Esc201A: Introduction to Electronics

Solution-Quiz-2A, October 2016

Total points: 15 marks

Duration: 45 minutes

1. Calculate the small signal gain v_o/v_i of the circuit in Figure 1(a) in terms of R_L, g_{m1}, and g_{m2}. Assume that both of the MOSFETS are operating in the saturation region, and that the small-signal models of the MOSFETS are as shown in Figure 1(b).

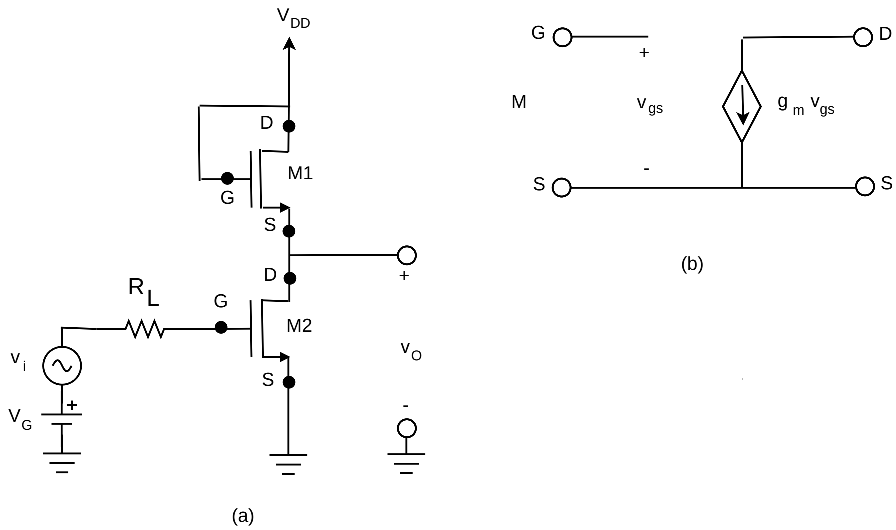

Figure 1: Figure for Problem 1

Solution: Current through MOSFET $M1$ is given by

$$i_{D1} = g_{m1}v_{gs1} = -g_{m1}v_o$$

Current through MOSFET $M2$ is given by

$$i_{D2} = g_{m2}v_{gs2} \approx g_{m2}v_i$$

The approximation can be made because current flowing into gate is nearly zero.

In the circuit we see that

$$i_{D1} = i_{D2}$$

$$\implies -g_{m1}v_o = g_{m2}v_i$$

Hence, we have

$$\implies \frac{v_o}{v_i} = \frac{-g_{m2}}{g_{m1}}$$

2. Determine the expression for v_O in terms of v_I for the circuit in Figure 2. Assume that the MOSFET is operating in the saturation region with $i_{DS} = (K/2)(v_{GS} - V_t)^2$, and that $v_I > 0$.

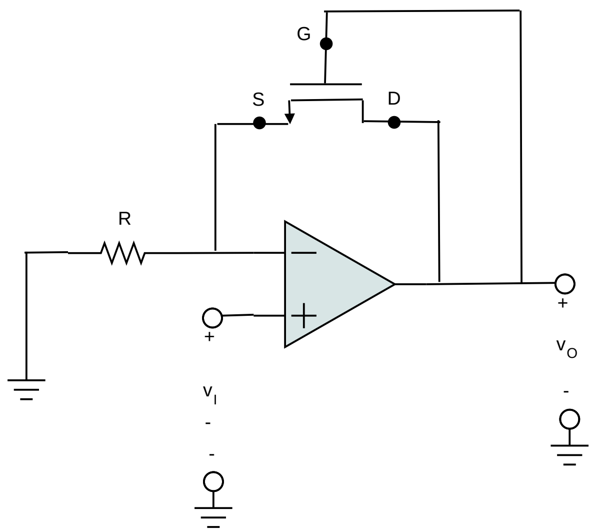

Figure 2: Figure for Problem 2

Solution: From $V_+ = V_- = v_I$, we see that $v_{DS} = v_{GS} = v_o - v_I = v_o$.

We can write

$$\frac{v_I - 0}{R} = i_{DS} = \frac{K}{2}(v_{GS} - V_t)^2 = \frac{K}{2}(v_o - v_I - V_t)^2$$

$$\implies \sqrt{\frac{2v_I}{RK}} = v_o - v_I - V_t$$

or

$$v_o = \sqrt{\frac{2v_I}{RK}} + v_I + V_t$$

3. Look at the op-amp circuit shown in Figure 4. An input signal $v_i = A\sin(\omega t)$ is applied to the circuit and an output $v_o(t) = V_o\sin(\omega t + \theta)$ is observed. Determine the value of V_o and θ in terms of A, R, C_1, C_2 and ω.

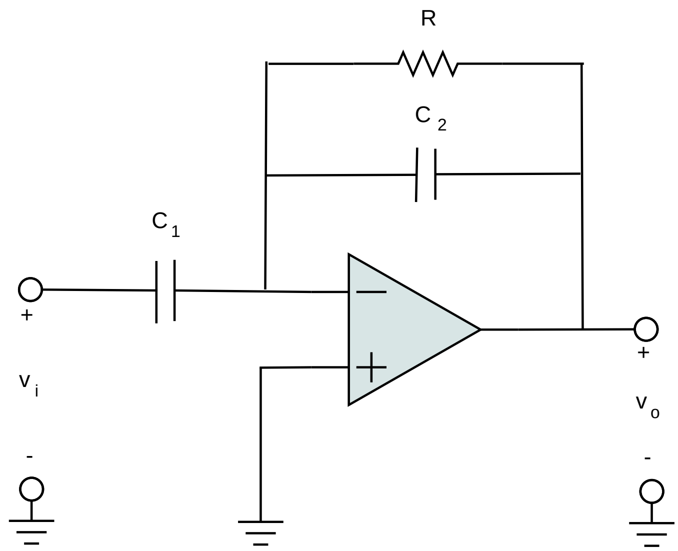

Figure 3: Figure for Problem 3

Solution: From the circuit, we can write

$$\frac{v_o(j\omega)}{v_i(j\omega)} = \frac{-Z_2\,(j\omega)}{Z_1\,(j\omega)}$$

where

$$Z_2\,(j\omega) = R\,||\,\frac{1}{j\omega C_2} = \frac{R}{1 + j\omega C_2 R} \quad and \quad Z_1\,(j\omega) = \frac{1}{j\omega C_1}$$

Substituting these values,

$$\begin{aligned}
\frac{v_o(j\omega)}{v_i(j\omega)} &= -\frac{j\omega C_1 R}{1 + j\omega C_2 R} = -\frac{j\omega C_1 R\,(1 - j\omega C_2 R)}{(1 + j\omega C_2 R)\,(1 - j\omega C_2 R)} = -\frac{\omega^2 R^2 C_1 C_2 + j\omega C_1 R}{1 + \omega^2 R^2 C_2^2} \\
&= -\frac{\sqrt{\omega^4 R^4 C_1^2 C_2^2 + \omega^2 C_1^2 R^2}}{\left(1 + \omega^2 R^2 C_2^2\right)} \;\angle \tan^{-1}\left(\frac{\omega C_1 R}{\omega^2 R^2 C_1 C_2}\right) \\
&= -\frac{\omega C_1 R\,\sqrt{\omega^2 R^2 C_2^2 + 1}}{\left(1 + \omega^2 R^2 C_2^2\right)} \;\angle \tan^{-1}\left(\frac{1}{\omega R C_2}\right) \\
&= \frac{\omega C_1 R}{\sqrt{\omega^2 R^2 C_2^2 + 1}} \;\angle \tan^{-1}\left(\frac{1}{\omega R C_2}\right) + \pi
\end{aligned}$$

When $v_i(t) = A\,\sin(\omega t)$ and $v_o(t) = V_0\,\sin(\omega t + \theta)$, we can write

$$V_0 = \frac{A\omega C_1 R}{\sqrt{\omega^2 R^2 C_2^2 + 1}}$$

and

$$\theta = \tan^{-1}\left(\frac{1}{\omega R C_2}\right) + \pi$$

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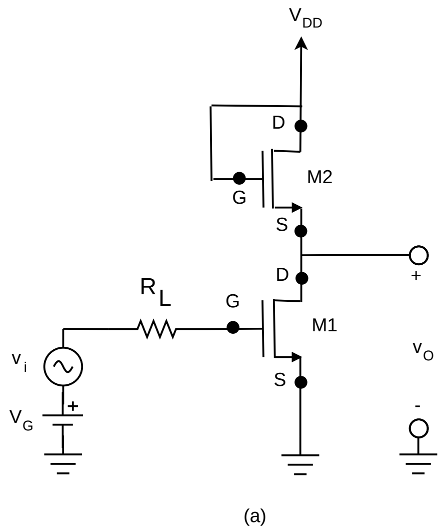
Solution-Quiz-2B, October 2016

Total points: 15 marks

Duration: 45 minutes

1. Calculate the small signal gain v_o/v_i of the circuit in Figure 1(a) in terms of R_L, g_{m1}, and g_{m2}. Assume that both of the MOSFETS are operating in the saturation region, and that the small-signal models of the MOSFETS are as shown in Figure 1(b).



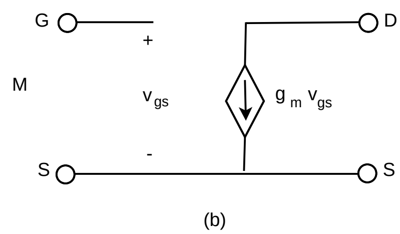


Figure 1: Figure for Problem 1

Solution: Current through MOSFET $M2$ is given by

$$i_{D2} = g_{m2}v_{gs2} = -g_{m2}v_o$$

Current through MOSFET $M1$ is given by

$$i_{D1} = g_{m1}v_{gs1} \approx g_{m1}v_i$$

The approximation can be made because current flowing into gate is nearly zero.

In the circuit we see that

$$i_{D1} = i_{D2}$$

$$\implies -g_{m2}v_o = g_{m1}v_i$$

Hence, we have

$$\implies \frac{v_o}{v_i} = \frac{-g_{m1}}{g_{m2}}$$

2. Determine the expression for v_O in terms of v_I for the circuit in Figure 2. Assume that the MOSFET is operating in the saturation region with $i_{DS} = (K/2)(v_{GS} - V_t)^2$, and that $v_I < 0$.

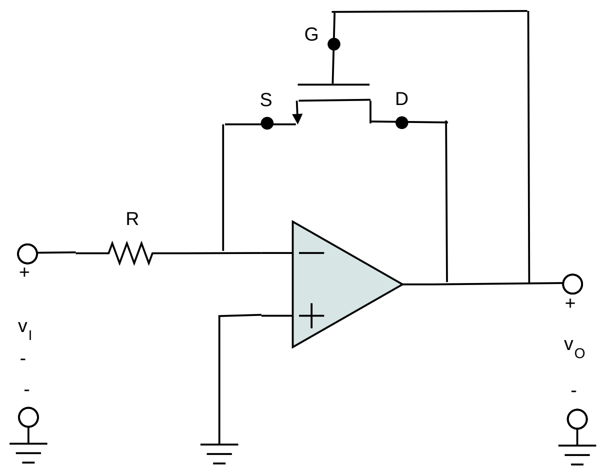


Figure 2: Figure for Problem 2

Solution: From $V_+ = V_- = 0$, we see that $v_{DS} = v_o - 0 = v_o$.

We can write

$$\frac{0 - v_I}{R} = i_{DS} = \frac{K}{2}\left(v_{GS} - V_t\right)^2$$

$$\implies v_{GS} = \sqrt{\frac{-2v_I}{RK}} + V_t$$

Also, from the circuit, $v_{DS} = v_{GS} = v_o$. Thus,

$$v_o = \sqrt{\frac{-2v_I}{RK}} + V_t$$

3. Look at the op-amp circuit shown in Figure 4. An input signal $v_i = A\sin(\omega t)$ is applied to the circuit and an output $v_o(t) = V_o \sin(\omega t + \theta)$ is observed. Determine the value of V_o and θ in terms of $A, R, C_\mathrm{A}, C_\mathrm{B}$ and ω.

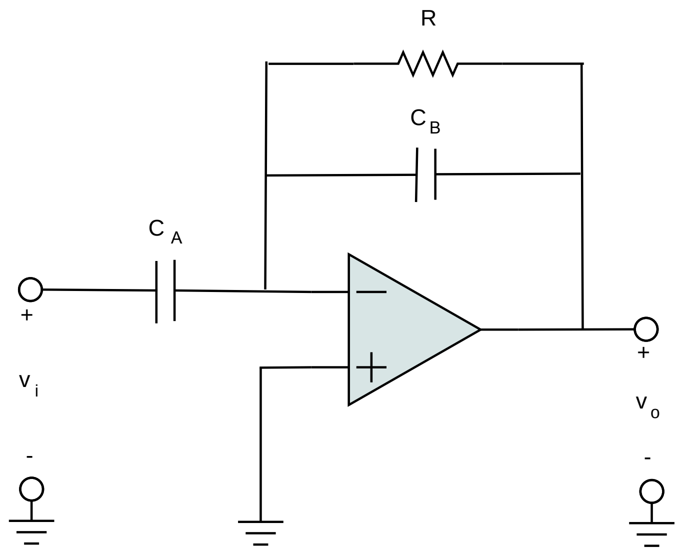


Figure 3: Figure for Problem 3

Solution: From the circuit, we can write

$$\frac{v_o(j\omega)}{v_i(j\omega)} = \frac{-Z_2\,(j\omega)}{Z_1\,(j\omega)}$$

where

$$Z_2\,(j\omega) = R\,||\,\frac{1}{j\omega C_\mathrm{B}} = \frac{R}{1+j\omega C_\mathrm{B}R} \quad \textit{and} \quad Z_1\,(j\omega) = \frac{1}{j\omega C_\mathrm{A}}$$

Substituting these values,

$$\begin{aligned}
\frac{v_o(j\omega)}{v_i(j\omega)} &= -\frac{j\omega C_\mathrm{A}R}{1+j\omega C_\mathrm{B}R} = -\frac{j\omega C_\mathrm{A}R\,(1-j\omega C_\mathrm{B}R)}{(1+j\omega C_\mathrm{B}R)\,(1-j\omega C_\mathrm{B}R)} = -\frac{\omega^2 R^2 C_\mathrm{B} C_\mathrm{A} + j\omega C_\mathrm{A} R}{1+\omega^2 R^2 C_\mathrm{B}^2} \\
&= -\frac{\sqrt{\omega^4 R^4 C_\mathrm{A}^2 C_\mathrm{B}^2 + \omega^2 C_\mathrm{A}^2 R^2}}{\left(1+\omega^2 R^2 C_\mathrm{B}^2\right)} \;\angle \tan^{-1}\left(\frac{\omega C_\mathrm{A} R}{\omega^2 R^2 C_\mathrm{A} C_\mathrm{B}}\right) \\
&= -\frac{\omega C_\mathrm{A} R\sqrt{\omega^2 R^2 C_\mathrm{B}^2 + 1}}{\left(1+\omega^2 R^2 C_\mathrm{B}^2\right.} \;\angle \tan^{-1}\left(\frac{1}{\omega R C_\mathrm{B}}\right) \\
&= \frac{\omega C_\mathrm{A} R}{\sqrt{\omega^2 R^2 C_\mathrm{B}^2 + 1}} \;\angle \tan^{-1}\left(\frac{1}{\omega R C_\mathrm{B}}\right) + \pi
\end{aligned}$$

When $v_i(t) = A\,\sin\,(\omega t)$ and $v_o(t) = V_0\,\sin\,(\omega t + \theta)$, we can write

$$V_0 = \frac{A\omega C_\mathrm{A} R}{\sqrt{\omega^2 R^2 C_\mathrm{B}^2 + 1}}$$

and

$$\theta = \tan^{-1}\left(\frac{1}{\omega R C_{\mathrm{B}}}\right) + \pi$$