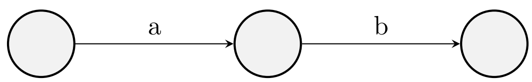
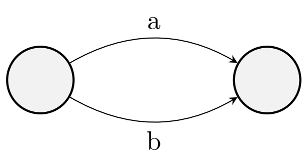
Conversions between RE, ε-NFA an DFA

1 Regular expressions (RE) to ε-NFA

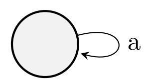
Regular expressions are used to generate patterns of strings. The set of strings generated by a RE is called the language of the RE. An RE consists of *characters*, *variables*, ε, $\emptyset$. The following operators can be performed on these.



(a) Concatenation (ab)

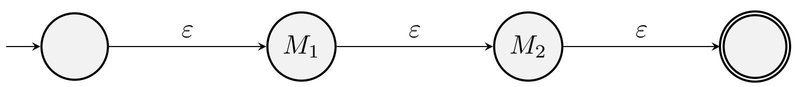


(b) Union ($a|b$, $a+b$, $a \cup b$)

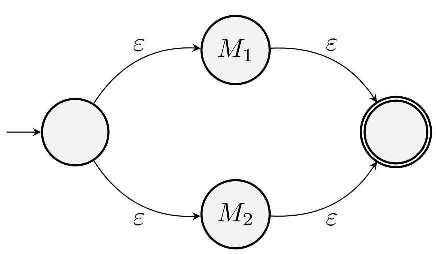


(c) Kleene Star/Closure (a^*)

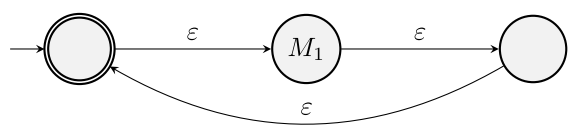
Let M_k be the automaton that recognizes the language generated by regular expression R_k. The operators on such expressions:



(d) Concatenation (R_1R_2)



(e) Union ($R_1|R_2$, $R_1 + R_2$, $R_1 \cup R_2$)



(f) Kleene Star/Closure (R_1^*)

1.1 Exercises

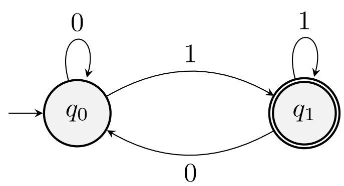
Give the ε-NFA-s for the following RE-s over alphabet $\Sigma = \{0, 1\}$:

1. $R_1 = 0^*1$
2. $R_2 = 0^* \cup 1^*$

3. $R_3 = 0\Sigma^*1$
4. $R_4 = (11)^*$
5. $R_5 = R_1^*0$

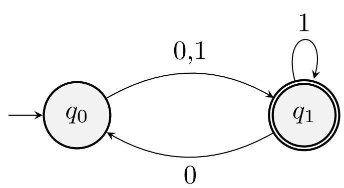
2 DFA to RE

2.1 State elimination



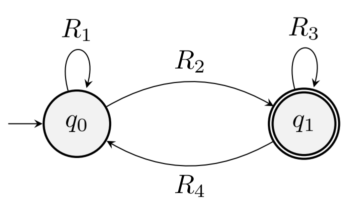
$R_1 = (0 \cup 11^*0)^*11^*$

DFA 1 and its RE



$R_2 = ((0 \cup 1)1^*0))^*(0 \cup 1)1^*$

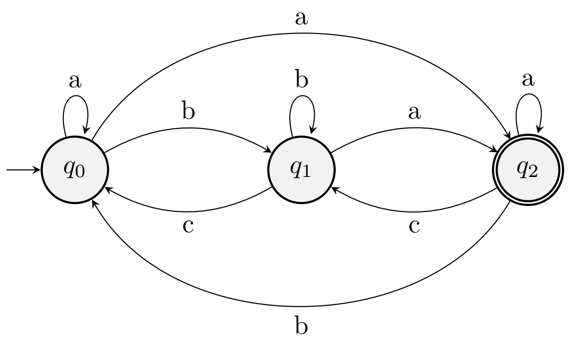
DFA 2 and its RE



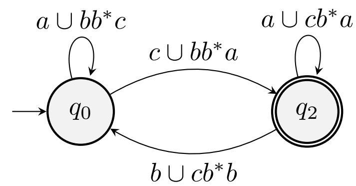
$R = (R_1 \cup R_2R_3^*R_4)^*R_2R_3^*$

DFA to RE - general

Use state elimination to determine the RE of the following DFA



(n) DFA 4



(o) DFA 4 with an eliminated state. The individual expressions can be combined into one RE similarly to the general example above.

2.2 k-path algorithm

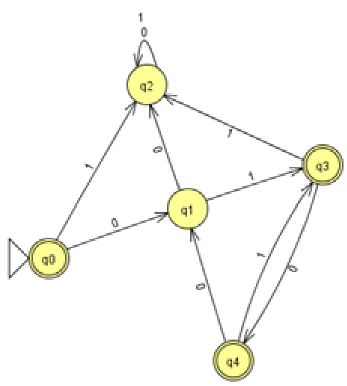
Generates regular expressions between pairs of nodes in an iterative way, every iteration has a limit on which nodes can be passed while generating the expression.

An initial table is created that shows the RE between each (p, q) state pair in the case when you only consider direct transitions (you don't move through any other staten than p or q, and don't loop). Then, an iterative algorithm is executed starting with $k = 0$. In each iteration, you fill in table R^k with the help of R^{k-1}, but in this new table, you also allow transitioning through state q_k when you build the REs.

General formula of the iteration:

$$R^k_{i,j} = R^{k-1}_{i,j} + R^{k-1}_{i,k}(R^{k-1}_{k,k})^* R^{k-1}_{k,j}.$$

Execute the algorithm for the following DFA:



R^0	q_0	q_1	q_2	q_3	q_4
q_0	ε	0	1	$\emptyset$	$\emptyset$
q_1	$\emptyset$	ε	0	1	$\emptyset$
q_2	$\emptyset$	$\emptyset$	ε+0+1	$\emptyset$	$\emptyset$
q_3	$\emptyset$	$\emptyset$	1	ε	0
q_4	$\emptyset$	0	$\emptyset$	1	ε

R^1	q_0	q_1	q_2	q_3	q_4
q_0	ε	0	1+00	01	$\emptyset$
q_1	$\emptyset$	ε	0	1	$\emptyset$
q_2	$\emptyset$	$\emptyset$	ε+0+1	$\emptyset$	$\emptyset$
q_3	$\emptyset$	$\emptyset$	1	ε	0
q_4	$\emptyset$	0	00	1+01	ε

R^2	q_0	q_1	q_2	q_3	q_4
q_0	ε	0	(1 + 00)(0+1)*	01	$\emptyset$
q_1	$\emptyset$	ε	(0+1)*	1	$\emptyset$
q_2	$\emptyset$	$\emptyset$	(0+1)*	$\emptyset$	$\emptyset$
q_3	$\emptyset$	$\emptyset$	1(0+1)*	ε	0
q_4	$\emptyset$	0	00(0+1)*	1+01	ε

Example for applying the formula:
$R^2_{0,2}$ = (1+00)+(1+00)(ε+0+1)*(ε+0+1) = (1+00)(0+1)*

R^3	q_0	q_1	q_2	q_3	q_4
q_0	ε	0	(1+00+011)(0+1)*	01	010
q_1	$\emptyset$	ε	(0+11)(0+1)*	1	10
q_2	$\emptyset$	$\emptyset$	(0+1)*	$\emptyset$	$\emptyset$
q_3	$\emptyset$	$\emptyset$	1(0+1)*	ε	0
q_4	$\emptyset$	0	(00+11+011)(0+1)*	1+01	ε+10+010

Examples for applying the formula:
$R^3_{0,2}$ = (1+00)(0+1)*+01ε*1(0+1)* = (1+00+011)(0+1)*
$R^3_{1,2}$ = 0(0+1)*+1ε*1(0+1)* = (0+11)(0+1)*
$R^3_{4,2}$ = 00(0+1)*+(1+01)ε*1(0+1)* = (00+11+011)(0+1)*

R^4 is our final table. We do not have to calculate everything, as we only want to know the REs that start in the initial state and end in an accepting state.
These are:
$RE = R^4_{0,0} + R^4_{0,3} + R^4_{0,4}$

$R^4_{0,0} = \varepsilon$
$R^4_{0,3}$ = 01+010(ε+10+010)*(1+01)
$R^4_{0,4}$ = 010+010(ε+10+010)*(ε+10+010) = 010(10+010)*