4C15/NE02: MEMS Design
Lecture 6

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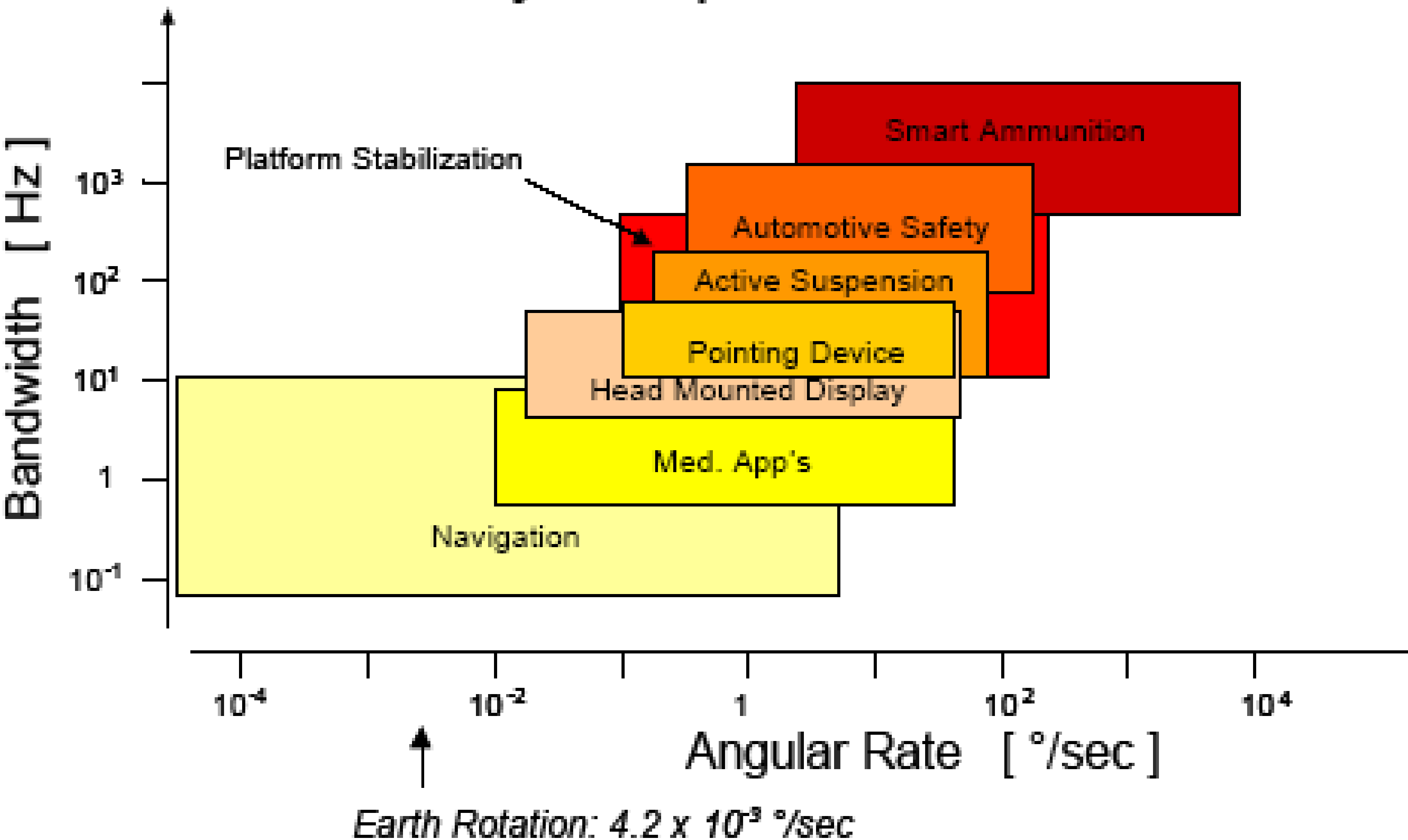
Gyroscope Performance



Vibratory rate gyroscope

- Two orthogonal vibratory modes
  - One driven at large amplitude
  - The other senses the rotation

$$\omega_x = \sqrt{\frac{2k_x}{m}} \qquad \omega_y = \sqrt{\frac{2k_y}{m}}$$

Assume sinusoidal motion in x

and assume some damping in each directio

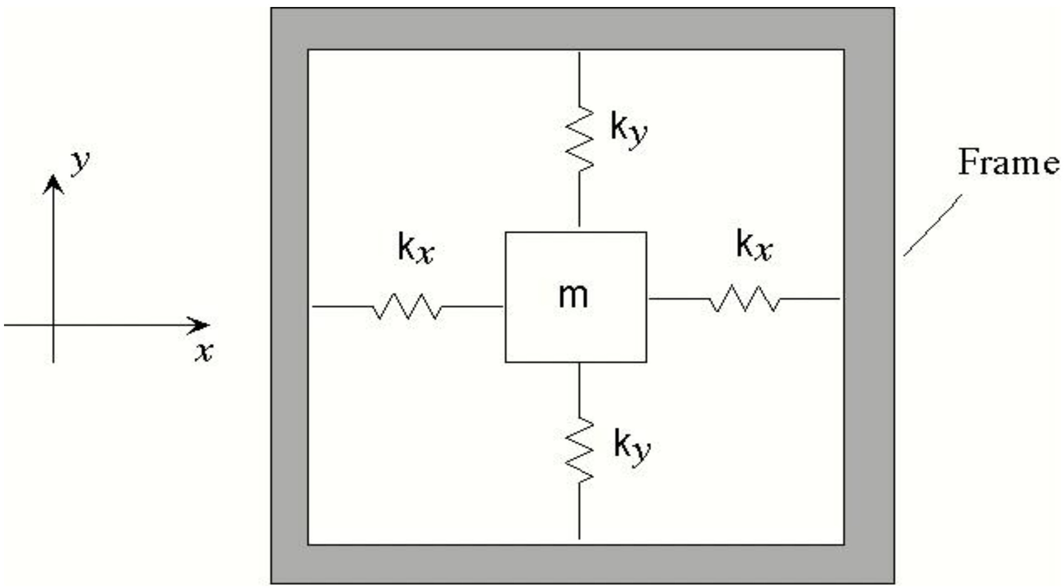
$$x = x_o \sin \omega_d t$$

$$m\ddot{y} + b_y \dot{y} + m\left(\omega_y^2 - \Omega^2\right)y = -2m\Omega\dot{x} - m\dot{\Omega}x$$

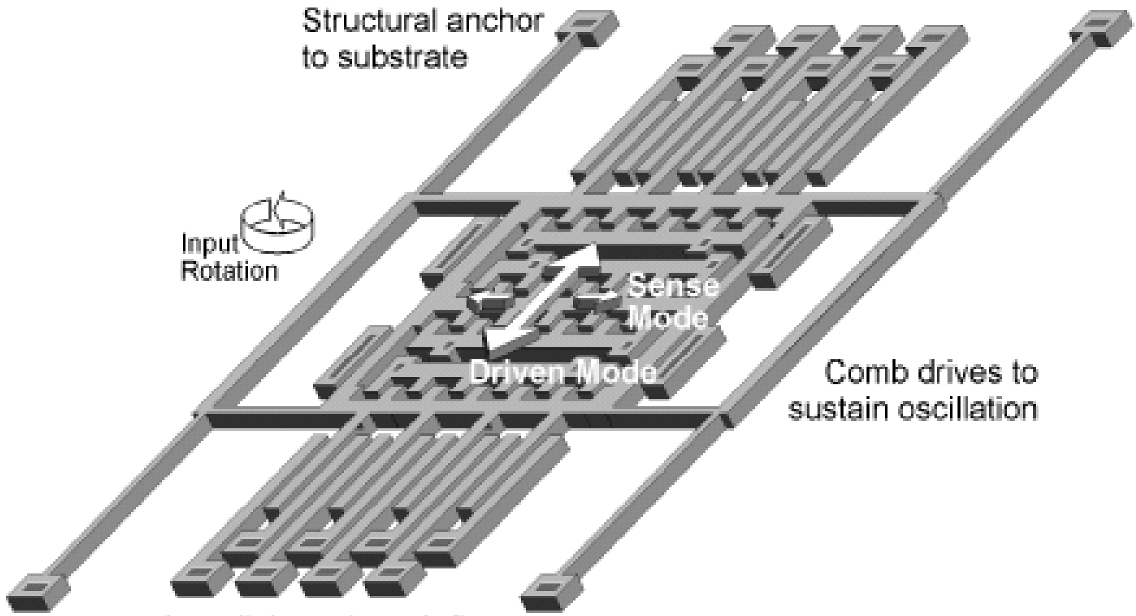
Can typically Ω^2 **ignore term**

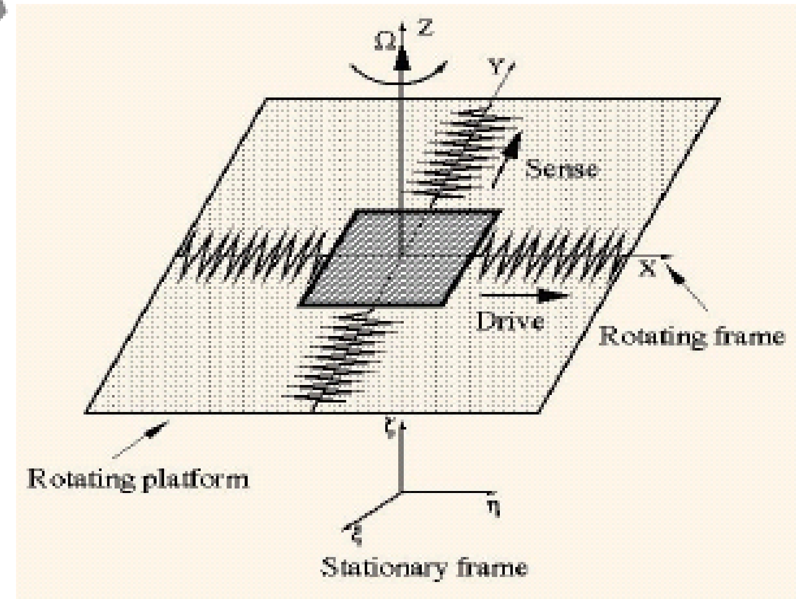
$$\Downarrow$$

$$\boxed{\ddot{y} + \frac{\omega_y}{Q_y}\dot{y} + \omega_y^2 y = -2\Omega\dot{x} - \dot{\Omega}x}$$



Vibratory Gyroscope ... How Does It Work?





Ref. W. Clark et al., '96

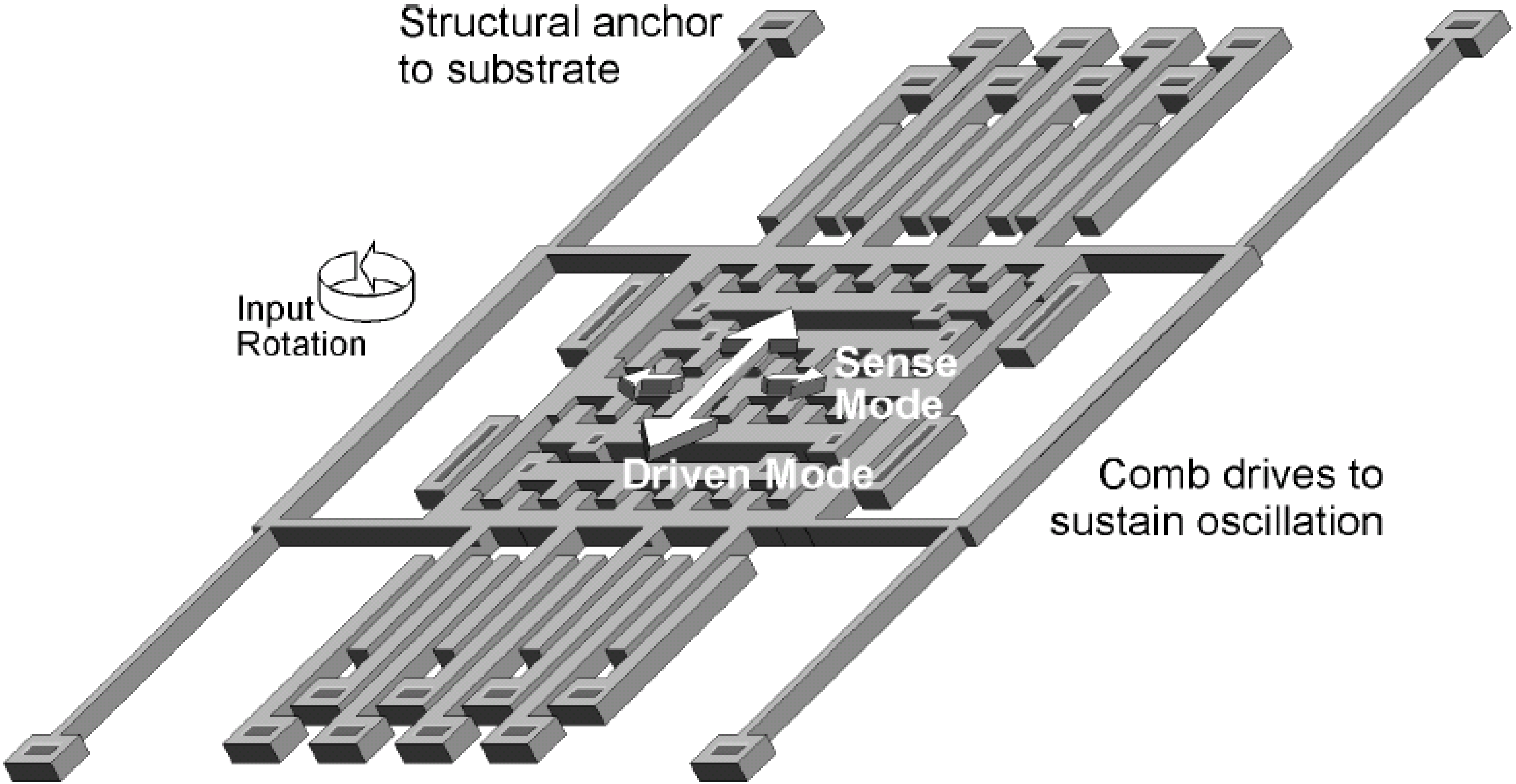
$$m\ddot{x} = -k \cdot x$$

$$m\ddot{y} = -k \cdot y - ma_c$$

<u>Drive mode</u>: $x = X_0 \cdot \sin(\omega \cdot t)$

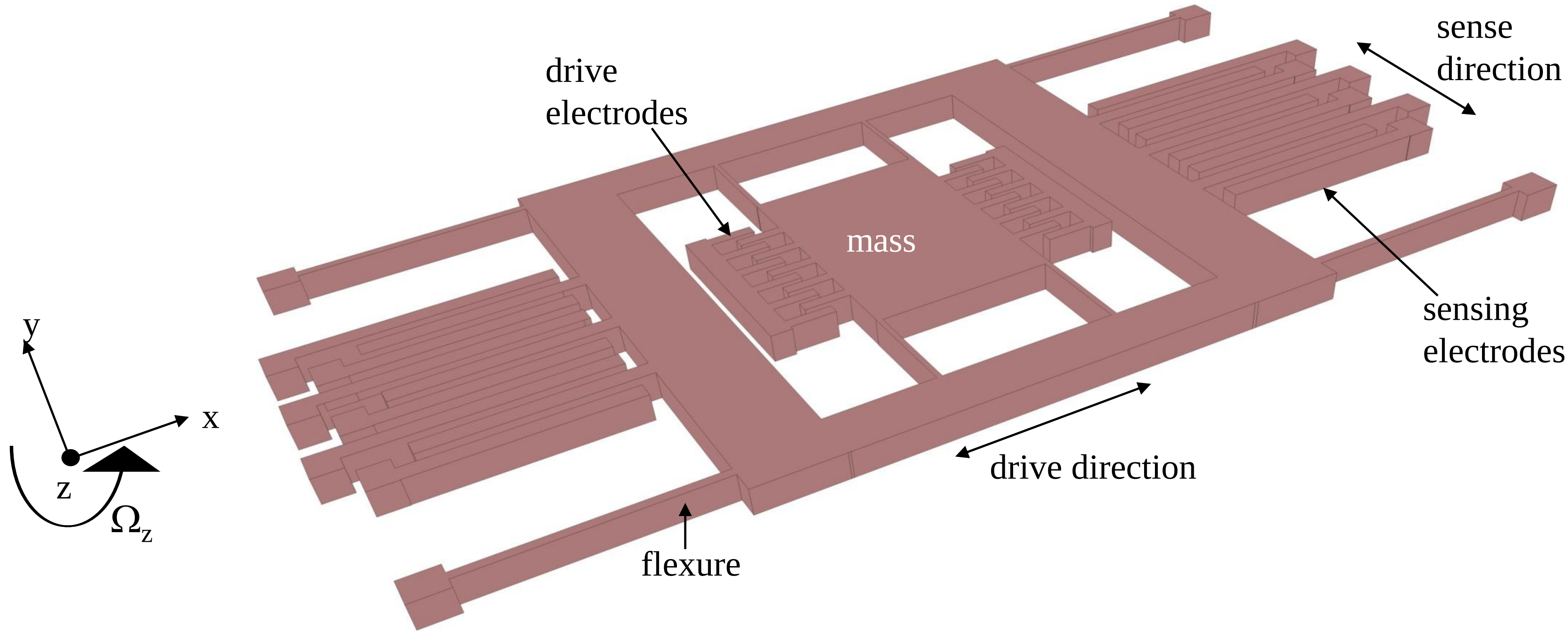
<u>Sense mode</u>: $a_c = 2 \cdot \Omega_z X_0 \cdot \cos(\omega \cdot t)$

Early Z-axis gyroscope prototype



Ref. W. Clark et al., '96

Vibratory MEMS Rate Gyros

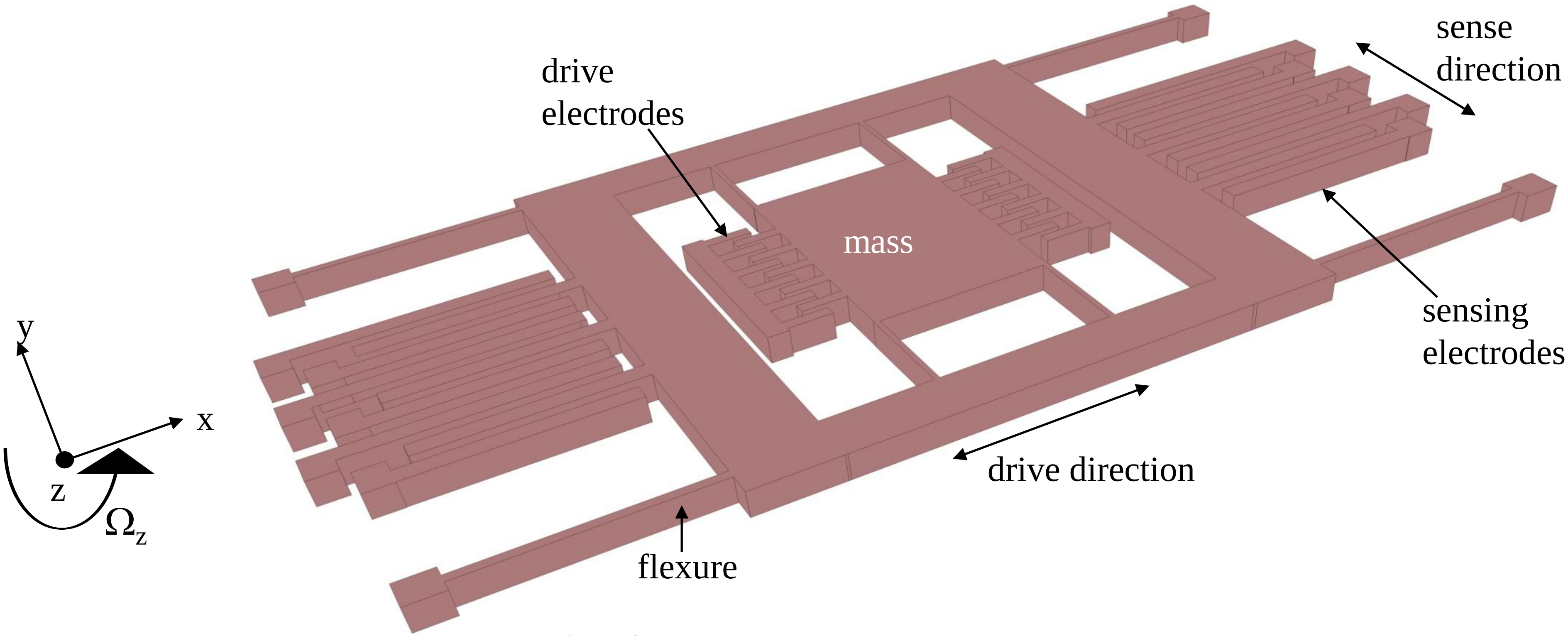


Drive mode $x = A\sin(\omega t)$

Sense mode $F_c = 2m\Omega \times v$

$$= 2mA\Omega_z \omega \cos(\omega t)$$

Vibratory Rate Gyroscopes

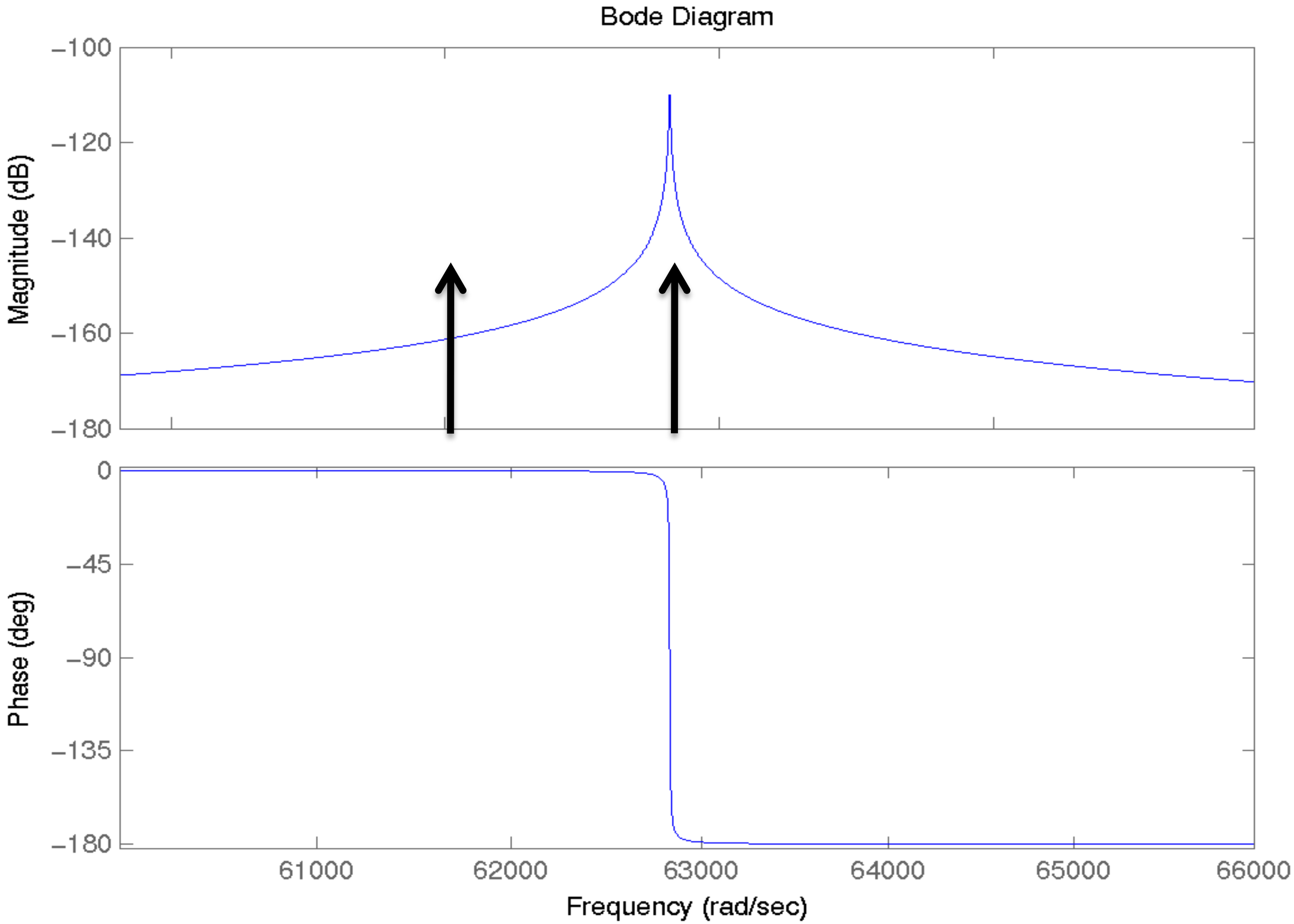


Drive mode $$x = A\sin(\omega t)$$

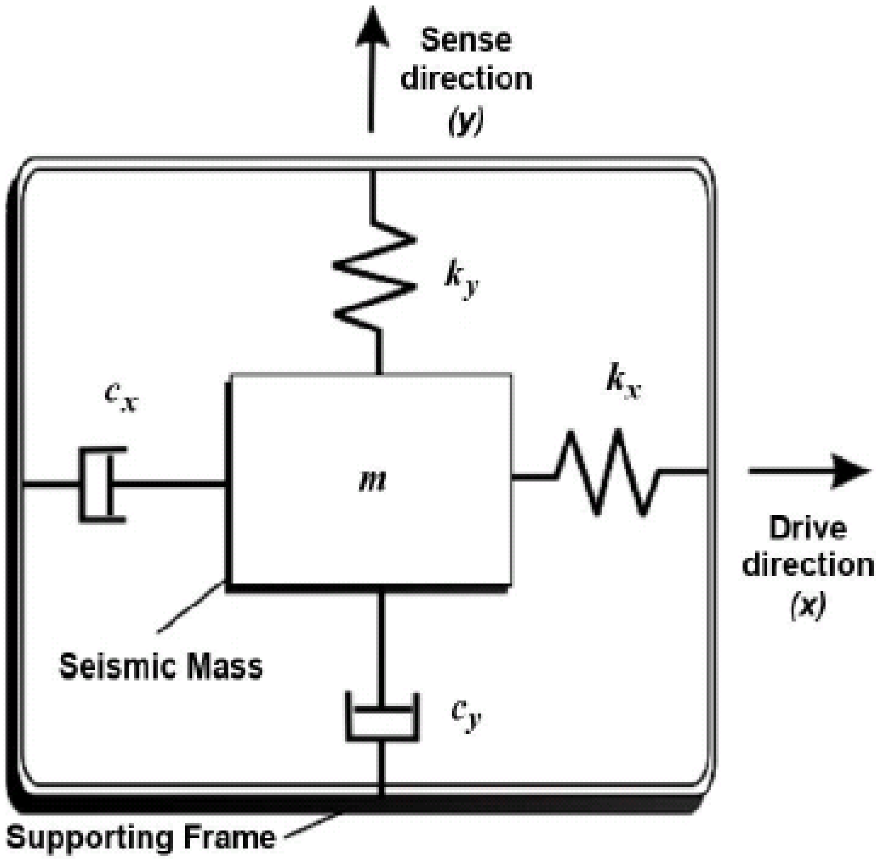
Sense mode $$F_c = 2m\Omega \times v$$
$$= 2mA\Omega_z \omega \cos(\omega t)$$

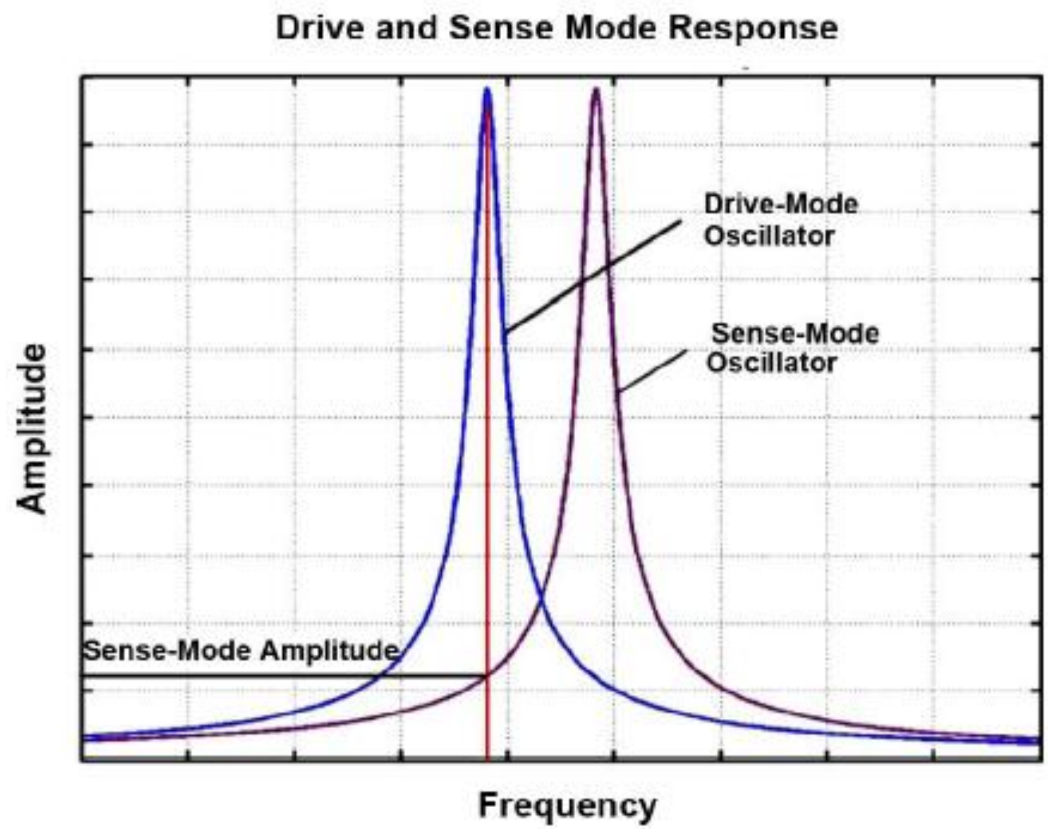
$$\left|\frac{y}{\Omega_z}\right| \propto \frac{2A\omega_x}{\sqrt{\left(\omega_y^2 - \omega_x^2\right)^2 + \left(\frac{\omega_x \omega_y}{Q_y}\right)^2}}$$

Bode Plot (Sense Output)



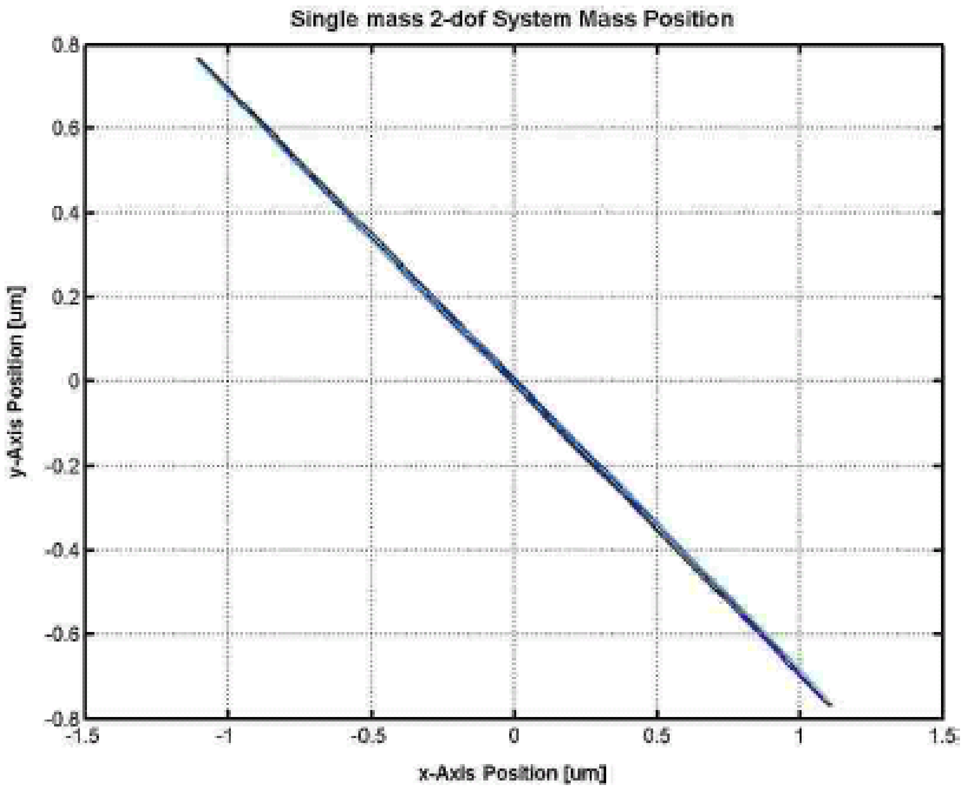
Coupled oscillator model

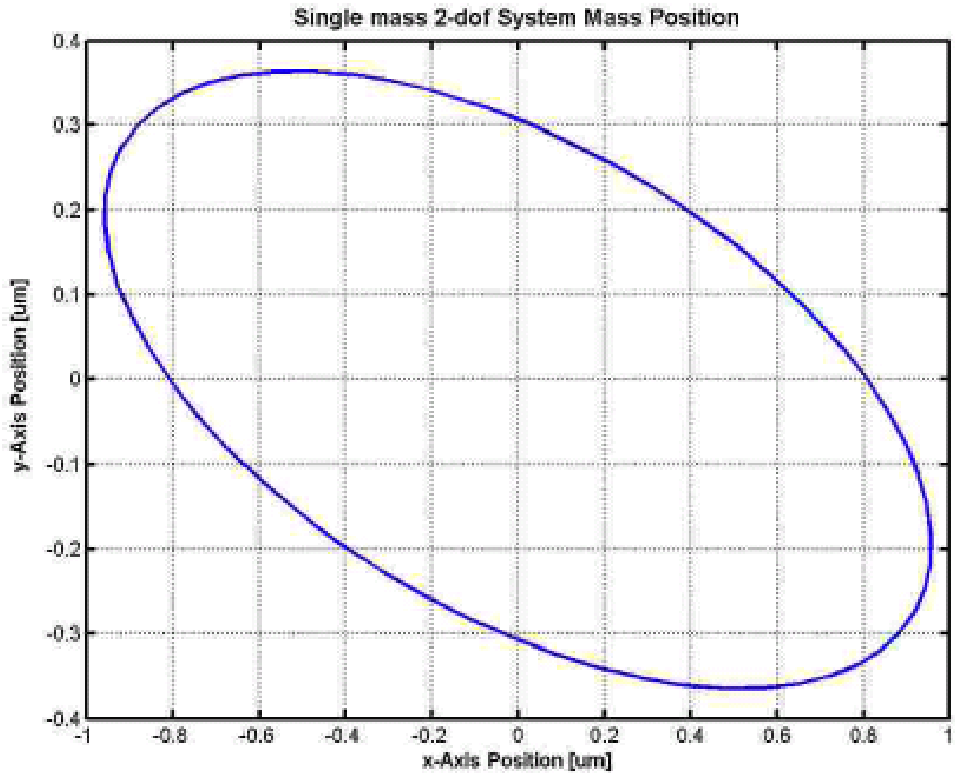




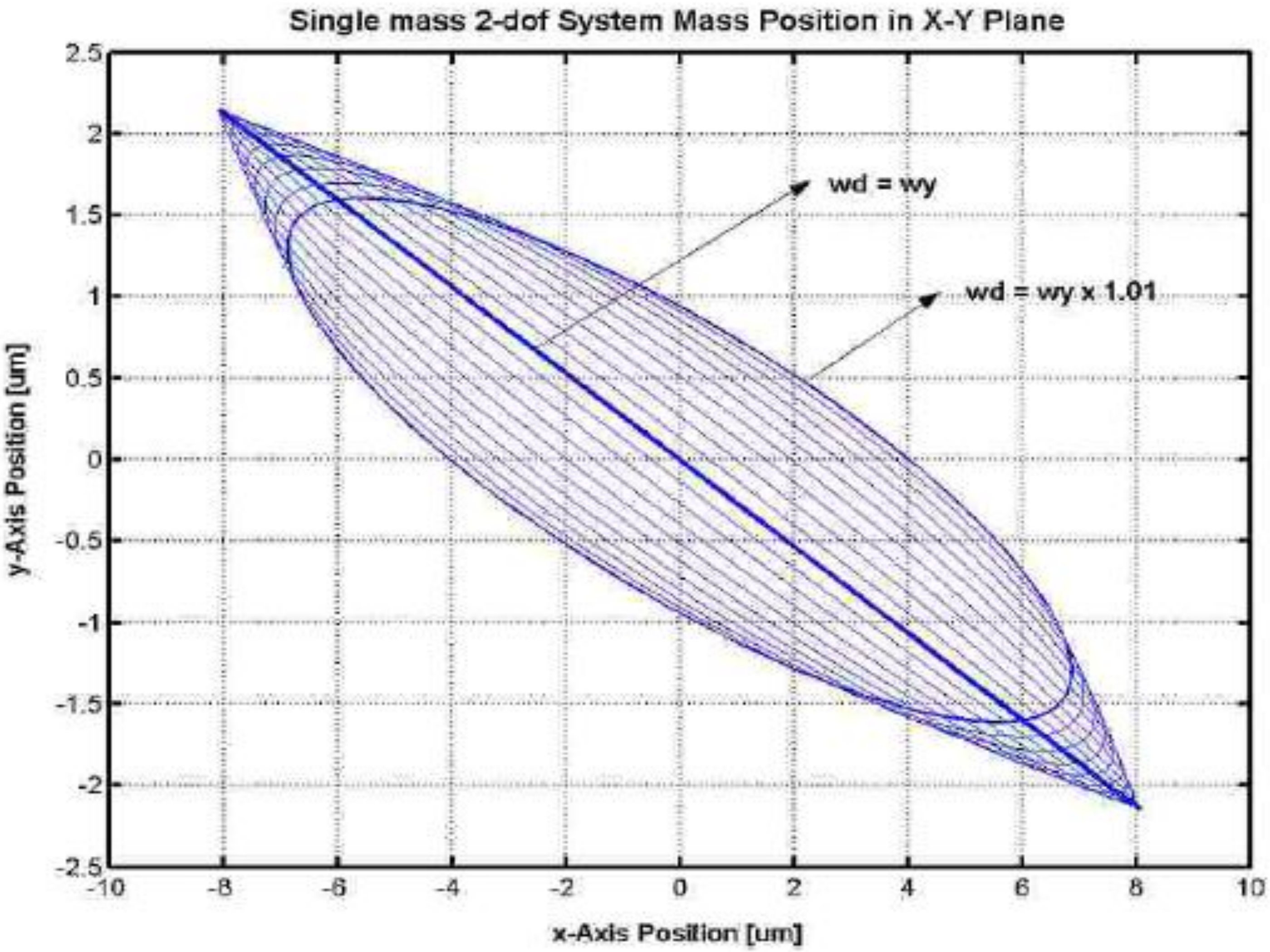
Vibratory Rate Gyroscopes

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}\begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} + \begin{bmatrix} b_{xx} & b_{xy} \\ b_{yx} & b_{yy} \end{bmatrix}\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} + \begin{bmatrix} k_{xx} & k_{xy} \\ k_{yx} & k_{yy} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} F_x^c \\ F_y^c \end{bmatrix} + \begin{bmatrix} 0 & -2m\Omega_z \\ 2m\Omega_z & 0 \end{bmatrix}\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} + \begin{bmatrix} E_x \\ E_y \end{bmatrix}$$



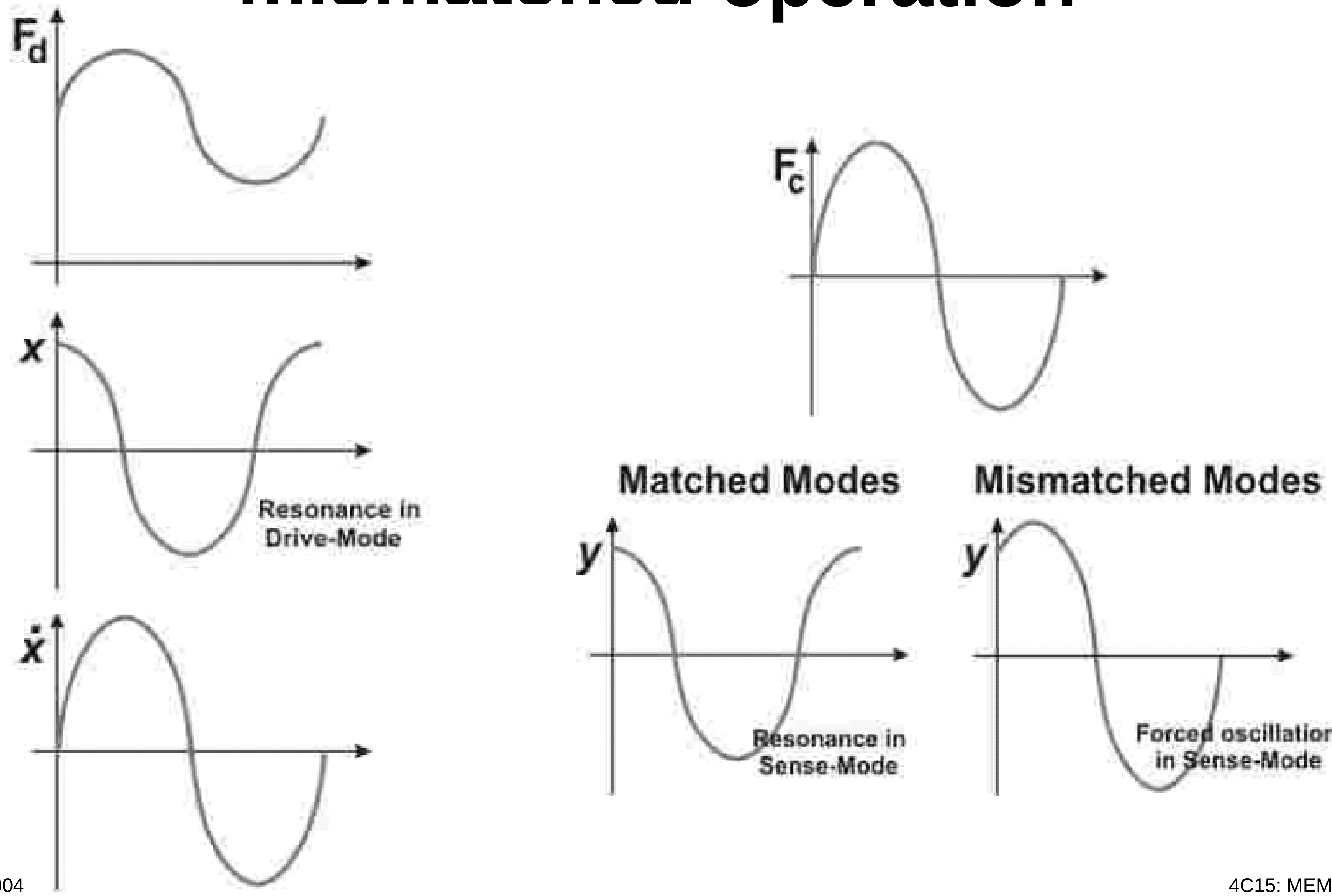


Mode-mismatched operation



$$\omega_x - \omega_y \sim \Delta$$

Mode-matched and mode-mismatched operation



Bandwidth/Sensitivity Analysis

- Assume sinusoidal motion in drive mode
- Sense mode excitation has two terms:
  - A term proportional to angular rotation rate
  - A term proportional to angular acceleration
- Sinusoidal-steady-state analysis reveals a bandwidth issue
  - Matching drive and sense frequencies may lead to insensitivity to changes in rate
  - Offsetting them a little gives a controlled bandwidth to the dynamic response

Analysis Details

Assume a sinusoidally varying rotation rate,
so that there is angular acceleration to deal with

$$\Omega = \Omega_{\mathbf{o}} \cos \omega_a t \;\Rightarrow\; RHS = x_o \Omega_o \left\{ \left(\omega_d + \frac{\omega_a}{2} \right) \sin\left[(\omega_d + \omega_a) t \right] + \left(\omega_d - \frac{\omega_a}{2} \right) \sin\left[(\omega_d - \omega_a) t \right] \right\}$$

The angular acceleration produces SIDEBANDS, meaning the signal has finite bandwidth

We can do sinusoidal-steady-state analysis by assuming $RHS = e^{j\omega_t t}$

The complex amplitude of the y-motion is

$$\hat{\mathbf{Y}} = \frac{\mathbf{1}}{\omega_{\mathbf{y}}^2 - \omega_{\mathbf{t}}^2 + j\delta_y \omega_t} \quad \textbf{where} \quad \delta_y = \omega_y / Q_y$$

If we define $\delta_{\omega} = \omega_y - \omega_t$

$$\hat{\mathbf{Y}} \cong \left(\frac{\mathbf{1}}{2\delta_{\omega} + j\delta_y} \right) \frac{A}{\omega_t}$$

We further define $\Delta_{\omega} = \omega_x - \omega_y$

$$\hat{\mathbf{Y}} \cong \left(\frac{\mathbf{1}}{2(\Delta_{\omega} \pm \omega_a) + j\delta_y} \right) \frac{A}{\omega_t}$$

Steady Rotation

$$\text{Assume } RHS = -2j\omega_d x_o \Omega_o e^{j\omega_d t}$$

If drive and sense frequencies very closely matched,

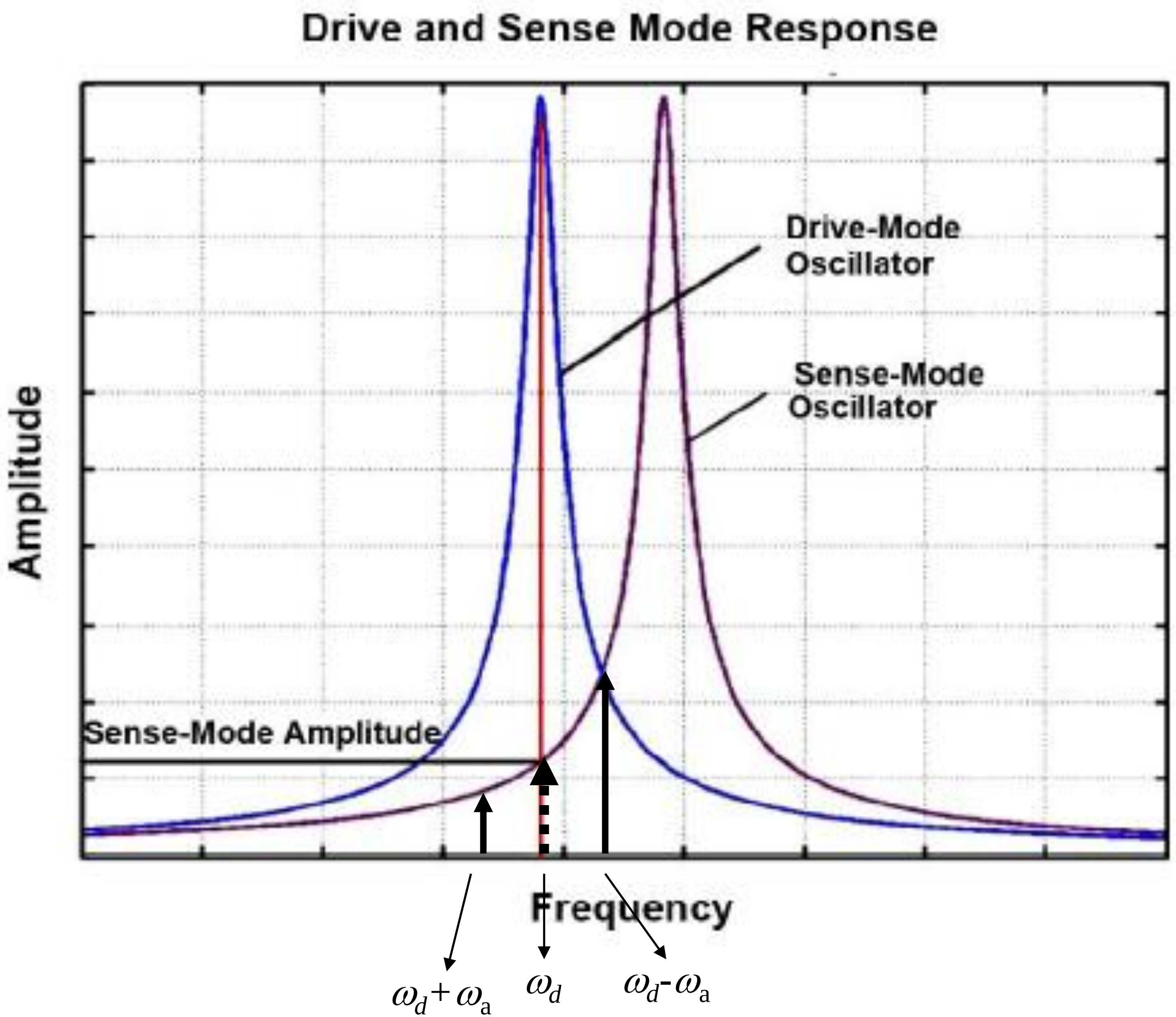
so that $\Delta_\omega << \delta_y$

$$\Rightarrow \quad \hat{Y} \cong -\left(\frac{2}{j\delta_y}\right)\left(\frac{j\omega_d x_o \Omega_o}{\omega_d}\right) = -\frac{2}{\delta_y} x_o \Omega_o$$

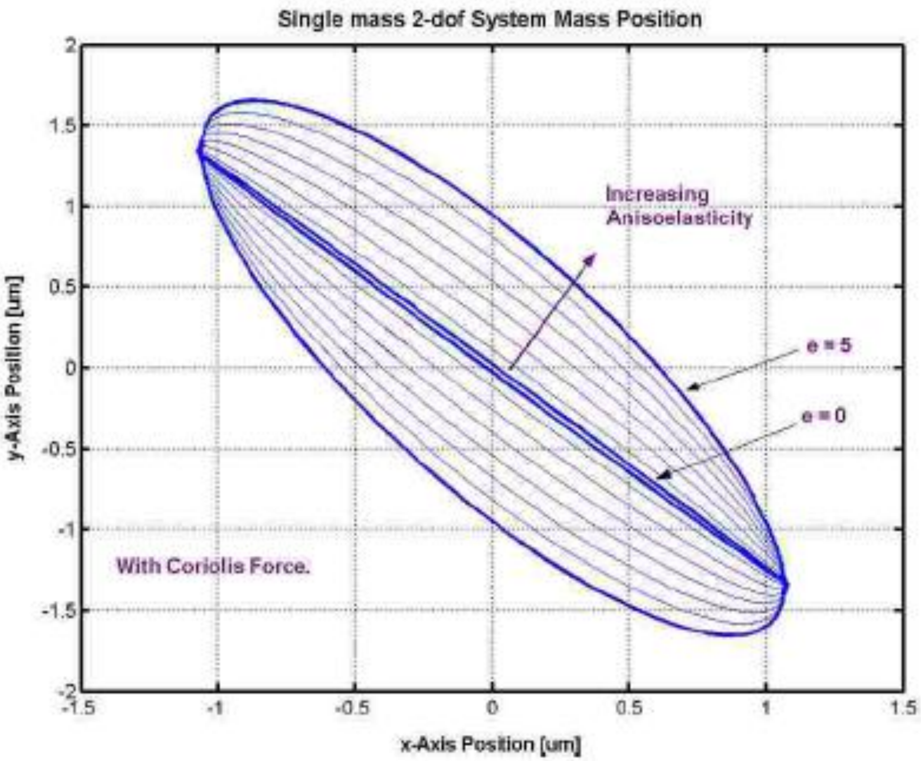
If, instead, $\Delta_\omega >> \delta_y$

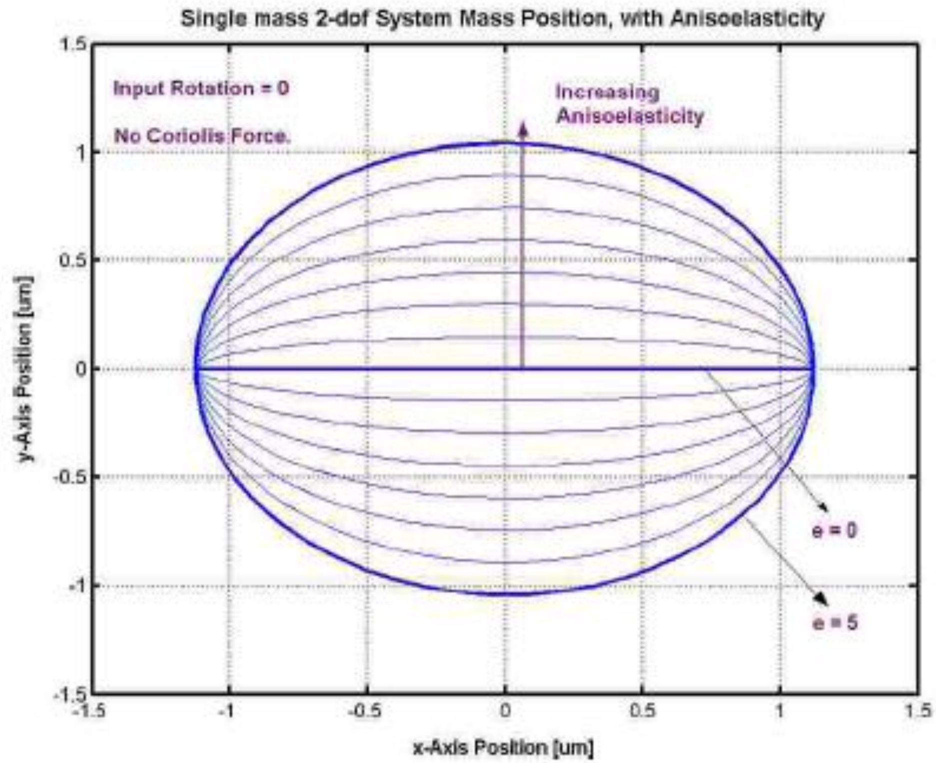
$$\hat{Y} \cong -\frac{j}{\Delta_\omega} x_o \Omega_o$$

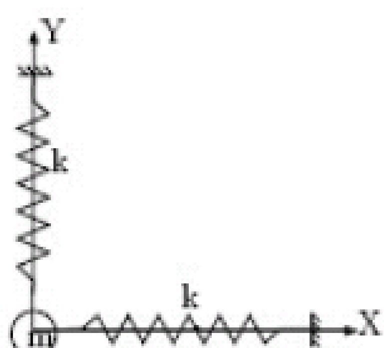
Mode matching



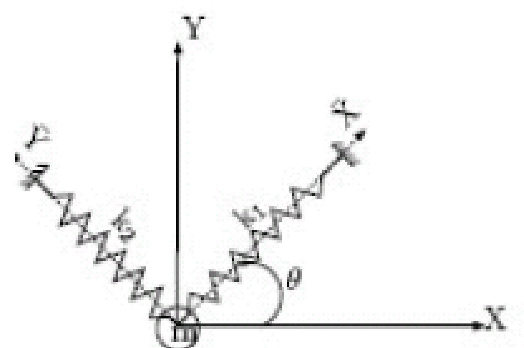
Quadrature Error





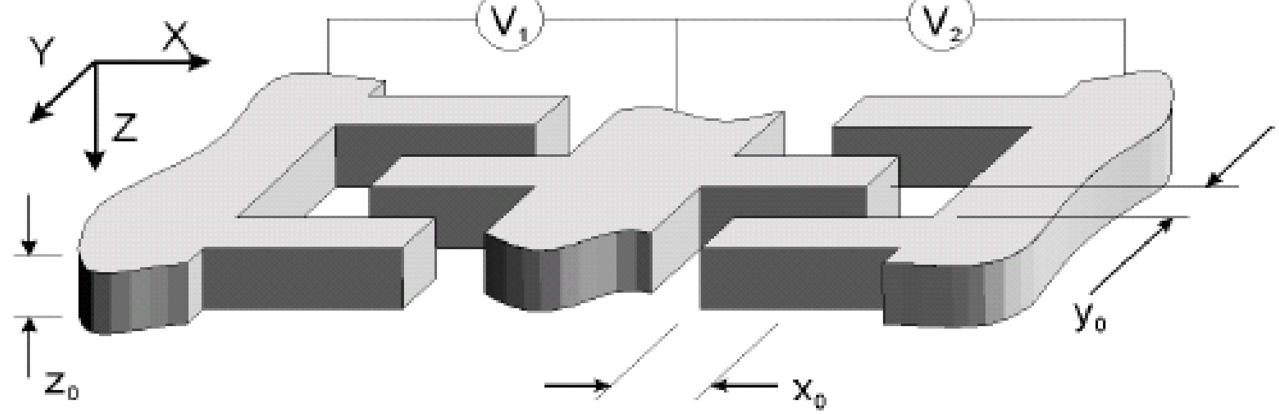


"Ideal" Gyroscope



"Non-ideal" Gyroscope

Calculations on Drive forces



$$V_1 = v_{ac} + V_{DC}$$

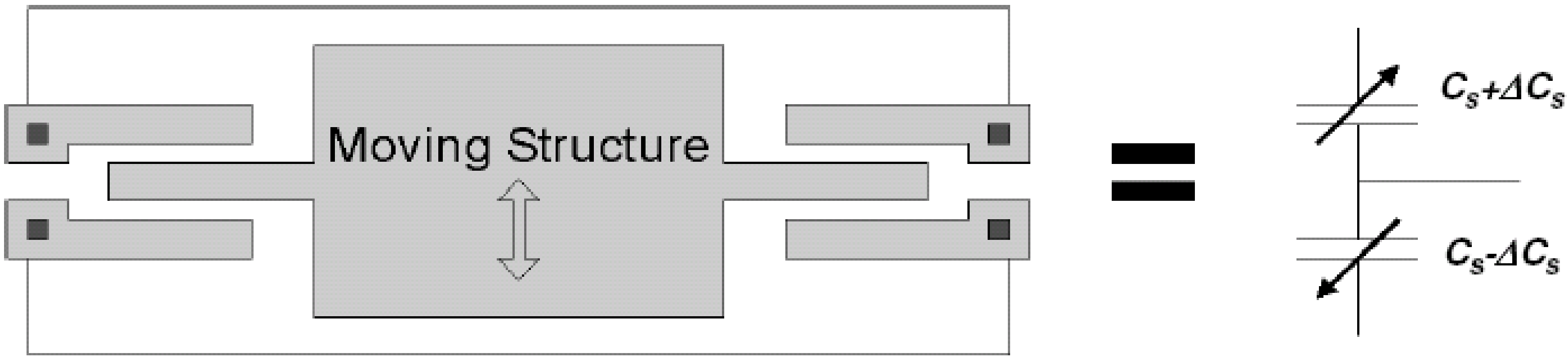
$$V_2 = -v_{ac} + V_{DC}$$

$$F_x = \frac{d}{dx}\left(\frac{1}{2}CV^2\right) = \frac{4N\varepsilon_0 z_0}{y_0} V_{DC} v_{ac}$$

- For: $Q = 1000; X_0 = 5\mu m; k_x = 1N/m \Rightarrow F_x = 5nN$

  $N = 100; V_{DC} = 1V; v_{ac} = 100mV$

Capacitive position sensing



$$\frac{y}{\Omega_z} = \frac{2X_0\omega_x}{\omega_y^2 - \omega_x^2}$$

$$\Delta C_s = \frac{y}{y_0} \cdot C_s$$

- Charge sensing implemented

Some calculations

$$m = 1\mu g\,; \omega_x = 2\pi(10kHz); \frac{\omega_y}{\omega_x} = 1.1; X_0 = 5\mu m; \Omega_z = 1\deg/\min$$

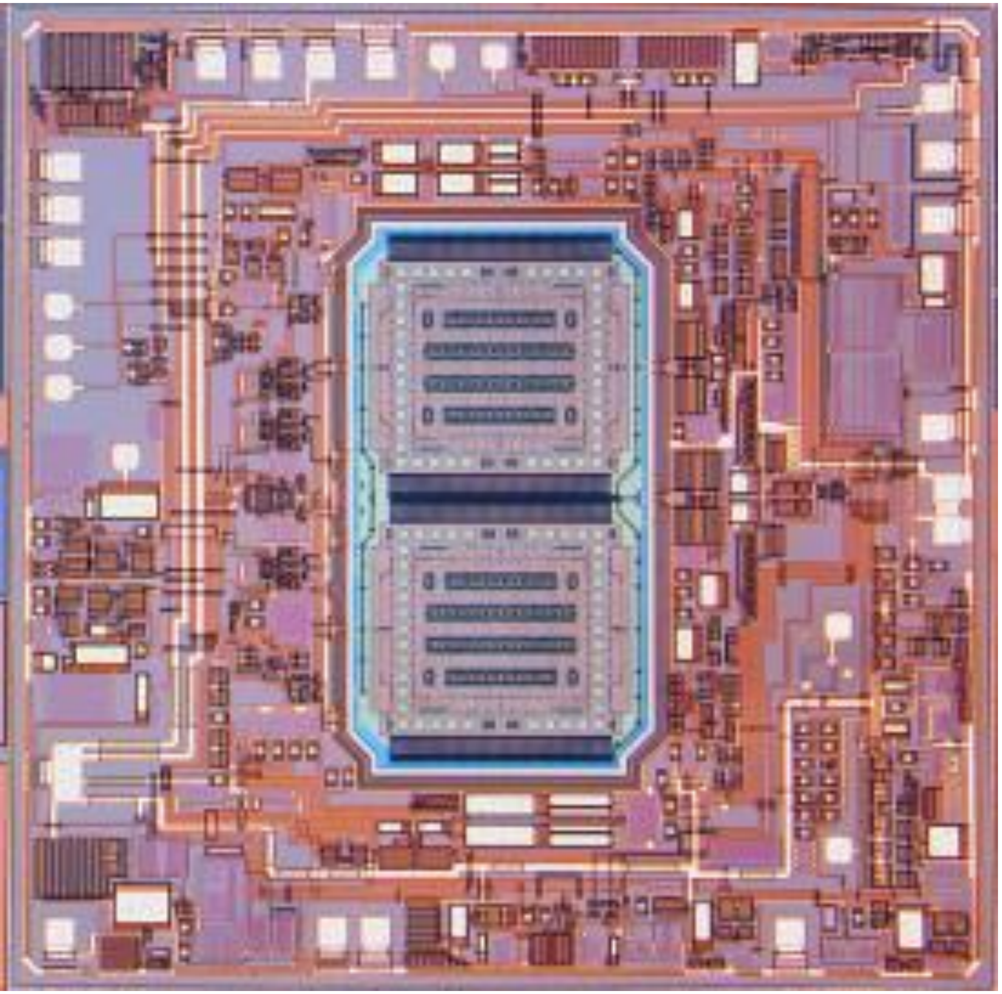
$$\frac{y}{\Omega_z} = \frac{2X_0\omega_x}{\omega_y^2 - \omega_x^2}$$

$$y = 0.22\,pm$$

$$\Delta C_s = 0.03aF$$

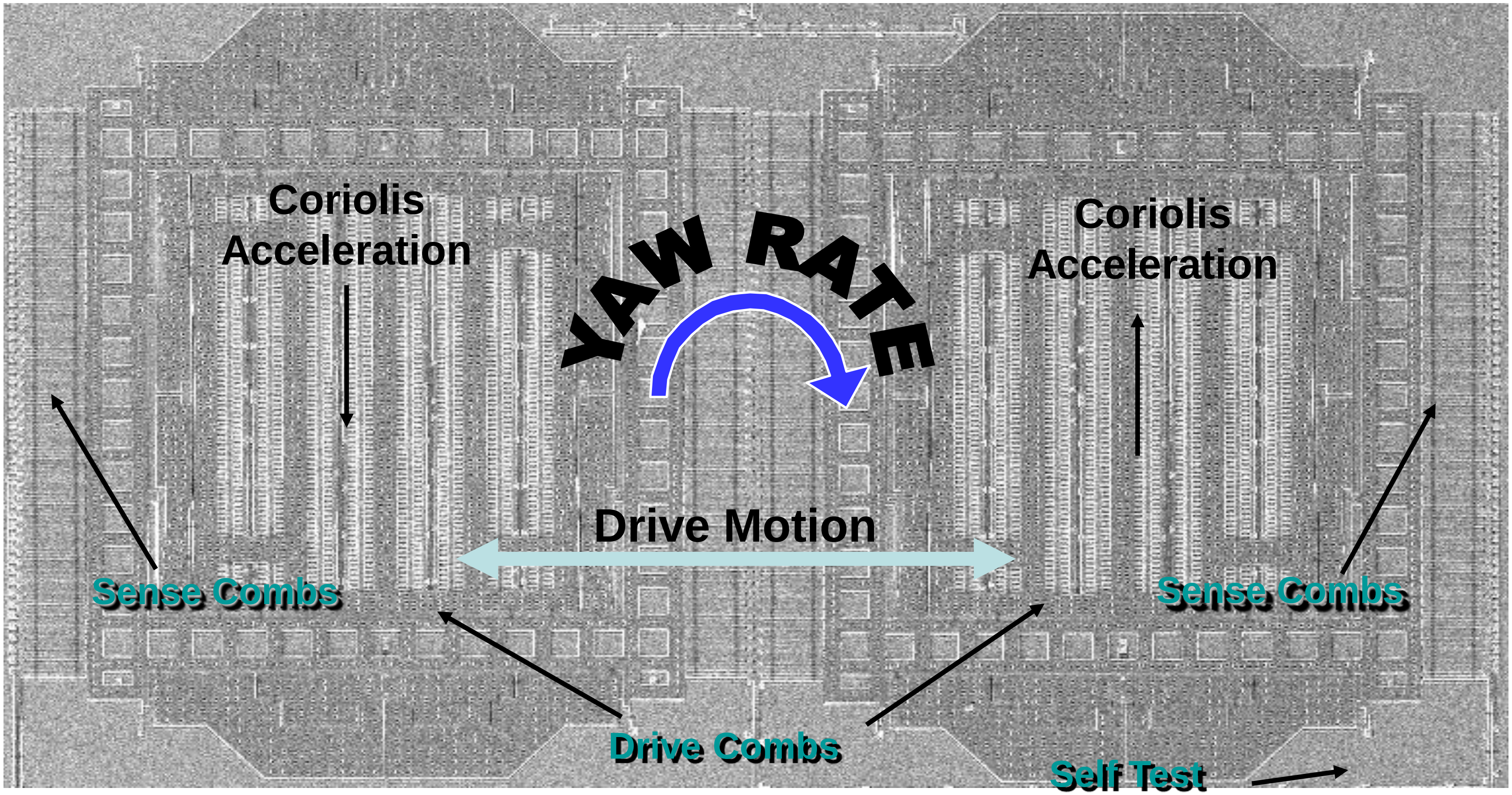
$$V_{out} = 1\mu V!$$

Analog Devices ADXRS150



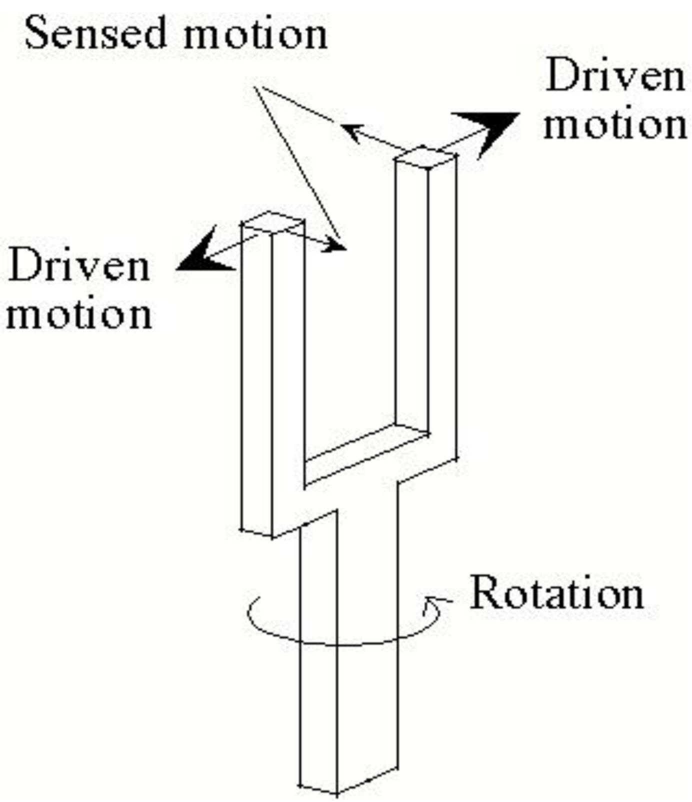
- Number of comb fingers: 4500
- Proof mass displacement: 10μm
- Full scale Coriolis-induced displacement = 20 Å
- Sense capacitance ~ 1000 fF
- Minimum detectable capacitance change 12 zF = 0.012 aF
- Nominal sense gap = 1.6 μm
- Minimum displacement: 16 fm !

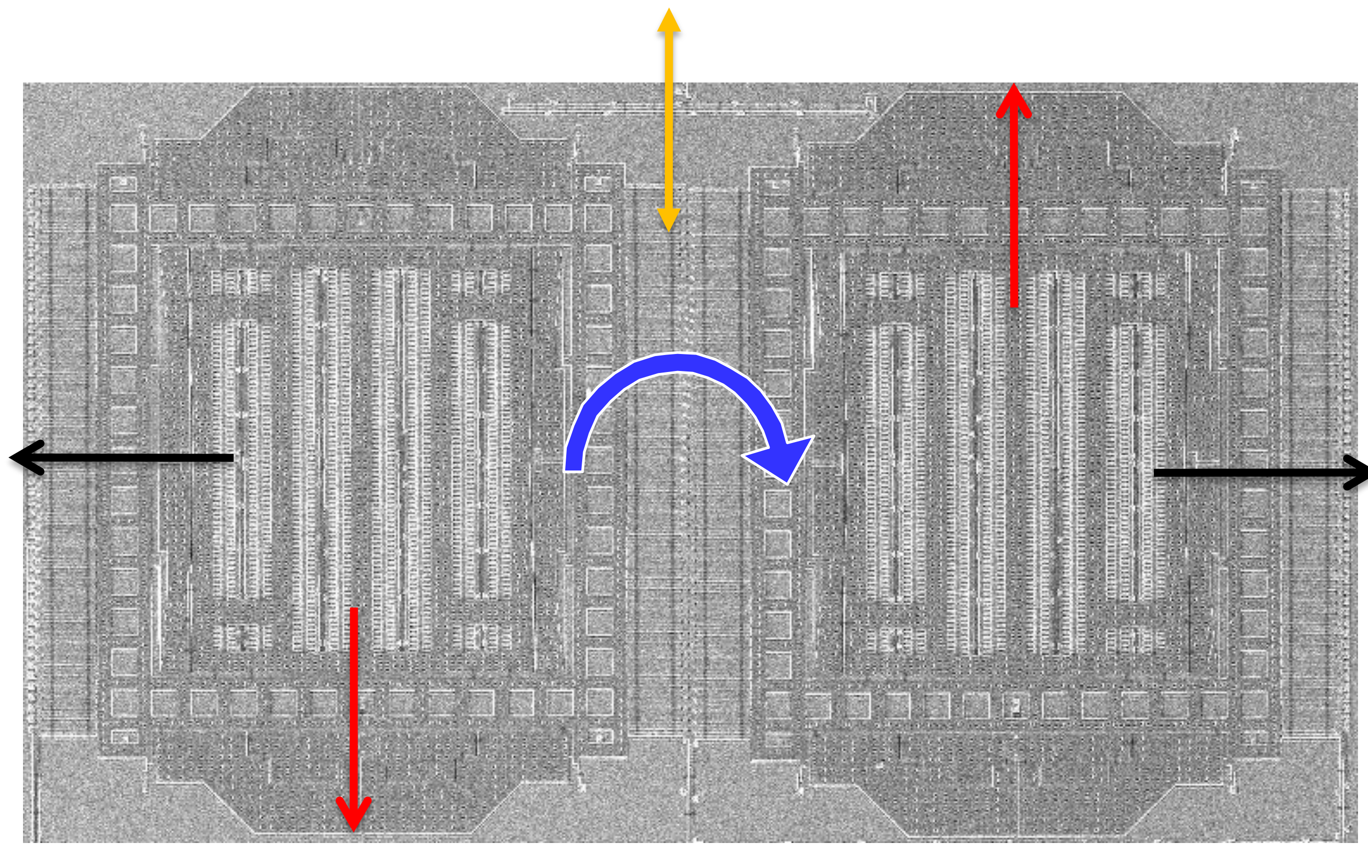
Gyro Sensor Configuration



J. Geen, Analog Devices Inc.

Operation of the ADXRS150





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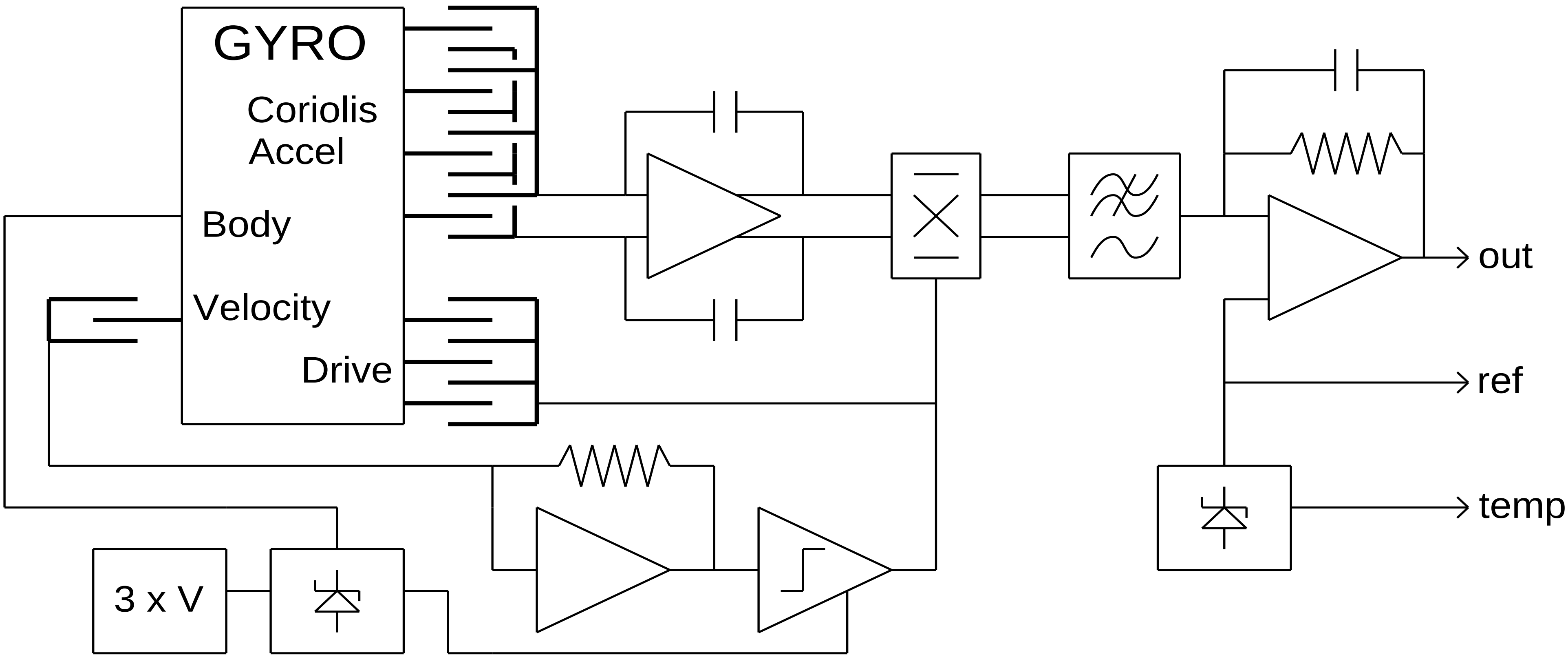
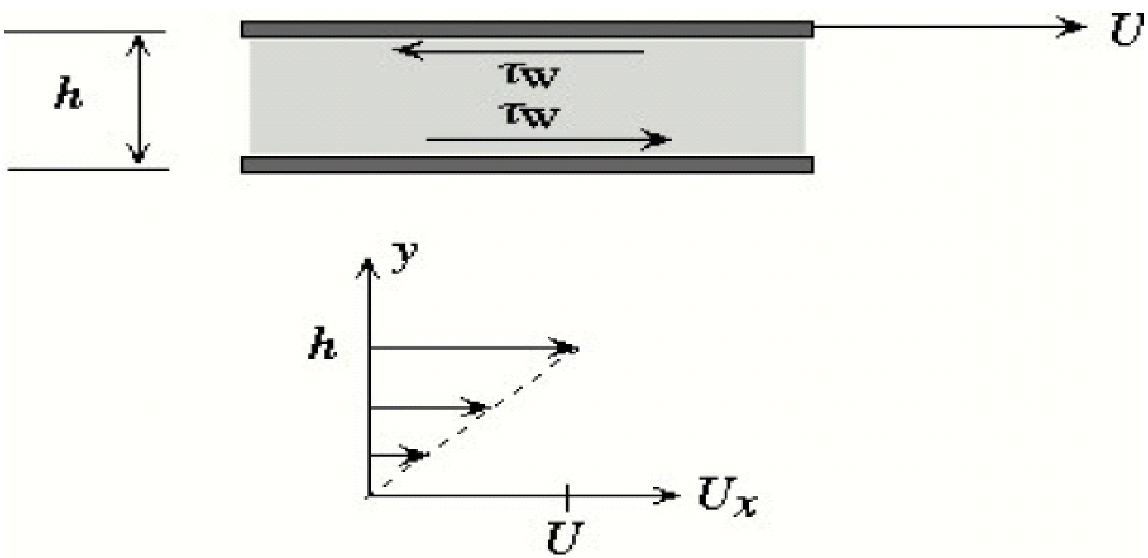
Planar Vibratory Gyros Electronics Block Diagram



Complementary phase of differential drive and feedback omitted for clarity

Air Damping – Couette Flow

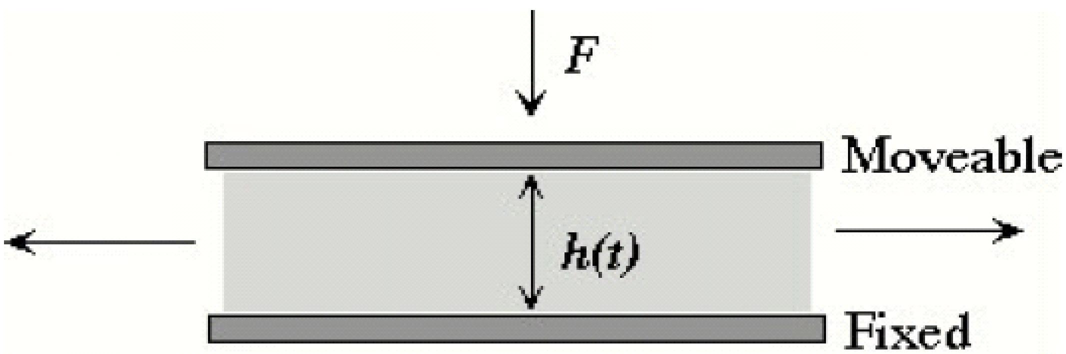
- Steady viscous flow
- The two plates move parallel to each other.
- Linear velocity profile of fluid between the plate
- Comb fingers moving parallel to one another or lateral motion of a mass moving parallel to a substrate.



$$b = \frac{\eta A}{h}$$

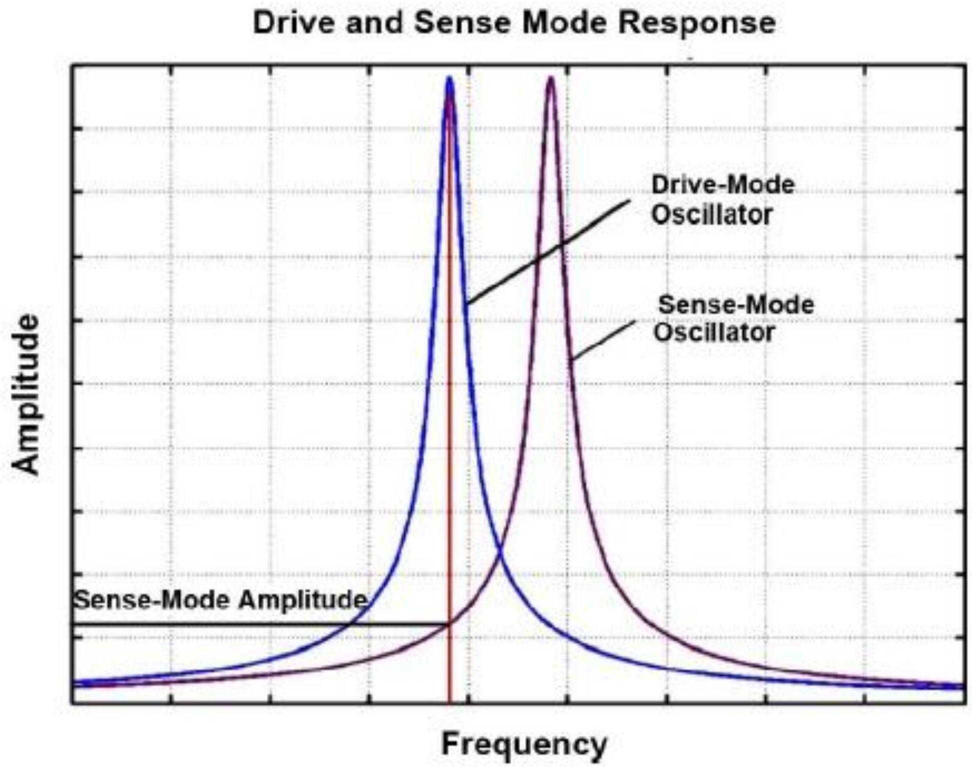
Squeezed-Film Air Damping

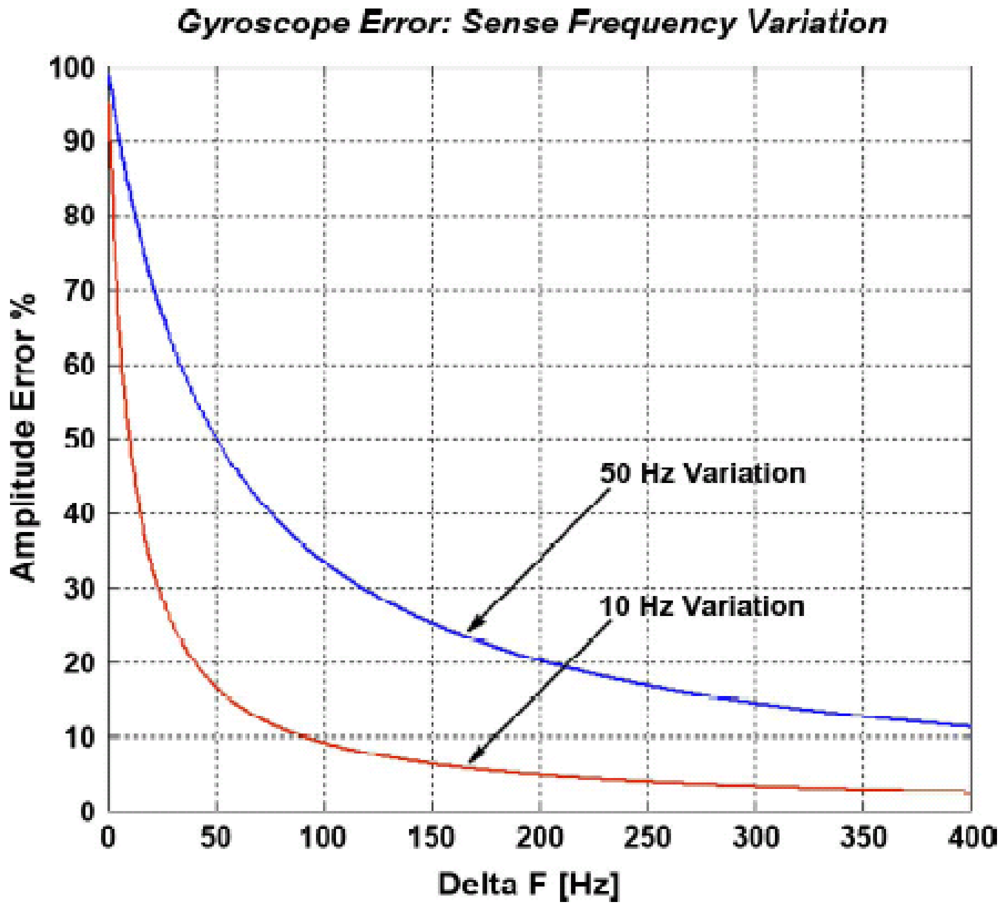
- The result of motion against a fluid boundary
  - If the fluid is incompressible, there can be a large pressure rise, so large back forces result
  - If the fluid is compressible, it takes finite motion to create a pressure rise
- In either case, the dissipation due to viscous flow provides a damping mechanism for the motion
- At high frequencies or for highly viscous fluids, this can also provide a spring



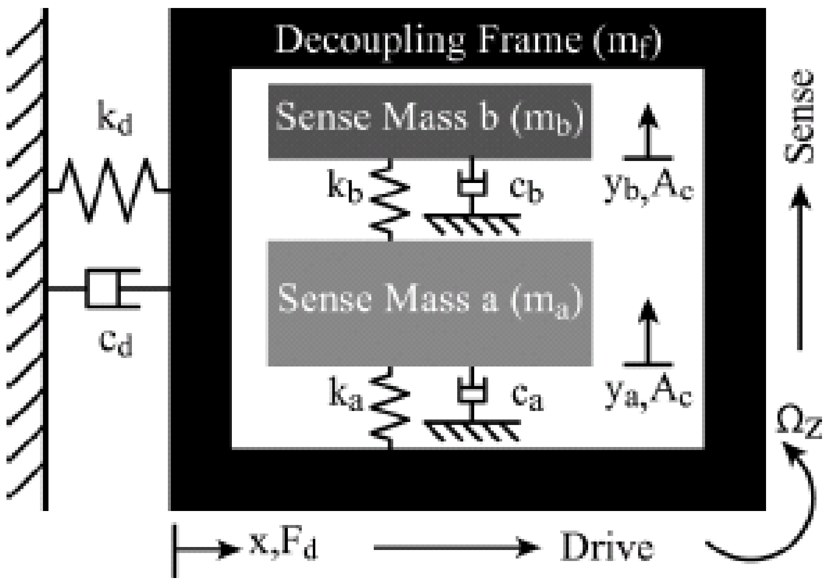
$$b = \frac{96\ \eta L W^{3}}{\pi^{4} h_{o}^{3}}$$

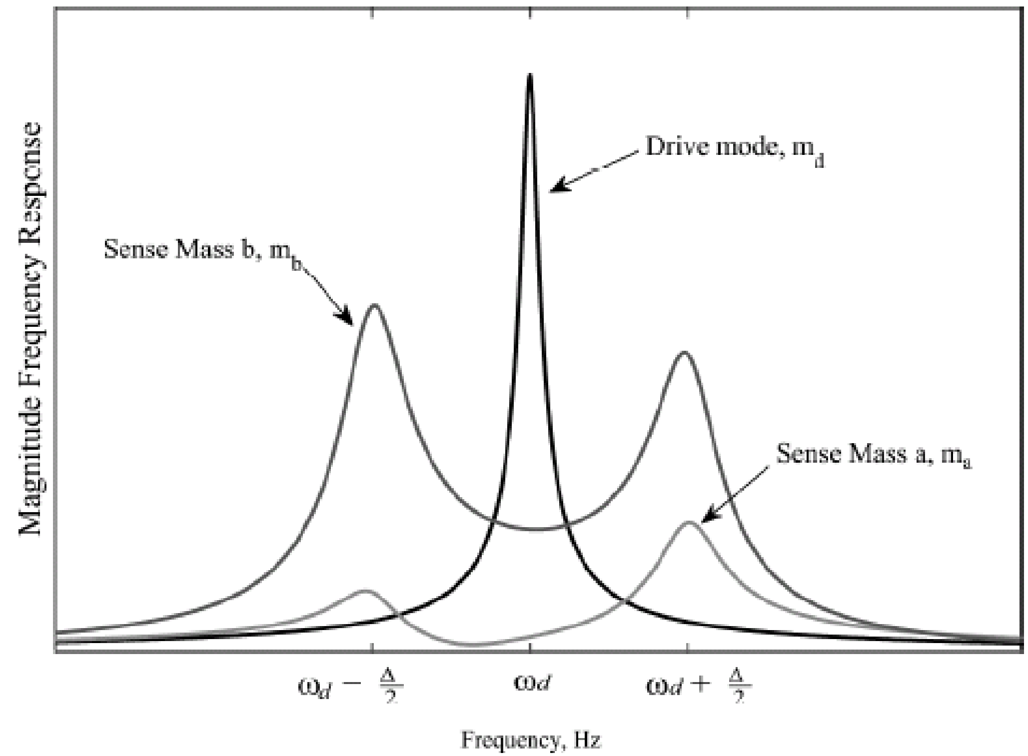
Sensitivity / Robustness trade-off





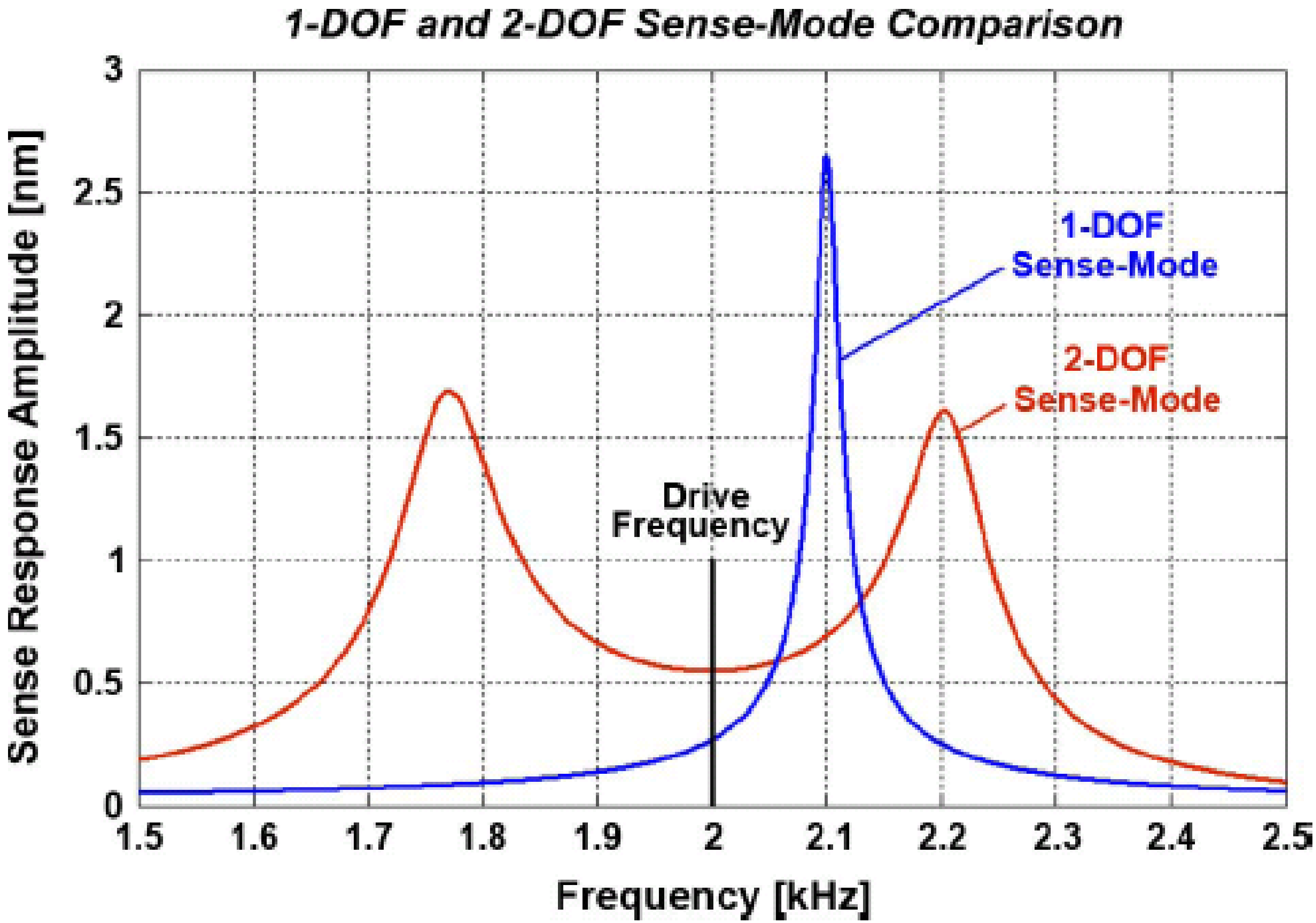
Multi-DOF Gyroscope



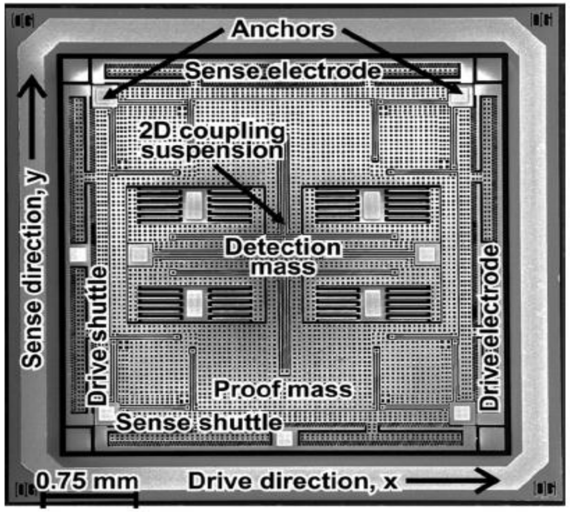


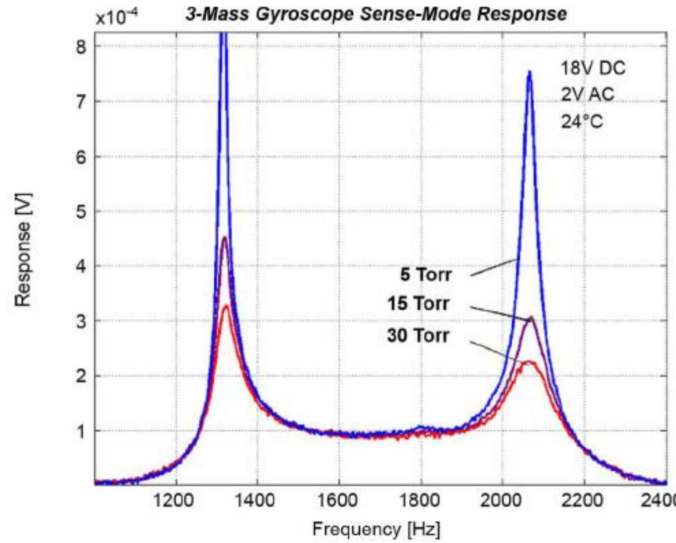
- Additional degrees of freedom allows the construction of a *band-pass filter* response for the sense mode.
- By selecting the drive frequency within the pass-band, an improved trade-off between sensitivity, bandwidth and robustness is possible.

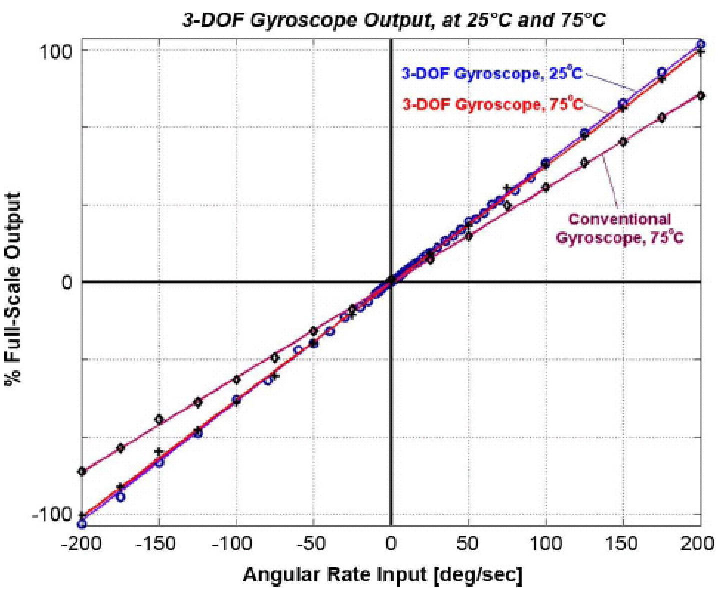
Comparison between 1-DOF and 2-DOF sense mode

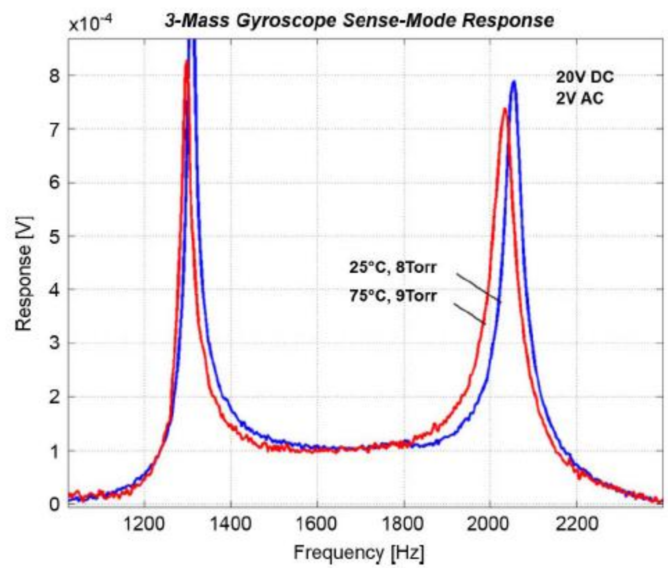


Fabricated MDOF gyroscope



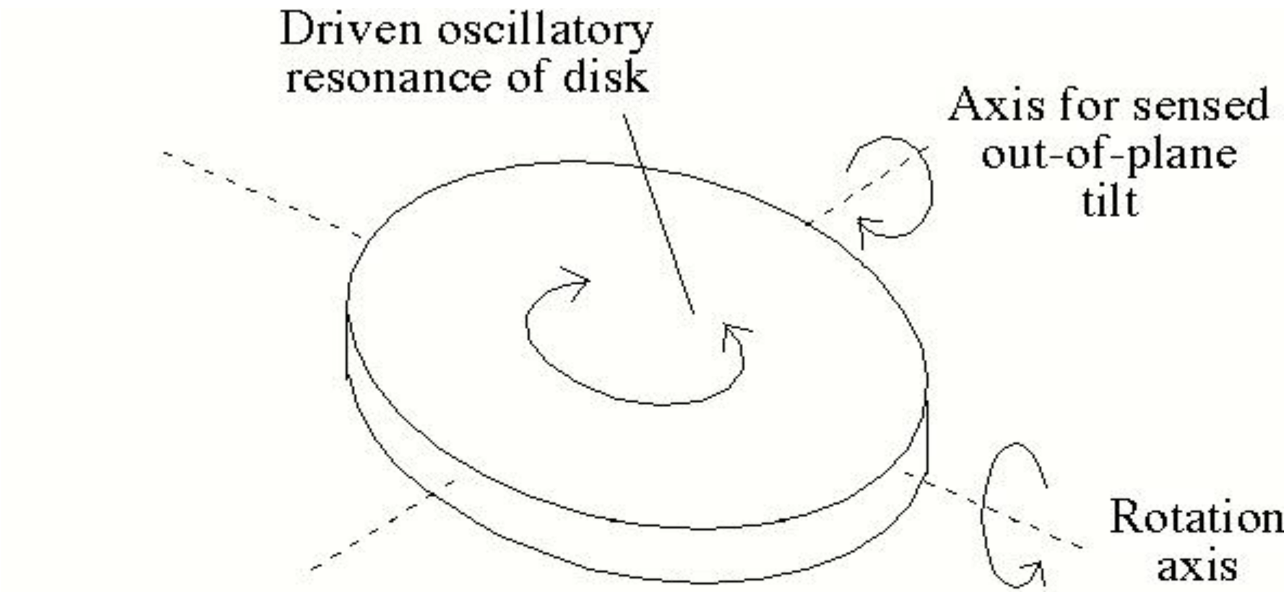




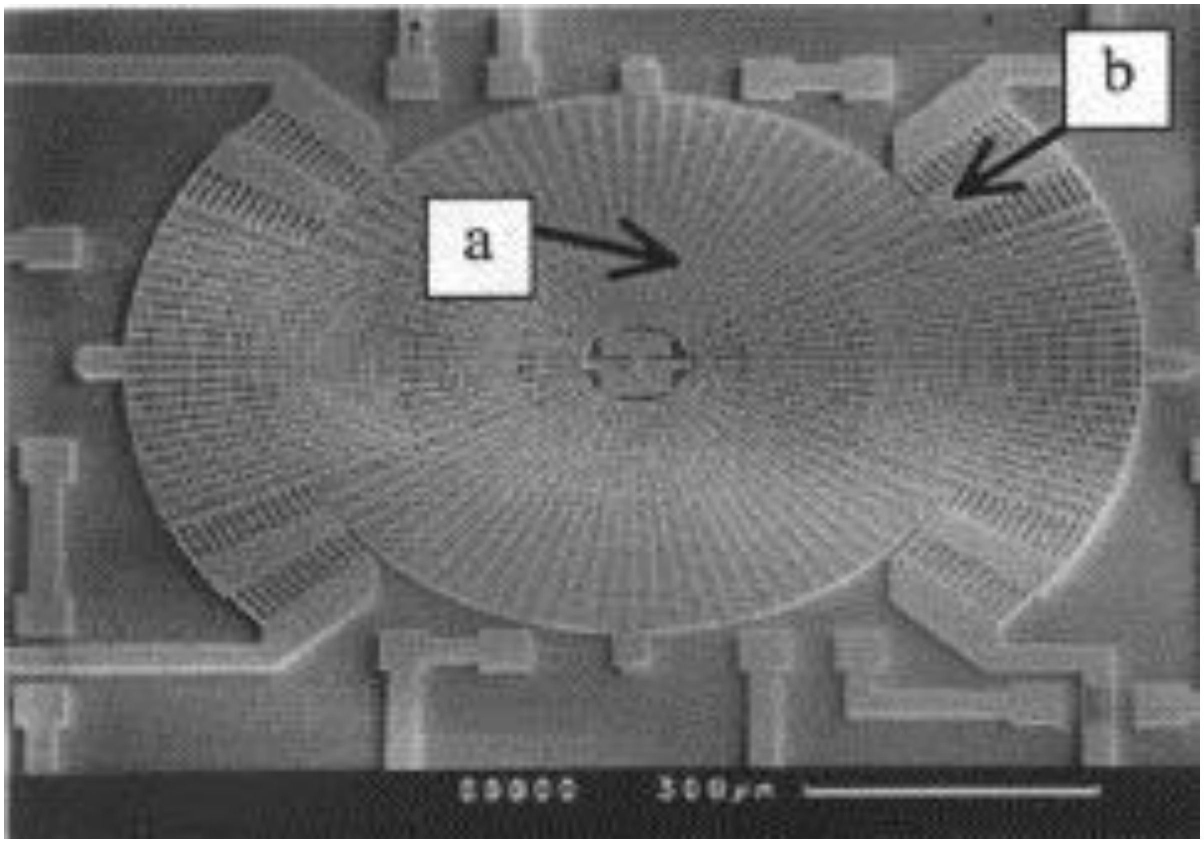


Generalized gyroscopic modes

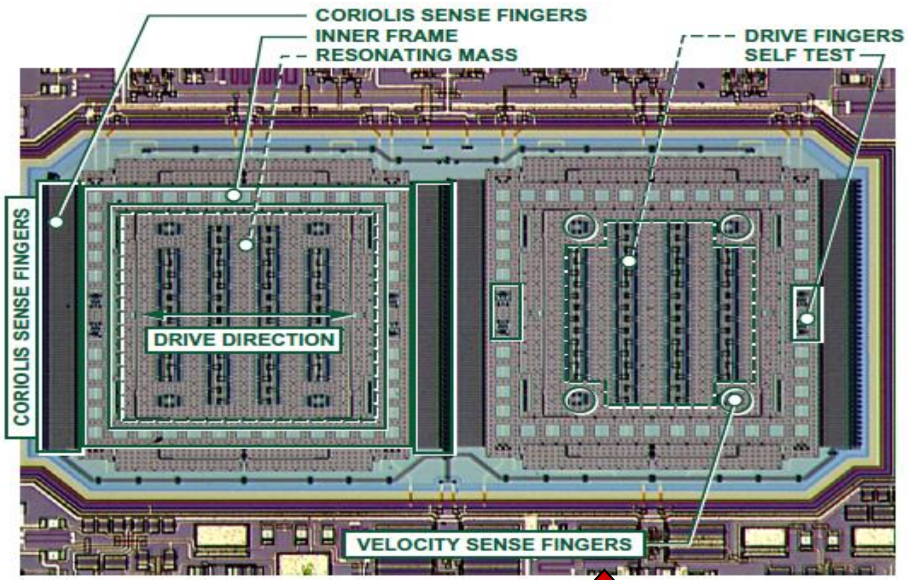
- Any two orthogonal modes will work

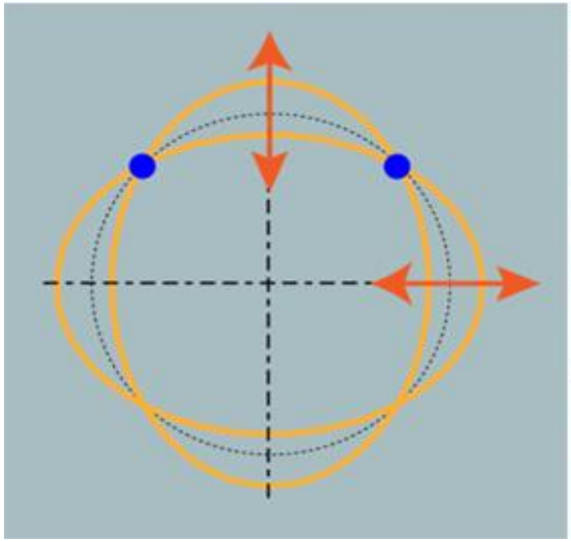


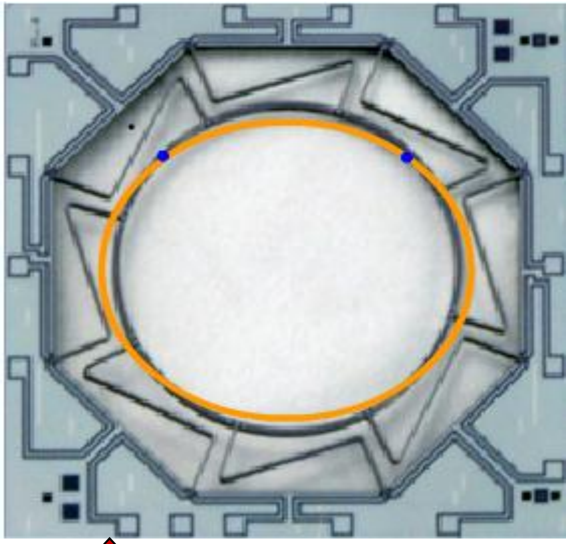
Bosch rotating-disk gyro



Commercial Microgyroscopes







Manufacturer	ADI	Bosch	Silicon Sensing Systems		InvenSense
Spec./Prod.	ADXRS450	SMG061	SiPRS01	CRS09-11	ISZ-1215
Range	±300°/sec	±240°/sec	±110°/sec	±200°/sec	±67°/sec
Sensitivity	80 LSB/°/sec	7mV/°/sec	18.18mV/°/sec	10mV/°/sec	15mV/°/sec
BW	80Hz	30Hz	50Hz	55Hz	–
Noise	0.015°/sec/rt-Hz	1.5°/sec/rt-Hz	0.35°/sec/rt-Hz	6°/hr/rt-Hz	–
Nonlinearity	0.25%	0.50%	1.00%	0.10%	0.50%