Function Approximation Methods-II

CS771: Introduction to Machine Learning

Purushottam Kar

Recap

Recap

$$\hat{\mathbf{w}} = \arg\min \sum_{i=1}^{n} \left(y^i - \langle \mathbf{w}, \mathbf{x}^i \rangle\right)^2 + \lambda \cdot \sum_{i=1}^{d} \mathbf{w}_i \log \mathbf{w}_i$$

$$\text{s.t.} \mathbf{w}_i \geq 0$$

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Loss Function

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Regularizer

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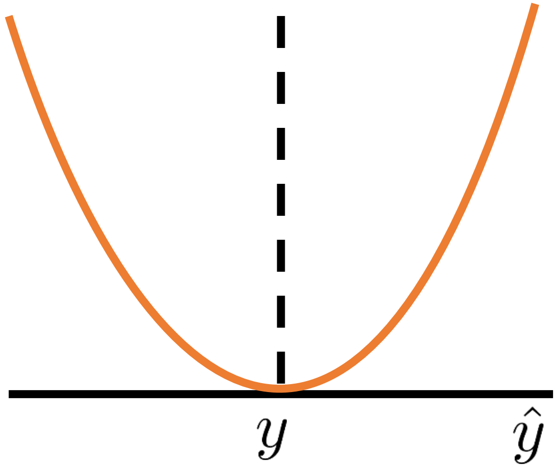
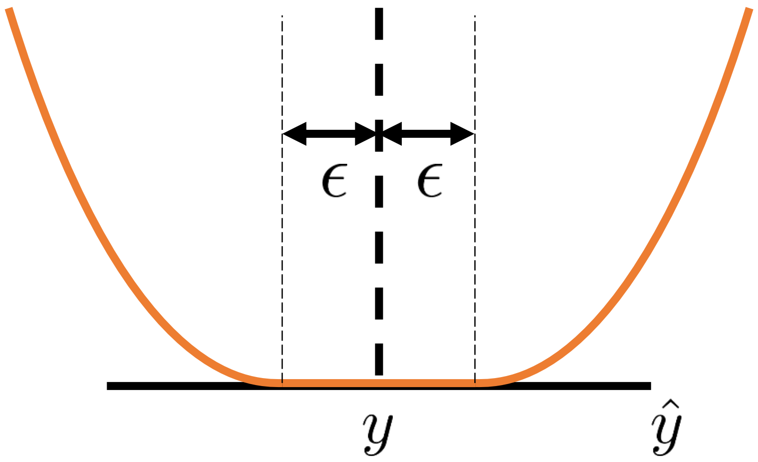
Constraint

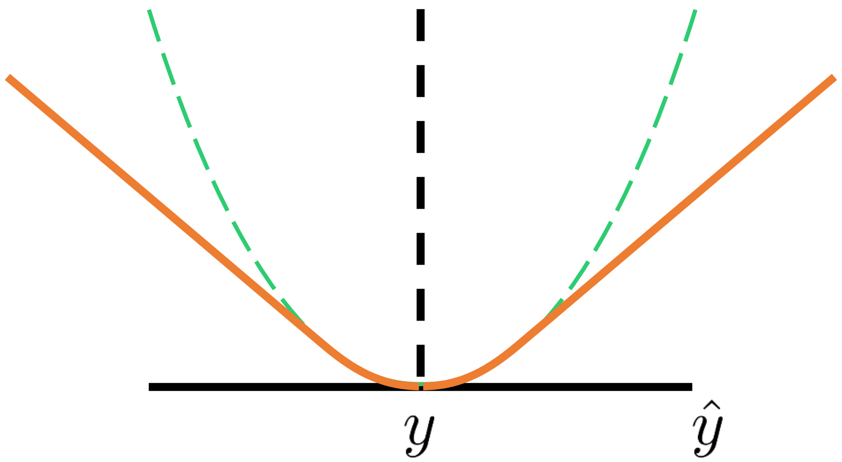
Recap



Square loss



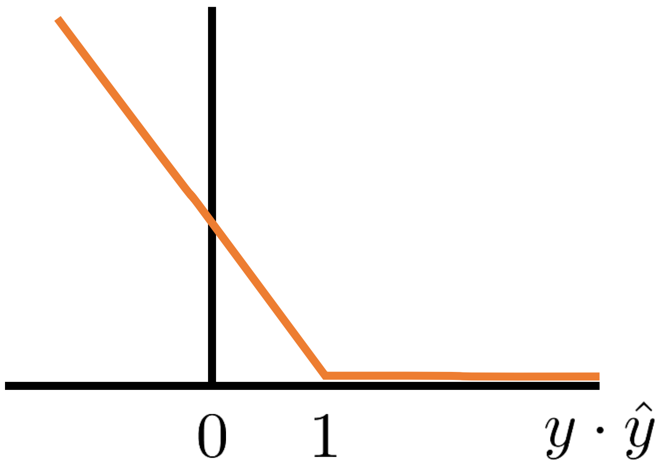
Vapnik's ϵ-insensitive loss



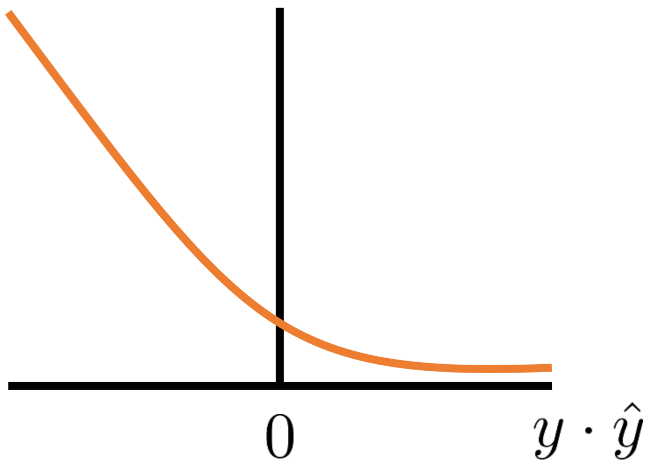
Huber loss

$$\hat{\mathbf{w}} = \arg\min \sum_{i=1}^{n} \ell(y^i, \langle \mathbf{w}, \mathbf{x}^i \rangle)$$

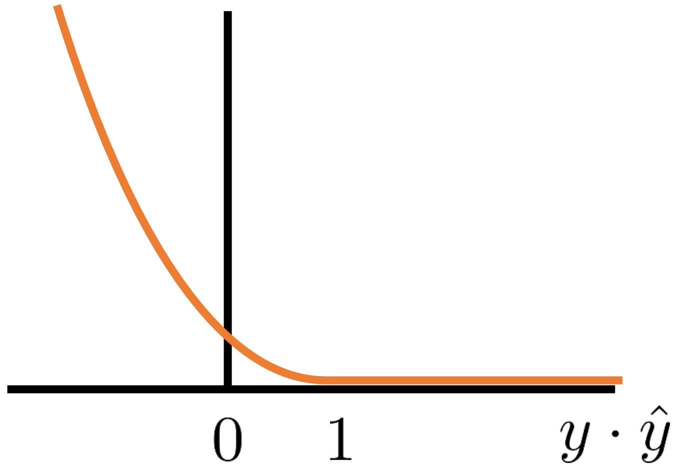
Recap



Hinge loss



Logistic loss



Squared Hinge loss

$$\hat{\mathbf{w}} = \arg\min \sum_{i=1}^{n} \ell(y^i, \langle \mathbf{w}, \mathbf{x}^i \rangle)$$

Recap: Looking into the Hinge Loss

$\boxed{\text{Binary Classification}}$

$$\hat{y}^i = \text{sign}(\langle \mathbf{w}, \mathbf{x}^i \rangle)$$

Recap: Looking into the Hinge Los

Can use non-linear functions too!

Binary Classification

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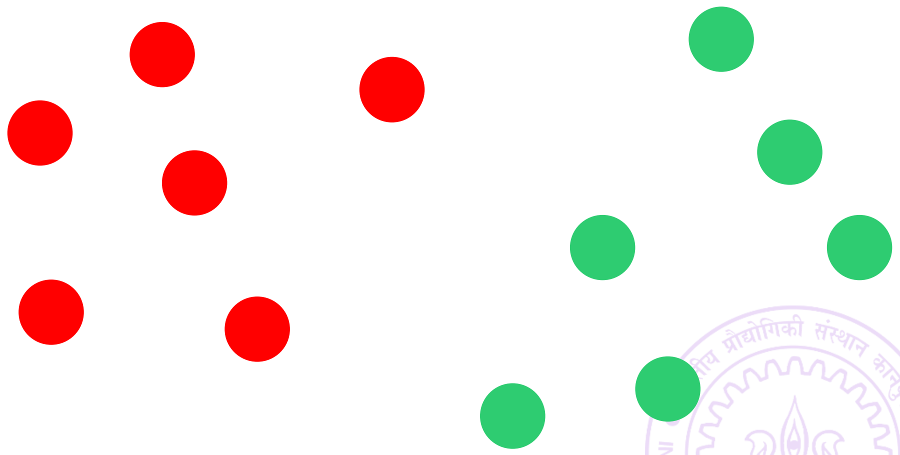

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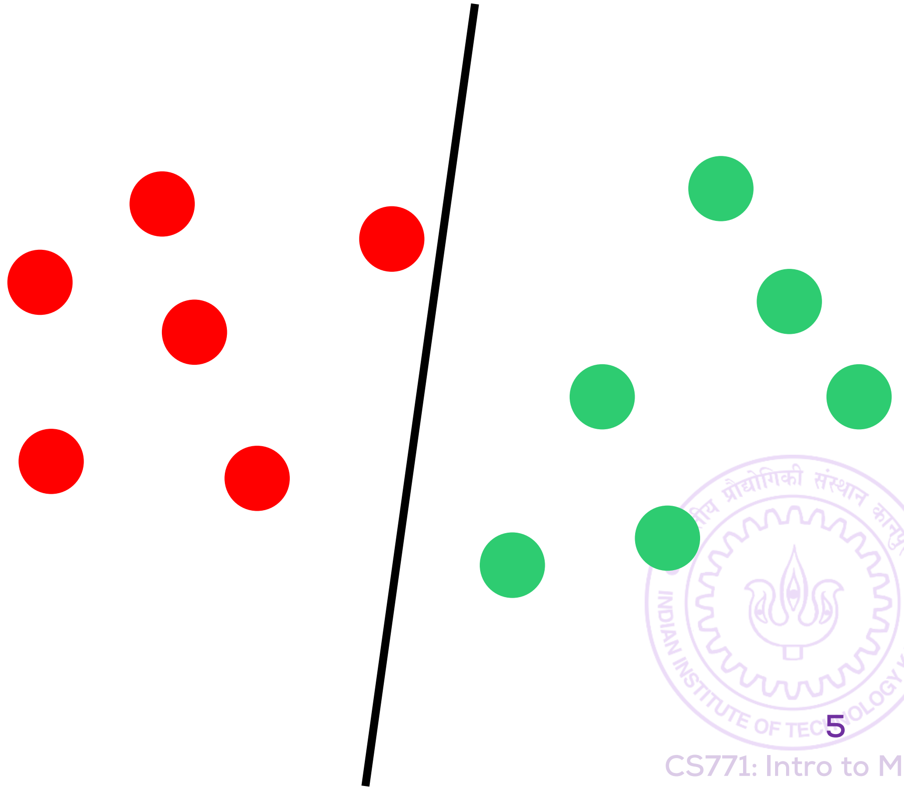

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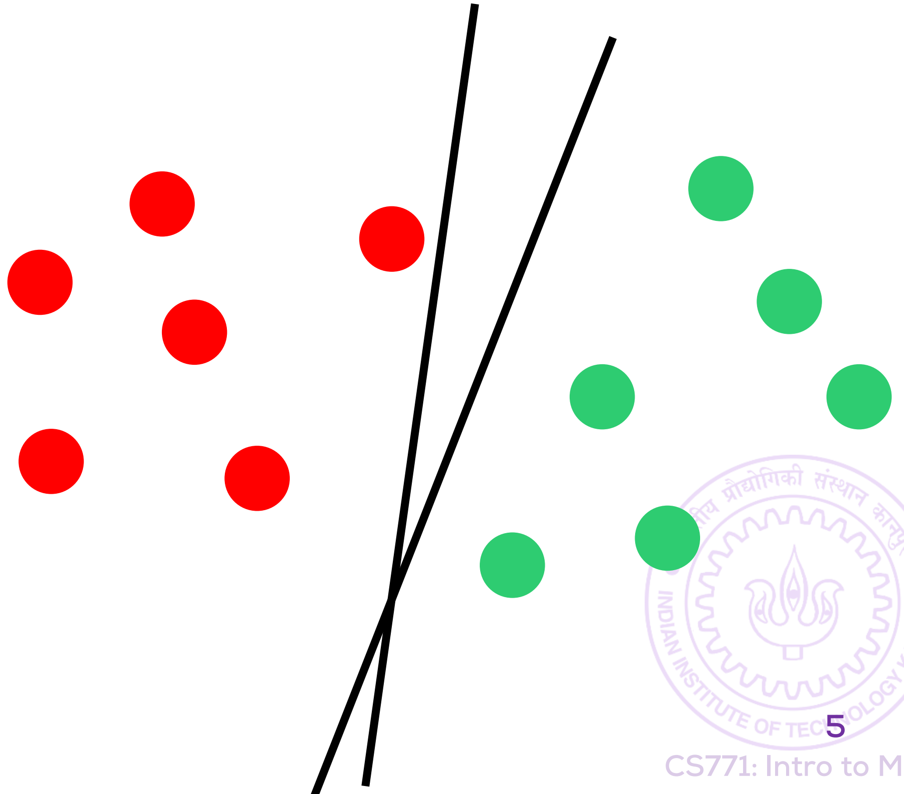

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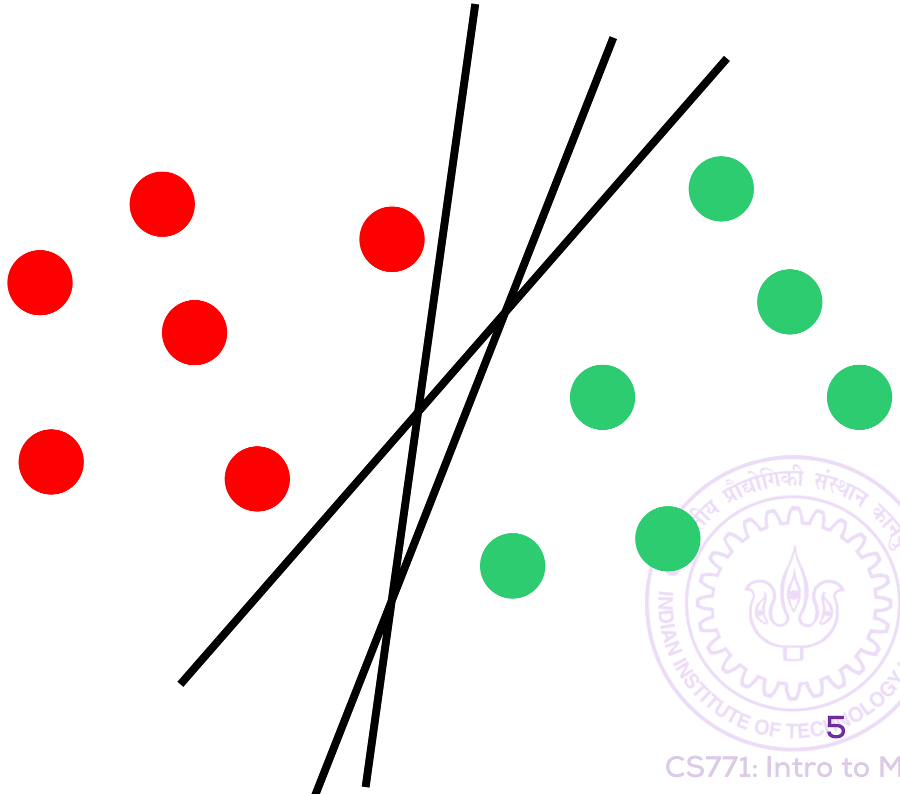

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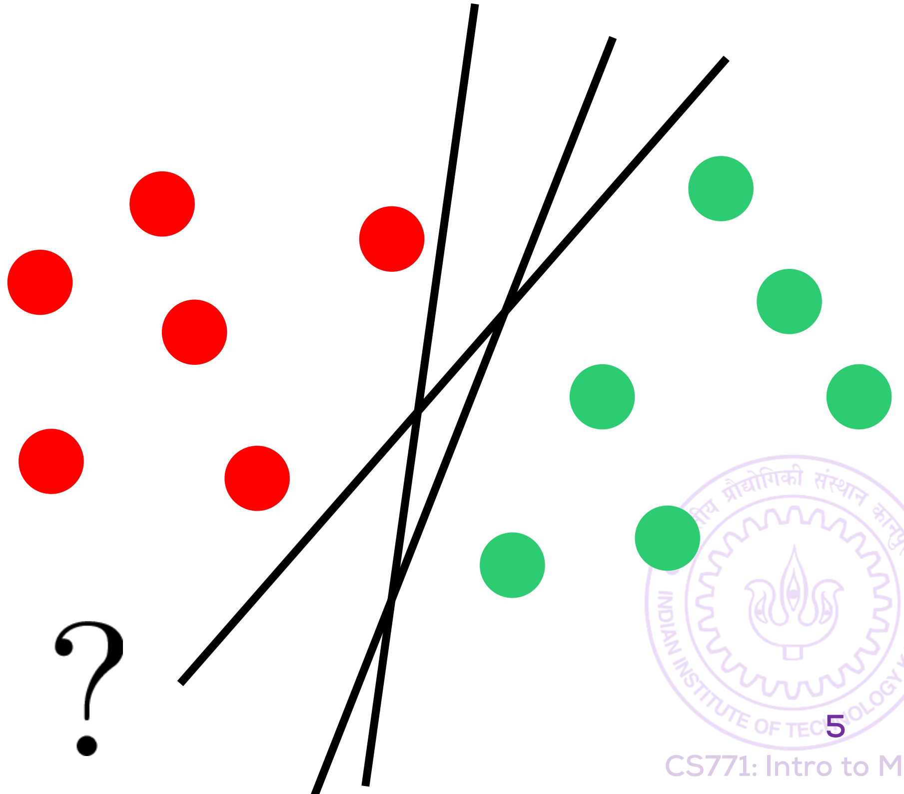

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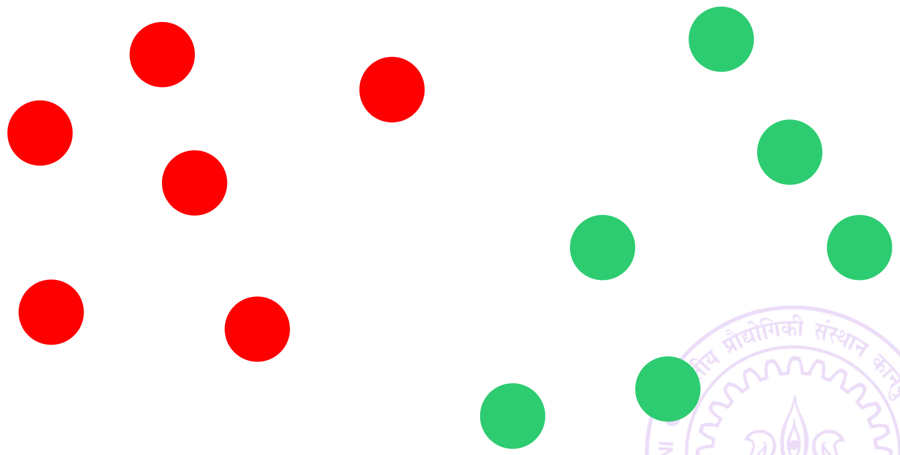

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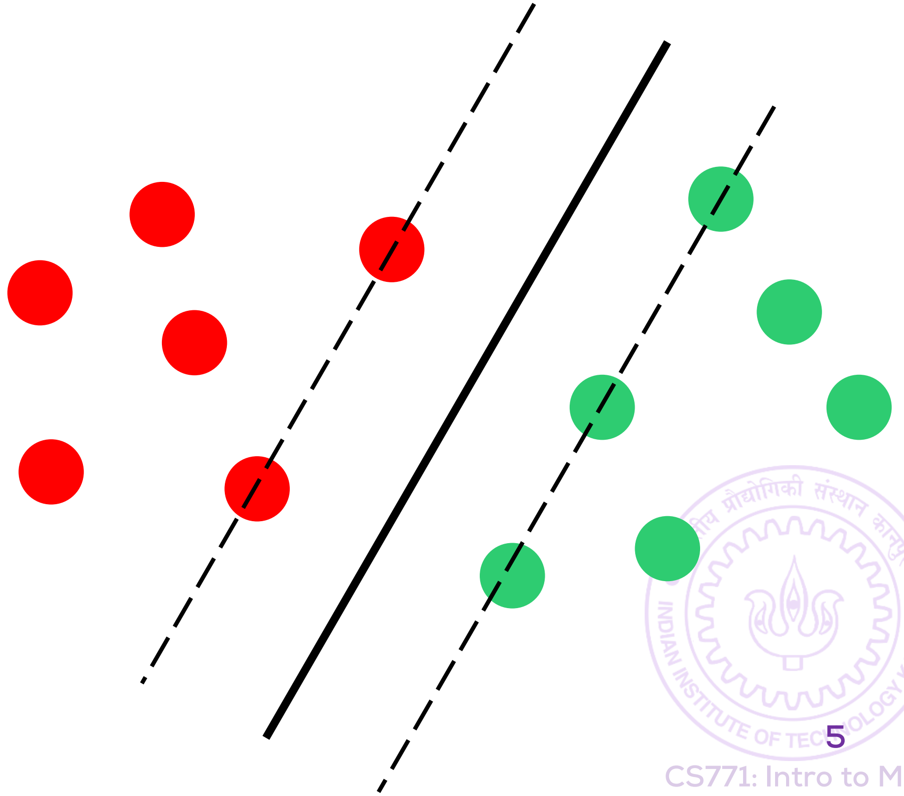

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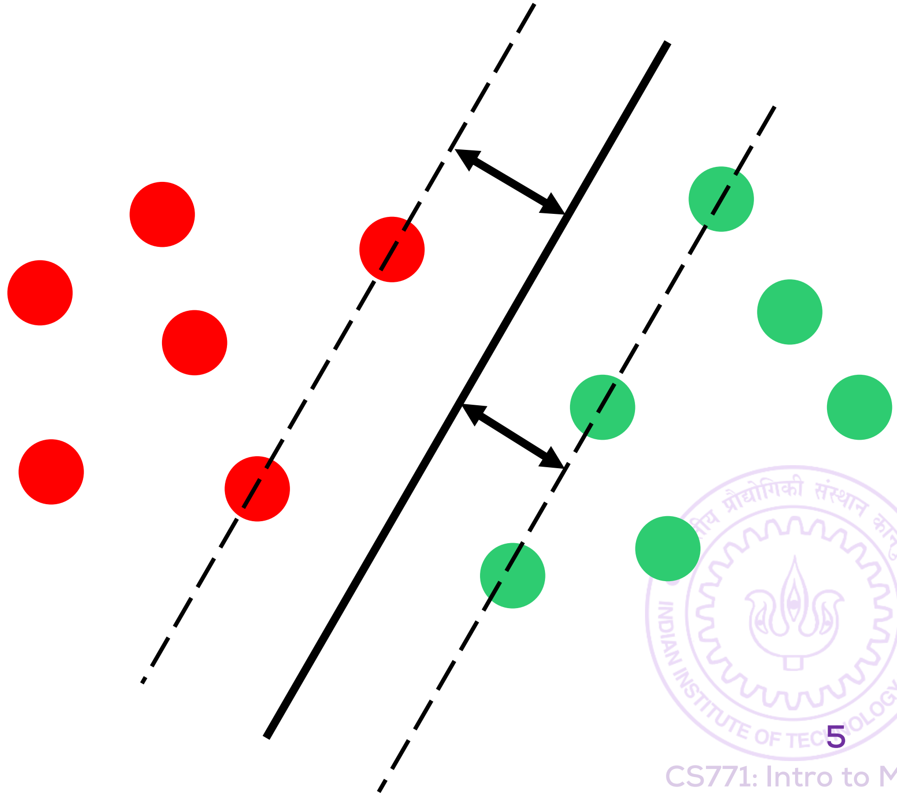

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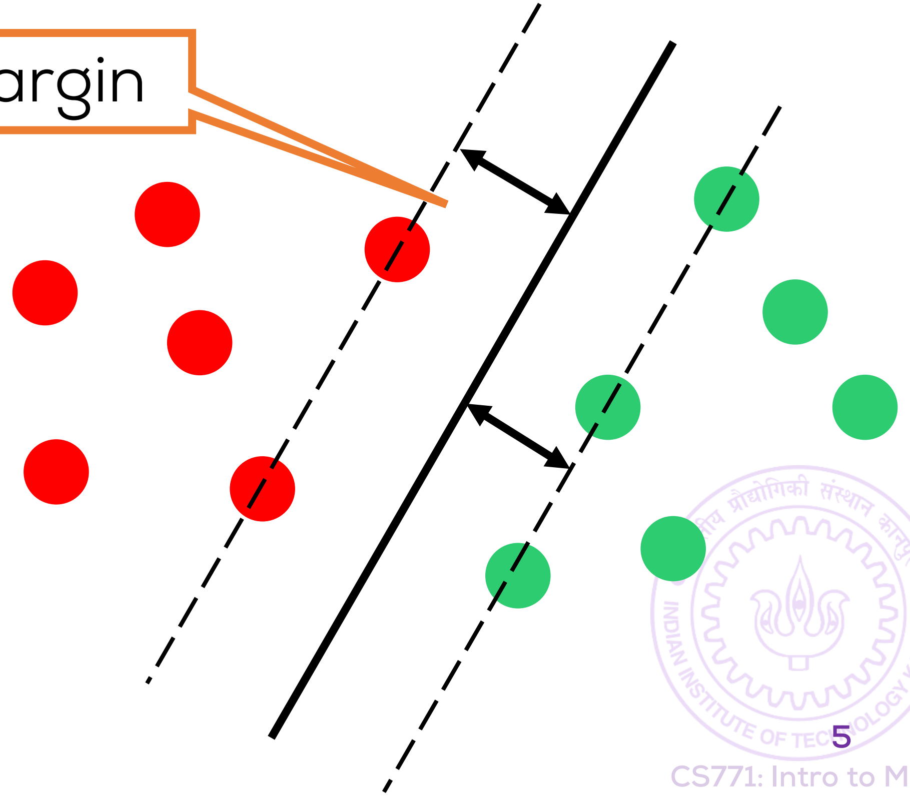
Margin



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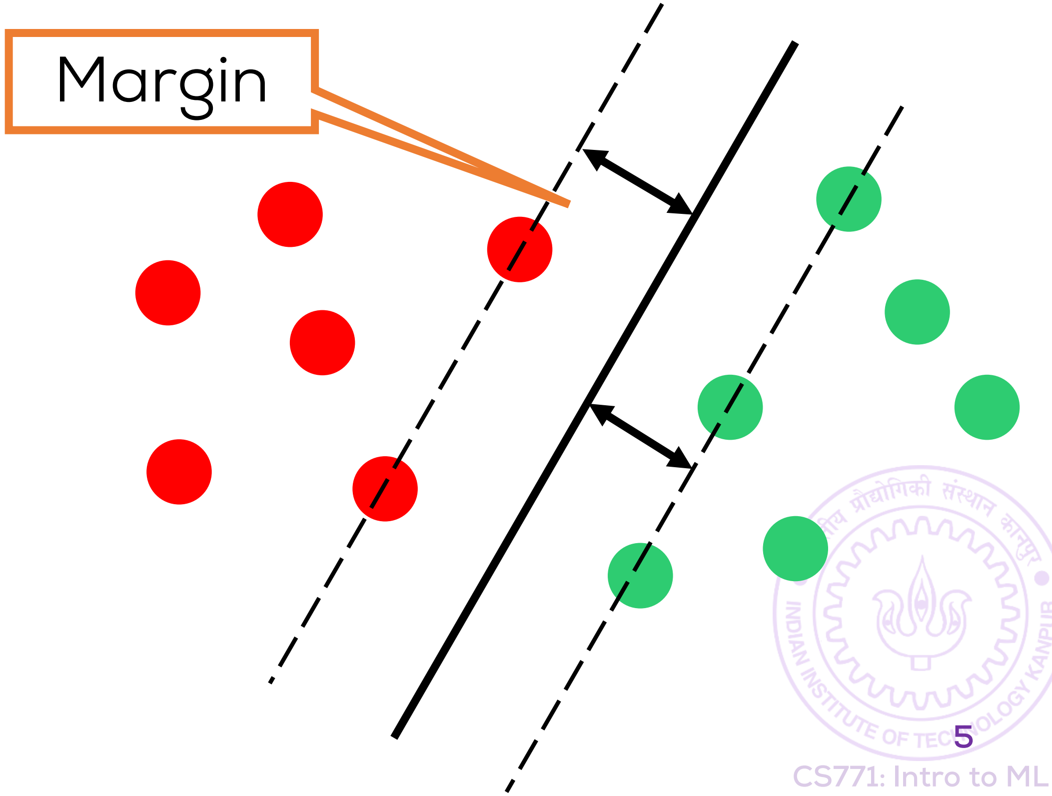

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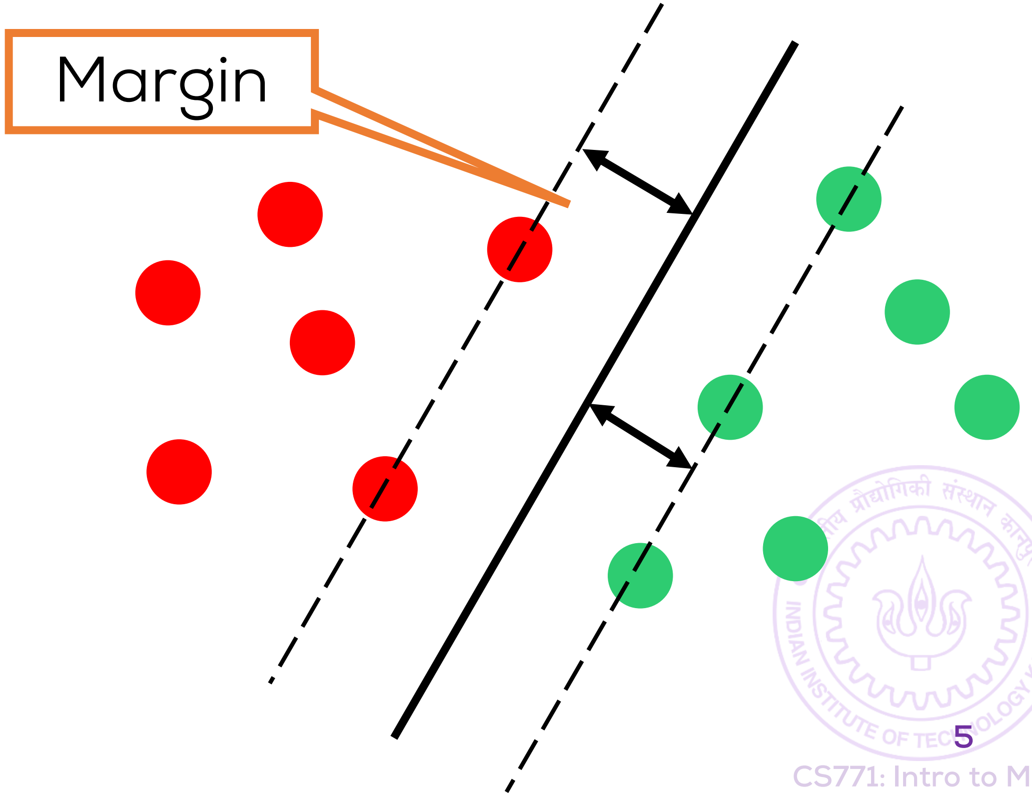

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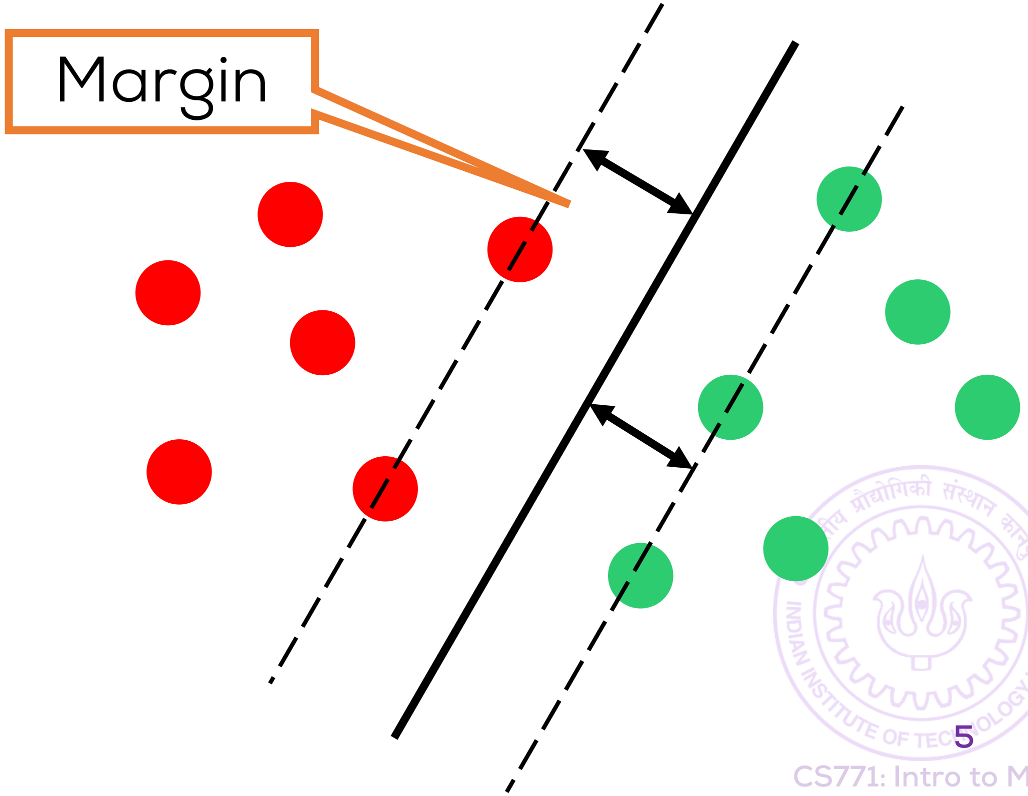
Margin

Why 1?



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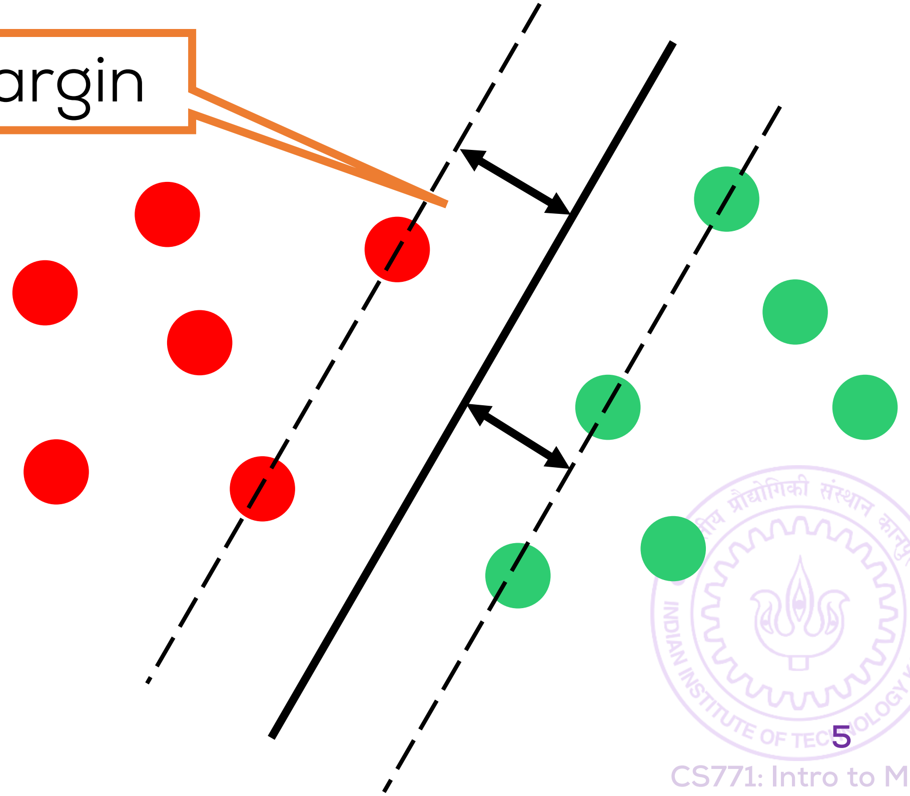
Regularization parameter

Margin

Why 1?

Margin



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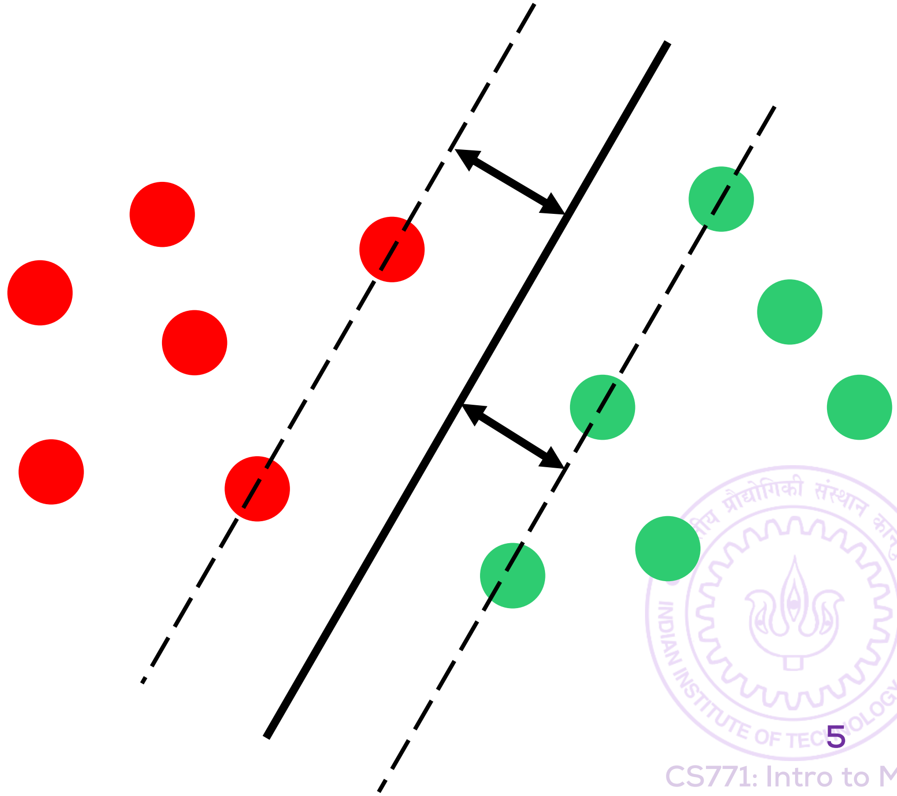

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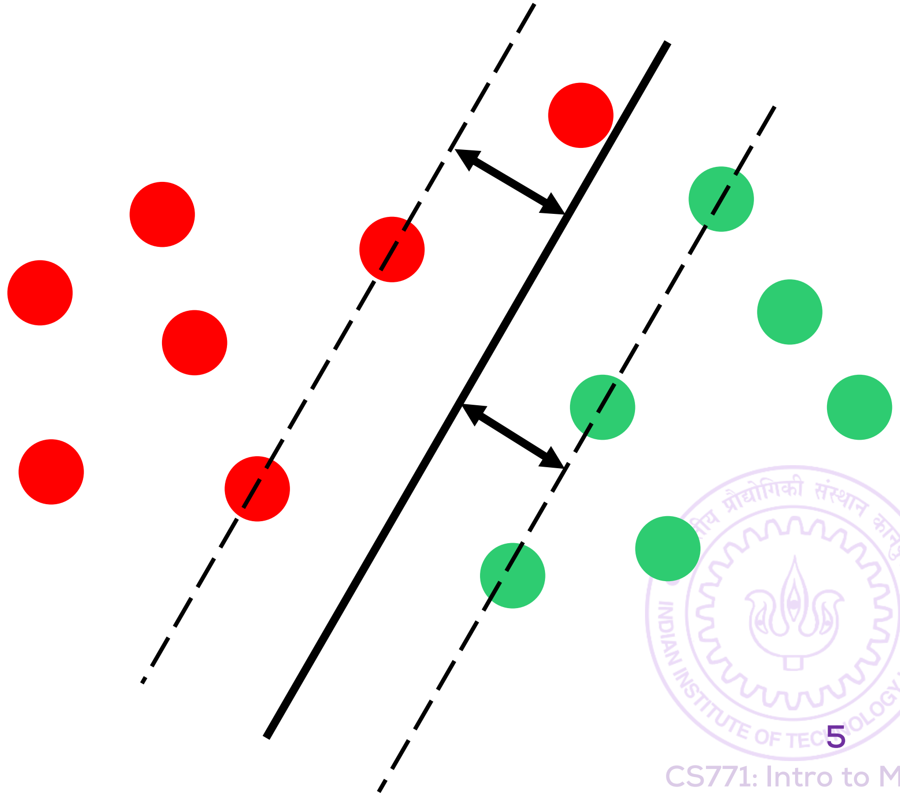

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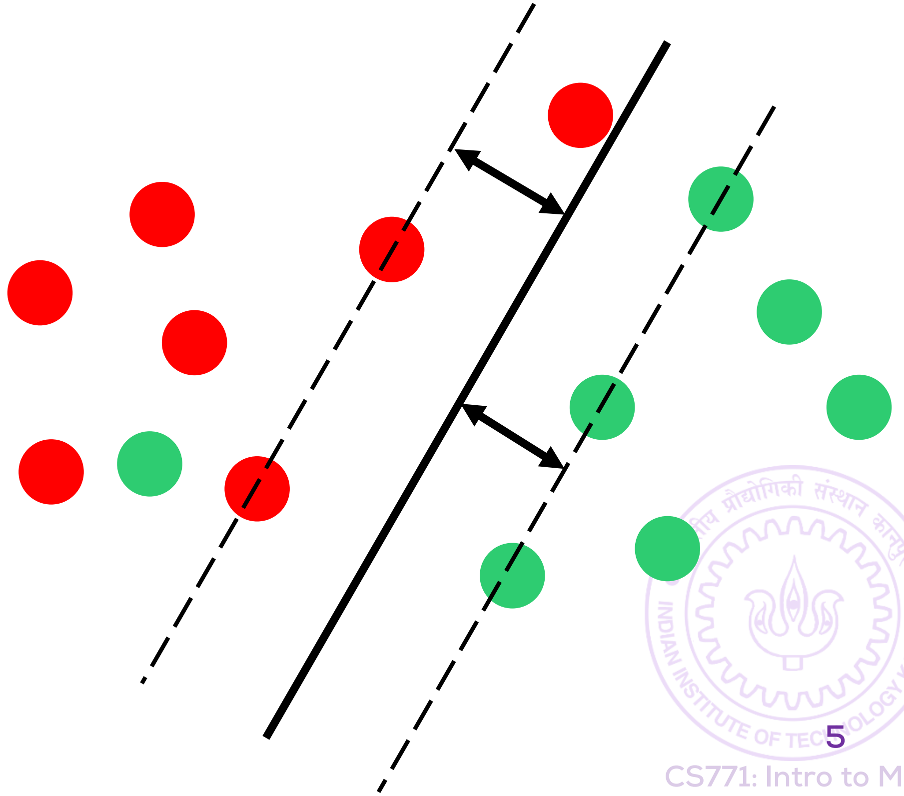

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Slack variable

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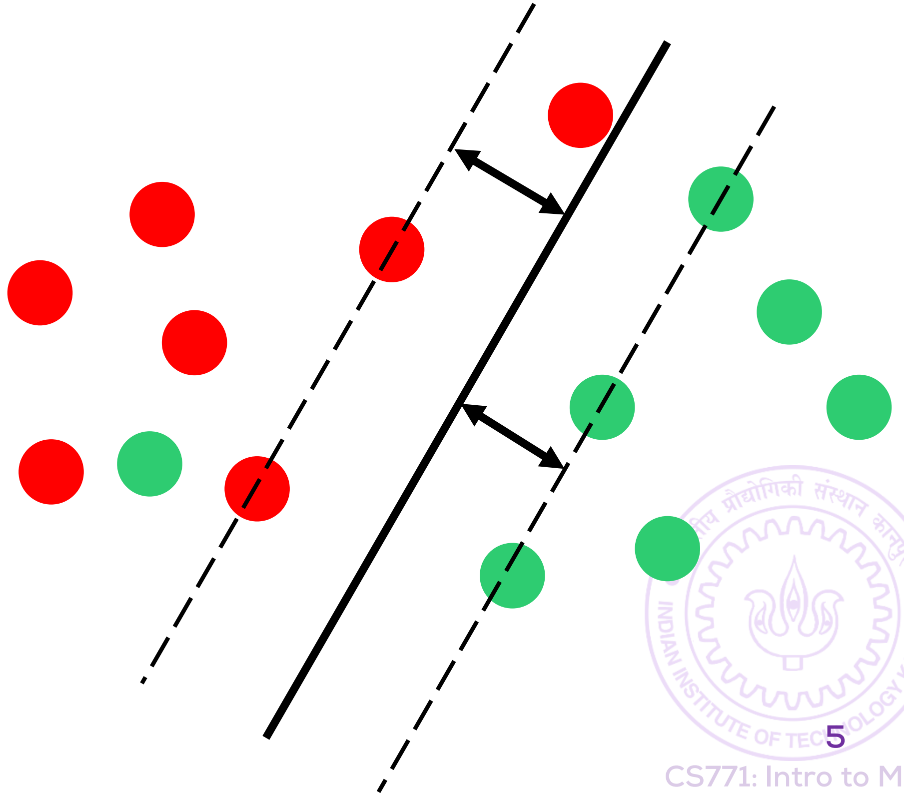
$$\text{s.t. } y^i \langle \mathbf{w}, \mathbf{x}^i \rangle \geq 1 - \xi_i$$



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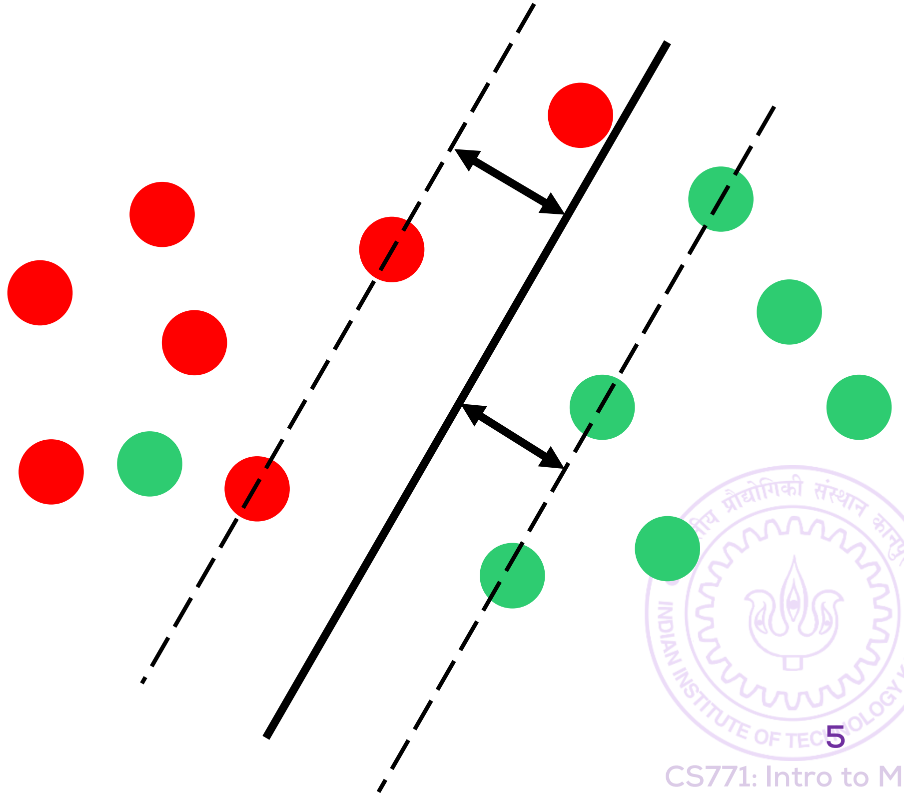
$$\xi_i \geq 0$$

More params to optimize



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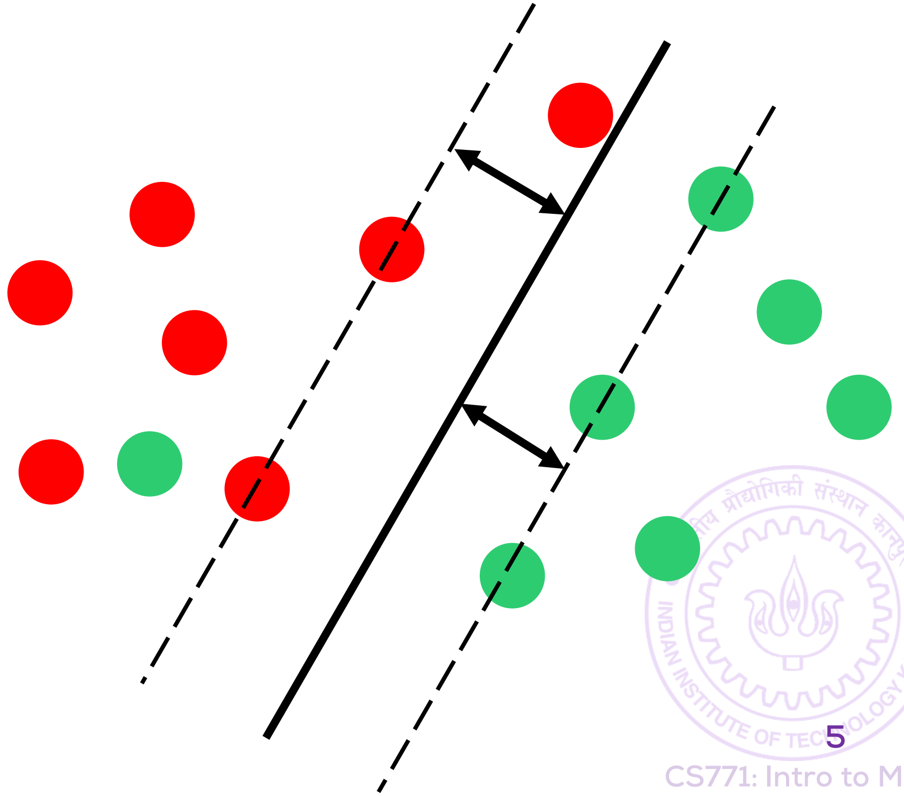

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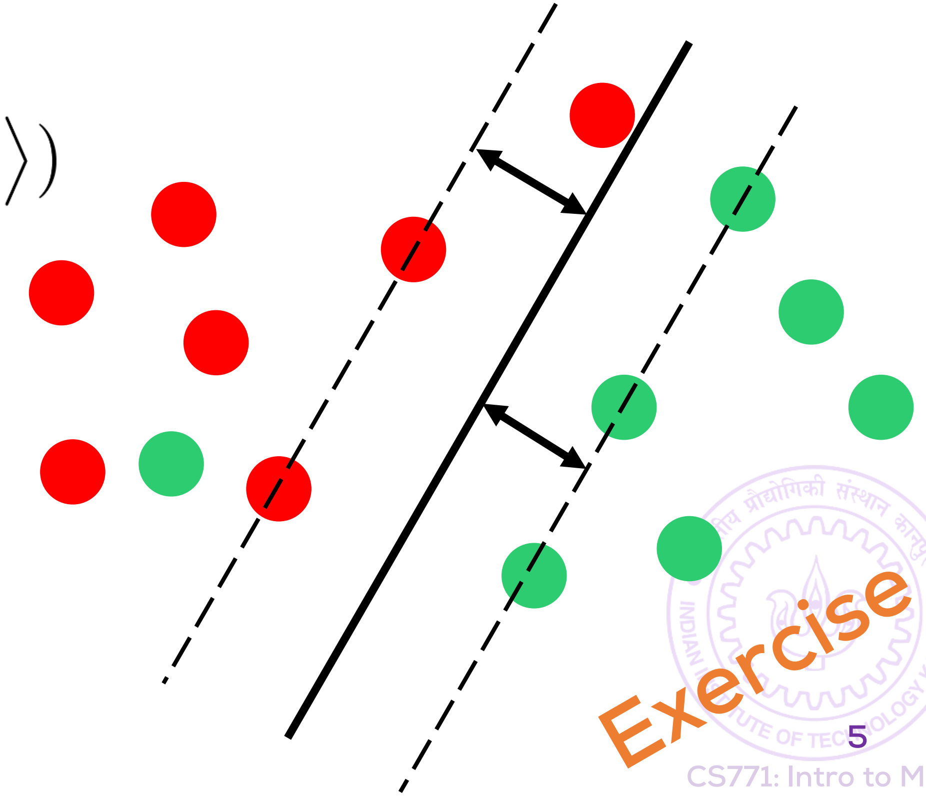

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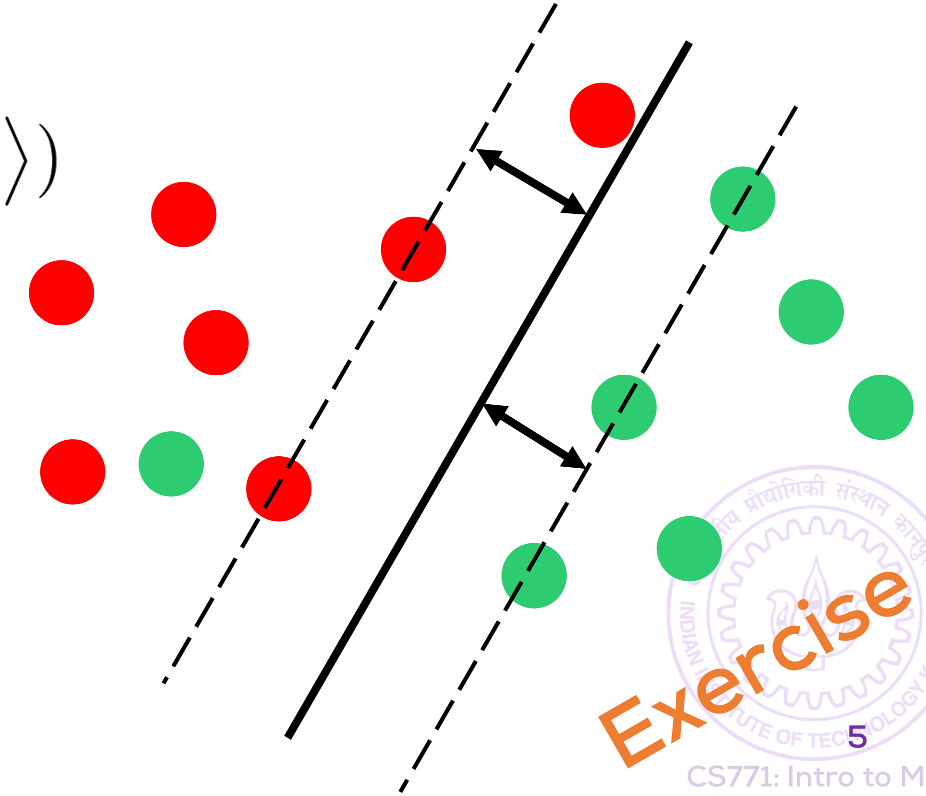

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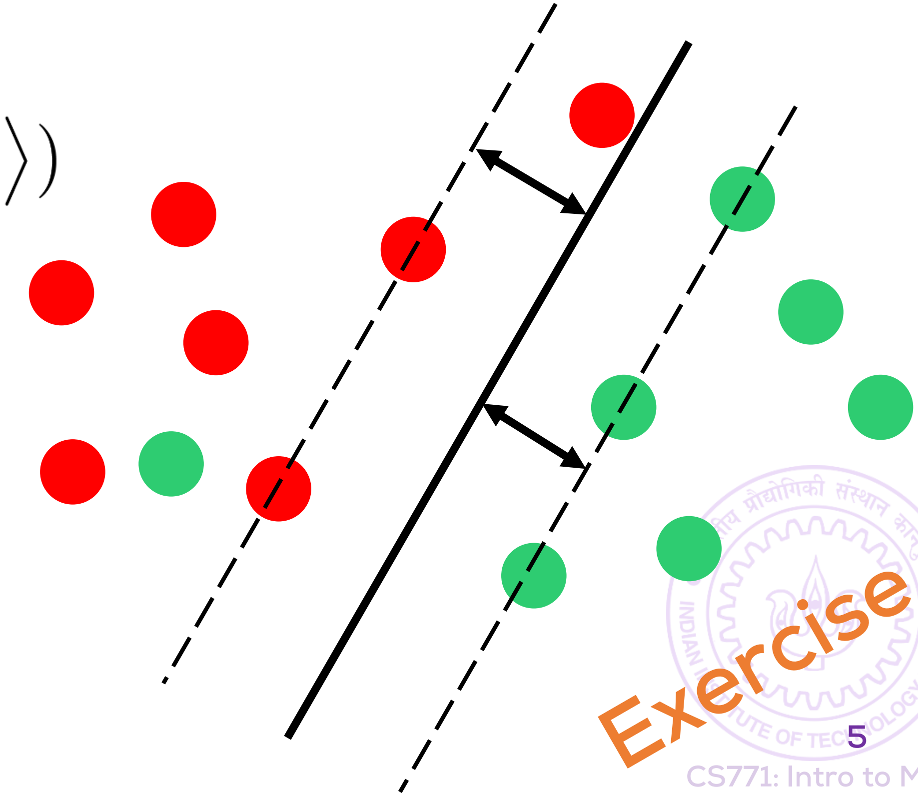
Large Margin Classifier



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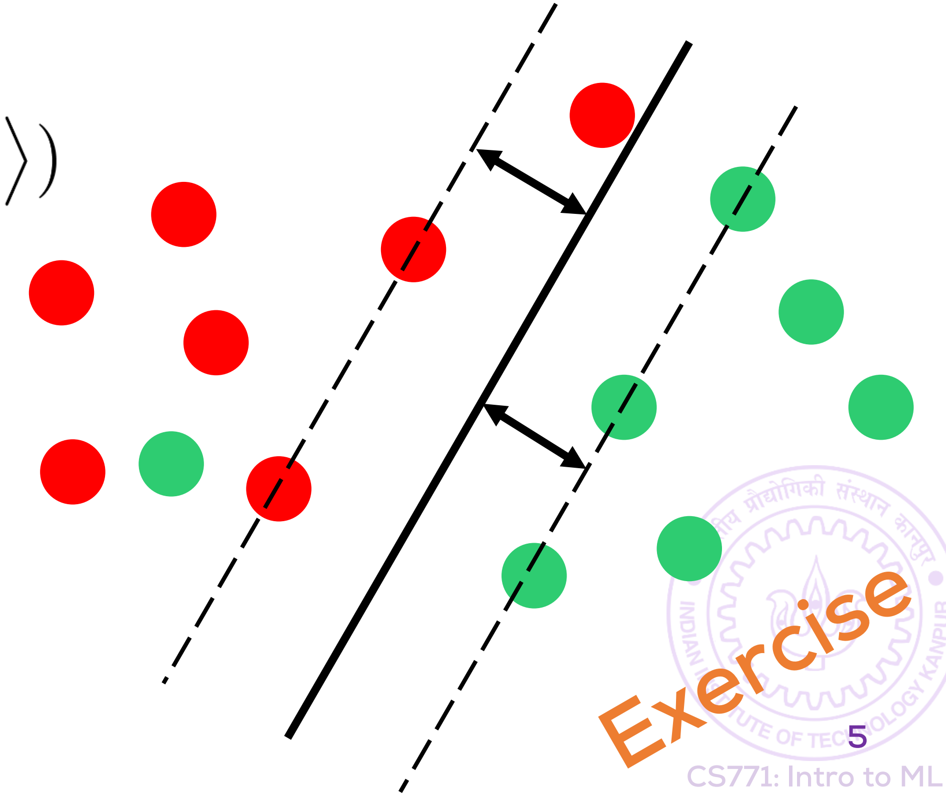
Large Margin Classifier

SVM



Exercise

Loss Functions for Structured ML problems

Multi-class and Multi-label classification

Multi-classification Loss Functions

$\boxed{\text{One-vs-All (OVA)}}$

$$\hat{y}^i = \arg\max_{j \in [K]} \langle \mathbf{w}^j, \mathbf{x}^i \rangle$$

Multi-classification Loss Functions

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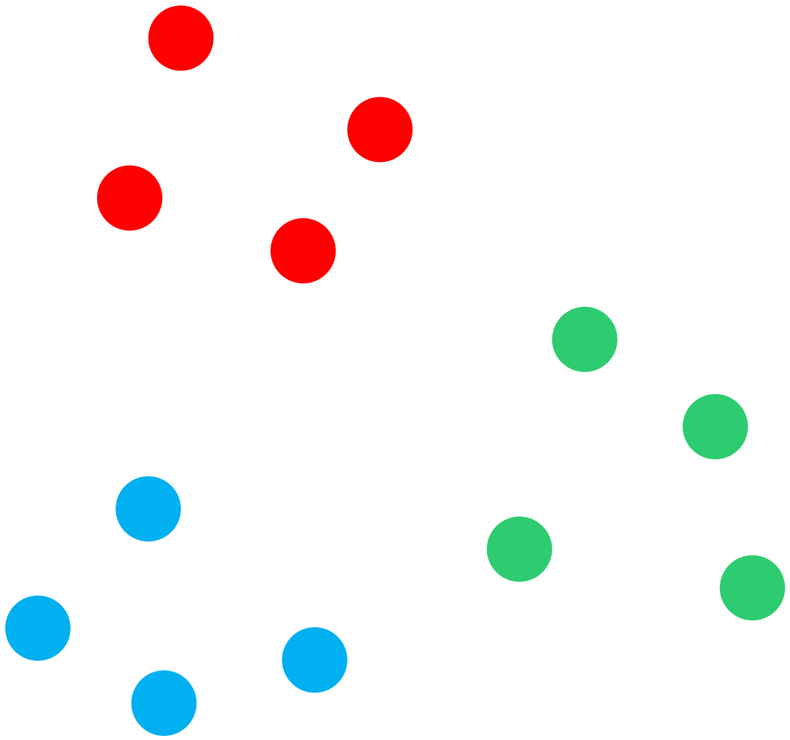

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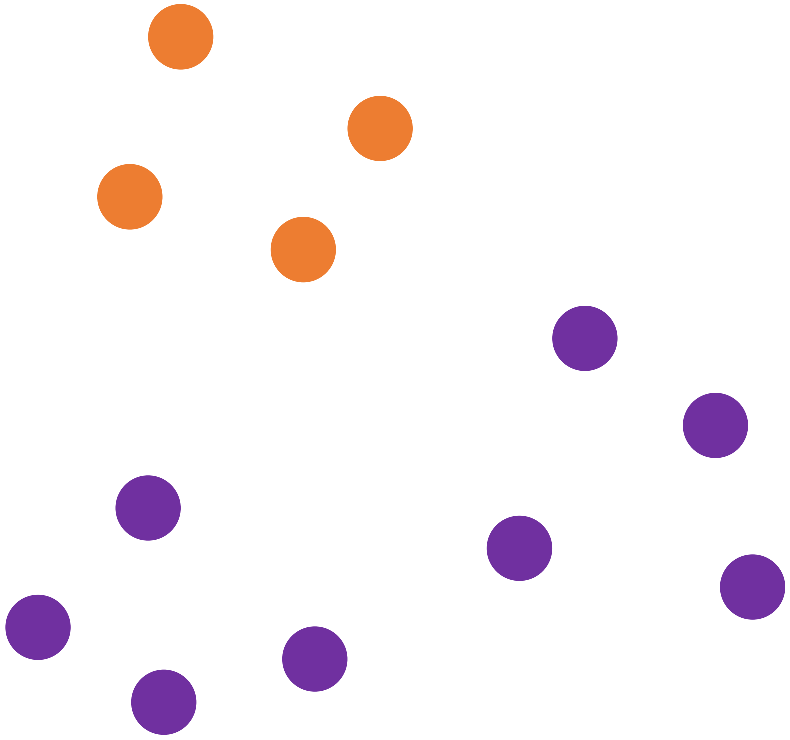

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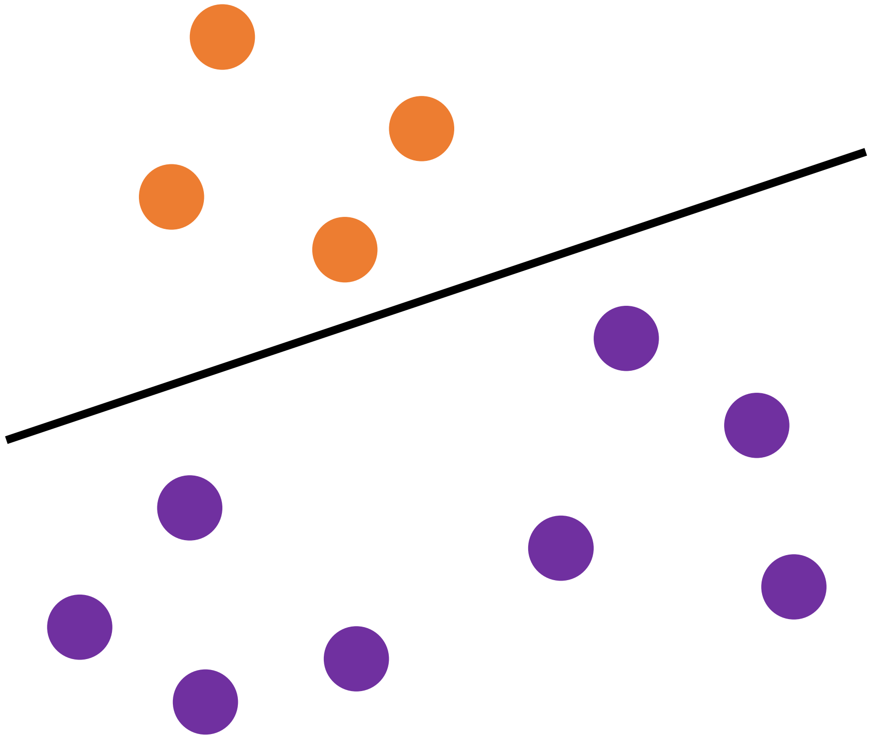

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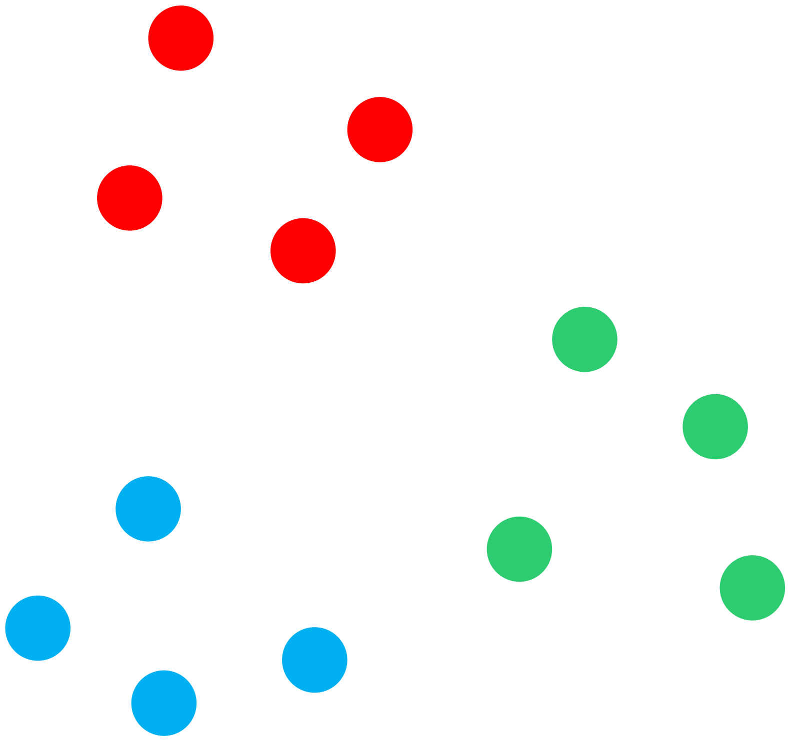

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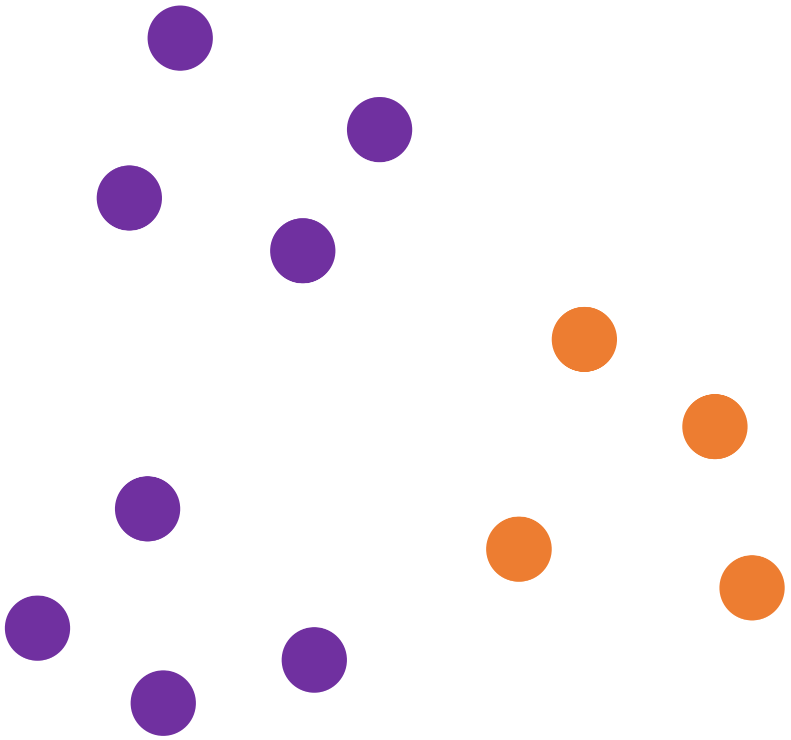
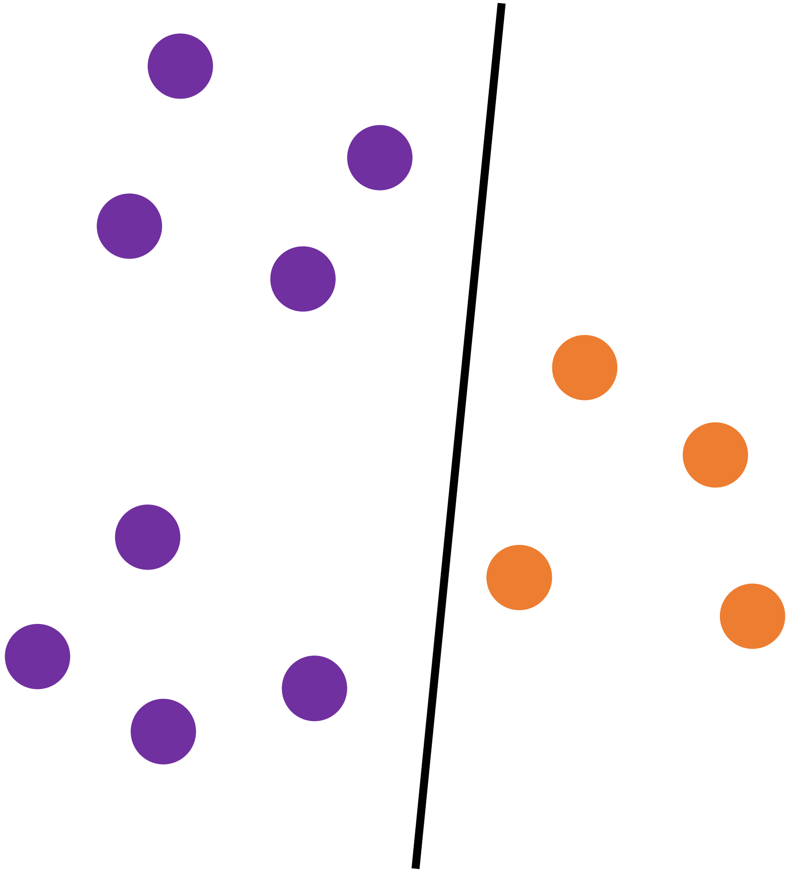


Multi-classification Loss Functions

One-vs-All (OVA)

$$\hat{y}^i = \arg\max_{j \in [K]} \langle \mathbf{w}^j, \mathbf{x}^i \rangle$$



Multi-classification Loss Functions

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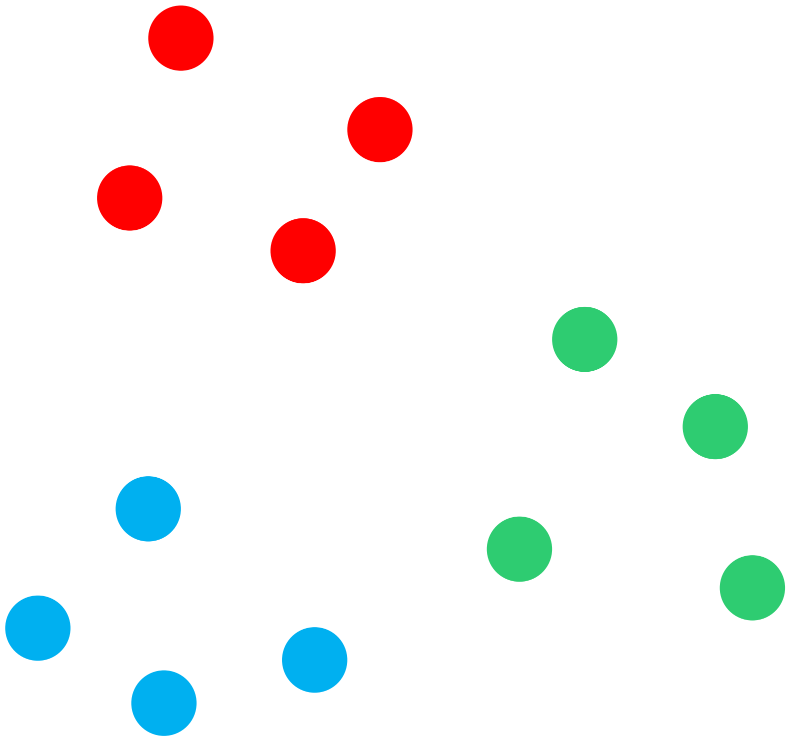

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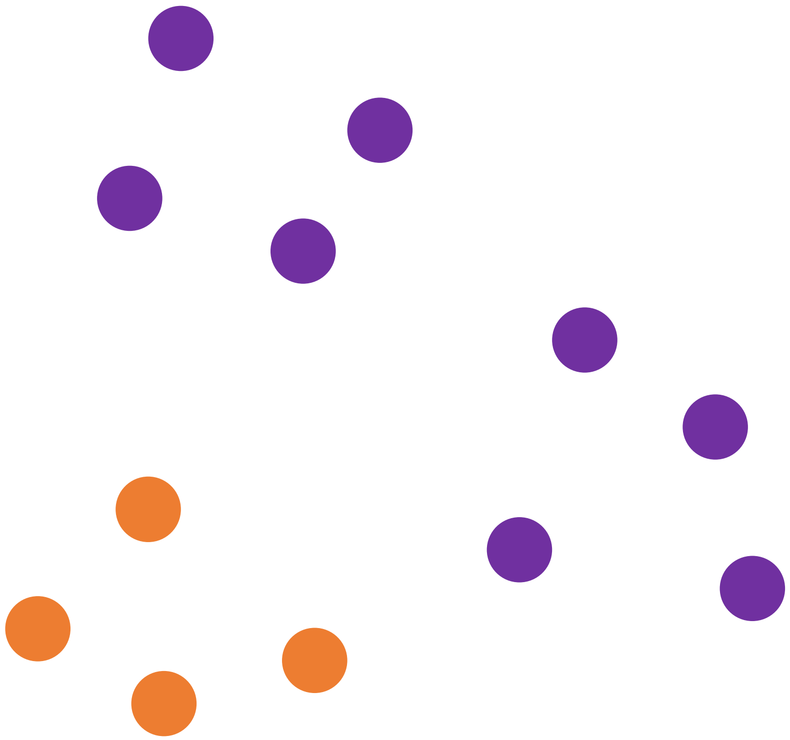

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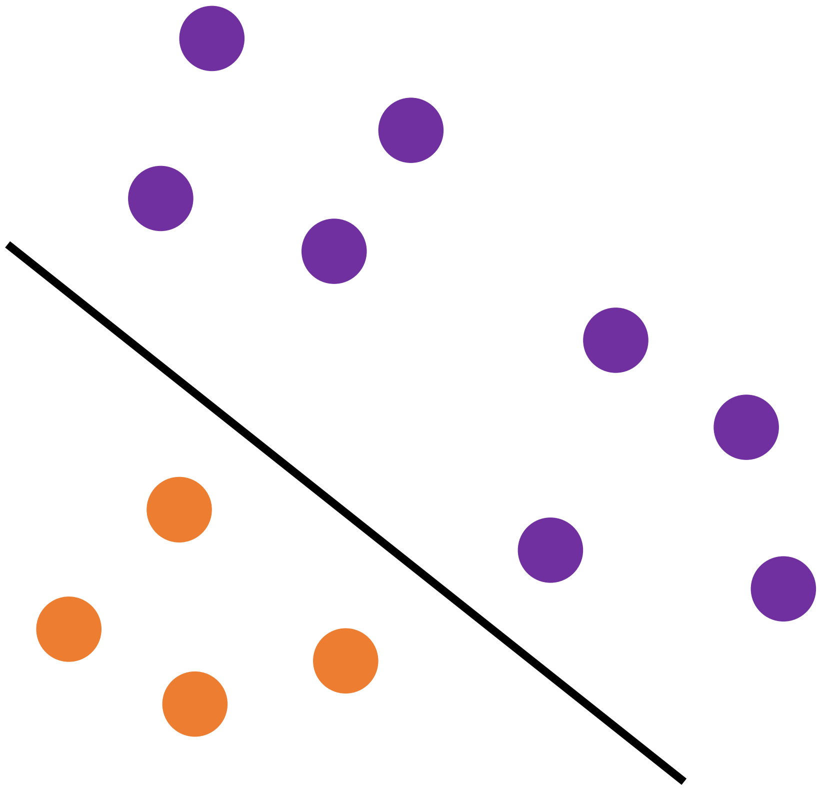

Multi-classification Loss Functions

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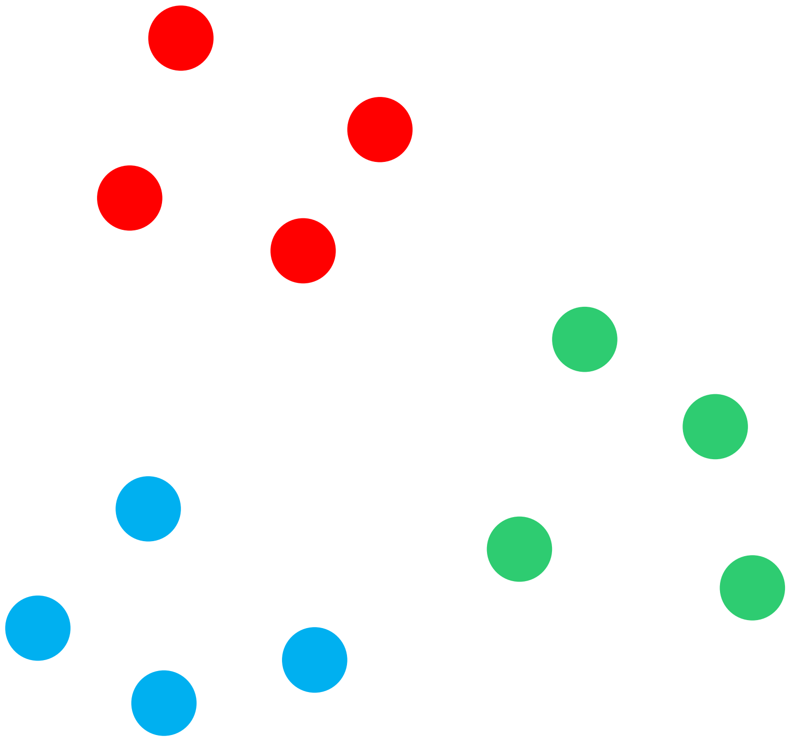

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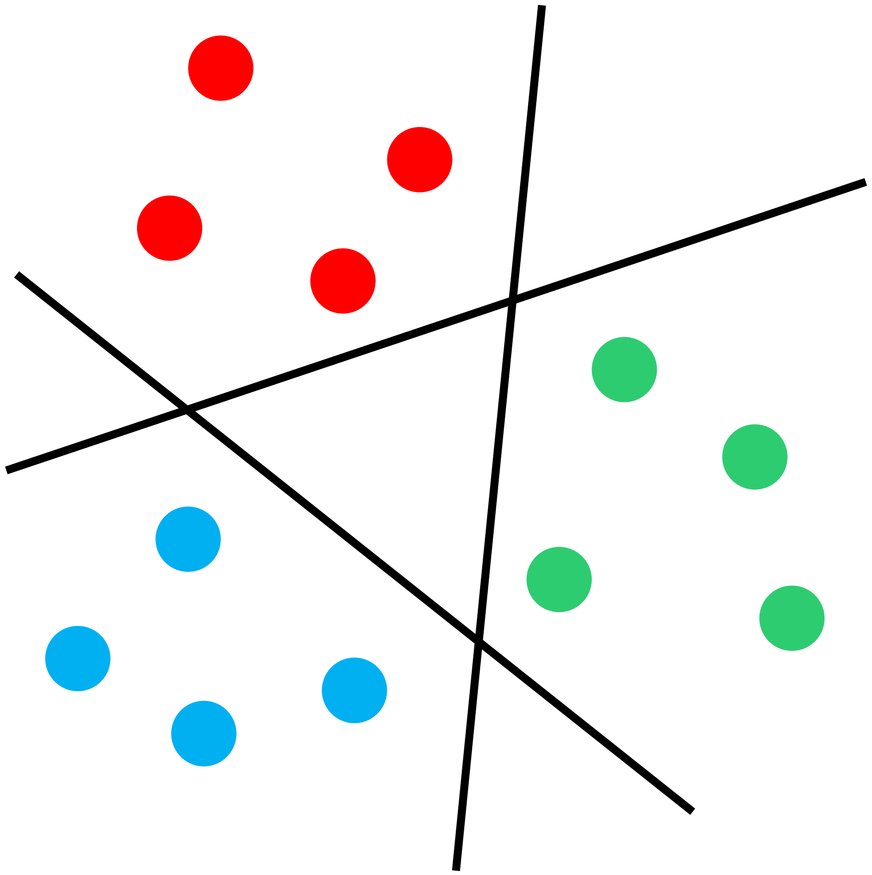

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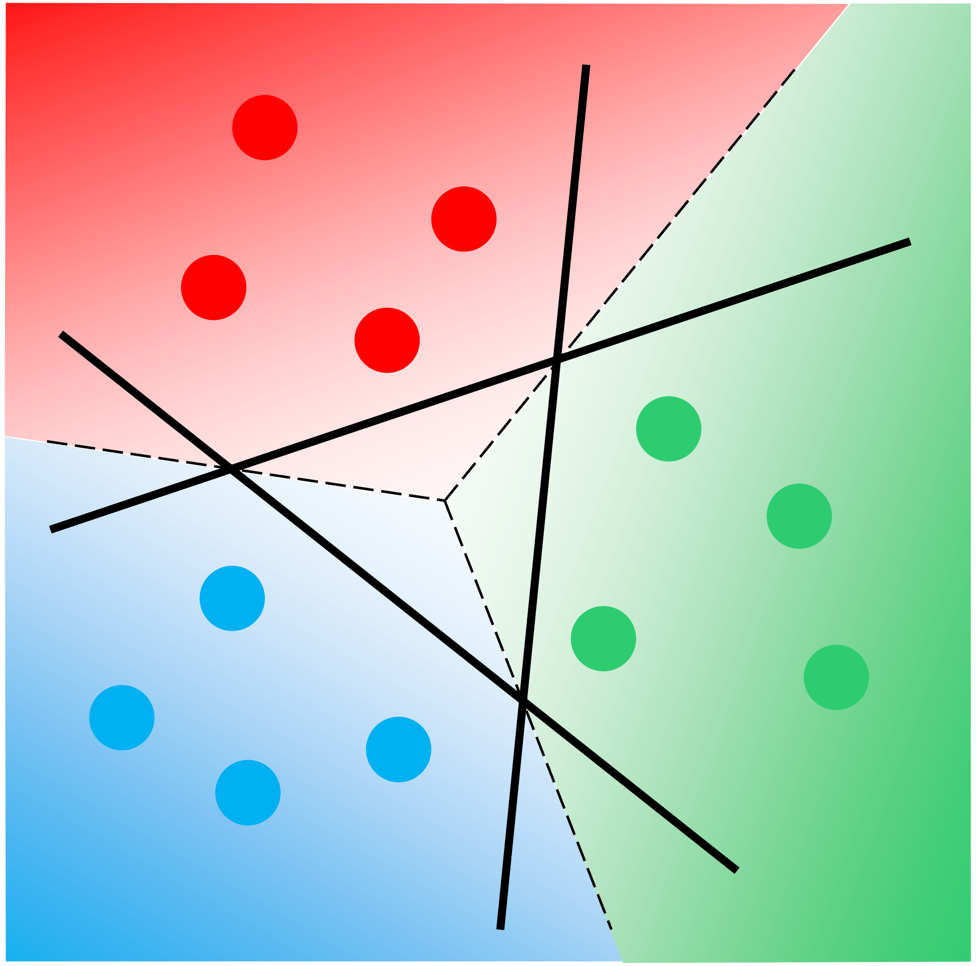

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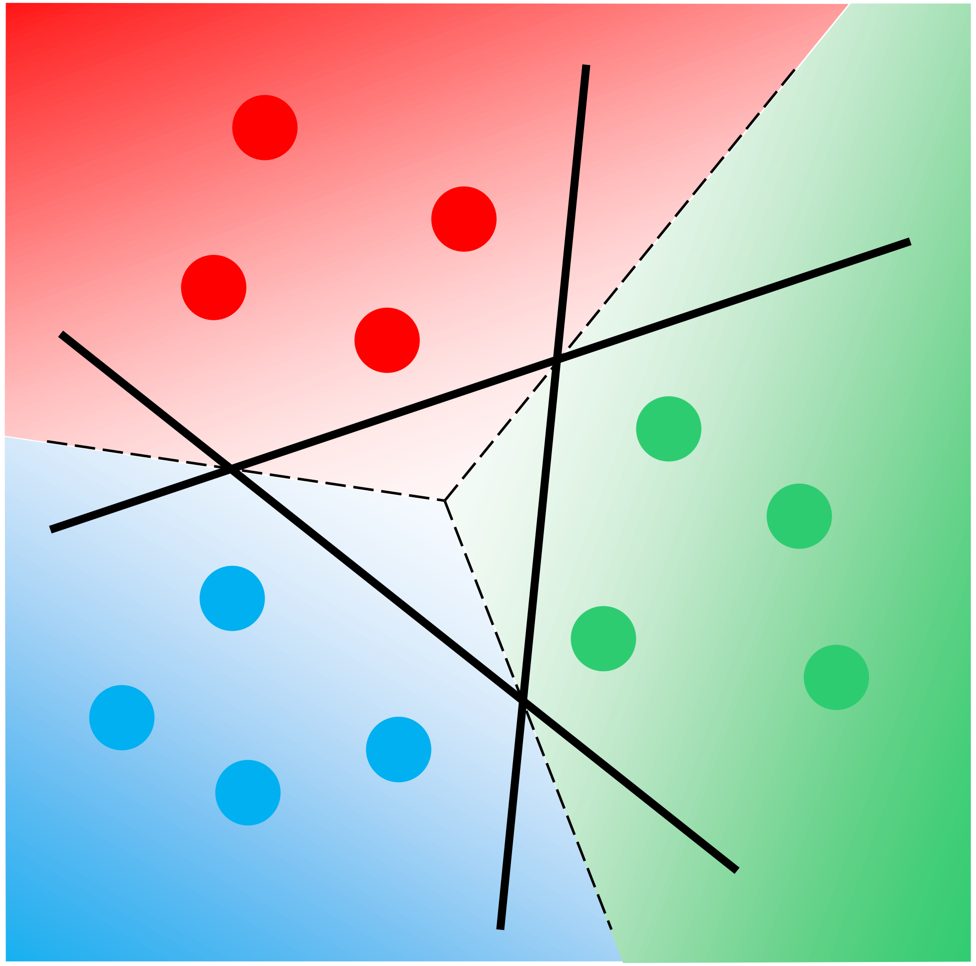

$$\hat{\mathbf{w}}^j = \underset{\mathbf{w}}{\arg\min} \; \sum_{i=1}^{n} \ell(y^{i,(j)}, \langle \mathbf{w}, \mathbf{x}^i \rangle)$$

$$y^{i,(j)} = \begin{cases} 1 & ; y^i = j \\ -1 & ; y^i \neq j \end{cases}$$

Multi-classification Loss Functions

One-vs-All (OVA)

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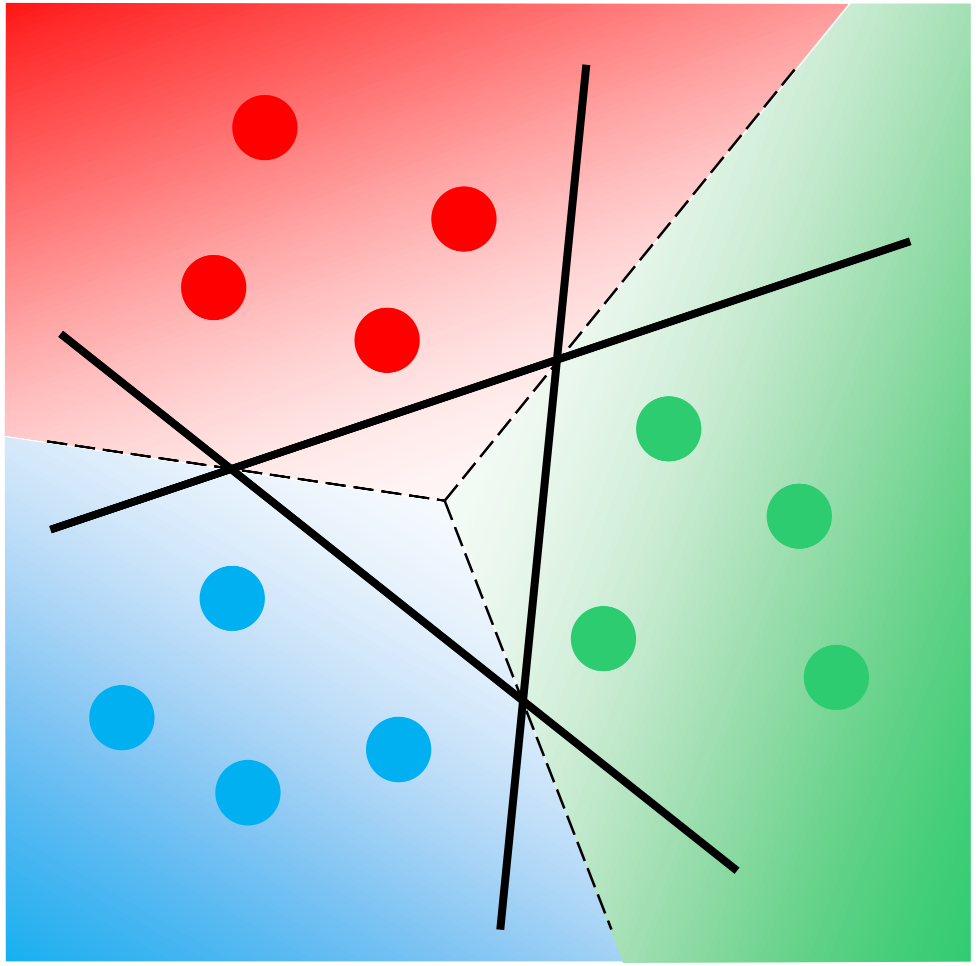

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$$y^{i,(j)} = \begin{cases} 1 & ; y^i = j \\ -1 & ; y^i \neq j \end{cases}$$

hinge, logistic etc

Multi-classification Loss Functions

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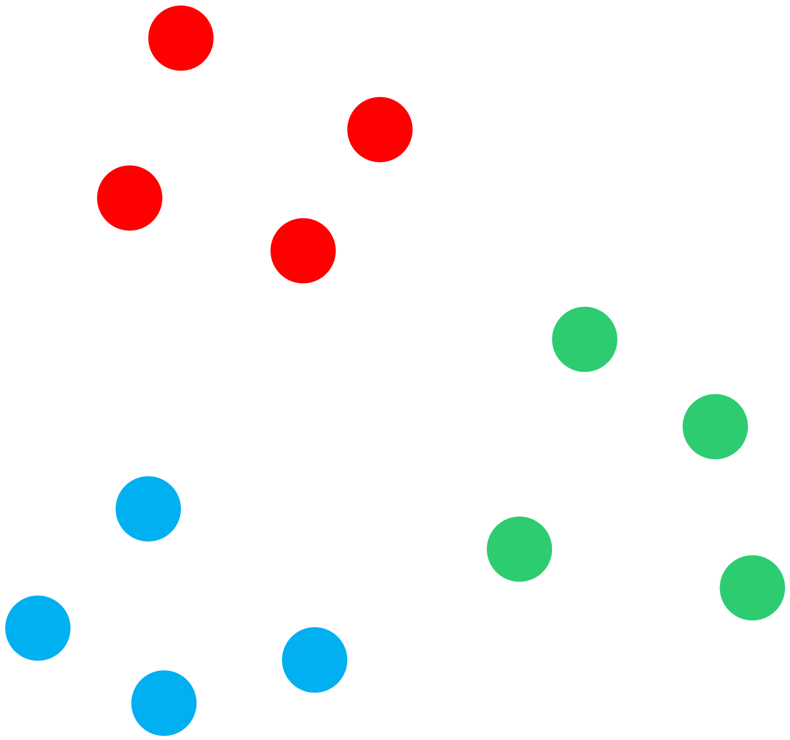

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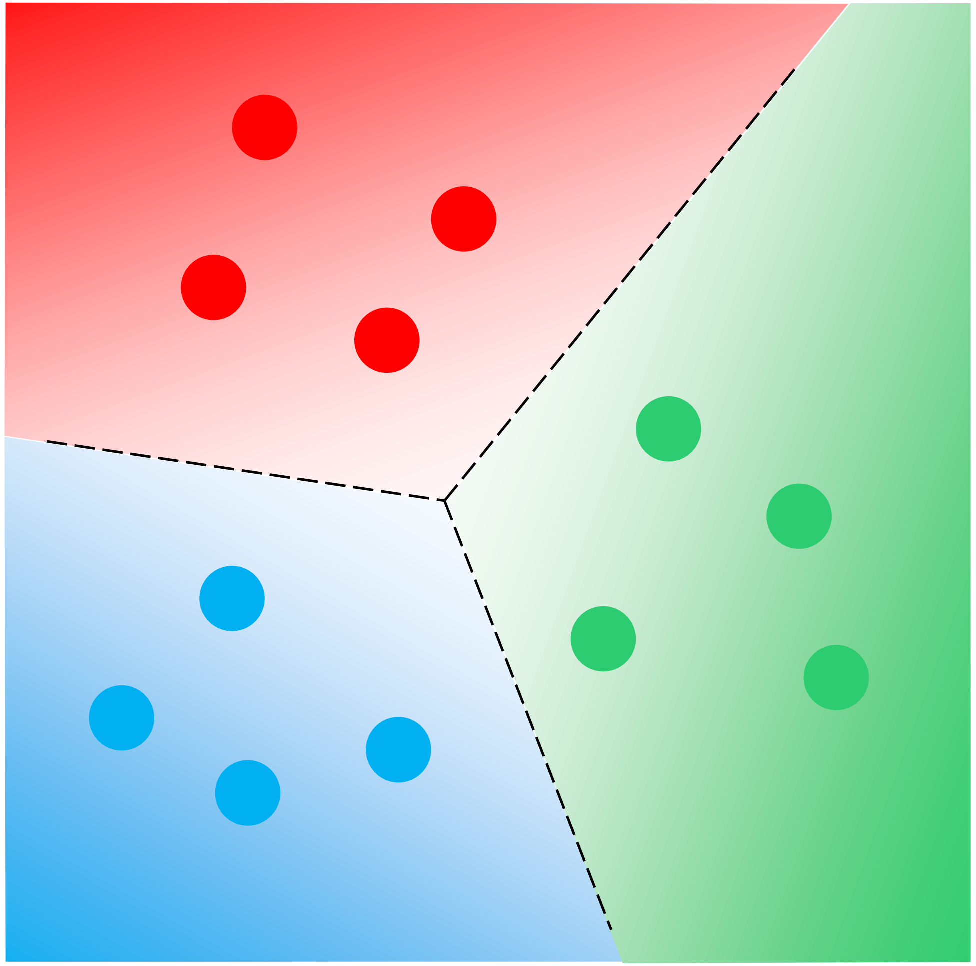
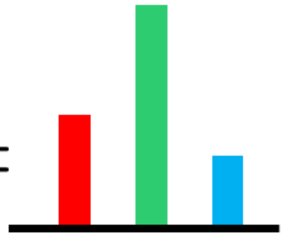


Multi-classification using MLE

- $K > 2$ classes – need more detailed parameters
- For each point, its label profile is a vector $\boldsymbol{\eta}(\mathbf{x}) =$ 

$$\mathbb{P}\left[y^i = k \,|\, \mathbf{x}^i, \{\mathbf{w}^l\}_{1,\ldots,K}\right] \propto \exp(\langle \mathbf{w}^k, \mathbf{x}^i \rangle)$$

$$\mathbb{P}\left[y^i = k \,|\, \mathbf{x}^i, \{\mathbf{w}^l\}_{1,\ldots,K}\right] = \frac{\exp(\langle \mathbf{w}^k, \mathbf{x}^i \rangle)}{\sum_{l=1}^{K} \exp(\langle \mathbf{w}^l, \mathbf{x}^i \rangle)}$$

- Likelihood function is multinomial instead of binomial

$$\mathbb{P}\left[\mathbf{y} \,|\, \mathbf{X}, \mathbf{w}\right] = \prod_{i=1}^{n} \hat{\boldsymbol{\eta}}^i_{y^i}(\mathbf{x}) \qquad \hat{\boldsymbol{\eta}}^i_k(\mathbf{x}) = \frac{\exp(\langle \mathbf{w}^k, \mathbf{x}^i \rangle)}{\sum_{l=1}^{K} \exp(\langle \mathbf{w}^l, \mathbf{x}^i \rangle)}$$

Softmax Regression

Multi-classification Loss Functions

One-vs-All (OVA)

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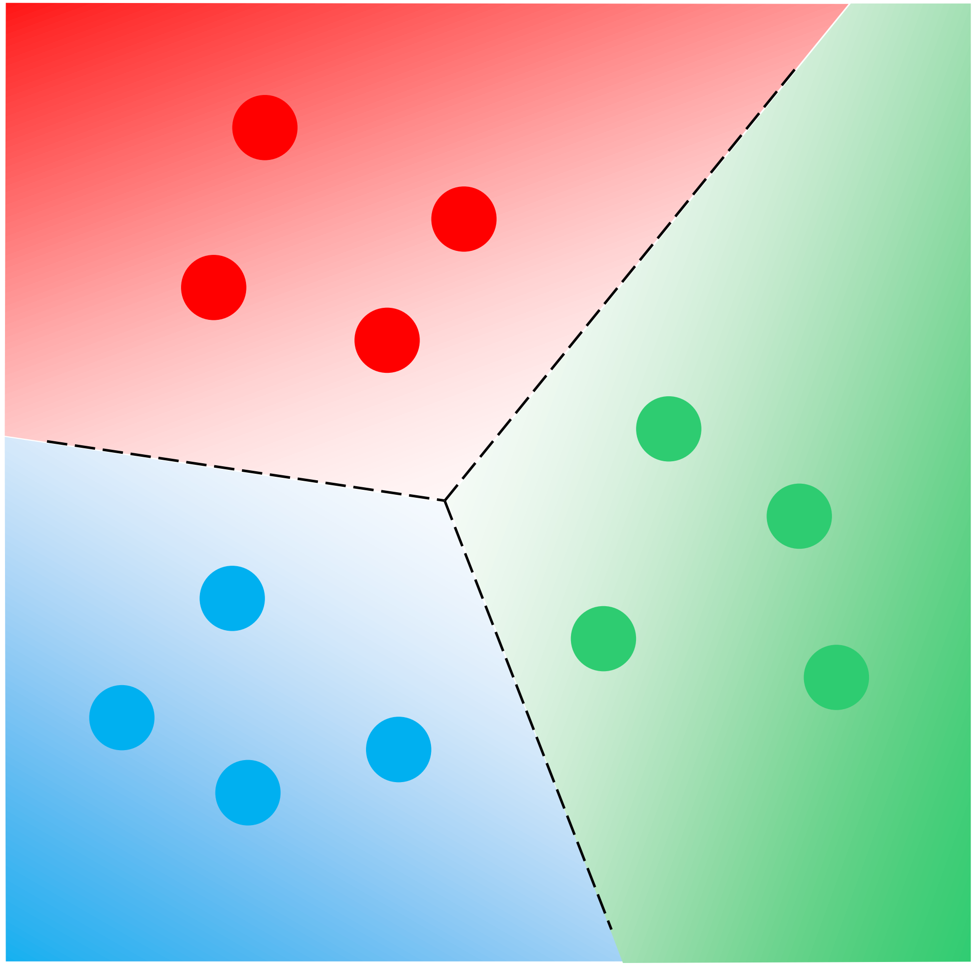

Multi-classification Loss Functions

One-vs-All (OVA)

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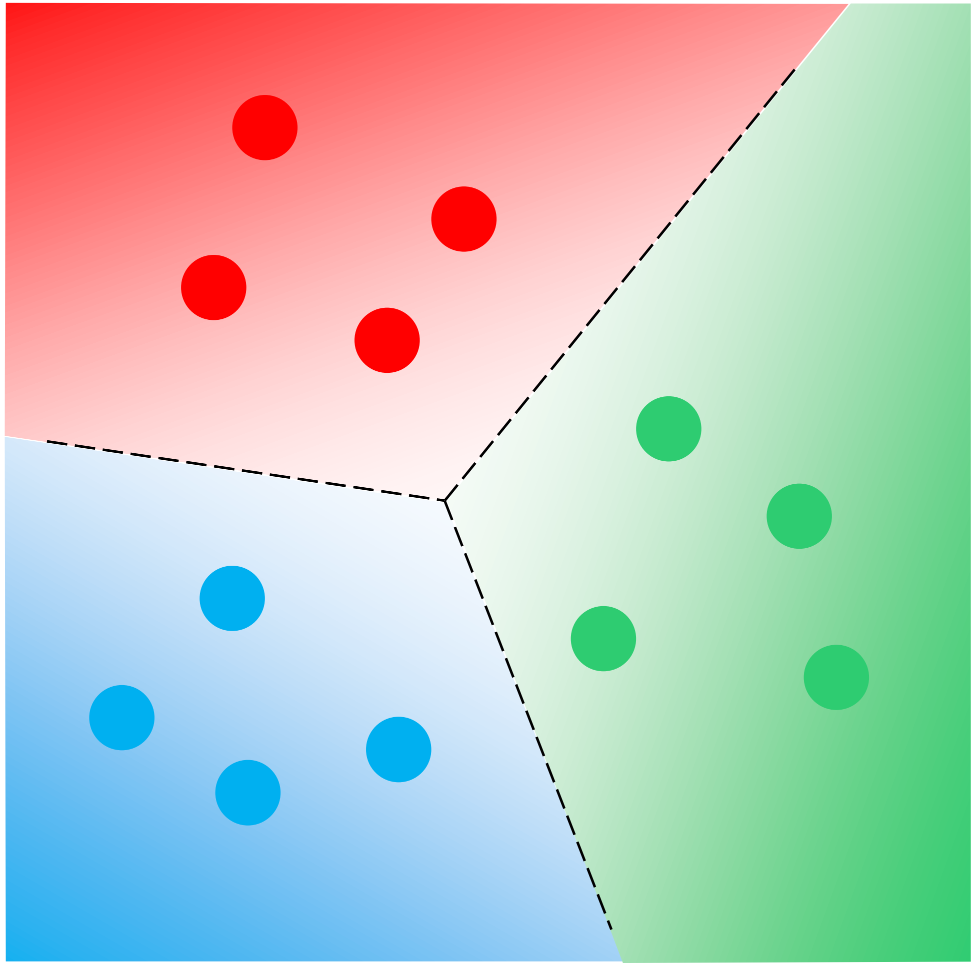
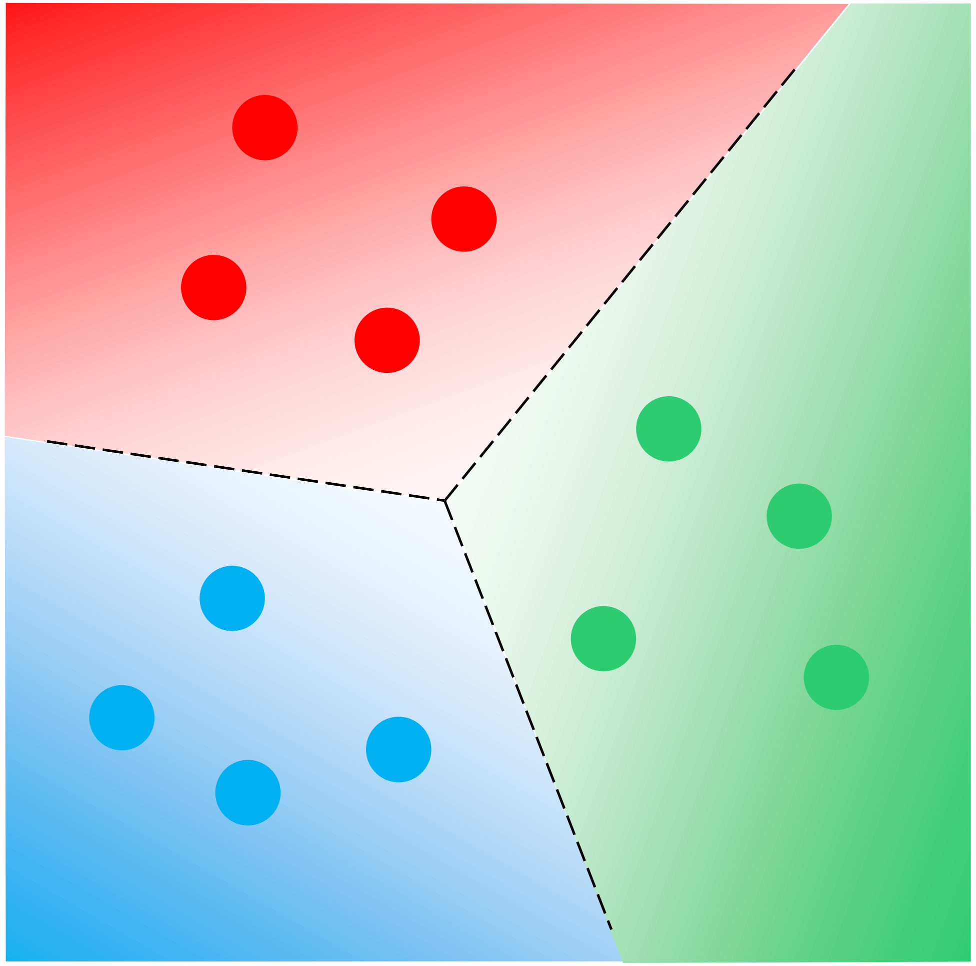


Multi-classification Loss Functions

One-vs-All (OVA)



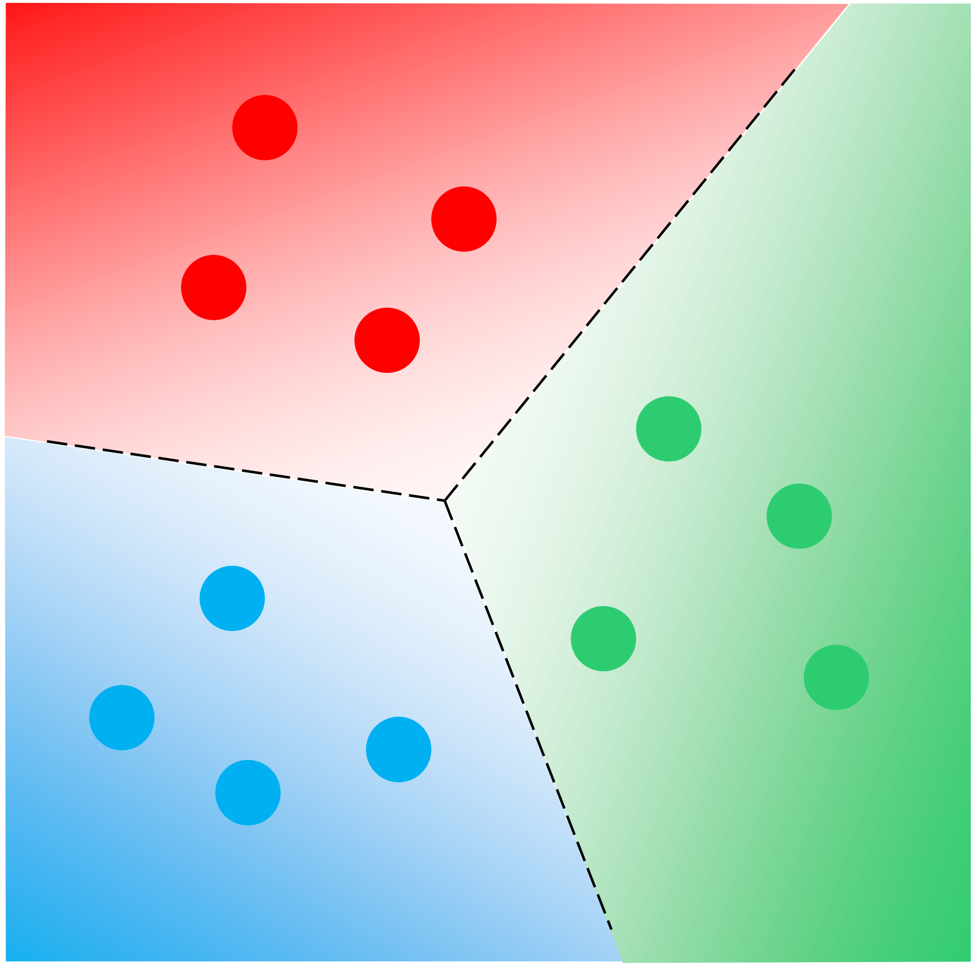
$$\hat{y}^i = \underset{j \in [K]}{\arg\max} \; \langle \mathbf{w}^j, \mathbf{x}^i \rangle$$

$$\mathbf{W} = [\mathbf{w}^1, \mathbf{w}^2, \ldots, \mathbf{w}^K]$$

$$\widehat{\mathbf{W}}_{\text{MLE}} = \underset{\mathbf{W}}{\arg\min} \; \sum_{i=1}^{n} \ell_{\text{sm}}(y^i, \langle \mathbf{W}, \mathbf{x}^i \rangle)$$

Multi-classification Loss Functions

One-vs-All (OVA)



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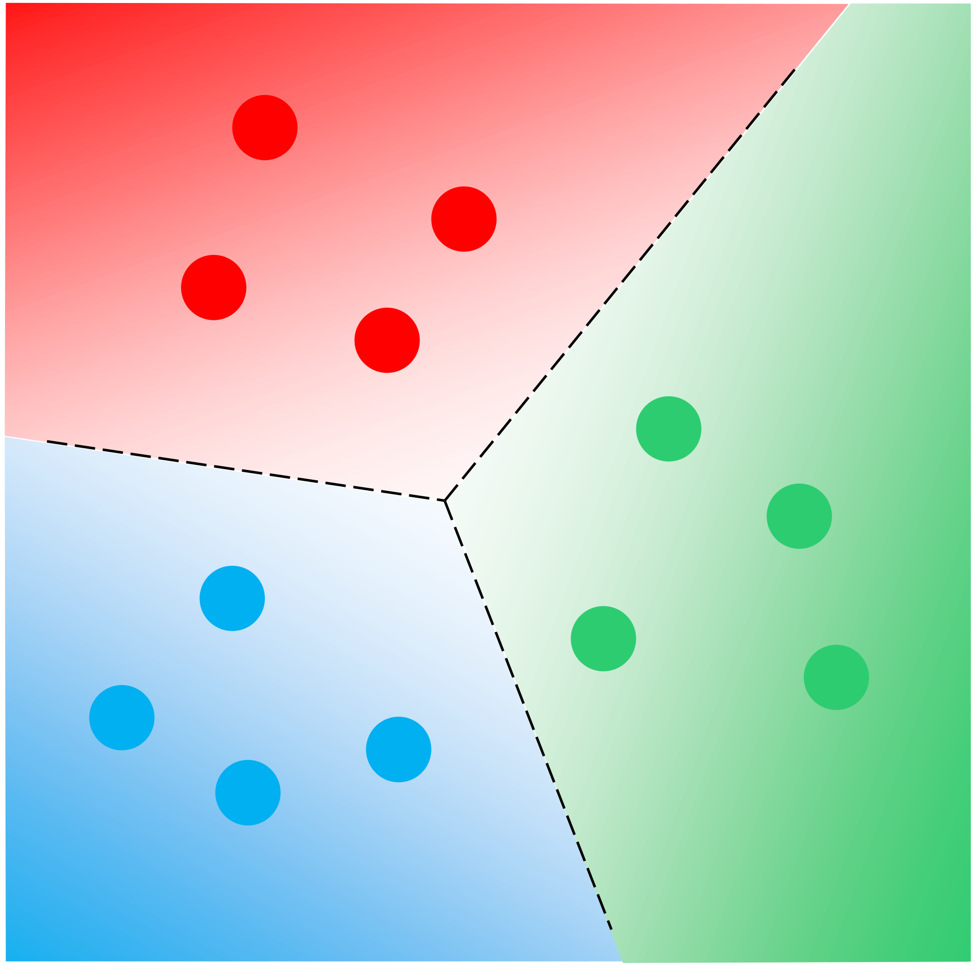
Multi-classification Loss Functions

One-vs-All (OVA)



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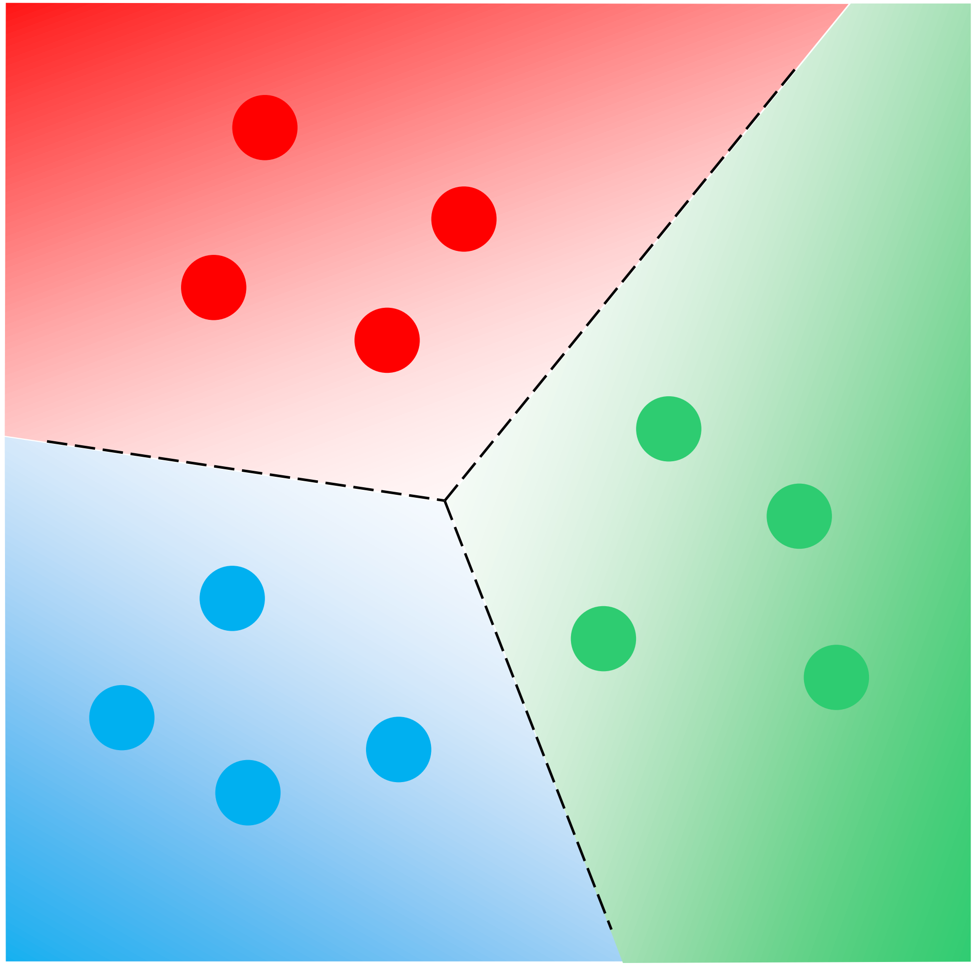
$$\ell_{\mathrm{sm}}(y, \{\boldsymbol{\eta}_j\}) = -\log \frac{\exp(\boldsymbol{\eta}_y)}{\sum_{k=1}^{K} \exp(\boldsymbol{\eta}_k)}$$

Multi-classification Loss Functions

One-vs-All (OVA)



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Softmax loss function

Multi-classification Loss Functions

One-vs-All (OVA)

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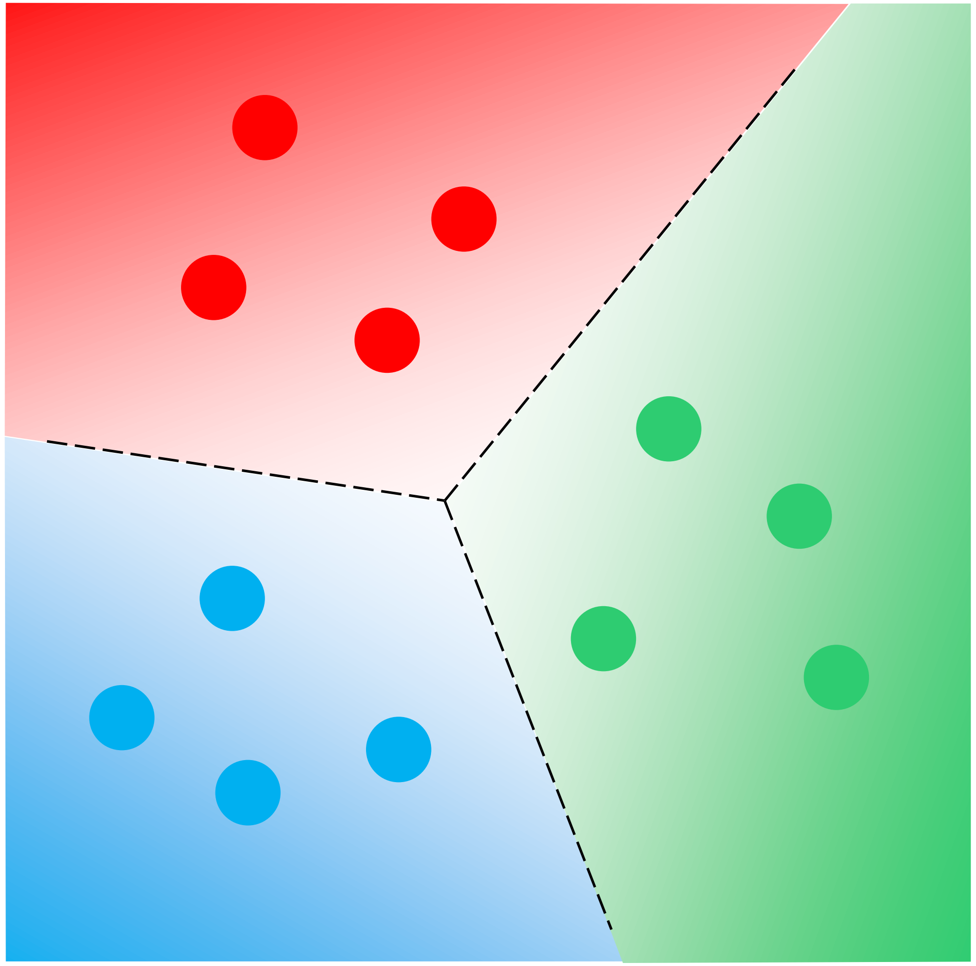

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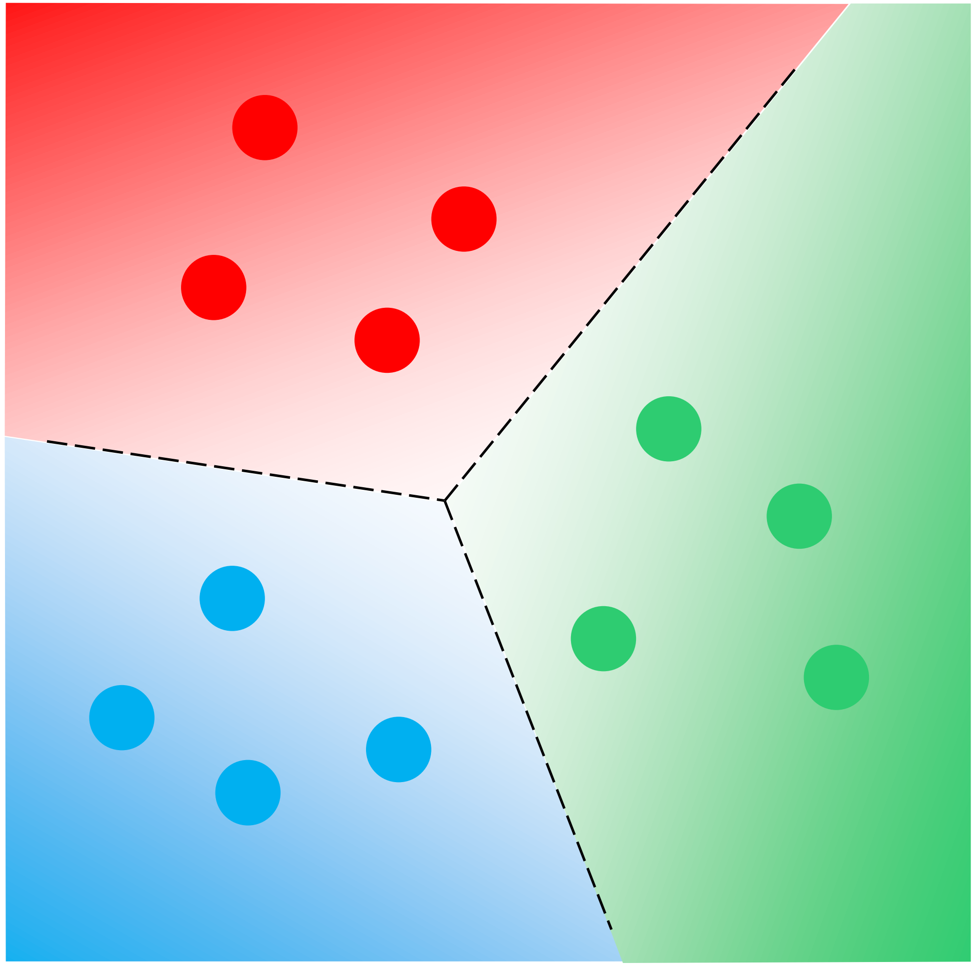
Multi-classification Loss Functions

One-vs-All (OVA)



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$$\text{s.t.} \left\langle \mathbf{w}^{y^i}, \mathbf{x}^i \right\rangle \geq \left\langle \mathbf{w}^k, \mathbf{x}^i \right\rangle + 1$$

Multi-classification Loss Functions

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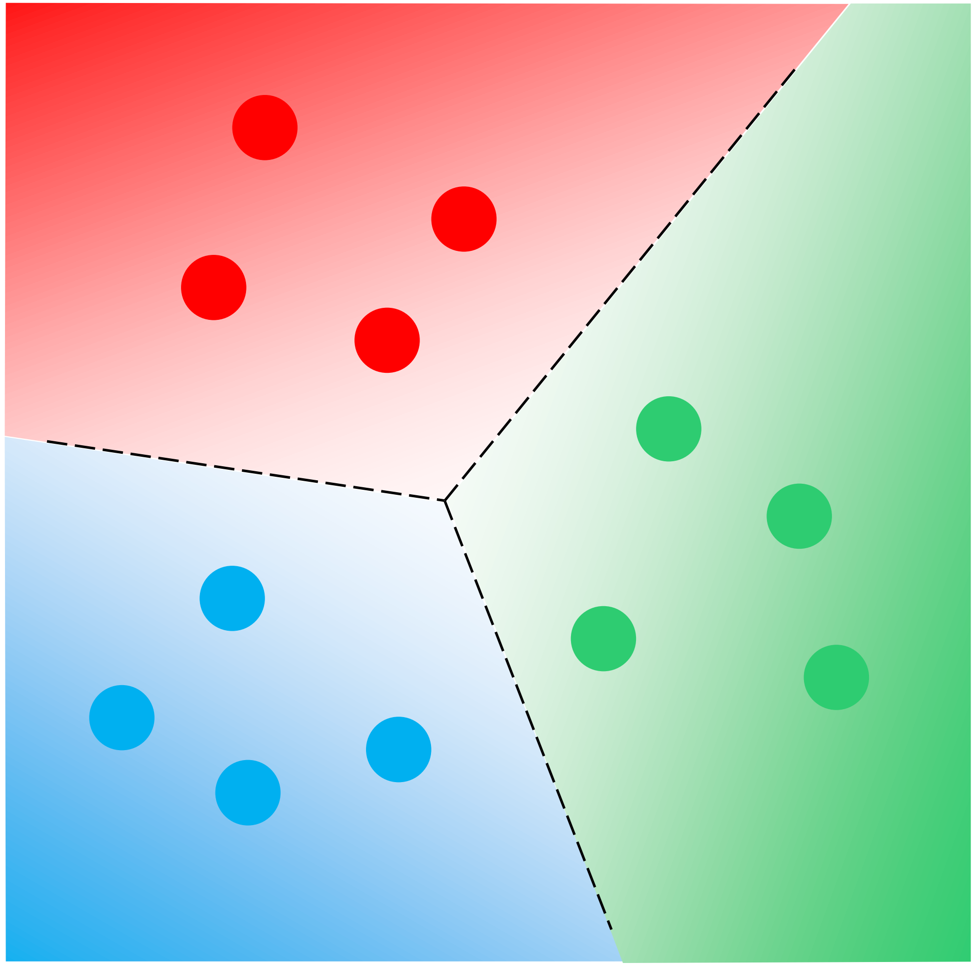

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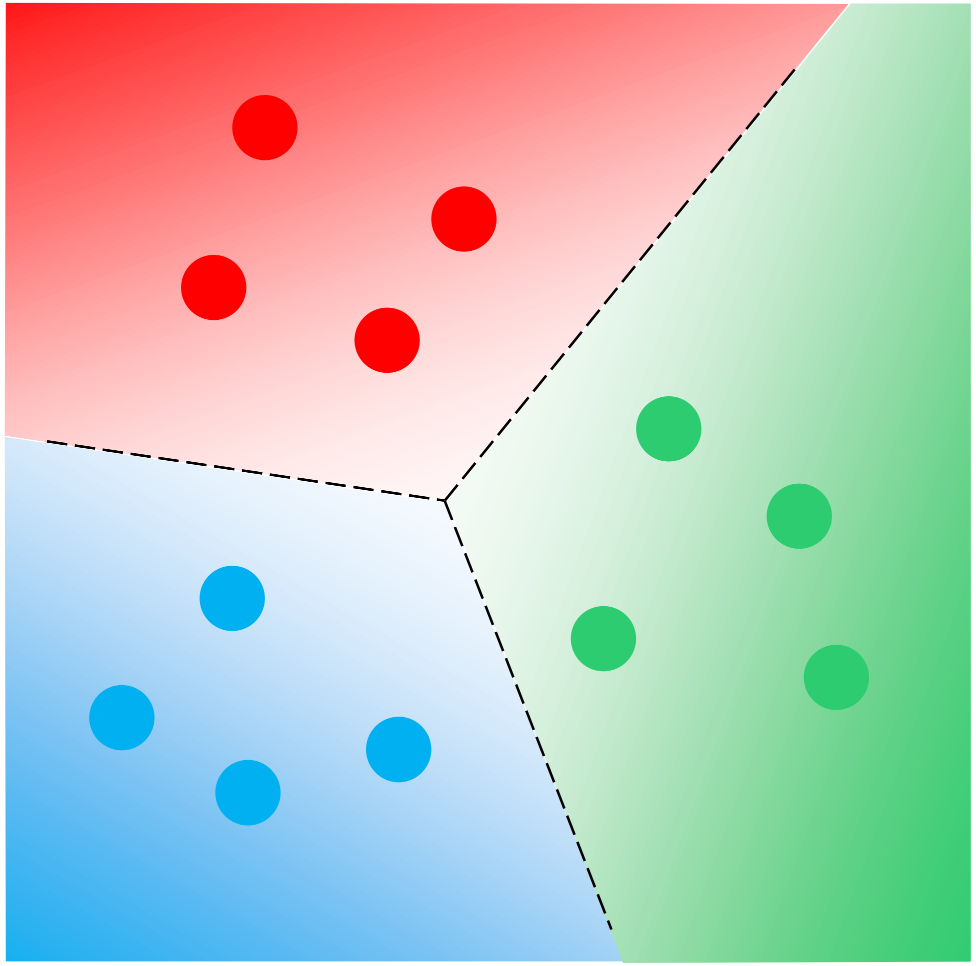
$$\forall k \neq y^i$$

Multi-classification Loss Functions

One-vs-All (OVA)



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$$\mathbf{W} = \left[\mathbf{w}^1, \mathbf{w}^2, \ldots, \mathbf{w}^K\right]$$

Slack variable

$$\widehat{\mathbf{W}} = \arg\min_{\mathbf{W}} \sum_{k=1}^{K} \left\|\mathbf{w}^k\right\|_2^2$$

$$\text{s.t.} \left\langle \mathbf{w}^{y^i}, \mathbf{x}^i \right\rangle \geq \left\langle \mathbf{w}^k, \mathbf{x}^i \right\rangle + 1 - \xi_i$$

$$\forall k \neq y^i$$

Multi-classification Loss Functions

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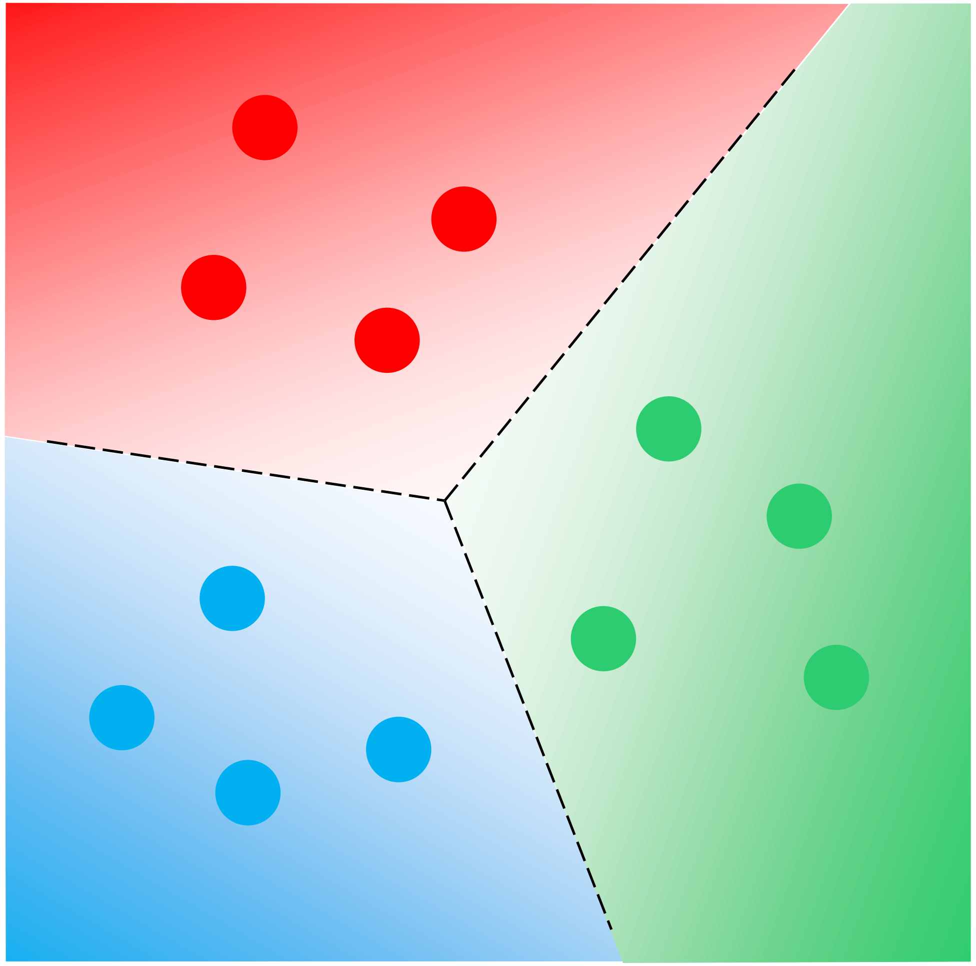

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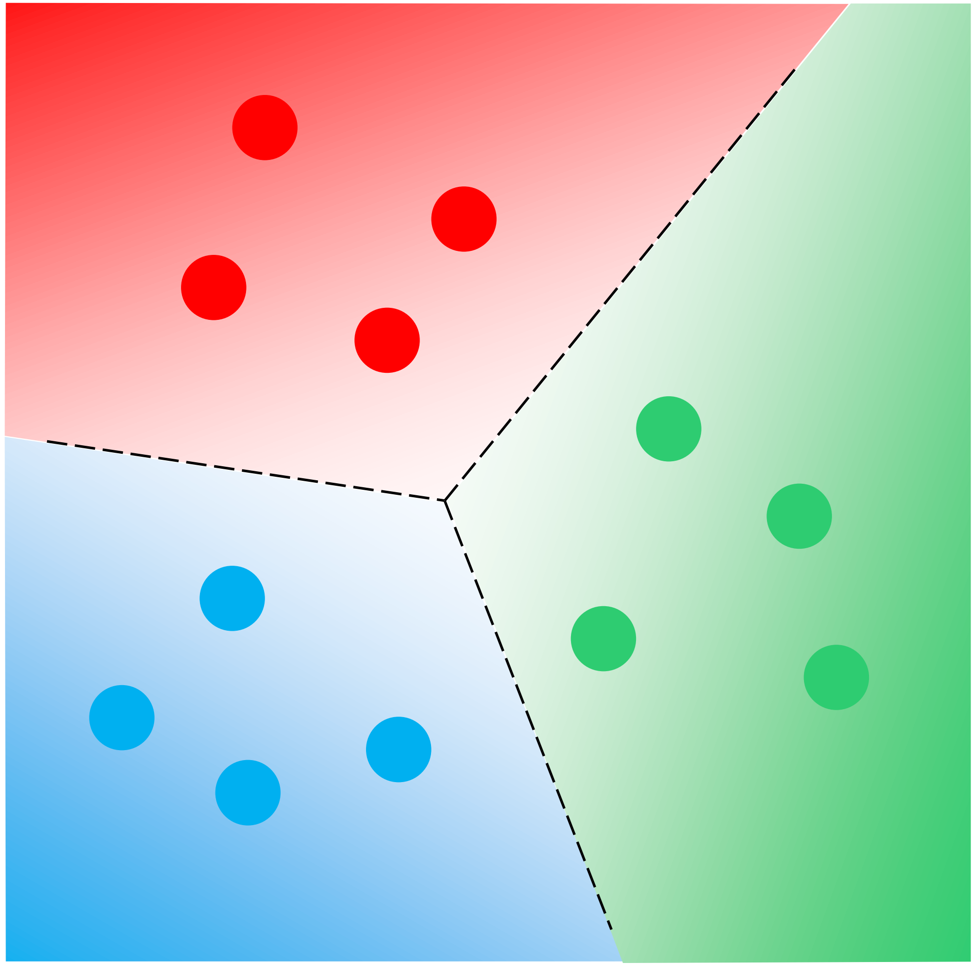
$$\xi_i \geq 0$$

Multi-classification Loss Functions

One-vs-All (OVA)



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Slack variable

$$\widehat{\mathbf{W}} = \arg\min_{\mathbf{W}} \sum_{k=1}^{K} \left\|\mathbf{w}^k\right\|_2^2 + \sum_{i=1}^{n} \xi_i$$

$$\text{s.t. } \left\langle \mathbf{w}^{y^i}, \mathbf{x}^i \right\rangle \geq \left\langle \mathbf{w}^k, \mathbf{x}^i \right\rangle + 1 - \xi_i \quad \forall k \neq y^i$$

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Multi-classification Loss Functions

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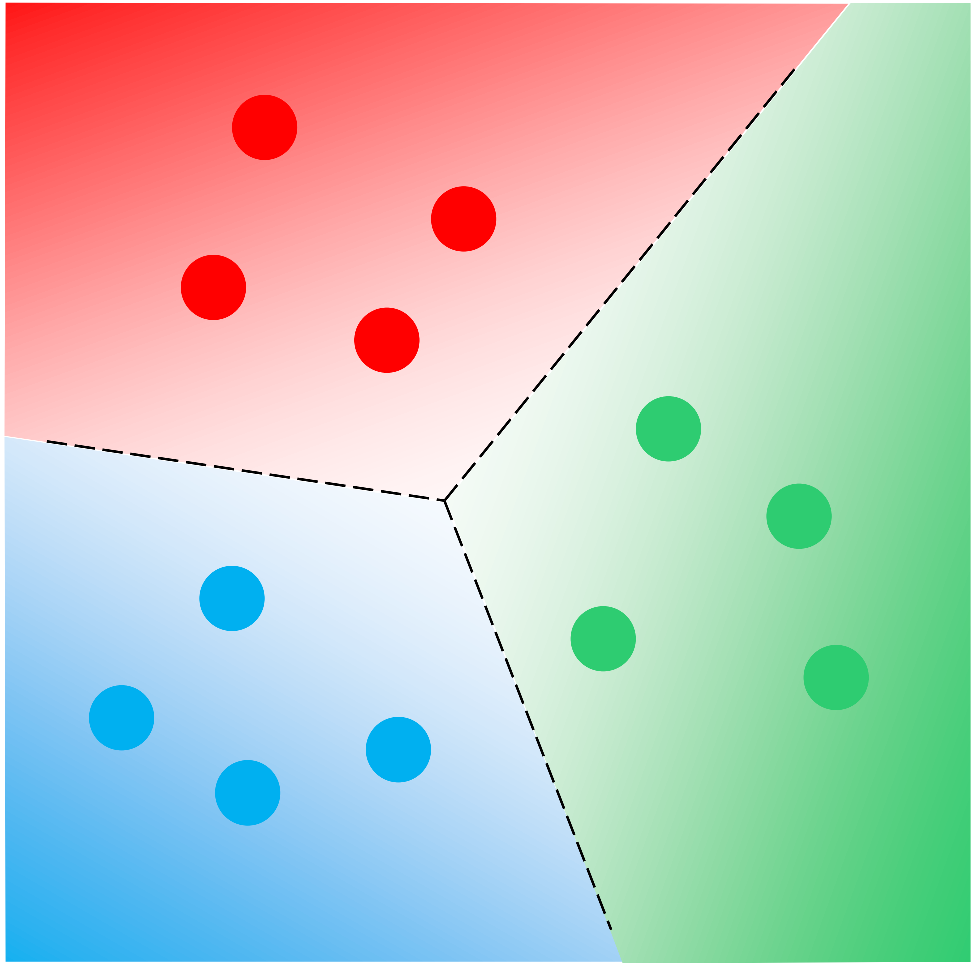

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$$\forall k \neq y^i$$

$$\xi_i \geq 0$$

Multi-classification Loss Functions

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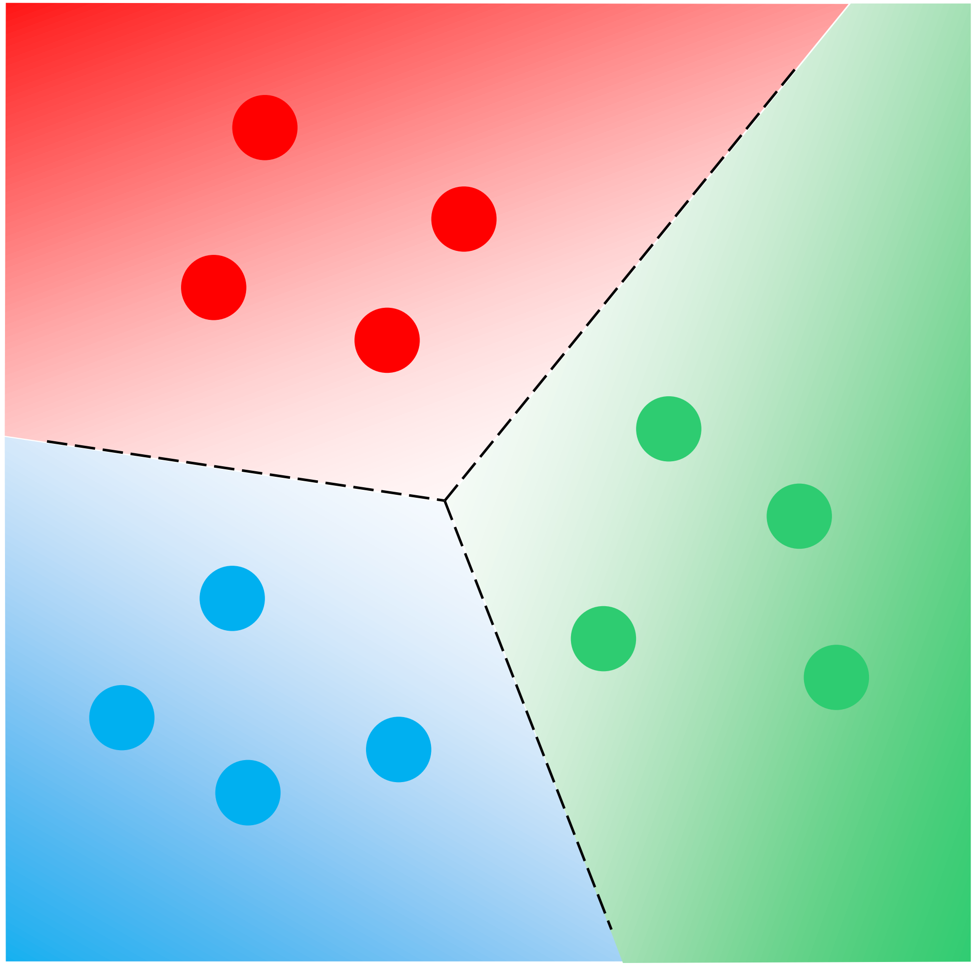

$$\mathbf{W} = [\mathbf{w}^1, \mathbf{w}^2, \ldots, \mathbf{w}^K]$$

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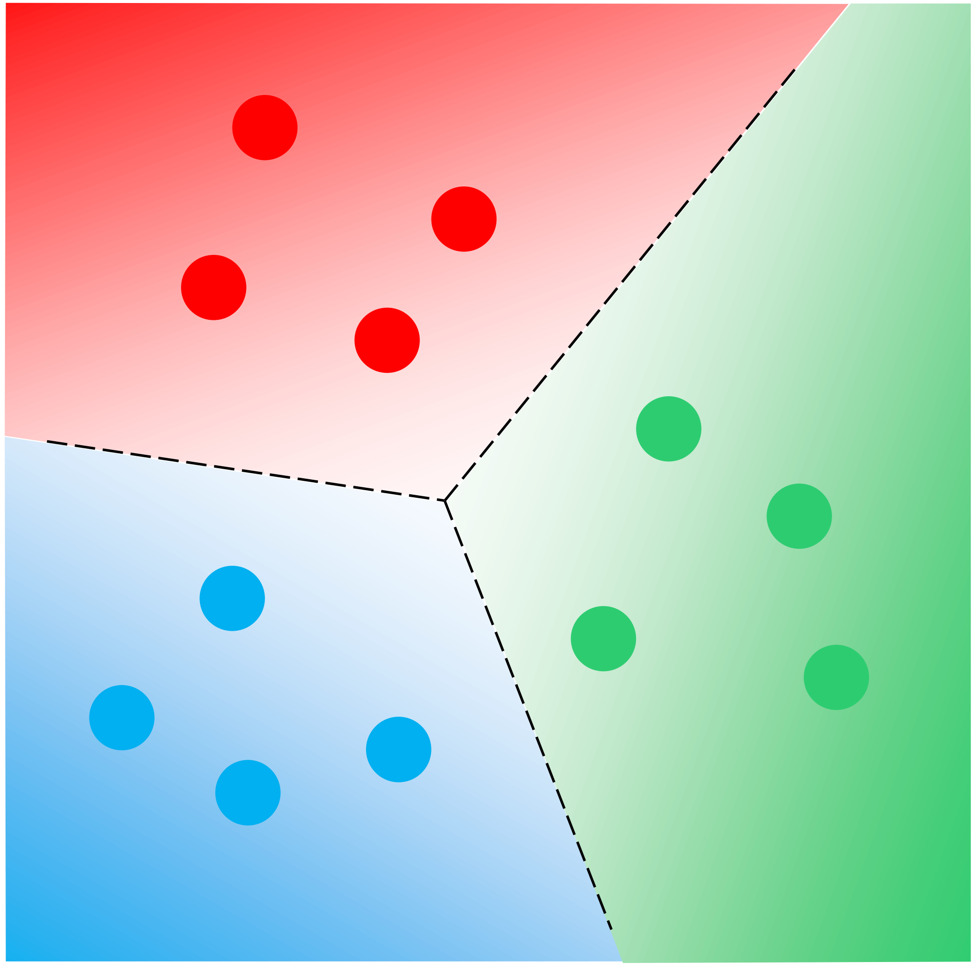
Multi-classification Loss Functions

One-vs-All (OVA)



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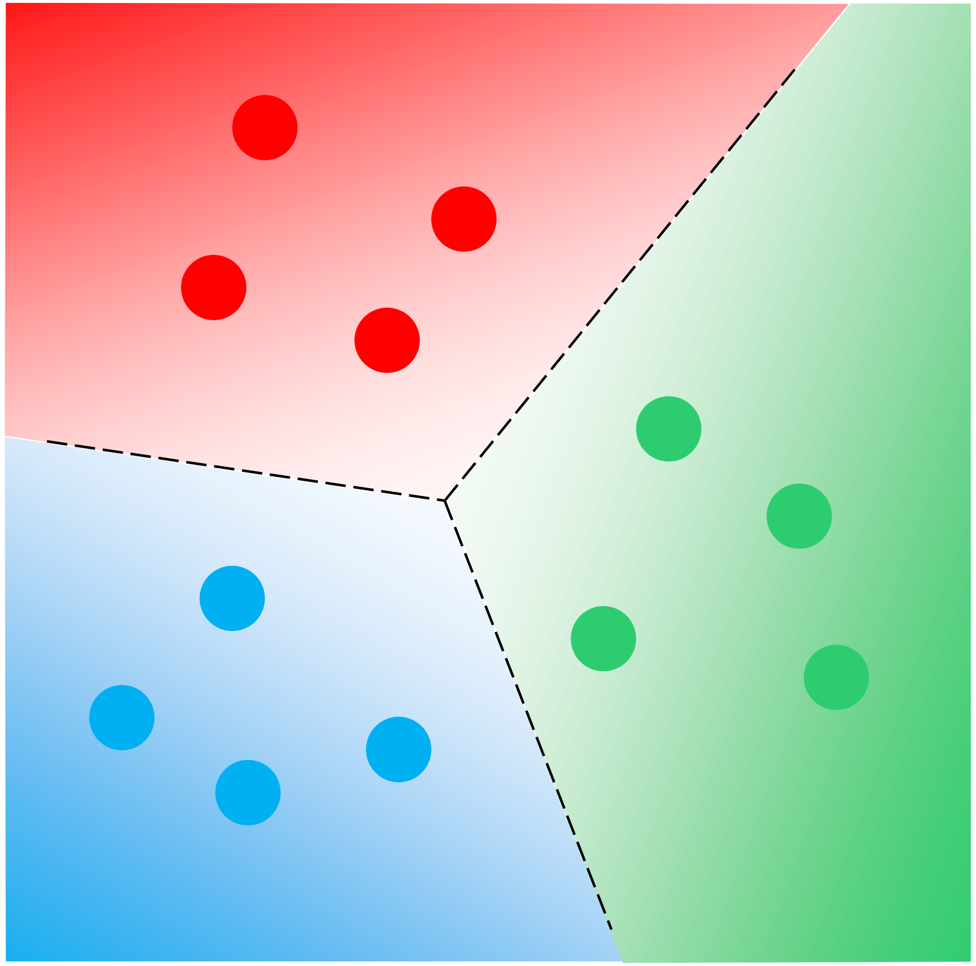
Multi-classification Loss Functions

One-vs-All (OVA)



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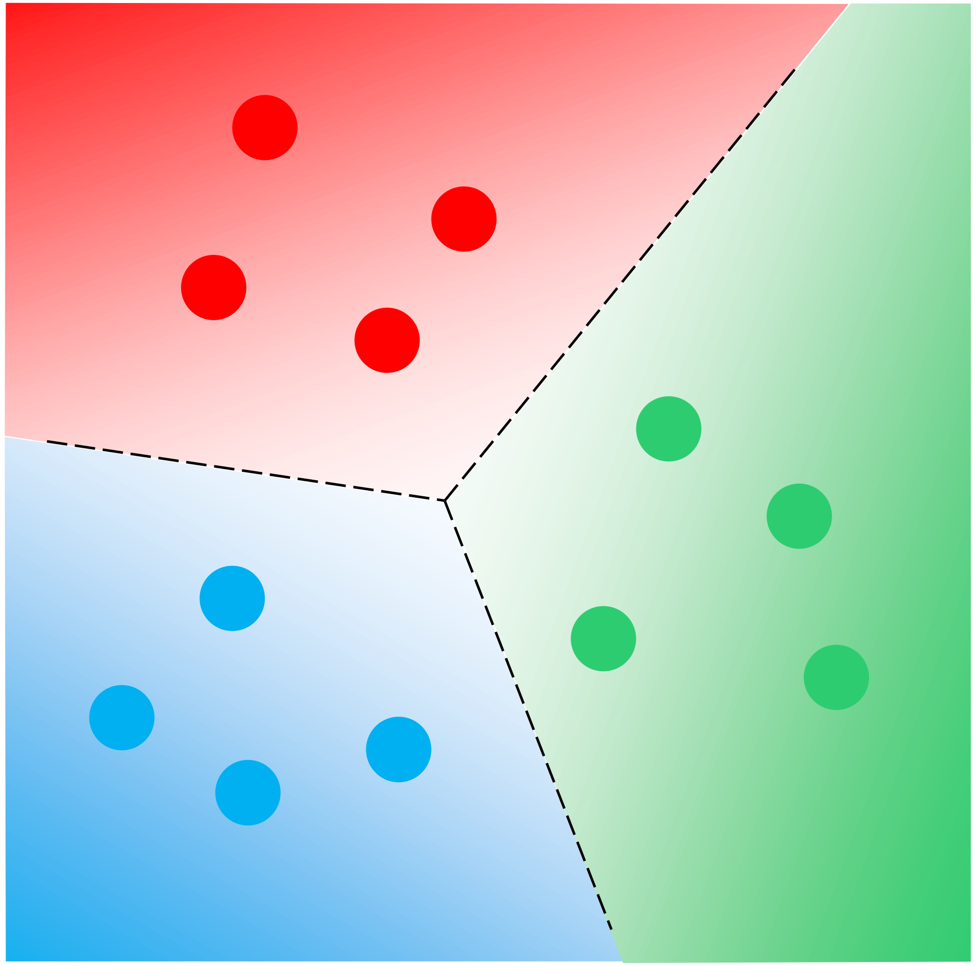
$$\ell_{\text{cs}}(y, \{\boldsymbol{\eta}_j\}) = [1 + \max_{k \neq y} \boldsymbol{\eta}_k - \boldsymbol{\eta}_y]_+$$

Multi-classification Loss Functions

One-vs-All (OVA)



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Exercise

Multi-classification Loss Functions

One-vs-All (OVA)

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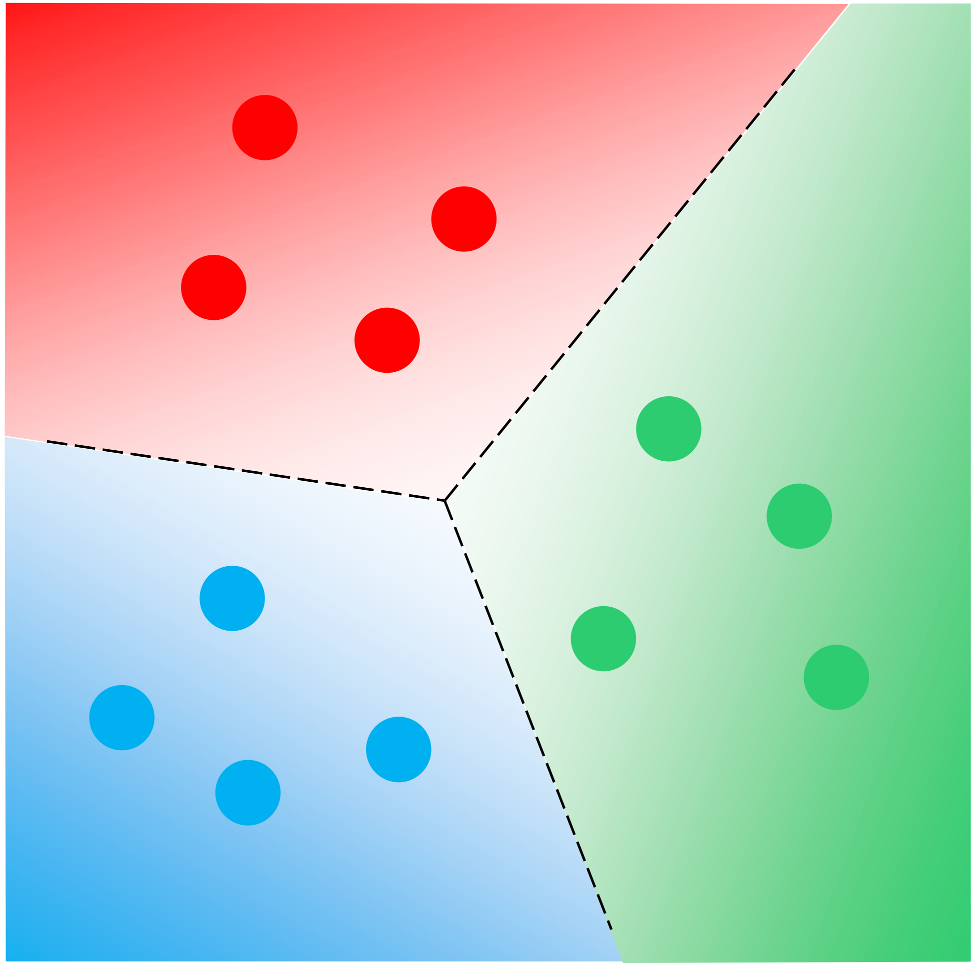

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$$\widehat{\mathbf{W}} = \arg\min_{\mathbf{W}} \sum_{k=1}^{K} \left\|\mathbf{w}^k\right\|_2^2 + \sum_{i=1}^{n} \ell_{\text{cs}}(y^i, \langle \mathbf{W}, \mathbf{x}^i \rangle)$$

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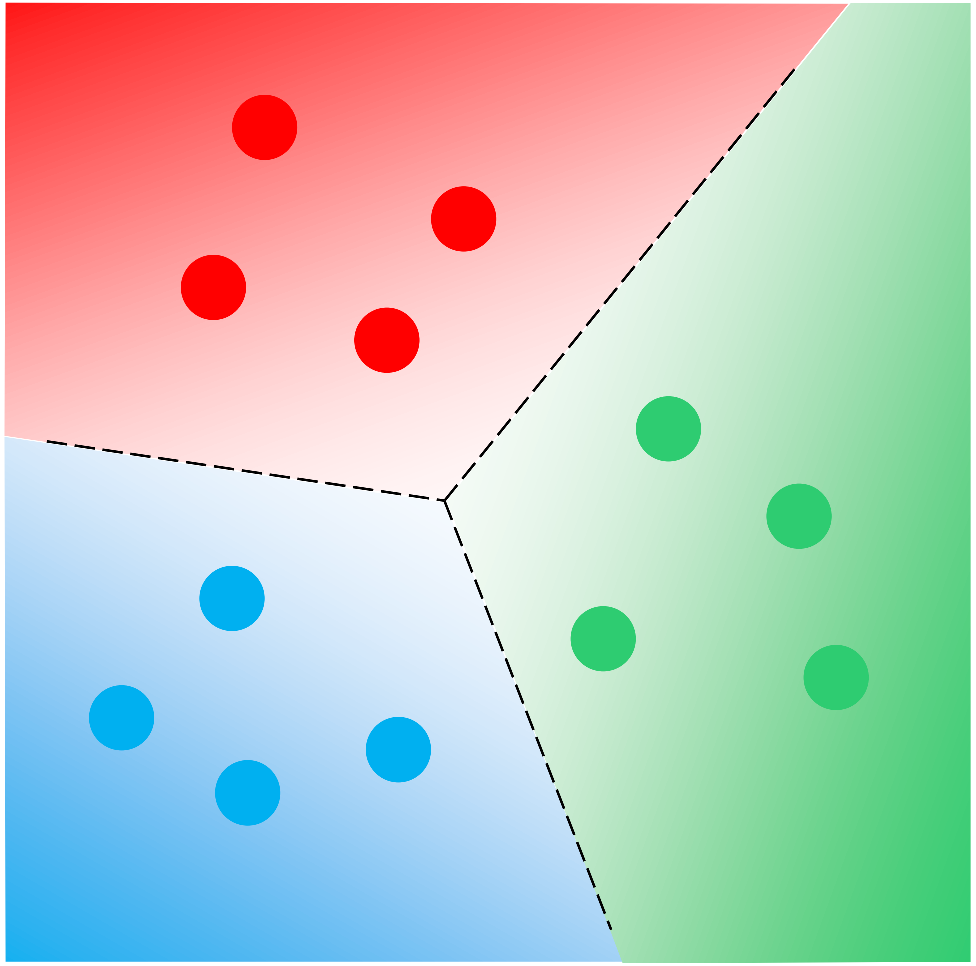
Exercise

Crammer-Singer loss function

Multi-classification Loss Functions

One-vs-All (OVA)



$$\hat{y}^i = \arg\max_{j \in [K]} \langle \mathbf{w}^j, \mathbf{x}^i \rangle$$

$$\mathbf{W} = [\mathbf{w}^1, \mathbf{w}^2, \ldots, \mathbf{w}^K]$$

$$\widehat{\mathbf{W}} = \arg\min_{\mathbf{W}} \sum_{k=1}^{K} \|\mathbf{w}^k\|_2^2 + \sum_{i=1}^{n} \ell_{\text{cs}}(y^i, \langle \mathbf{W}, \mathbf{x}^i \rangle)$$

$$\langle \mathbf{W}, \mathbf{x} \rangle = \boldsymbol{\eta} = [\boldsymbol{\eta}_1, \boldsymbol{\eta}_2, \ldots, \boldsymbol{\eta}_K]$$

$$\ell_{\text{cs}}(y, \{\boldsymbol{\eta}_j\}) = [1 + \max_{k \neq y} \boldsymbol{\eta}_k - \boldsymbol{\eta}_y]_+$$

Exercise

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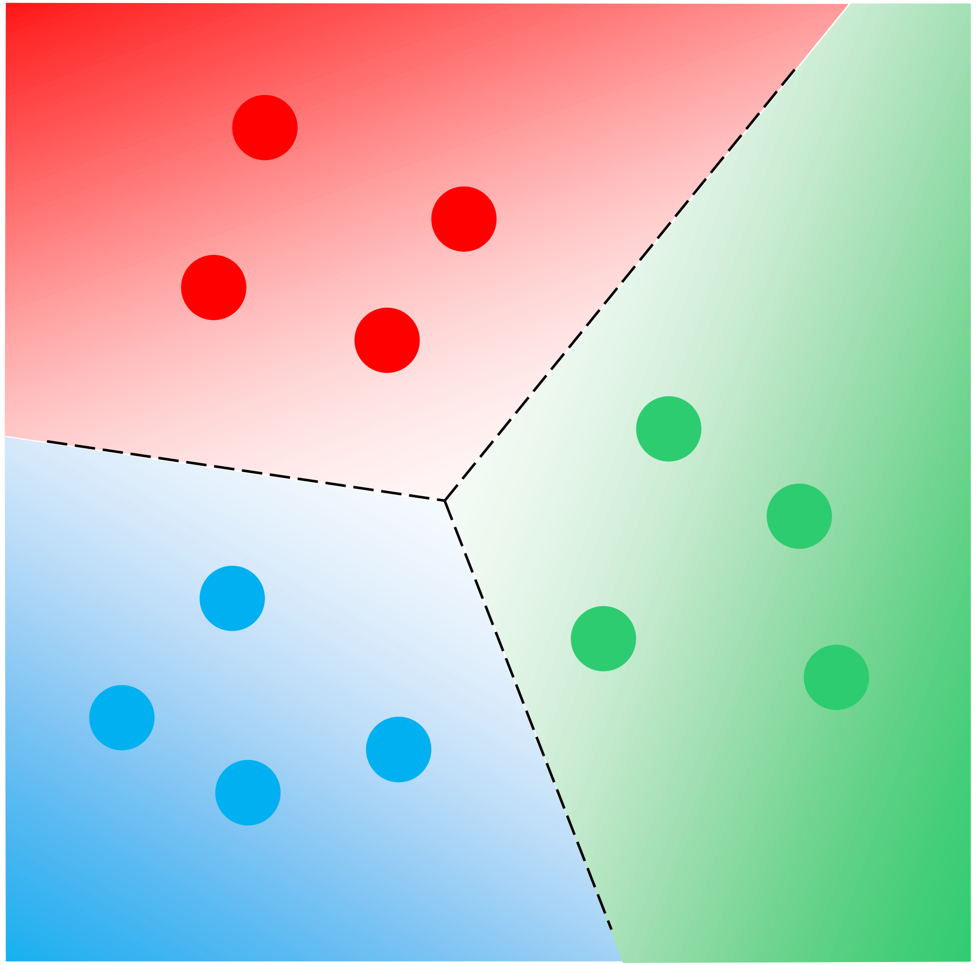

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$$\ell_{\text{cs}}(y, \{\boldsymbol{\eta}_j\} \quad + \max_{k \neq y} \boldsymbol{\eta}_k - \boldsymbol{\eta}_y]_+$$

Exercise

More powerful regularizers?

$\|\mathbf{W}\|_F^2, \|\mathbf{W}\|_*, \|\mathbf{W}\|_0, \|\mathbf{W}^\top\|_{2,0}$

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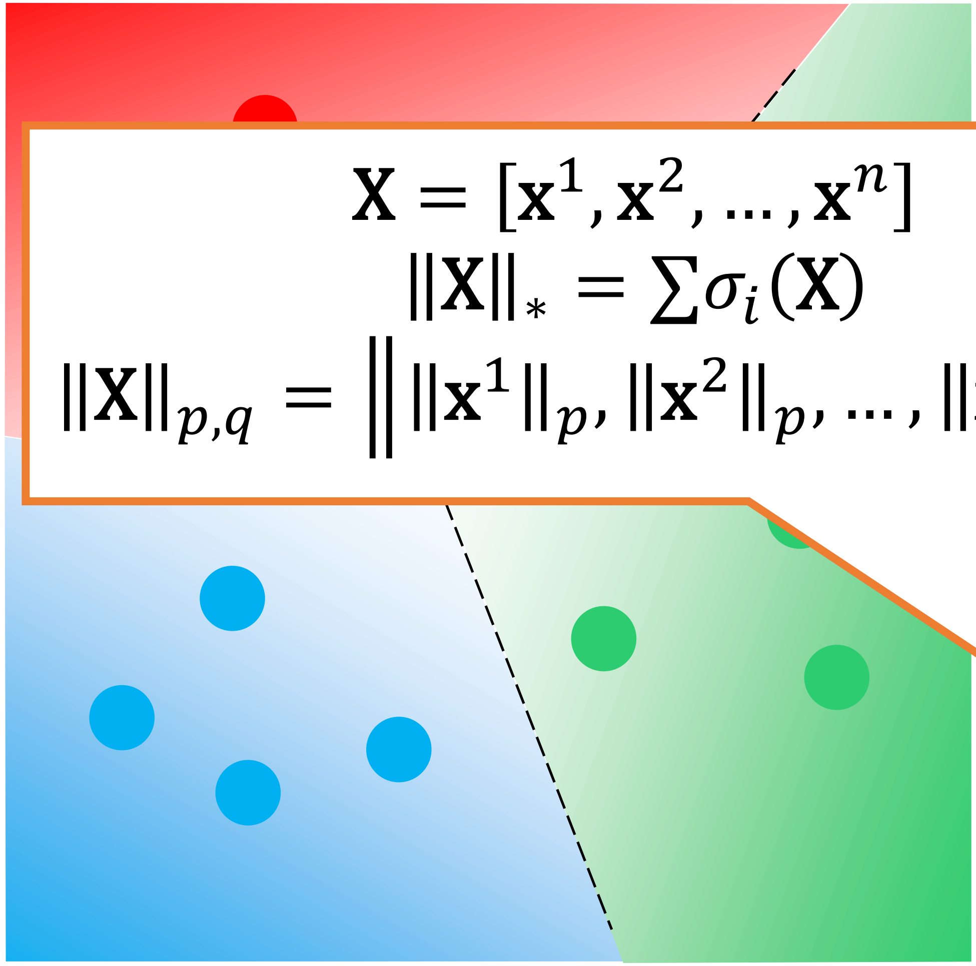

$$\mathbf{X} = [\mathbf{x}^1, \mathbf{x}^2, \ldots, \mathbf{x}^n]$$

$$\|\mathbf{X}\|_* = \sum \sigma_i(\mathbf{X})$$

$$\|\mathbf{X}\|_{p,q} = \Big\| \|\mathbf{x}^1\|_p, \|\mathbf{x}^2\|_p, \ldots, \|\mathbf{x}^n\|_p \Big\|_q$$

$$\text{rg}\min_{\mathbf{W}} \sum_{k=1}^{K} \|\mathbf{w}^k\|_2^2 + \sum_{i=1}^{n} \ell_{\text{cs}}(y^i, \langle \mathbf{W}, \mathbf{x}^i \rangle)$$

$$\mathbf{W}, \mathbf{x}\rangle = \boldsymbol{\eta} \quad [\boldsymbol{\eta}_1, \boldsymbol{\eta}_2, \ldots, \boldsymbol{\eta}_K]$$

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$$\hat{y}^i = \arg\max_{j \in [K]} \sum_{k \neq j} \left\langle \mathbf{w}^{j,k}, \mathbf{x}^i \right\rangle$$

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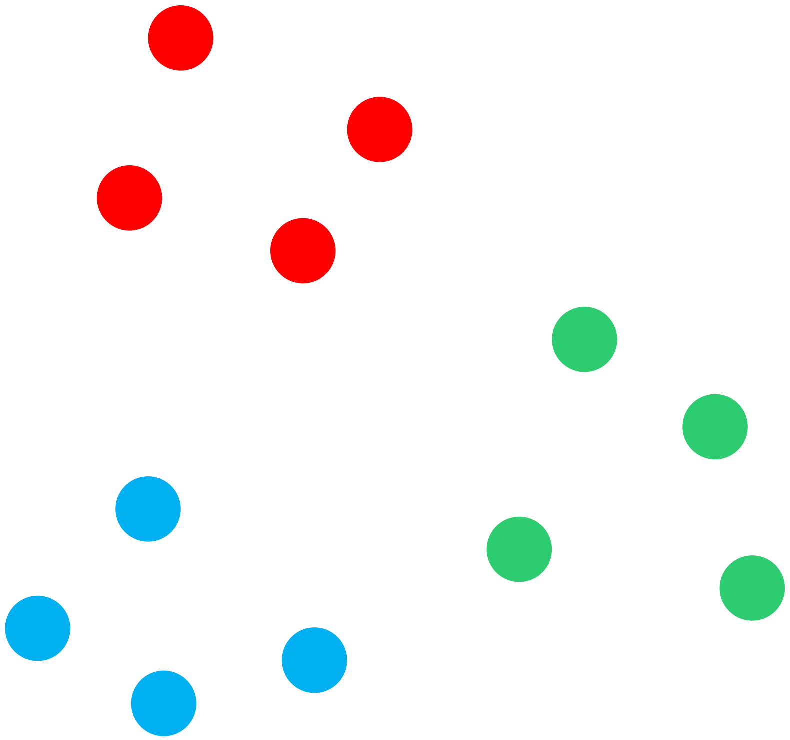

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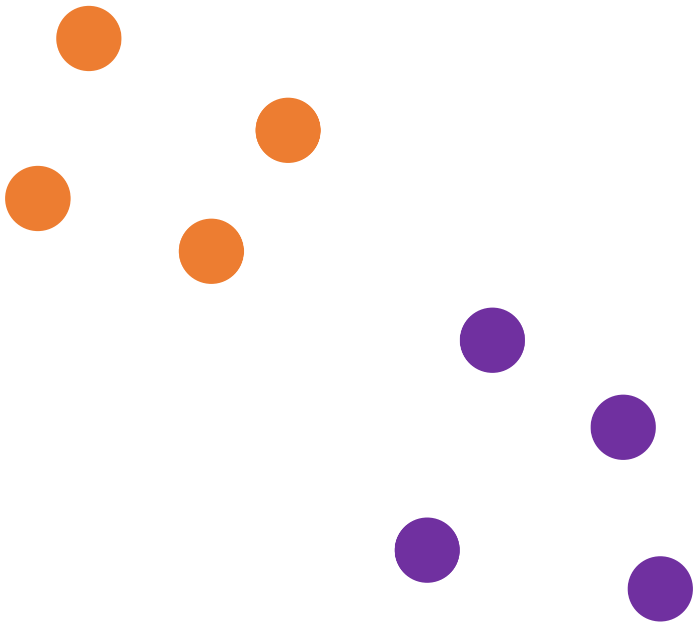

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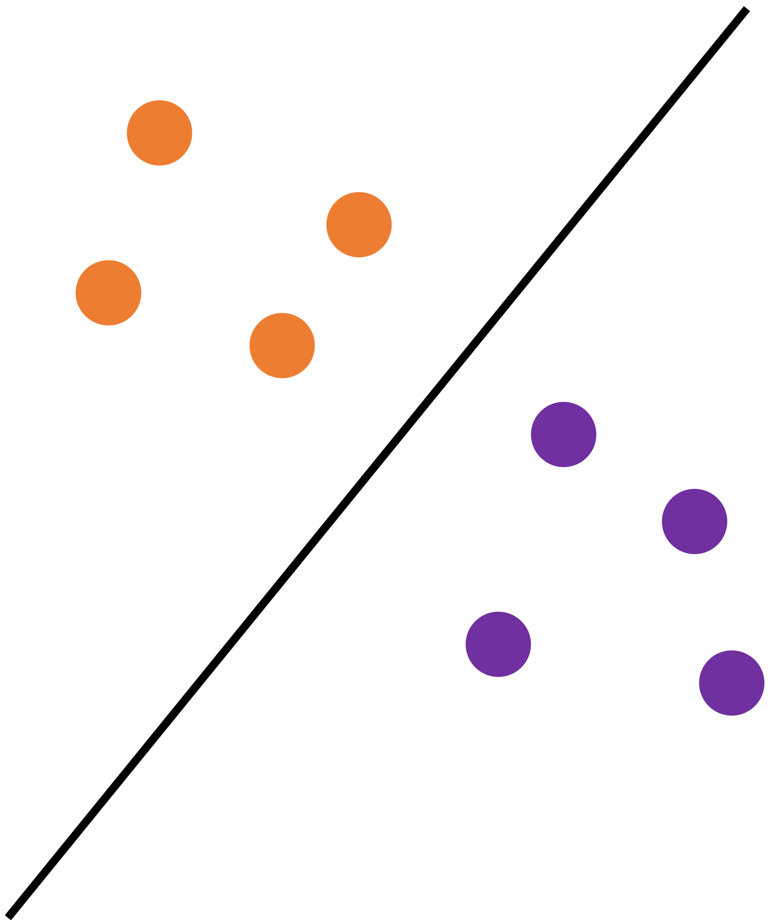

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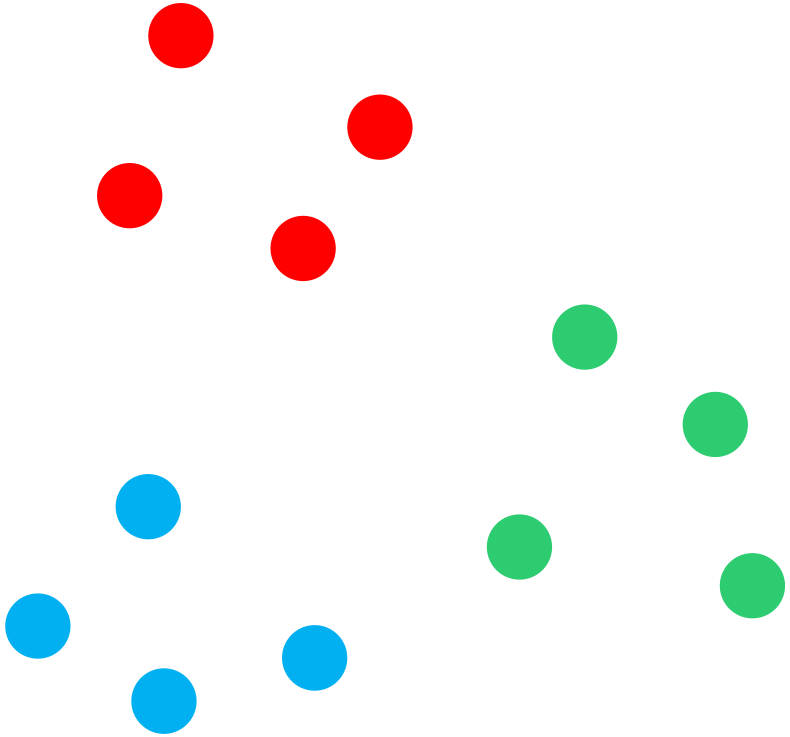

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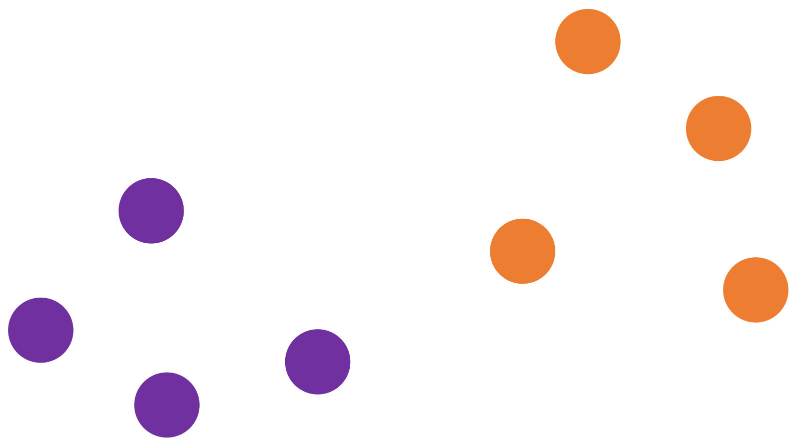

Multi-classification Loss Functions

$\boxed{\text{All-vs-All (AVA)}}$

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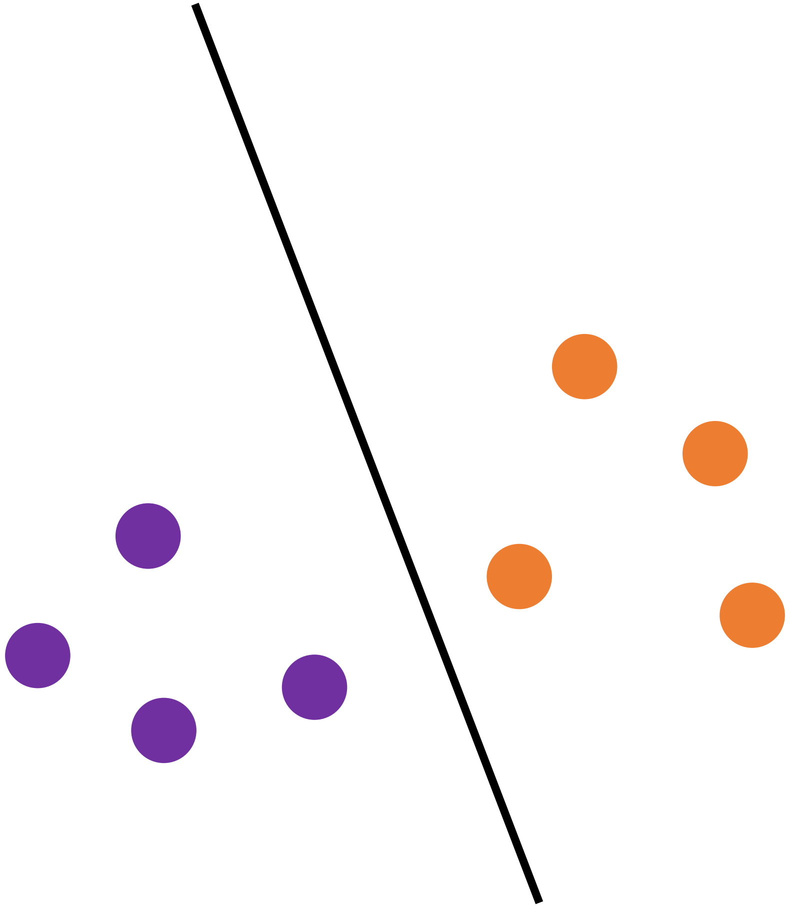

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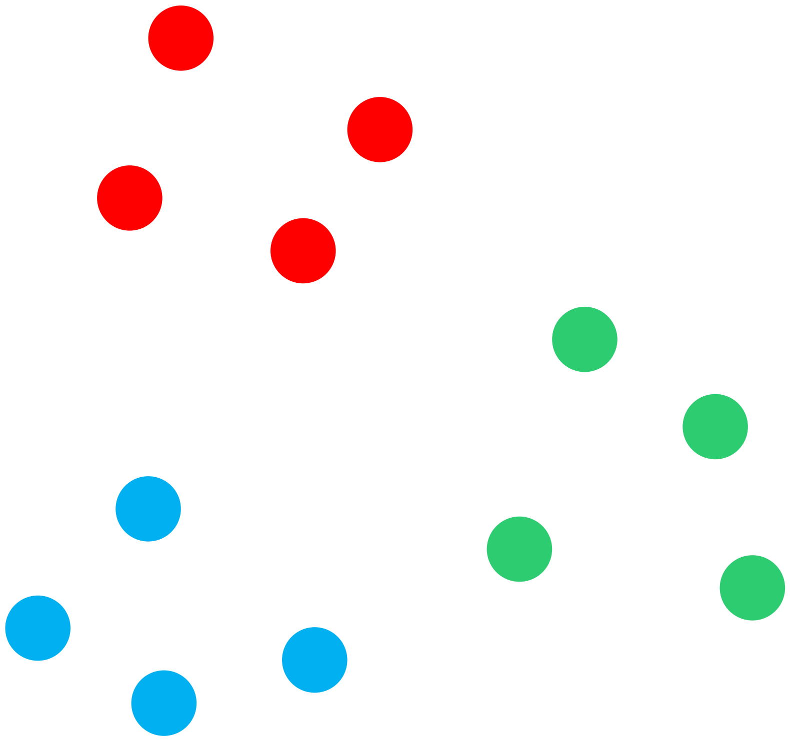

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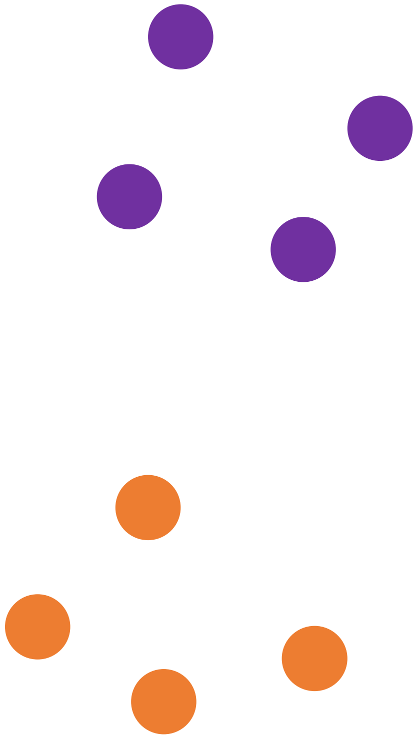

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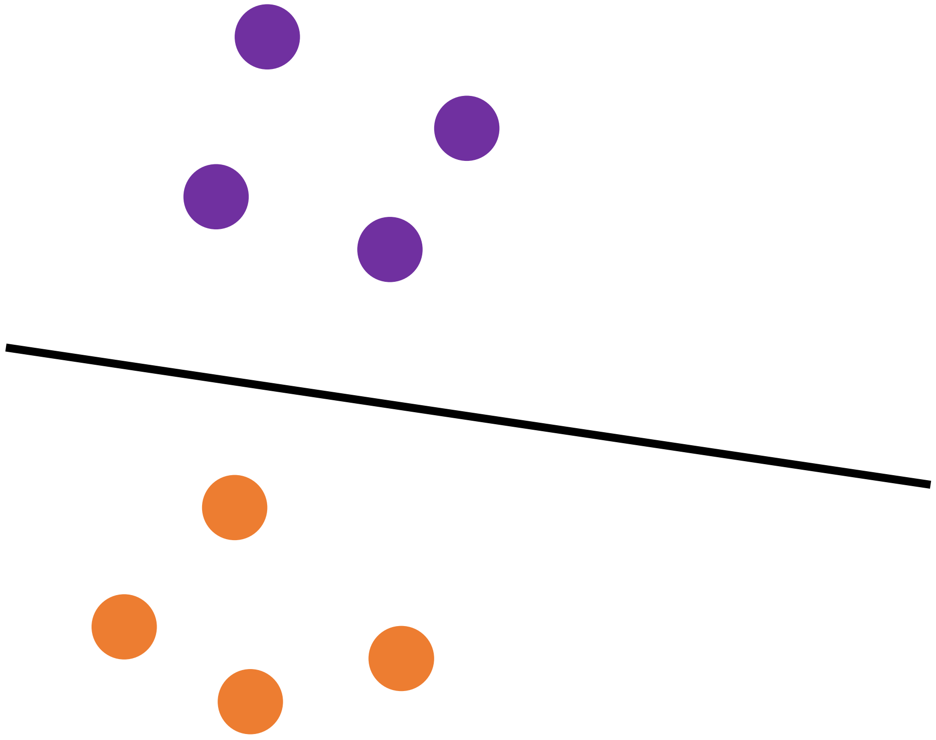

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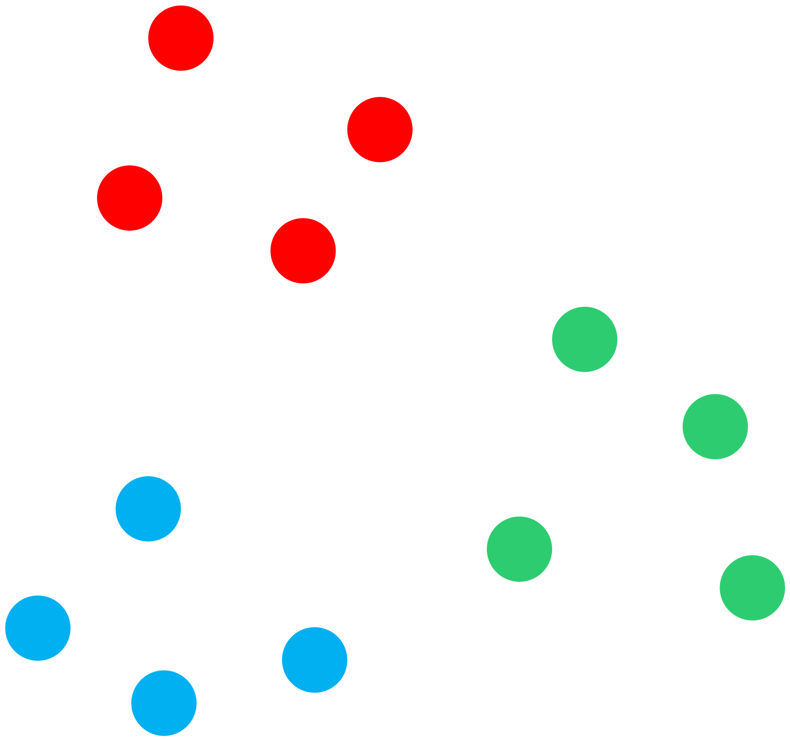

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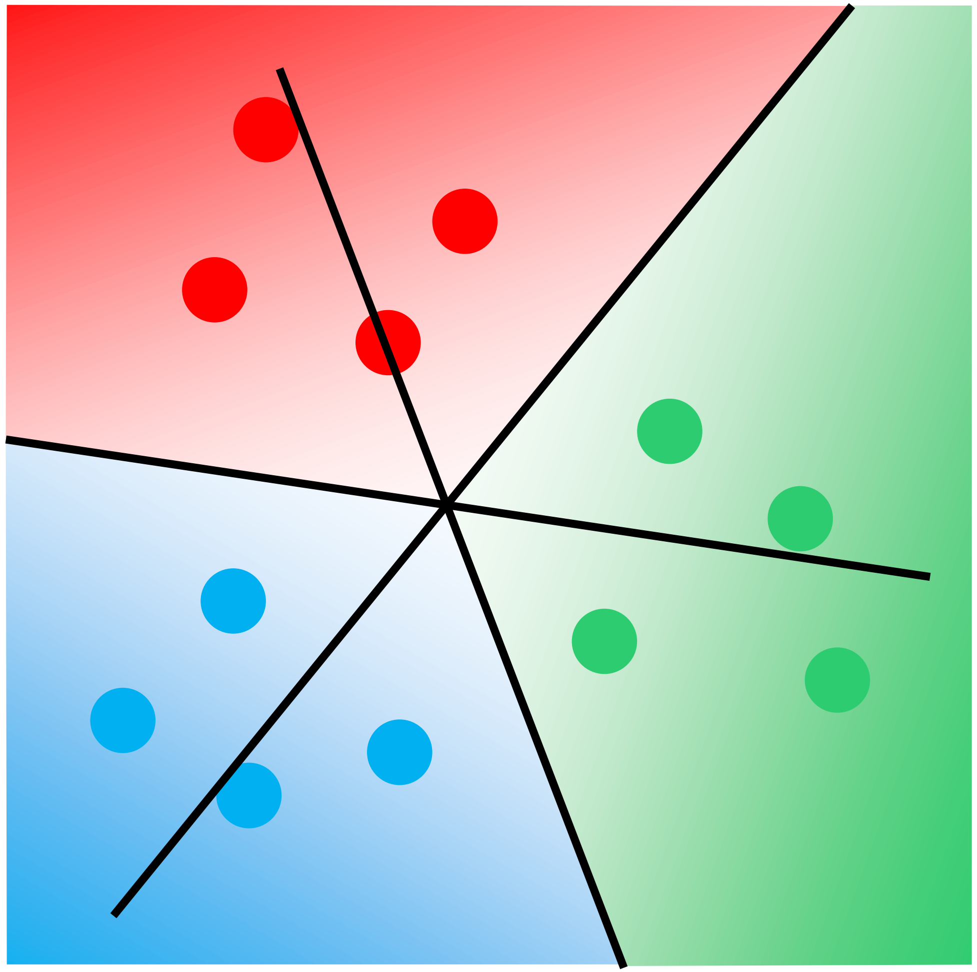

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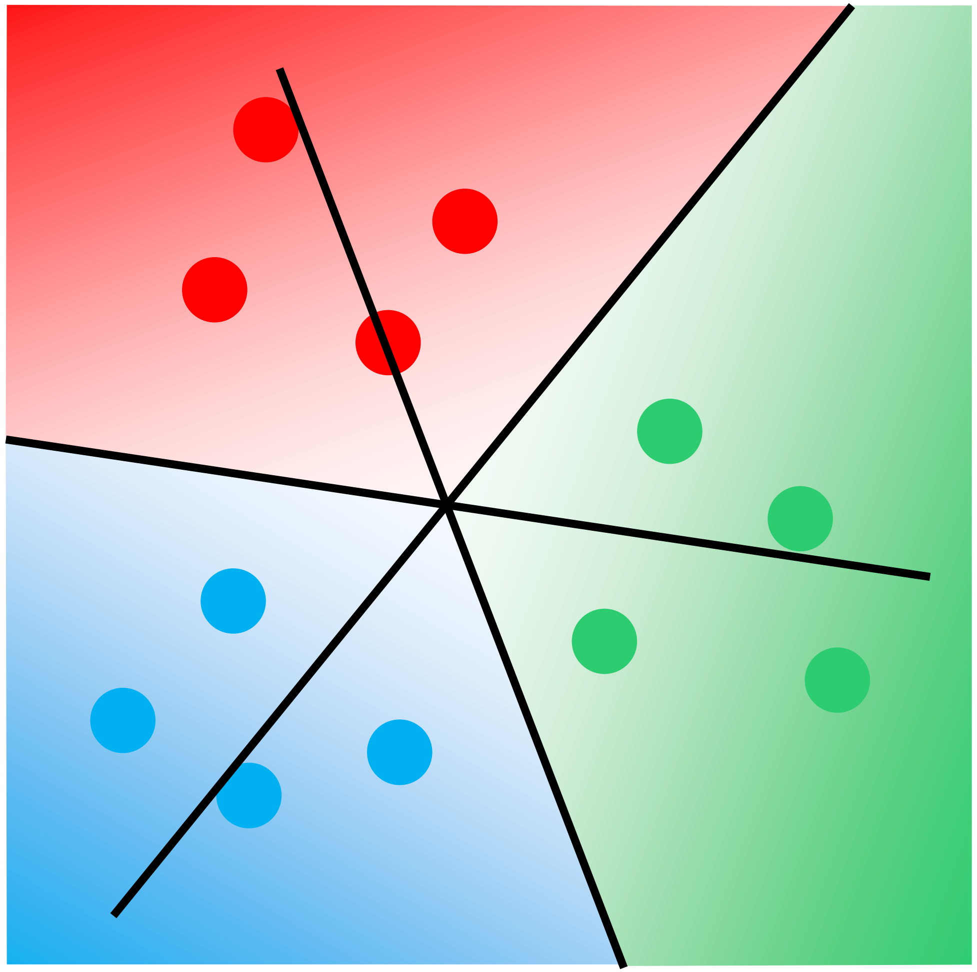

Expensive!

Multi-classification Loss Functions

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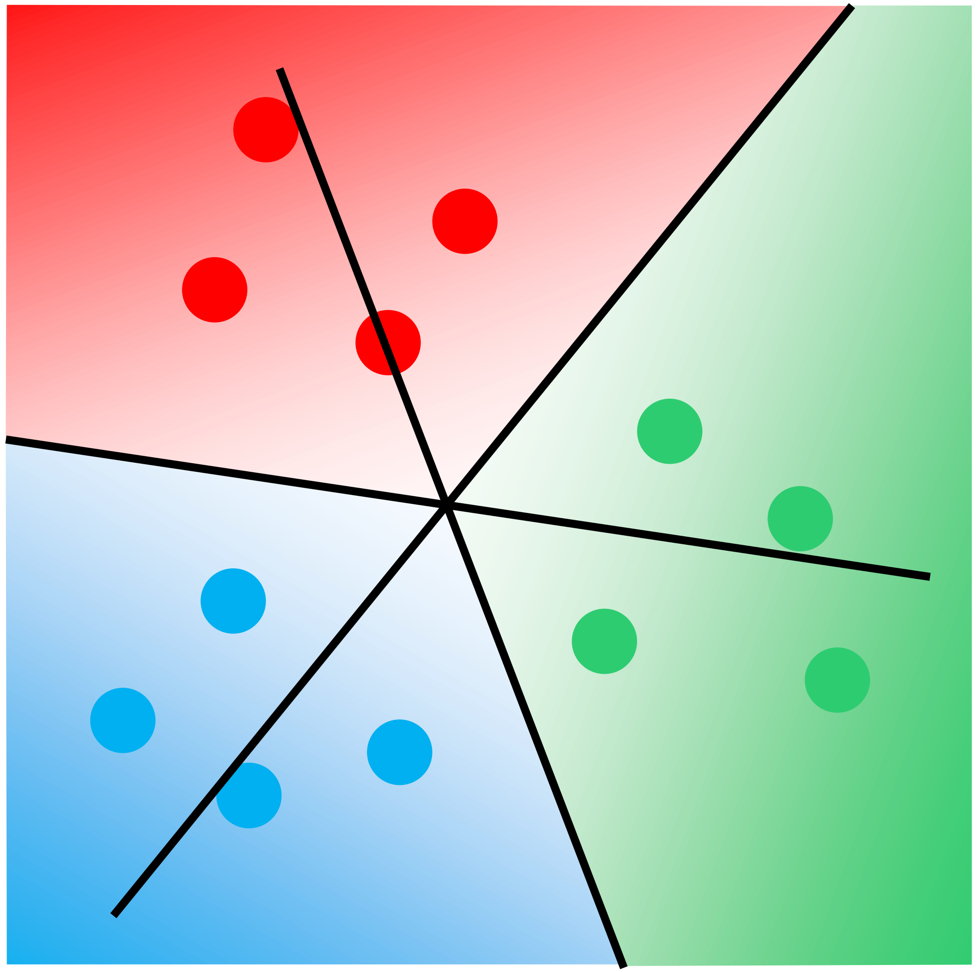

Expensive!

Other techniques: use DT to eliminate classes, error correcting codes

Multi-classification Loss Functions

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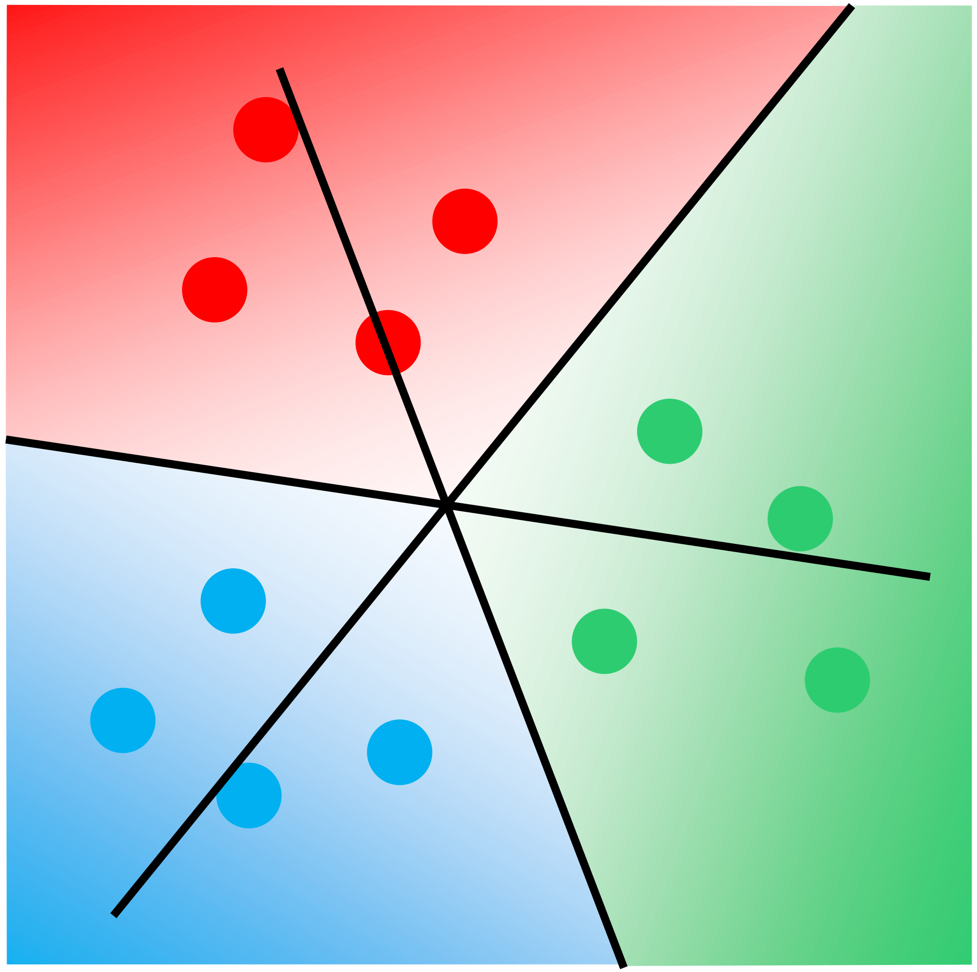


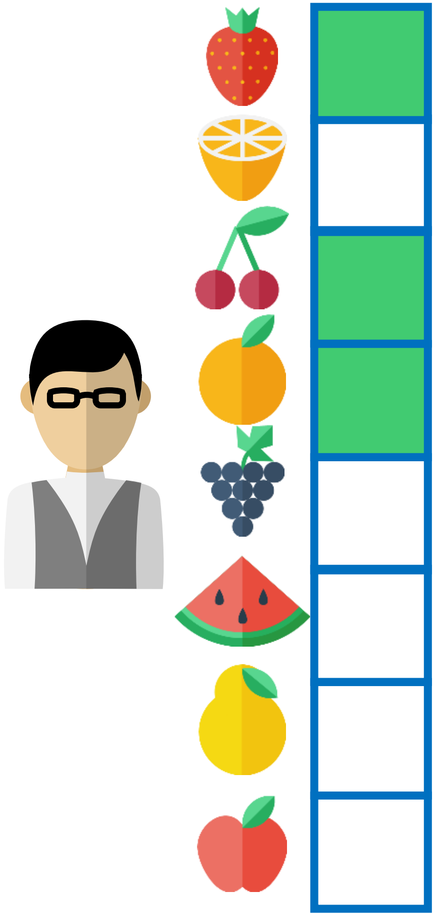
Expensive!

Other techniques: use DT to eliminate classes, error correcting codes

Exercise: Develop training techniques for AVA classifiers

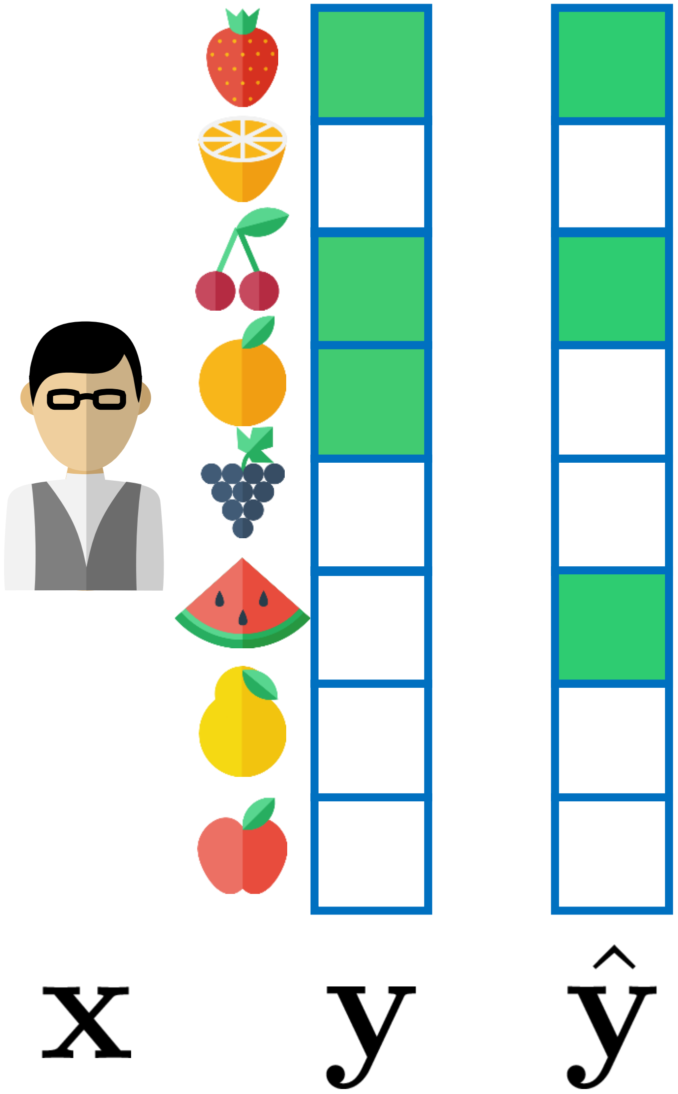
Multi-label Classification Loss Functions

Multi-label Classification Loss Functions

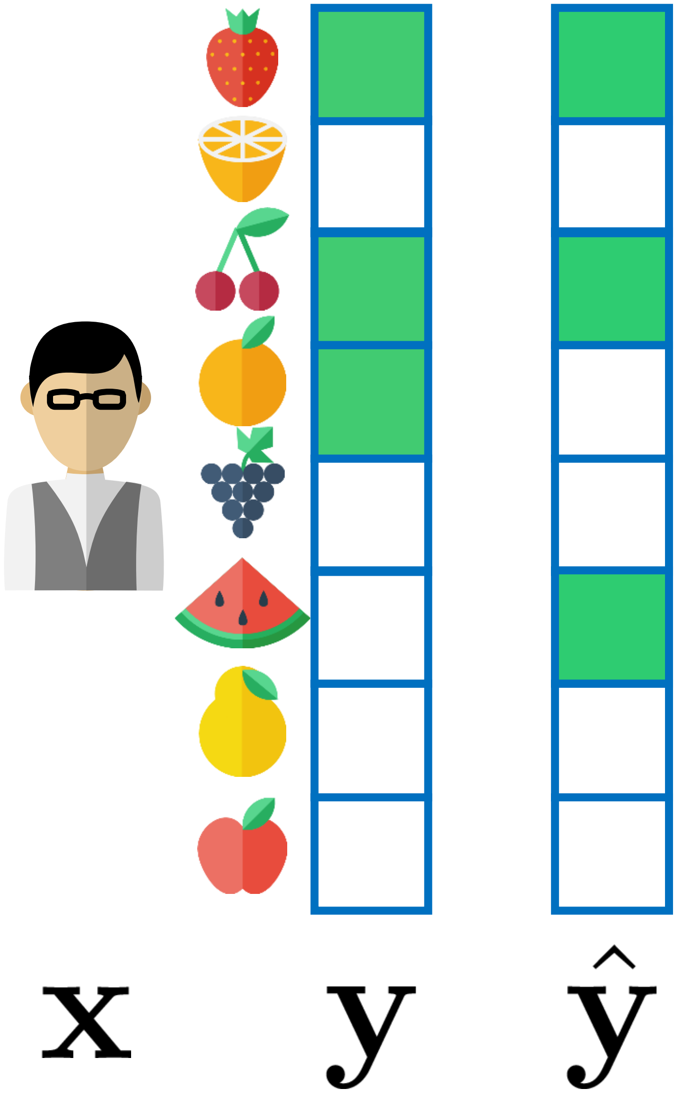


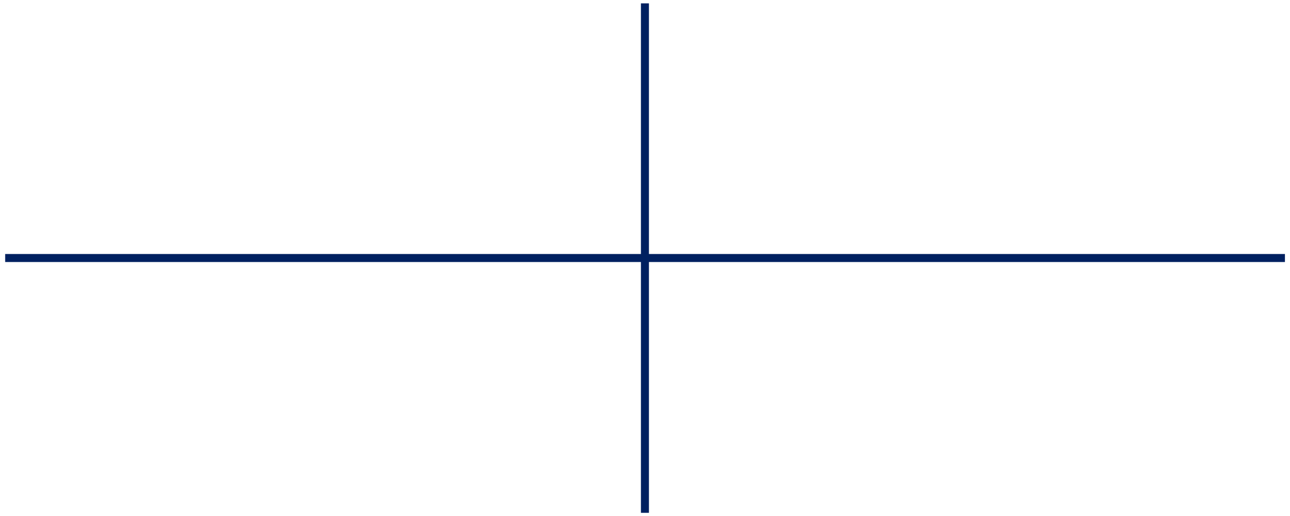
$\mathbf{x}$ $\mathbf{y}$

Multi-label Classification Loss Functions



Multi-label Classification Loss Functions





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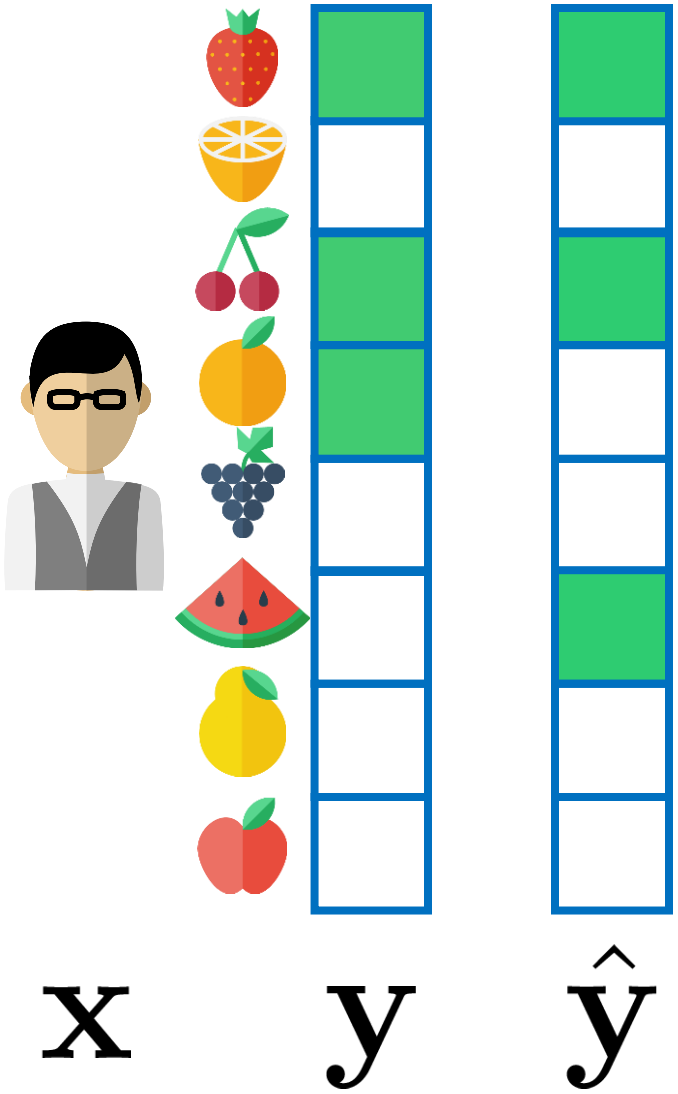
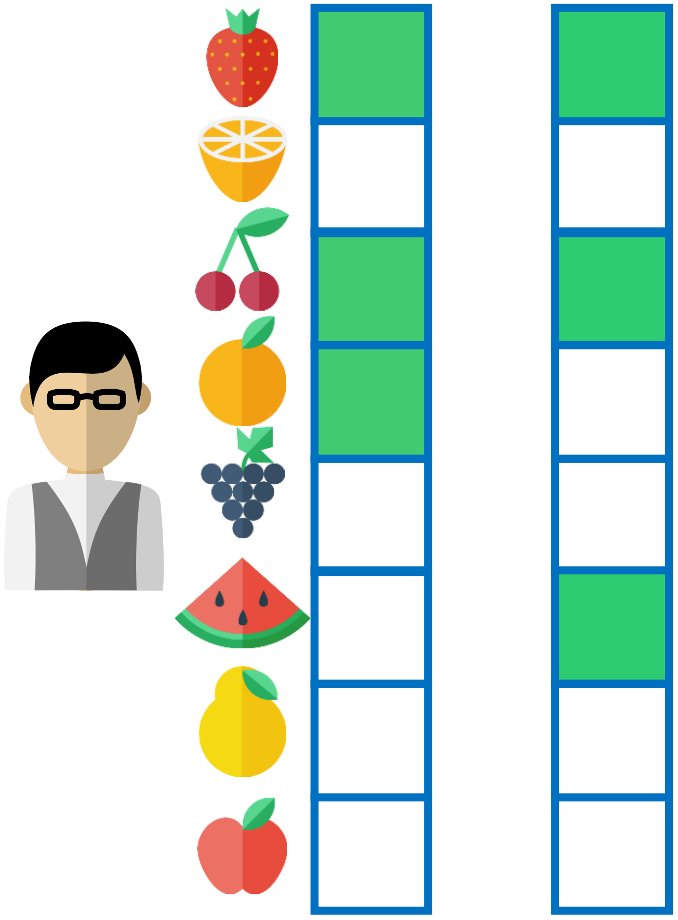


$$\left|\left\{ i : \begin{array}{l} \hat{\mathbf{y}}_i = 1 \\ \mathbf{y}_i = 1 \end{array} \right\}\right|$$

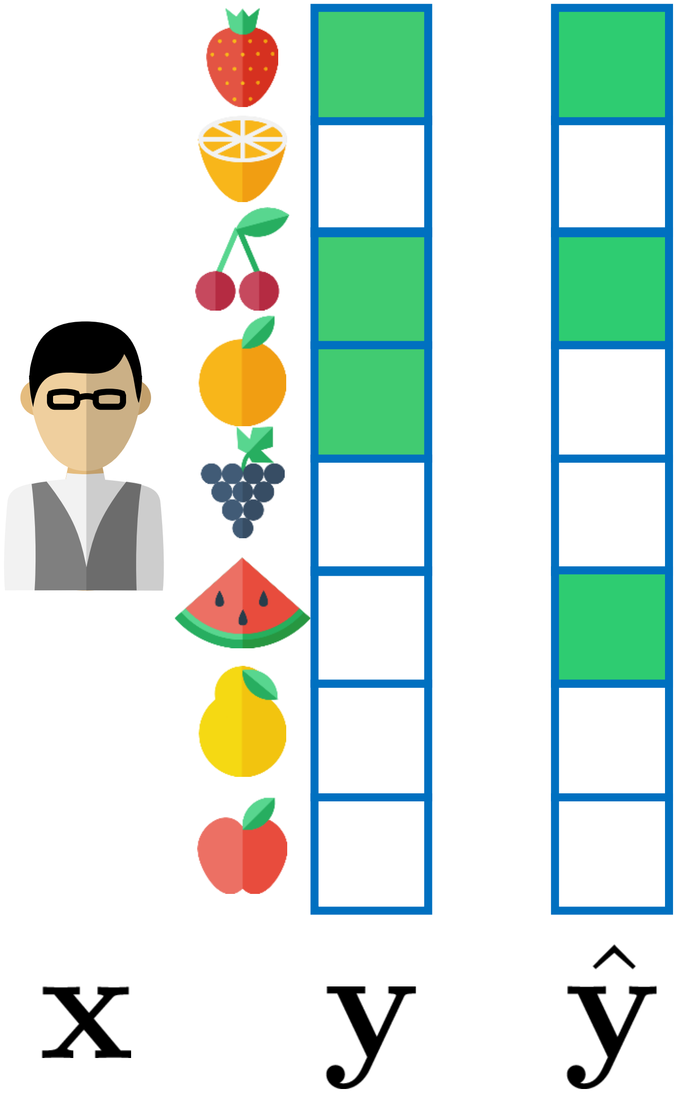
Multi-label Classification Loss Functions



$\mathbf{x}$ $\quad$ $\mathbf{y}$ $\quad$ $\hat{\mathbf{y}}$

$\left\lvert\left\{ i : \begin{array}{c} \hat{\mathbf{y}}_i = 1 \\ \mathbf{y}_i = 1 \end{array} \right\}\right\rvert$	$\left\lvert\left\{ i : \begin{array}{c} \hat{\mathbf{y}}_i = 1 \\ \mathbf{y}_i = -1 \end{array} \right\}\right\rvert$

Multi-label Classification Loss Functions



$$\left|\left\{ i : \begin{array}{c} \hat{\mathbf{y}}_i = 1 \\ \mathbf{y}_i = 1 \end{array} \right\}\right| \qquad \left|\left\{ i : \begin{array}{c} \hat{\mathbf{y}}_i = 1 \\ \mathbf{y}_i = -1 \end{array} \right\}\right|$$

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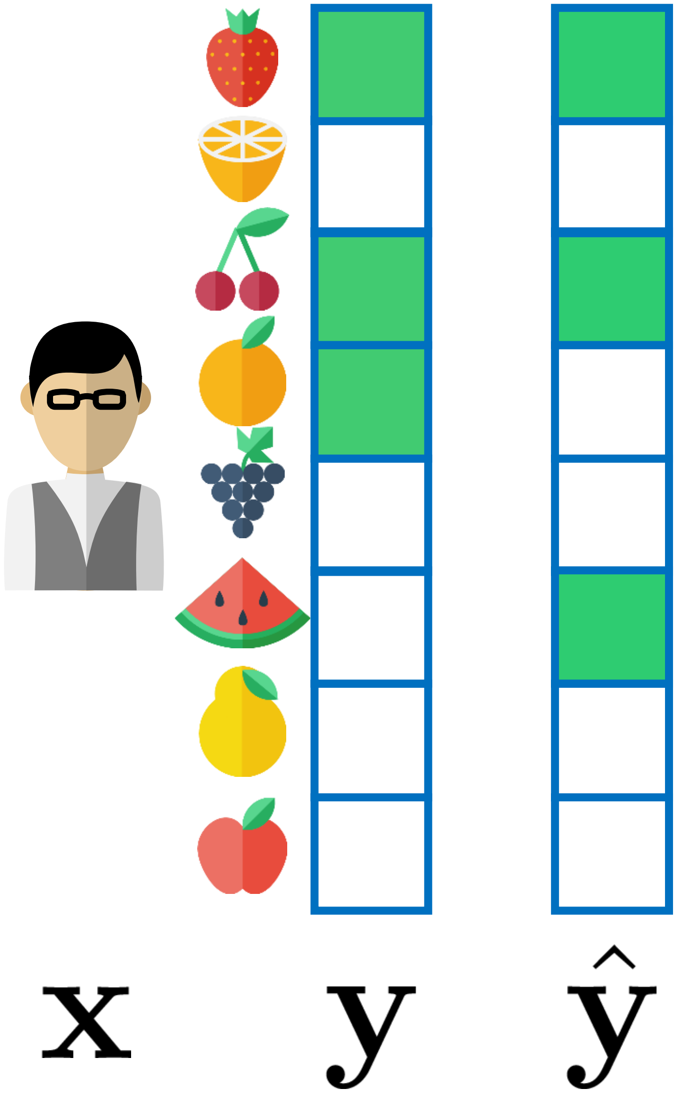
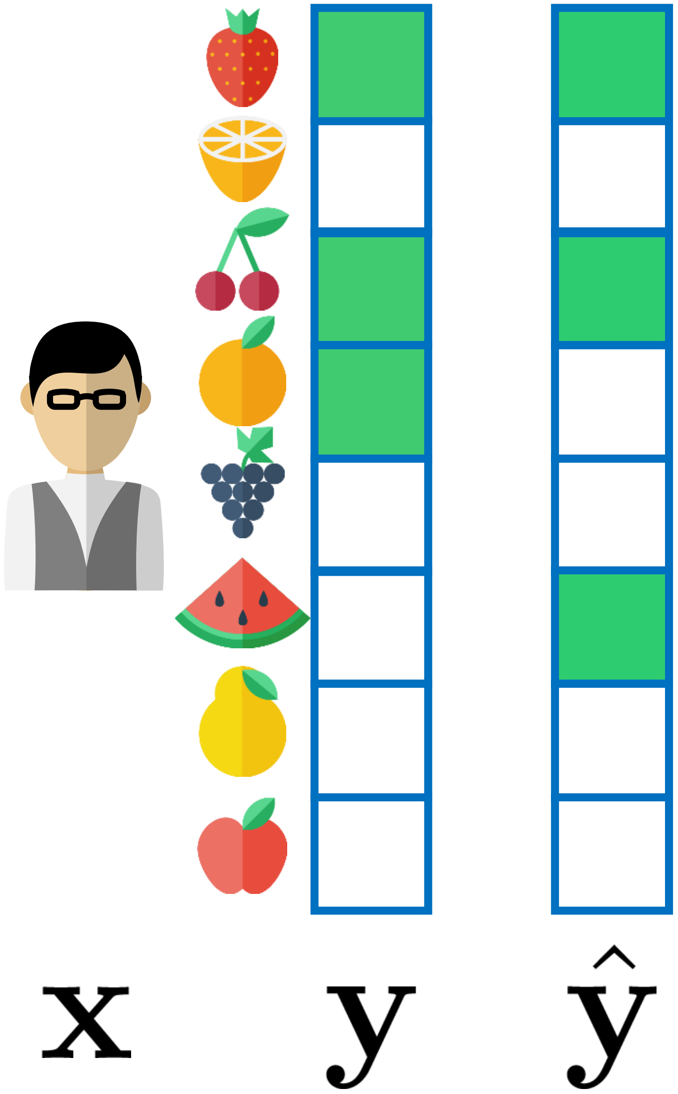
Multi-label Classification Loss Functions



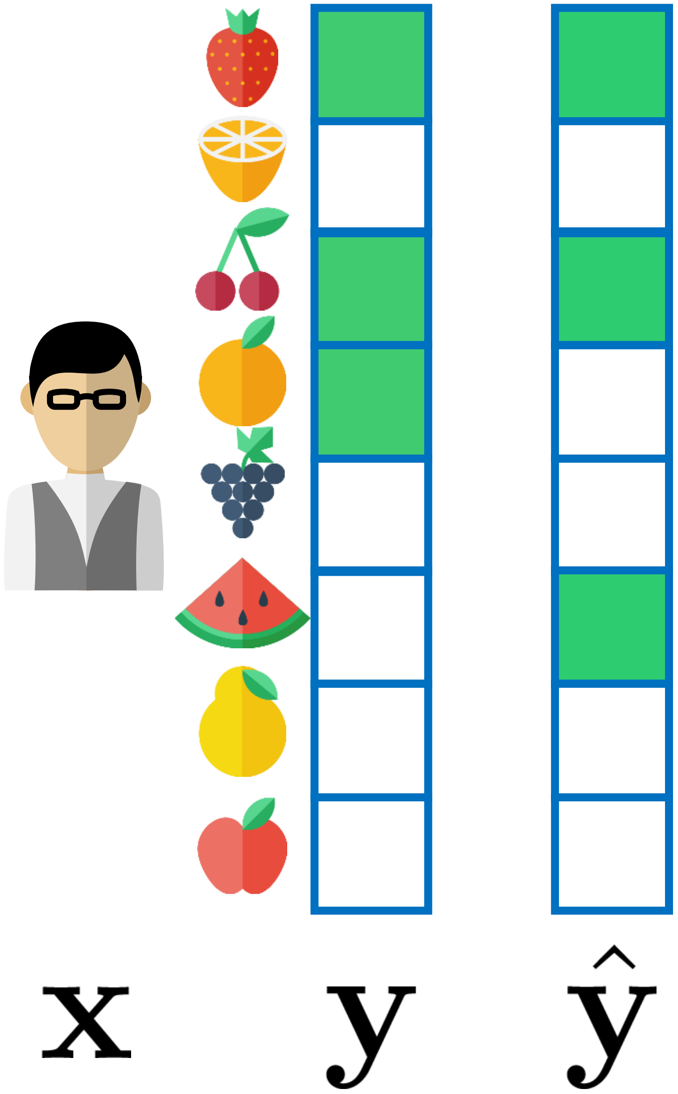
$$
\begin{array}{c|c}
\left|\left\{ i : \begin{array}{c} \hat{\mathbf{y}}_i = 1 \\ \mathbf{y}_i = 1 \end{array} \right\}\right| & \left|\left\{ i : \begin{array}{c} \hat{\mathbf{y}}_i = 1 \\ \mathbf{y}_i = -1 \end{array} \right\}\right| \\
\hline
\left|\left\{ i : \begin{array}{c} \hat{\mathbf{y}}_i = -1 \\ \mathbf{y}_i = 1 \end{array} \right\}\right| & \left|\left\{ i : \begin{array}{c} \hat{\mathbf{y}}_i = -1 \\ \mathbf{y}_i = -1 \end{array} \right\}\right|
\end{array}
$$

Multi-label Classification Loss Functions



a: $\left\lvert\left\{ i : \begin{matrix} \hat{\mathbf{y}}_i = 1 \\ \mathbf{y}_i = 1 \end{matrix} \right\}\right\rvert$	b: $\left\lvert\left\{ i : \begin{matrix} \hat{\mathbf{y}}_i = 1 \\ \mathbf{y}_i = -1 \end{matrix} \right\}\right\rvert$
c: $\left\lvert\left\{ i : \begin{matrix} \hat{\mathbf{y}}_i = -1 \\ \mathbf{y}_i = 1 \end{matrix} \right\}\right\rvert$	d: $\left\lvert\left\{ i : \begin{matrix} \hat{\mathbf{y}}_i = -1 \\ \mathbf{y}_i = -1 \end{matrix} \right\}\right\rvert$

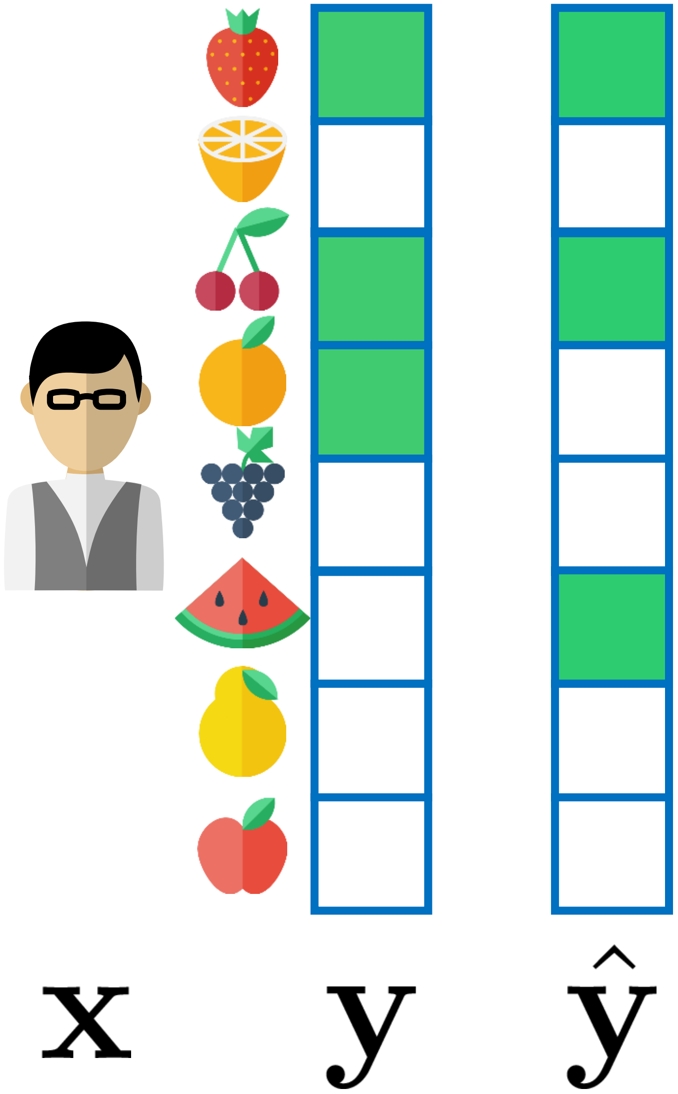
Multi-label Classification Loss Functions



$\left\lvert\left\{i : \begin{array}{c}\hat{\mathbf{y}}_i = 1 \\ \mathbf{y}_i = 1\end{array}\right\}\right\rvert$ a	$\left\lvert\left\{i : \begin{array}{c}\hat{\mathbf{y}}_i = 1 \\ \mathbf{y}_i = -1\end{array}\right\}\right\rvert$ b
$\left\lvert\left\{i : \begin{array}{c}\hat{\mathbf{y}}_i = -1 \\ \mathbf{y}_i = 1\end{array}\right\}\right\rvert$ c	$\left\lvert\left\{i : \begin{array}{c}\hat{\mathbf{y}}_i = -1 \\ \mathbf{y}_i = -1\end{array}\right\}\right\rvert$ d

$$\ell_{\text{Hamming}}(\mathbf{y}, \hat{\mathbf{y}}) = \frac{b + c}{a + b + c + d}$$

Multi-label Classification Loss Functions



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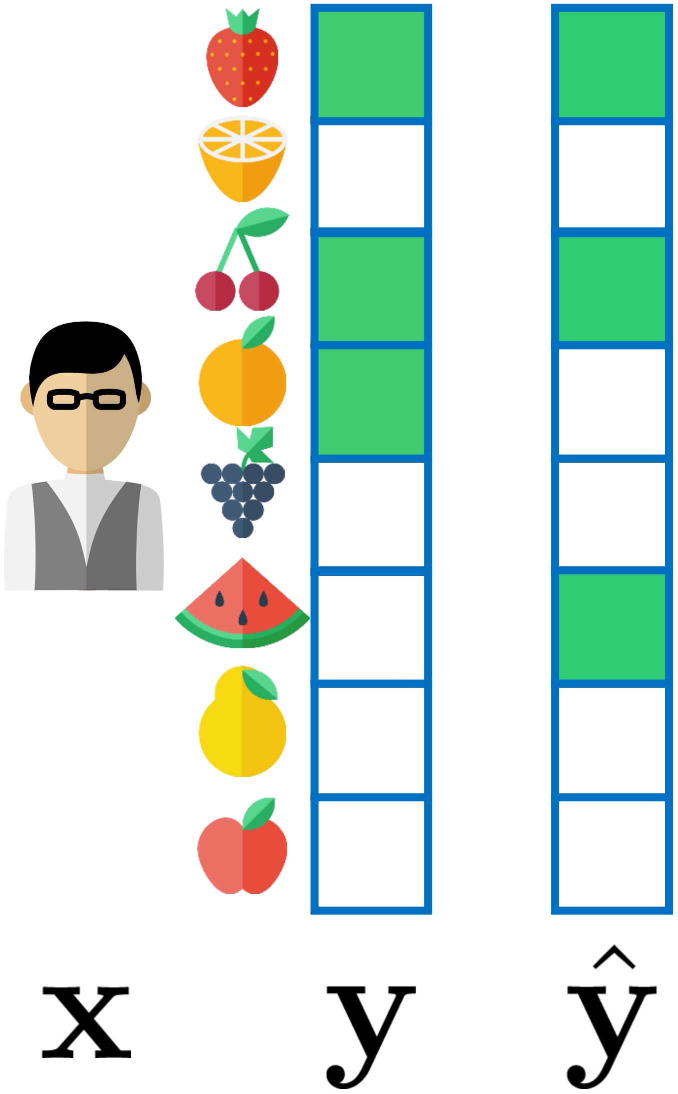
$$\ell_{\text{Hamming}}(\mathbf{y}, \hat{\mathbf{y}}) = \frac{b + c}{a + b + c + d}$$

$$r_{\text{Precision}}(\mathbf{y}, \hat{\mathbf{y}}) = \frac{a}{a + b}$$

Multi-label Classification Loss Functions



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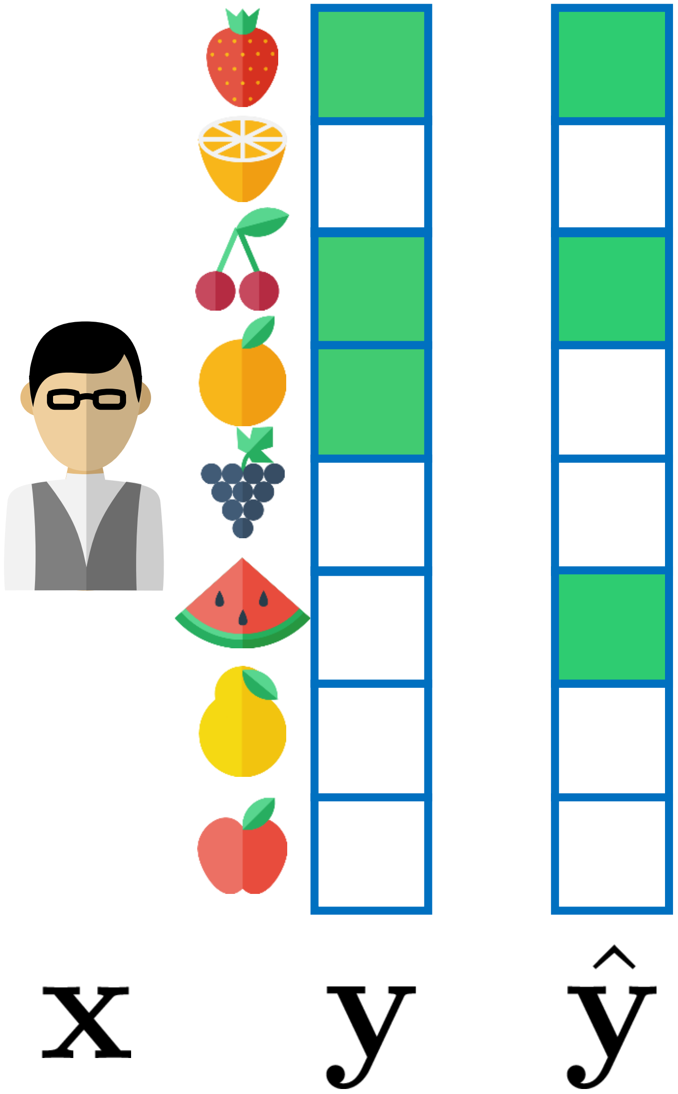
$$\ell_{\text{Hamming}}(\mathbf{y}, \hat{\mathbf{y}}) = \frac{b+c}{a+b+c+d}$$

$$r_{\text{Precision}}(\mathbf{y}, \hat{\mathbf{y}}) = \frac{a}{a+b} \qquad r_{\text{Recall}}(\mathbf{y}, \hat{\mathbf{y}}) = \frac{a}{a+c}$$

Multi-label Classification Loss Functions



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$$r_{\text{Precision}}(\mathbf{y}, \hat{\mathbf{y}}) = \frac{a}{a+b} \qquad r_{\text{Recall}}(\mathbf{y}, \hat{\mathbf{y}}) = \frac{a}{a+c}$$

$$F(\mathbf{y}, \hat{\mathbf{y}}) = \frac{2 r_{\text{Prec}} \cdot r_{\text{Rec}}}{r_{\text{Prec}} + r_{\text{Rec}}} = \frac{2a}{2a+b+c}$$

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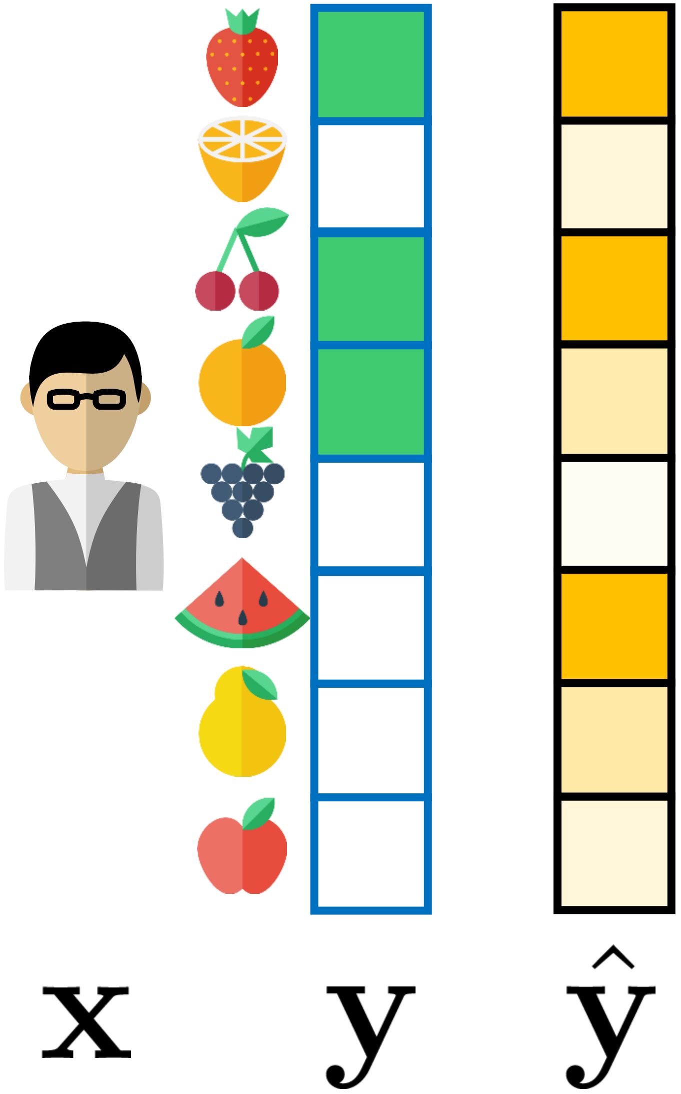

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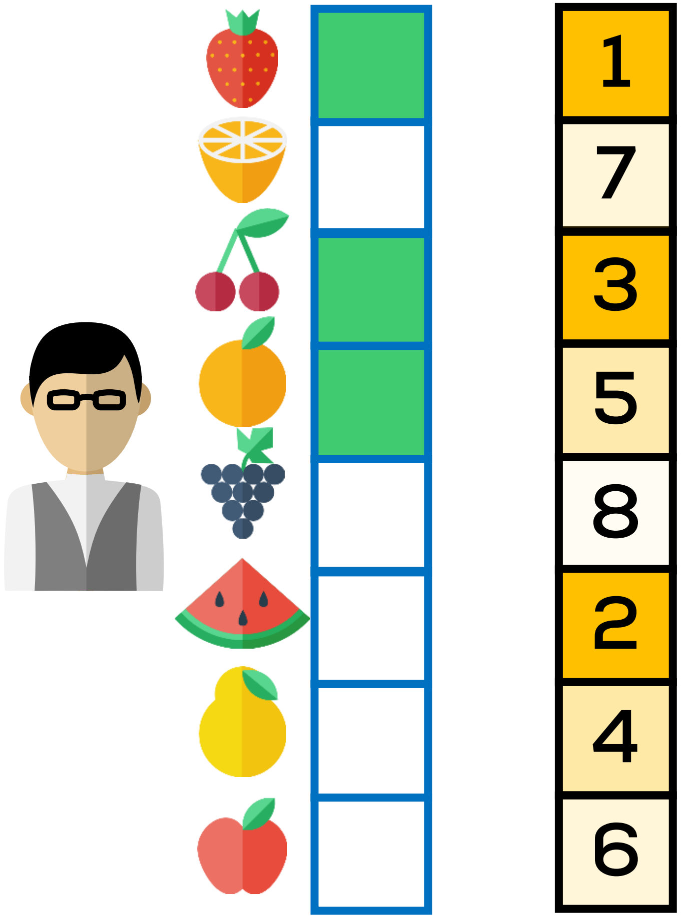
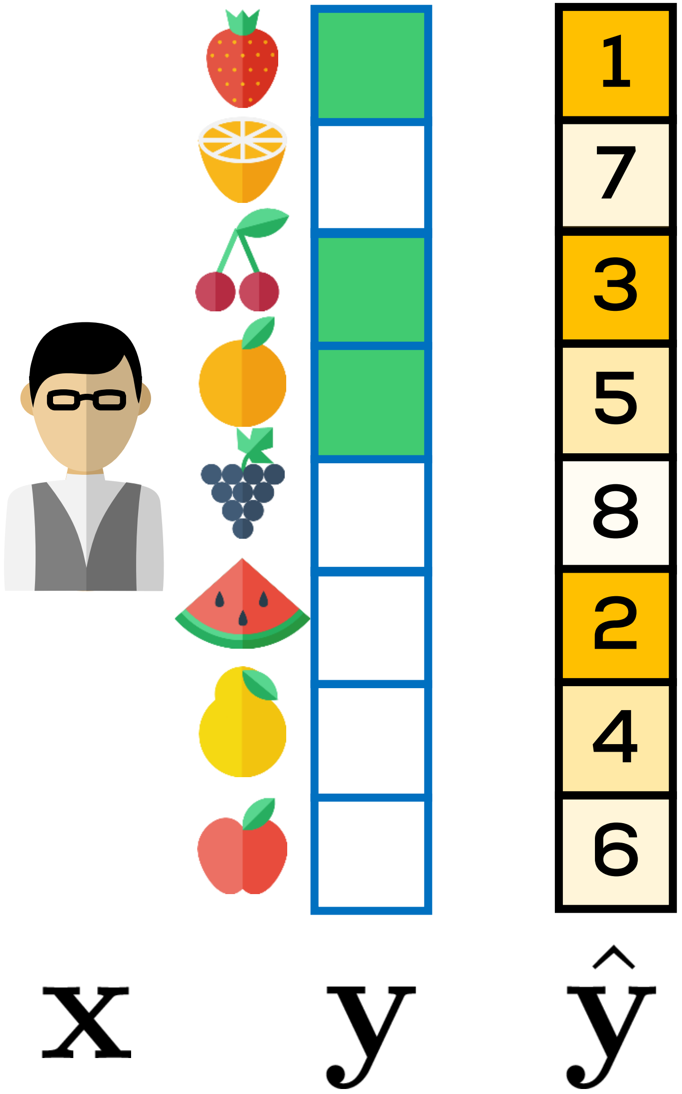


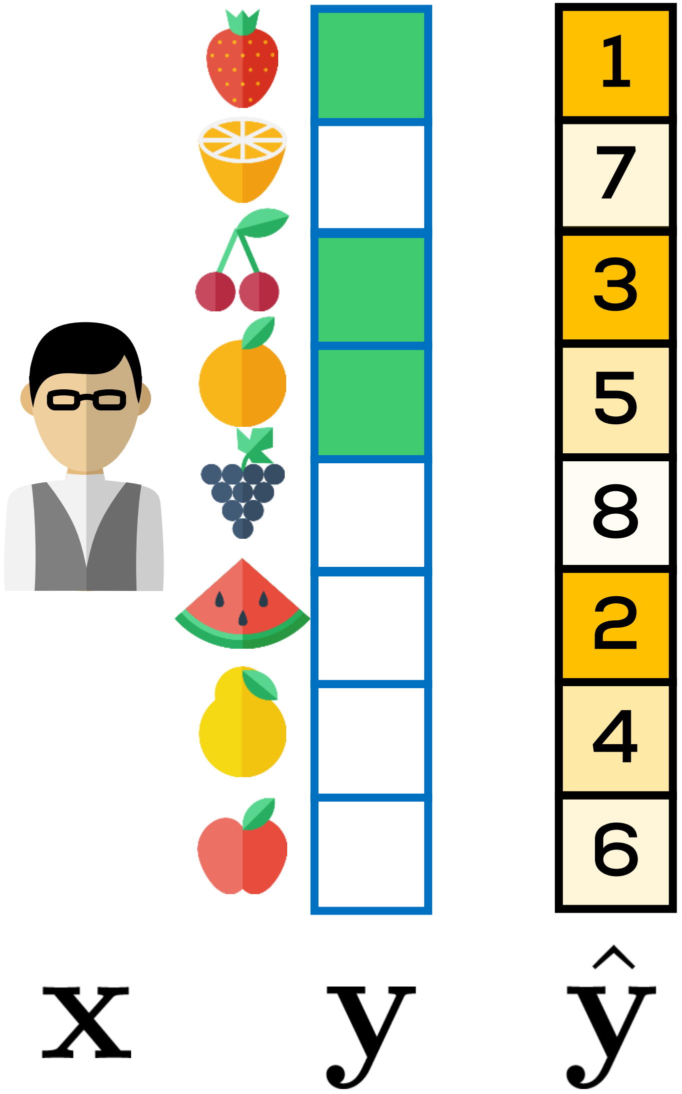
$$\mathbf{x} \qquad \mathbf{y} \qquad \hat{\mathbf{y}}$$

Multi-label Classification Loss Functions



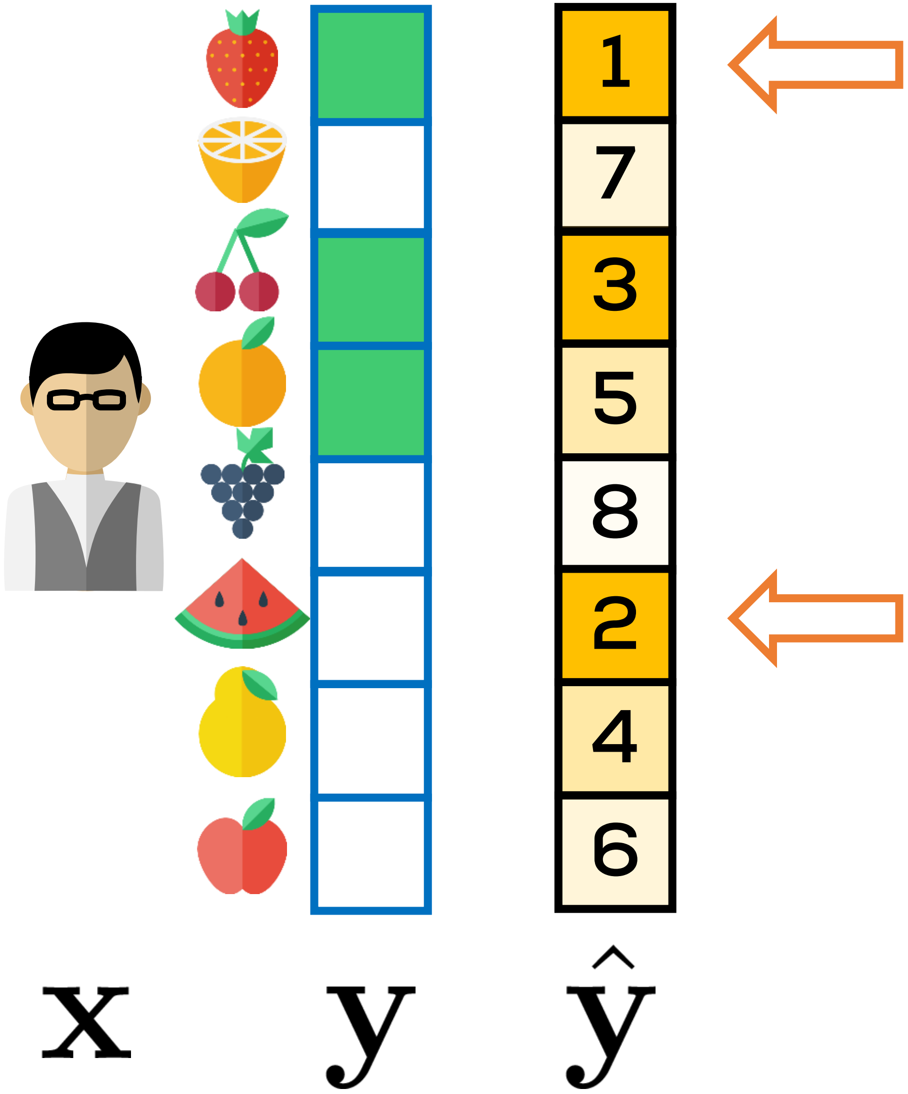
Precision@k

Multi-label Classification Loss Functions



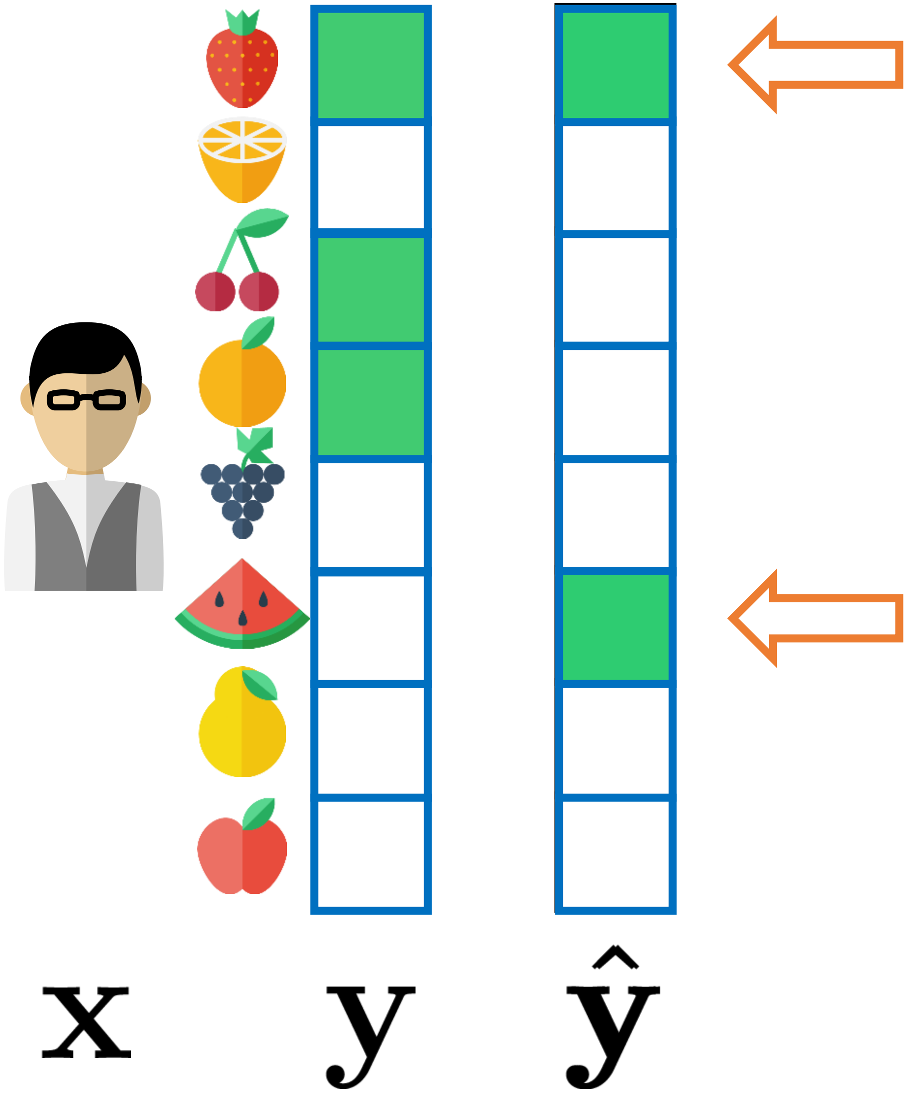
Precision@2

Multi-label Classification Loss Functions



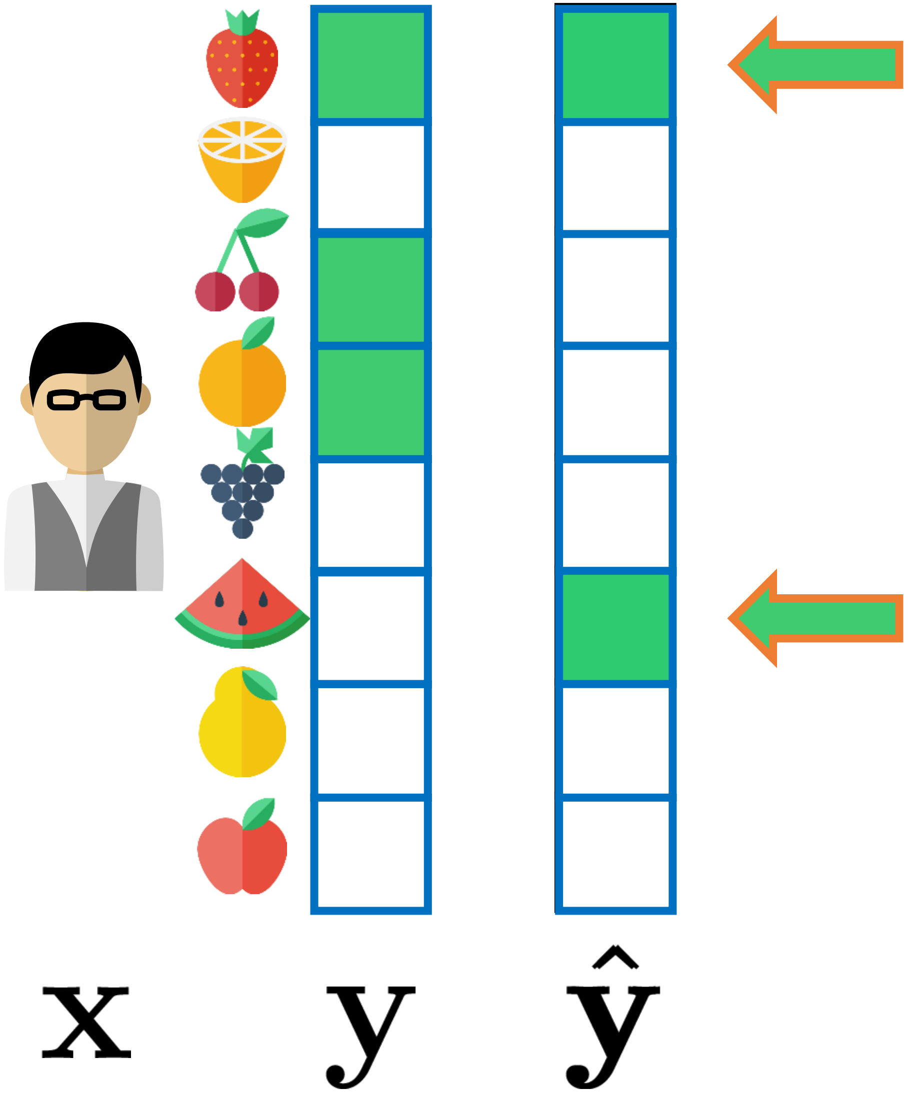
Precision@2

Multi-label Classification Loss Functions



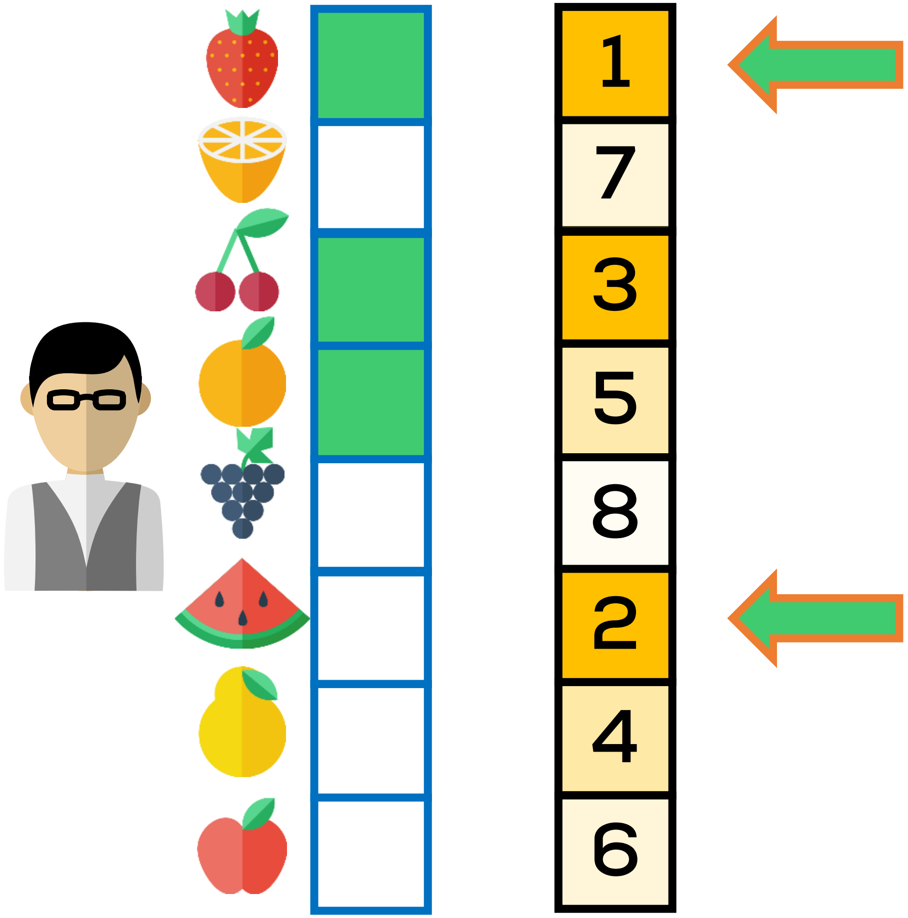
Precision@2

Multi-label Classification Loss Functions



Precision@2

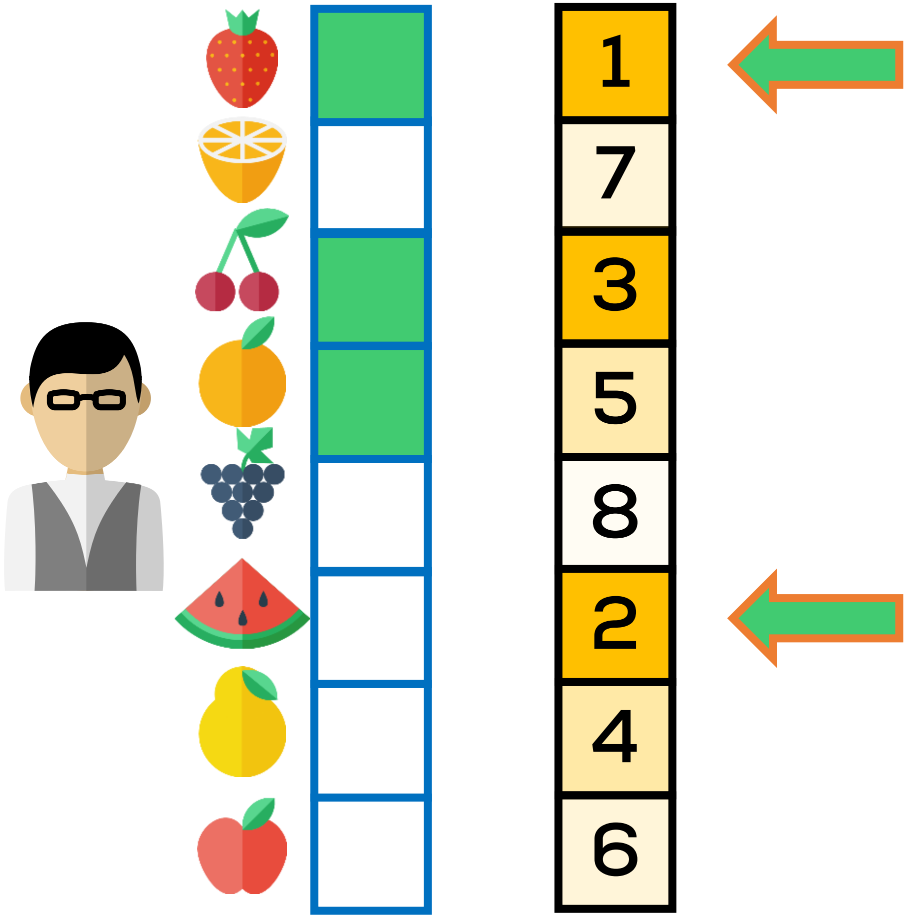
Multi-label Classification Loss Functions



$\mathbf{x}$ $\mathbf{y}$ $\hat{\mathbf{y}}$

Precision@2

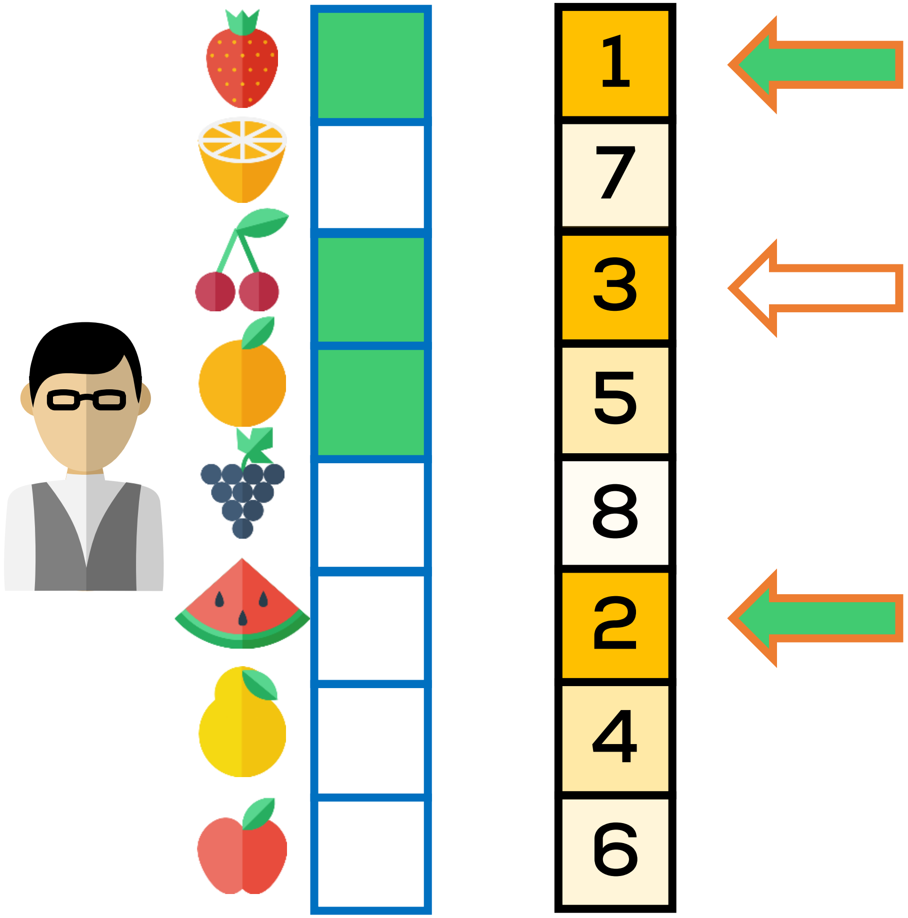
Multi-label Classification Loss Functions



$\mathbf{x}$ $\mathbf{y}$ $\hat{\mathbf{y}}$

Precision@3

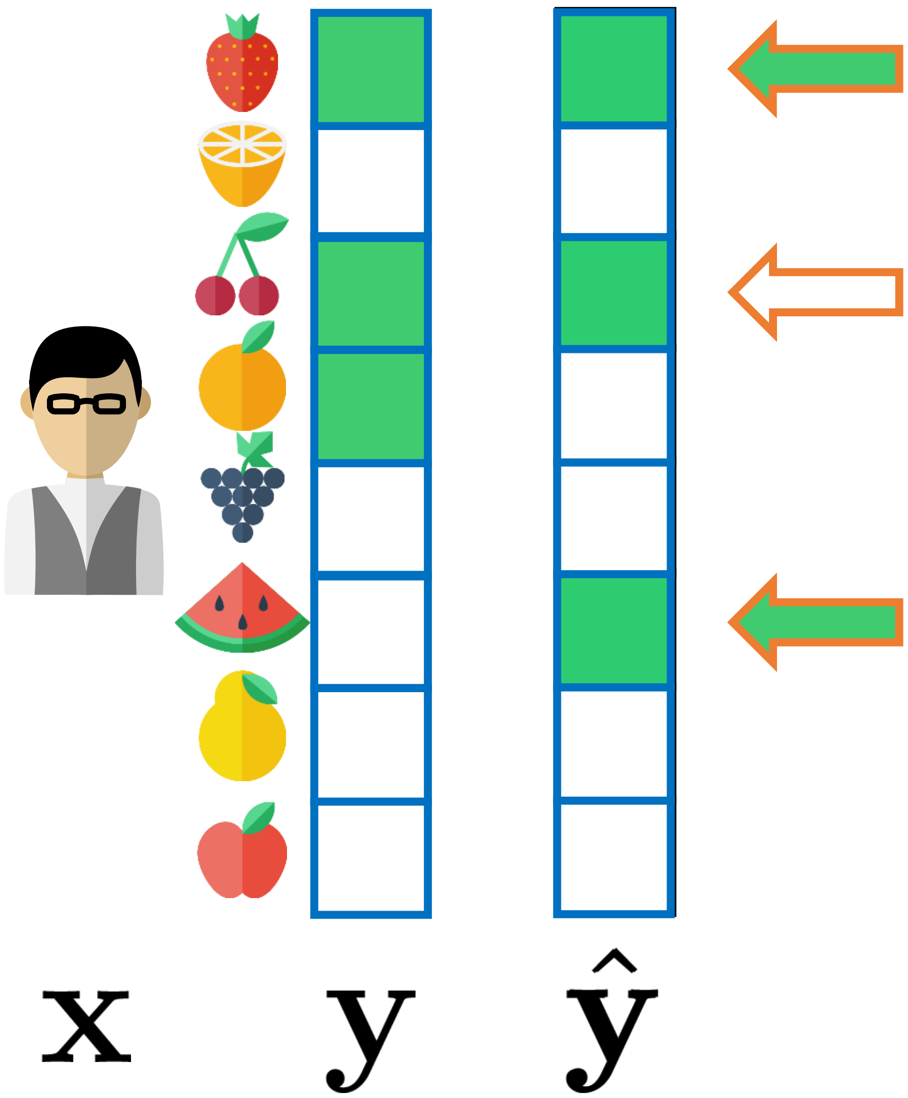
Multi-label Classification Loss Functions



$$\mathbf{x} \qquad \mathbf{y} \qquad \hat{\mathbf{y}}$$

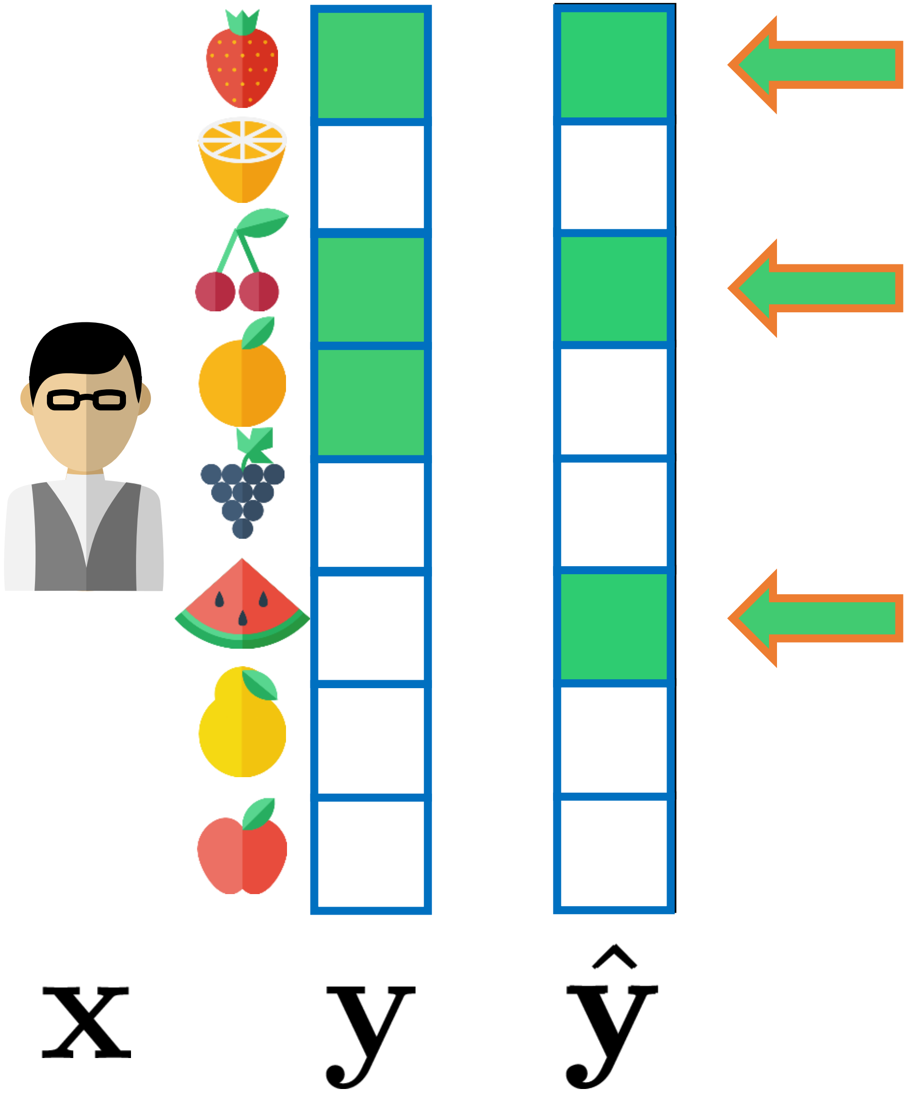
Precision@3

Multi-label Classification Loss Functions



Precision@3

Multi-label Classification Loss Functions



Precision@3

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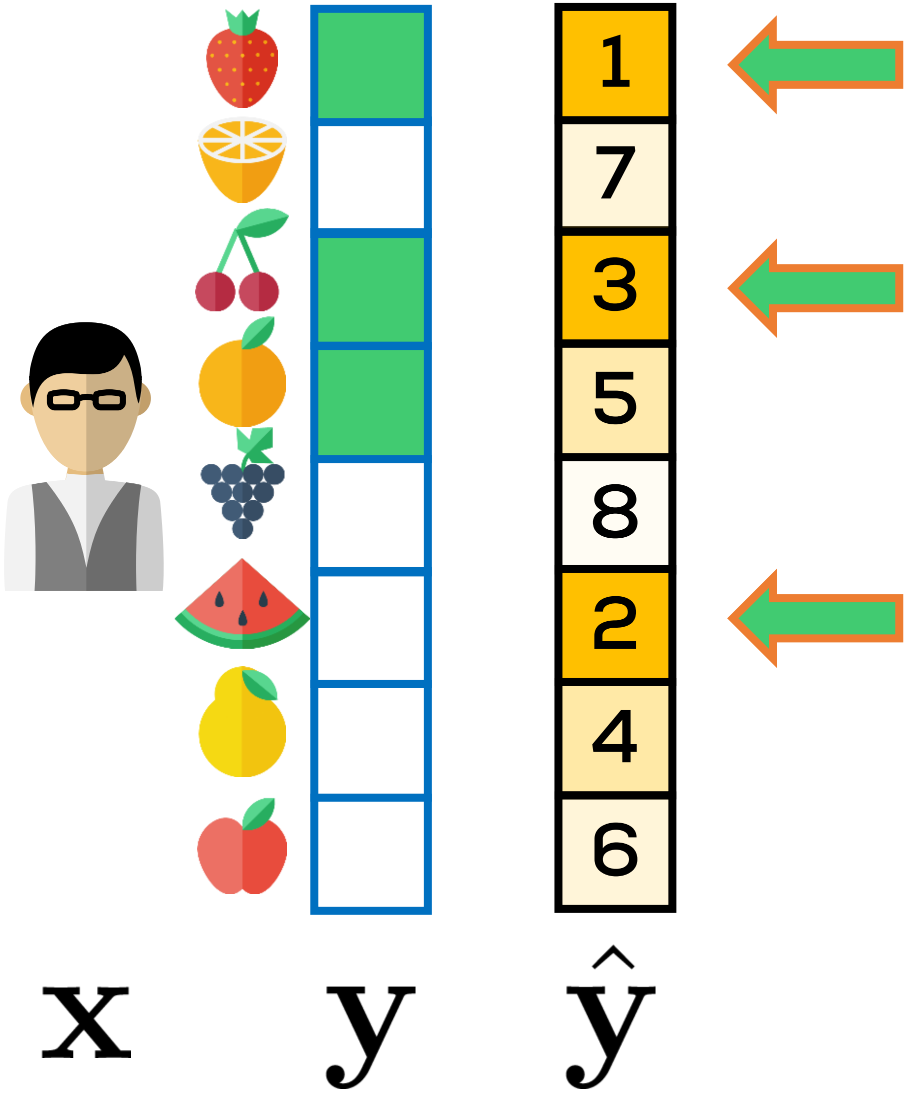

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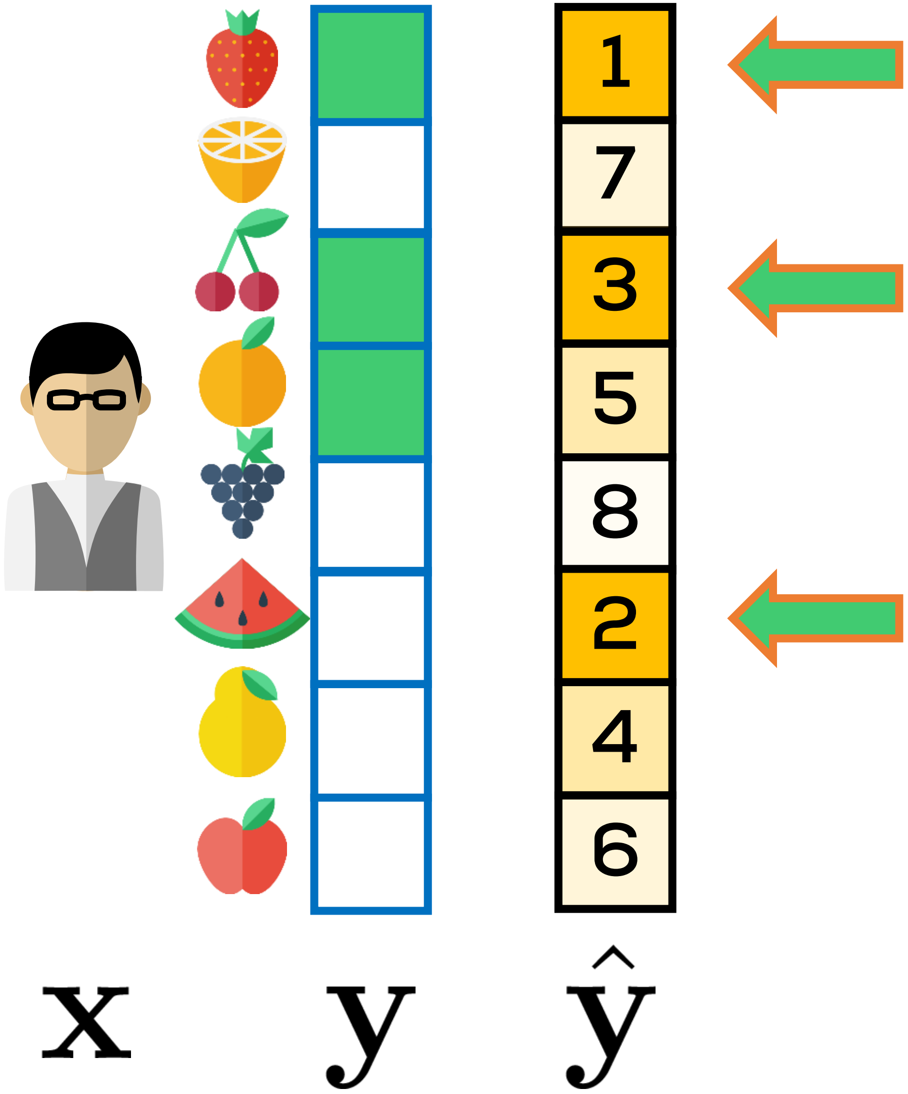
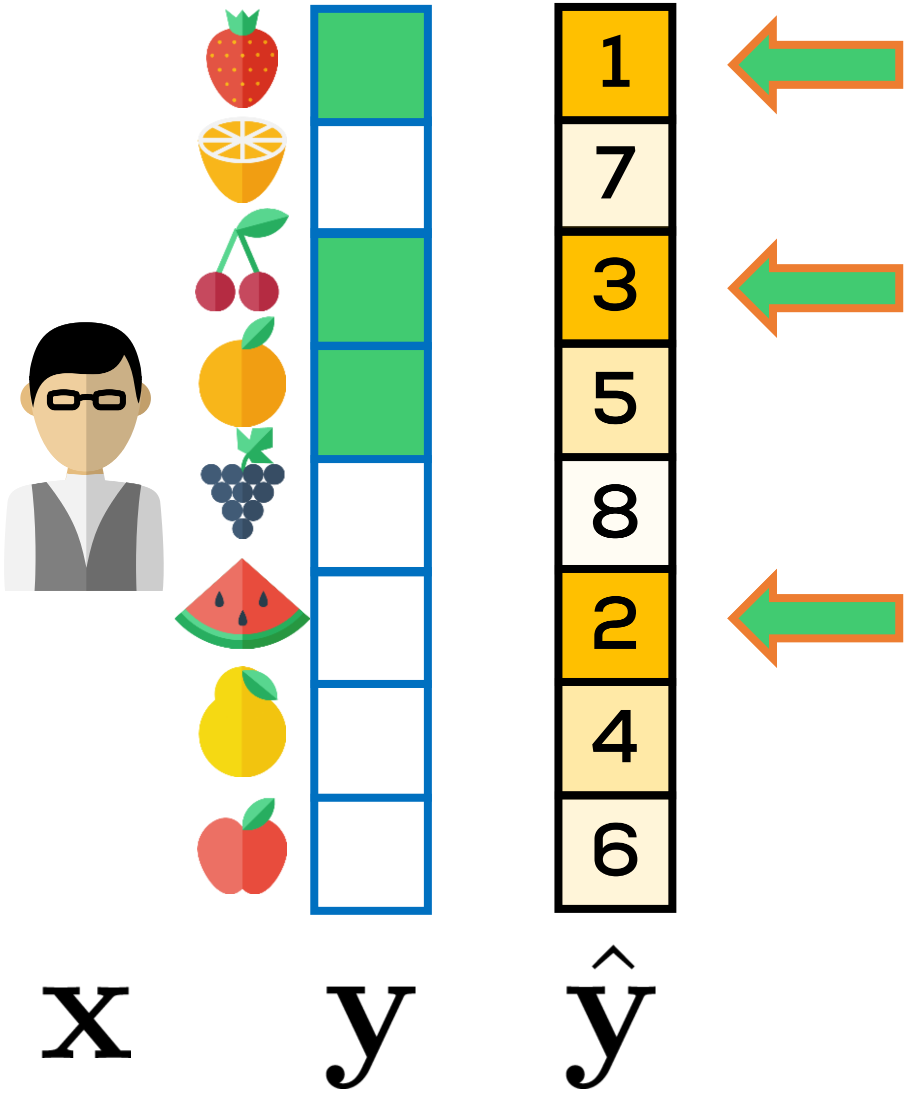
Multi-label Classification Loss Functions



Precision@3

Recall@k

Multi-label Classification Loss Functions



Precision@3

Recall@k

Many, many, more
NDCG, AUC, MRR, MAP

How to optimize these losses?

... aka where is the rest of the Math?

Some Fundamentals

Functional Analysis, Optimization Theory

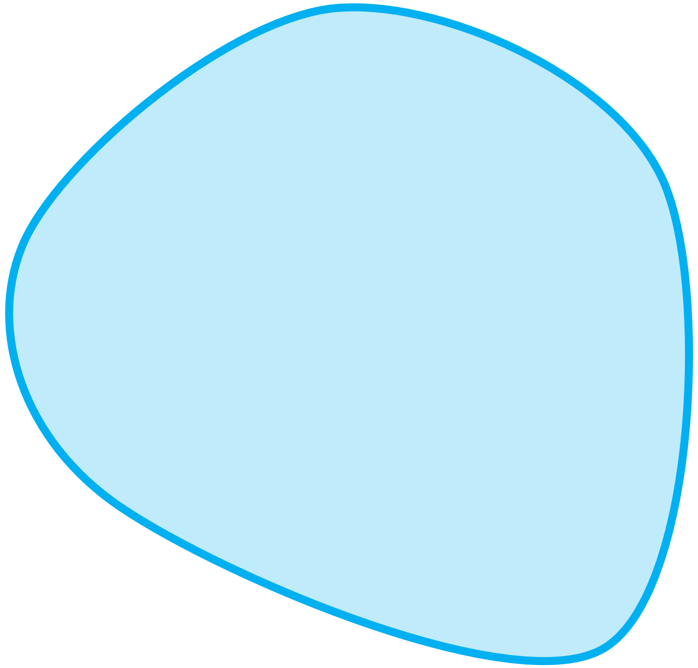
Convex Sets

Convex Sets

$$\mathcal{C} \subseteq \mathbb{R}^d$$

Convex Sets

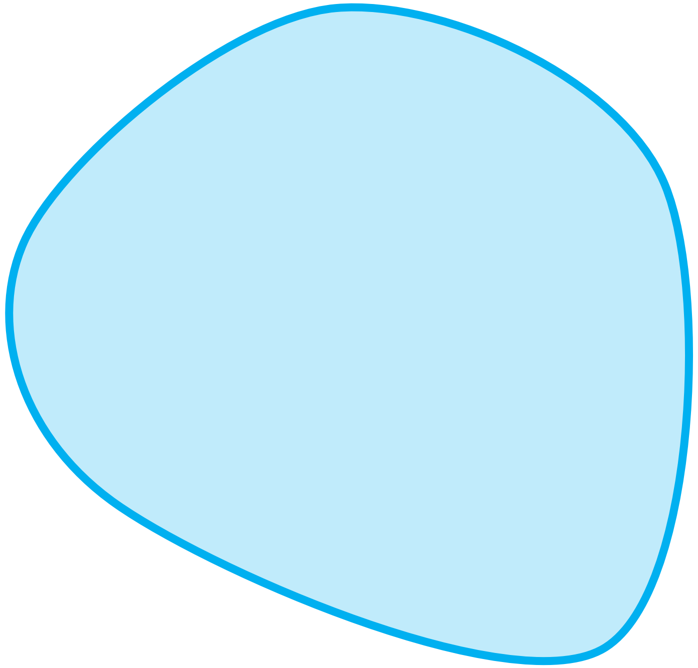
$$\mathcal{C} \subseteq \mathbb{R}^d$$



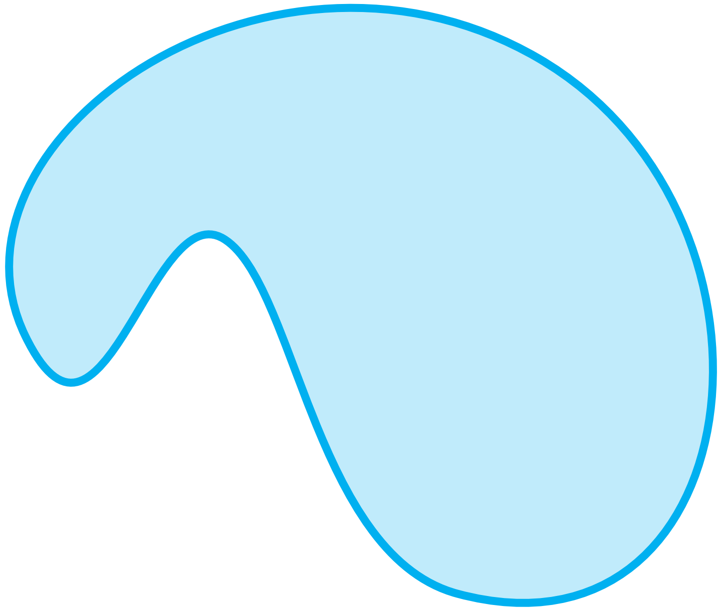
CONVEX SET

Convex Sets

$$\mathcal{C} \subseteq \mathbb{R}^d$$



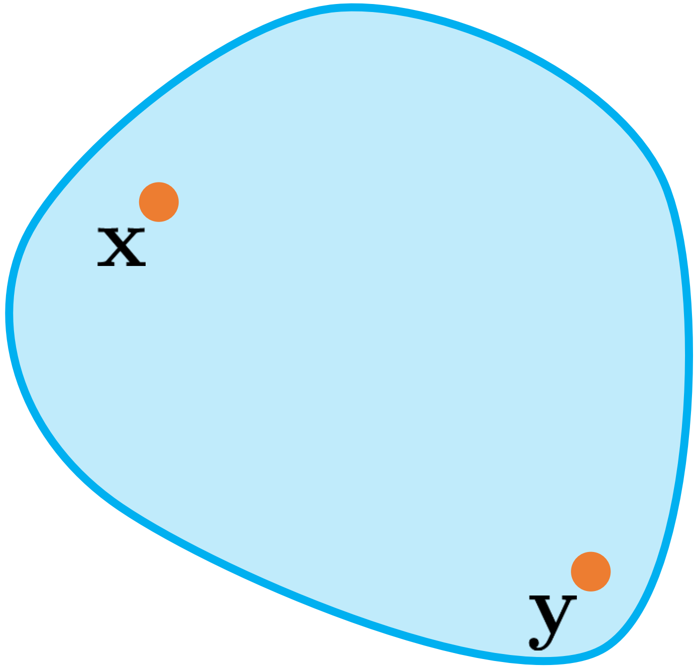
CONVEX SET



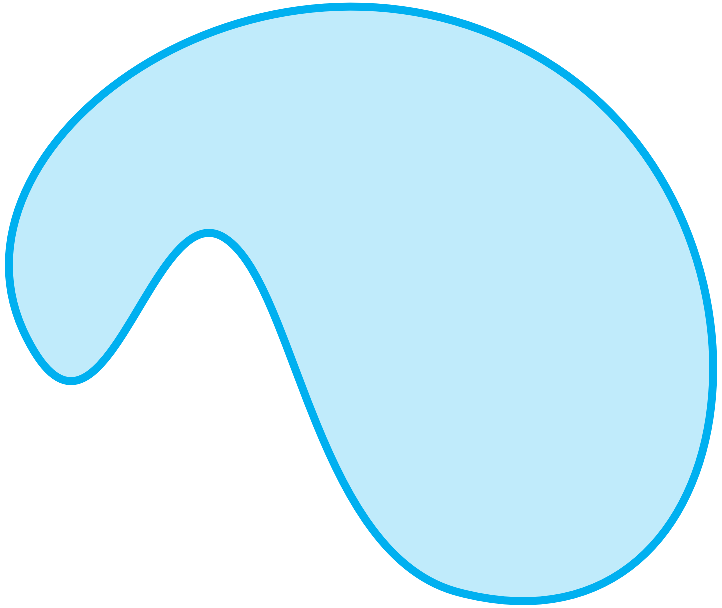
NON-CONVEX SET

Convex Sets

$$\mathcal{C} \subseteq \mathbb{R}^d$$



CONVEX SET



NON-CONVEX SET

$$\forall \mathbf{x}, \mathbf{y} \in \mathcal{C}$$

Convex Sets

$$\mathcal{C} \subseteq \mathbb{R}^d$$

$$\forall \mathbf{x}, \mathbf{y} \in \mathcal{C}$$
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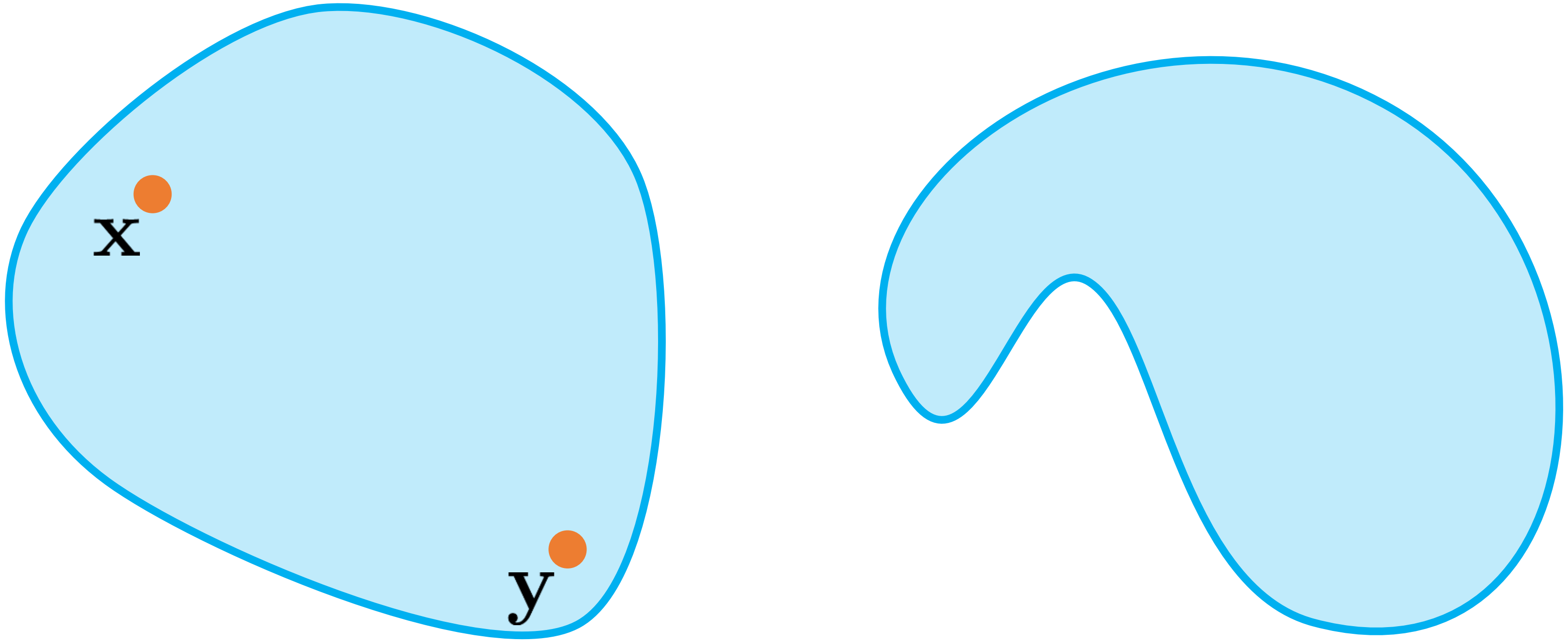

CONVEX SET

NON-CONVEX SET

Convex Sets

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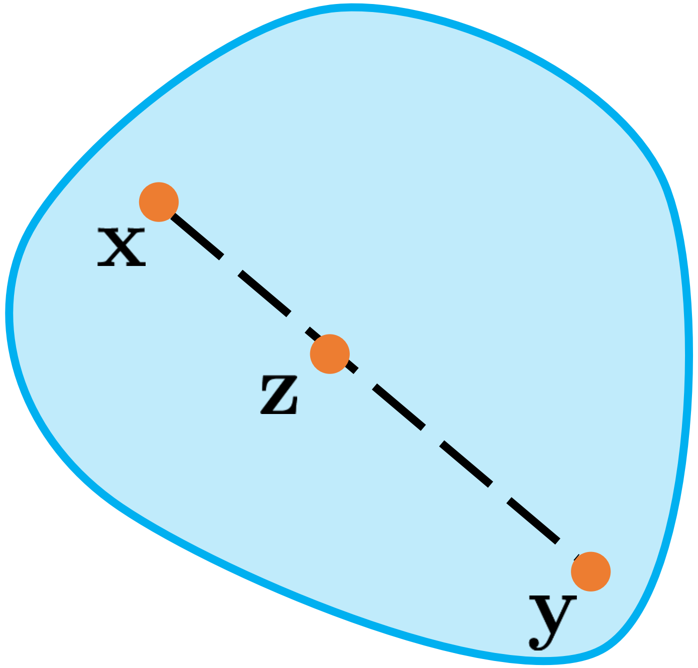
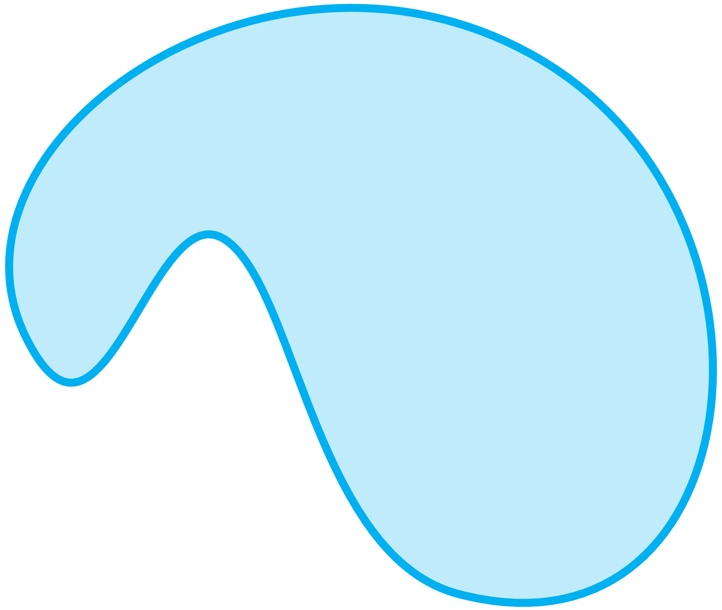


CONVEX SET



NON-CONVEX SET

$$\forall \mathbf{x}, \mathbf{y} \in \mathcal{C}$$
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$$\mathbf{z} = \lambda \cdot \mathbf{x} + (1 - \lambda) \cdot \mathbf{y}$$

Convex Sets

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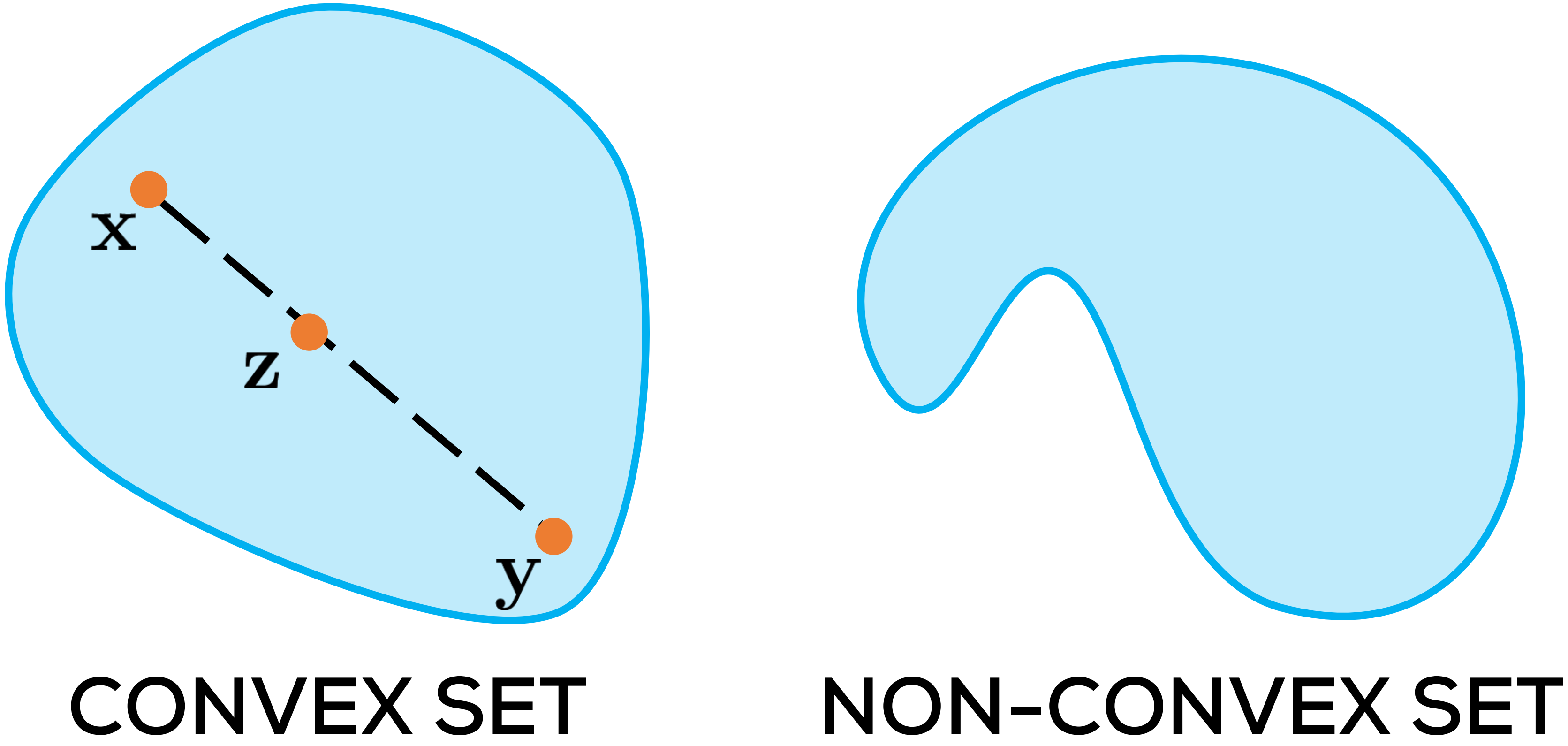

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Convex combination

Convex Sets

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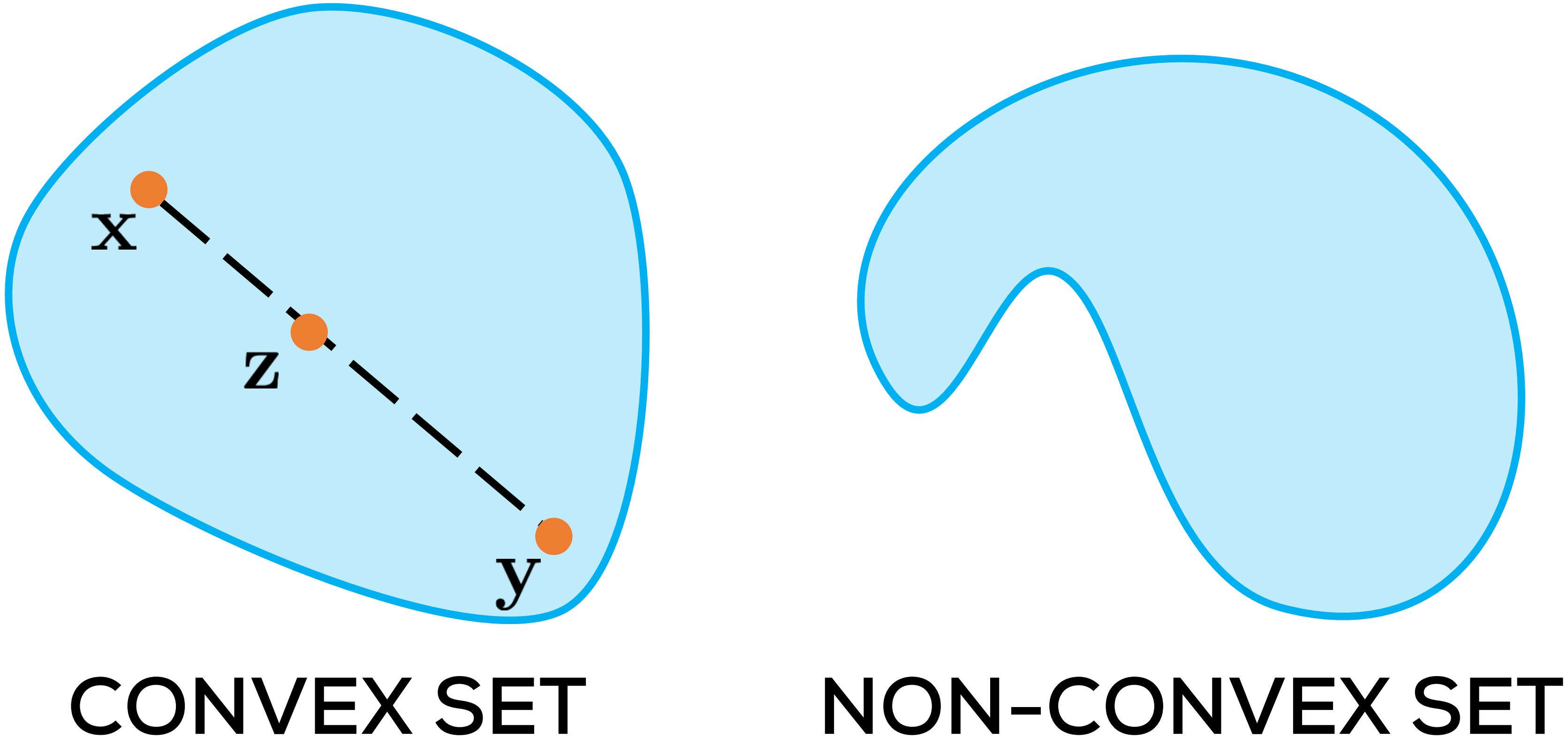

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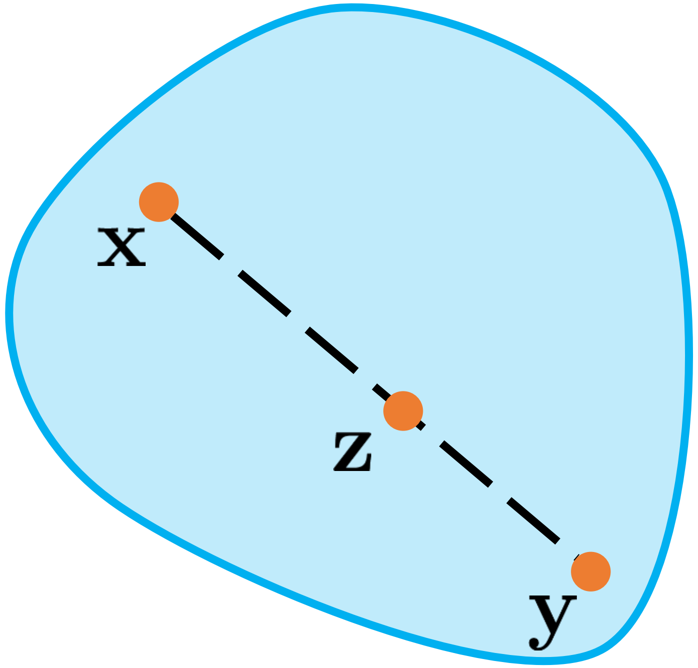

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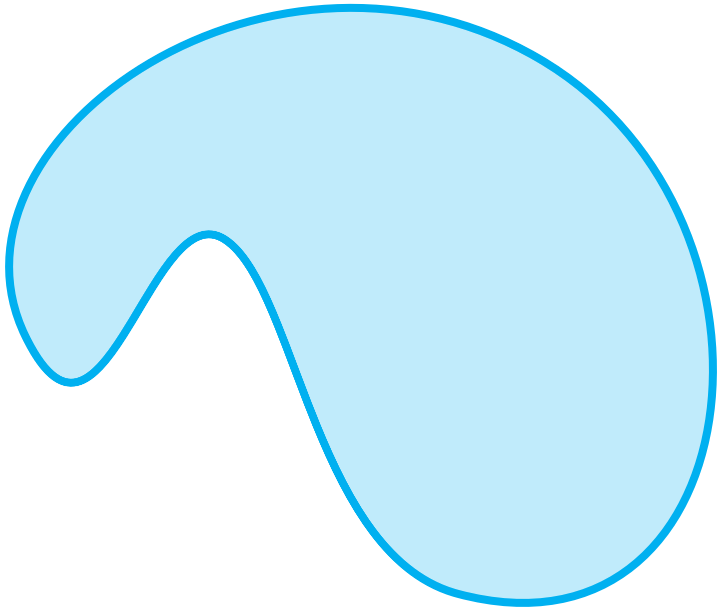

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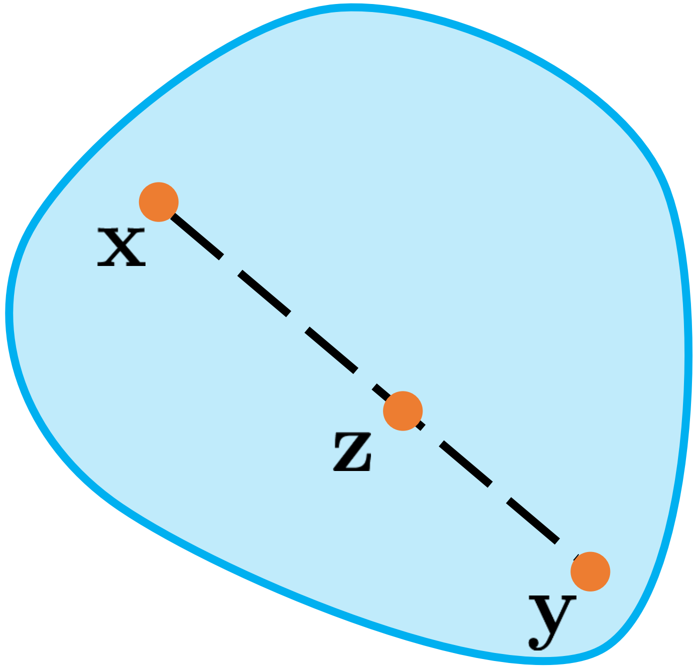
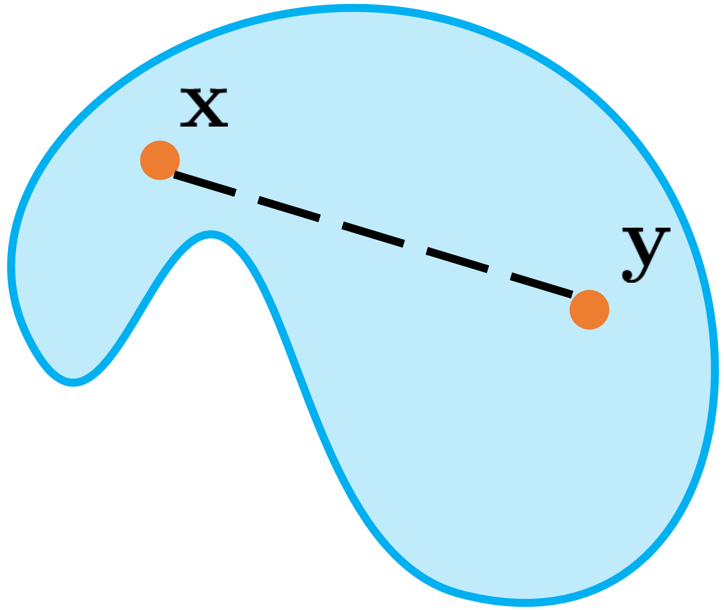


CONVEX SET



NON-CONVEX SET

$$\forall \mathbf{x}, \mathbf{y} \in \mathcal{C}$$
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$$\mathbf{z} = \lambda \cdot \mathbf{x} + \underbrace{(1 - \lambda) \cdot \mathbf{y}}_{\text{Convex combination}} \in \mathcal{C}$$

Convex Sets

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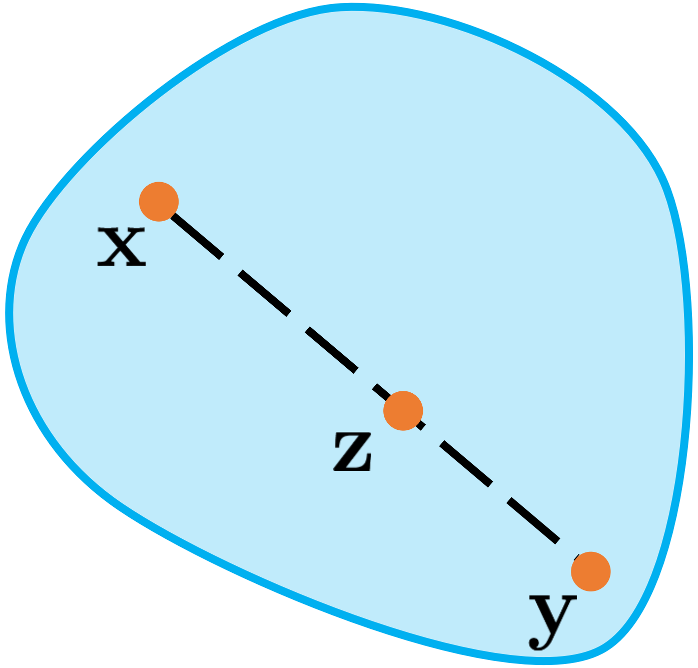

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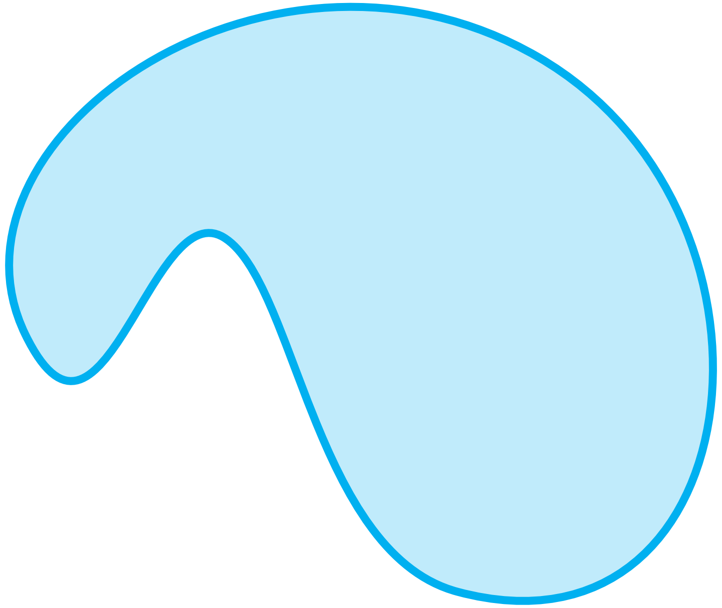

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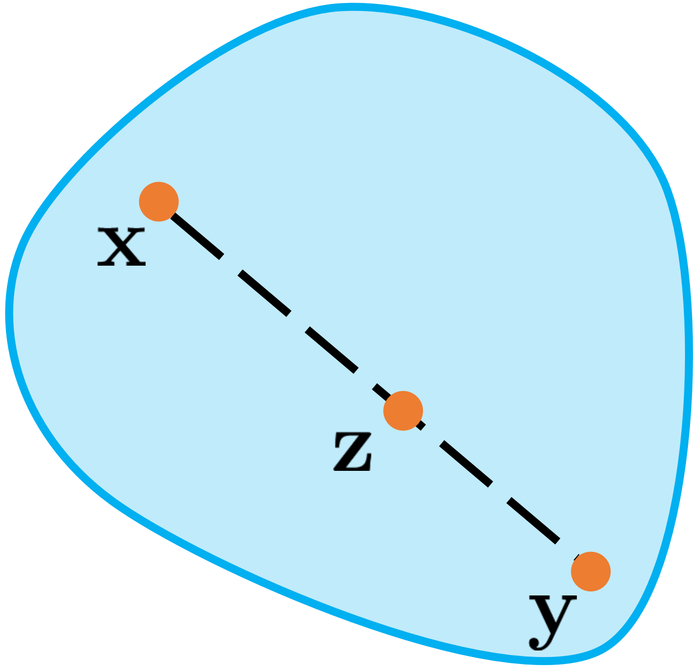

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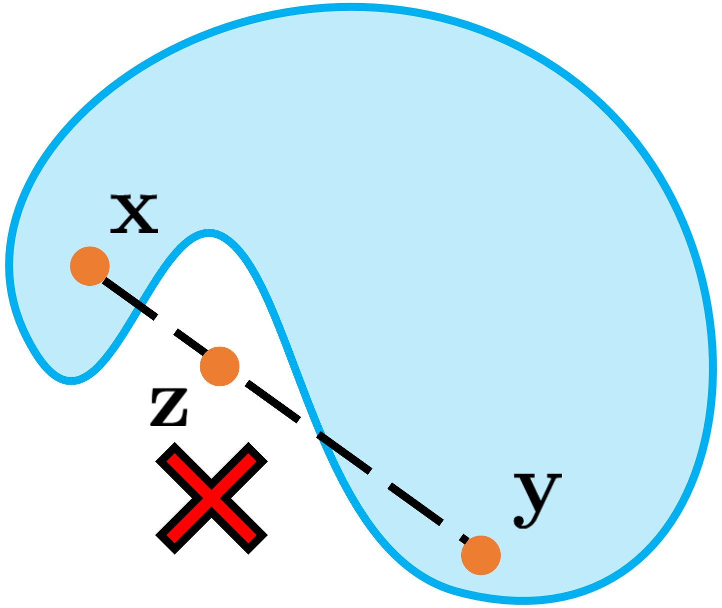

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Convex combination

Convex Functions

Convex Functions

$$f : \mathbb{R}^d \to \mathbb{R}$$

Convex Functions

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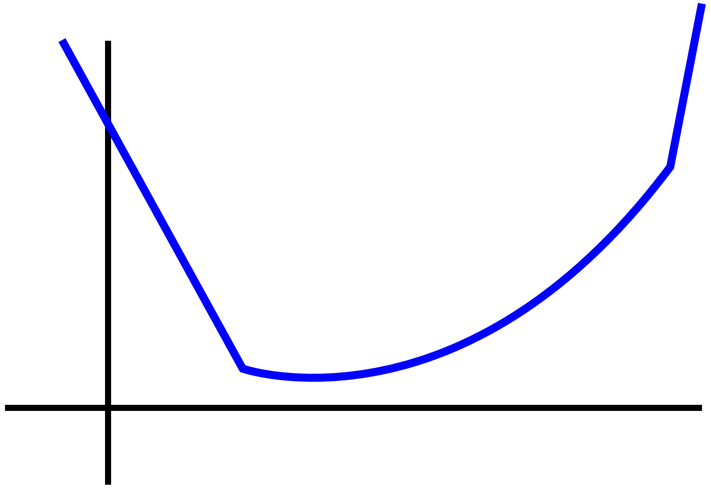

CONVEX FUNCTION

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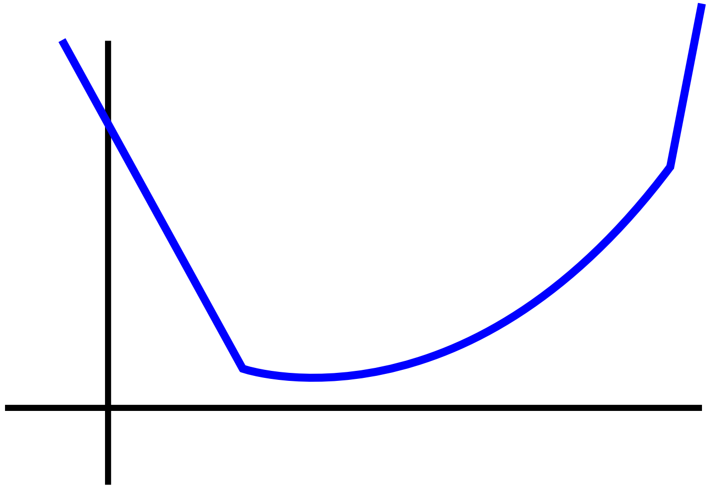

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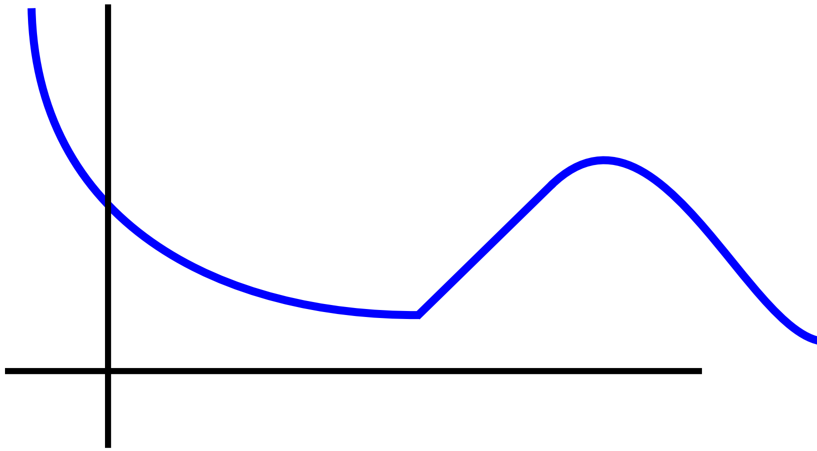

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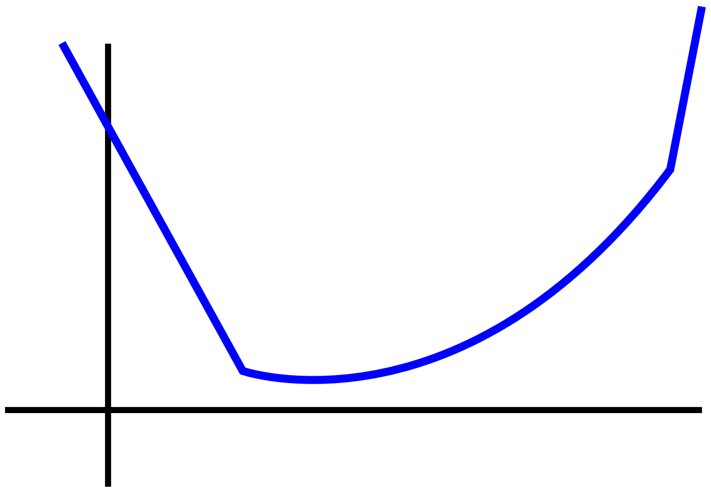
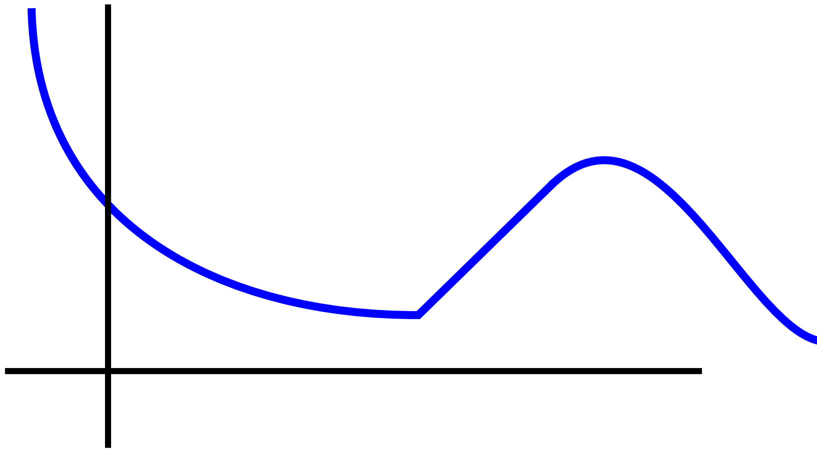


CONVEX FUNCTION



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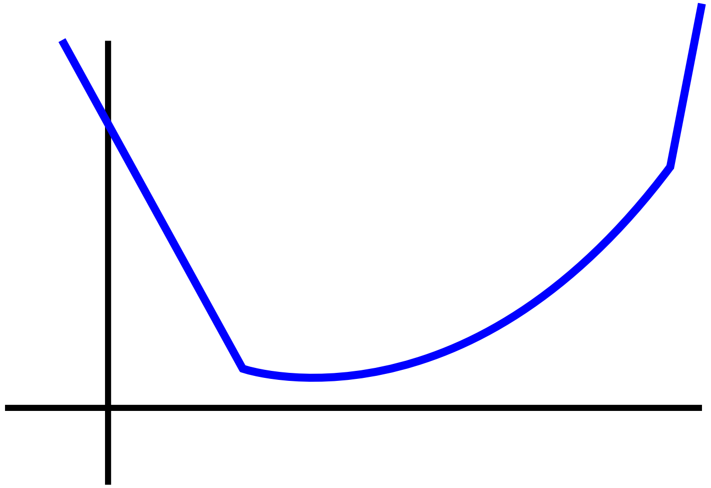

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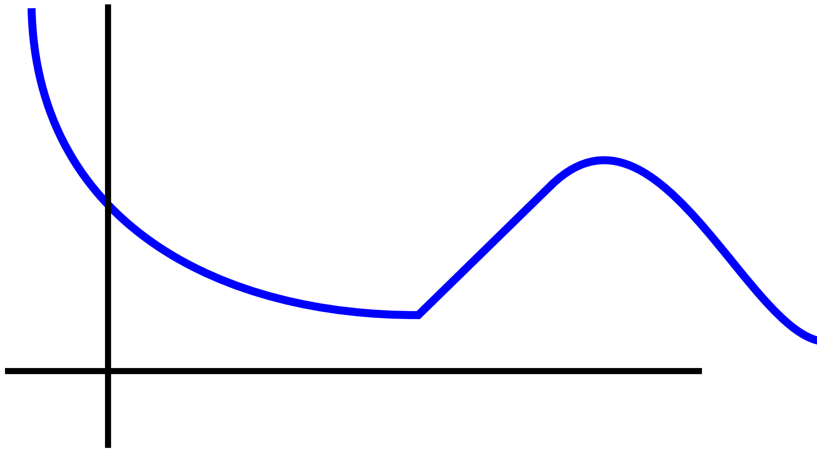

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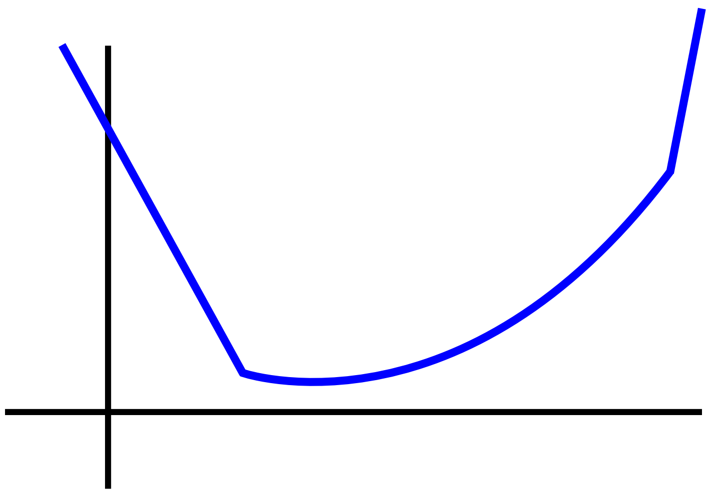

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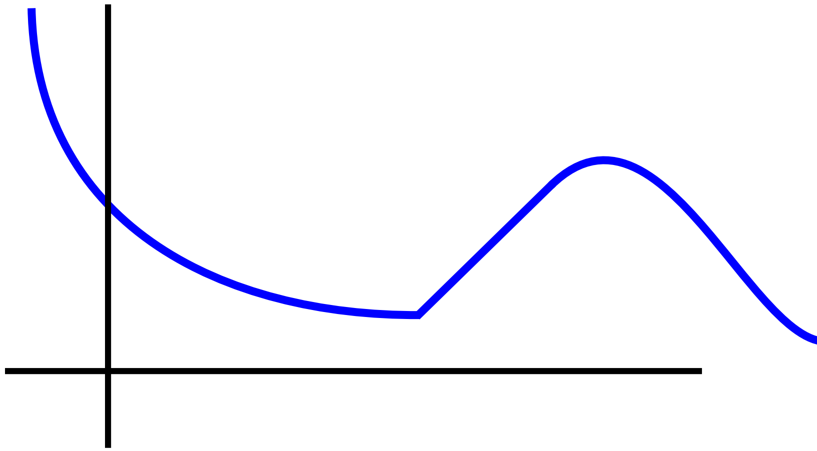

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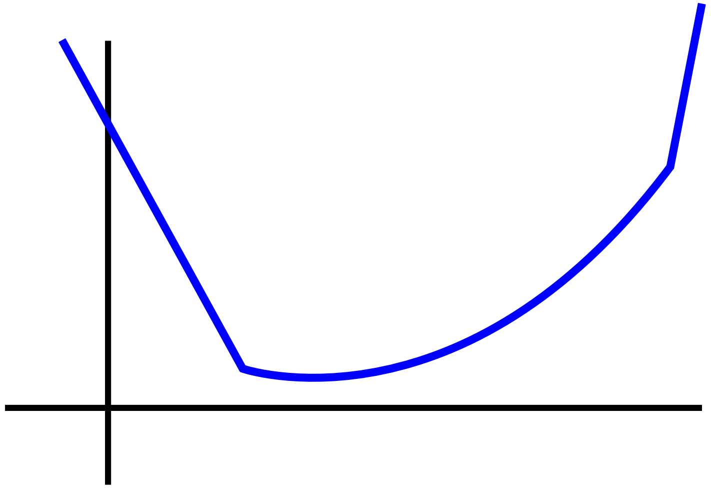

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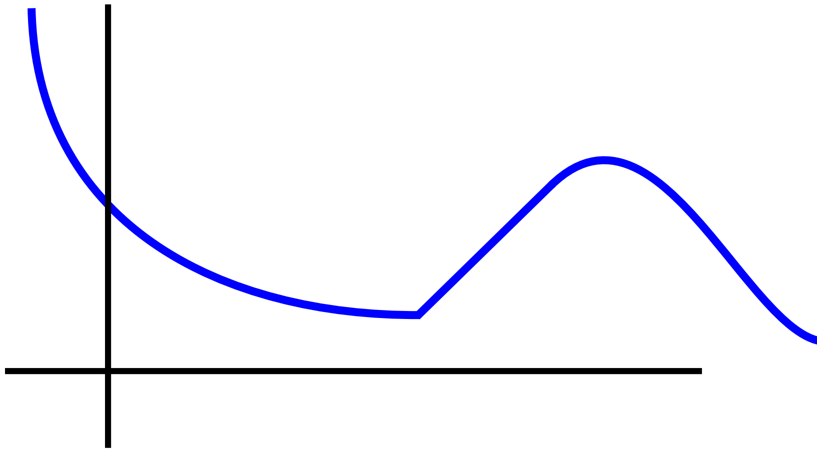

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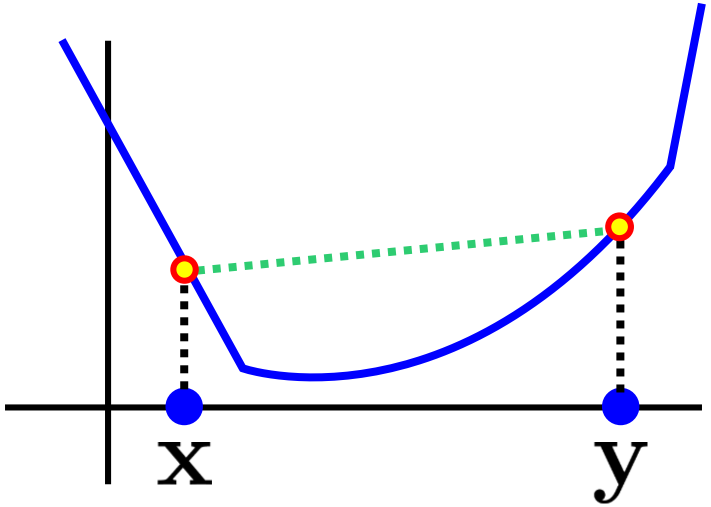

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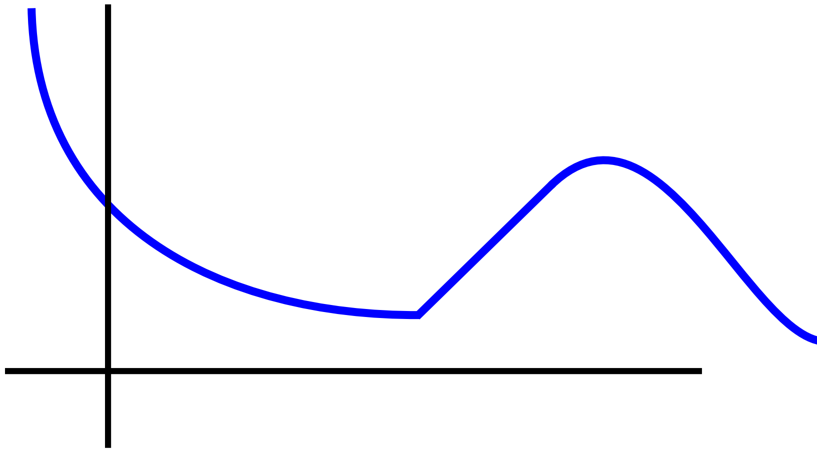

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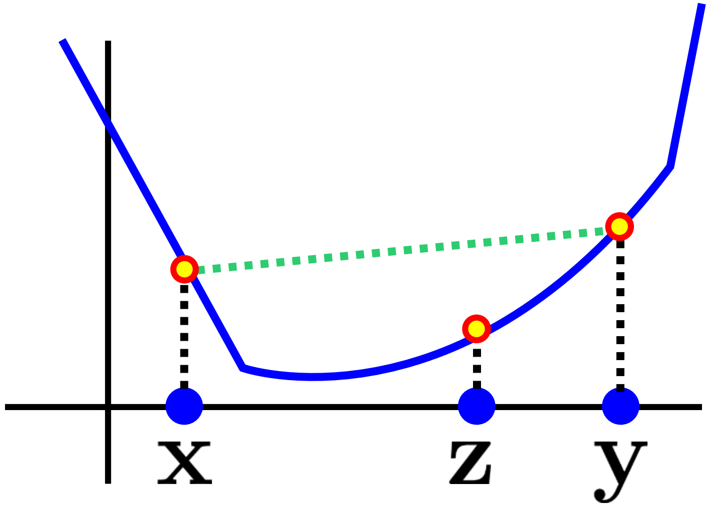

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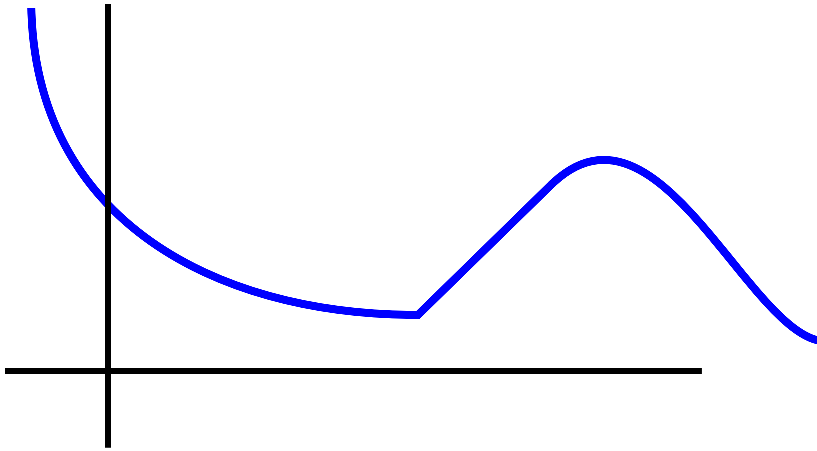

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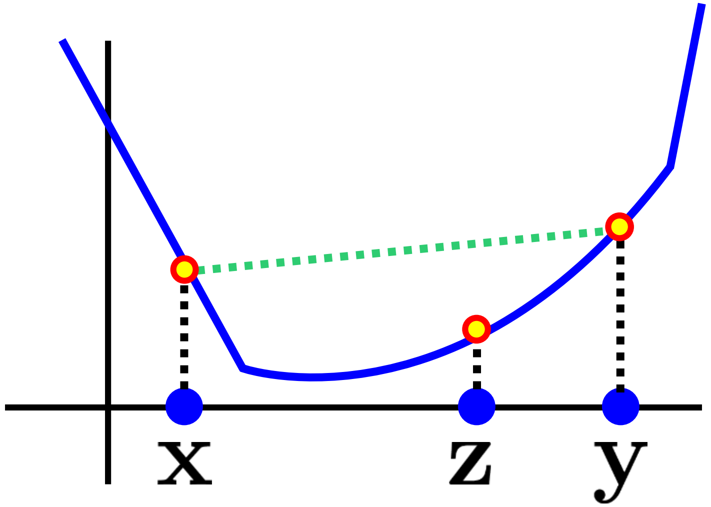

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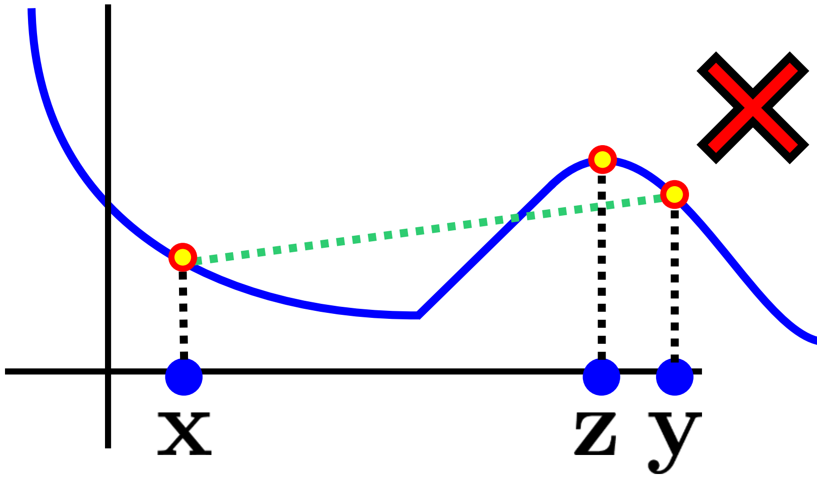

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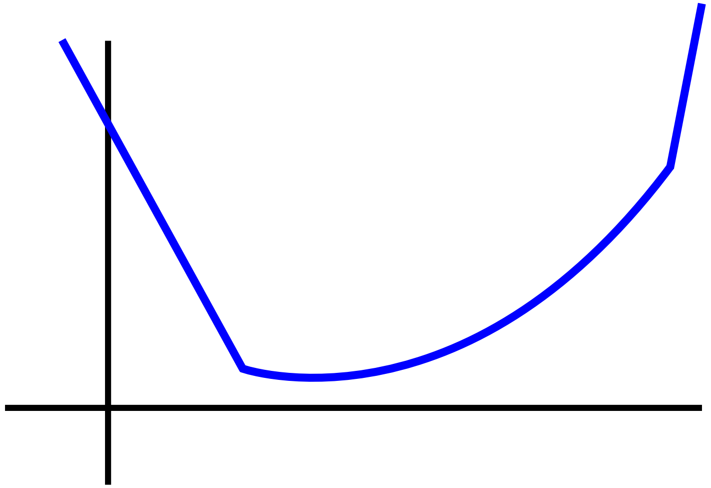

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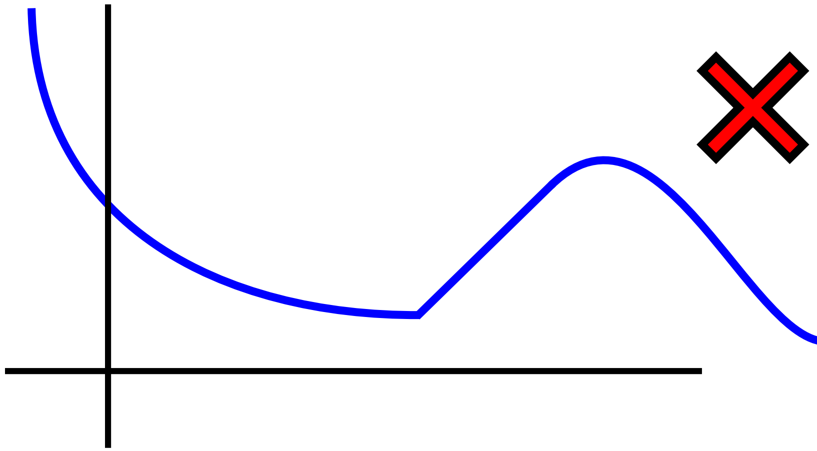

NON-CONVEX FUNCTION

Convex Functions

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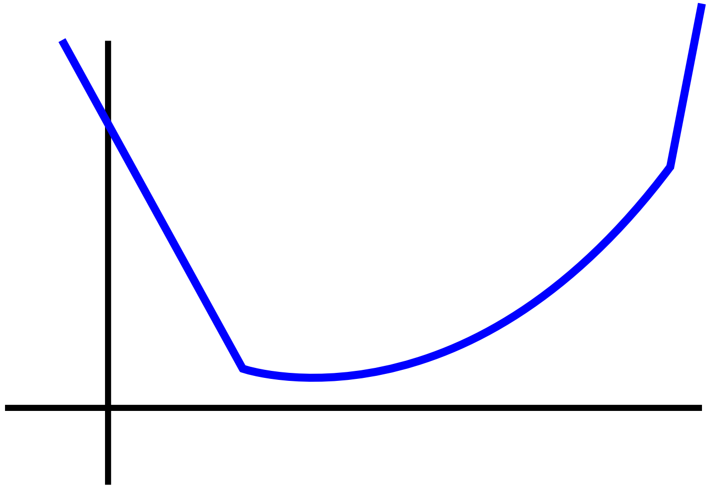
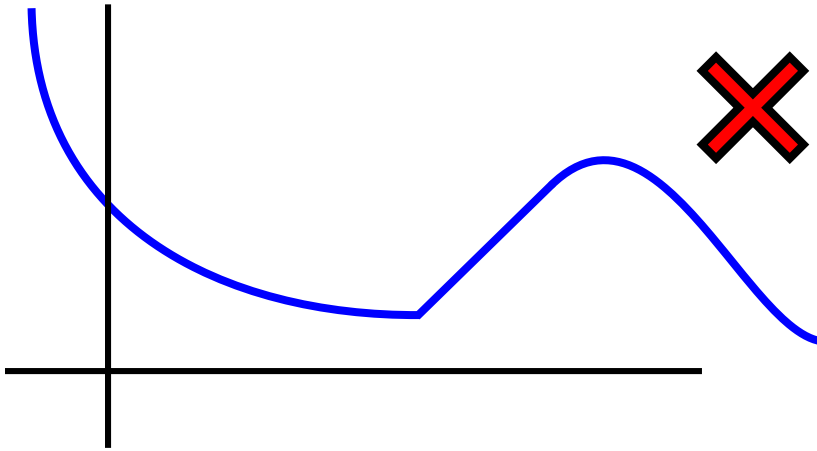


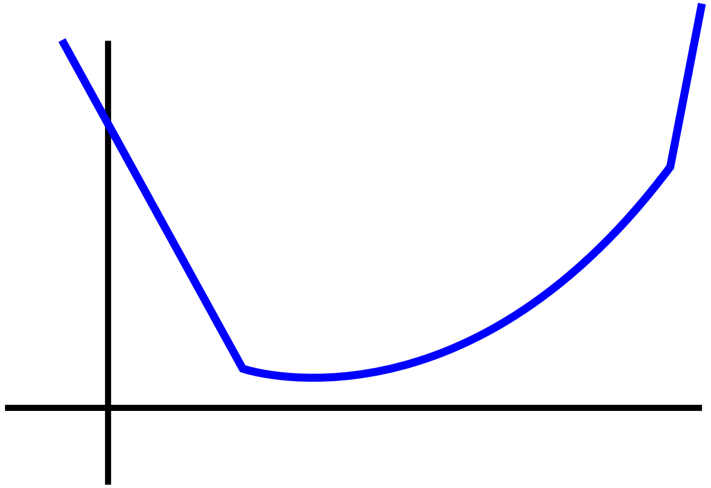


$\forall \mathbf{x}, \mathbf{y}$

Differentiable function!

Convex Functions

$$f : \mathbb{R}^d \to \mathbb{R}$$



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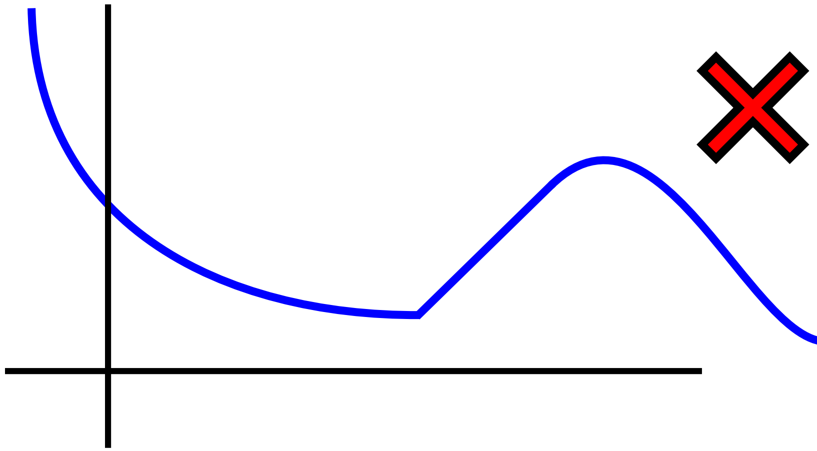


NON-CONVEX FUNCTION

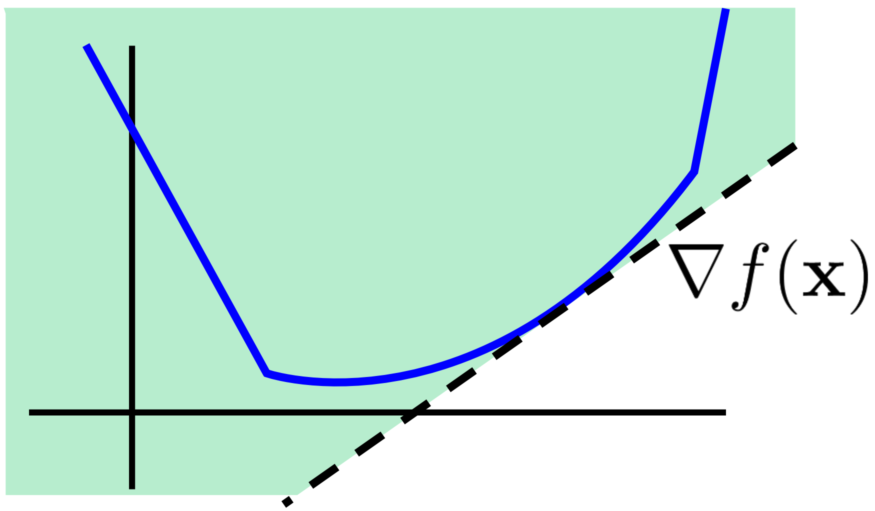
$\forall \mathbf{x}, \mathbf{y}$

Differentiable function!

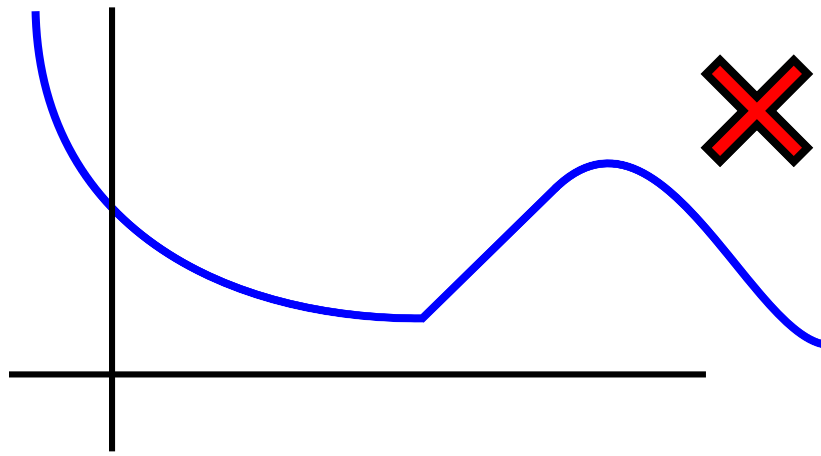
$$f(\mathbf{y}) \geq f(\mathbf{x}) + \langle \nabla f(\mathbf{x}), \mathbf{y} - \mathbf{x} \rangle$$

Convex Functions

$$f : \mathbb{R}^d \to \mathbb{R}$$



CONVEX FUNCTION



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Convex Functions

$$f : \mathbb{R}^d \to \mathbb{R}$$

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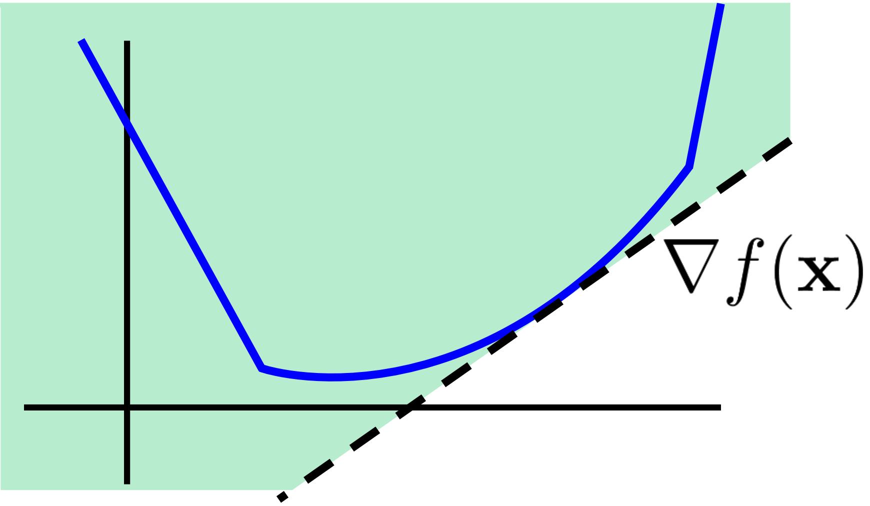

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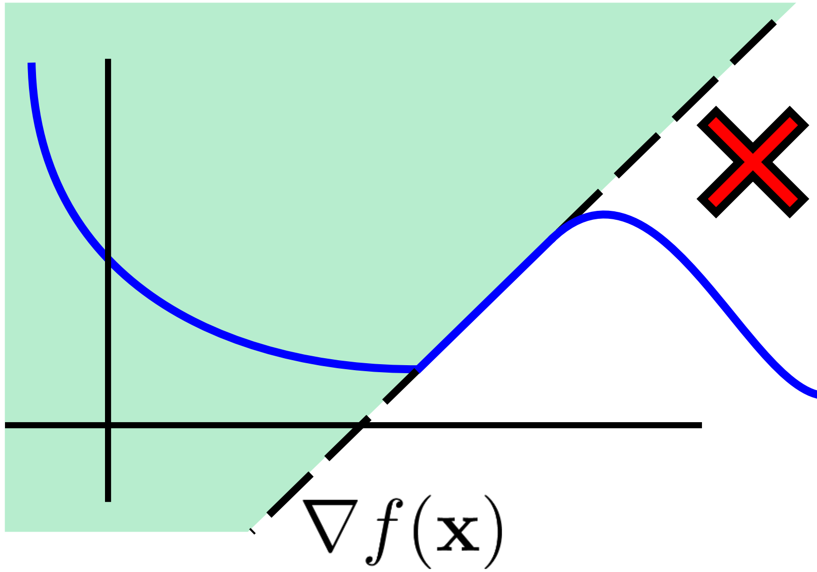
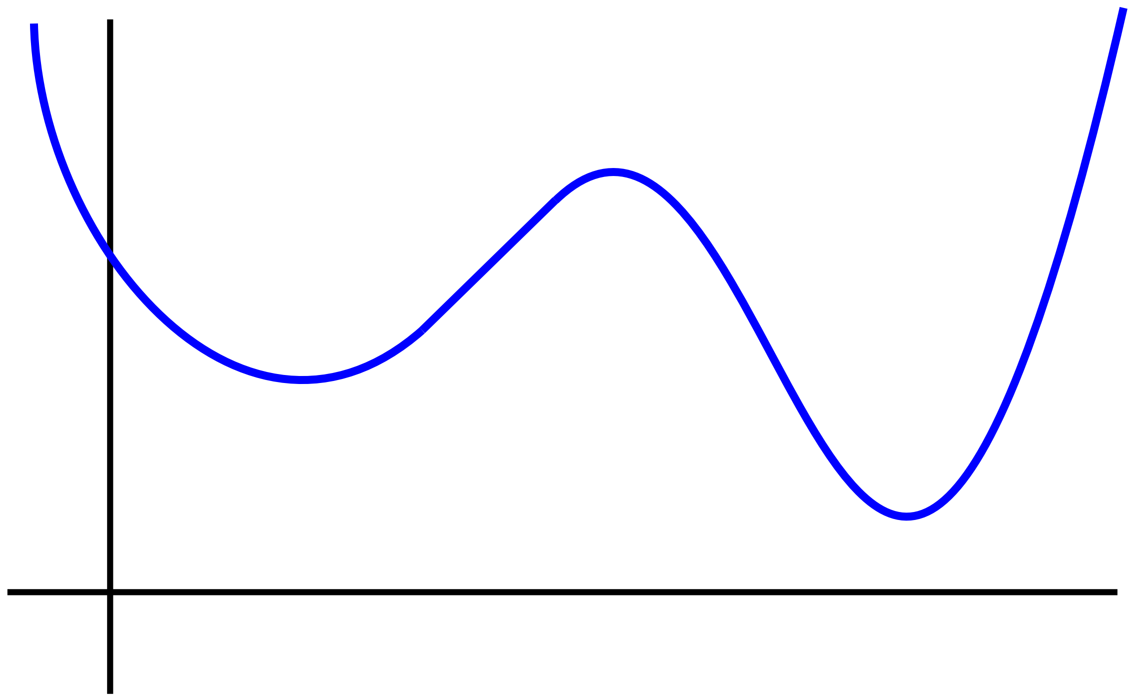


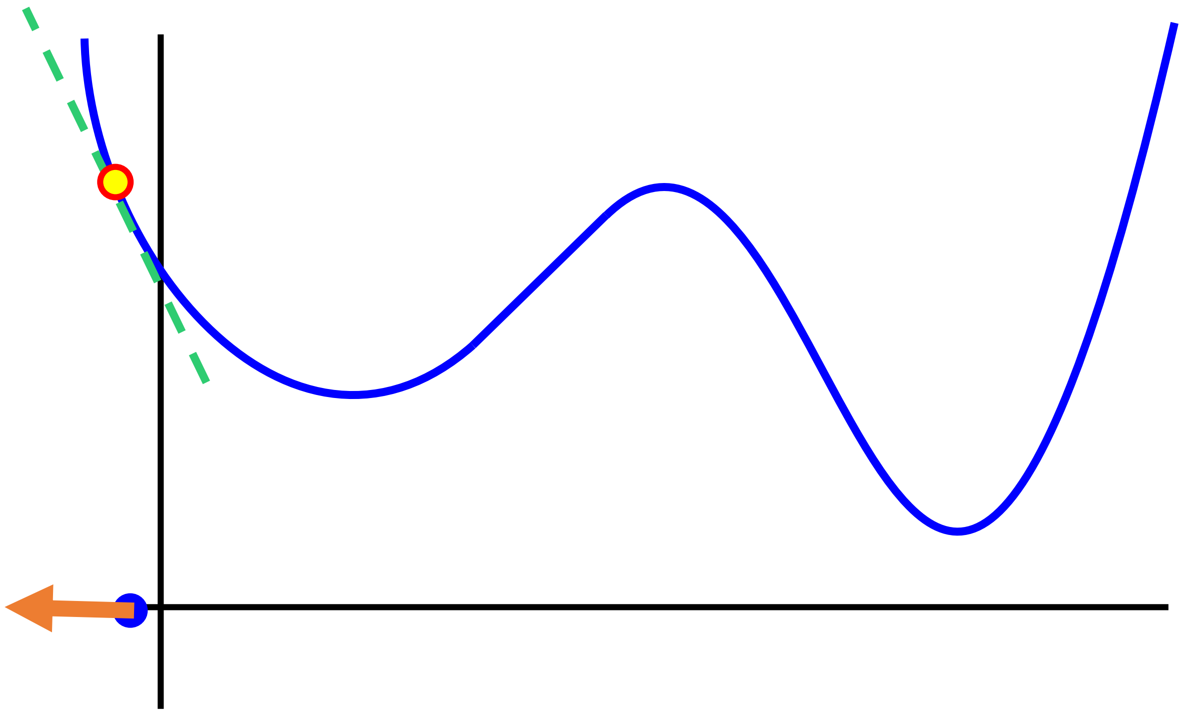
NON-CONVEX FUNCTION

Gradients

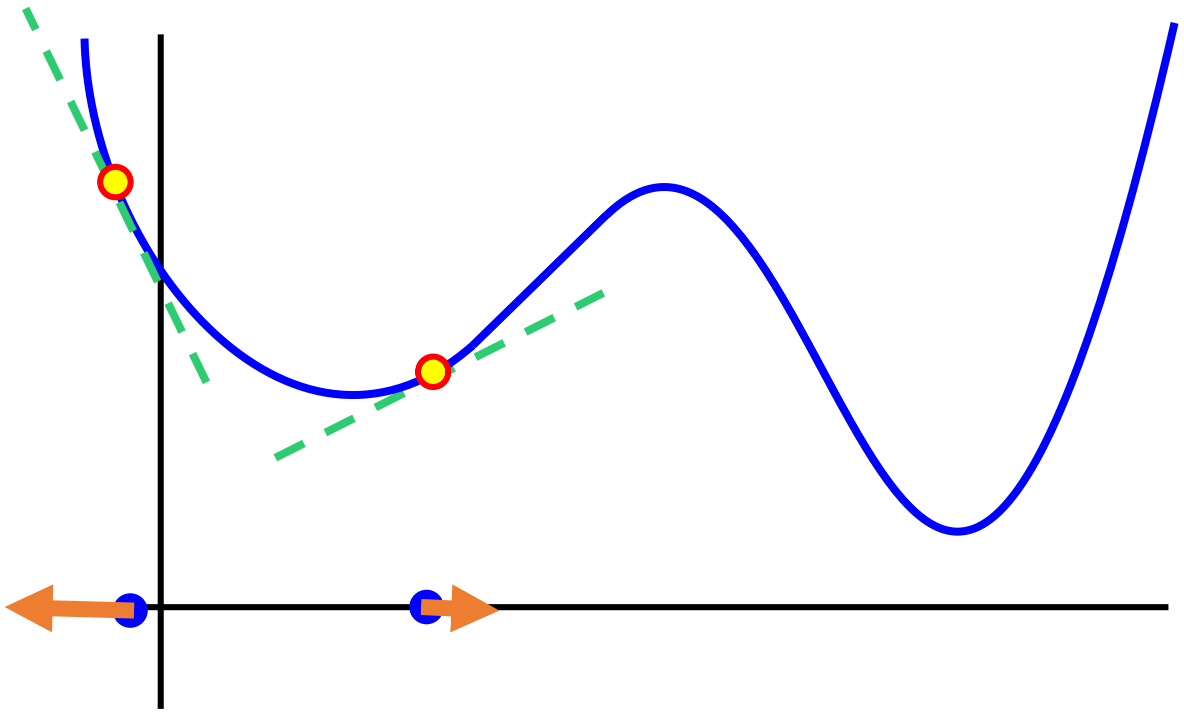
Gradients



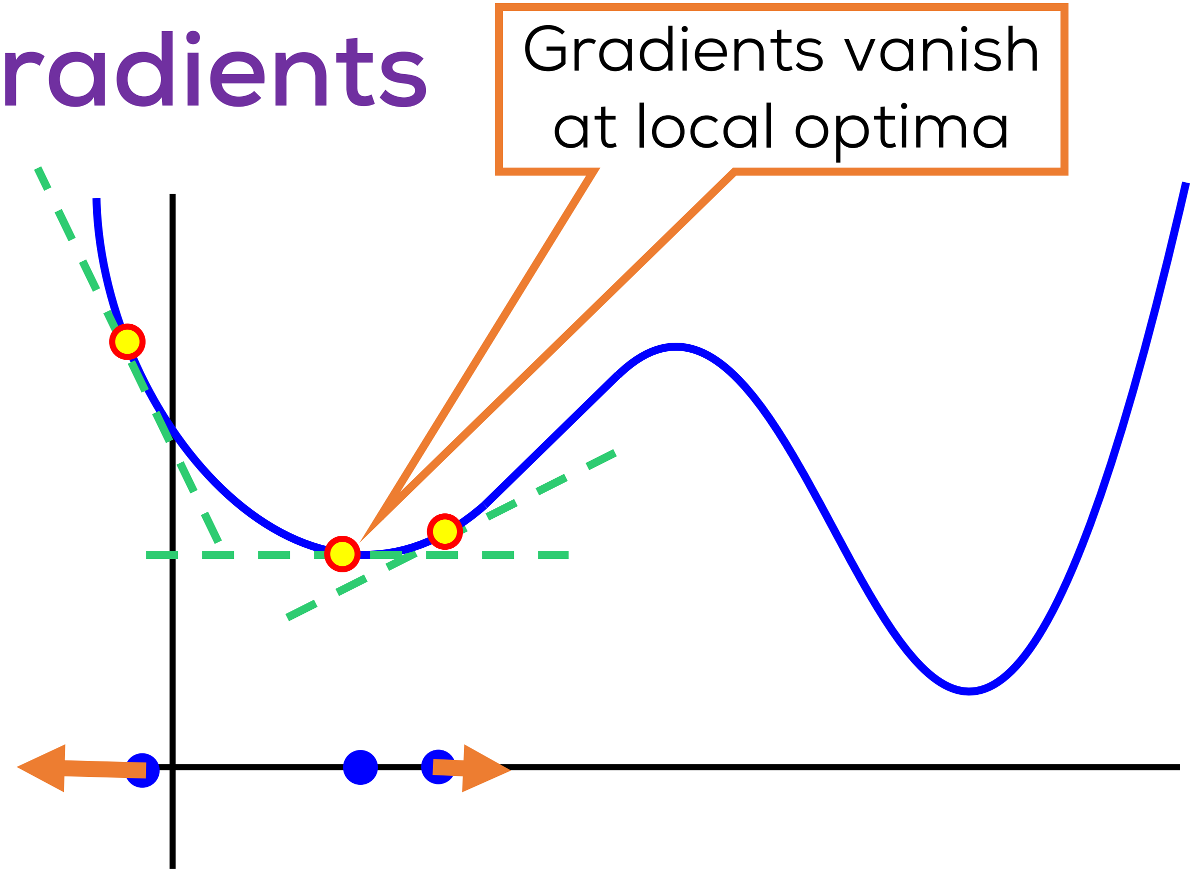
Gradients



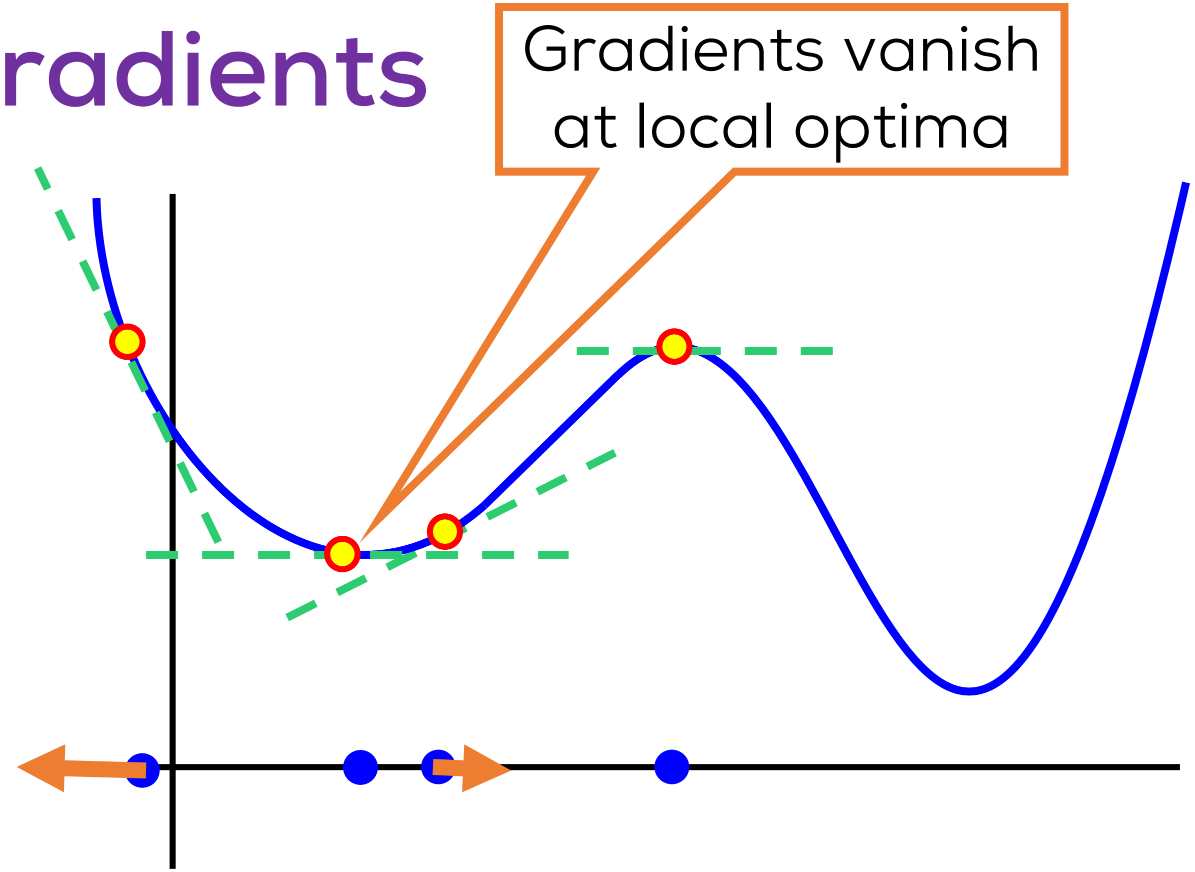
Gradients



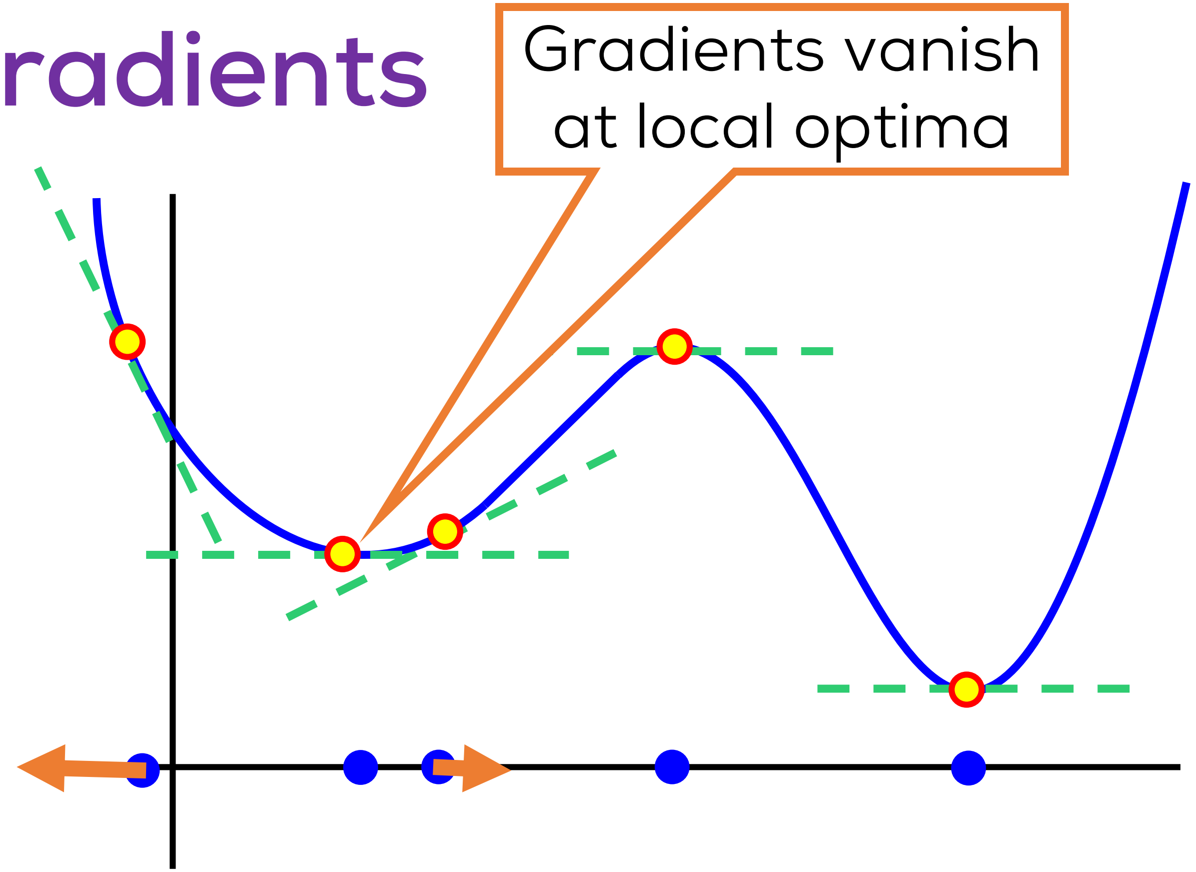
Gradients



Gradients



Gradients



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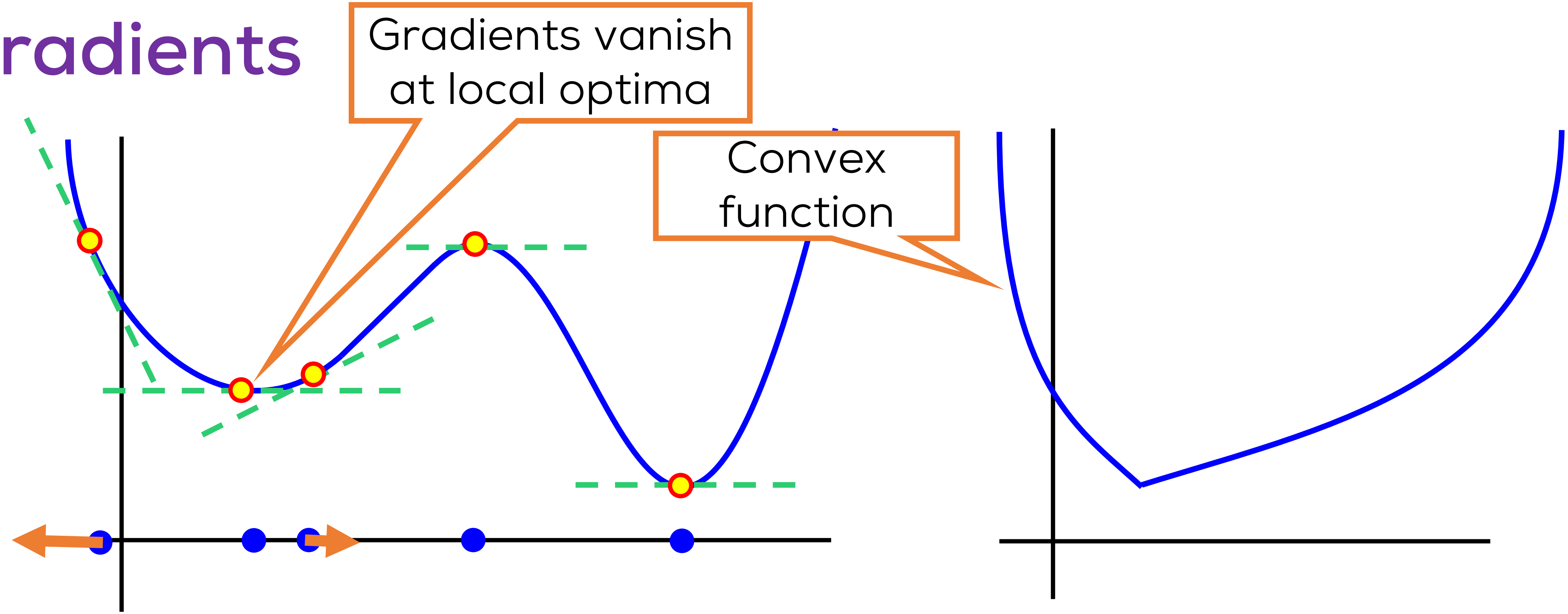
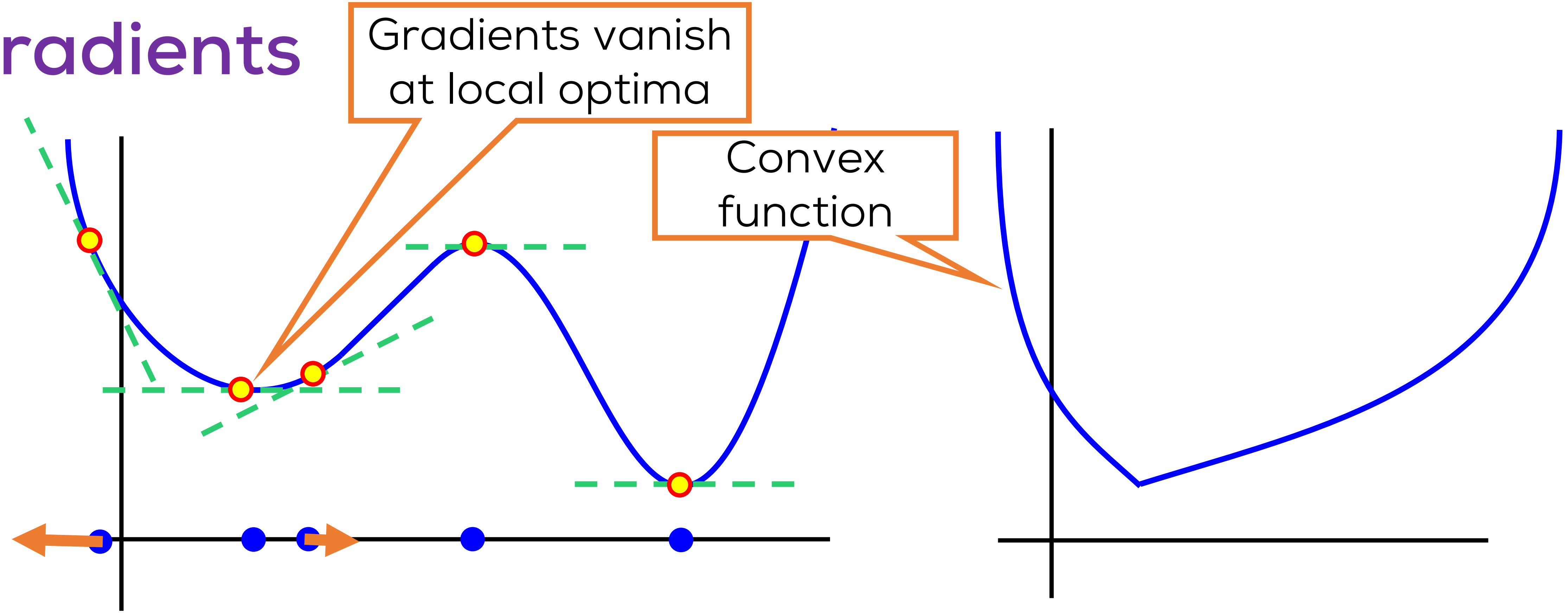


Gradients



$$\partial f(\mathbf{x}) = \{\mathbf{g} : f(\mathbf{y}) \geq f(\mathbf{x}) + \langle \mathbf{g}, \mathbf{y} - \mathbf{x} \rangle\}$$

$$\partial f(\mathbf{x}) = \{\nabla f(\mathbf{x})\} \text{ if } f \text{ differentiable}$$

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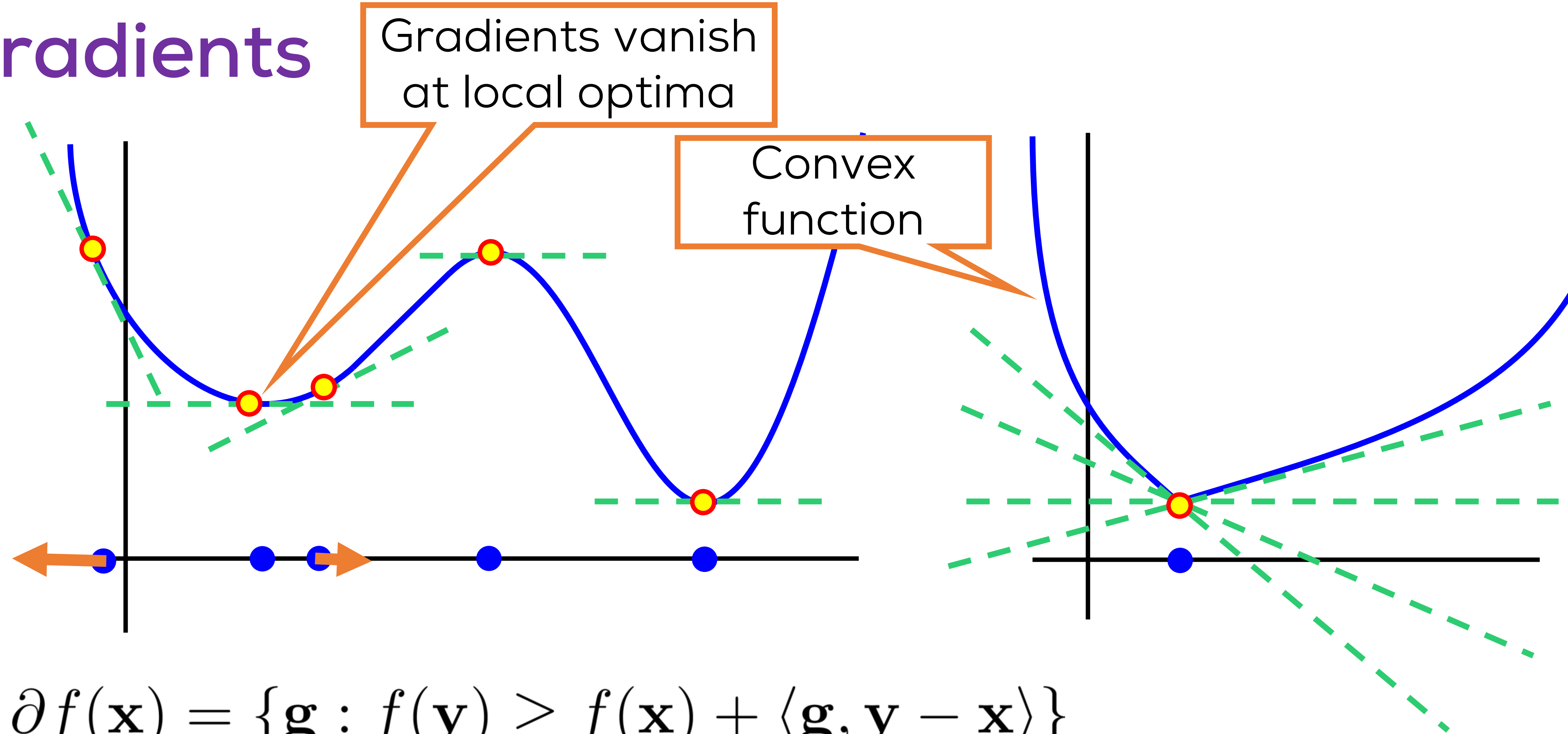

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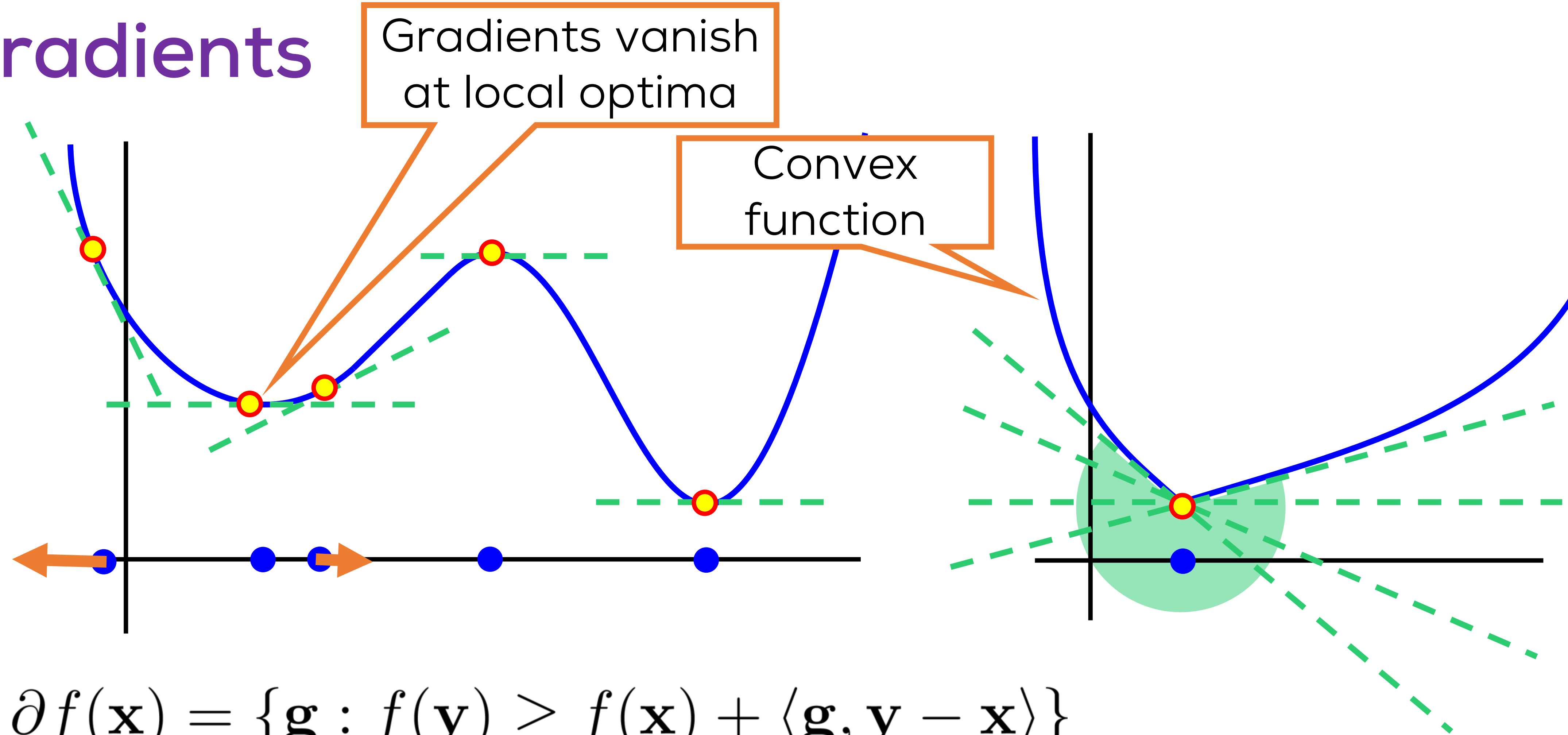

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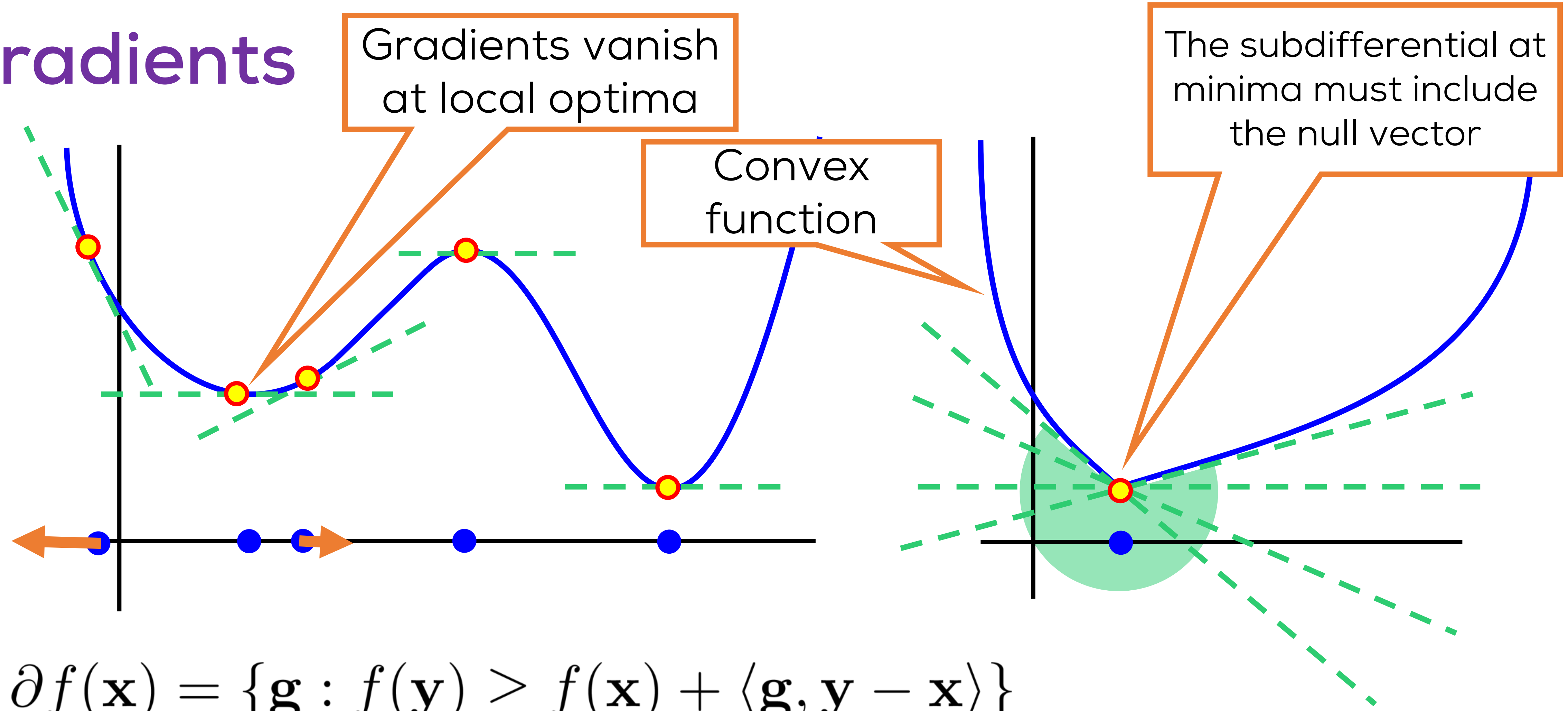

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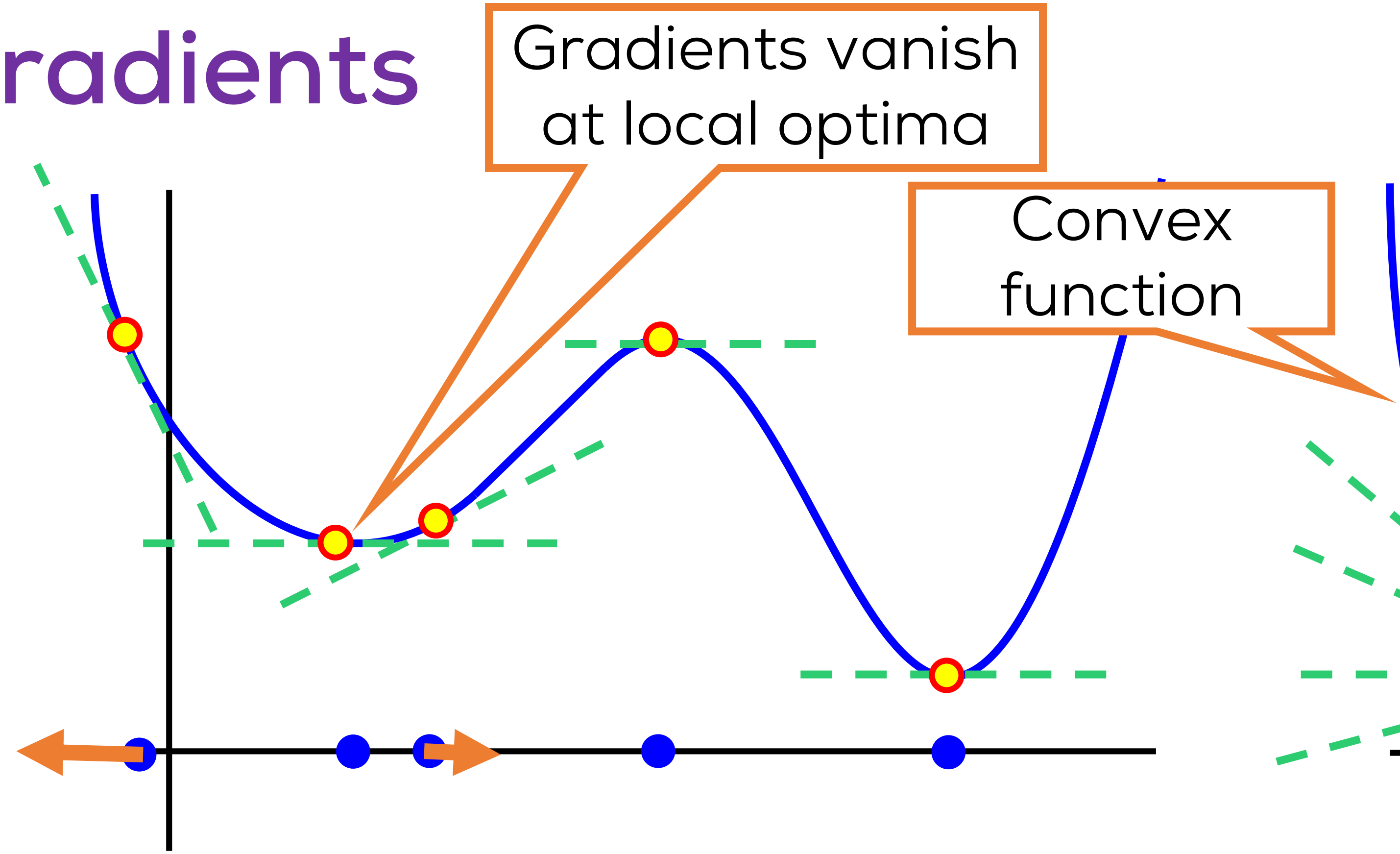

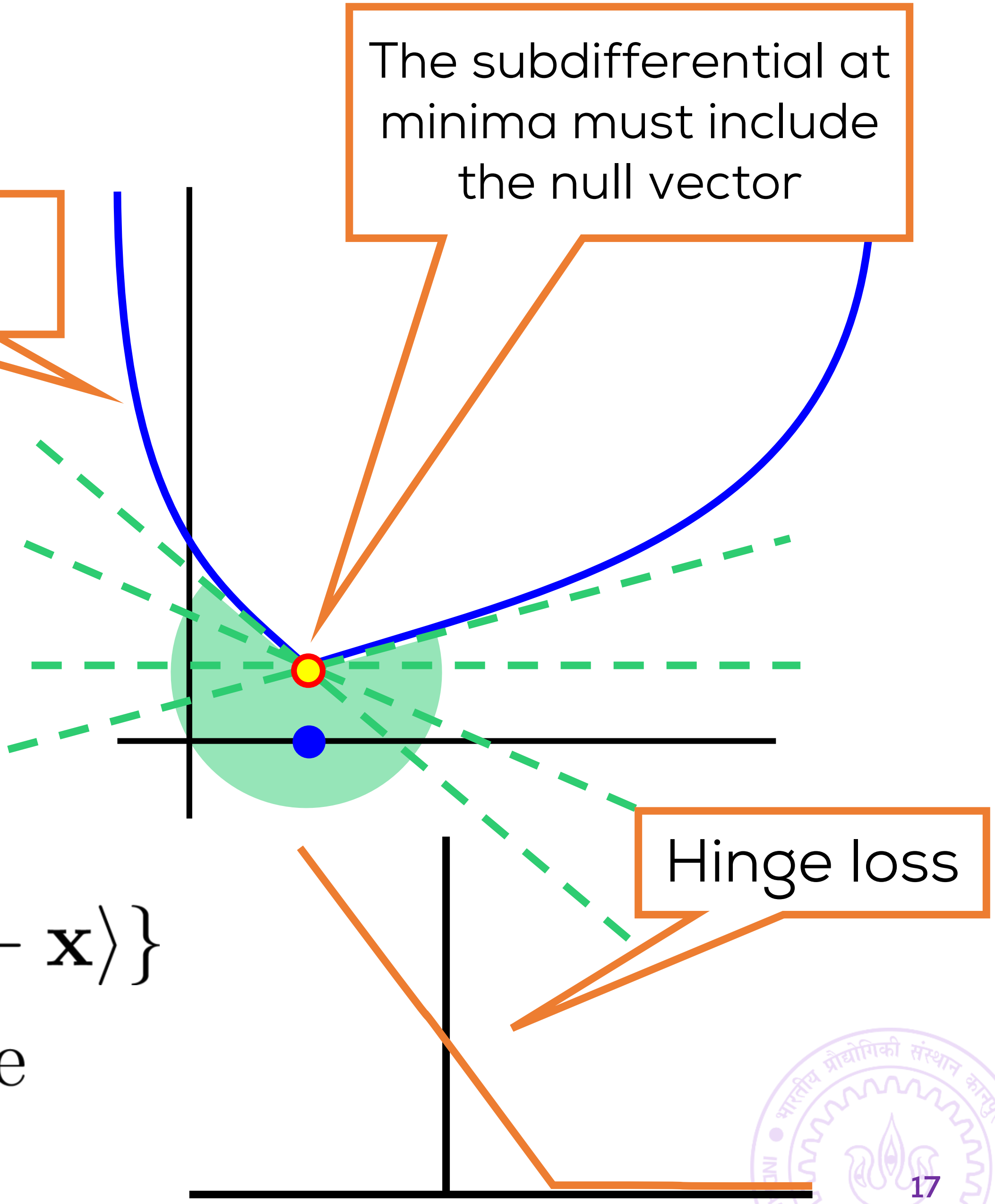

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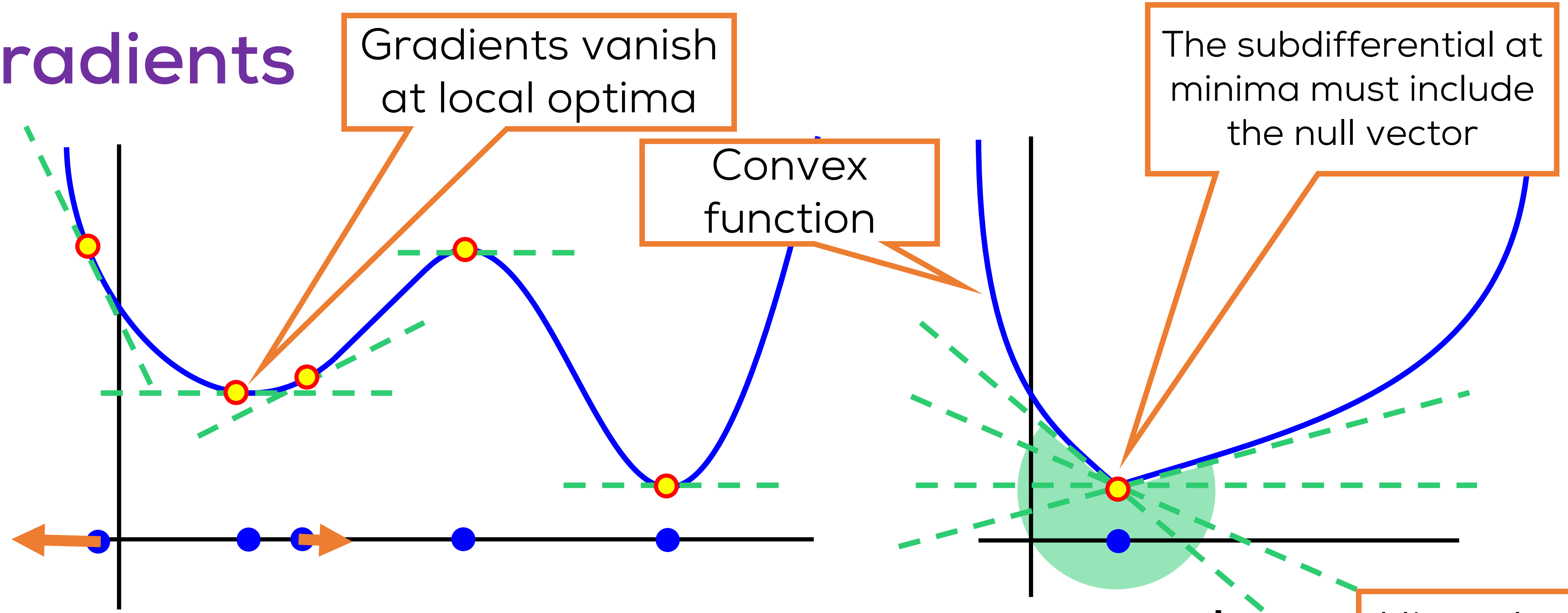

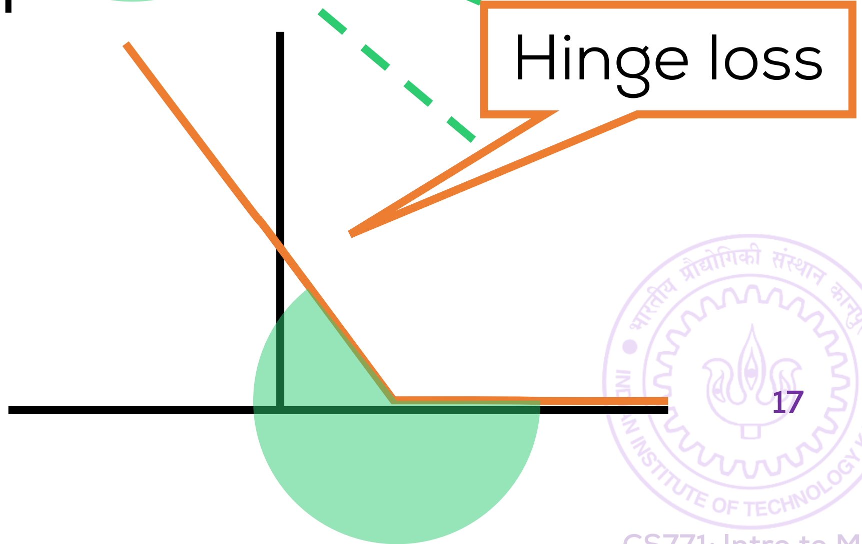

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So …

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$$\hat{\mathbf{w}} = \arg\min_{\mathbf{w}\in\mathbb{R}^d} \; f(\mathbf{w}) + r(\mathbf{w})$$

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Empirical
Loss function

So ...

$$\hat{\mathbf{w}} = \underset{\mathbf{w} \in \mathbb{R}^d}{\arg\min}\; f(\mathbf{w}) + r(\mathbf{w})$$

Empirical Loss function

Regularizer

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Examples

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Empirical Loss function

Regularizer

Examples

$$f(\mathbf{w}) = \sum_{i=1}^{n} \left(y^i - \langle \mathbf{w}, \mathbf{x}^i \rangle\right)^2$$

$$r(\mathbf{w}) = \lambda \cdot \|\mathbf{w}\|_1$$

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Empirical Loss function

Regularizer

Examples

$$f(\mathbf{w}) = \sum_{i=1}^{n} \left(y^i - \langle \mathbf{w}, \mathbf{x}^i \rangle\right)^2 \qquad f(\mathbf{w}) = \sum_{i=1}^{n} \log\left(1 + \exp(-y^i \langle \mathbf{w}, \mathbf{x}^i \rangle)\right)$$

$$r(\mathbf{w}) = \lambda \cdot \|\mathbf{w}\|_1 \qquad r(\mathbf{w}) = \lambda \cdot \|\mathbf{w}\|_2^2$$

Use Optimality Condition

Only works for the simplest of problems

First-order Optimality

$$\hat{\mathbf{w}} = \arg\min_{\mathbf{w} \in \mathbb{R}^d} \; f(\mathbf{w}) + r(\mathbf{w})$$

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$$2\sum_{i=1}^{n}(\langle \mathbf{w}, \mathbf{x}^i \rangle - y^i) \cdot \mathbf{x}_i + 2\lambda \cdot \mathbf{w} = \mathbf{0}$$

First-order Optimality

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$$\left(\sum_{i=1}^{n} (\mathbf{x}^i)(\mathbf{x}^i)^\top + \lambda \cdot I\right)\mathbf{w} = \sum_{i=1}^{n} y^i \mathbf{x}^i$$

First-order Optimality

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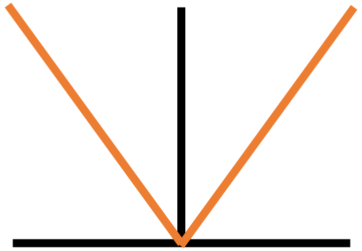

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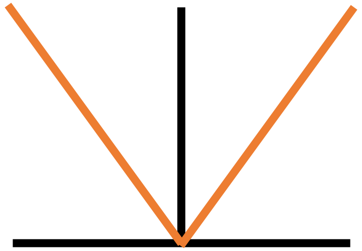

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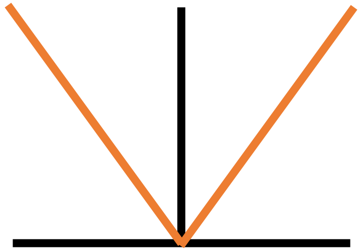

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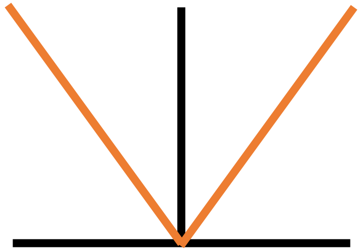

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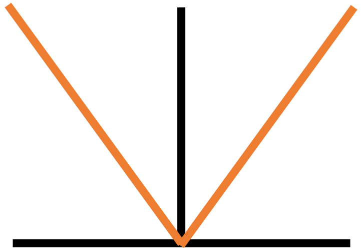

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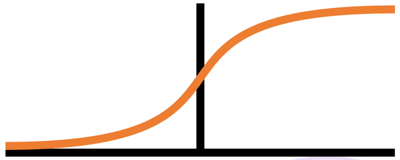

$$\sum_{i=1}^{n} (1 - \sigma(y^i \left\langle \mathbf{w}, \mathbf{x}^i \right\rangle)) y^i \cdot \mathbf{x}^i + 2\lambda \cdot \mathbf{w} = \mathbf{0}$$

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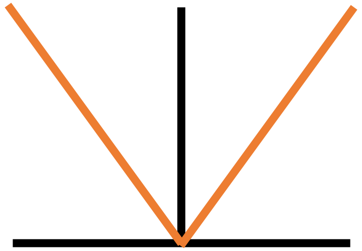

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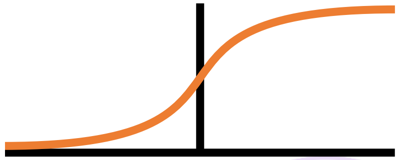
?



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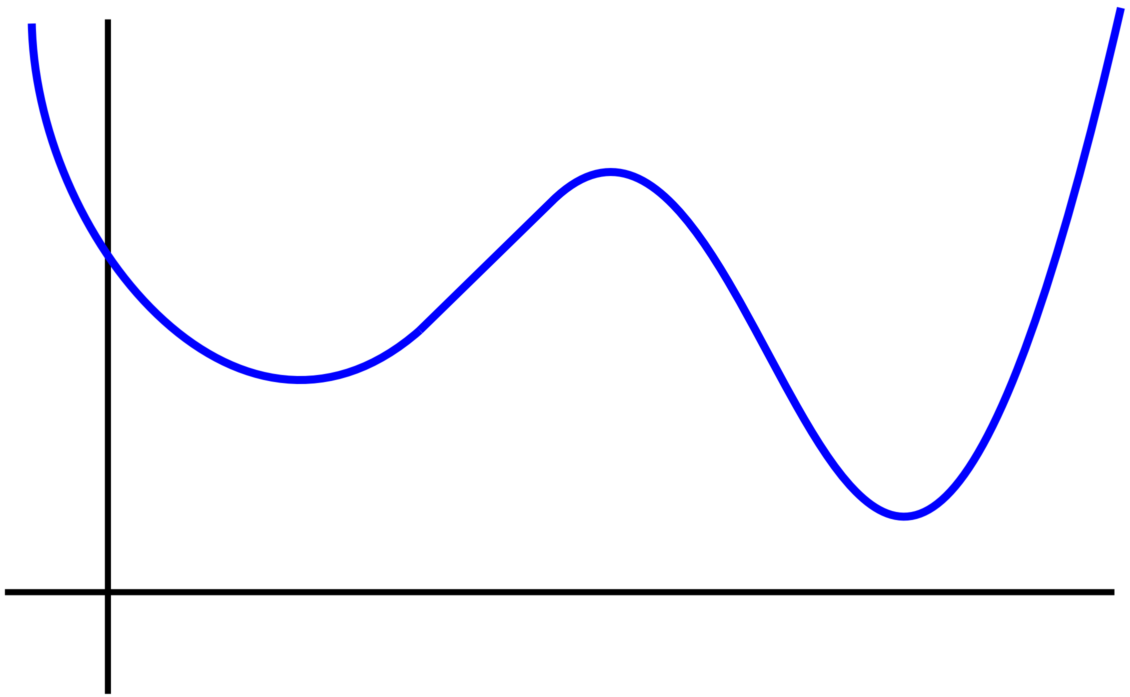
Use (sub)Gradient Descent

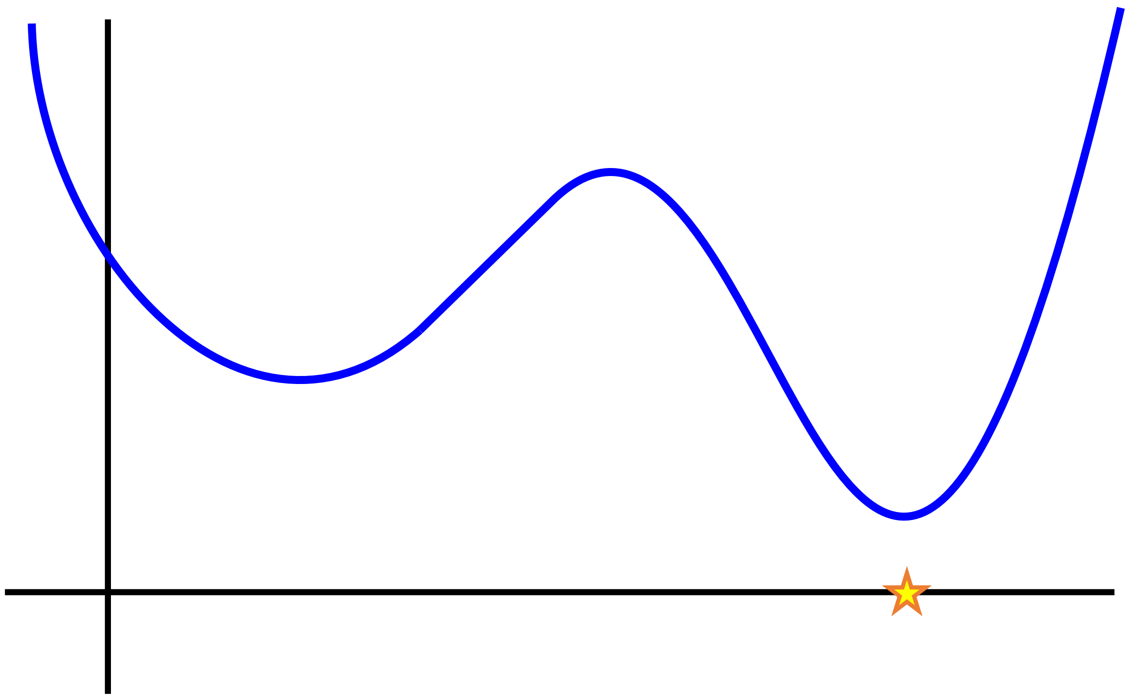
Use this technique before anything else

Gradient Descent

Gradient Descent



Gradient Descent



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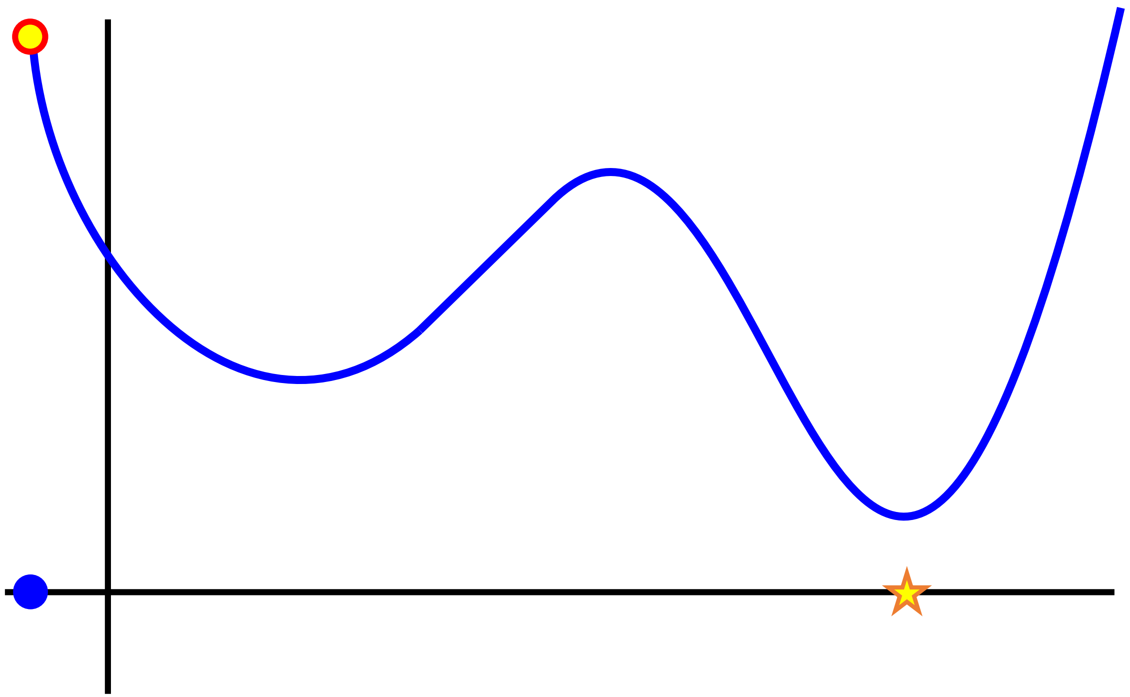

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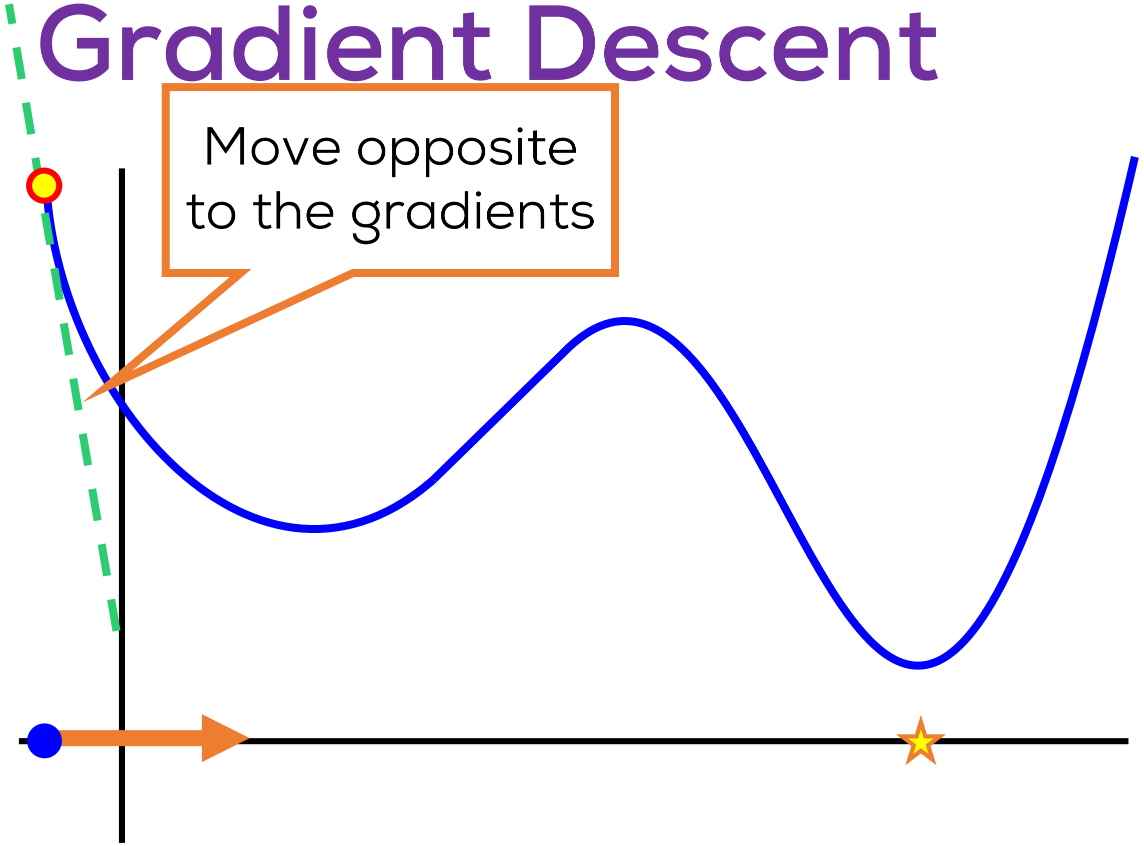

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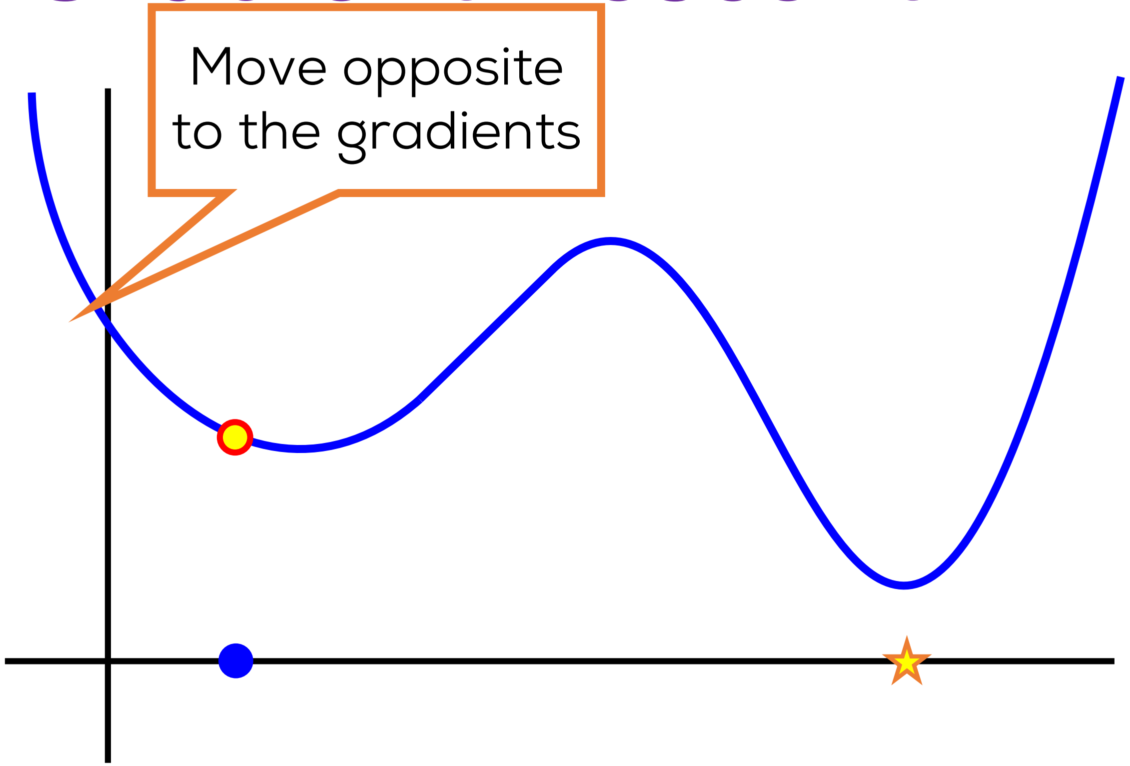

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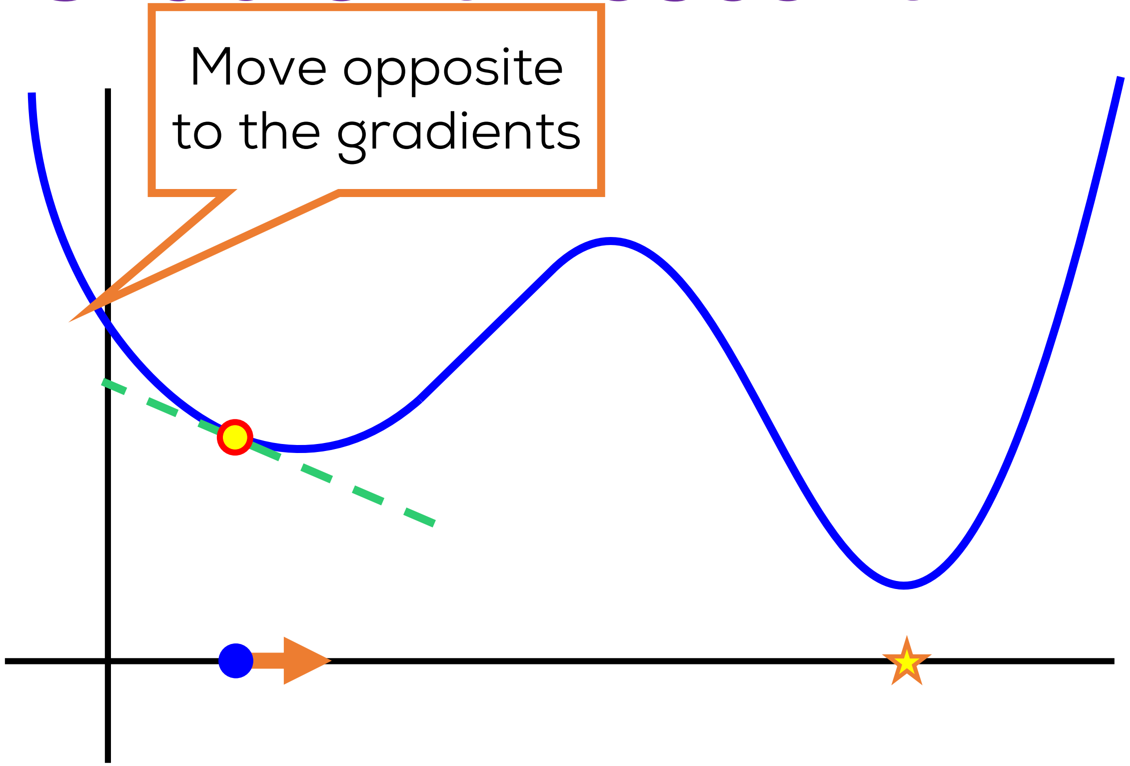

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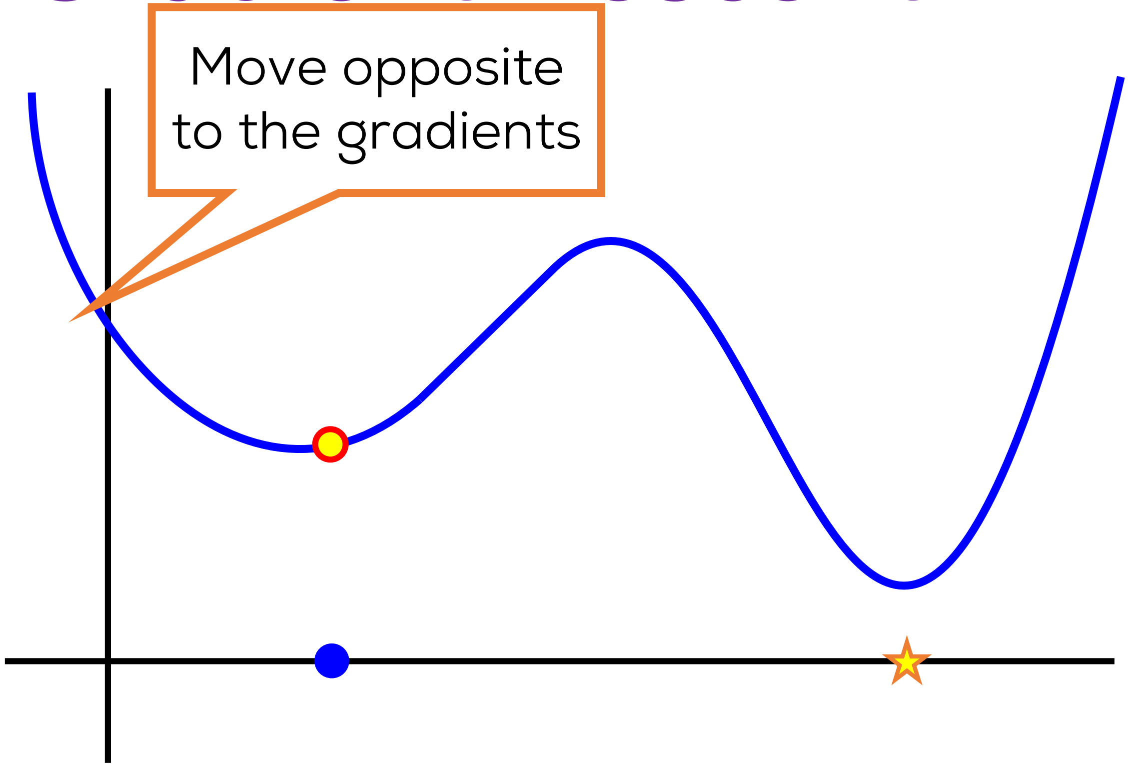

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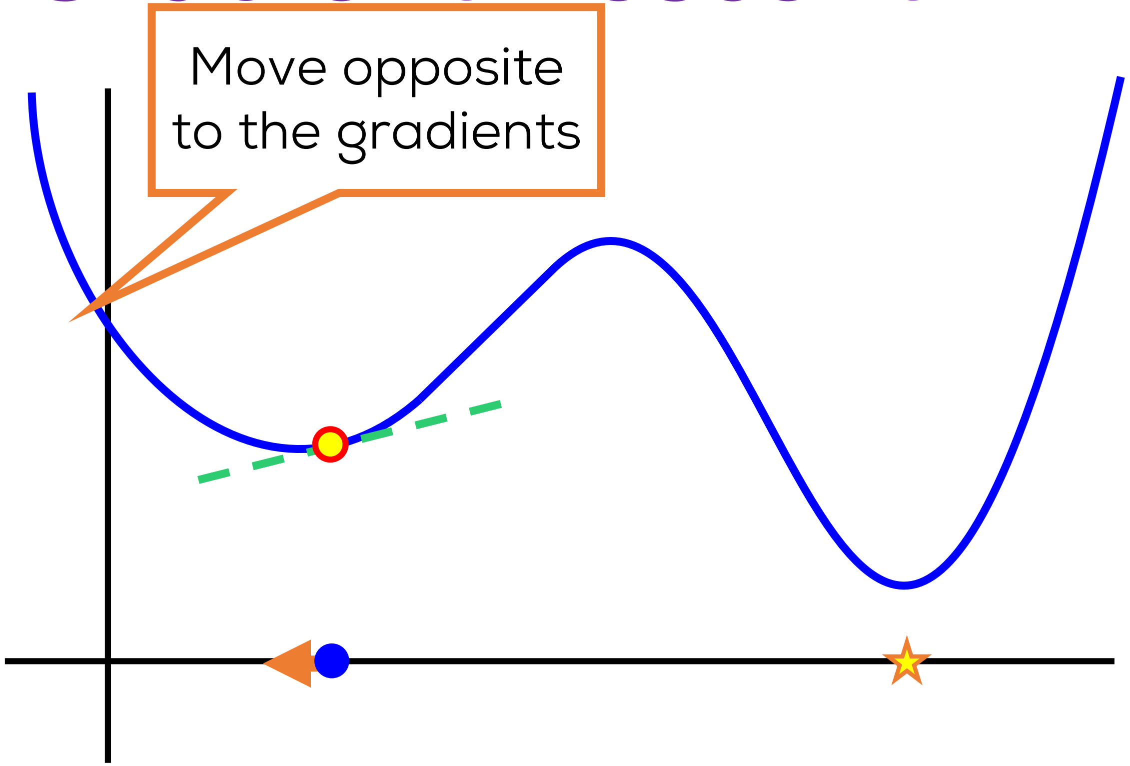

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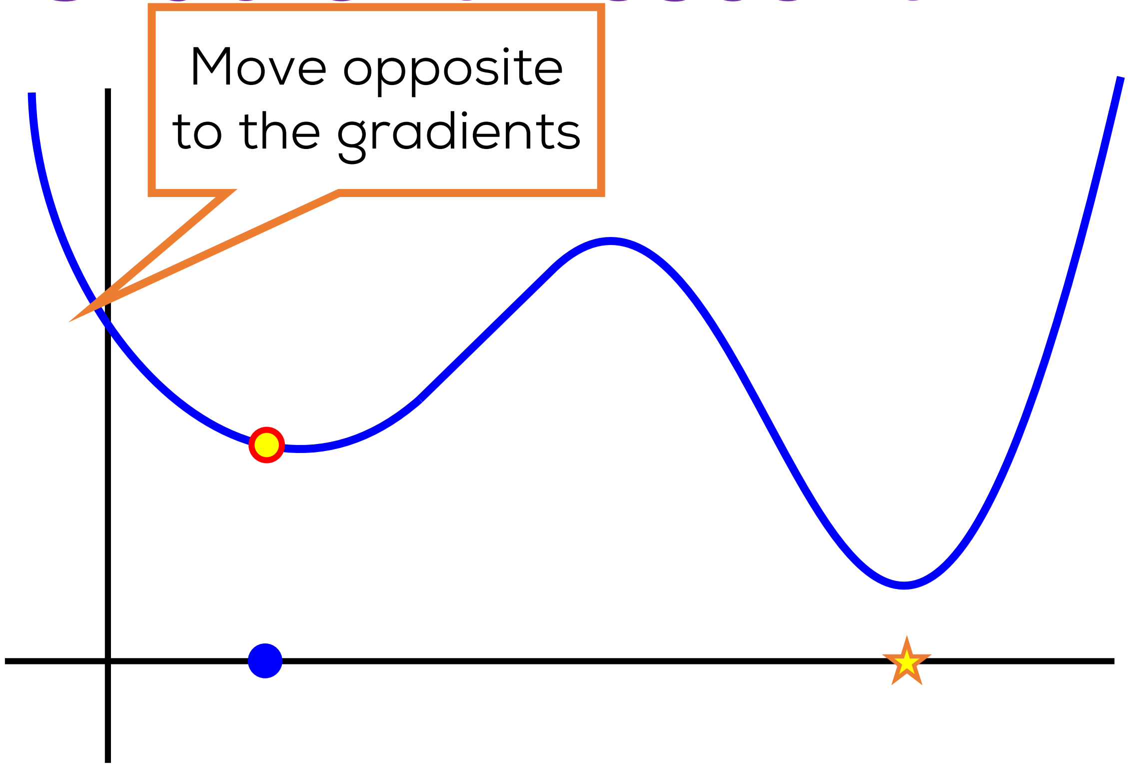

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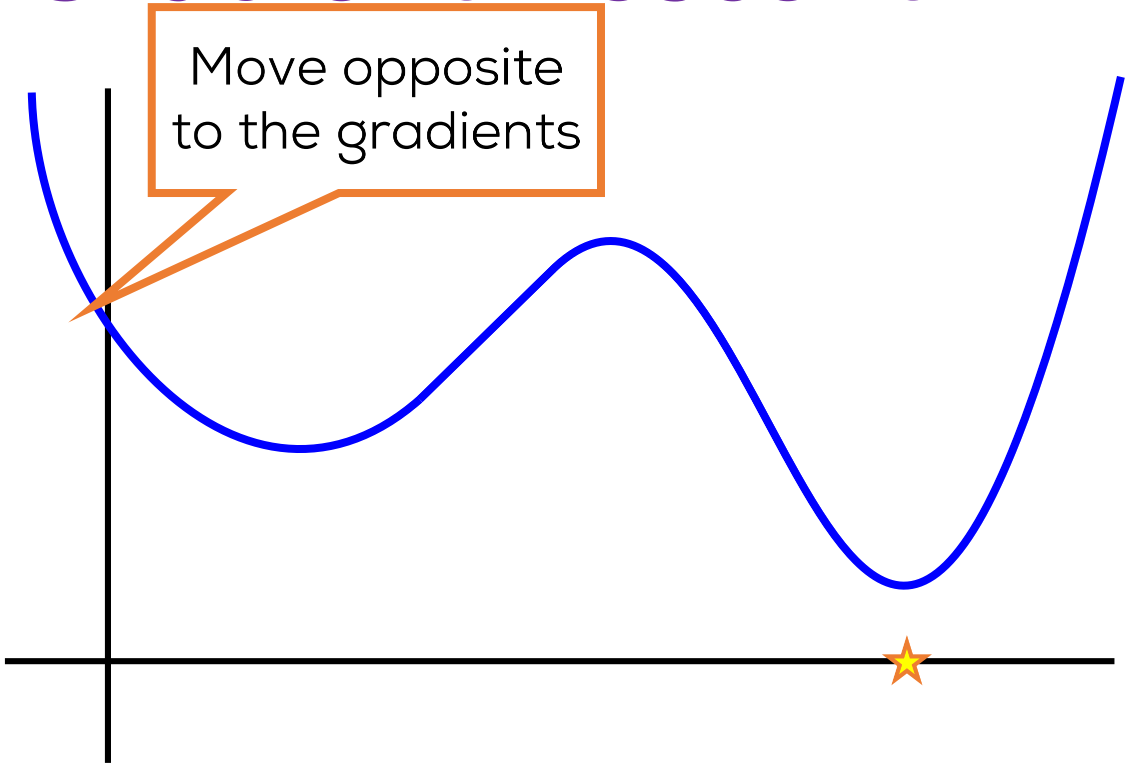

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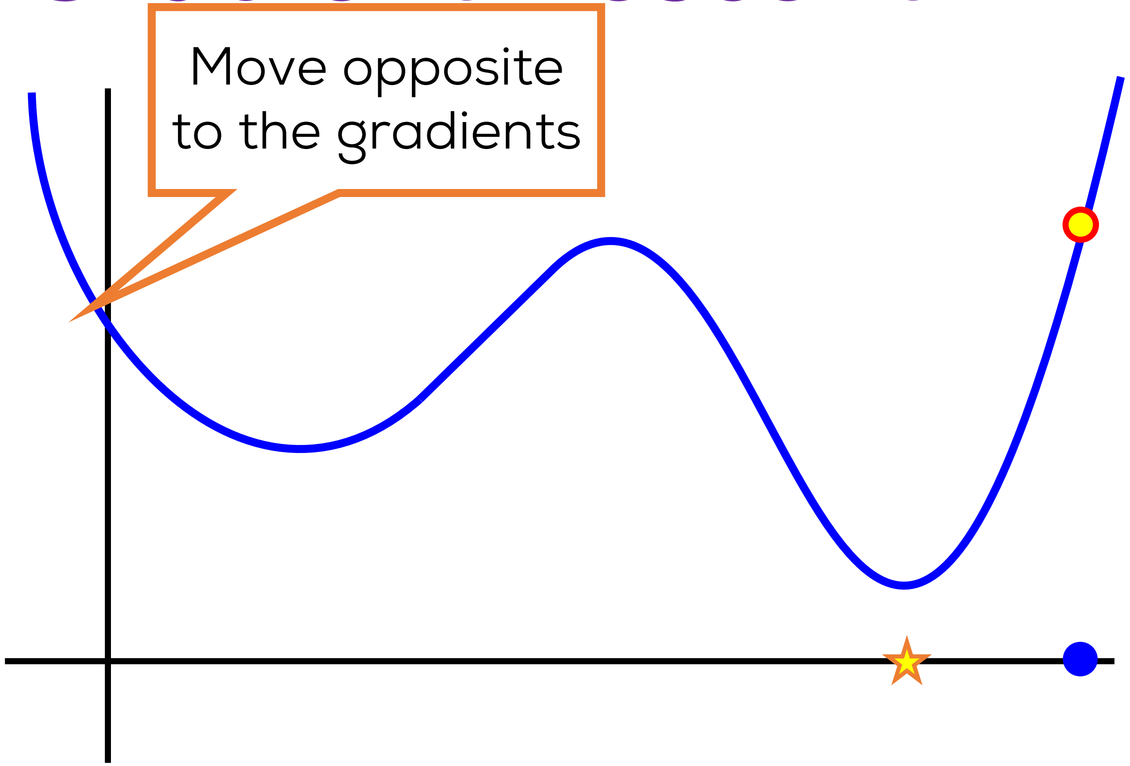

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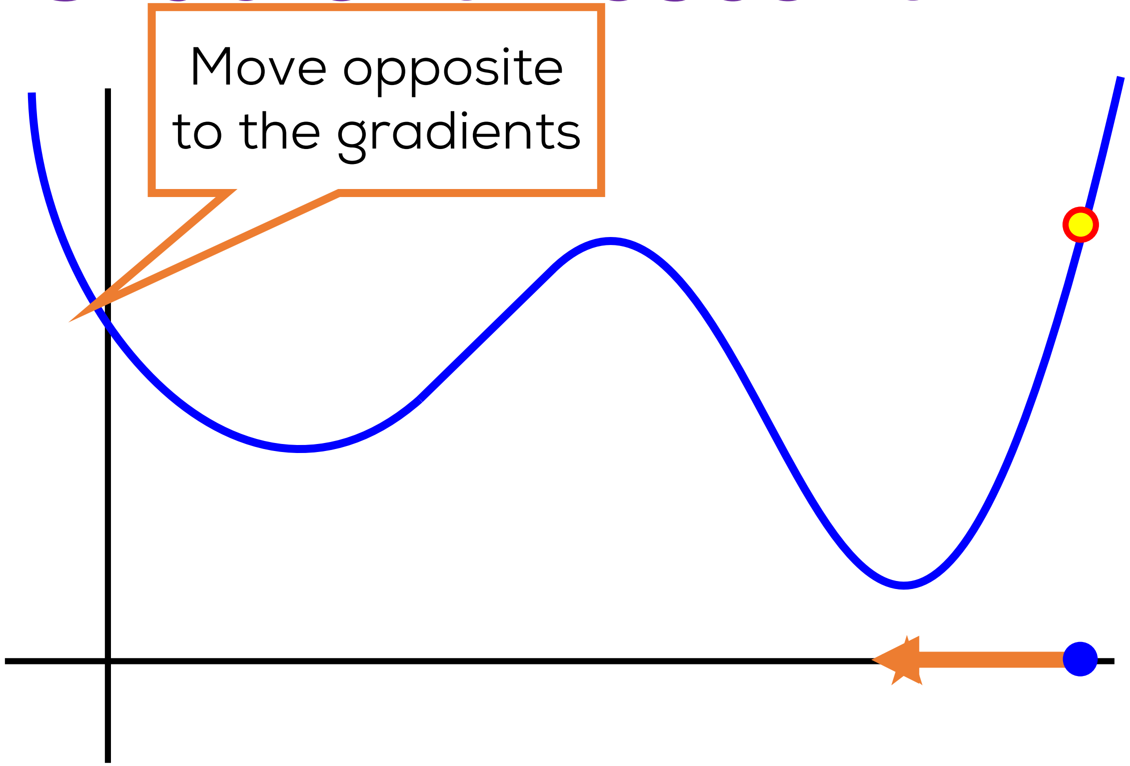

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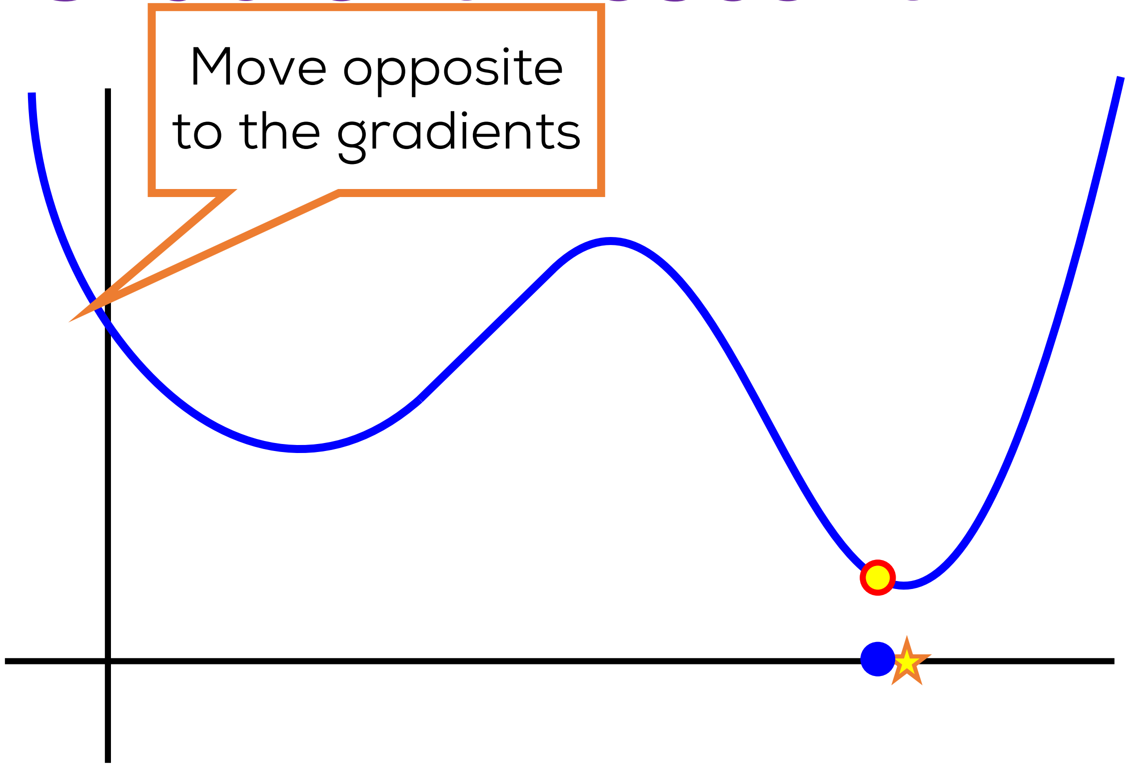

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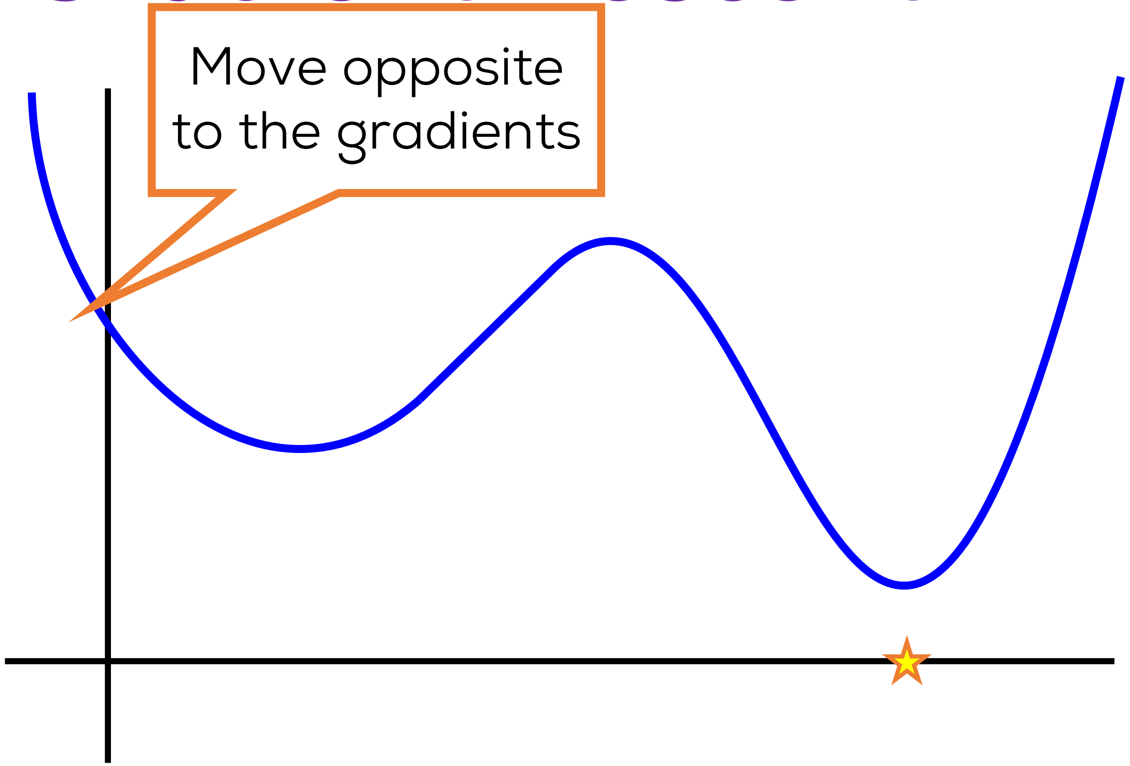

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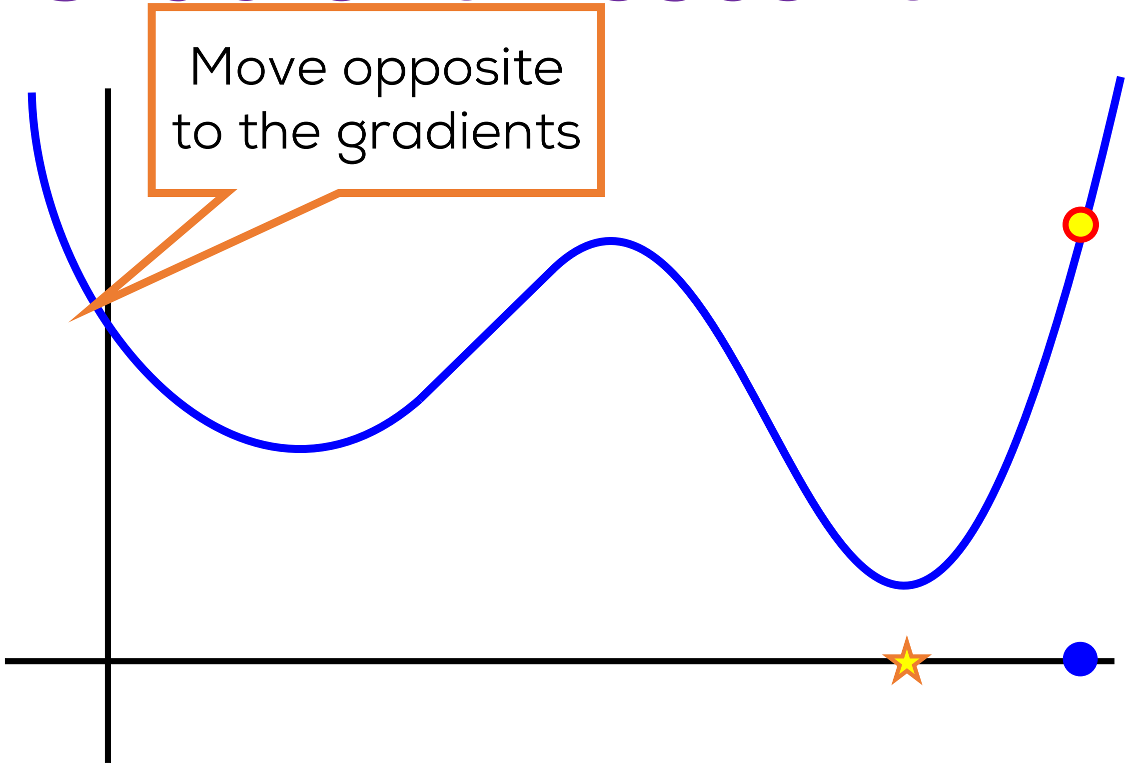

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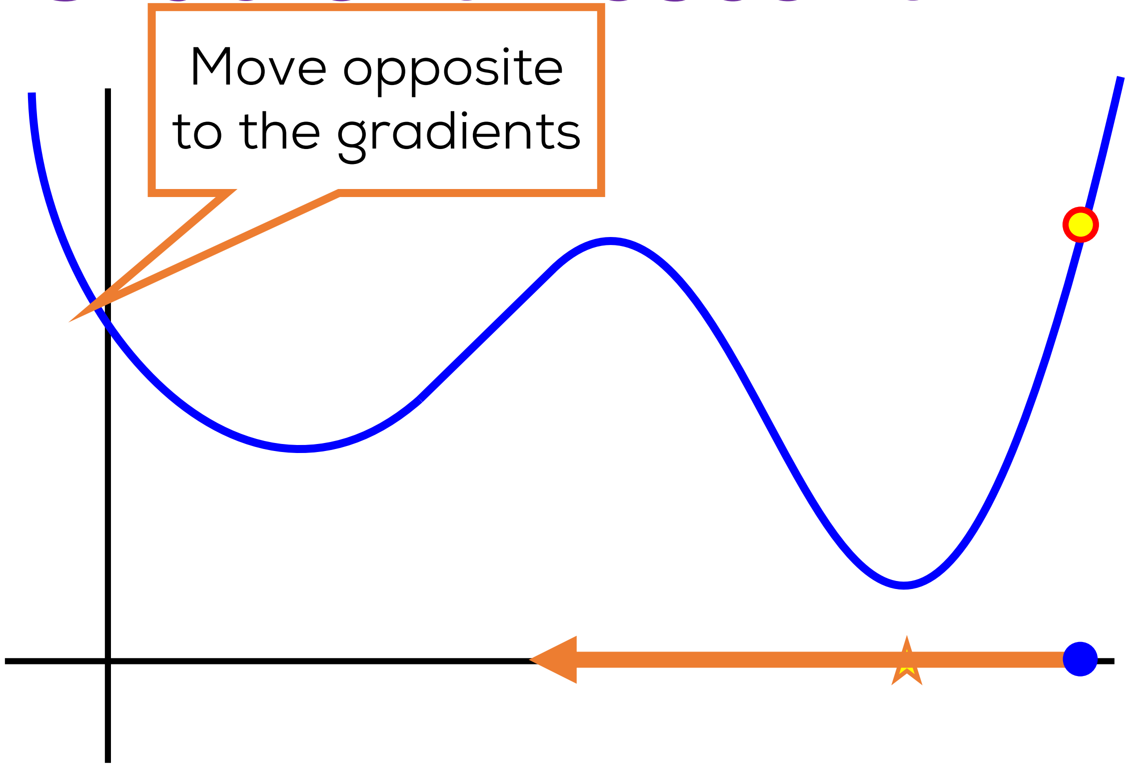

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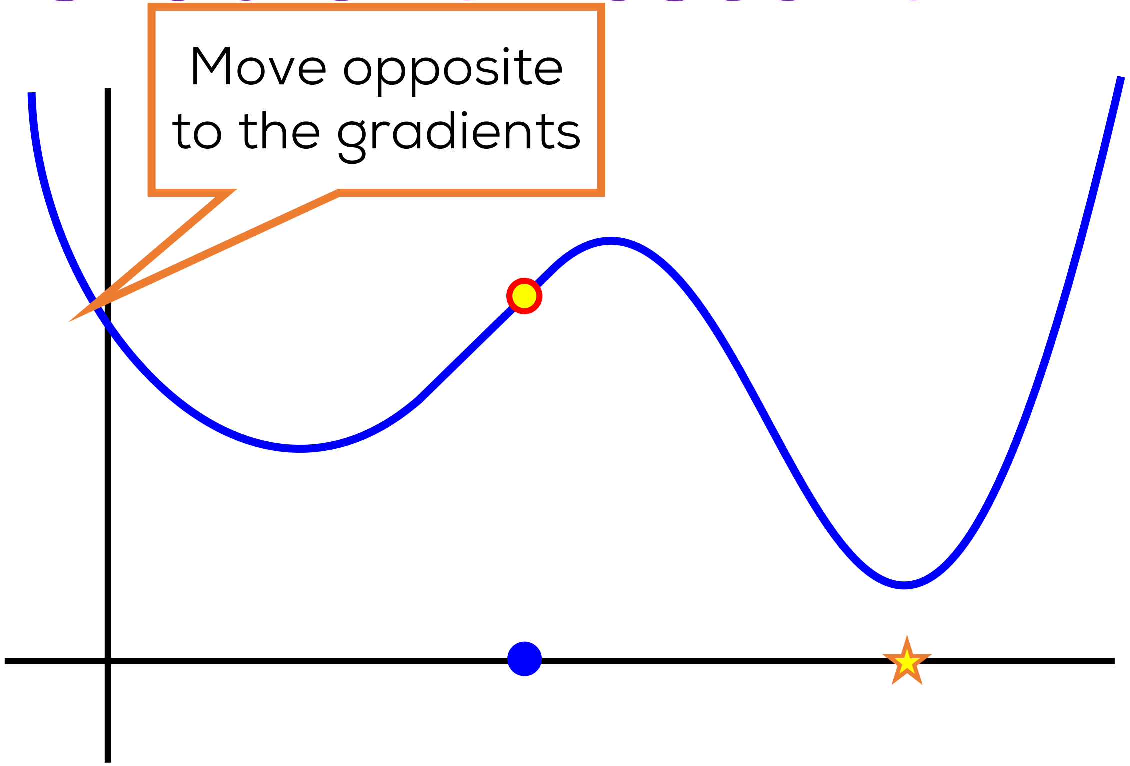

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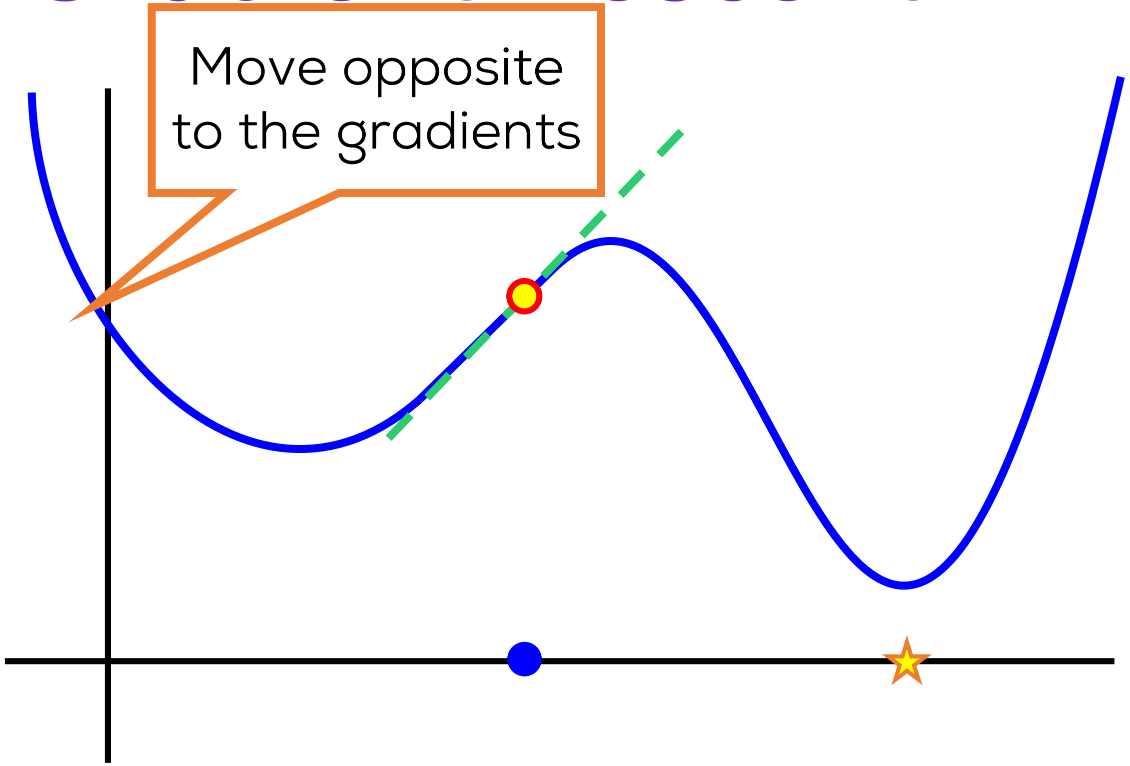

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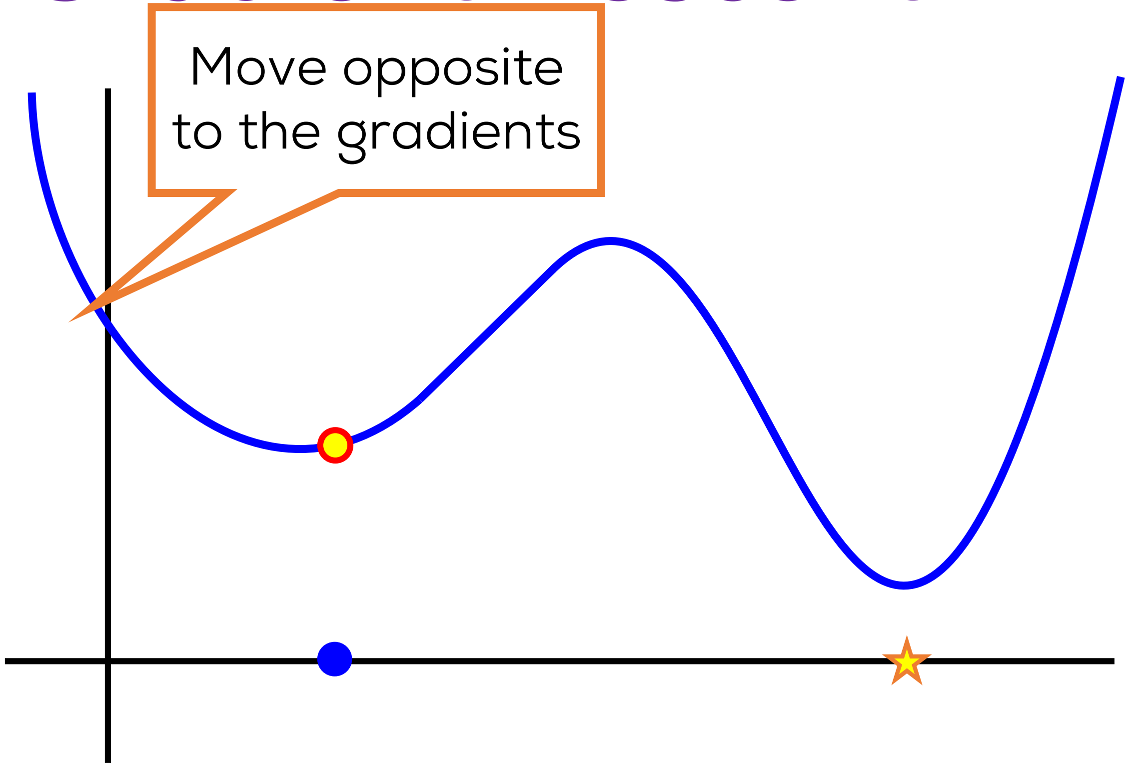
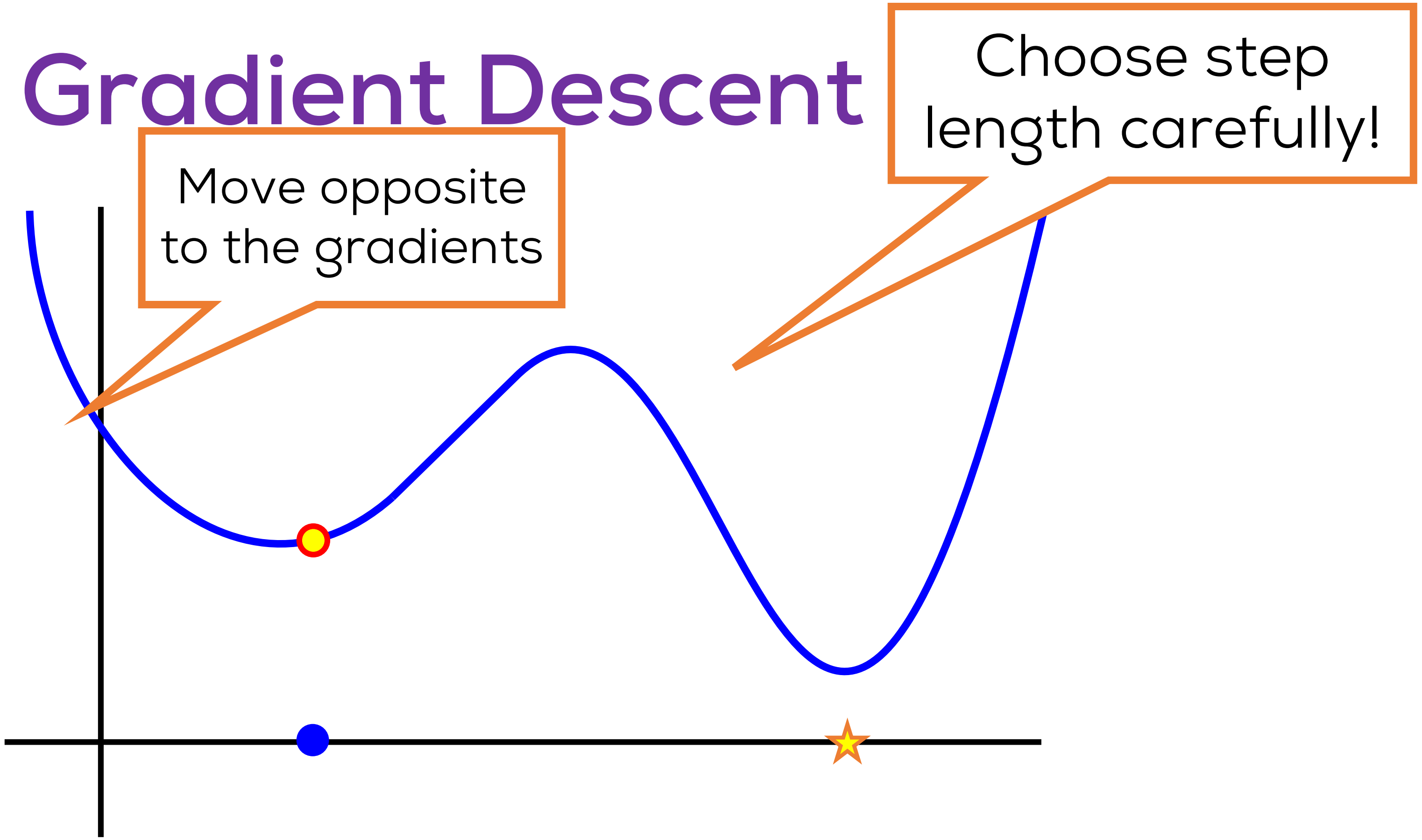
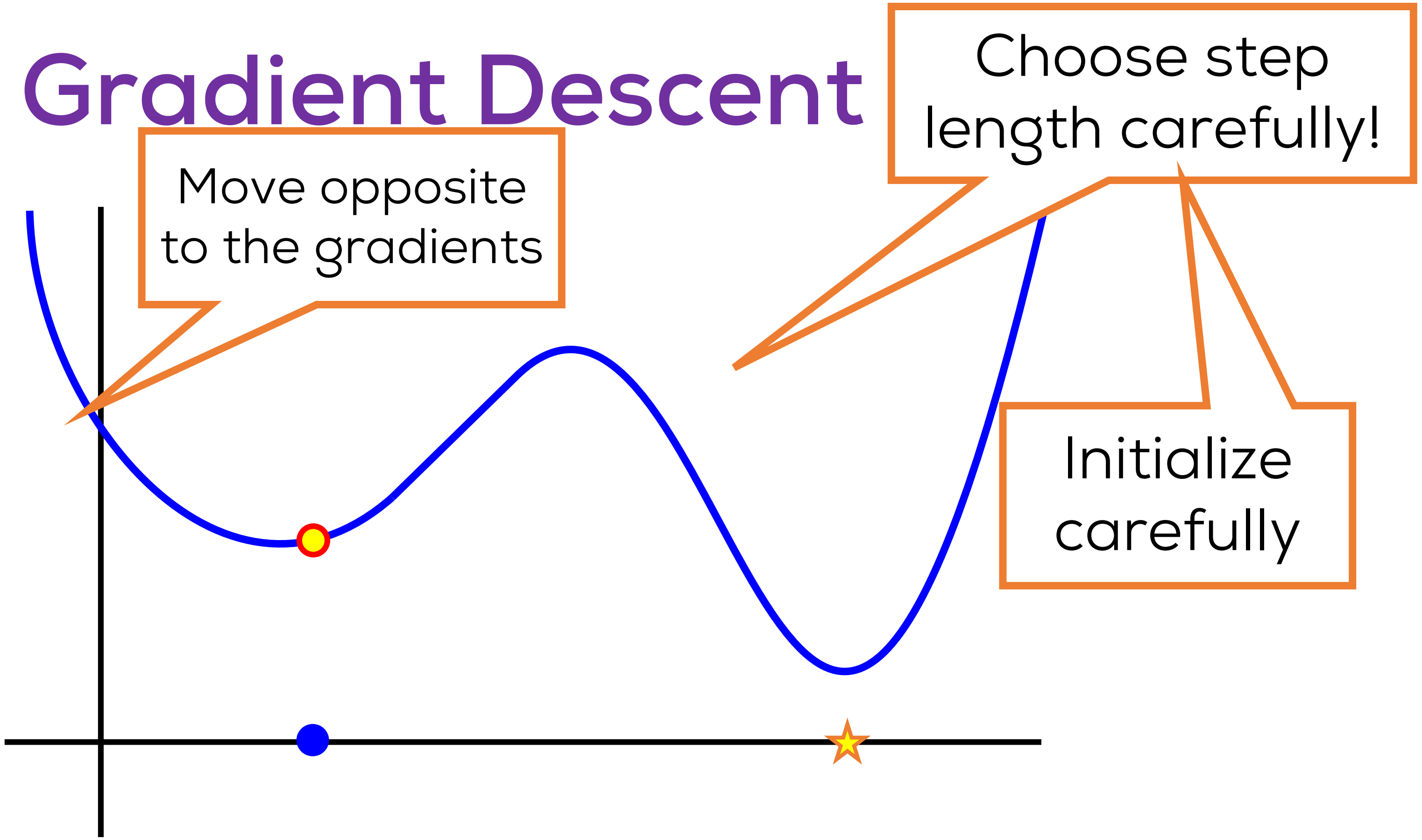


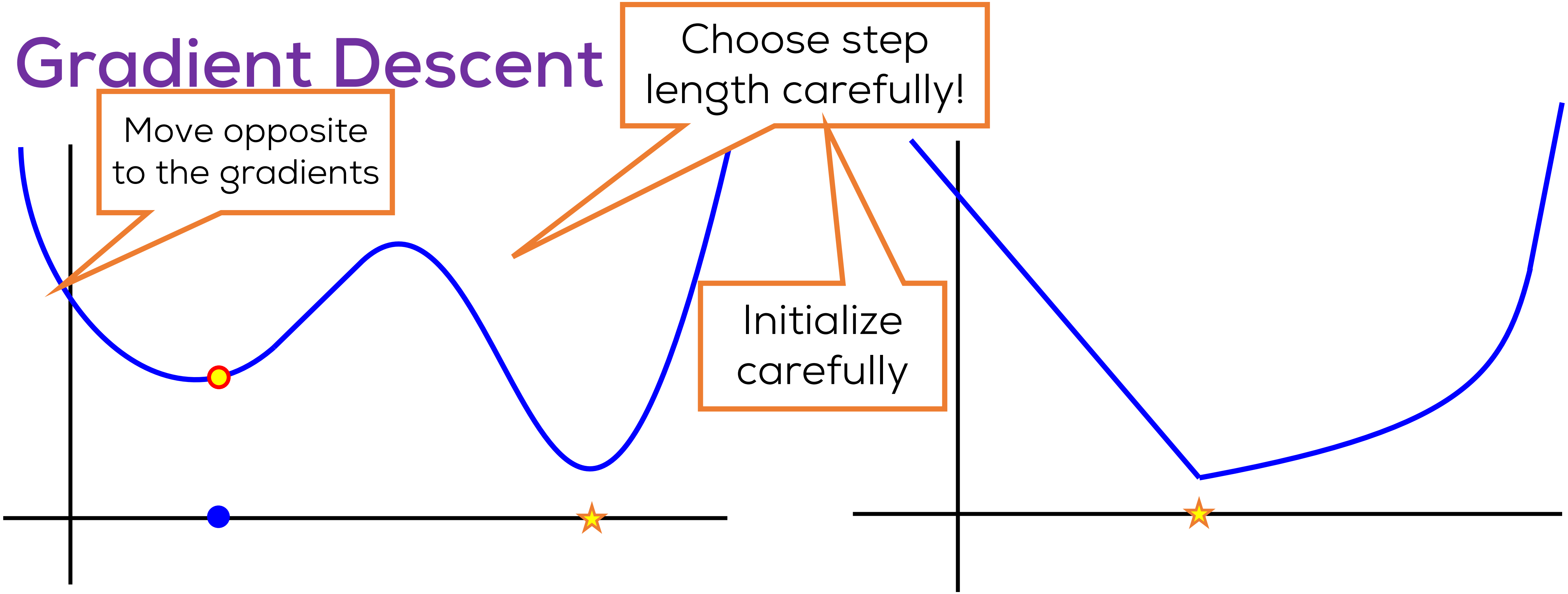
Gradient Descent



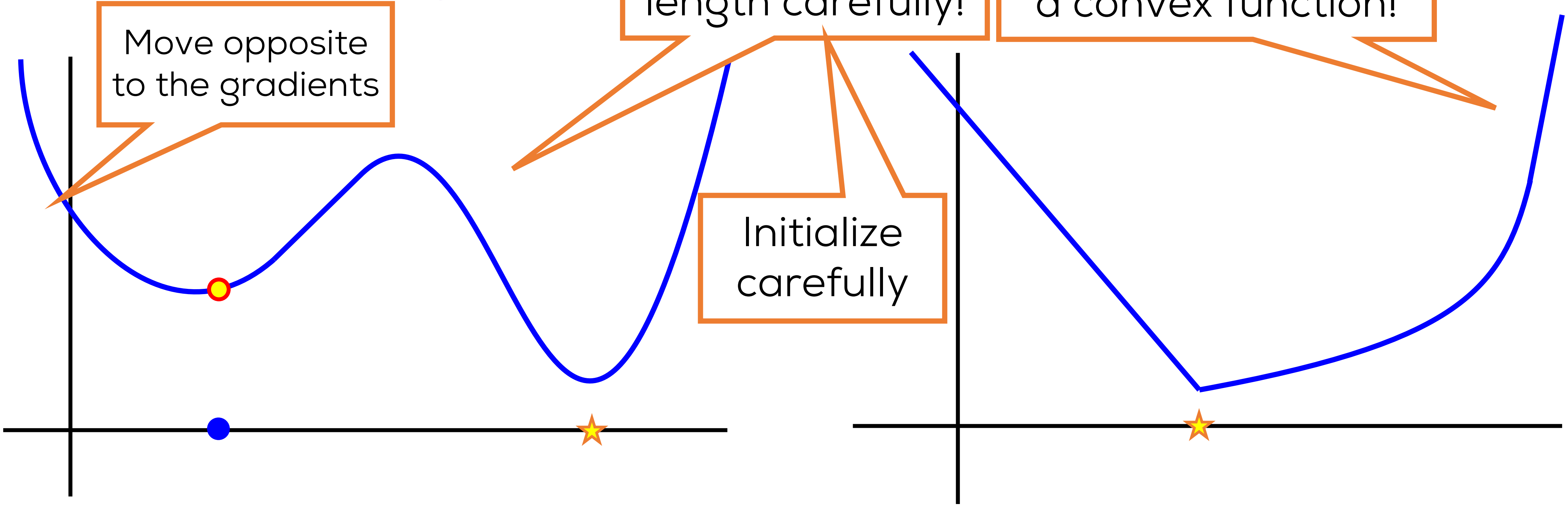
Gradient Descent



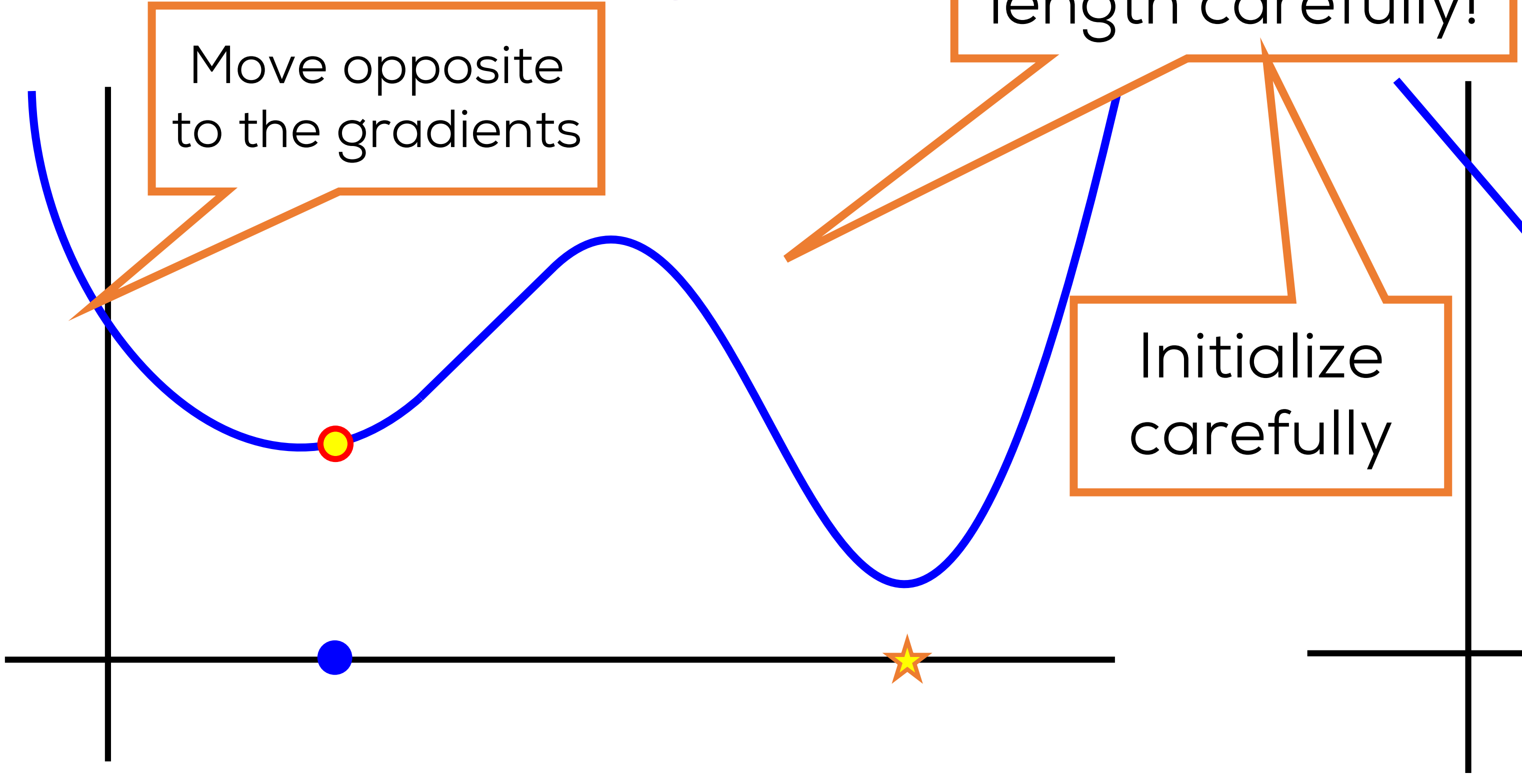
Gradient Descent

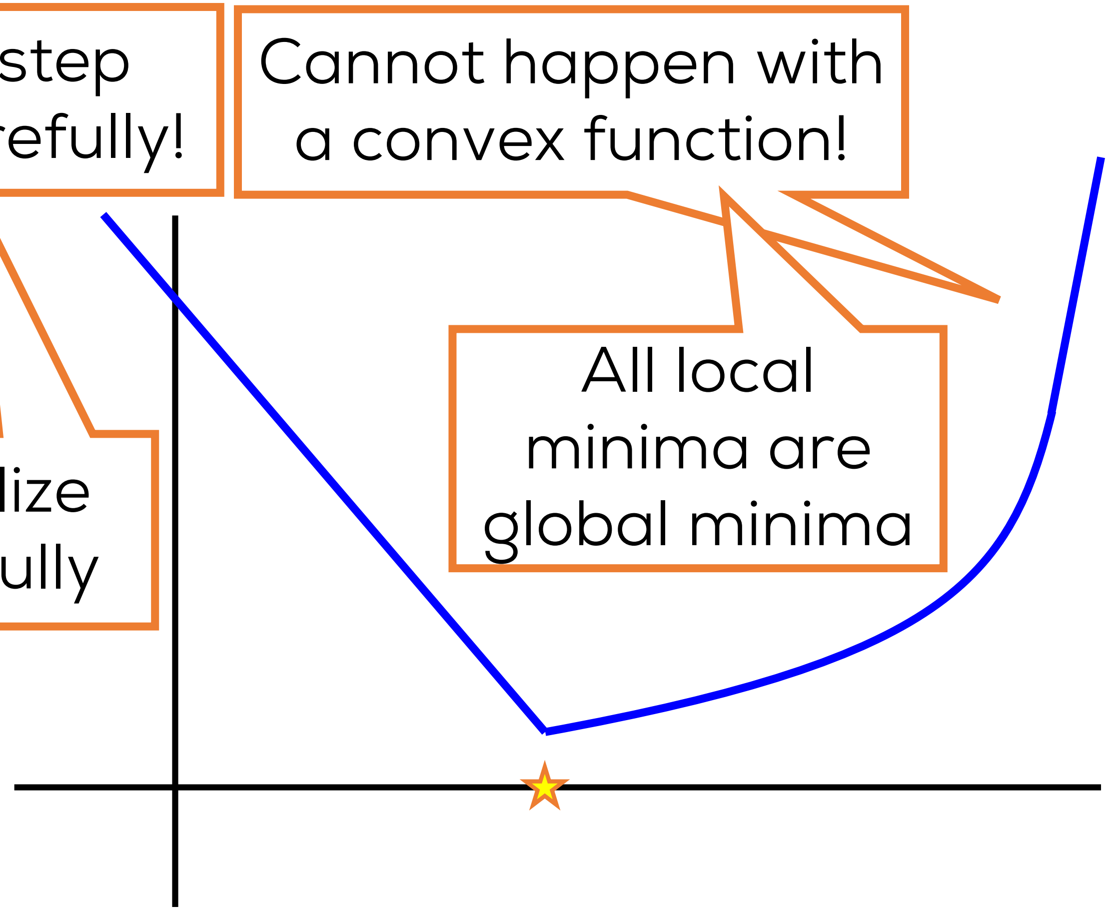


Gradient Descent

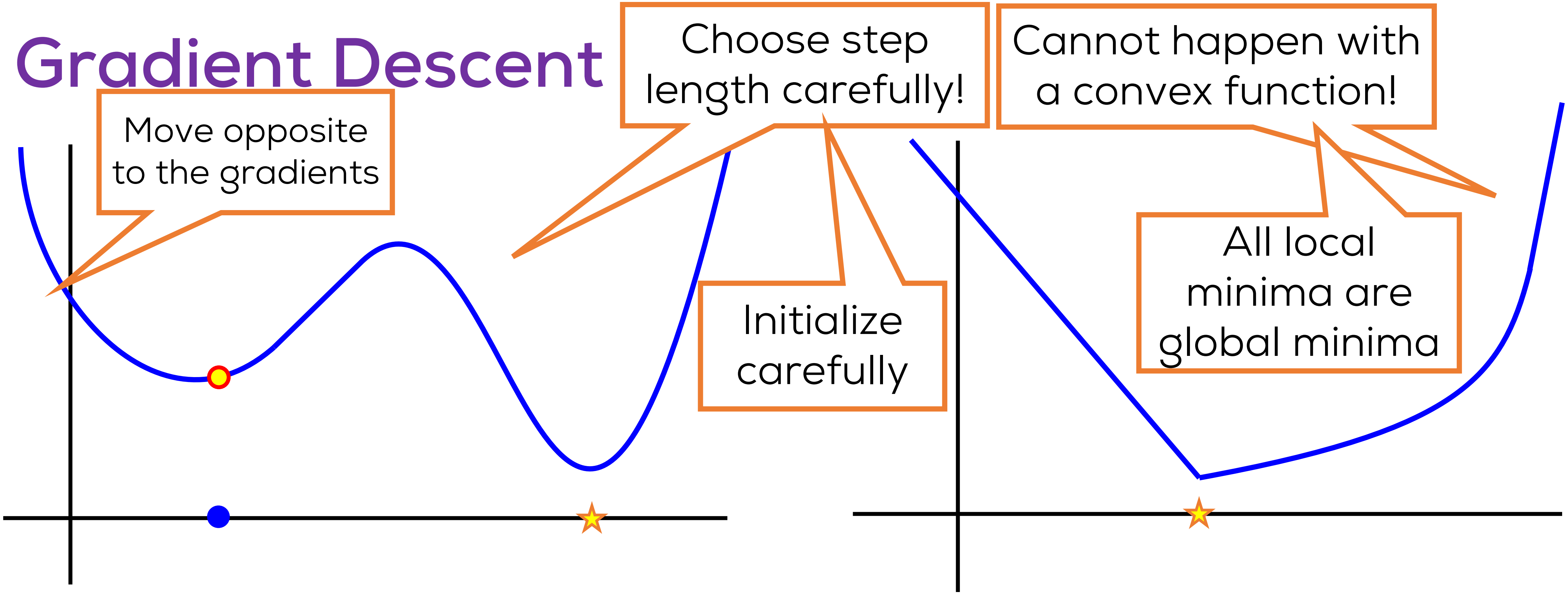


Gradient Descent





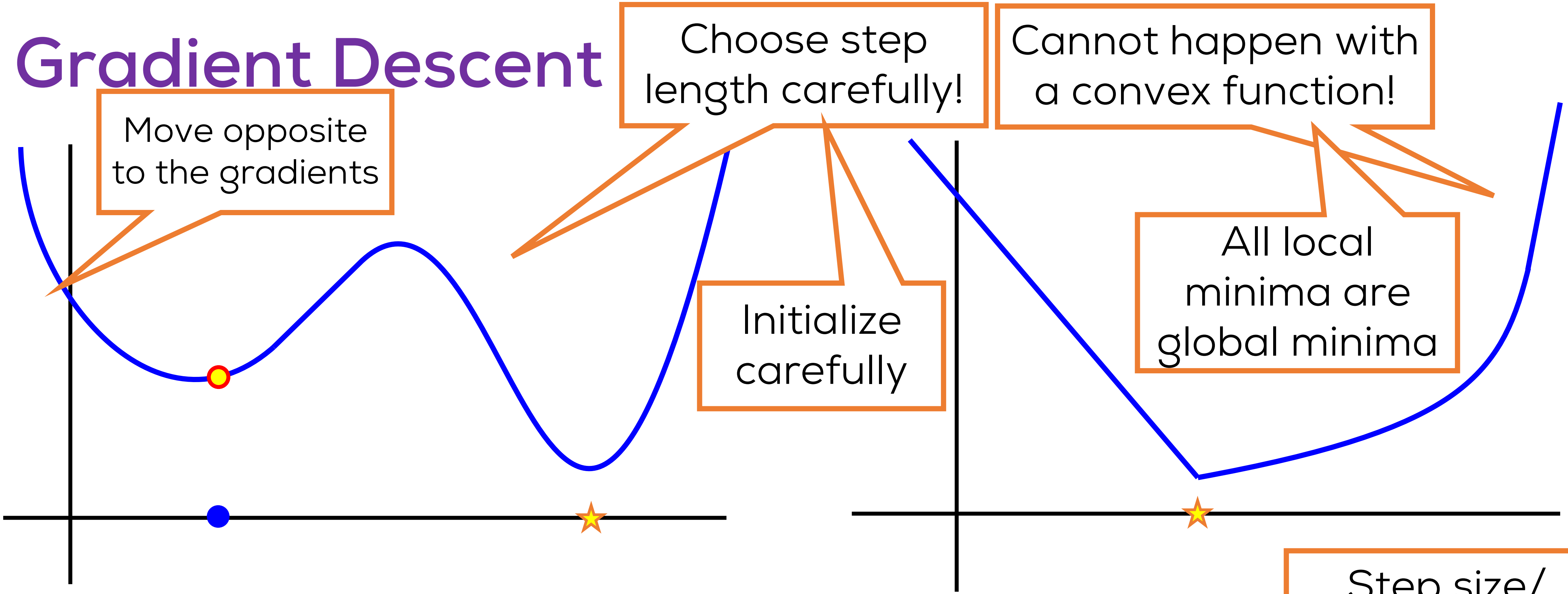
Gradient Descent



GRADIENT DESCENT

1. Initialize $\mathbf{w}^0$
2. Update $\mathbf{w}^{t+1} \leftarrow \mathbf{w}^t - \eta_t \cdot \mathbf{g}^t$
3. Repeat until convergence

Gradient Descent



Step size/
learning rate

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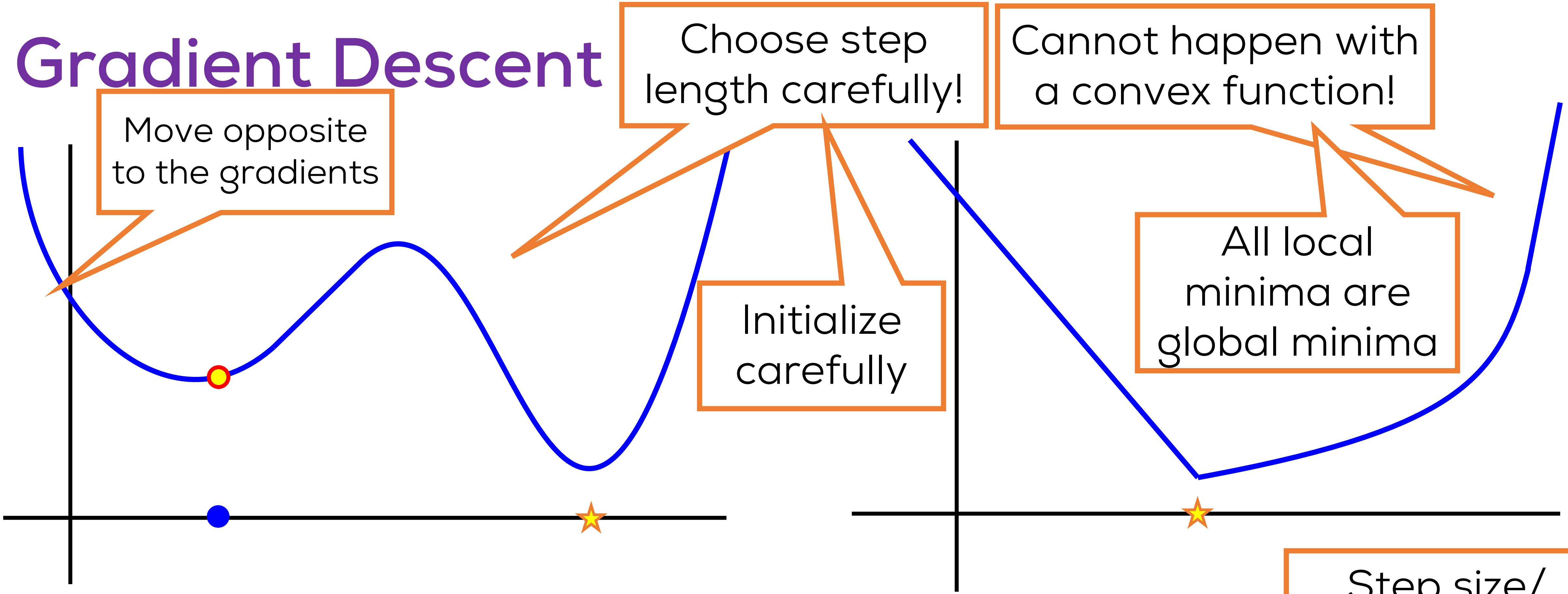

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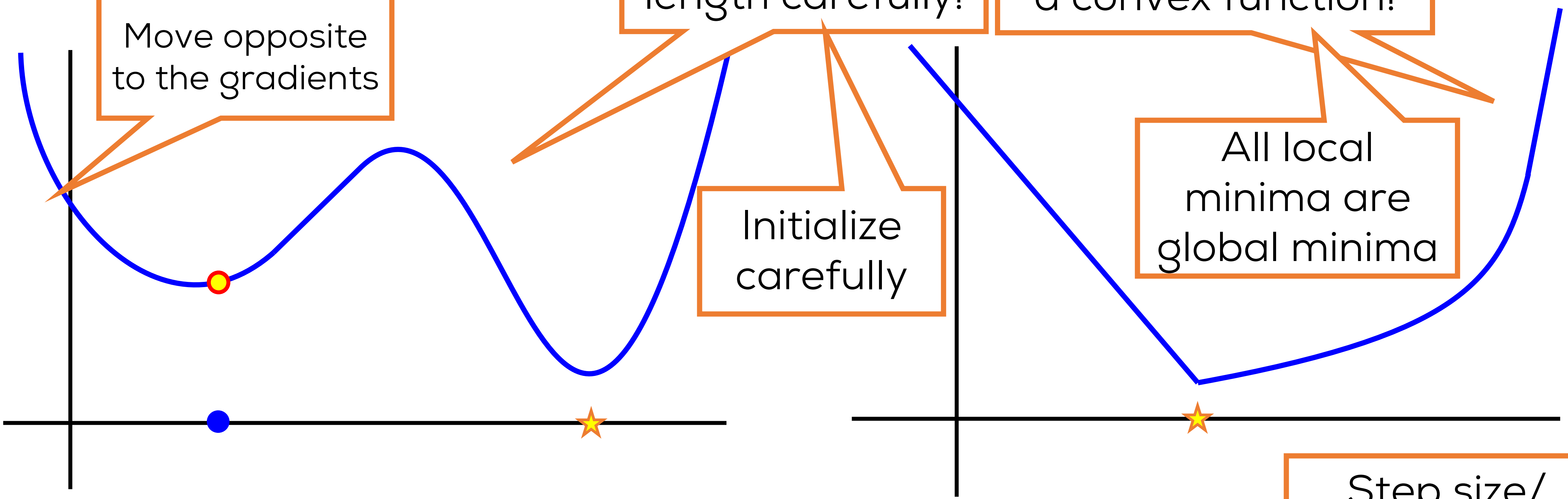
Step size/ learning rate

Tuned carefully with CV, Adam, Adagrad

Gradient Descent



Many convergence criteria – length of gradient, performance threshold, dual criteria

GRADIENT DESCENT

1. Initialize $\mathbf{w}^0$
2. Update $\mathbf{w}^{t+1} \leftarrow \mathbf{w}^t - \eta_t \cdot \mathbf{g}^t$
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Step size/ learning rate

Tuned carefully CV, Adam, Adagrad

Please give your Feedback

http://tinyurl.com/ml17-18afb