

IffyLang : Syntax and Evaluation

The following defines Functional Iffy:

Syntax:

(Bool) $b ::= x \mid 1 \mid 0 \mid b \wedge b \mid b \vee b \mid \text{if } b \text{ then } b \text{ else } b \mid \text{fun } x \Rightarrow b \mid \text{app } b_1 b_2$

Single-step reduction:

$$\begin{array}{c}
 \frac{}{\text{app}(\text{fun } x \Rightarrow b) b' \rightsquigarrow [b'/x]b} \text{ BETA} \qquad \frac{b \rightsquigarrow b'}{(\text{fun } x \Rightarrow b) \rightsquigarrow (\text{fun } x \Rightarrow b')} \text{ FUN} \\
 \\
 \frac{b_1 \rightsquigarrow b'_1}{\text{app } b_1 b_2 \rightsquigarrow \text{app } b'_1 b_2} \text{ APP1} \qquad \frac{b_2 \rightsquigarrow b'_2}{\text{app } b_1 b_2 \rightsquigarrow \text{app } b_1 b'_2} \text{ APP2} \qquad \frac{}{(1 \wedge 1) \rightsquigarrow 1} \text{ ANDTRUE} \\
 \\
 \frac{}{(0 \wedge 1) \rightsquigarrow 0} \text{ ANDFALSE1} \qquad \frac{}{(1 \wedge 0) \rightsquigarrow 0} \text{ ANDFALSE2} \qquad \frac{}{(0 \wedge 0) \rightsquigarrow 0} \text{ ANDFALSE} \\
 \\
 \frac{b_1 \rightsquigarrow b'_1}{(b_1 \wedge b_2) \rightsquigarrow (b'_1 \wedge b_2)} \text{ AND1} \qquad \frac{b_2 \rightsquigarrow b'_2}{(b_1 \wedge b_2) \rightsquigarrow (b_1 \wedge b'_2)} \text{ AND2} \qquad \frac{}{(1 \vee 1) \rightsquigarrow 1} \text{ ORTRUE} \\
 \\
 \frac{}{(0 \vee 1) \rightsquigarrow 1} \text{ ORTRUE2} \qquad \frac{}{(1 \vee 0) \rightsquigarrow 1} \text{ ORTRUE1} \qquad \frac{}{(0 \vee 0) \rightsquigarrow 0} \text{ ORFALSE} \\
 \\
 \frac{b_1 \rightsquigarrow b'_1}{(b_1 \vee b_2) \rightsquigarrow (b'_1 \vee b_2)} \text{ OR1} \qquad \frac{b_2 \rightsquigarrow b'_2}{(b_1 \vee b_2) \rightsquigarrow (b_1 \vee b'_2)} \text{ OR2} \\
 \\
 \frac{}{\text{if } 1 \text{ then } b_1 \text{ else } b_2 \rightsquigarrow b_1} \text{ IFTRUE} \qquad \frac{}{\text{if } 0 \text{ then } b_1 \text{ else } b_2 \rightsquigarrow b_2} \text{ IFFALSE} \\
 \\
 \frac{b \rightsquigarrow b'}{\text{if } b \text{ then } b_1 \text{ else } b_2 \rightsquigarrow \text{if } b' \text{ then } b_1 \text{ else } b_2} \text{ IF1} \qquad \frac{b_1 \rightsquigarrow b'_1}{\text{if } b \text{ then } b_1 \text{ else } b_2 \rightsquigarrow \text{if } b \text{ then } b'_1 \text{ else } b_2} \text{ IF2} \\
 \\
 \frac{b_2 \rightsquigarrow b'_2}{\text{if } b \text{ then } b_1 \text{ else } b_2 \rightsquigarrow \text{if } b \text{ then } b_1 \text{ else } b'_2} \text{ IF3}
 \end{array}$$

Mutli-step reduction:

$$\frac{}{b \rightsquigarrow^* b} \quad \text{REFL} \qquad \frac{b_1 \rightsquigarrow b_2}{b_1 \rightsquigarrow^* b_2} \quad \text{STEP} \qquad \frac{b_1 \rightsquigarrow^* b_2 \quad b_2 \rightsquigarrow^* b_3}{b_1 \rightsquigarrow^* b_3} \quad \text{MULT}$$