Final Exam Programming Languages Concepts (CSCI 3030)

Submit a single PDF file of your solution set on D2L.

All questions must be in order.

All assignments not adhering to this will not be graded.

- $0. \,$ Complete the midterm exam questions on Learn OCaml.
- 1. Derive the following:

 $\emptyset \vdash \mathsf{Fun}(\mathbb{N}, n.\mathsf{Rec}(n, \mathsf{Num}(0), m.r.m)) ? \mathbb{N}$

Type Checking vs Type Construction

Type Checking

 $\Gamma \vdash \mathsf{True} : \mathbb{B}$

 $\Gamma \vdash \mathsf{False} : \mathbb{B}$

 $\frac{\Gamma \vdash t_2 : T}{\Gamma \vdash t_1 : \mathbb{B} \qquad \Gamma \vdash t_3 : T}$ $\frac{\Gamma \vdash t_1 : \mathbb{B} \qquad \Gamma \vdash t_3 : T}{\Gamma \vdash \mathsf{If}(t_1, t_2, t_3) : T}$

 $\Gamma \vdash \mathsf{Num}\, n : \mathbb{N}$

 $\frac{\Gamma \vdash t : \mathbb{N}}{\Gamma \vdash \mathsf{Suc}\, t : \mathbb{N}}$

 $\Gamma \vdash t_2 : T$

 $\underline{\Gamma \vdash t_1 : \mathbb{B} \qquad \Gamma \vdash t_3 : T} \qquad \underline{\Gamma \vdash t_1 : \mathbb{N}} \qquad \underline{\Gamma, m : \mathbb{N}, r : T \vdash t_3 : T}$

 $\Gamma \vdash \mathsf{Rec}(t_1, t_2, m \,.\, r \,.\, t_3) : T$

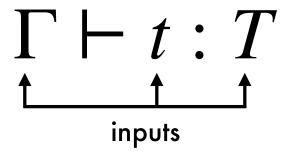
 Γ , $x : T \vdash x : T$

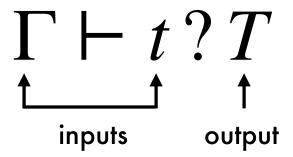
 $\frac{\Gamma, x : T \vdash t : T}{\Gamma \vdash \mathsf{Fun}(x \, . \, t) : T}$

 $\frac{\Gamma \vdash t_1 : T_1}{\Gamma, x : T_1 \vdash t_2 : T_2}$ $\frac{\Gamma \vdash \mathsf{Let}(t_1, x . t_2) : T_2}{\Gamma \vdash \mathsf{Let}(t_1, x . t_2) : T_2}$

 $\frac{\Gamma \vdash t_1 : \operatorname{Fun}(T_1, T_2)}{\Gamma \vdash t_2 : T_1}$ $\frac{\Gamma \vdash t_2 : T_1}{\Gamma \vdash \operatorname{App}(t_1, t_2) : T_2}$

Bidirectional Typing





Bidirectional Typing: Checking

$$\Gamma, x: T \vdash x: T$$

$$\begin{array}{c|c} \hline \Gamma \vdash \mathsf{True} : \mathbb{B} & \hline \Gamma \vdash \mathsf{Num}(n) : \mathbb{N} \\ \hline \hline \Gamma \vdash \mathsf{False} : \mathbb{B} & \hline \Gamma \vdash t : \mathbb{N} \\ \hline \hline \Gamma \vdash \mathsf{False} : \mathbb{B} & \hline \Gamma \vdash \mathsf{Suc}(t) : \mathbb{N} \\ \hline \hline \Gamma \vdash t_1 : \mathbb{B} & \Gamma \vdash t_2 : T \\ \hline \Gamma \vdash \mathsf{If}(t_1, t_2, t_3) : T & \hline \Gamma \vdash t_1 : \mathbb{N} & \Gamma, m : \mathbb{N}, r : T \vdash t_3 : T \\ \hline \Gamma \vdash \mathsf{Rec}(t_1, t_2, m . r . t_3) : T \\ \hline \end{array}$$

$$\begin{array}{c} \Gamma, x: T_{1} \vdash t: T_{2} \\ \hline \Gamma \vdash \operatorname{Fun}(x \cdot t) : \operatorname{Fun}(T_{1}, T_{2}) \\ \hline \Gamma \vdash t_{1} ? T_{1} \\ \hline \Gamma, x: T_{1} \vdash t_{2} : T_{2} \\ \hline \Gamma \vdash \operatorname{Let}(t_{1}, x \cdot t_{2}) : T_{2} \\ \hline \Gamma \vdash t_{1} ? \operatorname{Fun}(T_{1}, T_{2}') \\ \hline \Gamma \vdash t_{2} : T_{1} \qquad T_{2}' = T_{2} \\ \hline \Gamma \vdash \operatorname{App}(t_{1}, t_{2}) : T_{2} \end{array}$$

Bidirectional Typing: Synthesis

$$\Gamma, x: T \vdash x?T$$

$$\begin{array}{c|c} \Gamma \vdash \mathsf{True}\,?\,\mathbb{B} & \overline{\Gamma} \vdash \mathsf{Num}(n)\,?\,\mathbb{N} \\ \hline \Gamma \vdash \mathsf{False}\,?\,\mathbb{B} & \overline{\Gamma} \vdash t:\,\mathbb{N} \\ \hline \Gamma \vdash t_2\,?\,T & \Gamma \vdash t_2\,?\,T \\ \hline \Gamma \vdash t_1:\,\mathbb{B} & \Gamma \vdash t_3:\,T & \Gamma \vdash t_1:\,\mathbb{N} & \Gamma,m:\,\mathbb{N},r:\,T \vdash t_3:\,T \\ \hline \Gamma \vdash \mathsf{If}(t_1,t_2,t_3)\,?\,T & \Gamma \vdash \mathsf{Rec}(t_1,t_2,m.\,r.\,t_3)\,?\,T \end{array}$$

$$\begin{array}{c|c} \Gamma, x : T_{1} \vdash t \,?\, T_{2} \\ \Gamma \vdash \mathsf{Fun}(T_{1}, x \,.\, t) \,?\, \mathsf{Fun}(T_{1}, T_{2}) \\ \hline \Gamma \vdash t_{1} \,?\, T_{1} \\ \hline \Gamma, x : T_{1} \vdash t_{2} \,?\, T_{2} \\ \hline \Gamma \vdash \mathsf{Let}(t_{1}, x \,.\, t_{2}) \,?\, T_{2} \\ \hline \\ \Gamma \vdash t_{1} \,?\, \mathsf{Fun}(T_{1}, T_{2}) \\ \hline \Gamma \vdash t_{2} : T_{1} \\ \hline \Gamma \vdash \mathsf{App}(t_{1}, t_{2}) \,?\, T_{2} \\ \end{array}$$