

termvar, *x*, *y*
funcname, **name**
indecies, *i*, *j*

$types, T, A, B, C$	$::=$ $ $ Bool $ $ Nat	Types
Γ	$::=$ $ $ \emptyset $ $ $x_1 : T_1, \dots, x_i : T_i$	Typing Context
$program, p$	$::=$ $ $ func name $(x_1 : T_1, \dots, x_i : T_i) \rightarrow T\{body\}$ $ $ (p)	Programs S
$body$	$::=$ $ $ $asgn; t$ $ $ t	Function Bodies Body with assignments Body without assignments
lv	$::=$ $ $ let $ $ var	Assignment Tags Use x at least once Use x any number of times
$assignemnts, asgn$	$::=$ $ $ $lv_1 x_1 : T_1 = b_1; \dots; lv_j x_j : T_j = b_j$	Variable Assignments
t	$::=$ $ $ x $ $ 0 $ $ succ t $ $ match $t_1\{0 \rightarrow t_2; \text{succ } x \rightarrow t_3\}$ $ $ T $ $ F $ $ if b_1 then t_1 else t_2 $ $ name (b_1, \dots, b_i) $ $ return b $ $ $EC[t]$ $ $ (t)	Terms A variable Zero Successor Natural Number Pattern Match Logical true Logical false Pattern Matching for booleans Function application Return of a term Plugging the hole in EC gives S
b	$::=$ $ $ x $ $ 0 $ $ succ b $ $ match $b_1\{0 \rightarrow t_2; \text{succ } x \rightarrow t_3\}$ $ $ T $ $ F $ $ if b_1 then t_2 else t_3 $ $ name (b_1, \dots, b_i) $ $ $EC[b]$ $ $ (b)	Basic Terms A variable Zero Successor Natural Number Pattern Match Logical true Logical false Pattern Matching for booleans Function application Plugging the hole in EC gives S
nat	$::=$	Natural Number Values

	$\begin{array}{ l} 0 \\ \text{succ } nat \end{array}$	
v	$\begin{array}{ l} ::= \\ \text{T} \\ \text{F} \\ 0 \\ \text{succ } nat \end{array}$	Values
Δ	$\begin{array}{ l} ::= \\ p_1 \\ \Delta_1, \Delta_2 \end{array}$	Contexts of Function Definitions
TP	$\begin{array}{ l} ::= \\ \emptyset \\ b, TP \end{array}$	List of Term Parameters Empty List Term Argument
CP	$\begin{array}{ l} ::= \\ \emptyset \\ EC, TP \\ b, CP_2 \end{array}$	Evaluation Contexts for Parameters Empty List Context Evaluation Argument Term Argument
$evalctx, EC$	$\begin{array}{ l} ::= \\ \square \\ T \\ F \\ 0 \\ \text{match } EC \{0 \rightarrow t_1; \text{succ } x \rightarrow t_2\} \\ \text{if } EC \text{ then } t_2 \text{ else } t_3 \\ \text{name}(CP) \\ \text{return } b \\ (EC) \end{array}$	Evaluation Contexts The hole (location of the evaluation point)

S

$\boxed{\Delta_1 \vdash \Delta_2}$ Type Checking for Definitions

$$\begin{array}{c}
 \Delta; \Gamma, x_1 : T_1, \dots, x_i : T_i \vdash b_1 : A_1 \dots \Delta; \Gamma, x_1 : T_1, \dots, x_i : T_i, y_1 : A_1, \dots, y_{j-1} : A_{j-1} \vdash b_j : A_j \\
 \Delta_1; x_1 : T_1, \dots, x_i : T_i, y_1 : A'_1, \dots, y_j : A'_j \vdash t : T \\
 \hline
 \Delta_1 \vdash \text{func name}(x_1 : T_1, \dots, x_i : T_i) \rightarrow T\{lv \ y_1 : A_1 = b_1; \dots; lv \ y_j : A_j = b_j; t\} \quad \text{DT_ASGN} \\
 \\
 \frac{\Delta_1; x_1 : T_1, \dots, x_i : T_i \vdash t : T}{\Delta_1 \vdash \text{func name}(x_1 : T_1, \dots, x_i : T_i) \rightarrow T\{t\}} \quad \text{DT_NASGN} \\
 \\
 \frac{\Delta_1 \vdash p_1 \quad \Delta_1, p_1 \vdash \Delta_2}{\Delta_1 \vdash p_1, \Delta_2} \quad \text{DT_EXT}
 \end{array}$$

$\boxed{\Delta; \Gamma \vdash t : T}$ Type Checking for Terms

$$\begin{array}{c}
 \frac{x : T \in \Gamma}{\Delta; \Gamma \vdash x : T} \quad \text{T_VAR} \\
 \\
 \frac{}{\Delta; \Gamma \vdash 0 : \text{Nat}} \quad \text{T_ZERO} \\
 \\
 \frac{\Delta; \Gamma \vdash t : \text{Nat}}{\Delta; \Gamma \vdash \text{succ } t : \text{Nat}} \quad \text{T_SUCC}
 \end{array}$$

$$\frac{\Delta; \Gamma \vdash t_1 : \text{Nat} \quad \Delta; \Gamma \vdash t_2 : T \quad \Delta; \Gamma, x : \text{Nat} \vdash t_3 : T}{\Delta; \Gamma \vdash \text{match } t_1 \{0 \rightarrow t_2; \text{succ } x \rightarrow t_3\} : T} \quad \text{T_MATCH}$$

$$\frac{}{\Delta; \Gamma \vdash \top : \text{Bool}} \quad \text{T_TRUE}$$

$$\frac{}{\Delta; \Gamma \vdash \text{F} : \text{Bool}} \quad \text{T_FALSE}$$

$$\frac{\Delta; \Gamma \vdash t_1 : T \quad \Delta; \Gamma \vdash t_2 : T \quad \Delta; \Gamma \vdash b_1 : \text{Bool}}{\Delta; \Gamma \vdash \text{if } b_1 \text{ then } t_1 \text{ else } t_2 : T} \quad \text{T_IF}$$

$$\frac{\Delta; \Gamma \vdash b'_1 : T_1, \dots, \Delta; \Gamma \vdash b'_i : T_i \quad (\text{func name}(x_1 : T_1, \dots, x_i : T_i) \rightarrow T\{l_{v_1} y_1 : A_1 = b_1; \dots; l_{v_j} y_j : A_j = b_j; t\}) \in \Delta}{\Delta; \Gamma \vdash \text{name}(b'_1, \dots, b'_i) : T} \quad \text{T_APP}$$

$$\frac{\Delta; \Gamma \vdash b : T}{\Delta; \Gamma \vdash \text{return } b : T} \quad \text{T_RETURN}$$

$\Delta \vdash t_1 \rightsquigarrow t_2$ Single-Step Reduction for Terms

$$\frac{}{\Delta \vdash \text{match } 0 \{0 \rightarrow t_1; \text{succ } x \rightarrow t_2\} \rightsquigarrow t_1} \quad \text{S_BASE}$$

$$\frac{}{\Delta \vdash \text{match } (\text{succ } \text{nat}) \{0 \rightarrow t_1; \text{succ } x \rightarrow t_2\} \rightsquigarrow [\text{nat}/x]t_2} \quad \text{S_STEP}$$

$$\frac{}{\Delta \vdash \text{if } \top \text{ then } t_1 \text{ else } t_2 \rightsquigarrow t_1} \quad \text{S_IFT}$$

$$\frac{}{\Delta \vdash \text{if } \text{F} \text{ then } t_1 \text{ else } t_2 \rightsquigarrow t_2} \quad \text{S_IFF}$$

$$\frac{}{\Delta \vdash \text{return } b \rightsquigarrow b} \quad \text{S_RETURN}$$

$$\frac{(\text{func name}(x_1 : T_1, \dots, x_i : T_i) \rightarrow T\{l_{v_1} y_1 : A_1 = b_1; \dots; l_{v_j} y_j : A_j = b_j; t\}) \in \Delta}{\Delta \vdash \text{name}(v_1, \dots, v_i) \rightsquigarrow [v_1, \dots, v_i/x_1, \dots, x_i][b_1, \dots, b_j/y_1, \dots, y_j]t} \quad \text{S_BETA}$$

$\Delta \vdash t_1 \rightsquigarrow^* t_2$ Multi-Step Reduction for Terms

$$\frac{\Delta \vdash t_1 \rightsquigarrow t_2}{\Delta \vdash EC[t_1] \rightsquigarrow^* EC[t_2]} \quad \text{STEP}$$

$$\frac{\Delta \vdash t_1 \rightsquigarrow t_2 \quad \Delta \vdash EC[t_2] \rightsquigarrow^* EC[t_3]}{\Delta \vdash EC[t_1] \rightsquigarrow^* EC[t_3]} \quad \text{MULT}$$

Definition rules: 20 good 0 bad
Definition rule clauses: 36 good 0 bad