MICRO/NANOTECHNOLOGY ENTERPRISE MPHIL COURSE

MODULE NE.02

MEMS DESIGN – COURSEWORK HANDOUT 2

OBJECTIVE: To design a silicon micro-electro-mechanical *z*-axis vibratory rate gyroscope for a Brownian noise limited resolution of less than 1 deg/min/$\sqrt{Hz}$ over an operational bandwidth of 100 Hz.

INTRODUCTION

Micro-electro-mechanical vibratory rate gyroscopes are widely used for automotive (e.g. smart braking, active suspension) and hand-held devices (e.g. wireless mouse, camcorder image stabilisation). A single-axis vibratory rate gyroscope can be minimally abstracted as a two degree-of-freedom vibratory system. The operating principle of a vibratory gyroscope is based on coupling the vibrations of one degree-of-freedom to the other through the Coriolis force. In a typical implementation, the proof mass is set into periodic constant amplitude oscillation about one axis (drive axis) and in response to an applied rotation rate about an orthogonal direction (rotation axis), the Coriolis effect couples the motion of the proof mass along the drive axis to a third orthogonal axis (sense axis). The displacement of the proof mass in response to the Coriolis force can be measured optically or capacitively to obtain an estimate of the applied rotation rate.

REPORT

Your report should describe the key details of the design procedure and clearly justify all design decisions with analytical arguments wherever possible. Your report must also address all the questions posed in this handout, include any relevant plots, and describe any insight gained from the design process that may lead to enhanced versions of the starting design. It is recommended that you restrict the main body of your report to approximately 10 pages. The completed report should be labelled with your name and college and handed in to Rebecca Pritchard at the MPhil office in the Department of Materials Science. The deadline for all reports is 12 noon on March 22, 2013.

DEVICE DESCRIPTION

A top view of the starting design for a silicon micromachined *z*-axis vibratory microscope is shown in Fig. 1. Process and design constraints are described in the table on pages 2-3. The proof mass is set into constant amplitude oscillation along the drive direction (*y*-axis) using lateral comb drive actuators. The device is configured to respond to rotation rate about an axis orthogonal to the fabrication plane (*z*-axis). The displacement of the proof mass along the sense direction (*x*-axis) is measured optically. A clamped-end flexure arrangement is used to provide the required compliance along the sense and drive direction. The flexures are dimensioned of minimum width (2 μm) and a length of no longer than 500 μm. The maximum amplitude of motion along the drive direction is limited to 2% of the length of each supporting *y*-flexure to limit the bending stress in the flexures. The resonant frequency for drive motion of the proof mass is designed to be 10 kHz to provide rejection to environmental vibration. The drive and sense mode frequencies are mismatched by a factor of 10%. The device is packaged to operate at atmospheric pressure and it is assumed that the noise floor is limited by the Brownian noise of the structure. The design objective is to work out the dimensions of the proof mass and supporting flexures for the micromechanical gyroscope and the number and construction of the lateral comb drives required for actuation along the drive direction to meet the noise floor specification. You may assume that the process comprises of a single structural layer 6 μm thick and that the structure is suspended 2 μm above the substrate.

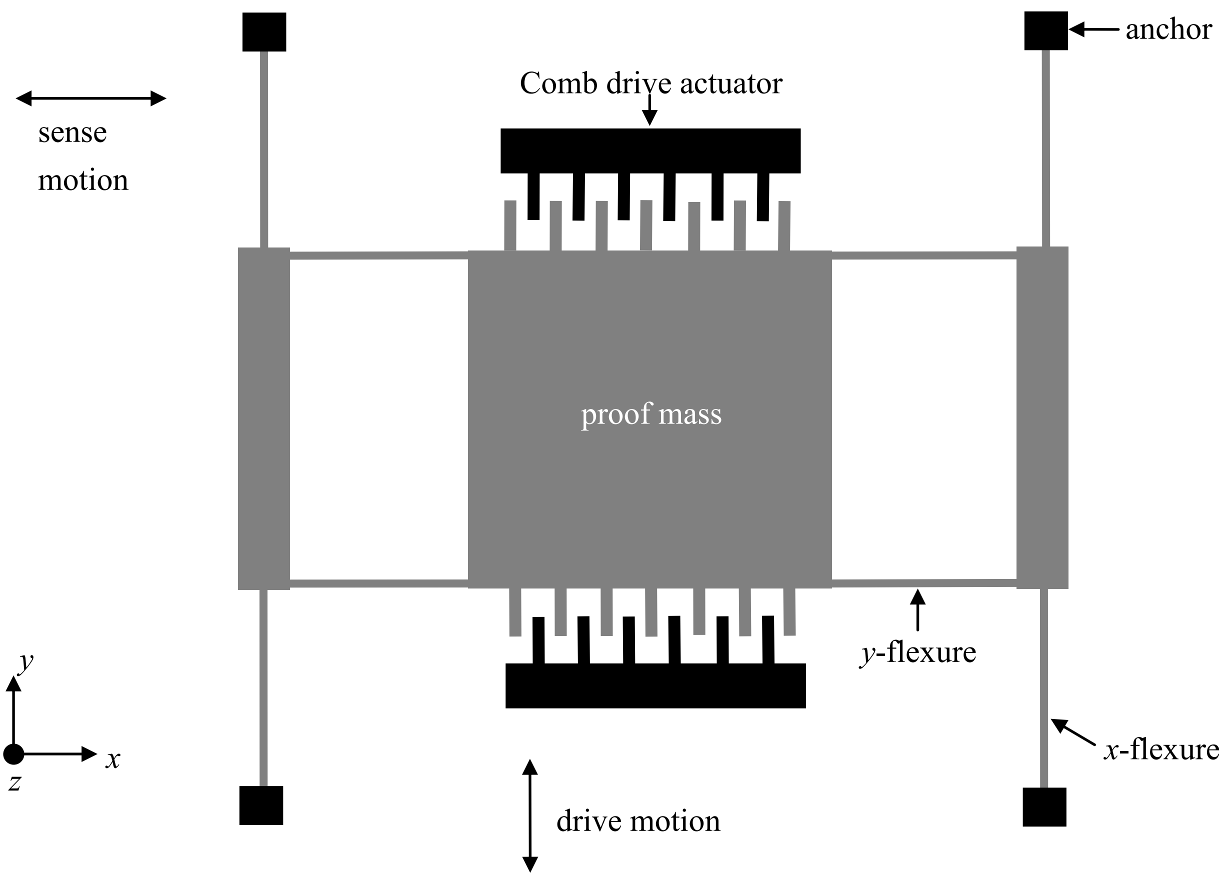

Figure 1: Schematic top view or layout of the micromechanical gyroscope.

DESIGN PROCEDURE

1. You are required to design a z-axis micromechanical gyroscope. Technological and design parameters are listed in the table shown below:

Parameter	**Value**
Material	Silicon
Process	SOI MEMS

Maximum chip design area	2 mm x 2 mm
Minimum dimension of proof mass	500 µm x 500 µm
Minimum beam width	2 µm
Structural thickness	6 µm
Maximum flexure length	500 µm
Minimum electrode gap	0.5 µm
Vertical gap between structure and substrate	2 µm
Drive Mode resonant frequency	10 kHz
Sense Mode resonant frequency	11 kHz
Maximum DC voltage	10 V
Maximum AC voltage	1 V
Operating Bandwidth	100 Hz
Brownian noise floor	< 1 deg/min/$\sqrt{Hz}$

2. **Hand analysis – Derivations**:

(a) Write down an expression for the spring constant of the structure along the drive and sense directions as a function of geometrical and material parameters.

(b) The proof mass is set into periodic oscillations along the drive direction at resonance. By describing the motion of the proof mass as a two-degree of freedom vibratory system, derive an expression for the displacement of the structure along the sense axis for an applied rotation rate Ω_z along the z-axis. Describe the bandwidth-sensitivity trade-off for a vibratory rate gyroscope based on your analysis.

(c) By equating the magnitude of the Coriolis force along the y-axis to the magnitude of the Brownian force acting along the same axis, work out an expression for Brownian noise of a gyroscope.

(d) Starting from an expression for stored energy in a capacitor, work out an expression for the actuator force generated by a lateral comb drive actuator as a function of geometry and material parameters, the number of elements in the array and the applied voltage between the actuator and the moving mass.

3. **Hand analysis – Numerical estimates:**

(a) Start with the expression derived in 2(c) for Brownian noise limited sensor resolution. If the resonant frequency along the drive and sense directions is fixed, this implies that a lower noise floor is achievable by maximising the drive mode displacement. Hence, maximise the displacement along the drive mode by considering the constraint that sets a relation between the length of the flexures and the driven displacement (to minimise bending stresses and mechanical nonlinearities) and a fixed resonant frequency along the drive direction. Note: Flexure lengths are constrained as indicated in the table.

(b) For the displacement above, choose flexure lengths for motion along the drive direction. Note: Beam widths and structural thickness has been constrained as indicated in the table.

(c) Given values for the drive mode resonant frequency, determine a value for the proof mass and the area of the mass (configured as a rectangular element). This should be larger than the lower limit specified in the table. Otherwise, you will have to iterate between the value of displacement, flexure length and mass to obtain reasonable results.

(d) Dimension the flexures for compliance along the sense direction to meet the resonant frequency constraint in this direction. Why is the sense mode resonant frequency designed to be higher than the drive mode resonant frequency?

(e) Estimate the quality factor for resonant motion along the drive mode by considering Couette flow between the proof mass and the substrate as the dominant damping mechanism. Note that the device is packaged to operate at atmospheric pressure. The viscosity of air at room temperature is 1.8×10^{-5} kg/m-s.

(f) Hence, estimate the magnitude of the actuator force required to achieve the deflection along the drive direction. How many comb fingers are required to construct a comb drive actuator to realise the required magnitude of driven displacement given the voltage constraints?

(g) The comb fingers add to the damping in the drive direction as well. Assuming a Couette flow model for damping due to overlapping comb fingers, recalculate the Quality factor for motion along the drive mode. You may have to iterate between the value for number of comb fingers and calculated displacement. *Hint: Operating at voltages lower than the specified maximum AC voltage will allow you to converge more quickly to the desired answer.*

(h) Note that the actuated mass is now distributed between the comb fingers and the rectangular proof mass. Obtain dimensions for the rectangular proof mass element.

(i) Work out a value for Quality factor for motion along the sense mode. You can assume that Couette flow between the mass and the substrate and a squeeze film flow model between adjacent comb fingers are the dominant damping mechanisms for motion along this direction.

(j) Using the value of Quality factor for motion along the sense mode, estimate the Brownian noise limited resolution? Have you meet the required specification of 1 deg/min/$\sqrt{Hz}$. If not, you will have to iterate on your design choices.

(k) Estimate the magnitude of the deflection for motion along the sense direction due to the Coriolis force for an applied rotation rate of 1 deg/sec about the *z*-axis. Estimate the ratio of the displacement due to the Coriolis force to the displacement along the drive direction for this rotation rate of 1 deg/sec. You will note that this ratio is extremely small. Explain how this might impact device performance.

(l) In actual practice, the design space is less constrained and the number of objectives far greater. Give examples of what these additional objectives might be for a z-axis micro-electro-mechanical vibratory rate gyroscope and describe how this will impact the design process.

4. Comment in your report on how you might improve upon this starting design for the measurement of z-axis rotation rate. Also discuss the impact of manufacturing imperfections on device performance and how this impact might be mitigated through design.

Reference: H. Xie and G. Fedder, "Integrated Micromechanical Gyroscopes", *Journal of Aerospace Engineering*, **16**(2), pp. 65-75, April 2003.