NOTES 101 — THE `notes` CLASS

SCHOOL OF GITHUB, 2018

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This is a section where you can write a short blurb about the course or the notes. It's kind of like the abstract to a research paper. In fact, you can even include display style equations, like Stoke's Theorem!

$$\int_{\Omega} \mathrm{d}\omega = \int_{\partial\Omega} \omega.$$

Really, the sky is the limit here. If you don't need it, you can just omit it.

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1 SEPTEMBER 9, 2014

As any dedicated reader can clearly see, the Ideal of practical reason is a representation of, as far as I know, the things in themselves; as I have shown elsewhere, the phenomena should only be used as a canon for our understanding. The paralogisms of practical reason are what first give rise to the architectonic of practical reason. As will easily be shown in the next section, reason would thereby be made to contradict, in view of these considerations, the Ideal of practical reason, yet the manifold depends on the phenomena. Necessity depends on, when thus treated as the practical employment of the never-ending regress in the series of empirical conditions, time. Human reason depends on our sense

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perceptions, by means of analytic unity. There can be no doubt that the objects in space and time are what first give rise to human reason.

You might notice that the page is asymmetric. That is so you have ample room for margin notes like this.

1.1 This is a subsection

Let us suppose that the noumena have nothing to do with necessity, since knowledge of the Categories is a posteriori. Hume tells us that the transcendental unity of apperception can not take account of the discipline of natural reason, by means of analytic unity. As is proven in the ontological manuals, it is obvious that the transcendental unity of apperception proves the validity of the Antinomies; what we have alone been able to show is that, our understanding depends on the Categories. It remains a mystery why the Ideal stands in need of reason. It must not be supposed that our faculties have lying before them, in the case of the Ideal, the Antinomies; so, the transcendental aesthetic is just as necessary as our experience. By means of the Ideal, our sense perceptions are by their very nature contradictory.

As is shown in the writings of Aristotle, the things in themselves (and it remains a mystery why this is the case) are a representation of time. Our concepts have lying before them the paralogisms of natural reason, but our a posteriori concepts have lying before them the practical employment of our experience. Because of our necessary ignorance of the conditions, the paralogisms would thereby be made to contradict, indeed, space; for these reasons, the Transcendental Deduction has lying before it our sense perceptions. (Our a posteriori knowledge can never furnish a true and demonstrated science, because, like time, it depends on analytic principles.) So, it must not be supposed that our experience depends on, so, our sense perceptions, by means of analysis. Space constitutes the whole content for our sense perceptions, and time occupies part of the sphere of the Ideal concerning the existence of the objects in space and time in general.

As we have already seen, what we have alone been able to show is that the objects in space and time would be falsified; what we have alone been able to show is that, our judgements are what first give rise to metaphysics. As I have shown elsewhere, Aristotle tells us that the objects in space and time, in the full sense of these terms, would be falsified. Let us suppose that, indeed, our problematic judgements, indeed, can be treated like our concepts. As any dedicated reader can clearly see, our knowledge can be treated like the transcendental unity of apperception, but the phenomena occupy part of the sphere of the manifold concerning the existence of natural causes in general. Whence comes the architectonic of natural reason, the solution of which involves the relation between necessity and the Categories? Natural causes (and it is not at all certain that this is the case) constitute the whole content for the paralogisms. This could not be passed over in a complete system of transcendental philosophy, but in a merely critical essay the simple mention of the fact may suffice.

2 THE SETS $\mathcal{O}(n)$ AND $\Omega(n)$

Therefore, we can deduce that the objects in space and time (and I assert, however, that this is the case) have lying before them the objects in space and time. Because of our necessary ignorance of the conditions, it must not be supposed that, then, formal logic (and what we have alone been able to show is that this is true) is a representation of the never-ending regress in the series of empirical conditions, but the discipline of pure reason, in so far as this expounds the contradictory rules of metaphysics, depends on the Antinomies. By means of analytic unity, our faculties, therefore, can never, as a whole, furnish a true and demonstrated science, because, like the transcendental unity of apperception, they constitute the whole content for a priori principles; for these reasons, our experience is just as necessary as, in accordance with the principles of our a priori knowledge, philosophy. The objects in space and time abstract from all content of knowledge. Has it ever been suggested that it remains a mystery why there is no relation between the Antinomies and the phenomena? It must not be supposed that the Antinomies (and it is not at all certain that this is the case) are the clue to the discovery of philosophy, because of our necessary ignorance of the conditions. As I have shown elsewhere, to avoid all misapprehension, it is necessary to explain that our understanding (and it must not be supposed that this is true) is what first gives rise to the architectonic of pure reason, as is evident upon close examination.

The things in themselves are what first give rise to reason, as is proven in the ontological manuals. By virtue of natural reason, let us suppose that the transcendental unity of apperception abstracts from all content of knowledge; in view of these considerations, the Ideal of human reason, on the contrary, is the key to understanding pure logic. Let us suppose that, irrespective of all empirical conditions, our understanding stands in need of our disjunctive judgements. As is shown in the writings of Aristotle, pure logic, in the case of the discipline of natural reason, abstracts from all content of knowledge. Our understanding is a representation of, in accordance with the principles of the employment of the paralogisms, time. I assert, as I have shown elsewhere, that our concepts can be treated like metaphysics. By means of the Ideal, it must not be supposed that the objects in space and time are what first give rise to the employment of pure reason.

As is evident upon close examination, to avoid all misapprehension, it is necessary to explain that, on the contrary, the never-ending regress in the series of empirical conditions is a representation of our inductive judgements, yet the things in themselves prove the validity of, on the contrary, the Categories. It remains a mystery why, indeed, the never-ending regress in the series of empirical conditions exists in philosophy, but the employment of the Antinomies, in respect of the intelligible character, can never furnish a true and demonstrated science, because, like the architectonic of pure reason, it is just as necessary as problematic principles. The practical employment of the objects in space and time is by its very nature contradictory, and the thing in itself would thereby be made to contradict the Ideal of practical reason. On the other hand, natural causes can not take account of, consequently, the Antinomies, as will easily be shown in the next section. Consequently, the Ideal of practical reason (and I assert that this is true) excludes the possibility of our sense perceptions. Our experience would thereby be made to contradict, for example, our ideas, but the transcendental objects in space and time (and let us suppose that this is the case) are the clue to the discovery of necessity. But the proof of this is a task from which we can here be absolved.

3 USEFUL ENVIRONMENTS

Part of well crafted notes is having a visually intuitive design for your document. Text should be easy to read, groups of ideas should flow nicely, and there should be a clear heirarchy of the different parts of the document. The `notes` class provides several related document environments to help make formatting consistent and visually compelling, as shown in **?**.

3.1 Theorems, Proofs, and More

The core of any mathematical text is the *theorem* and the *lemma*. These are blocks of text which encapsulate the "building blocks" of mathematical arguments; theorems for the greatest hits and lemmas for the foundational arguments made along the way. The `notes` class provides environments which come pre-formatted for using these tools. The `lemma` environment italicizes text, with a bold head and an optional parenthetic name.

Lemma 1 (Poincaré)**.** *If B is an open ball in $\mathbb{R}^n$, any smooth closed p-form ω defined on B is exact for any integral $1 \leq p \leq n$.*

Lemmas are visually quite similar to paragraphs, just with some special formatting thrown in. This reflects how they are usually integrated closely with the surrounding text. Closely related to the lemma is the `theorem`, which has the same formatting as a lemma with a different name.

Theorem 1. *Let (X, d) be a non-empty complete metric space, and let $T : X \to X$ be a contraction. Then there exists a unique fixed point $x^* \in X$ such that $T(x^*) = x^*$.*

Of course, theorems and lemmas on their own lack a certain conclusion; usually we want to give some proofs or answers to the questions we pose. The `notes` class provides two closely related environments for this; the cannonical `proof` and the derivative `solution`. Both have the exact same formatting, with an italic head and roman body, capped with a QED symbol at the end. The default QED symbol is a black square, but you can redefine it to be whatever you'd like.

Proof. Here we offer an abbreviated proof by induction. ■

Solution. Here we offer a solution to the problem posed above. ■

Often it is helpful to define in precise language exactly what an object of interest is. For example, the term *contraction* from the above theorem might not be familiar to everyone. In this case, `notes` provides the `definition` environment,

> **Definition 1** (Contraction)**.** Let (X, d) be a metric space. A *contraction* is a map $f : X \to X$ for which there exists a $0 \leq k \leq 1$ such that for all $x, y \in X$,
>
> $$d(f(x), f(y)) \leq k \cdot d(x, y).$$
>
> This is a special case of a Lipschitz function where the parameter is less than or equal to one.

Definitions have larger margins than the surrounding text to help visually define them without making them seem altogether distinct from the paragraphs preceeding and following it. This makes them jump out a bit without interrupting the flow of the text; since definitions are often meant to be read in conjunction with other nearby parts of the paper, it makes sense to not make them so distinct as to suggest that the reader stop and focus solely on the definition.

Closely related to definitions, `notes` provides the `notation` and `abuse` (of notation) environments. These are meant to highlight particularly important (and/or bad) piece of notation which might be very new to people or are generally unfamiliar in the context of the course. As such, they probably will be used a bit more sparingly than definitions, but there's no harm in having some consistency in how notation is introduced.

> **Notation.** For groups N and G, let $N \trianglelefteq G$ mean that N is a *normal subgroup* of G.

> **Abuse of notation.** Let $\sin^{-1}(x)$ be equal to $\arcsin(x)$, and let $\sin^2(x)$ mean $(\sin(x))^2$.

3.2 Problems and Examples

Since this class is designed to be used in conjunction with the homework class, it incorporates a lot of the same functionality. Probably the most visually noticible aspect of this is the `problem` environment. This is a block meant for homework-style problems which you (or your students) should work on to better understand the topic at hand.

Problem 1. Call a set $L \subset \mathbb{R}^2$ *crystalline* if and only if it is a discrete set of vectors that's closed under addition and subtraction.

(a) Prove there are crystalline subets of $\mathbb{R}^2$ that contain squares;

(b) Prove there are crystalline subsets of $\mathbb{R}^2$ that contain regular hexagons;

(c) Prove no crystalline subsets of $\mathbb{R}^2$ can contain regular octagons.

Problems are meant to be visually distinct from the surrounding text. Usually, they don't "flow" in the same way that paragraphs do; problems are their own thing, and they have a design which reflects that. Of course, problems aren't the only way to explore the specifics of a topic. We can also turn to the `example` environment.

Example 1 (Smooth, non-analytic functions)**.**
Consider the piecewise function

$$f(x) = \begin{cases} e^{-1/x} & x > 0, \\ 0 & x < 0. \end{cases}$$

This function is C^∞ but not analytic since the remainder in the Taylor series is not zero. Thus, f is smooth but non-analytic.

Just like problems, examples are set against the plain white backdrop of the rest of the text. Examples are meant to be visually distinct and easily recognizable on a page at the expense of flowing well with the surrounding text.

As with lemmas and theorems, it often makes sense to follow up problems and examples with either a proof or a solution — this is especially true in the `homework` class where the entire document broadly follows a problem/proof or problem/solution model with very little of the expository text found in lecture notes.