Midterm Exam
Programming Languages Concepts (CSCI 3030)

Submit a single PDF file of your solution set on D2L.
All questions must be in order.
All assignments not adhering to this will not be graded.

0. Complete the midterm exam questions on Learn OCaml.

1. Suppose we have the following context-free grammar:

$$b ::= 0 \mid 1 \mid \mathsf{if}\ b\ \mathsf{then}\ b\ \mathsf{else}\ b$$

   (a) Derive the string $\mathsf{if}\ 0\ \mathsf{then}\ 1\ \mathsf{else}\ 0$ using the above context-free grammar.

   (b) Consider the judgment $b_1 \rightsquigarrow b_2$ which stands for "b_1 evaluates in exactly one step to b_2". It is defined by:

$$\frac{}{\mathsf{if}\ 1\ \mathsf{then}\ b_1\ \mathsf{else}\ b_2 \rightsquigarrow b_1}\ \text{T} \qquad \frac{}{\mathsf{if}\ 0\ \mathsf{then}\ b_1\ \mathsf{else}\ b_2 \rightsquigarrow b_2}\ \text{F}$$

   Then we define a second judgment $b_1 \rightsquigarrow^+ b_2$ which stands for "b_1 evaluates in at least one step to b_2". It is defined by:

$$\frac{b_1 \rightsquigarrow b_2}{b_1 \rightsquigarrow^+ b_2}\ \textsc{Step} \qquad \frac{b_1 \rightsquigarrow b_2 \qquad b_2 \rightsquigarrow^+ b_3}{b_1 \rightsquigarrow^+ b_3}\ \textsc{Mult}$$

   Using these two judgments and their rules derive $\mathsf{if}\ 0\ \mathsf{then}\ 1\ \mathsf{else}\ (\mathsf{if}\ 1\ \mathsf{then}\ 0\ \mathsf{else}\ 1) \rightsquigarrow^+ 0$.

   Hint: to start use the mult rule where b_1 is $\mathsf{if}\ 0\ \mathsf{then}\ 1\ \mathsf{else}\ (\mathsf{if}\ 1\ \mathsf{then}\ 0\ \mathsf{else}\ 1)$ and b_3 is 0, and then by looking at the conclusions of rules for the $b_1 \rightsquigarrow b_2$ judgment choose a b_2, then continue.