Longest Common Subsequence

Let us define $c[i, j]$ to be the length of an LCS of the sequences X_i and Y_j. If either $i = 0$ or $j = 0$, one of the sequences has length 0, so the LCS has length 0. The optimal substructure of the LCS problem gives the recursive formula.

$$c[i, j] = \begin{cases} 0 & \text{if } i = 0 \text{ or } j = 0, \\ c[i-1, j-1] + 1 & \text{if } i, j > 0 \text{ and } x_i = y_j, \\ \max\left(c[i, j-1], c[i-1, j]\right) & \text{if } i, j > 0 \text{ and } x_i \neq y_j. \end{cases}$$