Assignment - 1

1. Simplify the following logical expression using logical equivalence : $(p \land q) \lor (\neg p \land q)$

→ Truth Table

p	q	$p \land q$	$\neg p$	$\neg p \land q$	$(p \land q) \lor (\neg p \land q)$
T	T	T	F	F	T
T	F	F	F	F	F
F	T	F	T	T	T
F	F	F	T	F	F

The column for $(p \land q) \lor (\neg p \land q)$ matches q in all cases.

2. Show that $(p \to q) \leftrightarrow (\neg p \land q)$ is a tautology.

→

p	q	$p \to q$	$\neg p$	$\neg p \lor q$	$(p \to q) \leftrightarrow (\neg p \land q)$
T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

3. Prove or Disprove
$(p \lor q) \land (\neg p \lor \neg q)$ is a contradiction

→

p	q	$p \lor q$	$\neg p$	$\neg q$	$\neg p \lor \neg q$	$(p \lor q) \land (\neg p \lor \neg q)$
T	T	T	F	F	F	F
T	F	T	F	T	T	T
F	T	T	T	F	T	T
F	F	F	T	T	T	F

4. Define a Relation RR on the set $A = \{1, 2, 3\}$ as $R = \{(1,1), (2,2), (3,3), (1,2)\}$ is R Reflexive ?
R symmetric ?
R Transitive ?

symmetric :-

- A Relation

  R we have $(1, 2) \in R$

  $(2, 1) \in R$

- R is symmetric

Transitive :-

- $(1, 2) \in R$

  $(2, 1) \in R$ since

  $(1, 1) \in R$

- R is transitive

- R is an equivalence relation

6. prove by induction that $2^n > n$ for all integers $n \geq 1$

→ Base Case ($n = 1$) :

for $n = 1$,

$2^n = 2^1 = 2$, & $n = 1$

clearly $2 > 1$

Assume that the statement is true for some $n = k$, i.e,

$2^k > k$

we need to prove that $2^{k+1} > k + 1$.

using the inductive hypothesis $2^k > k$,

consider 2^{k+1}

$2^{k+1} = 2 \cdot 2^k$

from the inductive hypothesis, $2^k > k$

substituting this

$2^{k+1} = 2 \cdot 2^k > 2 \cdot k$

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Now, we compare $2.k$ with $k+1$

$$2.k \geq k+1 \quad \text{for } k \geq 1$$

This is true because $2.k - k - 1 = k - 1 \geq 0$ when $k \geq 1$.

Thus, $2^{k+1} > k+1$.

By mathematical induction, $2^n > n$ is true for all integers $n \geq 1$.

7. prove that $3^n + 2^n$ is divide by 5 for all $n \geq 1$.

→ Base case ($n = 1$):

for $n = 1$

$$3^1 + 2^1 = 3 + 2 = 5$$

∴ 5 is divide by 5, the base case holds

Inductive Hypothesis:

Assume that for some $n = k$, the statement is true, i.e, $3^k + 2^k \equiv 0 \pmod 5$

This means

$$3^k + 2^k = 5m \quad (\text{for some integer } m)$$

we need to prove that $3^{k+1} + 2^{k+1} \equiv 0 \pmod 5$

consider $3^{k+1} + 2^{k+1}$;

$$3^{k+1} + 2^{k+1} = 3 \cdot 3^k + 2 \cdot 2^k.$$

taking modula 5, we use the inductive hypothesis

$$3^k + 2^k \equiv 0 \pmod 5$$

$$3^{k+1} + 2^{k+1} = (3 \cdot 3^k) + (2 \cdot 2^k)$$

Now, reduce 3 mod 5 & 2 mod 5

$$3^k \equiv -2k \pmod 5$$

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we want to prove that $3^{k+1} + 2^{k+1} = 0$

from the expression:

$$3^{k+1} + 2^{k+1} = 3.3^k + 2.2^k$$

taking modulo 5 & using properties of modular arithmetic

$$3^{k+1} + 2^{k+1} = 3.(3^k \bmod 5) + 2.(2^k \bmod 5) (\bmod 5)$$

$$3^k + 2^k = 0 \ (\bmod 5)$$

thus, $3^k = -2^k$

substitute $3^k = -2^k$ into $3^{k+1}, 2^{k+1}$

$$3^{k+1} + 2^{k+1} = 3.(-2)^k + 2.2^k (\bmod 5)$$

factor 2^k out

$$3^{k+1} + 2^{k+1} = (-3+2) \cdot 2^k$$

simplify $-3+2 = -1 \Rightarrow (-1) \cdot 2^k = -2^k$

since $-2^k + 2^{k+1} = 0$, the statement holds for $k+1$

8. prove that $n(n^2+5)$ is divide by 6 for all integers n using PMI.

→ Base case $(n=1)$:

for $n = 1$

$$n(n^2+5) = 1(1^2+5) = 1.6 = 6$$

clearly, 6 is divisible by 6.

$$n = k$$

i.e, $k = k^2 + 5$ is divisible by 6.

This means, $k(k^2+5) = 6m$ for some integer m.

$-n = k+1$, i.e $(k+1)((k+1)^2+5)$ is divisible by 6

Expand $(k+1)((k+1)^2+5) = (k+1)(k^2+2k+1+5) =$
$(k+1)(k^2+2k+6)$

Distribute $(k+1)$: $(k+1)(k^2+2k+6)$

$= (k+1)(k^2+2k) + 6(k+1)$

factor : $(k+1)(k^2+2k+6)$

$= k(k^2+5) + (2k^2+2k) + 6(k+1)$

$\Rightarrow$ first term : $k(k^2+5)$ is divisible by 6.

second term : $2k^2+2k = 2k(k+1)$, since k and $k+1$ are consecutive integers, one of them is divisible by 2 and one is divisible by 3. Thus, product is divisible by 6.