Vertex Cover

$S \subseteq V$ is called a vertex cover if for any edge $ab \in E$ either $a \in S$ or $b \in S$. Minimum vertex cover is one with smallest number of vertices. This problem is NP-hard.

Approximation Algorithm

If you choose nodes with maximum degree first. Be careful with a greedy algorithm because it might choose a and b first and then end up with degree 4 rather than minimal of degree 3.

Need to pick the minimum number of vertices that cover all the edges.

Let $G = (V, E)$ be a graph. Let $M \subseteq E$. We say that M is matching if no two edges in M have an end point in common. For example a hexagon's maximum matching has cardinality of three.

Let $M \subseteq E$ be matching. We say M is "maximal" if for any e not in M maximum matching $M \cup \{e\}$ is not maximal matching.

Let S be a vertex cover and M be matching then $|S^*| \geq |M|$.

If this is maximal matching then you can pick up endpoints for vertex cover. Must pick up all a's and b's.

Let $M = \{a_1, b_1, a_2, b_2, ..., a_k, b_k\}$ be a maximal matching then $S = \{a_1, .., a_k, b_1, ..., b_k\}$ is a vertex cover. Note that $|S| = 2|M|$
$2|S^*| \geq |S| = 2|M| \geq |S^*|$ which is an upper bound.