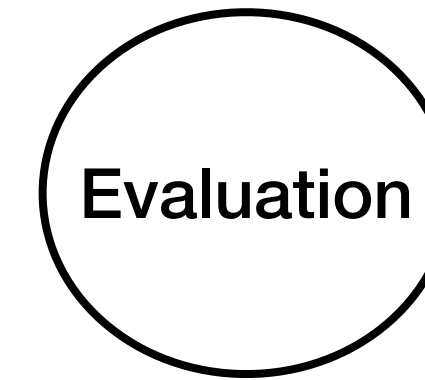
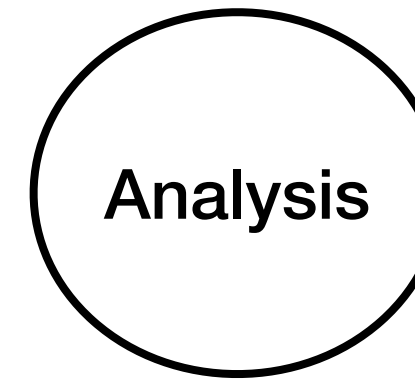


Core Design Concepts Discussed:



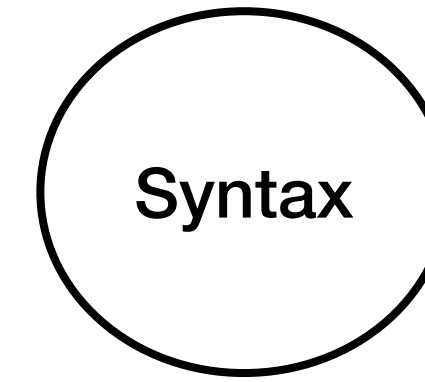
Static and Dynamic Semantics

Analysis and Evaluation

Harley Eades III

IffyLang

Core Design Concepts:



(Terms) $t \rightarrow x \mid T \mid F \mid \text{if}(t_1, t_2, t_3) \mid \text{let}(t_1, x . t_2)$

Free and Bound Variables

Core Design Concepts:

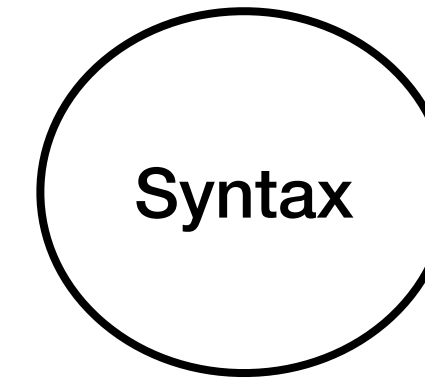


Syntax

$$\text{let}(t_1, x . t_2)$$

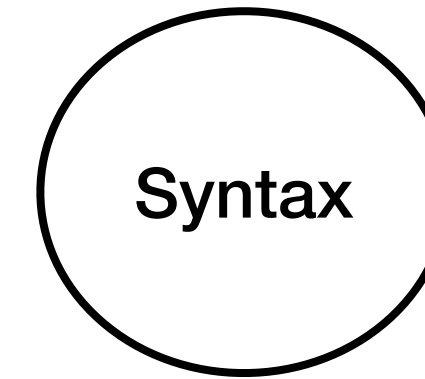
Free and Bound Variables

Core Design Concepts:



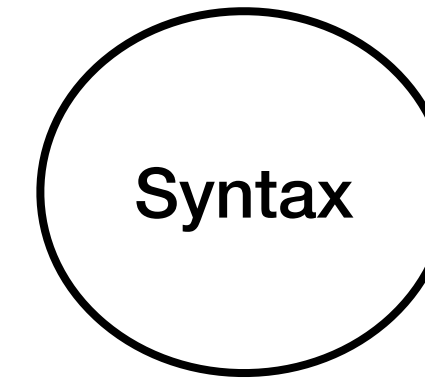
$\text{let}(t_1, x . t_2)$

A diagram illustrating the binding of the variable x in the expression $\text{let}(t_1, x . t_2)$. A horizontal line with two upward-pointing arrows connects the x to the t_2 , with the word "bound" centered below the line.



Free and Bound Variables

(Terms) $t \rightarrow x \mid T \mid F \mid \text{if}(t_1, t_2, t_3) \mid \text{let}(t_1, x . t_2)$



Free and Bound Variables

if(*y*, T, F)

Free and Bound Variables



Syntax

Bound variable : A variable associated with a binder.

Free variable : A variable with no associated binder.

Free and Bound Variables

Core Design Concepts:

Syntax

Bound variable : A variable associated with a binder.

let(T, x . if(x, F, x))

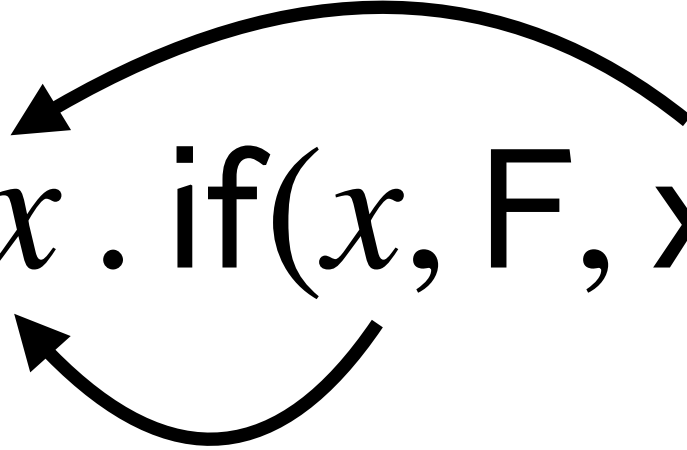
Free variable : A variable with no associated binder.

Free and Bound Variables

Bound variable :

A variable associated with a binder.

let(T, x . if(x , F, x))

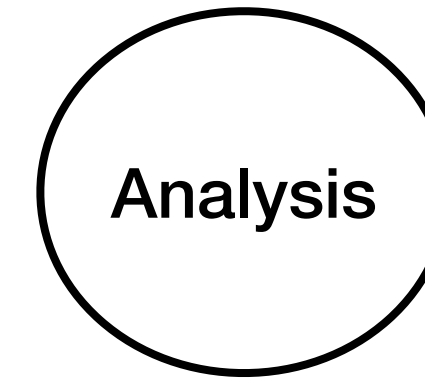


Free variable :

A variable with no associated binder.

let(T, x . if(y , F, x))





Free Variable Function

$$\text{FV}(\text{T}) = \emptyset$$

$$\text{FV}(\text{if}(t_1, t_2, t_3)) = \text{FV}(t_1) \cup \text{FV}(t_2) \cup \text{FV}(t_3)$$

$$\text{FV}(\text{F}) = \emptyset$$

$$\text{FV}(\text{let}(t_1, x . t_2)) = (\text{FV}(t_1) \cup \text{FV}(t_2)) - \{x\}$$

$$\text{FV}(x) = \{x\}$$

$$\text{FV}(\text{let}(x, x . \text{if}(x, T, T))$$

$$= (\text{FV}(\{x\}) \cup \text{FV}(\text{if}(x, T, T))) - \{x\}$$

$$= (\text{FV}(\{x\}) \cup (\text{FV}(x) \cup \text{FV}(T) \cup \text{FV}(T))) - \{x\}$$

$$= (\{x\} \cup (\{x\} \cup \emptyset \cup \emptyset)) - \{x\}$$

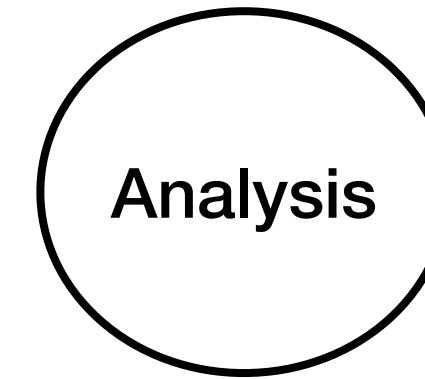
$$= (\{x\} \cup \{x\}) - \{x\}$$

$$= \{x\} - \{x\}$$

$$= \emptyset$$

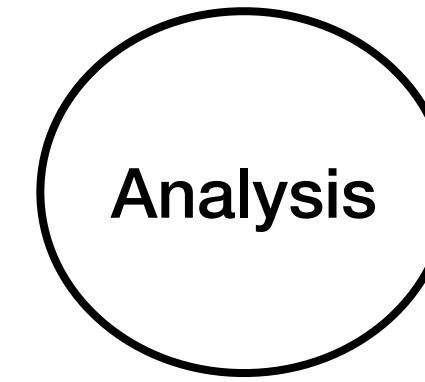
$$\begin{aligned}
& \text{FV}(\text{let}(x, y . \text{if}(y, T, T)) \\
&= (\text{FV}(x) \cup \text{FV}(\text{if}(y, T, T))) - \{y\} \\
&= (\text{FV}(x) \cup (\text{FV}(y) \cup \text{FV}(T) \cup \text{FV}(T)))) - \{y\} \\
&= (\{x\} \cup (\{y\} \cup \emptyset \cup \emptyset)) - \{y\} \\
&= (\{x\} \cup \{y\}) - \{y\} \\
&= \{x, y\} - \{y\} \\
&= \{x\}
\end{aligned}$$

α -Conversion



The renaming of bound variables so that they are distinct from the set of free variables.

α -Equivalence



$t_1 =_{\alpha} t_2$ if and only if t_1 can be α -converted into t_2

α -Equivalence Classes

Core Design Concepts:

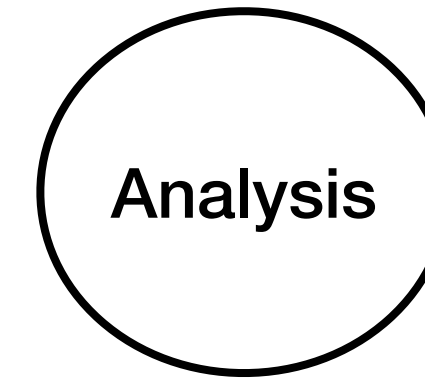


Analysis

$$[t] = \{t' \mid t =_{\alpha} t'\}$$

We will now be working up to alpha-equivalence.

All operations, judgments, etc will be defined on α -equivalence classes.



Free Variable Function

$$FV(T) = \emptyset$$

$$FV(\text{if}(t_1, t_2, t_3)) = FV(t_1) \cup FV(t_2) \cup FV(t_3)$$

$$FV(F) = \emptyset$$

$$FV(\text{let}(t_1, x . t_2)) = (FV(t_1) \cup FV(t_2)) - \{x\}$$

$$FV(x) = \{x\}$$

Substitution

$$[t_1/x]\top = \top$$

$$[t_1/x]\text{F} = \text{F}$$

$$[t_1/x]y = y, \text{ if } x \neq y$$

$$[t_1/x]x = t_1$$

$$[t_1/x]\text{if}(t_2, t_3, t_4) = \text{if}([t_1/x]t_2, [t_1/x]t_3, [t_1/x]t_4)$$

$$[t_1/x]\text{let}(t_2, y . t_3) = \text{let}([t_1/x]t_2, y . [t_1/x]t_3)$$

Variable Capture

Core Design Concepts:



Analysis

$$[\text{if}(x, T, T)/x]\text{let}(T, x . x)$$
$$= \text{let}([\text{if}(x, T, T)/x]T, x . [\text{if}(x, T, T)/x]x)$$
$$= \text{let}(T, x . \text{if}(x, T, T))$$

Substitution

$$[t_1/x]\top = \top$$

$$[t_1/x]\text{F} = \text{F}$$

$$[t_1/x]y = y, \text{ if } x \neq y$$

$$[t_1/x]x = t_1$$

$$[t_1/x]\text{if}(t_2, t_3, t_4) = \text{if}([t_1/x]t_2, [t_1/x]t_3, [t_1/x]t_4)$$

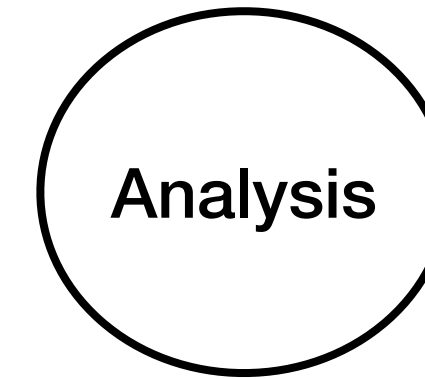
$$[t_1/x]\text{let}(t_2, y . t_3) = \text{let}([t_1/x]t_2, y . t_3), \text{ if } x = y$$

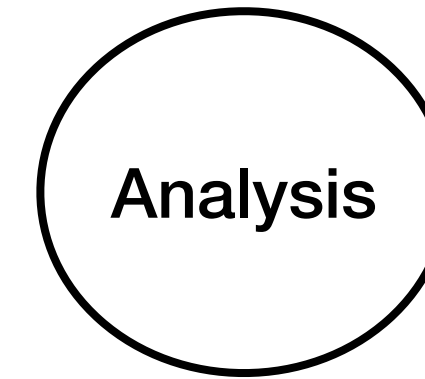
$$[t_1/x]\text{let}(t_2, y . t_3) = \text{let}([t_1/x]t_2, y . [t_1/x]t_3), \text{ if } x \neq y$$

Variable Capture

$[if(x, T, T)/x]let(T, x . x)$
 $= ?$

Core Design Concepts:





Variable Capture

$[if(x, T, T)/z]let(T, y . y)$

$= let([if(x, T, T)/z]T, y . [if(x, T, T)/z]y)$

$= let(T, y . y)$

Capture Avoiding Substitution

Analysis

$$[t_1/x]\top = \top$$

$$[t_1/x]\text{if}(t_2, t_3, t_4) = \text{if}([t_1/x]t_2, [t_1/x]t_3, [t_1/x]t_4)$$

$$[t_1/x]\text{F} = \text{F}$$

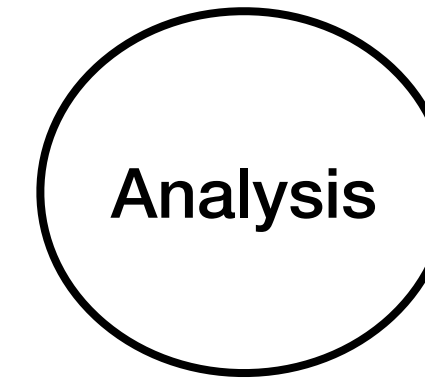
$$[t_1/x]\text{let}(t_2, y . t_3) = \text{let}([t_1/x]t_2, y . [t_1/x]t_3), \text{ if } x \neq y$$

$$[t_1/x]y = y, \text{ if } x \neq y$$

$$[t_1/x]x = t_1$$

Static Semantics

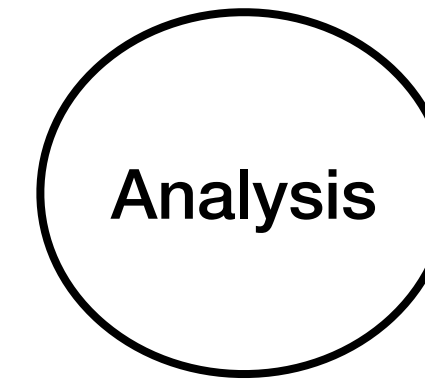
Core Design Concepts:



Assigning a program a type.

Hypothetical Judgment

Core Design Concepts:



Hypothetical Judgments:

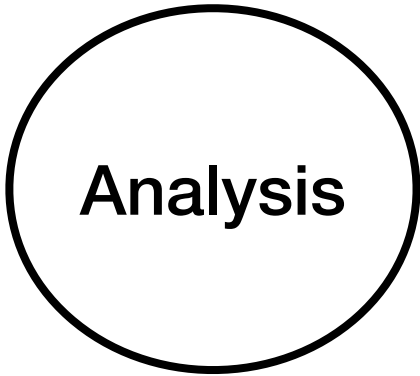
$$J_1, \dots, J_i \vdash J$$

Typing Context:

$$\Gamma \rightarrow \emptyset \mid \Gamma, J$$

Static Semantics: Typing

Core Design Concepts:



$$\frac{x_j \in \{x_1, \dots, x_i\}}{x_1 : \mathbb{B}, \dots, x_i : \mathbb{B} \vdash x_j : \mathbb{B}} \text{Var}$$

$$\frac{}{\Gamma \vdash \top : \mathbb{B}} \text{True}$$

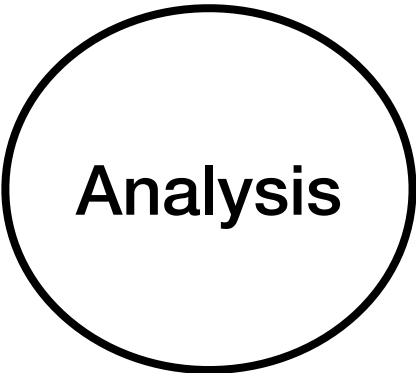
$$\frac{}{\Gamma \vdash \text{F} : \mathbb{B}} \text{False}$$

$$\frac{\Gamma \vdash t_1 : \mathbb{B} \quad \Gamma \vdash t_2 : \mathbb{B} \quad \Gamma \vdash t_3 : \mathbb{B}}{\Gamma \vdash \text{if}(t_1, t_2, t_3) : \mathbb{B}} \text{If}$$

$$\frac{\Gamma \vdash t_1 : \mathbb{B} \quad \Gamma, x : \mathbb{B} \vdash t_2 : \mathbb{B}}{\Gamma \vdash \text{let}(t_1, x . t_2) : \mathbb{B}} \text{If}$$

Static Semantics: Typing

Core Design Concepts:



$$\frac{}{x_1 : \mathbb{B}, \dots, x_i : \mathbb{B} \vdash x_j : \mathbb{B}}$$

Var

$$\frac{\Gamma \vdash t_1 : \mathbb{B} \quad \Gamma \vdash t_2 : \mathbb{B} \quad \Gamma \vdash t_3 : \mathbb{B}}{\Gamma \vdash \text{if}(t_1, t_2, t_3) : \mathbb{B}}$$

If

$$\frac{}{\Gamma \vdash \text{True} : \mathbb{B}}$$

True

$$\frac{\Gamma \vdash t_1 : \mathbb{B} \quad \Gamma, x : \mathbb{B} \vdash t_2 : \mathbb{B}}{\Gamma \vdash \text{let}(t_1, x . t_2) : \mathbb{B}}$$

If

$$\frac{}{\Gamma \vdash \text{False} : \mathbb{B}}$$

False

If $\emptyset \vdash t : \mathbb{B}$, then $\text{FV}(t) = \emptyset$

Dynamic Semantics

(Environment) $\delta \rightarrow \emptyset \mid \delta, x \Downarrow v$

$$\frac{}{x_1 \Downarrow v_1, \dots, x_i \Downarrow v_i \vdash x_j \Downarrow v_j} \text{Var}$$

$$\frac{}{\delta \vdash \top \Downarrow \top} \text{True}$$

$$\frac{}{\delta \vdash \text{F} \Downarrow \text{F}} \text{False}$$

$$\frac{\delta \vdash t_1 \Downarrow \top \quad \delta \vdash t_2 \Downarrow v_2}{\delta \vdash \text{if}(t_1, t_2, t_3) \Downarrow v_2} \text{IfTrue}$$

$$\frac{\delta \vdash t_1 \Downarrow \text{F} \quad \delta \vdash t_3 \Downarrow v_3}{\delta \vdash \text{if}(t_1, t_2, t_3) \Downarrow v_3} \text{IfFalse}$$

$$\frac{\delta \vdash t_1 \Downarrow v_1 \quad \delta, x \Downarrow v_1 \vdash t_2 \Downarrow v_2}{\delta \vdash \text{let}(t_1, x . t_2) \Downarrow v_2} \text{Let}$$