Student: Anastasija Temova
Student ID: 89221055

Homework 3

A. Using the definition of limit, show that $\lim_{n\to\infty} 1/n^5 = 0$ and that $\lim_{n\to\infty} 1/n^{1/5} = 0$

We need to prove that $\forall \varepsilon > 0 \ \exists n_0 \ \forall n > n_0 \ \left|\frac{1}{n^5} - 0\right| < \varepsilon$

Let $\varepsilon > 0$ $\exists n_0$ is a natural number so that $n_0 \geq \sqrt[5]{\frac{1}{\varepsilon}}$

and let $n > n_0$, then:

$$\Rightarrow |a_n - L| = \left|\frac{1}{n^5} - 0\right| = \frac{1}{n^5} < \frac{1}{n_0^5} \leq \varepsilon$$

For $\frac{1}{n_0^5} \leq \varepsilon$

$$n_0^5 \geq \frac{1}{\varepsilon}$$

$$n_0 \geq \sqrt[5]{\frac{1}{\varepsilon}} \quad \blacksquare$$

Let $\varepsilon > 0$ $\exists n_0$ is a natural number so that $n_0 \geq \frac{1}{\varepsilon^5}$

and let $n > n_0$, then:

$$\Rightarrow |a_n - L| = \left|\frac{1}{n^{1/5}} - 0\right| = \frac{1}{n^{1/5}} < \frac{1}{n_0^{1/5}} \leq \varepsilon$$

For $n_0^{1/5} \geq \frac{1}{\varepsilon} \quad /()^5$

$$n_0 \geq \frac{1}{\varepsilon^5} \quad \blacksquare$$