termvar, x, y
funcname, `name`
indecies, i, j

types, T, A, B, C	::=		Types
	|	Bool	
	|	Nat	
	|	$[A]$	
Γ	::=		Typing Context
	|	$\emptyset$	
	|	$x_1 : T_1, \ldots, x_i : T_i$	
program, p	::=		Programs
	|	$\mathsf{func}\,\mathtt{name}(x_1 : T_1, \ldots, x_i : T_i) \to T\{body\}$	
	|	(p)	S
body	::=		Function Bodies
	|	$asgn; t$	Body with assignments
	|	t	Body without assignments
lv	::=		Assignment Tags
	|	let	Use x at least once
	|	var	Use x any number of times
assignemnts, *asgn*	::=		Variable Assignments
	|	$lv_1\, x_1 : T_1 = b_1; \ldots; lv_j\, x_j : T_j = b_j$	
t	::=		Terms
	|	x	A variable
	|	0	Zero
	|	$\mathsf{succ}\, t$	Successor
	|	$\mathsf{match}\, t_1\{0 \to t_2; \mathsf{succ}\, x \to t_3\}$	Natural Number Pattern Matc
	|	$[]$	
	|	$t_1 :: t_2$	
	|	$\mathsf{match}\, t_1\{[] \to t_2; x :: y \to t_3\}$	List Pattern Matching
	|	T	Logical true
	|	F	Logical false
	|	$\mathsf{if}\, b_1\, \mathsf{then}\, t_1\, \mathsf{else}\, t_2$	Pattern Matching for booleans
	|	$\mathtt{name}(b_1, \ldots, b_i)$	Function application
	|	$\mathsf{return}\, b$	Return of a term
	|	$EC[t]$	Plugging the hole in EC gives
	|	(t)	S
b	::=		Basic Terms
	|	x	A variable
	|	0	Zero
	|	$\mathsf{succ}\, b$	Successor
	|	$\mathsf{match}\, b_1\{0 \to t_2; \mathsf{succ}\, x \to t_3\}$	Natural Number Pattern Matc
	|	$[]$	
	|	$b_1 :: b_2$	
	|	$\mathsf{match}\, b_1\{[] \to t_2; x :: y \to t_3\}$	List Pattern Matching
	|	T	Logical true

	$\mid$	F	Logical false
	$\mid$	$\mathsf{if}\ b_1\ \mathsf{then}\ t_2\ \mathsf{else}\ t_3$	Pattern Matching for booleans
	$\mid$	$\mathtt{name}(b_1, \ldots, b_i)$	Function application
	$\mid$	$EC[b]$	Plugging the hole in EC gives a term.
	$\mid$	(b)	S
nat	::=		Natural Number Values
	$\mid$	0	
	$\mid$	$\mathsf{succ}\ nat$	
$list$	::=		List of Values
	$\mid$	$[]$	
	$\mid$	$v :: list$	
v	::=		Values
	$\mid$	T	
	$\mid$	F	
	$\mid$	0	
	$\mid$	$\mathsf{succ}\ nat$	
	$\mid$	$[]$	
	$\mid$	$v :: list$	
Δ	::=		Contexts of Function Definitions
	$\mid$	p_1	
	$\mid$	Δ_1, Δ_2	
TP	::=		List of Term Parameters
	$\mid$	$\emptyset$	Empty List
	$\mid$	b, TP	Term Argument
CP	::=		Evaluation Contexts for Parameters
	$\mid$	$\emptyset$	Empty List
	$\mid$	EC, TP	Context Evaluation Argument
	$\mid$	b, CP_2	Term Argument
$evalctx,\ EC$	::=		Evaluation Contexts
	$\mid$	$\square$	The hole (location of the evaluation point)
	$\mid$	T	
	$\mid$	F	
	$\mid$	0	
	$\mid$	$[]$	
	$\mid$	$\mathsf{match}\ EC\{[] \to t_1; x :: y \to t_2\}$	
	$\mid$	$\mathsf{if}\ EC\ \mathsf{then}\ t_2\ \mathsf{else}\ t_3$	
	$\mid$	$\mathtt{name}(CP)$	
	$\mid$	$\mathsf{return}\ b$	
	$\mid$	(EC)	S

$\boxed{\Delta_1 \vdash \Delta_2}$ Type Checking for Definitions

$$\frac{\Delta;\Gamma, x_1 : T_1, \ldots, x_i : T_i \vdash b_1 : A_1 \ \ldots \ \Delta;\Gamma, x_1 : T_1, \ldots, x_i : T_i, y_1 : A_1, \ \ldots \ , y_{j-1} : A_{j-1} \vdash b_j : A_j \quad \Delta_1; x_1 : T_1, \ldots, x_i : T_i, y_1 : A'_1, \ \ldots \ , y_j : A'_j \vdash t : T}{\Delta_1 \vdash \mathsf{func}\,\mathtt{name}(x_1 : T_1, \ldots, x_i : T_i) \to T\{lv\ y_1 : A_1 = b_1; \ldots; lv\ y_j : A_j = b_j; t\}} \ \text{DT\_Asgn}$$

$$\frac{\Delta_1; x_1 : T_1, \ldots, x_i : T_i \vdash t : T}{\Delta_1 \vdash \mathsf{func}\,\mathtt{name}(x_1 : T_1, \ldots, x_i : T_i) \to T\{t\}} \ \text{DT\_NAsgn}$$

$$\frac{\Delta_1 \vdash p_1 \quad \Delta_1, p_1 \vdash \Delta_2}{\Delta_1 \vdash p_1, \Delta_2} \ \text{DT\_Ext}$$

$\boxed{\Delta;\Gamma \vdash t : T}$ Type Checking for Terms

$$\frac{x : T \in \Gamma}{\Delta;\Gamma \vdash x : T} \ \text{T\_Var}$$

$$\frac{}{\Delta;\Gamma \vdash 0 : \mathsf{Nat}} \ \text{T\_Zero}$$

$$\frac{\Delta;\Gamma \vdash t : \mathsf{Nat}}{\Delta;\Gamma \vdash \mathsf{succ}\ t : \mathsf{Nat}} \ \text{T\_Succ}$$

$$\frac{\Delta;\Gamma \vdash t_1 : \mathsf{Nat} \quad \Delta;\Gamma \vdash t_2 : T \quad \Delta;\Gamma, x : \mathsf{Nat} \vdash t_3 : T}{\Delta;\Gamma \vdash \mathsf{match}\ t_1\{0 \to t_2;\ \mathsf{succ}\ x \to t_3\} : T} \ \text{T\_NatMatch}$$

$$\frac{}{\Delta;\Gamma \vdash [] : [A]} \ \text{T\_Empty}$$

$$\frac{\Delta;\Gamma \vdash t_1 : A \quad \Delta;\Gamma \vdash t_2 : [A]}{\Delta;\Gamma \vdash t_1 :: t_2 : [A]} \ \text{T\_Cons}$$

$$\frac{\Delta;\Gamma \vdash t_1 : [A] \quad \Delta;\Gamma \vdash t_2 : T \quad \Delta;\Gamma, x : A, y : [A] \vdash t_3 : T}{\Delta;\Gamma \vdash \mathsf{match}\ t_1\{[] \to t_2; x :: y \to t_3\} : T} \ \text{T\_ListMatch}$$

$$\frac{}{\Delta;\Gamma \vdash \mathsf{T} : \mathsf{Bool}} \ \text{T\_True}$$

$$\frac{}{\Delta;\Gamma \vdash \mathsf{F} : \mathsf{Bool}} \ \text{T\_False}$$

$$\frac{\Delta;\Gamma \vdash t_1 : T \quad \Delta;\Gamma \vdash t_2 : T \quad \Delta;\Gamma \vdash b_1 : \mathsf{Bool}}{\Delta;\Gamma \vdash \mathsf{if}\ b_1\ \mathsf{then}\ t_1\ \mathsf{else}\ t_2 : T} \ \text{T\_If}$$

$$\frac{\Delta;\Gamma \vdash b'_1 : T_1, \ldots, \Delta;\Gamma \vdash b'_i : T_i \quad (\mathsf{func}\,\mathtt{name}(x_1 : T_1, \ldots, x_i : T_i) \to T\{lv_1\ y_1 : A_1 = b_1; \ldots; lv_j\ y_j : A_j = b_j; t\}) \in \Delta}{\Delta;\Gamma \vdash \mathtt{name}(b'_1, \ldots, b'_i) : T} \ \text{T\_App}$$

$$\frac{\Delta;\Gamma \vdash b : T}{\Delta;\Gamma \vdash \mathsf{return}\ b : T} \ \text{T\_Return}$$

$\boxed{\Delta \vdash t_1 \rightsquigarrow t_2}$ Single-Step Reduction for Terms

$$\frac{}{\Delta \vdash \mathsf{match}\ 0\{0 \to t_1;\ \mathsf{succ}\ x \to t_2\} \rightsquigarrow t_1} \ \text{S\_Base}$$

$$\frac{}{\Delta \vdash \mathsf{match}\ (\mathsf{succ}\ nat)\{0 \to t_1;\ \mathsf{succ}\ x \to t_2\} \rightsquigarrow [nat/x]t_2} \ \text{S\_Step}$$

$$\frac{}{\Delta \vdash \mathsf{match}\ []\{[] \to t_1; x :: y \to t_2\} \rightsquigarrow t_1} \ \text{S\_ListBase}$$

$$\frac{}{\Delta \vdash \mathsf{match}\,(v :: list)\{[] \to t_3; x :: y \to t_4\} \rightsquigarrow [v/x][list/y]t_2}\ \text{S\_ListStep}$$

$$\frac{}{\Delta \vdash \mathsf{if}\ \mathsf{T}\ \mathsf{then}\ t_1\ \mathsf{else}\ t_2 \rightsquigarrow t_1}\ \text{S\_IfT}$$

$$\frac{}{\Delta \vdash \mathsf{if}\ \mathsf{F}\ \mathsf{then}\ t_1\ \mathsf{else}\ t_2 \rightsquigarrow t_2}\ \text{S\_IfF}$$

$$\frac{}{\Delta \vdash \mathsf{return}\ b \rightsquigarrow b}\ \text{S\_Return}$$

$$\frac{(\mathsf{func}\,\mathtt{name}(x_1 : T_1, \ldots, x_i : T_i) \to T\{lv_1\ y_1 : A_1 = b_1; \ldots; lv_j\ y_j : A_j = b_j; t\}) \in \Delta}{\Delta \vdash \mathtt{name}(v_1, \ldots, v_i) \rightsquigarrow [v_1, \ldots, v_i/x_1, \ldots, x_i][b_1, \ldots, b_j/y_1, \ldots, y_j]t}\ \text{S\_Beta}$$

$\boxed{\Delta \vdash t_1 \rightsquigarrow^* t_2}$ Multi-Step Reduction for Terms

$$\frac{\Delta \vdash t_1 \rightsquigarrow t_2}{\Delta \vdash EC[t_1] \rightsquigarrow^* EC[t_2]}\ \text{Step}$$

$$\frac{\Delta \vdash t_1 \rightsquigarrow t_2 \quad \Delta \vdash EC[t_2] \rightsquigarrow^* EC[t_3]}{\Delta \vdash EC[t_1] \rightsquigarrow^* EC[t_3]}\ \text{Mult}$$

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Definition rules:         25 good    0 bad
Definition rule clauses:  44 good    0 bad
```