Data Modelling Methods-II

CS771: Introduction to Machine Learning

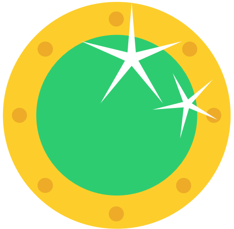
Purushottam Kar

Outline of today's discussion

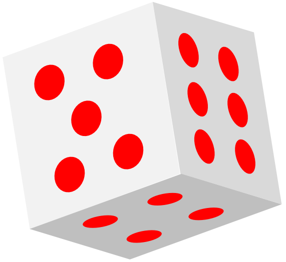
- Revise Naïve Bayes method
- Feature modelling methods for unlabelled data
- Gaussian mixture Models (GMMs)
- The alternating optimization approach to learning GMMs
- The k-means approach to learning GMMs

The Multinoulli Distribution

- Also known as categorical distribution
- Generalizes the Bernoulli distribution
  - Bernoulli models a coin with two outcomes (call them head and tail)
  - … using one “bias” parameter p (taken to be the probability of heads)



- Multinoulli models a K-sided dice
  - … using a K-dimensional vector π
  - π_k is taken to denote the probability of the k-th side turning up
  - $\pi_k \geq 0$ and $\sum_{k=1}^{K} \pi_k = 1$



- Conjugate prior for Bernoulli: Beta $\mathbb{P}\left[p;\alpha,\beta\right] = \frac{1}{B(\alpha,\beta)} p^{\alpha-1}(1-p)^{\beta-1}$
- Conjugate prior for Multinoulli: Dirichlet $\mathbb{P}\left[\boldsymbol{\pi};\boldsymbol{\alpha}\right] = \frac{1}{B(\boldsymbol{\alpha})} \prod_{i=1}^{L} \boldsymbol{\pi}_i^{\boldsymbol{\alpha}_i - 1}$

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HEC	DoSA	Supervisor

HallPresi	Awesome talk at eSumm
HEC	**Urgent! Mess bill overdue**
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$\mathbf{x}$

talk	meet	project	wake	meeting	wallet	keys	awesome	lost	trip	lost	mess	report

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$$y \in \{ \bullet , \bullet , \bullet \}$$

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Bag of words feature can record just occurrence or count

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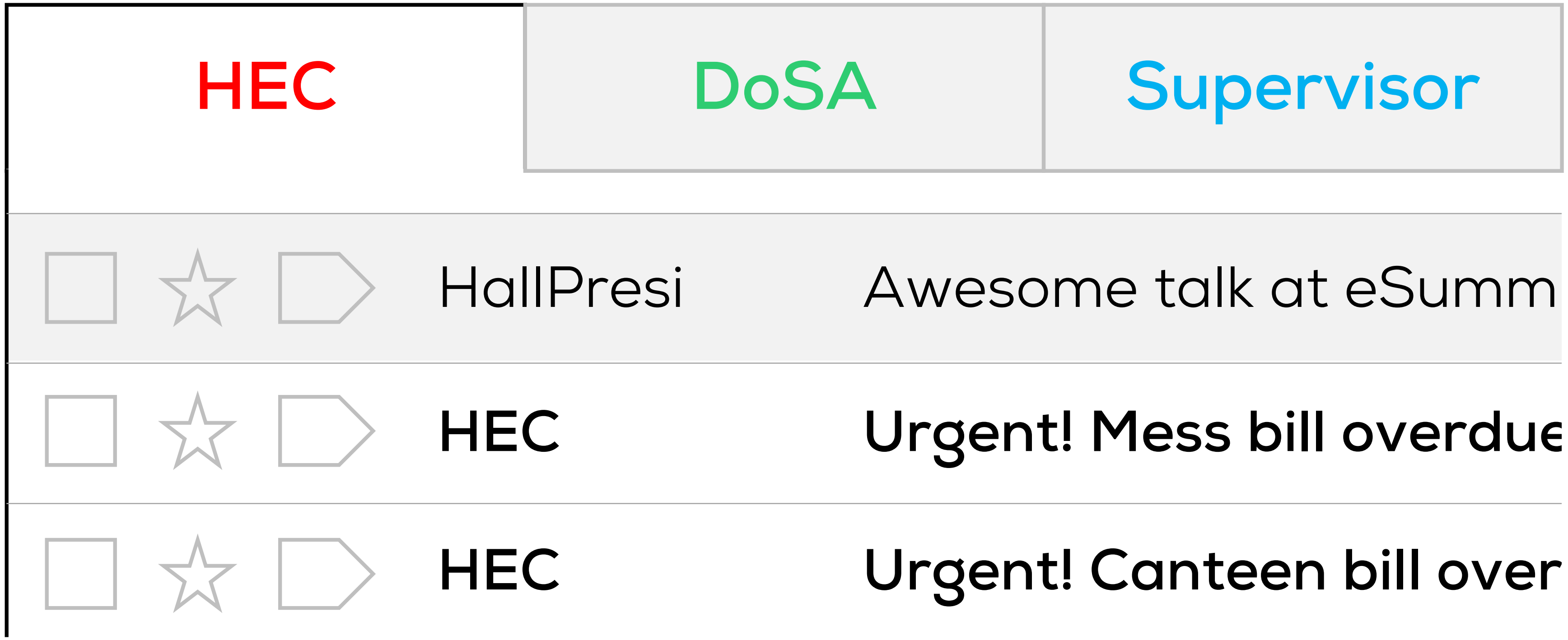

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1	0	0	0	0	0	0	1	0	0	0	0	0
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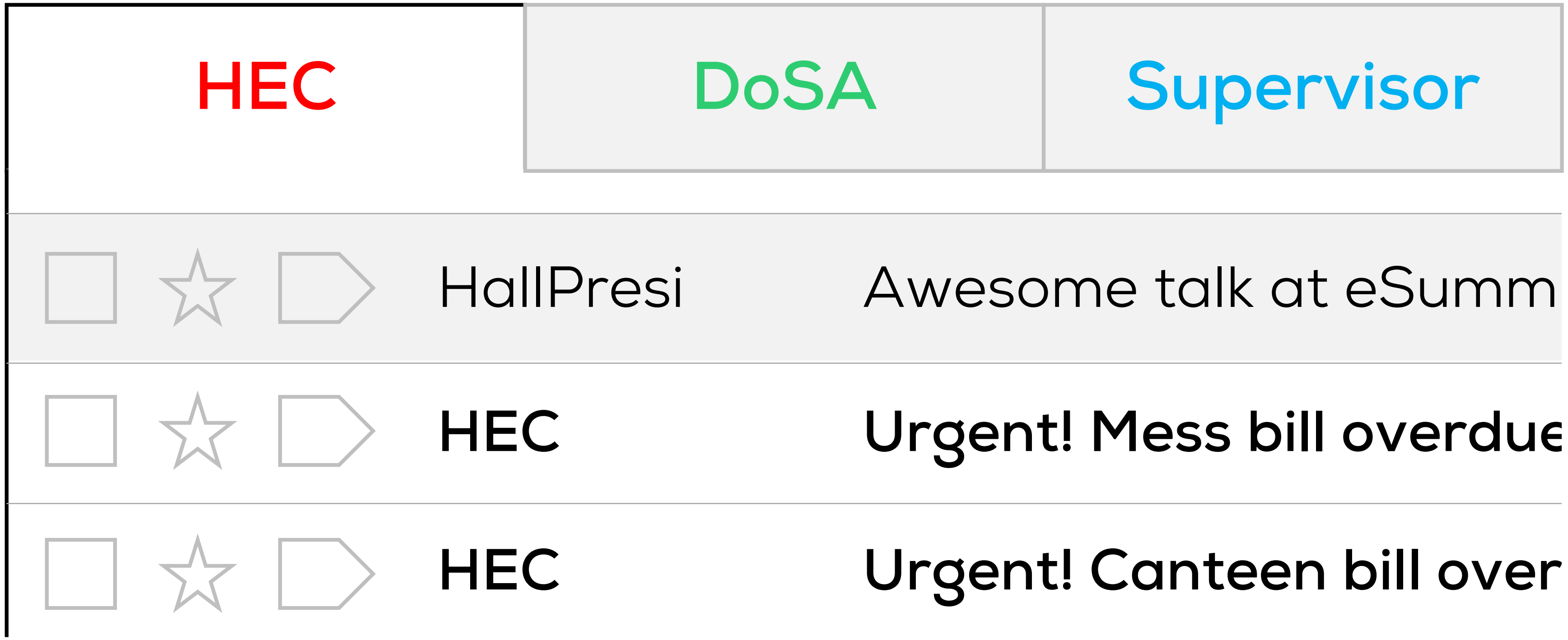

Choice of words crucial – stemming, throw away articles etc

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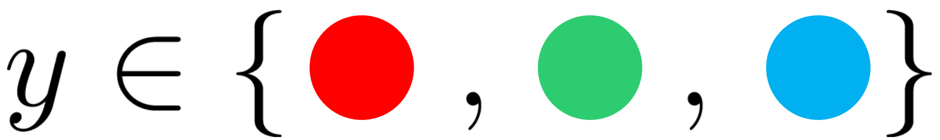

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Commonly used in NLP

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Usually very high dimensional

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$y \in \{ \text{red}, \text{green}, \text{blue} \}$

Usually very high dimensional

Usually very very sparse

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$$\mathbb{P}[\bullet] = \frac{|\#\text{emails from supervisor}|}{|\#\text{total emails}|}$$

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$$\mathbb{P}[\mathbf{x}^t, \bullet] = \mathbb{P}[\bullet] \cdot \prod_{j=1}^{d} \mathbb{P}[\mathbf{x}_i^t \mid \bullet]$$

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$\mathbb{P}[awesome = 0|\bullet]$
$= 1 - \mathbb{P}[awesome = 1|\bullet]$

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$$\mathbb{P}[{\color{cyan}\bullet}] = \frac{|\#\text{emails from supervisor}|}{|\#\text{total emails}|}$$

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$$\mathbb{P}[awesome = 0|{\color{cyan}\bullet}]$$
$$= 1 - \mathbb{P}[awesome = 1|{\color{cyan}\bullet}]$$

At test time …

$$\mathbb{P}[\mathbf{x}^t, {\color{cyan}\bullet}] = \mathbb{P}[{\color{cyan}\bullet}] \cdot \prod_{j=1}^{d} \mathbb{P}[\mathbf{x}_i^t | {\color{cyan}\bullet}]$$

$$\hat{y}^t = \arg\max\{\mathbb{P}[\mathbf{x}^t, {\color{red}\bullet}], \mathbb{P}[\mathbf{x}^t, {\color{green}\bullet}], \mathbb{P}[\mathbf{x}^t, {\color{cyan}\bullet}]\}$$

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At test time …

$$\mathbb{P}[\mathbf{x}^t, \textcolor{cyan}{\bullet}] = \mathbb{P}[\textcolor{cyan}{\bullet}] \cdot \prod_{j=1}^{d} \mathbb{P}\left[\mathbf{x}_i^t \mid \textcolor{cyan}{\bullet}\right]$$

Will give the same result as

$$\arg\max\{\mathbb{P}[\textcolor{red}{\bullet} \mid \mathbf{x}^t], \mathbb{P}[\textcolor{green}{\bullet} \mid \mathbf{x}^t], \mathbb{P}[\textcolor{cyan}{\bullet} \mid \mathbf{x}^t]\}$$

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App: Automatic Email Generator!

Class proportions

- Choose a category from {HEC, DoSA,Supervisor}
  - Toss a 3-sided "coin" aka categorical/multinoulii distribution using $\mathbb{P}[\bullet]$
  - Say we chose DoSA
- For each word in your dictionary of d words, toss a Bernoulli coin to decide whether to include that word in the mail or not
  - For $j \in [d]$, toss a coin that lands heads with probability $\mathbb{P}\left[x_j = 1 \mid \bullet\right]$
- Collect all words for which the toss landed heads
- Compose an email using only those words (and maybe a few articles, prepositions etc)
- Congratulations, you can now ask the dean to stop sending you emails – you will generate them yourself!

Already learnt from training data!

The generative story for unlabelled data??

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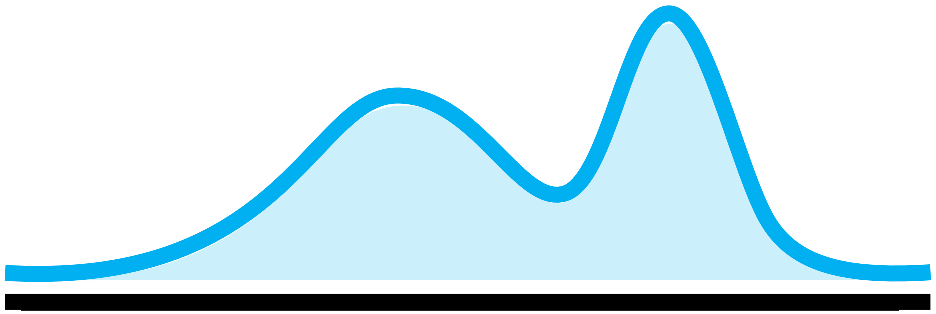

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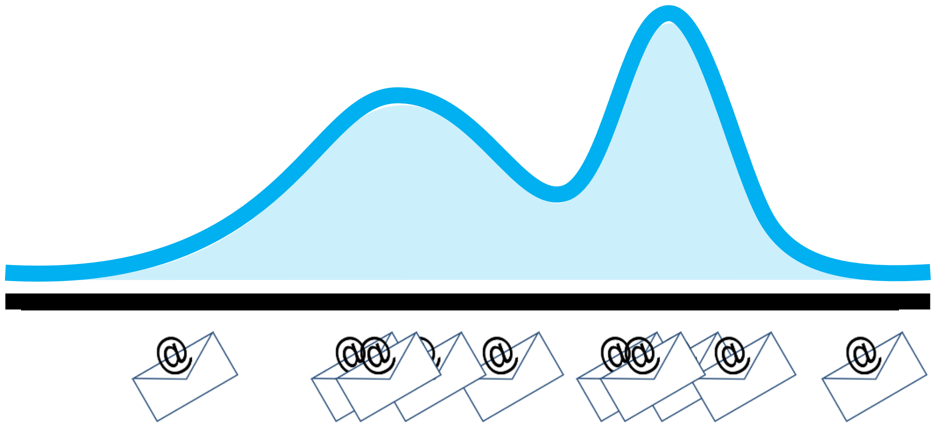

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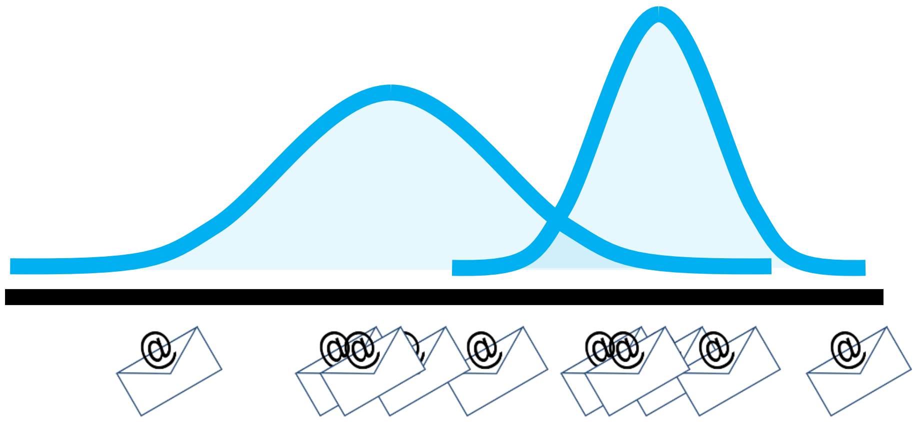

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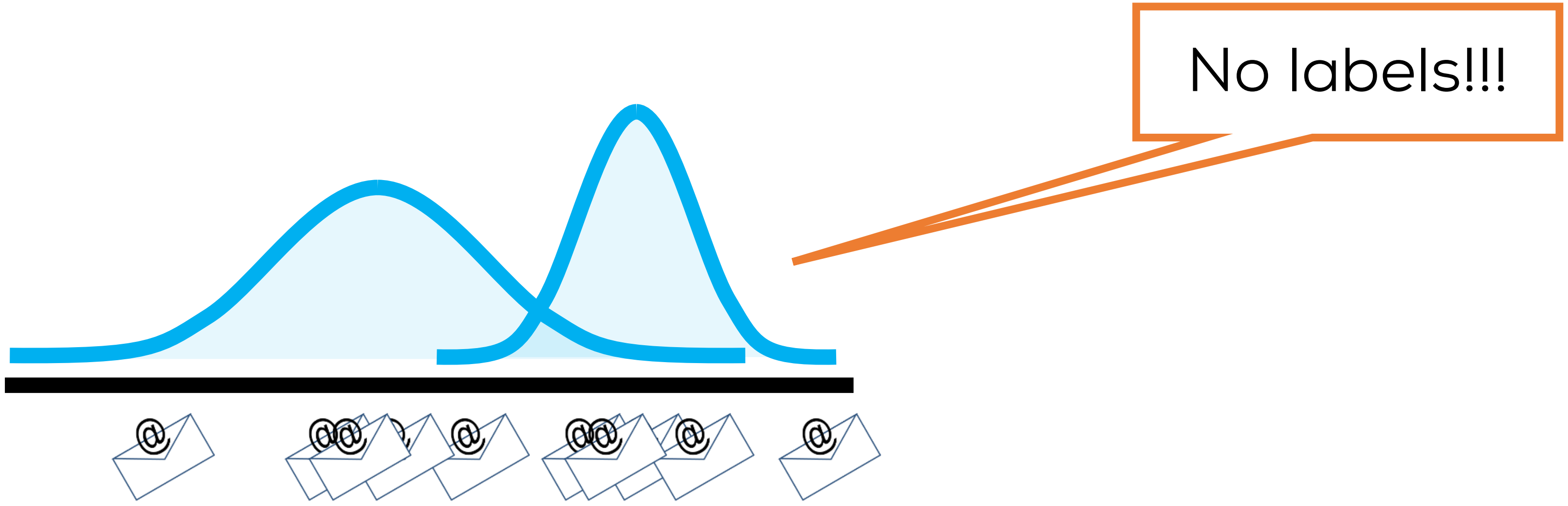

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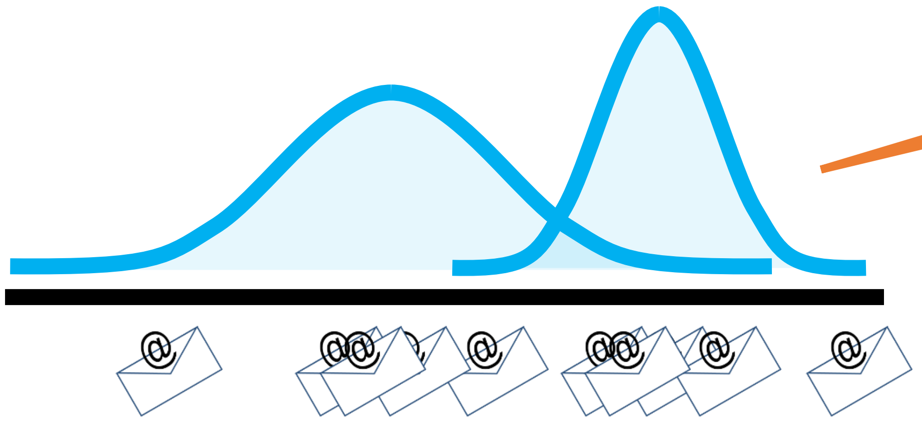

No labels!!!

Can we still recover the two Gaussian components in the mixture??

How to grade a CS771 exam??

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gradescope

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Q1. $\int x = ?$

5 marks

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5 marks

$\frac{x^2}{2} + b$

$x^2 + d$

$x^2/2$

$x^2/2$

$x^2 + C$

$\frac{x^2}{2} + c$

$x^2/2$

$x^2/2 + a$

$x^2 + \epsilon$

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Q1. $\int x = ?$

5 marks

$\frac{x^2}{2} + a$

$\frac{x^2}{2} + b\frac{x^2}{2} + c$

$x^2/2$

$x^2/2 \quad x^2/2$

$x^2 + cx^2 + d$

$x^2 + \varepsilon$

How to grade a CS771 exam??

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Q1. $\int x = ?$

5 marks

$\frac{x^2}{2}$ $\quad x^2/2$ $\quad X^2/2$

$\frac{X^2}{2} + a$

$\frac{x^2}{2} + b$ $\quad \frac{x^2}{2} + c$

$x^2 + c$ $\quad X^2 + d$

$X^2 + \varepsilon$

How to grade a CS771 exam??

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Q1. $\int x = ?$

5 marks

$x^2/2$ $x^2/2$ $x^2/2$

4/5

$\frac{x^2}{2} + a$ $\frac{x^2}{2} + b$ $\frac{x^2}{2} + c$

5/5

$x^2 + c$ $x^2 + d$ $x^2 + e$

2/5

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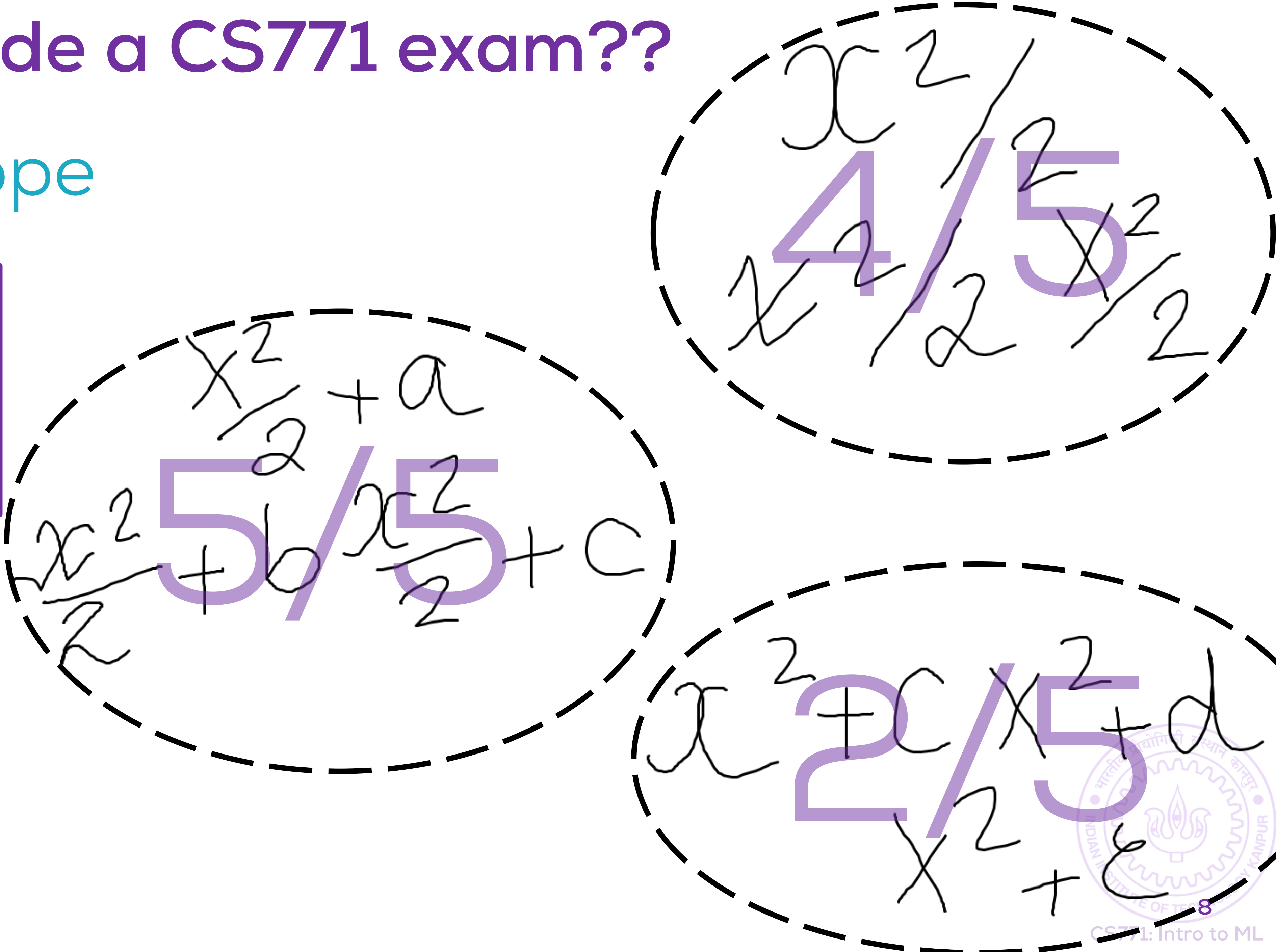
5 marks



Clustering!

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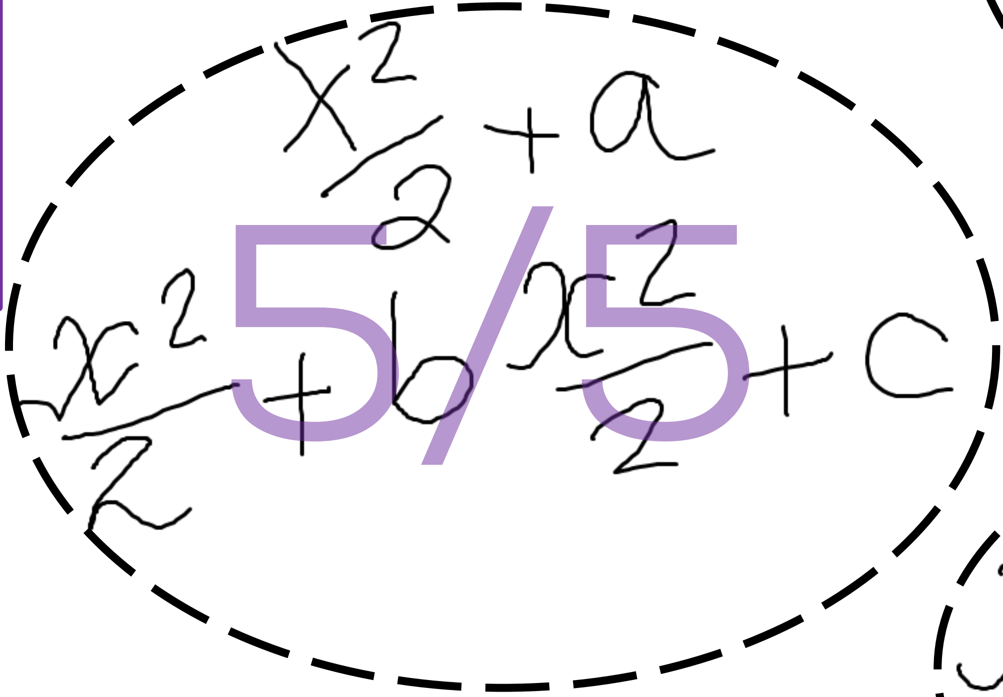
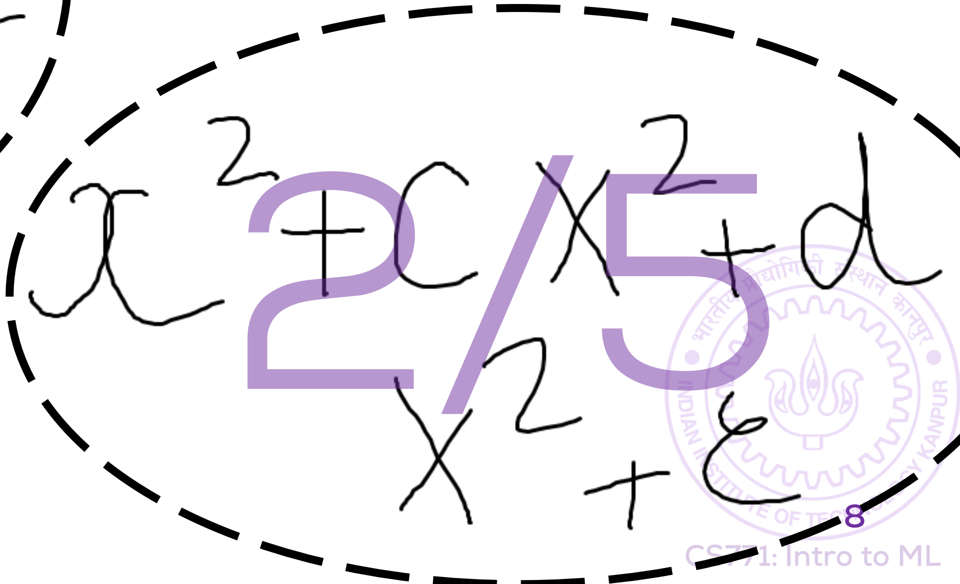
5 marks





Clustering!

Like classification without labels ☺

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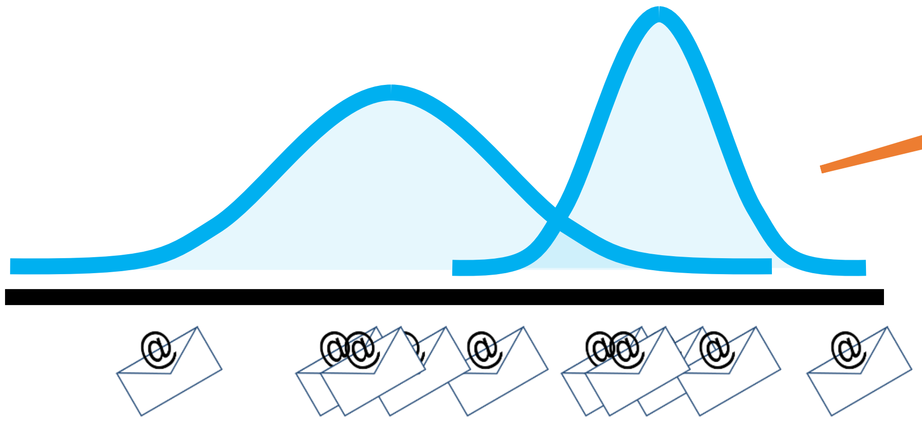

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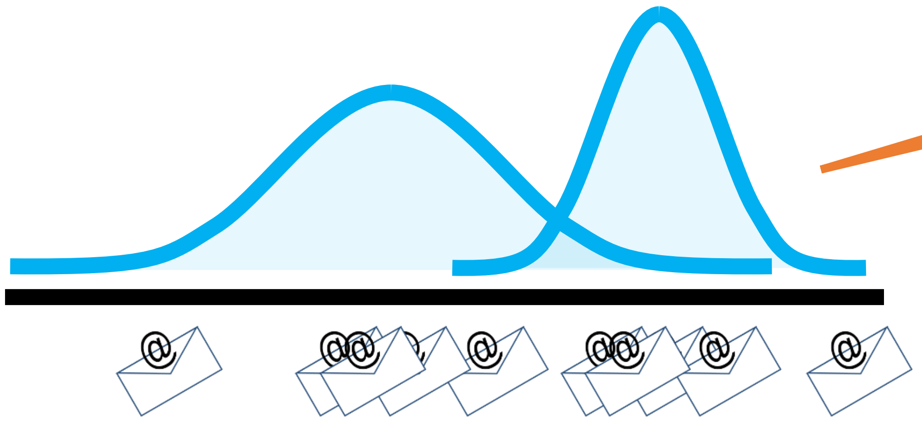

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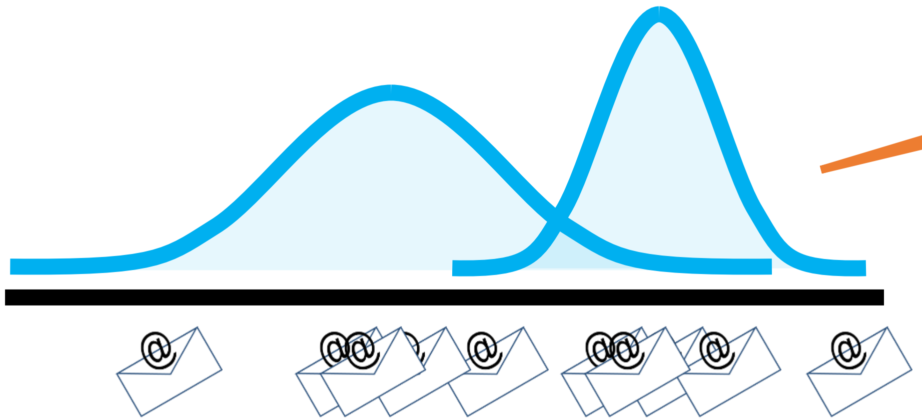

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Gaussians can be a hyper-parameter K you can tune

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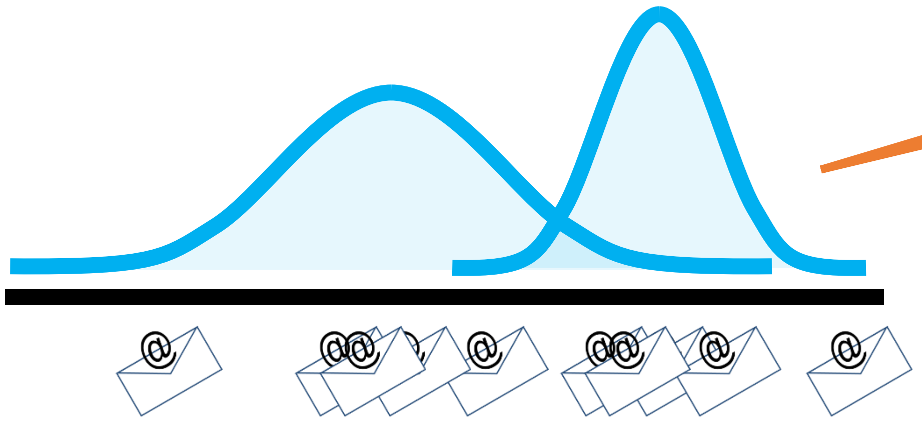

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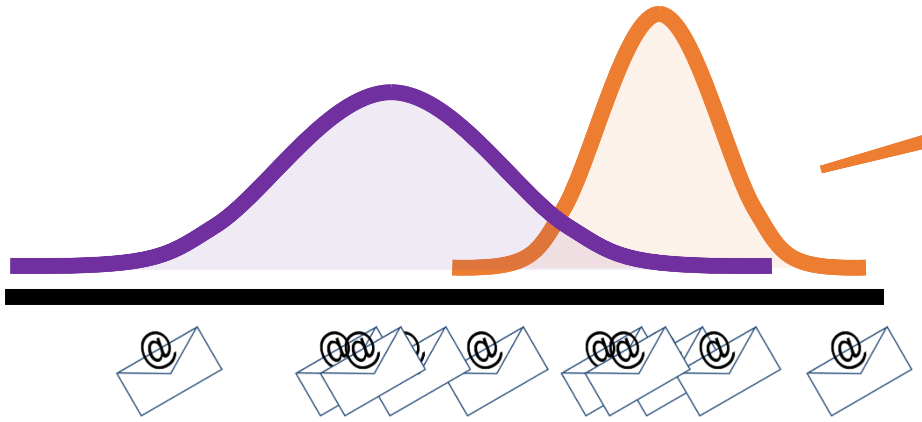

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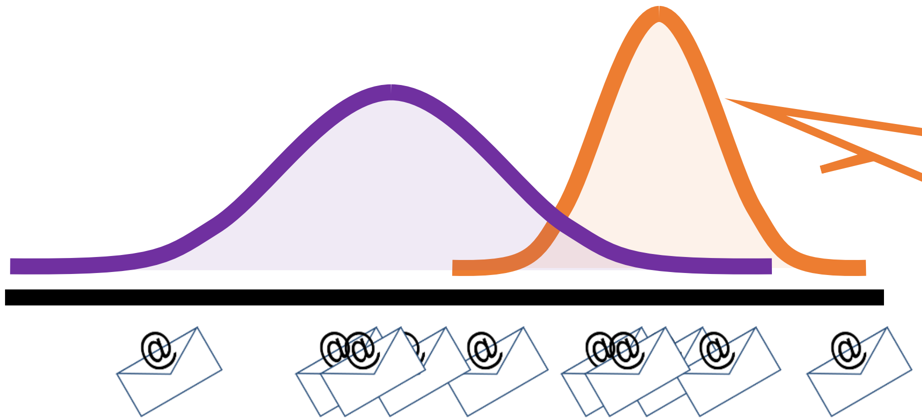

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Give the Gaussians colors to help identifying them

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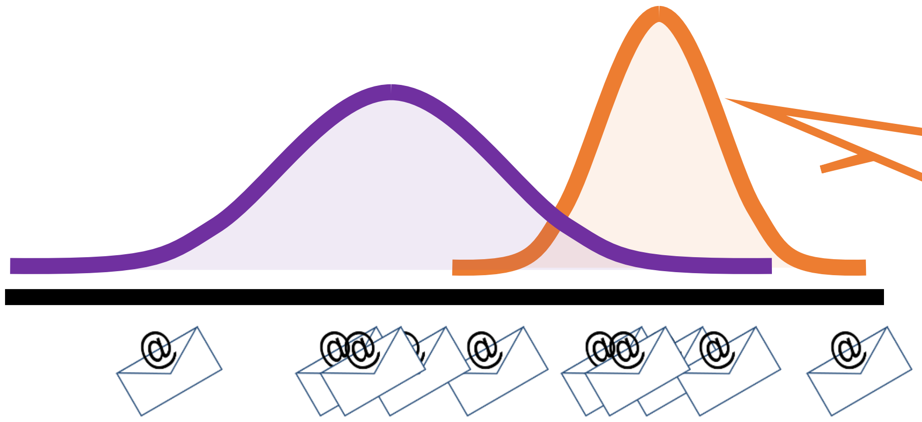

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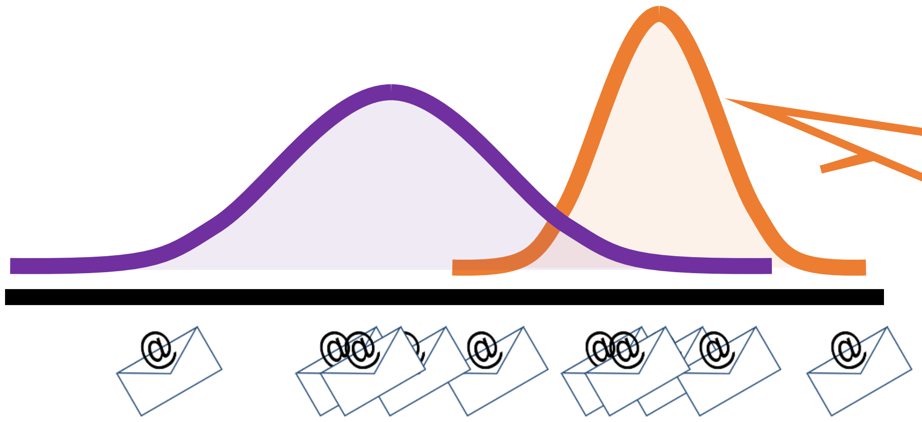

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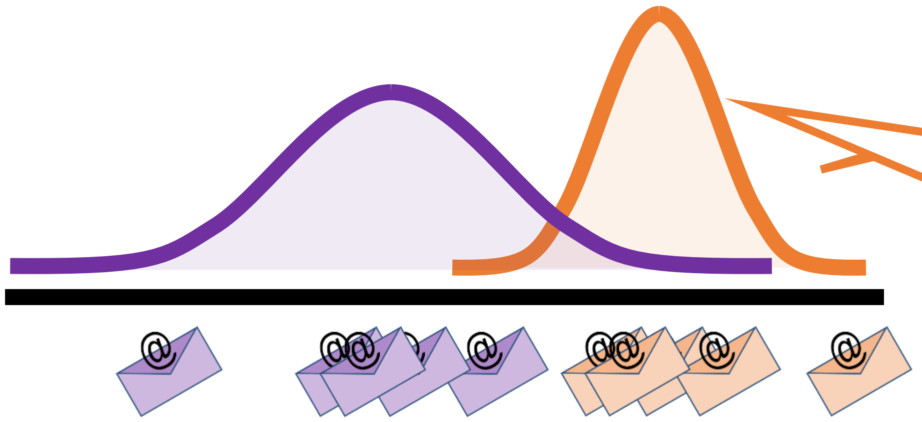

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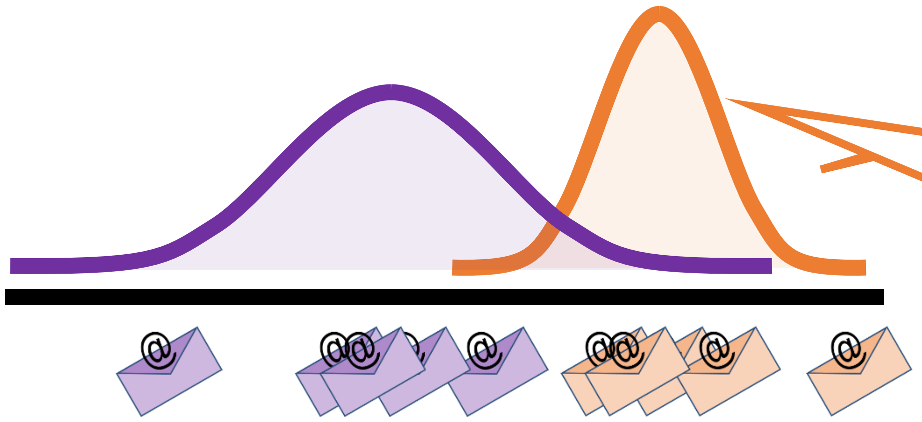

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Can estimate the class proportions, and means and variances of these 1D Gaussians using training data!

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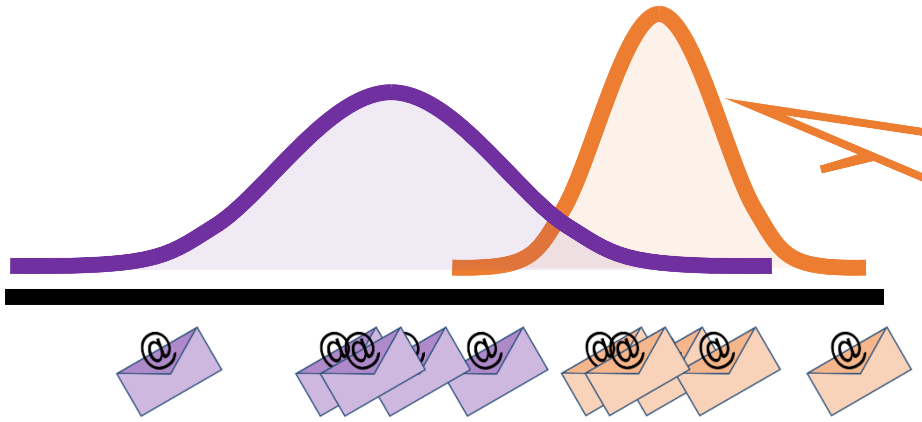

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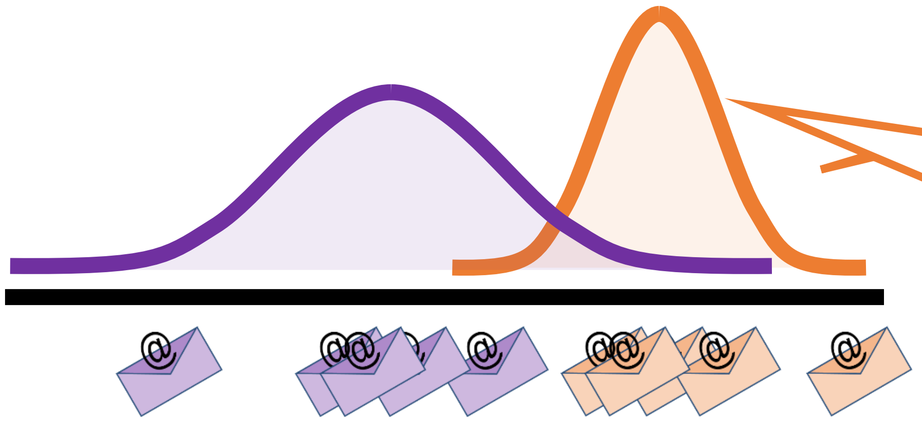

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Read **[DAU]** Sections 9.1-9.5

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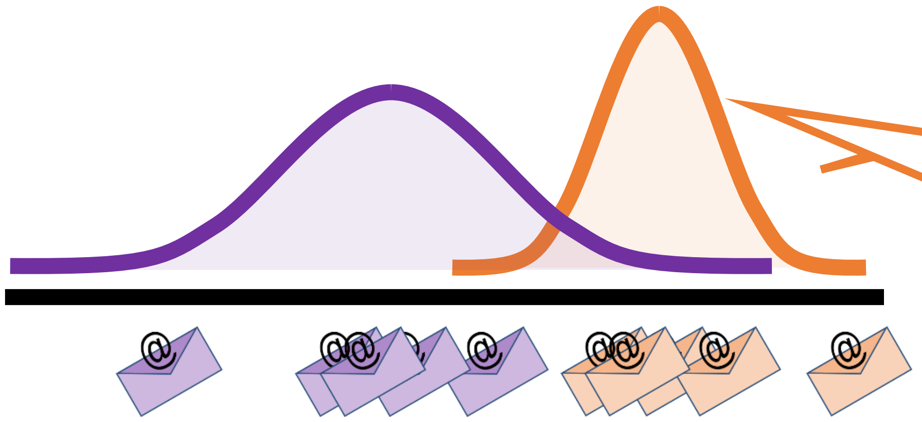

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Better take a look now ☺

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$\langle$detour$\rangle$

Learning a mixture of Gaussians in presence of labels

The generative story for labelled data

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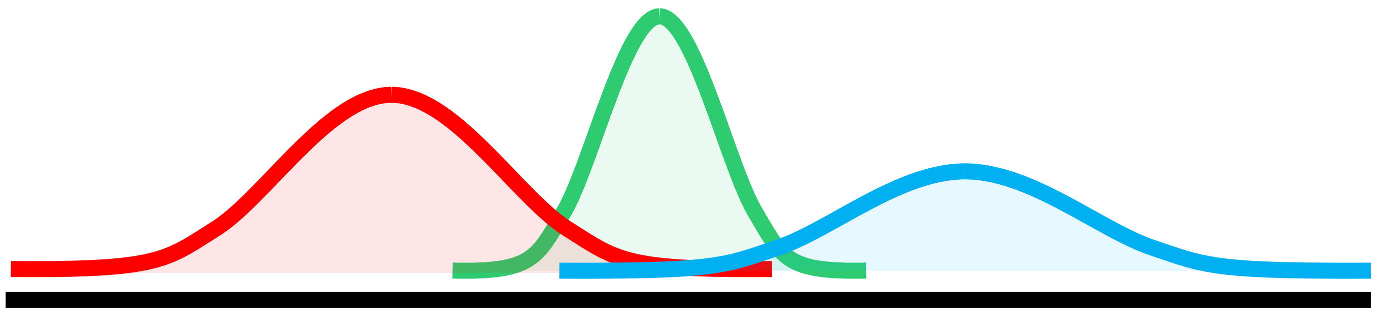

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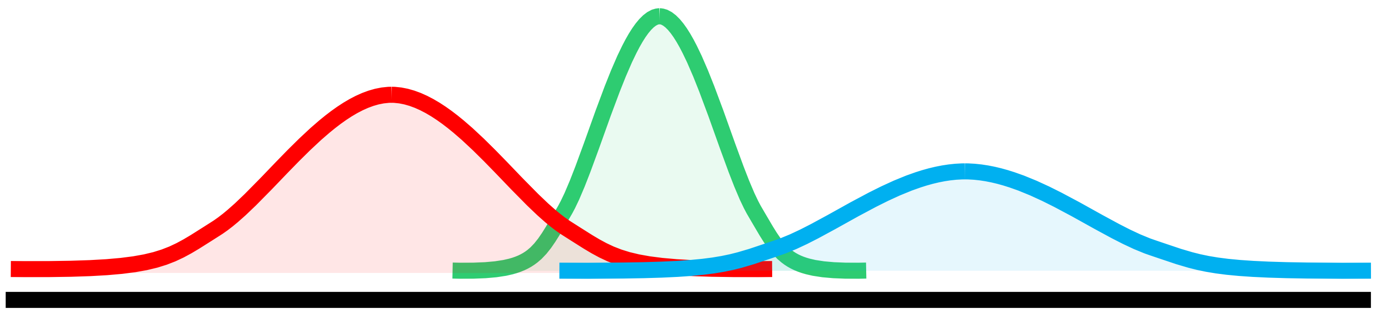

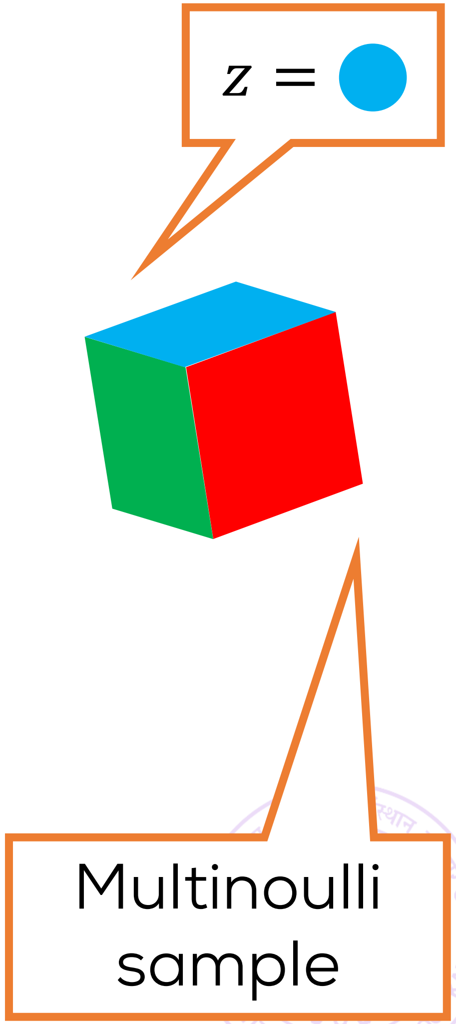

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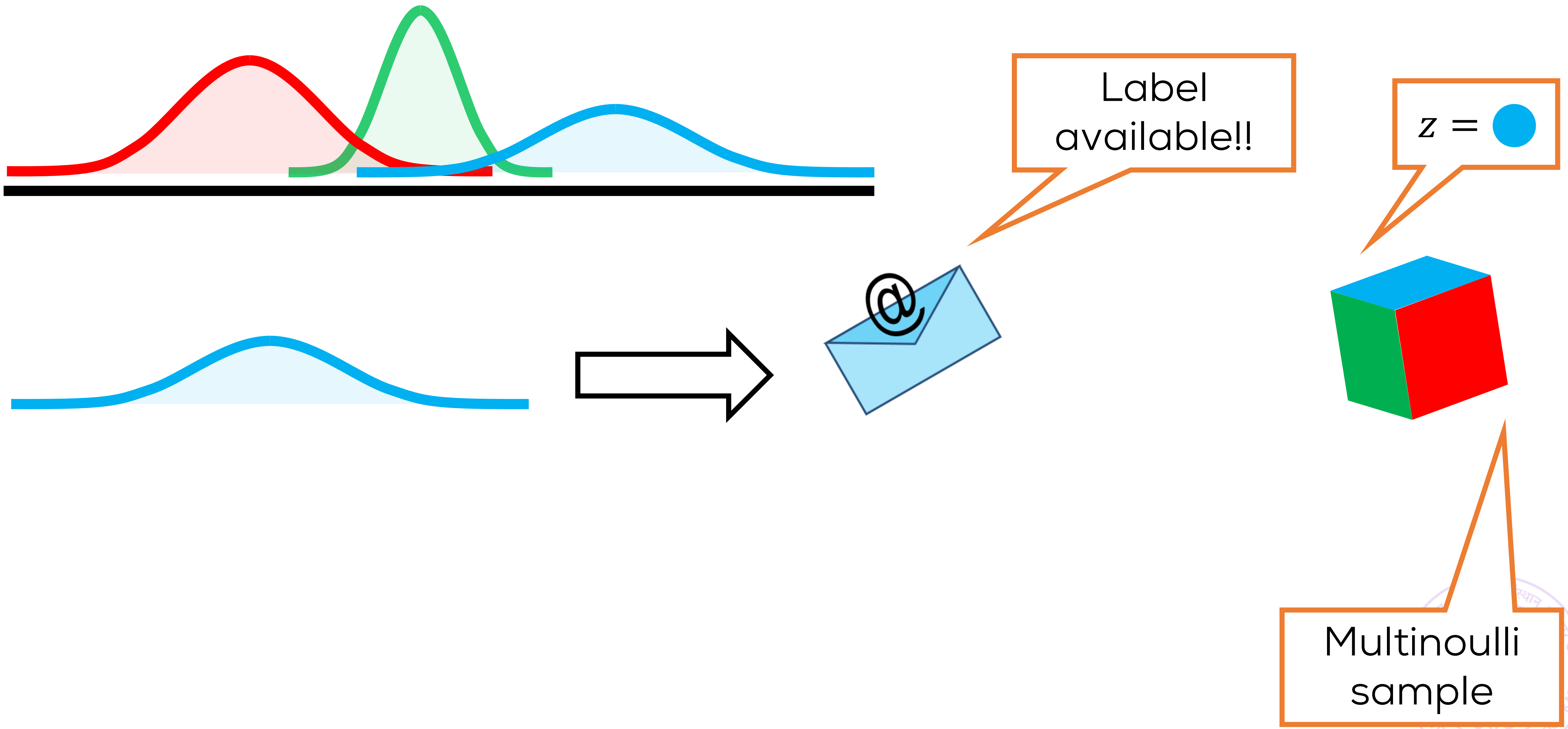

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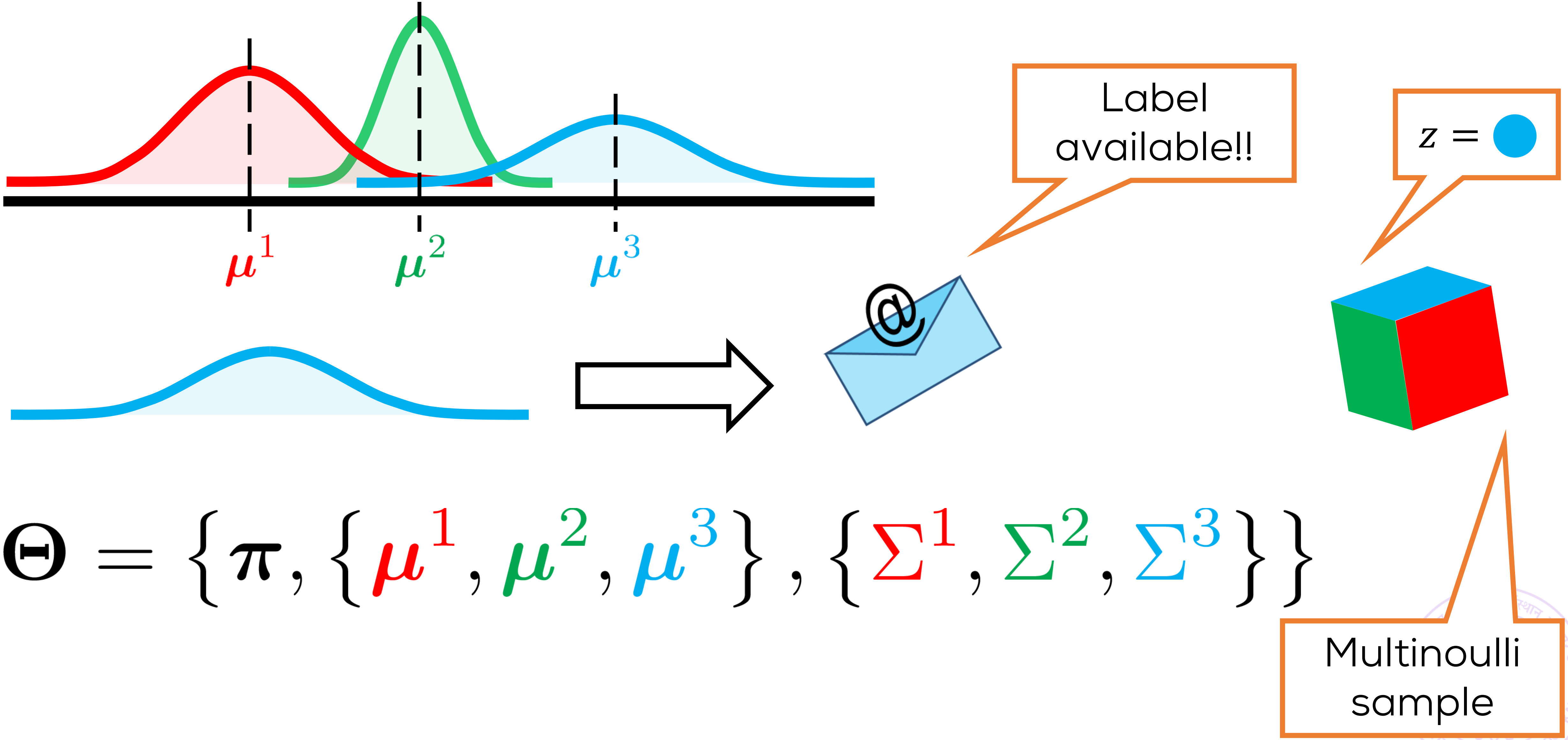

$$\Theta = \{\pi, \{\mu^1, \mu^2, \mu^3\}, \{\Sigma^1, \Sigma^2, \Sigma^3\}\}$$

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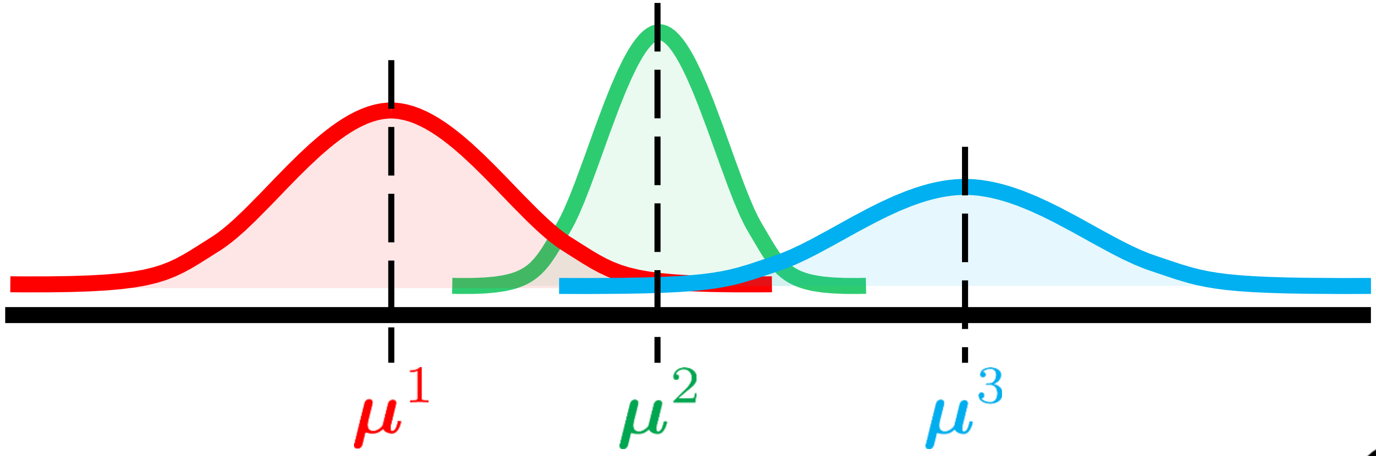

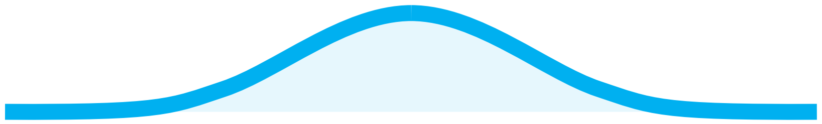

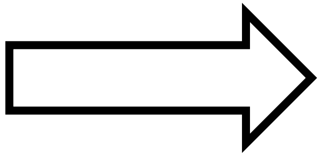

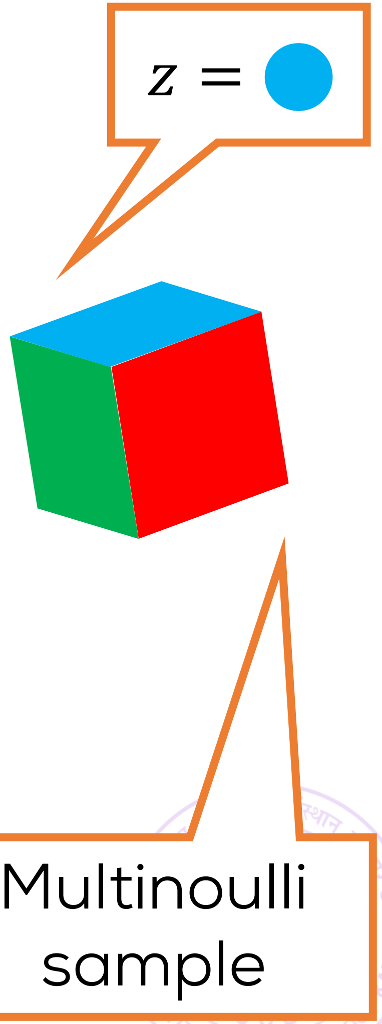

$$\boldsymbol{\Theta} = \{\boldsymbol{\pi}, \{\boldsymbol{\mu}^1, \boldsymbol{\mu}^2, \boldsymbol{\mu}^3\}, \{\Sigma^1, \Sigma^2, \Sigma^3\}\}$$

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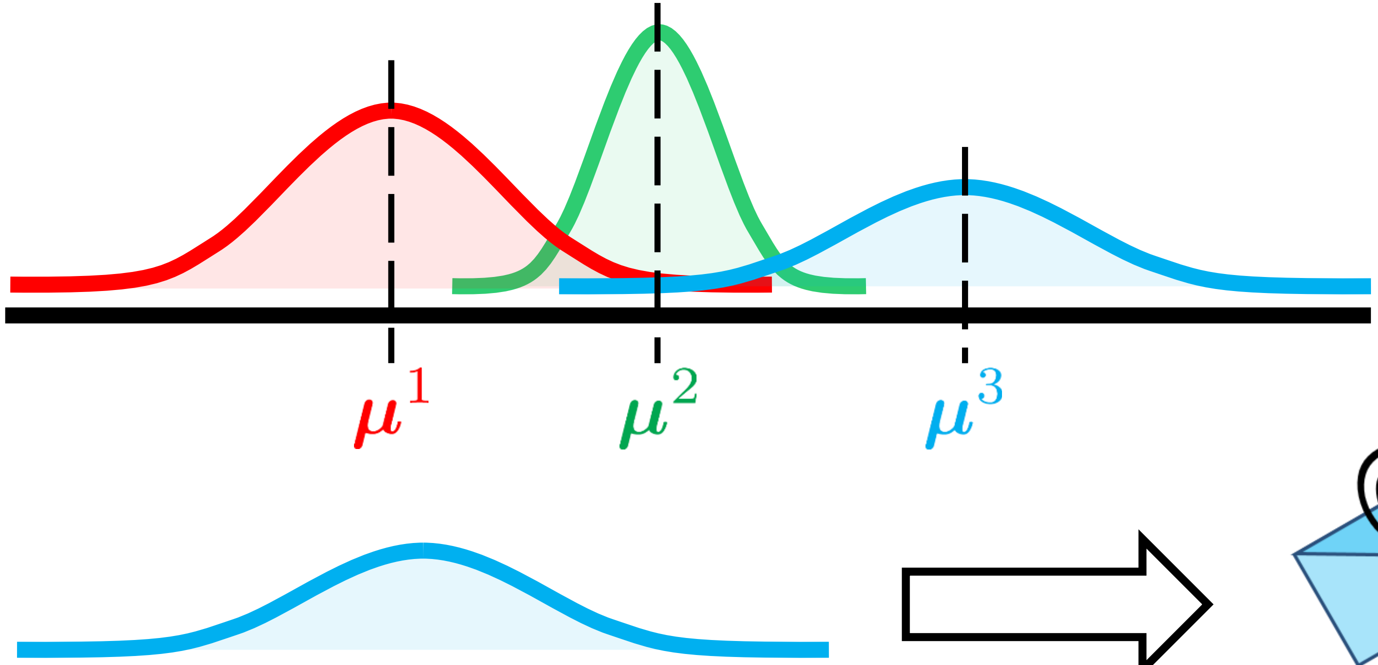

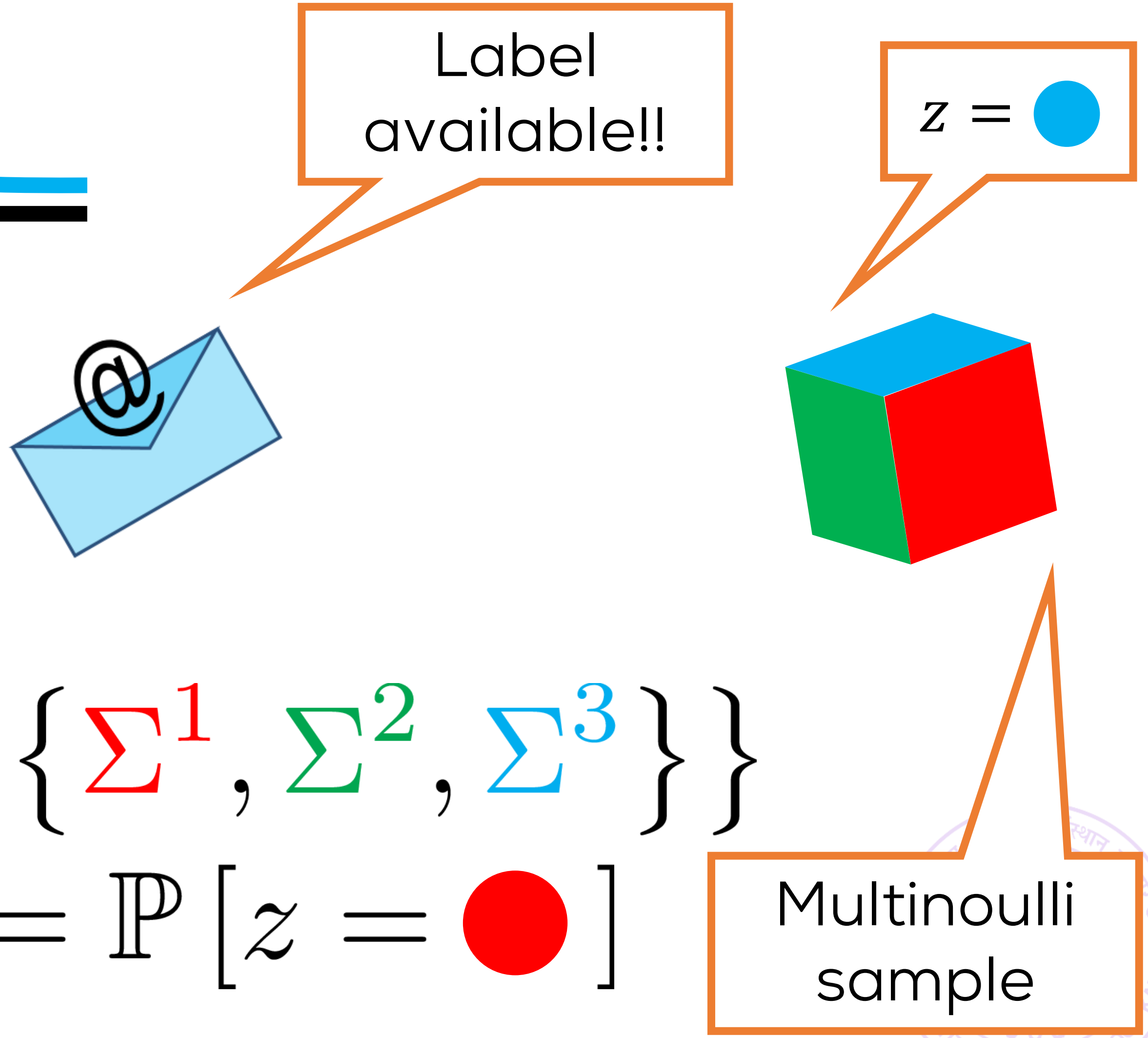

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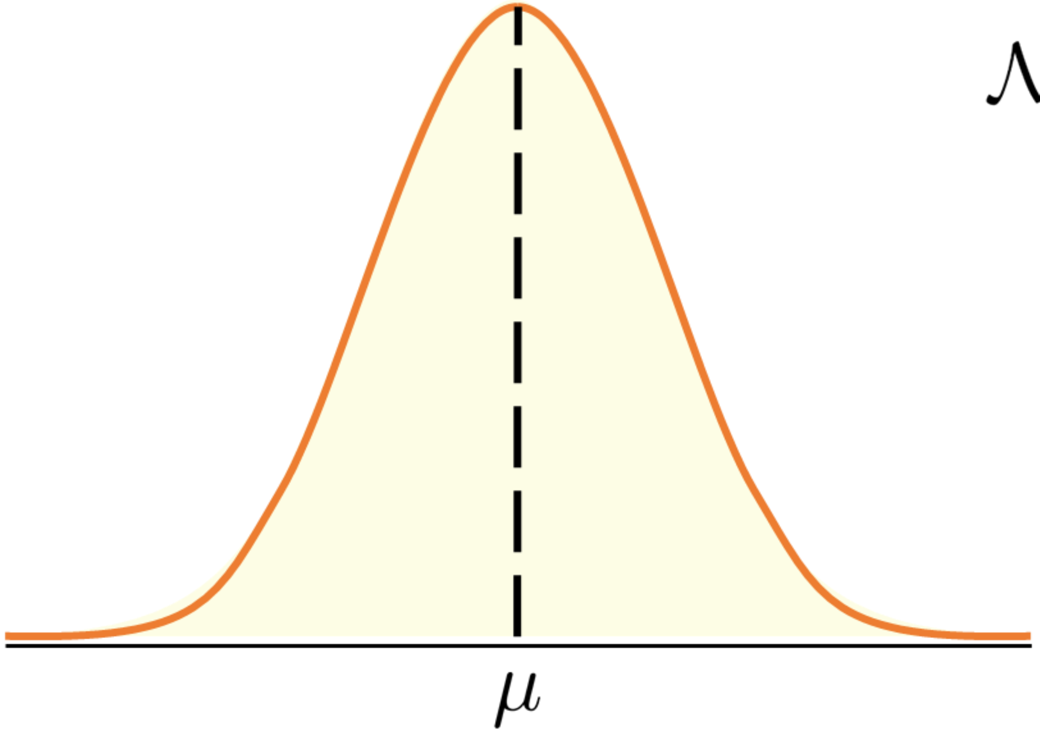
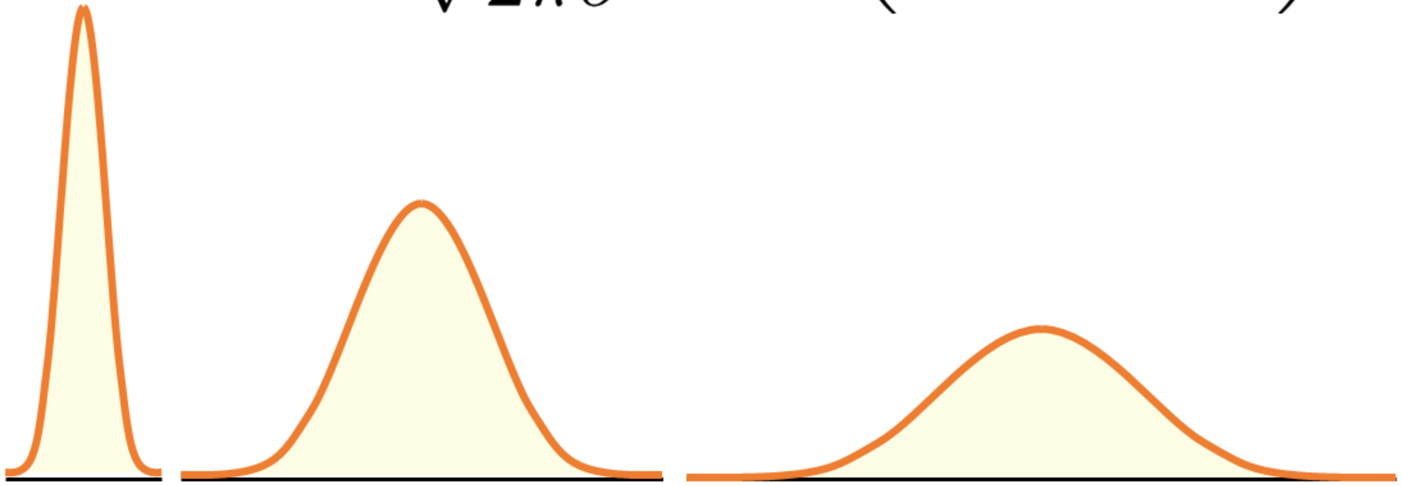
The Gaussian Distribution



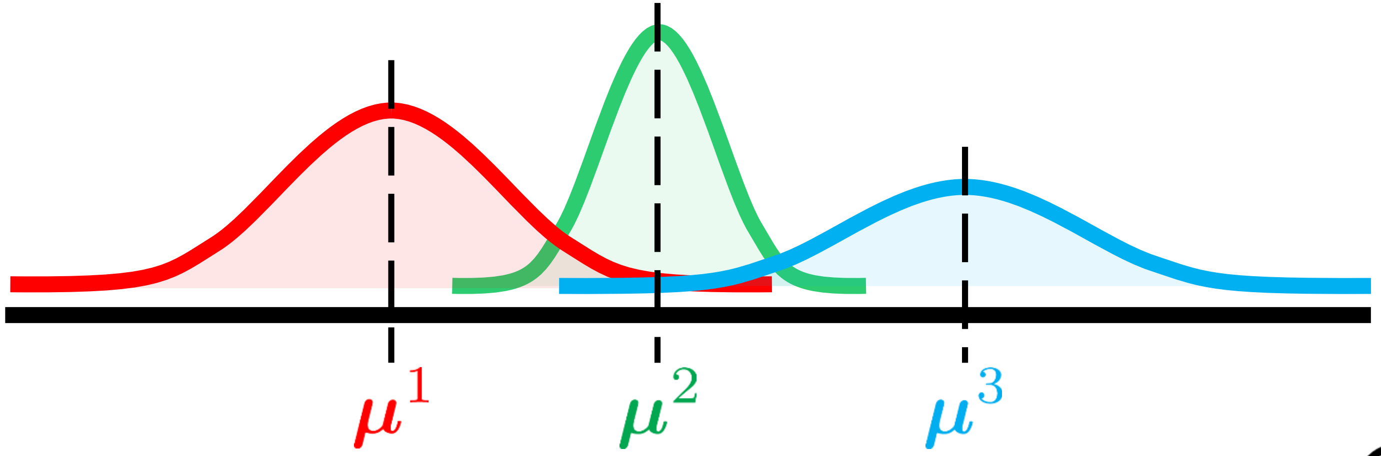
$$\mathcal{N}(t;\mu,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(t-\mu)^2}{2\sigma^2}\right)$$

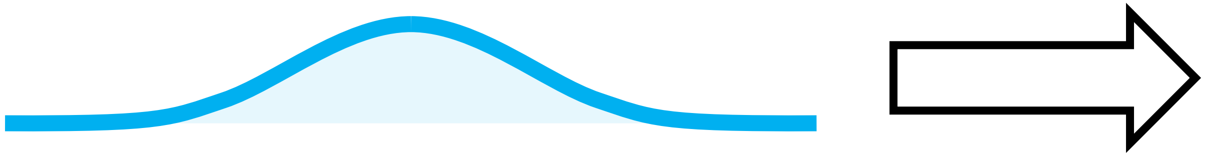


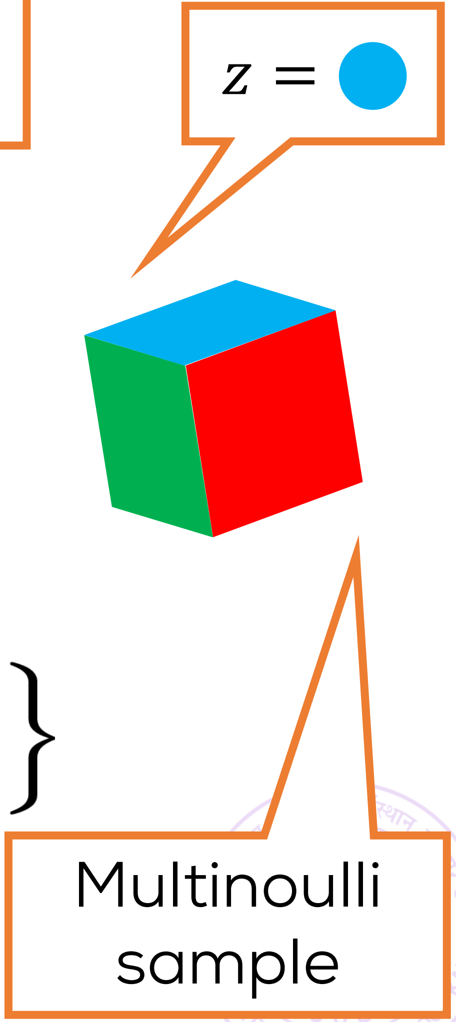
Multivariate Gaussian

$$\mathcal{N}(\mathbf{v};\boldsymbol{\mu},\Sigma) = \frac{1}{\sqrt{(2\pi)^d\,|\Sigma|}} \exp\left(-\frac{1}{2}(\mathbf{v}-\boldsymbol{\mu})^\top\Sigma^{-1}(\mathbf{v}-\boldsymbol{\mu})\right)$$

The generative story for labelled data







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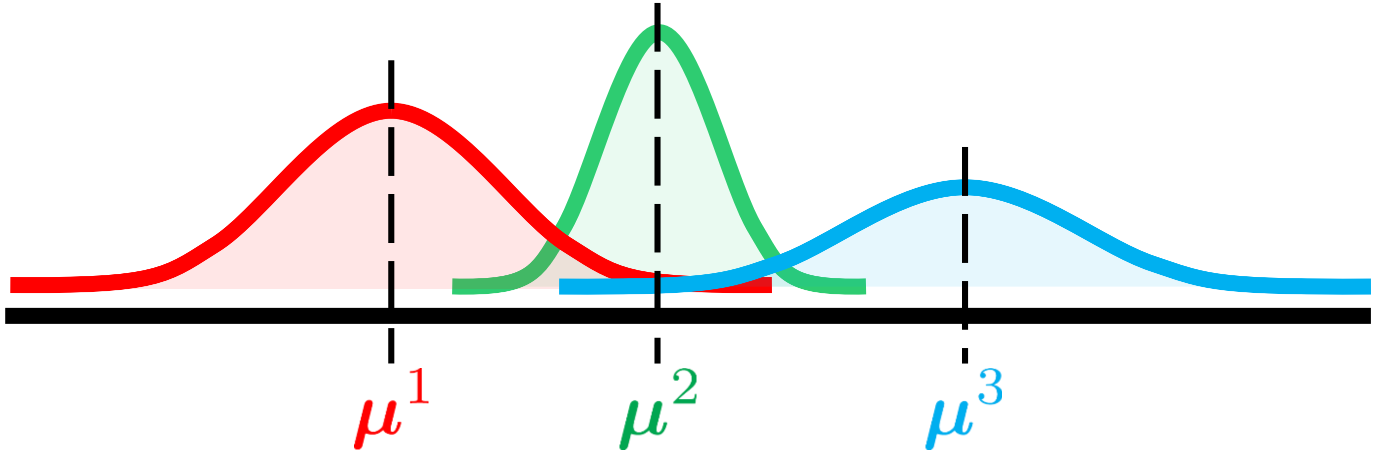

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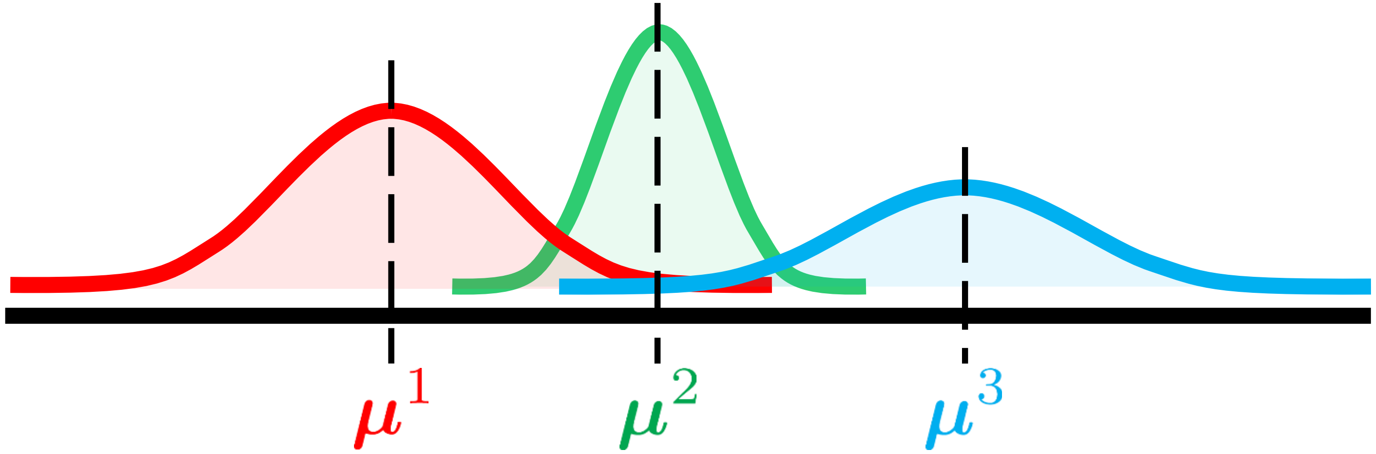

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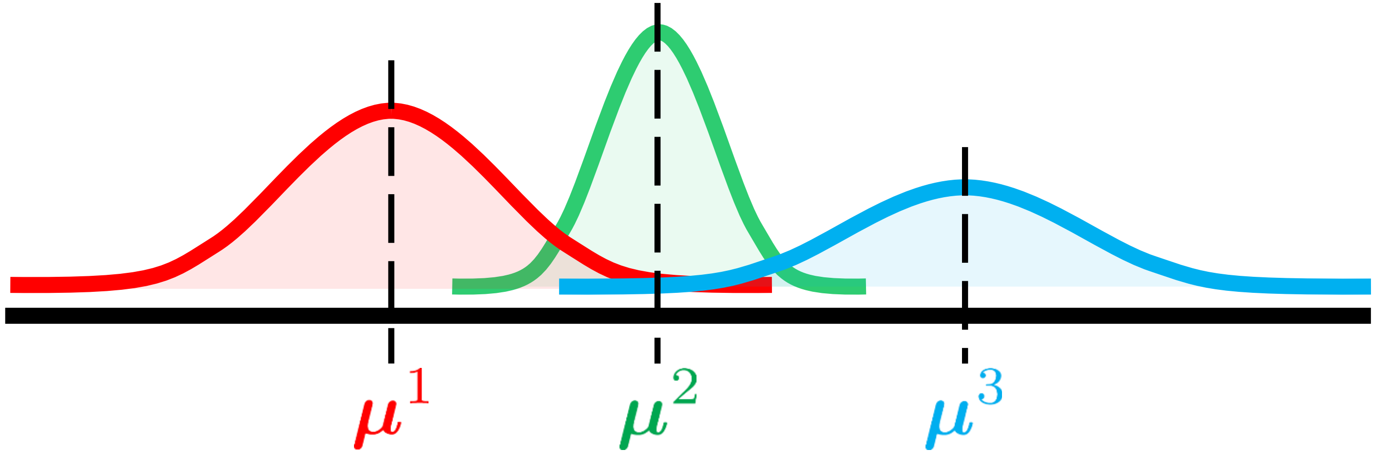

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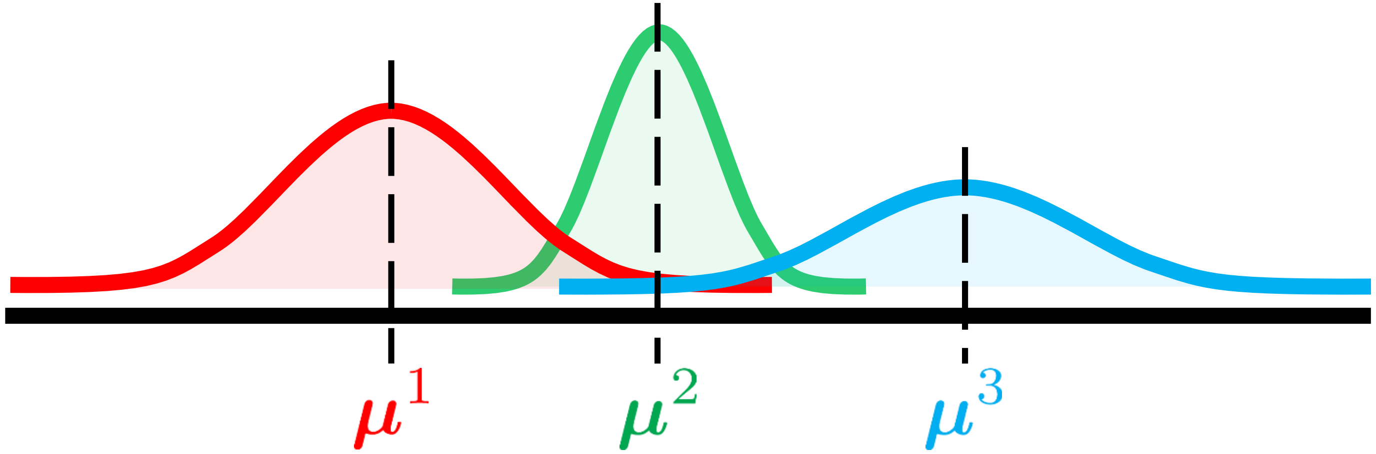

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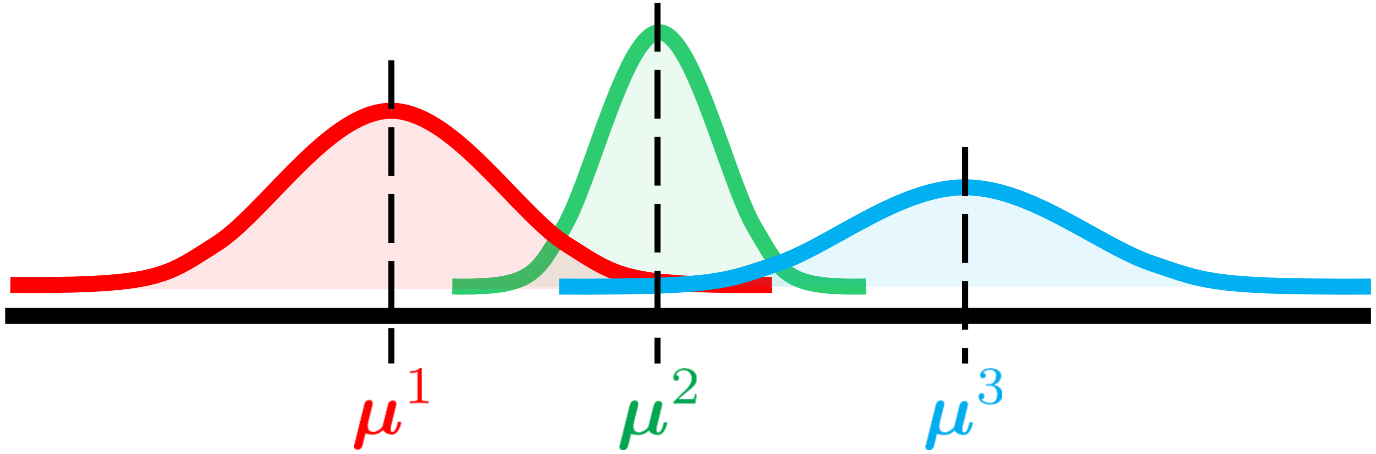

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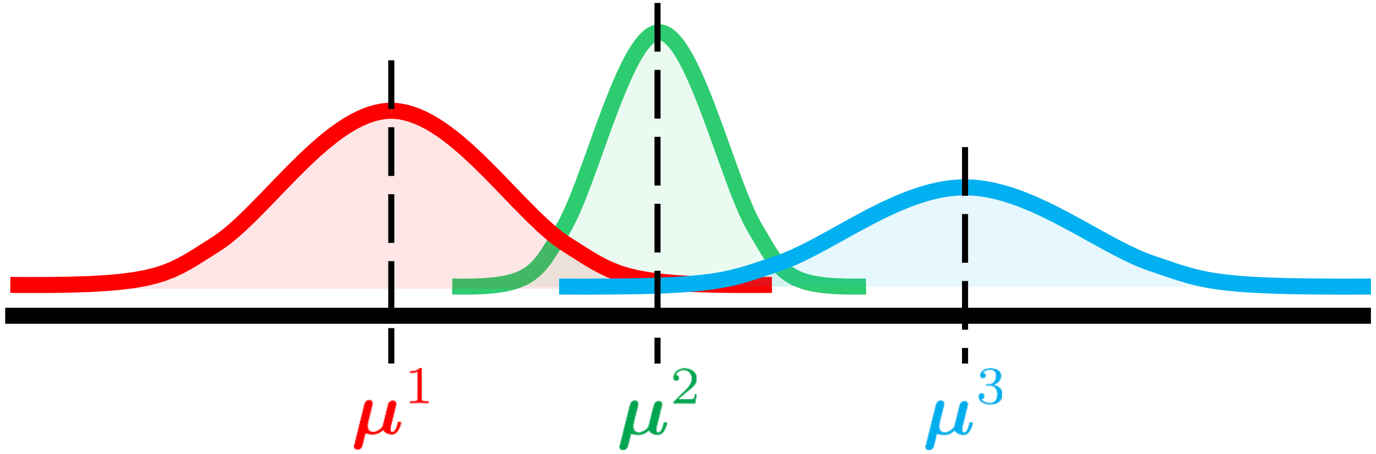

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Take log and apply 1st order optimality

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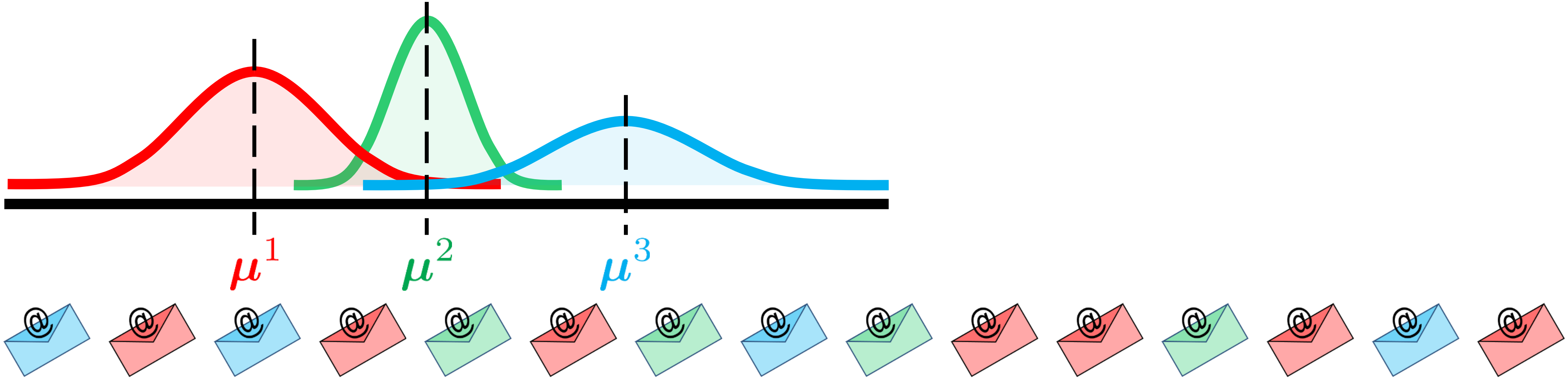

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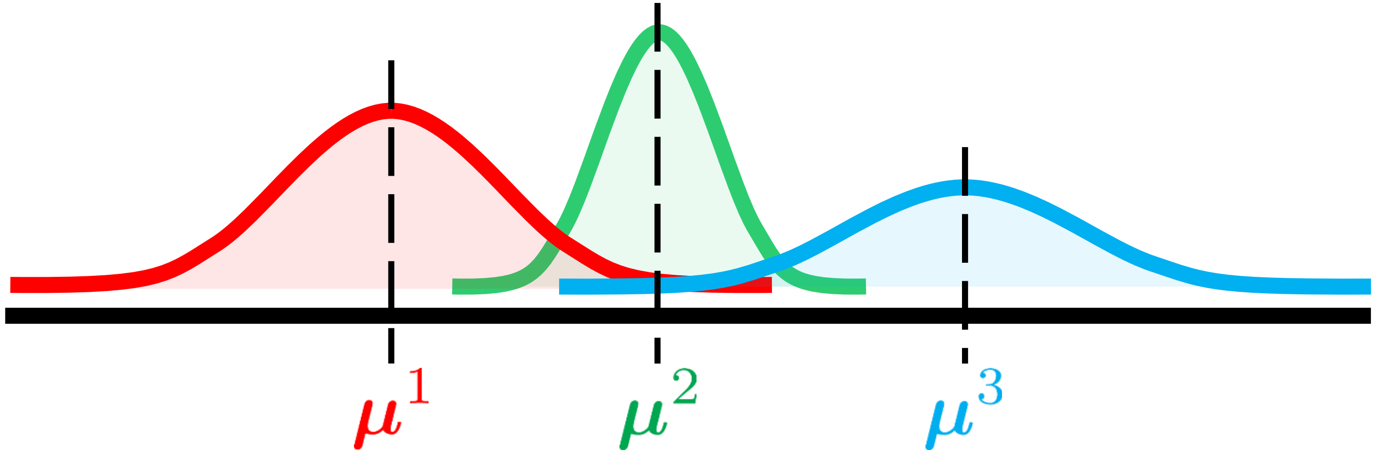

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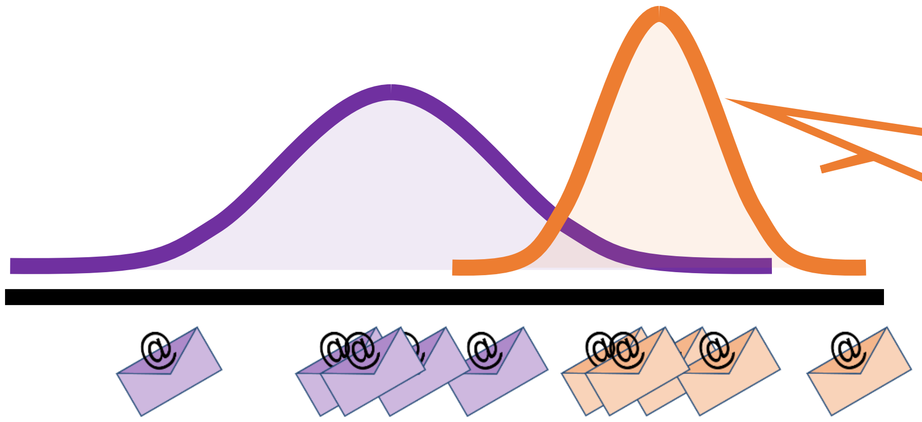
Take log and apply 1st order optimality

Read [DAU] Sections 9.1-9.5

⟨/detour⟩

The generative story for unlabelled data??



No labels!!!

Can we still recover the two Gaussian components in the mixture??

Give the Gaussians colors to help identifying them

We do not know which email is purple and which is orange – no labels!!

Better take a look now ☺

Saw how to do this for Bernoulli distributions last time.

Hmm … what if someone magically labelled the emails with the color

Can estimate the class proportions, and means and variances of these 1D Gaussians using training data!

Read **[DAU]** Sections 9.1-9.5

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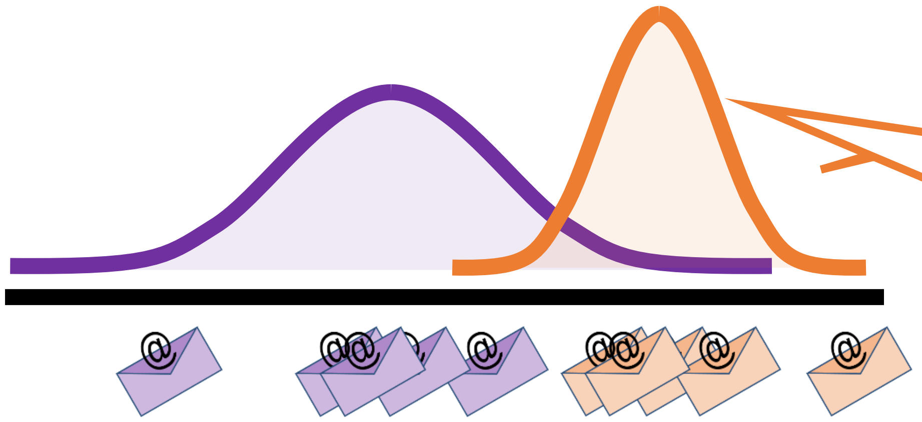

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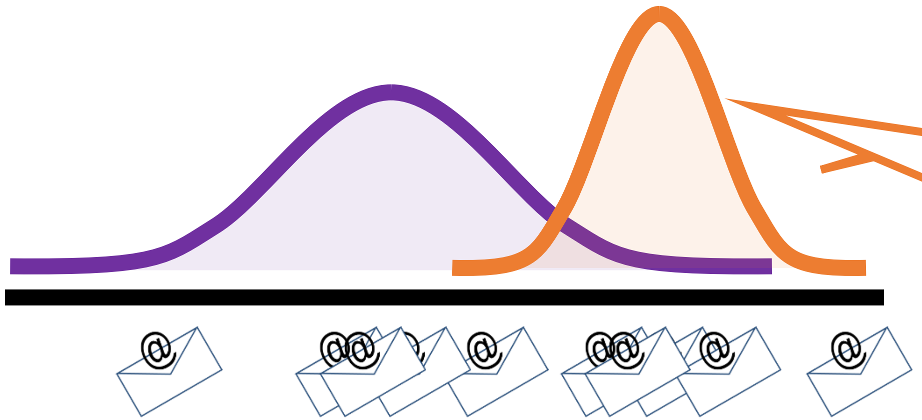

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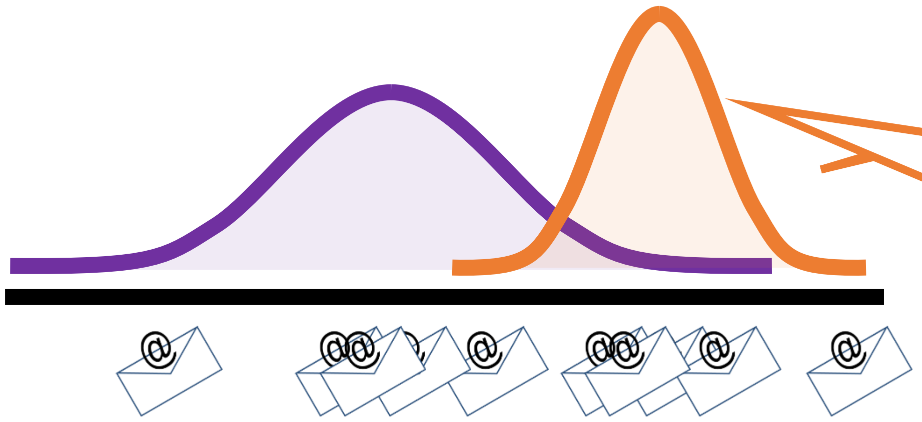

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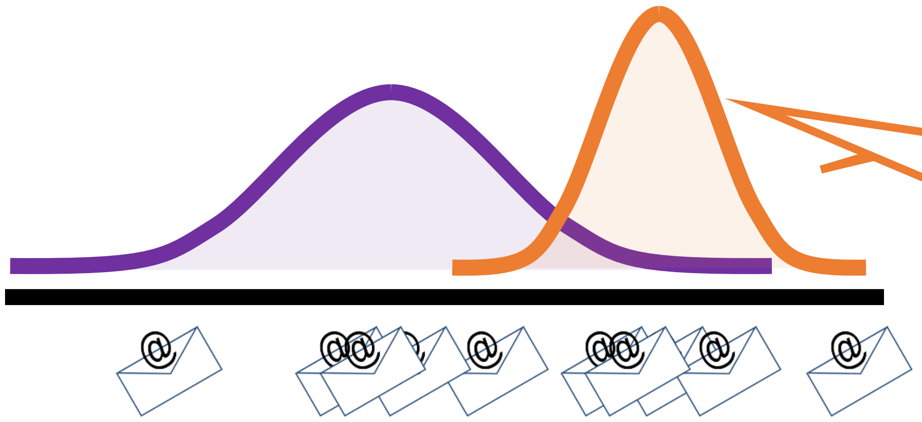

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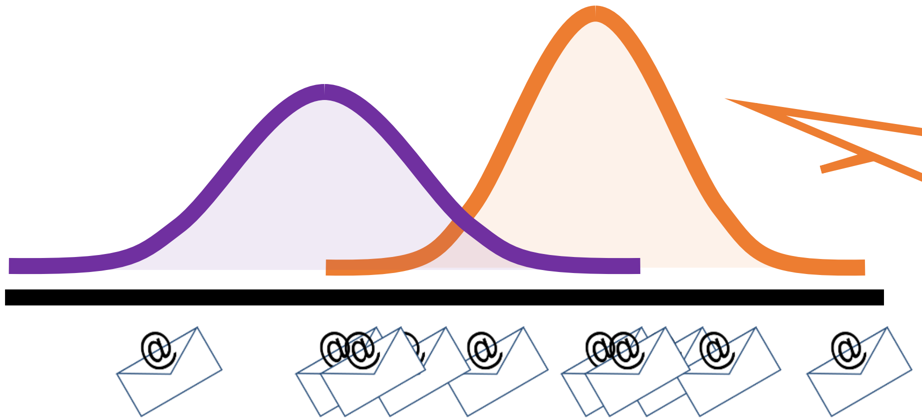

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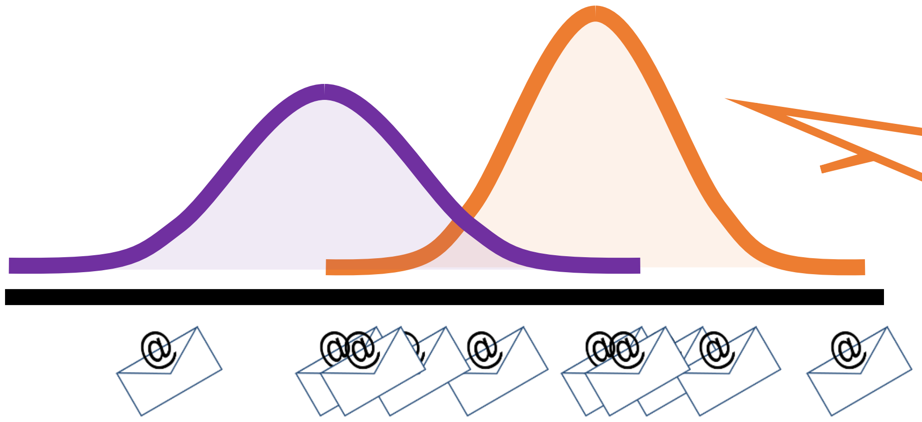

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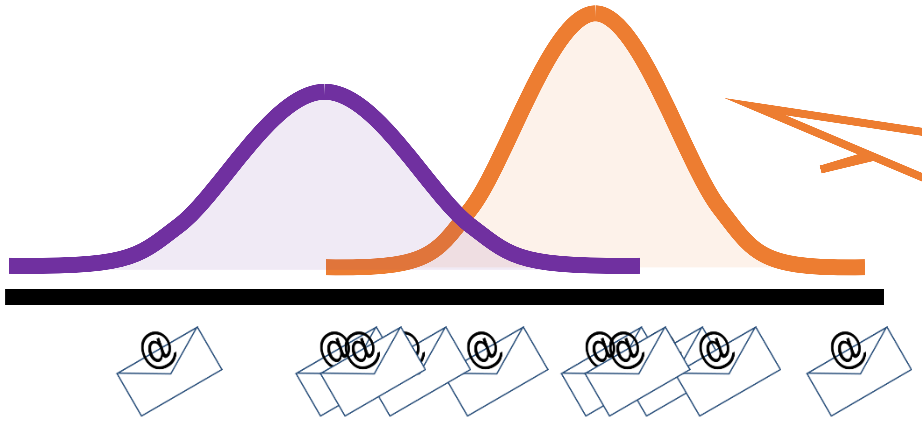

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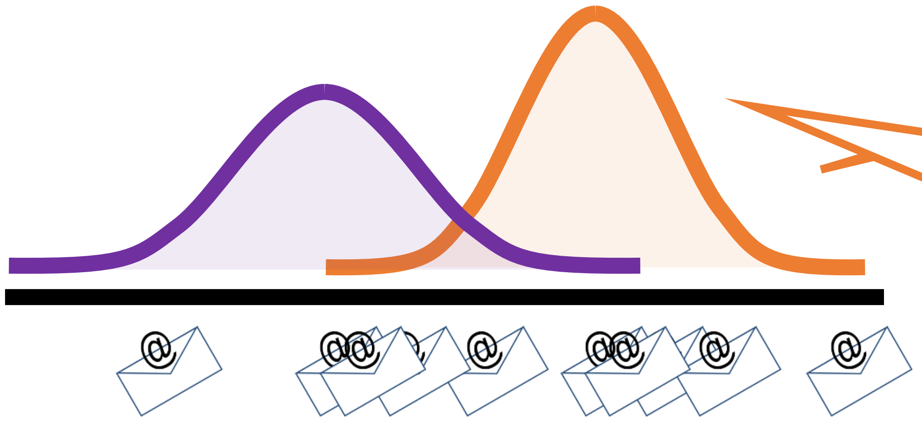

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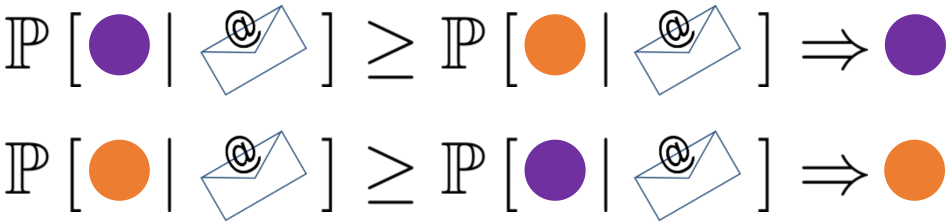

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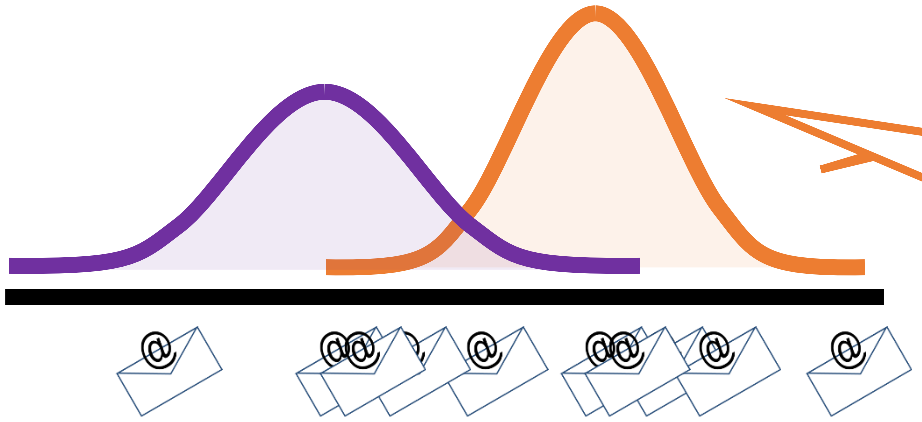

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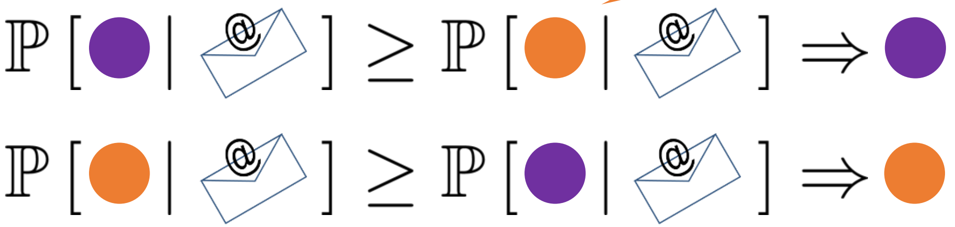

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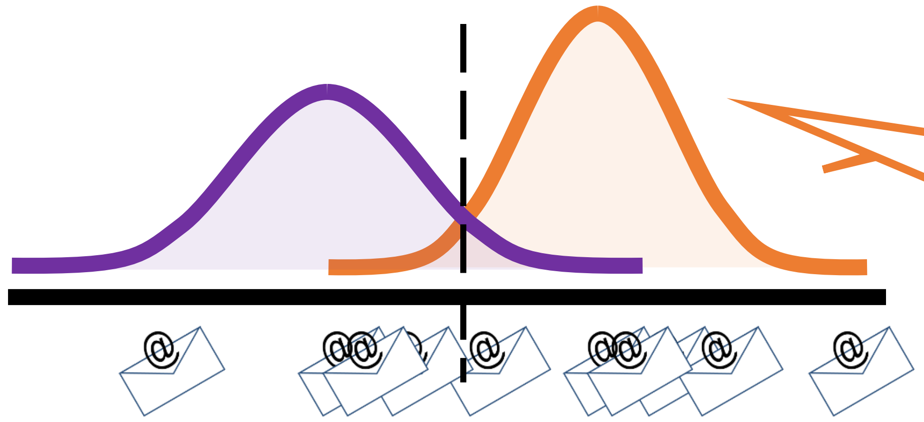

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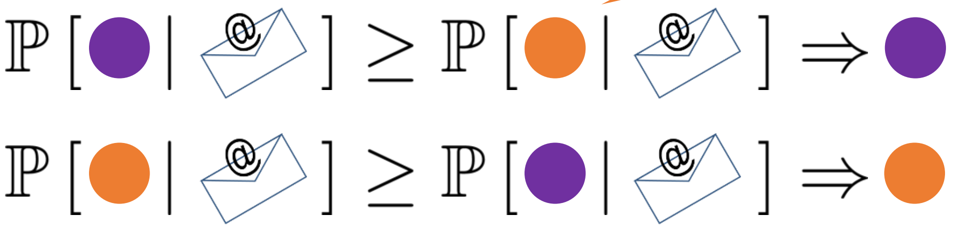

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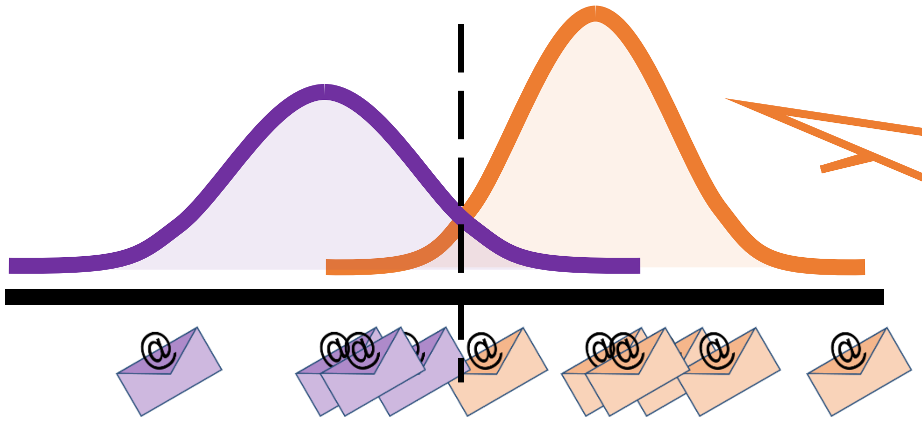

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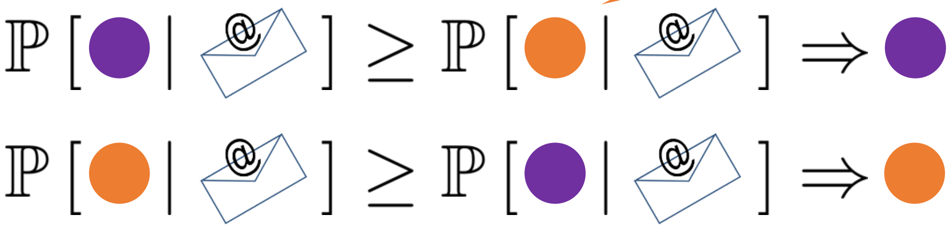

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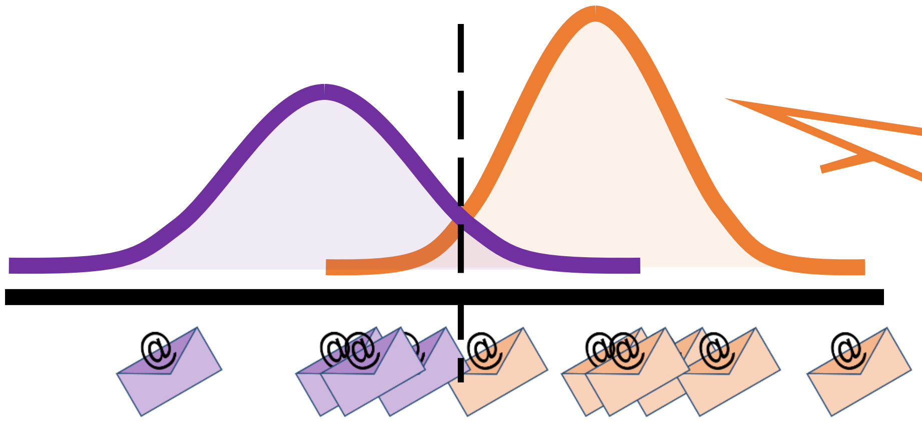

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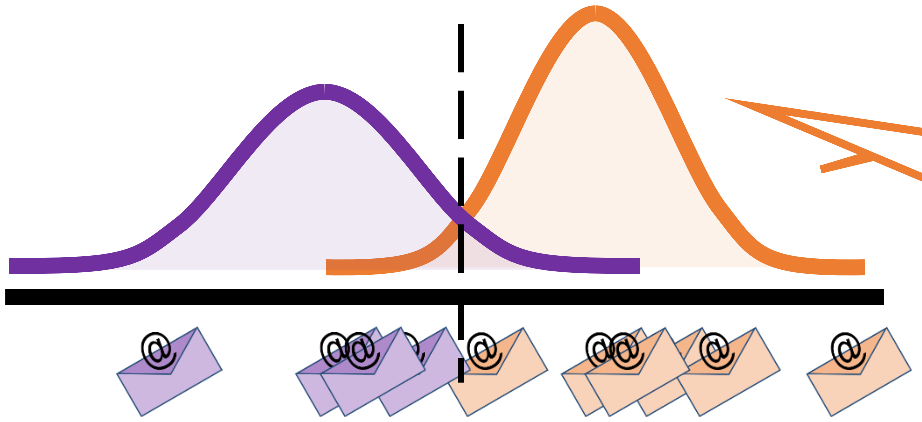

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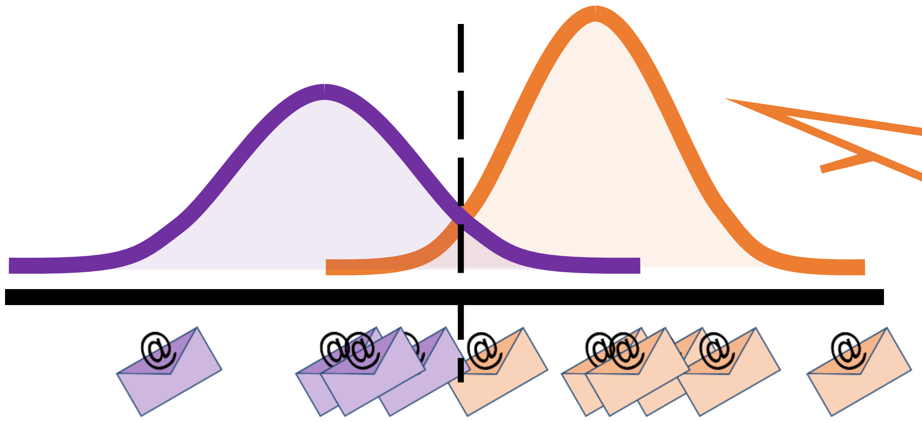

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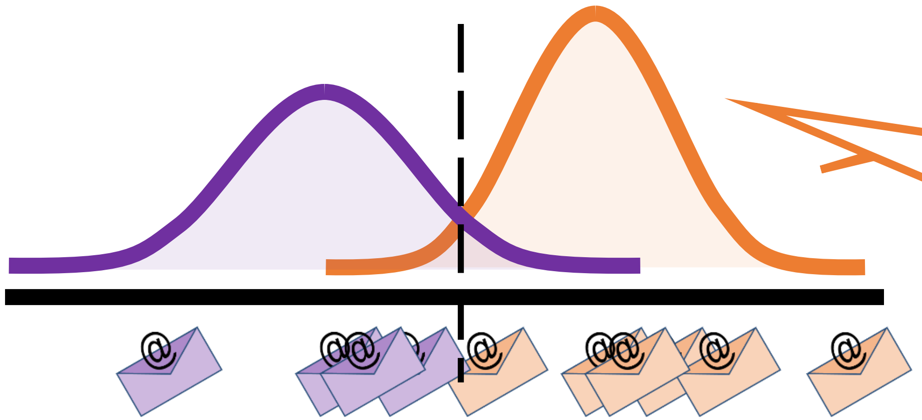

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In practice: you will learn slightly “less wrong” ones ☺

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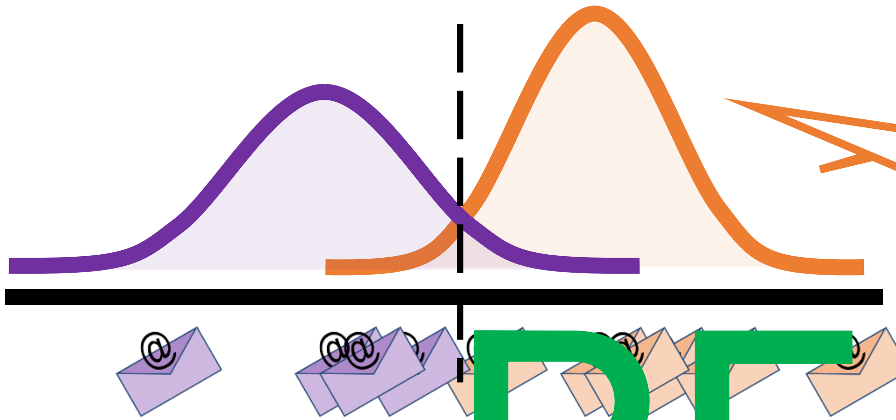

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What if I use these “magical” labels to learn two Gaussians?

Won't I just learn the wrong ones that generated the labels?

In practice: you will learn slightly “less wrong” ones ☺

REPEAT!

The generative story for unlabelled data

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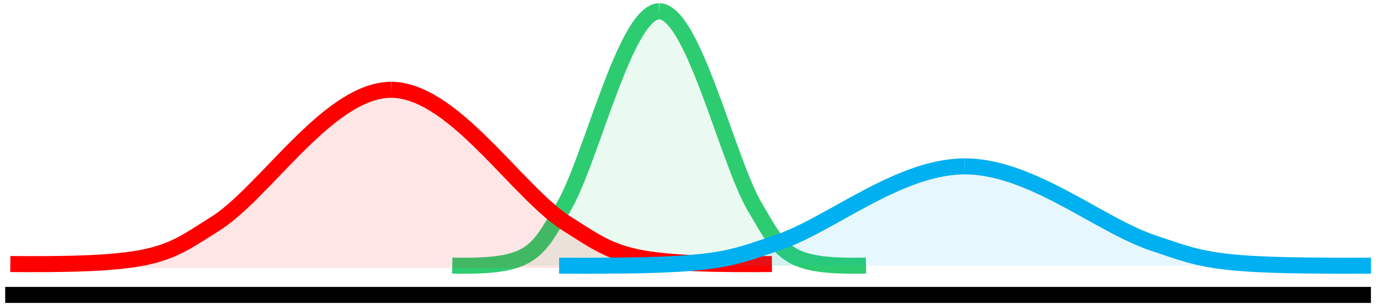

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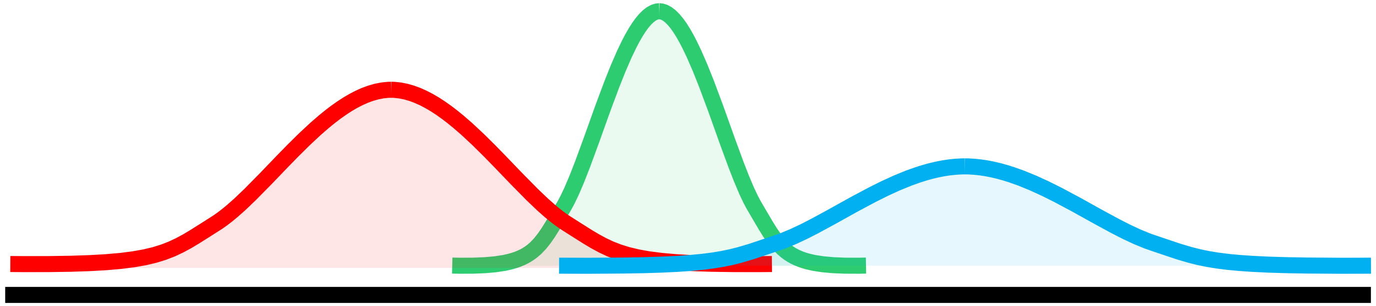

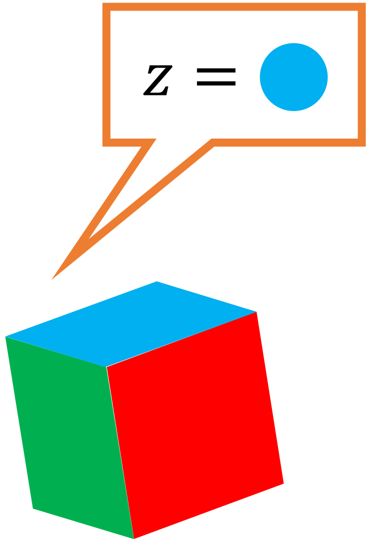

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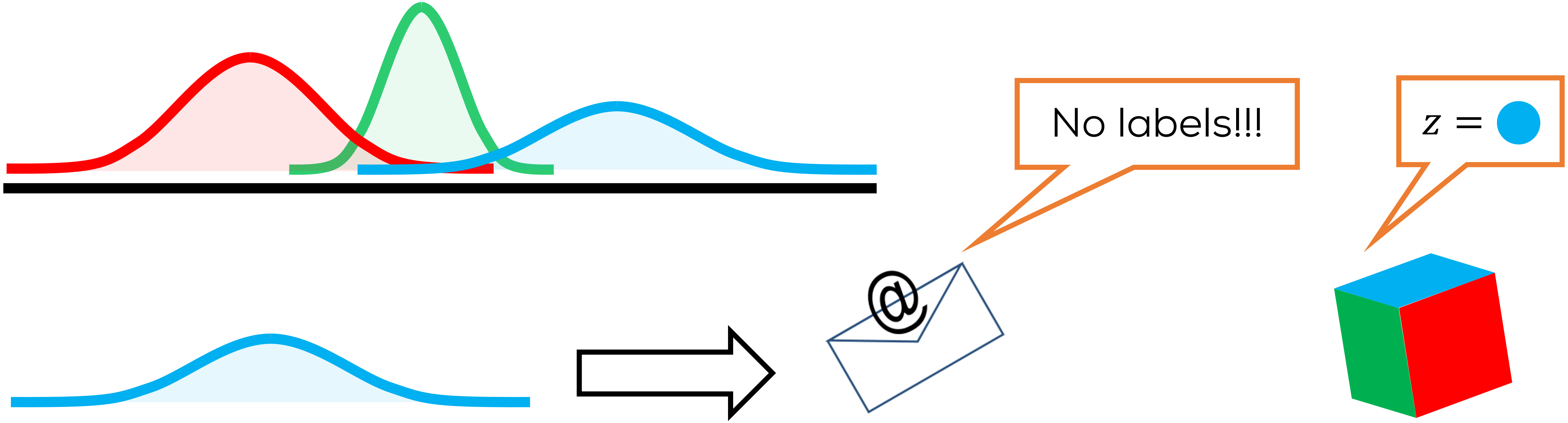

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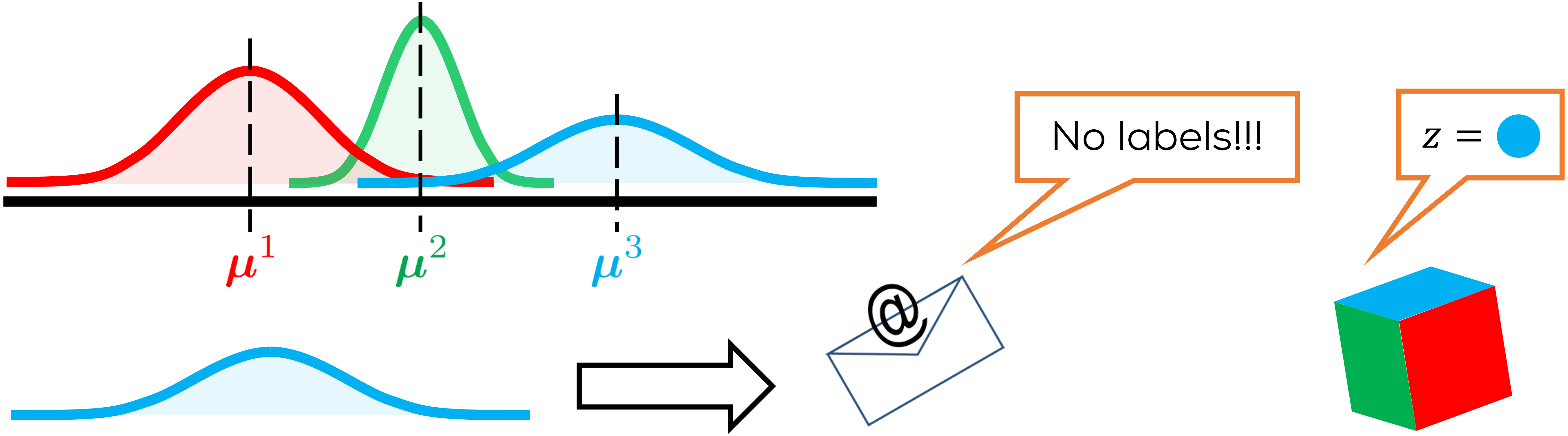

$$\boldsymbol{\Theta} = \left\{ \boldsymbol{\pi}, \left\{ \boldsymbol{\mu}^1, \boldsymbol{\mu}^2, \boldsymbol{\mu}^3 \right\}, \left\{ \Sigma^1, \Sigma^2, \Sigma^3 \right\} \right\}$$

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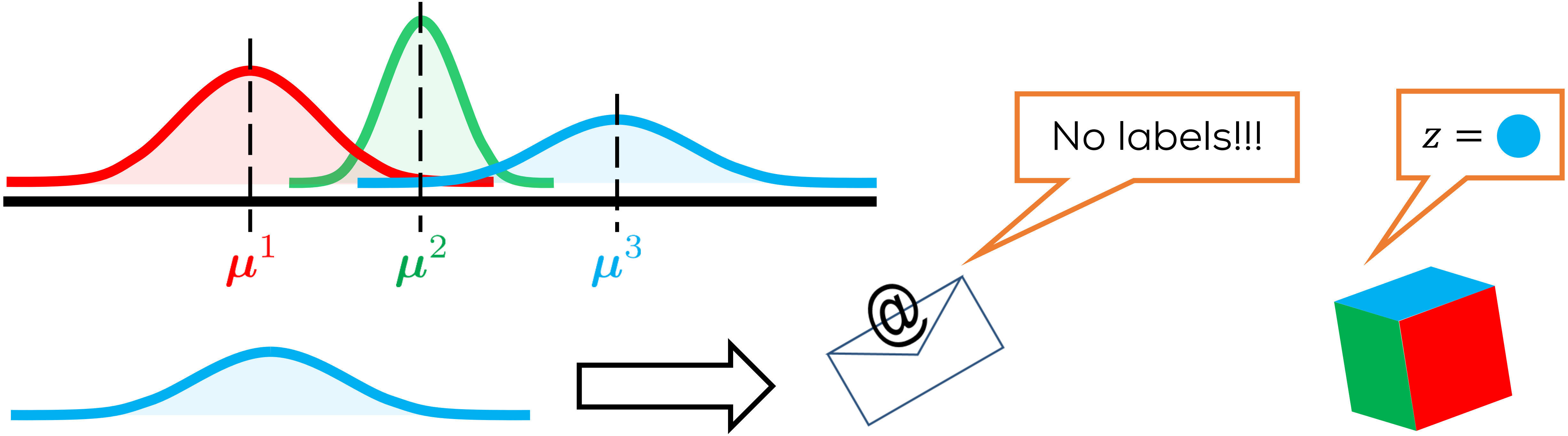

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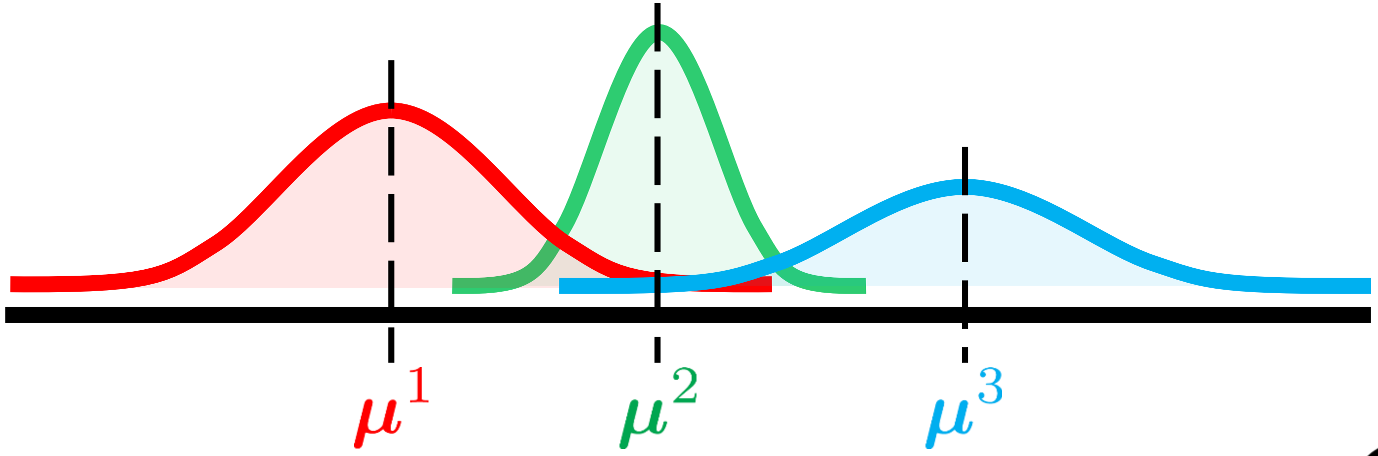

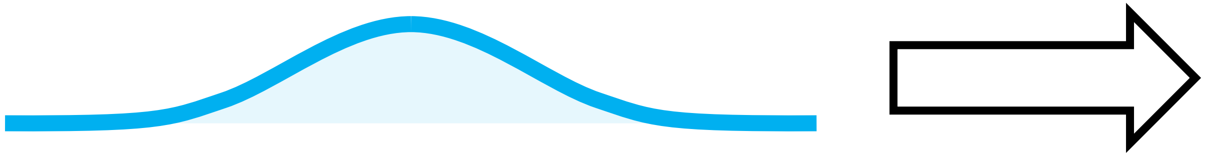

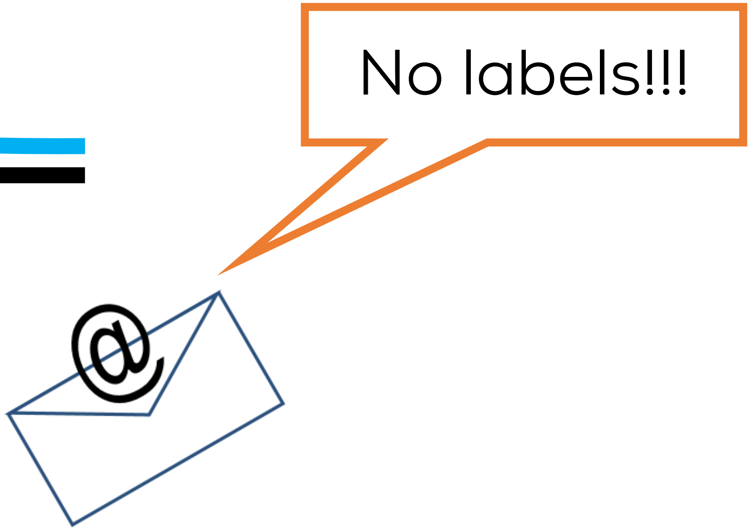

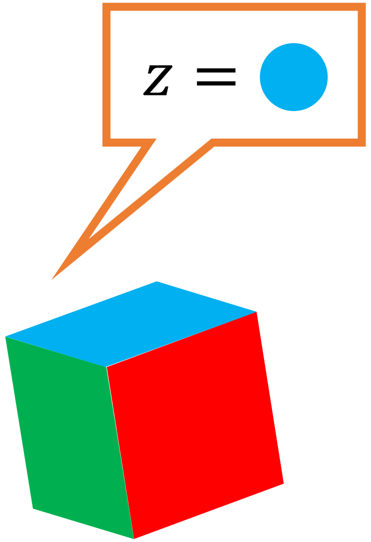

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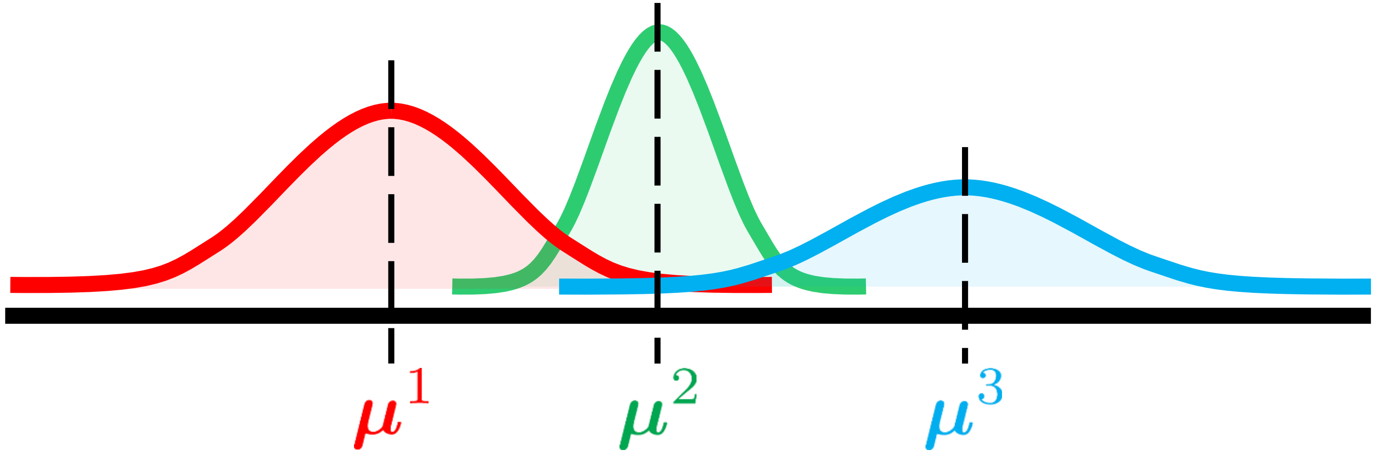
Just as before!

The generative story for unlabelled data



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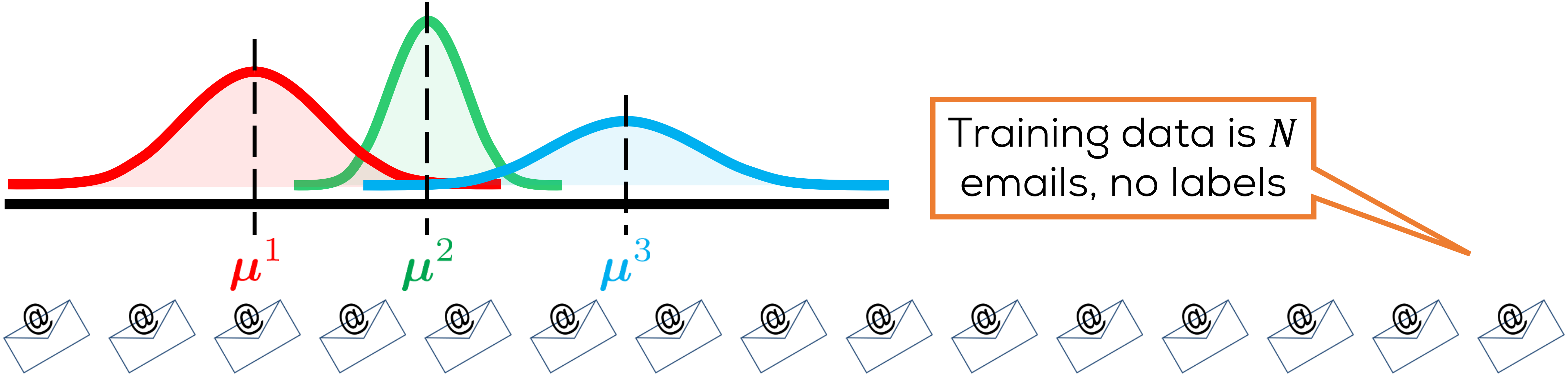

Training data is N emails, no labels

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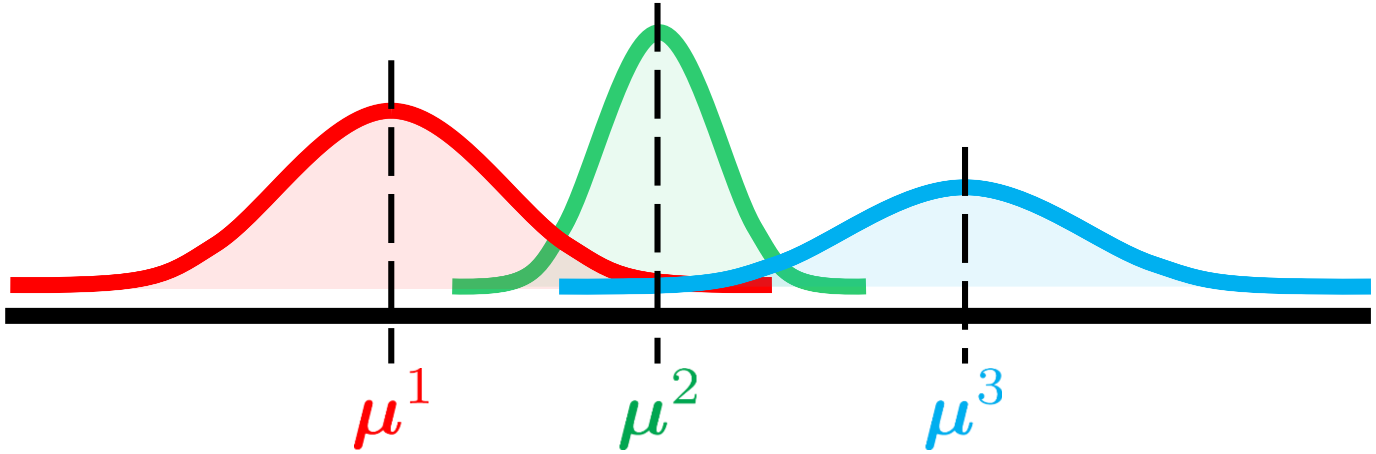

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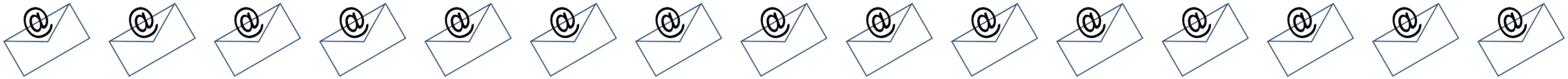

$$\mathbb{P}\left[\mathbf{x}^i \mid \mathbf{\Theta}\right]$$

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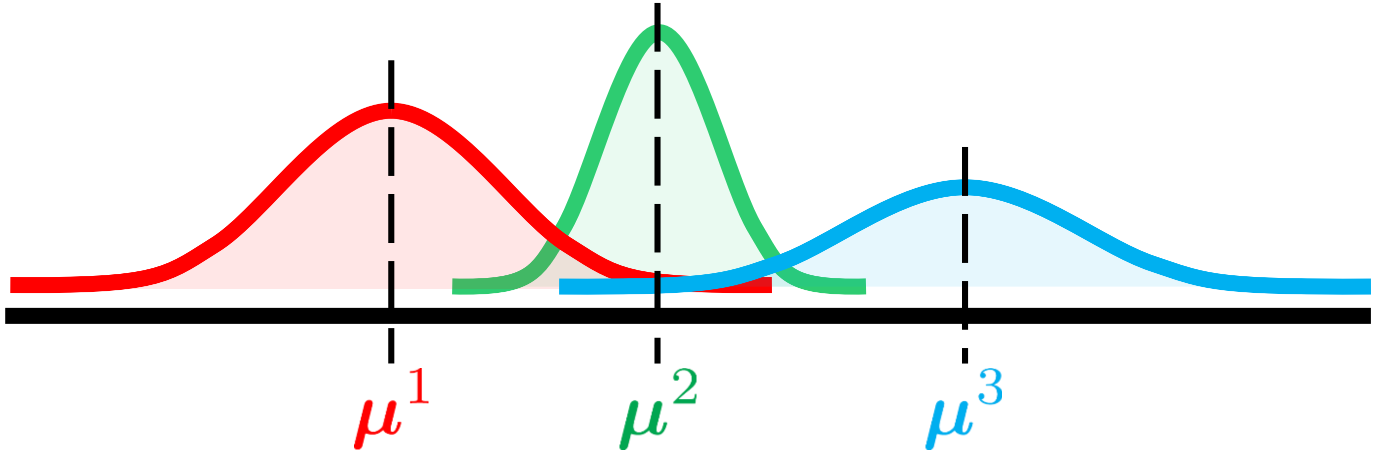

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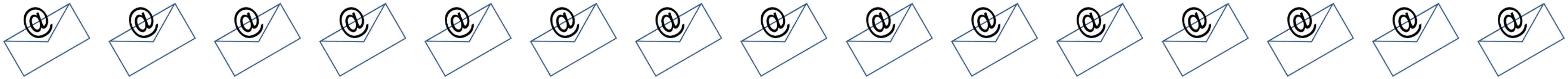

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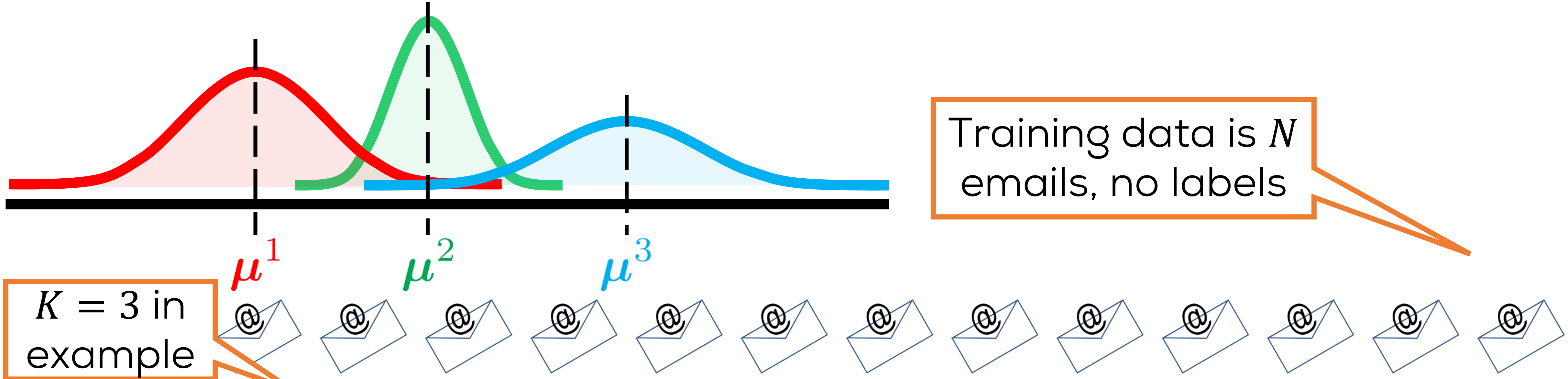

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$K = 3$ in example

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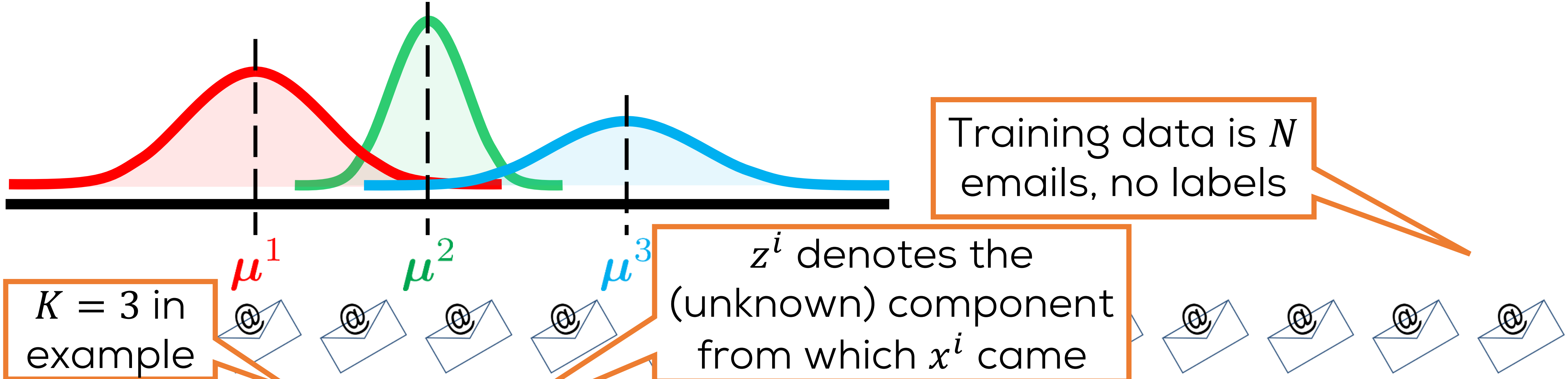

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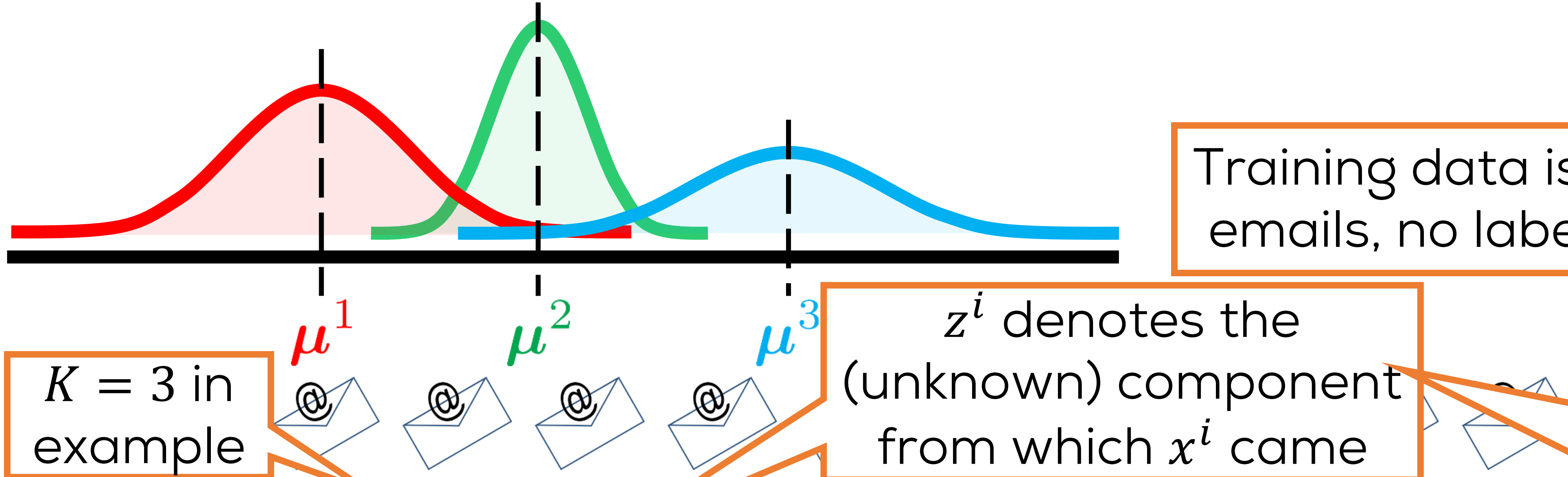

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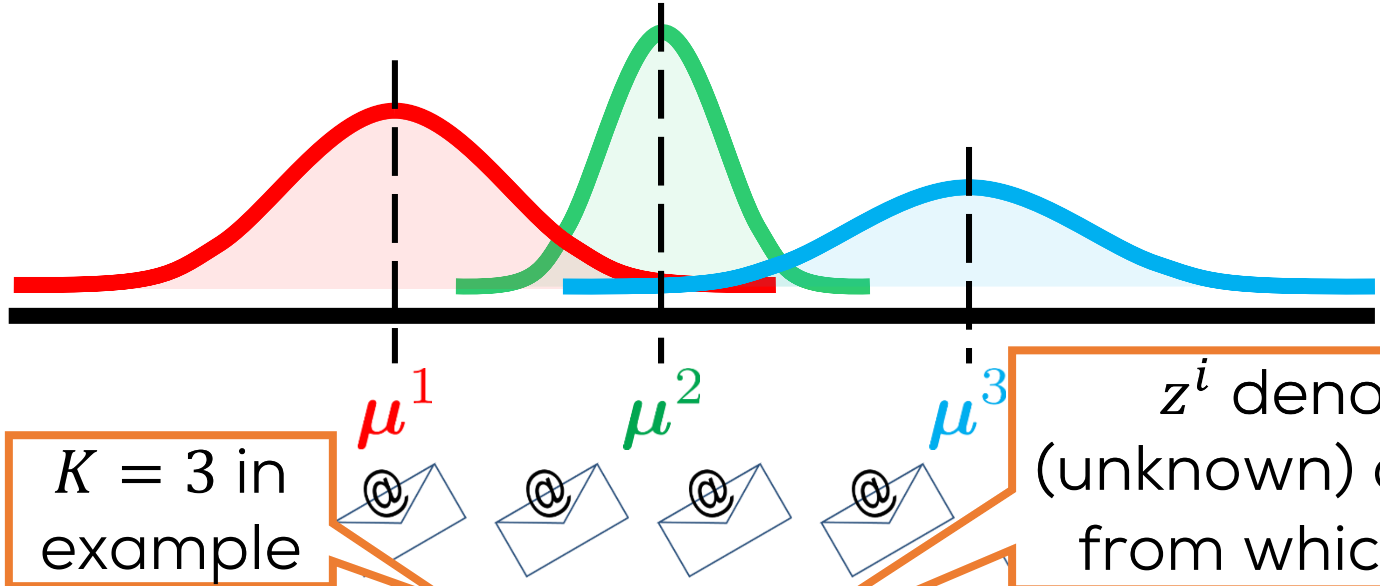

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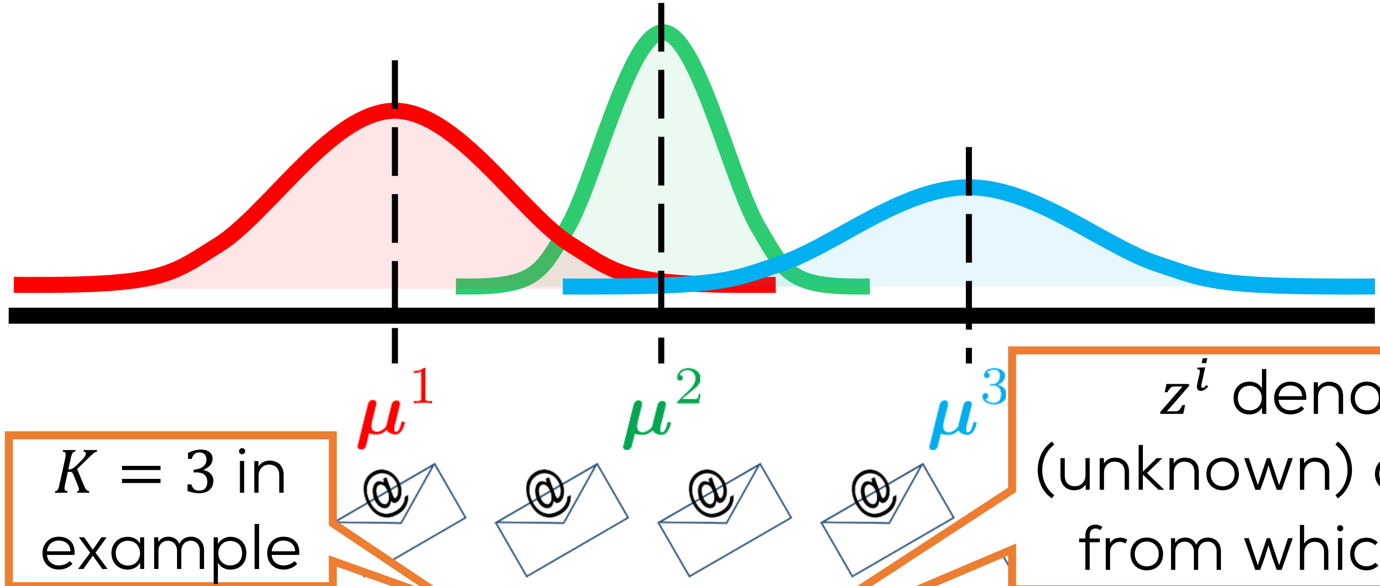

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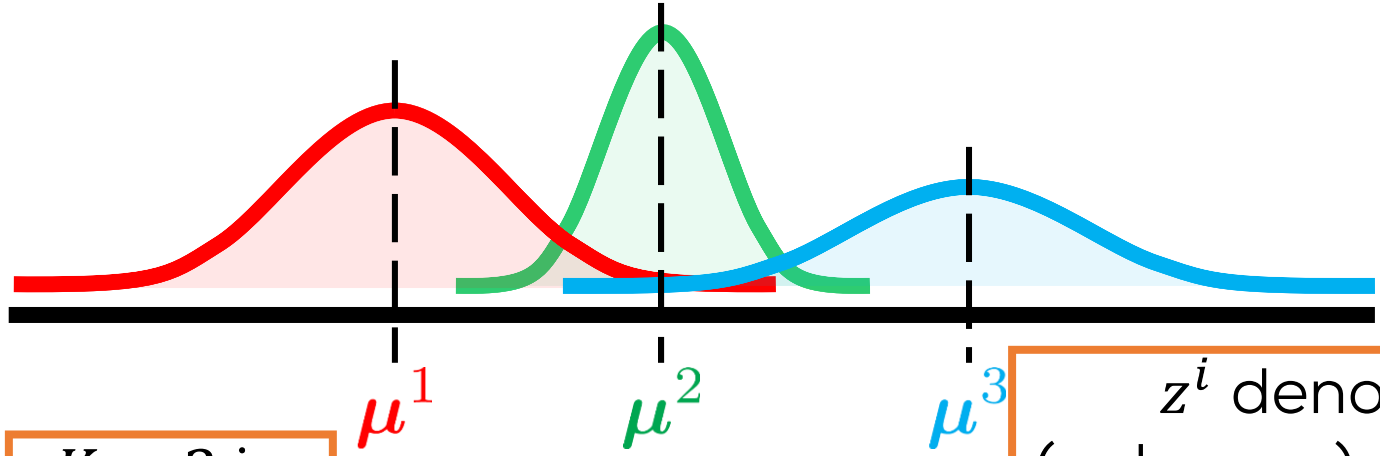

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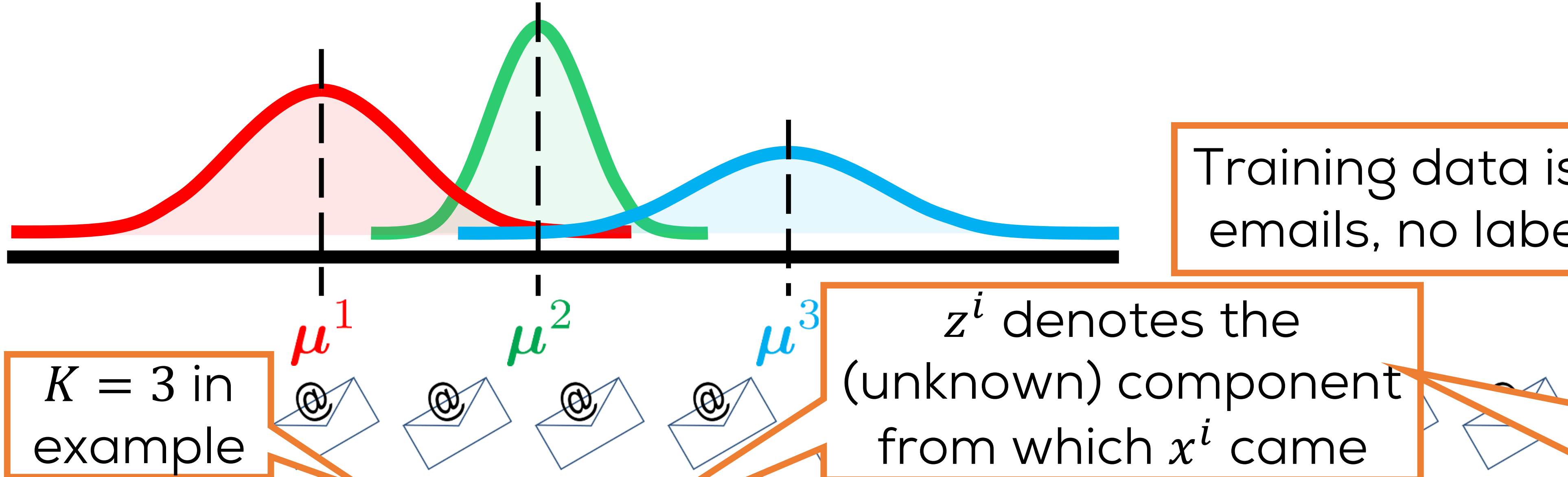

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Gaussian Mixture Model (GMM) with K components

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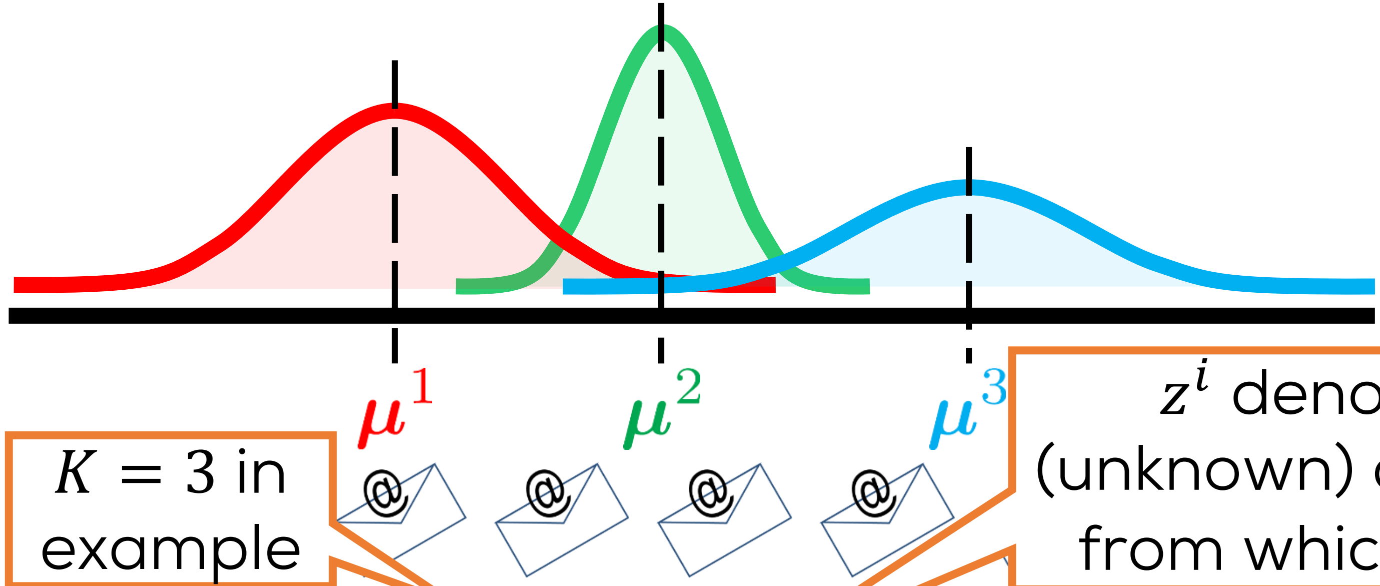

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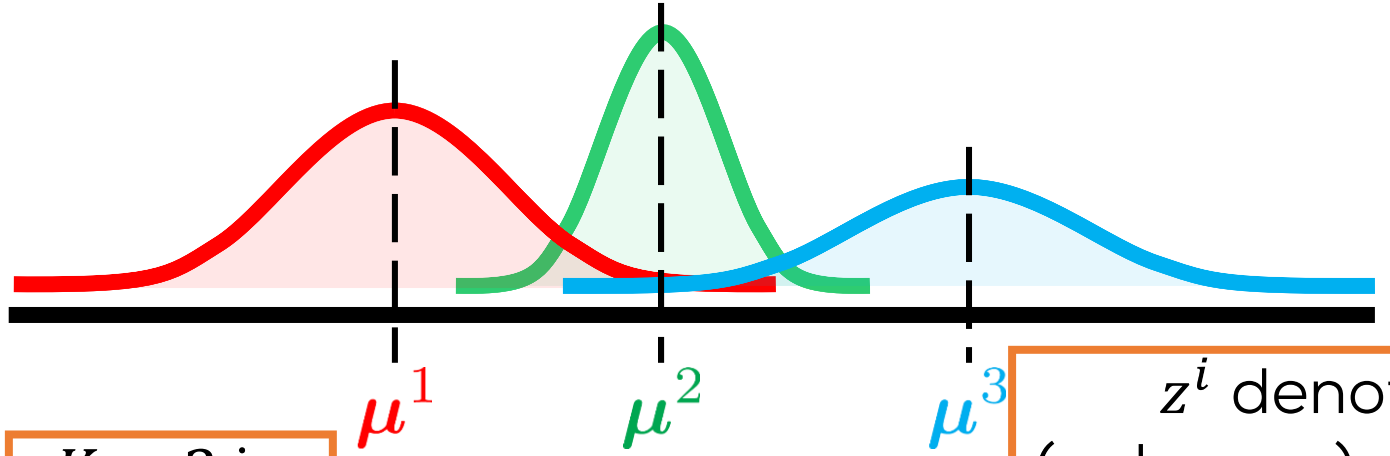

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Gaussian Mixture Model (GMM) with K components

Goal: incomplete data, learn $\boldsymbol{\mu}^k, \Sigma^k, \mathbb{P}[z = k]$

$\mathbb{P}[z^i = k]$ prior prob. of $\mathbf{x}^i$ coming from k-th component

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Cannot apply first order Optimality to get solution

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Horribly non-convex problem. Initialization matters!

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Things were so nice when we had the labels 😐

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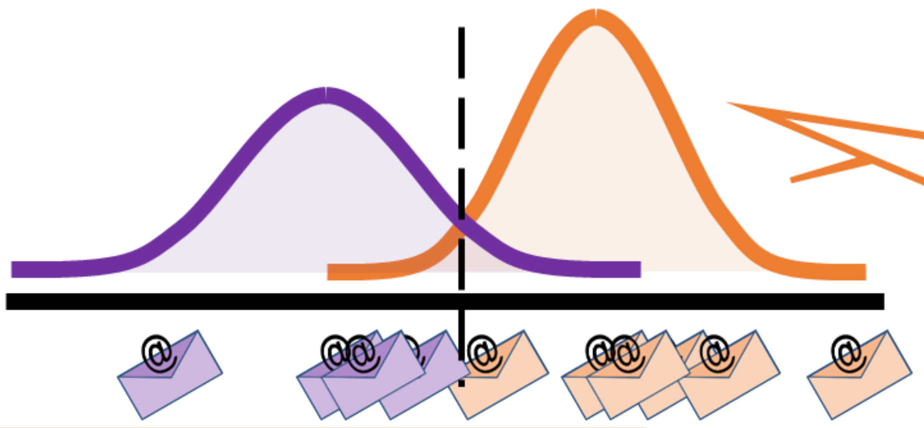

No labels!!!

Give the Gaussians colors to help identifying them

Can we still recover the two Gaussian components in the mixture??

We do not know which email is purple and which is orange – no labels!!

Hmm … what if someone magically labelled the emails with the color

How to get these magical labels??

Hmm … what if someone gave me a purple and an orange Gaussian, maybe slightly wrong ones

Can use these to label emails!

What if I use these “magical” labels to learn two Gaussians?

Won’t I just learn the wrong ones that generated the labels?

In practice: you will learn slightly “less wrong” ones ☺

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A Ray of Hope

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ALTERNATING OPTIMIZATION

1. Initialize $\mathbf{\Theta}^0$
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Various ways of updating z^i

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Looks like block coordinate descent with $\mathbf{\Theta}, \{z^i\}$ being two blocks of "coordinates"

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Hard Assignment

The K-means algorithm

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3. Update $\mathbf{\Theta}^{t+1} = \arg\max_{\mathbf{\Theta}} \mathbb{P}\left[X, \{z^{i,t}\} \mid \mathbf{\Theta}\right]$
4. Repeat until convergence

Various ways of updating z^i

Can use a method like the one discussed in the "detour"!

A Ray of Hope

$$\hat{\mathbf{\Theta}}_{\mathrm{MLE}} = \arg\max_{\mathbf{\Theta}} \mathbb{P}\left[X \mid \mathbf{\Theta}\right]$$

ALTERNATING OPTIMIZATION

1. Initialize $\mathbf{\Theta}^0$
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4. Repeat until convergence

$$z^{i,t} = \arg\max_{k \in [K]} \mathbb{P}\left[k \,|\, \mathbf{x}^i, \mathbf{\Theta}^t\right]$$

A Ray of Hope

$$\hat{\boldsymbol{\Theta}}_{\text{MLE}} = \arg\max_{\boldsymbol{\Theta}} \mathbb{P}\left[X \mid \boldsymbol{\Theta}\right]$$

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4. Repeat until convergence

$$z^{i,t} = \arg\max_{k \in [K]} \mathbb{P}\left[k \mid \mathbf{x}^i, \boldsymbol{\Theta}^t\right]$$

$$\boldsymbol{\Theta}^t = \left\{\boldsymbol{\pi}^t, \left\{\boldsymbol{\mu}^{1,t}, \boldsymbol{\mu}^{2,t}, \boldsymbol{\mu}^{3,t}\right\}, \left\{\Sigma^{1,t}, \Sigma^{2,t}, \Sigma^{3,t}\right\}\right\}$$

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4. Repeat until convergence

Bayes Rule!

$$z^{i,t} = \arg\max_{k \in [K]} \mathbb{P}\left[k \mid \mathbf{x}^i, \mathbf{\Theta}^t\right] = \arg\max_{k \in [K]} \mathbb{P}\left[k \mid \mathbf{\Theta}^t\right] \cdot \mathbb{P}\left[\mathbf{x}^i \mid k, \mathbf{\Theta}^t\right]$$

$$\mathbf{\Theta}^t = \left\{\boldsymbol{\pi}^t, \left\{\boldsymbol{\mu}^{1,t}, \boldsymbol{\mu}^{2,t}, \boldsymbol{\mu}^{3,t}\right\}, \left\{\Sigma^{1,t}, \Sigma^{2,t}, \Sigma^{3,t}\right\}\right\}$$

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4. Repeat until convergence

Bayes Rule!

$$z^{i,t} = \arg\max_{k \in [K]} \mathbb{P}\left[k \mid \mathbf{x}^i, \mathbf{\Theta}^t\right] = \arg\max_{k \in [K]} \boldsymbol{\pi}_k^t \cdot \mathbb{P}\left[\mathbf{x}^i \mid k, \mathbf{\Theta}^t\right]$$

$$\mathbf{\Theta}^t = \left\{\boldsymbol{\pi}^t, \left\{\boldsymbol{\mu}^{1,t}, \boldsymbol{\mu}^{2,t}, \boldsymbol{\mu}^{3,t}\right\}, \left\{\Sigma^{1,t}, \Sigma^{2,t}, \Sigma^{3,t}\right\}\right\}$$

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$$\hat{\boldsymbol{\Theta}}_{\text{MLE}} = \underset{\boldsymbol{\Theta}}{\arg\max}\ \mathbb{P}\left[X \mid \boldsymbol{\Theta}\right]$$

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4. Repeat until convergence

Bayes Rule!

$$z^{i,t} = \underset{k \in [K]}{\arg\max}\ \mathbb{P}\left[k \mid \mathbf{x}^i, \boldsymbol{\Theta}^t\right] = \underset{k \in [K]}{\arg\max}\ \boldsymbol{\pi}_k^t \cdot \mathcal{N}(\mathbf{x}^i \mid \boldsymbol{\mu}^{k,t}, \Sigma^{k,t})$$

$$\boldsymbol{\Theta}^t = \left\{\boldsymbol{\pi}^t, \left\{\boldsymbol{\mu}^{1,t}, \boldsymbol{\mu}^{2,t}, \boldsymbol{\mu}^{3,t}\right\}, \left\{\Sigma^{1,t}, \Sigma^{2,t}, \Sigma^{3,t}\right\}\right\}$$

Towards the K-Means Algorithm

ALTERNATING OPTIMIZATION

1. Initialize $\boldsymbol{\Theta}^0$
2. For $i \in [n]$, update $z^{i,t}$ using $\boldsymbol{\Theta}^t$
    1. Let $z^{i,t} = \arg\max_k \boldsymbol{\pi}_k^t \cdot \mathcal{N}\left(\mathbf{x}^i \mid \boldsymbol{\mu}^{k,t}, \Sigma^{k,t}\right)$
3. Update $\boldsymbol{\Theta}^{t+1} = \arg\max_{\boldsymbol{\Theta}} \mathbb{P}\left[X, \left\{z^{i,t}\right\} \mid \boldsymbol{\Theta}\right]$
    1. Let $\boldsymbol{\pi}_k^{t+1} = \frac{n_k^t}{n}$, where $\mathrm{n}_k^t = \left|\left\{i: \, z^{i,t} = k\right\}\right|$
    2. Let $\boldsymbol{\mu}^{k,t+1} = \frac{1}{n_k^t} \sum_{i: z^{i,t} = k} \mathbf{x}^i$
    3. Let $\Sigma_k^{t+1} = \frac{1}{n_k^t} \sum_{i: z^{i,t} = k} \left(\mathbf{x}^i - \boldsymbol{\mu}^{k,t+1}\right)\left(\mathbf{x}^i - \boldsymbol{\mu}^{k,t+1}\right)^\top$
4. Repeat until convergence

A few simplifications

- Fix $\boldsymbol{\pi}_k^t = \frac{1}{K}$ for all iterations. Don't update it.
- Fix $\boldsymbol{\Sigma}^{k,t} = I$ for all iterations. Don't update it.

K-MEANS/LLOYD'S ALGORITHM

1. Initialize means $\{\boldsymbol{\mu}^{k,0}\}_{k=1,\ldots K}$
2. For $i \in [n]$, update $z^{i,t}$ using $\boldsymbol{\mu}^{k,t}$
   1. Let $z^{i,t} = \arg\max_k \mathcal{N}\left(\mathbf{x}^i \mid \boldsymbol{\mu}^{k,t}, I\right)$
3. Update $\boldsymbol{\mu}^{k,t+1} = \frac{1}{n_k^t} \sum_{i:z^{i,t}=k} \mathbf{x}^i$
4. Repeat until convergence

A few simplifications

- Fix $\boldsymbol{\pi}_k^t = \frac{1}{K}$ for all iterations. Don't update it.
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The K-Means Objective

The K-Means Objective

$$\hat{\mathbf{\Theta}}_{\mathrm{km}} = \underset{\substack{\{\boldsymbol{\mu}^k\}_{k=1,\ldots,K} \\ \{z^i\}_{i=1,\ldots,n}}}{\arg\min} \sum_{k=1}^{K} \sum_{i:z^i=k} \left\|\mathbf{x}^i - \boldsymbol{\mu}^k\right\|_2^2$$

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For cluster k, we have cluster center $\boldsymbol{\mu}^k$

The K-Means Objective

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For cluster k,

$$\sum_{i:z^i=k} \left\| \mathbf{x}^i - \boldsymbol{\mu}^k \right\|_2^2$$

is a measure of how much variance is in the cluster

For cluster k, we have cluster center $\boldsymbol{\mu}^k$

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For cluster k,

$$\sum_{i:z^i=k} \left\| \mathbf{x}^i - \boldsymbol{\mu}^k \right\|_2^2$$

is a measure of how much variance is in the cluster

Want to minimize sum of variance across all clusters. Want tight clusters

For cluster k, we have cluster center $\boldsymbol{\mu}^k$

The K-Means Objective

$$\hat{\boldsymbol{\Theta}}_{\mathrm{km}} = \underset{\substack{\{\boldsymbol{\mu}^k\}_{k=1,\ldots,K} \\ \{z^i\}_{i=1,\ldots,n}}}{\arg\min} \sum_{k=1}^{K} \sum_{i:z^i=k} \left\|\mathbf{x}^i - \boldsymbol{\mu}^k\right\|_2^2$$

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$$\hat{\mathbf{\Theta}}_{\mathrm{km}} = \arg\min_{\substack{\{\boldsymbol{\mu}^k\}_{k=1,\ldots,K} \\ \{z^i\}_{i=1,\ldots,n}}} \sum_{k=1}^{K} \sum_{i:z^i=k} \left\|\mathbf{x}^i - \boldsymbol{\mu}^k\right\|_2^2$$

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   1. Let $z^{i,t} = \arg\min_k \left\|\mathbf{x}^i - \boldsymbol{\mu}^{k,t}\right\|_2^2$
3. Update $\boldsymbol{\mu}^{k,t+1} = \frac{1}{n_k^t} \sum_{i:z^{i,t}=k} \mathbf{x}^i$
4. Repeat until convergence

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$$\hat{\Theta}_{\text{km}} = \underset{\substack{\{\boldsymbol{\mu}^k\}_{k=1,\dots,K} \\ \{z^i\}_{i=1,\dots,n}}}{\arg\min} \sum_{k=1}^{K} \sum_{i:z^i=k} \left\| \mathbf{x}^i - \boldsymbol{\mu}^k \right\|_2^2$$

Alternates between updating $\{z^i\}$ and $\{\mu^k\}$

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An FA approach to solving a data modelling task!

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The K-Means Objective

NP-hard problem!

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Very scalable but sensitive to initialization!

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4. Repeat until convergence

k-means++ initialization

1. Sample $i_1 \sim [n]$, let $\boldsymbol{\mu}^{1,0} = \mathbf{x}^{i_1}$
2. For k = 2,…K
   - Sample $i_k \propto$ min distance from $\{\boldsymbol{\mu}^{1,0}, \ldots, \boldsymbol{\mu}^{k-1,0}\}$
   - Let $\boldsymbol{\mu}^{k,0} = \mathbf{x}^{i_k}$

K-Means Algorithm in action!

K-MEANS/LLOYD'S ALGORITHM

1. Initialize means $\{\boldsymbol{\mu}^{k,0}\}_{k=1\ldots K}$
2. For $i \in [n]$, update $z^{i,t}$ using $\boldsymbol{\mu}^{k,t}$

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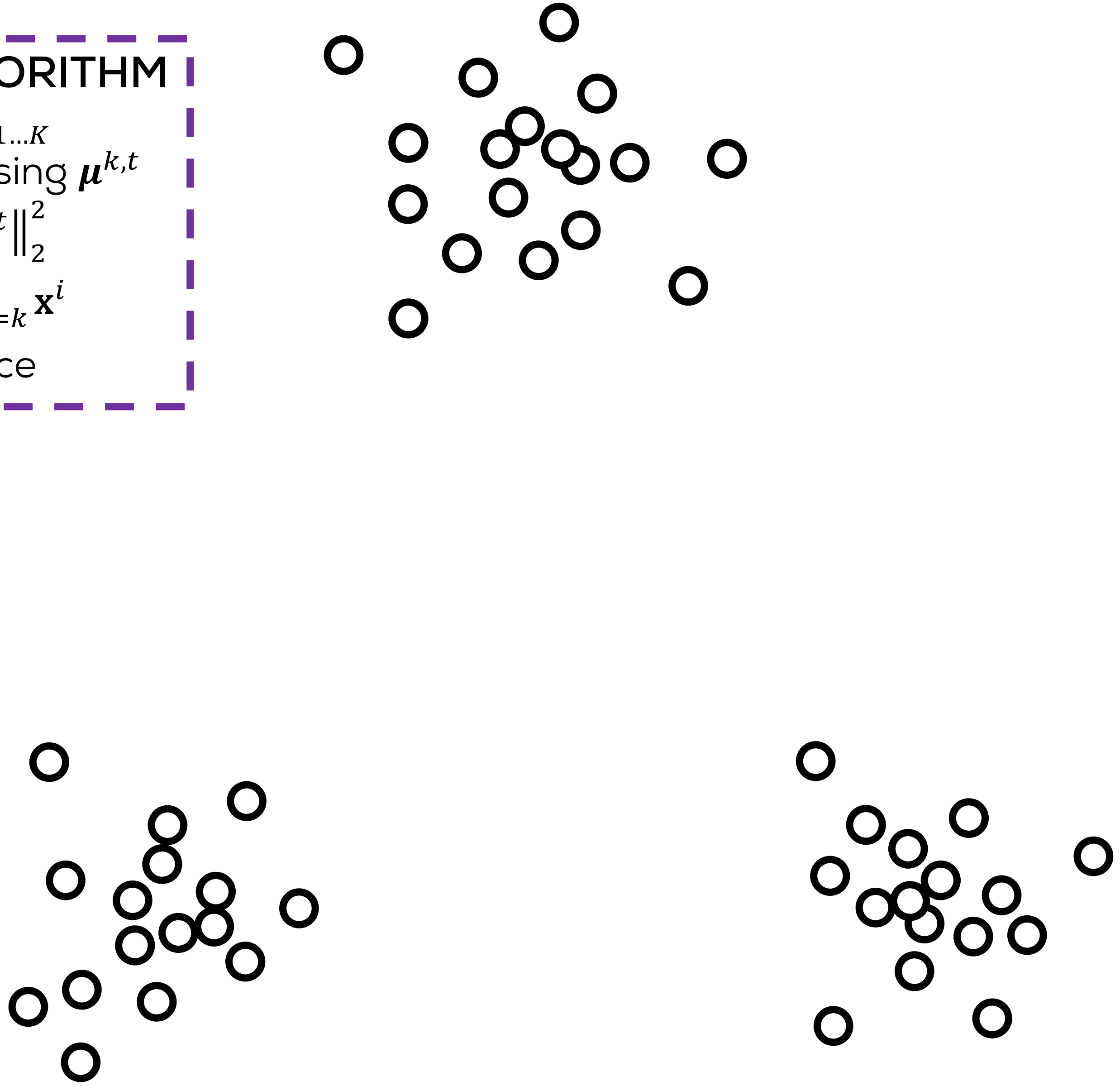

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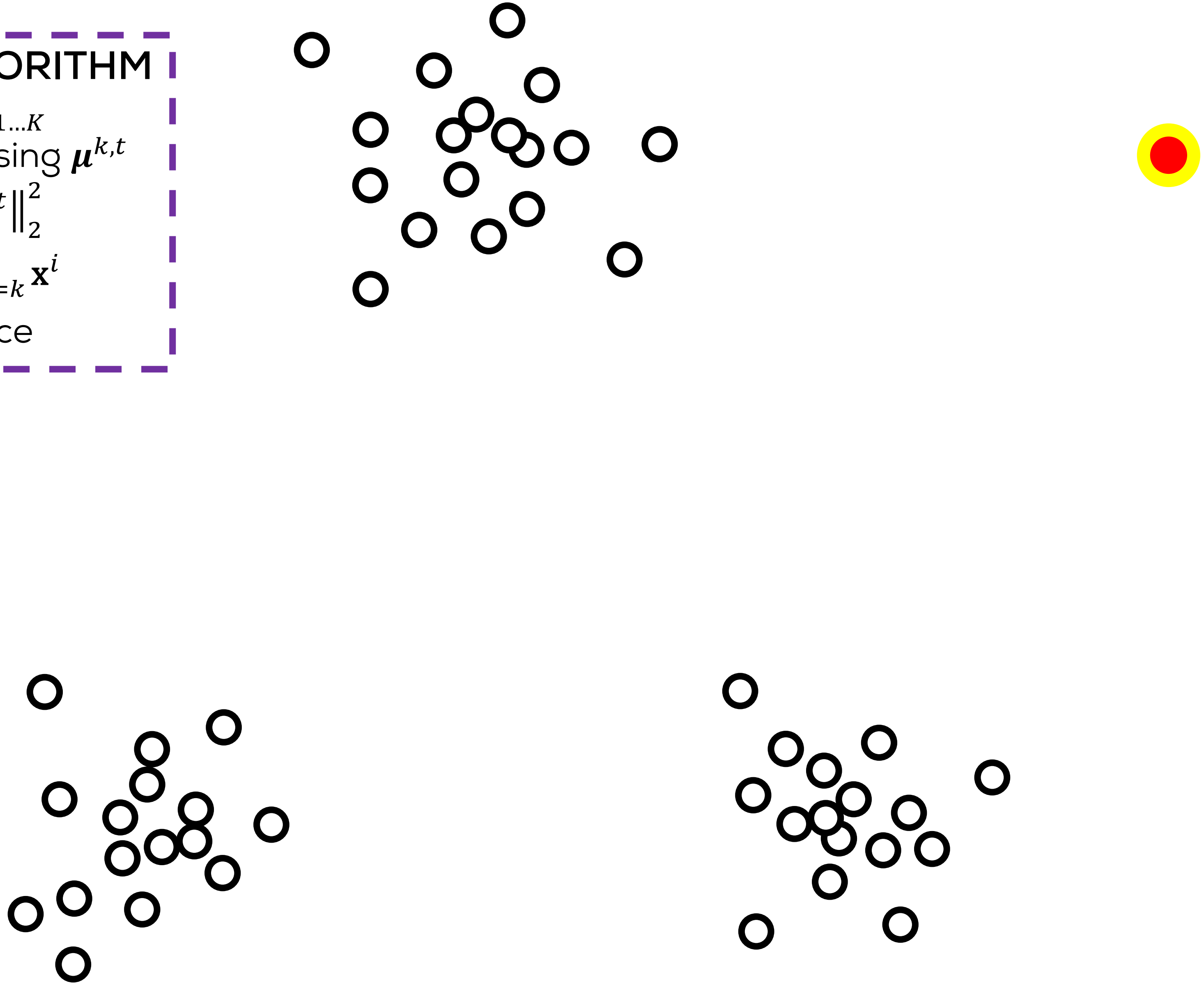

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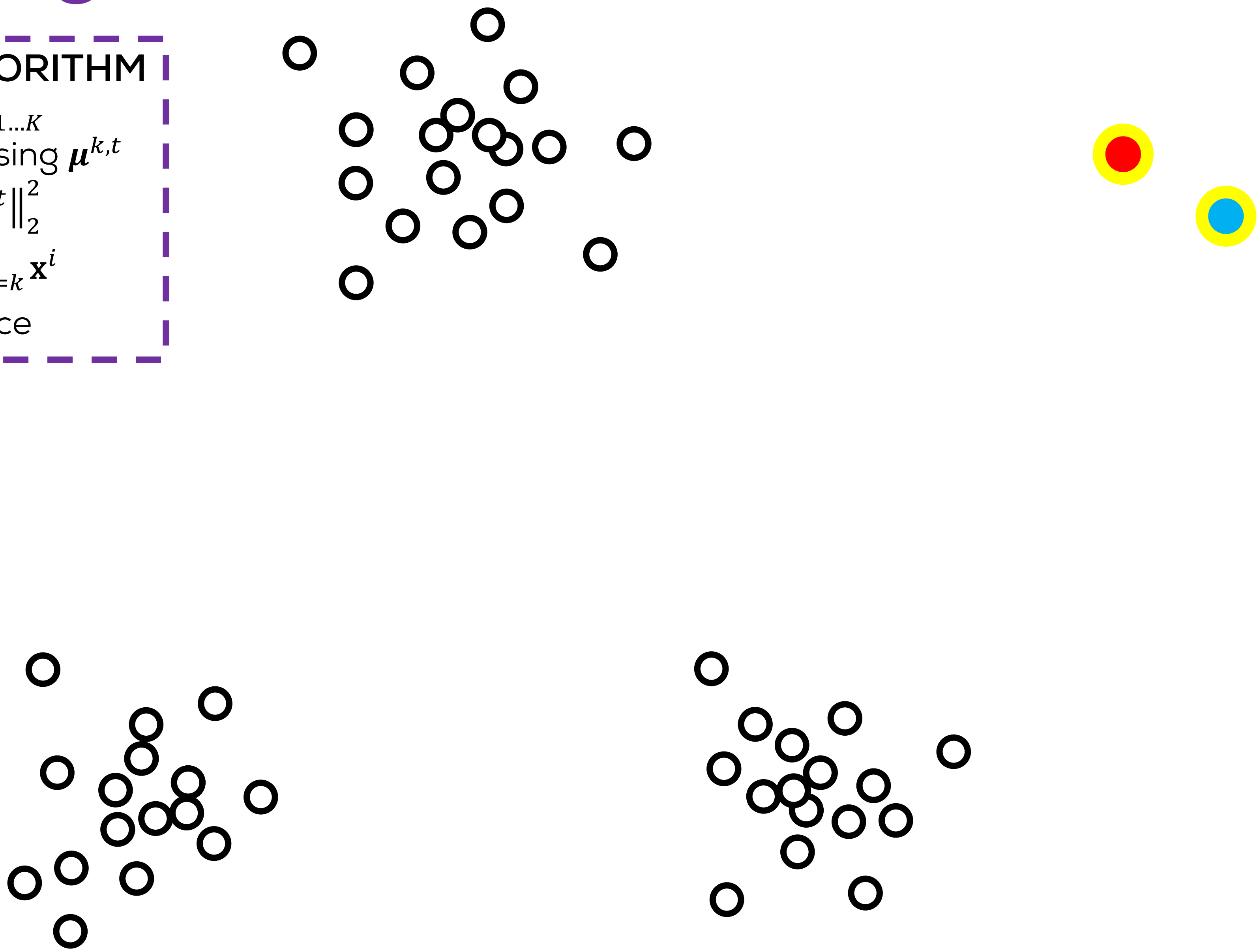

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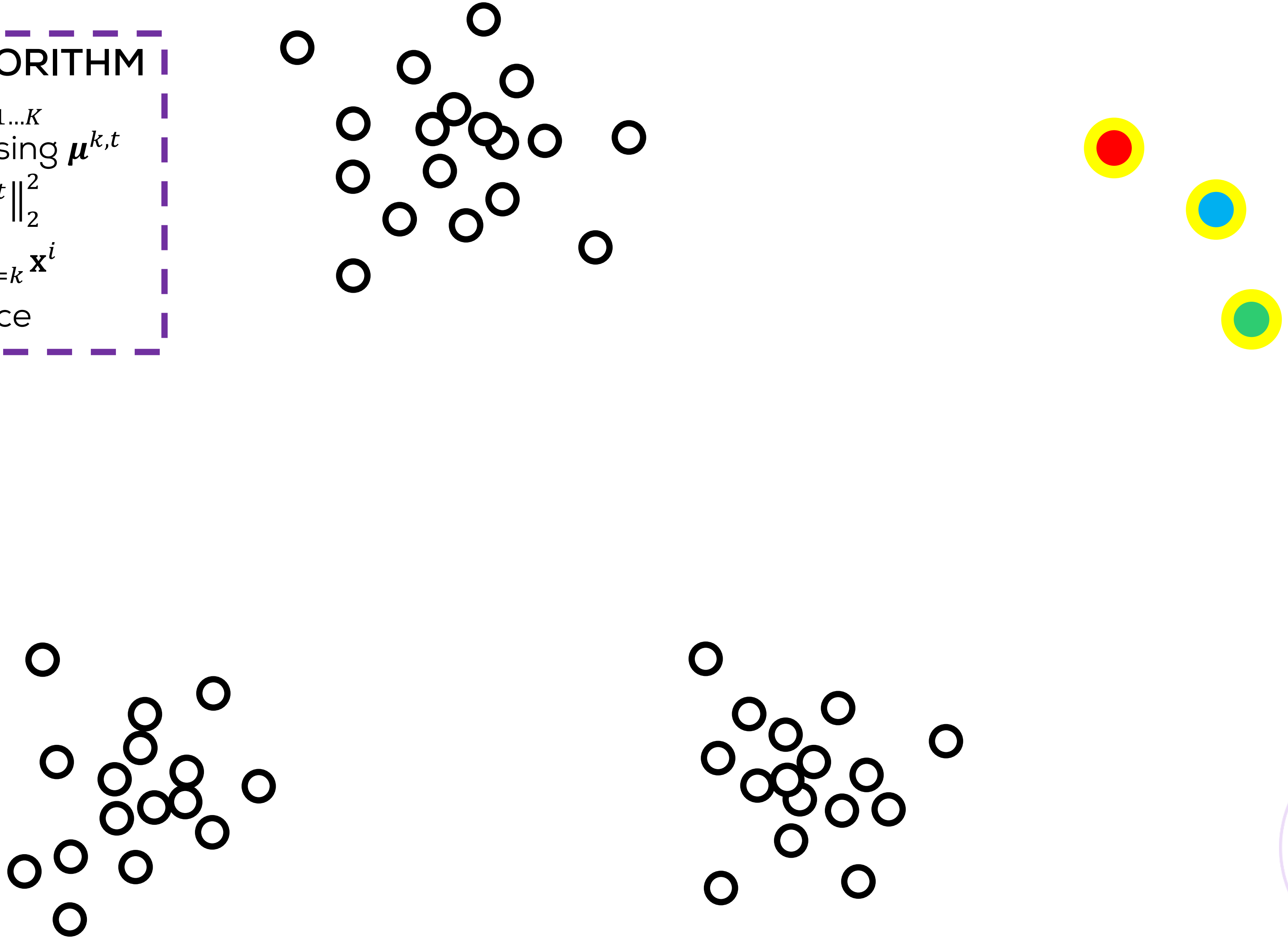

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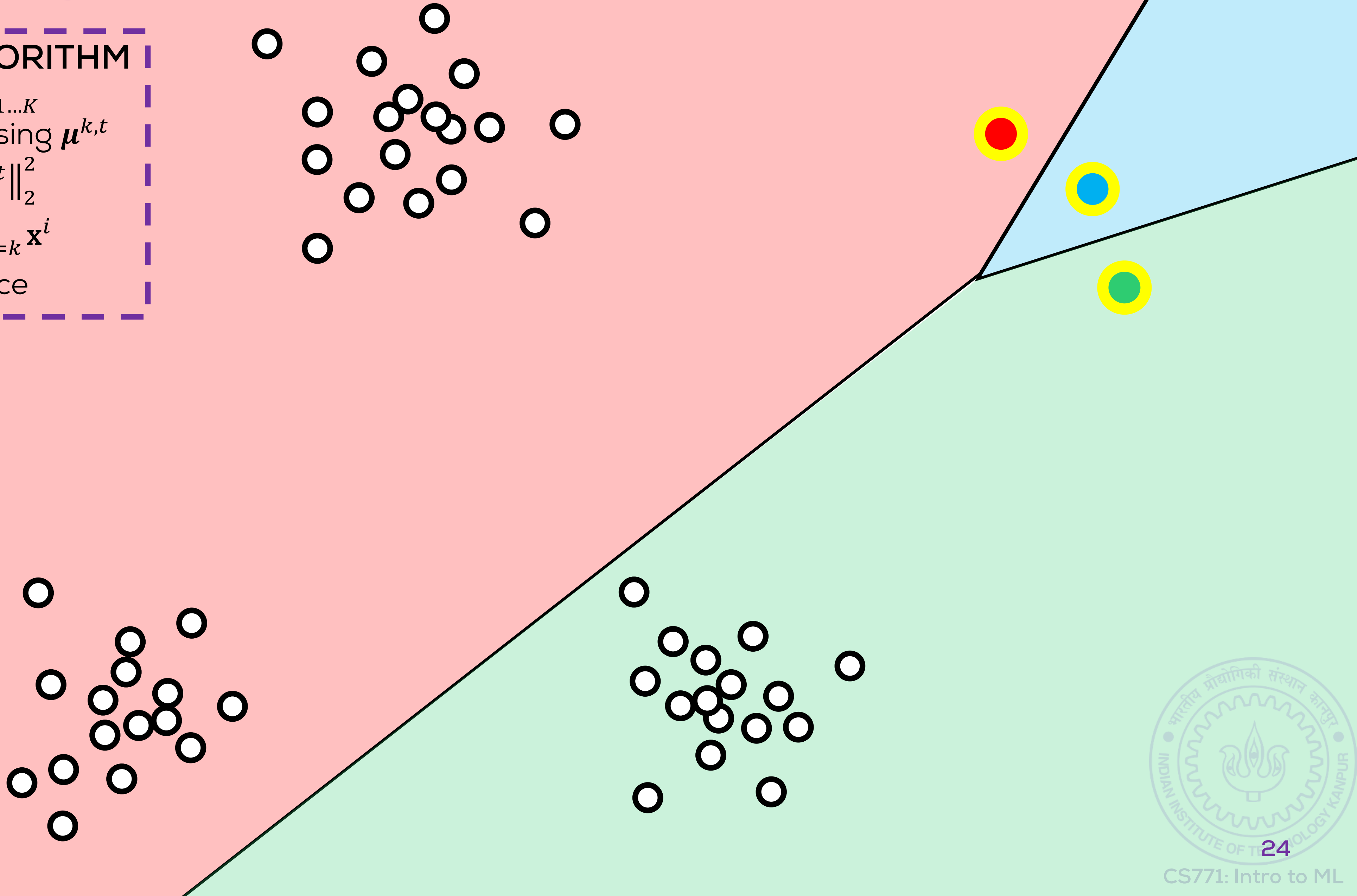

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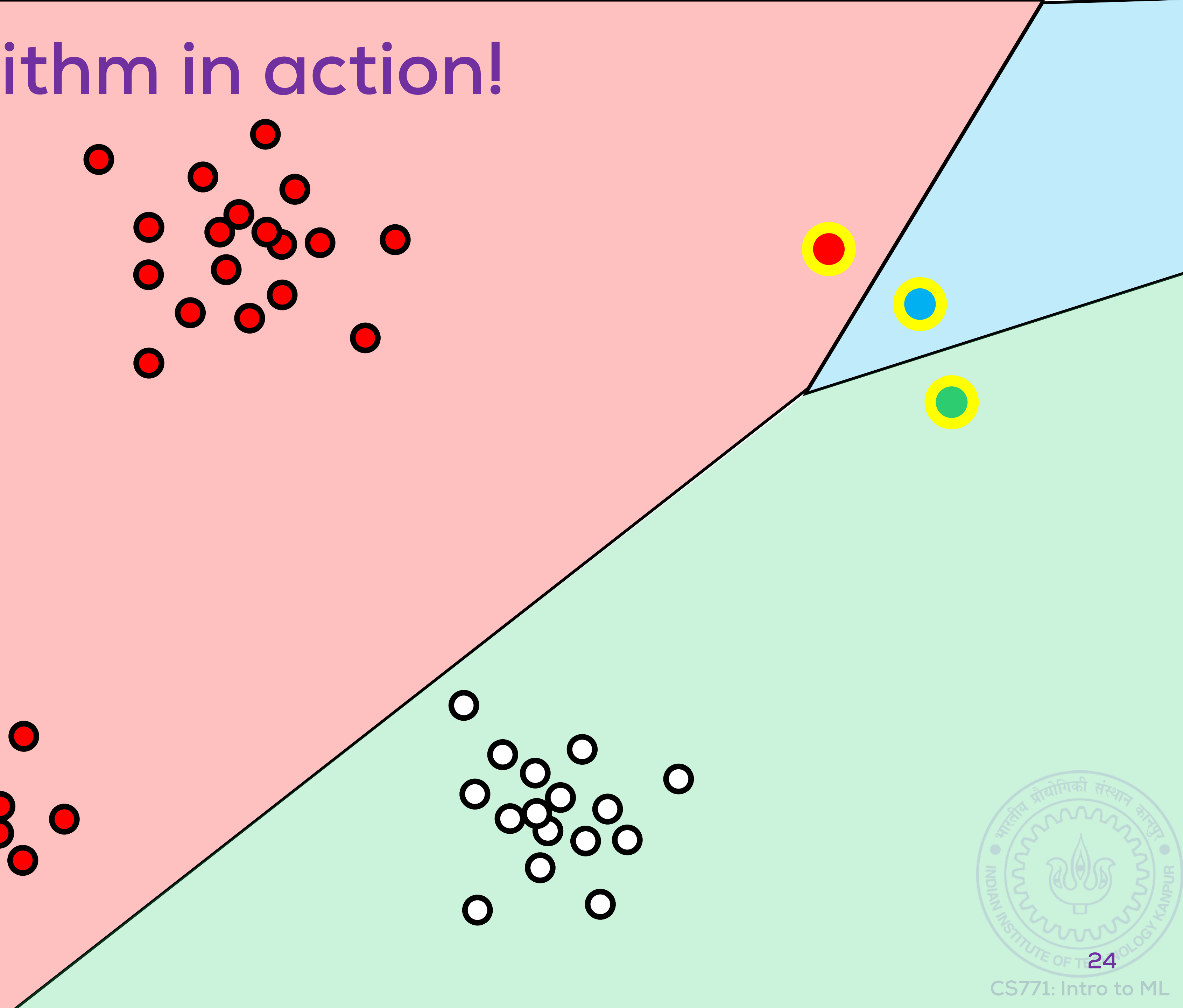

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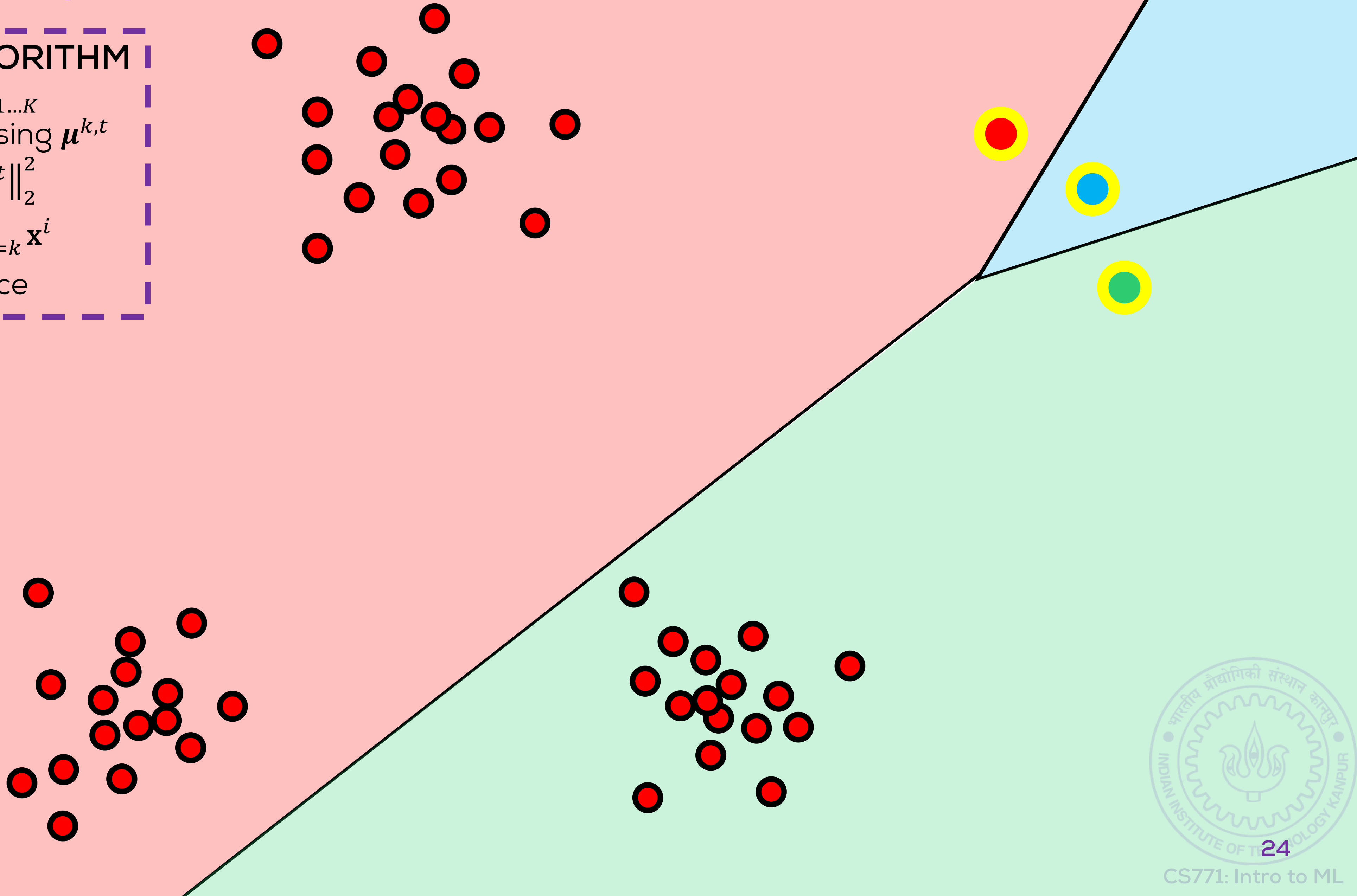

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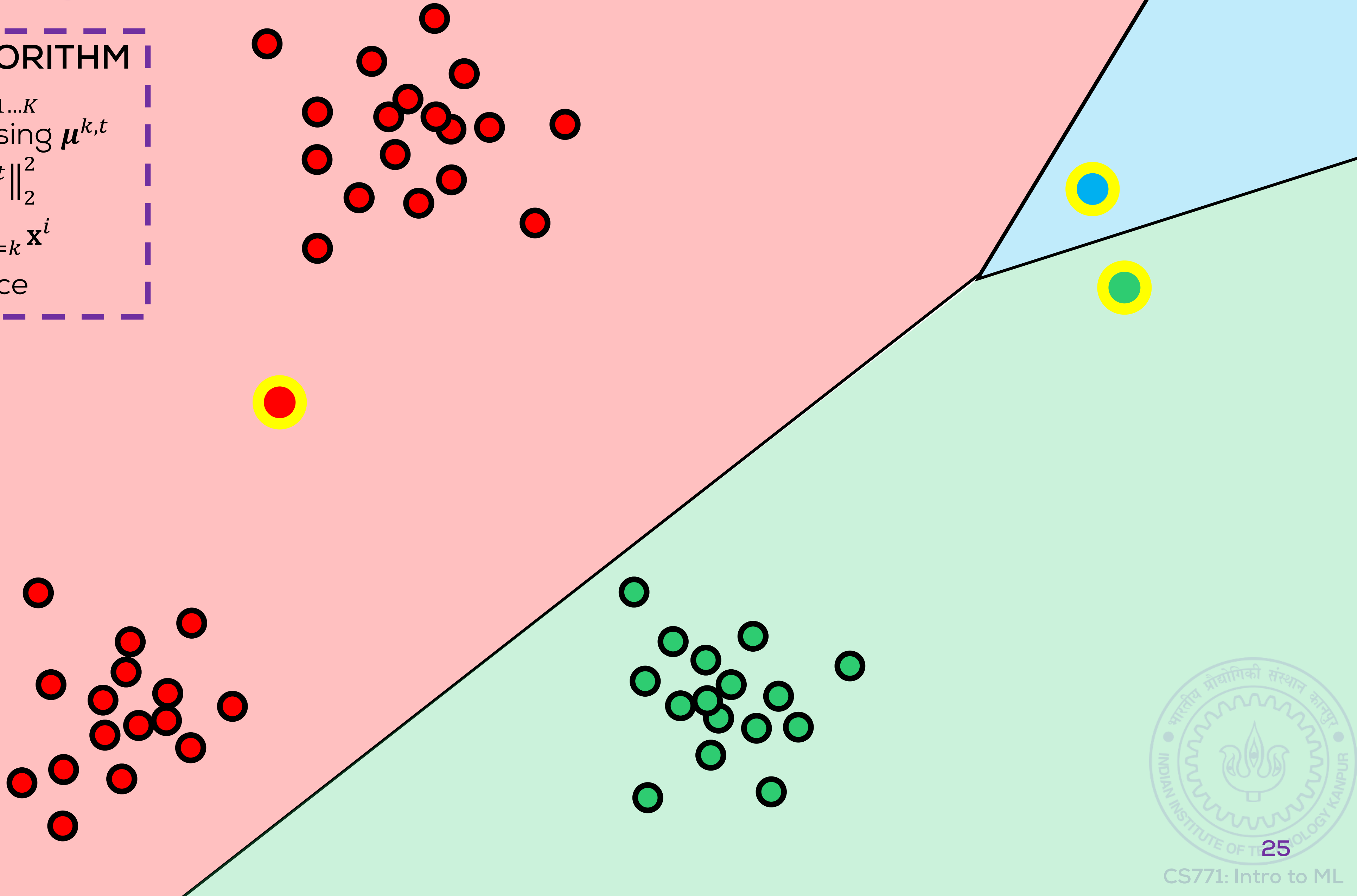

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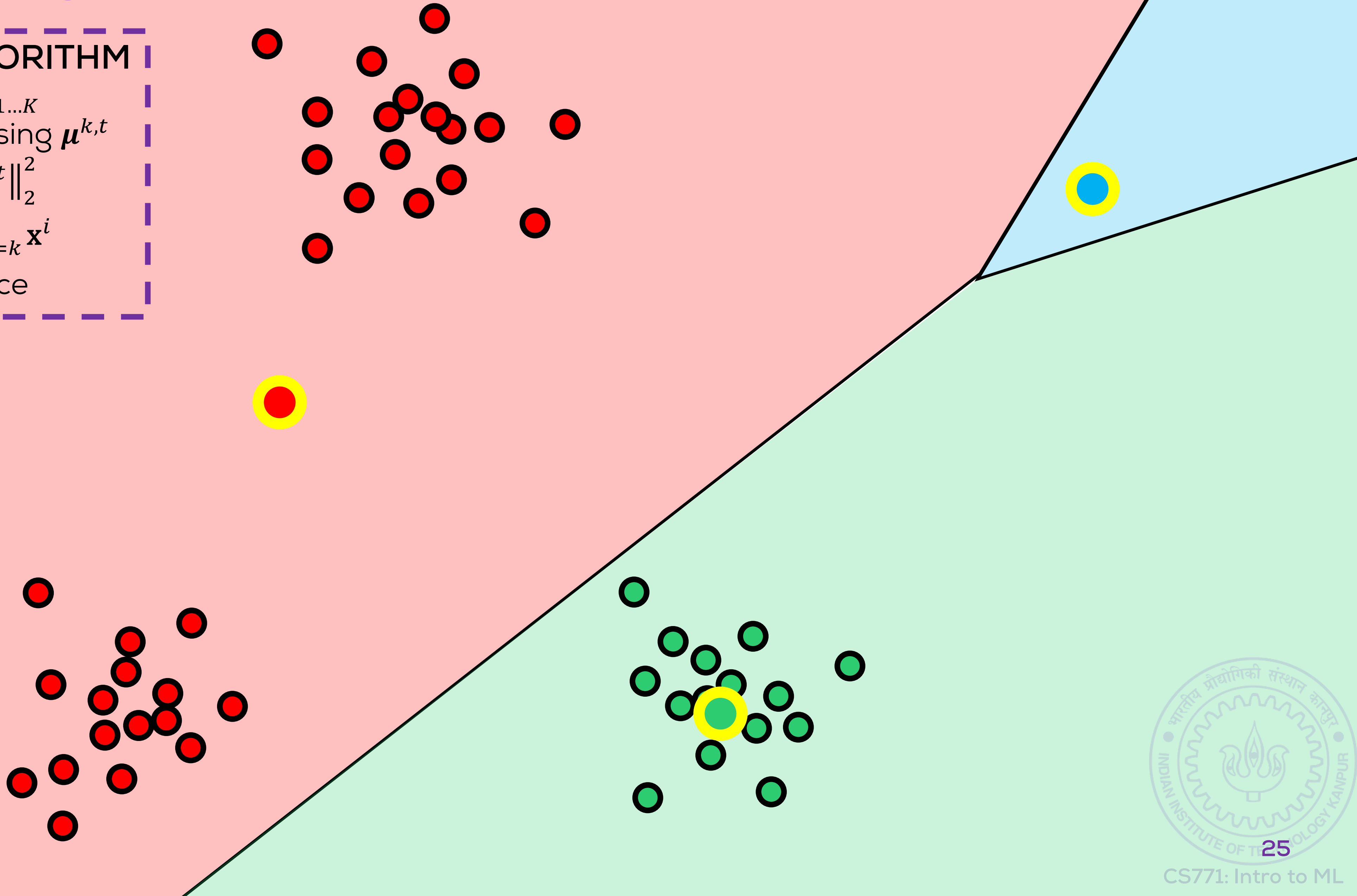

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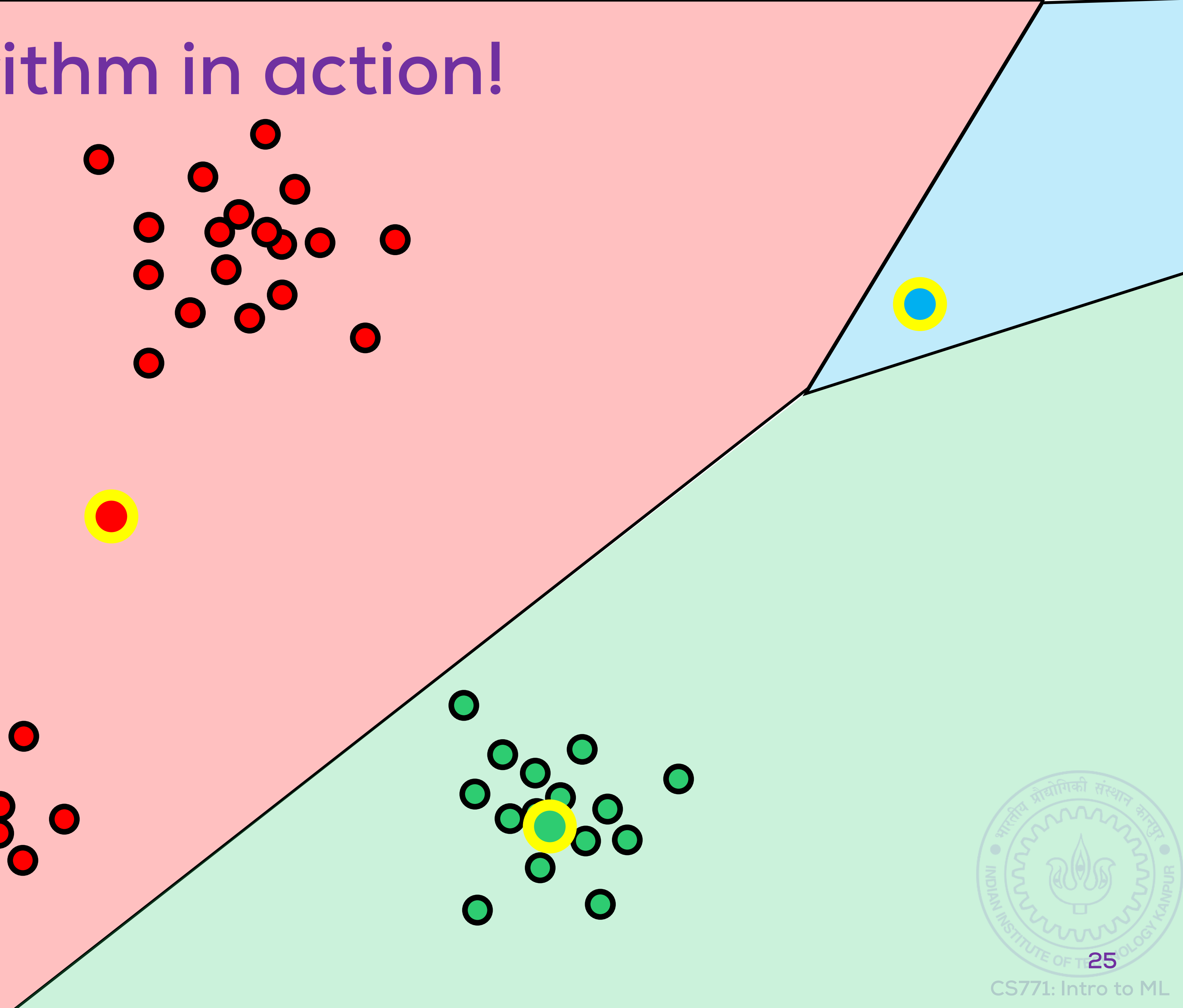

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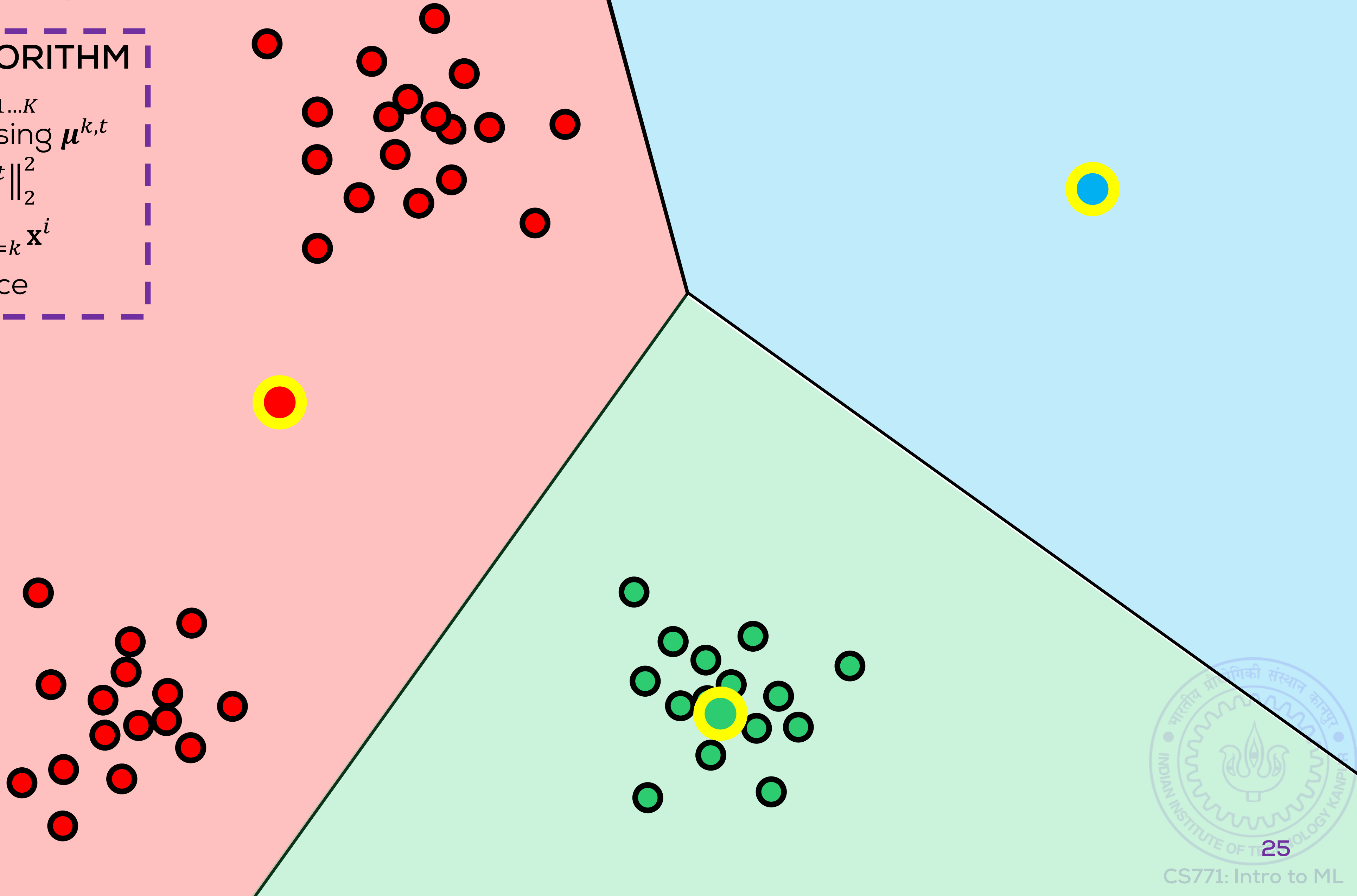

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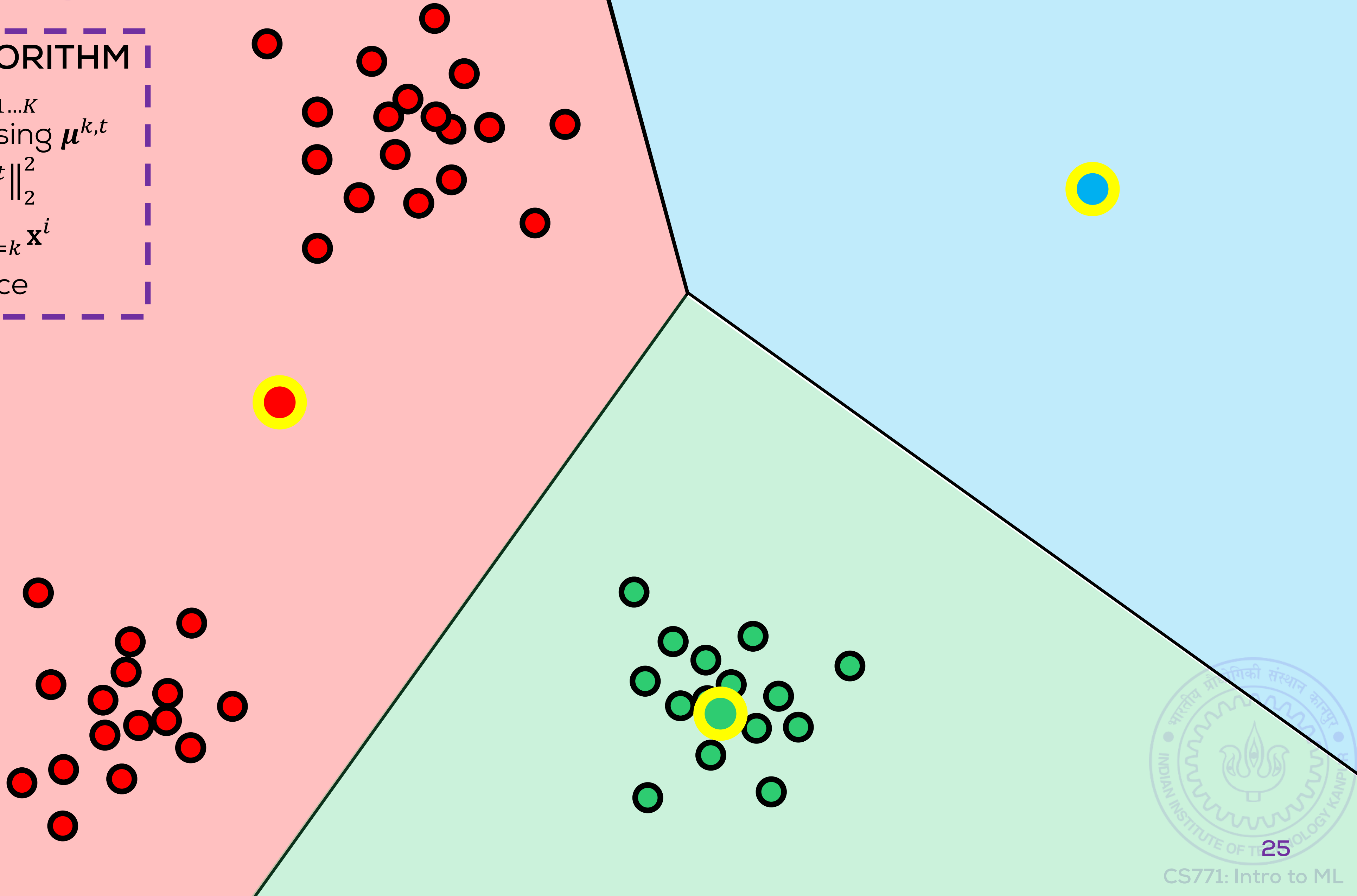

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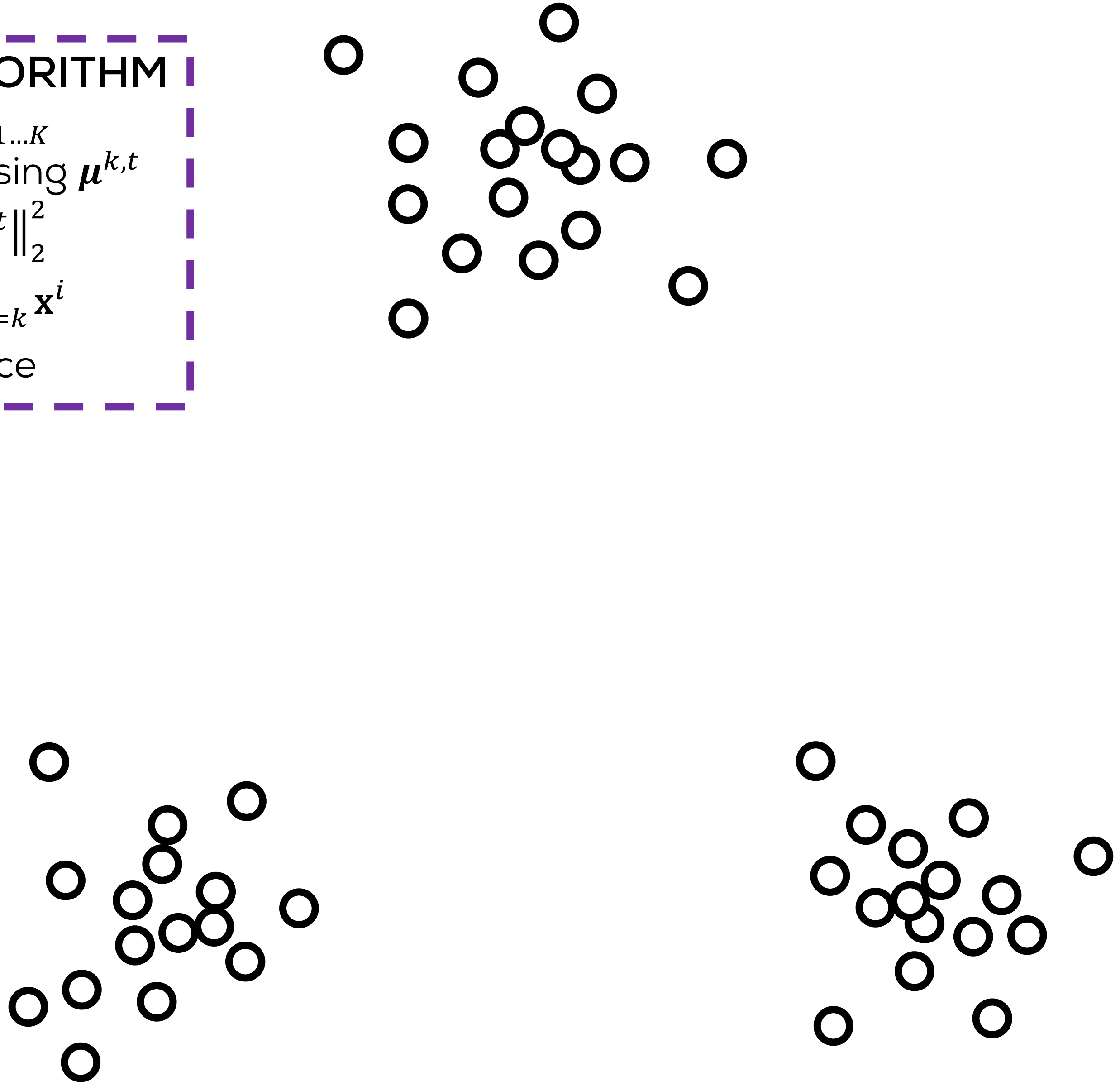

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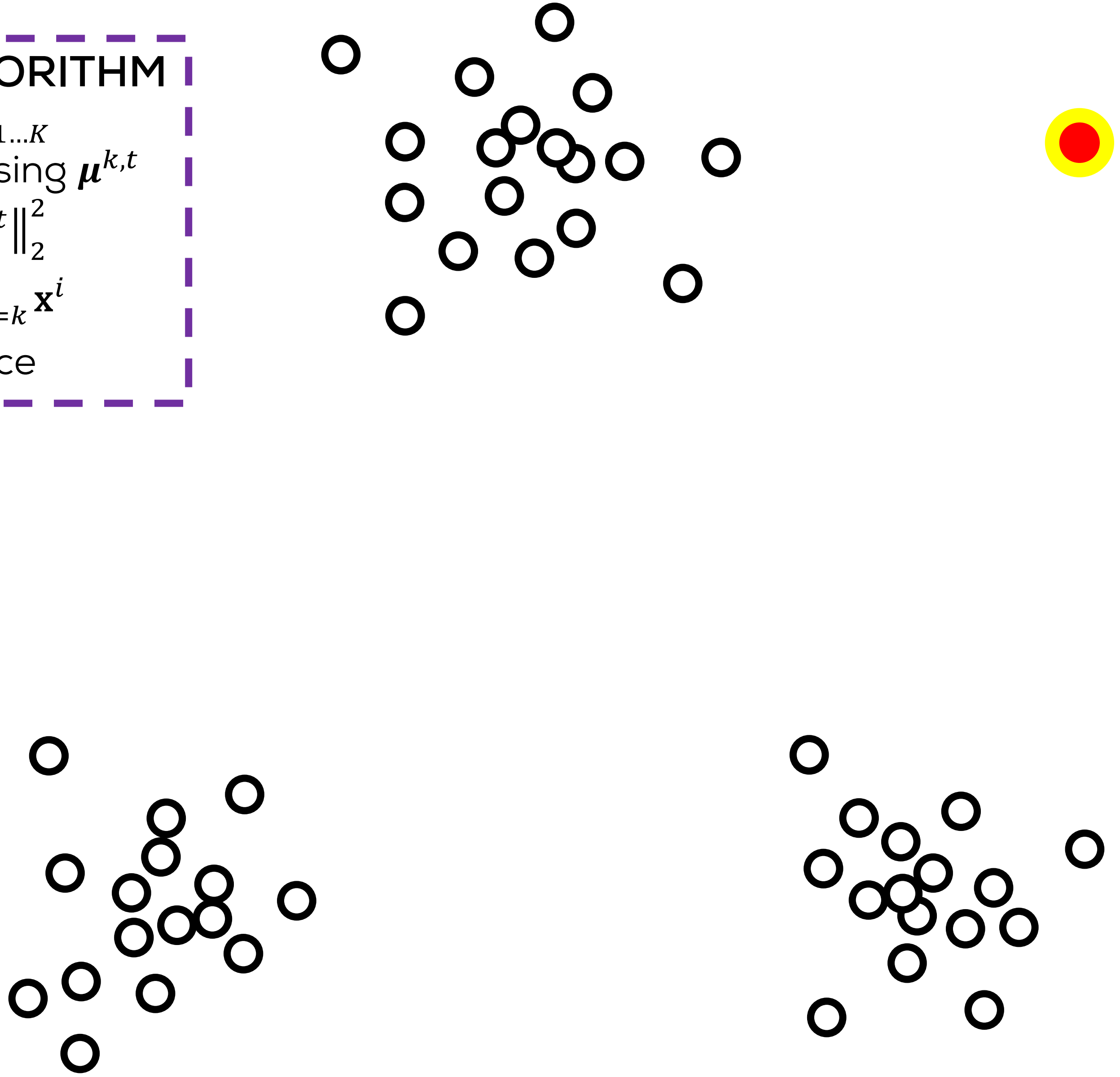

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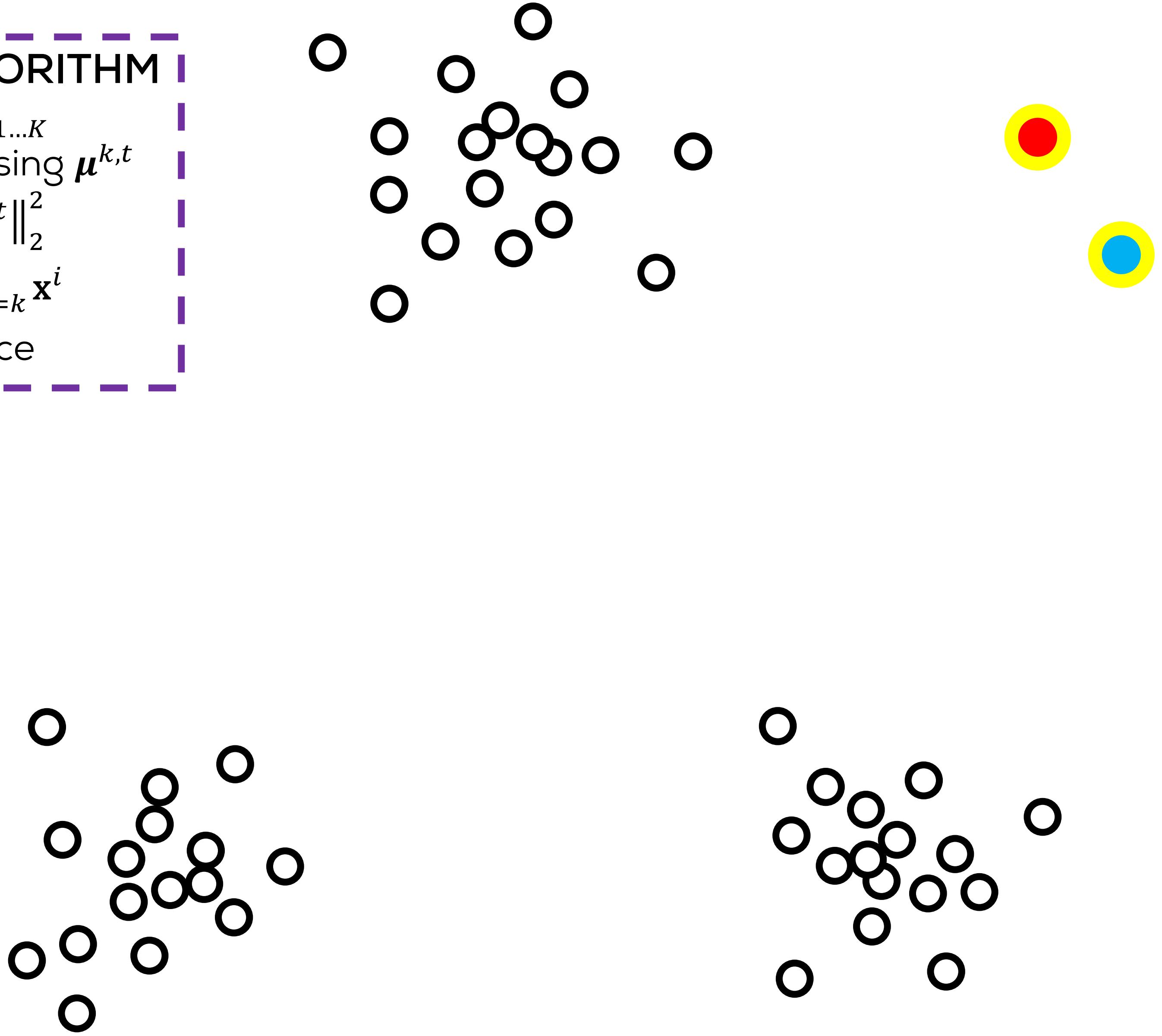

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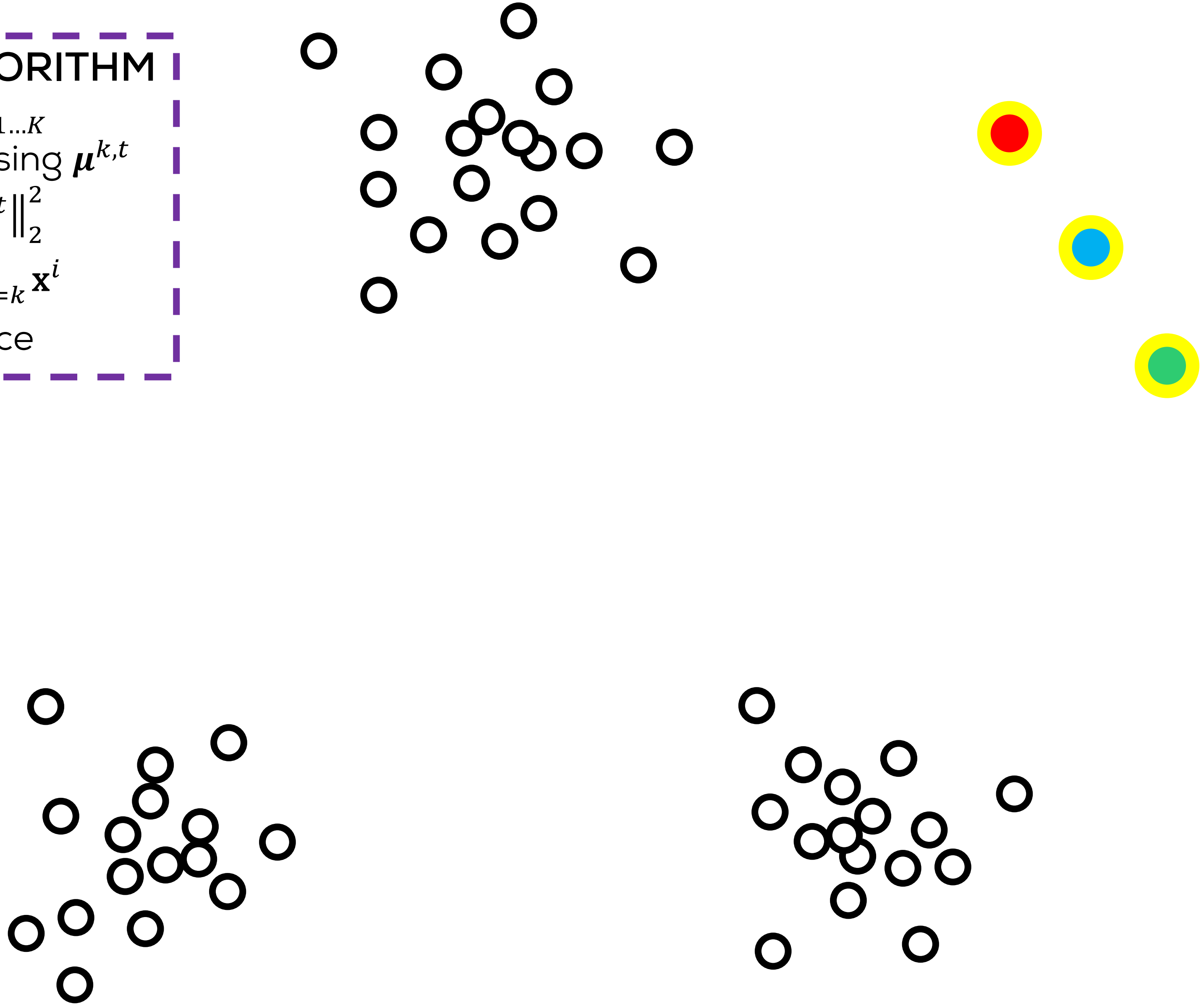

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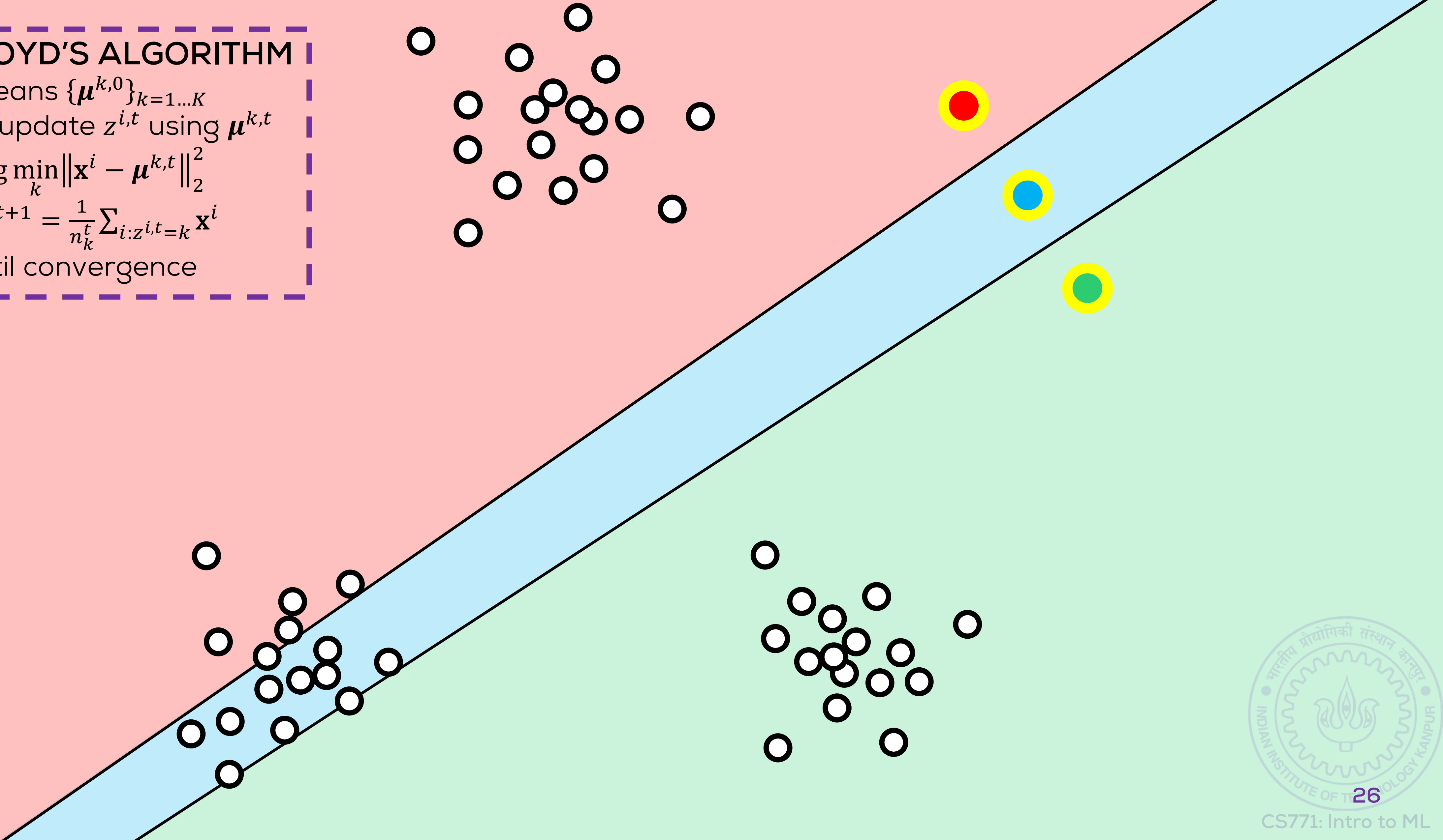

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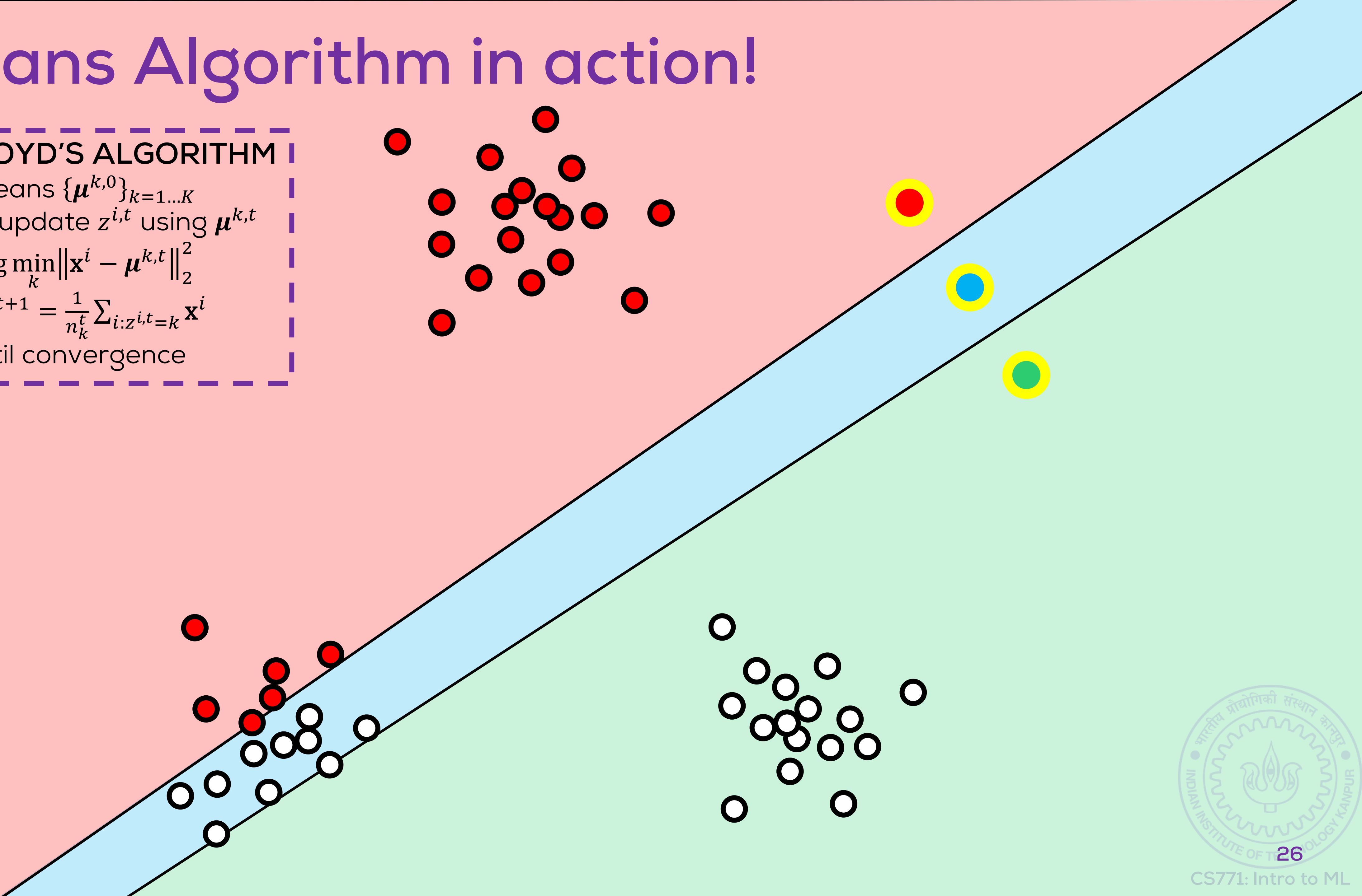

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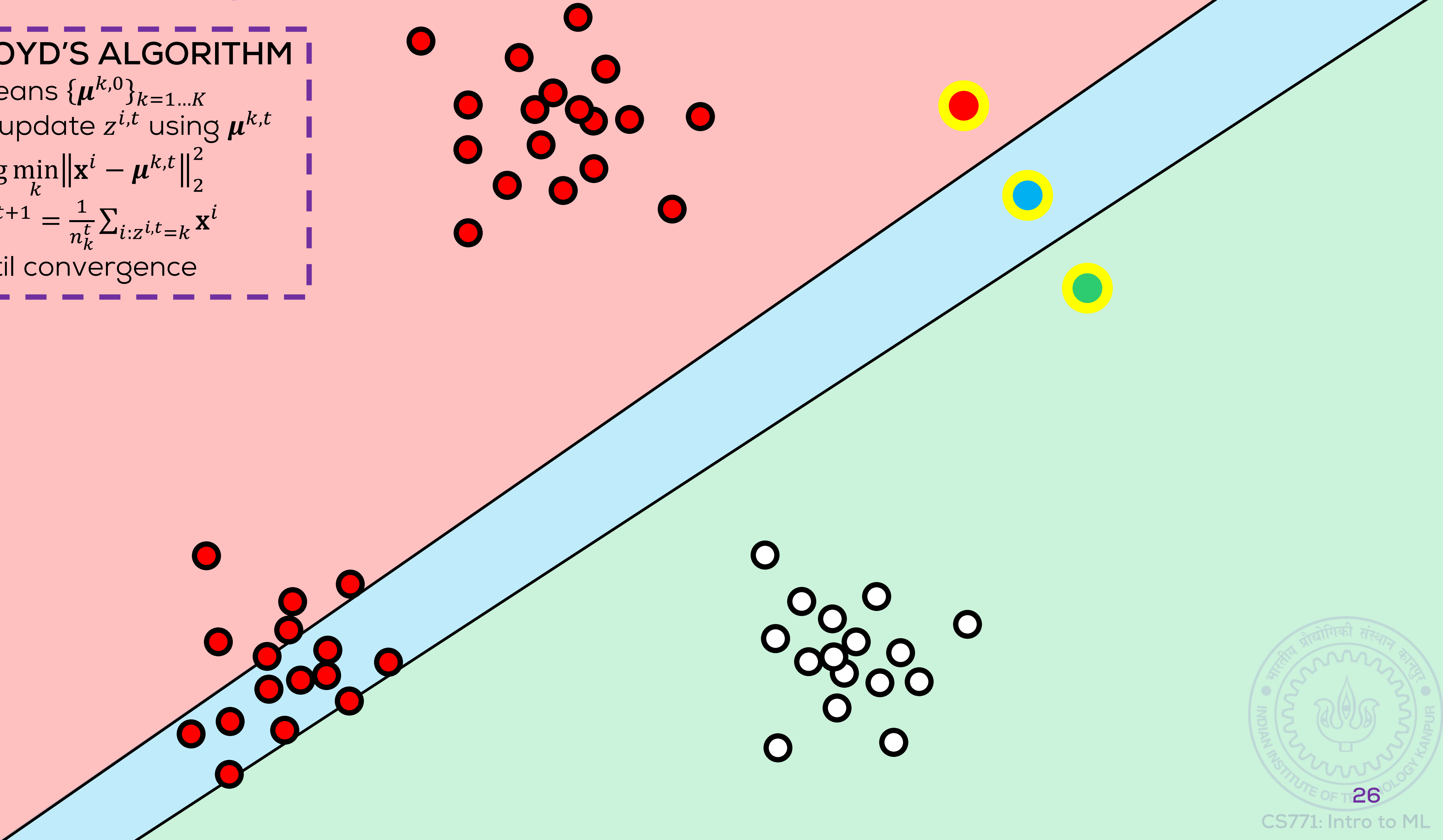

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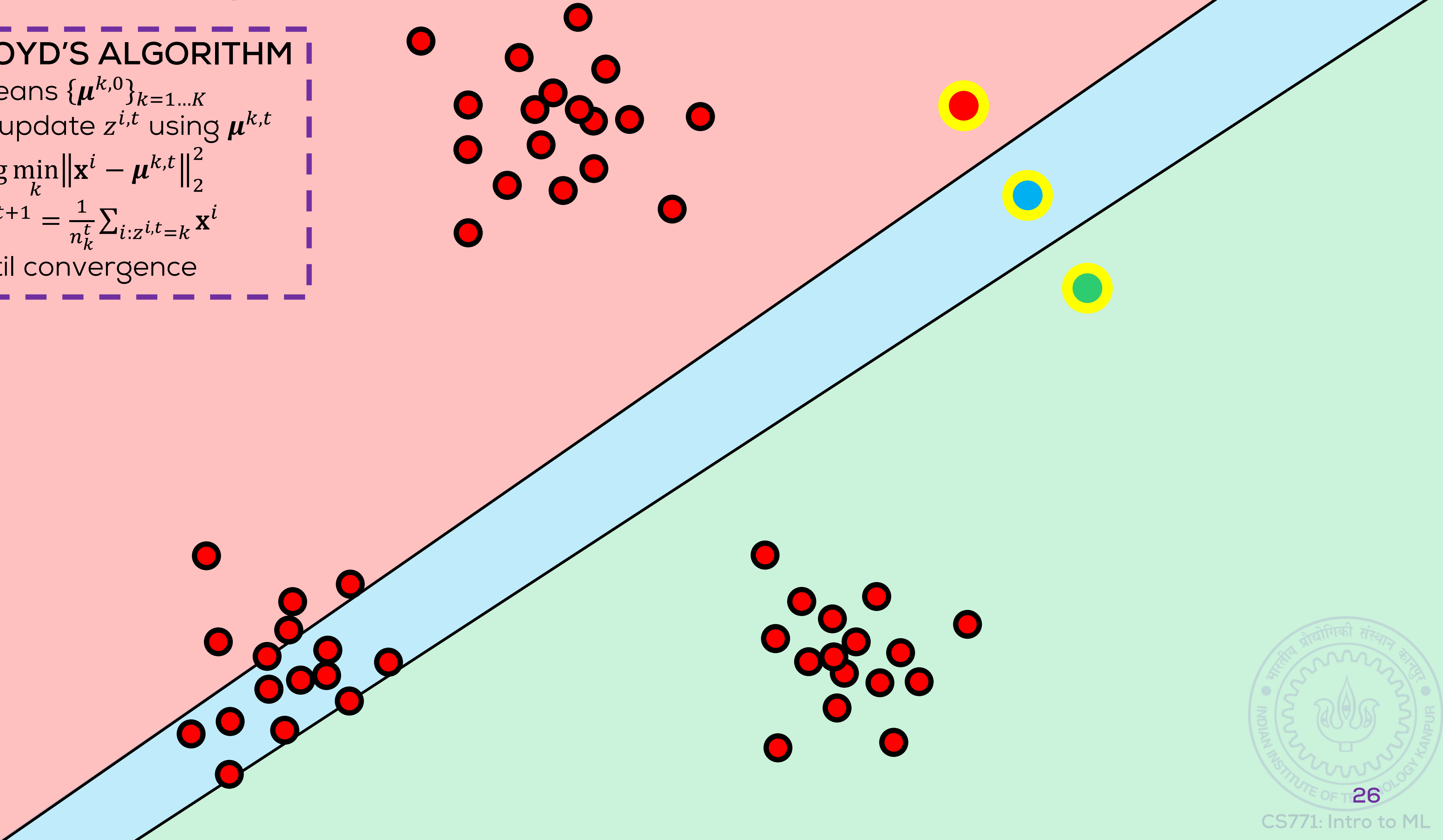

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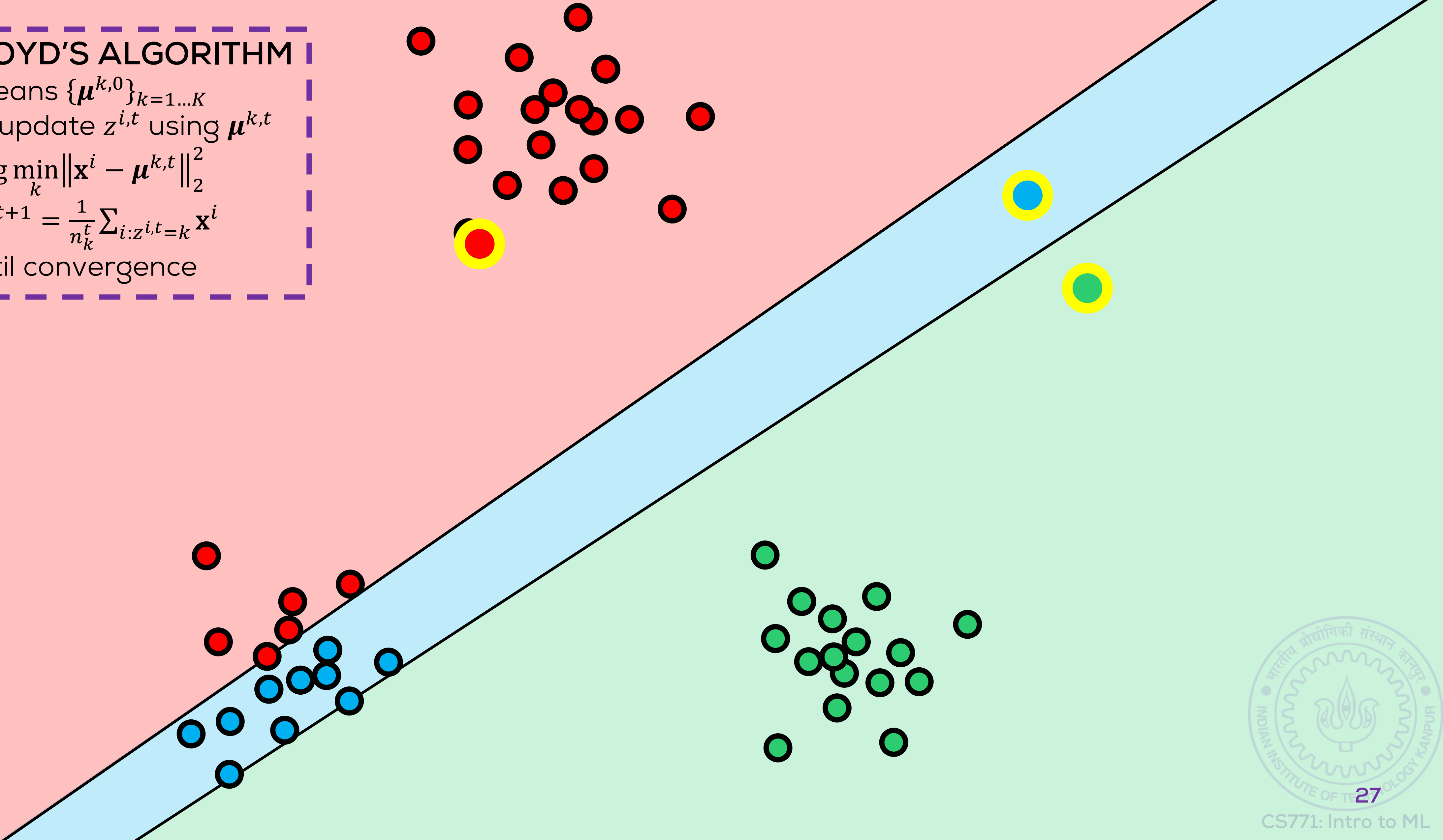

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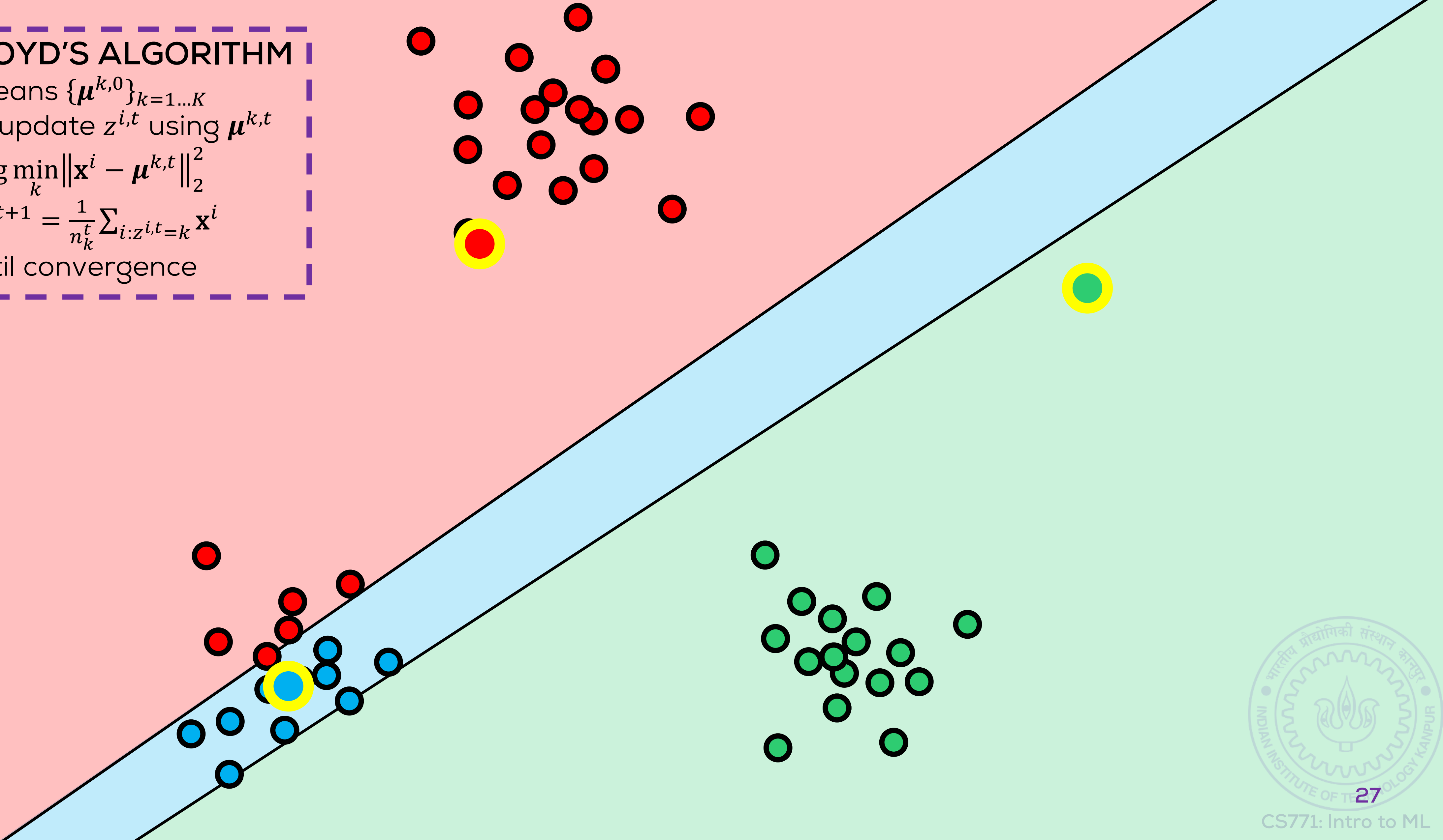

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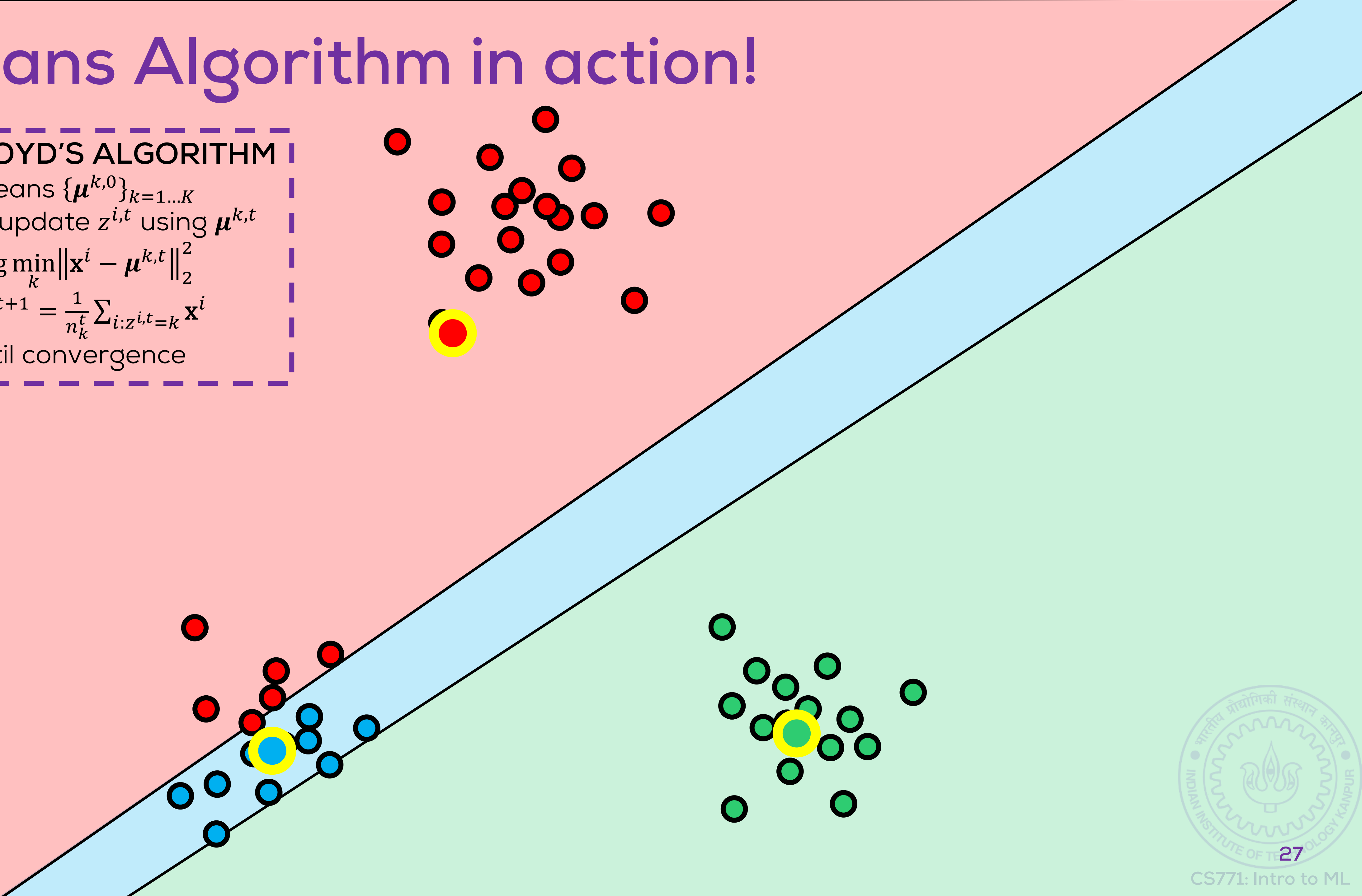

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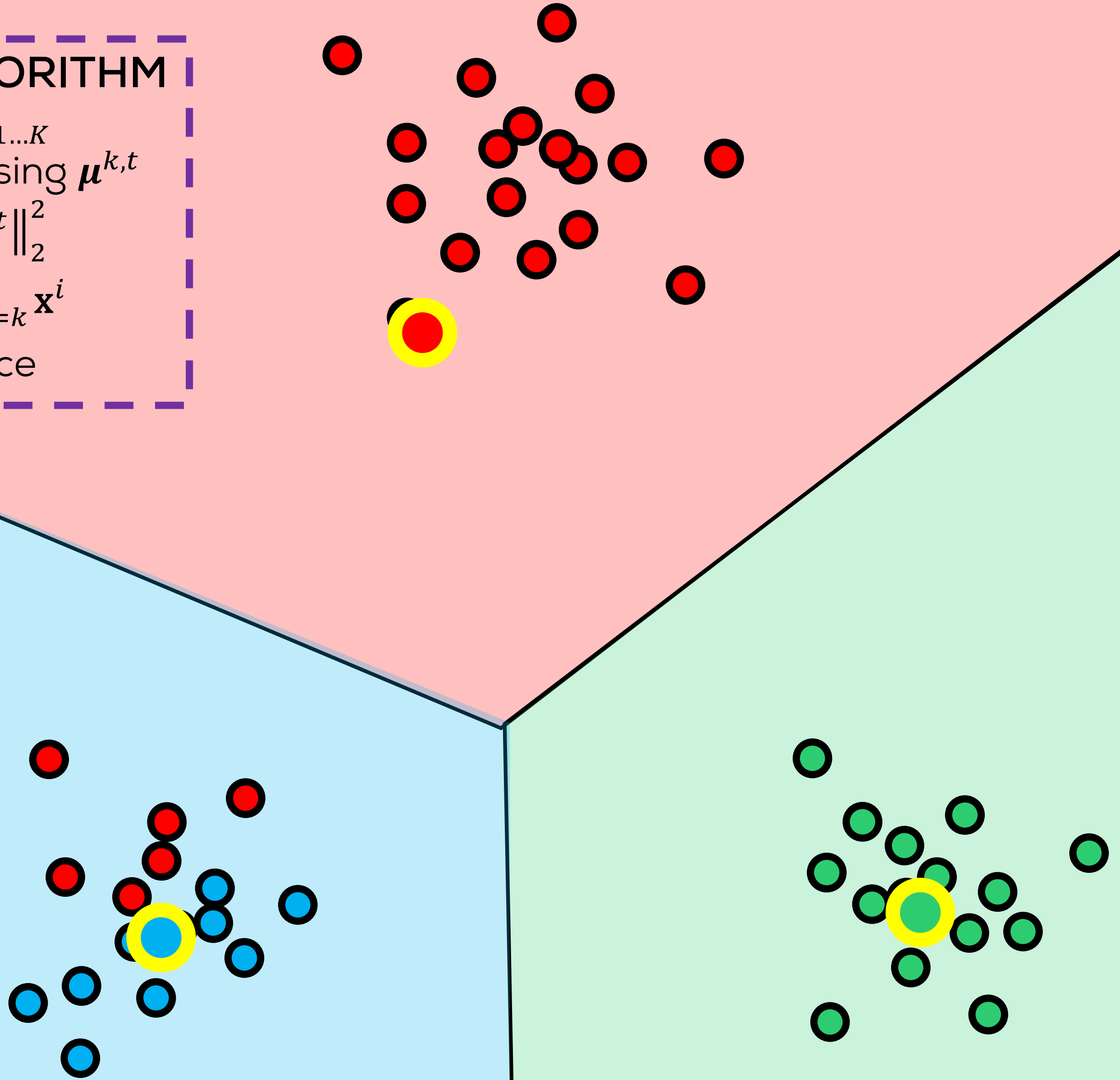

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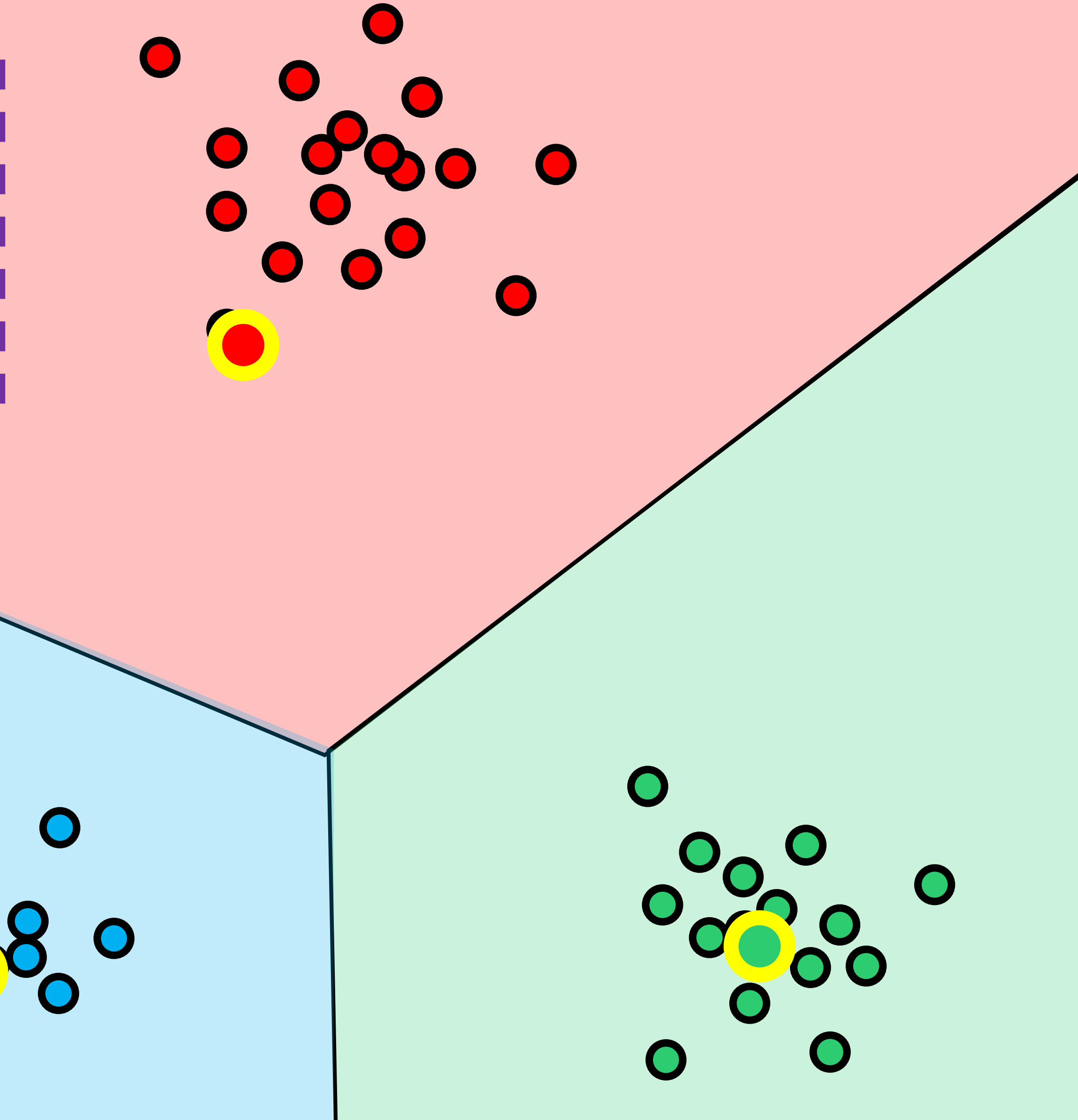

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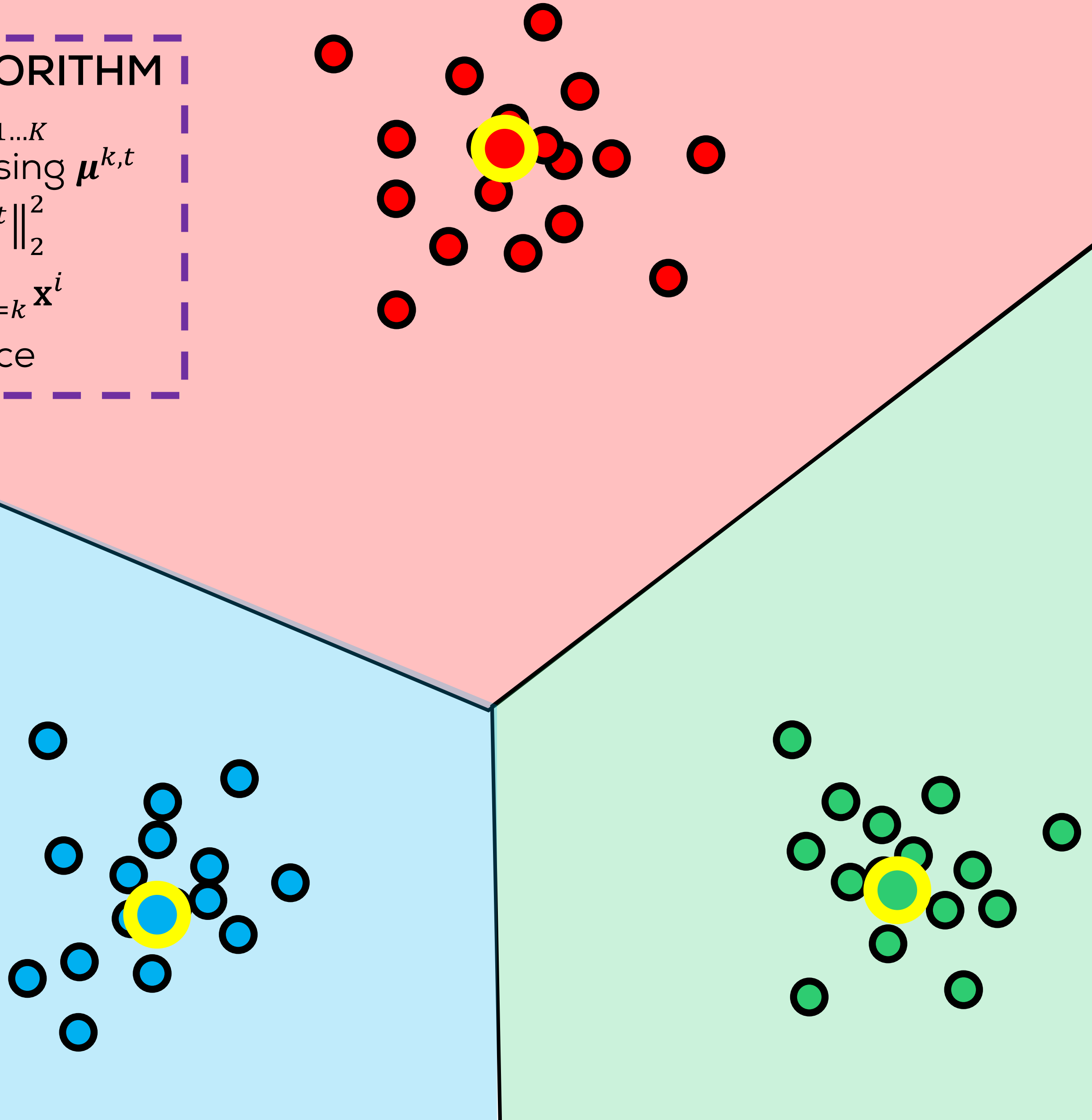

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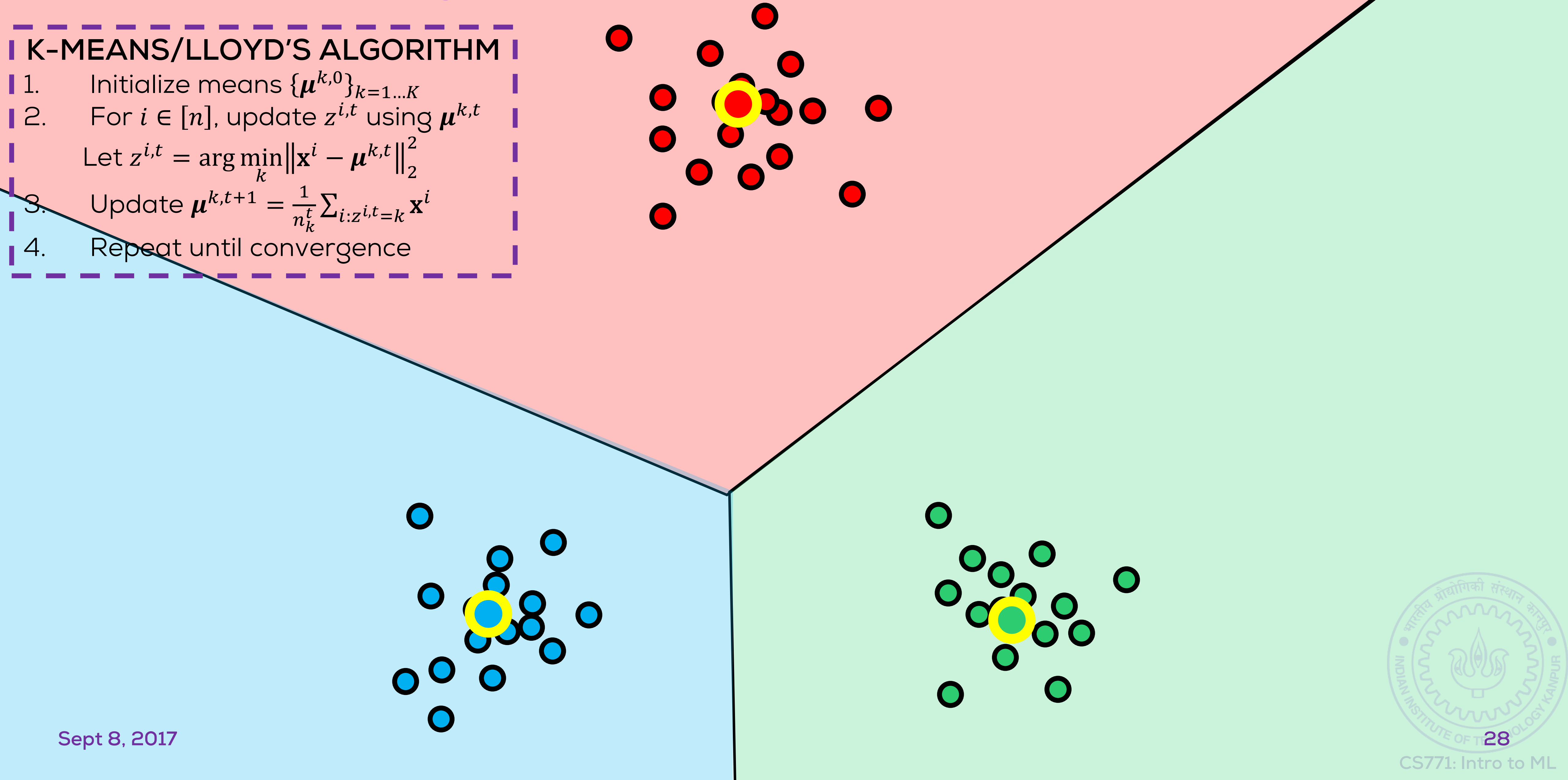

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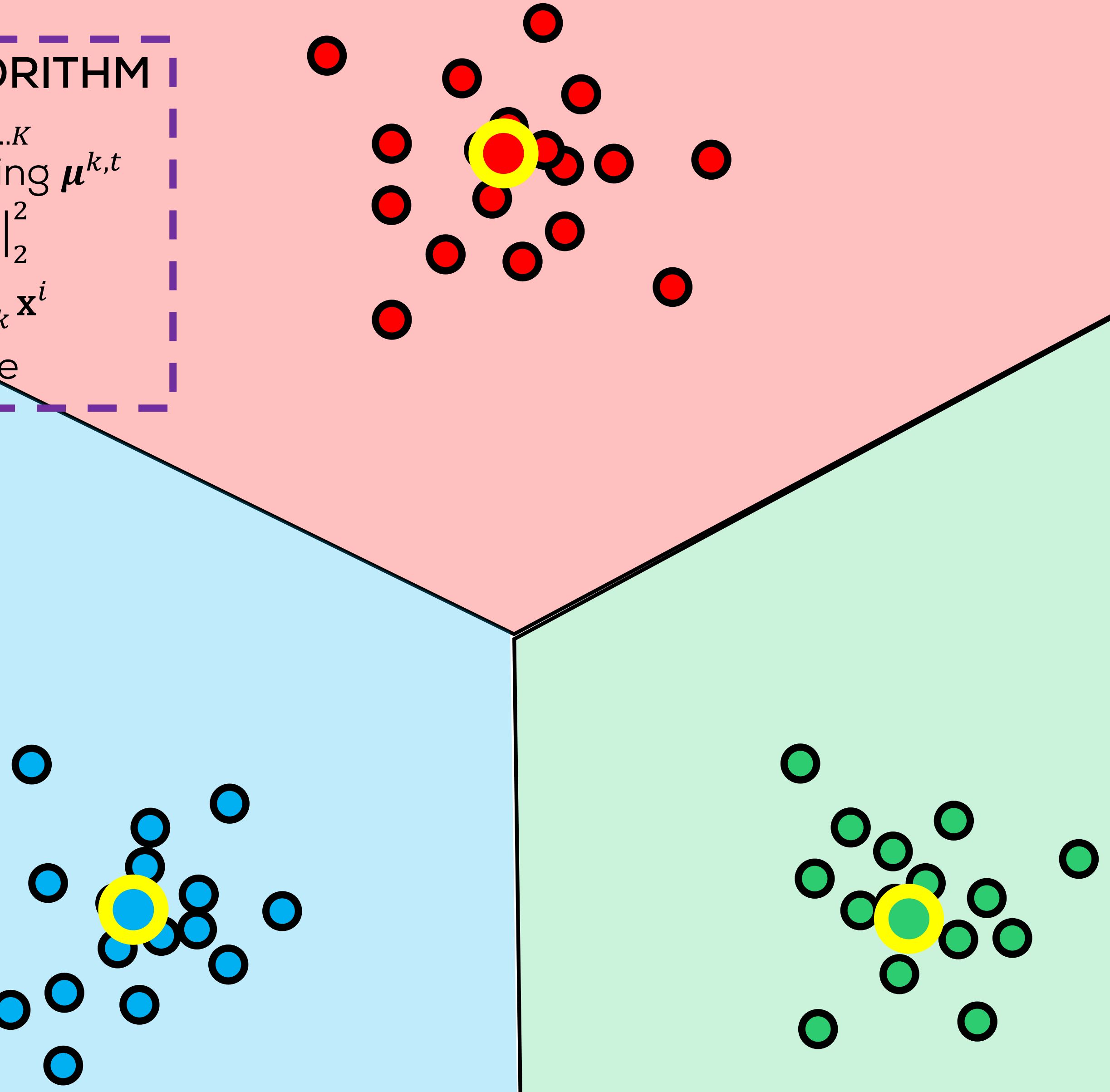

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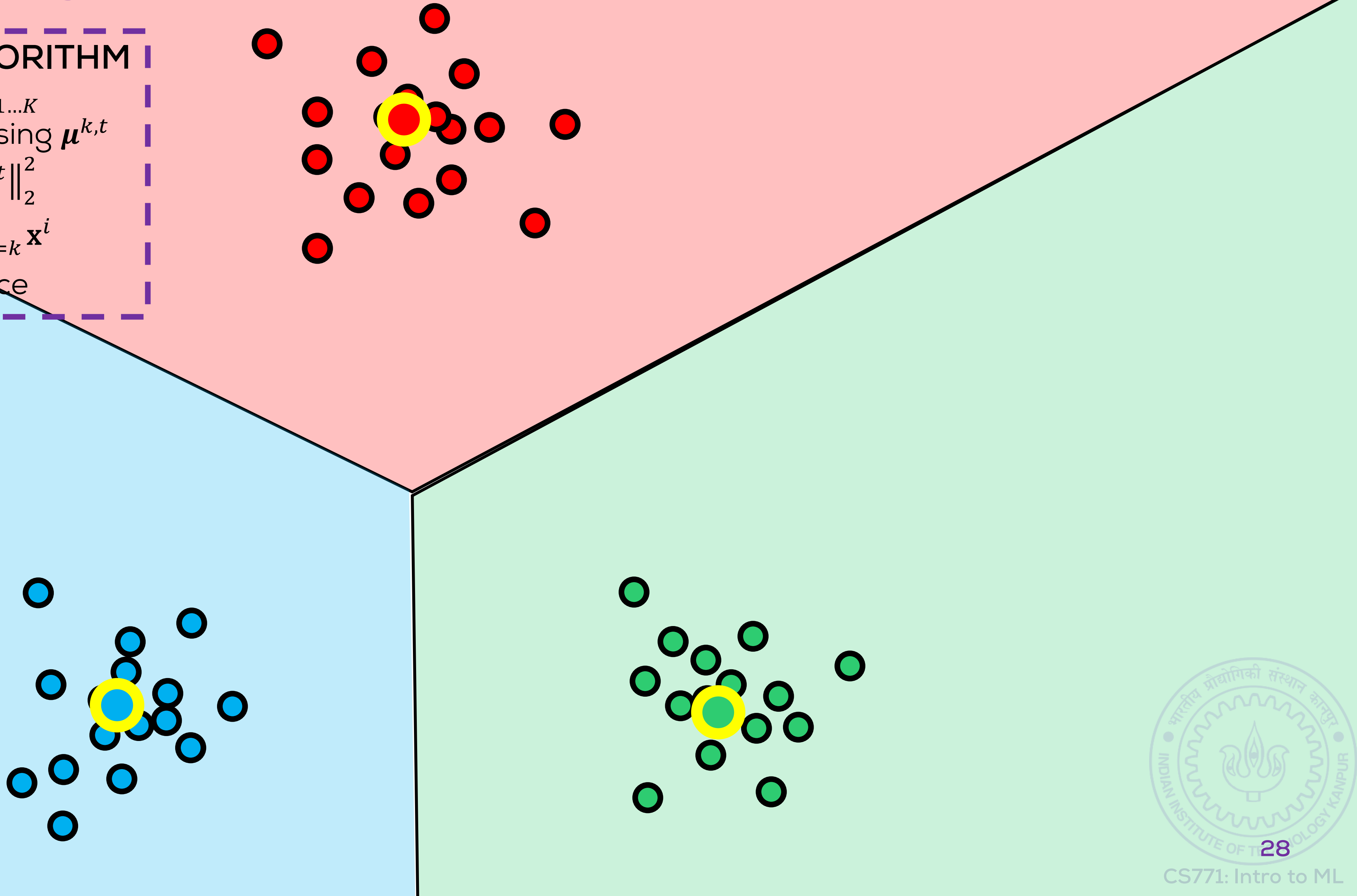

Stuck!!! … but at the global optimum ☺

Generating data from GMM learnt using Lloyd

- K clusters with means $\boldsymbol{\mu}^1, \boldsymbol{\mu}^2, \ldots, \boldsymbol{\mu}^K$ learnt using k-means
- To generate a data point from this GMM
  1. Select a cluster $k \sim [K]$ uniformly at random
  2. Select a point from the Gaussian $\mathcal{N}(\boldsymbol{\mu}^k, I)$

K-means uses $\boldsymbol{\pi}_k = \frac{1}{K}$

K-means use $\boldsymbol{\Sigma}^k = I$

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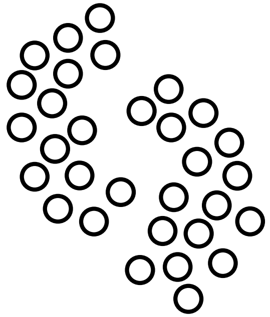

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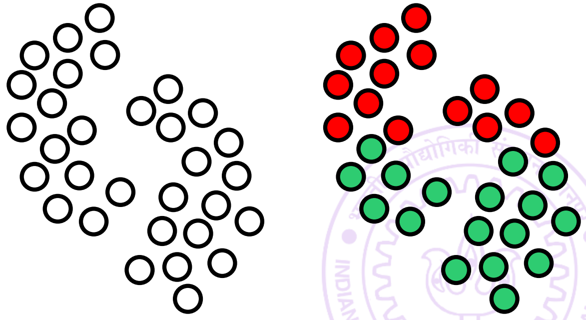

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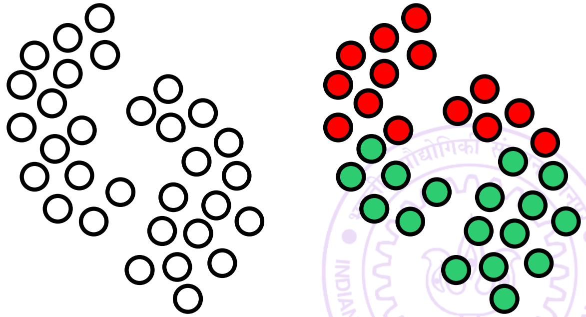

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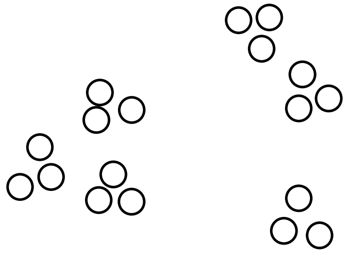

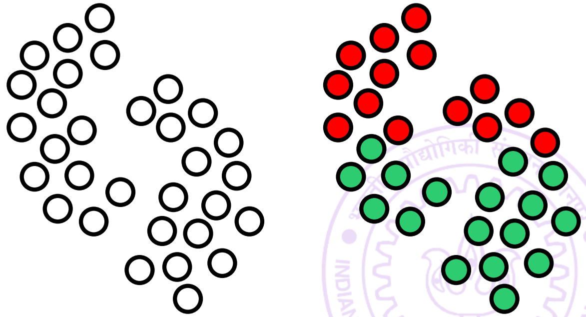

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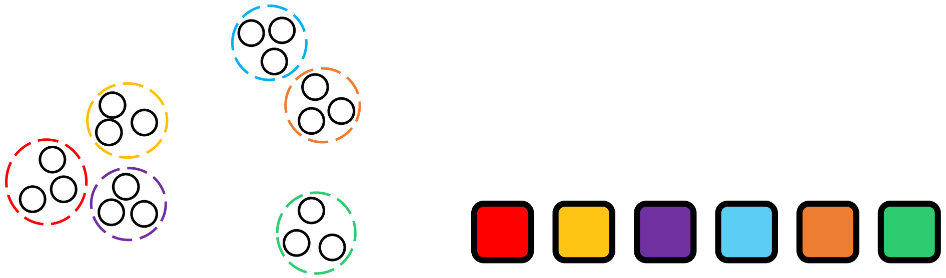

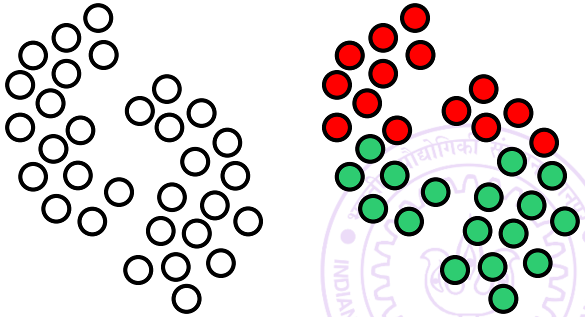

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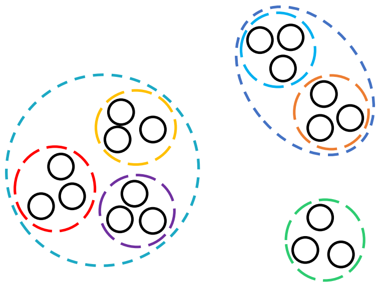
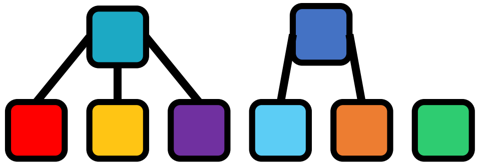
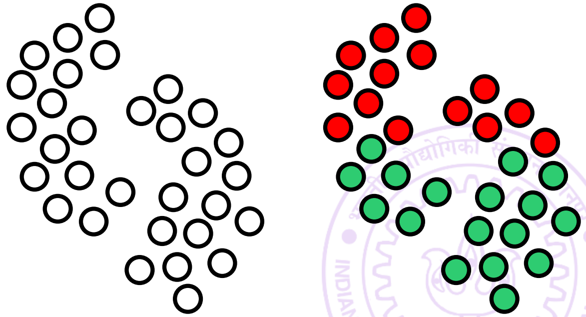
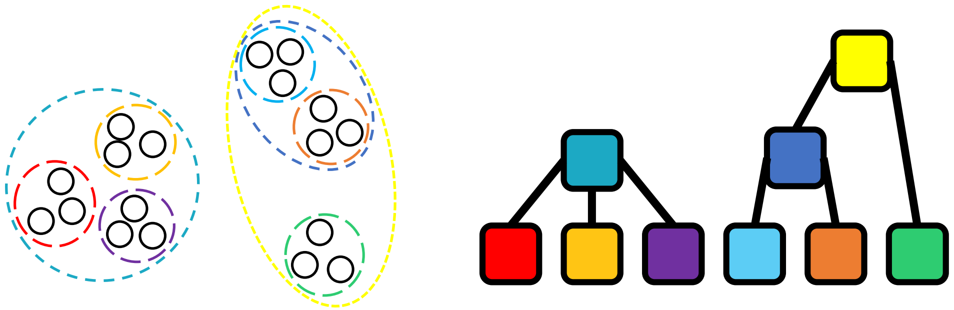


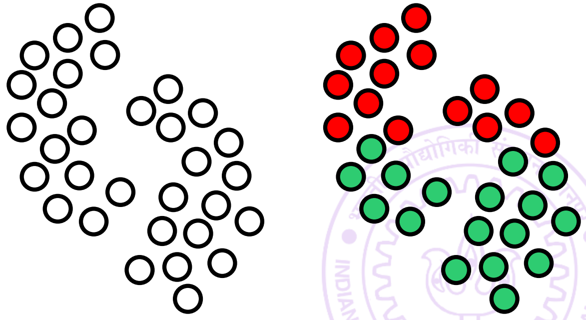




The K-means clustering algorithm

- Extremely popular
- Helps make sense of data
- Can speed up k-NN computation quite a bit (assignment ☺)
- Submission deadline: September 10, 2017, 2359 hrs, IST
- Works well if all clusters are balanced (comparable)
- Brittle in high dimensions and if many many small clusters
- Learns clusters that are ball shaped, does not do well on non-convex shaped clusters
- Hierarchical clustering





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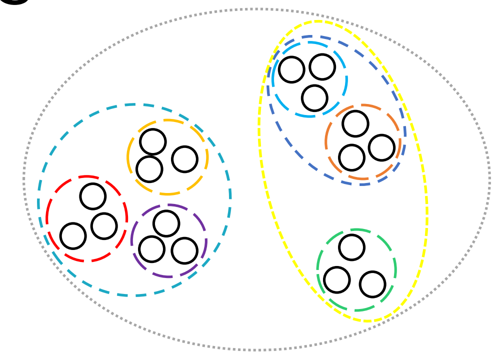

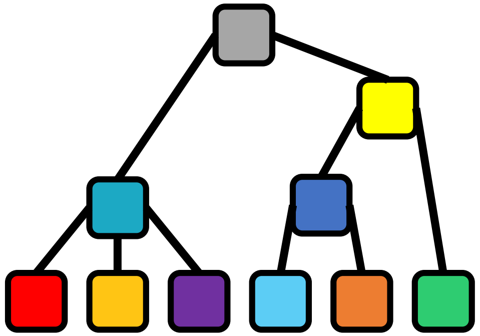

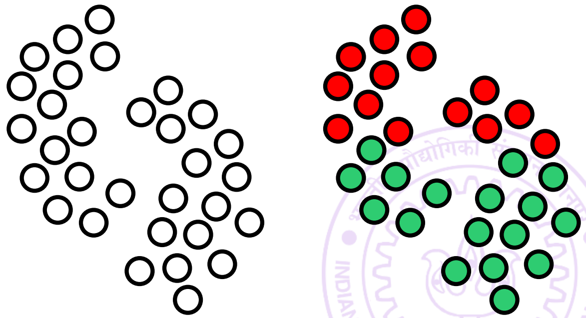

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