ESO 208A

Computational Method in Engineering

Lecture 05

Summary

- Computational methods cannot be studied in isolation of the problem

"The purpose of computing is insight, not numbers", Hamming

- Significant digits/figures are the numbers that one can use with confidence

Example: d = 100±1 m, t = 3.0±0.1 s, v = d/t = ?

- True error = True value – Measured/Computed value
  - approximate error
  - error bound

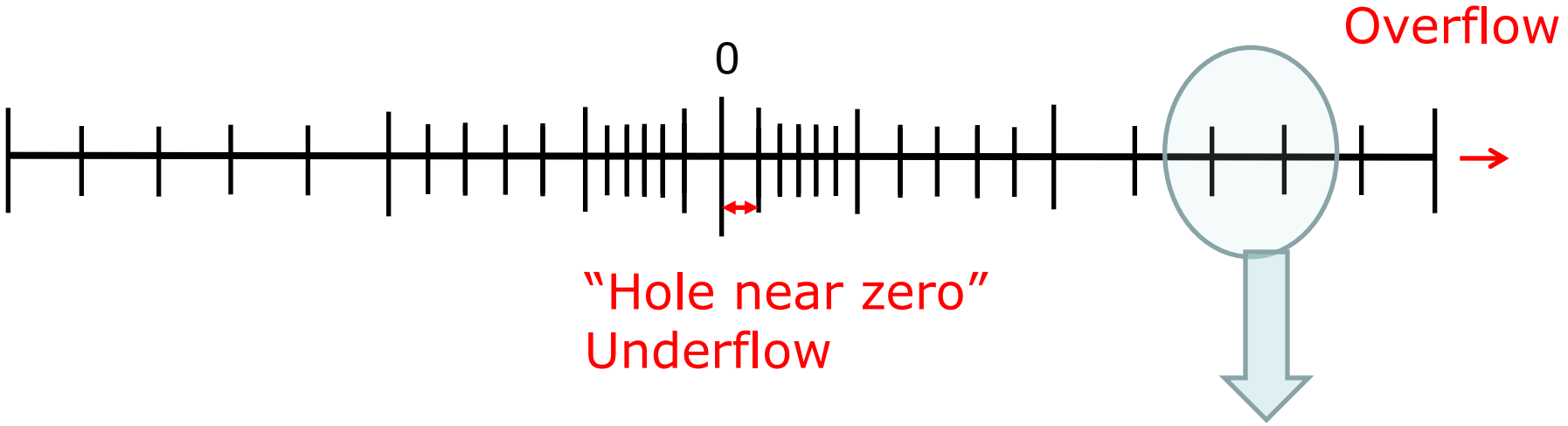
True error is never known

Summary

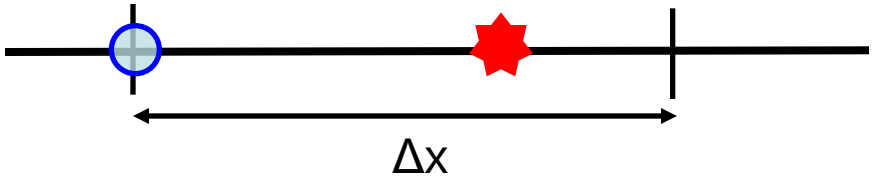
- Types of error
  - Model error
  - Data error
  - **Truncation error**
  - **Round-off error**

Computers are finite

Floating point number representation



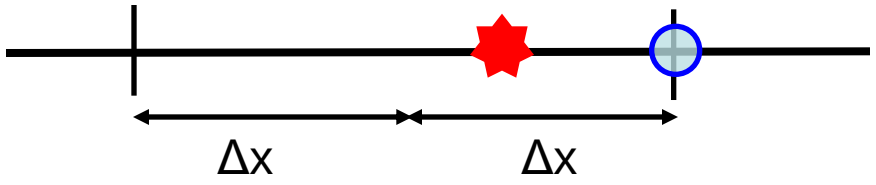
Chopping



Round-off Errors

$$\left|\frac{\Delta x}{x}\right| \le u = 0.5 \times b^{1-t}$$

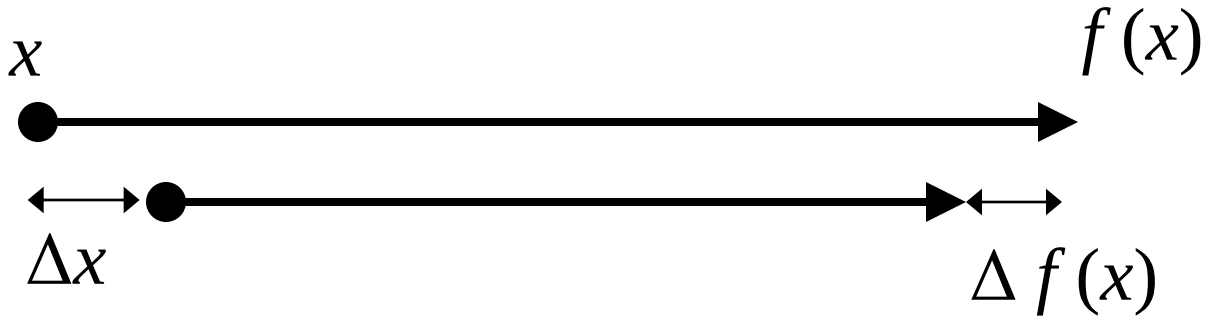
Rounding



Real number in Maths and Computer are not the same

Round-off errors can be avoided subtraction of nearly equal nos.

Forward error analysis



Condition number of the problem

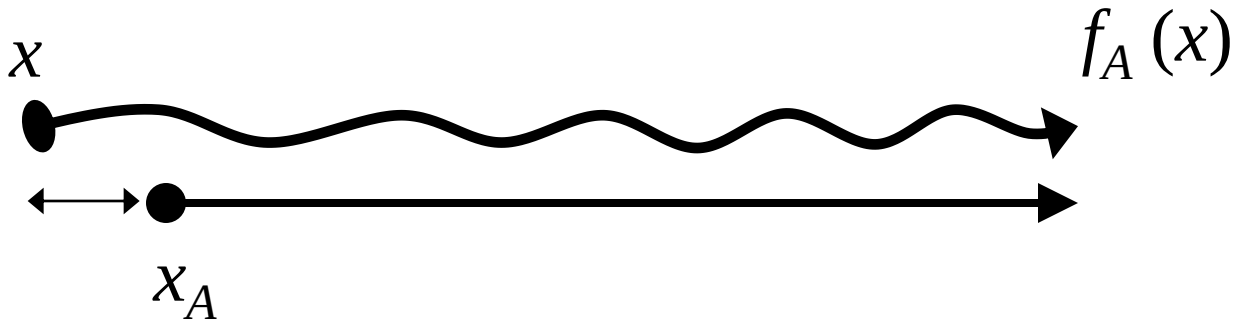
$$C_p = \frac{\text{Relative error in } f(x)}{\text{Relative error in } x} = \frac{\Delta f(x)/f(x)}{\Delta x/x} = \left|\frac{xf'(x)}{f(x)}\right|$$

$C_p \leq 1$ - well-conditioned problem

$C_p > 1$ - ill-conditioned problem

Characteristic of the problem

Backward error analysis



Condition number of the algorithm

$$\left|\frac{x - x_A}{x}\right| \leq C_A u \qquad u \text{ is machine precision}$$

Characteristic of the numerical stability of the algorithm

small C_A - stable algorithm

large C_A - instable algorithm

Total error in Output

$$\left| \frac{f(x) - f_A(x + \Delta x)}{f(x)} \right| \leq C_p \left(\frac{\Delta x}{x} + C_A u \right)$$

Announcements

- First computer assignment
- Tutorial on Matlab
- Access to Brihaspati: Course TA: pramods@