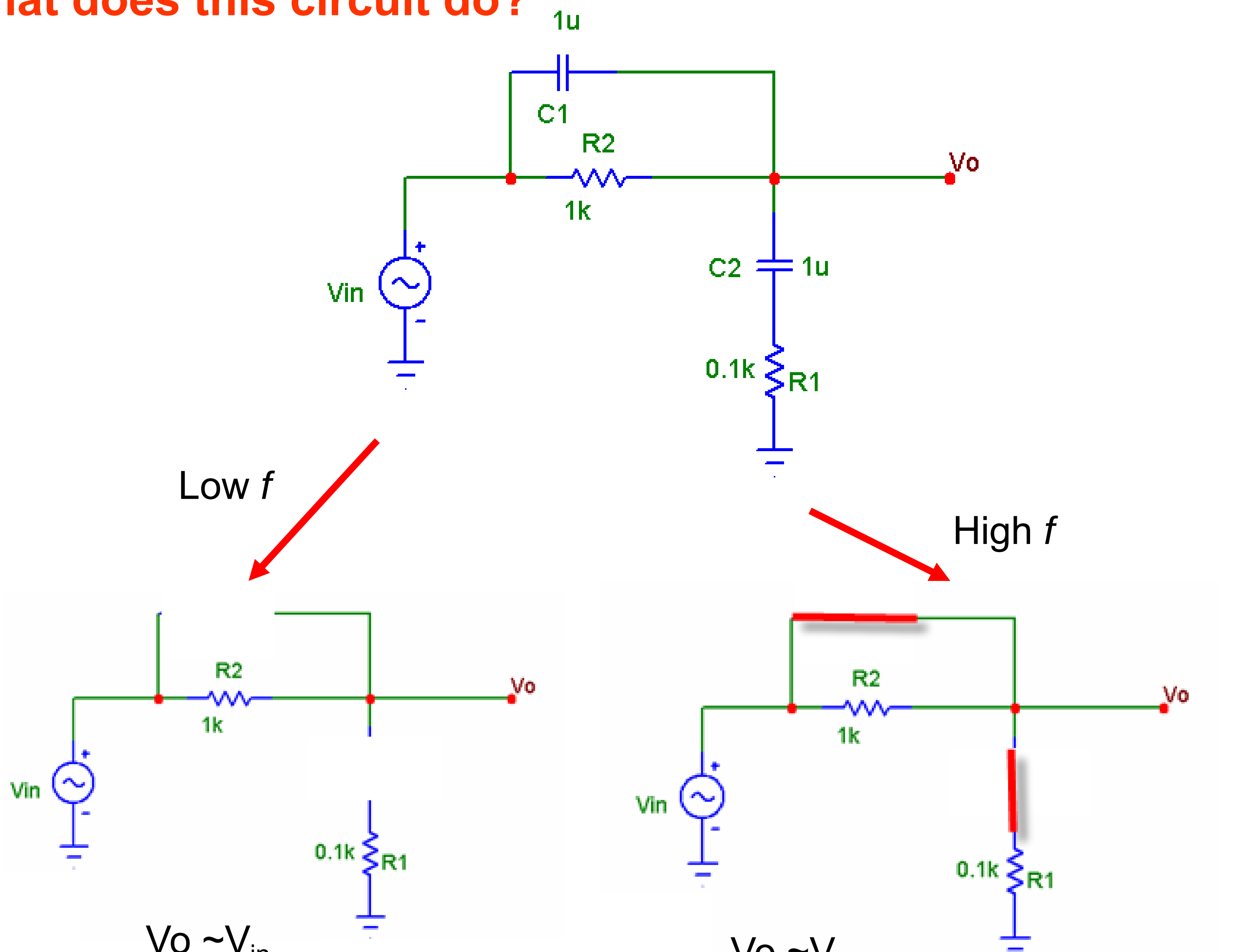
Bandstop Filter



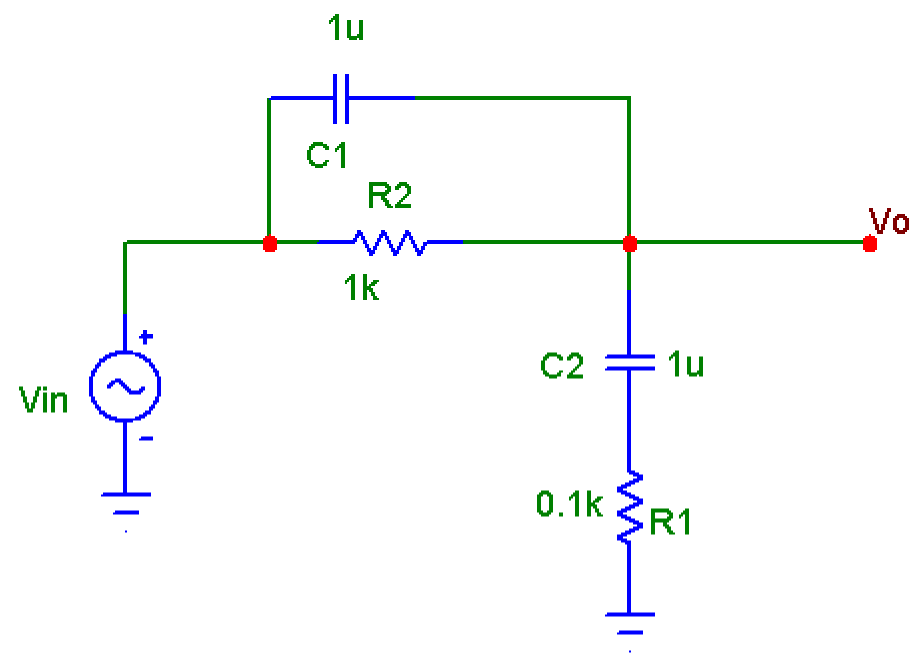
Will this work?

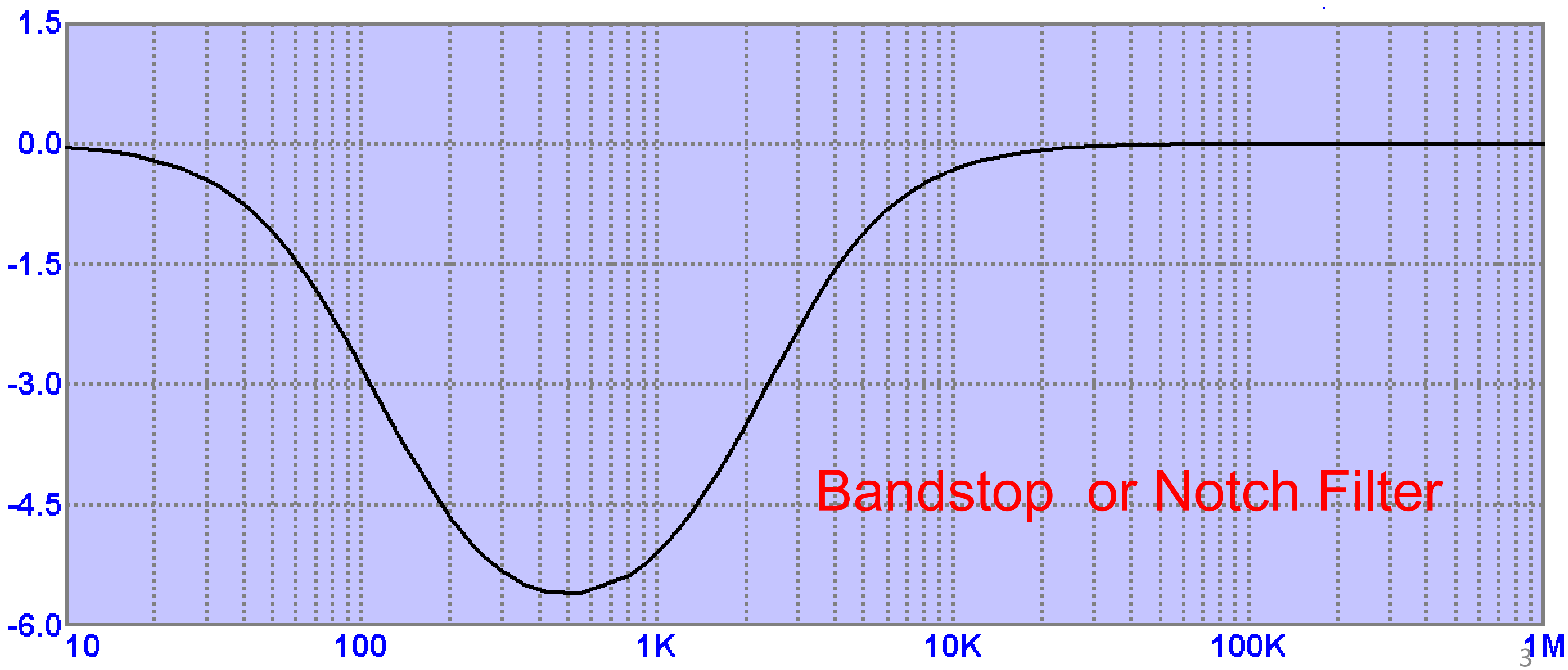
What does this circuit do?



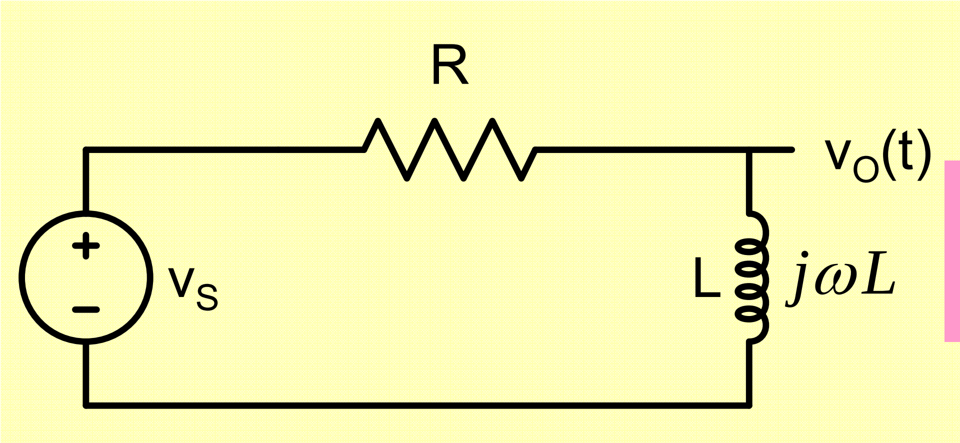
Vo ~V_{in} Vo ~V_{in}

What does this circuit do?





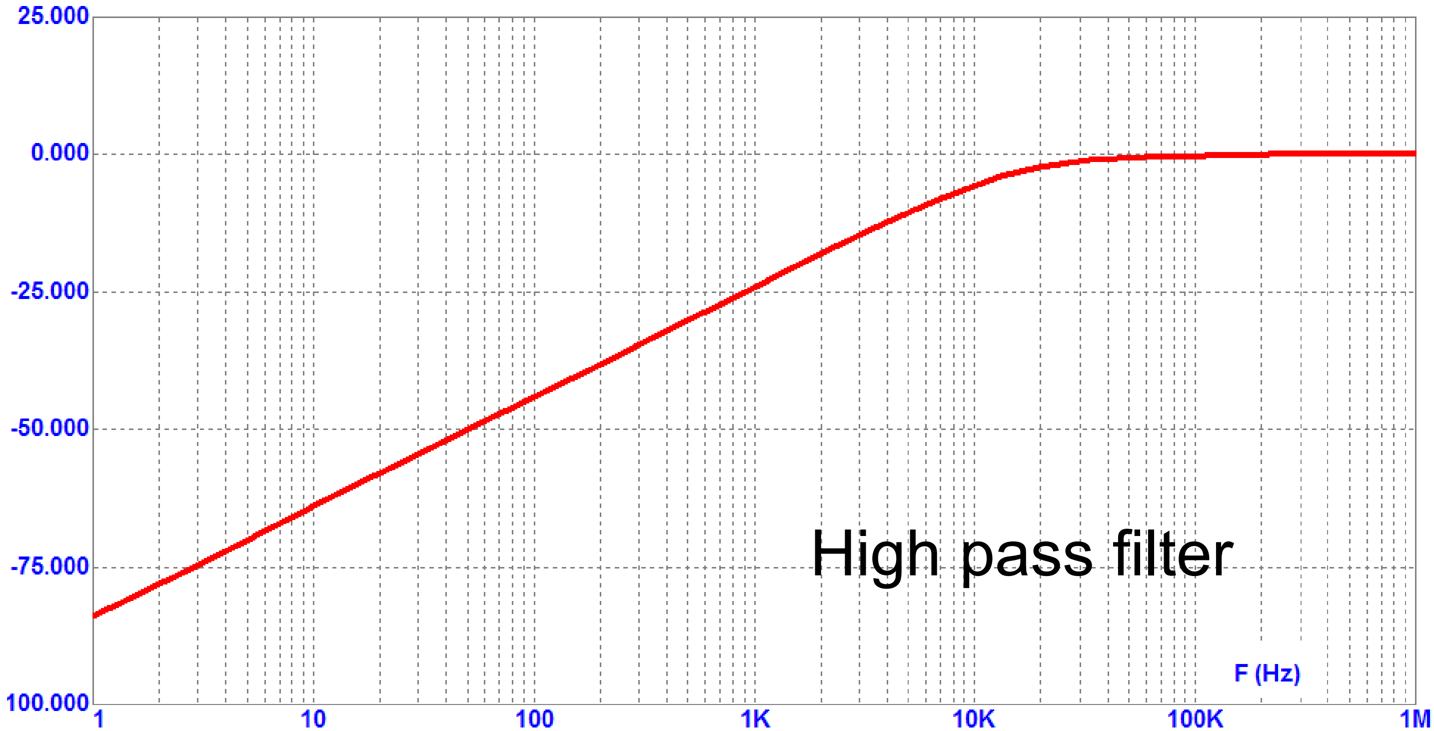
R-L Circuits (Filters)



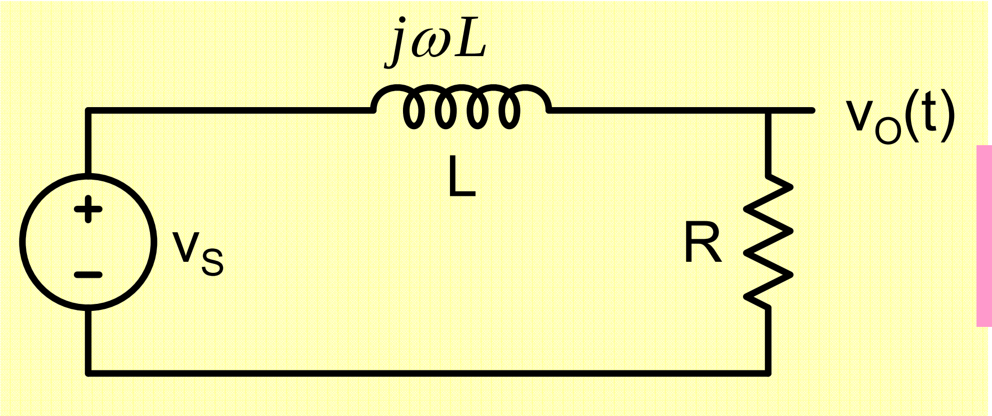
$$H(\omega) = \frac{V_O(\omega)}{V_S(\omega)}$$

$$H(\omega) = \frac{j\omega L}{R + j\omega L} = \frac{j(\omega/\omega_{3dB})}{1 + j(\omega/\omega_{3dB})}$$

$$\omega_{3dB} = \frac{R}{L}$$



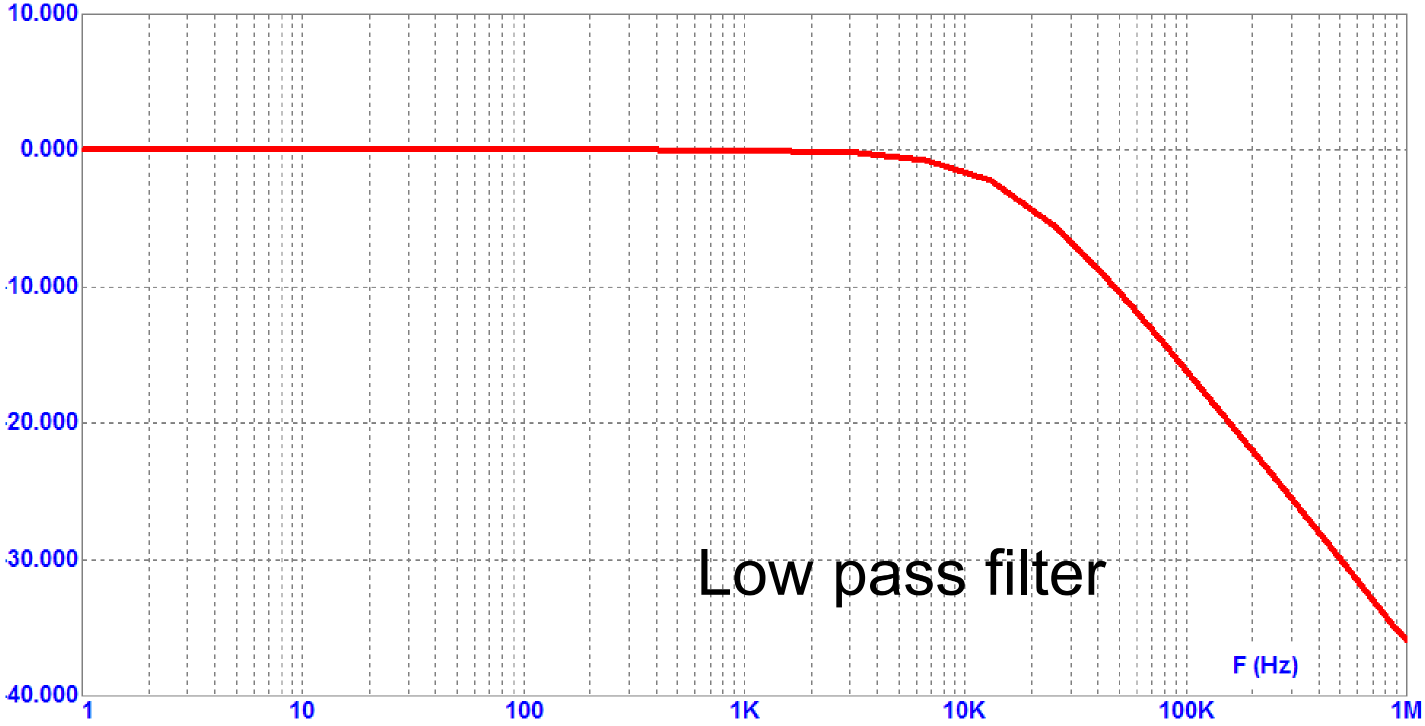
R-L Circuits



$$H(\omega) = \frac{V_O(\omega)}{V_S(\omega)}$$

$$H(\omega) = \frac{R}{R + j\omega L} = \frac{1}{1 + j(\omega / \omega_{3dB})}$$

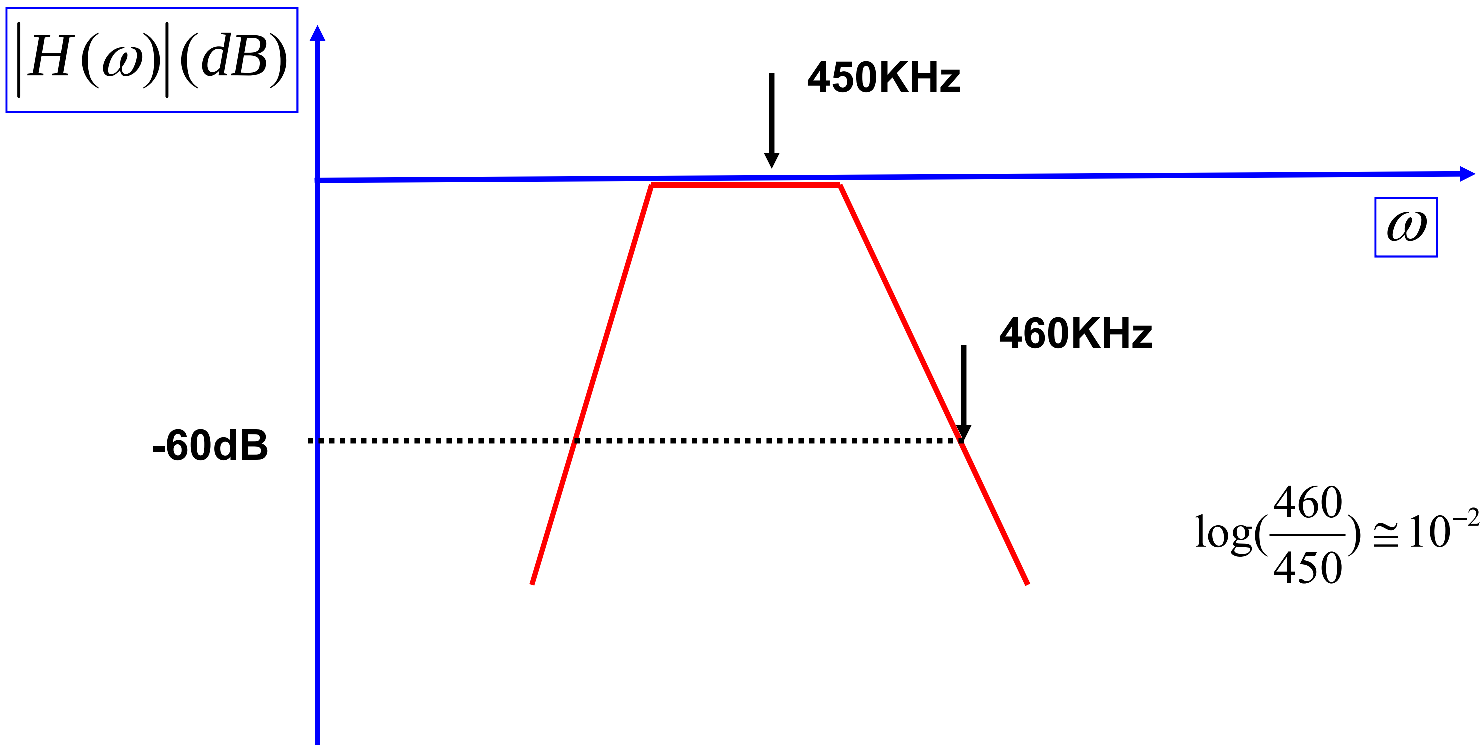
$$\omega_{3dB} = \frac{R}{L}$$



Amplitude Modulated (AM) Radio

Different radio channels are separated by very narrow frequency interval.

For example, one may want to receive a 450KHz signal but reject 460KHz or 440KHz



$$\log\left(\frac{460}{450}\right) \cong 10^{-2}\, decades$$

This implies an attenuation of -6000 dB/decade !!

Resonance

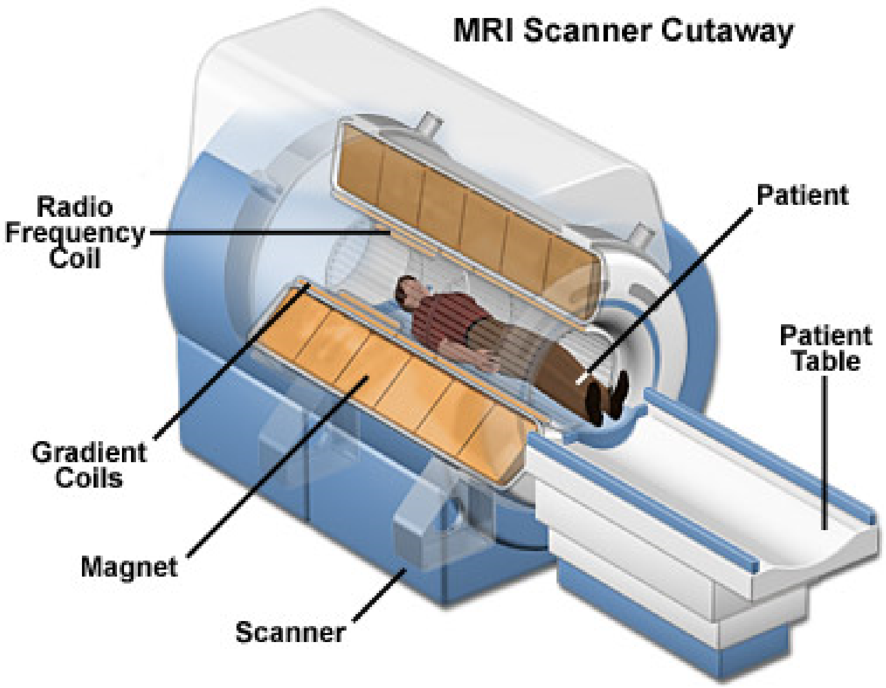
Galloping Gertie, the first Tacoma Narrows Bridge in Tacoma Washington, United States. The bridge opened on July 1, 1940 and from the start became notorious for its movement during windy days, earning the nickname "Galloping Gertie". The wind-induced collapse occurred on November 7, 1940, due partially to a physical phenomenon known as **mechanical resonance**…..wikepedia

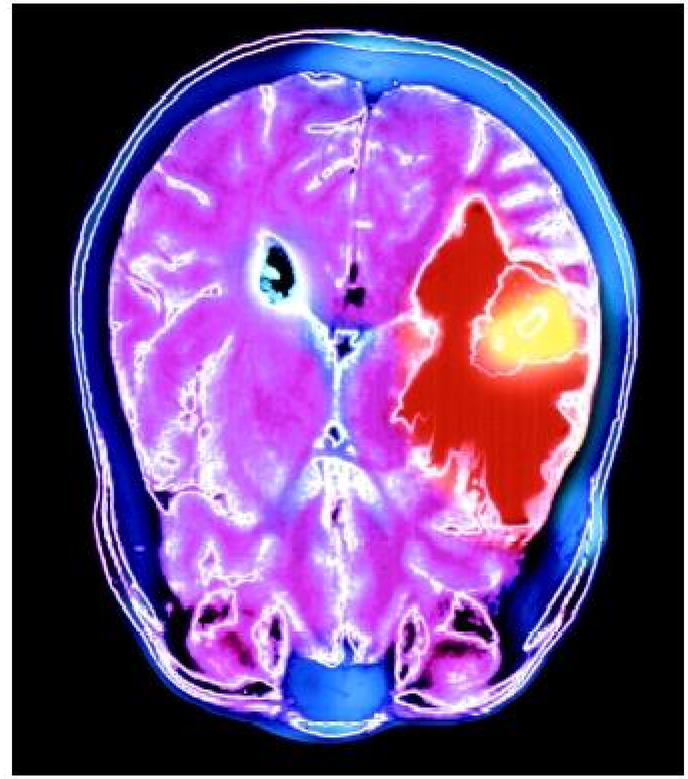
Resonance describes when a vibrating system or external force drives another system to oscillate with greater amplitude at a specific preferential frequency.

At resonant frequencies, small periodic driving forces have the ability to produce large amplitude oscillations. This is because the system stores vibrational energy.

Resonant systems can be used to generate vibrations of a specific frequency (e.g., musical instruments), or pick out specific frequencies from a complex vibration containing many frequencies (e.g., filters).

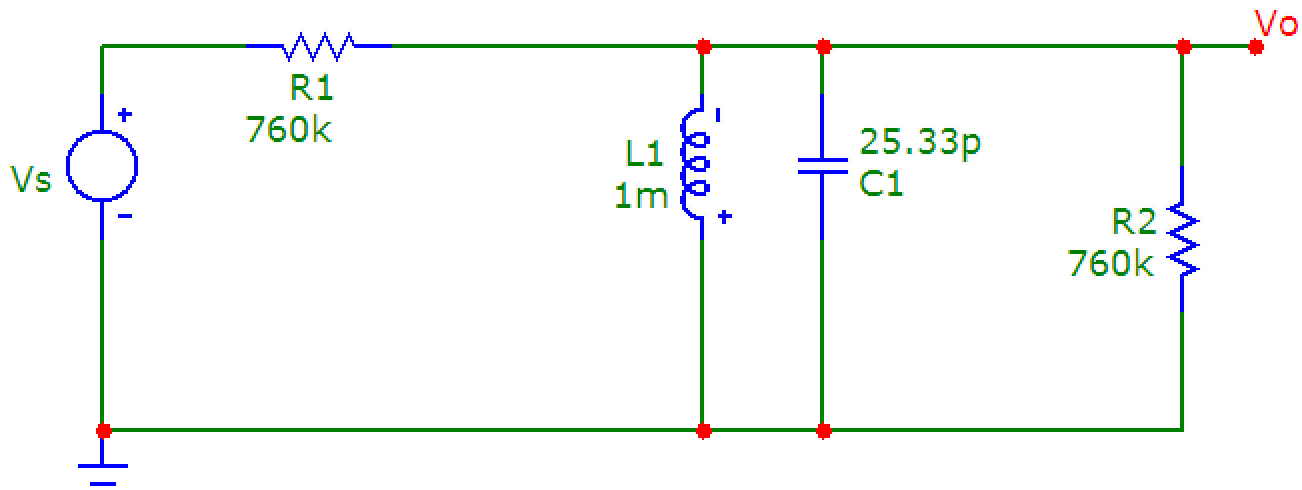
Source: Wikipedia

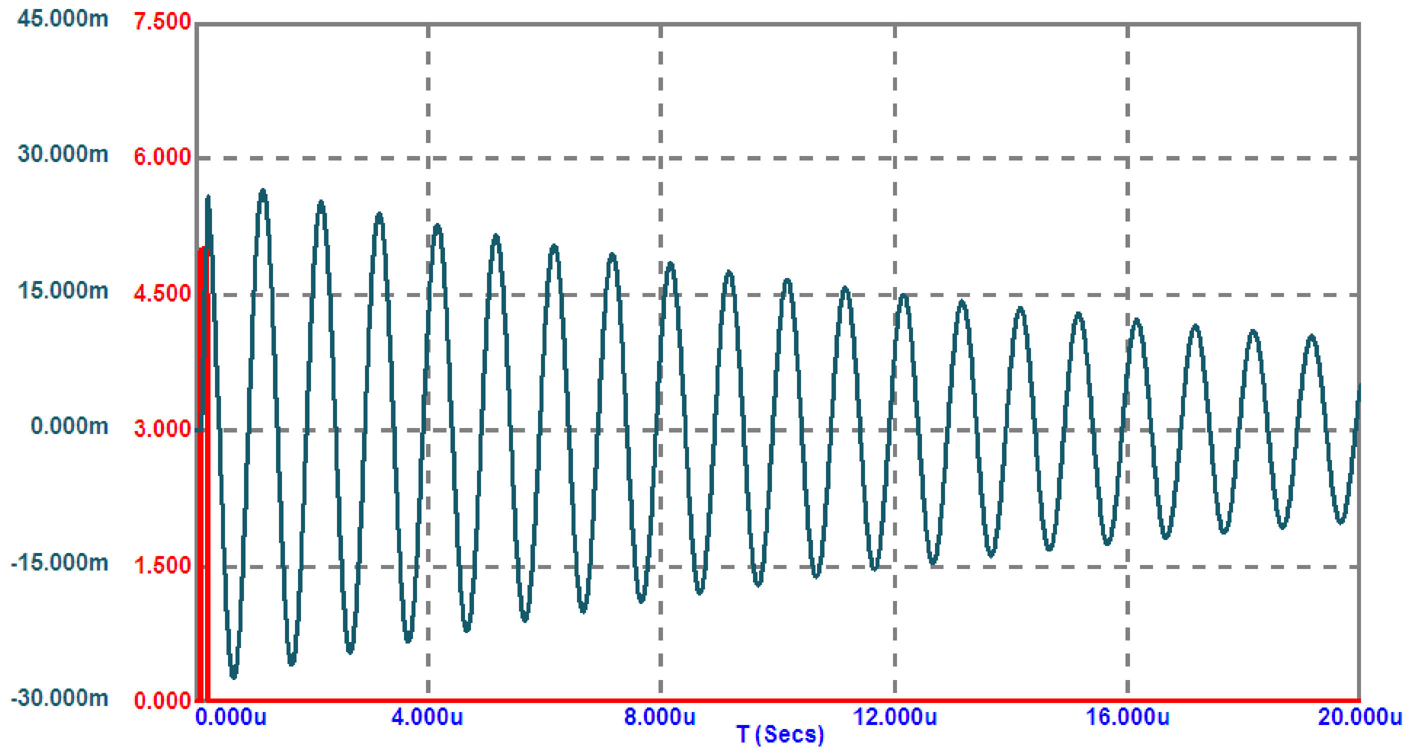




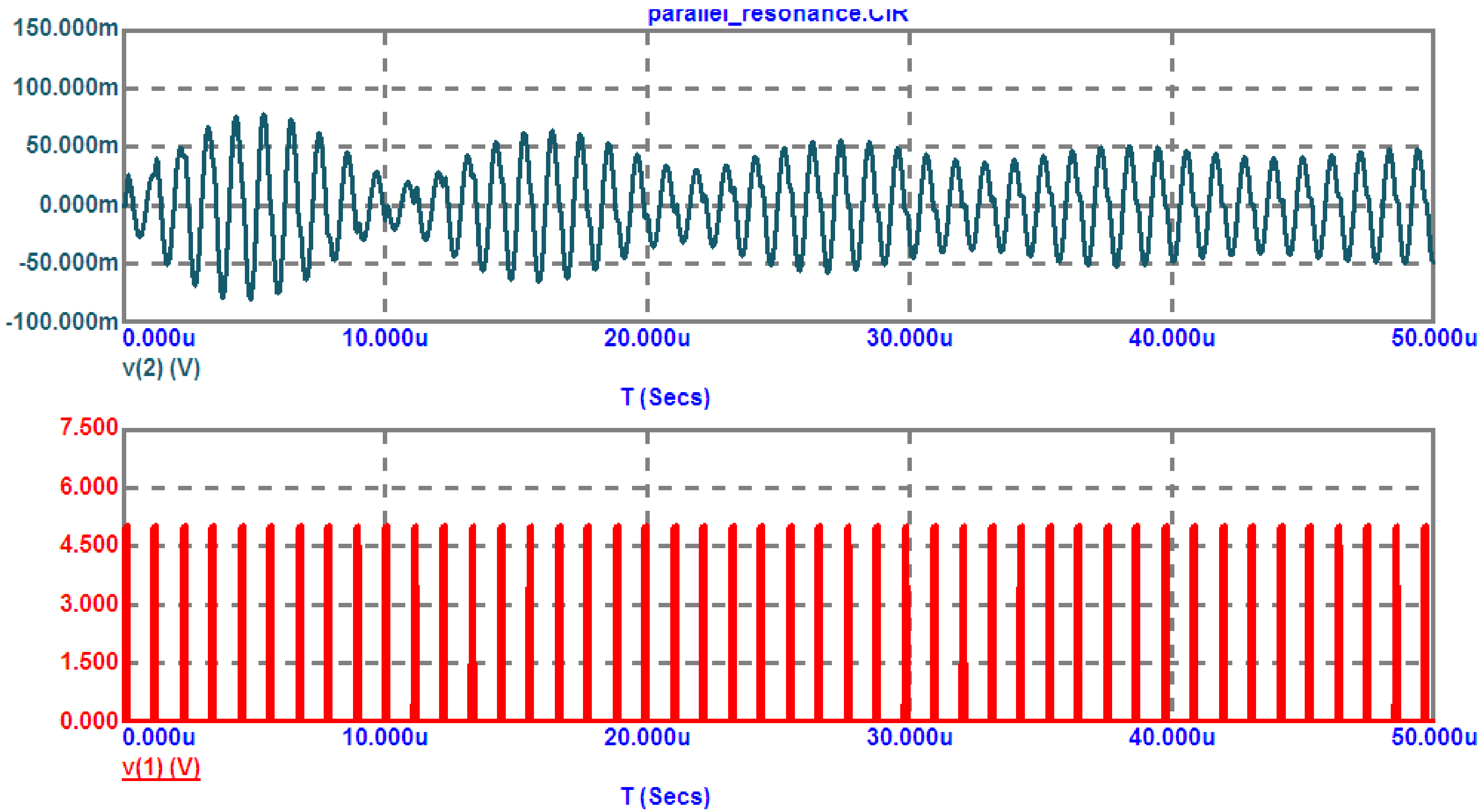
Nuclear magnetic resonance

Resonance



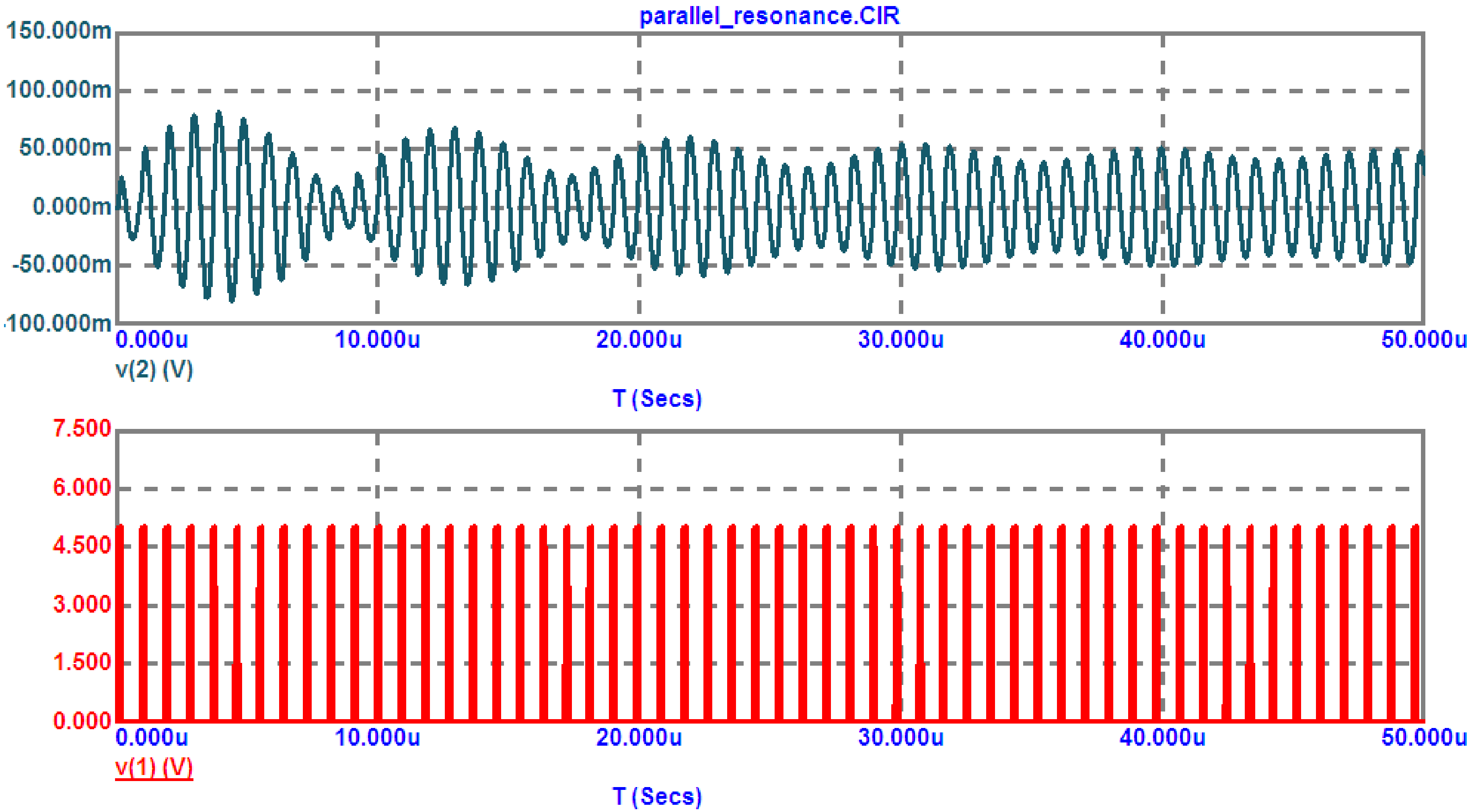


A small disturbance leads to oscillatory behavior

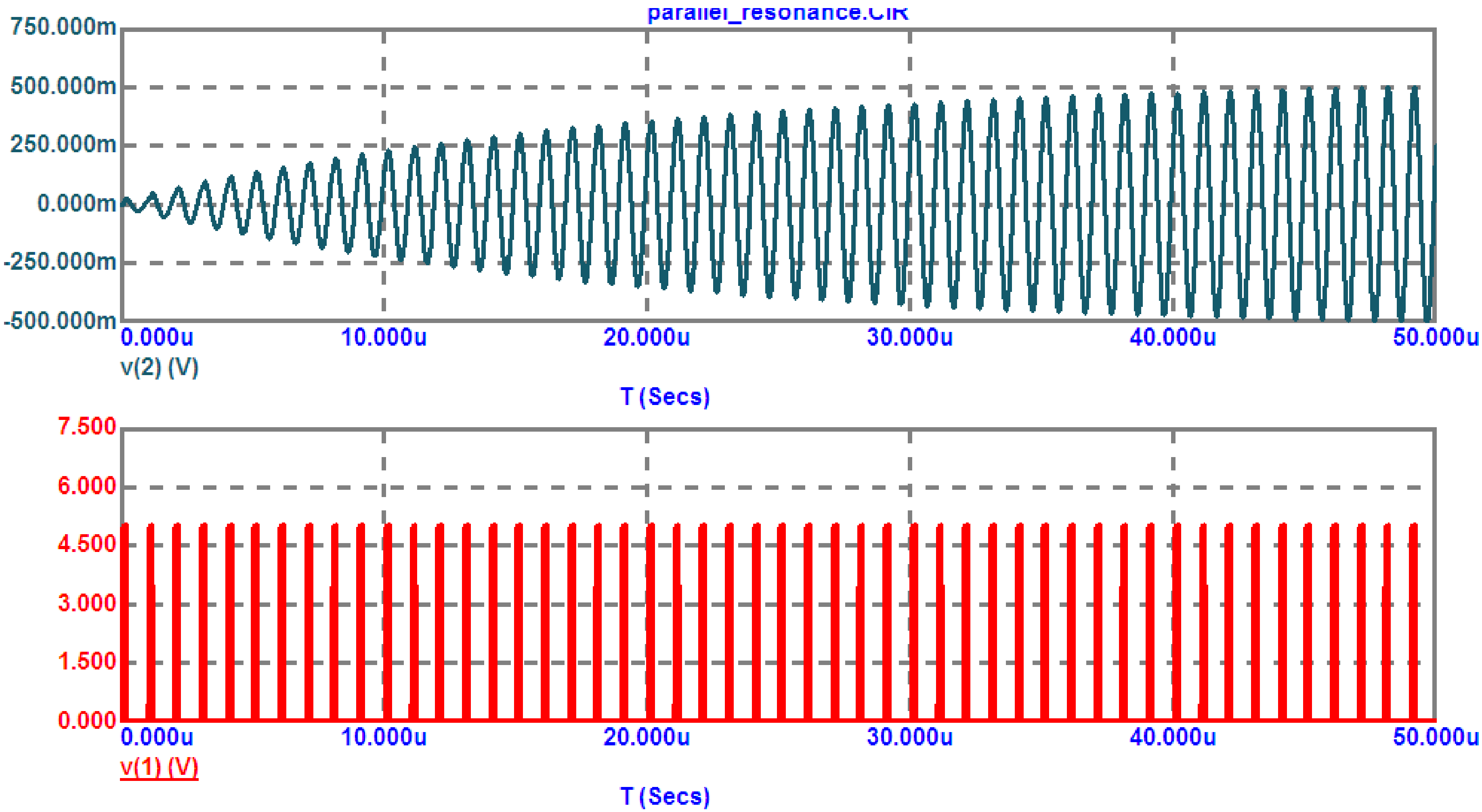


parallel_resonance.CIR
150.000m
100.000m
50.000m
0.000m
-50.000m
-100.000m
0.000u
10.000u
20.000u
30.000u
40.000u
50.000u
v(2) (V)
T (Secs)
7.500
6.000
4.500
3.000
1.500
0.000
0.000u
10.000u
20.000u
30.000u
40.000u
50.000u
v(1) (V)
T (Secs)

T = 1.1μs


parallel_resonance.CIR
150.000m
100.000m
50.000m
0.000m
-50.000m
-100.000m
0.000u
10.000u
20.000u
30.000u
40.000u
50.000u
v(2) (V)
T (Secs)
7.500
6.000
4.500
3.000
1.500
0.000
0.000u
10.000u
20.000u
30.000u
40.000u
50.000u
v(1) (V)
T (Secs)

$T = 0.9\mu s$

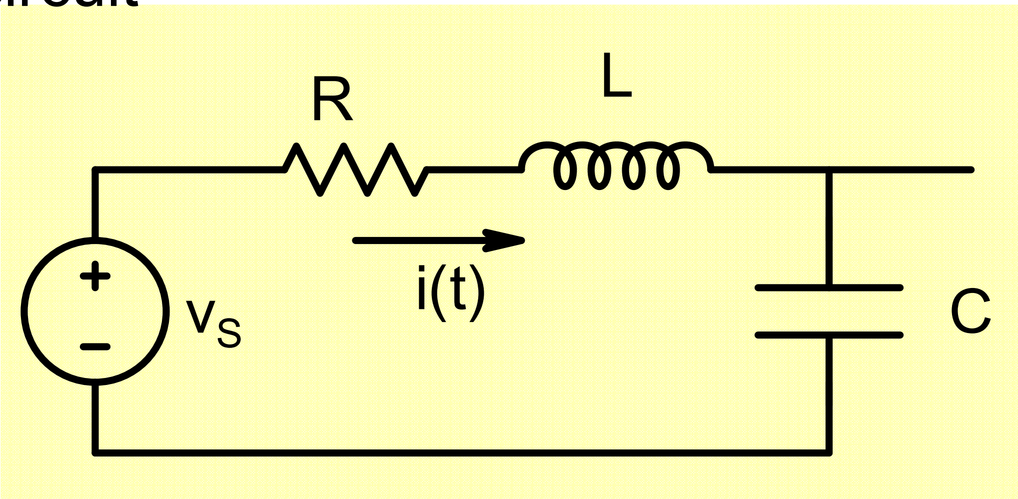


T = 1μs

The amplitude is 10 times larger even though input magnitude is same !

Series Resonant Circuit

Resonance is a condition in which capacitive and inductive reactance cancel each other to give rise to a purely resistive circuit



$$Z_{eq} = R + j\omega L - j\frac{1}{\omega C}$$

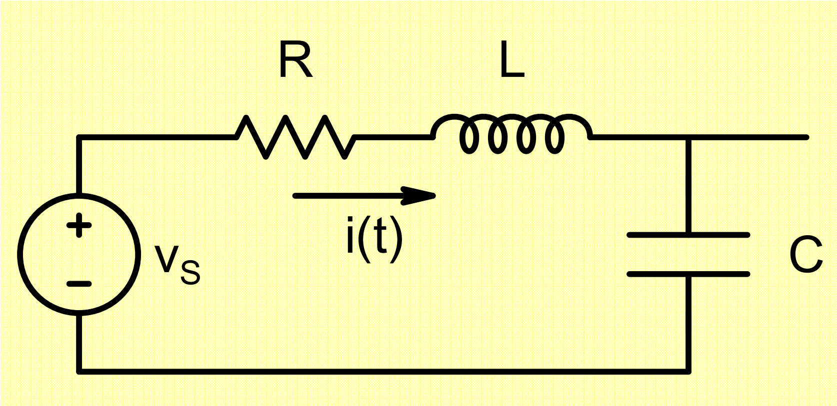
Resonant frequency:

$$j\omega_O L - j\frac{1}{\omega_O C} = 0 \Rightarrow \omega_O = \frac{1}{\sqrt{LC}}$$

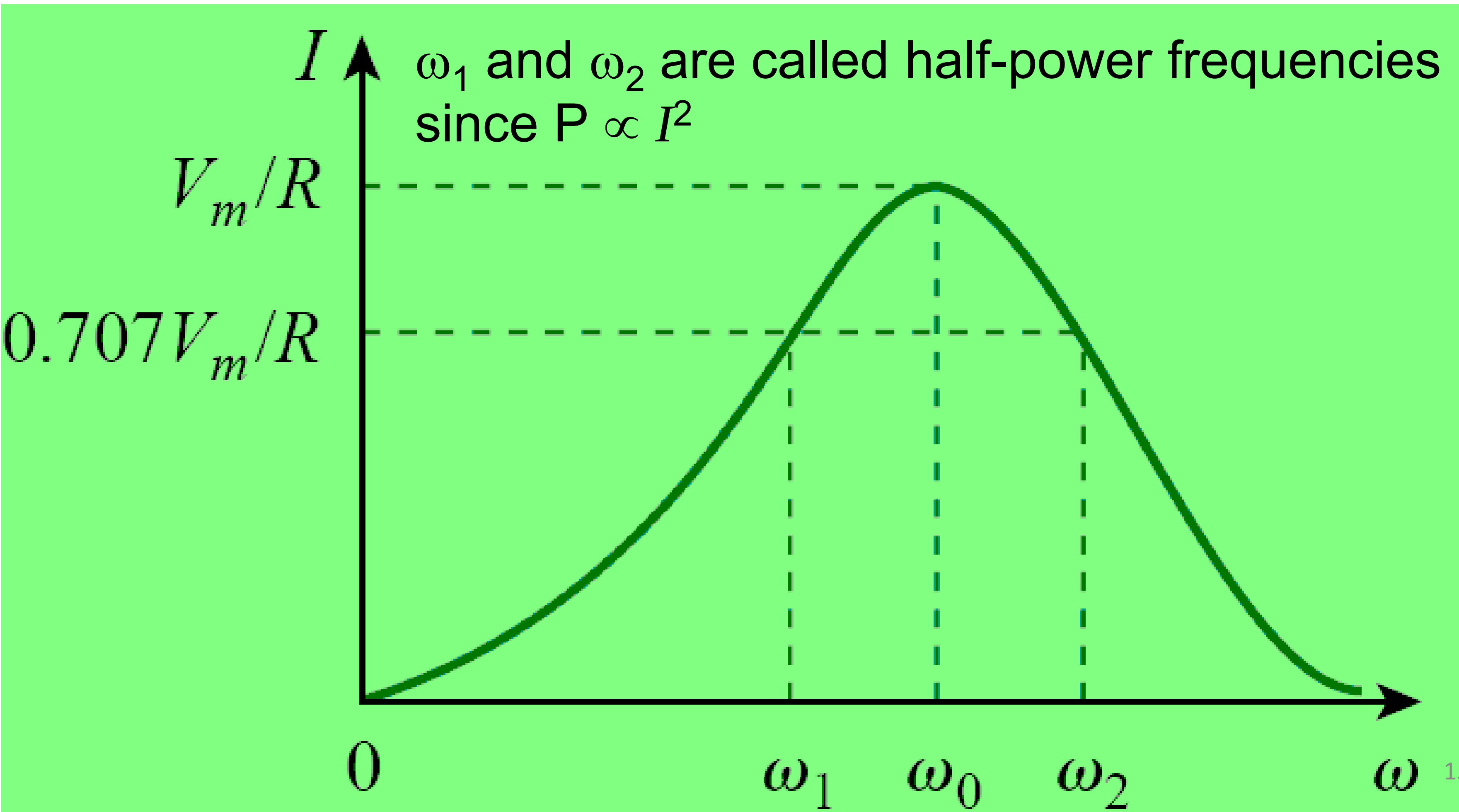
$$f_O = \frac{1}{2\pi\sqrt{LC}}$$

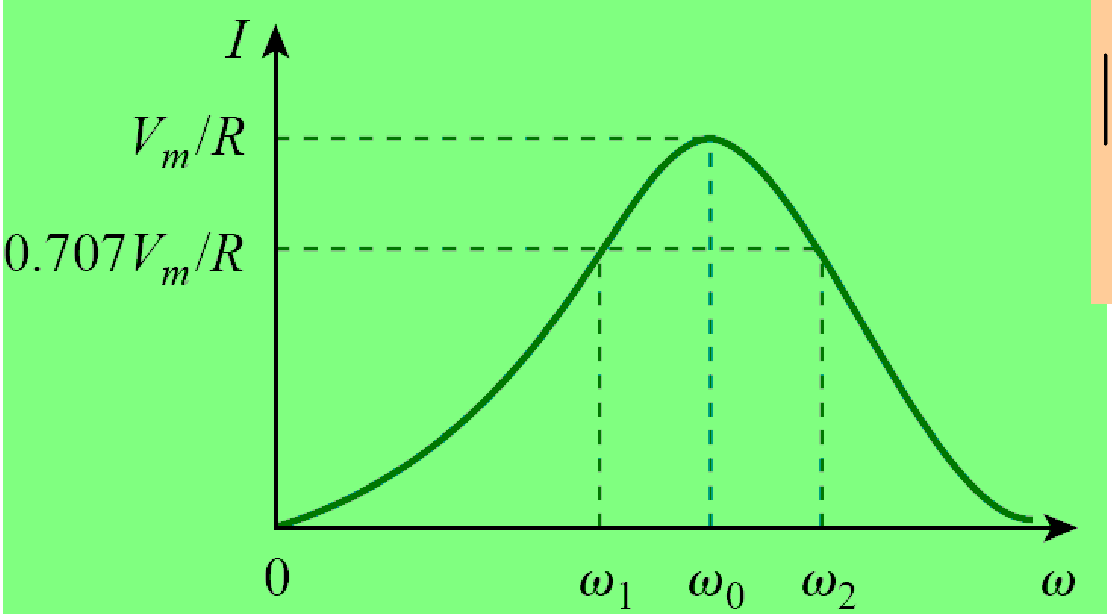
$$Z_{eq} = R$$

Current and voltage are in phase (power factor is unity) and current is maximum !



$$|I(\omega)| = \frac{V_m}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$





$$\left|I(\omega)\right| = \frac{V_m}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

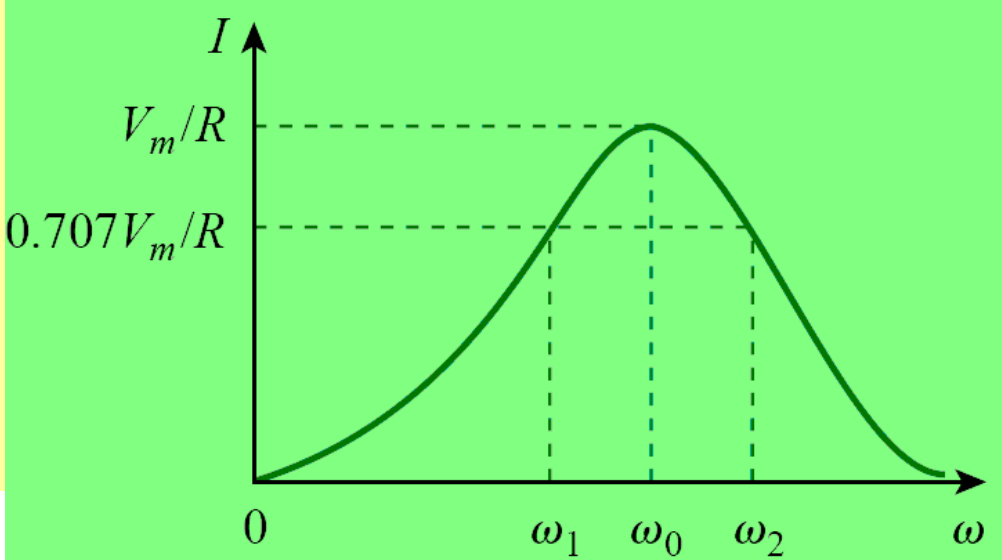
$$\left|I(\omega_1)\right| = \frac{V_m}{\sqrt{R^2 + (\omega_1 L - \frac{1}{\omega_1 C})^2}} = \frac{V_m}{\sqrt{2}R}$$

$$\left|I(\omega_2)\right| = \frac{V_m}{\sqrt{R^2 + (\omega_2 L - \frac{1}{\omega_2 C})^2}} = \frac{V_m}{\sqrt{2}R}$$

ω_1 and ω_2 are called half-power frequencies since $P \propto I^2$

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$



$$\omega_o = \sqrt{\omega_1 \omega_2}$$

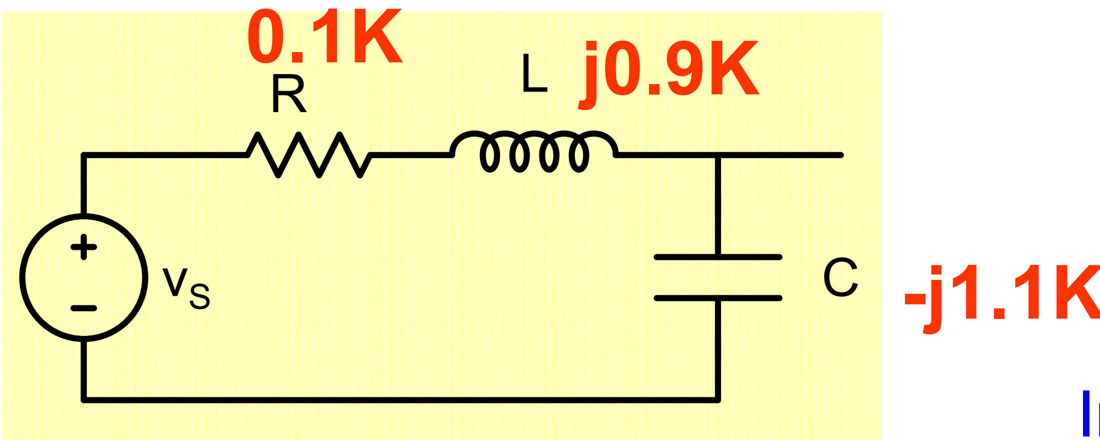
$$B = \omega_2 - \omega_1 = \frac{R}{L}$$

Quality (Q) factor: Sharpness of resonance

$$Q = 2\pi \frac{\text{Peak Stored Energy}}{\text{Energy dissipated in one period at resonance}}$$

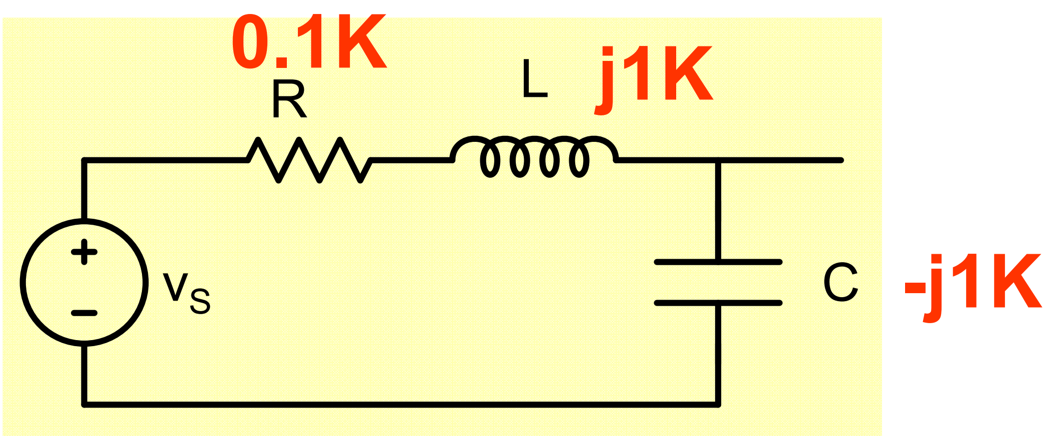
$$Q = 2\pi \times \frac{\frac{1}{2} L \times I_m^2}{\frac{1}{2} I_m^2 R \times T_o} = \frac{\omega_o L}{R}$$

$$\omega_o = \frac{1}{\sqrt{LC}} \Rightarrow Q = \frac{1}{\omega_o CR}$$



$Z=0.1K-j0.2K$

Impedance is in kΩ

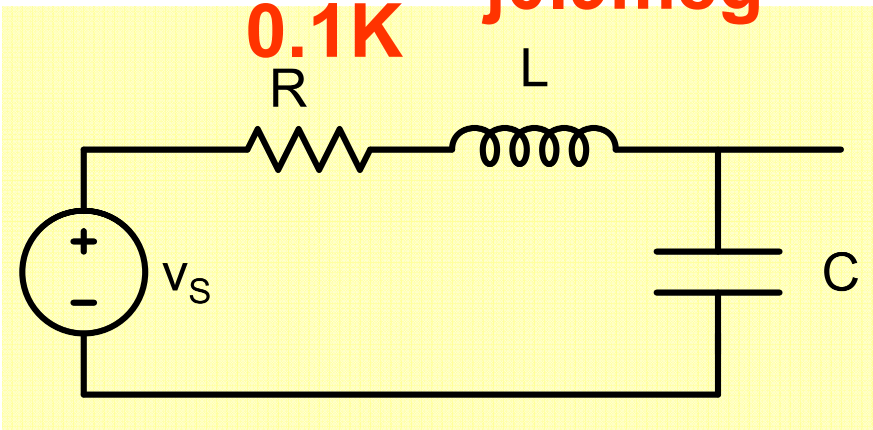


$Z=0.1K$

Impedance is in kΩ

Not very large change in impedance as we approach resonance !

0.1K **j0.9meg**

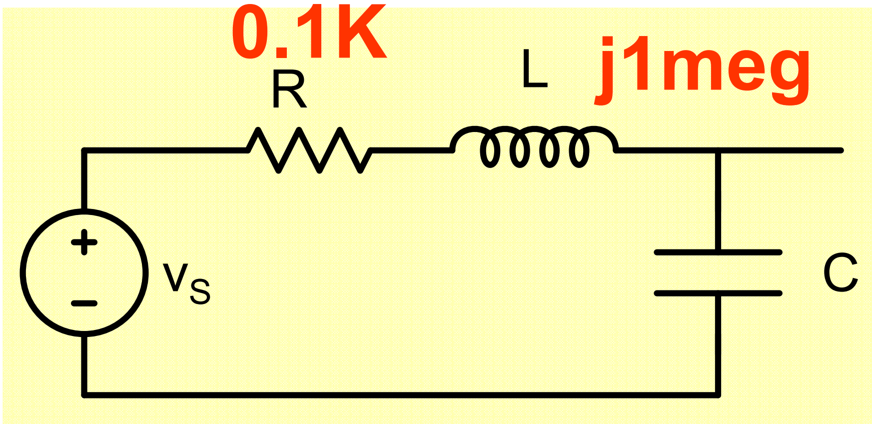


-j1.1meg

Z=0.1K-j0.2meg

Impedance is in MΩ

0.1K **j1meg**



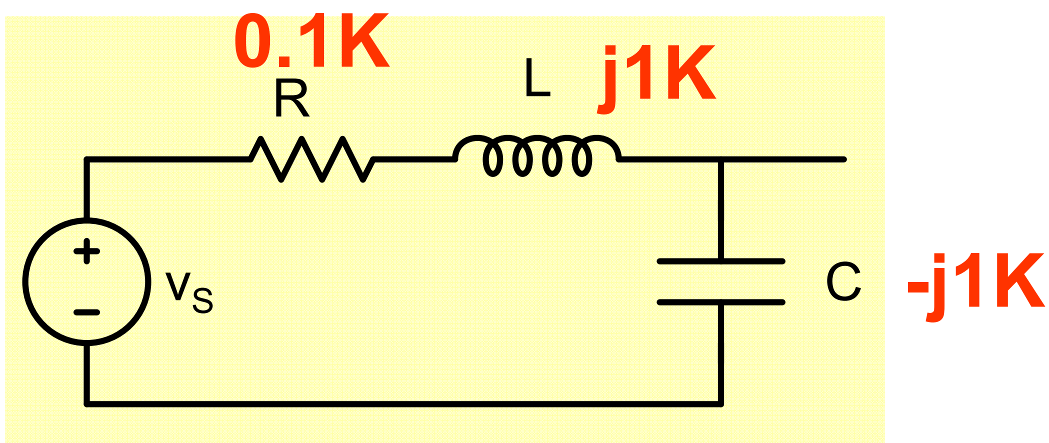
-j1meg

Z=0.1K

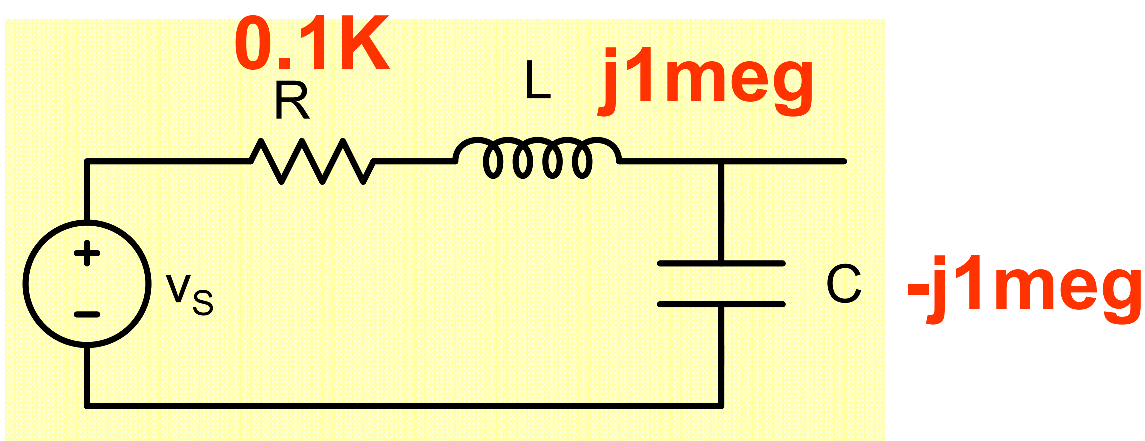
Impedance is in kΩ

very large change in impedance as we approach resonance !
Implying high quality factor

Quality factor Q



Z=0.1K



Z=0.1K

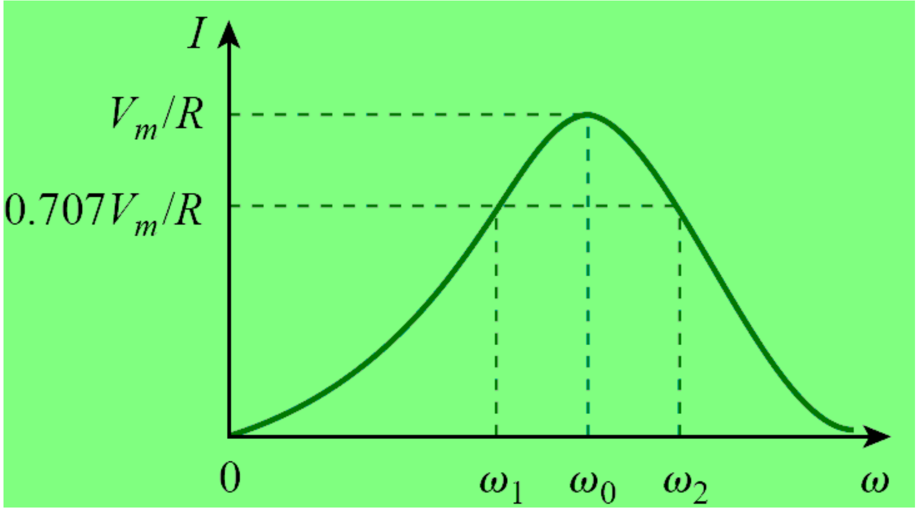
$$Q = \frac{\omega_O L}{R} \quad or \quad Q = \frac{1/\omega_O C}{R}$$

$$Q = \frac{\omega_O L}{R}$$

$$B = \omega_2 - \omega_1 = \frac{R}{L}$$

$$Q = \frac{\omega_O}{B} = \frac{\omega_O}{\Delta\omega}$$

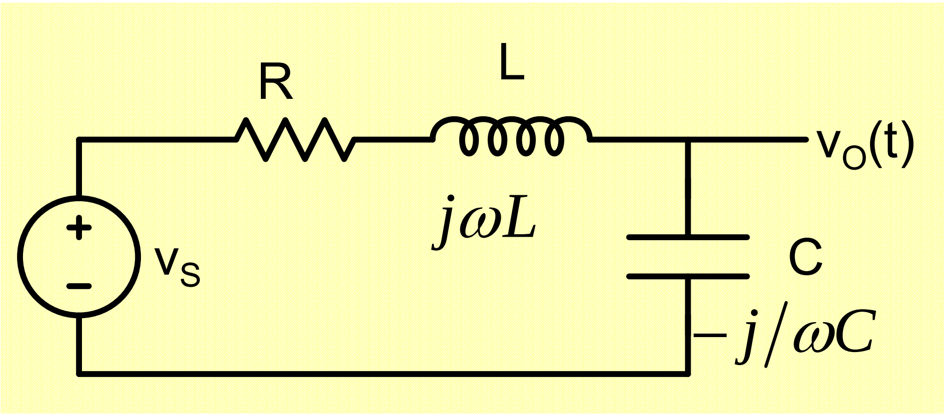
Hence Q represents sharpness of resonance



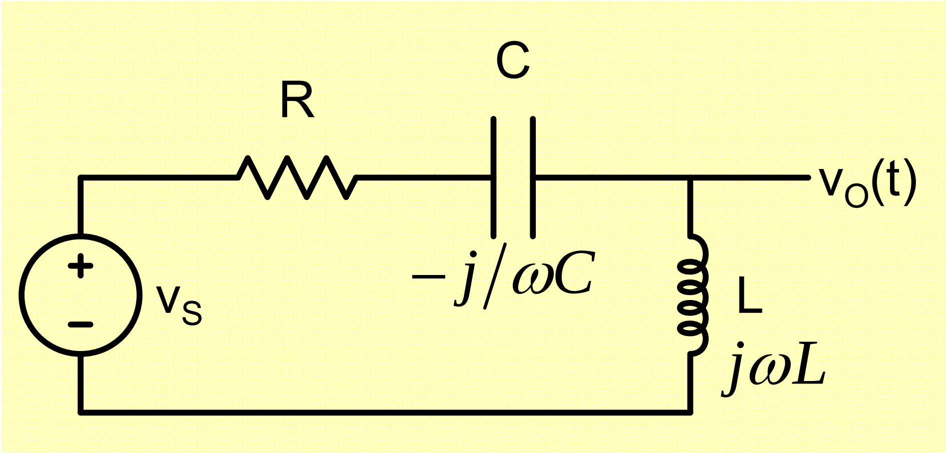
For high Q circuits:

$$\omega_1 \simeq \omega_0 - \frac{B}{2}, \qquad \omega_2 \simeq \omega_0 + \frac{B}{2}$$

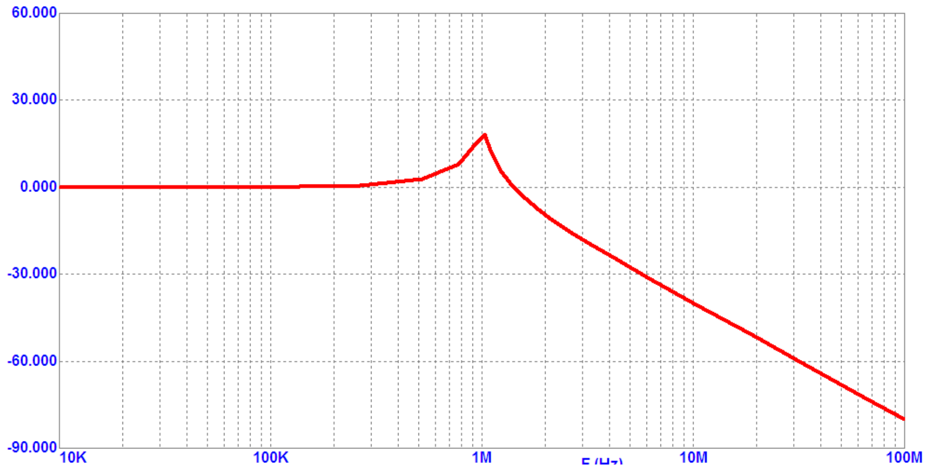
R-L-C filters



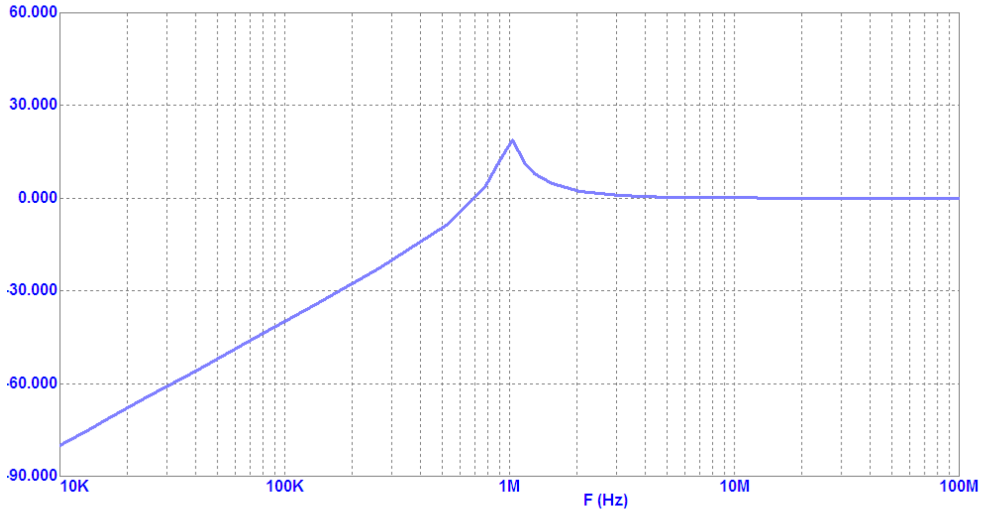
R
L
$j\omega L$
C
$-j/\omega C$
v_S
$v_O(t)$



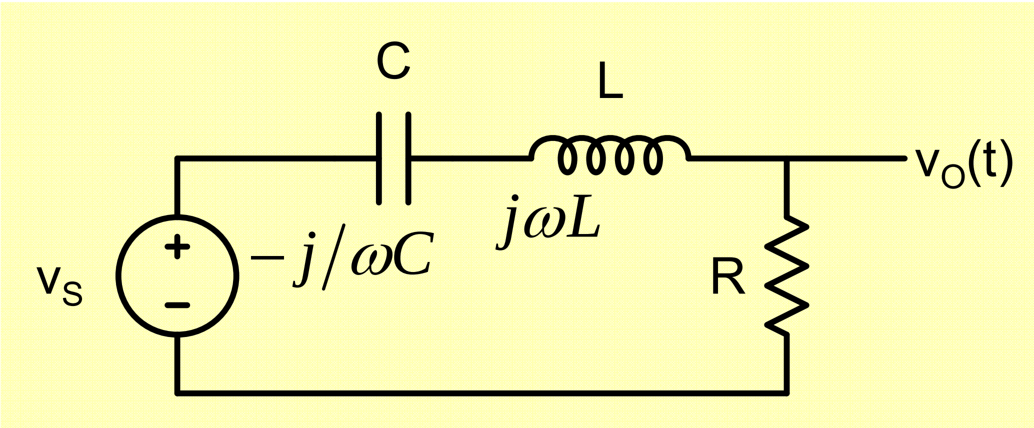
R
C
$-j/\omega C$
L
$j\omega L$
v_S
$v_O(t)$

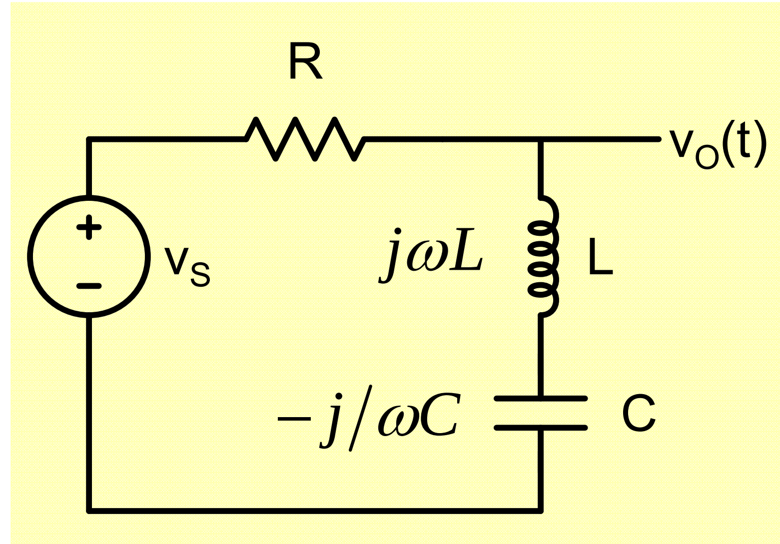


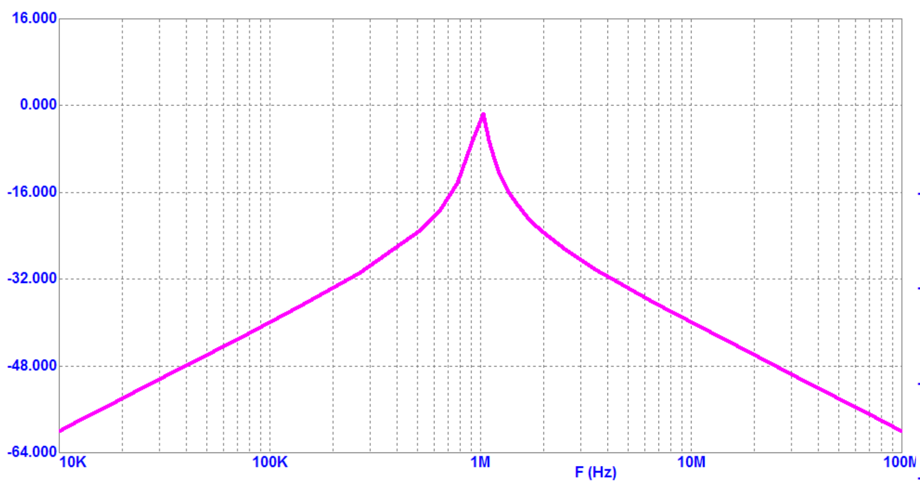
60.000
30.000
0.000
-30.000
-60.000
-90.000
10K
100K
1M
10M
100M

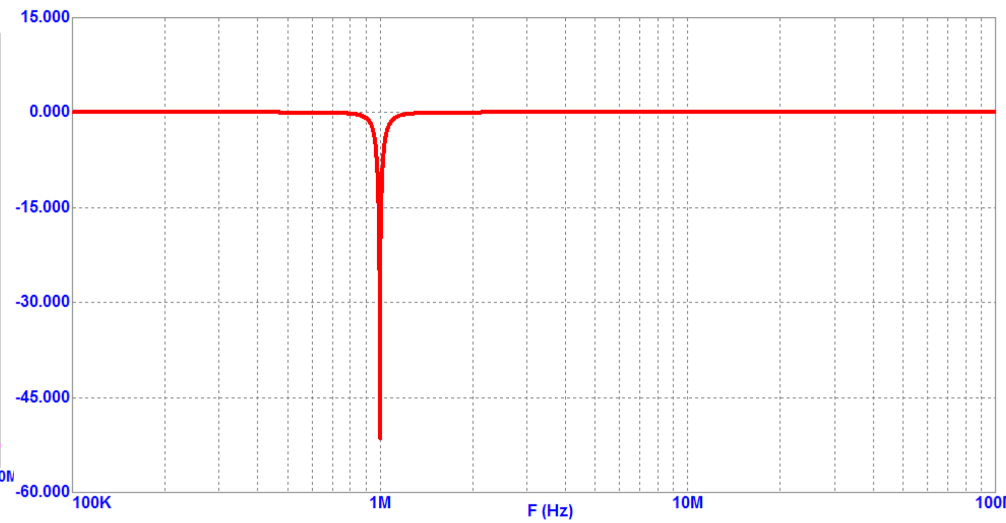


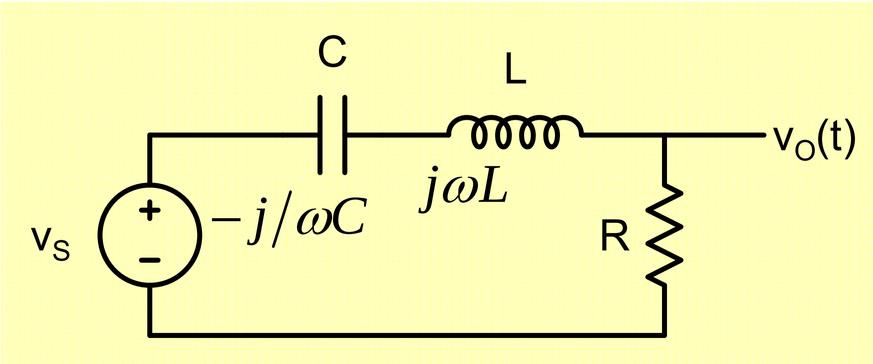
60.000
30.000
0.000
-30.000
-60.000
-90.000
10K
100K
1M
10M
100M
F (Hz)

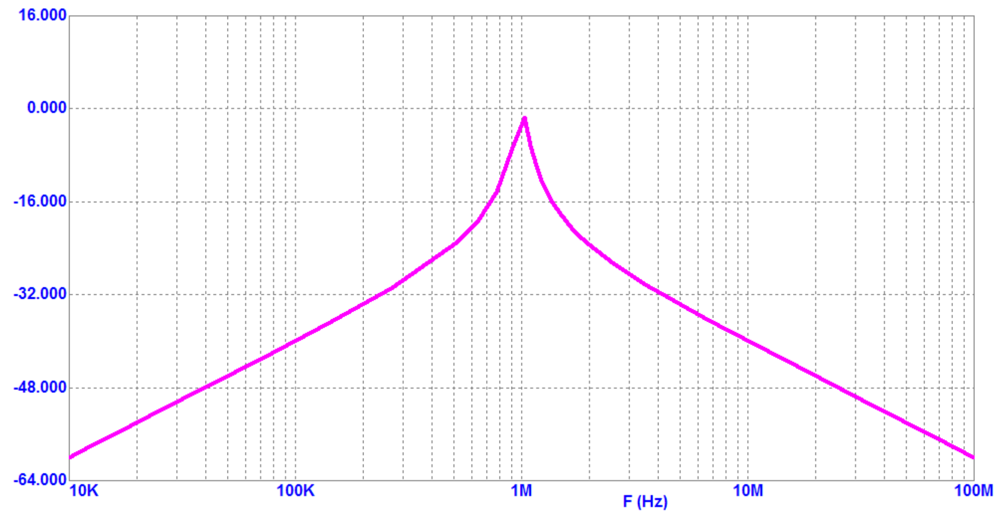

C
L
$v_O(t)$
$j\omega L$
v_S
$-j/\omega C$
R


R
$v_O(t)$
v_S
$j\omega L$
L
$-j/\omega C$
C


16.000
0.000
-16.000
-32.000
-48.000
-64.000
10K
100K
1M
F (Hz)
10M


15.000
0.000
-15.000
-30.000
-45.000
-60.000
100K
1M
F (Hz)
10M
100M





How much Q do we need to pass 450KHz but reject 460KHz by 60dB?

$$\left|H(\omega)\right| = \left|\frac{V_O(\omega)}{V_{IN}(\omega)}\right| = \frac{R}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

Assuming V_{IN} = 1V and noting that $Q = \omega_O L/R$

$$\left|V_O(\omega)\right| = \frac{1}{\sqrt{1 + Q^2(\frac{\omega^2}{\omega_O^2} - 1)^2}}$$

For $\omega = \omega_O$, V_O = 1 so the signal simply passes through !

$$\omega_O = 2 \times \pi \times 450 \times 10^3 = 2.8 \times 10^6 \, rad/s$$

$$\left|V_O(\omega)\right| = \frac{1}{\sqrt{1+Q^2\left(1-\frac{\omega_O^2}{\omega^2}\right)^2}}$$

$$\omega_0 = 2\pi\times 450\times 10^3 = 2.827\times 10^6\, rad/s$$

$$\omega = 2\pi\times 460\times 10^3 = 2.89\times 10^6\, rad/s$$

For an attenuation of -60dB or 10^{-3} at ω: **Q=23,000**

Such a large value of Q is not easy to get !

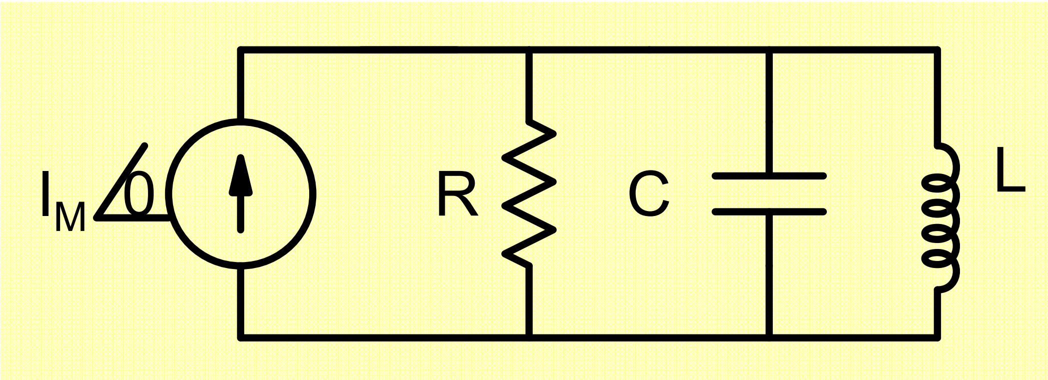
Example: for Q = 10^4 at 450KHz

$$Q = \frac{\omega_o L}{R}$$

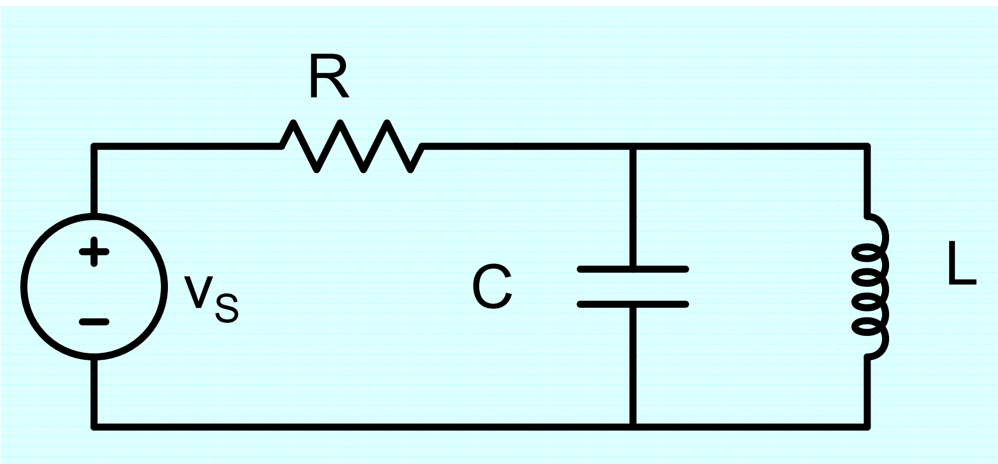
Suppose $L = 10^{-3} H ;\Rightarrow R = 0.28\Omega;\Rightarrow C = 125\, pF$

Suppose $L = 0.1H ;\Rightarrow R = 28\Omega;\Rightarrow C = 1.25\, pF$

Parallel Resonance



$$Y_{eq} = \frac{1}{R} + j\omega C - j\frac{1}{\omega L}$$

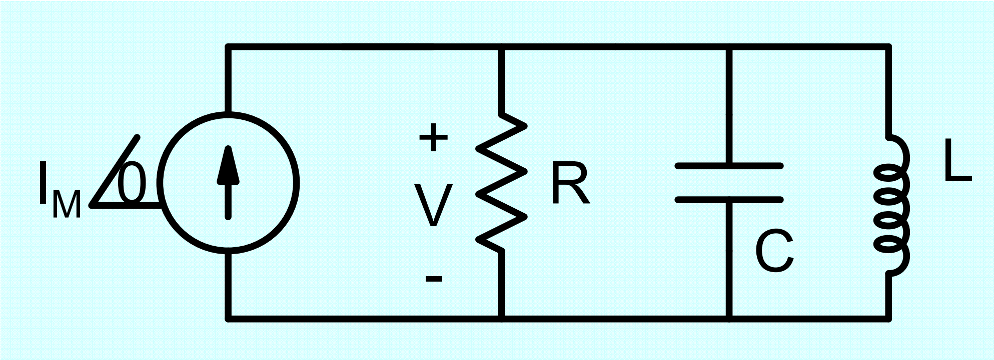


Resonant frequency:

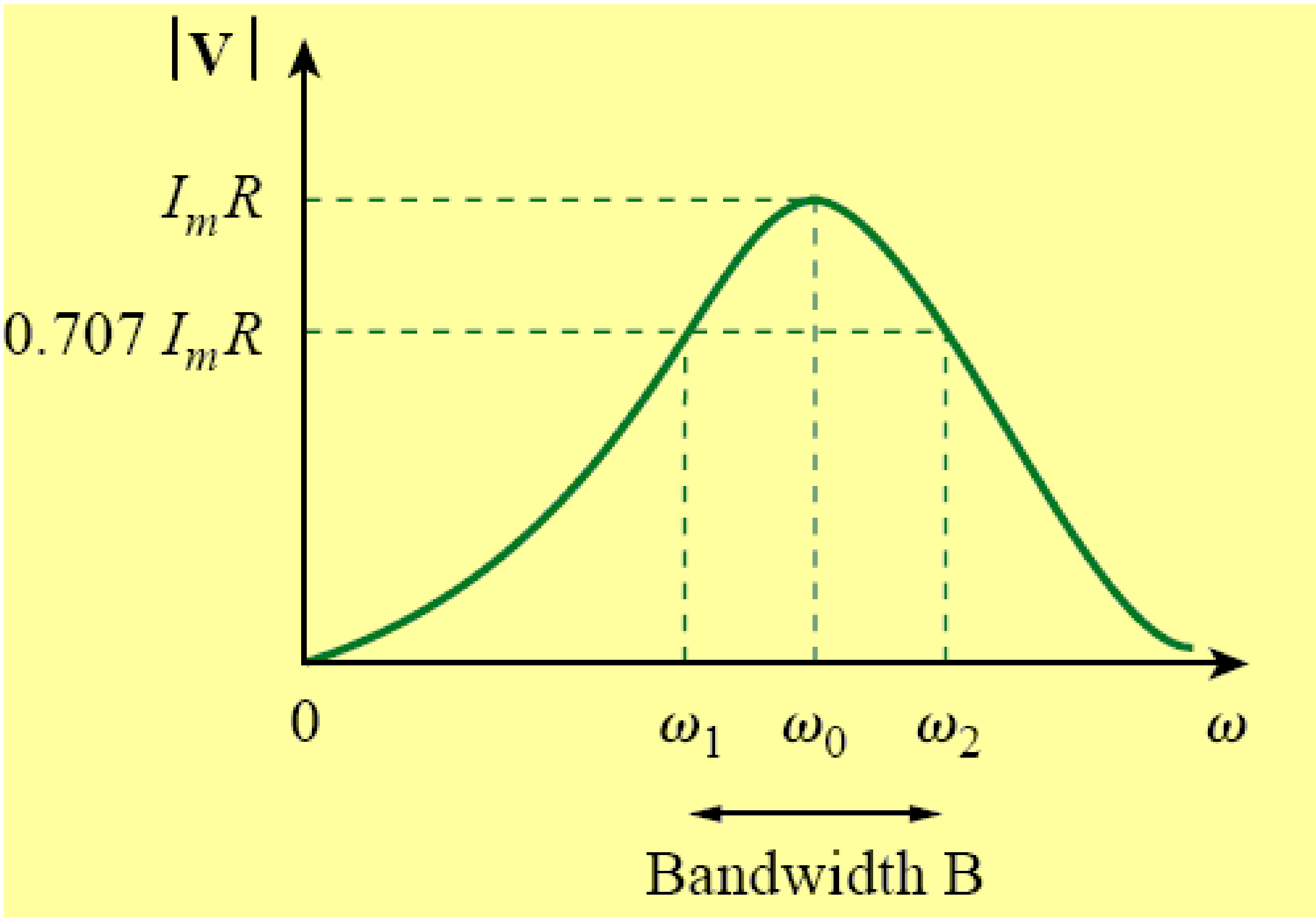
$$j\omega_o C - j\frac{1}{\omega_o L} = 0 \Rightarrow \omega_o = \frac{1}{\sqrt{LC}}$$

$$f_o = \frac{1}{2\pi\sqrt{LC}}$$

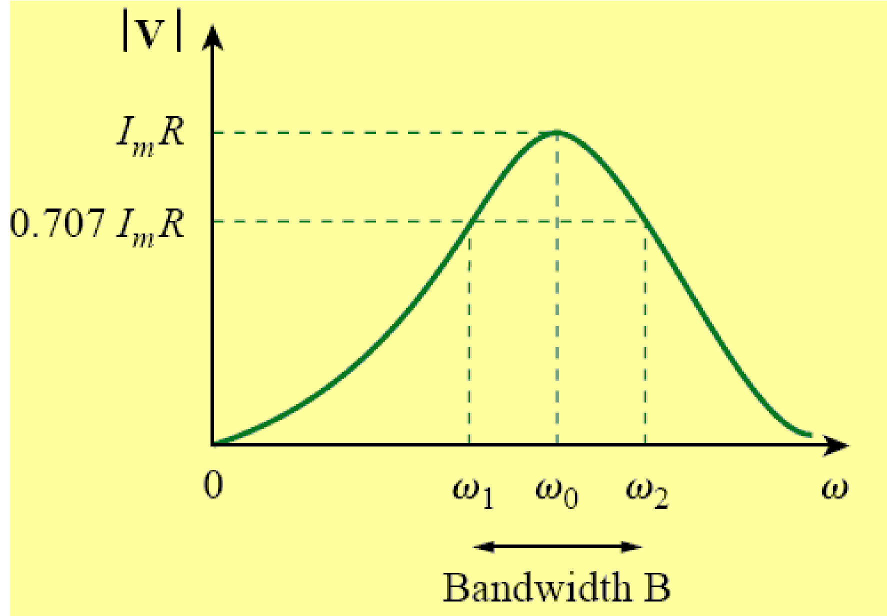
$$Z_{eq} = R$$



$$\left|V(\omega)\right| = \frac{I_m R}{\sqrt{1+\dfrac{R^2C^2}{L^2}\left(\omega L-\dfrac{1}{\omega C}\right)^2}}$$



$$|V(\omega)| = \frac{I_m R}{\sqrt{1+\frac{R^2C^2}{L^2}(\omega L - \frac{1}{\omega C})^2}}$$



$$\omega_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

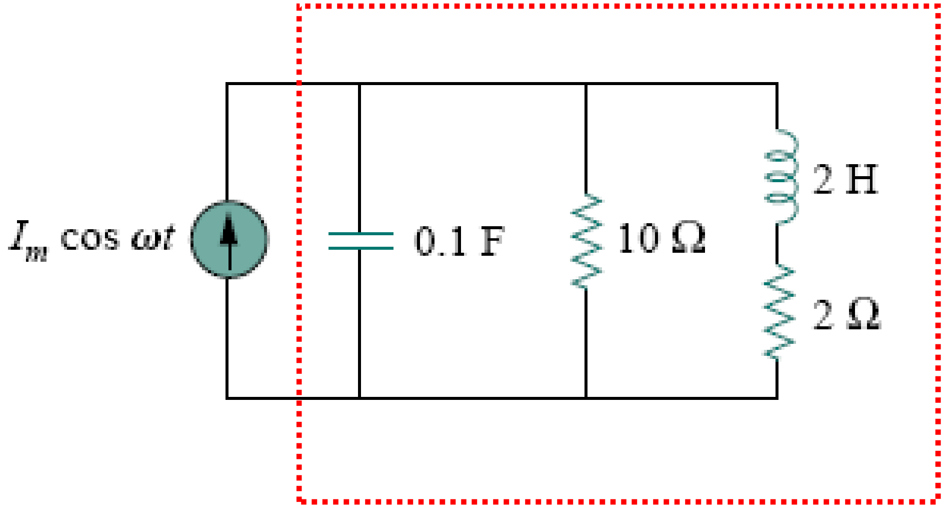
$$B = \omega_2 - \omega_1 = \frac{1}{RC}$$

$$Q = \frac{\omega_0}{B} = \omega_0 RC = \frac{R}{\omega_0 L}$$

$$\omega_1 = \omega_0\sqrt{1+\left(\frac{1}{2Q}\right)^2} - \frac{\omega_0}{2Q}, \qquad \omega_2 = \omega_0\sqrt{1+\left(\frac{1}{2Q}\right)^2} + \frac{\omega_0}{2Q}$$

For high Q: $$\omega_1 \simeq \omega_0 - \frac{B}{2}, \qquad \omega_2 \simeq \omega_0 + \frac{B}{2}$$

What is the resonant frequency ?



$$\mathbf{Y} = j\omega 0.1 + \frac{1}{10} + \frac{1}{2 + j\omega 2} = 0.1 + j\omega 0.1 + \frac{2 - j\omega 2}{4 + 4\omega^2}$$

At resonance, $\mathrm{Im}(\mathbf{Y}) = 0$

$$\omega_0 0.1 - \frac{2\omega_0}{4 + 4\omega_0^2} = 0 \qquad \Longrightarrow \qquad \omega_0 = 2 \text{ rad/s}$$

$$f_O = \frac{\omega_o}{2\pi}$$