CSE340: Theory of Computation (Problem Set)

Question 1. Construct DFAs for the following languages.

1. $L_1 = \{w \in \{0,1\}^* | \#_0(w) \text{ is even and } \#_1(w) \text{ is odd}\}$
2. $L_2 = \{w \in \{0\}^* | |w| \text{ is divisible by 2 or 7}\}$
3. $L_3 = \{w \in \{0,1\}^* | w \text{ is divisible by 5}\}$

Remark. $\#_0(w)$ denotes the number of occurrences of 0 in w. Similarly $\#_1(w)$.

Question 2. Consider the following language

$$L = \{w \in \{0,1\}^* \mid \text{the 3rd last symbol of } w \text{ is } 1\}$$

Construct a DFA for the above language. What can you say about the size (i.e. no. of states) of the DFA compared to the NFA? Consider the language

$$L_k = \{w \in \{0,1\}^* \mid \text{the } k\text{-th last symbol of } w \text{ is } 1\}$$

What is the smallest sized NFA that can accept L_k (as a function of k)? What about the smallest sized DFA?

Question 3. Solve problem 1.5 from chapter 1 in the textbook.

Question 4. For a language $L \subseteq \Sigma^*$, define

$$\text{SecondHalves}(L) = \{y \mid \exists x \text{ such that } |x| = |y|, xy \in L\}.$$

Prove that if L is regular, SecondHalves(L) is also regular.

Question 5. For a language L, let

$$\text{MiddleThirds}(L) = \{y \mid \exists x, z \text{ and } |x| = |y| = |z| \text{ and } xyz \in L\}$$

For example, MiddleThirds$(\{\epsilon, a, ab, bab, bbab, aabbab\}) = \{\epsilon, a, bb\}$.
Prove that if L is regular, MiddleThirds(L) is also regular.

Question 6. Given $L \subseteq \{0,1\}^*$, define

$$L' = \{xy \mid x1y \in L\}.$$

Show that if L is regular then L' is also regular.

Question 7. For a language A, let

$$A'' = \{xz \mid \exists y \text{ and } |x| = |y| = |z| \text{ and } xyz \in A\}$$

Show that even if A is regular, A'' is not necessarily regular.

Question 8. Show that the following languages are not regular.

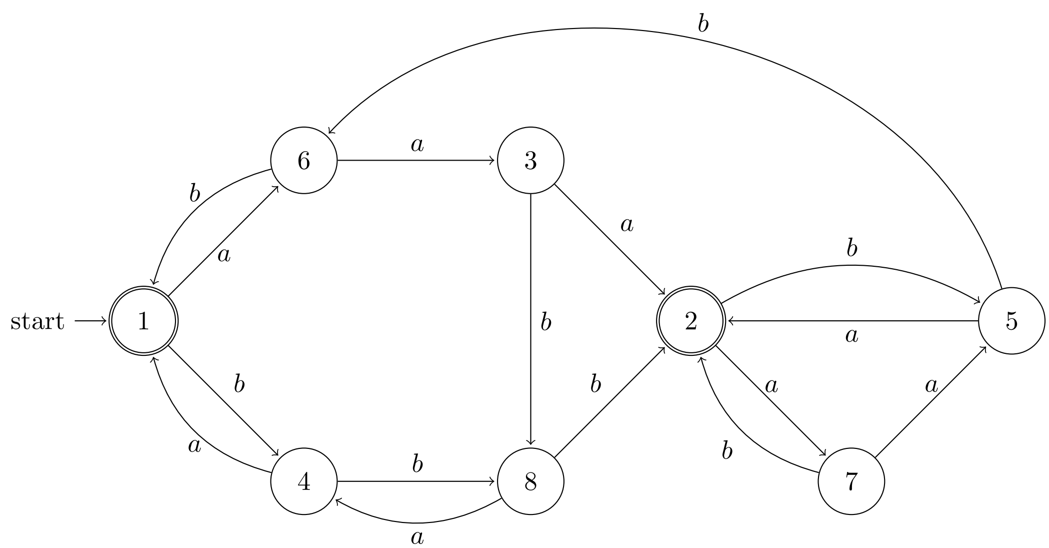
1. $\{0^{n^2}1^n \mid n \geq 0\}$
2. $\{0^n1^m \mid n > m\}$
3. $\{ww \mid w \in \{0,1\}^*\}$
4. $\{a^ib^jc^k \mid i \neq 2 \text{ or } j = k\}$

Question 9. Verify that $\approx$ (defined in lecture 7) is an equivalence relation.

Question 10. Show that the δ' (define in lecture 7) is well defined. In other words, if $[p] = [q]$, then $[\delta(p,a)] = [\delta(q,a)]$ for all $a \in \Sigma$.

Question 11. Can you collapse the quotient DFA any further? What happens if you try to do so?

Question 12. Minimize the following DFA.



Question 13.

$$\begin{aligned} S &\longrightarrow ASB \mid \epsilon \\ A &\longrightarrow a \\ B &\longrightarrow bb \end{aligned}$$

The language generated by the above grammar is

$$L = \{a^nb^{2n} \mid n \geq 0\}$$

which is not regular. What happens if we add the production rule

$$B \longrightarrow \epsilon$$

to the above grammar?

Question 14. Prove Theorem 4 from lecture 8.

Question 15. Give an example of an unambiguous grammar that has at least 2 derivations for some string.

Question 16. Solve problem 2.14 from textbook.

Question 17. Prove that the following languages are not context-free.

1. $L_1 = \{a^n b^m c^n d^m \mid n, m \geq 0\}$
2. $L_2 = \{0^n 1^{n^2} \mid n \geq 0\}$
3. $L_3 = \{0^n \mid n \text{ is prime}\}$

Question 18. Construct PDA for the following languages

(i) $L_1 = \{w \in \{0,1\}^* \mid \#_0(w) = \#_1(w)\}$

(ii) $L_2 = \{0^{2n} 1^{3n} \mid n \geq 0\}$

Question 19. Construct PDA for the following languages

(i) $L_1 = \{a^i b^j c^k \mid j \leq i + k \leq 2j\}$

(ii) $L_2 = \{a^i b^j \mid i \neq j\}$

(iii) $L_3 = L(a^* b^* c^*) \setminus \{a^n b^n c^n \mid n \geq 0\}$

(iv) $L_4 = \overline{L}$, where $L = \{ww \mid w \in \{a,b\}^*\}$

Question 20. Show that CFLs are closed under homomorphism and inverse inverse homomorphism. (*Hint*: For homomorphism start with a CFG and for inverse homomorphism start with a PDA.)

Question 21. Construct a DPDA for the language $L_1 = \{0^n 1^n \mid n \geq 0\}$.

Question 22. Show that there is a CFL that is not a DCFL and has an unambiguous grammar.