

Final Exam

Programming Languages Concepts (CSCI 3030)

Submit a single PDF file of your solution set on D2L.

All questions must be in order.

All assignments not adhering to this will not be graded.

0. Complete the midterm exam questions on Learn OCaml.

1. Derive the following:

$$\emptyset \vdash \text{Fun}(\mathbb{N}, n.\text{Rec}(n, \text{Num}(0), m.r.m)) ? \mathbb{N}$$

Type Checking vs Type Construction

Type Checking

$$\frac{}{\Gamma \vdash \text{True} : \mathbb{B}}$$

$$\frac{}{\Gamma \vdash \text{False} : \mathbb{B}}$$

$$\frac{\Gamma \vdash t_1 : \mathbb{B} \quad \Gamma \vdash t_2 : T \quad \Gamma \vdash t_3 : T}{\Gamma \vdash \text{If}(t_1, t_2, t_3) : T}$$

$$\frac{}{\Gamma \vdash \text{Num } n : \mathbb{N}}$$

$$\frac{\Gamma \vdash t : \mathbb{N}}{\Gamma \vdash \text{Suc } t : \mathbb{N}}$$

$$\frac{\Gamma \vdash t_1 : \mathbb{N} \quad \Gamma \vdash t_2 : T \quad \Gamma, m : \mathbb{N}, r : T \vdash t_3 : T}{\Gamma \vdash \text{Rec}(t_1, t_2, m . r . t_3) : T}$$


$$\frac{}{\Gamma, x : T \vdash x : T}$$

$$\frac{\Gamma, x : T \vdash t : T}{\Gamma \vdash \text{Fun}(x . t) : T}$$

$$\frac{\Gamma \vdash t_1 : T_1 \quad \Gamma, x : T_1 \vdash t_2 : T_2}{\Gamma \vdash \text{Let}(t_1, x . t_2) : T_2}$$


$$\frac{\Gamma \vdash t_1 : \text{Fun}(T_1, T_2) \quad \Gamma \vdash t_2 : T_1}{\Gamma \vdash \text{App}(t_1, t_2) : T_2}$$

Bidirectional Typing

$$\Gamma \vdash t : T$$


inputs

The diagram shows a horizontal line with three upward-pointing arrows. The first arrow points to the symbol Γ , the second arrow points to the term t , and the third arrow points to the type T . This indicates that Γ , t , and T are all inputs to the typing judgment.

$$\Gamma \vdash t ? T$$


inputs output

The diagram shows two horizontal lines. The first line has two upward-pointing arrows: the first points to Γ and the second points to t . The second line has one upward-pointing arrow pointing to T . The label 'inputs' is centered under the first line, and the label 'output' is centered under the second line. This indicates that Γ and t are inputs, and T is the output of the backward typing judgment.

Bidirectional Typing: Checking

$$\overline{\Gamma, x : T \vdash x : T}$$

$$\frac{\Gamma, x : T_1 \vdash t : T_2}{\Gamma \vdash \text{Fun}(x.t) : \text{Fun}(T_1, T_2)}$$

$$\frac{\Gamma \vdash t_1 ? T_1 \quad \Gamma, x : T_1 \vdash t_2 : T_2}{\Gamma \vdash \text{Let}(t_1, x.t_2) : T_2}$$

$$\frac{\Gamma \vdash t_1 ? \text{Fun}(T_1, T'_2) \quad \Gamma \vdash t_2 : T_1 \quad T'_2 = T_2}{\Gamma \vdash \text{App}(t_1, t_2) : T_2}$$

$$\overline{\Gamma \vdash \text{True} : \mathbb{B}}$$

$$\overline{\Gamma \vdash \text{False} : \mathbb{B}}$$

$$\frac{\Gamma \vdash t_1 : \mathbb{B} \quad \Gamma \vdash t_2 : T \quad \Gamma \vdash t_3 : T}{\Gamma \vdash \text{If}(t_1, t_2, t_3) : T}$$

$$\overline{\Gamma \vdash \text{Num}(n) : \mathbb{N}}$$

$$\frac{\Gamma \vdash t : \mathbb{N}}{\Gamma \vdash \text{Suc}(t) : \mathbb{N}}$$

$$\frac{\Gamma \vdash t_1 : \mathbb{N} \quad \Gamma \vdash t_2 : T \quad \Gamma, m : \mathbb{N}, r : T \vdash t_3 : T}{\Gamma \vdash \text{Rec}(t_1, t_2, m.r.t_3) : T}$$

Bidirectional Typing: Synthesis

$$\begin{array}{c} \frac{}{\Gamma \vdash \text{True} ? \mathbb{B}} \\ \frac{}{\Gamma \vdash \text{False} ? \mathbb{B}} \\ \frac{\Gamma \vdash t_1 : \mathbb{B} \quad \Gamma \vdash t_2 ? T \quad \Gamma \vdash t_3 : T}{\Gamma \vdash \text{If}(t_1, t_2, t_3) ? T} \end{array} \quad \begin{array}{c} \frac{}{\Gamma \vdash \text{Num}(n) ? \mathbb{N}} \\ \frac{\Gamma \vdash t : \mathbb{N}}{\Gamma \vdash \text{Suc}(t) ? \mathbb{N}} \\ \frac{\Gamma \vdash t_1 : \mathbb{N} \quad \Gamma \vdash t_2 ? T \quad \Gamma, m : \mathbb{N}, r : T \vdash t_3 : T}{\Gamma \vdash \text{Rec}(t_1, t_2, m . r . t_3) ? T} \end{array} \quad \begin{array}{c} \frac{}{\Gamma, x : T \vdash x ? T} \\ \frac{\Gamma, x : T_1 \vdash t ? T_2}{\Gamma \vdash \text{Fun}(T_1, x . t) ? \text{Fun}(T_1, T_2)} \\ \frac{\Gamma \vdash t_1 ? T_1 \quad \Gamma, x : T_1 \vdash t_2 ? T_2}{\Gamma \vdash \text{Let}(t_1, x . t_2) ? T_2} \\ \frac{\Gamma \vdash t_1 ? \text{Fun}(T_1, T_2) \quad \Gamma \vdash t_2 : T_1}{\Gamma \vdash \text{App}(t_1, t_2) ? T_2} \end{array}$$