Vertex Cover

$S \subseteq V$ is called a vertex cover if for any edge $ab \in E$ either $a \in S$ or $b \in S$. Minimum vertex cover is one with smallest number of vertices. This problem is NP-hard.

Approximation Algorithm

If you choose nodes with maximum degree first. Be careful with a greedy algorithm because it might choose a and b first and then end up with degree 4 rather than minimal of degree 3.

Need to pick the minimum number of vertices that cover all the edges.

Look in book at set cover approximation algorithm. Not on test.

Let $G = (V, E)$ be a graph. Let $M \subseteq E$. We say that M is matching if no two edges in M have an end point in common. For example a hexagon's maximum matching has cardinality of three.

Let $M \subseteq E$ be matching. We say M is "maximal" if for any e not in M maximum matching $M \cup \{e\}$ is not maximal matching.

Let S be a vertex cover and M be matching then $|S^*| \geq |M|$.

If this is maximal matching then you can pick up endpoints for vertex cover. Must pick up all a's and b's.

Let $M = \{a_1, b_1, a_2, b_2, ..., a_k, b_k\}$ be a maximal matching then $S = \{a_1, .., a_k, b_1, ..., b_k\}$ is a vertex cover. Note that $|S| = 2|M|$
$2|S^*| \geq |S| = 2|M| \geq |S^*|$ which is an upper bound.