

Selected exercises 02

1. Give the Dedekind cuts in $\mathbb{R}^{\geq 0}$ corresponding to the following. Your definition should not refer to the elements themselves.
(a) $\sqrt{7}$ (c) $2 + \sqrt[3]{5}$ (e) $\sqrt{2} + 1$ (g) $3 - \sqrt{2}$
(b) $\sqrt[3]{11}$ (d) $\sqrt[4]{8}$ (f) $4 - \sqrt{7}$ (h) $3 - \sqrt{3}$
2. Prove that the following are not rational numbers
(a) $\sqrt{7}$ (b) $\sqrt{3}$ (c) $\sqrt{2} + \sqrt{17}$ (d) $\sqrt{3} + \sqrt{13}$
3. Prove that the square root of a positive integer is either an integer or an irrational.
4. Decide if following sets are bounded from above/below.
(a) $S = \{1, 2, 3\}$ (c) $S = \{0\} \cup \{x \mid x > 0\}$ (e) $S = \{x^3 \mid x \in \mathbb{Z}\}$
(b) $S = \{x \mid x \geq 5\}$ (d) $S = \{x^2 \mid x < -2\}$ (f) $S = \{x^2 - x \mid x \geq 1\}$
5. Let $F = \{a - b\sqrt{5} \mid a, b \in \mathbb{Q}\}$. Prove that F is a field. (*Hint:* use that \mathbb{R} is a field.)
6. Let $n \in \mathbb{N}$, $n > 0$. Prove that $\sqrt{n + \sqrt{n}}$ is irrational.
7. Prove that the set of remainders modulo 5 is a field.