CS201: MATHEMATICS FOR COMPUTER SCIENCE - I
NITIN SAXENA

ASSIGNMENT 4

POINTS: 35

DATE GIVEN: 30-SEP-2016 DUE: 07-OCT-2016(6PM)

<u>Rules</u>:

- You are strongly encouraged to work *independently*.
- Write the solutions on your own and honorably *acknowledge* the sources if any.
  `http://cse.iitk.ac.in/pages/AntiCheatingPolicy.html`
- The "0 point" questions are optional.
- Submit your solutions, before time, to your TAs as per the roll numbers: Amit Sinhababu (12000–150130), Pranav Bisht (150131–150365), Ashish Dwivedi (150366–150600), Pulkit Kariryaa (150601–150840).

Let K_n be the complete graph on $n \in \mathbb{N}_{>1}$ vertices.

Question 1: *[2+2+3 points]* How many subgraphs does K_n have?
When will K_n have an Eulerian circuit?
What are the eigenvalues of the adjacency matrix of K_n?

Let G be a graph, A be its adjacency matrix, and d be its maximum degree.

Question 2: *[4+4 points]* Show that the maximum eigenvalue of A is at most d.
Show that the minimum eigenvalue of A is at least $-d$.

Question 3: *[5 points]* Show that the number of walks of length m between vertex i and vertex j is the (i,j)-th entry of A^m.

Question 4: *[4+4 points]* Define *graph product* of G_1 and G_2 as the graph $G_1 \hat{\otimes} G_2$ with vertex set $V_1 \times V_2$ as:

$((u,i),(v,j))$ is an edge in $G_1 \hat{\otimes} G_2$ if,

$$(u = v \text{ and } (i,j) \in E(G_2)) \text{ or } ((u,v) \in E(G_1) \text{ and } i = j).$$

Show that $\chi(G) \leq t$ iff $\alpha(G \hat{\otimes} K_t) = |V(G)|$.

(Note: $\chi(\cdot)$ is the chromatic number and $\alpha(\cdot)$ is the stability number.)

Question 5: *[2+5 points]* What is the chromatic number of a bipartite graph?

Show that a regular bipartite graph has a *perfect* matching.

Question 6: *[0 points]* How many non-isomorphic graphs are there of degree 3 with 7 vertices?

Question 7: *[0 points]* Find all graphs on 4 vertices which are isomorphic to their complement.

Question 8: *[0 points]* Let $K_{m,n}$ be the complete bipartite graph. When is there a Hamiltonian circuit?

Question 9: *[0 points]* Show that for a connected planar graph: $|E| \leq 3|V| - 6$.

Question 10: *[0 points]* Extend the ideas of alternating path and Hall's theorem to *general* graphs.

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