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Publisher: Institute for Operations Research and the Management Sciences (INFORMS)

INFORMS is located in Maryland, USA



INFORMS Transactions on Education

Publication details, including instructions for authors and subscription information: http://pubsonline.informs.org

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Michael F. Gorman,

To cite this article:

Michael F. Gorman, (2012) Analytics, Pedagogy and the Pass the Pigs Game. INFORMS Transactions on Education 13(1):57-64. http://dx.doi.org/10.1287/ited.1120.0088

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Analytics, Pedagogy and the Pass the Pigs Game

Michael F. Gorman

Operations Management and Decision Sciences, Department of Management Information Systems, School of Business Administration, University of Dayton, Dayton, Ohio 45469, michael.gorman@udayton.edu

T he Pass the Pigs® game provides an opportunity to combine multiple analytical skills sets and problem-solving capabilities in an enjoyable and challenging application. In this paper, I describe how I structure a classroom exercise to help students work through developing, analyzing, and testing strategies for the game. The game utilizes multiple analytics and decision making tools, such as problem framing, data collection and preparation, probability, optimization, heuristics, expert systems, and simulation.

Key words: active learning; classroom games; teaching management; teaching modeling; statistics;

optimization; heuristics; Monte Carlo simulation

History: Received November 2011; accepted February 2012.

Introduction

Pass the Pigs® created by David Moffat and marketed by Hasbro is simple enough a child can play it, but it is amazingly helpful in distilling in an undergraduate's or MBA's mind key concepts in optimization fundamentals, probability, expected-value decision making, risk analysis, and simulation, among other topics. The exercise is designed for use in upper-level undergraduate decision science classes and introductory management science MBA classes.

Below, I give an account of how I utilize the game in these classes to help students gain an appreciation of OR/MS concepts in places where they might not expect to find them. Of course, I describe how the conversation typically goes; but each time I cover the subject the conversation follows its own path.

Originally, I gave this problem as a homework assignment with all required information to "solve" it but found that students were surprisingly not well equipped to deal with solving the problem on their own; it takes some guidance to get through the challenges of converting a game into an OR problem, much less solving it. Instead, I find that introduction of the subject in the classroom as a subject of discussion is more enlightening, productive, and fun for the students. As a result of the class session discussing the game, students appreciate the potential for applying integrated analytical techniques in relatively unexpected places.

I usually cover the game in a single class session at the end of the semester as a review and as an opportunity to integrate these numerous topics in a single session. I typically lead the conversation using a question and answer format, spending between one and a half and two hours on the subject. Alternative approaches might allow for teams to develop and share strategies, approaches, and results (though I suspect this approach would take longer, it would force more students to think about the concepts more deeply). Another approach might be to introduce the game at the beginning of the semester, and as each concept is introduced (e.g., objective functions, problem framing, expected values, etc.), revisit the game and extend the prior analysis to include the new concept, but this approach might feel somewhat disjointed to the student.

Previous Literature

Generally, games have long been recognized as a good way to motivate topics in education (Garris et al. 2002). Ben-Zvi and Carton (2007) discuss the use of games as a motivating way to introduce, apply, and critically evaluate advanced topics. Wood (2007), for example, describes the use of simulation to introduce and motivate supply chain topics. In Wood's taxonomy, big picture, integrative topics at the end of a course can be used to tie together several topics. This game achieves many of those same objectives in the span of a single class.

Only one prior research article by Kern (2006) discusses *Pass the Pigs*. Kern focuses on a theoretical presentation of the statistical treatment of the game, conducting Bayesian analysis based on unknown prior probabilities of various roll outcomes. Neller and Presser (2004) analyze optimal and heuristic



playing strategies of a game called "Pig," a similar but noncommercial game, based on dice. Two studies, Shi (2000) and Neller et al. (2006), have explored the pedagogical value of Pig, including probability assessment and strategy development. They offer suggestions of how the game can be instructional at varying levels of education, focusing on its mathematical properties.

Other probabilistic games have also been suggested as an interesting context for teaching analytical concepts. A game called "Hog" has been proposed as a teaching tool by Bohan and Schultz (1996) and Feldman and Morgan (2003). Similar dice games such as "Farkle" and "Greed" might also be fun and instructive games for the classroom, but to my knowledge, there are no academic articles on the subjects. More complex games have also been covered in the literature, such as a baseball board game "Strat-O-Matic®" (Cochran 2005), The Price is Right television game show (Biesterfield 2001), and Let's Make a Deal game show (Taras and Grossman 2002).

This presentation is geared towards business students, unlike prior research that primarily focused on mathematics and statistics students and statistics education. I do not focus on a single aspect of the game (such as probability); but rather, I present the integrative framework of multiple analytical approaches in the game. Finally, the presentation here is more focused on how to best present and motivate the material to the students in an interactive session, rather than presenting the more theoretical underpinnings of the game.

Game Introduction

To start off, I present the basic information for *Pass the Pigs*:

In the game "Pass the Pigs," a player tosses two small plastic pigs that land in various configurations, scoring a designated number of points for each configuration. Each subsequent roll's points accumulate for each roll of the turn. Each turn, a player can roll the pigs as often as they like until they "pig out," meaning they rolled a configuration that forces them to pass the pigs to an opponent and they lose all the points they have accumulated to that point in the turn. Alternatively, at any time they may "pass the pigs" voluntarily, saving all of the points they have earned in that turn.

I show the pigs in my hand, but they are too small for a classroom setting to be effective. Thankfully, there are on-line versions of the game that are conducive to presentation.³ Although not presented to the students, full instructions are available online as well.⁴

Starting the Conversation

Rather than spell out all of the rules of the game, I find it useful to give the students the modicum of information above and ask, simply, "So, what is the best strategy in *Pass the Pigs?*" Students are often stunned, simply because they do not know where to begin. The conversation is free-form, and often awkward and messy. Almost always, it starts with someone (sarcastically?) muttering under their breath, "I try to win."

"Sure you do!" I answer, "But how?"

The Basic Objective

Of course, trying to win is the objective of most (all?) games, but this one can be viewed in a highly numerical context. It usually takes some coaching to get students to view the game in the context of course material. Getting the students to view the objective in more numerical terms is not natural for them; guiding questions such as, "How do you win?" help them identify the numerical objective—to score points. We quickly conclude the objective is to maximize the points scored. The word "maximize" starts to help the students' minds bridge from a child's game to an optimization problem. Even though we have not fully identified the true objective, we are moving in the right direction.

The Data Required to Support a Good Decision

At some point students will realize they need to ask questions to solve the problem. That in itself is a lesson; sometimes all the data is not readily available. My question for them is, "What do you need to know to play this game well?"

Students are quick to comment that they don't know how the scoring works. Students have seen the pigs, but they have not learned the scoring possibilities of each roll. I present Table 1 as the possible scores for each roll. Most often, a pig lands on its side—a "sider," but often scoring roles such as a "razorback" or "trotter" occurs. When scoring roles such as



¹ http://en.wikipedia.org/wiki/Farkle.

² http://en.wikipedia.org/wiki/Greed_(dice_game).

³ At the time of this writing (December 2011), via Google search of "Pass the Pigs online" I found the same application at numerous sites, among others: http://www.passthepigs.com/; http://www.fetchfido.co.uk/games/pigs/pigs.htm; http://www.censusonline.net/games/pigs/passthepigs.html.

It is unclear what the commercial arrangements are with these sites and the owners of the Pass the Pigs copyright and trademark, so these applications might not be available persistently.

⁴ http://www.hasbro.com/common/instruct/passthepigs.pdf.

Table 1 Scoring Possibilities for Pass the Pigs^a

If you roll a	You will get	If you roll a	You will get
Razorback	5 points	Double Razorback	20 points
Trotter	5 points	Double Trotter 45	20 points
Snouter	10 points	Double Snouter	40 points
Leaning Jowler	15 points	Double Leaning Jowler	60 points
Pig Out	0 points	Sider Or	1 point
Oinker	Back to zero	Mixed Combo	Combined score

^aReproduced from http://www.censusonline.net/games/pigs/passthepigs.html.

these occur in tandem, scores are doubled, as shown in Table 1. I describe Table 1 as a great example of a sample space—the exhaustive and mutually exclusive universe of every possible outcome from a roll.

The Probability Distribution

Of course, this information is necessary, but not sufficient to help develop a winning strategy. Short of starting to roll the pigs and see what happens (the experiential approach), students have a hard time identifying what other information they might need to develop a strategy.

I ask the question, "Why do you think different rolls are worth different point values?" to get them thinking along the right lines. The students intuit (both from the point value and the intuition from the physical shape of the pigs) that some individual pig landing positions are more likely than others. I provide the data in Table 2 and ask how it might help their decision making.

Ioint Probabilities

It is not a big surprise that students don't know how to use this information; in this format it isn't useful. Although each pig's final resting position is

Table 2 Probability of Each Pig Roll

Pig position	Probability (%)
Side (no dot)	34.9
Side (dot)	30.2
Razorback	22.4
Trotter	8.8
Snouter	3.0
Leaning Jowler	0.6

independent of the other, the total score has interdependency; scoring depends not on one pig, but two. ("Doubles" double the scoring; two "Leaning Jowlers" are not worth 2*15=30, but rather 60. Thus, the value of the score of a roll can't be determined with one pig.) Of course, the positions the pigs themselves end up in are independent, so the probabilities of the combinations of any two configurations can be easily calculated as the product of the two marginal probabilities. The juxtaposition of these two concepts provides another lesson in probability and independence. It is worth emphasizing for the students that calculating scores is dependent on both pigs' positions, but the pigs' positions are independent of each other.

I demonstrate the calculation of joint probabilities in Excel, resulting in Table 3. Similarly, I convert the information in Table 1 into a similar format in Table 4.

Table 3 Joint Probabilities of Any Combination of Two Pigs

	Side (no dot) 34.90 (%)	Side (dot) 30.20 (%)	Razorback 22.40 (%)	Trotter 8.80 (%)	Snouter 3.00 (%)	Leaning Jowler 0.70 (%)
Side (no dot) 34.90 (%)	12.180	10.540	7.818	3.071	1.047	0.244
Side (dot) 30.20 (%)	10.540	9.120	6.765	2.658	0.906	0.211
Razorback 22.40 (%)	7.818	6.765	5.018	1.971	0.672	0.157
Trotter 8.80 (%)	3.071	2.658	1.971	0.774	0.264	0.062
Snouter 3.00 (%)	1.047	0.906	0.672	0.264	0.090	0.021
Leaning Jowler 0.70 (%)	0.244	0.211	0.157	0.062	0.021	0.005



Table 4 Scoring Value of Any Pig Configuration

	Side (no dot) 34.90 (%)	Side (dot) 30.20 (%)	Razorback 22.40 (%)	Trotter 8.80 (%)	Snouter 3.00 (%)	Leaning Jowler 0.70 (%)
Side (no dot) 34.90 (%)	1	0	5	5	10	15
Side (dot) 30.20 (%)	0	1	5	5	10	15
Razorback 22.40 (%)	5	5	20	10	10	20
Trotter 8.80 (%)	5	5	10	20	15	20
Snouter 3.00 (%)	10	10	15	15	40	25
Leaning Jowler 0.70 (%)	15	15	20	20	25	60

Simplifying the State Space

I lead the students to realize this state space is unnecessarily large and complicated for the problem; I ask the students if they care if they just rolled a "Razorback" or a "Trotter"; the result is five points. They realize that they don't care about 36 different permutations of pigs. How many outcomes are there in each roll "experiment"? Students count nine numerical outcomes that are relevant to the problem (0, 1, 5, 10, 15, 20, 25, 40, 60). By summing the probabilities associated with each score, we arrive at Table 5. Although this simplification is not necessary to solve the problem, it is valuable to demonstrate opportunities to simplify a problem without losing the fidelity of the model to accurately depict the problem. Furthermore, it is surprising to students that the most likely outcome is a five-point roll.

Omission of an Irrelevant Outcome

Occasionally, a student will notice one outcome from Table 1 is not represented in Tables 3 and 4—the dreaded "Oinker" in which the pigs are touching and all points earned to that point in the *game* are lost. (If no student mentions it, I bring it up.) How can we legitimately ignore this possibility and arrive at the right solution?

Table 5 Numerical Outcomes and Their Probabilities

Score	Probability (%)		
0	21.080		
1	21.301		
5	40.622		
10	8.520		
15	2.111		
20	6.229		
25	0.042		
40	0.090		
60	0.005		

I ask students to posit how it might be okay to ignore this possibility. From a practical perspective, it is impossible to a priori predict the probability of an oinker; this depends on the roller more than the pig. Intuitively, it is very hard to estimate this probability and the data would be hard to come by, making it impractical to try. I use this example as a case for misguided data collection; the cost of data collection is high, the value of the data is low (in fact, the value is zero, as described below).

Most importantly, this outcome is irrelevant to the optimal strategy in any case. Students rarely comprehend this point. How is it irrelevant? Although an oinker is a devastating outcome of a roll, the presence of an oinker does not affect the decision to roll, thus it is irrelevant. It often comes as a surprise to the students that such a catastrophic event does not affect the playing strategy, but because passing the pigs doesn't "insure" against an oinker, the presence of the possibility does not affect the decision to roll. This is another great point of discussion.

Determining the Decision Variable

Many students have a hard time defining the decision variable in this game. When I ask, "What is the decision variable?" I often get in response, "How many rolls to take each turn." I ask for a rationale, and hear responses such as "You want to pick a number of rolls that is low enough that you probably won't pig out." I then ask, "Where in the description of the rules does a player declare the number of rolls they plan to take?"

Others speculate that setting a reasonable goal for a turn and achieving it might be the objective, which is close to the right answer, but for the wrong reason. Setting an arbitrary goal is not optimizing, and is certainly not the decision variable in this game. Later in the class we will in fact develop a heuristic stopping rule based on optimization behavior, which is akin to setting a "goal" for each roll. However closely related, I consider a "goal" not based on analytics as incorrect; in a sense, the right idea based on the wrong logic. (Similarly, a math teacher of mine would give an example of simplifying the fraction 16/64 to 1/4 by "cancelling the sixes"; right answer, but wrong method.)

I ask them to imagine the pigs are in their hands; what are their choices? Students come to realize the decision at each roll in the turn is simply to roll or pass the pigs.

Calculation of Expected Costs and Benefits

Given an objective of maximizing points, and the decision to roll or not, what criteria are applied to



support the decision to roll or not? An open-ended question is usually met with wide-open eyes and gaping mouths. To encourage the analytical thinking process, I ask the student to imagine it is their first roll in a turn; should they roll? Of course, they respond, why not? There's nothing to lose. Next situation: You just rolled a "double leaning jowler" worth 60 points; do you roll again? Of course not. Why not? Is there a constraint in this problem? (No) You want more points, right? The realization is made; the cost of a roll is the risk of losing accumulated points. To reinforce this idea, I ask the obvious: Why roll? To score points, of course. Why stop rolling? To save your points from being lost.

The problem is that the student must make a decision before the outcome is known. I give them the following information, which generally is not easily intuited by students: The decision must be made based on expected value—the probability of each outcome times its point value.

Surprisingly, students are confused to some degree about what an "outcome" is, because they have a hard time separating a positive outcome from a negative one. Although the problem can be solved while combining point scoring and pig out possibilities, I suggest that it is easier to separate out the positive from the negative outcomes. Starting with the positive outcomes, what is the expected value of a roll on your first roll of a turn, with nothing to lose? The calculation of the probability of all outcomes times the value of the roll in Table 5 yields an expected value of 4.7. I explain that this expected benefit is constant for every roll of the game.

What of the negatives? There is a 21% chance of pigging out, but the cost increases with each roll of a turn. Some students have a hard time with this, because they try to predict the outcome of upcoming rolls. Outcomes of future rolls are irrelevant, as are the number of past rolls; one need only know the number of accumulated points to describe the state space and make a decision. Students left to their own devices will create things such as a massively complicated probability tree to describe a turn, but such complexity is unwarranted and misleading. It is an elegant outcome of this discussion that the required "input data" to make a decision is so simple, yet optimum-seeking.

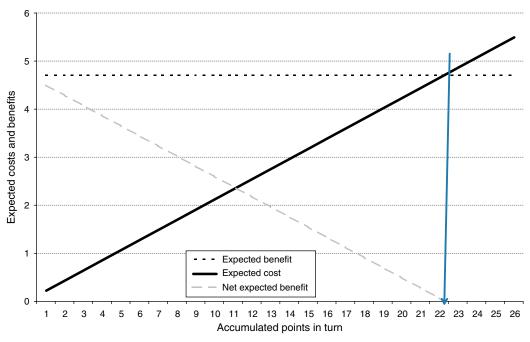
Marginal Analysis

The problem requires marginal analysis; for each roll in a turn, the number of points at risk changes. Similar to the lesson on the decision variable, the roller doesn't decide how many times to roll in advance, but rather makes the decision on the margin to roll or not. With a 21% chance of pigging out, the expected loss is 0.21 per point accumulated. Figure 1 shows the increasing cost and constant benefit of each roll. Optimally, a roller maximizes the expected points of each turn by stopping when the expected cost exceeds the expected benefit of each roll.

Resulting Heuristic Rule

The result of this optimization exercise is a simple heuristic: Roll at 22 and stop at 23 accumulated points. I compare this rule to the "two-second rule"







of driving, a very simple and easy-to-follow rule to follow based on a relatively complex set of data and calculations. (In the case of the two-second rule, the calculations are based on reaction time, deceleration rates, and the like.) From this example, students see how simply optimization can translate to implementation. However, as discussed later, this heuristic is a good starting point, but not optimal if the objective is to win, rather than maximize the expected benefit of each role.

Testing the Heuristic in a Stochastic Environment

Once the rule is established, it can be put to the test online at one of the sites listed above playing *Pass the Pigs* against an automated opponent. This is where the lecture inevitably "falls apart;" I often lose against the computer using my heuristic strategy. Students chortle; maybe the professor isn't so smart. How can this scientifically derived optimum fail? Of course, this loss represents another chance to learn.

Evaluation of the Strategy

I ask, simply, "Is our strategy wrong?" Many recognize significant randomness in the game and realize that sometimes the computer gets "lucky" (or I got unlucky!), and they are willing to forgive the heuristic from failing. Of course, randomness plays a big role in any game of chance. This strategy produces, on average in the long run, the highest average expected points.

However, there is a greater lesson here that transcends analytical techniques; never judge a decision by its outcome! A decision should be evaluated by the information and analysis at the time of the decision. Based on the best information at the time of the rolls, the decisions were sound, regardless of how it turned out. Most students don't like this logic; they are accustomed to evaluating the outcome of decisions. To simplify the discussion (or in the case where I have actually won), I ask the students to consider the case where they pig out after earning a single point on that turn. Is that a bad decision? Of course not; it makes sense to keep going when only one point has been earned. Expected-value decision making is a long-run concept; using this rule, on average, will lead to the largest average number of points. In any given instance, we might not be so lucky. A single game outcome is basically a sample size of one.

Monte Carlo Simulation for More Advanced Decision Evaluation

I ask the students, "If you can't evaluate a decision based on a single outcome, how can you evaluate it,

Table 6 Expected Score of 10 Turns Under Various Stopping Rules

Stopping rule	Mean	Std dev	Std dev mean
10	80.30	24.29	1.78
11	89.61	27.31	2.00
12	95.76	32.13	2.35
13	91.83	29.93	2.19
14	93.57	31.08	2.27
15	93.83	33.44	2.45
16	95.01	36.84	2.69
17	97.19	33.84	2.47
18	95.92	37.99	2.78
19	99.73	36.99	2.71
20	98.71	37.56	2.75
21	100.02	38.17	2.79
22	99.12	41.55	3.04
23	96.09	43.15	3.16
24	98.24	40.25	2.94
25	99.35	45.50	3.33
26	97.52	47.40	3.47
27	97.43	53.89	3.94
28	103.95	48.65	3.56
29	98.95	47.21	3.45
30	98.74	49.16	3.60

^{*}Sample size is 187 simulated games.

or test it empirically?" A student might sheepishly venture that you have to try it many times to see if it works out in the long run. It would take too long to play 100 games of *Pass the Pigs* online, record the data, and analyze it. But it doesn't take too long to create a mock-up of the game in Excel using RAND(), some IF functions, and a data table. (This work can be completed ahead of class time, and presented to the class in just a minute or two, or can be a half-hour exercise in spreadsheet development.)

Importantly, however, students recognize the power of Monte Carlo simulation to evaluate decisions under risk and further come to realize that it is important to assess decisions based on both mean (expected value) and standard deviation (risk) of the outcome. Table 6 shows an example of mean and standard deviation of results with various stopping rules. The students can see (even with a large sample size of games, in this (somewhat arbitriarily chosen) case 187 games under each stopping rule) that although a stopping rule of 23 is consistent with good performance, because of the very high variance of game outcomes (exactly what makes it fun!), by no means is the best performance assured. That is, mean scores with a stopping rule anywhere from 20 to 25 might not be statistically significantly different.

Students might complain that all of this replication leads to a situation where there is still uncertainty of the optimal stopping rule. They are correct. Because of the inherent randomness in the game, a very large sample size is required to discern which rule is, in



fact, best. A discussion about the value of collecting or creating more data—a larger sample size—might ensue. To be "more sure," more data must be collected. In this case, increasing the sample size is easy and low cost; in others, it might not be.

Students might also notice that the risk of each stopping rule (measured by standard deviation) rises with the stopping point of each turn; this is intuitive, as the roller is more likely to end up with zero with a pig out, or a larger total number of points before they pass the pigs. But it also leads to a discussion of risk tolerance; if one considers the regret of giving up 20 accumulated points should a pig out occur, it might be "optimal" (from a utility sense) to stop rolling before the magic number of 23 is achieved.

Objective Revisited

There are some students who might feel that there is something wrong with the heuristic, and that it could be improved. And they are right. (Most do not challenge the original objective until they witness its flaw in a live environment when either ahead or behind the opponent, and the strategy does not change.) While the expected-value approach leads to the highest points on average, the objective is, in fact, not to score high on average, but to win.⁵

I tell the students that even though the expected-value approach is "wrong" in the sense that it does not capture all of the complexity of the actual game and might fail in some cases, it is still worth using in many situations and provides insight into sound strategy. In other words, sometimes a simple strategy is pretty good, even if it isn't "right." Sometimes, a simple model that captures most problem elements pretty well is worth considering and using. In practice, this is often the case; a simple model that works most of the time might be sufficient to guide behavior and is more likely to be easily implemented.

More Advanced Heuristics/ Decision Approaches

Why is the expected-value approach suboptimal for this game? To develop a strategy for the game that increases the probability of winning (rather than maximizing the expected value of each role), a player needs to consider his or her position relative to the competition. Students recognize in some cases a player should be more conservative to nurse a lead, or play more aggressively to recover from a deficit.

Markov Chain Analysis: Technically, the game is a stochastic Markov game in which each player tries to maximize his expected probability of winning. Intuitively, the players' strategies dynamically depend on

⁵ In the physical game, players play to 100; in the online game, each player gets ten turns and the higher total score wins. The optimal strategy will vary with different definitions of a win.

their ability to get in a superior position relative to the opponent. Such analysis might be productive in an upper-level operations research course.

Expert System/Heuristic Approach: Although analyzing Markov games is too complex for undergraduate business or MBA students, a more sophisticated expert system/heuristic approach can be developed to capture the essence of these complexities. For example, students can easily volunteer observations such as:

- Players should take more rolls if "far" behind or the opponent is "about" to win;
- Players should be more conservative if in the opposite position;
- The roll strategy should change "near" the end of the game—focusing more on the probability of winning rather than the expected value of each roll.
- I lead the students to understand that "far," "about," and "near" are simply new parameters in a new and improved "expert system" (if-test) based heuristic. Of course, now the definition of "win" must be clarified. For simplicity, we adopt the online game convention of the higher score out of ten turns so the number of turns in a game is not stochastic.
- Default strategy is to roll until 23 or more is scored;
- If opponent is ahead by more than *X*, then roll until *Z* is scored in a turn;
- If opponent is behind by more than *XX*, then roll until *ZZ* is scored in a turn;
- If round ten (last turn), roll until ahead of opponent, then stop.

Of course, more sophisticated expert systems can be derived, such as adopting a modified strategy by round of the game other than the last. It is worth noting at this point, as I often do with my students, that often, more sophisticated models can be created, but some consideration of the benefits of such modeling should be weighed against their incremental complication and ease of implementation. Despite this caveat, we can test various parameterizations using Monte Carlo simulation (as above, the spreadsheet is developed in advance, with appropriate cells created with the X and Z parameters) to estimate which is most successful on average, and which carry the largest variances. This is a great example of searching for optimal parameters in a stochastic environment. Because there are so many possible options to explore here, it makes a good extra credit assignment for the interested student.

Conclusion

At the end of the class, the students have a new appreciation for the use of various analytical techniques in something as simple as *Pass the Pigs*. I tell them that when I explained these concepts (in basic



form—the "stop at 23" rule) to my kids, they latched on to the simple decision rule, used it, and then completely stopped playing the once-cherished game. All the fun was gone and I was at fault. After everyone gets a laugh, I explain quite simply that business decision making is not a game and should not be left to a "gut feeling;" it's not supposed to be exciting—it's business by the numbers.

Pass the Pigs has great potential to bring these numerous analytical concepts to light in an integrated, memorable and enjoyable way.

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