Core Design Concepts Discussed:

Analysis

Evaluation

Static and Dynamic Semantics

Analysis and Evaluation

Harley Eades III

Core Design Concepts:

Syntax

IffyLang

(Terms) $t \to x \mid \mathsf{T} \mid \mathsf{F} \mid \mathsf{if}(t_1, t_2, t_3) \mid \mathsf{let}(t_1, x\,.\,t_2)$

Core Design Concepts:

Syntax

Free and Bound Variables

$$\mathrm{let}(t_1, x \,.\, t_2)$$

Core Design Concepts:

Syntax

Free and Bound Variables

$$\mathrm{let}(t_1, \underbrace{x \,.\, t_2}_{\text{bound}})$$

Core Design Concepts:

Syntax

Free and Bound Variables

(Terms) $t \to x$ | T | F | if(t_1, t_2, t_3) | let$(t_1, x \,.\, t_2)$

Core Design Concepts:

Syntax

Free and Bound Variables

$$\mathrm{if}(y, \mathrm{T}, \mathrm{F})$$

Core Design Concepts:

Syntax

Free and Bound Variables

Bound variable : A variable associated with a binder.

Free variable : A variable with no associated binder.

Core Design Concepts:

Syntax

Free and Bound Variables

Bound variable : A variable associated with a binder.

$$\mathrm{let}(\mathrm{T}, x\,.\,\mathrm{if}(x, \mathrm{F}, \mathrm{x}))$$

Free variable : A variable with no associated binder.

Free and Bound Variables

Bound variable :

A variable associated with a binder.

$$\mathrm{let}(\mathrm{T}, x\,.\,\mathrm{if}(x, \mathrm{F}, \mathrm{x}))$$

Free variable :

A variable with no associated binder.

$$\mathrm{let}(\mathrm{T}, x\,.\,\mathrm{if}(y, \mathrm{F}, \mathrm{x}))$$

Core Design Concepts:

Analysis

Free Variable Function

$\mathsf{FV}(\mathsf{T}) = \varnothing$

$\mathsf{FV}(\mathsf{F}) = \varnothing$

$\mathsf{FV}(x) = \{x\}$

$\mathsf{FV}(\mathsf{if}(t_1, t_2, t_3)) = \mathsf{FV}(t_1) \cup \mathsf{FV}(t_2) \cup \mathsf{FV}(t_3)$

$\mathsf{FV}(\mathsf{let}(t_1, x\,.\,t_2)) = (\mathsf{FV}(t_1) \cup \mathsf{FV}(t_2)) - \{x\}$

$$
\begin{aligned}
&\mathsf{FV}(\mathsf{let}(x, x\,.\,\mathsf{if}(x, \mathsf{T}, \mathsf{T})) \\
&\quad = (\mathsf{FV}(\{x\}) \cup \mathsf{FV}(\mathsf{if}(x, \mathsf{T}, \mathsf{T})))) - \{x\} \\
&\quad = (\mathsf{FV}(\{x\}) \cup (\mathsf{FV}(x) \cup \mathsf{FV}(\mathsf{T}) \cup \mathsf{FV}(\mathsf{T}))))) - \{x\} \\
&\quad = (\{x\} \cup (\{x\} \cup \varnothing \cup \varnothing)) - \{x\} \\
&\quad = (\{x\} \cup \{x\}) - \{x\} \\
&\quad = \{x\} - \{x\} \\
&\quad = \varnothing
\end{aligned}
$$

$$
\begin{aligned}
&\mathsf{FV}(\mathsf{let}(x, y\,.\,\mathsf{if}(y, \mathsf{T}, \mathsf{T}))\\
&\quad = (\mathsf{FV}(x) \cup \mathsf{FV}(\mathsf{if}(y, \mathsf{T}, \mathsf{T}))) - \{y\}\\
&\quad = (\mathsf{FV}(x) \cup (\mathsf{FV}(y) \cup \mathsf{FV}(\mathsf{T}) \cup \mathsf{FV}(\mathsf{T}))) - \{y\}\\
&\quad = (\{x\} \cup (\{y\} \cup \varnothing \cup \varnothing)) - \{y\}\\
&\quad = (\{x\} \cup \{y\}) - \{y\}\\
&\quad = \{x, y\} - \{y\}\\
&\quad = \{x\}
\end{aligned}
$$

Core Design Concepts:

Analysis

α-Conversion

The renaming of bound variables so that they are distinct from the set of free variables.

Core Design Concepts:

Analysis

α-Equlivalence

$t_1 =_\alpha t_2$ if and only if t_1 can be α-converted into t_2

α-Equlivalence Classes

Core Design Concepts:

Analysis

$$[t] = \{t' \mid t =_\alpha t'\}$$

We will now be working up to alpha-equivalence.

All operations, judgments, etc will be defined on α-equivalence classes.

Core Design Concepts:

Analysis

Free Variable Function

$$\mathsf{FV}(\mathsf{T}) = \varnothing$$

$$\mathsf{FV}(\mathsf{F}) = \varnothing$$

$$\mathsf{FV}(x) = \{x\}$$

$$\mathsf{FV}(\mathsf{if}(t_1, t_2, t_3)) = \mathsf{FV}(t_1) \cup \mathsf{FV}(t_2) \cup \mathsf{FV}(t_3)$$

$$\mathsf{FV}(\mathsf{let}(t_1, x\,.\,t_2)) = (\mathsf{FV}(t_1) \cup \mathsf{FV}(t_2)) - \{x\}$$

Substitution

$[t_1/x]\mathsf{T} = \mathsf{T}$

$[t_1/x]\mathsf{F} = \mathsf{F}$

$[t_1/x]y = y$, if $x \neq y$

$[t_1/x]x = t_1$

$[t_1/x]\mathsf{if}(t_2, t_3, t_4) = \mathsf{if}([t_1/x]t_2, [t_1/x]t_3, [t_1/x]t_4)$

$[t_1/x]\mathsf{let}(t_2, y\,.\,t_3) = \mathsf{let}([t_1/x]t_2, y\,.\,[t_1/x]t_3)$

Variable Capture

Core Design Concepts:

Analysis

$$
\begin{aligned}
&[\mathsf{if}(x, \mathsf{T}, \mathsf{T})/x]\mathsf{let}(\mathsf{T}, x\,.\,x) \\
&\quad = \mathsf{let}([\mathsf{if}(x, \mathsf{T}, \mathsf{T})/x]\mathsf{T}, x\,.\,[\mathsf{if}(x, \mathsf{T}, \mathsf{T})/x]x) \\
&\quad = \mathsf{let}(\mathsf{T}, x\,.\,\mathsf{if}(x, \mathsf{T}, \mathsf{T}))
\end{aligned}
$$

Substitution

$$[t_1/x]\mathsf{T} = \mathsf{T}$$

$$[t_1/x]\mathsf{F} = \mathsf{F}$$

$$[t_1/x]y = y, \text{ if } x \neq y$$

$$[t_1/x]x = t_1$$

$$[t_1/x]\mathsf{if}(t_2, t_3, t_4) = \mathsf{if}([t_1/x]t_2, [t_1/x]t_3, [t_1/x]t_4)$$

$$[t_1/x]\mathsf{let}(t_2, y\,.\,t_3) = \mathsf{let}([t_1/x]t_2, y\,.\,t_3), \text{ if } x = y$$

$$[t_1/x]\mathsf{let}(t_2, y\,.\,t_3) = \mathsf{let}([t_1/x]t_2, y\,.\,[t_1/x]t_3), \text{ if } x \neq y$$

Core Design Concepts:

Analysis

Variable Capture

$$[\mathsf{if}(x, \mathsf{T}, \mathsf{T})/x]\mathsf{let}(\mathsf{T}, x\,.\,x)$$

$$= ?$$

Core Design Concepts:

Analysis

Variable Capture

$$[\mathsf{if}(x, \mathsf{T}, \mathsf{T})/z]\mathsf{let}(\mathsf{T}, y\,.\,y)$$
$$= \mathsf{let}([\mathsf{if}(x, \mathsf{T}, \mathsf{T})/z]\mathsf{T}, y\,.\,[\mathsf{if}(x, \mathsf{T}, \mathsf{T})/z]y)$$
$$= \mathsf{let}(\mathsf{T}, y\,.\,y)$$

Core Design Concepts:

Analysis

Capture Avoiding Substitution

$[t_1/x]\mathsf{T} = \mathsf{T}$

$[t_1/x]\mathsf{F} = \mathsf{F}$

$[t_1/x]y = y$, if $x \neq y$

$[t_1/x]x = t_1$

$[t_1/x]\mathsf{if}(t_2, t_3, t_4) = \mathsf{if}([t_1/x]t_2, [t_1/x]t_3, [t_1/x]t_4)$

$[t_1/x]\mathsf{let}(t_2, y\,.\,t_3) = \mathsf{let}([t_1/x]t_2, y\,.\,[t_1/x]t_3)$, if $x \neq y$

Core Design Concepts:

Analysis

Static Semantics

Assigning a program a type.

Core Design Concepts:

Analysis

Hypothetical Judgment

Hypothetical Judgments:

$$J_1, \ldots, J_i \vdash J$$

Typing Context:

$$\Gamma \rightarrow \varnothing \mid \Gamma, J$$

Static Semantics: Typing

Core Design Concepts:

Analysis

$$\frac{x_j \in \{x_1, \ldots, x_i\}}{x_1 : \mathbb{B}, \ldots, x_i : \mathbb{B} \vdash x_j : \mathbb{B}}\ \text{Var}$$

$$\frac{}{\Gamma \vdash \mathsf{T} : \mathbb{B}}\ \text{True}$$

$$\frac{}{\Gamma \vdash \mathsf{F} : \mathbb{B}}\ \text{False}$$

$$\frac{\Gamma \vdash t_1 : \mathbb{B} \quad \Gamma \vdash t_2 : \mathbb{B} \quad \Gamma \vdash t_3 : \mathbb{B}}{\Gamma \vdash \mathsf{if}(t_1, t_2, t_3) : \mathbb{B}}\ \text{If}$$

$$\frac{\Gamma \vdash t_1 : \mathbb{B} \qquad \Gamma, x : \mathbb{B} \vdash t_2 : \mathbb{B}}{\Gamma \vdash \mathsf{let}(t_1, x \,.\, t_2) : \mathbb{B}}\ \text{If}$$

Static Semantics: Typing

Core Design Concepts:

Analysis

$$\frac{}{x_1 : \mathbb{B}, \ldots, x_i : \mathbb{B} \vdash x_j : \mathbb{B}} \text{ Var}$$

$$\frac{}{\Gamma \vdash \mathsf{T} : \mathbb{B}} \text{ True}$$

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$$\frac{\Gamma \vdash t_1 : \mathbb{B} \quad \Gamma \vdash t_2 : \mathbb{B} \quad \Gamma \vdash t_3 : \mathbb{B}}{\Gamma \vdash \mathsf{if}(t_1, t_2, t_3) : \mathbb{B}} \text{ If}$$

$$\frac{\Gamma \vdash t_1 : \mathbb{B} \qquad \Gamma, x : \mathbb{B} \vdash t_2 : \mathbb{B}}{\Gamma \vdash \mathsf{let}(t_1, x \,.\, t_2) : \mathbb{B}} \text{ If}$$

If $\varnothing \vdash t : \mathbb{B}$, then $\mathsf{FV}(t) = \varnothing$

Dynamic Semantics

(Environment) $\delta \to \varnothing \mid \delta, x \Downarrow v$

$$\frac{}{x_1 \Downarrow v_1, \ldots, x_i \Downarrow v_i \vdash x_j \Downarrow v_j}\ \text{Var}$$

$$\frac{}{\delta \vdash \mathsf{T} \Downarrow \mathsf{T}}\ \text{True}$$

$$\frac{}{\delta \vdash \mathsf{F} \Downarrow \mathsf{F}}\ \text{False}$$

$$\frac{\delta \vdash t_1 \Downarrow \mathsf{T} \qquad \delta \vdash t_2 \Downarrow v_2}{\delta \vdash \mathsf{if}(t_1, t_2, t_3) \Downarrow v_2}\ \text{IfTrue}$$

$$\frac{\delta \vdash t_1 \Downarrow \mathsf{F} \qquad \delta \vdash t_3 \Downarrow v_3}{\delta \vdash \mathsf{if}(t_1, t_2, t_3) \Downarrow v_3}\ \text{IfFalse}$$

$$\frac{\delta \vdash t_1 \Downarrow v_1 \qquad \delta, x \Downarrow v_1 \vdash t_2 \Downarrow v_2}{\delta \vdash \mathsf{let}(t_1, x\,.\,t_2) \Downarrow v_2}\ \text{Let}$$