Final Exam
Programming Languages Concepts (CSCI 3030)

Submit a single PDF file of your solution set on D2L.
All questions must be in order.
All assignments not adhering to this will not be graded.

0. Complete the midterm exam questions on Learn OCaml.
1. Derive the following:

$$\emptyset \vdash \mathsf{Fun}(\mathbb{N}, n.\mathsf{Rec}(n, \mathsf{Num}(0), m.r.m)) \,?\, \mathbb{N}$$

Type Checking vs Type Construction

Type Checking

$$\frac{}{\Gamma \vdash \mathsf{True} : \mathbb{B}} \qquad \frac{}{\Gamma \vdash \mathsf{False} : \mathbb{B}}$$

$$\frac{\Gamma \vdash t_1 : \mathbb{B} \qquad \Gamma \vdash t_2 : T \qquad \Gamma \vdash t_3 : T}{\Gamma \vdash \mathsf{If}(t_1, t_2, t_3) : T}$$

$$\frac{}{\Gamma \vdash \mathsf{Num}\, n : \mathbb{N}} \qquad \frac{\Gamma \vdash t : \mathbb{N}}{\Gamma \vdash \mathsf{Suc}\, t : \mathbb{N}}$$

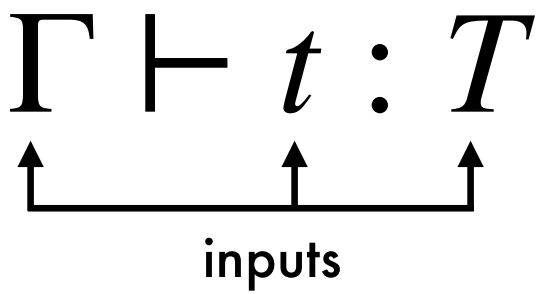
$$\frac{\Gamma \vdash t_1 : \mathbb{N} \qquad \Gamma \vdash t_2 : T \qquad \Gamma, m : \mathbb{N}, r : T \vdash t_3 : T}{\Gamma \vdash \mathsf{Rec}(t_1, t_2, m\,.\,r\,.\,t_3) : T}$$

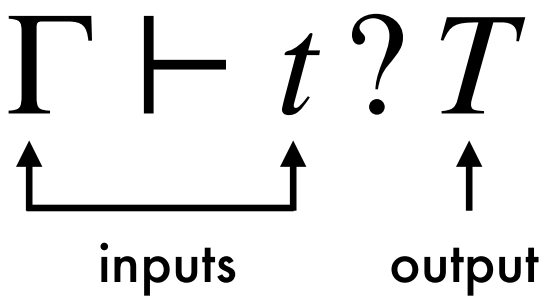
$$\frac{}{\Gamma, x : T \vdash x : T} \qquad \frac{\Gamma, x : T \vdash t : T}{\Gamma \vdash \mathsf{Fun}(x\,.\,t) : T}$$

$$\frac{\Gamma \vdash t_1 : T_1 \qquad \Gamma, x : T_1 \vdash t_2 : T_2}{\Gamma \vdash \mathsf{Let}(t_1, x\,.\,t_2) : T_2}$$

$$\frac{\Gamma \vdash t_1 : \mathsf{Fun}(T_1, T_2) \qquad \Gamma \vdash t_2 : T_1}{\Gamma \vdash \mathsf{App}(t_1, t_2) : T_2}$$

Bidirectional Typing





Bidirectional Typing: Checking

$$\frac{}{\Gamma, x : T \vdash x : T}$$

$$\frac{}{\Gamma \vdash \mathsf{True} : \mathbb{B}}$$

$$\frac{}{\Gamma \vdash \mathsf{False} : \mathbb{B}}$$

$$\frac{\Gamma \vdash t_1 : \mathbb{B} \qquad \Gamma \vdash t_2 : T \qquad \Gamma \vdash t_3 : T}{\Gamma \vdash \mathsf{If}(t_1, t_2, t_3) : T}$$

$$\frac{}{\Gamma \vdash \mathsf{Num}(n) : \mathbb{N}}$$

$$\frac{\Gamma \vdash t : \mathbb{N}}{\Gamma \vdash \mathsf{Suc}(t) : \mathbb{N}}$$

$$\frac{\Gamma \vdash t_1 : \mathbb{N} \qquad \Gamma \vdash t_2 : T \qquad \Gamma, m : \mathbb{N}, r : T \vdash t_3 : T}{\Gamma \vdash \mathsf{Rec}(t_1, t_2, m \,.\, r \,.\, t_3) : T}$$

$$\frac{\Gamma, x : T_1 \vdash t : T_2}{\Gamma \vdash \mathsf{Fun}(x \,.\, t) : \mathsf{Fun}(T_1, T_2)}$$

$$\frac{\Gamma \vdash t_1 \,?\, T_1 \qquad \Gamma, x : T_1 \vdash t_2 : T_2}{\Gamma \vdash \mathsf{Let}(t_1, x \,.\, t_2) : T_2}$$

$$\frac{\Gamma \vdash t_1 ?\ \mathsf{Fun}(T_1, T_2') \qquad \Gamma \vdash t_2 : T_1 \qquad T_2' = T_2}{\Gamma \vdash \mathsf{App}(t_1, t_2) : T_2}$$

Bidirectional Typing: Synthesis

$$\frac{}{\Gamma, x : T \vdash x \,?\, T}$$

$$\frac{}{\Gamma \vdash \mathsf{True} \,?\, \mathbb{B}}$$

$$\frac{}{\Gamma \vdash \mathsf{False} \,?\, \mathbb{B}}$$

$$\frac{\Gamma \vdash t_1 : \mathbb{B} \qquad \Gamma \vdash t_2 \,?\, T \qquad \Gamma \vdash t_3 : T}{\Gamma \vdash \mathsf{If}(t_1, t_2, t_3) \,?\, T}$$

$$\frac{}{\Gamma \vdash \mathsf{Num}(n) \,?\, \mathbb{N}}$$

$$\frac{\Gamma \vdash t : \mathbb{N}}{\Gamma \vdash \mathsf{Suc}(t) \,?\, \mathbb{N}}$$

$$\frac{\Gamma \vdash t_1 : \mathbb{N} \qquad \Gamma \vdash t_2 \,?\, T \qquad \Gamma, m : \mathbb{N}, r : T \vdash t_3 : T}{\Gamma \vdash \mathsf{Rec}(t_1, t_2, m \,.\, r \,.\, t_3) \,?\, T}$$

$$\frac{\Gamma, x : T_1 \vdash t \,?\, T_2}{\Gamma \vdash \mathsf{Fun}(T_1, x \,.\, t) \,?\, \mathsf{Fun}(T_1, T_2)}$$

$$\frac{\Gamma \vdash t_1 \,?\, T_1 \qquad \Gamma, x : T_1 \vdash t_2 \,?\, T_2}{\Gamma \vdash \mathsf{Let}(t_1, x \,.\, t_2) \,?\, T_2}$$

$$\frac{\Gamma \vdash t_1 \,?\, \mathsf{Fun}(T_1, T_2) \qquad \Gamma \vdash t_2 : T_1}{\Gamma \vdash \mathsf{App}(t_1, t_2) \,?\, T_2}$$