Function Approximation Methods-IV

CS771: Introduction to Machine Learning

Purushottam Kar

Stochastic Gradient Descent

$$\hat{\mathbf{w}} = \arg\min_{\mathbf{w}\in\mathbb{R}^d} \; f(\mathbf{w}) + r(\mathbf{w})$$

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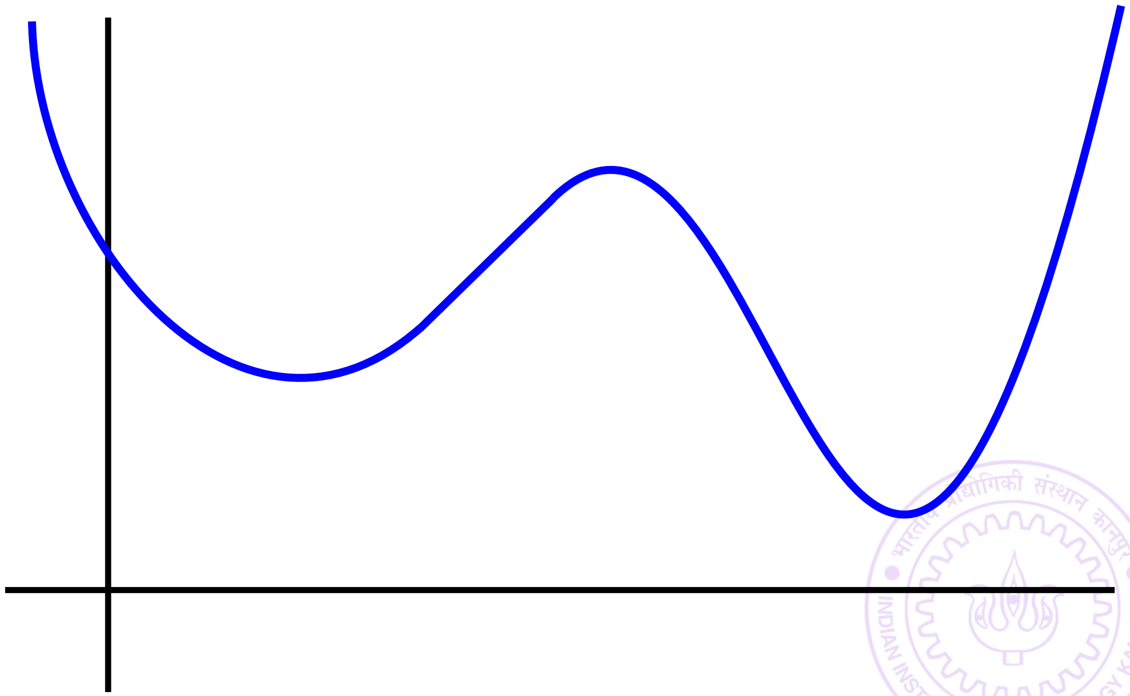

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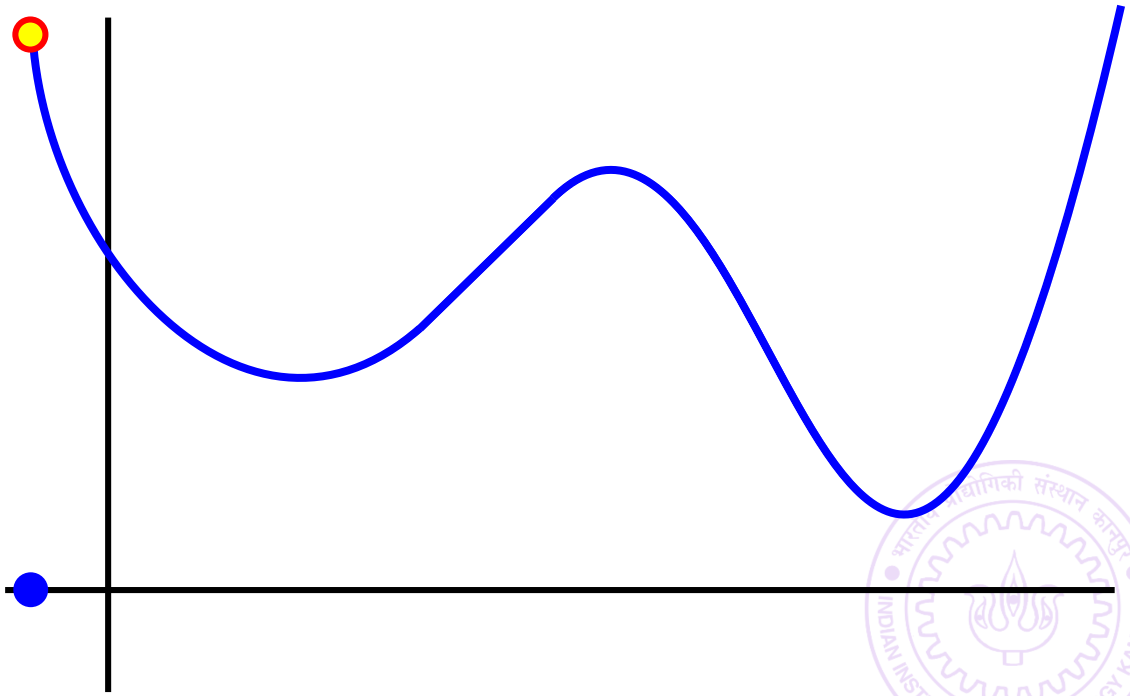

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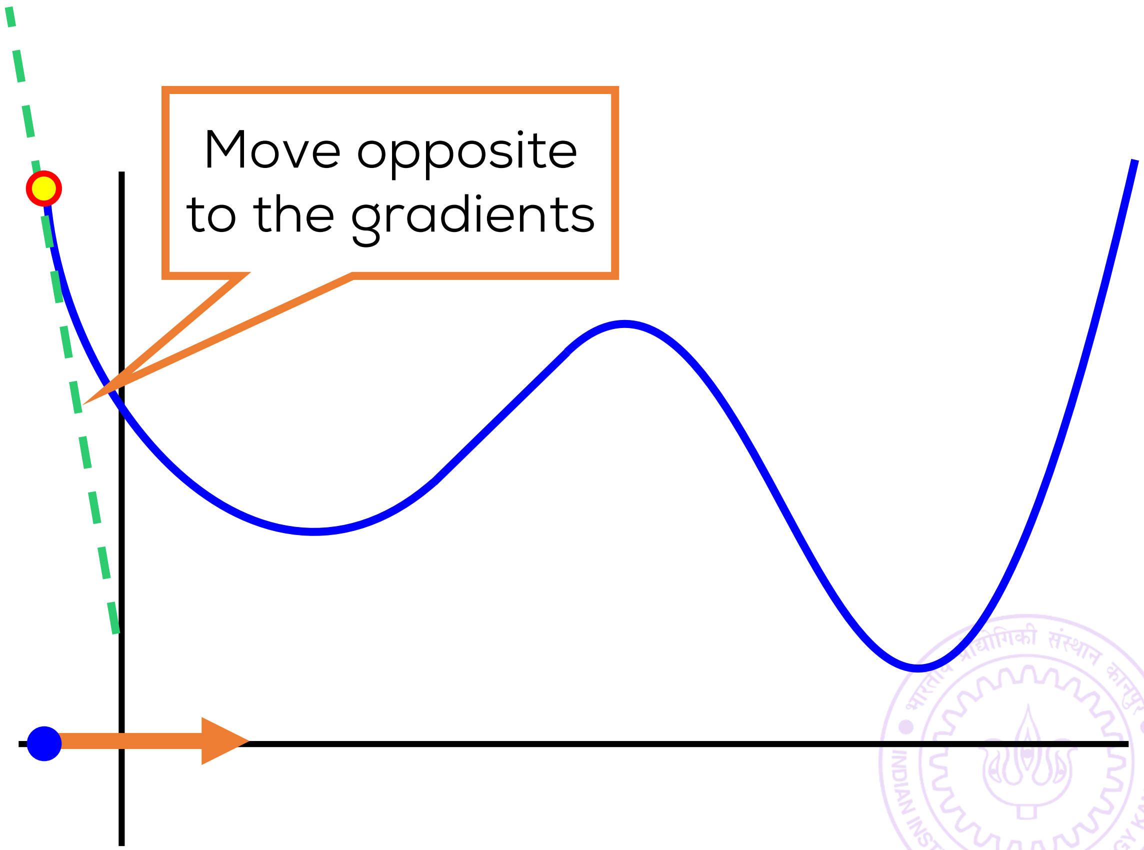

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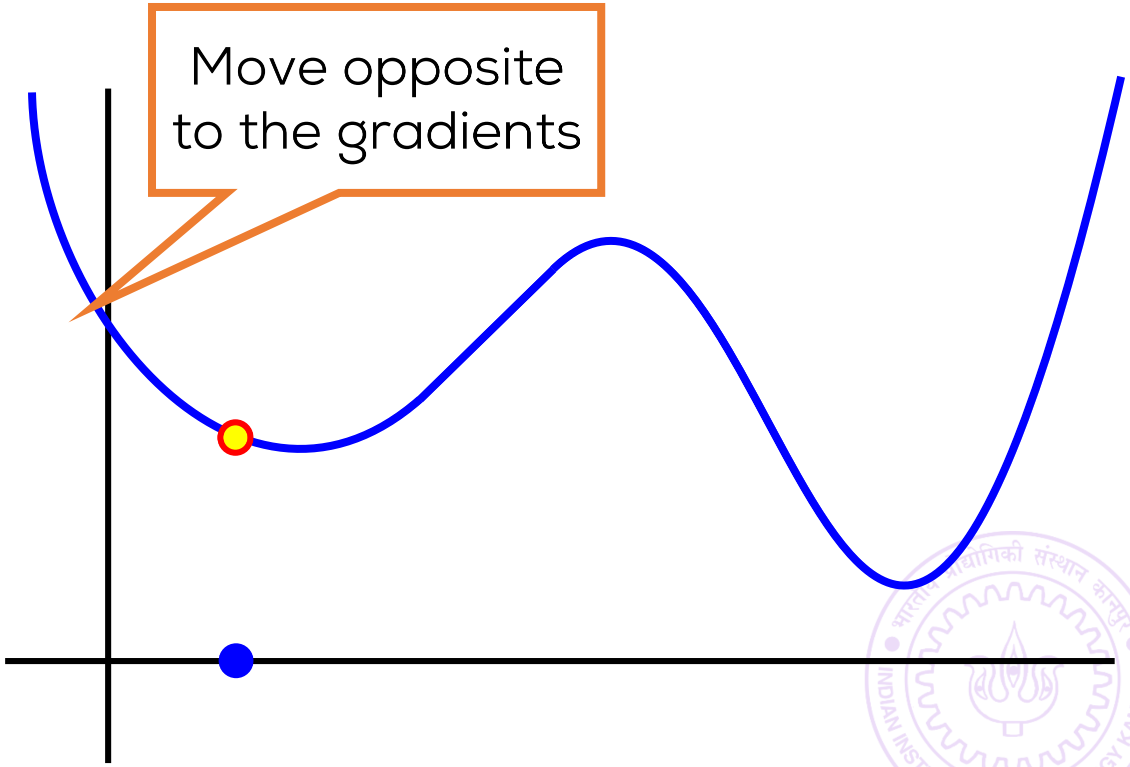

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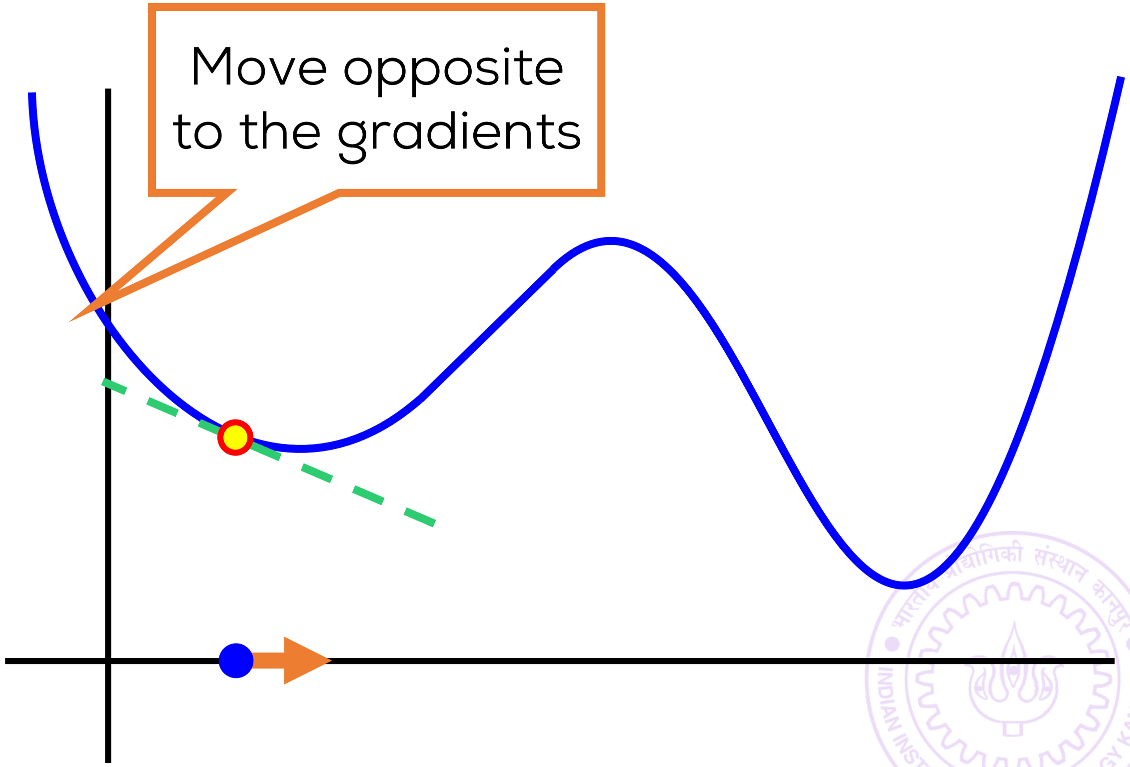

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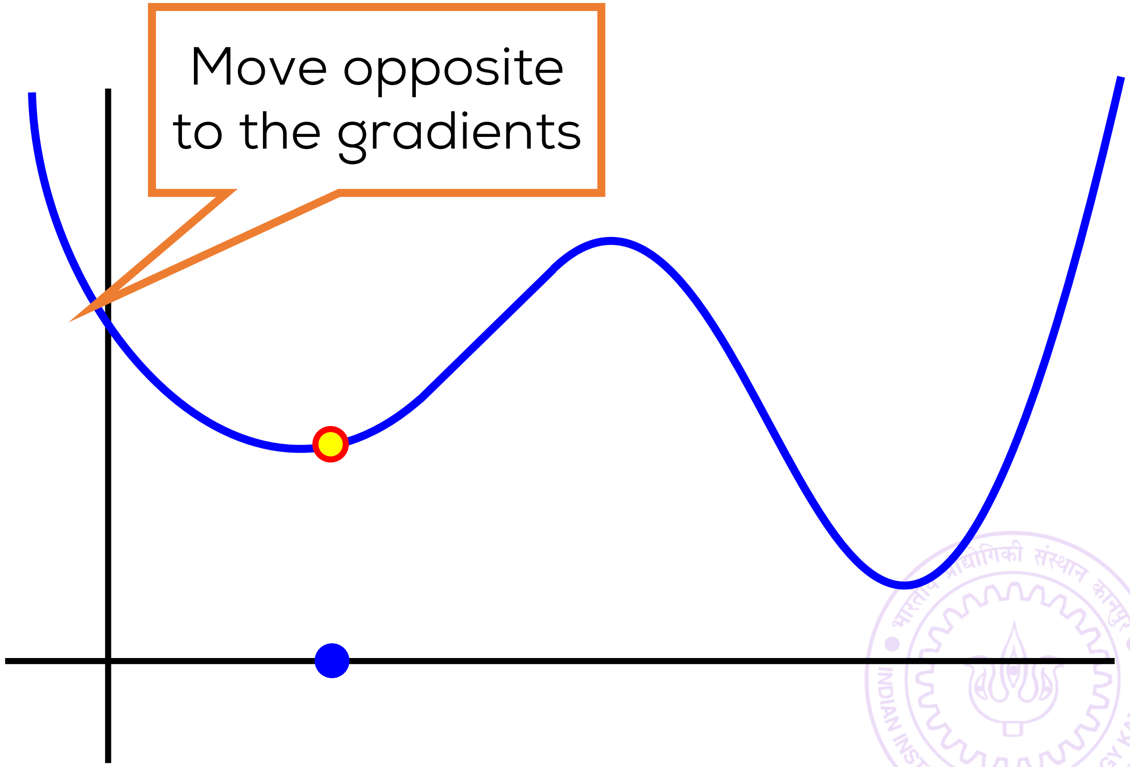

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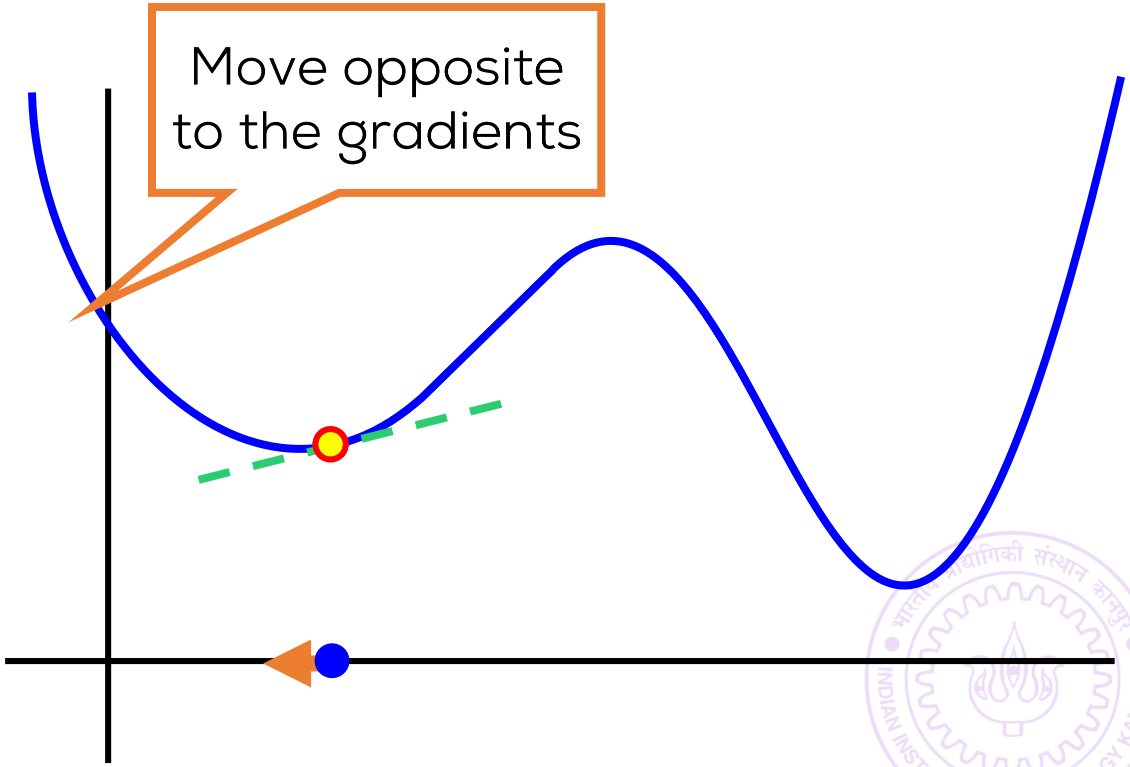

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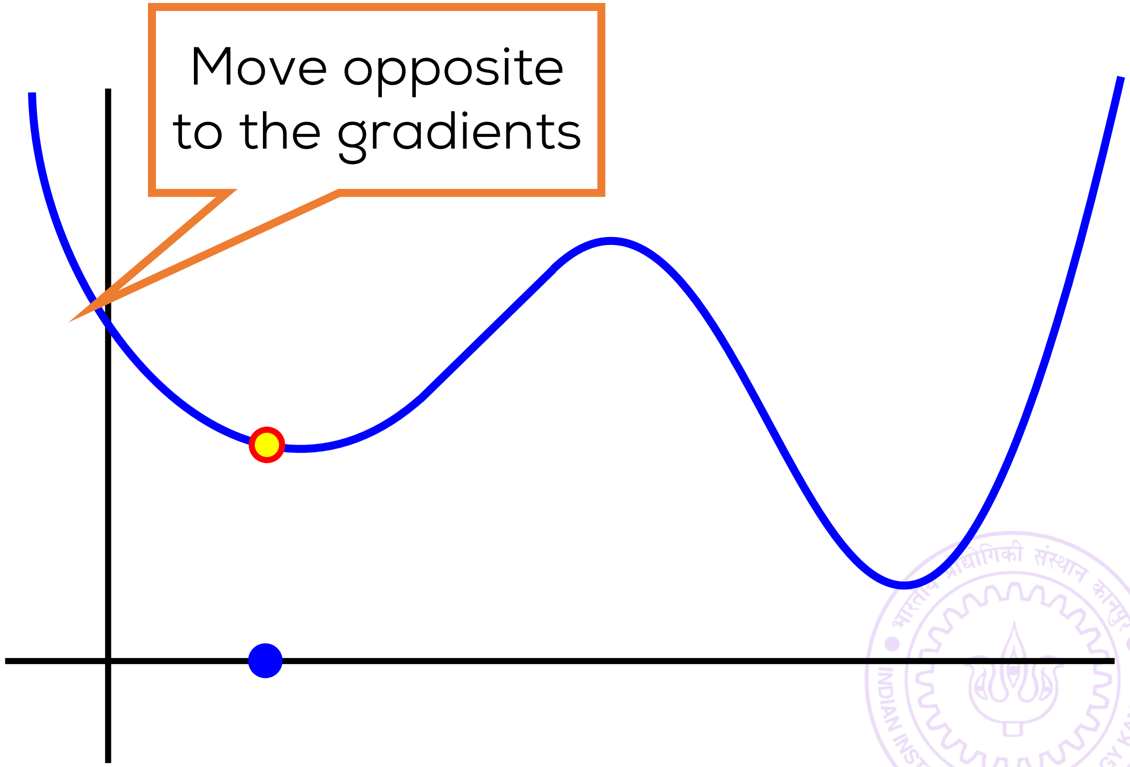

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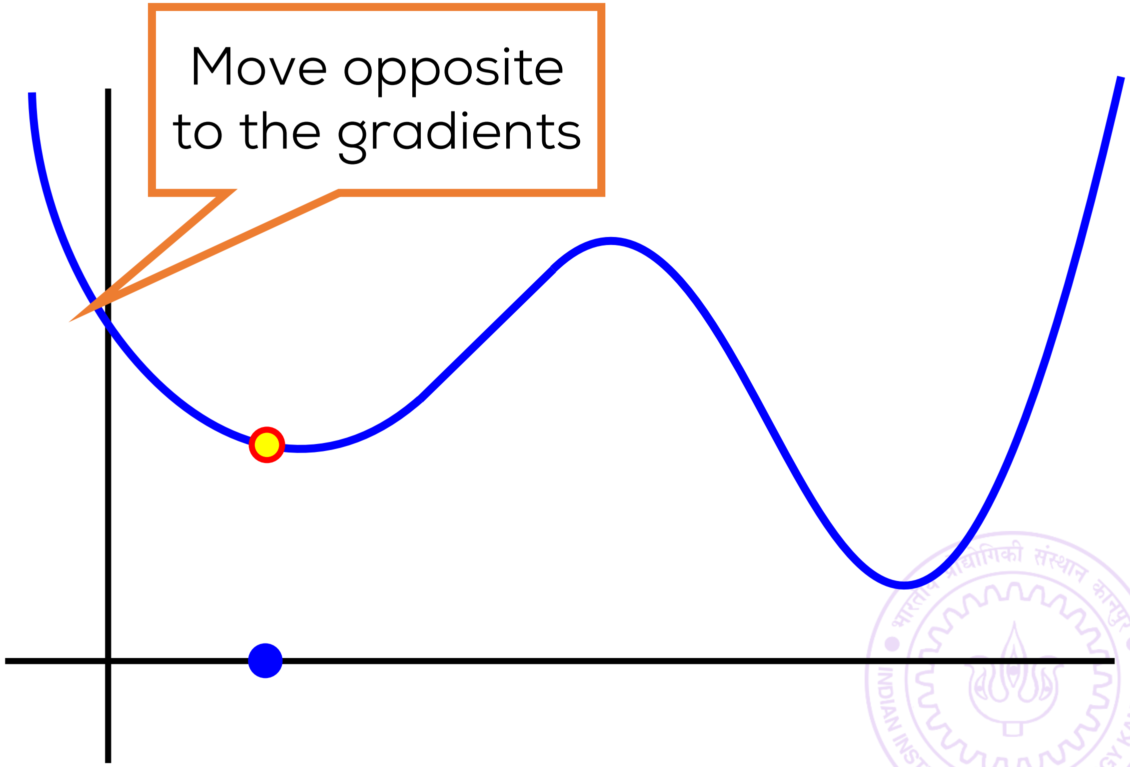

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Stochastic Gradient Descent

$$f(\mathbf{w}) = \frac{1}{n}\sum_{i=1}^{n} f_i(\mathbf{w}) = \frac{1}{n}\sum_{i=1}^{n} \ell(y^i, \langle \mathbf{w}, \mathbf{x}^i \rangle)$$

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Stochastic Gradient Descent

$O(nd)$ time

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STOCHASTIC GRADIENT DESCENT

1. Initialize $\mathbf{w}^0$
2. Select a unif. random point $I_t \in [n]$
3. Let $\mathbf{g}^t \leftarrow \nabla f_{I_t}(\mathbf{w}^t) + \nabla r(\mathbf{w}^t)$
4. Update $\mathbf{w}^{t+1} \leftarrow \mathbf{w}^t - \eta_t \cdot \mathbf{g}^t$
5. Repeat until convergence

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Only $O(d)$ time per iter!

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Step length
$\eta_t = C, \frac{C}{t}, \frac{C}{\sqrt{t}}$

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Step length $\eta_t = C, \frac{C}{t}, \frac{C}{\sqrt{t}}$

w.h.p. ϵ-optimal solution in $O\left(\frac{1}{\epsilon^2}\right)$ iterations

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$f(\mathbf{w}^t) + r(\mathbf{w}^t) \leq f(\mathbf{w}^*) + r(\mathbf{w}^*) + \epsilon$

Stochastic Gradient Descent

$O(d)$ time!!

$$\nabla f(\mathbf{w}) \approx \qquad \nabla f_i(\mathbf{w}) \approx \qquad \nabla \ell(y^i, \langle \mathbf{w}, \mathbf{x}^i \rangle)$$

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Only $O(d)$ time per iter!

ONLINE GRADIENT DESCENT

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2. Receive a data point $\mathbf{z}^t = (y^t, \mathbf{x}^t)$
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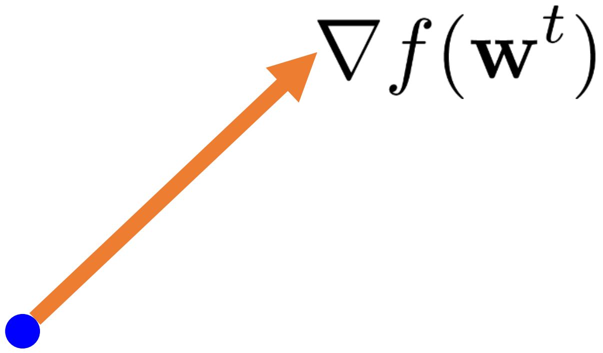
Mini-batch Gradient Descent



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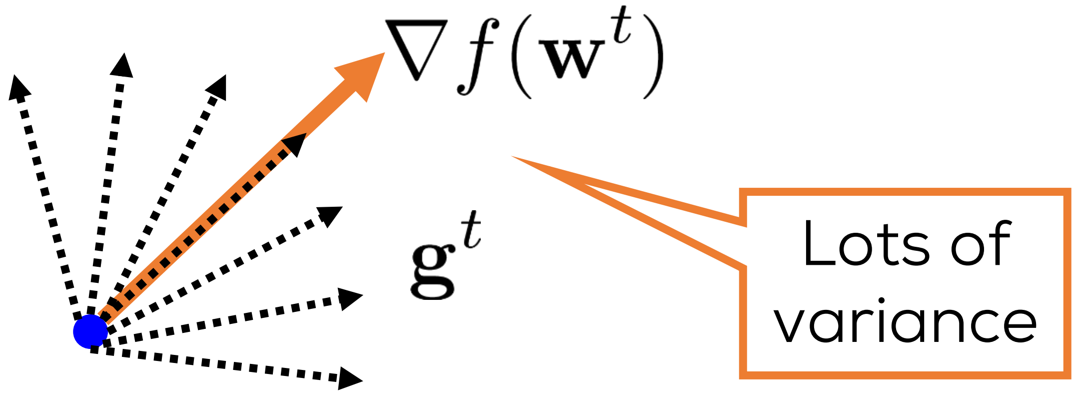

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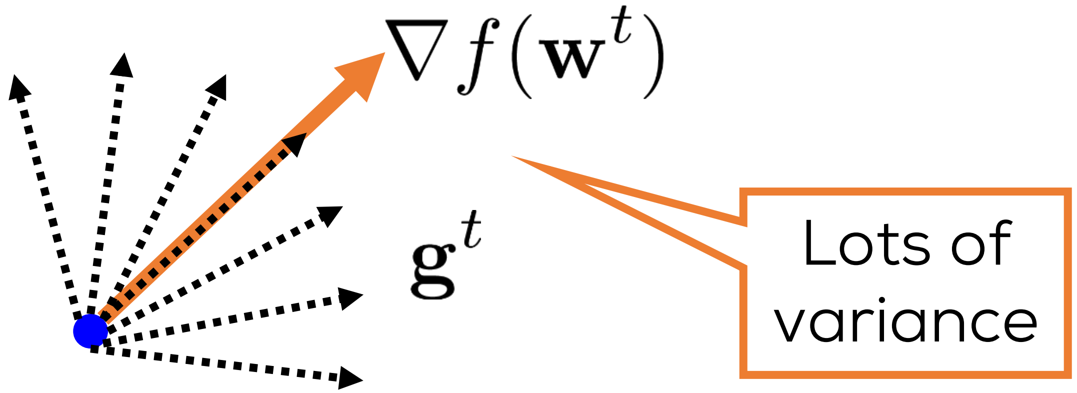

MINI-BATCH SGD

1. Initialize $\mathbf{w}^0$
2. Select B unif. random points $\{I_{t,1}, \ldots, I_{t,B}\} \in [n]$
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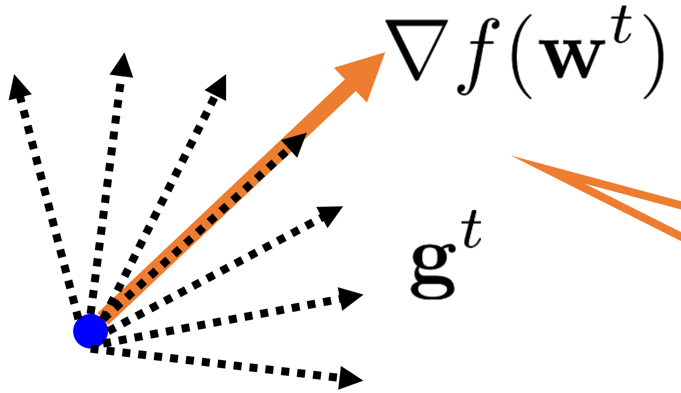

Lots of variance

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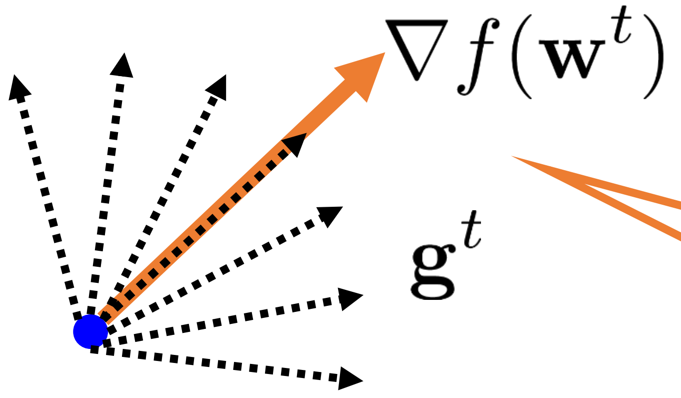
Without replacement?

Mini-batch Gradient Descent



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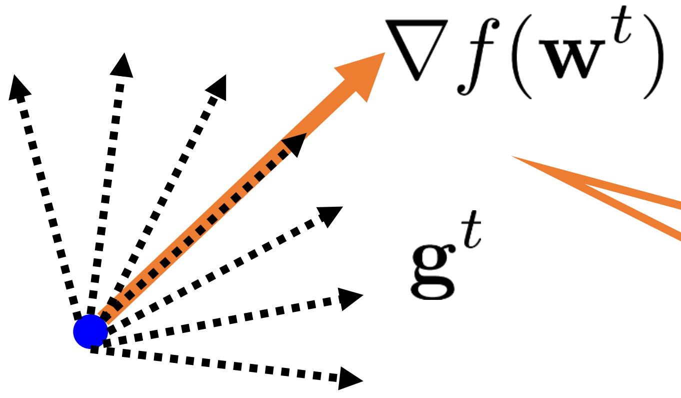
Without replacement?

$O(B \cdot d)$ time per iter!

Mini-batch Gradient Descent



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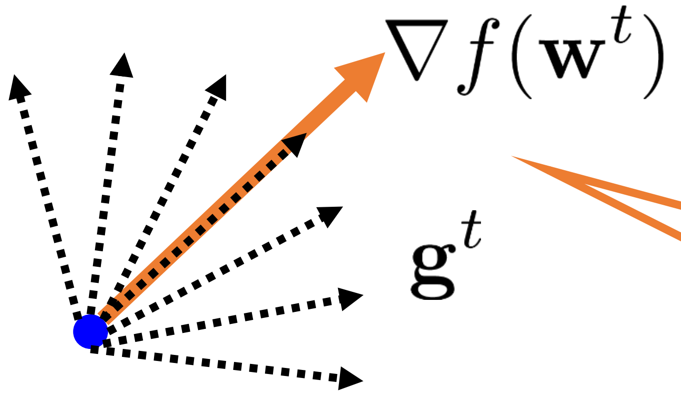
Mini-batch Gradient Descent



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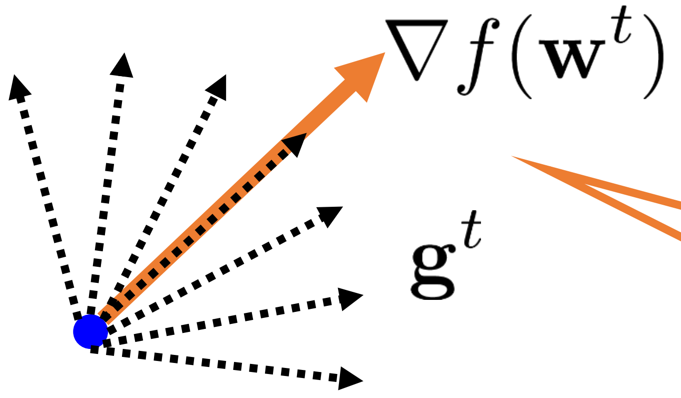
Mini-batch Gradient Descent



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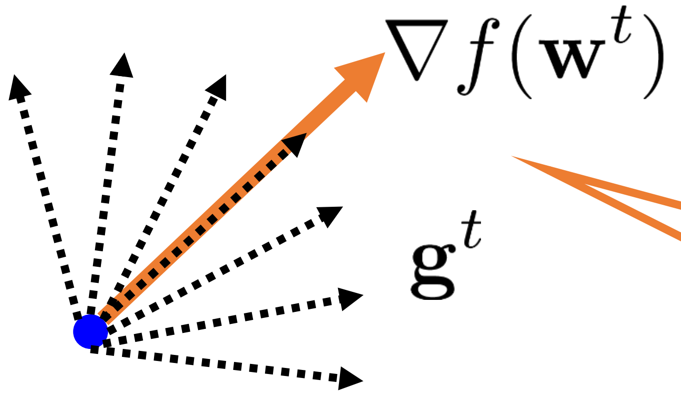

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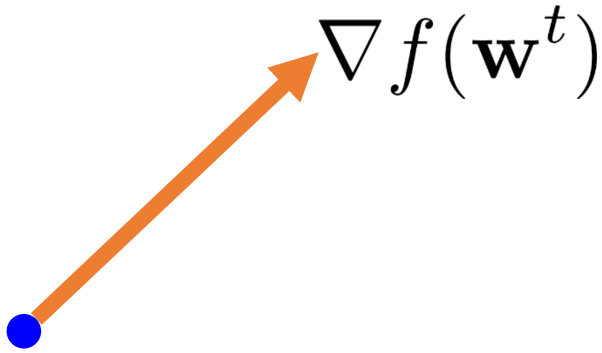

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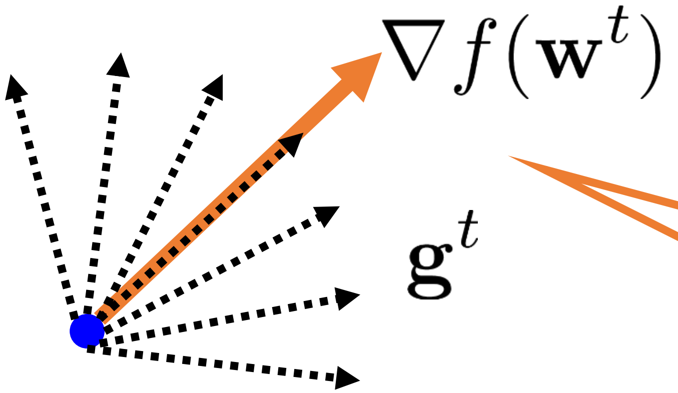
Mini-batch Gradient Descent



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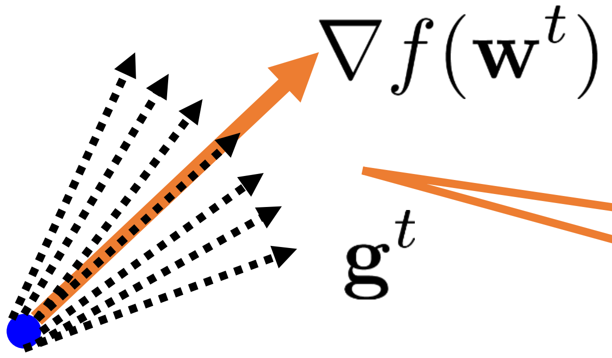

No variance if all points chosen i.e with $B = n$

Less variance with mini-batch

MINI-BATCH SGD

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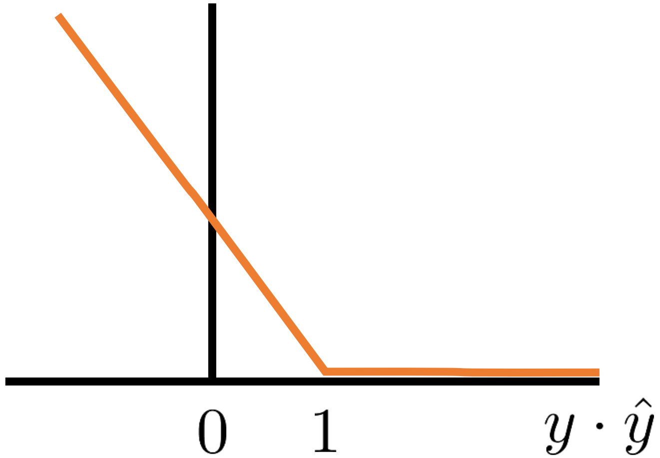
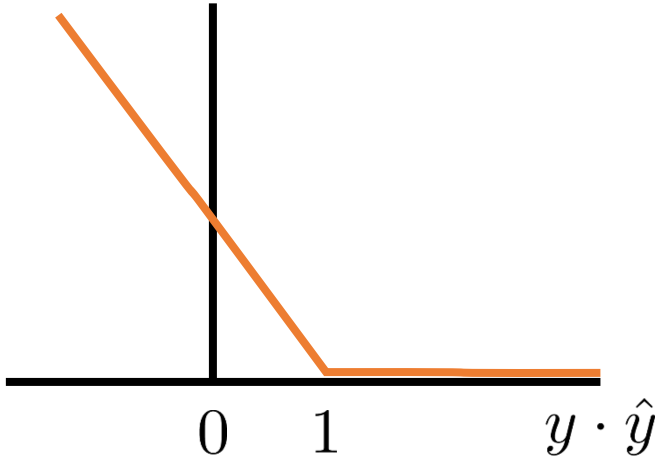
App: The Perceptron Rule via OGD



Hinge loss

App: The Perceptron Rule via OGD



Hinge loss

ONLINE GRADIENT DESCENT

1. Initialize $\mathbf{w}^0$
2. Receive a data point $\mathbf{z}^t = (y^t, \mathbf{x}^t)$
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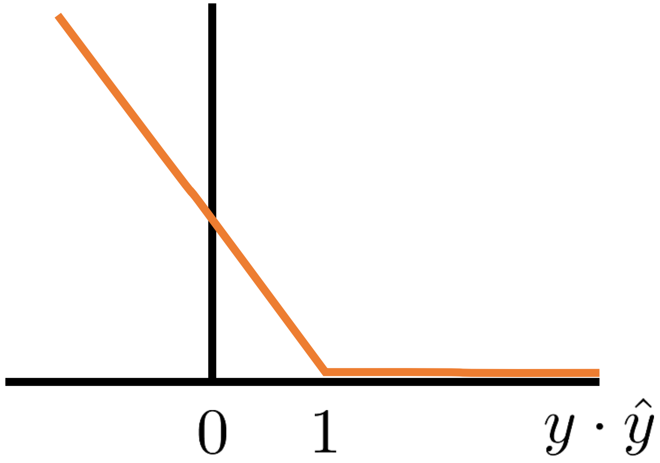

Hinge loss

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$y^t \in \{-1, +1\}$

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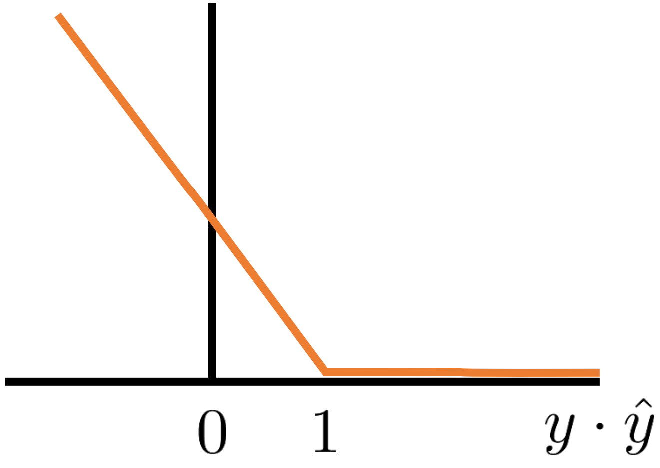

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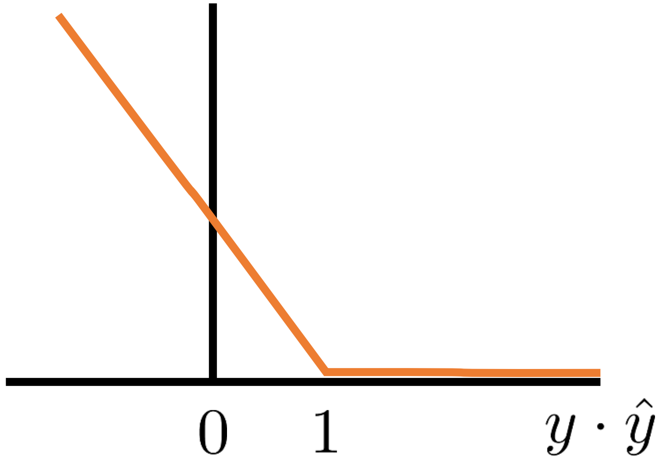

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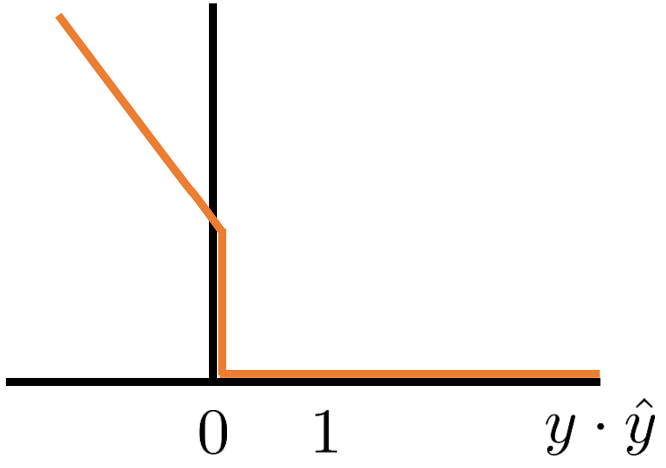

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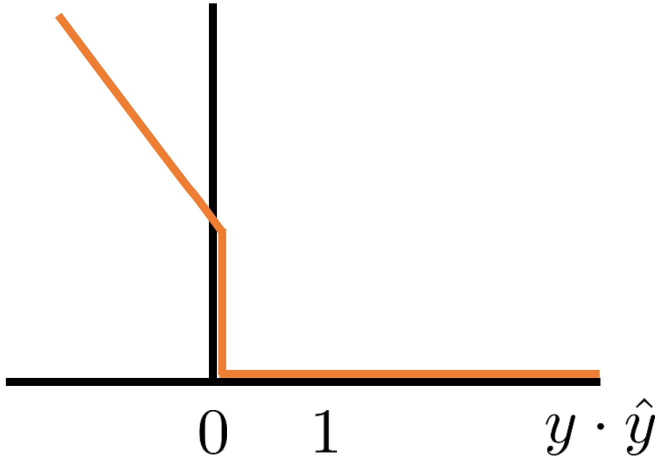

Truncated Hinge loss

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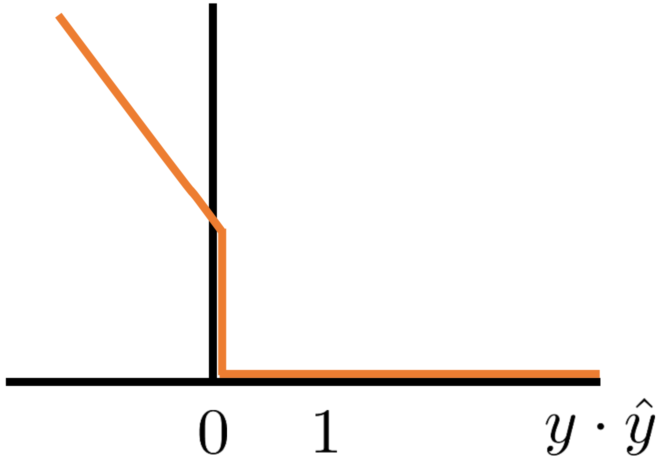

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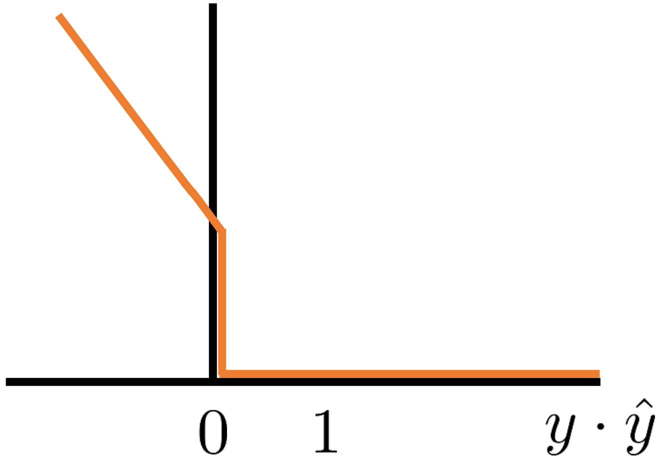

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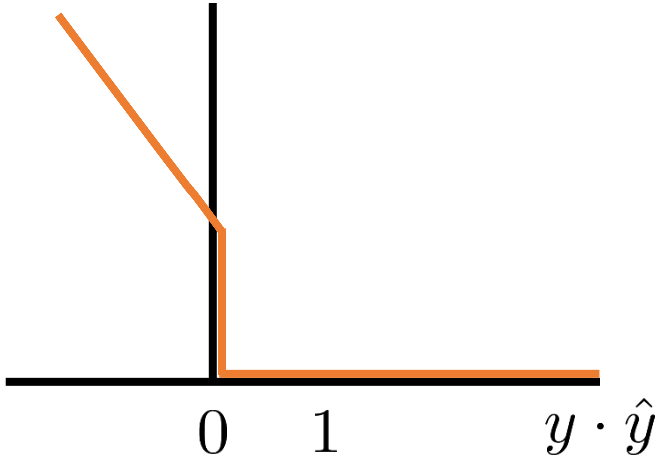

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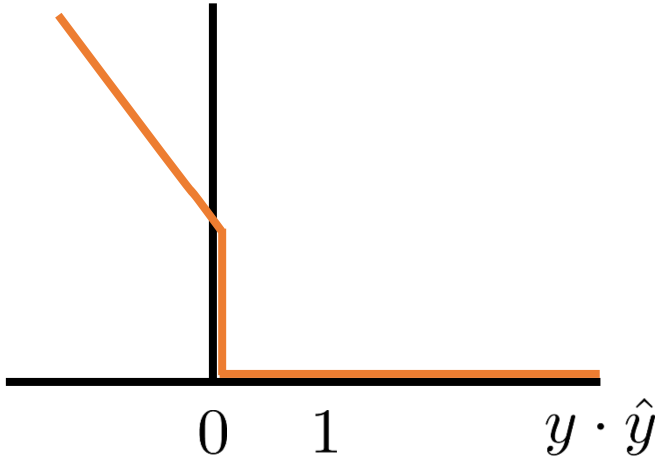

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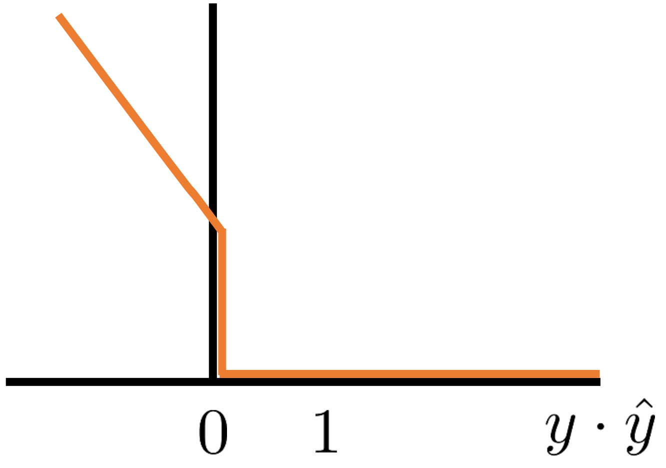

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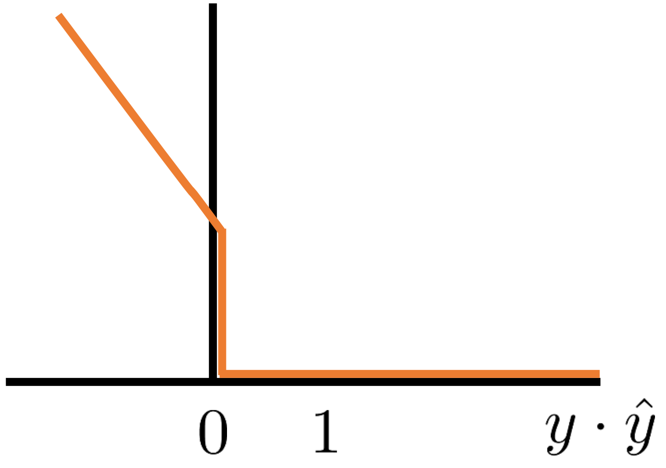

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Perceptron Logistic Regression??

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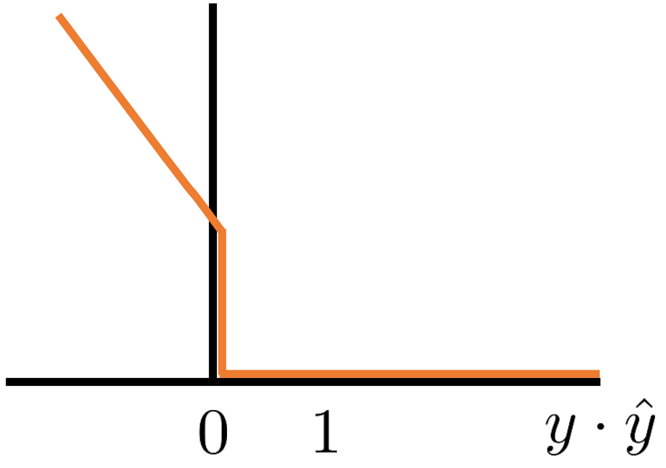

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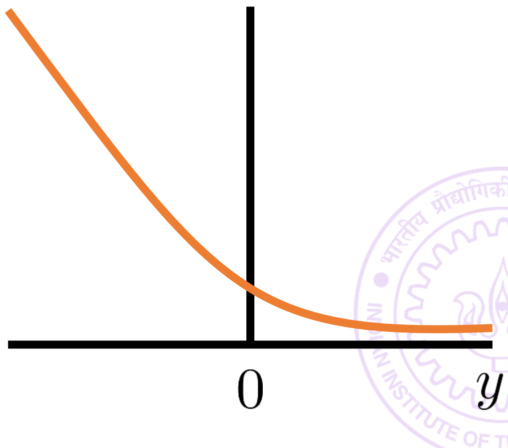
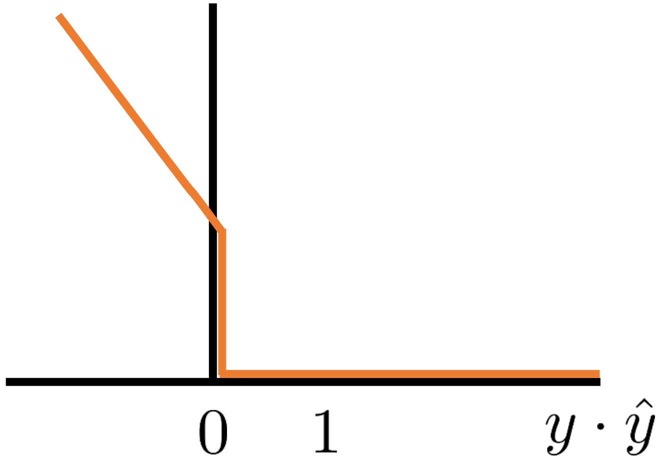


App: The Perceptron Rule via OGD



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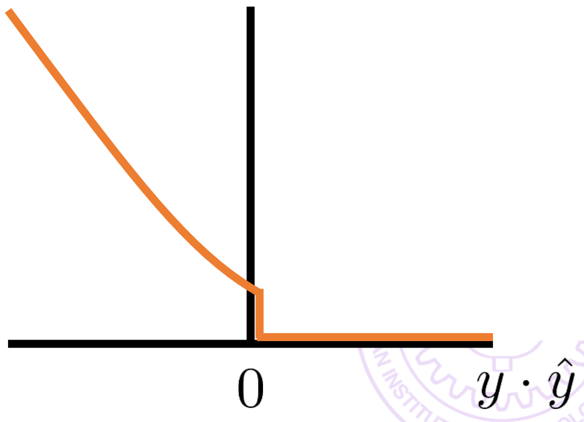

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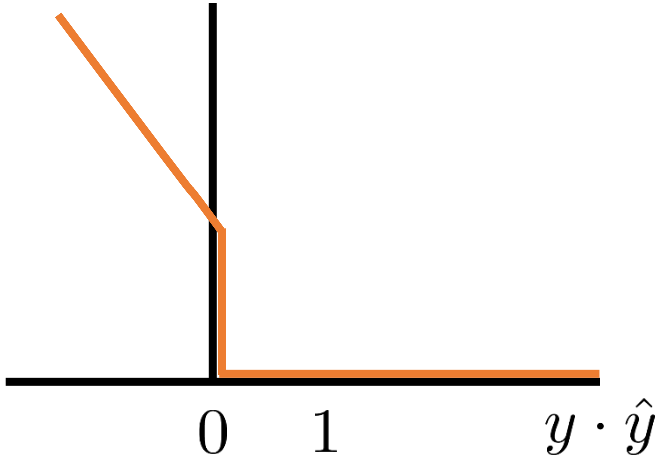

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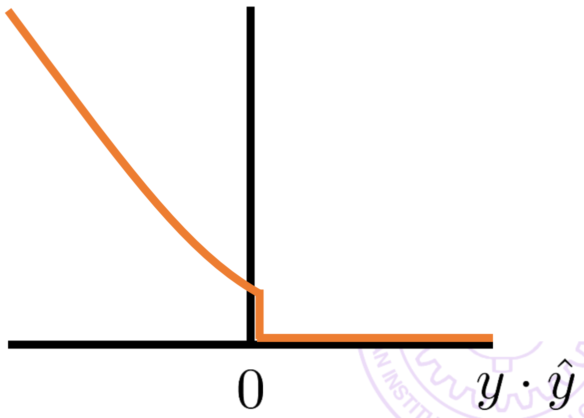
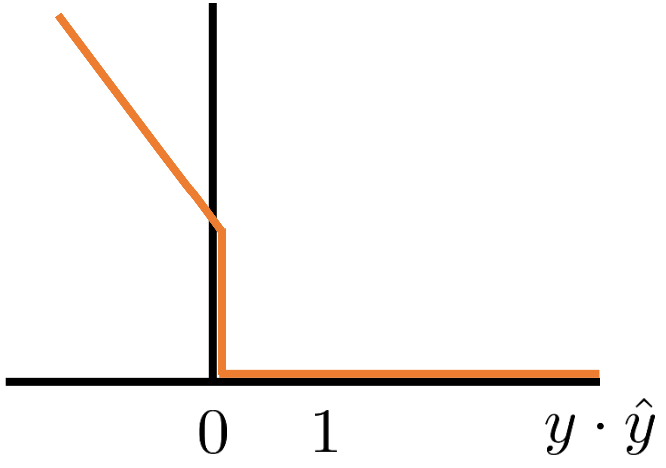
Perceptron Crammer Singer for multi-classification?



App: The Perceptron Rule via OGD

Can do stochastic perceptron too



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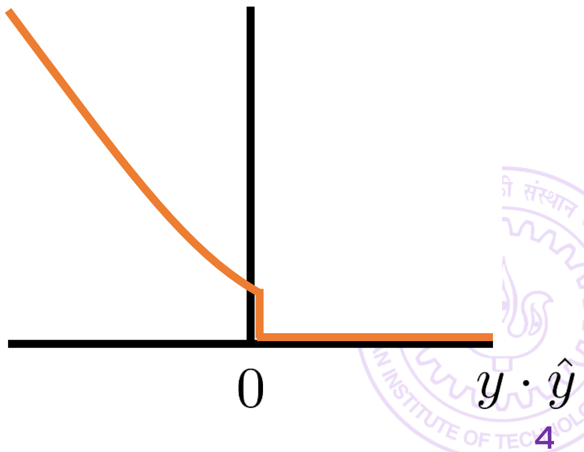

Coordinate Descent

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- Similar to gradient descent except update one variable at a time
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Somewhat like mini-batch SGD

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App: Linear Regression via CD

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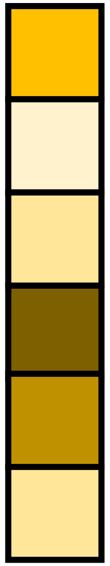
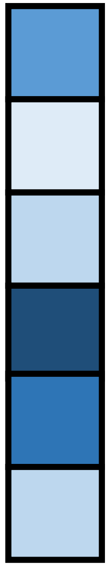
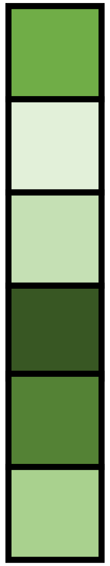


$$= 2\sum_{i=1}^{n}(\langle \mathbf{w}^t, \mathbf{x}^i \rangle - y^i) \cdot$$



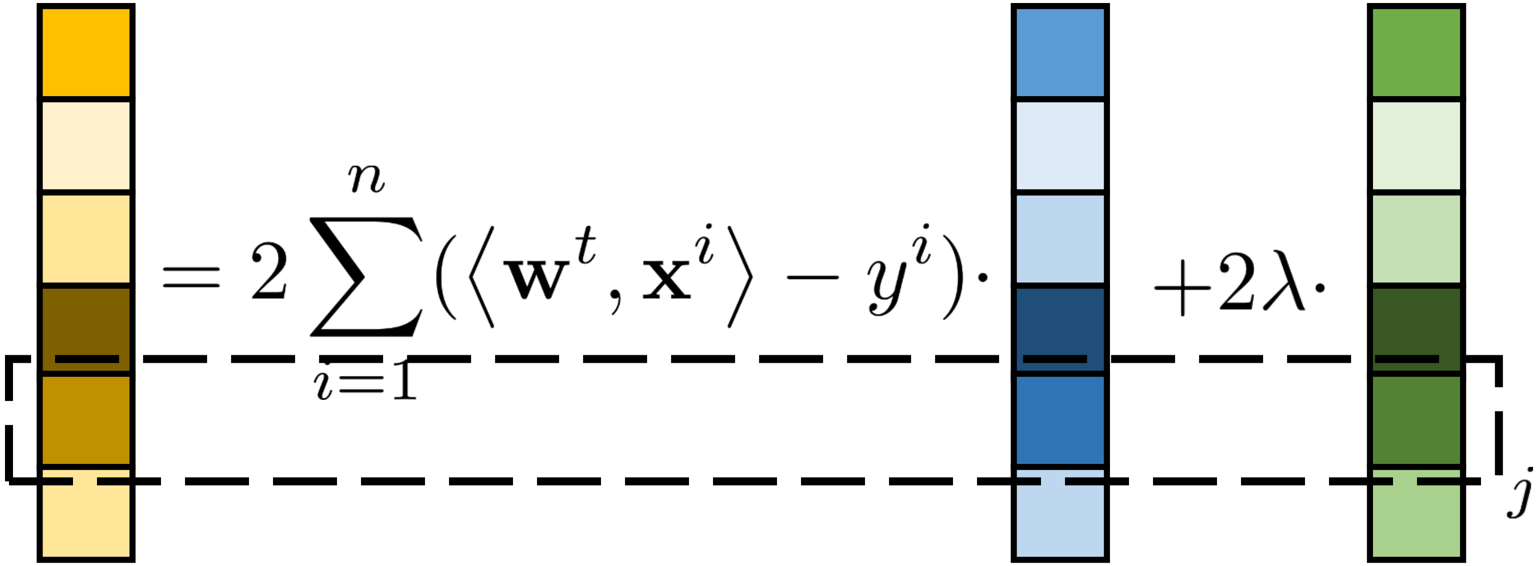
$$+2\lambda \cdot$$



App: Linear Regression via CD

$$\hat{\mathbf{w}} = \arg\min \sum_{i=1}^{n} \left(y^i - \langle \mathbf{w}, \mathbf{x}^i \rangle\right)^2 + \lambda \cdot \|\mathbf{w}\|_2^2$$

$$\mathbf{g}^t = 2\sum_{i=1}^{n} \left(\langle \mathbf{w}^t, \mathbf{x}^i \rangle - y^i\right) \cdot \mathbf{x}^i + 2\lambda \cdot \mathbf{w}^t$$



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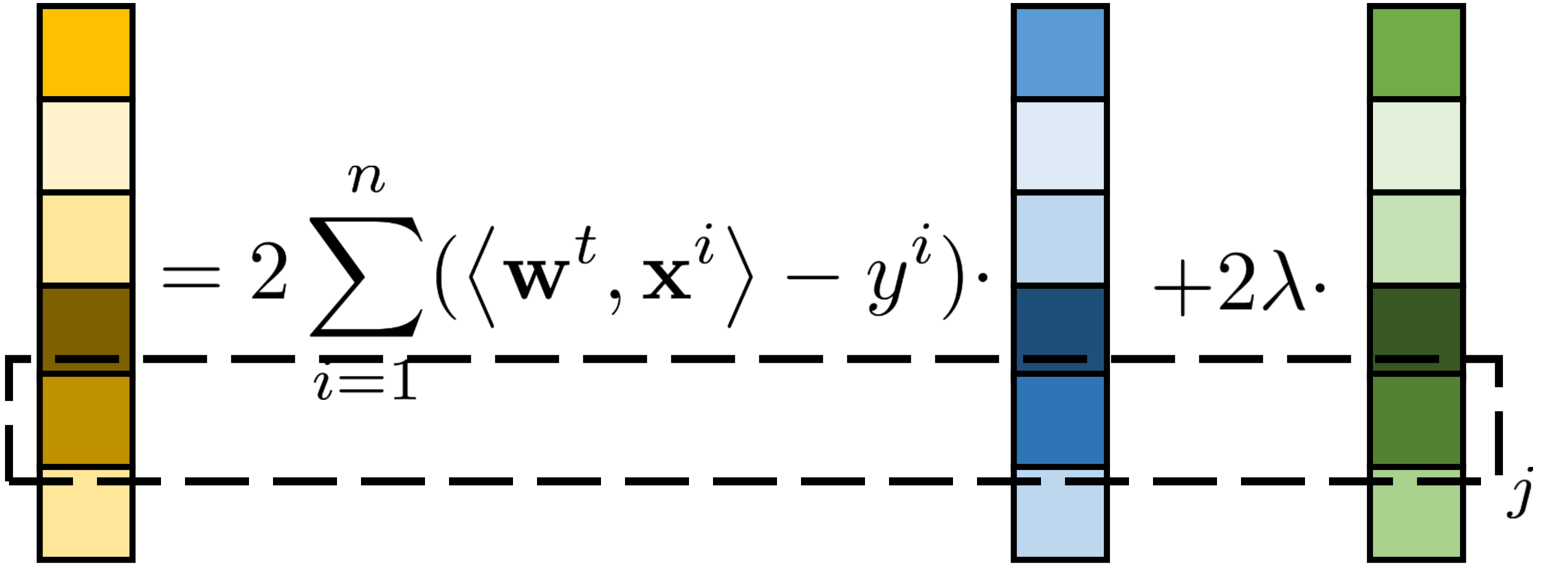

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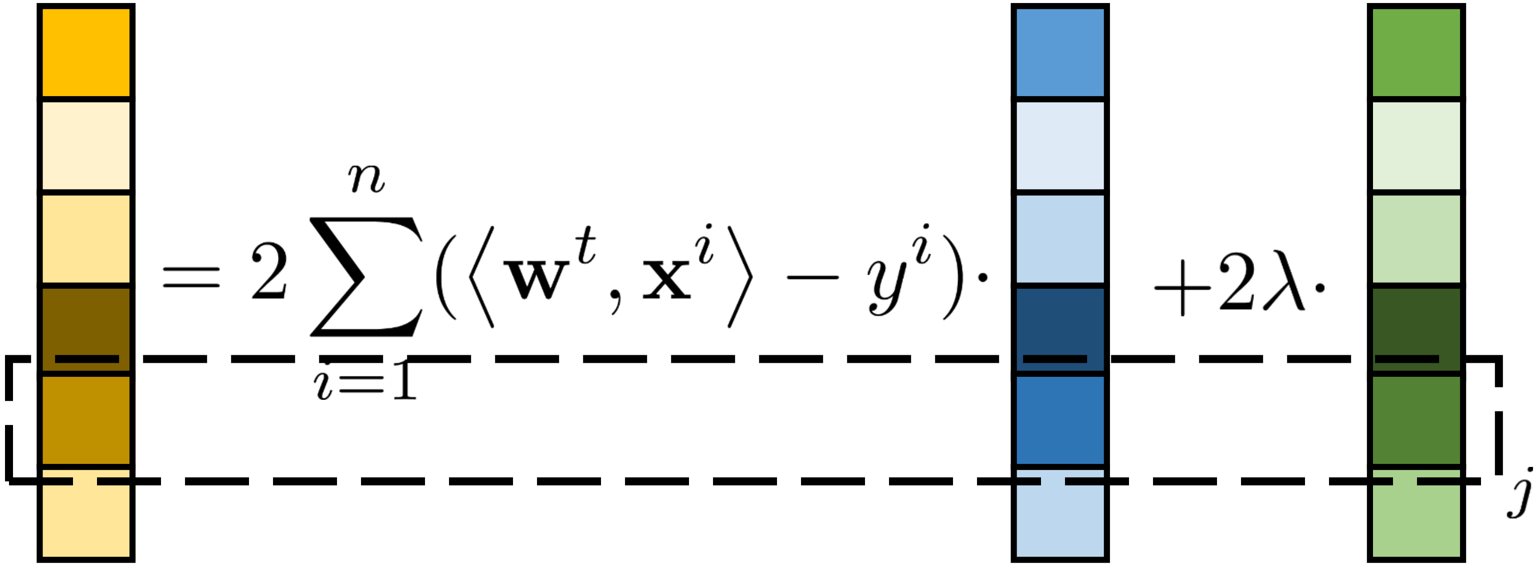

$$= 2\sum_{i=1}^{n}(\langle \mathbf{w}^t, \mathbf{x}^i \rangle - y^i) \cdot \quad +2\lambda \cdot$$

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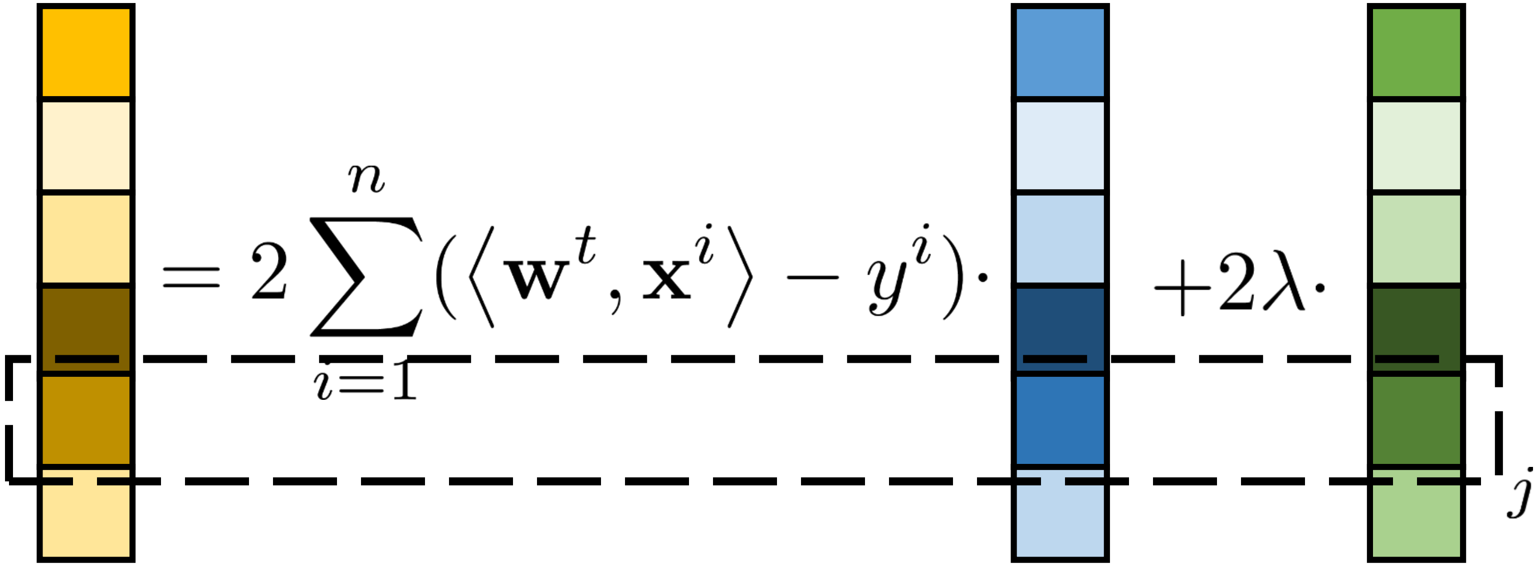

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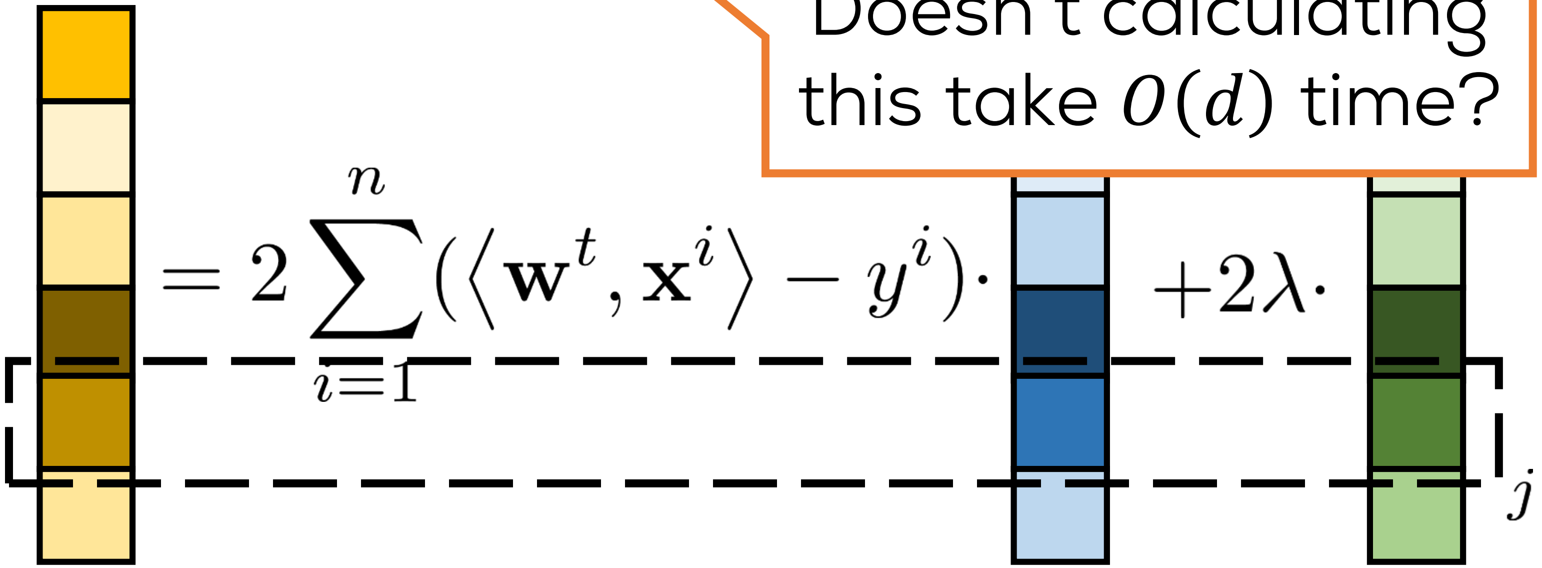
Doesn't calculating this take $O(d)$ time?



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$O(nd)$ time in total

Doesn't calculating this take $O(d)$ time?

$$= 2 \sum_{i=1}^{n} \left(\langle \mathbf{w}^t, \mathbf{x}^i \rangle - y^i\right) \cdot \qquad + 2\lambda \cdot$$

j

App: Linear Regression via CD

$$\hat{\mathbf{w}} = \arg\min \sum_{i=1}^{n} \left(y^i - \langle \mathbf{w}, \mathbf{x}^i \rangle\right)^2 + \lambda \cdot \|\mathbf{w}\|_2^2$$

$$\mathbf{g}_j^t = 2\sum_{i=1}^{n} (\langle \mathbf{w}^t, \mathbf{x}^i \rangle - y^i) \cdot \mathbf{x}_j^i + 2\lambda \cdot \mathbf{w}_j^t$$

$O(nd)$ time in total

Doesn't calculating this take $O(d)$ time?

Only if you are not careful

$$= 2\sum_{i=1}^{n} (\langle \mathbf{w}^t, \mathbf{x}^i \rangle - y^i) \cdot \quad + 2\lambda \cdot$$

j

App: Linear Regression via CD

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$O(nd)$ time in total

$$\mathbf{w}^0 = \mathbf{0}$$

App: Linear Regression via CD

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$O(nd)$ time in total

$$\mathbf{w}^0 = \mathbf{0}$$

$$\langle \mathbf{w}^0, \mathbf{x}^i \rangle = 0$$

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$O(nd)$ time in total

$$\mathbf{w}^0 = \mathbf{0}$$

$$\langle \mathbf{w}^0, \mathbf{x}^i \rangle = 0$$

$O(n)$ time

App: Linear Regression via CD

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$O(nd)$ time in total

$$\mathbf{w}^0 = \mathbf{0} \qquad \mathbf{w}^t$$

$$\langle \mathbf{w}^0, \mathbf{x}^i \rangle = 0$$

$O(n)$ time

App: Linear Regression via CD

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$\mathcal{O}(nd)$ time in total

$$\mathbf{w}^0 = \mathbf{0} \qquad \mathbf{w}^t$$

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$\mathcal{O}(n)$ time

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$O(nd)$ time in total

$$\mathbf{w}^0 = \mathbf{0} \qquad \mathbf{w}^t$$

$$\langle \mathbf{w}^0, \mathbf{x}^i \rangle = 0 \qquad \langle \mathbf{w}^t, \mathbf{x}^i \rangle$$

$O(n)$ time

Available

App: Linear Regression via CD

$$\hat{\mathbf{w}} = \arg\min \sum_{i=1}^{n} \left(y^i - \langle \mathbf{w}, \mathbf{x}^i \rangle\right)^2 + \lambda \cdot \|\mathbf{w}\|_2^2$$

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$$\mathbf{w}^0 = \mathbf{0} \qquad \mathbf{w}^t$$

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$O(n)$ time

Available

Assumed

App: Linear Regression via CD

$$\hat{\mathbf{w}} = \arg\min \sum_{i=1}^{n} \left(y^i - \langle \mathbf{w}, \mathbf{x}^i \rangle\right)^2 + \lambda \cdot \|\mathbf{w}\|_2^2$$

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$O(nd)$ time in total

$\mathbf{w}^0 = \mathbf{0}$ $\qquad$ $\mathbf{w}^t$ $\qquad$ $\mathbf{w}_{j_t}^{t+1} = \mathbf{w}_{j_t}^t - \eta_t \cdot \mathbf{g}_{j_t}^t$

$\langle \mathbf{w}^0, \mathbf{x}^i \rangle = 0$ $\qquad$ $\langle \mathbf{w}^t, \mathbf{x}^i \rangle$

$O(n)$ time

Available

Assumed

App: Linear Regression via CD

$$\hat{\mathbf{w}} = \arg\min \sum_{i=1}^{n} \left(y^i - \langle \mathbf{w}, \mathbf{x}^i \rangle\right)^2 + \lambda \cdot \|\mathbf{w}\|_2^2$$

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$O(nd)$ time in total

$$\mathbf{w}^0 = \mathbf{0} \qquad \mathbf{w}^t \qquad \mathbf{w}_{j_t}^{t+1} = \mathbf{w}_{j_t}^t - \eta_t \cdot \mathbf{g}_{j_t}^t$$

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$O(n)$ time

Available

Assumed

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$\mathcal{O}(nd)$ time in total

$$\mathbf{w}^0 = \mathbf{0} \qquad \mathbf{w}^t \qquad \mathbf{w}_{j_t}^{t+1} = \mathbf{w}_{j_t}^t - \eta_t \cdot \mathbf{g}_{j_t}^t$$

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$\mathcal{O}(n)$ time

Available

Already available

Assumed

App: Linear Regression via CD

$$\hat{\mathbf{w}} = \arg\min \sum_{i=1}^{n} \left(y^i - \langle \mathbf{w}, \mathbf{x}^i \rangle\right)^2 + \lambda \cdot \|\mathbf{w}\|_2^2$$

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$$\mathbf{w}^0 = \mathbf{0} \qquad \mathbf{w}^t \qquad \mathbf{w}_{j_t}^{t+1} = \mathbf{w}_{j_t}^t - \eta_t \cdot \mathbf{g}_{j_t}^t$$

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$\mathcal{O}(n)$ time

Available

Assumed

Already available

$\mathcal{O}(1)$ time per data point!

App: Linear Regression via CD

$$\hat{\mathbf{w}} = \arg\min \sum_{i=1}^{n} \left(y^i - \langle \mathbf{w}, \mathbf{x}^i \rangle\right)^2 + \lambda \cdot \|\mathbf{w}\|_2^2$$

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$\mathcal{O}(nd)$ time in total

$$\mathbf{w}^0 = \mathbf{0} \qquad \mathbf{w}^t \qquad \mathbf{w}_{j_t}^{t+1} = \mathbf{w}_{j_t}^t - \eta_t \cdot \mathbf{g}_{j_t}^t$$

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$\mathcal{O}(n)$ time

Available

Assumed

Already available

$\mathcal{O}(1)$ time per data point!

$\mathcal{O}(n)$ time in total

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$O(nd)$ time in total

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$O(n)$ time

Available

Assumed

Already available

$O(1)$ time per data point!

Induction complete!

$O(n)$ time in total

App: Linear Regression via CD

$$\hat{\mathbf{w}} = \arg\min \sum_{i=1}^{n} \left(y^i - \langle \mathbf{w}, \mathbf{x}^i \rangle\right)^2 + \lambda \cdot \|\mathbf{w}\|_2^2$$

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$O(n)$ time

Available

Assumed

Already available

$O(1)$ time per data point!

Induction complete!

$O(n)$ time in total

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$O(n)$ time

Available

Assumed

Already available

$O(1)$ time per data point!

Induction complete!

$O(n)$ time in total

Handling Constraints

The Tale of Two Techniques

The Tale of Two Techniques

$$\hat{\mathbf{w}} = \arg\min \sum_{i=1}^{n} \left(y^i - \langle \mathbf{w}, \mathbf{x}^i \rangle\right)^2$$

$$\text{s.t. } \|\mathbf{w}\|_2 \leq r$$

The Tale of Two Techniques

$$\hat{\mathbf{w}} = \arg\min \; f(\mathbf{w})$$
$$\text{s.t. } \mathbf{w} \in \mathcal{C}$$

The Tale of Two Techniques

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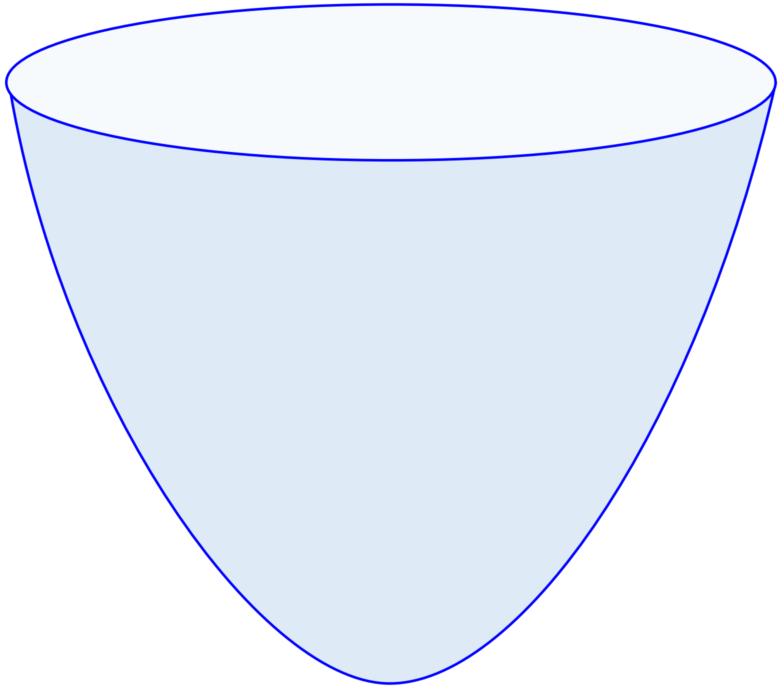

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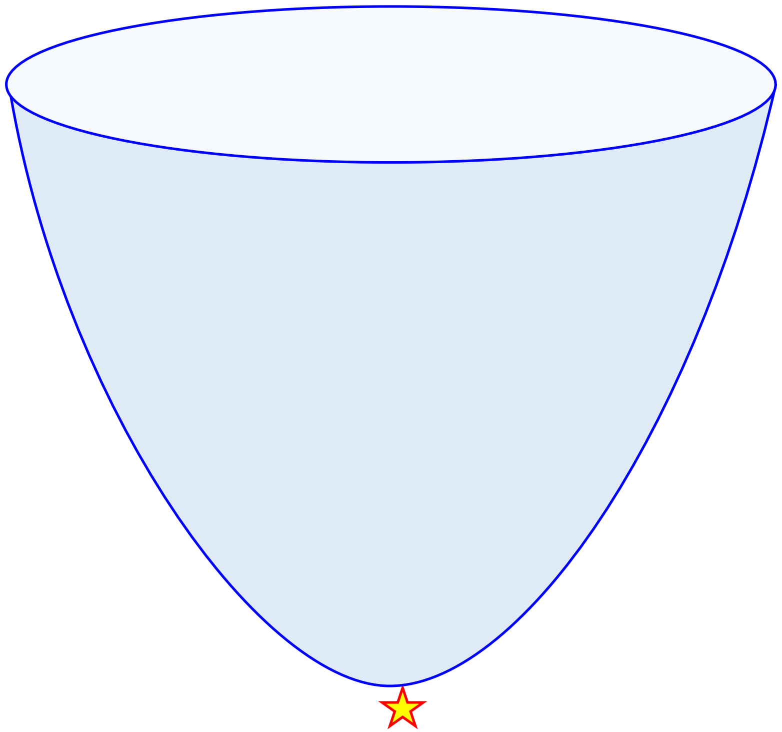

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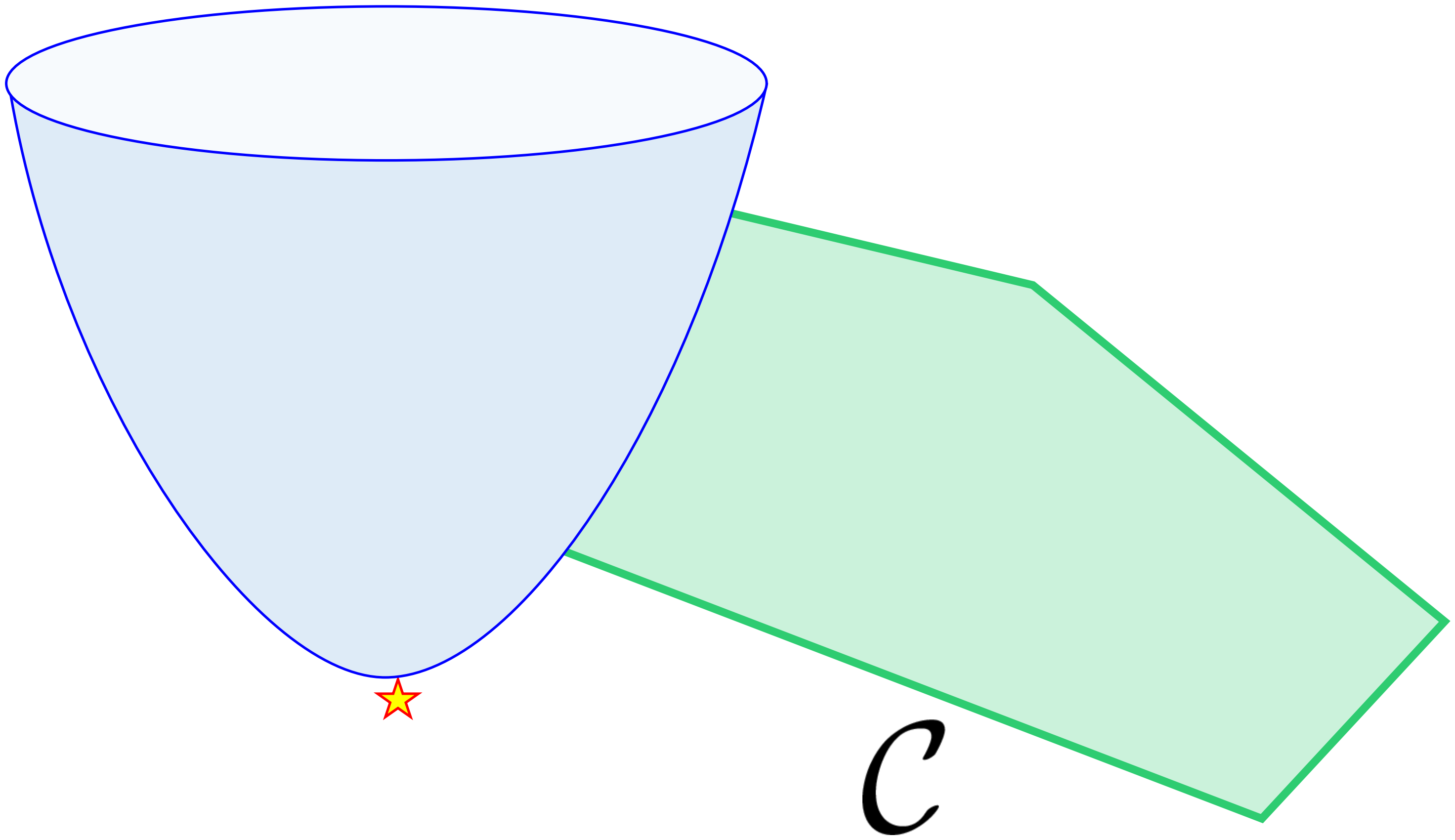

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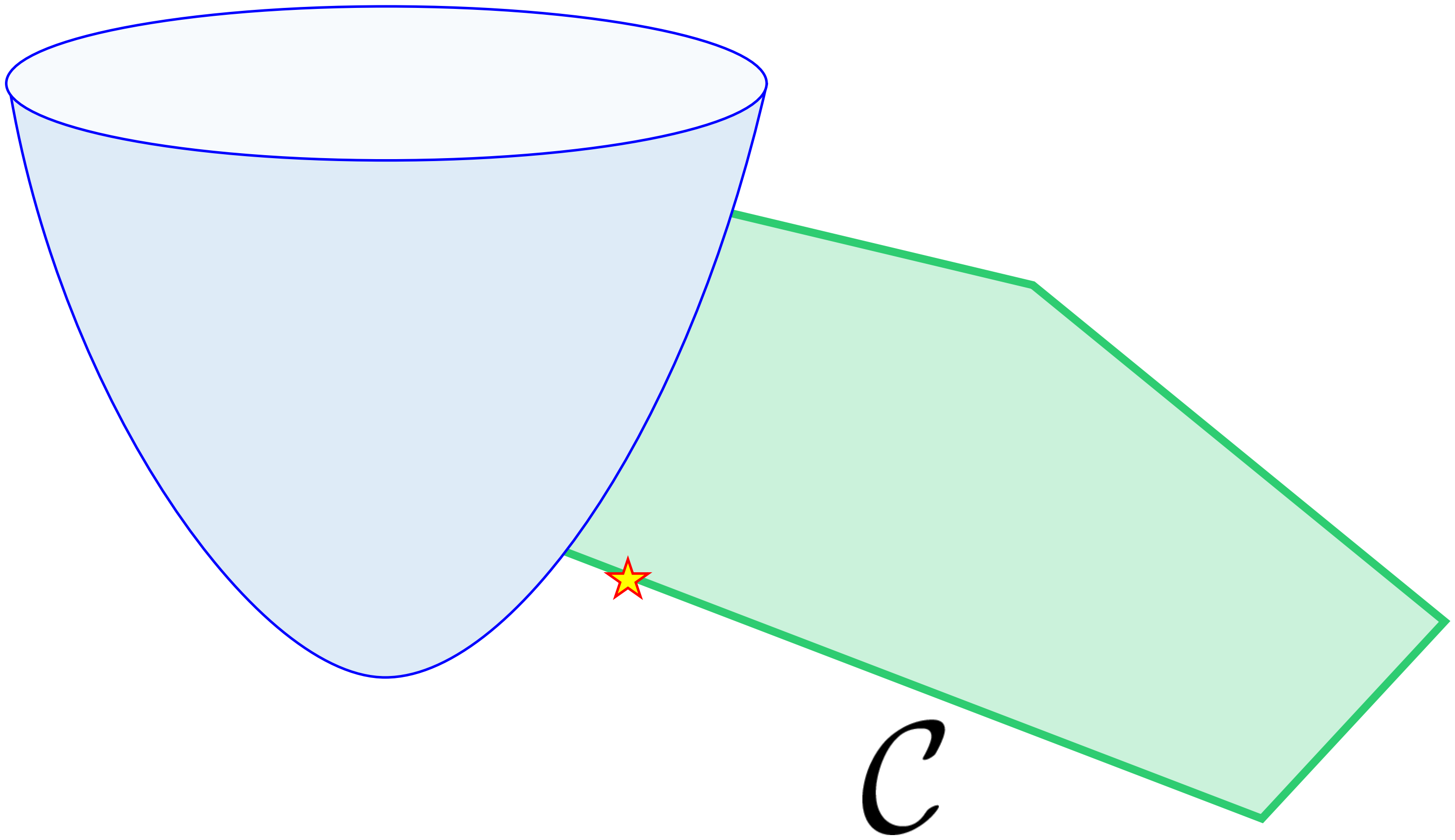

The Tale of Two Techniques

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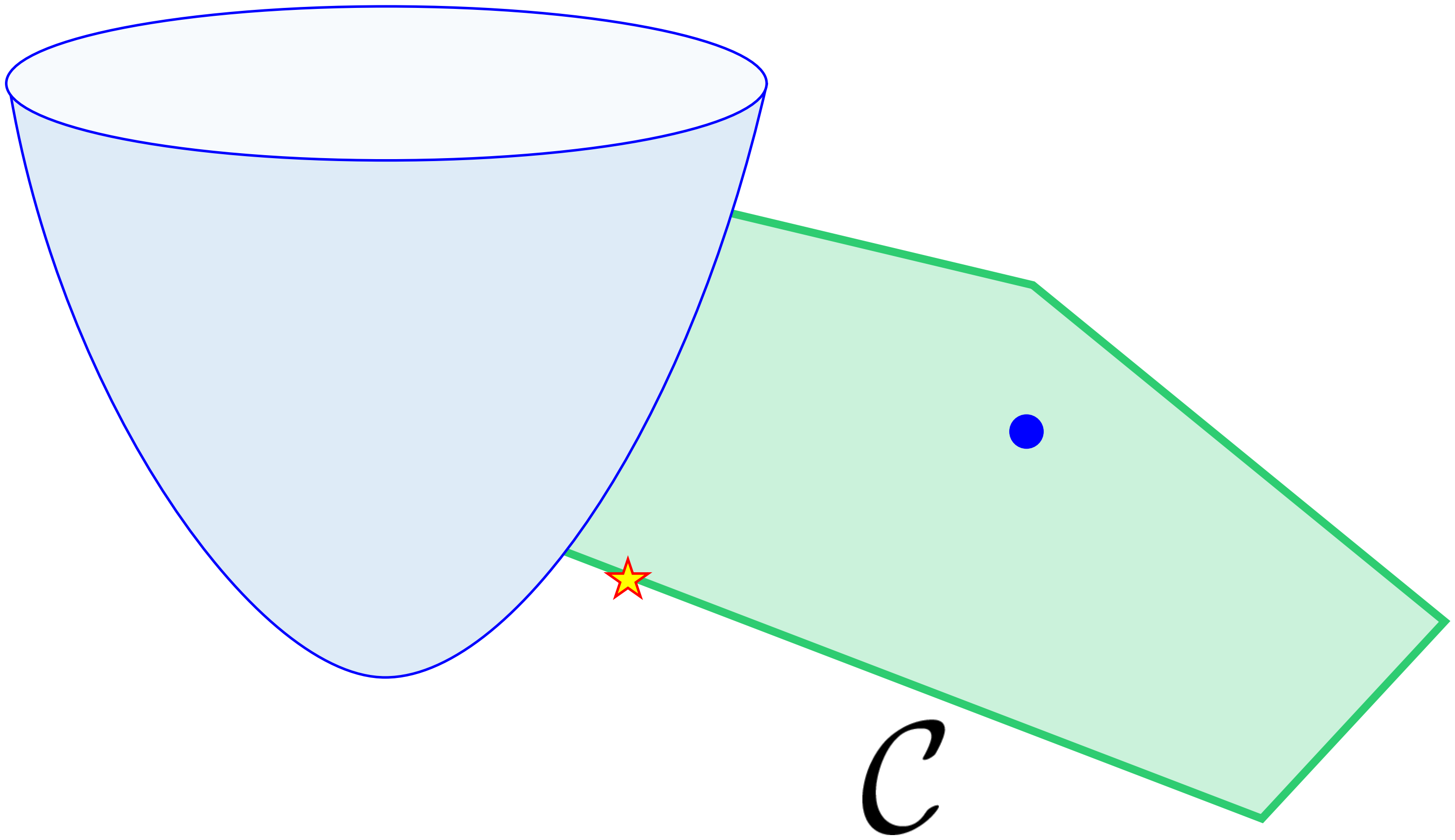

The Tale of Two Techniques

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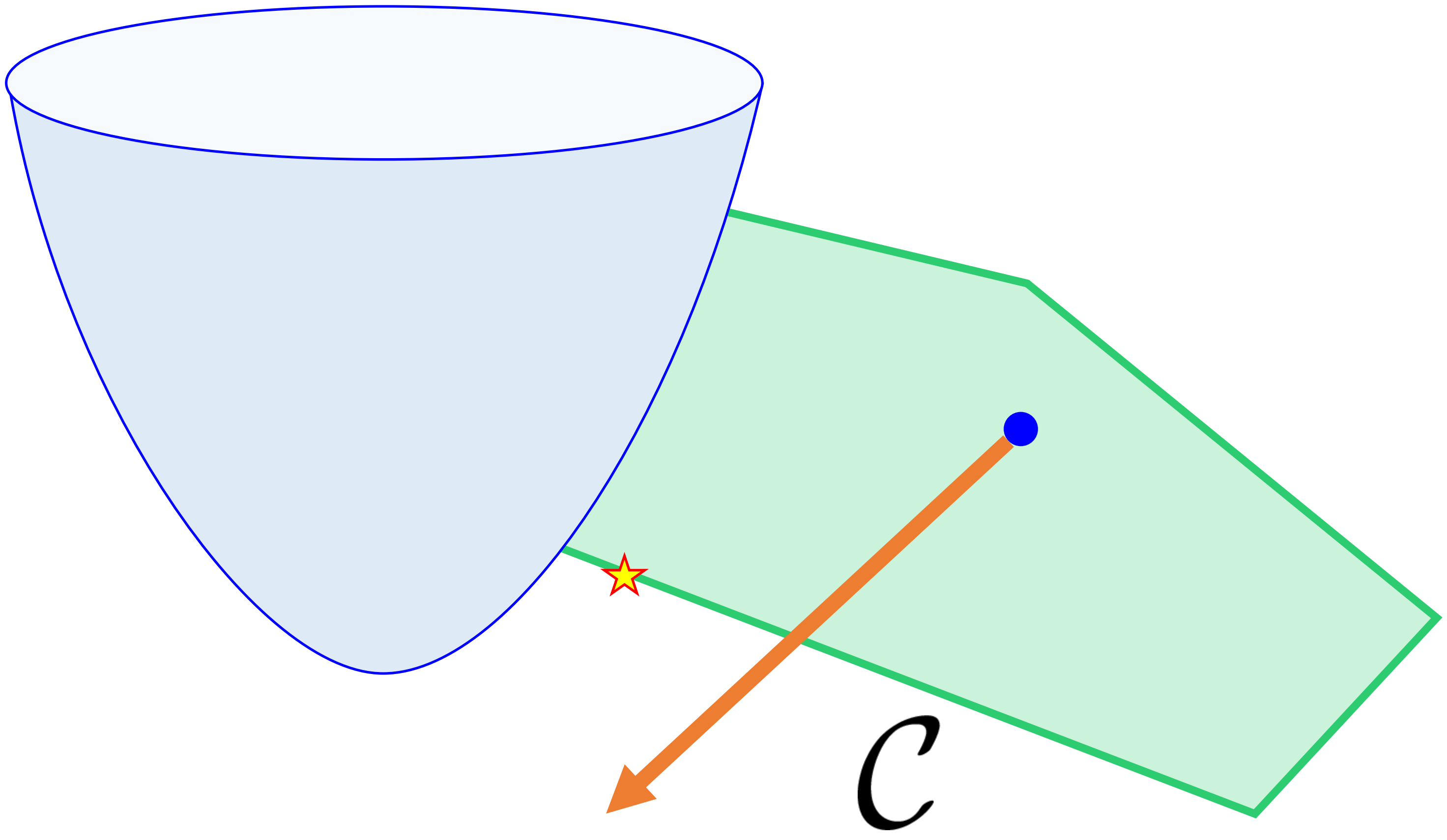

The Tale of Two Techniques

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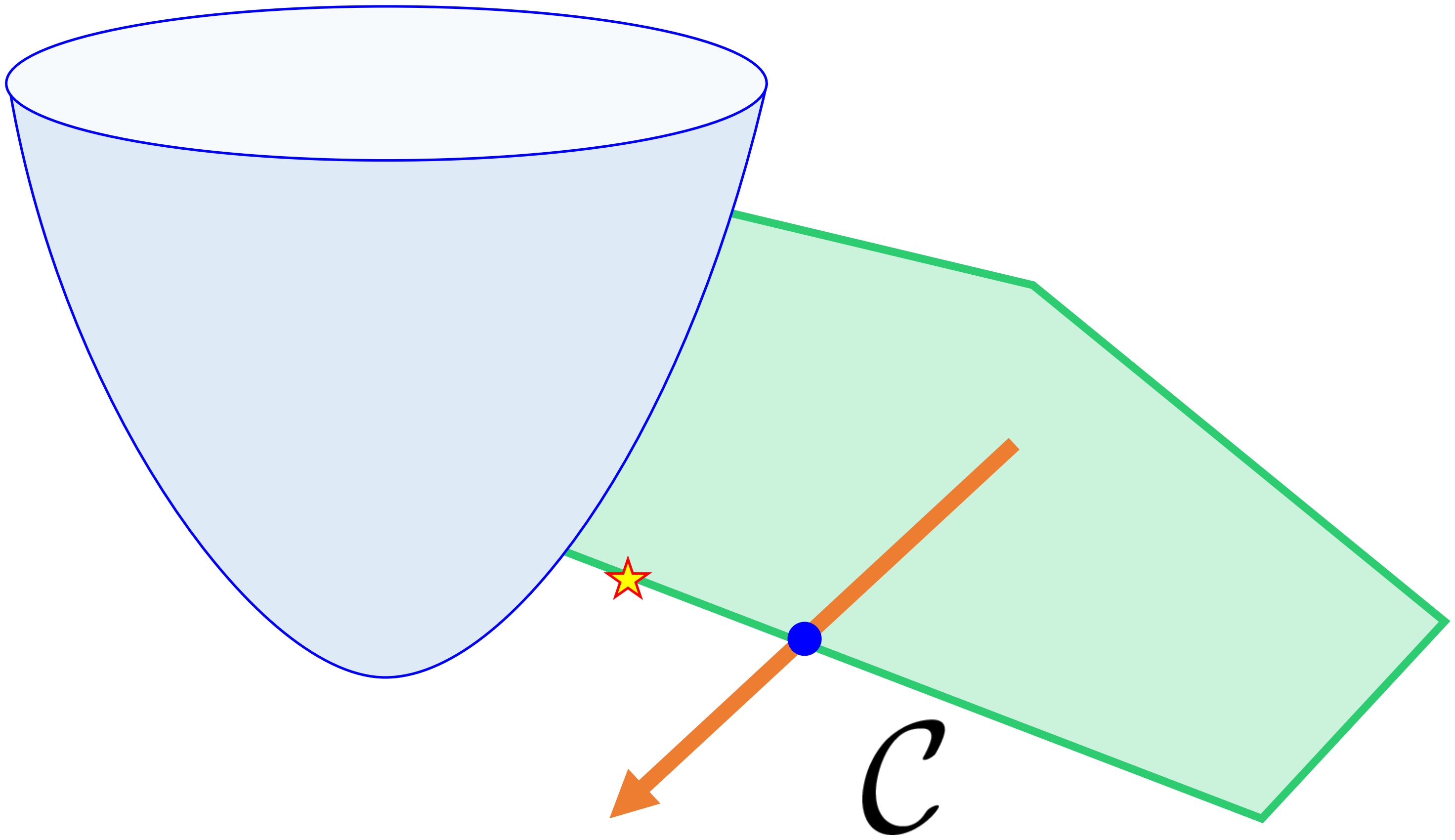
Interior Point Method



The Tale of Two Techniques

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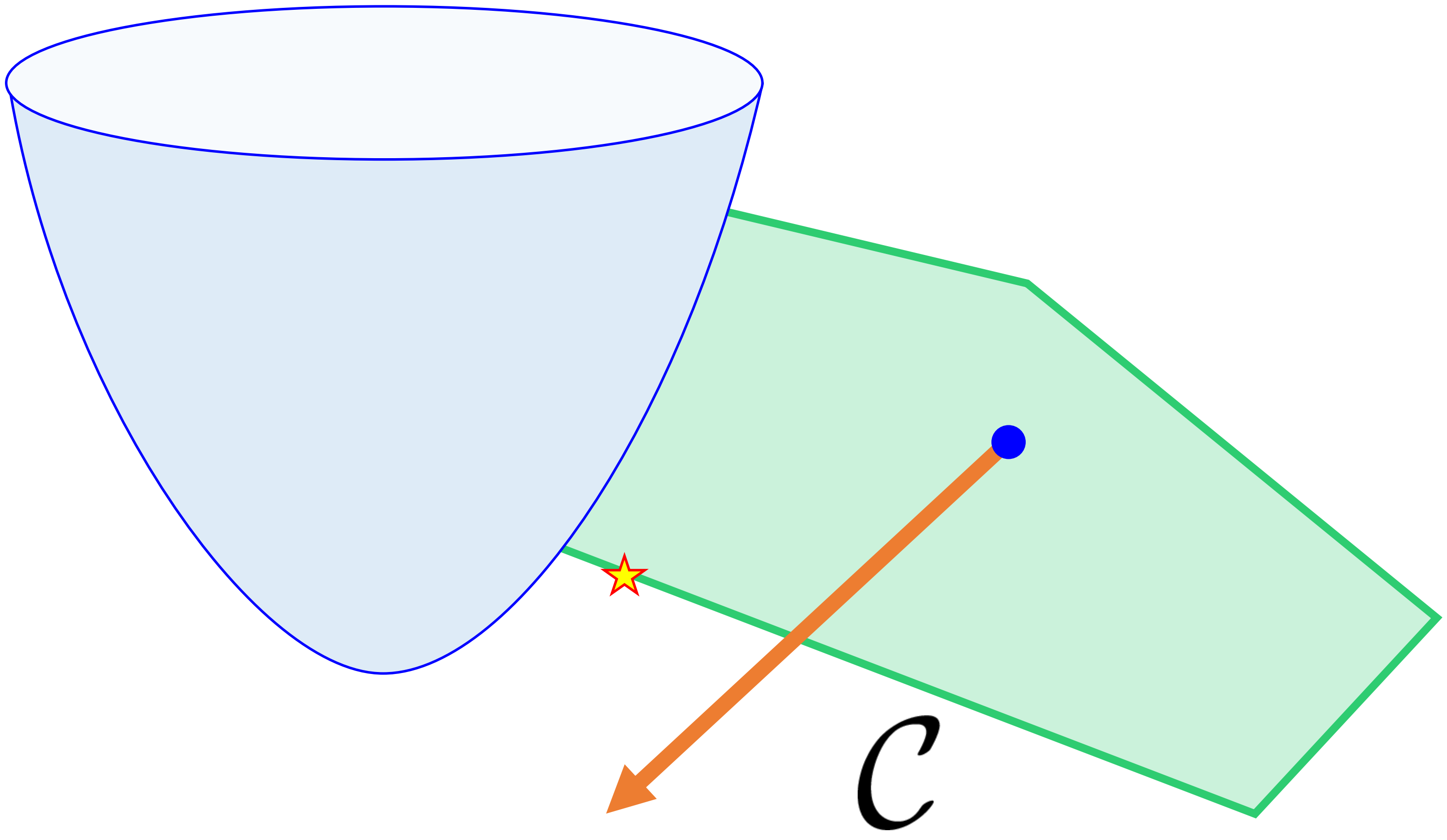
Interior Point Method



The Tale of Two Techniques

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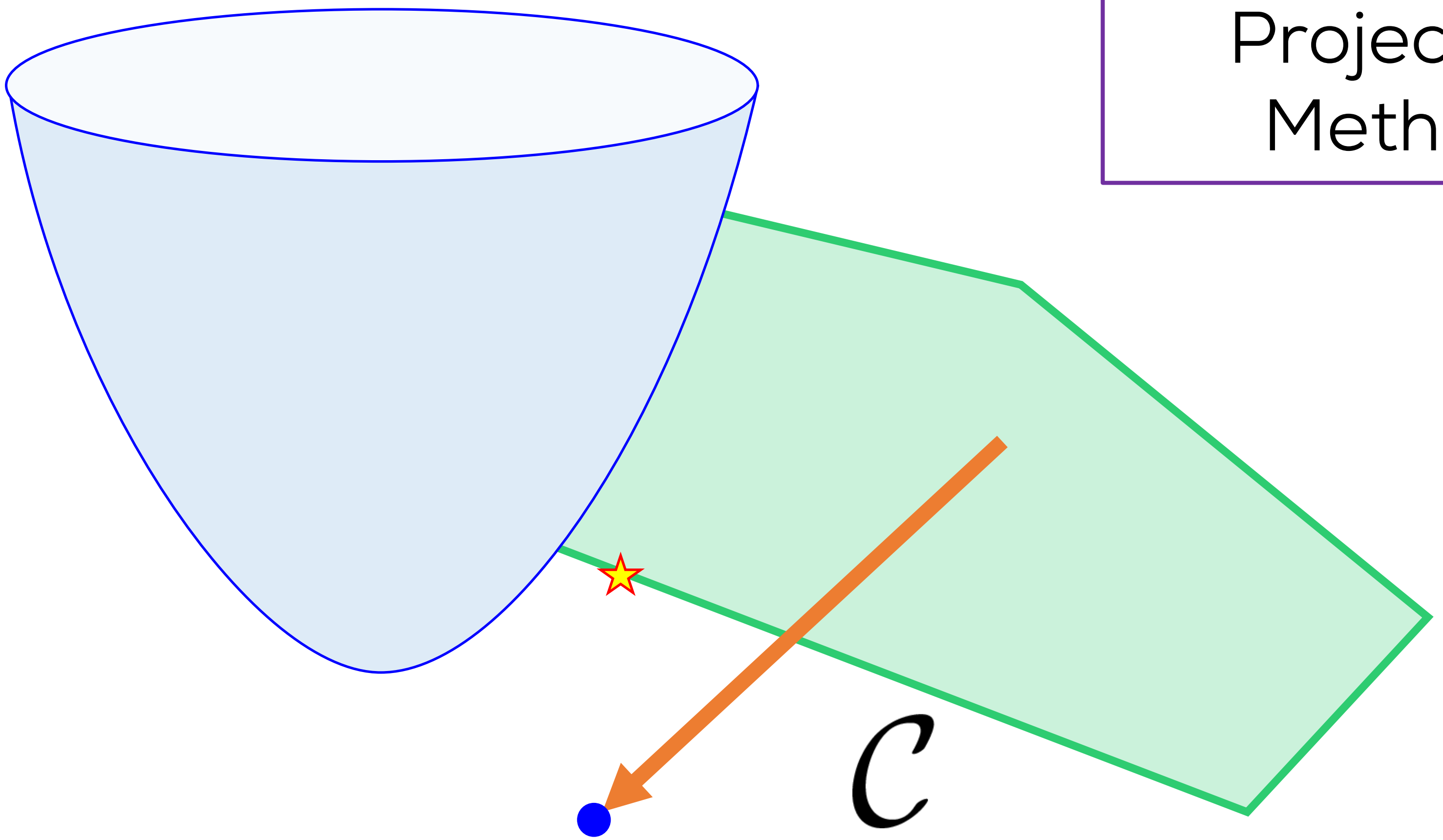
$$\text{s.t. } \mathbf{w} \in \mathcal{C}$$

Interior Point Method

Projected Method

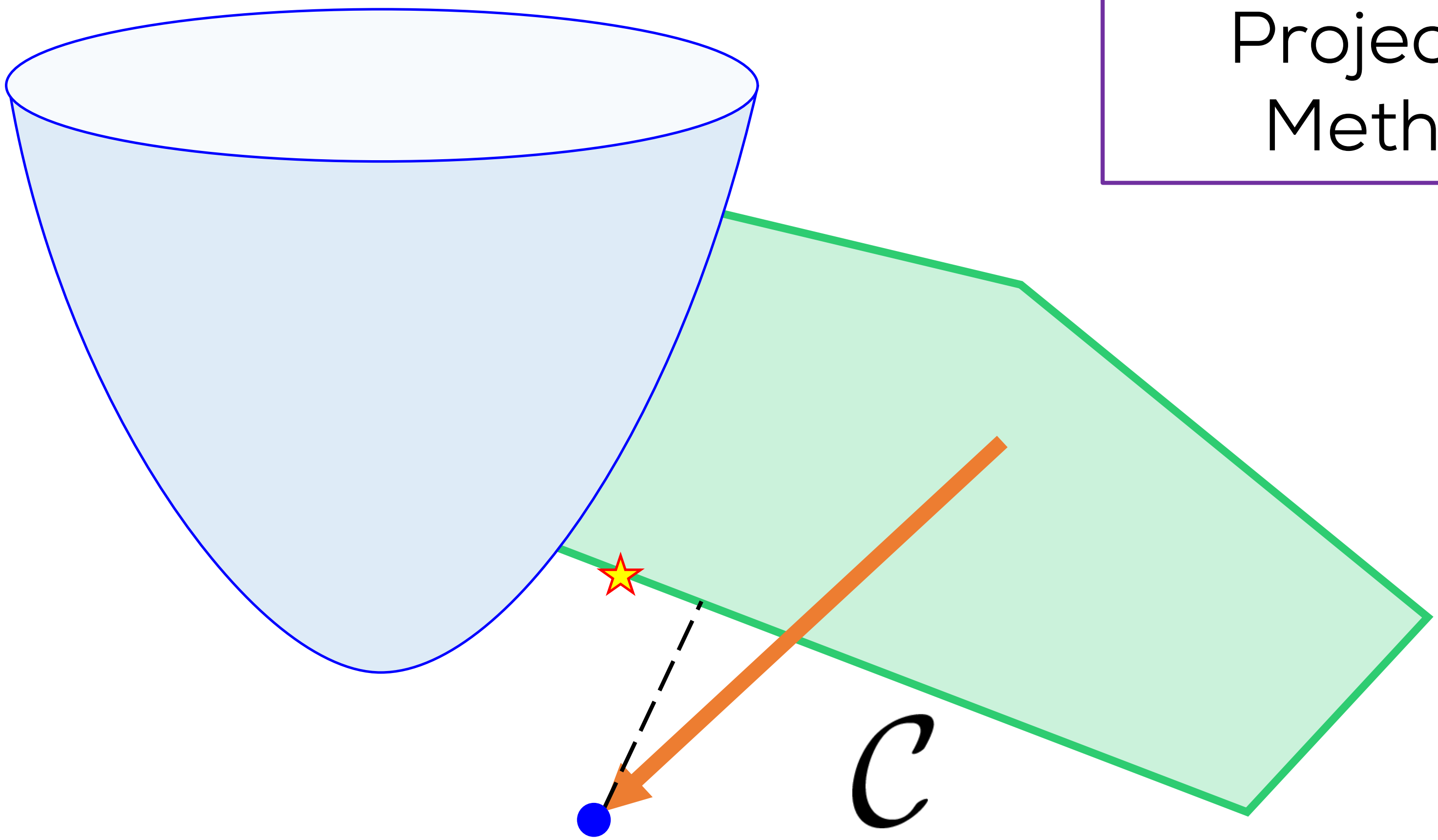


The Tale of Two Techniques

$$\hat{\mathbf{w}} = \arg\min \; f(\mathbf{w})$$
$$\text{s.t. } \mathbf{w} \in \mathcal{C}$$

Interior Point Method

Projected Method



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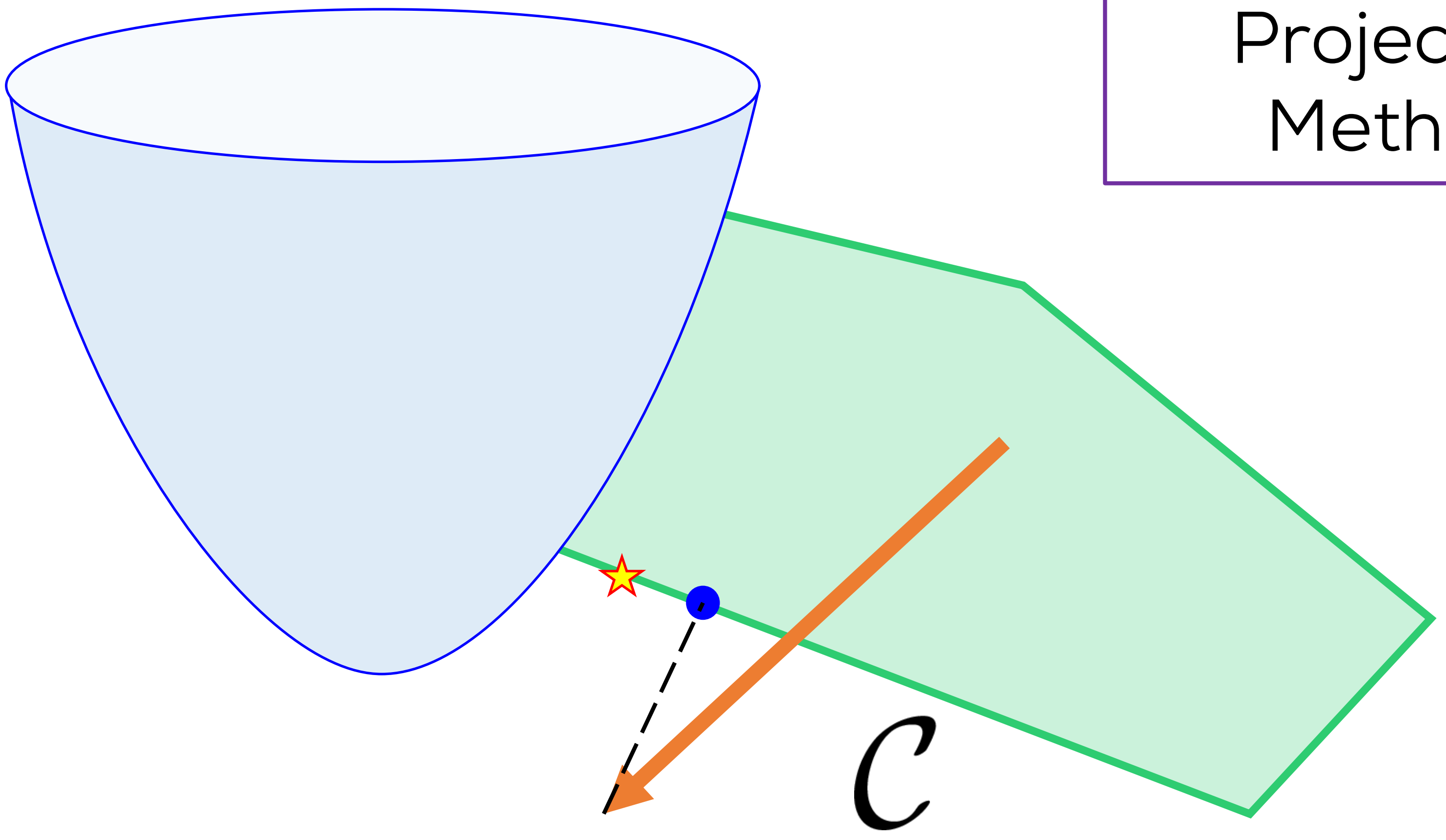
$$\text{s.t. } \mathbf{w} \in \mathcal{C}$$

Interior Point Method



Projected Method

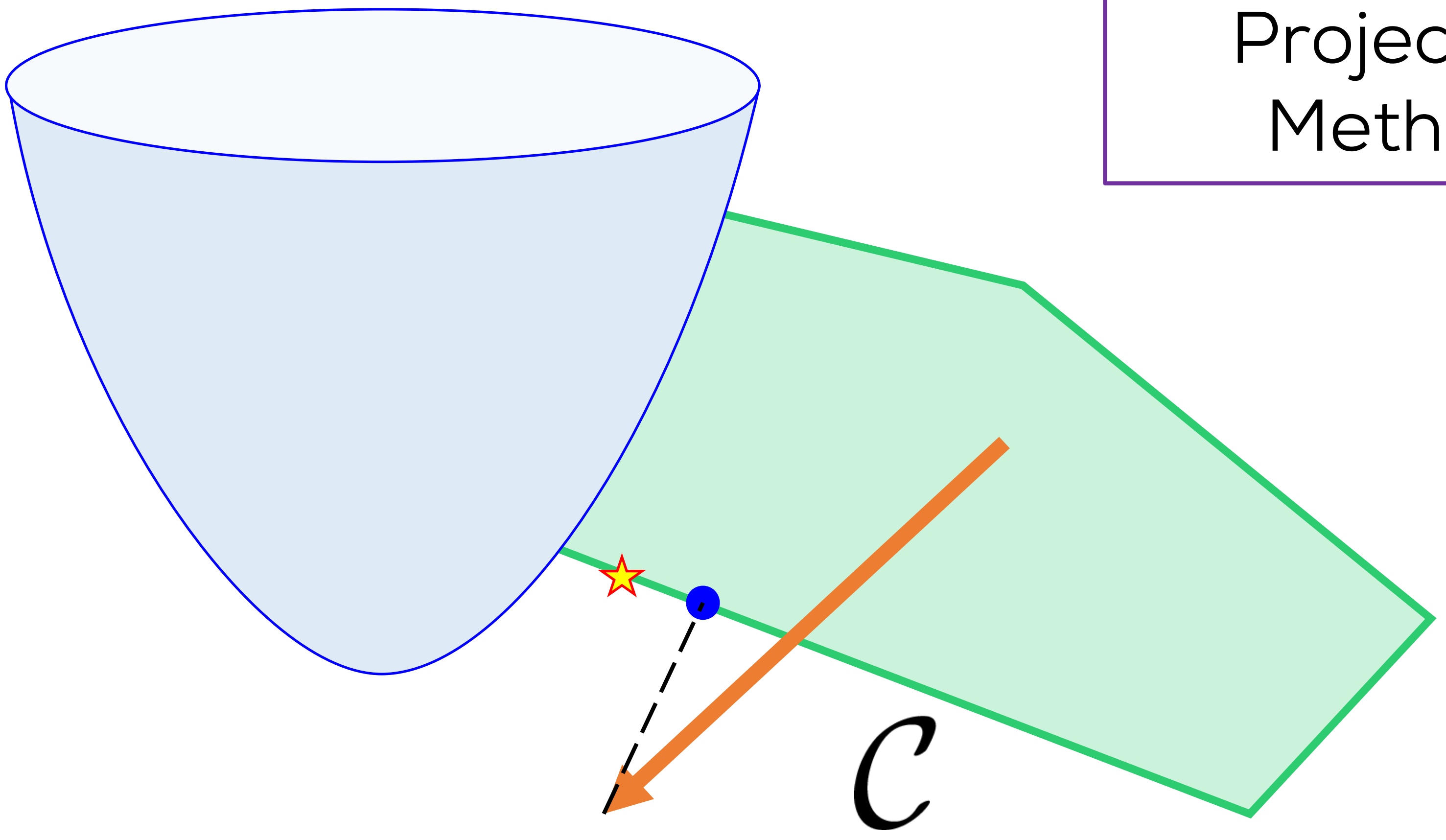
The Tale of Two Techniques

$$\hat{\mathbf{w}} = \arg\min \; f(\mathbf{w})$$
$$\text{s.t. } \mathbf{w} \in \mathcal{C}$$

Interior Point Method

Powerful but can be slow, difficult to implement

Projected Method



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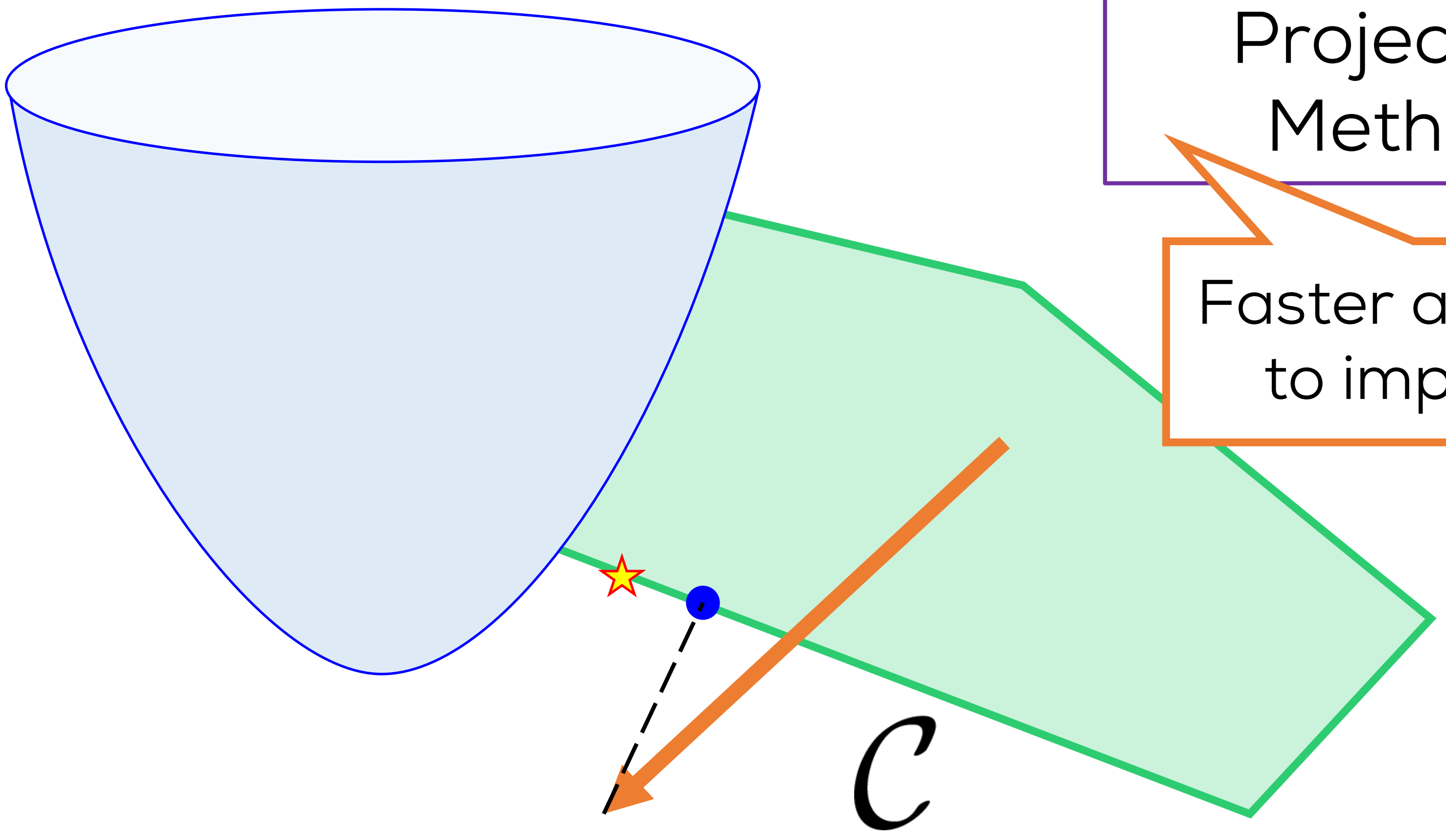


Projected Method

Faster and easier to implement

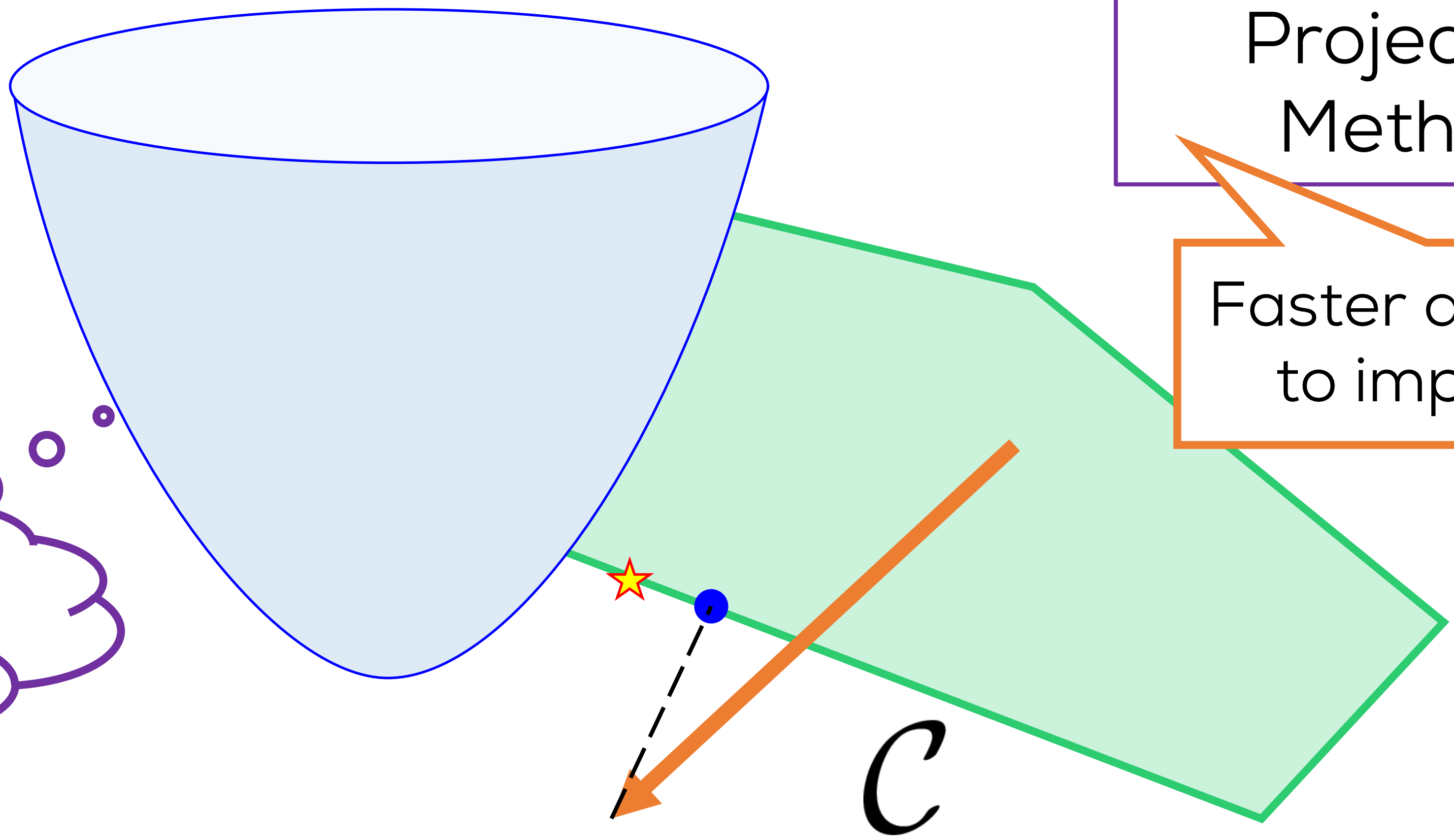
The Tale of Two Techniques

$$\hat{\mathbf{w}} = \arg\min \; f(\mathbf{w})$$
$$\text{s.t. } \mathbf{w} \in \mathcal{C}$$

Interior Point Method

Powerful but can be slow, difficult to implement

What if the set is non-convex?



Projected Method

Faster and easier to implement

The Tale of Two Techniques

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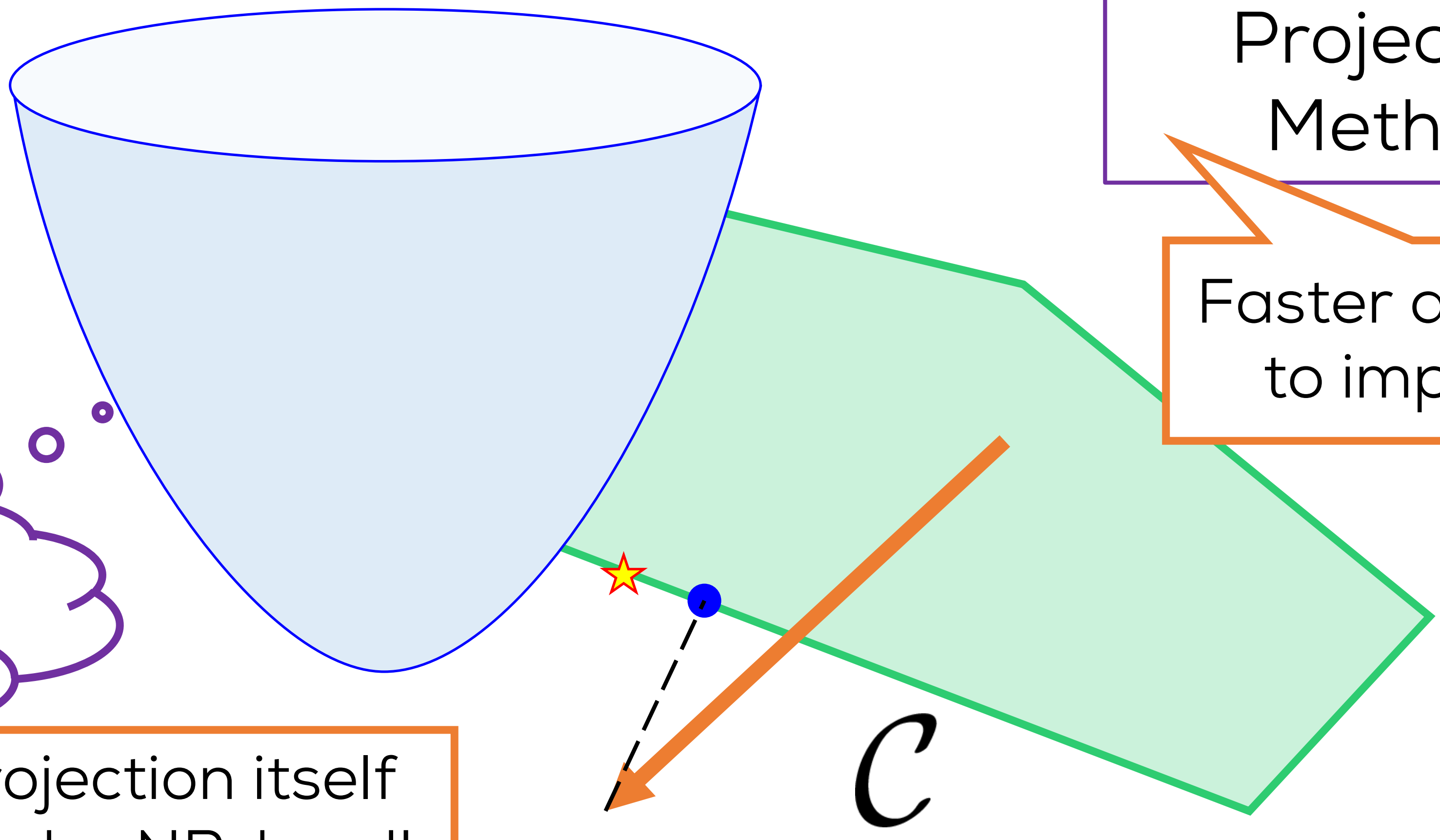


Projected Method

Faster and easier to implement

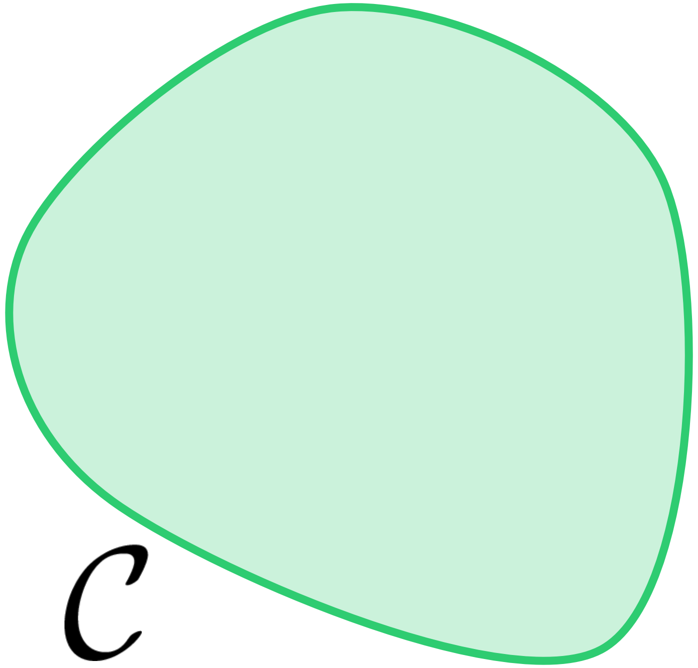
Projection itself can be NP-hard!

Projected Gradient Descent

Projected Gradient Descent

$$\arg\min \; f(\mathbf{w})$$
$$\text{s.t. } \mathbf{w} \in \mathcal{C}$$

Projected Gradient Descent



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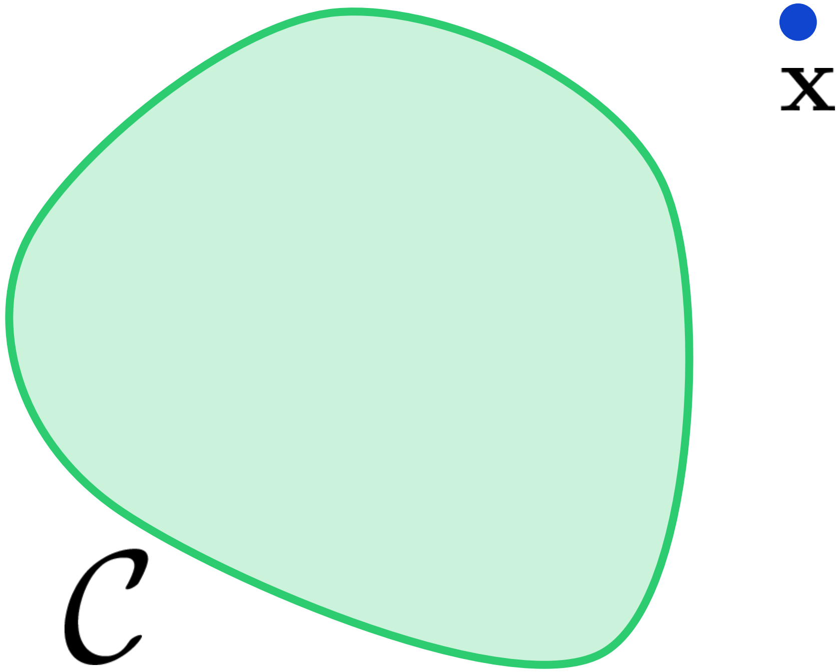
Projected Gradient Descent



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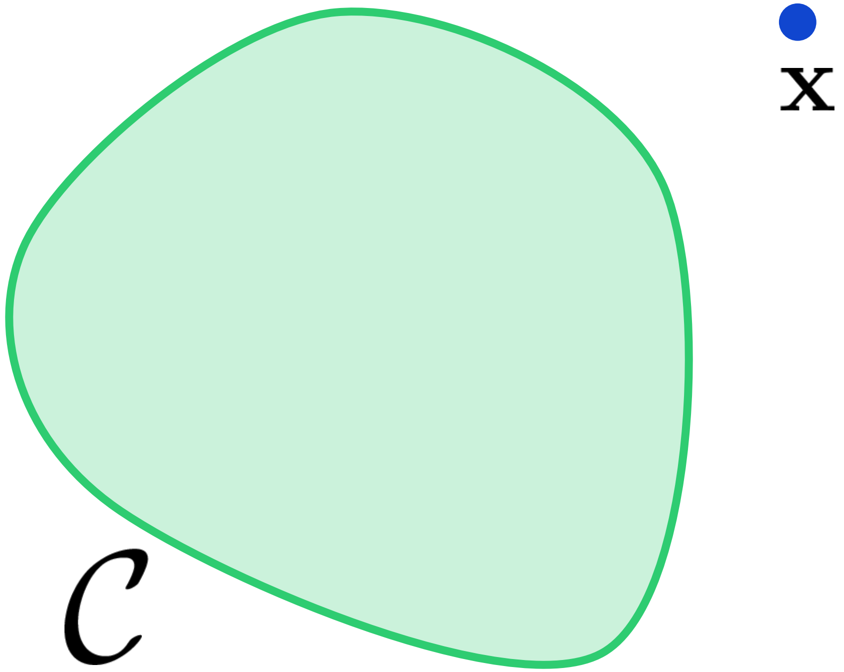
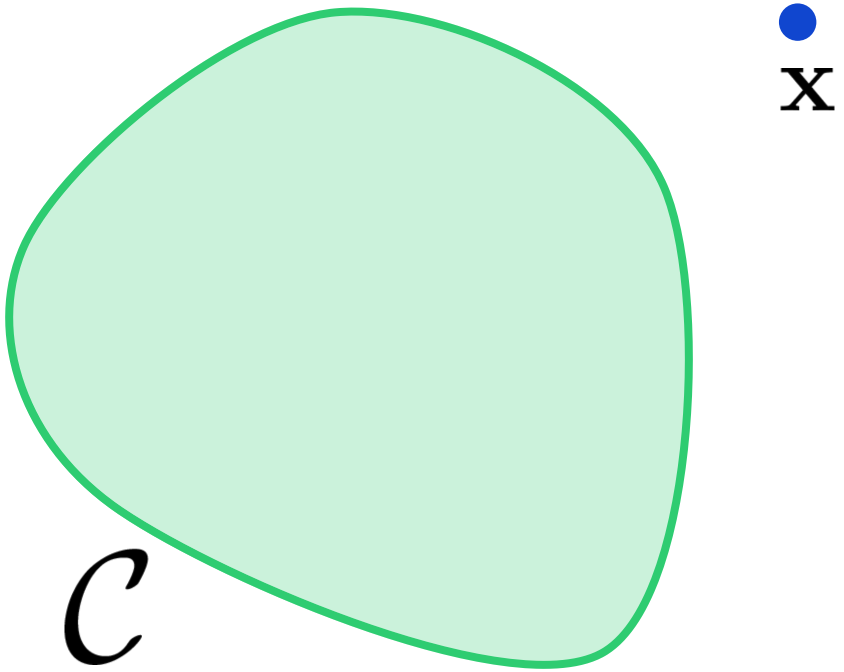
Projected Gradient Descent



$$\Pi_{\mathcal{C}}(\mathbf{x}) = \arg\min_{\mathbf{z}\in\mathcal{C}} \|\mathbf{z} - \mathbf{x}\|$$

$$\arg\min \; f(\mathbf{w}) \quad \text{s.t. } \mathbf{w} \in \mathcal{C}$$

Projected Gradient Descent

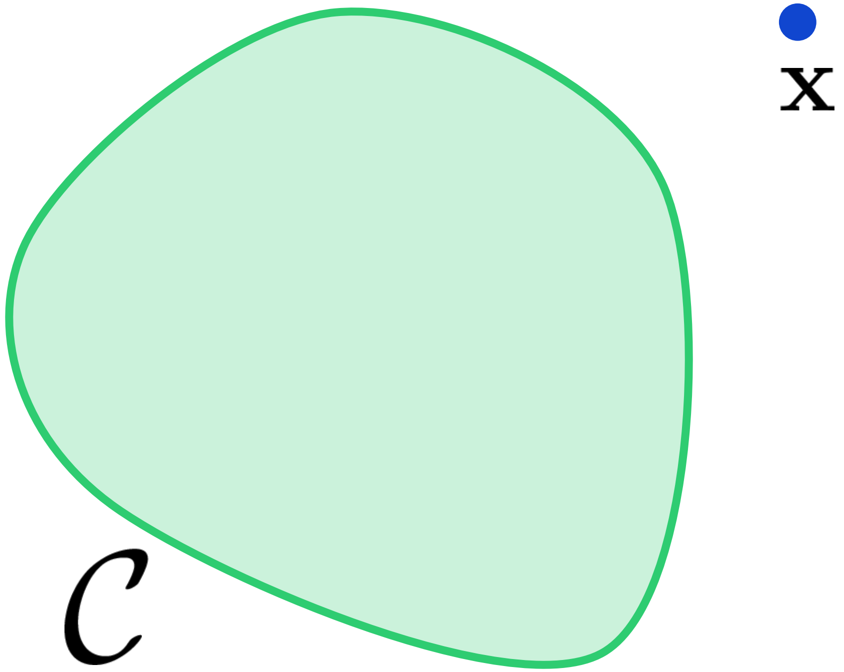


$$\Pi_{\mathcal{C}}(\mathbf{x}) = \arg\min_{\mathbf{z}\in\mathcal{C}} \|\mathbf{z}-\mathbf{x}\|_2$$

Can use other norms too!

$$\arg\min f(\mathbf{w}) \quad \text{s.t. } \mathbf{w} \in \mathcal{C}$$

Projected Gradient Descent



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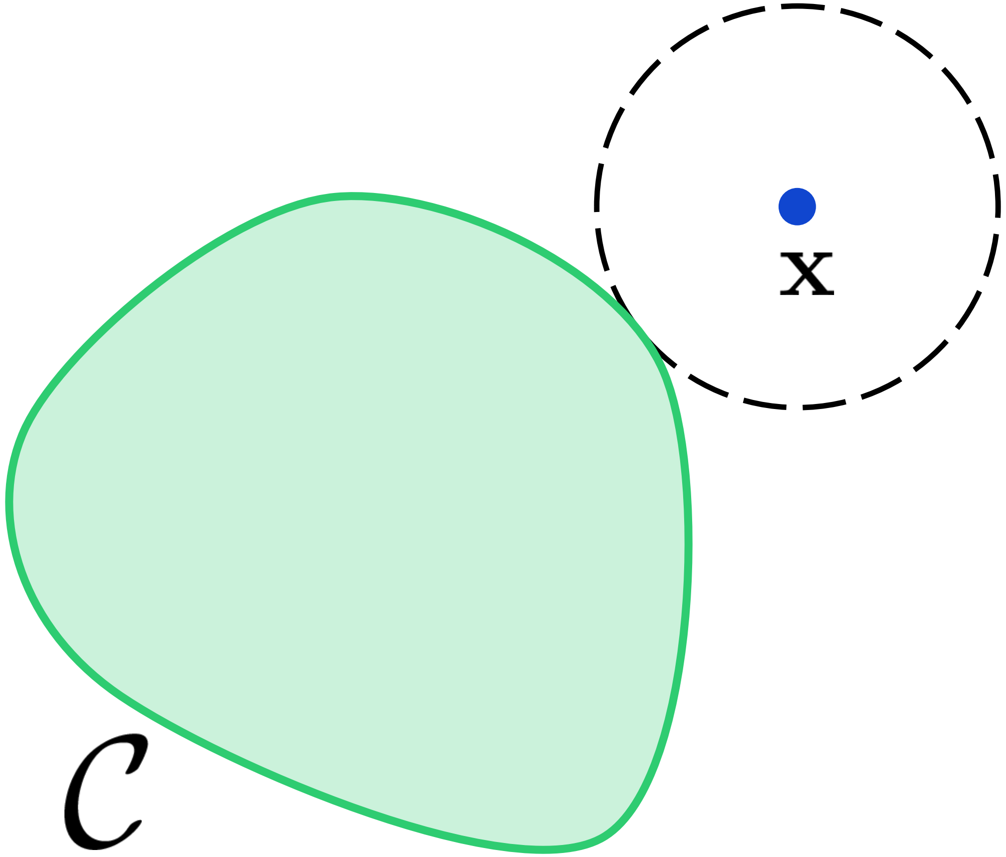

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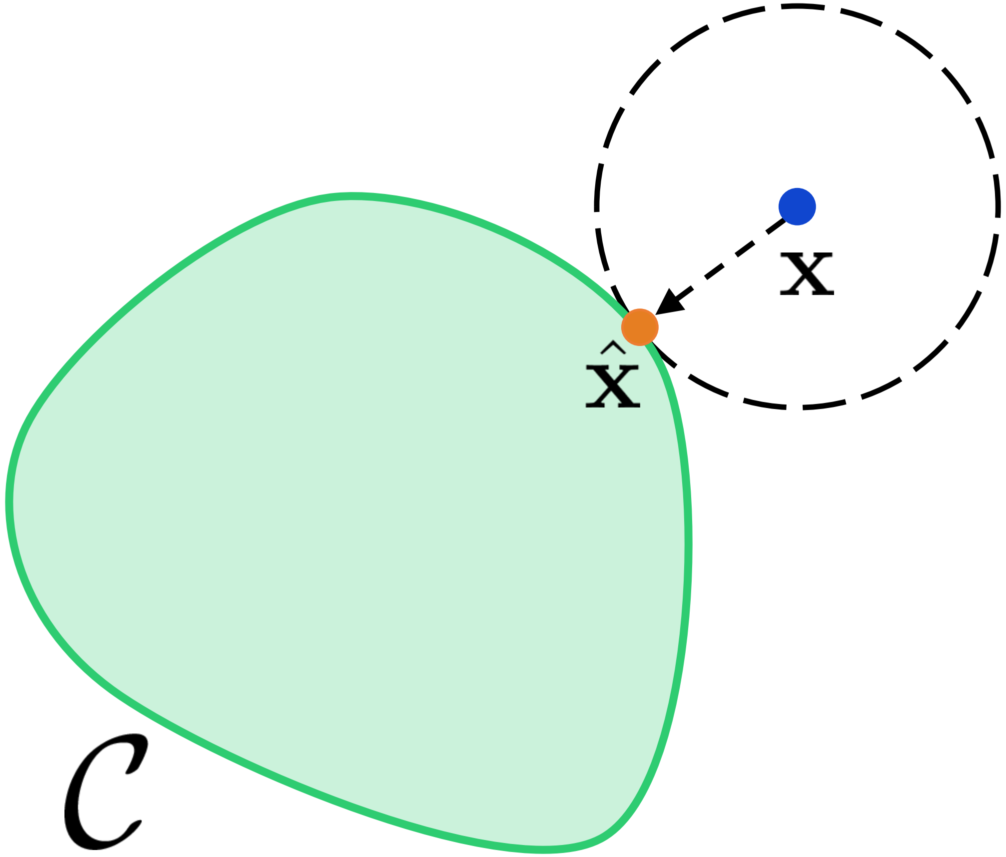

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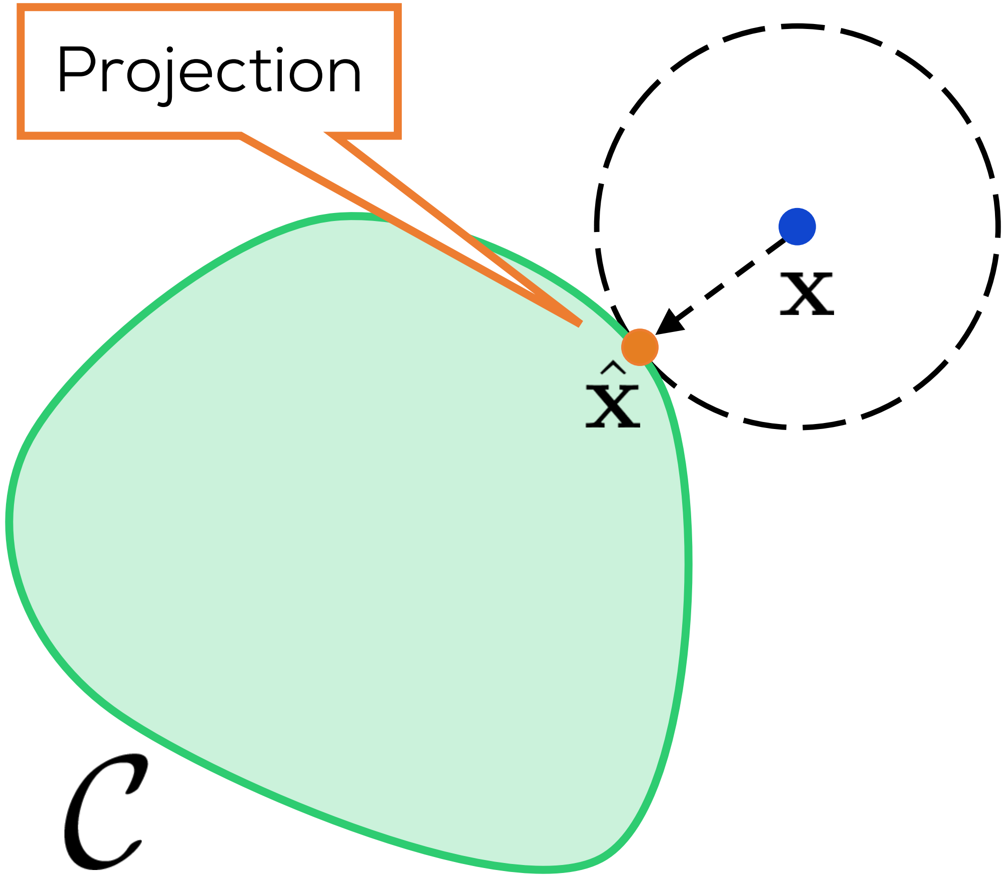
Projected Gradient Descent



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$$\arg\min \; f(\mathbf{w})$$
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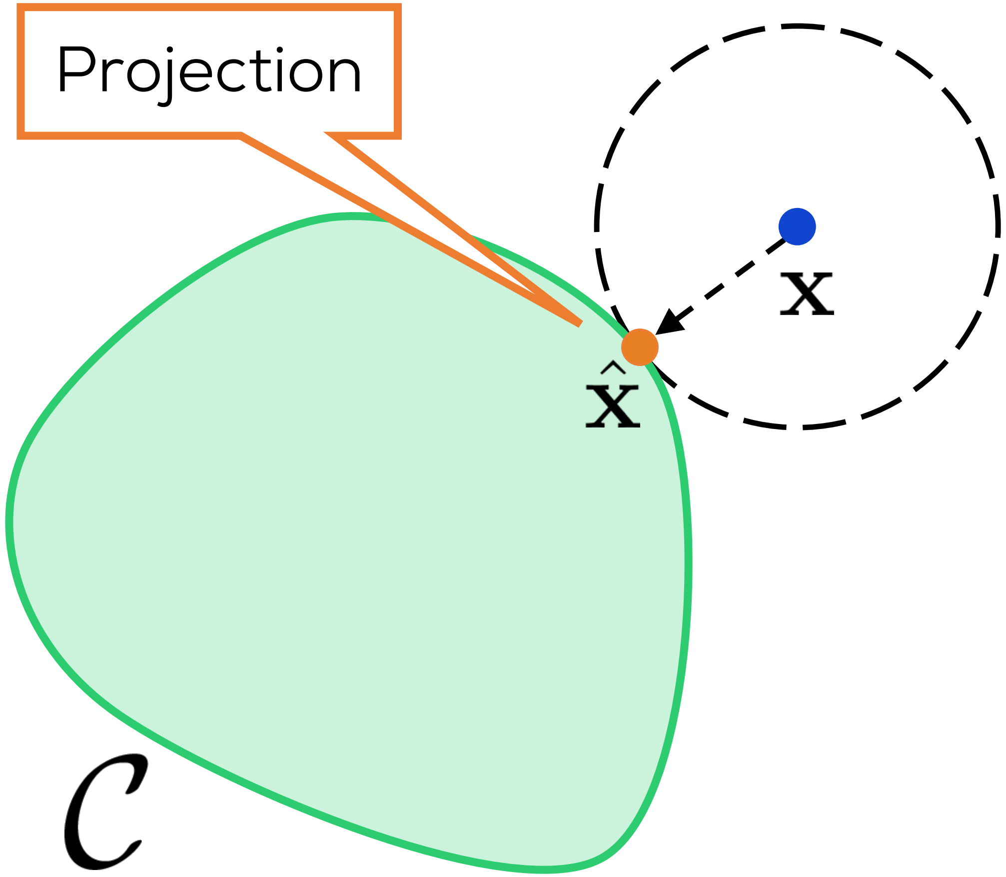
Projected Gradient Descent

Doesn't change the problem



$$\Pi_{\mathcal{C}}(\mathbf{x}) = \arg\min_{\mathbf{z}\in\mathcal{C}} \ \|\mathbf{z} - \mathbf{x}\|_2^2$$

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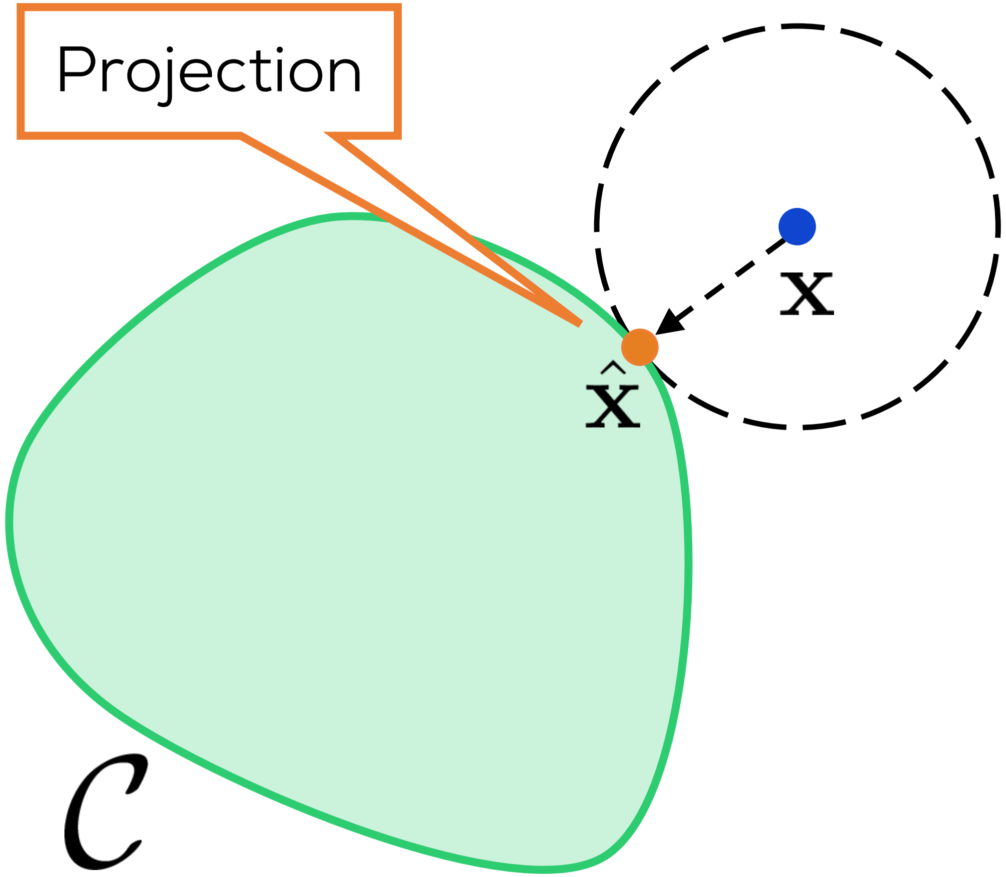
Projected Gradient Descent



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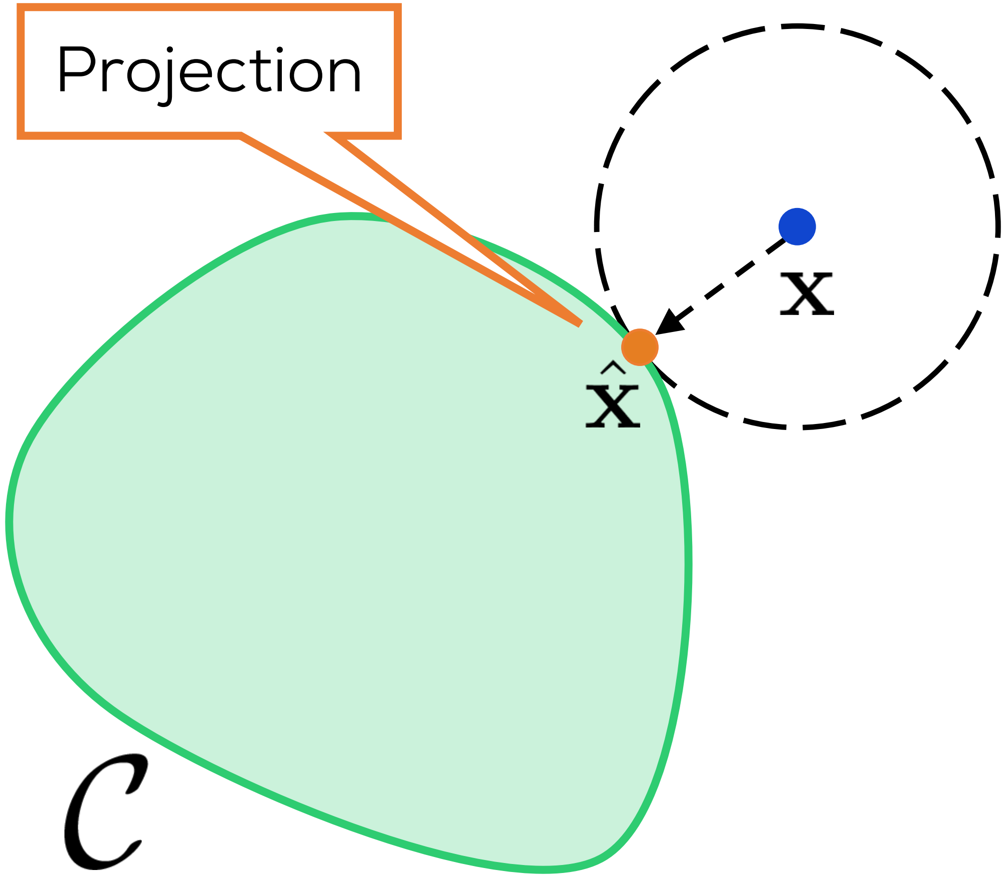

Projection is just another optimization problem!

$$\Pi_{\mathcal{C}}(\mathbf{x}) = \arg\min_{\mathbf{z}\in\mathcal{C}} \|\mathbf{z} - \mathbf{x}\|_2^2$$

$$\arg\min \; f(\mathbf{w})$$
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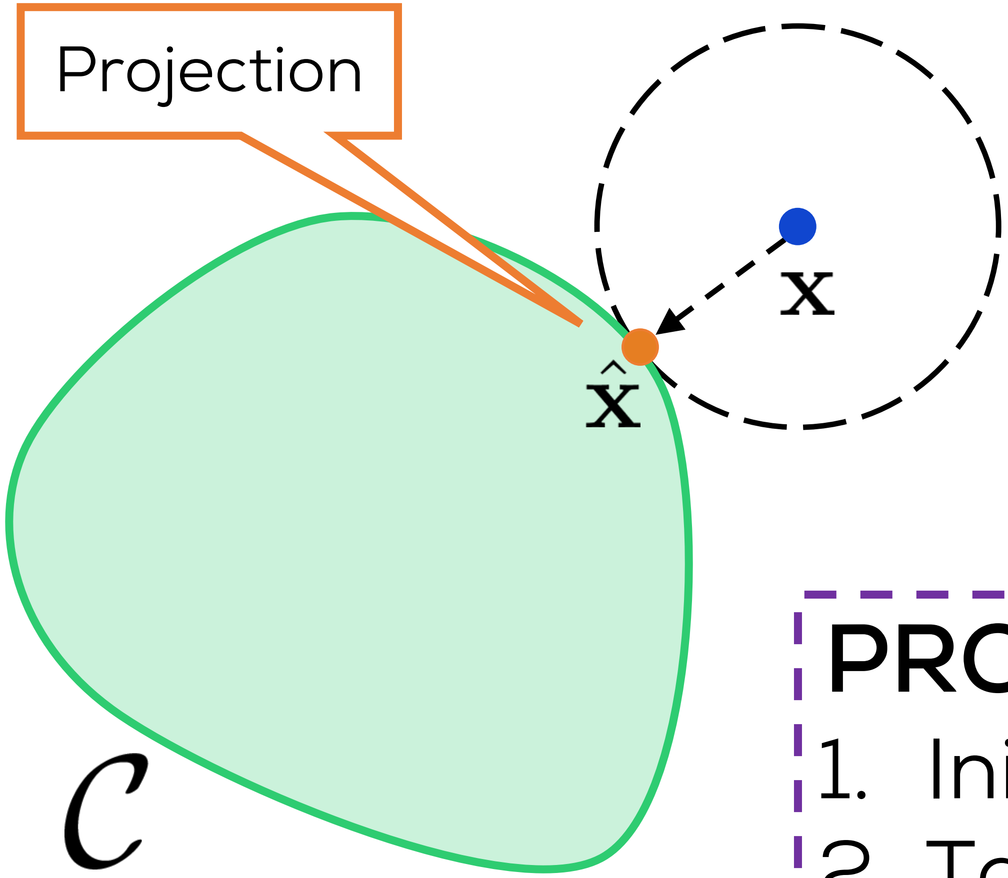
Projected Gradient Descent



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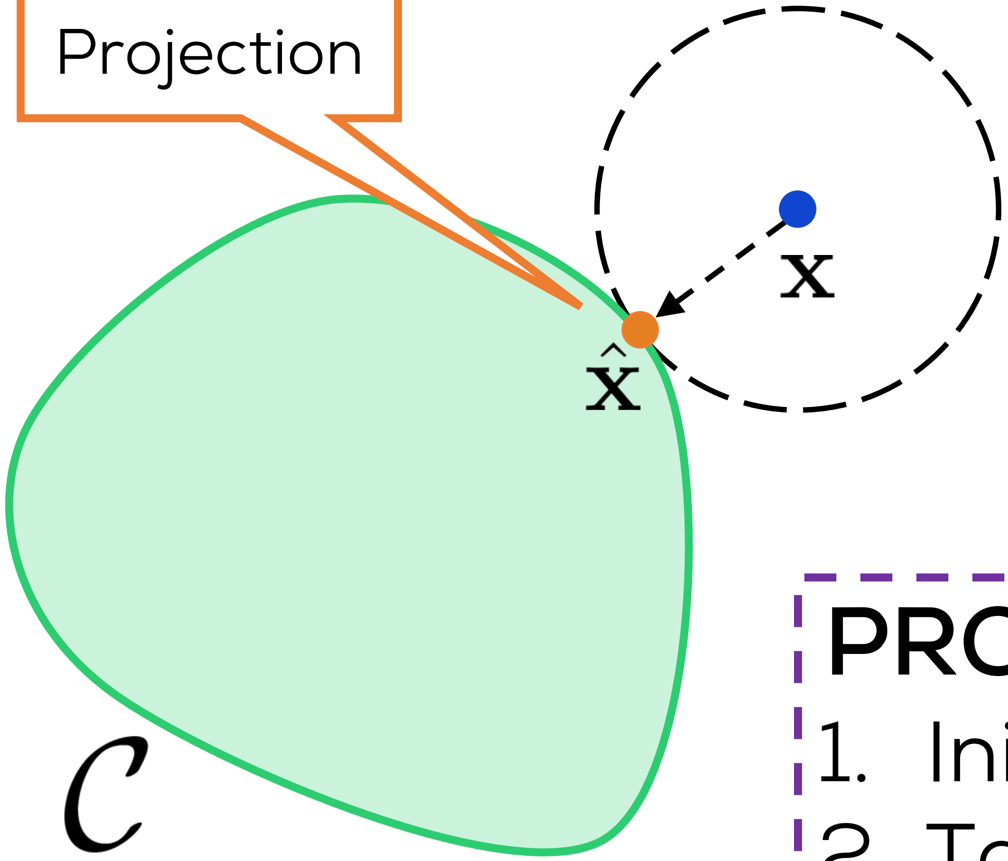
$$\arg\min \; f(\mathbf{w}) \quad \text{s.t. } \mathbf{w} \in \mathcal{C}$$

PROJECTED GRADIENT DESCENT

1. Initialize $\mathbf{w}^0$
2. Take gradient step $\mathbf{u}^{t+1} \leftarrow \mathbf{w}^t - \eta_t \cdot \mathbf{g}^t$
3. Satisfy constraint $\mathbf{w}^{t+1} \leftarrow \Pi_{\mathcal{C}}(\mathbf{u}^{t+1})$
4. Repeat until convergence

Projected Gradient Descent



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Projection is just another optimization problem!

$$\Pi_{\mathcal{C}}(\mathbf{x}) = \arg\min_{\mathbf{z}\in\mathcal{C}} \; \|\mathbf{z} - \mathbf{x}\|_2^2$$

Projected Stochastic Gradient Descent??

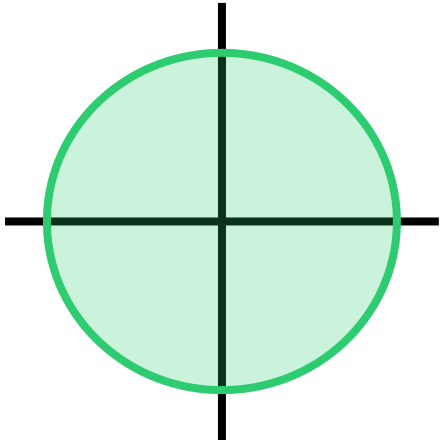
PROJECTED GRADIENT DESCENT

1. Initialize $\mathbf{w}^0$
2. Take gradient step $\mathbf{u}^{t+1} \leftarrow \mathbf{w}^t - \eta_t \cdot \mathbf{g}^t$
3. Satisfy constraint $\mathbf{w}^{t+1} \leftarrow \Pi_{\mathcal{C}}(\mathbf{u}^{t+1})$
4. Repeat until convergence

A few useful projections

A few useful projections

$$\mathcal{C} = \{\mathbf{x} : \|\mathbf{x}\|_2 \leq 1\}$$



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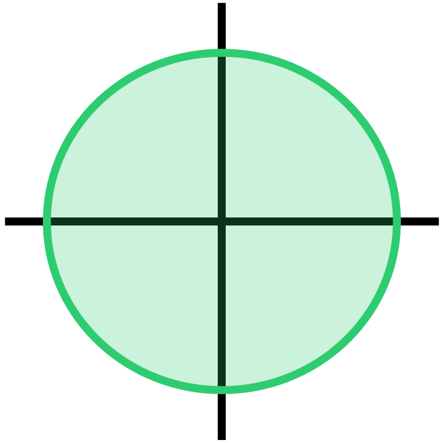
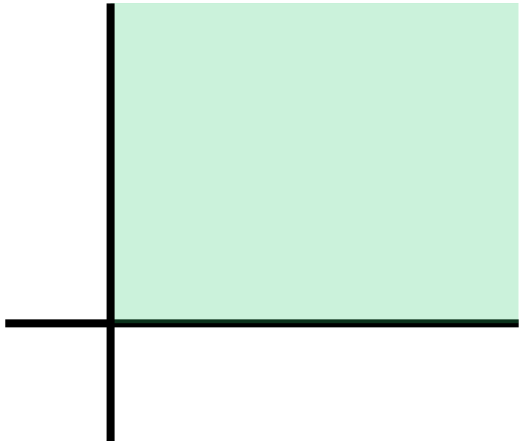


$$\mathcal{C} = \{\mathbf{x} : \mathbf{x}_i \geq 0\}$$



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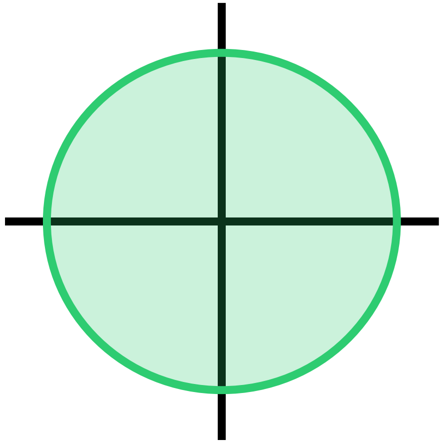

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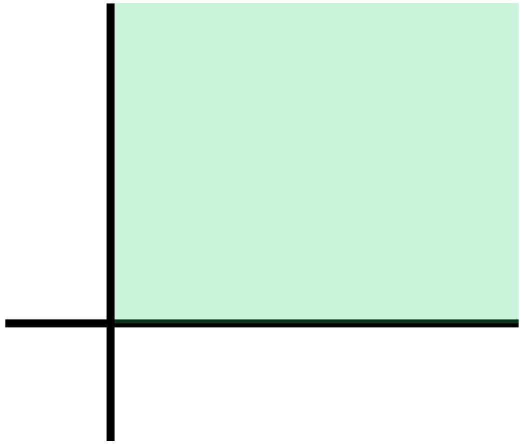

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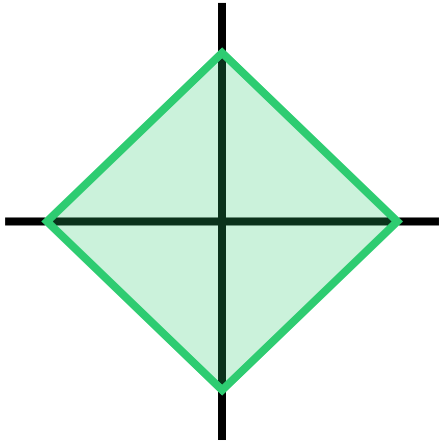

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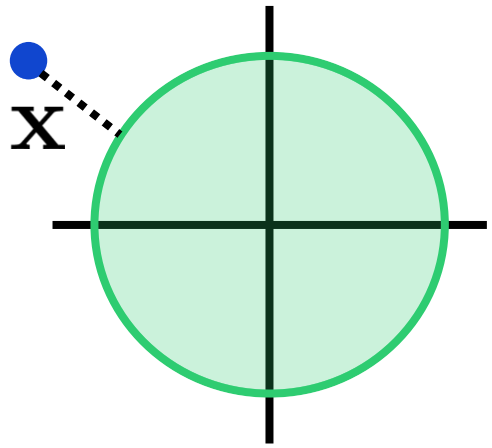

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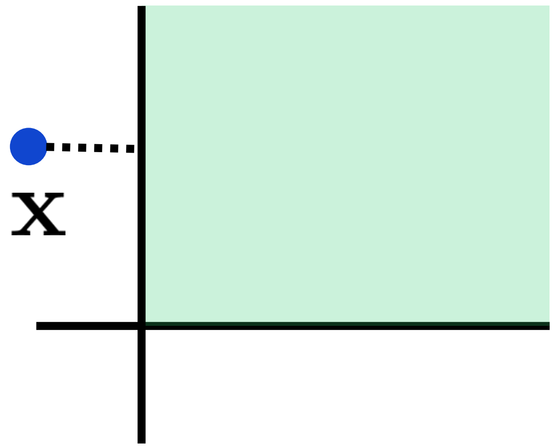

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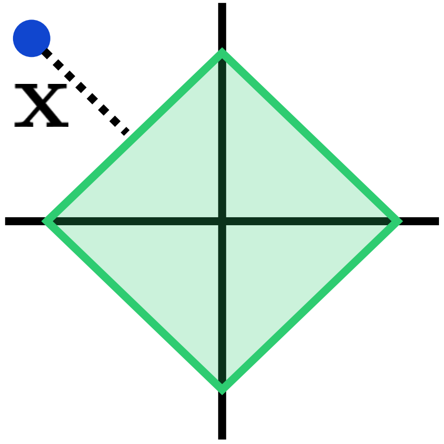

A few useful projections

$$\hat{\mathbf{x}} = \Pi_{\mathcal{C}}(\mathbf{x})$$

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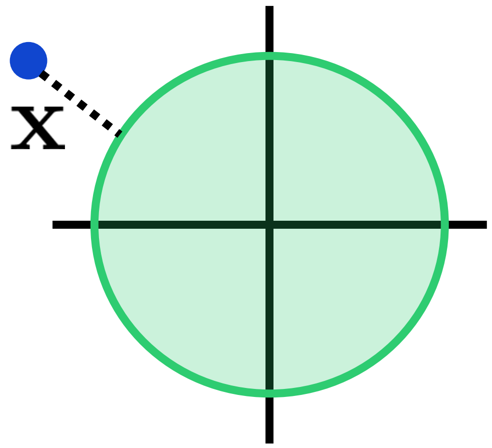

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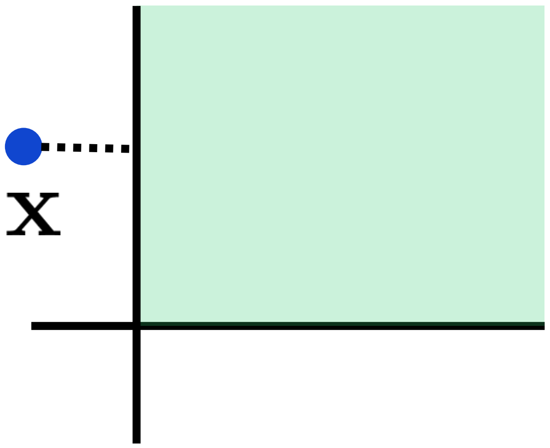

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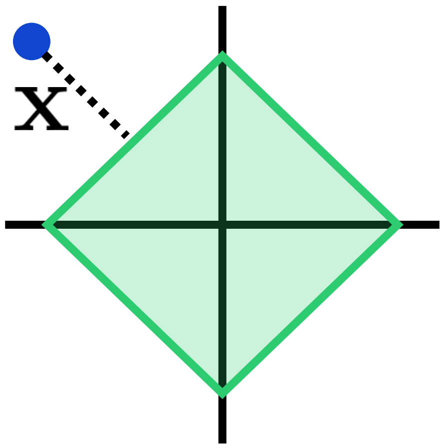

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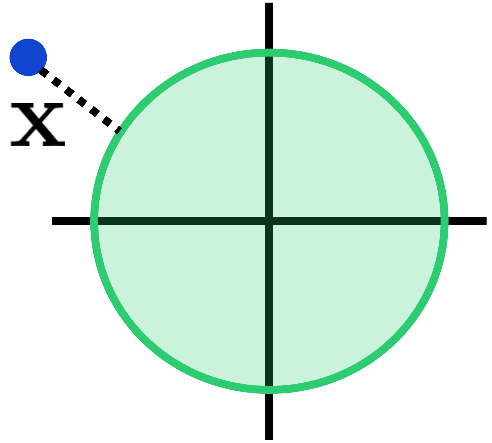

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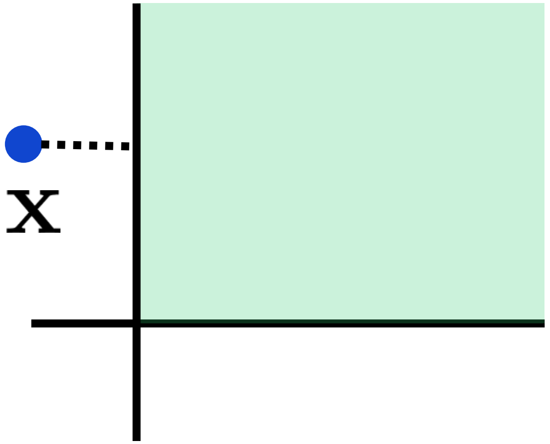

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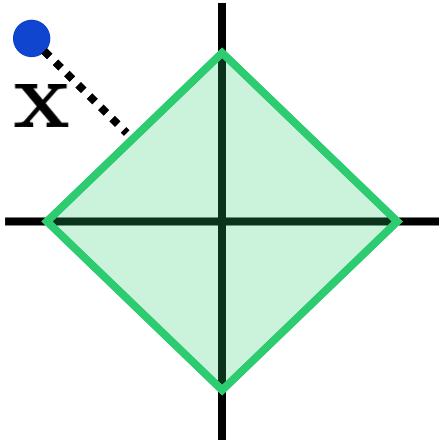

$$\hat{\mathbf{x}} = \begin{cases} \mathbf{x} & \text{if } \|\mathbf{x}\|_2 \leq 1 \\ \frac{\mathbf{x}}{\|\mathbf{x}\|_2} & \text{if } \|\mathbf{x}\|_2 > 1 \end{cases}$$

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$$\hat{\mathbf{x}} = \Pi_{\mathcal{C}}(\mathbf{x})$$

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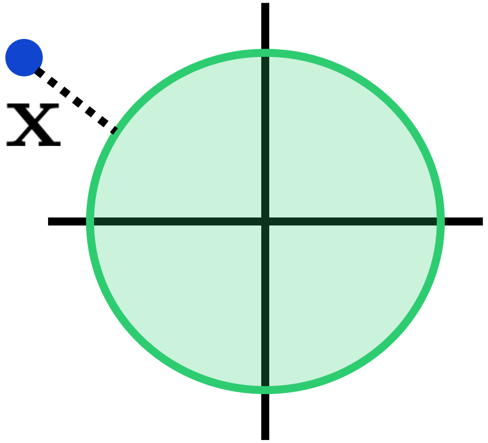

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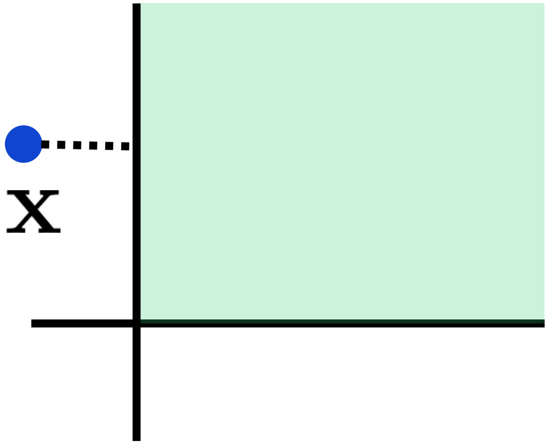

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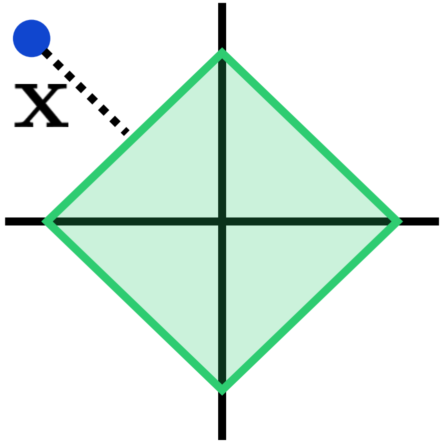

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What if $\mathcal{C} = \{\mathbf{x} : \|\mathbf{x}\|_2 \leq r\}$?

A few useful projections

$$\hat{\mathbf{x}} = \Pi_{\mathcal{C}}(\mathbf{x})$$

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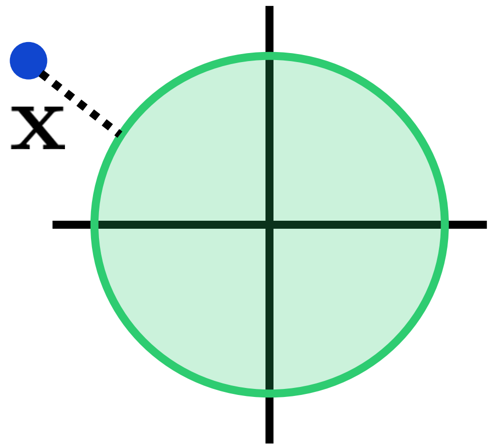

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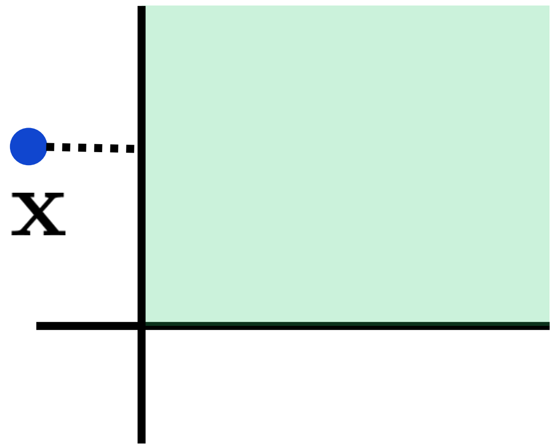
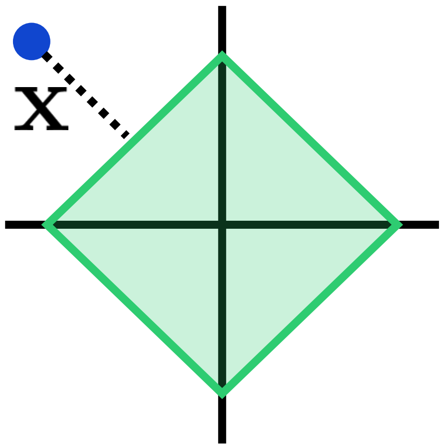


$$\hat{\mathbf{x}}_i = \begin{cases} \mathbf{x}_i & \text{if } \mathbf{x}_i \geq 0 \\ 0 & \text{if } \mathbf{x}_i < 0 \end{cases}$$

$$\mathcal{C} = \{\mathbf{x} : \|\mathbf{x}\|_1 \leq 1\}$$



What if $\mathcal{C} = \{\mathbf{x} : \|\mathbf{x}\|_2 \leq r\}$?

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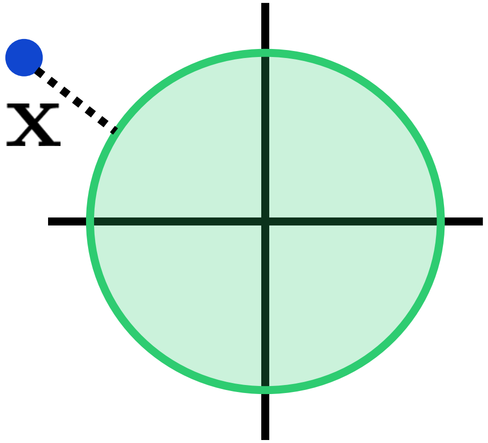

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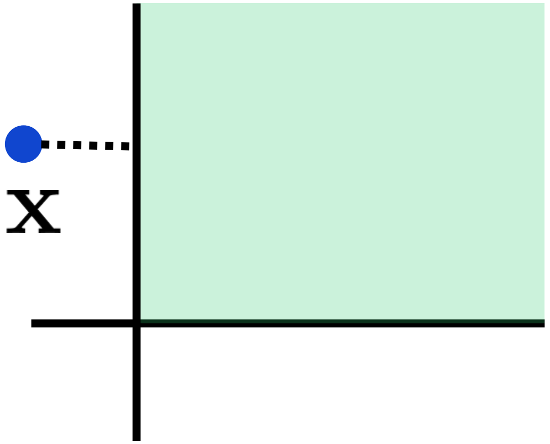

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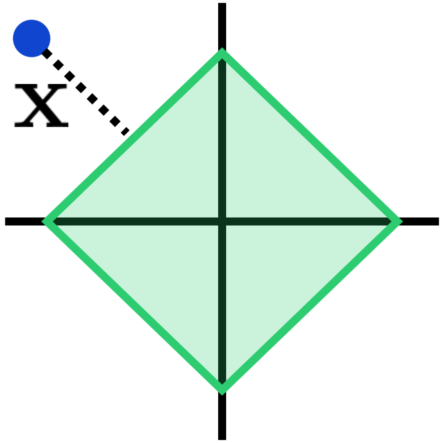


Find out!

What if $\mathcal{C} = \{\mathbf{x} : \|\mathbf{x}\|_2 \leq r\}$?

Some disclaimers

- GD can be slow in convergence
  - Step size tuning can affect performance quite a bit – Armijo, Adagrad
  - Momentum-based methods $\mathbf{w}^{t+1} \leftarrow \mathbf{w}^t - \eta_t \cdot \mathbf{g}^t - \gamma \cdot (\mathbf{w}^t - \mathbf{w}^{t-1})$
  - Second order methods – expensive but super fast!

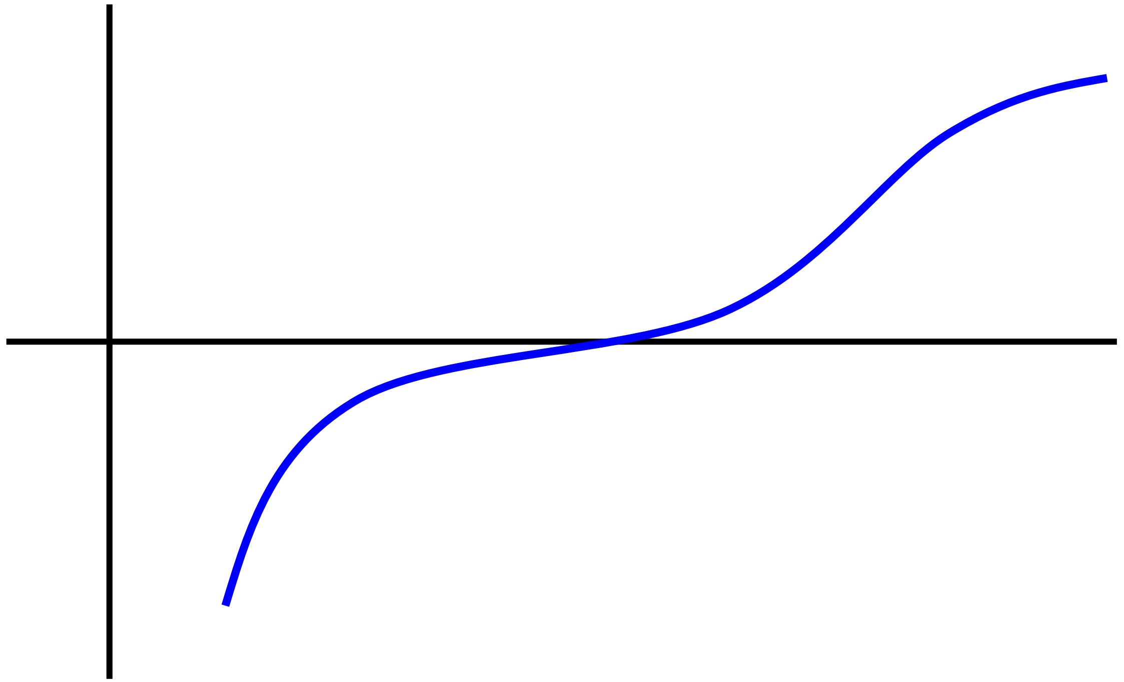
Momentum weight

- Exist superior methods for specific problems
  - RR (L_2 regularized ls regression) – conjugate gradient method
  - Lasso (L_1 regularized ls regression) – proximal gradient method
  - Logistic regression – trust region Newton method
- Details, convergence analysis beyond scope of this course
- CS774: Optimization Techniques
- Next: Newton method, duality, coordinate descent

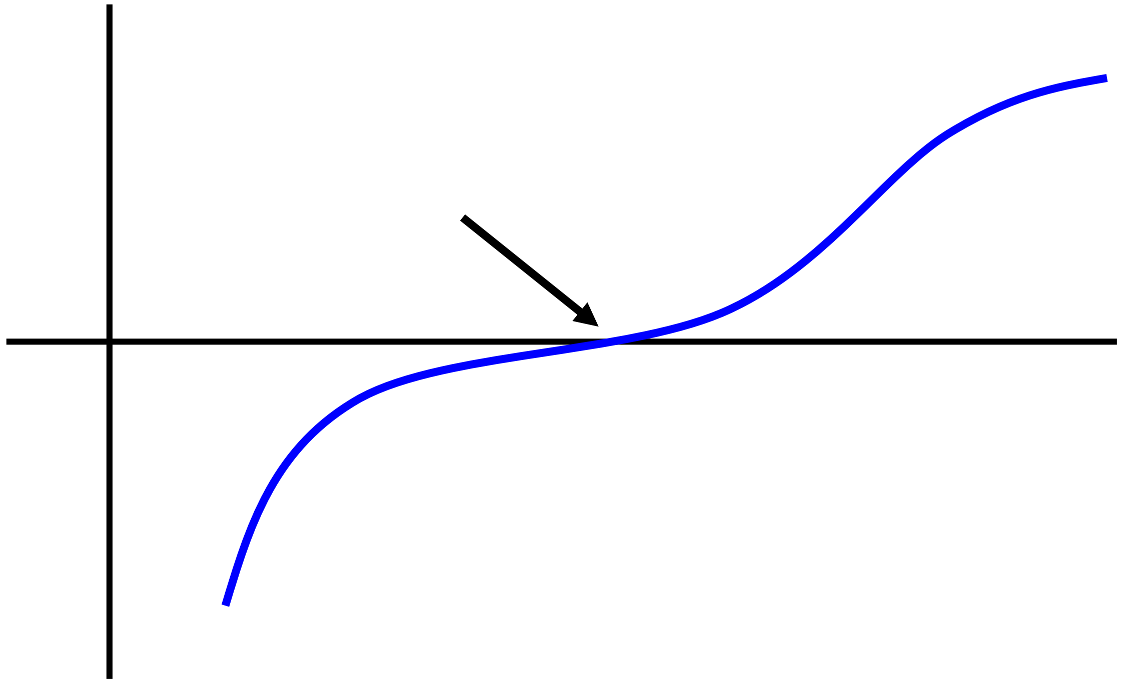
Newton Method

Descent with second order derivatives

Newton-Raphson Method for Root Finding

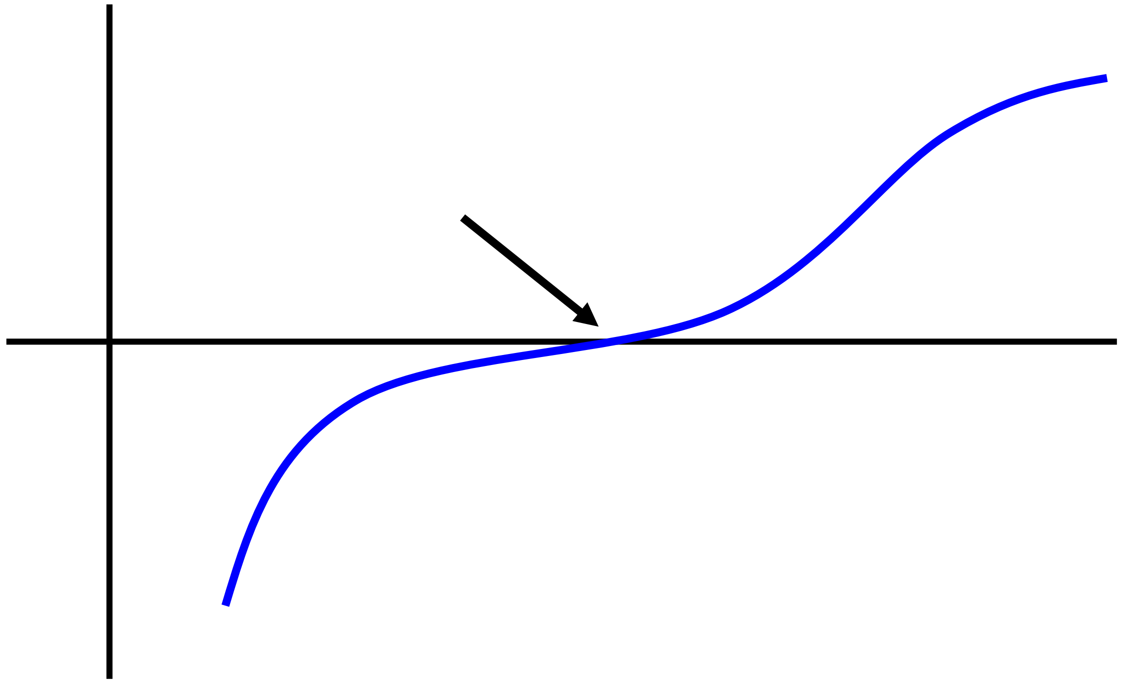


Newton-Raphson Method for Root Finding



$$x : f(x) = 0$$

Newton-Raphson Method for Root Finding

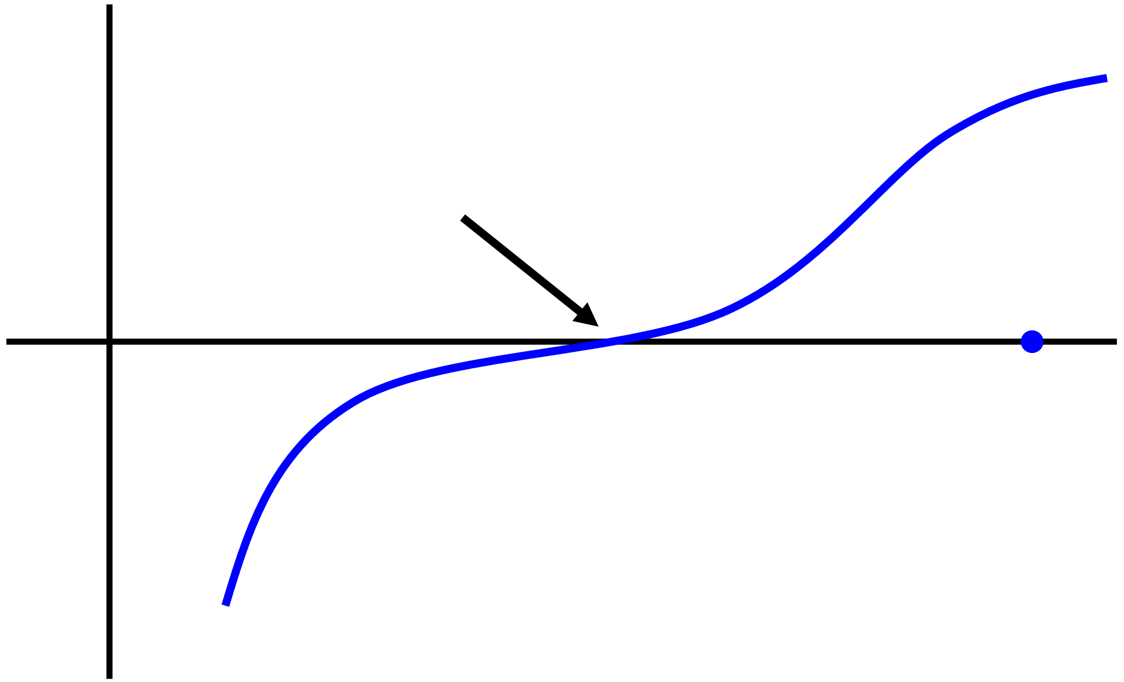


$$x : f(x) = 0$$

Finding roots of linear functions is easy

$f(x) = ax + b$, $x_0 = \frac{-b}{a}$

Newton-Raphson Method for Root Finding



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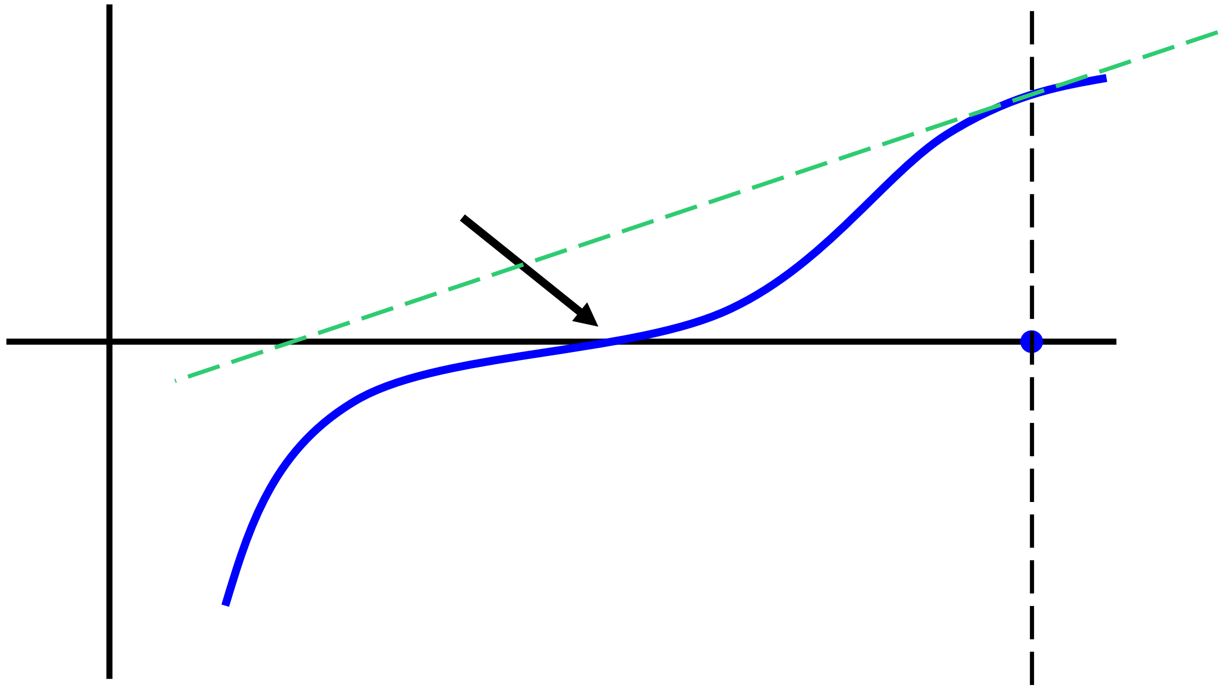
Finding roots of linear functions is easy

$f(x) = ax + b$, $x_0 = \frac{-b}{a}$

Newton-Raphson Method for Root Finding



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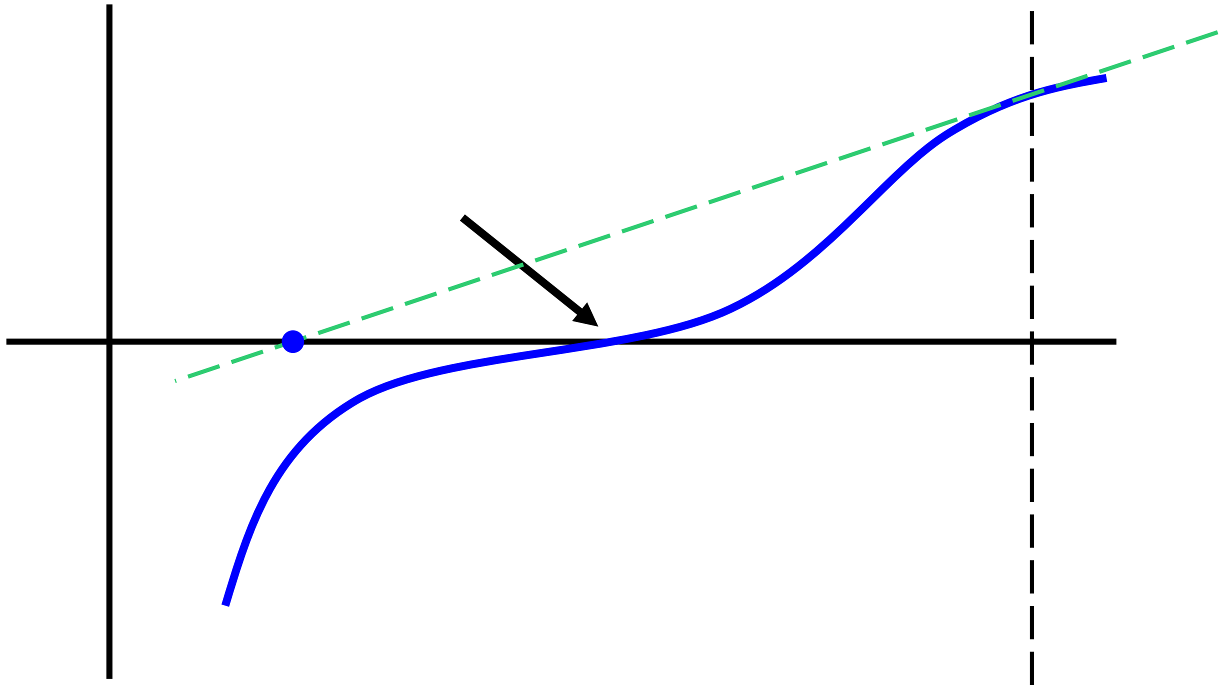

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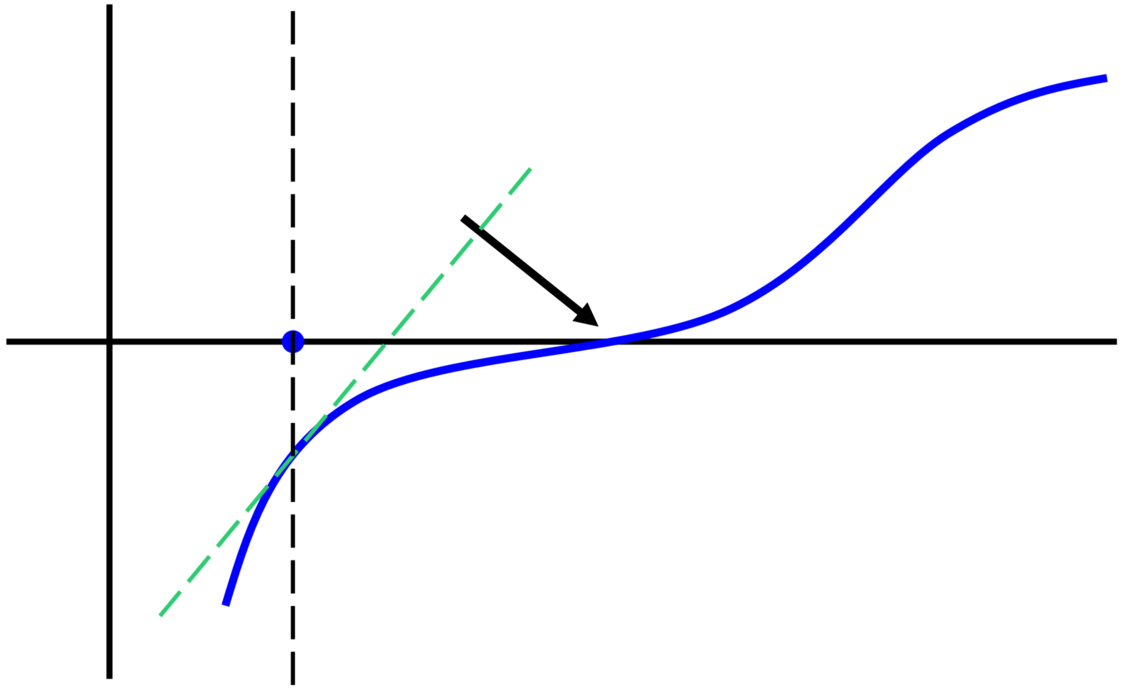

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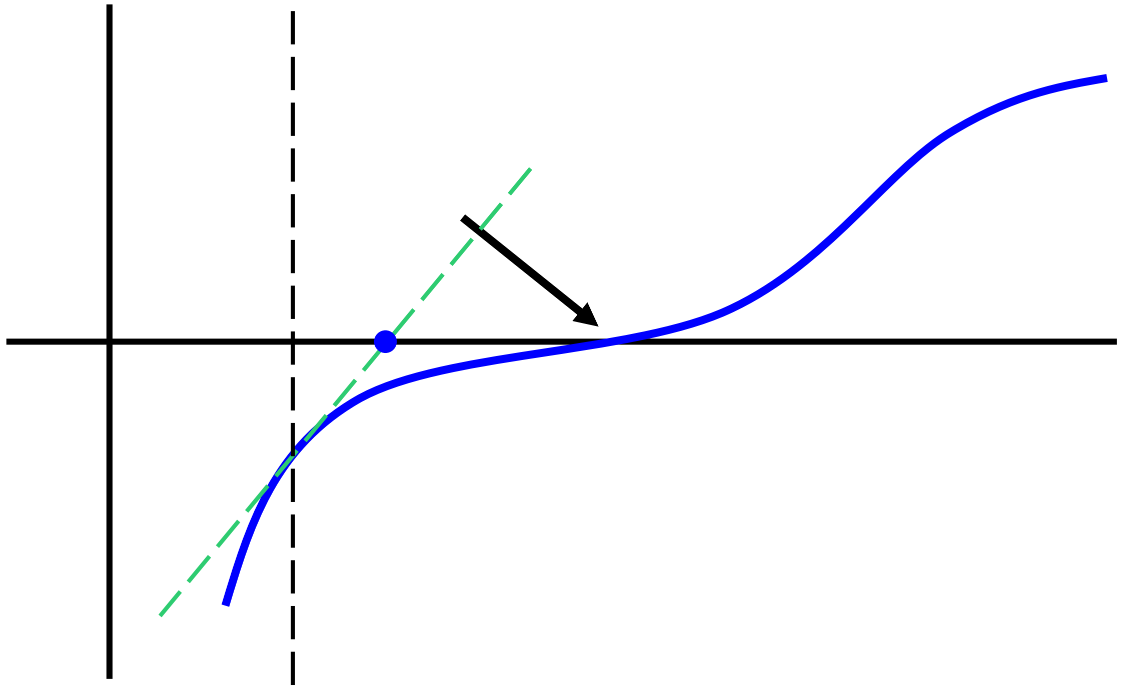

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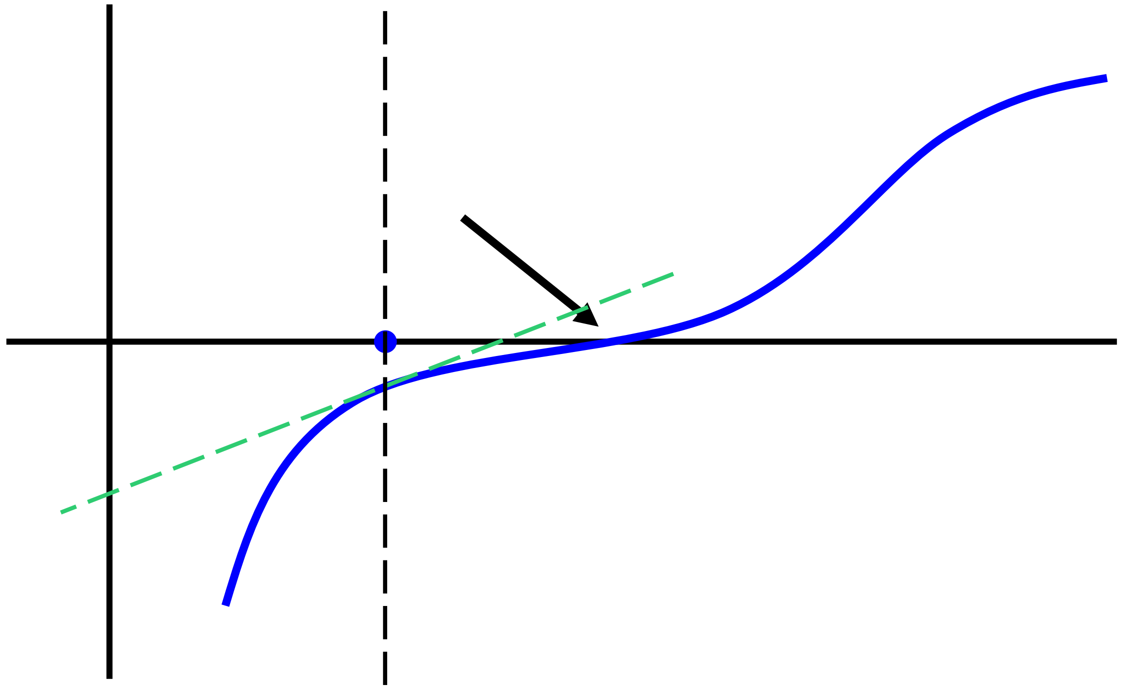

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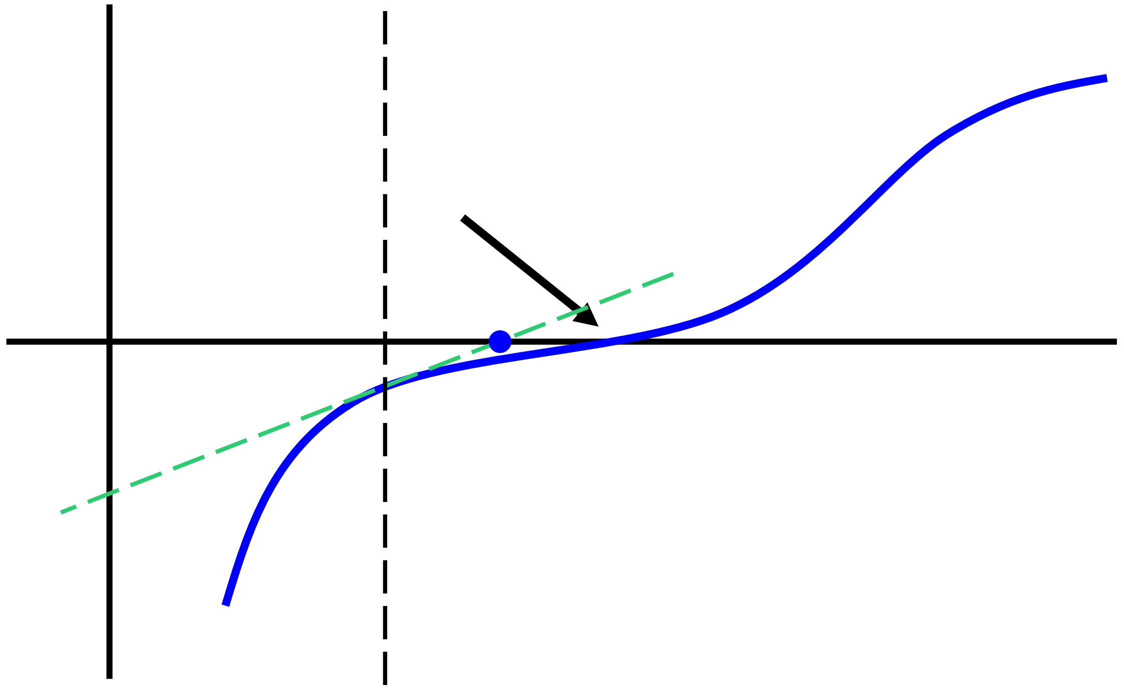

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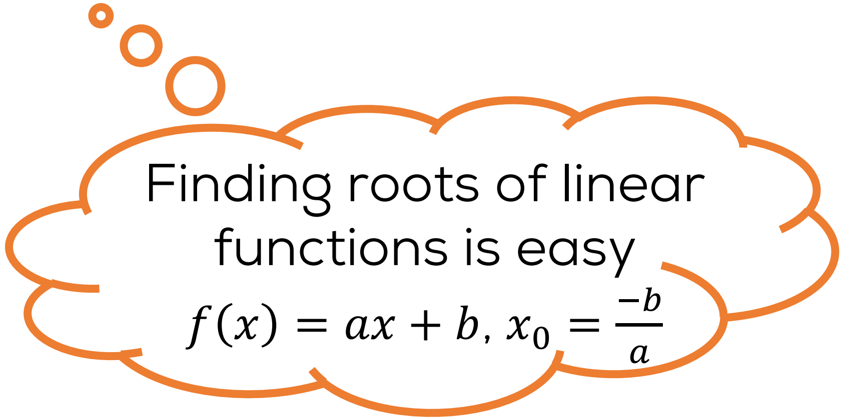

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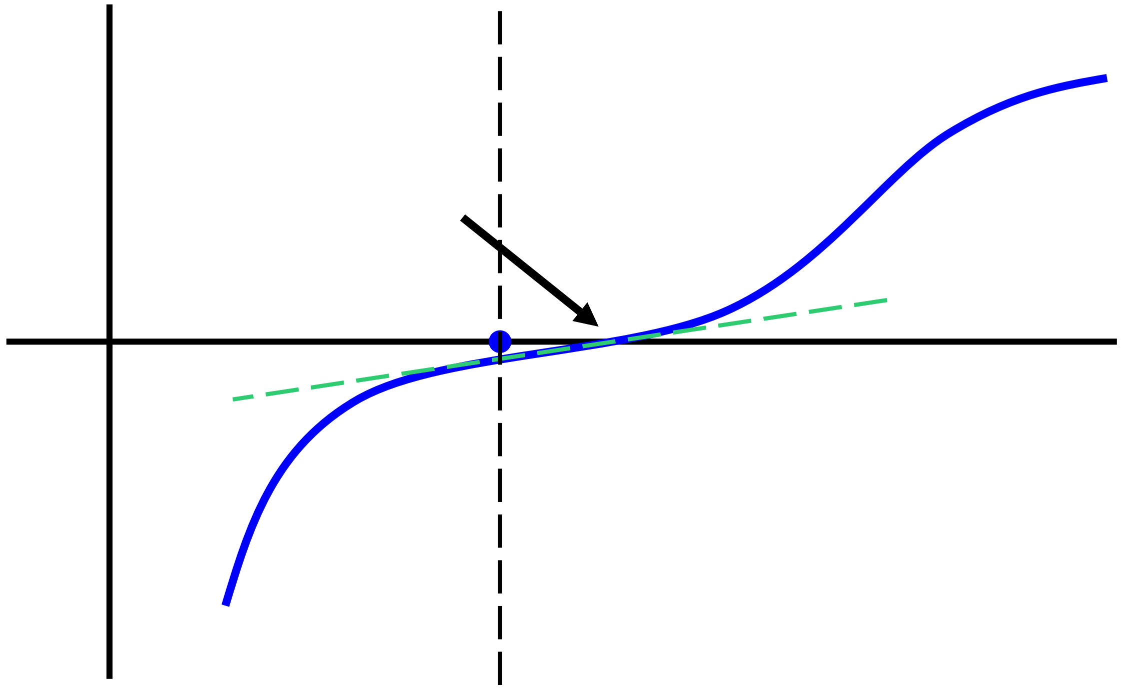

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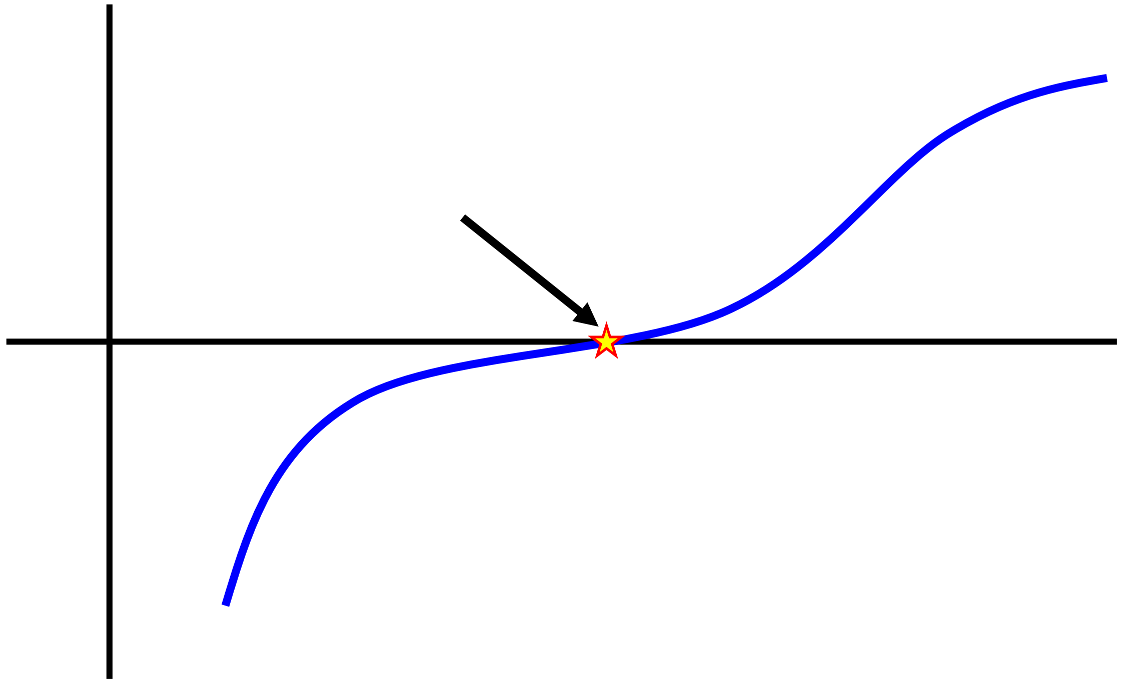

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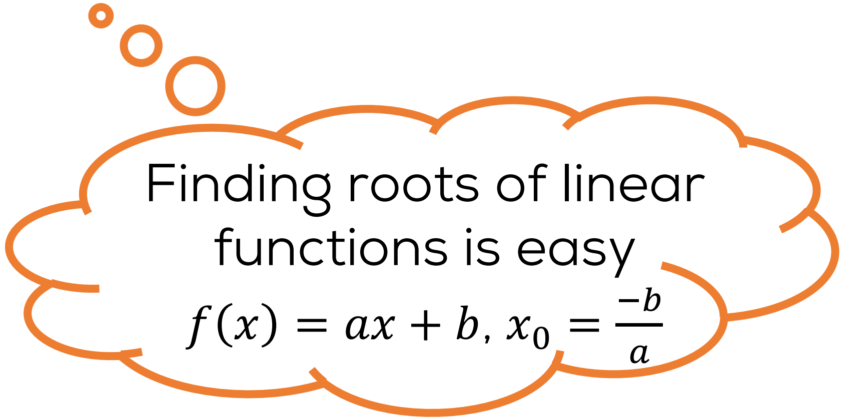

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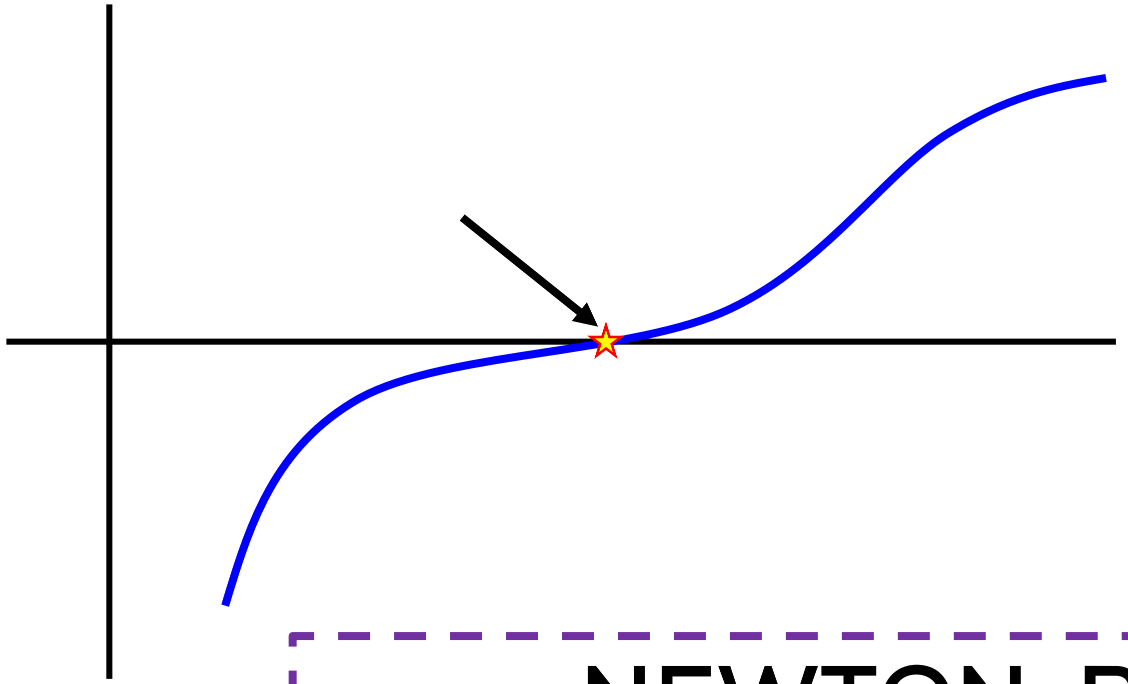

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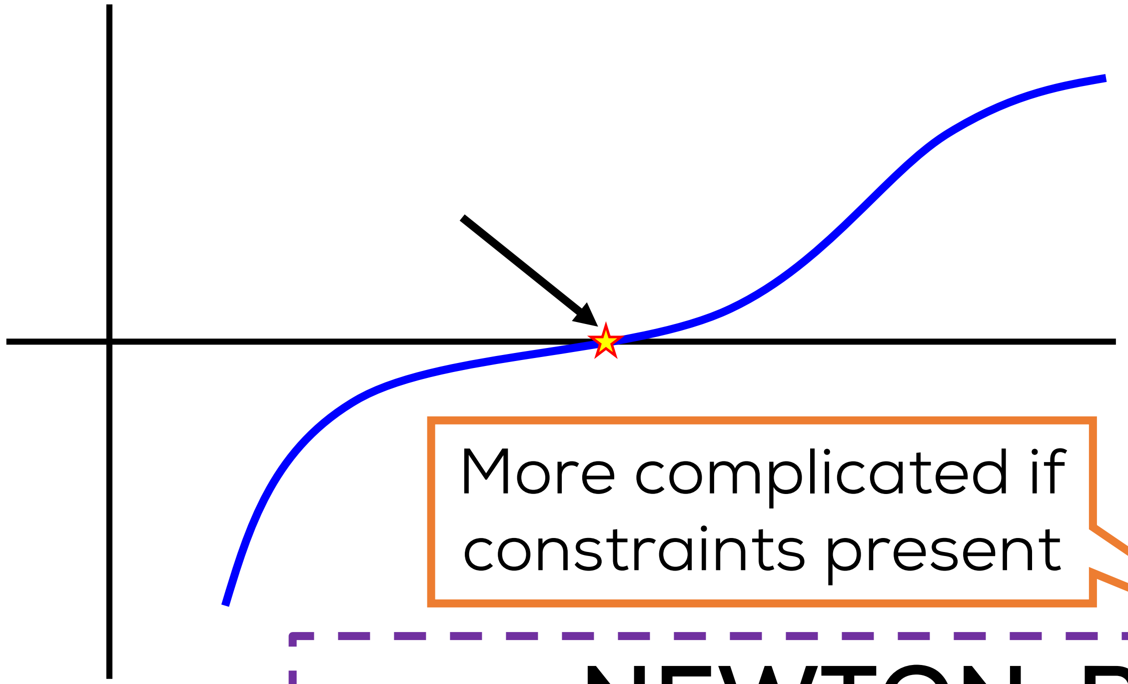

More complicated if constraints present

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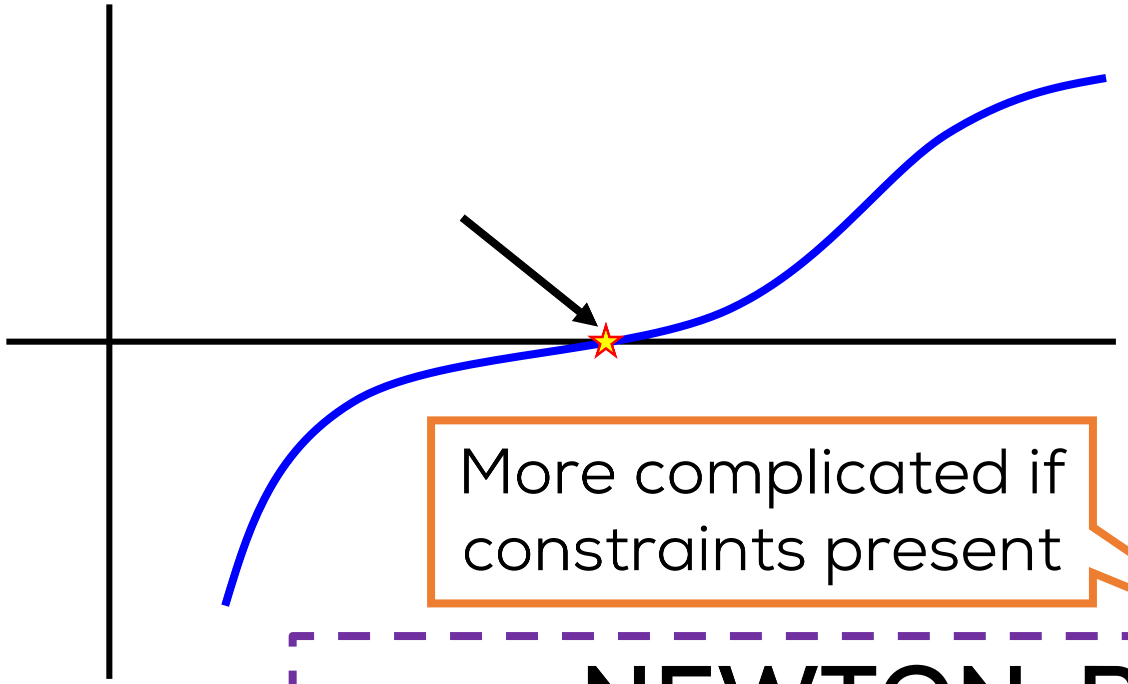

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More complicated if constraints present

No guarantees in general

NEWTON-RAPHSON METHOD

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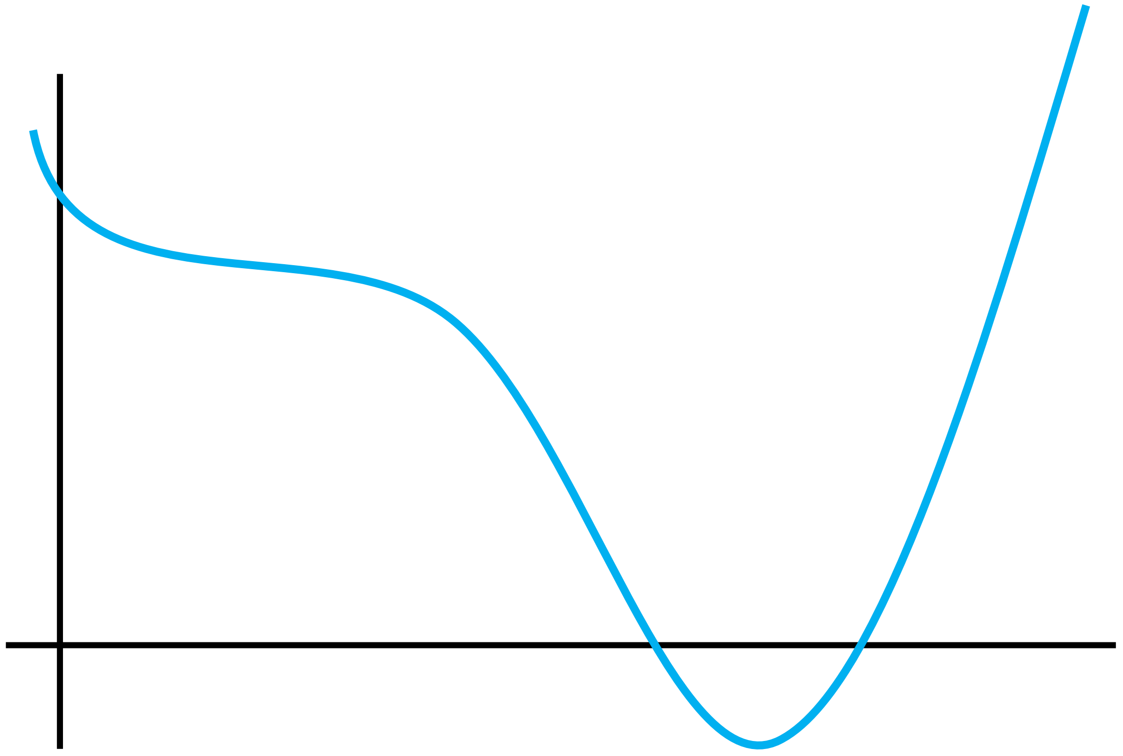

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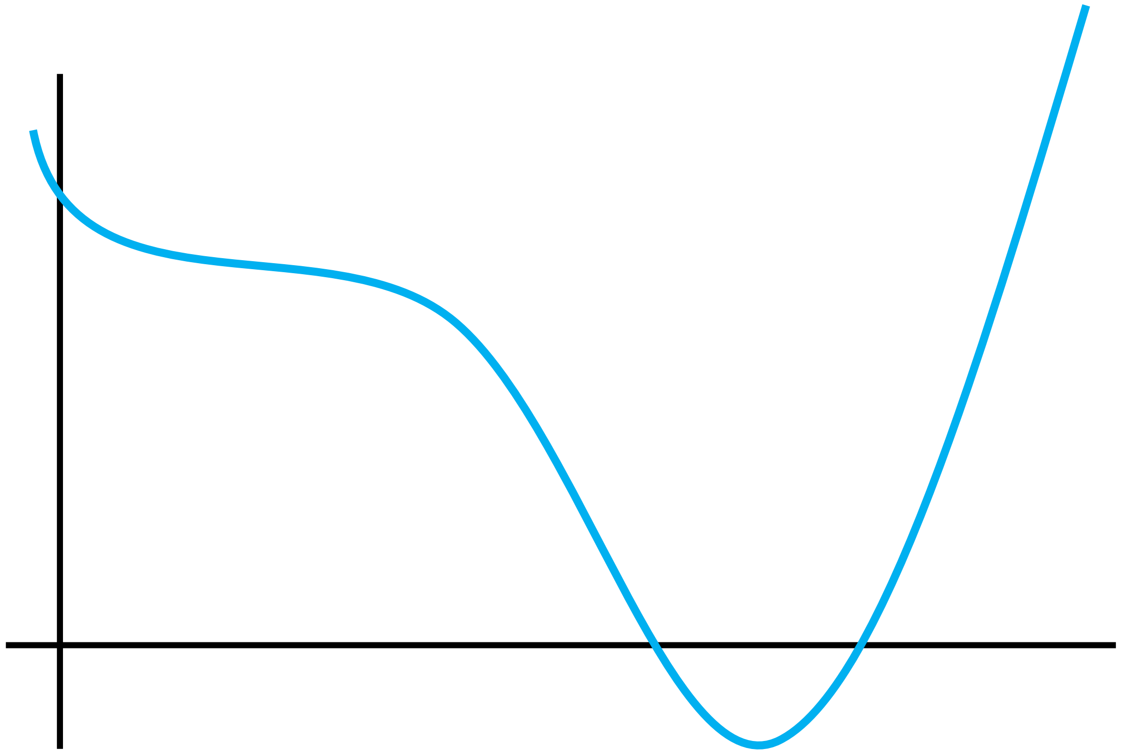
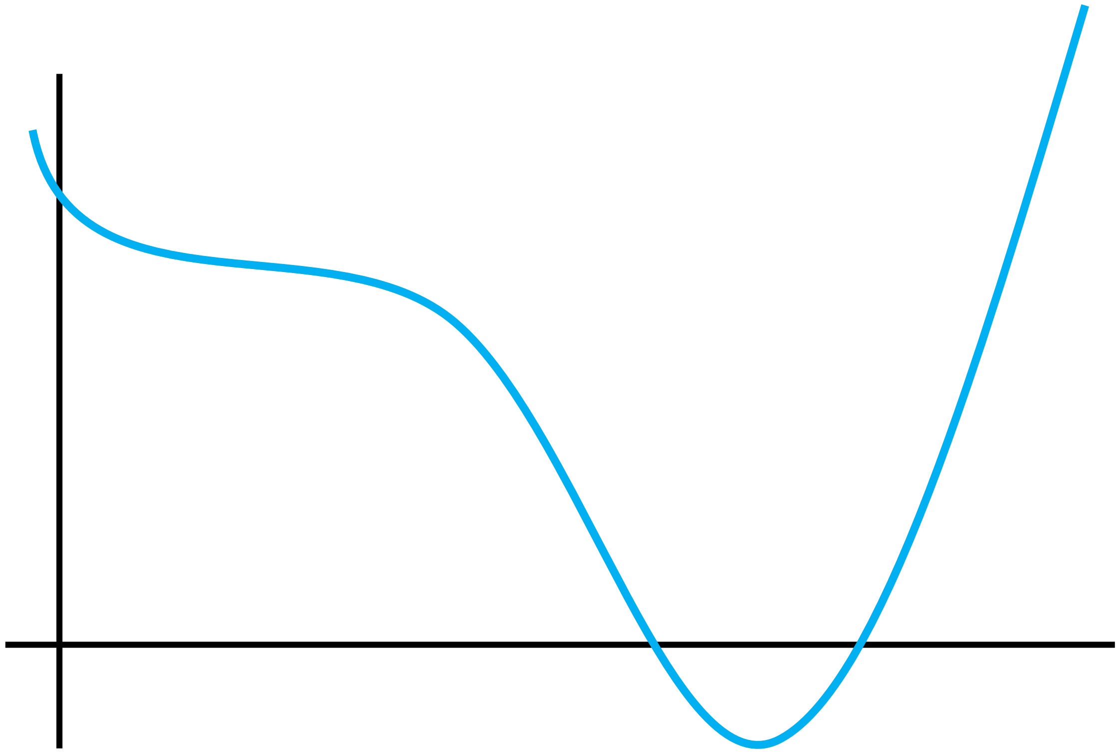


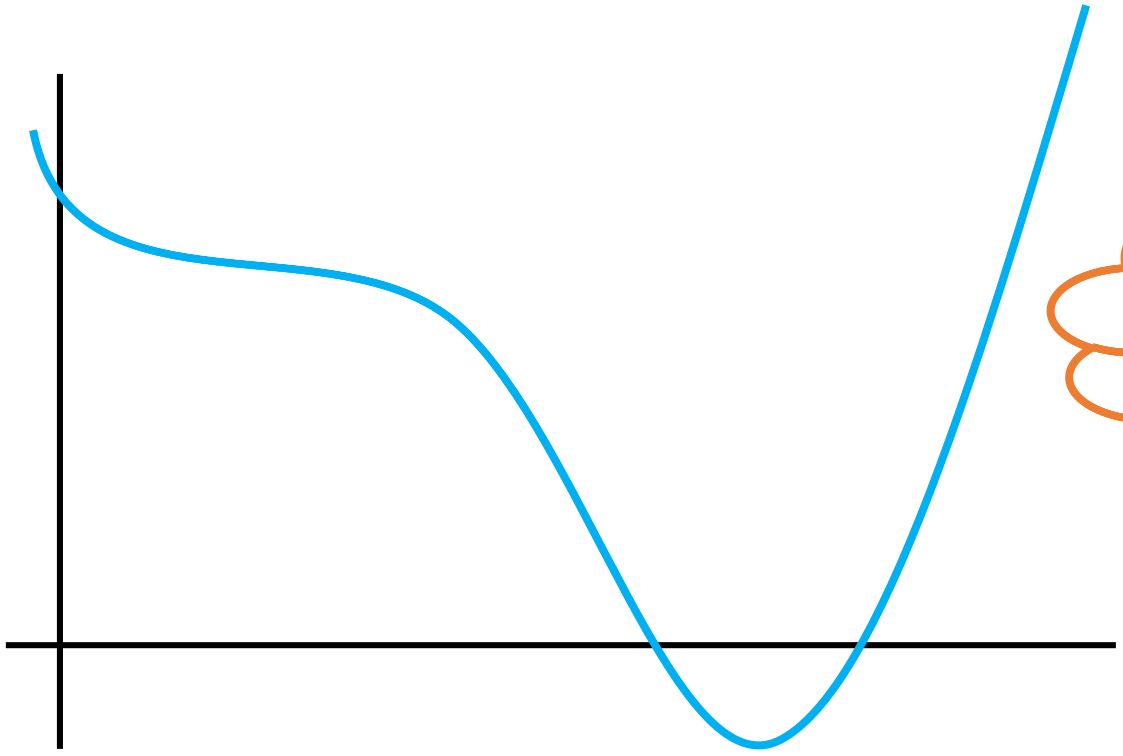
$$\mathbf{x} \in \arg\min f(\mathbf{x})$$

Newton Method



$$\mathbf{x} \in \arg\min f(\mathbf{x}) \subseteq \mathbf{x} : \nabla f(\mathbf{x}) = \mathbf{0}$$

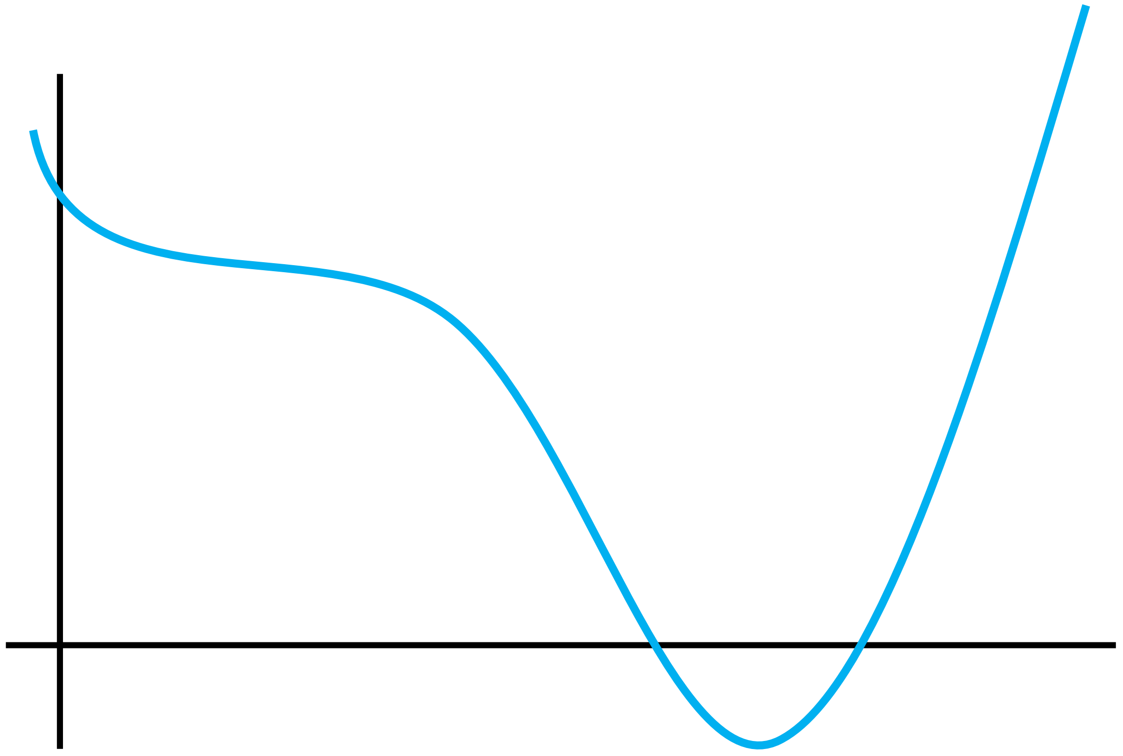
Newton Method



$$\mathbf{x} \in \arg\min f(\mathbf{x}) \subseteq \mathbf{x} : \nabla f(\mathbf{x}) = \mathbf{0}$$

Instead of finding minima, search for stable points instead i.e. $\nabla f(\mathbf{x}) = \mathbf{0}$

Newton Method



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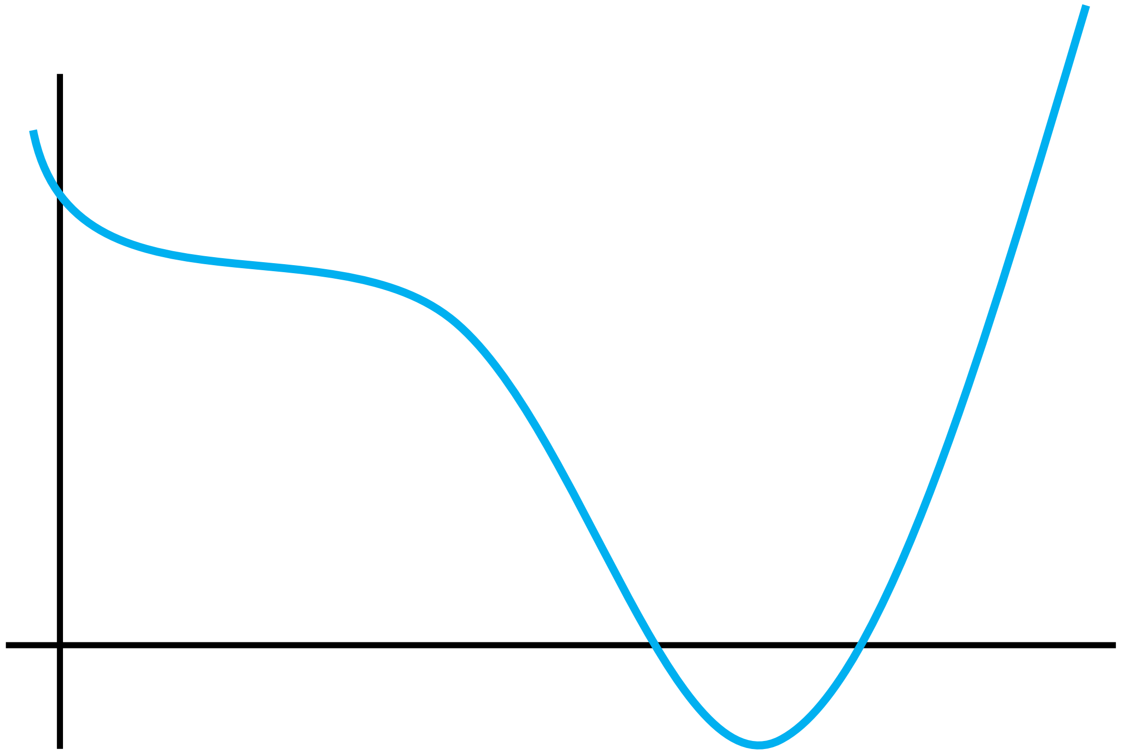

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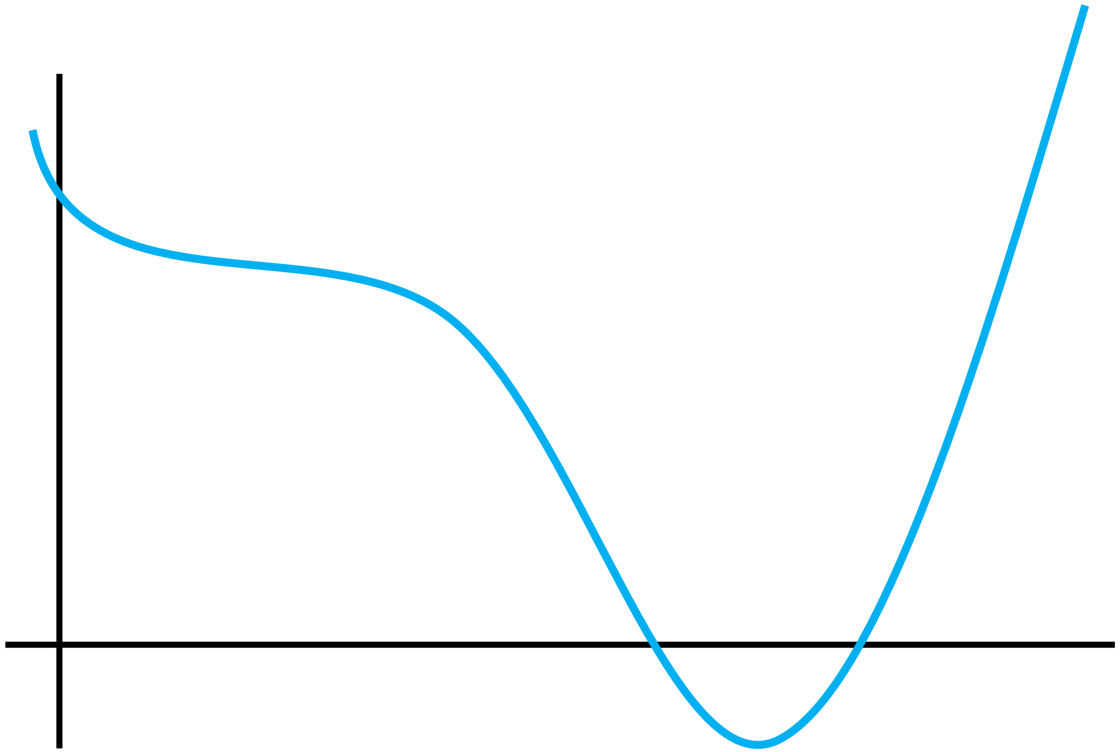
What is the gradient of $\nabla f(\cdot)$?

Newton Method



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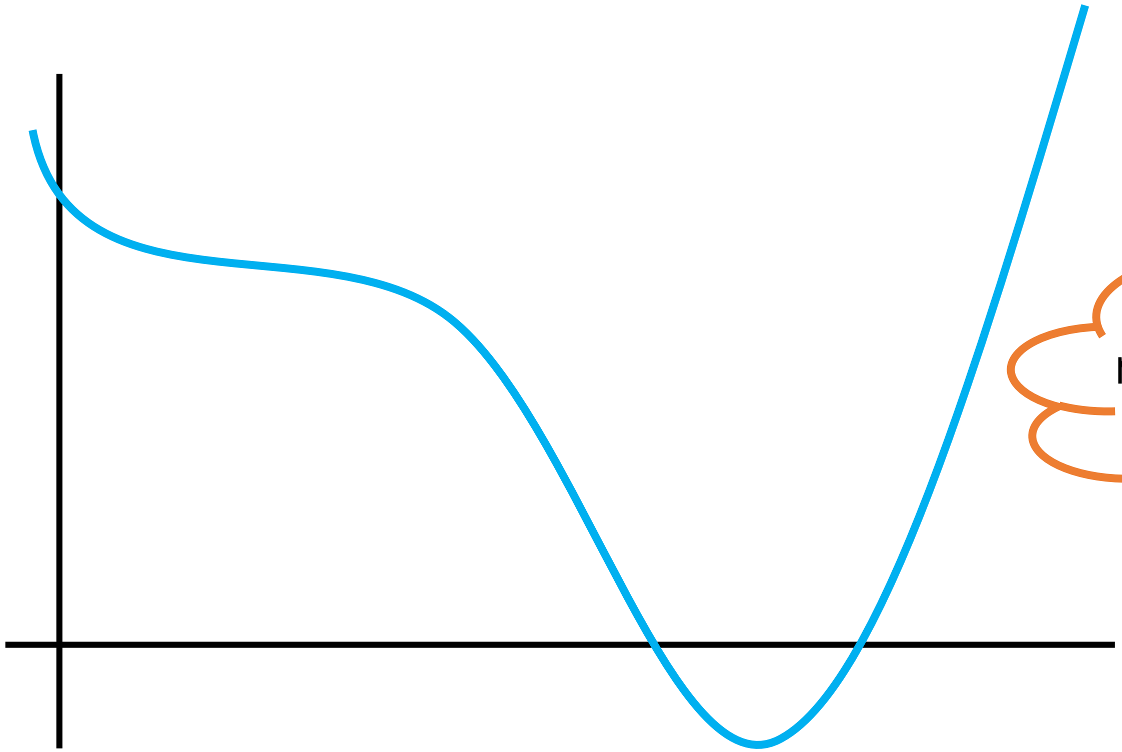

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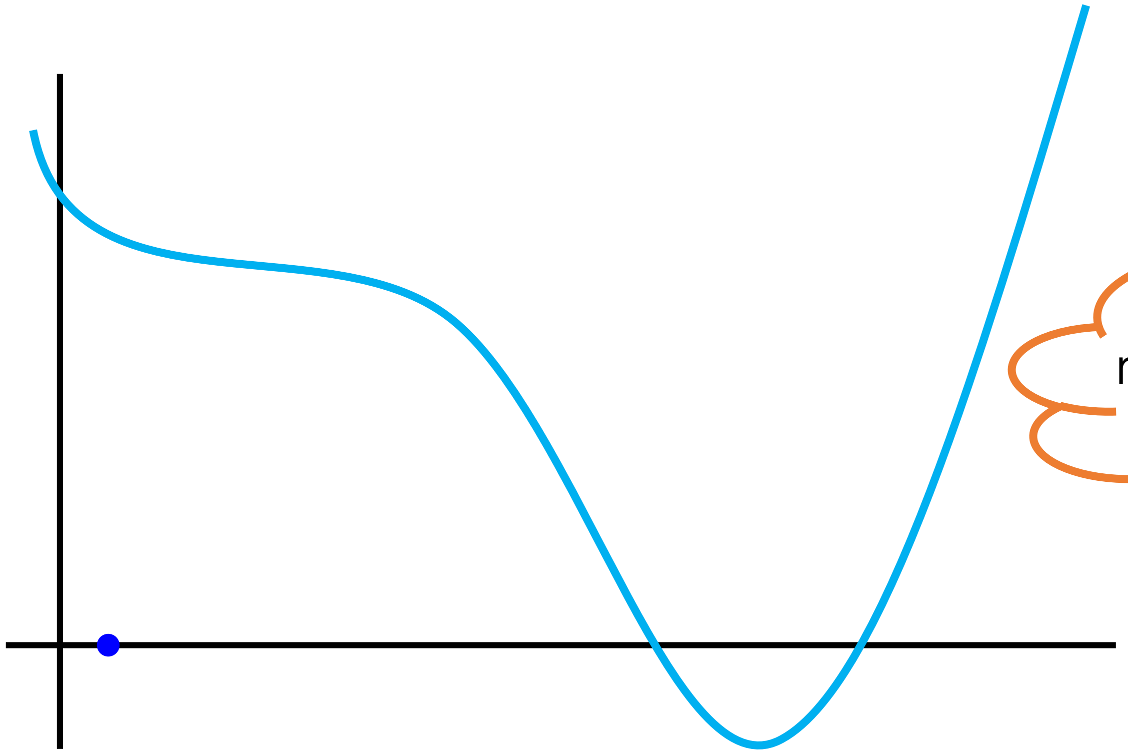
Better motivation: Finding minima of quadratic fn. easy
$f(\mathbf{x}) = \mathbf{x}^\top \mathbf{A}\mathbf{x} + \langle \mathbf{b}, \mathbf{x} \rangle + c$

Newton Method



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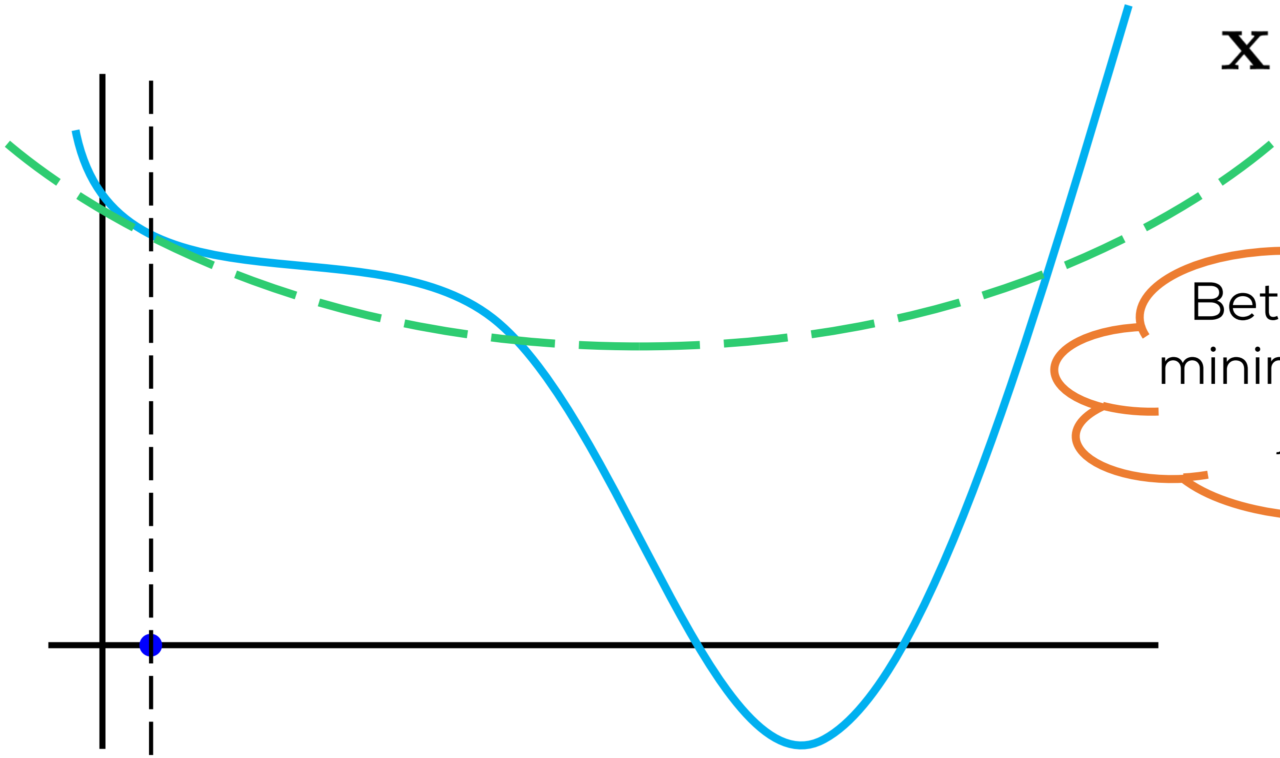

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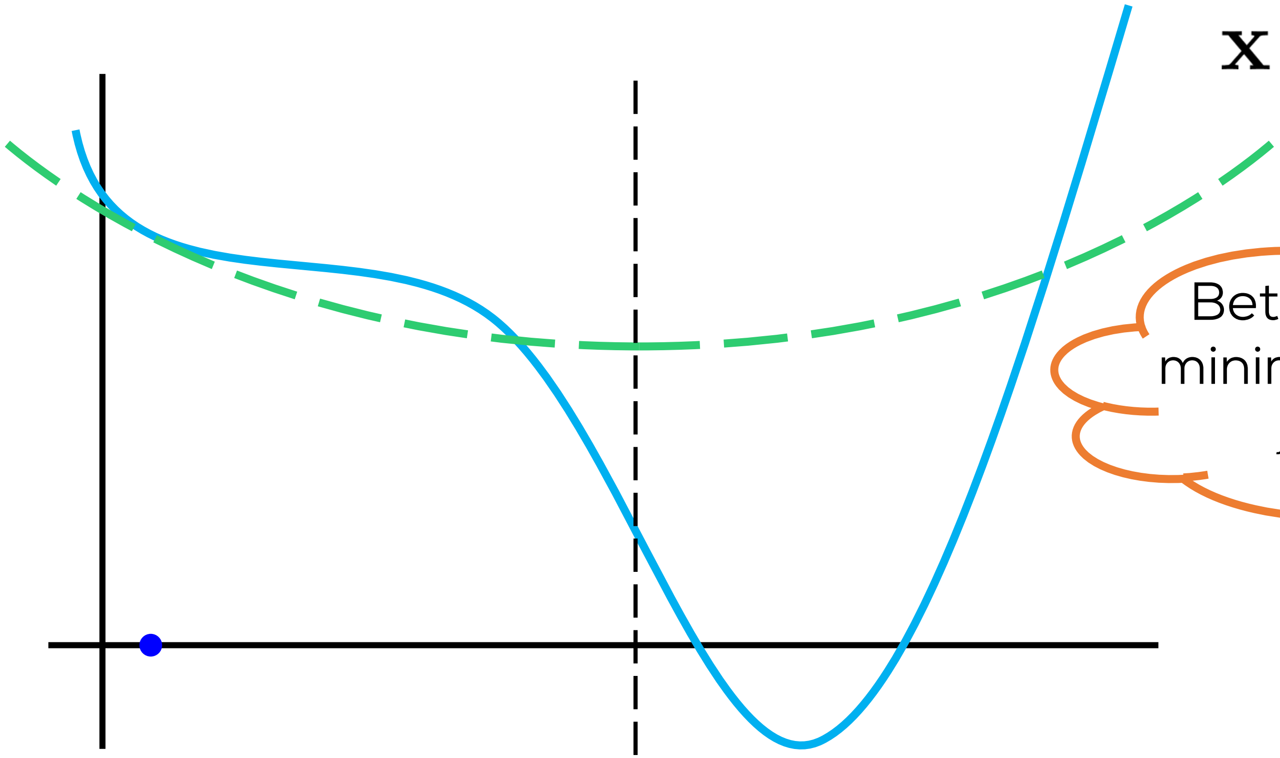

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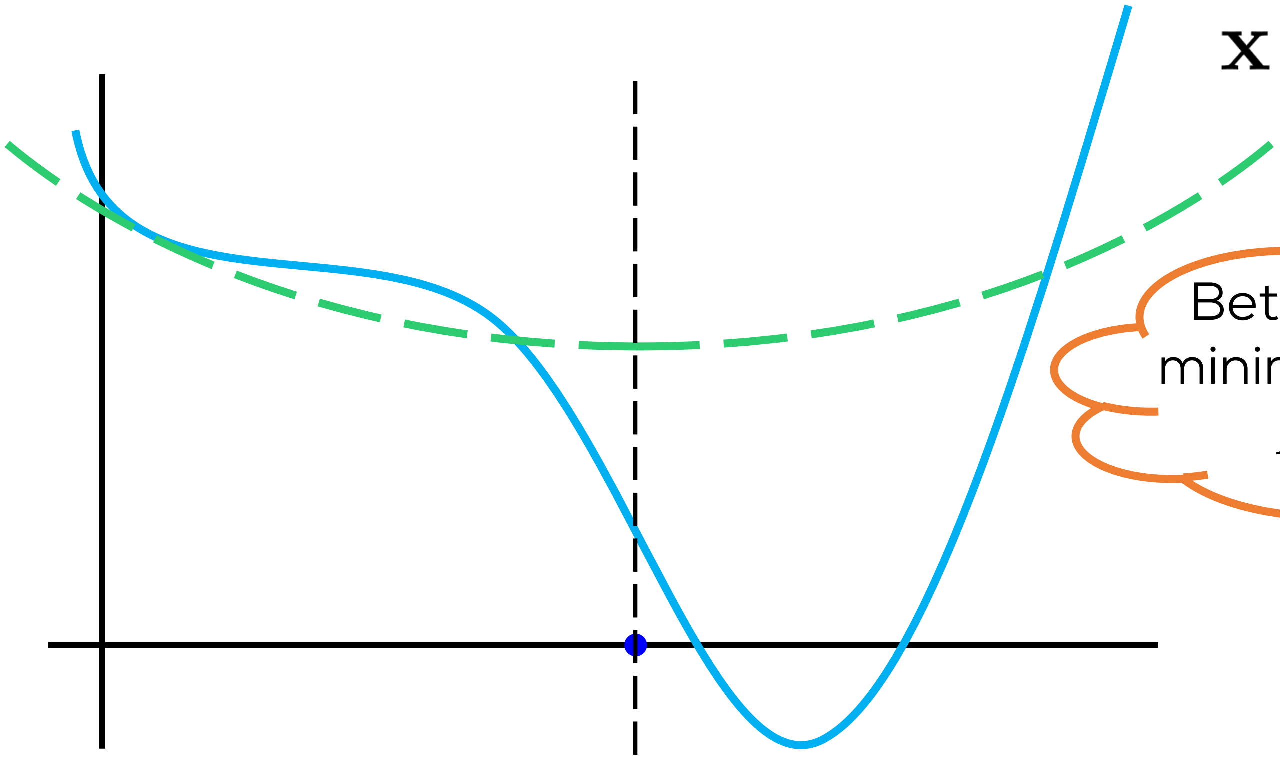

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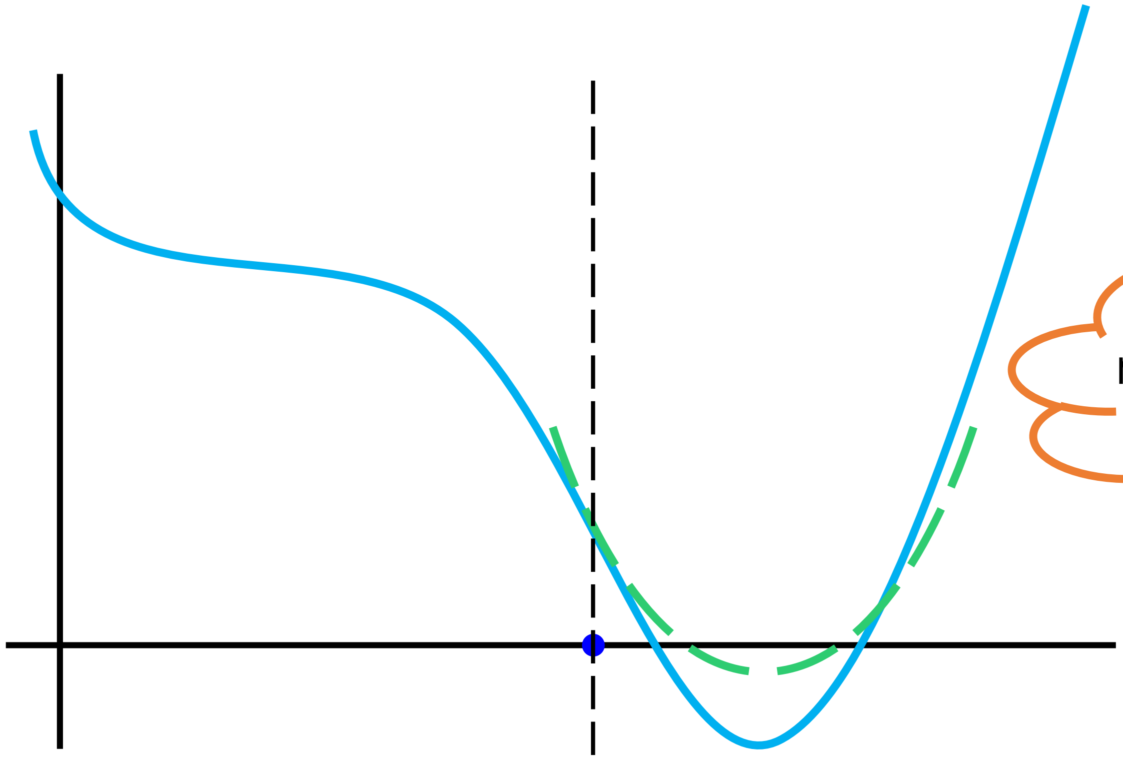

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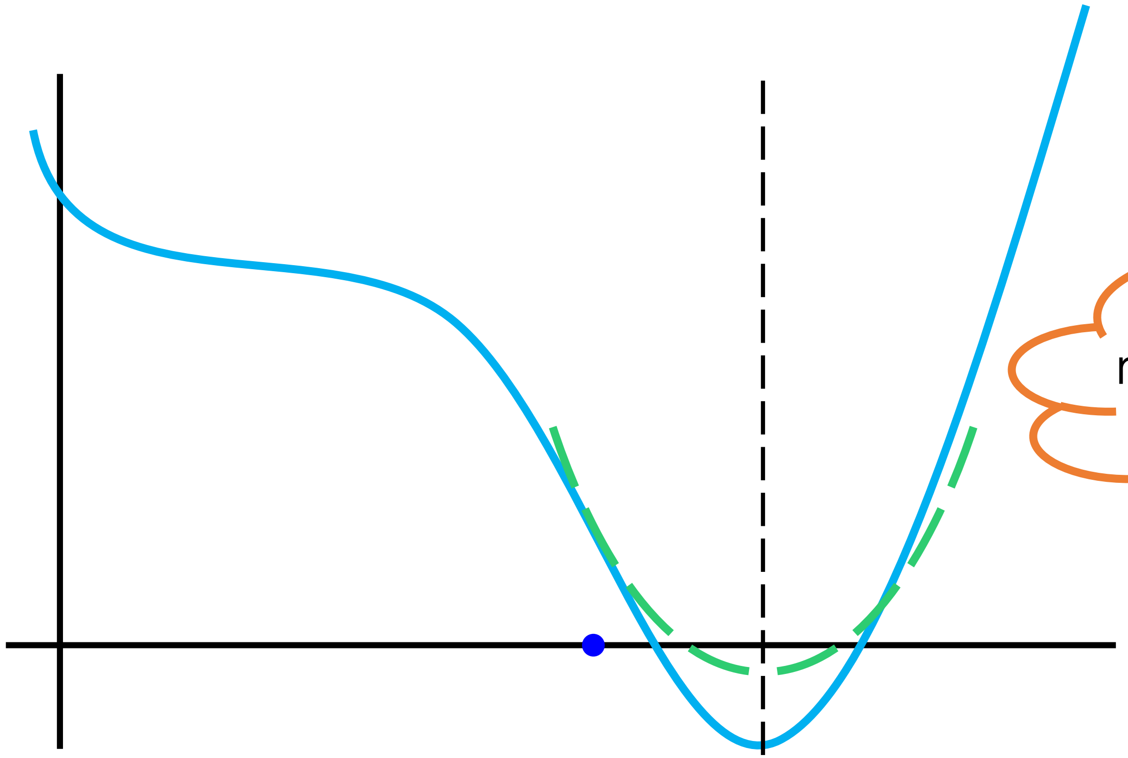

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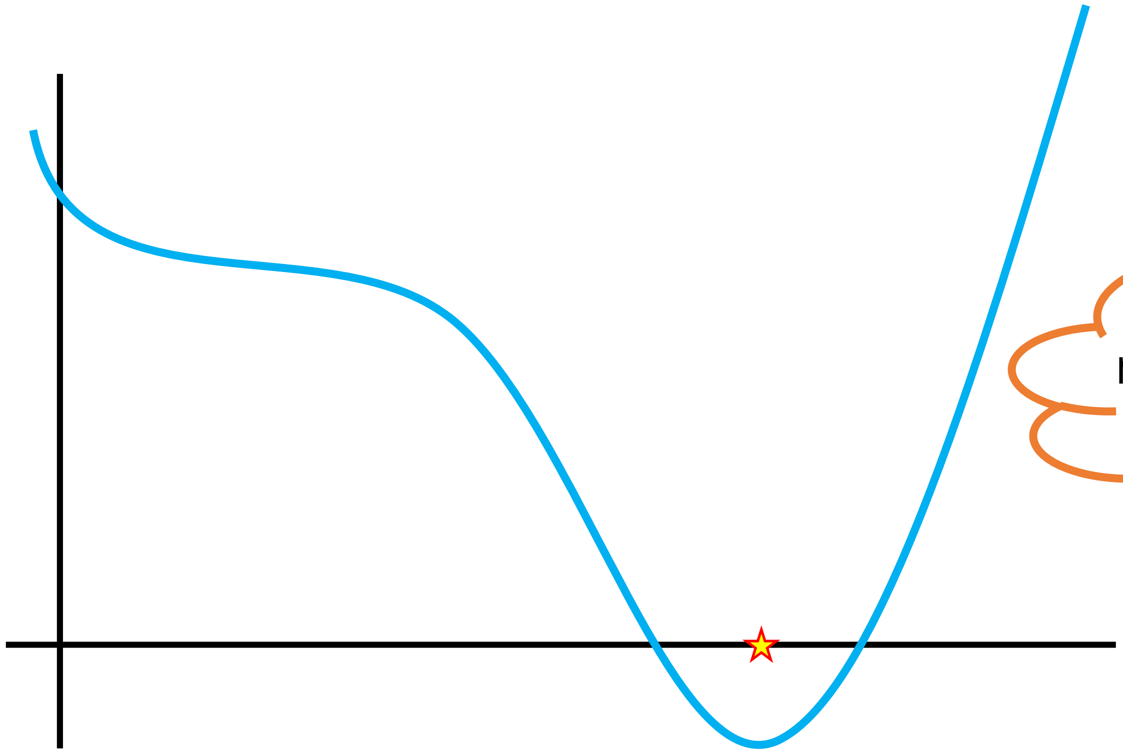

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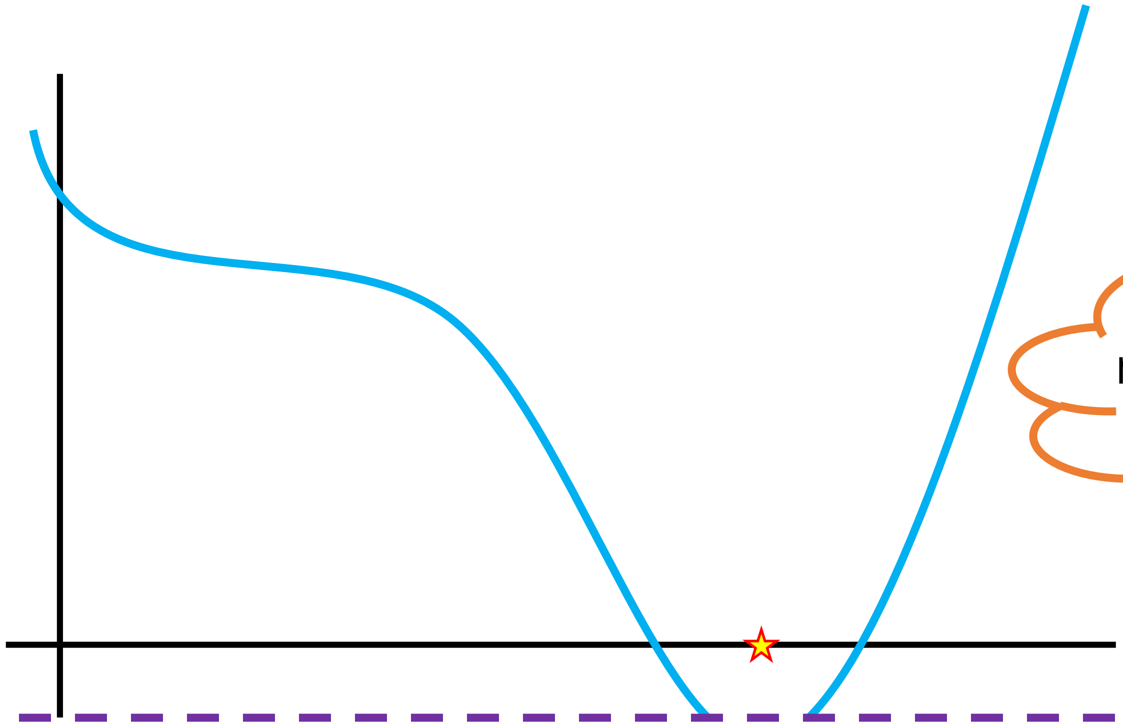

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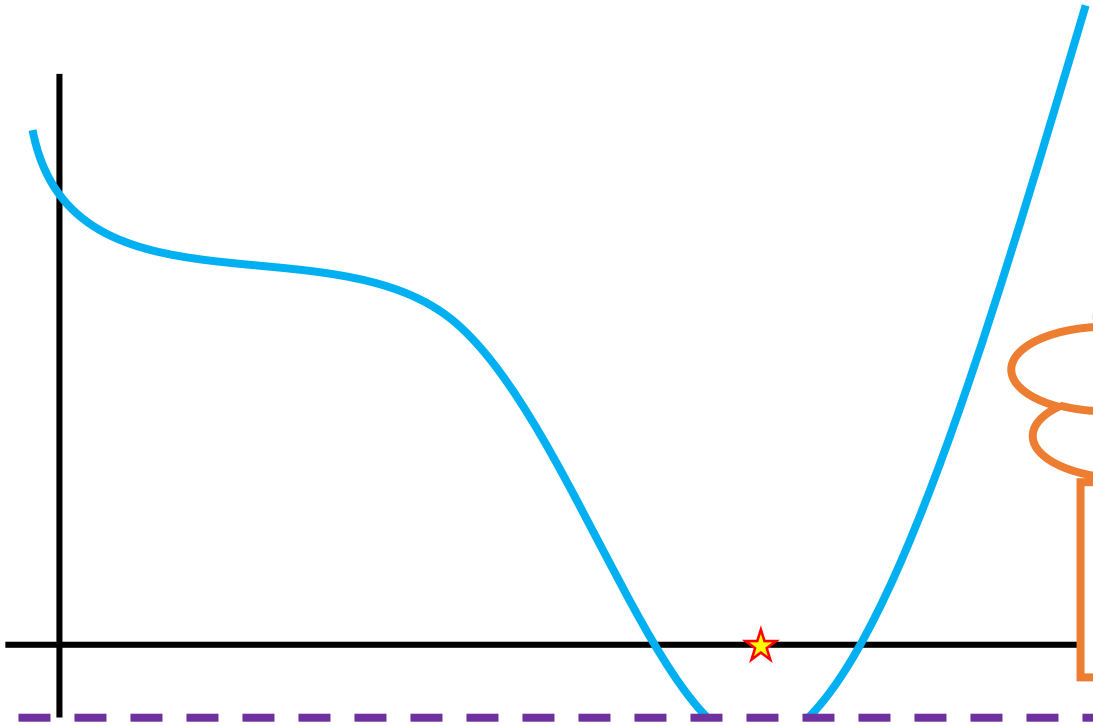

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$f(\cdot)$ and $\tilde{f}_t(\cdot)$ have the same gradient and Hessian at $\mathbf{x}^t$

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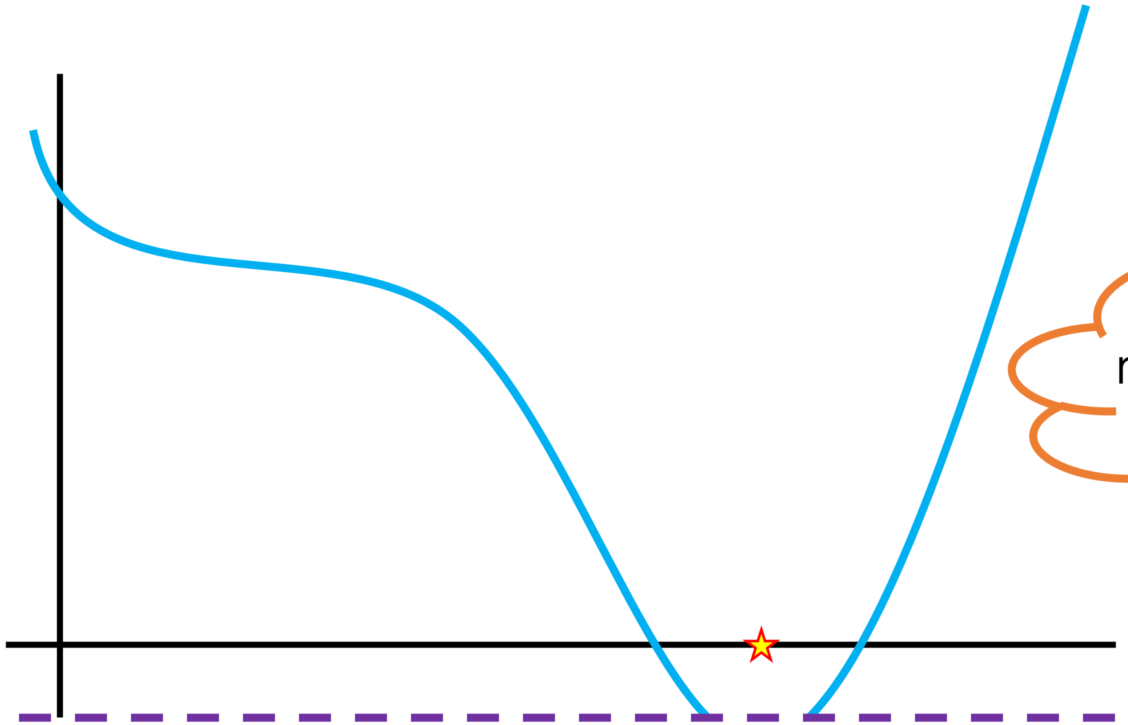

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Ridiculously fast for convex problems

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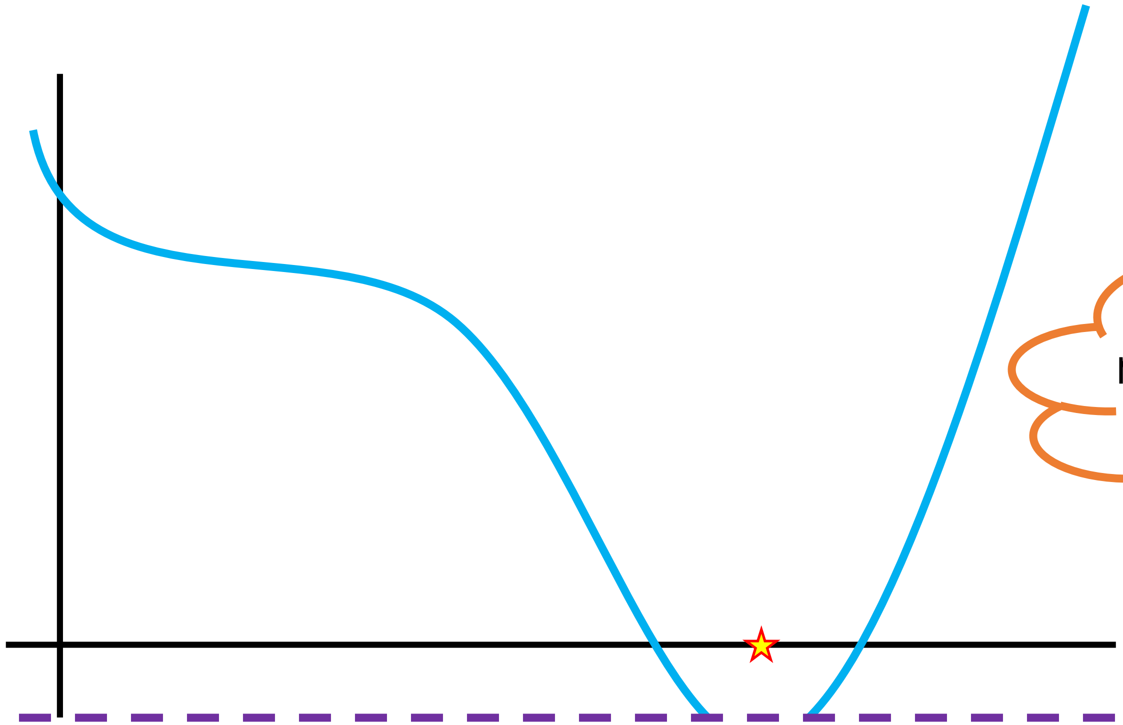

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$O\left(\log\log\frac{1}{\epsilon}\right)$ steps

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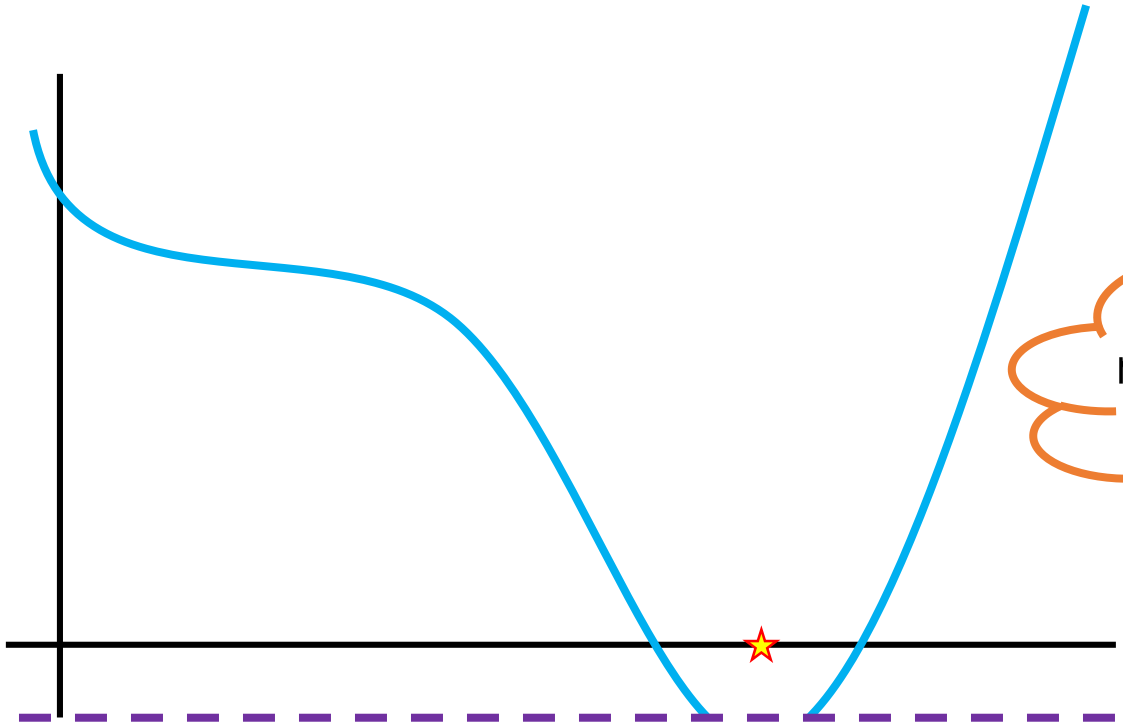

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Steps more costly $O(d^2)$

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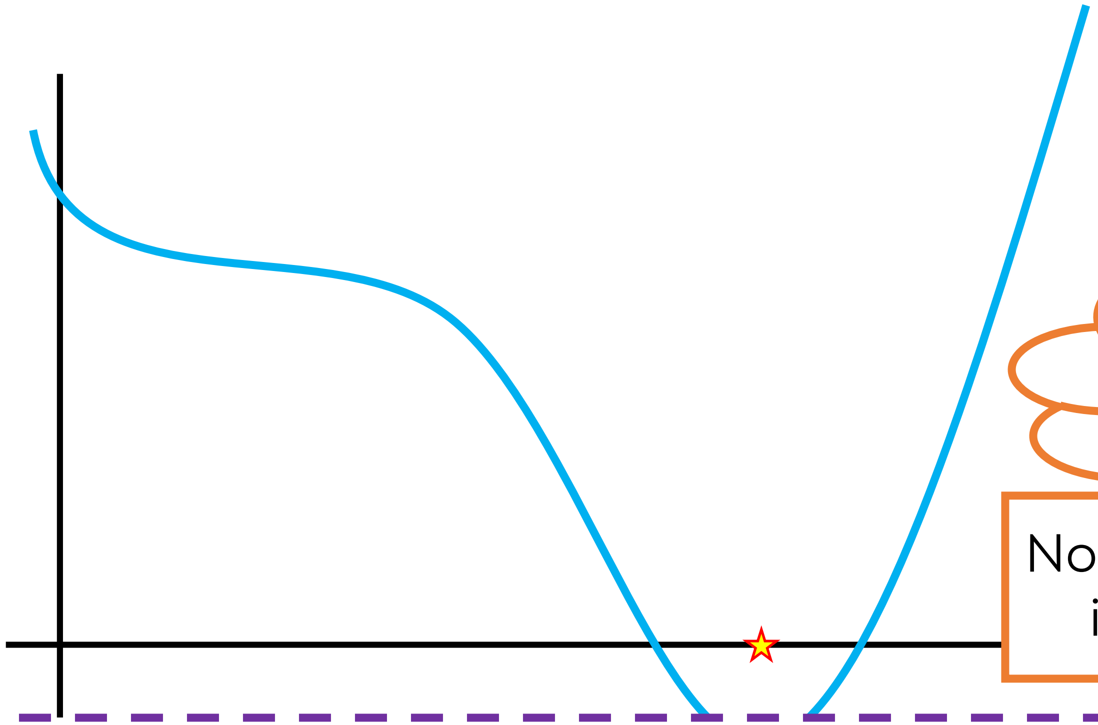

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$\mathcal{O}\left(\log\log\frac{1}{\epsilon}\right)$ steps

No guarantees in general

Steps more costly $\mathcal{O}(d^2)$

Ridiculously fast for convex problems

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Duality

How to take the problem you want to solve
… and convert it to a problem you can solve

Fenchel, Lagrange, Wolfe, Pontryagin

Duality

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The Lagrangian

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$$\hat{\mathbf{w}} = \arg\min_{\mathbf{w}\in\mathbb{R}^d} f(\mathbf{w})$$

$$\text{s.t. } g(\mathbf{w}) \leq 0$$

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$$\text{s.t.} \;\; g(\mathbf{w}) \le 0$$

$$\hat{\mathbf{w}} = \arg\min \sum_{i=1}^{n} \left(y^i - \langle \mathbf{w}, \mathbf{x}^i \rangle\right)^2$$

$$s.t. \; \|\mathbf{w}\|_2 \le r$$

The Lagrangian

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$$\hat{\mathbf{w}} = \arg\min_{\mathbf{w}\in\mathbb{R}^d} f(\mathbf{w})$$

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Want $g(\mathbf{w}) \geq 0$?
$-g(\mathbf{w}) \leq 0$

The Lagrangian

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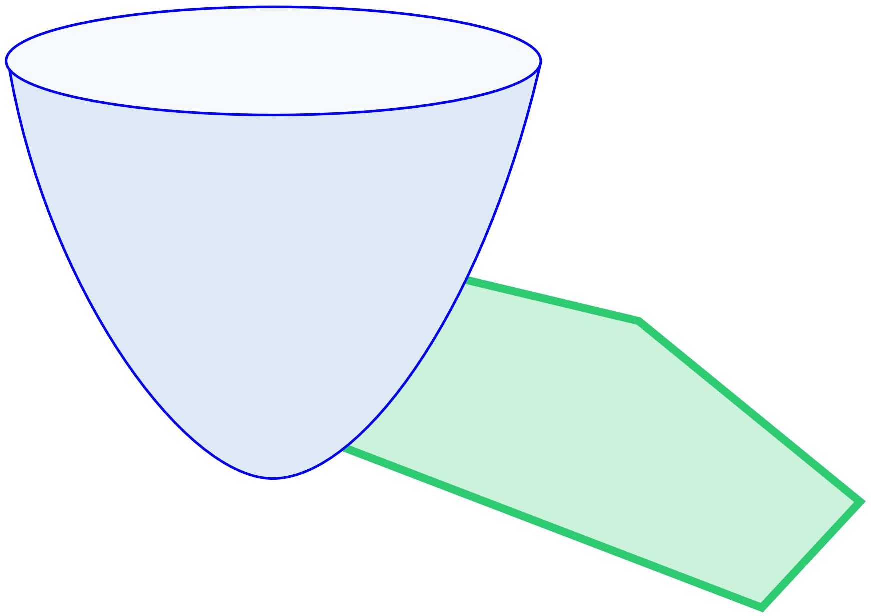
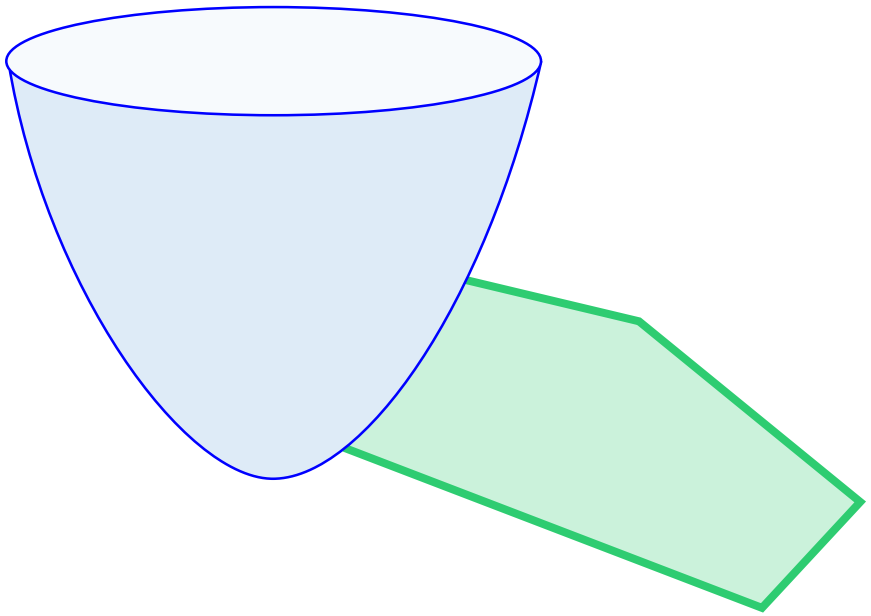
$$\hat{\mathbf{w}} = \arg\min_{\mathbf{w}\in\mathbb{R}^d} f(\mathbf{w})$$

$$\text{s.t. } g(\mathbf{w}) \leq 0$$



The Lagrangian

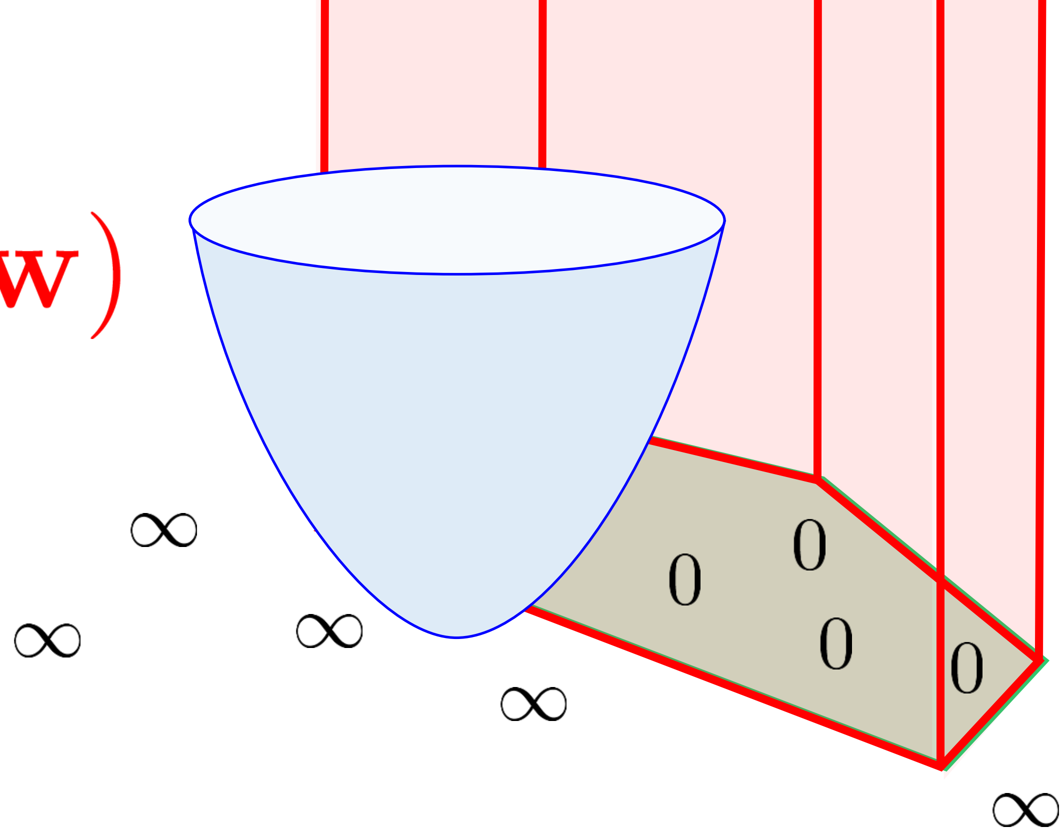
$$\hat{\mathbf{w}} = \arg\min_{\mathbf{w}\in\mathbb{R}^d} f(\mathbf{w}) + r(\mathbf{w})$$



The Lagrangian

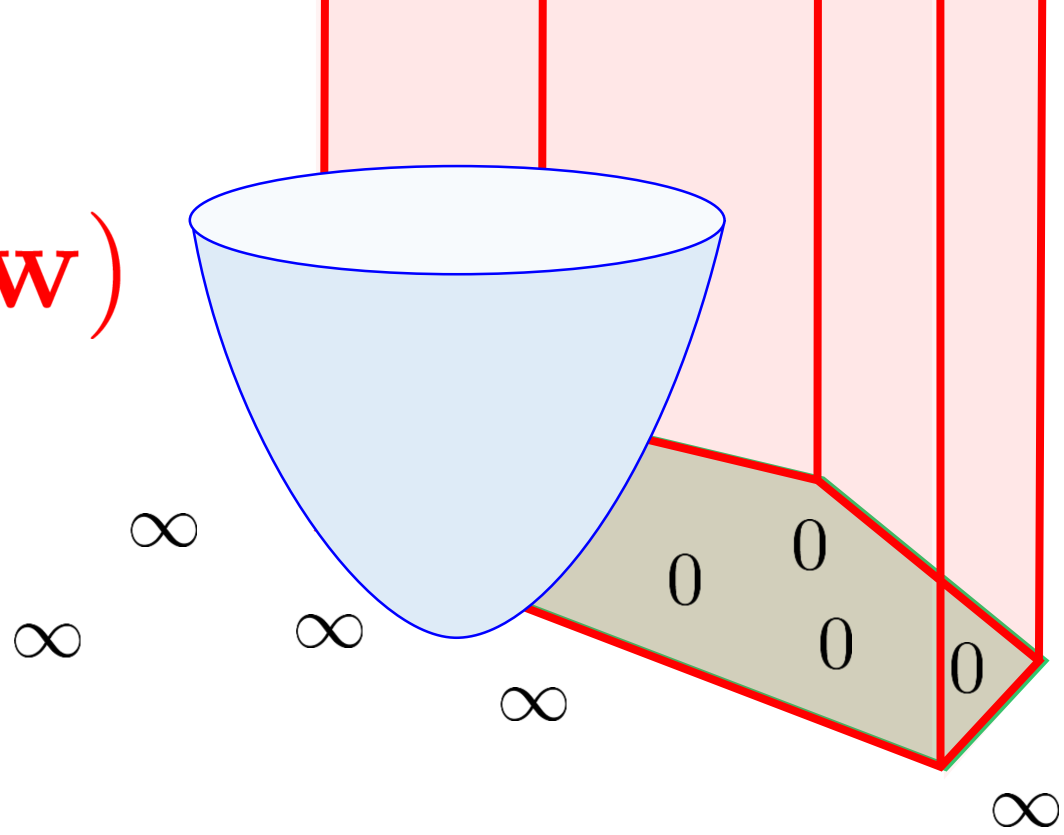
$$\hat{\mathbf{w}} = \arg\min_{\mathbf{w}\in\mathbb{R}^d} f(\mathbf{w}) + r(\mathbf{w})$$

Barrier Function



The Lagrangian

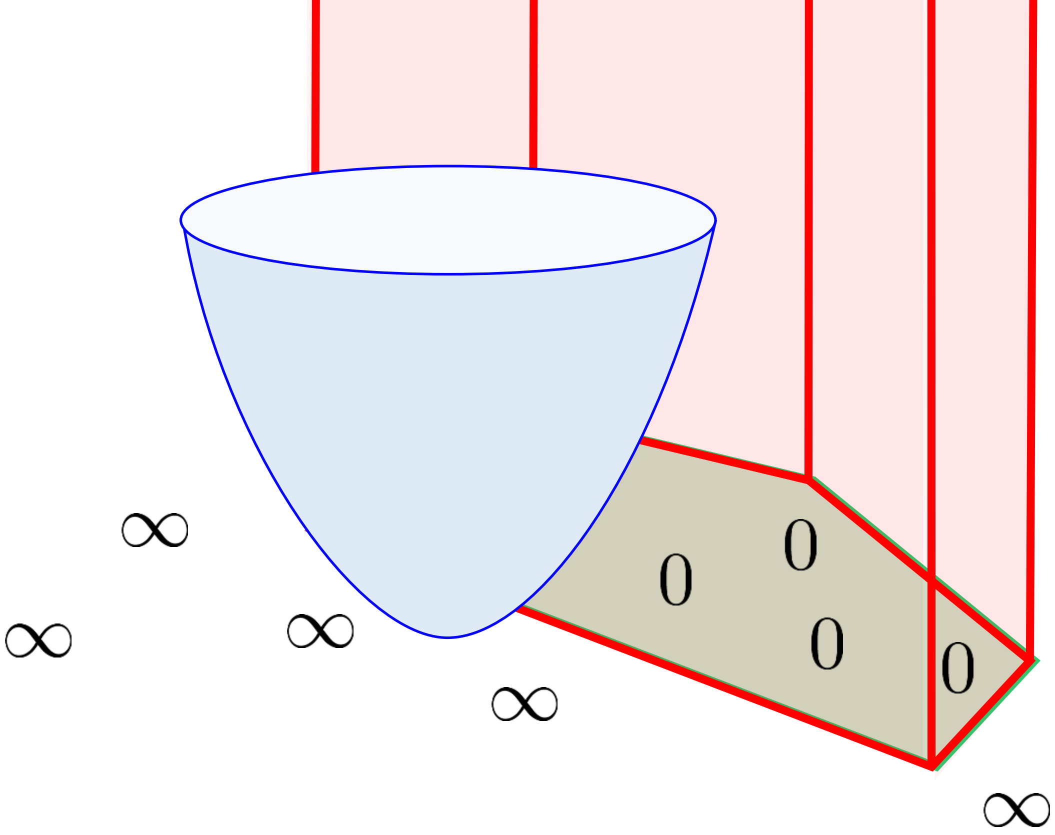
$$\hat{\mathbf{w}} = \arg\min_{\mathbf{w}\in\mathbb{R}^d} \; f(\mathbf{w}) + r(\mathbf{w})$$



The Lagrangian

$$\hat{\mathbf{w}} = \arg\min_{\mathbf{w}\in\mathbb{R}^d} f(\mathbf{w})$$

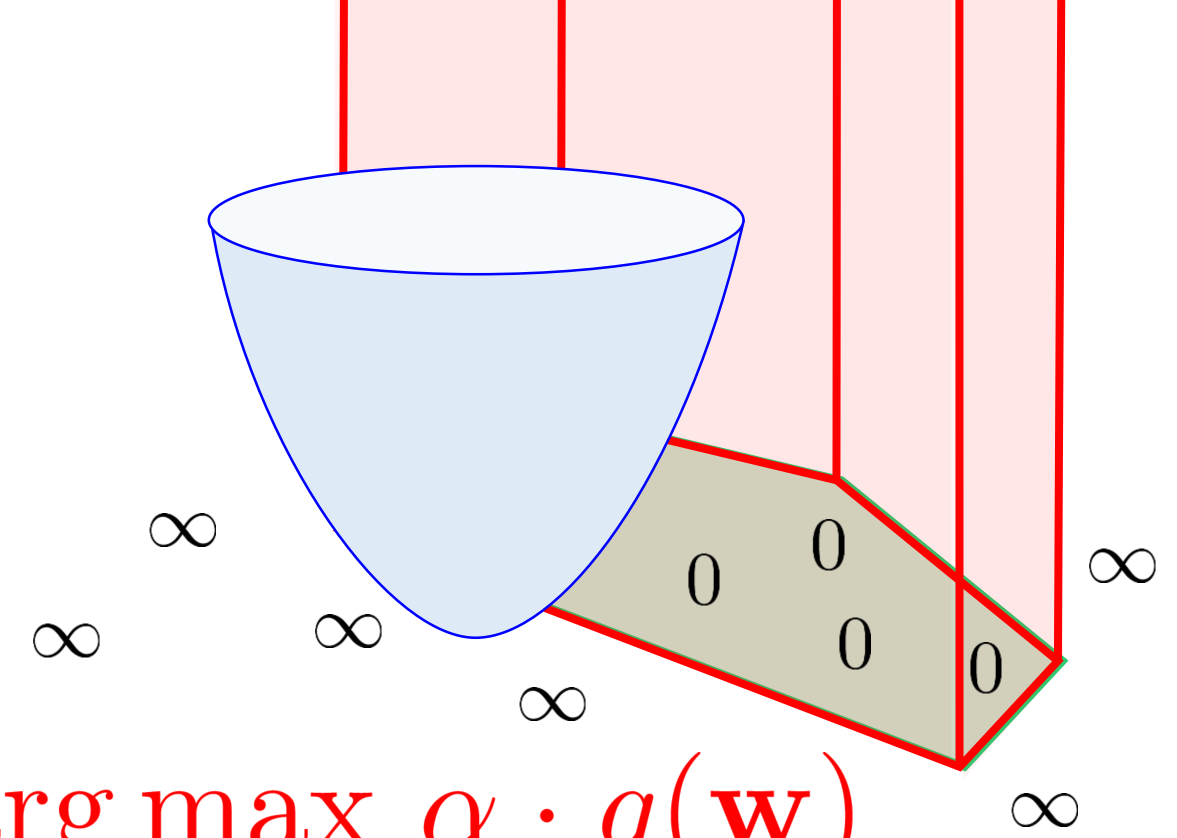
$$\text{s.t. } g(\mathbf{w}) \leq 0$$



The Lagrangian

$$\hat{\mathbf{w}} = \arg\min_{\mathbf{w} \in \mathbb{R}^d} f(\mathbf{w})$$

$$\text{s.t. } g(\mathbf{w}) \leq 0$$

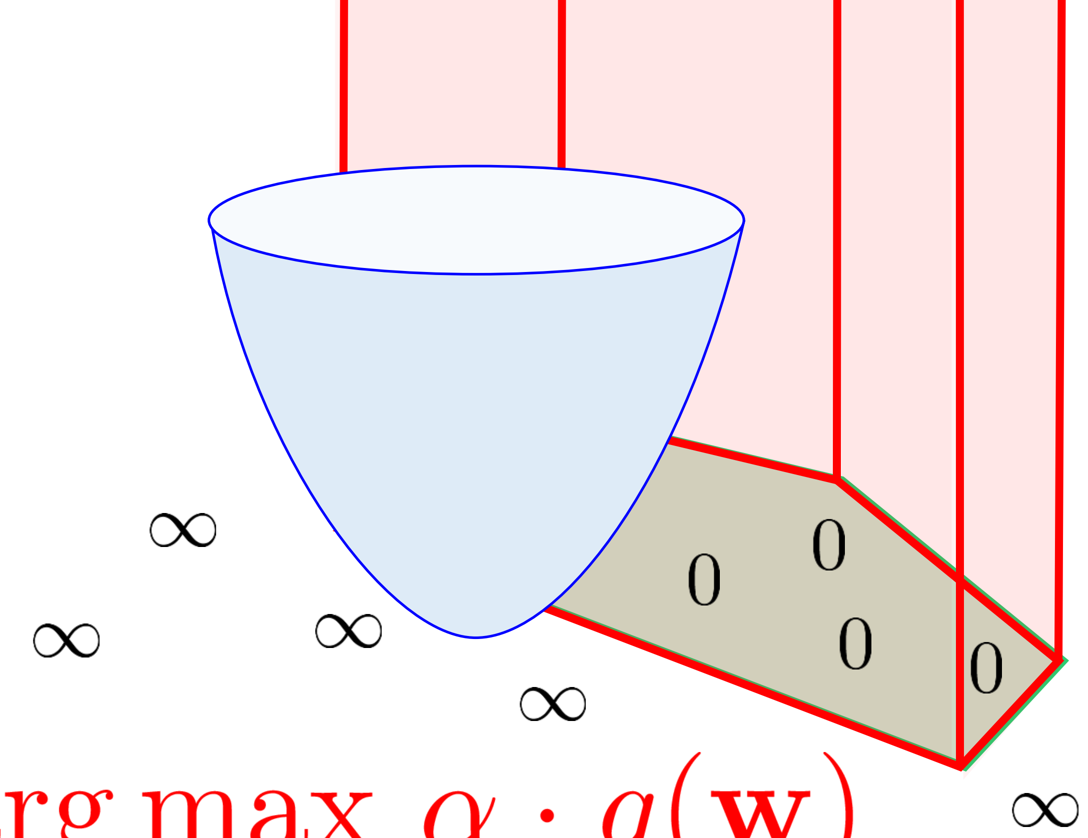


$$\arg\max_{\alpha \geq 0} \alpha \cdot g(\mathbf{w})$$

The Lagrangian

$$\hat{\mathbf{w}} = \underset{\mathbf{w} \in \mathbb{R}^d}{\arg\min}\; f(\mathbf{w})$$

$$\text{s.t. } g(\mathbf{w}) \leq 0$$



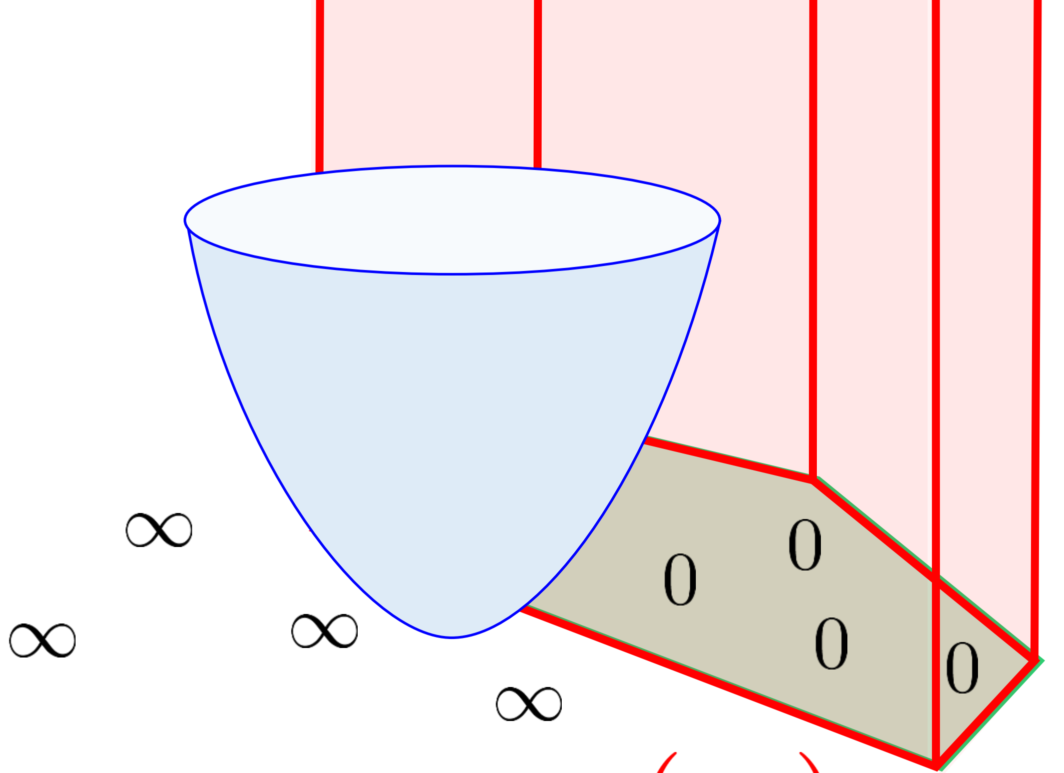
$$\underset{\alpha \geq 0}{\arg\max}\; \alpha \cdot g(\mathbf{w})$$

∞ if $g(\mathbf{w}) > 0$

The Lagrangian

$$\hat{\mathbf{w}} = \arg\min_{\mathbf{w}\in\mathbb{R}^d} f(\mathbf{w})$$

$$\text{s.t. } g(\mathbf{w}) \leq 0$$



$$\arg\max_{\alpha\geq 0} \alpha \cdot g(\mathbf{w})$$

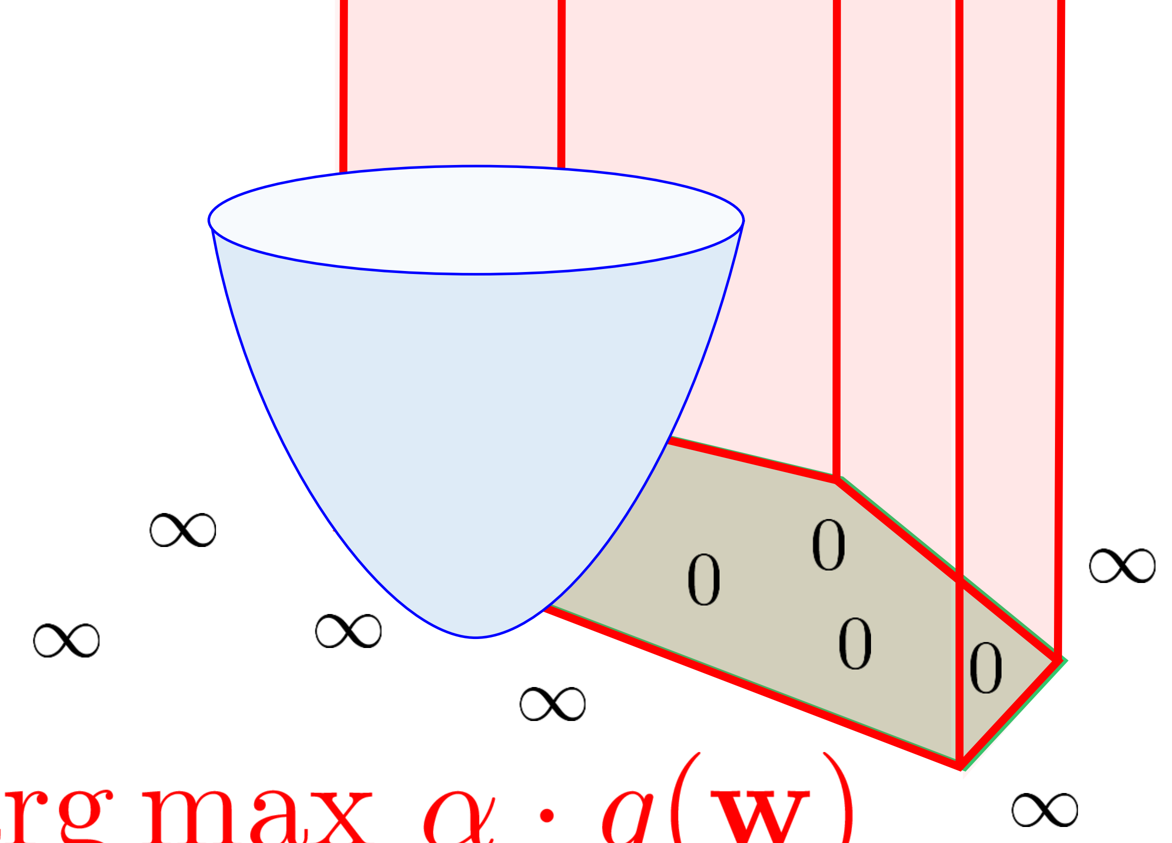
∞ if $g(\mathbf{w}) > 0$

0 if $g(\mathbf{w}) \leq 0$

The Lagrangian

$$\hat{\mathbf{w}} = \arg\min_{\mathbf{w}\in\mathbb{R}^d} \; f(\mathbf{w})$$

$$\text{s.t. } g(\mathbf{w}) \leq 0$$



$$f(\mathbf{w}) + \arg\max_{\alpha\geq0} \; \alpha \cdot g(\mathbf{w})$$

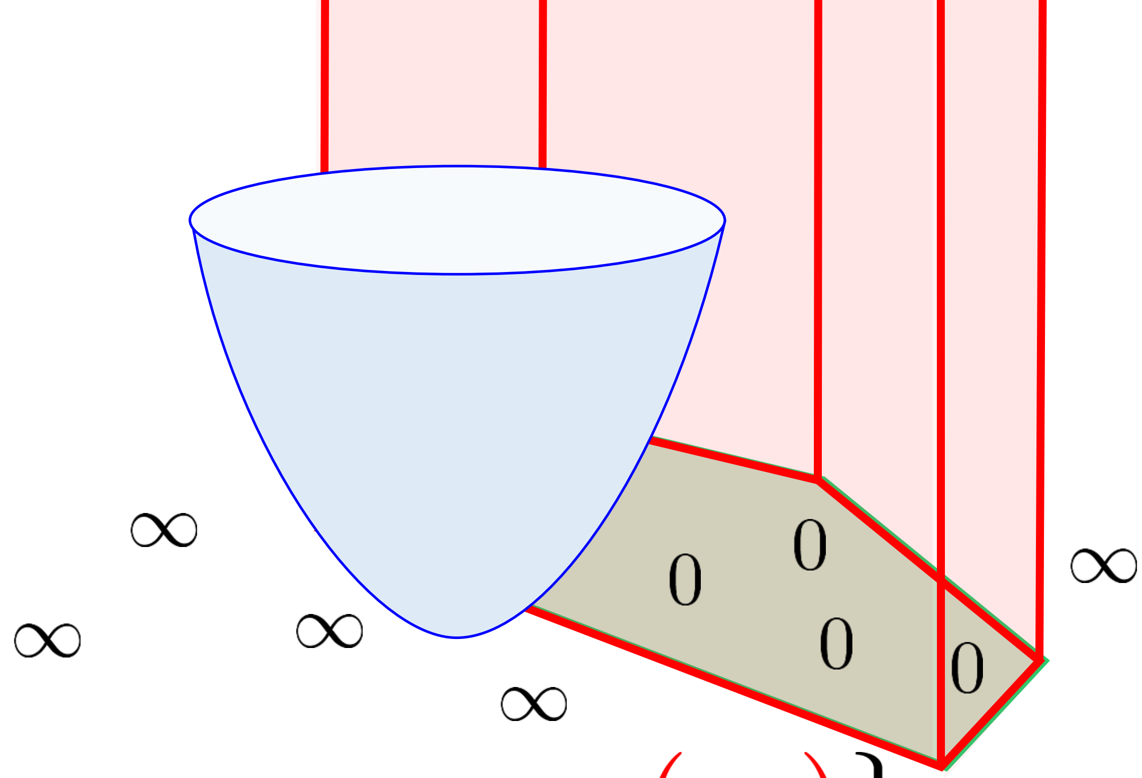
∞ if $g(\mathbf{w}) > 0$

0 if $g(\mathbf{w}) \leq 0$

The Lagrangian

$$\hat{\mathbf{w}} = \arg\min_{\mathbf{w}\in\mathbb{R}^d} f(\mathbf{w})$$

$$\text{s.t. } g(\mathbf{w}) \leq 0$$



$$\arg\min_{\mathbf{w}\in\mathbb{R}^d} \left\{ f(\mathbf{w}) + \arg\max_{\alpha\geq 0} \alpha \cdot g(\mathbf{w}) \right\}$$

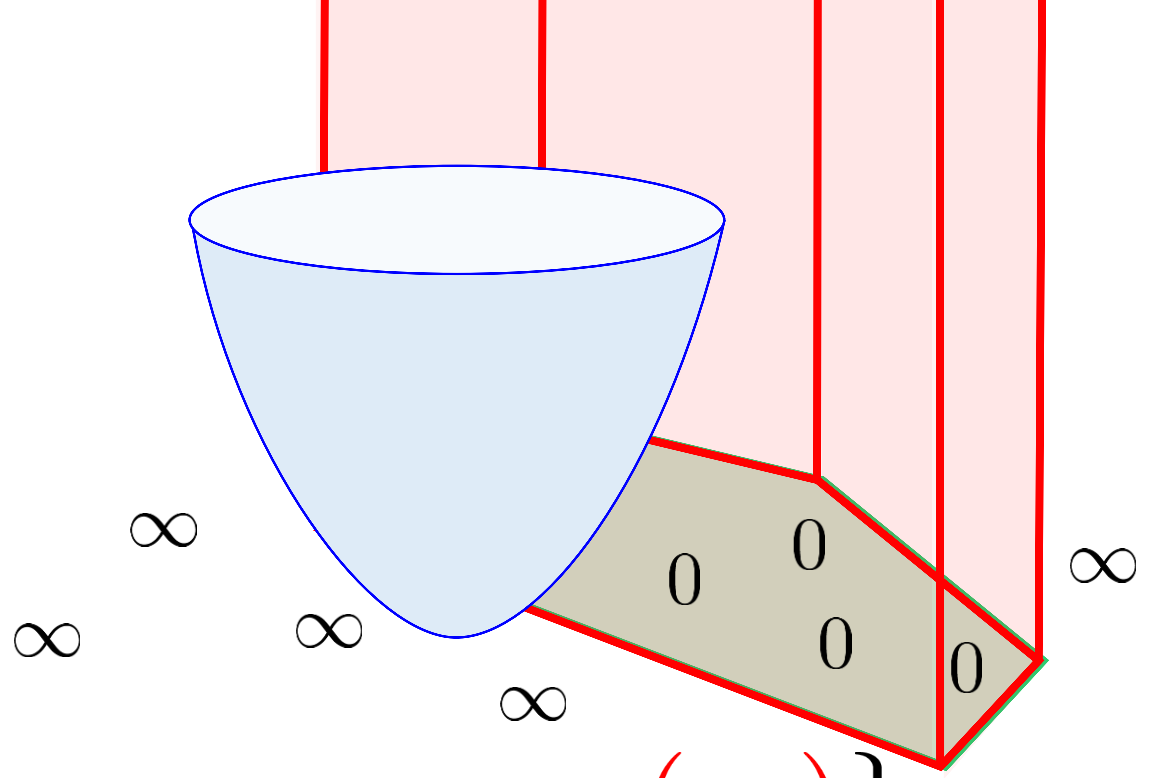
∞ if $g(\mathbf{w}) > 0$

0 if $g(\mathbf{w}) \leq 0$

The Lagrangian

$$\hat{\mathbf{w}} = \arg\min_{\mathbf{w}\in\mathbb{R}^d} f(\mathbf{w})$$

$$\text{s.t. } g(\mathbf{w}) \leq 0$$

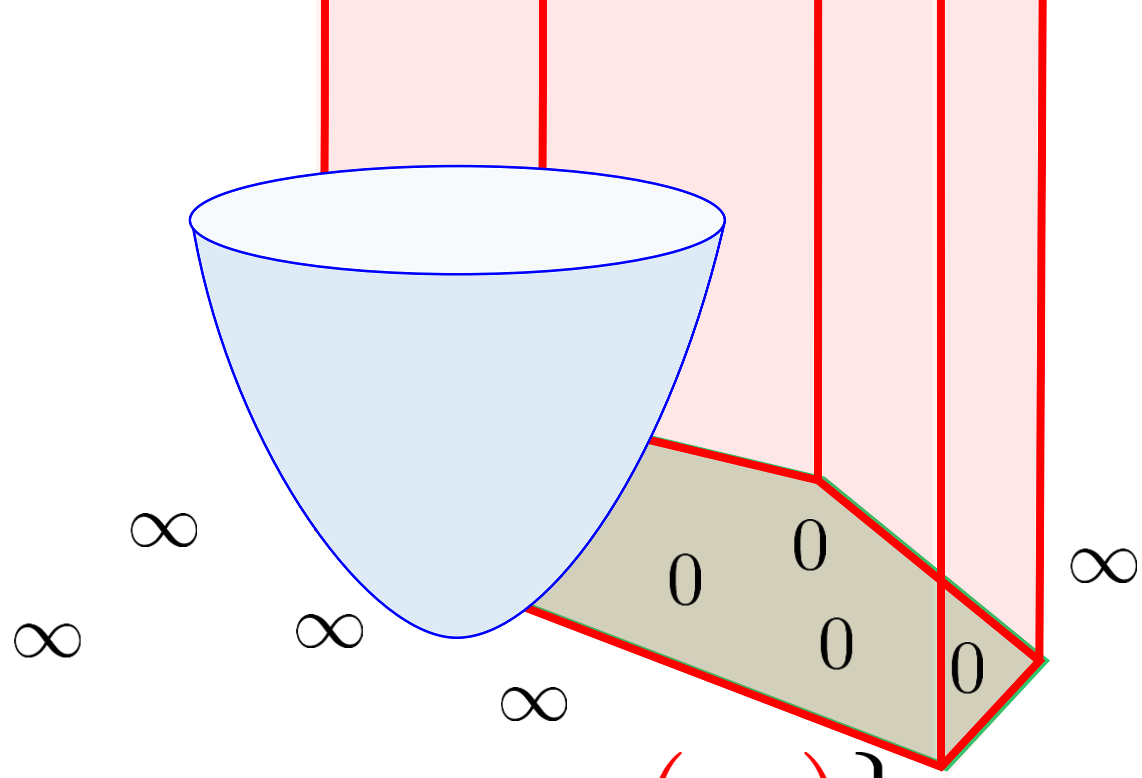


$$\hat{\mathbf{w}} = \arg\min_{\mathbf{w}\in\mathbb{R}^d} \left\{ f(\mathbf{w}) + \arg\max_{\alpha\geq 0} \alpha\cdot g(\mathbf{w}) \right\}$$

∞ if $g(\mathbf{w}) > 0$

0 if $g(\mathbf{w}) \leq 0$

The Lagrangian



$$\hat{\mathbf{w}} = \arg\min_{\mathbf{w}\in\mathbb{R}^d} f(\mathbf{w})$$

$$\text{s.t. } g(\mathbf{w}) \leq 0$$

$$\hat{\mathbf{w}} = \arg\min_{\mathbf{w}\in\mathbb{R}^d} \left\{ f(\mathbf{w}) + \arg\max_{\alpha\geq 0} \alpha\cdot g(\mathbf{w}) \right\} \quad \infty$$

∞ if $g(\mathbf{w}) > 0$

0 if $g(\mathbf{w}) \leq 0$

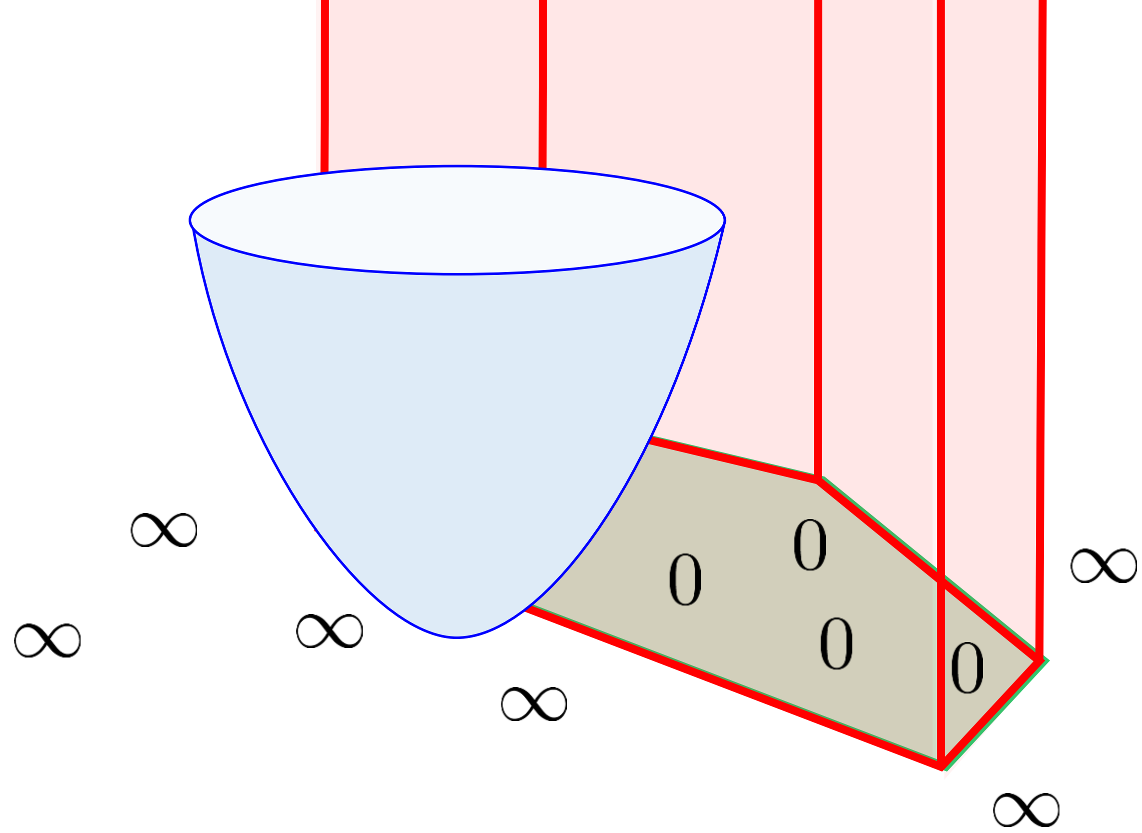
$$\hat{\mathbf{w}} = \arg\min_{\mathbf{w}\in\mathbb{R}^d} \left\{ \arg\max_{\alpha\geq 0} \left\{ f(\mathbf{w}) + \alpha\cdot g(\mathbf{w}) \right\} \right\}$$

Exercise

The Lagrangian

$$\hat{\mathbf{w}} = \arg\min_{\mathbf{w}\in\mathbb{R}^d} \; f(\mathbf{w})$$

$$\text{s.t. } g(\mathbf{w}) \leq 0$$

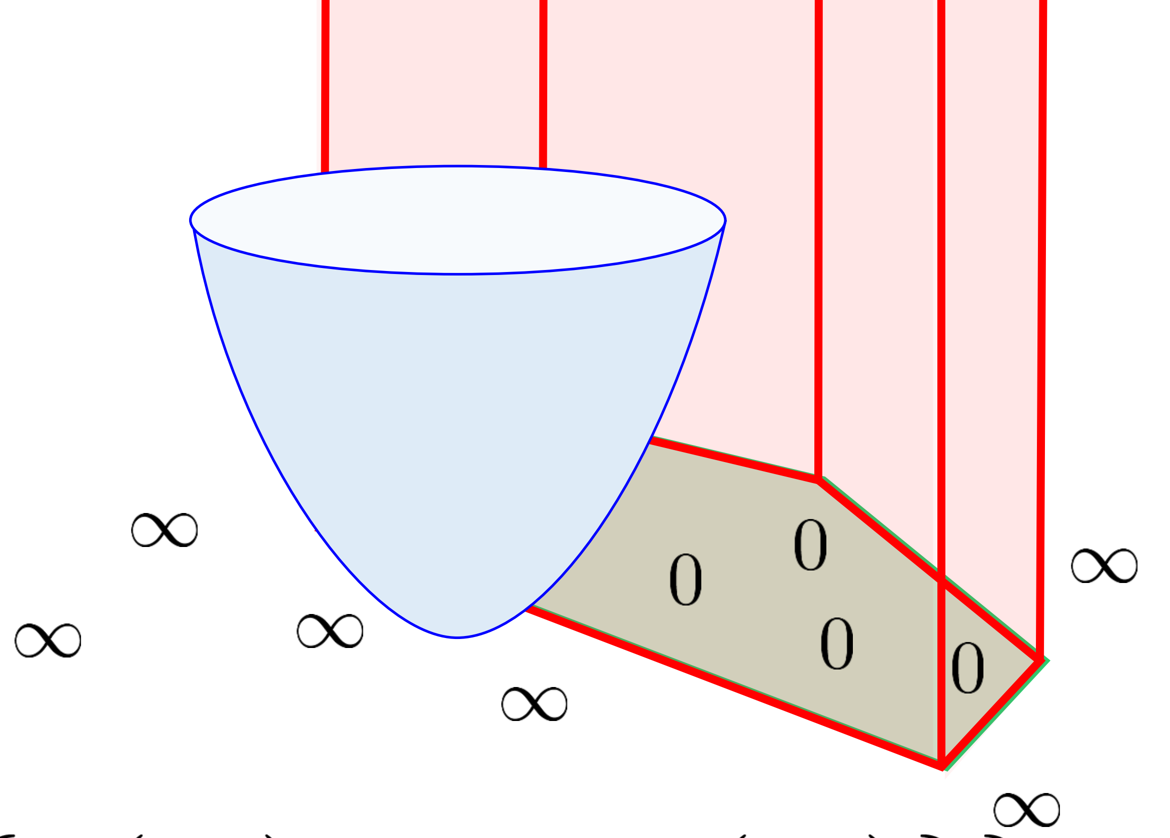


$$\hat{\mathbf{w}} = \arg\min_{\mathbf{w}\in\mathbb{R}^d} \left\{\arg\max_{\alpha\geq 0} \left\{f(\mathbf{w}) + \alpha \cdot g(\mathbf{w})\right\}\right\}$$

The Lagrangian

$$\hat{\mathbf{w}} = \arg\min_{\mathbf{w}\in\mathbb{R}^d} f(\mathbf{w})$$

$$\text{s.t. } g(\mathbf{w}) \leq 0$$



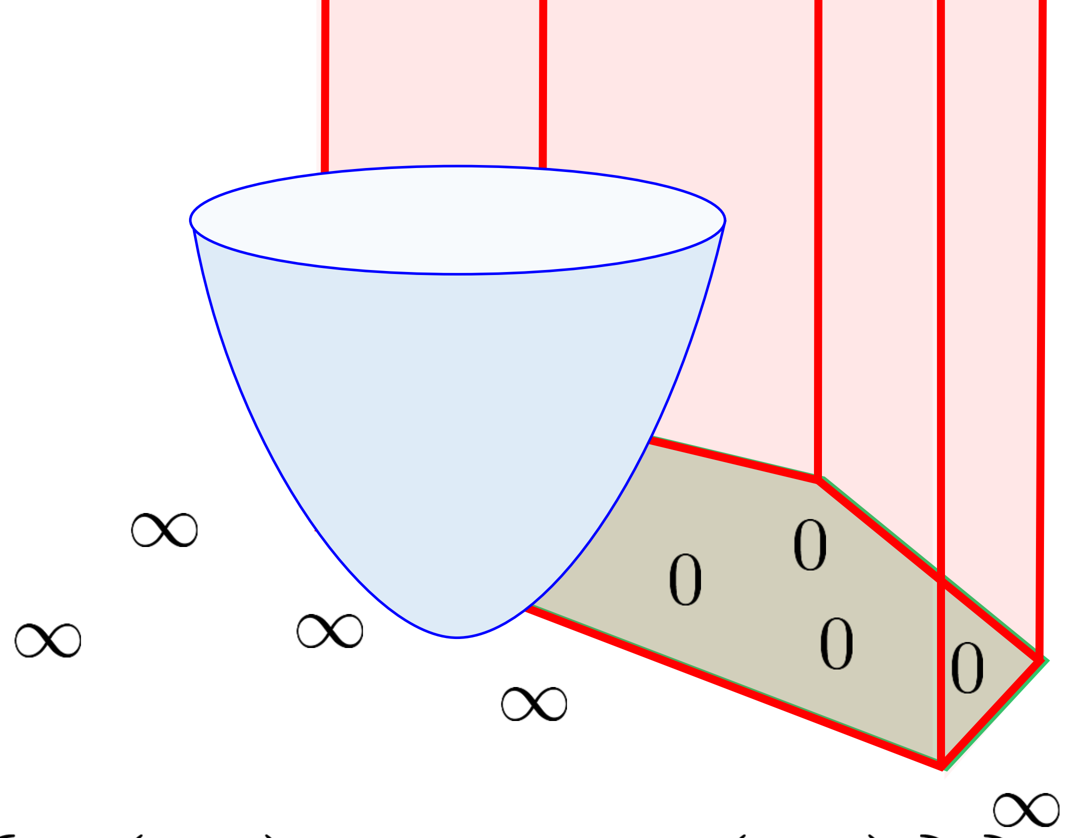
$$\hat{\mathbf{w}} = \arg\min_{\mathbf{w}\in\mathbb{R}^d} \left\{\arg\max_{\alpha\geq 0} \left\{f(\mathbf{w}) + \alpha \cdot g(\mathbf{w})\right\}\right\}$$

$$\mathcal{L}(\mathbf{w},\alpha) = f(\mathbf{w}) + \alpha \cdot g(\mathbf{w})$$

The Lagrangian

$$\hat{\mathbf{w}} = \arg\min_{\mathbf{w}\in\mathbb{R}^d} f(\mathbf{w})$$

$$\text{s.t. } g(\mathbf{w}) \leq 0$$



$$\hat{\mathbf{w}} = \arg\min_{\mathbf{w}\in\mathbb{R}^d} \left\{\arg\max_{\alpha\geq 0} \left\{f(\mathbf{w}) + \alpha\cdot g(\mathbf{w})\right\}\right\}$$

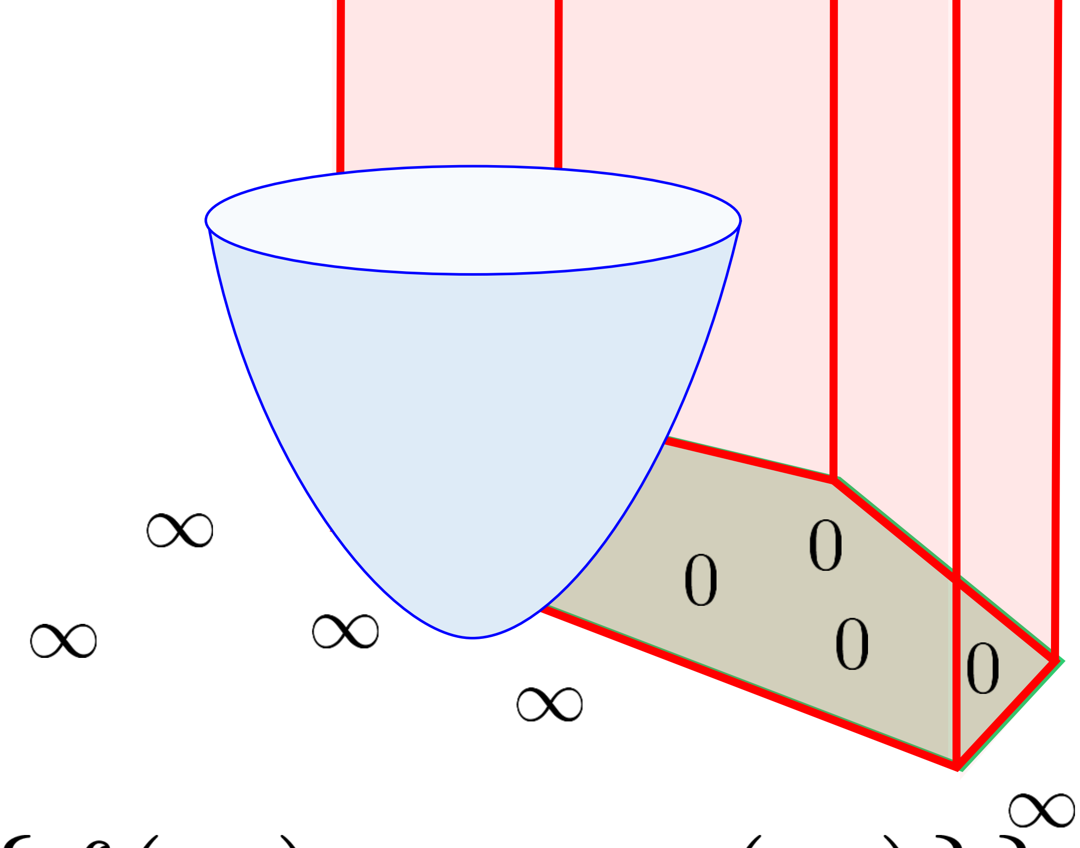
$$\mathcal{L}(\mathbf{w},\alpha) = f(\mathbf{w}) + \alpha\cdot g(\mathbf{w})$$

Lagrangian

The Lagrangian

$$\hat{\mathbf{w}} = \arg\min_{\mathbf{w}\in\mathbb{R}^d} f(\mathbf{w})$$

$$\text{s.t. } g(\mathbf{w}) \leq 0$$



$$\hat{\mathbf{w}} = \arg\min_{\mathbf{w}\in\mathbb{R}^d} \left\{\arg\max_{\alpha\geq 0} \{f(\mathbf{w}) + \alpha \cdot g(\mathbf{w})\}\right\}$$

Lagrange multiplier

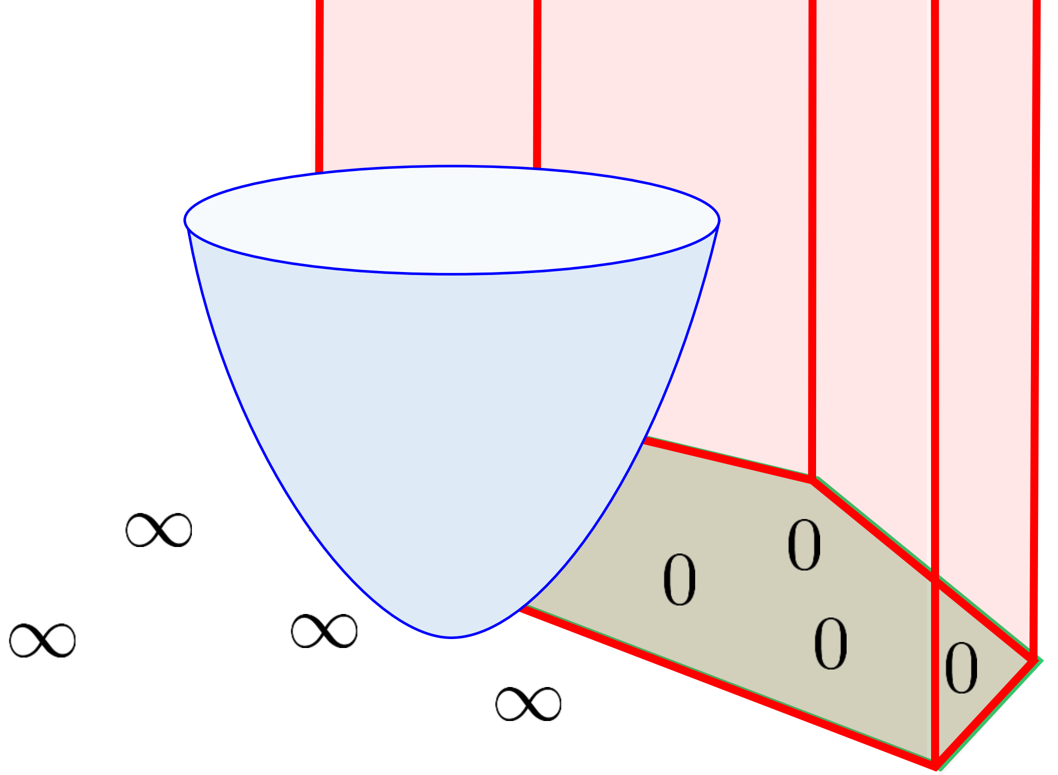
$$\mathcal{L}(\mathbf{w}, \alpha) = f(\mathbf{w}) + \alpha \cdot g(\mathbf{w})$$

Lagrangian

The Lagrangian

$$\hat{\mathbf{w}} = \arg\min_{\mathbf{w}\in\mathbb{R}^d} f(\mathbf{w})$$

$$\text{s.t. } g(\mathbf{w}) \leq 0$$



$$\hat{\mathbf{w}} = \arg\min_{\mathbf{w}\in\mathbb{R}^d} \{\arg\max_{\alpha\geq 0} \{ \quad \mathcal{L}(\mathbf{w},\alpha) \quad \}\}$$

∞

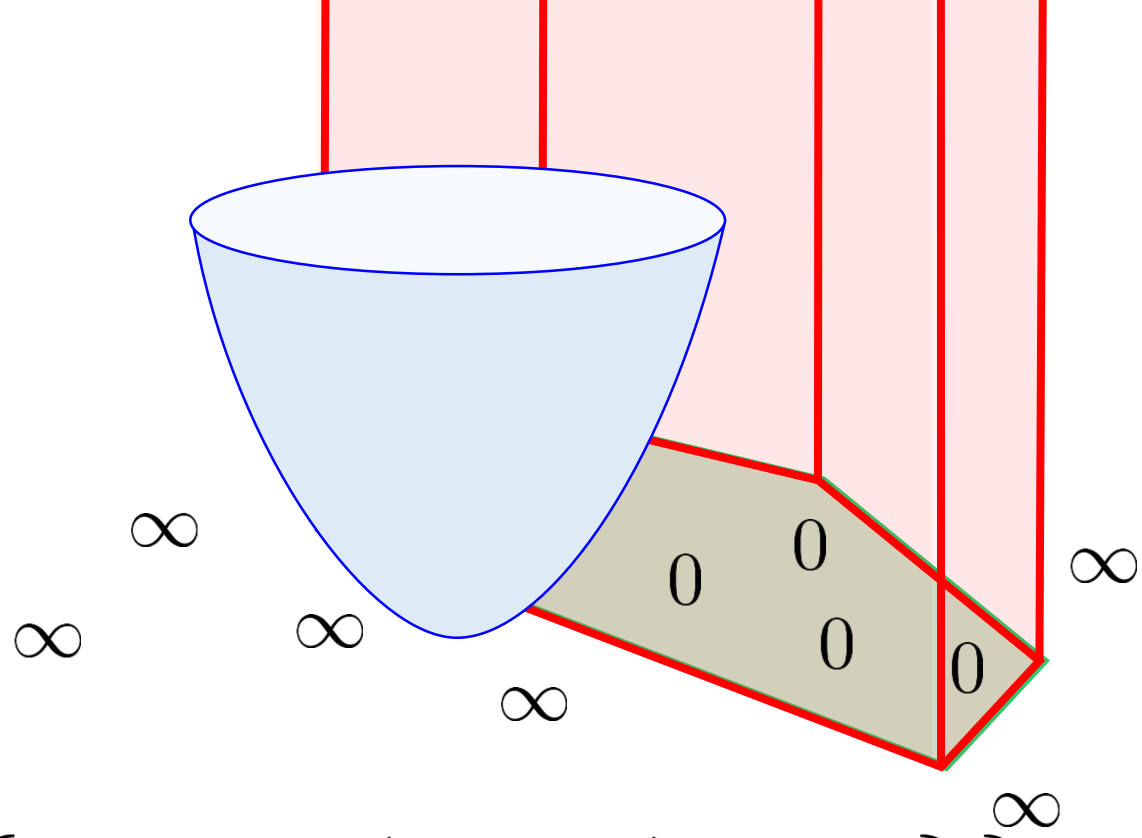
Lagrange multiplier

$$\mathcal{L}(\mathbf{w},\alpha) = f(\mathbf{w}) + \alpha \cdot g(\mathbf{w})$$

Lagrangian

The Lagrangian

$$\hat{\mathbf{w}} = \arg\min_{\mathbf{w}\in\mathbb{R}^d} f(\mathbf{w}) \quad \text{s.t. } g(\mathbf{w}) \le 0$$



$$\hat{\mathbf{w}}_{\mathrm{P}} = \arg\min_{\mathbf{w}\in\mathbb{R}^d} \left\{ \arg\max_{\alpha\ge 0} \left\{ \mathcal{L}(\mathbf{w},\alpha) \right\} \right\}$$

Lagrange multiplier

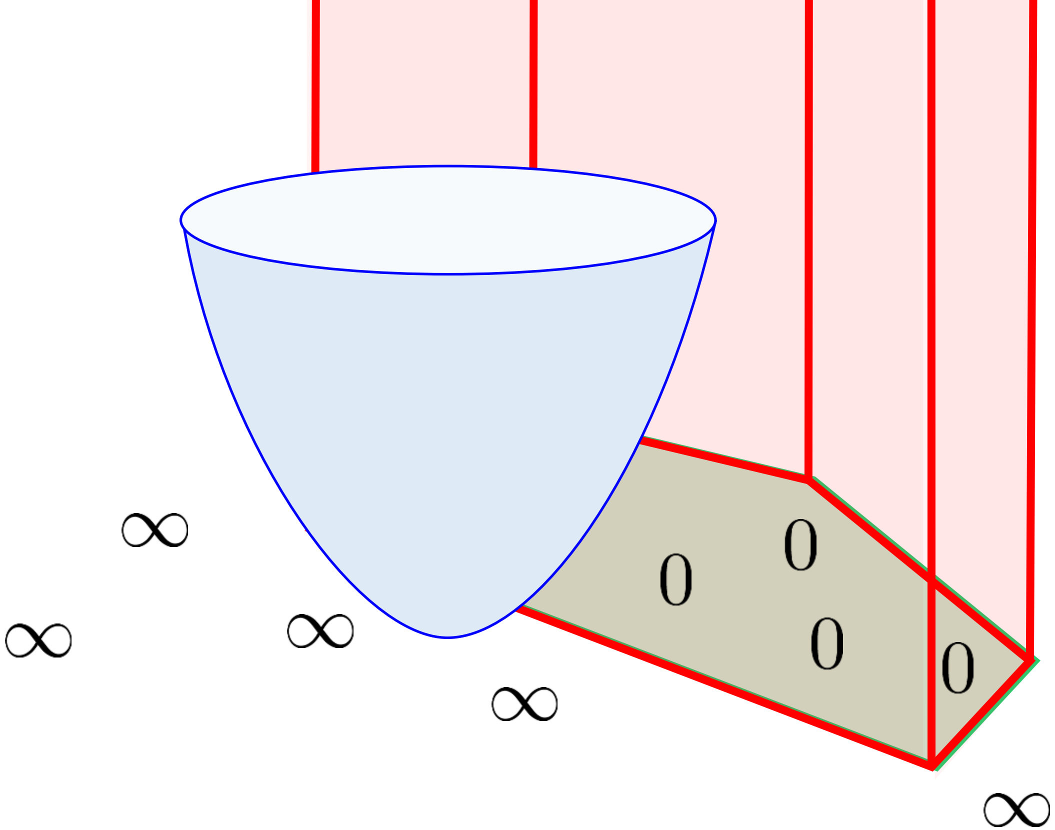
$$\mathcal{L}(\mathbf{w},\alpha) = f(\mathbf{w}) + \alpha \cdot g(\mathbf{w})$$

Lagrangian

The Lagrangian

$$\hat{\mathbf{w}} = \arg\min_{\mathbf{w}\in\mathbb{R}^d} f(\mathbf{w})$$

$$\text{s.t. } g(\mathbf{w}) \leq 0$$



$$\hat{\mathbf{w}}_P = \arg\min_{\mathbf{w}\in\mathbb{R}^d} \left\{ \arg\max_{\alpha\geq 0} \left\{ \mathcal{L}(\mathbf{w},\alpha) \right\} \right\}$$

Lagrange multiplier

Primal problem

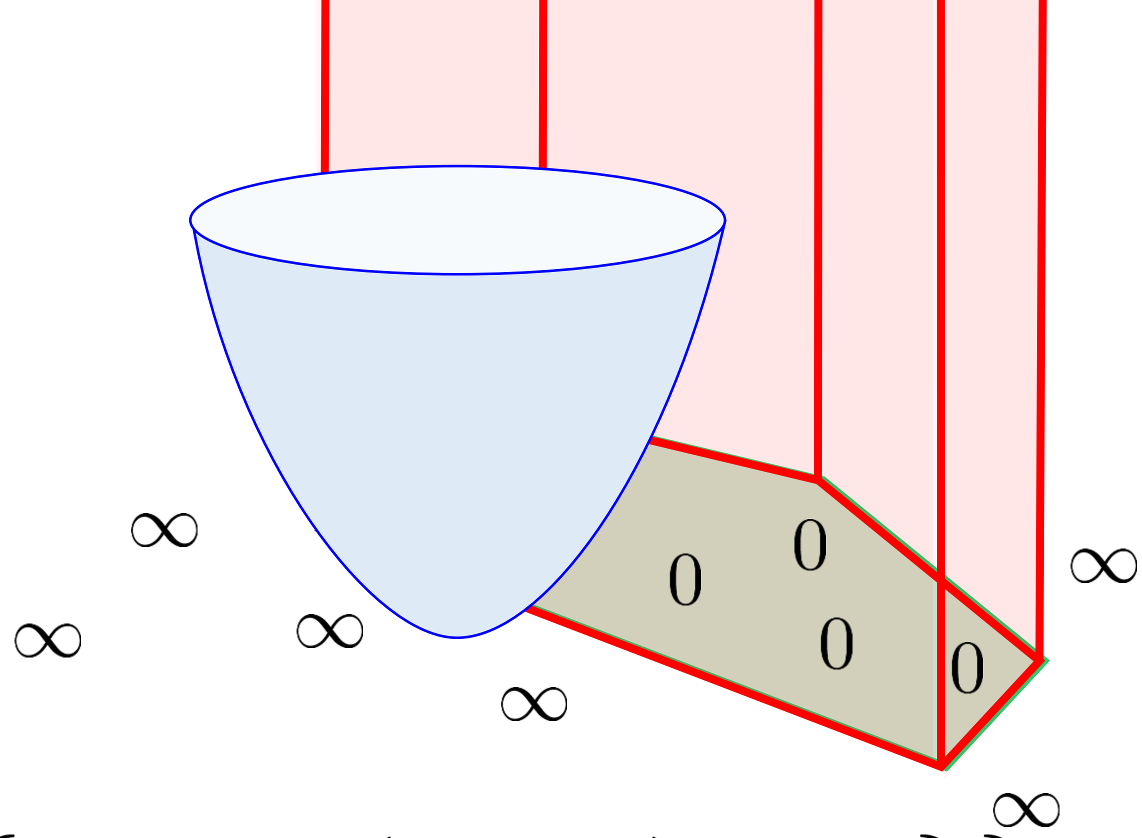
$$\mathcal{L}(\mathbf{w},\alpha) = f(\mathbf{w}) + \alpha \cdot g(\mathbf{w})$$

Lagrangian

The Lagrangian

$$\hat{\mathbf{w}}_{\mathrm{P}} = \arg\min_{\mathbf{w}\in\mathbb{R}^d} f(\mathbf{w})$$

$$\text{s.t. } g(\mathbf{w}) \leq 0$$



$$\hat{\mathbf{w}}_{\mathrm{P}} = \arg\min_{\mathbf{w}\in\mathbb{R}^d} \left\{\arg\max_{\alpha\geq 0} \left\{ \mathcal{L}(\mathbf{w},\alpha) \right\}\right\}$$

Lagrange multiplier

Primal problem

$$\mathcal{L}(\mathbf{w},\alpha) = f(\mathbf{w}) + \alpha \cdot g(\mathbf{w})$$

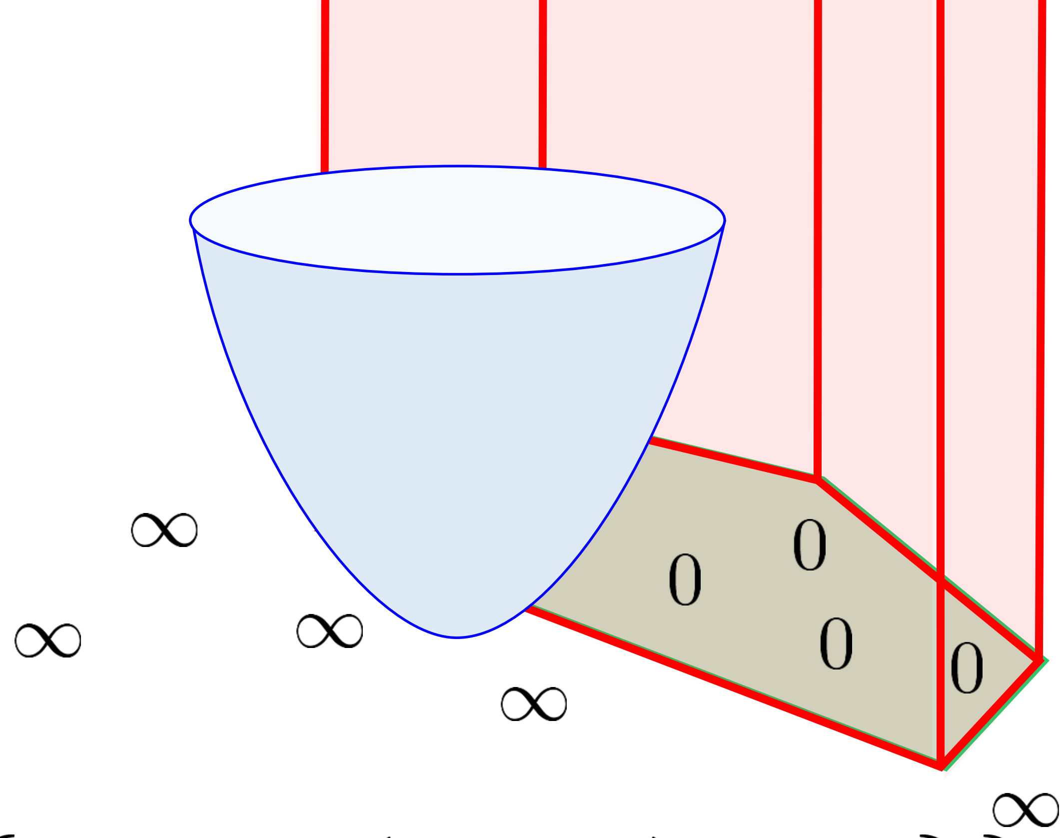
Lagrangian

The Lagrangian

$$\hat{\mathbf{w}}_P = \arg\min_{\mathbf{w}\in\mathbb{R}^d} f(\mathbf{w})$$

$$\text{s.t. } g(\mathbf{w}) \leq 0$$

Primal problem



$$\hat{\mathbf{w}}_P = \arg\min_{\mathbf{w}\in\mathbb{R}^d} \{\arg\max_{\alpha\geq 0} \{ \quad \mathcal{L}(\mathbf{w},\alpha) \quad \}\}$$

Lagrange multiplier

Primal problem

$$\mathcal{L}(\mathbf{w},\alpha) = f(\mathbf{w}) + \alpha \cdot g(\mathbf{w})$$

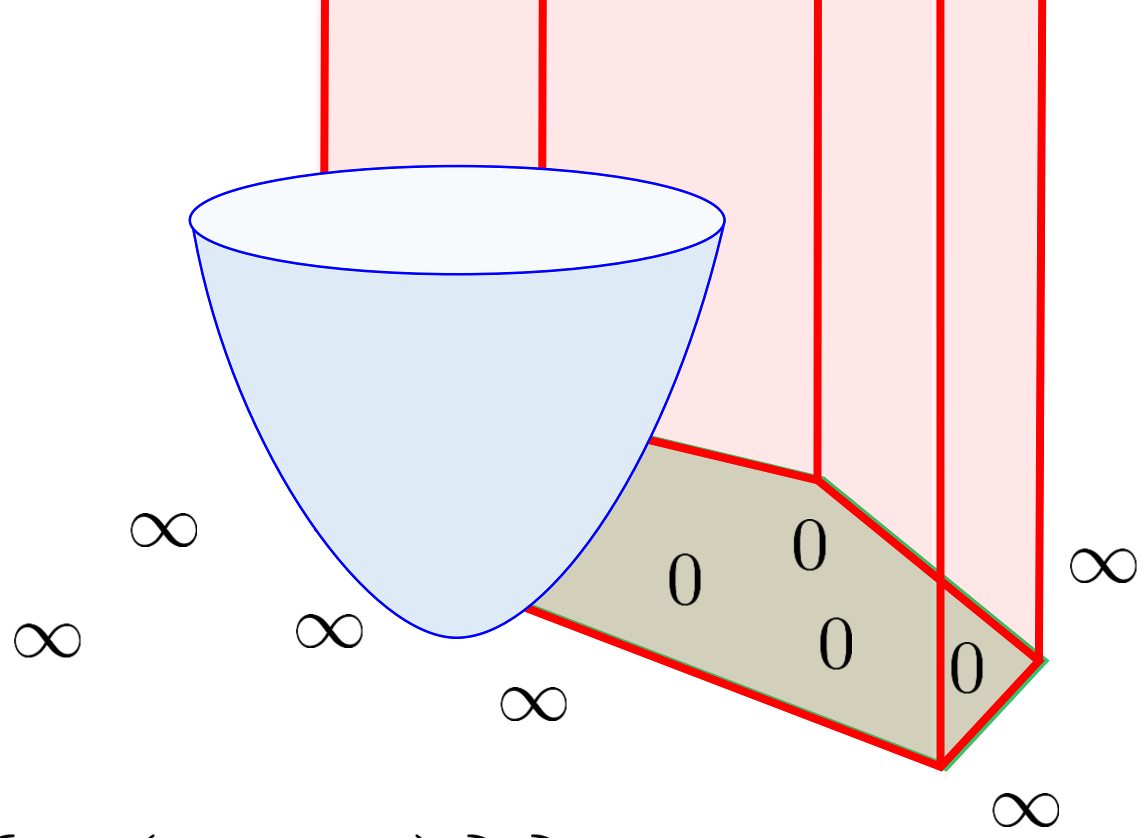
Lagrangian

The Lagrangian

$$\hat{\mathbf{w}}_P = \arg\min_{\mathbf{w}\in\mathbb{R}^d} f(\mathbf{w})$$

$$\text{s.t. } g(\mathbf{w}) \leq 0$$

Primal problem



$$\hat{\mathbf{w}}_P = \arg\min_{\mathbf{w}\in\mathbb{R}^d} \left\{\arg\max_{\alpha\geq 0} \{\mathcal{L}(\mathbf{w},\alpha)\}\right\}$$

Lagrange multiplier

Primal problem

$$\mathcal{L}(\mathbf{w},\alpha) = f(\mathbf{w}) + \alpha \cdot g(\mathbf{w})$$

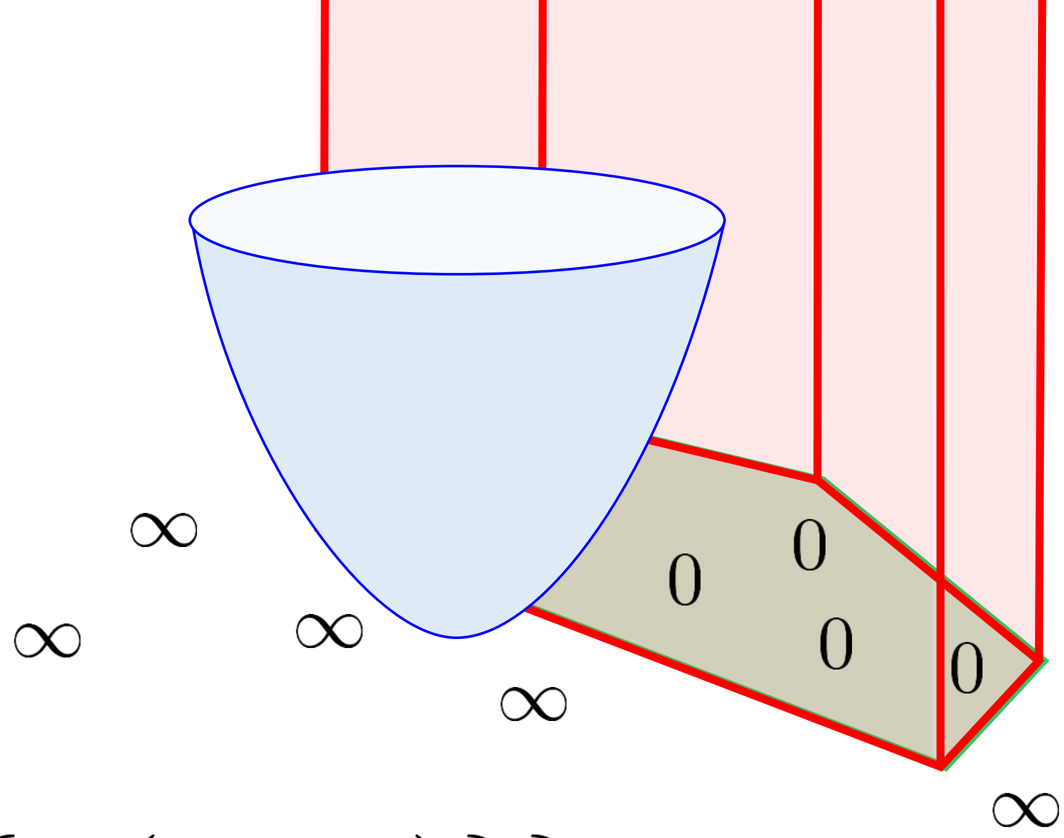
Lagrangian

The Lagrangian

$$\hat{\mathbf{w}}_{\mathrm{P}} = \arg\min_{\mathbf{w}\in\mathbb{R}^d} \; f(\mathbf{w})$$

$$\text{s.t. } g(\mathbf{w}) \leq 0$$

Primal problem



$$\hat{\mathbf{w}}_{\mathrm{P}} = \arg\min_{\mathbf{w}\in\mathbb{R}^d} \left\{\arg\max_{\alpha\geq 0} \left\{\mathcal{L}(\mathbf{w},\alpha)\right\}\right\}$$

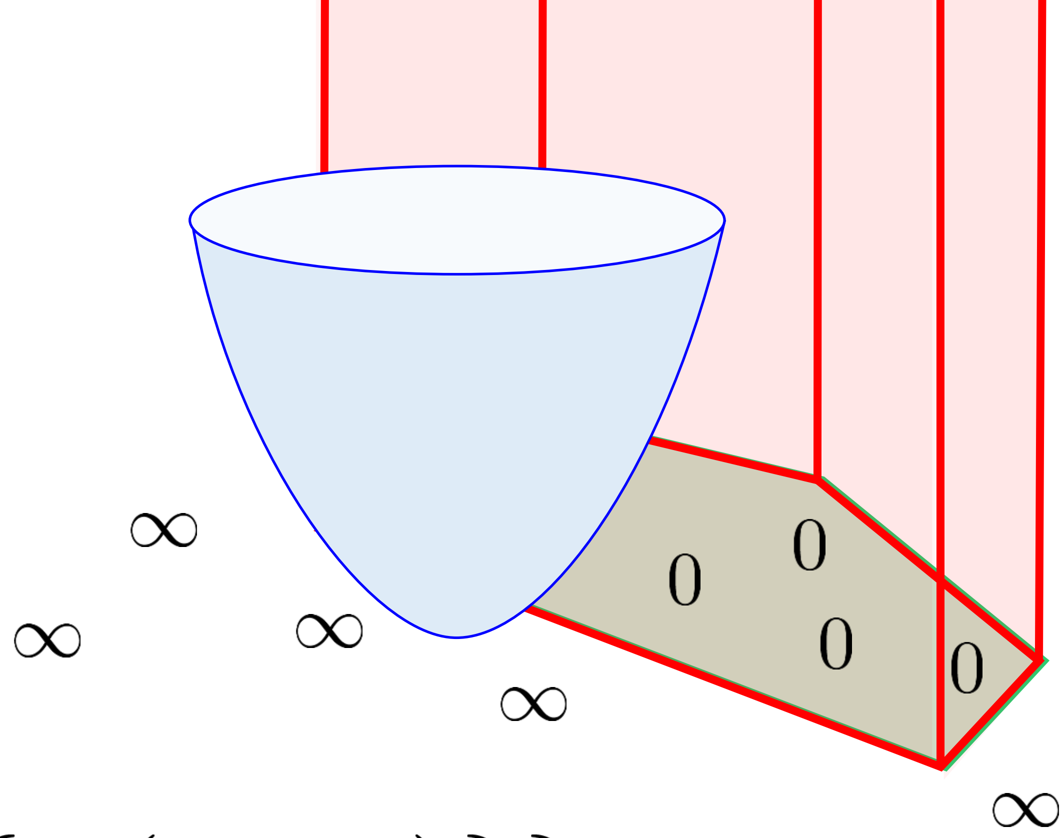
Primal problem

The Lagrangian

$$\hat{\mathbf{w}}_{\mathrm{P}} = \arg\min_{\mathbf{w}\in\mathbb{R}^d} f(\mathbf{w})$$

$$\text{s.t. } g(\mathbf{w}) \leq 0$$

Primal problem



$$\hat{\mathbf{w}}_{\mathrm{P}} = \arg\min_{\mathbf{w}\in\mathbb{R}^d} \left\{\arg\max_{\alpha\geq 0} \left\{\mathcal{L}(\mathbf{w},\alpha)\right\}\right\}$$

Primal problem

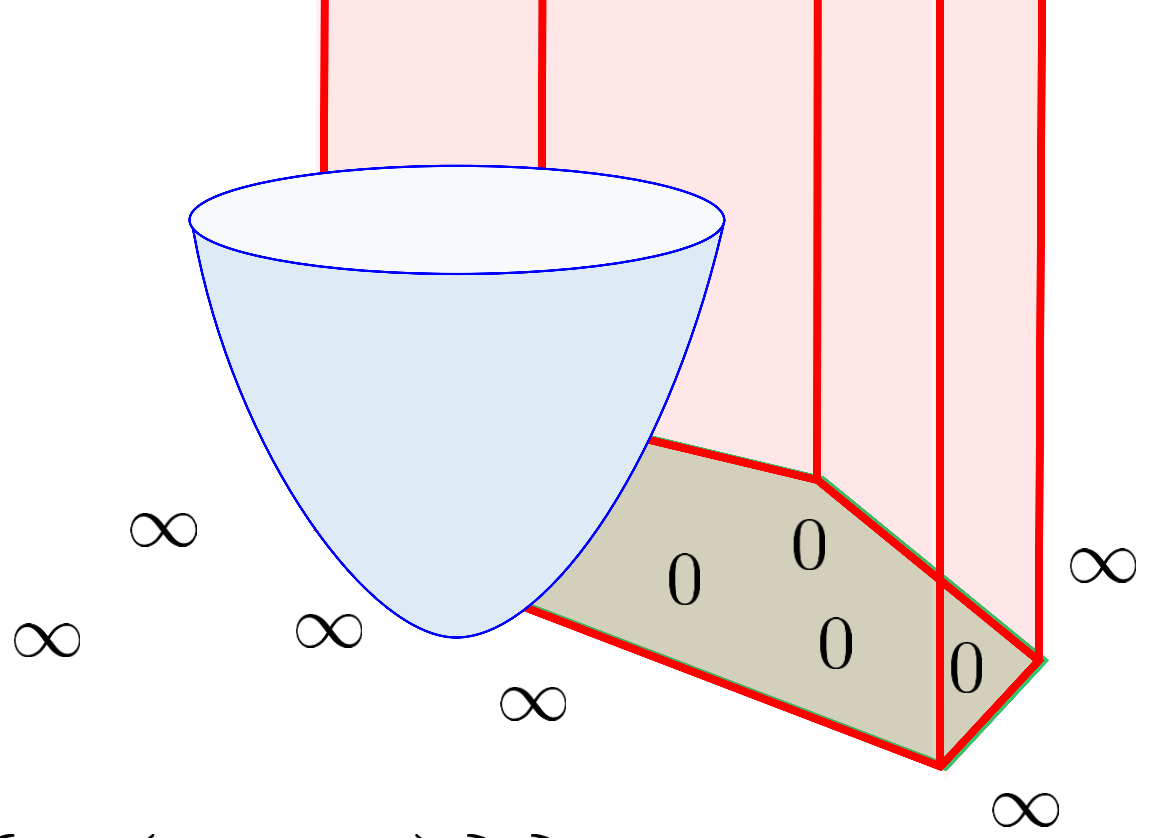
$$\hat{\mathbf{w}}_{\mathrm{D}} = \arg\max_{\alpha\geq 0} \left\{\arg\min_{\mathbf{w}\in\mathbb{R}^d} \left\{\mathcal{L}(\mathbf{w},\alpha)\right\}\right\}$$

The Lagrangian

$$\hat{\mathbf{w}}_{\mathrm{P}} = \arg\min_{\mathbf{w}\in\mathbb{R}^d} \; f(\mathbf{w})$$

$$\text{s.t. } g(\mathbf{w}) \leq 0$$

Primal problem



$$\hat{\mathbf{w}}_{\mathrm{P}} = \arg\min_{\mathbf{w}\in\mathbb{R}^d} \left\{ \arg\max_{\alpha\geq 0} \left\{ \mathcal{L}(\mathbf{w},\alpha) \right\} \right\}$$

Primal problem

$$\hat{\mathbf{w}}_{\mathrm{D}} = \arg\max_{\alpha\geq 0} \left\{ \arg\min_{\mathbf{w}\in\mathbb{R}^d} \left\{ \mathcal{L}(\mathbf{w},\alpha) \right\} \right\}$$

Dual problem

The Lagrange Dual Problem

$$\hat{\mathbf{w}}_{\mathrm{P}} = \underset{\mathbf{w}\in\mathbb{R}^d}{\arg\min}\; f(\mathbf{w})$$

$$\text{s.t. } g(\mathbf{w}) \leq 0$$

The Lagrange Dual Problem

$$\hat{\mathbf{w}}_{\mathrm{P}} = \arg\min_{\mathbf{w}\in\mathbb{R}^d} f(\mathbf{w})$$

$$\text{s.t. } g(\mathbf{w}) \leq 0$$

$$\mathcal{L}(\mathbf{w}, \alpha) = f(\mathbf{w}) + \alpha \cdot g(\mathbf{w})$$

The Lagrange Dual Problem

$$\hat{\mathbf{w}}_P = \arg\min_{\mathbf{w} \in \mathbb{R}^d} f(\mathbf{w})$$

$$\text{s.t. } g(\mathbf{w}) \leq 0$$

$$\mathcal{L}(\mathbf{w}, \alpha) = f(\mathbf{w}) + \alpha \cdot g(\mathbf{w})$$

$$\hat{\mathbf{w}}_D = \arg\max_{\alpha \geq 0} \left\{ \arg\min_{\mathbf{w} \in \mathbb{R}^d} \left\{ \mathcal{L}(\mathbf{w}, \alpha) \right\} \right\}$$

The Lagrange Dual Problem

$$\hat{\mathbf{w}}_{\mathrm{P}} = \arg\min_{\mathbf{w} \in \mathbb{R}^d} f(\mathbf{w})$$

$$\text{s.t. } g(\mathbf{w}) \leq 0$$

$$\hat{\mathbf{w}}_{\mathrm{P}} \overset{?}{=} \hat{\mathbf{w}}_{\mathrm{D}}$$

$$\mathcal{L}(\mathbf{w}, \alpha) = f(\mathbf{w}) + \alpha \cdot g(\mathbf{w})$$

$$\hat{\mathbf{w}}_{\mathrm{D}} = \arg\max_{\alpha \geq 0} \left\{ \arg\min_{\mathbf{w} \in \mathbb{R}^d} \left\{ \mathcal{L}(\mathbf{w}, \alpha) \right\} \right\}$$

The Lagrange Dual Problem

$$\hat{\mathbf{w}}_{\mathrm{P}} = \arg\min_{\mathbf{w} \in \mathbb{R}^d} f(\mathbf{w})$$

$$\text{s.t. } g(\mathbf{w}) \leq 0$$

$$\hat{\mathbf{w}}_{\mathrm{P}} \overset{?}{=} \hat{\mathbf{w}}_{\mathrm{D}}$$

$\hat{\mathbf{w}}_p = \hat{\mathbf{w}}_d$ for nice problems*

$$\mathcal{L}(\mathbf{w}, \alpha) = f(\mathbf{w}) + \alpha \cdot g(\mathbf{w})$$

$$\hat{\mathbf{w}}_{\mathrm{D}} = \arg\max_{\alpha \geq 0} \left\{ \arg\min_{\mathbf{w} \in \mathbb{R}^d} \left\{ \mathcal{L}(\mathbf{w}, \alpha) \right\} \right\}$$

The Lagrange Dual Problem

$$\hat{\mathbf{w}}_\mathrm{P} = \arg\min_{\mathbf{w}\in\mathbb{R}^d} f(\mathbf{w})$$

$$\text{s.t. } g(\mathbf{w}) \leq 0$$

$$\hat{\mathbf{w}}_\mathrm{P} \overset{?}{=} \hat{\mathbf{w}}_\mathrm{D}$$

$\hat{\mathbf{w}}_p = \hat{\mathbf{w}}_d$ for nice problems*

$$\mathcal{L}(\mathbf{w}, \alpha) = f(\mathbf{w}) + \alpha \cdot g(\mathbf{w})$$

E.g. if $f(\cdot)$ is convex and $\{\mathbf{w} : g(\mathbf{w}) \leq 0\}$ is convex

$$\hat{\mathbf{w}}_\mathrm{D} = \arg\max_{\alpha\geq 0} \{\arg\min_{\mathbf{w}\in\mathbb{R}^d} \{\mathcal{L}(\mathbf{w}, \alpha)\}\}$$

The Lagrange Dual Problem

$$\hat{\mathbf{w}}_{\mathrm{P}} = \arg\min_{\mathbf{w}\in\mathbb{R}^d} f(\mathbf{w})$$

$$\text{s.t. } g(\mathbf{w}) \leq 0$$

$$\hat{\mathbf{w}}_{\mathrm{P}} \stackrel{?}{=} \hat{\mathbf{w}}_{\mathrm{D}}$$

$\hat{\mathbf{w}}_p = \hat{\mathbf{w}}_d$ for nice problems*

E.g. if $f(\cdot)$ is convex and $\{\mathbf{w} : g(\mathbf{w}) \leq 0\}$ is convex

$$\mathcal{L}(\mathbf{w}, \alpha) = f(\mathbf{w}) + \alpha \cdot g(\mathbf{w})$$

$$\hat{\mathbf{w}}_{\mathrm{D}} = \arg\max_{\alpha\geq 0} \left\{\arg\min_{\mathbf{w}\in\mathbb{R}^d} \{\mathcal{L}(\mathbf{w}, \alpha)\}\right\}$$

$$\alpha_{\mathrm{D}} \cdot g(\hat{\mathbf{w}}_{\mathrm{D}}) = 0$$

The Lagrange Dual Problem

$$\hat{\mathbf{w}}_{\mathrm{P}} = \arg\min_{\mathbf{w}\in\mathbb{R}^d} f(\mathbf{w})$$

$$\text{s.t. } g(\mathbf{w}) \leq 0$$

$$\mathcal{L}(\mathbf{w}, \alpha) = f(\mathbf{w}) + \alpha \cdot g(\mathbf{w})$$

$$\hat{\mathbf{w}}_{\mathrm{D}} = \arg\max_{\alpha\geq 0} \left\{\arg\min_{\mathbf{w}\in\mathbb{R}^d} \left\{\mathcal{L}(\mathbf{w}, \alpha)\right\}\right\}$$

$$\alpha_{\mathrm{D}} \cdot g(\hat{\mathbf{w}}_{\mathrm{D}}) = 0$$

$$\hat{\mathbf{w}}_{\mathrm{P}} \stackrel{?}{=} \hat{\mathbf{w}}_{\mathrm{D}}$$

$\hat{\mathbf{w}}_p = \hat{\mathbf{w}}_d$ for nice problems*

E.g. if $f(\cdot)$ is convex and $\{\mathbf{w} : g(\mathbf{w}) \leq 0\}$ is convex

The Lagrange Dual Problem

$$\hat{\mathbf{w}}_{\mathrm{P}} = \arg\min_{\mathbf{w}\in\mathbb{R}^d} f(\mathbf{w})$$

$$\text{s.t. } g(\mathbf{w}) \leq 0$$

$$\hat{\mathbf{w}}_{\mathrm{P}} \overset{?}{=} \hat{\mathbf{w}}_{\mathrm{D}}$$

$\hat{\mathbf{w}}_p = \hat{\mathbf{w}}_d$ for nice problems*

E.g. if $f(\cdot)$ is convex and $\{\mathbf{w} : g(\mathbf{w}) \leq 0\}$ is convex

$$\mathcal{L}(\mathbf{w}, \alpha) = f(\mathbf{w}) + \alpha \cdot g(\mathbf{w})$$

$$\hat{\mathbf{w}}_{\mathrm{D}} = \arg\max_{\alpha\geq 0} \left\{\arg\min_{\mathbf{w}\in\mathbb{R}^d} \left\{\mathcal{L}(\mathbf{w}, \alpha)\right\}\right\}$$

The maximizer

$$\alpha_{\mathrm{D}} \cdot g(\hat{\mathbf{w}}_{\mathrm{D}}) = 0$$

The Lagrange Dual Problem

$$\hat{\mathbf{w}}_{\mathrm{P}} = \arg\min_{\mathbf{w} \in \mathbb{R}^d} f(\mathbf{w})$$

$$\text{s.t. } g(\mathbf{w}) \leq 0$$

$$\hat{\mathbf{w}}_{\mathrm{P}} \stackrel{?}{=} \hat{\mathbf{w}}_{\mathrm{D}}$$

$\hat{\mathbf{w}}_p = \hat{\mathbf{w}}_d$ for nice problems*

E.g. if $f(\cdot)$ is convex and $\{\mathbf{w} : g(\mathbf{w}) \leq 0\}$ is convex

$$\mathcal{L}(\mathbf{w}, \alpha) = f(\mathbf{w}) + \alpha \cdot g(\mathbf{w})$$

$$\hat{\mathbf{w}}_{\mathrm{D}} = \arg\max_{\alpha \geq 0} \left\{ \arg\min_{\mathbf{w} \in \mathbb{R}^d} \left\{ \mathcal{L}(\mathbf{w}, \alpha) \right\} \right\}$$

The maximizer

$$\alpha_{\mathrm{D}} \cdot g(\hat{\mathbf{w}}_{\mathrm{D}}) = 0$$

Complimentary slackness KKT Condition

Please give your Feedback

http://tinyurl.com/ml17-18afb