ESO 208A: Computational Methods in Engineering

Tutorial 7

Orthogonal basis functions

1. (a) Estimate a quadratic polynomial approximation of the following function by using Legendre polynomials as the basis functions.

$$f(x)=\frac{1}{1+x^2};\quad x\in(-1,1)$$

Legendre Polynomials

$$P_0(x)=1, P_1(x)=x, P_{n+1}(x)=\frac{2n+1}{n+1}xP_n(x)-\frac{n}{n+1}P_{n-1}(x);\ \langle P_n,P_j\rangle=\begin{cases}0 & \text{if } n\neq j\\ \dfrac{2}{2n+1} & \text{if } n=j\end{cases}$$

(b) Perform the second order Taylor series approximation of $f(x)$ at $x=0$.

(c) Estimate the true relative error in estimating $f(x)$ by the above two approximations at $x=-0.9,-0.5,\ 0,\ \text{and}\ 0.5$.

Interpolation

2. For the following data, interpolate the value of function $f(x)$ at $x=0.275$ by (a) Lagrange polynomials and (b) Newton's divided difference. Perform interpolation using first, second and third order polynomials.

x	$f(x)$
0.1	0.99010
0.2	0.96154
0.3	0.91743
0.4	0.86207
0.5	0.80000

If the actual function is $f(x)=\frac{1}{1+x^2}$, estimate the true relative error in interpolation by the different order polynomials.