ESO208A: Computer Assignment-5

Marks: 100 Due Date: Saturday, Nov 05, 2016

PART – I **(40)**

Write a computer program for Adaptive Quadrature with Simpson's $1/3^{rd}$ rule to evaluate one-dimensional integrals.

Input: (i) Function $f(x)$ to be integrated; (ii) lower limit, a, and upper limit, b, of integration domain; and (iii) the maxim allowable approximate error or tolerance (tol) for each sub-interval.

Output: The output from the program should be a

(i) the value of integral, I;
(ii) number of intervals, n, needed to achieve the desired accuracy; and
(iii) a figure showing the location of points where the function was evaluated and the corresponding function value.

PART – II **(60)**

Write a computer program for solving IVPs.

Input: (i) Ordinary differential equation to be solved $\frac{dy}{dx} = f(x, y)$; (ii) initial values x_0 and y_0; (iii) final value x_f (iv) interval size h; (v) maximum interval size h_{max}, constant α for refining h, and maximum tolerance tol for RK45 method.

Options: The user should have the option of selecting one or more of the following –

a. Euler's method
b. Midpoint method
c. 4^{th} order RK method
d. Runge-Kutta Fehlberg (RK45) method

Obtaining analytical solution is optional.

Output: The output from the program should be:

(a) A text file containing the values of x_i and corresponding y_i;
(b) A figure showing y vs x.

Submission

Due date: Saturday, 05 November by 5:00 pm

Submit a single zip folder in the Brihaspati server under Assignment-5. The name of the zip-folder should be “your roll-number_CA5” (e.g. If your roll no. is 99999, the folder name should be '99999_CA5.zip'). The folder should include -

(i) All the computer program file(s)
(ii) Input file for the test data and output files/figures for the test data generated by your program(s)

Test Data: Part I:

Sample input

$f(x) = 13(x - x^2)e^{-3x/2}$

$a = 0$

$b = 6$

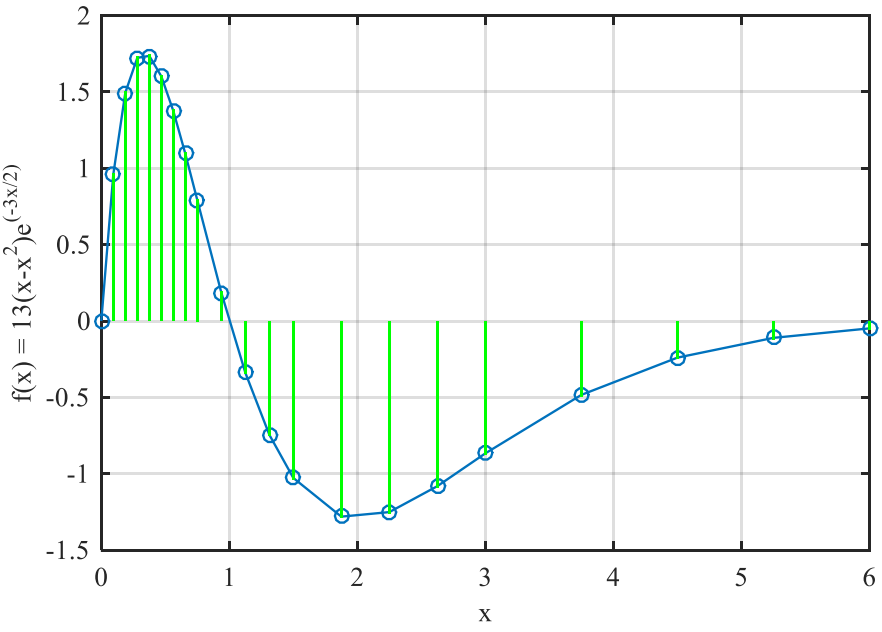
$tol = 10^{-2}$

Sample output files

I = -1.8845

$n = 20$

Sample Figure



Part II:

Sample input file

$$\frac{dy}{dx} = 5e^{-100(x-2)^2} - 0.5y$$

$x_0 = 0.0$

$y_0 = 0.5$

$x_f = 4.0$

$h = 0.2$

$h_{max} = 2.0$

$\alpha = 0.25$

$tol = 10^{-5}$

Sample output files

x	y_analytical	y_euler	y_midpoint	y_RK4
0.00000	0.50000	0.50000	0.50000	0.50000
0.20000	0.45242	0.45000	0.45250	0.45242
0.40000	0.40937	0.40500	0.40951	0.40937
0.60000	0.37041	0.36450	0.37061	0.37041
0.80000	0.33516	0.32805	0.33540	0.33516
1.00000	0.30327	0.29525	0.30354	0.30327
1.20000	0.27441	0.26572	0.27470	0.27441
1.40000	0.24829	0.23915	0.24861	0.24829
1.60000	0.22466	0.21523	0.22499	0.22466
1.80000	0.20534	0.19371	0.20374	0.20642
2.00000	0.61483	0.19265	0.55135	0.58950
2.20000	0.96673	1.17339	0.81685	0.92054
2.40000	0.87663	1.07437	0.73845	0.83578
2.60000	0.79321	0.96693	0.66830	0.75625
2.80000	0.71773	0.87024	0.60481	0.68428
3.00000	0.64942	0.78321	0.54736	0.61916
3.20000	0.58762	0.70489	0.49536	0.56024
3.40000	0.53170	0.63440	0.44830	0.50693
3.60000	0.48111	0.57096	0.40571	0.45869
3.80000	0.43532	0.51387	0.36717	0.41504
4.00000	0.39390	0.46248	0.33229	0.37554

x	*y_analytical*	*y_RK45*
0.00000	0.50000	0.50000
0.16489	0.46043	0.46043
0.33869	0.42211	0.42211
0.51179	0.38711	0.38711
1.25939	0.26638	0.26638
1.56481	0.22865	0.22865
1.73138	0.21044	0.21044
1.78524	0.20584	0.20584
1.83566	0.20850	0.20850
1.89873	0.25983	0.25982
1.97100	0.48171	0.48169
2.02542	0.73069	0.73067
2.08708	0.92678	0.92677
2.15500	0.97814	0.97812
2.24301	0.94795	0.94793
2.45793	0.85160	0.85165
3.36764	0.54038	0.54040
4.00000	0.39390	0.39391