1 Pushdown automata

1.1 Definition

A pushdown automaton (PDA) is a nondeterministic finite automaton with ε transitions. It also has a *stack* where it can store symbols.

A **pushdown automaton** can be defined as $P = (Q, \Sigma, \Gamma, \delta, q_0.Z_0, F)$, where:

- Q is the finite set of states
- Σ : input alphabet (finite set of symbols)
- Γ : stack alphabet (finite set of symbols)
- $q_0 \in Q$: starting state
- $Z_0 \in F$: starting stack symbol
- $F \subseteq Q$: set of accepting states
- $\delta : Q \times (\Sigma \cup \{\varepsilon\}) \times \Gamma \to \mathcal{P}_w(Q \times \Sigma^*)$

1.2 Instantaneous descriptors

Configurations of P: $C = Q \times \Sigma^* \times \Gamma^*$ A configuration can be represented as $(q, w, \gamma) \in C$, where

- q is the current state of P,
- w is the remaining input,
- γ is the string on the stack.

Suppose configuration $(q, aw, Z\gamma)$ and transition $\delta(q, a, Z) = \{..., (p, X), ...\}$. Then

$$(q, aw, Z\gamma) \vdash (p, w, X\gamma)$$

Accepting with final state:

$$L_f = \{w \in \Sigma^* \mid (q_0, w, Z_0) \vdash^* (q, \varepsilon, \delta),\ \text{where } q \in F, \delta \in \Sigma^*\}$$

Accepting with empty stack:

$$L_\emptyset = \{w \in \Sigma^* \mid (q_0, w, Z_0) \vdash^* (q, \varepsilon, \varepsilon),\ \text{where } q \in Q\}$$

1.3 PDA Exercises

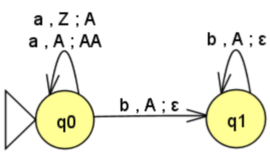
Provide a PDA for the following languages:

1. $L = \{a^n b^n \mid n \geq 0\}$
2. $L = \{a^n b^{2n} \mid n \geq 0\}$
3. $L = \{wcw^{-1} \mid w \in \{a, b\}^*\}$
4. $L = \{a^{2n} b^n \mid n \geq 0\}$
5. $L = \{a^n b^m | n < m\}$

$L = \{a^n b^n \mid n \geq 0\}$

Empty stack acceptance: $P = (Q, \Sigma, \Gamma, \delta, q_0, Z, \emptyset)$, where:

- $Q = \{q_0, q_1\}$
- $\Sigma = \{a, b\}$
- $\Gamma = \{Z, A\}$
- δ : defined by the following PDA



Final state acceptance: $P = (Q, \Sigma, \Gamma, \delta, q_0, Z, \{q_2\})$, where:

- $Q = \{q_0, q_1, q_2\}$
- $\Sigma = \{a, b\}$
- $\Gamma = \{Z, A\}$
- δ : defined by the following PDA

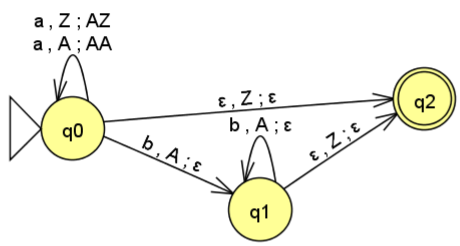
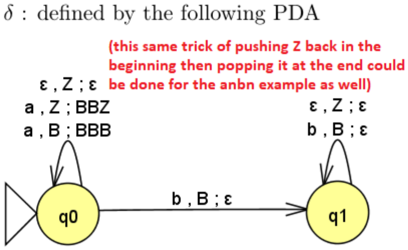


Figure 1: Exercise 1

$L = \{a^n b^{2n} \mid n \geq 0\}$

Empty stack acceptance: $P = (Q, \Sigma, \Gamma, \delta, q_0, Z, \emptyset)$, where:

- $Q = \{q_0, q_1\}$
- $\Sigma = \{a, b\}$
- $\Gamma = \{Z, B\}$
- δ : defined by the following PDA



Final state acceptance: $P = (Q, \Sigma, \Gamma, \delta, q_0, Z, \{q_2\})$, where:

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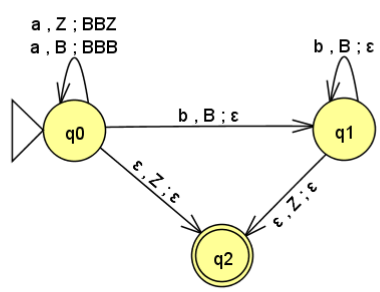
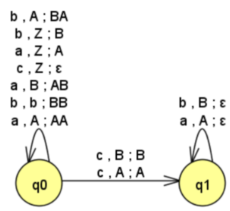


Figure 2: Exercise 2

$L = \{wcw^{-1} \mid w \in \{a, b\}^*\}$

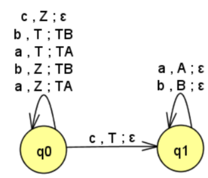
Empty stack acceptance: $P = (Q, \Sigma, \Gamma, \delta, q_0, Z, \emptyset)$, where:

- $Q = \{q_0, q_1\}$
- $\Sigma = \{a, b, c\}$
- $\Gamma = \{Z, A, B\}$
- δ : defined by the following PDA



Alternative empty stack acceptance: $P = (Q, \Sigma, \Gamma, \delta, q_0, Z, \emptyset)$, where:

- $Q = \{q_0, q_1\}$
- $\Sigma = \{a, b, c\}$
- $\Gamma = \{Z, A, B, T\}$
- δ : defined by the following PDA



Final state acceptance: $P = (Q, \Sigma, \Gamma, \delta, q_0, Z, \{q_2\})$, where:

- $Q = \{q_0, q_1, q_2\}$
- $\Sigma = \{a, b, c\}$
- $\Gamma = \{Z, A, B, T\}$
- δ : defined by the following PDA

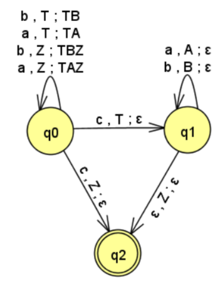
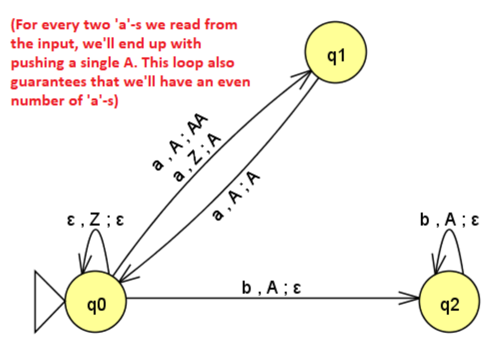


Figure 3: Exercise 3

$L = \{a^{2n}b^n \mid n \geq 0\}$

Empty stack acceptance: $P = (Q, \Sigma, \Gamma, \delta, q_0, Z, \emptyset)$, where:

- $Q = \{q_0, q_1, q_2\}$
- $\Sigma = \{a, b\}$
- $\Gamma = \{Z, A\}$
- δ : defined by the following PDA



Final state acceptance: $P = (Q, \Sigma, \Gamma, \delta, q_0, Z, \{q_3\})$, where:

- $Q = \{q_0, q_1, q_2, q_3\}$
- $\Sigma = \{a, b\}$
- $\Gamma = \{Z, A\}$
- δ : defined by the following PDA

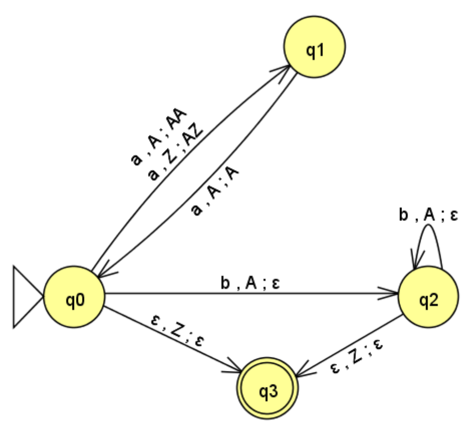
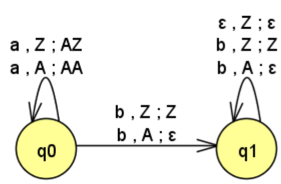


Figure 4: Exercise 4

$L = \{a^n b^m | n < m\}$

Empty stack acceptance: $P = (Q, \Sigma, \Gamma, \delta, q_0, Z, \emptyset)$, where:

- $Q = \{q_0, q_1\}$
- $\Sigma = \{a, b\}$
- $\Gamma = \{Z, A\}$
- δ : defined by the following PDA



Final state acceptance: $P = (Q, \Sigma, \Gamma, \delta, q_0, Z, \{q_2\})$, where:

- $Q = \{q_0, q_1, q_2\}$
- $\Sigma = \{a, b\}$
- $\Gamma = \{Z, A\}$
- δ : defined by the following PDA

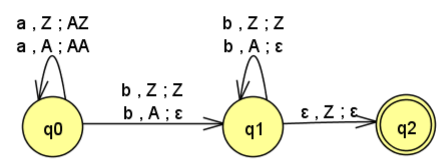


Figure 5: Exercise 5