CS201: MATHEMATICS FOR COMPUTER SCIENCE - I

NITIN SAXENA

ASSIGNMENT 1

POINTS: 70

DATE GIVEN: 05-AUG-2016 DUE: 13-AUG-2016(6PM)

Rules:

- You are strongly encouraged to work *independently*.
- Write the solutions on your own and honorably *acknowledge* the sources if any.
  `http://cse.iitk.ac.in/pages/AntiCheatingPolicy.html`
- Submit your solutions, before time, to your TAs as per the roll numbers: Amit Sinhababu (12000–150130), Pranav Bisht (150131–150365), Ashish Dwivedi (150366–150600), Pulkit Kariryaa (150601–150840).

Question 1: *[5 points]* Prove that contrapositivity works. I.e. show the equivalence of $p \Rightarrow q$ and $\neg q \Rightarrow \neg p$.

Question 2: *[5+5=10 points]* Show that $|\mathbb{N}| = |\mathbb{Z}| = |\mathbb{Q}|$.

Question 3: *[5+15=20 points]* Prove the classical result: $|\mathbb{N}| < |2^{\mathbb{N}}| = |\mathbb{R}|$.

(Something to further think about: Is there a set with cardinality strictly between $|\mathbb{N}|$ and $|2^{\mathbb{N}}|$?)

Question 4: *[5 points]* Let $f_n = f_{n-1} + f_{n-2}$ and $f_1 = f_2 = 1$. $\{f_n\}_n$ are called *Fibonacci numbers*.

Show that $f_1 + f_2 + \cdots + f_n = f_{n+2} - 1$.

Question 5: *[5+5=10 points]* Use induction to,

(1) Prove that for any finite set S, $|2^S| = 2^{|S|}$.

(2) Prove that the number of permutations on n elements is $n!$.

Question 6: *[20 points]* Let A be an infinite set. Compare the cardinalities of the following three sets:

$$A, A \cup \mathbb{N}, A \times \mathbb{N}\,.$$

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