# 1 Turing Machine

## 1.1 Definition

A **Turing machine** (TM) is defined as a 7-tuple $P = (Q, \Sigma, \Gamma, \delta, q_0.B, F)$, where:

- $Q$ is the finite set of states
- $\Sigma :$ input symbols (finite set of symbols)
- $\Gamma :$ tape symbols (finite set of symbols), $\Sigma \in \Gamma$
- $q_0 \in Q :$ starting state
- $B \in \Gamma :$ blank symbol ($B \notin \Sigma$)
- $F \subseteq Q :$ set of accepting states
- $\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times D$
  - D is the direction where the head moves (L, R)

Figure 1: A Turing machine

## 1.2 Instantaneous descriptors

Similarly to PDAs, the instantaneous descriptors (ID) also represent configurations. However, for TMs, we have to represent the entire (non-blank) content of the tape, the position of the head and the state of the TM as well.
Configurations of a TM look like $pa\alpha$, $\alpha pa\beta$ or $\alpha pB$.

Provide TMs for the following languages:

1. $L = \{u101v \mid u, v \in \{0,1\}^*\}$

2. $L = \{101w \mid w \in \{0,1\}^*\}$
3. $L = \{w101 \mid w \in \{0,1\}^*\}$

Provide TMs for the following languages:

1. $L = \{awb \mid w \in \{a,b\}^*\}$
2. $L = \{a^n b^n \mid n \geq 0\}$
3. $L = \{ww^{-1} \mid w \in \{a,b\}^*\}$
4. $L = \{ww \mid w \in \{a,b\}^*\}$
5. $L = \{a^{2n} \mid n \geq 0\}$
6. $L = \{a^{2n} b^n \mid n \geq 0\}$
7. $L = \{a^n b^n c^n \mid n \geq 0\}$

Provide a 2-tape TM for the following language:

1. $L = \{a^m b^n \mid m, n \geq 1, n\%m = 0\}$