Scores	
1.	
2.	
3.	
4.	
5.	
6.	
7.	
Total:	

ANA-I Foundations of Analysis
Final Examination B – 15 Feb 2022

Name ________________________________

General Instructions: Please answer the following, showing all your work and writing neatly. You may have 1 handwritten A4-sized sheet of paper, but no other notes, books, or calculators.
110 total points.

1. (6 points each) Calculate the following limits, or explain why they diverge. You may use any theorems we have proved in class or on homework.

   (a) $\displaystyle\lim_{n\to\infty} \frac{n^3+4n-4}{1-4n^3}$

   (b) $\displaystyle\lim_{x\to 2-} \frac{x^2-4}{x-2}$

   (c) $\displaystyle\lim_{x\to -2} \frac{x^2-4}{x-2}$

   (d) $\displaystyle\lim_{x\to -2} \frac{x^2+4}{x+2}$

   (e) $\displaystyle\lim_{x\to\infty} \frac{3\sin x^2 - 6\cos(x/\sqrt{2})}{x-6}$

2. (6 points each) Series

   (a) Find the exact value that $\displaystyle\sum_{n=1}^{\infty} \frac{2^n-1}{3^{n+2}}$ converges to, or else conclude that the series diverges.

   (b) Determine whether $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \cdot n^2+3}{n^4-4}$ converges absolutely, converges conditionally, or diverges.

3. (6 points each) Examples. Justify your answers briefly.

   (a) Give an example of a metric on $\mathbb{R}^2$.

   (b) Is the set of all points (x, y) in $\mathbb{R}^2$ such that $y = x^2$ sequentially compact? Explain why or why not.

   (c) Verify that the product of the Dedekind cut for 2 and that for 3 is the Dedekind cut for 6.

   (d) Explain why a strictly decreasing function is injective.

(-see reverse side-)

4. (10 points) Let a_n be recursively defined by $a_1 = 3/4$, $a_{n+1} = \dfrac{n}{n+2} \cdot a_n^2$ for $n \geq 1$. Find $\lim\limits_{n\to\infty} a_n$.

5. (12 points) Show directly from definition that if $\lim_{x\to 2-} f(x) = 1$ and $\lim_{x\to 2-} g(x) = 4$, then $\lim_{x\to 2-} f(x) \cdot g(x) = 4$.

6. (12 points) Estimate the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{2^n - 5n}$ to within an accuracy of 0.5.

7. Prove that there is no sequence of real numbers whose set of accumulation points is exactly $\mathbb{Q}$.