Extended Kalman Filter for Tracking a Three-Wheeled Robot

An Extended Kalman Filter (EKF) is to be designed for tracking the position and orientation of a three-wheeled robot that is moving on a plane. A schematic drawing of the robot is shown in Figure 1.

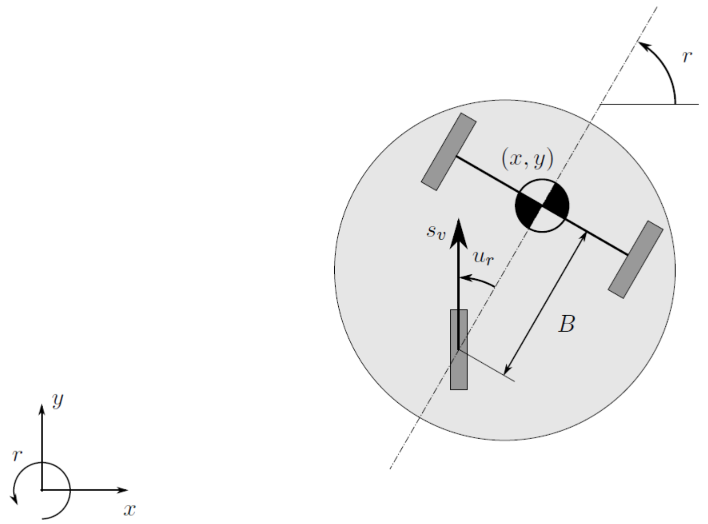

Figure 1: Top view of the three-wheeled robot and relevant physical quantities: (x, y) is the position of the robot's center of gravity, r its orientation, s_v denotes the (linear) velocity of the driving wheel, and u_r is the angle of the driving wheel.

The robot can command the angular velocity of its driving wheel ($u_v(t)$; in rad/s) and the driving wheel angle ($u_r(t)$; in rad). The translational speed s_t (in m/s) of the vehicle is

$$s_t(t) = s_v(t)\cos(u_r(t)) \tag{1}$$

with $s_v(t)$ being the driving wheel (linear) velocity,

$$s_v(t) = W\,u_v(t) \tag{2}$$

where W is the radius of the driving wheel (in m). The wheel radius is not known perfectly; it is modeled as a continuous random variable according to

$$W = W_0 + \gamma$$

with the known nominal wheel radius W_0 and the uniformly distributed random variable $\gamma \in [-\overline{\gamma}, \overline{\gamma}]$.

The rotational speed $s_r(t)$ (in rad/s) of the vehicle is

$$s_r(t) = \frac{-s_v(t)\sin(u_r(t))}{B} \tag{3}$$

where B is the wheel base (distance in m between the main axle and the driving wheel), see Figure 1.

The kinematic equations of the robot can be written as follows:

$$\dot{x}(t) = s_t(t)\cos\big(r(t)\big) \tag{4}$$
$$\dot{y}(t) = s_t(t)\sin\big(r(t)\big) \tag{5}$$
$$\dot{r}(t) = s_r(t) \tag{6}$$

where $(x(t), y(t))$ is the position of the robot (in m) and $r(t)$ its orientation (in rad).

The robot's commands are given at discrete time instants $t_0 = 0, t_1 = T, t_2 = 2T, \ldots,$ where T is the constant sampling time in seconds. The robot's commands are kept constant over the sampling interval; that is, for example, $u_r(t) = u_r[k]$ for $t \in [kT, (k+1)T)$ [1].

The robot starts at $(x(0), y(0)) = (x_0, y_0)$ with orientation $r(0) = r_0$. The initial position is uniformly distributed with $x_0, y_0 \in [-\overline{p}, \overline{p}]$, and the probability density function of r_0 is triangular (see Figure 2) with width $2\overline{r}$, centered at zero: $r \sim \text{Tri}(\overline{r})$.

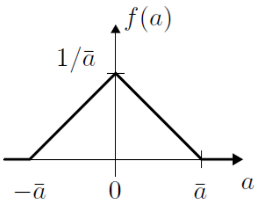

Figure 2: Triangular probability density function, $a \sim \text{Tri}(\overline{a})$.

At any sampling instant kT, the robot may receive noise measurements of its orientation and its distance from the origin, i.e.

$$z_r[k] = r[k] + w_r[k]$$
$$z_d[k] = \sqrt{x^2[k] + y^2[k]} + w_d[k]$$

with $w_r[k] \sim \mathcal{N}\left(0, \sigma_r^2\right)$ and $w_d[k] \sim \text{Tri}(\overline{w}_d)$, where $a \sim \mathcal{N}(0, \sigma^2)$ denotes a normally distributed continuous random variable a with zero mean and variance σ^2. At any sampling instant kT, measurements may be available from one, two, or none of the sensors.

All random variables γ, r_0, x_0, y_0, $\{w_r[\cdot]\}$, $\{w_d[\cdot]\}$ are mutually independent.

[1] Recall the notation from lecture 8, where $x[k] := x(kT)$.

Estimator Design Part 1

The objective is to design and tune a hybrid EKF to estimate the position and orientation of the three-wheeled robot. The estimator will be executed at the time instants $t_0 = 0, t_1 = T, t_2 = 2T, \ldots$. At time $t_k = kT$, the estimator has access to the time t_k, the control inputs $u_v[k-1]$ and $u_r[k-1]$, and possibly the measurements $z_d[k]$ and/or $z_r[k]$. Furthermore, the constants W_0, B, $\overline{\gamma}$, $\overline{w}_d$, σ_r, $\overline{p}$, and $\overline{r}$ are known to the estimator. The orientation, the position, and the wheel radius are estimator states. The motion of the robot is corrupted by process noise, the properties of which are unknown.

Design a hybrid EKF using the given problem data, and tune its performance by changing the process noise characteristics appropriately. We will evaluate your solution by executing it for several runs using the parameters given in the provided MATLAB files. Over all runs, the average root mean square error in the x and y position trajectory will be computed. You will receive full marks on this part if the average error is below 0.6m.

Estimator Design Part 2

The objective is to design a hybrid EKF for a different robot of similar type. For this second robot, a model of the process noise is available. This model takes non-idealities in the actuation mechanism (for example, wheel slip or imperfect command tracking) into account. That is, Equation (2) is *replaced* by

$$s_v(t) = W\, u_v(t)\,\big(1 + v_v(t)\big) \tag{7}$$

where $v_v(t)$ is assumed to be continuous-time white noise with constant power spectral density Q_v over all frequencies. Equations (1) and (3) are *replaced* by

$$s_t(t) = s_v(t)\cos(u_r(t) + v_r(t)) \quad \text{and} \quad s_r(t) = \frac{-s_v(t)\sin(u_r(t) + v_r(t))}{B}. \tag{8}$$

with $v_r(t)$ assumed to be continuous-time white noise with constant power spectral density Q_r over all frequencies. The remainder of the system description remains unchanged. The process noise signals $\{v_v(\cdot)\}$ and $\{v_r(\cdot)\}$ are mutually independent, and independent of all other random variables. Additionally to the information of the EKF in design part 1 above, this EKF has access to the constants Q_v and Q_r.