Floyd-Warshall

Let d_{ij}^k be the least cost shortest path from i to j that does not contain vertices $k+1, ..., n$ or has intermediate vertices $k = 1, ..., n$ then $d_{ij}^0 = w(i, j)$.

Note that d_{ij}^n is the least cost shortest path fron i to j.

Case 1: vertex k is not on path p.

$$d_{ij}^k = d_{ij}^{k-1}$$

Case 2: vertex k is on the path p.

$$d_{ij}^k = d_{ik}^{k-1} + d_{kj}^{k-1}$$

$$d_{ij}^0 = w(i, j)$$

$$d_{ij}^k = \min\left(d_{ij}^{k-1}, d_{ik}^{k-1} + d_{kj}^{k-1}\right) \tag{1}$$

Compute $d^0, d^1, ..., d^n$ in that order and store them where d^k, where $k = 1, ..., n$, is an $n \times n$ matrix.

Compute d^k from d^{k-1} using (1).

Time complexity $\mathrm{O}(n^3)$.