TCS II

Formal Languages and Computability
2021/22

2nd Midterm (A)

10th June 2022

Solutions

1. Assignment:

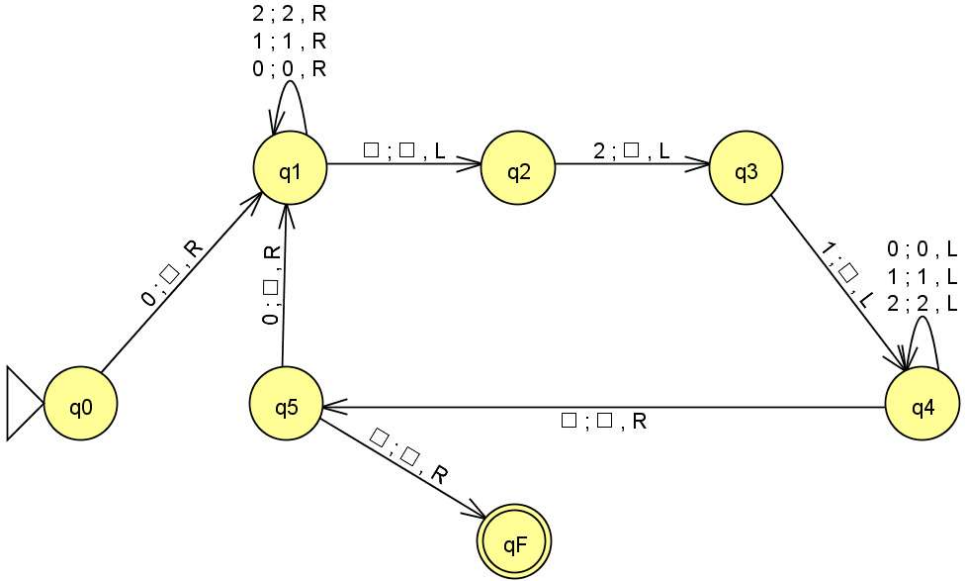
Question (a) (TM for the given language)

TM for language L_1:

M = <Q, Σ, Γ, q_0, B, F, δ>

Q = {q_0, q_1, q_2, q_3, q_4, q_5, q_F} (**1 point**)
Σ = {0, 1, 2} (**1 point**)
Γ = {0, 1, 2, □} (**1 point**)
q_0 = q_0 (**1 point**)
B = □ (**1 point**)
F = {q_F} (**1 point**)

δ (graph): (**14 points**)



δ (transitions):

δ(q_0, 0) = (q_1, □, R)
δ(q_1, 0) = (q_1, 0, R)
δ(q_1, 1) = (q_1, 1, R)
δ(q_1, 2) = (q_1, 2, R)
δ(q_1, □) = (q_2, □, L)
δ(q_2, 2) = (q_3, □, L)
δ(q_3, 1) = (q_4, □, L)
δ(q_4, 0) = (q_4, 0, L)
δ(q_4, 1) = (q_4, 1, L)
δ(q_4, 2) = (q_4, 2, L)
δ(q_4, □) = (q_5, □, R)
δ(q_5, 0) = (q_1, □, R)
δ(q_5, □) = (q_F, □, R)

δ (transition table):

Q \ Σ	0	1	2	□
→ q_0	(q_1, □, R)	/	/	/
q_1	(q_1, 0, R)	(q_1, 1, R)	(q_1, 2, R)	(q_2, □, L)
q_2	/	/	(q_3, □, L)	/
q_3	/	(q_4, □, L)	/	/
q_4	(q_4, 0, L)	(q_4, 1, L)	(q_4, 2, L)	(q_5, □, R)
q_5	(q_1, □, R)	/	/	(q_F, □, R)
* q_F	/	/	/	/

Question (b) (IDs)

001212 ∈ L(M) with IDs (**10 points**)

$q_0001212 \vdash q_101212 \vdash 0q_11212 \vdash 01q_1212 \vdash 012q_112 \vdash 0121q_12 \vdash 01212q_1 \vdash 0121q_22 \vdash 012q_31 \vdash 01q_42$

$\vdash 0q_412 \vdash q_4012 \vdash q_4\square012 \vdash q_5012 \vdash q_112 \vdash 1q_12 \vdash 12q_1 \vdash 1q_22 \vdash q_31 \vdash q_4 \vdash q_5 \vdash q_F$

2. Assignment:

Question (a) (turn the given CFG into CNF)

Given the CFG:

S → XYZ | YZ
X → **x**X | **y**
Y → **x** | X
Z → **z**
W → **x**X | **yy** | **z**

2. step – remove unit productions: (**2 points**)

S → XYZ | YZ
X → **x**X | **y**
Y → **x** | **x**X | **y**
Z → **z**
W → **x**X | **yy** | **z**

1. step – remove ε-productions: (**1 point**)
there are no such productions.

3. step – remove variables that don't produce strings of all terminals: (**1 point**)
there are no such variables.

4. step – remove »unreachable« variables:
variable W is »unreachable« (**2 points**)

S → XYZ | YZ
X → **x**X | **y**
Y → **x** | **x**X | **y**
Z → **z**

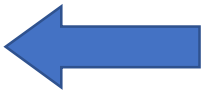
5. step – Chomsky Normal Form (CNF):

a) Terminals → Variables: (**2 points**)

S → XYZ | YZ
X → AX | **y**
Y → **x** | AX | **y**
Z → **z**
A → **x**

b) »Shortening«: (**2 points**)

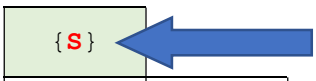
S → XB | YZ
X → AX | **y**
Y → **x** | AX | **y**
Z → **z**
A → **x**
B → YZ



CNF

Question (b) (use the CYK algorithm)

Is **xxyxz** in the language of the given CFG? (**15 points**)



Yes. The string **xxyxz** is in the language of the given CFG.

{S}				
{ }	{S}			
{X,Y}	{ }	{S}		
{ }	{X,Y}	{ }	{S,B}	
{Y,A}	{Y,A}	{X,Y}	{Y,A}	{Z}
x	**x**	**y**	**x**	**z**

3. Assignment:

Question (a) (reduction from MPCP to PCP)

MPCP to PCP reduction: (**10 points**)

idx_{MPCP}	MPCP	PCP	idx_{PCP}
1	(a,ab)	(a*,*a*b)	1
2	(b,ca)	(b*,*c*a)	2
3	(ca,a)	(c*a*,*a)	3
4	(abc,c)	(a*b*c*,*c)	4
		(*a*,*a*b)	S1
		($,*$)	F

Question (b) (solution to MPCP – and PCP)

The solution to the given MPCP is the index sequence 1, 2, 3, 1, 4 (shown below): (**7 points**)

Pair index	1	2	3	1	4	concatenated string
1st element of pair	a	b	ca	a	abc	→ abcaaabc
2nd element of pair	ab	ca	a	ab	c	→ abcaaabc

The solution to the related PCP is the index sequence S1, 2, 3, 1, 4, F (shown below): (**3 points**)

Pair index	S1	2	3	1	4	F	concatenated string
1st element of pair	*a*	b*	c*a*	a*	a*b*c*	$	→ *a*b*c*a*a*a*b*c*$
2nd element of pair	*a*b	*c*a	*a	*a*b	*c	*$	→ *a*b*c*a*a*a*b*c*$

4. Assignment:

Question (a) (conversion to 3-CNF)

There are 3 Boolean variables, so direct conversion to 3-CNF is possible (using the truth table).

Truth table: (**5 points**)

A	B	C	A(–C) + B
T	T	T	T
T	T	F	T
T	F	T	F
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	F
F	F	F	F

Falsifying variable assignments: (**5 points**)

(**A** = T, **B** = F, **C** = T),

(**A** = F, **B** = F, **C** = T),

(**A** = F, **B** = F, **C** = F).

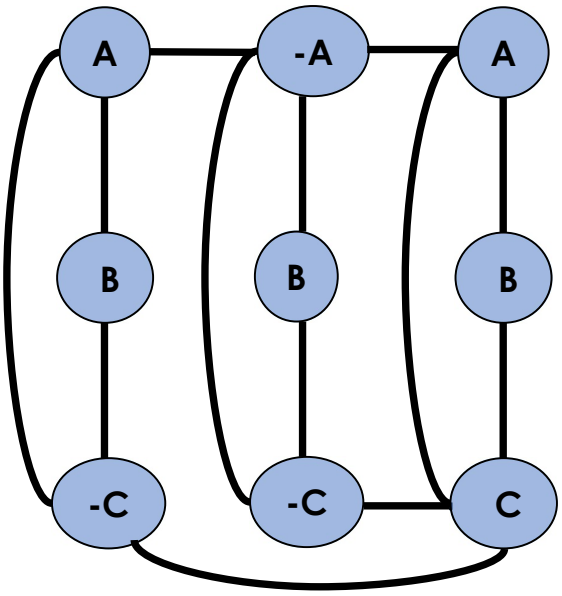
Resulting expression in 3-CNF: (**5 points**)

(–A + B + –C) (A + B + –C) (A + B + C)

Question (b) (reduction from 3-SAT to VC)

Graph form the 3-CNF expression
(order of clauses changed to ease connecting variables with their negations): (**5 points**)

(A + B + –C) (–A + B + –C) (A + B + C)



Select a truth assignment (**A** = T, **B** = T, **C** = T) and pick a true node in each column:

All the non-selected nodes form the node cover (**green nodes**) (**5 points**)

Budget = 6