

# Floyd-Warshall

Let  $d_{ij}^k$  be the least cost shortest path from  $i$  to  $j$  that does not contain vertices  $k + 1, \dots, n$  or has intermediate vertices  $k = 1, \dots, n$  then  $d_{ij}^0 = w(i, j)$ .

Note that  $d_{ij}^n$  is the least cost shortest path from  $i$  to  $j$ .

Case 1: vertex  $k$  is not on path  $p$ .

$$d_{ij}^k = d_{ij}^{k-1}$$

Case 2: vertex  $k$  is on the path  $p$ .

$$d_{ij}^k = d_{ik}^{k-1} + d_{kj}^{k-1}$$

$$d_{ij}^0 = w(i, j)$$

$$d_{ij}^k = \min \left( d_{ij}^{k-1}, d_{ik}^{k-1} + d_{kj}^{k-1} \right) \quad (1)$$

Compute  $d^0, d^1, \dots, d^n$  in that order and store them where  $d^k$ , where  $k = 1, \dots, n$ , is an  $n \times n$  matrix.

Compute  $d^k$  from  $d^{k-1}$  using (1).

Time complexity  $O(n^3)$ .