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# Homework 3

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Intuition - Scanning from left to right you connect each point to its closest neighbor. As the scan of the graph occurs you pick up a new point and connect it to the vertex closest to the point that is already in the bitonic tour.

If $Z$ represents the cost of the shortest bitonic path from i, through all verticies, and $1 \leq i \leq n$, $j > i$ then

$$Z_{i,j} = d(i, i+1) + Z_{i+1,j} \text{ where } j \neq i+1$$

which represents the addition of a new point to the tour, and

$$Z_{i,i+1} = \min_{k>i+1} d(i,k) + Z_{i+1,k}$$

It takes O($i$) to compute $Z_{i,j}$. So time complexity is $\Theta(\sum_{i=1}^{n} i)$ which is $\Theta(n^2)$.

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For any $x \in T$. Let $W_x$ be the value of an optimal solution if we force $x$ in the the solution. Let $W'_x$ be the value of the optimal solution for subtree rooted at $x$ if we force x out of the solution. Let $O_x$ be a value of an optimal solution for a subtree rooted at $x$. We want to compute $O$.

If $x$ is not a leaf,

$$O_x = \max(W_x, W'_x) \tag{1}$$

Let $C(x)$ be the children of $x$,

$$W_x = w(x) + \sum_{y \in C(x)} W'_y \tag{2}$$

$$W'_x = \sum_{y \in C(x)} O_y \tag{3}$$

If $x$ is a leaf,

$$W_x = W(x)$$

$$W'_x = O$$

The algorithm is post order in which recurrence relations (1), (2), (3) are used to compute $O_x, W_x, W'_x$ for all $x \in V$. Time complexity is O($n$) where $n = |v|$.

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You could order the jobs $a$ by deadline $d$. From here you place the maximum profit $p$ in the matrix and optimize your path through the jobs. Which would yield O($n^3$) running time based upon the size of the table $n^2$ and then the recovery of the optimal path $n$.

In table $T$,

$$T[i,t] = \max \begin{cases} T[i-1,t] & \\ T[i-1,t-t_i] + p_i & \text{if } t \leq d_i \\ T[i-1,t-t_i] & \text{if } t > d_i \end{cases}$$

## Problem 4