

Tutorial 9

Numerical integration

1. Estimate the following integral by using –

- (a) $O(h^8)$ Romberg integration and determine true and approximate percentage errors;
- (b) two- and three-point Gauss-Legendre formulas and determine true percentage error.

$$I = \int_{-2}^2 x e^{-x} dx$$

2. Estimate the following improper integral

$$I = \int_{-2}^{\infty} x e^{-x} dx$$

Numerical differentiation

3. The location of an object at various times was measured as follows:

Time (t ; s)	0	1	2	3	4	5	6	7	8	9
Distance (x ; m)	1	1.55	2.32	3.58	5.79	9.68	16.49	28.22	48.2	81.92

Estimate the speed and acceleration of the object at 5 seconds by using - (i) Forward difference, $O(h^2)$ (ii) Backward difference, $O(h^2)$ (iii) Central difference, $O(h^2)$ and (iv) Richardson extrapolation, $O(h^6)$ using three central differences of $O(h^2)$. Estimate the true percentage error if the object location is given by $x = e^{0.5t} - 0.1t^2$.