Satisfiability exercises

1. Transform the following BE into a 3-CNF:
   $x(y + -z)(-x + y + z + v)$

   Solution:
   The above expression is a CNF. It can be transformed into a 3-CNF. Depending on the number of literals in each clause, new variables should be introduced based on the following steps:

   - If our clause C consists of a **single literal** l, we have to introduce new variables v_1, v_2. The we will replace C with four new clauses:
     $(l + v_1 + v_2)(l + -v_1 + v_2)(l + v_1 + -v_2)(l + -v_1 + -v_2)$
     In the case of our clause with only x, we should simply use the above formula and substitute x for all instances of l
   - If our clause C consists of **two literals** l_1, l_2 (or simply $C = (l_1 + l_2)$) we only have to introduce a single new variable v_1, and replace C with the following two clauses: $(l_1 + l_2 + v_1)(l_1 + l_2 + -v_1)$
     In the clause of our clause $(y + -z)$, we should simply introduce v_1, and replace the clause with the following:
     $(y + -z + v_1)(y + -z + -v_1)$
   - If our clause C consists of **three literals**, we do not change it. We have no clauses with 3 literals.
   - If our clause C consists of $k \geq 4$ **literals** (or $C = (l_1 + l_2 + ... + l_k)$), he have to introduce $k - 3$ new variables $v_1, v_2, ..., v_{k-3}$, then replace C with the following series of clauses:
     $(l_1 + l_2 + v_1)(l_3 + -v_1 + v_2)(l_4 + -v_2 + v_3)...(l_{k-2} + -v_{k-4} + v_{k-3})(l_{k-1} + l_k + -v_{k-3})$
     In the case of our clause $(-x + y + z + v)$, as $k = 4$, we should introduce one new variable v_1, and replace the clause with the following:
     $(-x + y + v_1)(z + v + -v_1)$

   Combining all the cases above, the transformed 3-CNF is the following:
   $(x + v_1 + v_2)(x + -v_1 + v_2)(x + v_1 + -v_2)(x + -v_1 + -v_2)(y + -z + v_1)(y + -z + -v_1)(-x + y + v_1)(z + v + -v_1)$

2. Transform the following BE into a 3-CNF, then give a minimal VC for its underlying graph:
   $xyz + (-x) + (x - yz)$

3. Transform the following BE into a 3-CNF, then give a minimal VC for its underlying graph:
   $xy + yz(y + z)$