Example 6.12 Cruise control using Jacobian linearization
Consider again the cruise control system from Example 6.11 with θ taken as a constant θ_e. We can write the dynamics as a first-order, nonlinear differential equation:

$$\begin{aligned}\frac{dx}{dt} &= f(x,u) = \frac{\alpha_n}{m} uT(\alpha_n x) - gC_r \operatorname{sgn}(x) - \frac{1}{2}\frac{\rho C_d A}{m} x|x| - g\sin\theta_e,\\ y &= h(x,u) = x,\end{aligned}$$

where $x = v$ is the velocity of the vehicle and u is the throttle. We use the velocity as the output of the system (since this is what we are trying to control).

If we linearize the dynamics of the system about an equilibrium point $x = v_e > 0$, $u = u_e$, using equation (6.35) and the previous formula we obtain

$$\begin{aligned} A &= \left.\frac{\partial f}{\partial x}\right|_{(x_e,u_e)} = \frac{u_e\alpha_n^2 T'(\alpha_n x_e) - \rho C_d A|x_e|}{m}, & B &= \left.\frac{\partial f}{\partial u}\right|_{(x_e,u_e)} = \frac{\alpha_n T(\alpha_n x_e)}{m},\\ C &= \left.\frac{\partial h}{\partial x}\right|_{(x_e,u_e)} = 1 & D &= \left.\frac{\partial h}{\partial u}\right|_{(x_e,u_e)} = 0,\end{aligned}$$

where we have used the fact that $\operatorname{sgn}(x) = 1$ for $x > 0$. This matches the results in Example 6.11, remembering that we have used x as the system state (vehicle velocity). ∇