1 Testing membership in a CFL

We can decide the if a string w is the member of a context free language L. An efficient way to do this is the dynamic-programming (table filling) CYK (Cocke-Younger-Kasami) Algorithm.

How can we decide if string $w = a_1a_2a_3a_4a_5$ is in L? We need the **CNF** of L to do this, and fill out the following table.

x_{15}				
x_{14}	x_{25}			
x_{13}	x_{24}	x_{35}		
x_{12}	x_{23}	x_{34}	x_{45}	
x_{11}	x_{22}	x_{33}	x_{44}	x_{55}
a_1	a_2	a_3	a_4	a_5

- Each row of the table corresponds to a particular length of substrings of w.
- x_{ij} corresponds to the subset of variables that can generate substring $a_ia_{i+1}...a_j$
- We start by computing substrings of length 1.
- Substrings of length k will be generated by combining substrings of length $k-l$ and l (for all l values).
- The string w is a member of L, if $S \in x_{15}$

Exercise 1: Use the CYK algorithm to determine if $baaba \in L(G)$ holds, where the productions of your grammar G are

$S \to AB \mid BC$
$A \to BA \mid a$
$B \to CC \mid b$
$C \to AB \mid a$

Exercise 2: Use the CYK algorithm to determine if $cabab \in L(G)$ holds, where the productions of your grammar G are

$S \to AB \mid b$
$A \to CB \mid AA \mid a$
$B \to AS \mid b$
$C \to BS \mid c$

Exercise 3: Use the CYK algorithm to determine if $abaab \in L(G)$ holds, where the productions of your grammar G are

$S \to AB \mid SS \mid a$
$A \to BS \mid CD \mid b$
$B \to DD \mid b$
$C \to DE \mid a \mid b$
$D \to a$
$E \to SS$

Exercise 3: Use the CYK algorithm to determine if *she eats a fork with a fish* $\in L(G)$ holds, where $G = \{\{S, VP, PP, NP, V, P, N, D\}, \{a, eats, fish, fork, with\}, R, S\}$, and the productions of R are

$$
\begin{aligned}
S &\to NP\ VP \\
VP &\to VP\ PP \\
VP &\to eats \\
PP &\to P\ NP \\
NP &\to D\ N \\
V &\to eats \\
P &\to with \\
N &\to fish \\
N &\to fork \\
D &\to a
\end{aligned}
$$