

# Beyond Classical Search

"Artificial Intelligence: A Modern Approach", Chapter 4

# Outline

- Local search algorithms
  - Hill-climbing search
  - Simulated annealing search
  - Local beam search
  - Genetic algorithms
- Searching in more complex environments
  - Non-deterministic environments
  - Partially observable environments
  - Unknown environments

# Problem types

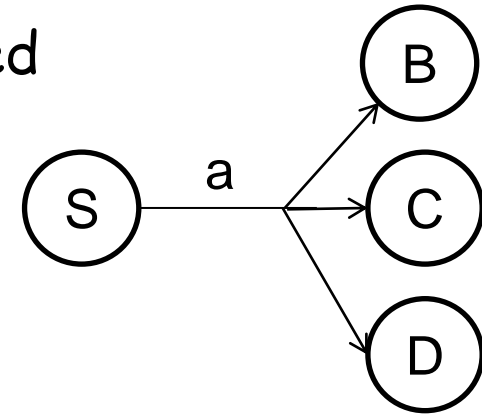
- Deterministic and fully observable (single-state problem)
  - Agent knows exactly its state even after a sequence of actions
  - Solution is a sequence
- Non-observable or sensor-less (conformant problem)
  - Agent's percepts provide no information at all
  - Solution is a sequence
- Nondeterministic and/or partially observable (contingency problem)
  - Percepts provide new information about current state
  - Solution can be a contingency plan (tree or strategy) and not a sequence
  - Often interleave search and execution
- Unknown state space (exploration problem)

# Non-deterministic or partially observable

- Perception become useful
  - Partially observable
    - To narrow down the set of possible states for the agent
  - Non-deterministic
    - To show which outcome of the action has occurred
- Future percepts can not be determined in advance
- Solution is a contingency plan
  - A tree composed of nested if-then-else statements
  - What to do depending on what percepts are received

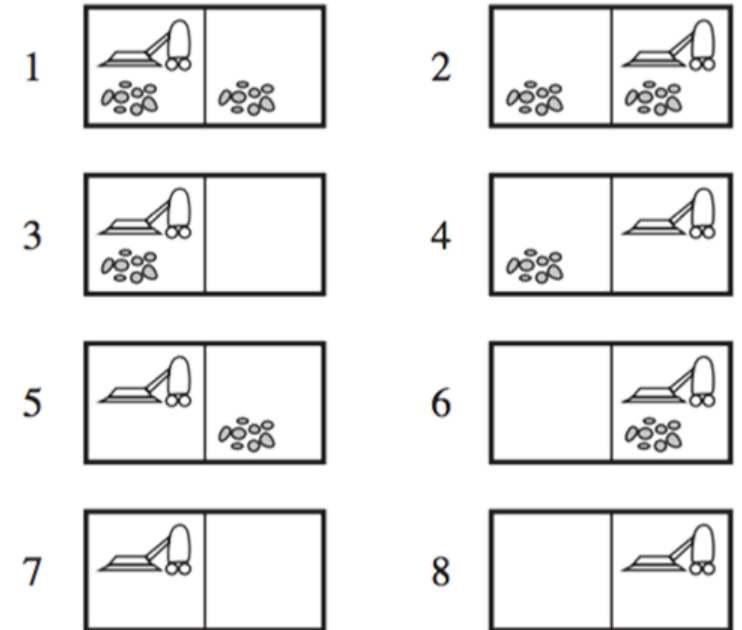
# Searching with non-deterministic actions

- In **non-deterministic** environments, the result of an action can vary
  - Future percepts can specify which outcome has occurred
- Generalizing the **transition function**
  - Results:  $S \times A \rightarrow 2^S$  instead of Results:  $S \times A \rightarrow S$
- Search tree will be an AND-OR tree
  - Solution will be a sub-tree containing a contingency plan (nested if-then-else statements)



# The erratic vacuum world

- States
  - $\{1, 2, \dots, 8\}$
- Actions
  - {Left, Right, Suck}
- Goal
  - $\{7\}$  or  $\{8\}$
- Non-deterministic
  - When sucking a dirty square, it cleans it and sometimes cleans up dirt in an adjacent square
    - $\text{Results}\{1, \text{suck}\} = \{5, 7\}$
  - When sucking a clean square, it sometimes deposits dirt on the carpet



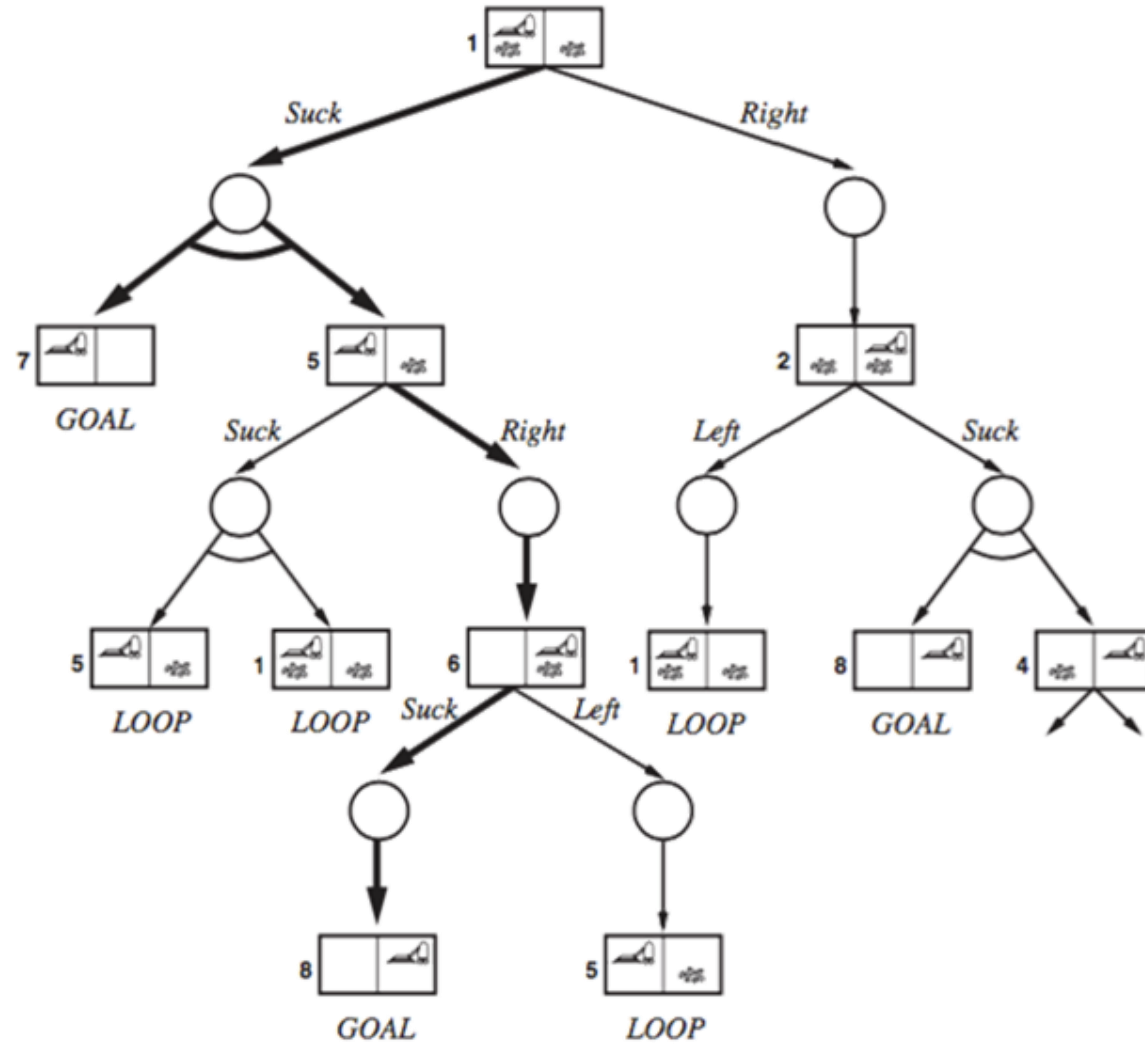
**State=1**

Suck

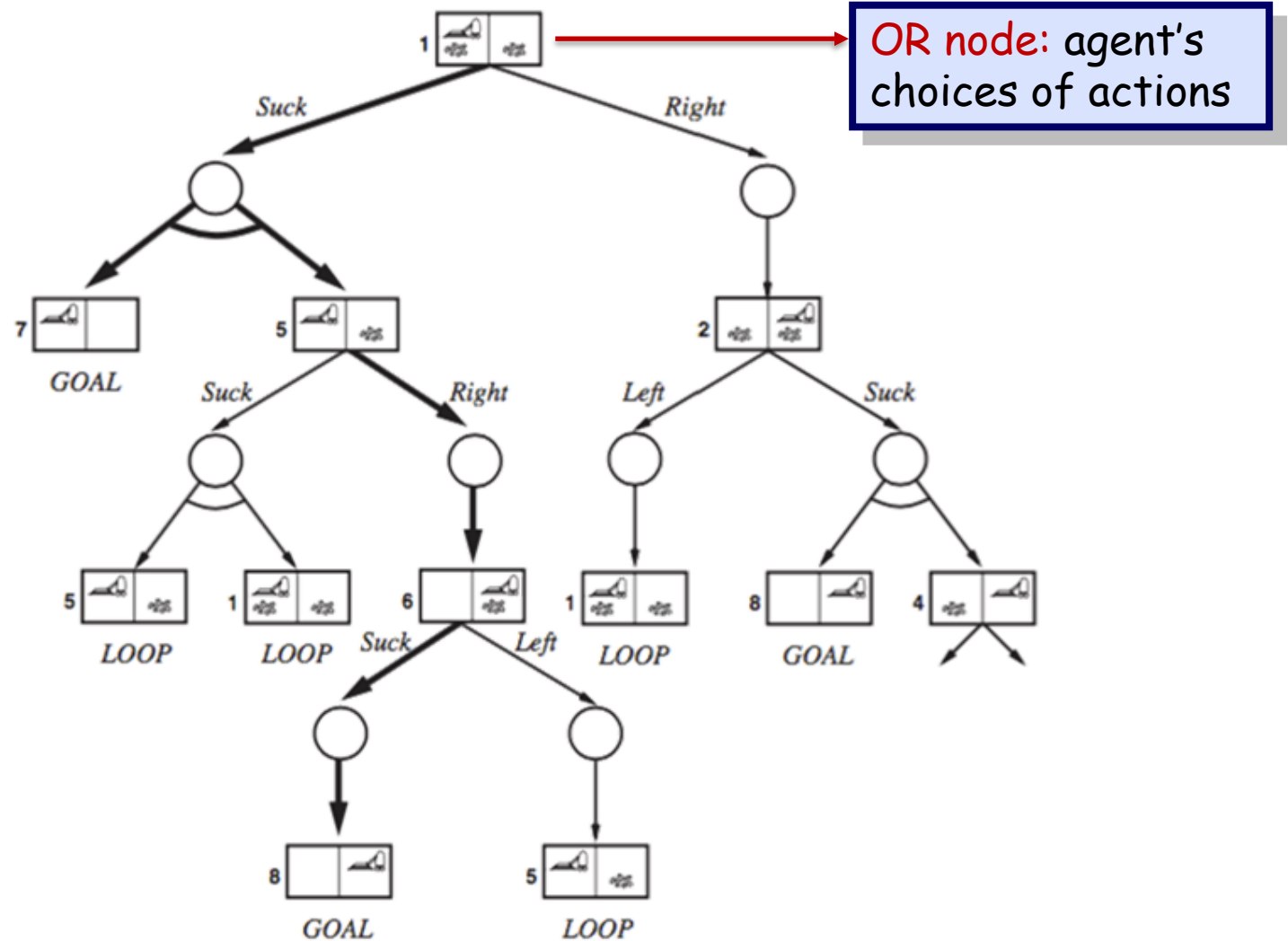
if State=5 then [Right, Suck]

else []

# AND-OR search tree



# AND-OR search tree

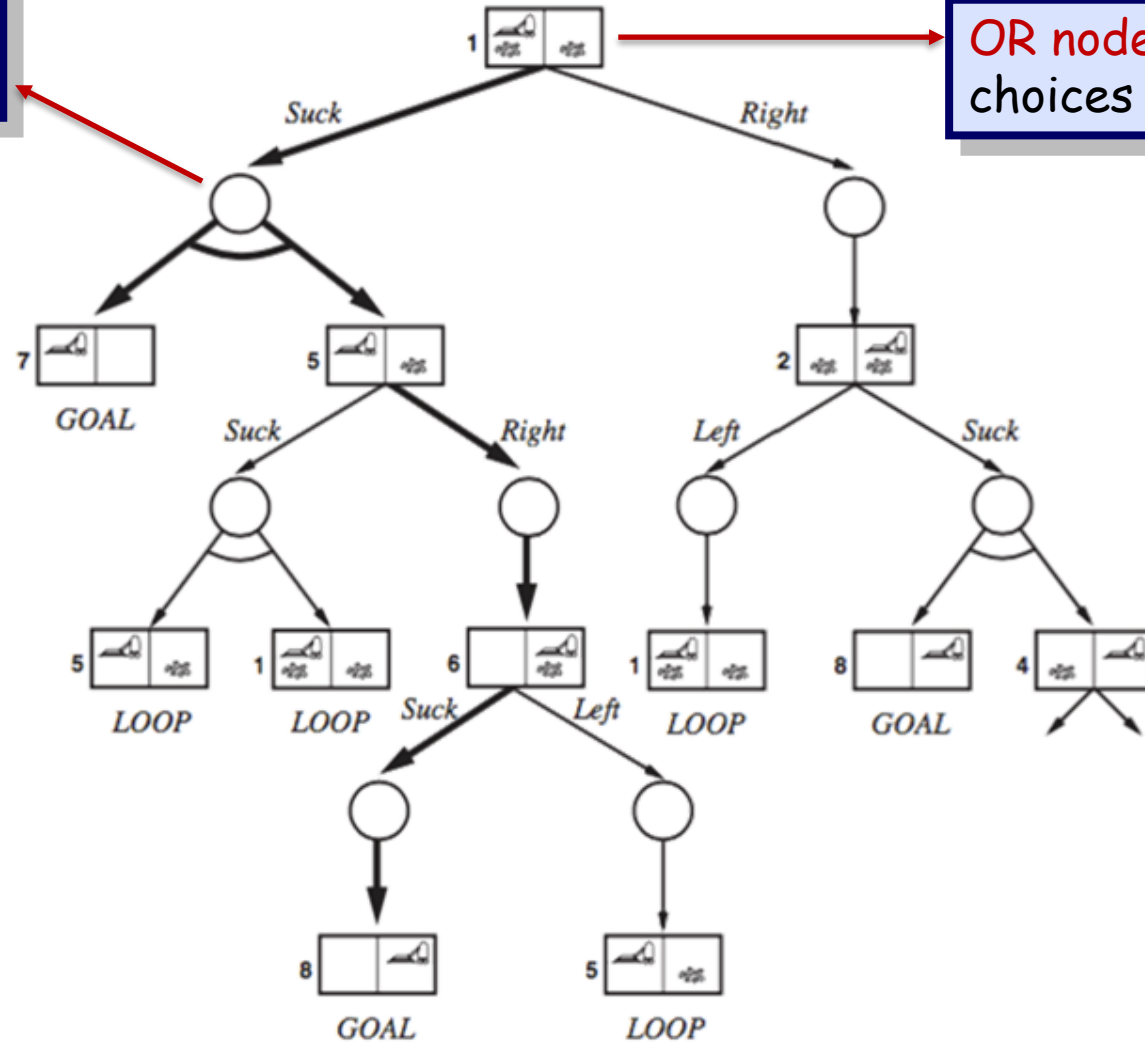




# AND-OR search tree

**AND node:** environment's choice of outcome

**OR node:** agent's choices of actions



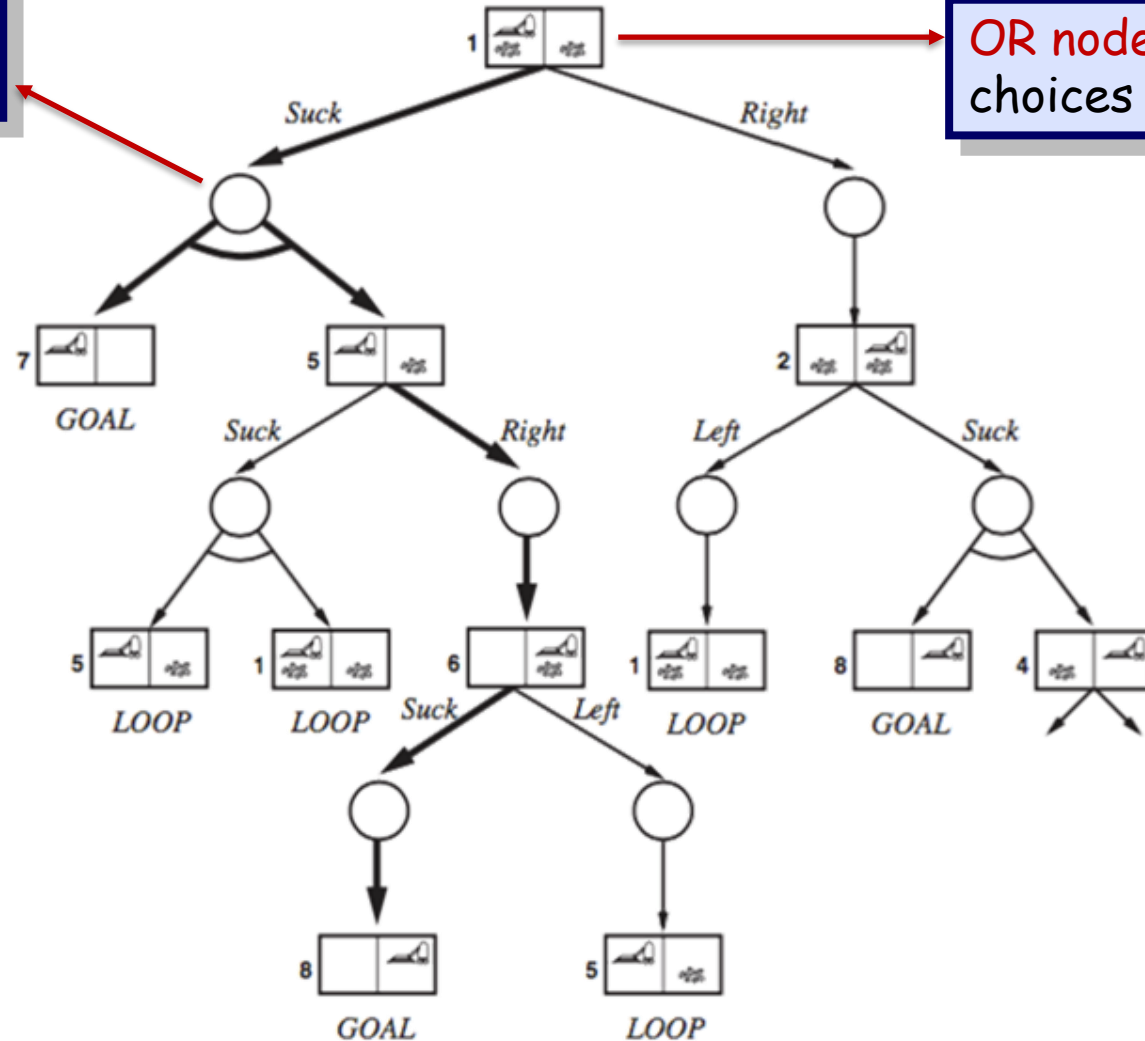
# AND-OR search tree

**AND node:** environment's choice of outcome

OR node: agent's choices of actions

**Solution** for AND-OR search problem is a **sub-tree** that:

- specifies **one action** at each **OR** node
- includes **every outcome** at each **AND** node
- has a goal node at every leaf



## AND-OR search tree

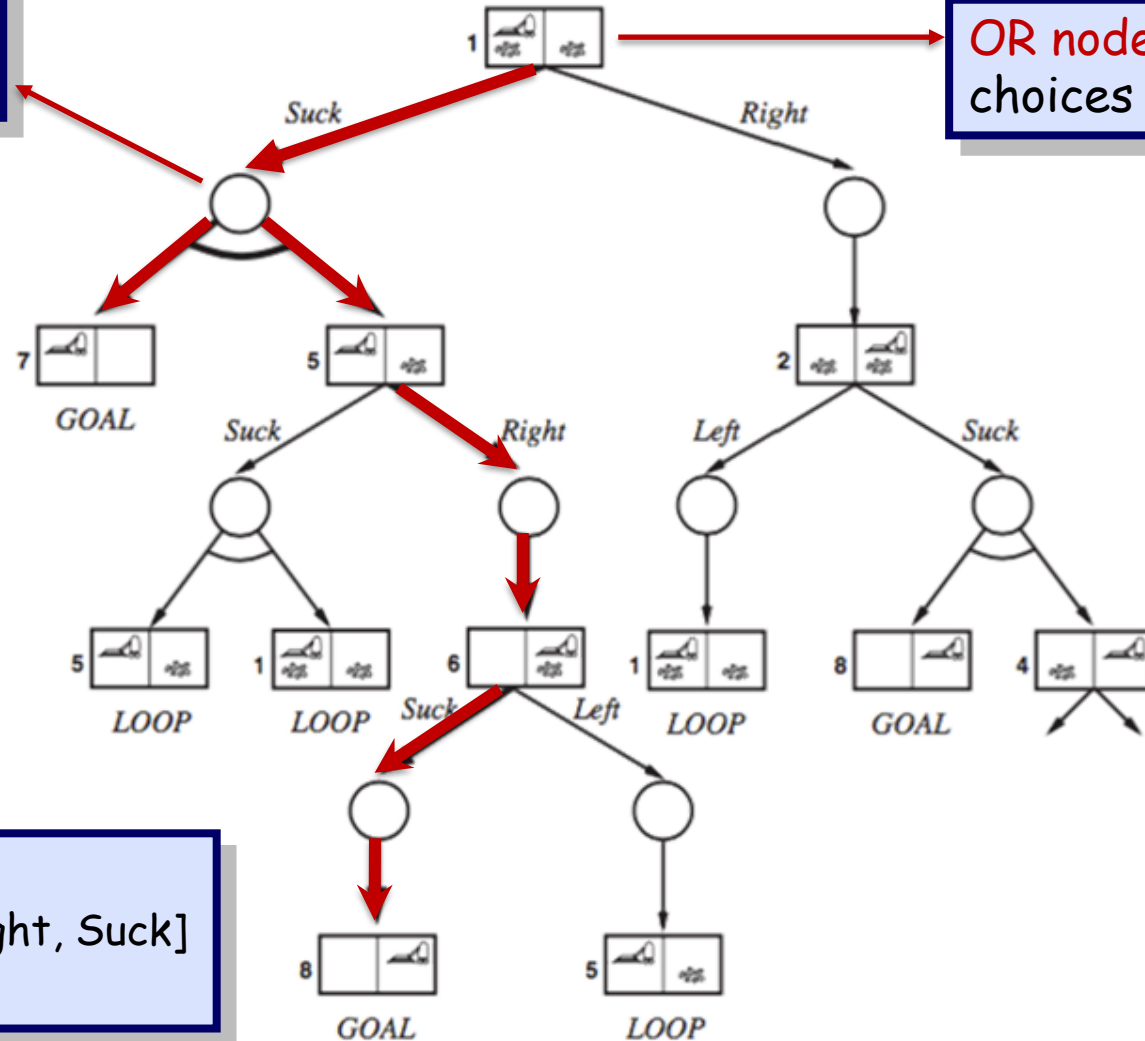
**AND node:** environment's choice of outcome

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**Solution** for AND-OR search problem is a **sub-tree** that:

- specifies **one action** at each **OR** node
- includes **every outcome** at each **AND** node
- has a goal node at every leaf

```
Suck
if State=5 then [Right, Suck]
else []
```



# AND-OR depth first graph search

```
function AND-OR-GRAPH-SEARCH(problem) returns a conditional plan or failure  
  OR-SEARCH(problem.INITIAL-STATE, problem, [])
```

```
function OR-SEARCH(state, problem, path) returns a conditional plan or failure  
  if problem.GOAL-TEST(state) then return the empty plan  
  if state is on path then return failure  
  for each action in problem.ACTIONS(state) do  
    plan=AND-SEARCH(RESULTS(state, action), problem, [state | path])  
    if plan  $\neq$  failure then return [action | plan]  
  return failure
```

```
function AND-SEARCH(state, problem, path) returns a conditional plan or failure  
  for each  $s_i$  in states do  
    plani = OR-SEARCH( $s_i$ , problem, path)  
    if plani = failure then return failure  
  return [if  $s_1$  then plan1 else if  $s_2$  then plan2 else ... if  $s_{n-1}$  then plann-1 else plann]
```

OR-Search(1, [])

OR-Search(1, [])

suck



AND-Search({7,5}, [1])

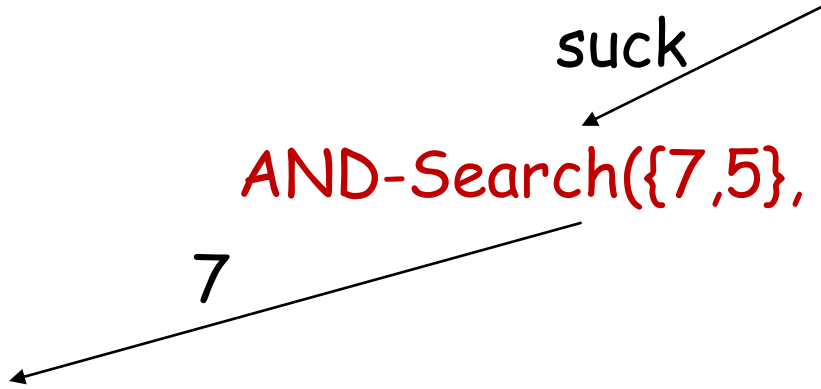
OR-Search(1, [])

suck

AND-Search({7,5}, [1])

7

OR-Search(7, [1])



OR-Search(1, [])

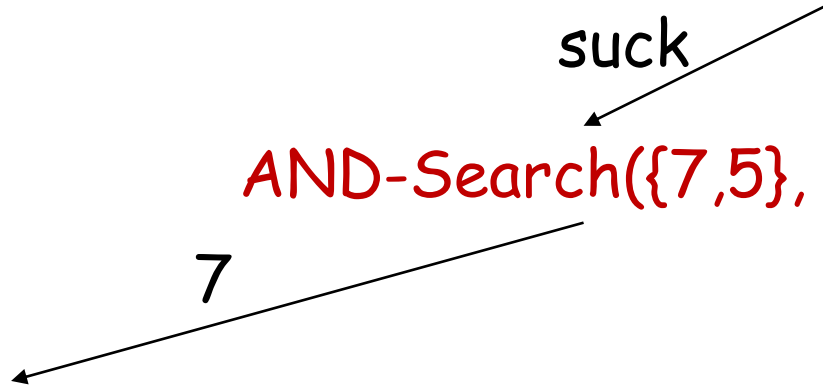
suck

AND-Search({7,5}, [1])

7

OR-Search(7, [1])

[]





OR-Search(1, [])

suck

AND-Search({7,5}, [1])

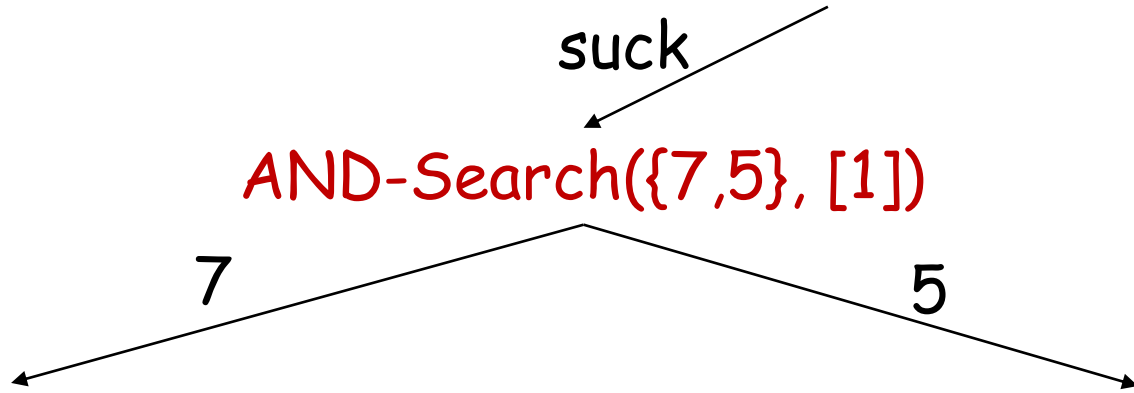
7

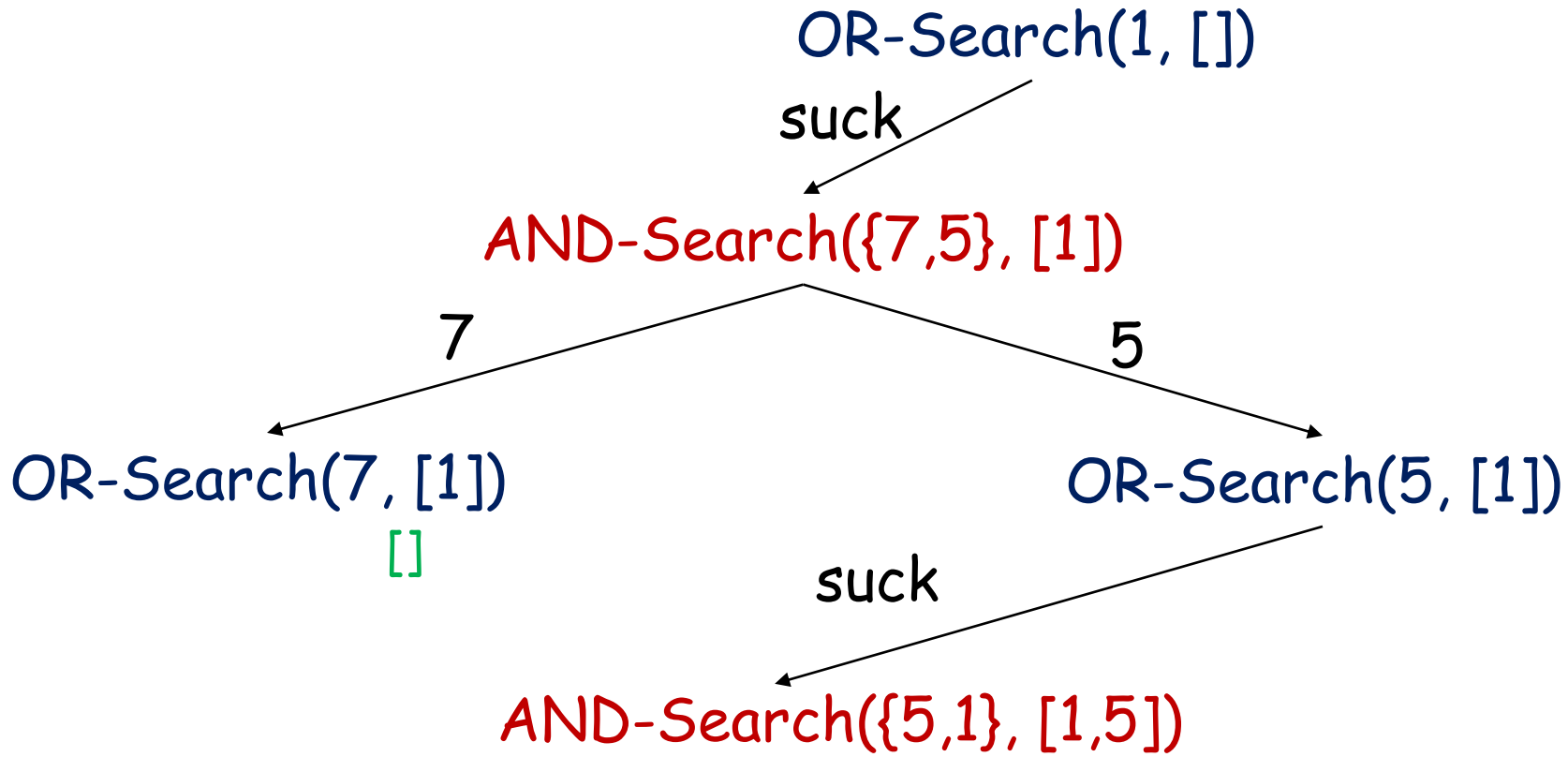
5

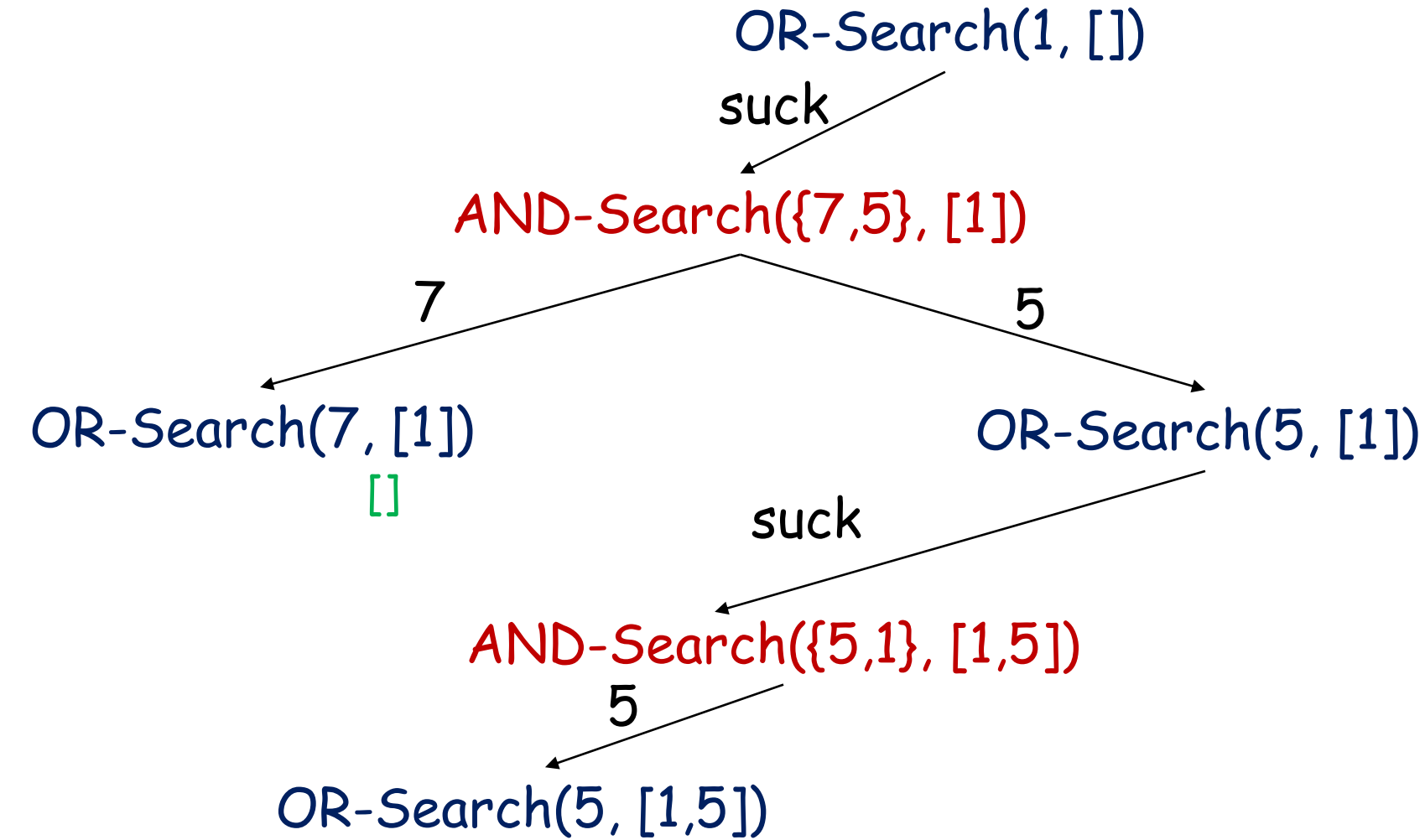
OR-Search(7, [1])

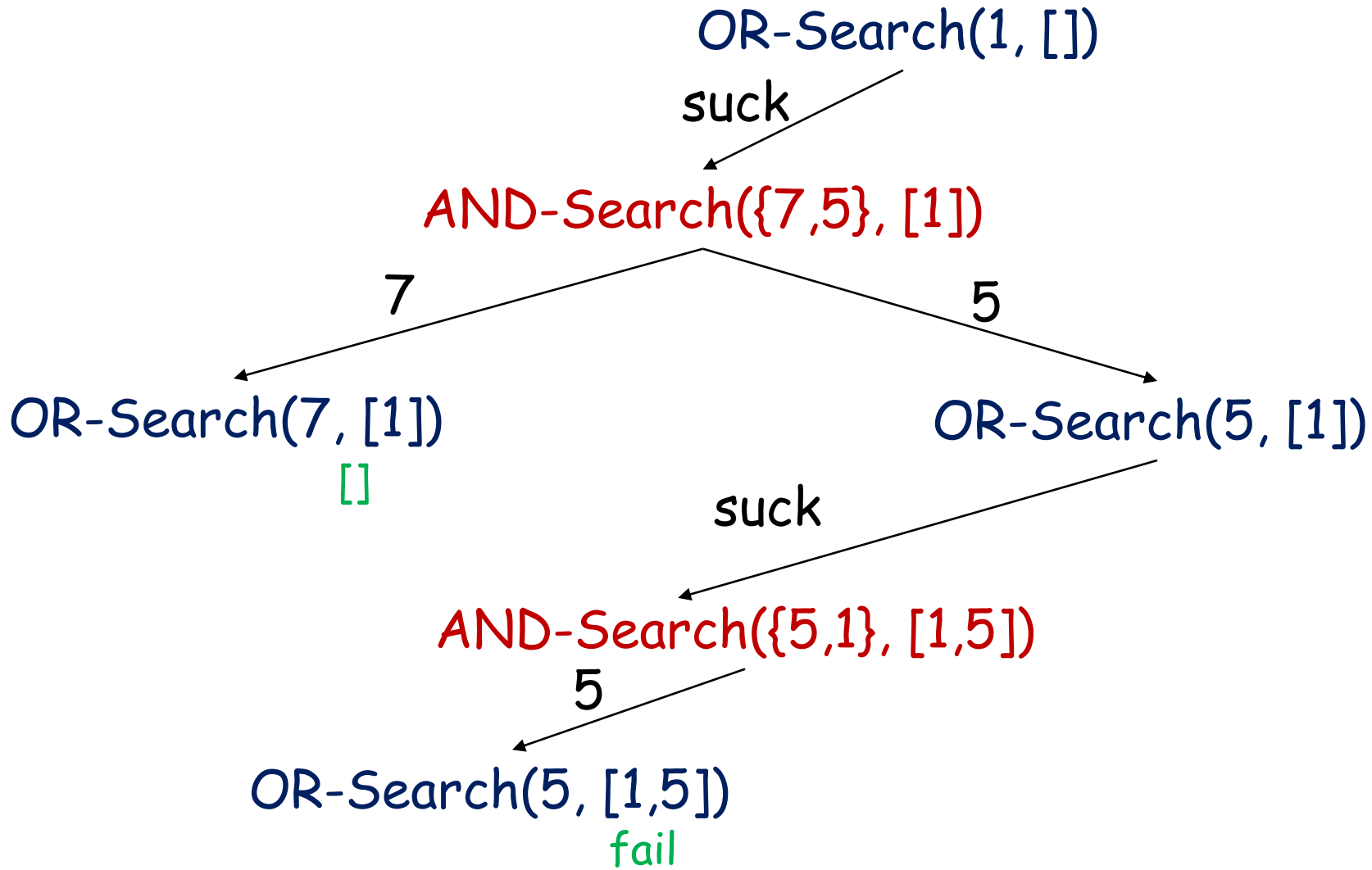
[]

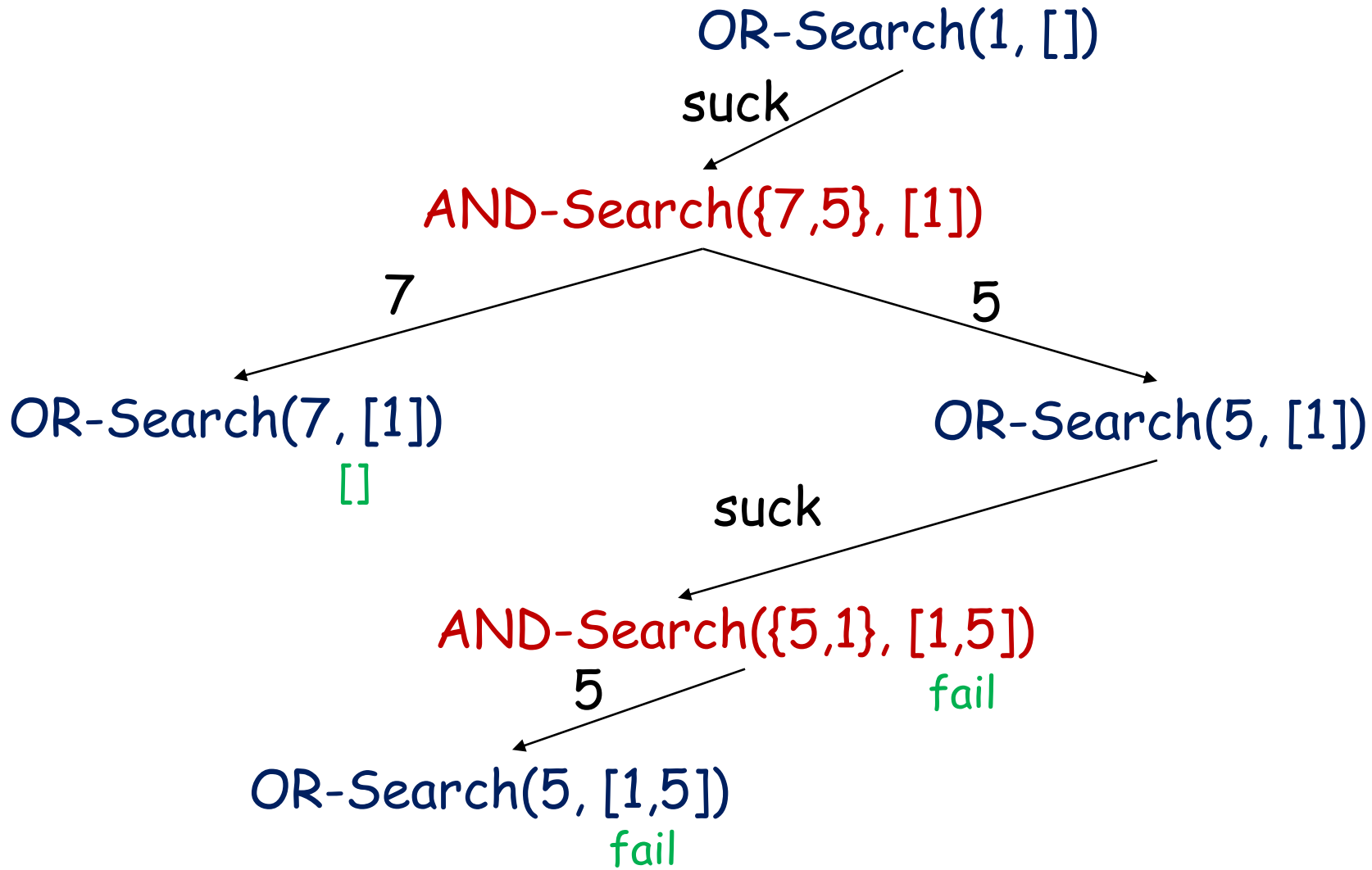
OR-Search(5, [1])

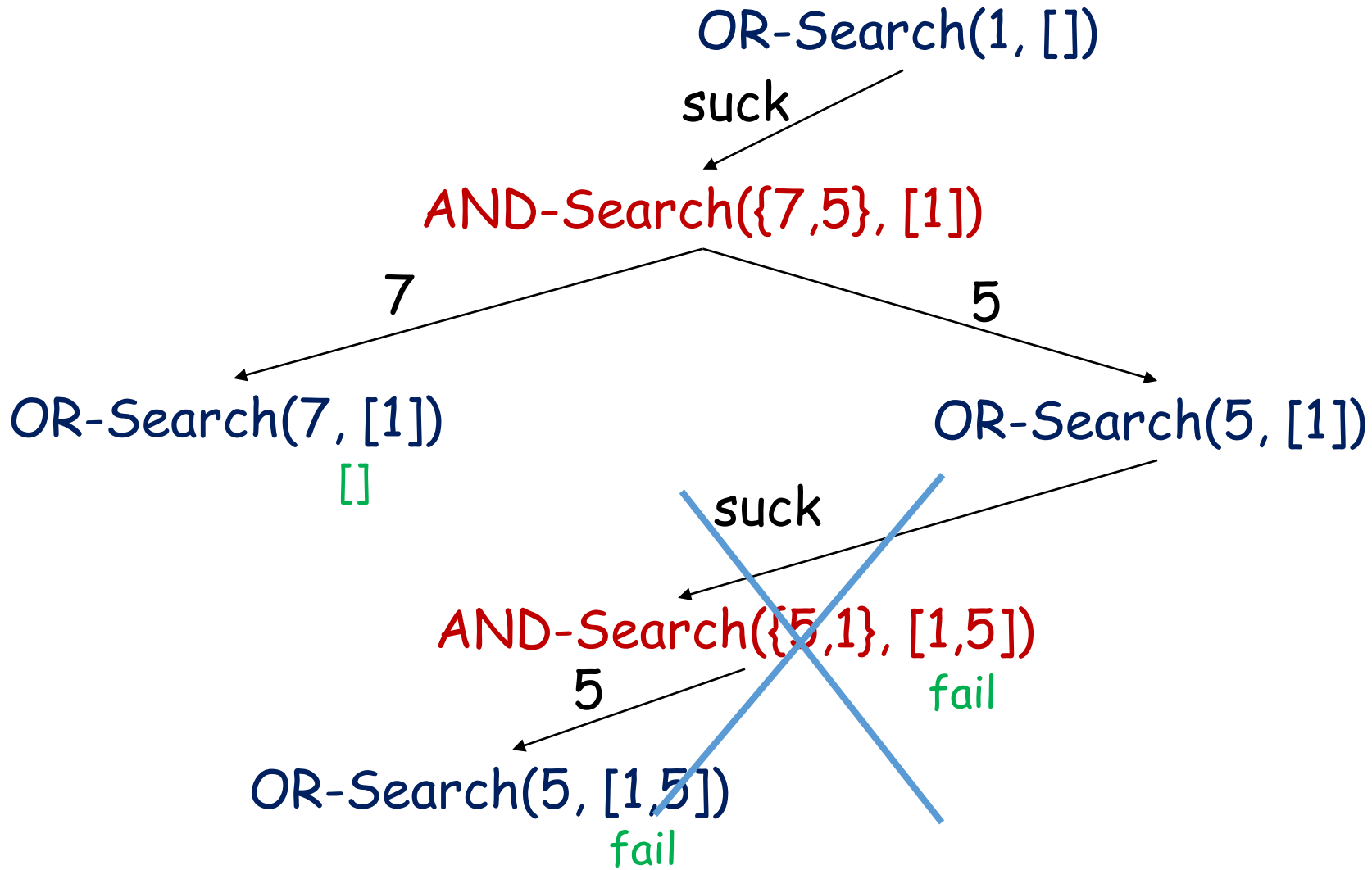


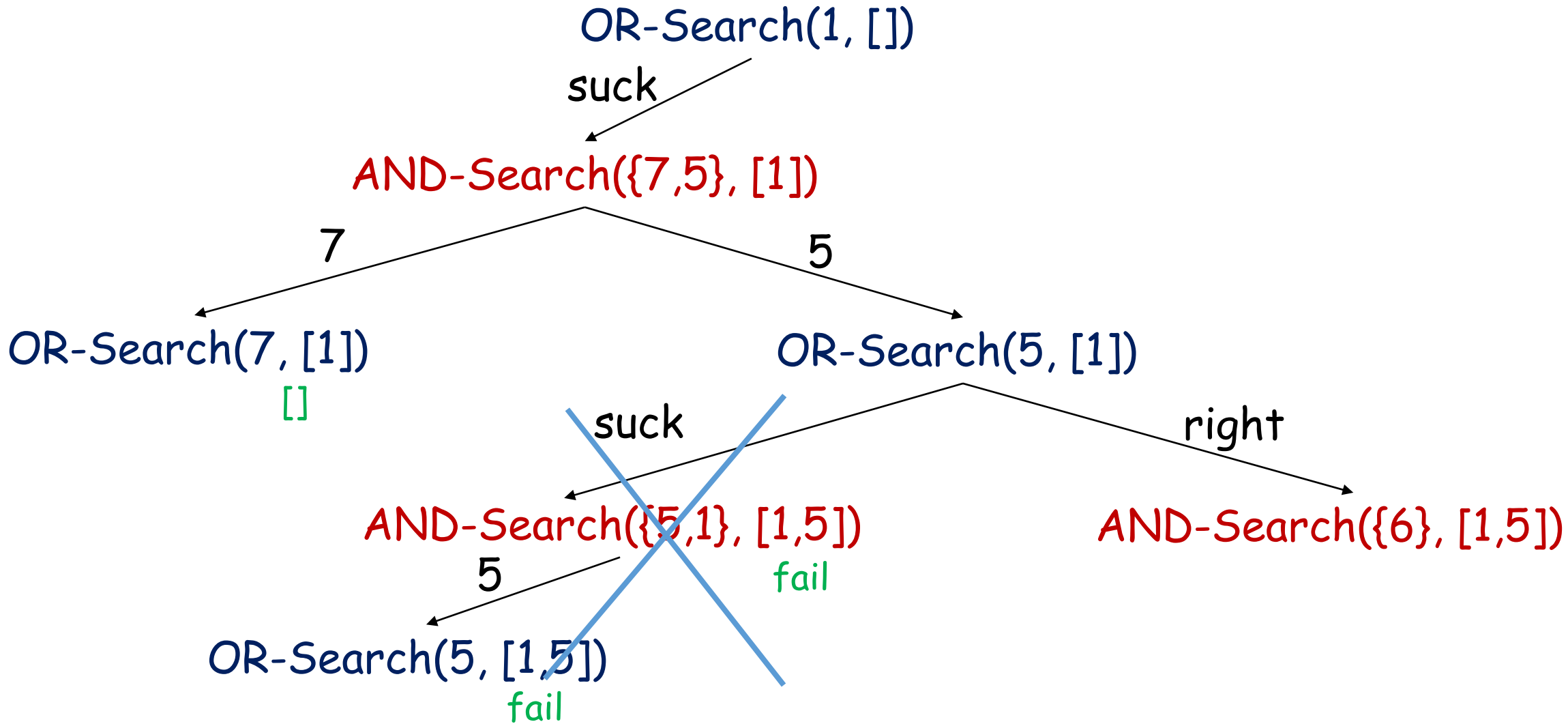


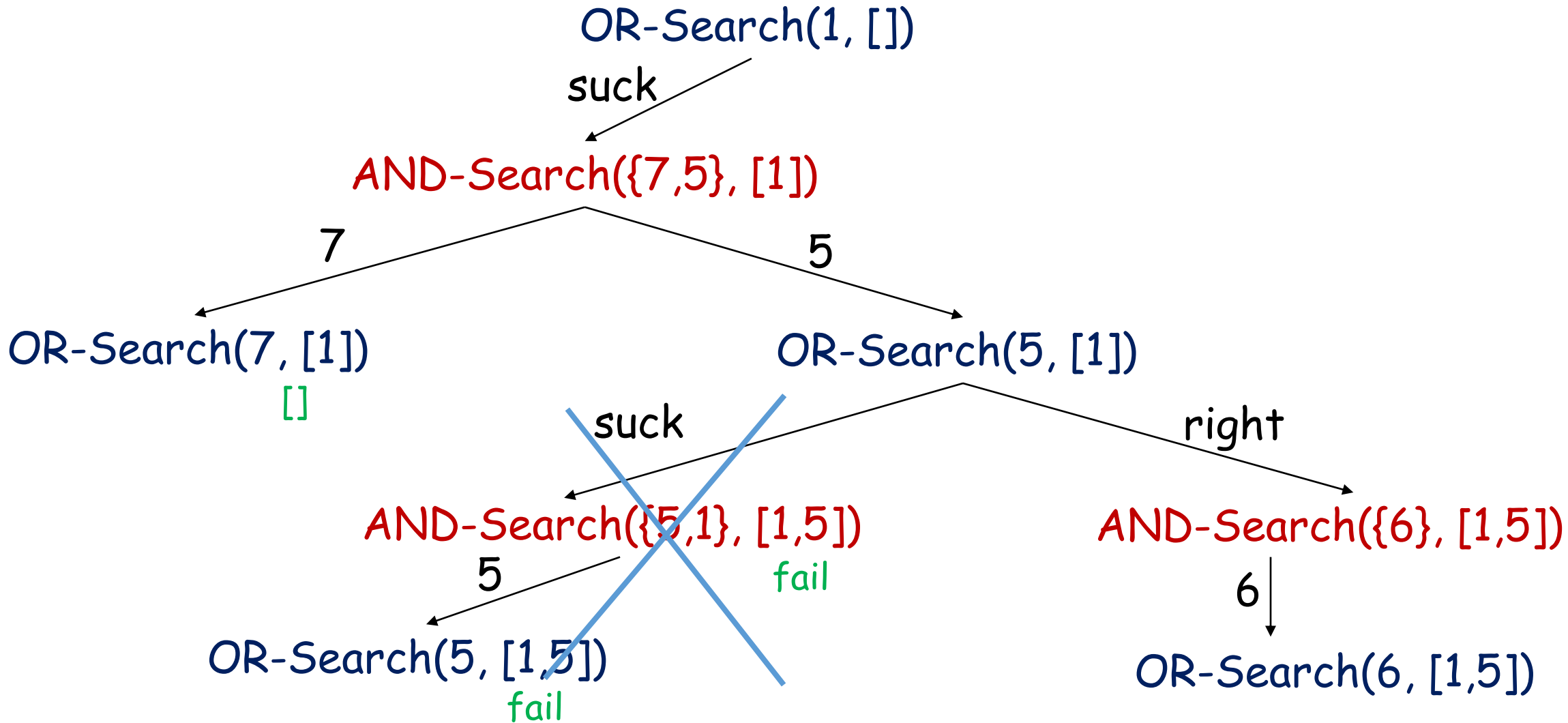




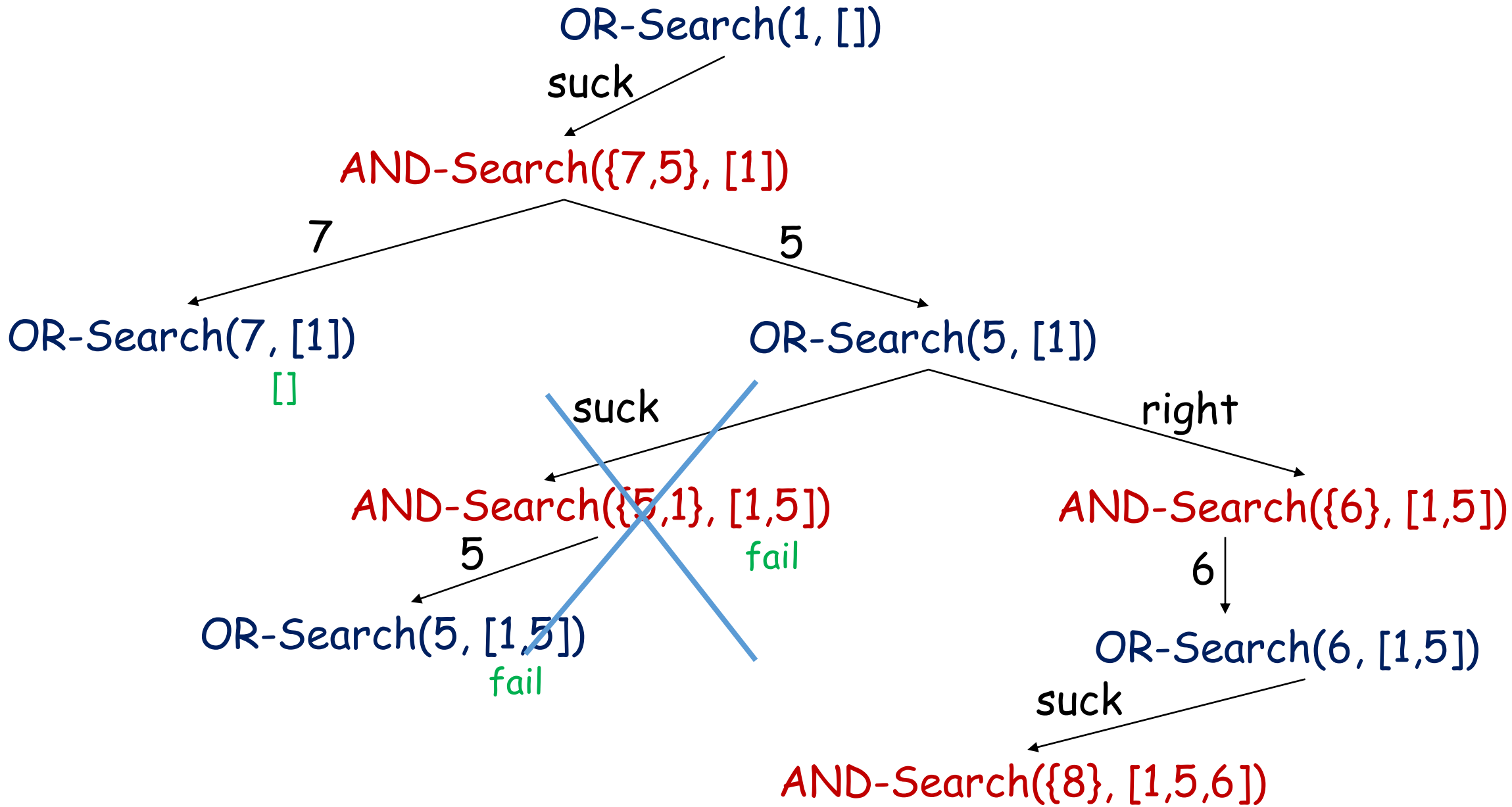


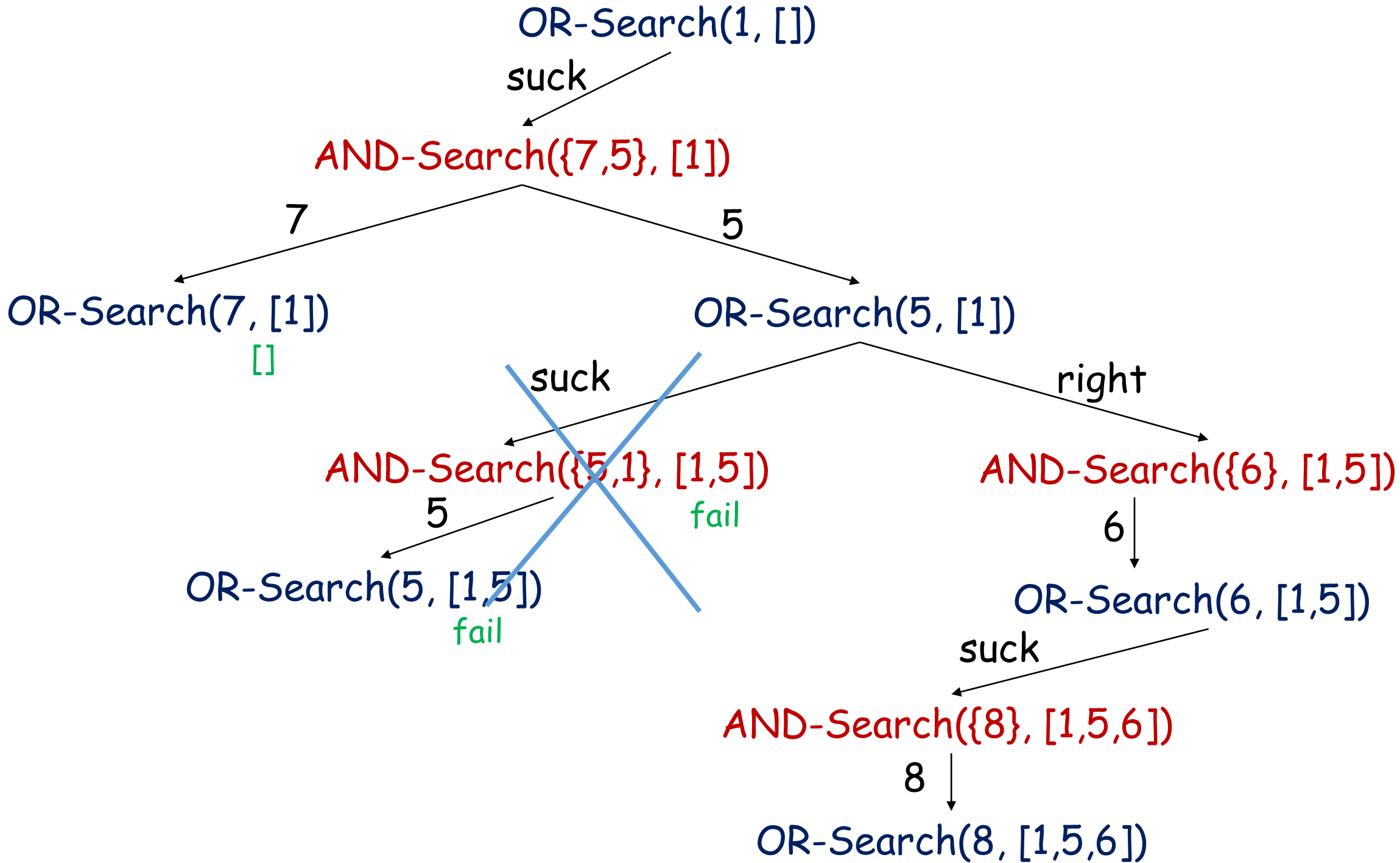


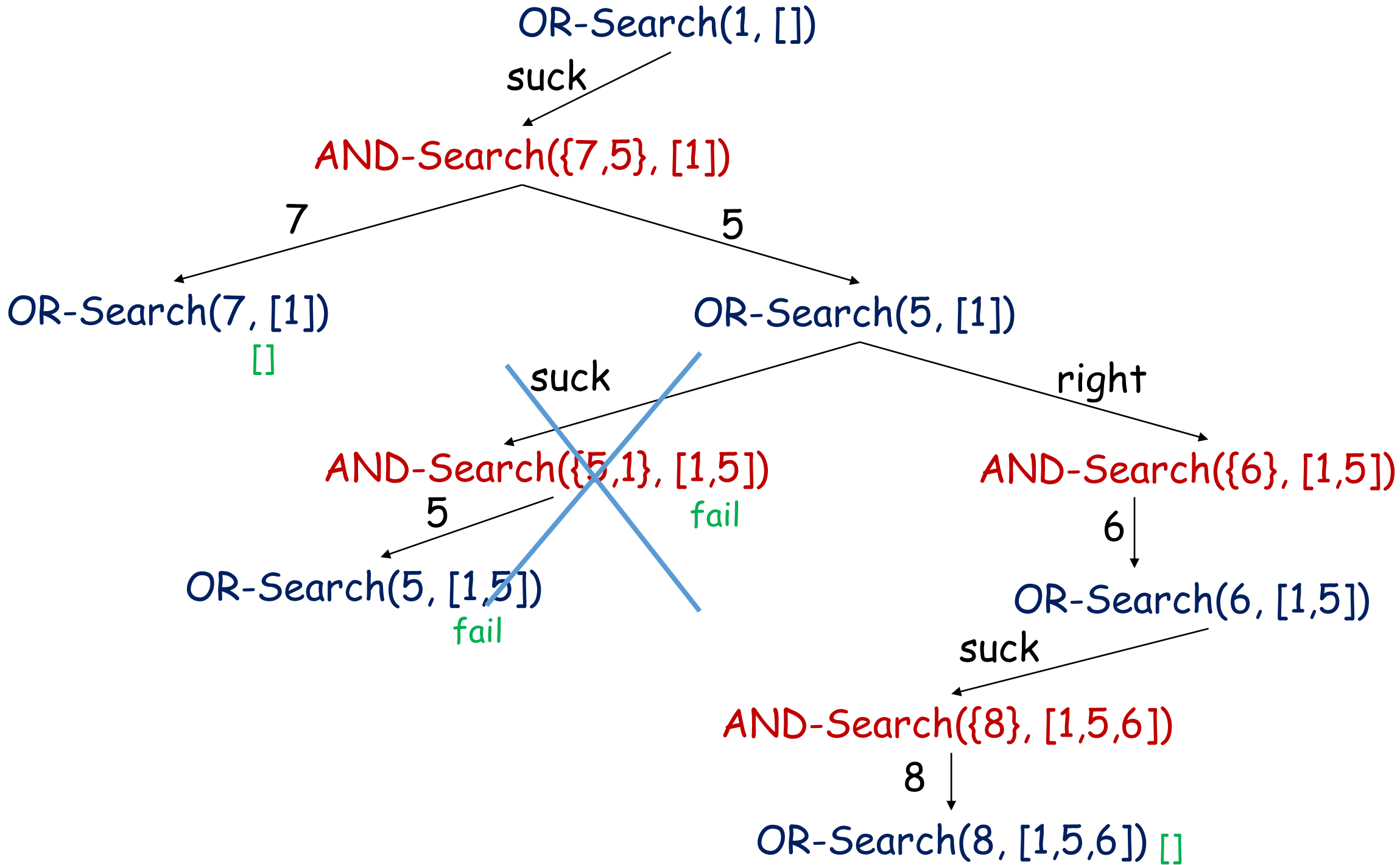


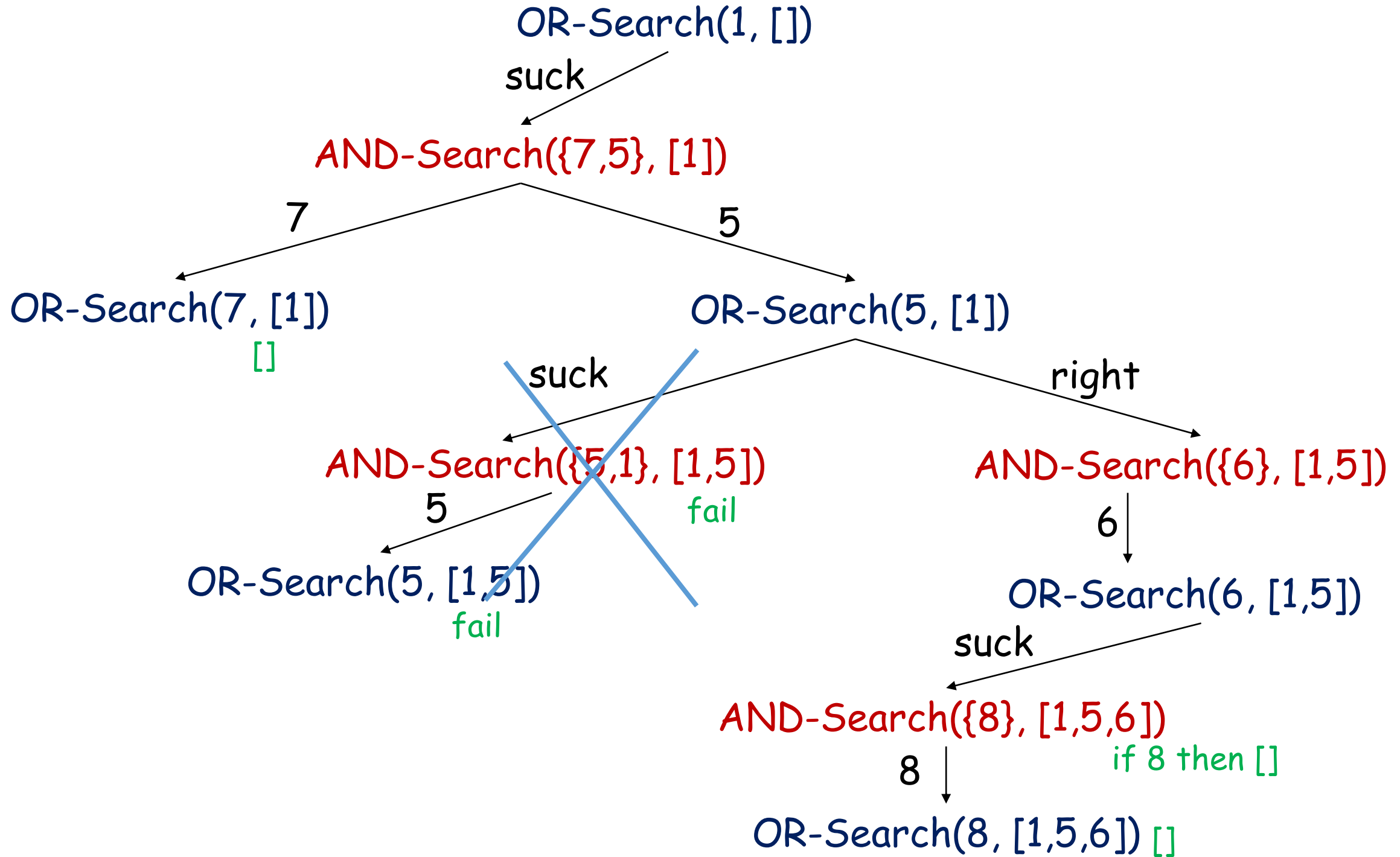


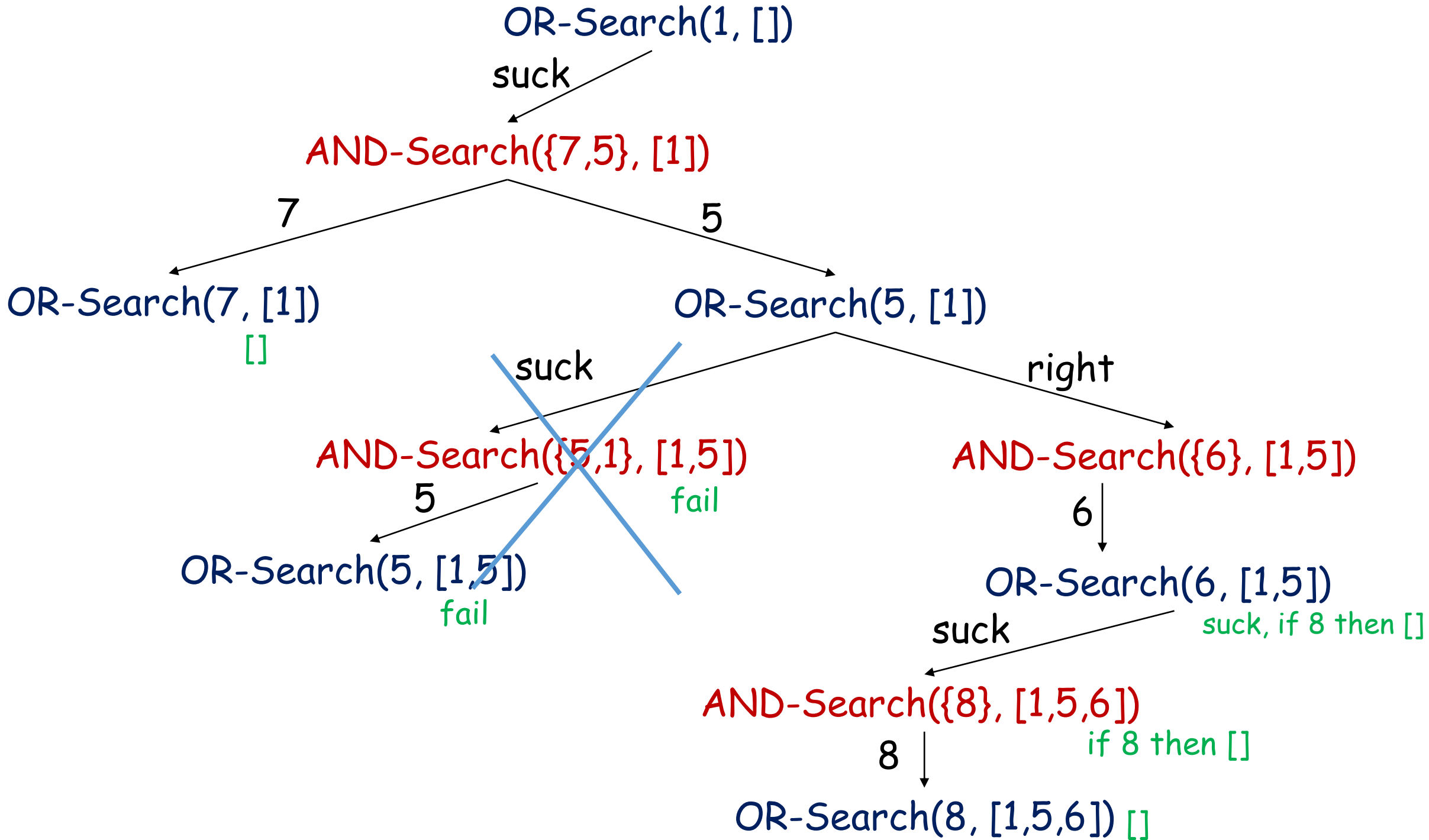


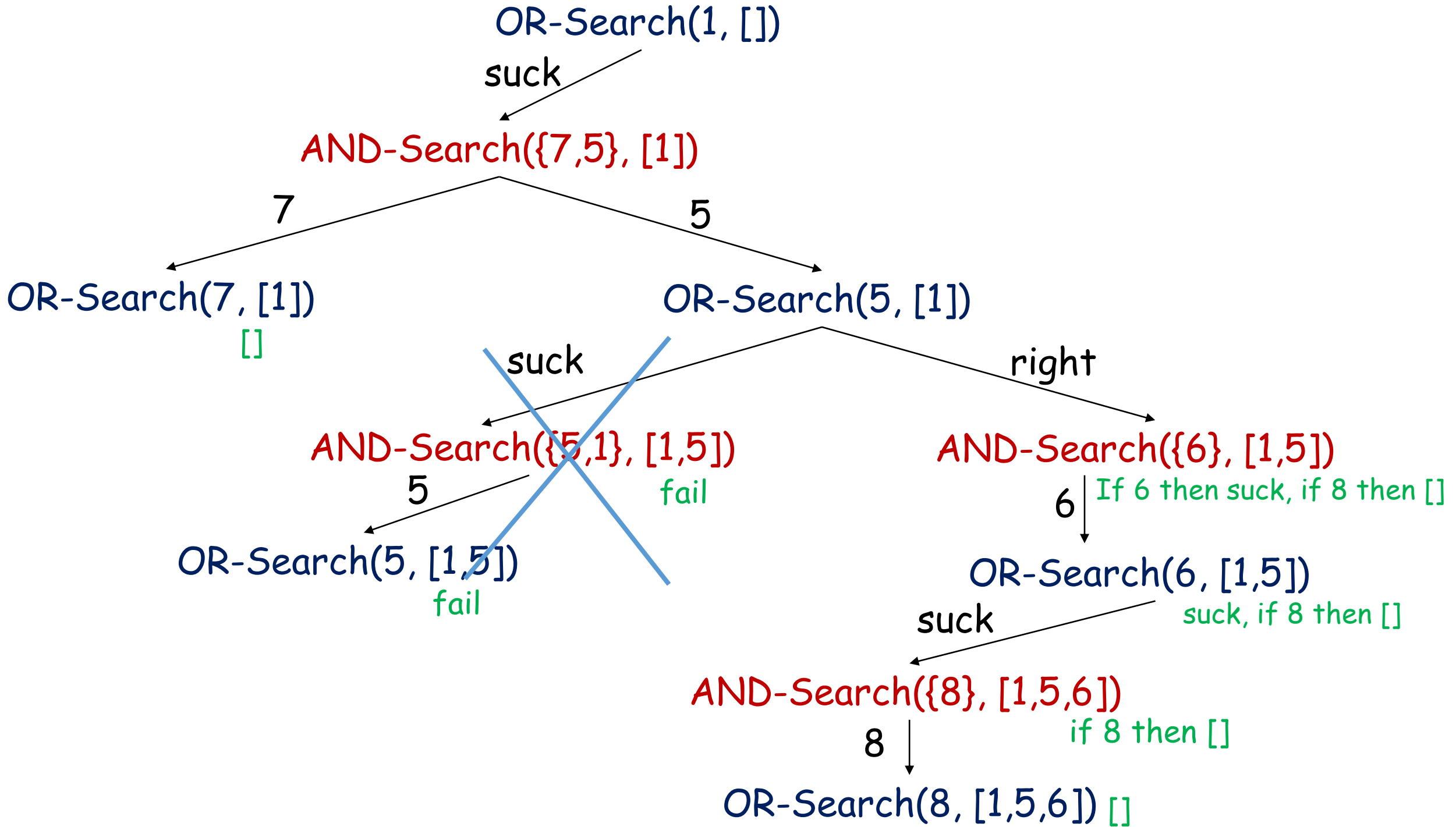


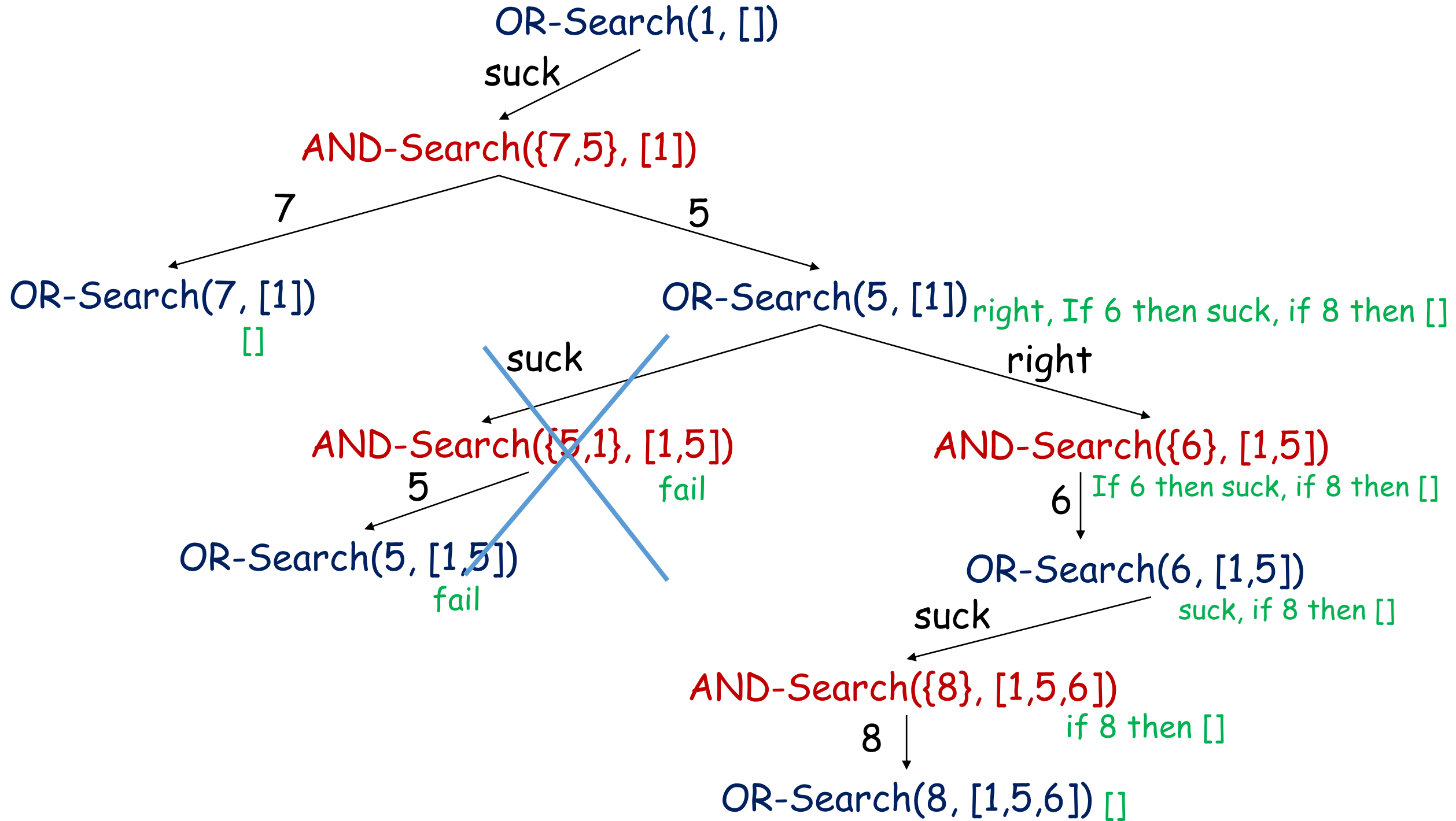


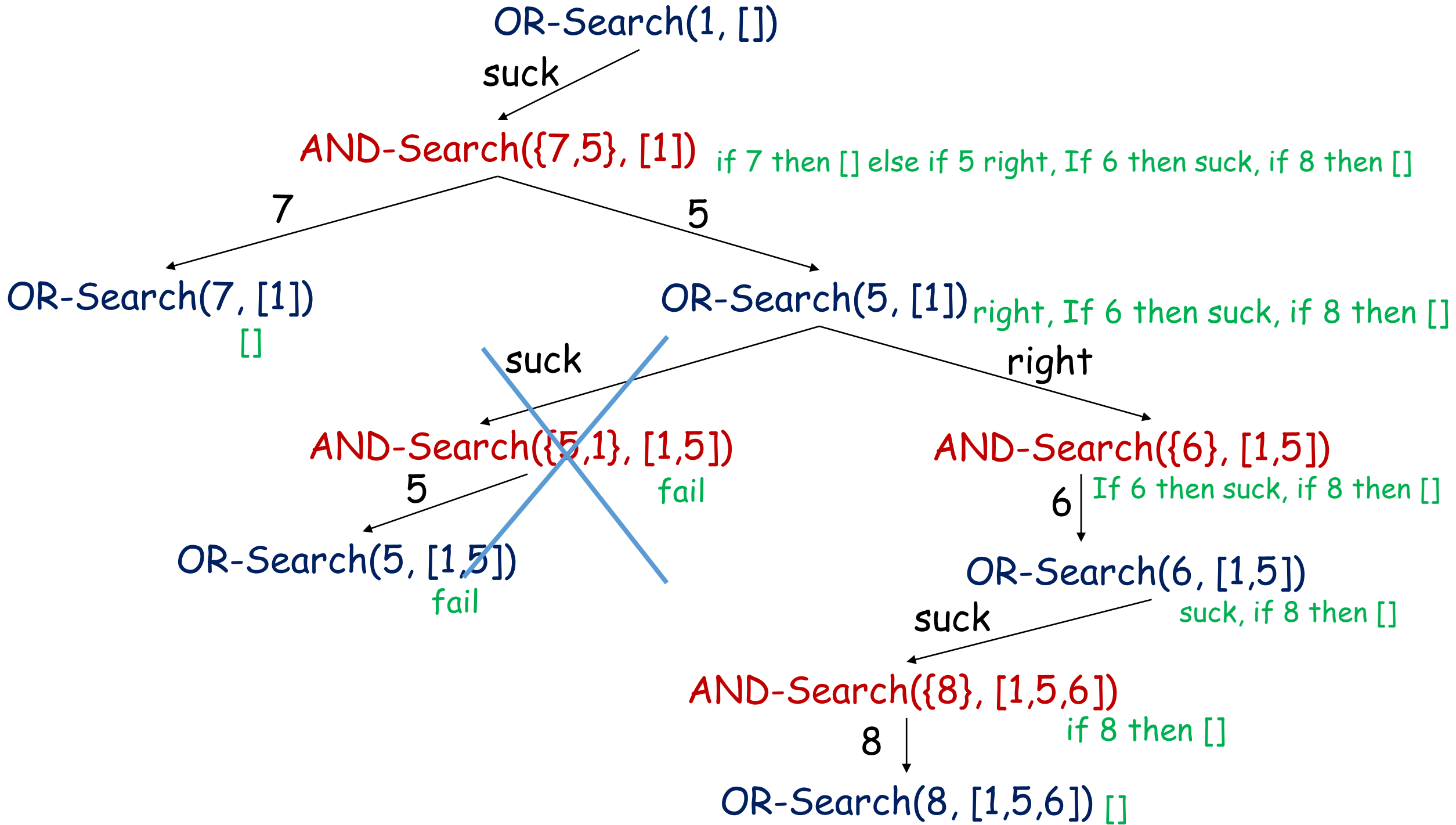




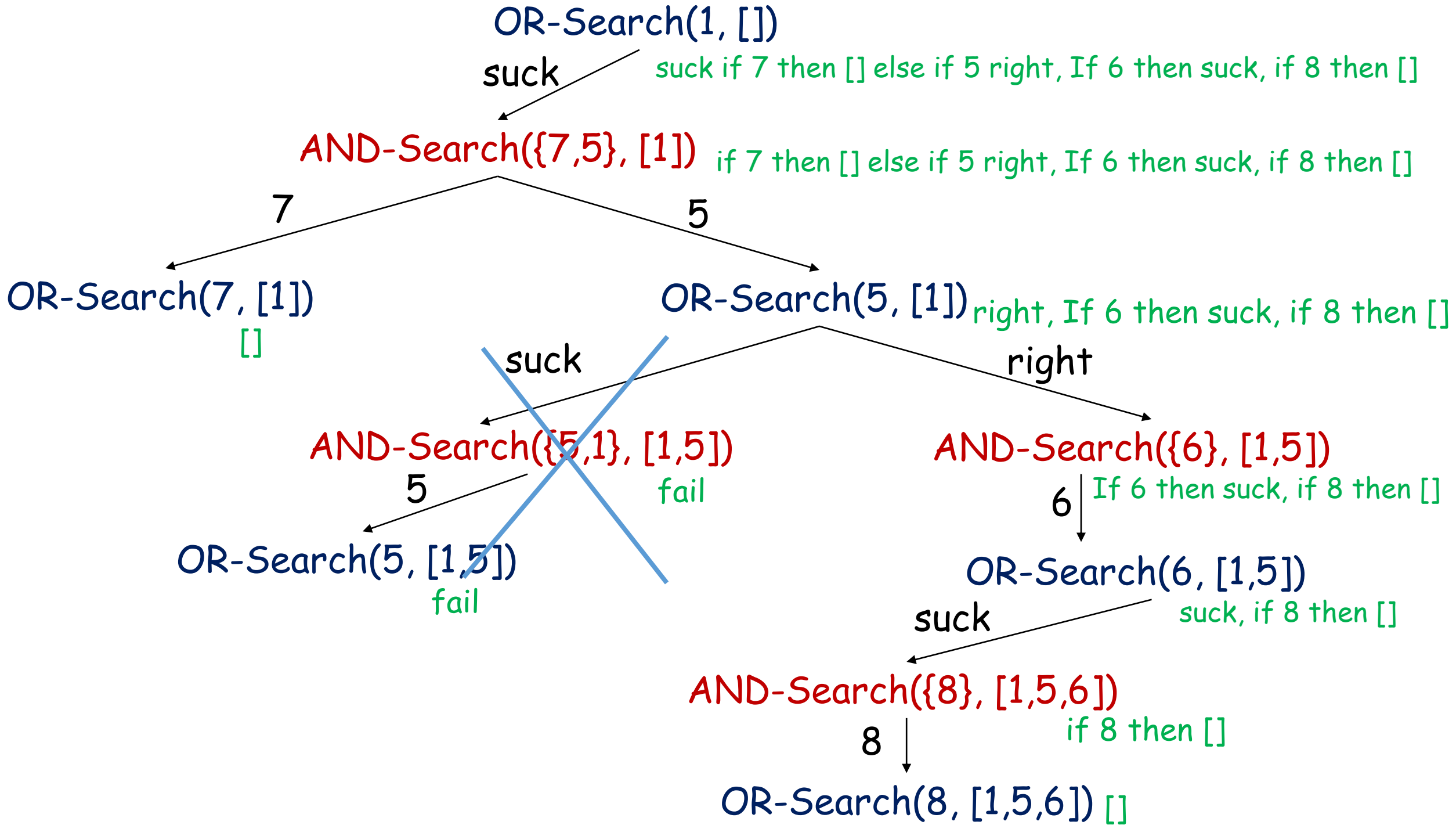












# AND-OR depth first graph search

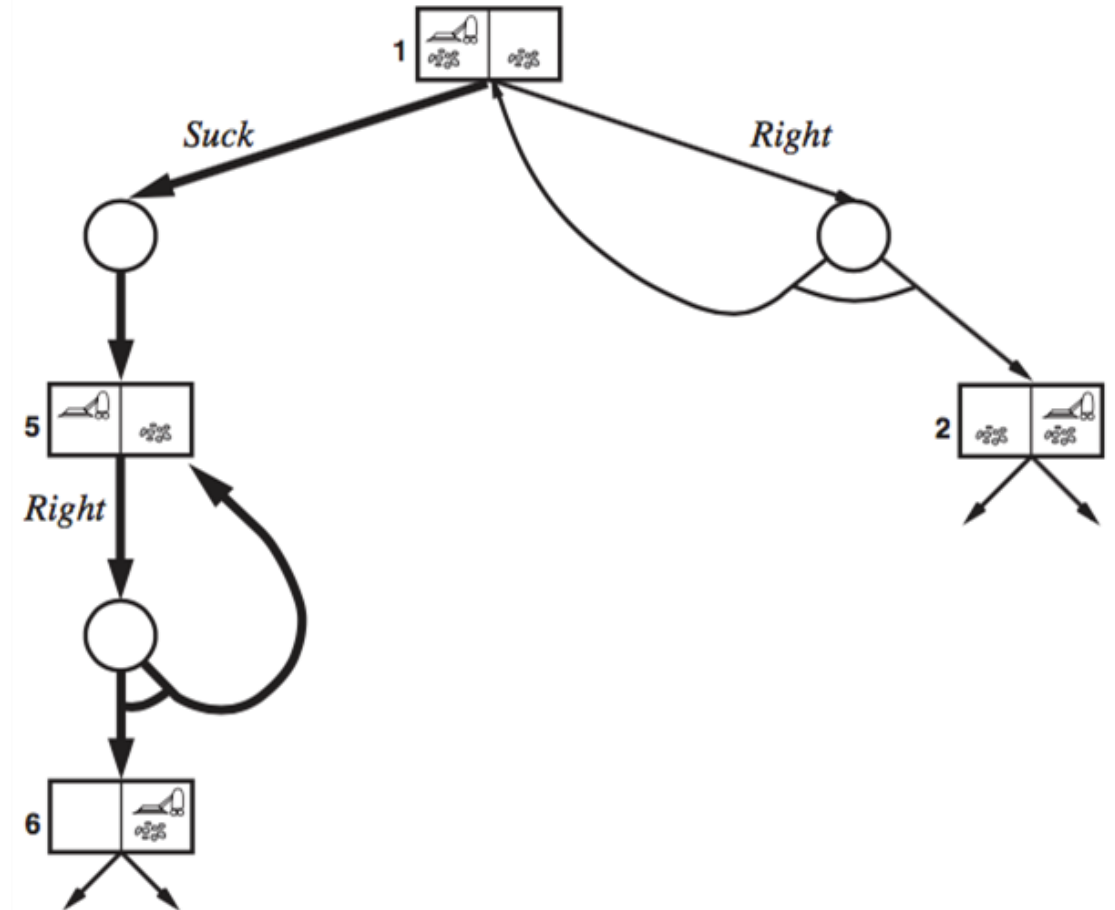
- **Cycles arise** often in non-deterministic problems
- Algorithm **returns with failure** when the current state is identical to one of ancestors
  - If there is a non-cyclic path, the earlier consideration of the state is sufficient
- Termination is guaranteed in finite state spaces
  - Every path reaches a goal, a dead-end, or a repeated state

# Cycles

- Slippery vacuum world
  - Left and Right actions sometimes fail (leaving the agent in the same location)
  - No acyclic solution
    - Solution?
      - Cyclic plan: keep on trying an action until it works

[Suck, **L1**: Right, **if** state = 5 **then** **L1** **else** Suck]

[Suck, **while** state = 5 **do** Right, Suck]

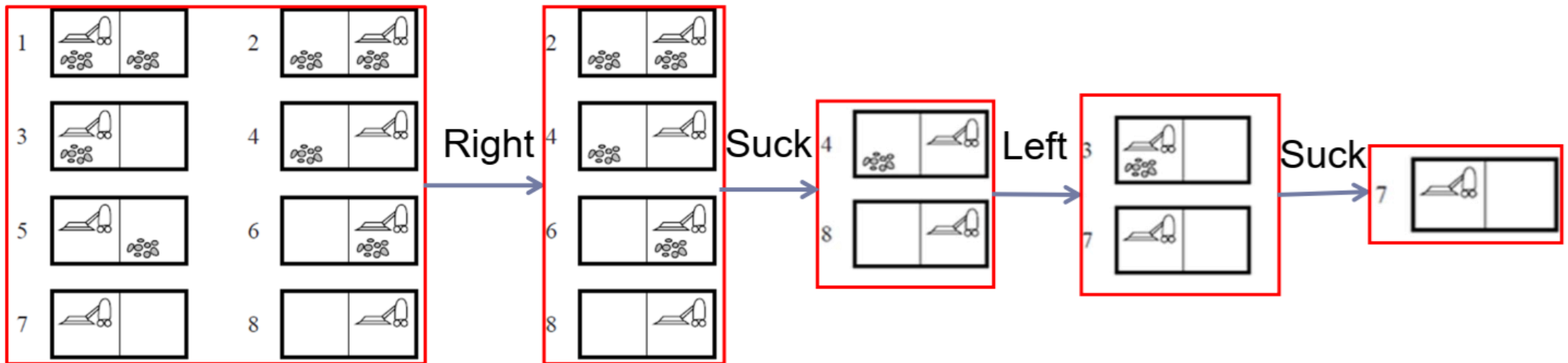


# Searching with partial observations

- The agent does not always know its exact state
  - Agent is in one of several possible states and thus an action may lead to one of several possible outcomes
- Belief state
  - Agent's current **belief** about the **possible states**, given the sequence of actions and observations up to that point

# Searching with unobservable states

- Sensor-less or conformant problem
- Vacuum world example
  - Initial state
    - belief = {1, 2, 3, 4, 5, 6, 7, 8}
  - Action sequence (conformant plan)
    - [Right, Suck, Left, Suck]



# Sensor-less problem formulation

- Belief state space (instead of physical state space)
  - It is fully observable
  - Solution is a sequence of actions (even in non-deterministic environment)
- Physical problem
  - $N$  states,  $ACTIONS_p$ ,  $RESULTS_p$ ,  $GOAL\_TEST_p$ ,  $STEP\_COST_p$
- Sensor-less problem
  - Up to  $2^N$  states,  $ACTIONS$ ,  $RESULTS$ ,  $GOAL\_TEST$ ,  $STEP\_COST$

# Sensor-less problem formulation

- States
  - Every possible set of physical states,  $2^N$
- Initial State
  - Usually the set of all physical states
- Actions
  - $ACTIONS(b) = \bigcup_{s \in b} ACTIONS_p(s)$ 
    - Illegal actions? i.e.,  $b = \{s_1, s_2\}$ ,  $ACTIONS_p(s_1) \neq ACTIONS_p(s_2)$
    - Illegal actions have no effect on the env. (union of physical actions)
    - Illegal actions are not legal at all (intersection of physical actions)

# Sensor-less problem formulation

## ■ Transposition model

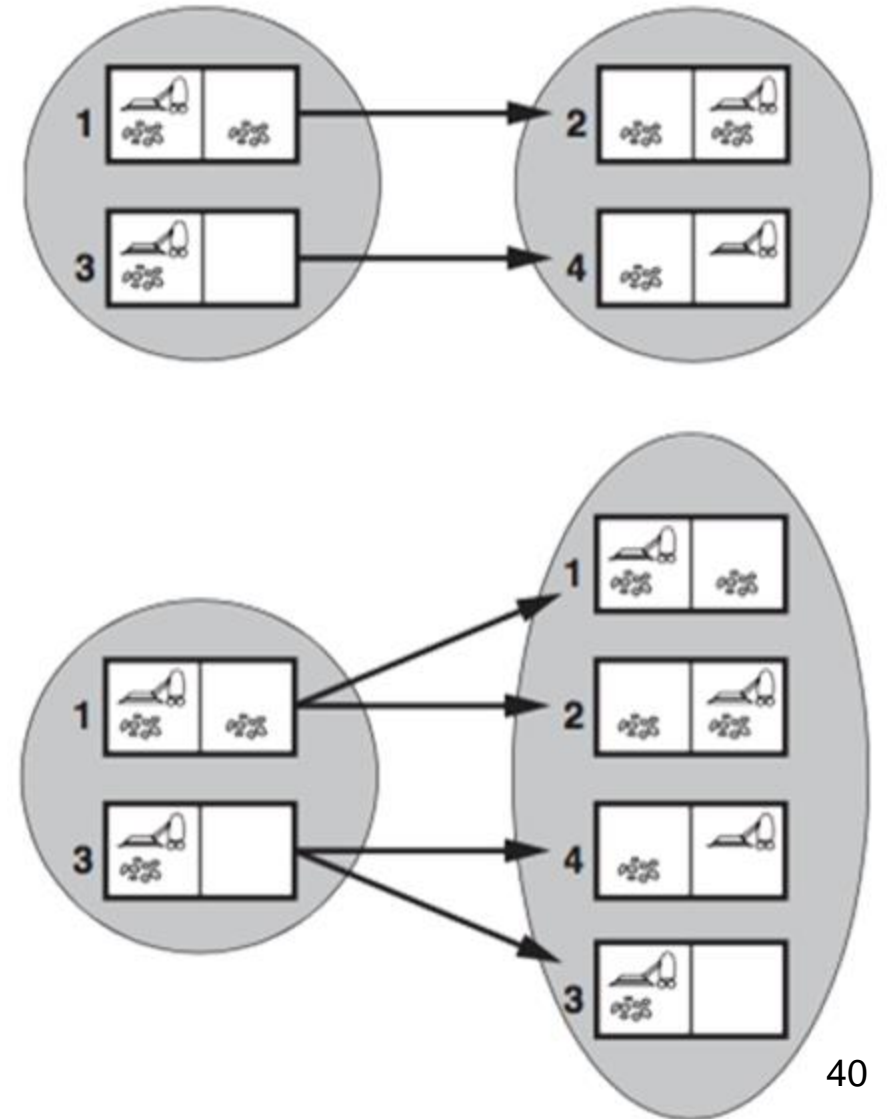
- $b' = \text{PREDICT}(b, a)$ 
  - Deterministic actions
    - $b' = \{s' : s' = \text{RESULTS}_p(s, a) \text{ and } s \in b\}$
  - Nondeterministic actions
    - $b' = \bigcup_{s \in b} \text{RESULTS}_p(s, a)$

## ■ Goal test

- Goal is satisfied when all the physical states in the belief state satisfy *GOAL\_TEST*

## ■ Step cost

- $\text{STEP\_COST}_p$  if the cost of an action is the same in all states

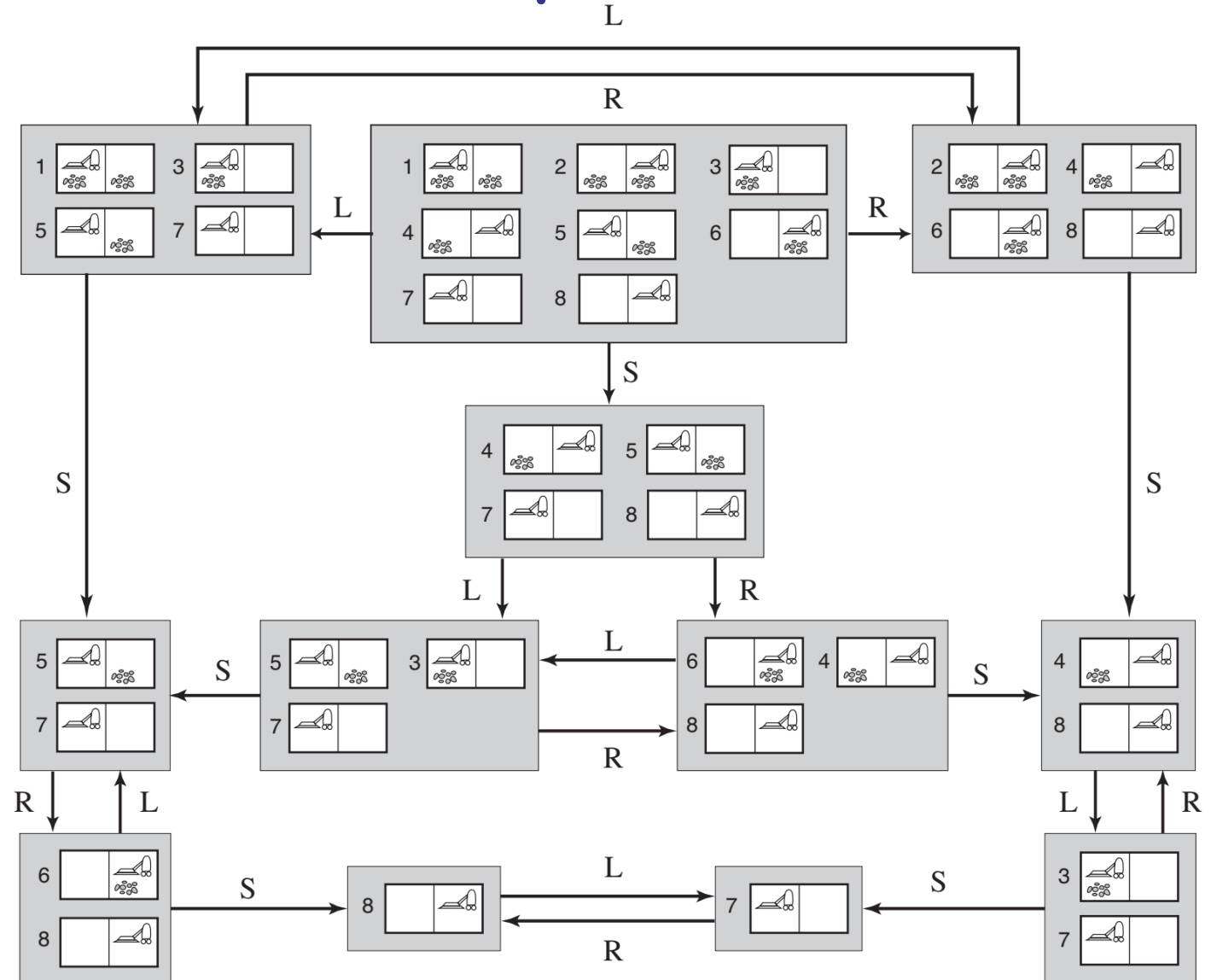




# Vacuum world example

## ■ Belief-state space for sensor-less deterministic vacuum world

- Total number of possible belief states?  $2^8$
- Number of reachable belief states? 12



# Sensor-less problem: searching

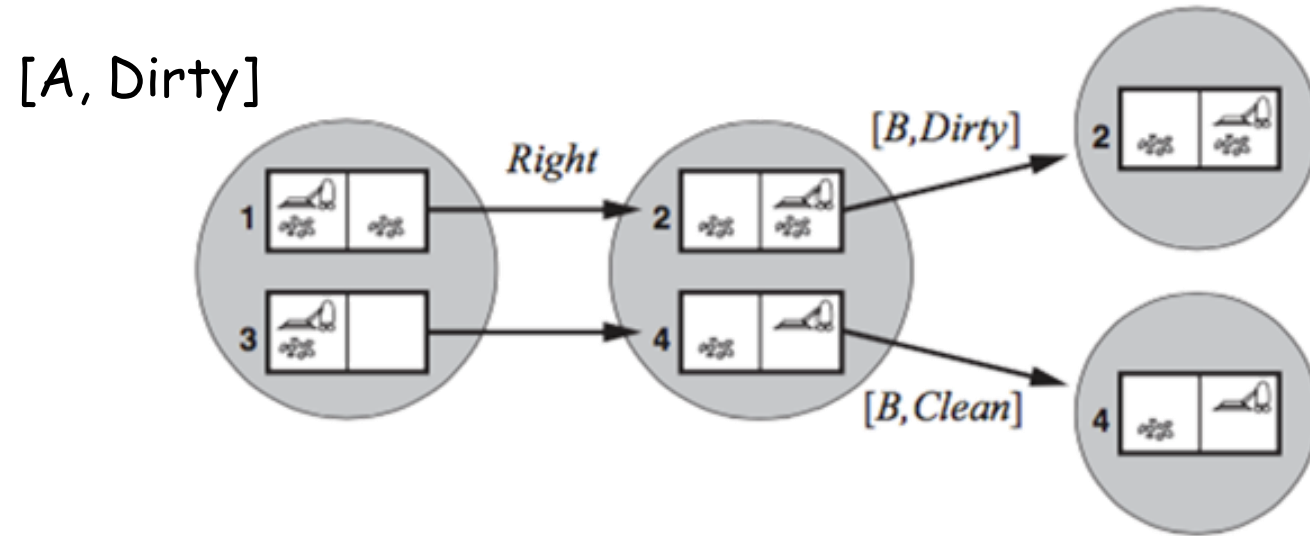
- In general, we can use any standard search algorithm
- Searching in these spaces is not usually feasible (scalability)
  - **Problem1:** No. of reachable belief states
    - Pruning (subsets or supersets) can reduce this difficulty
    - Branching factor and solution depth in the belief-state space and physical state space are not usually such different
  - **Problem2:** (main difficulty): No. of physical states in each belief state
    - Using a **compact state representation** (like formal representation)
    - Incremental belief-state search: Search for solutions by **considering physical states incrementally** (not whole belief space) to **quickly detect failure** if we reach an unsolvable physical state

# Searching with partial observations

- Similar to sensor-less, **after each action** the new belief state must be **predicted**
- After each perception the belief state is **updated**
  - E.g., local sensing vacuum world
    - After each perception, the belief state can contain at most two physical states.
- We must plan for different **possible perceptions**

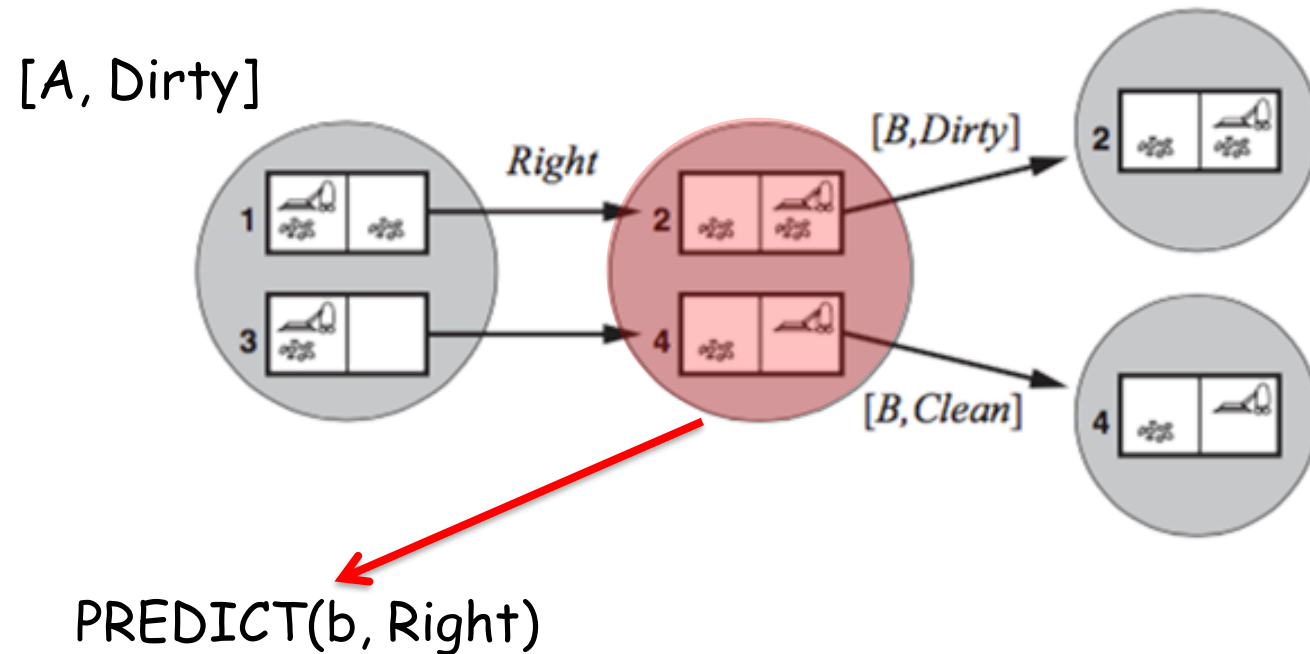
# Vacuum world with A position sensor and local dirt sensor

- Deterministic world



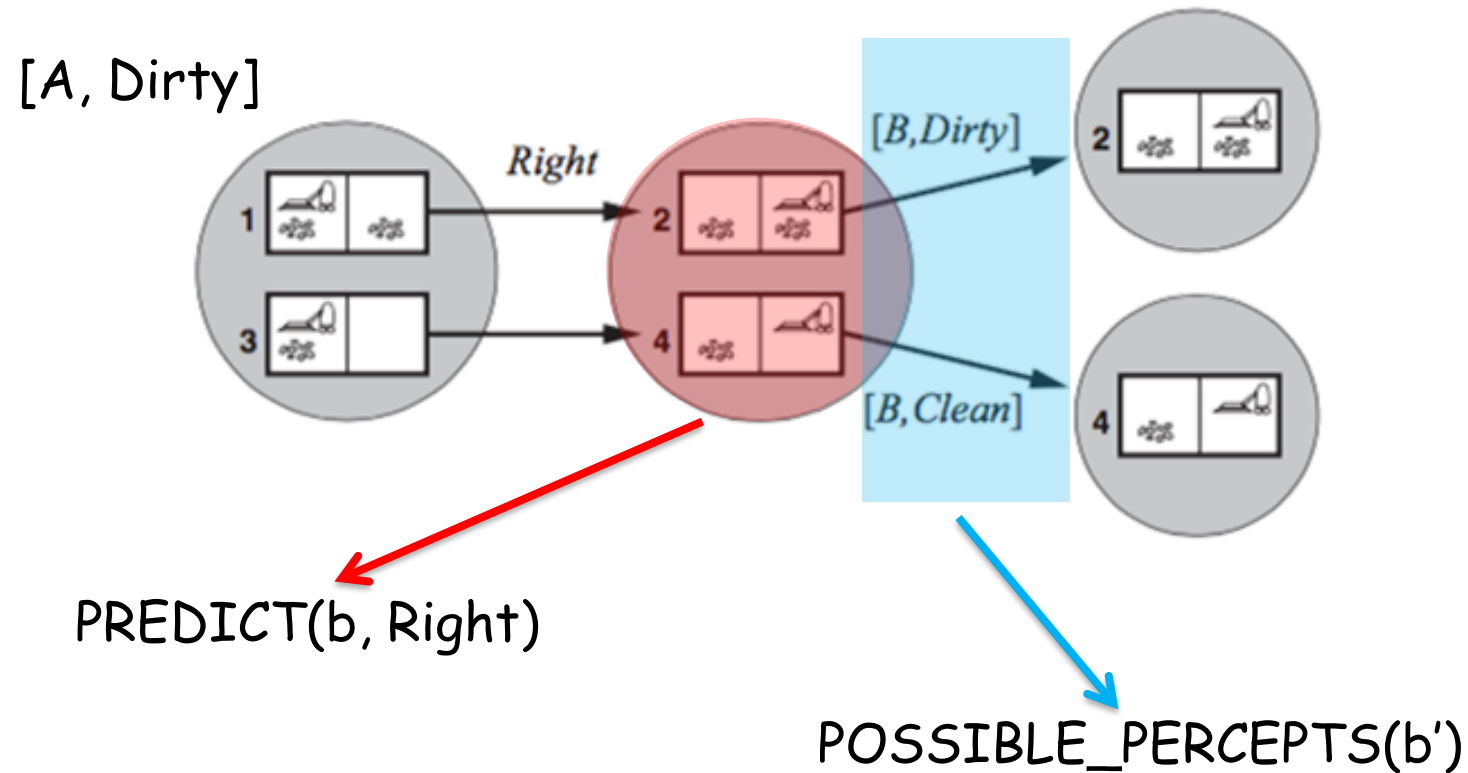
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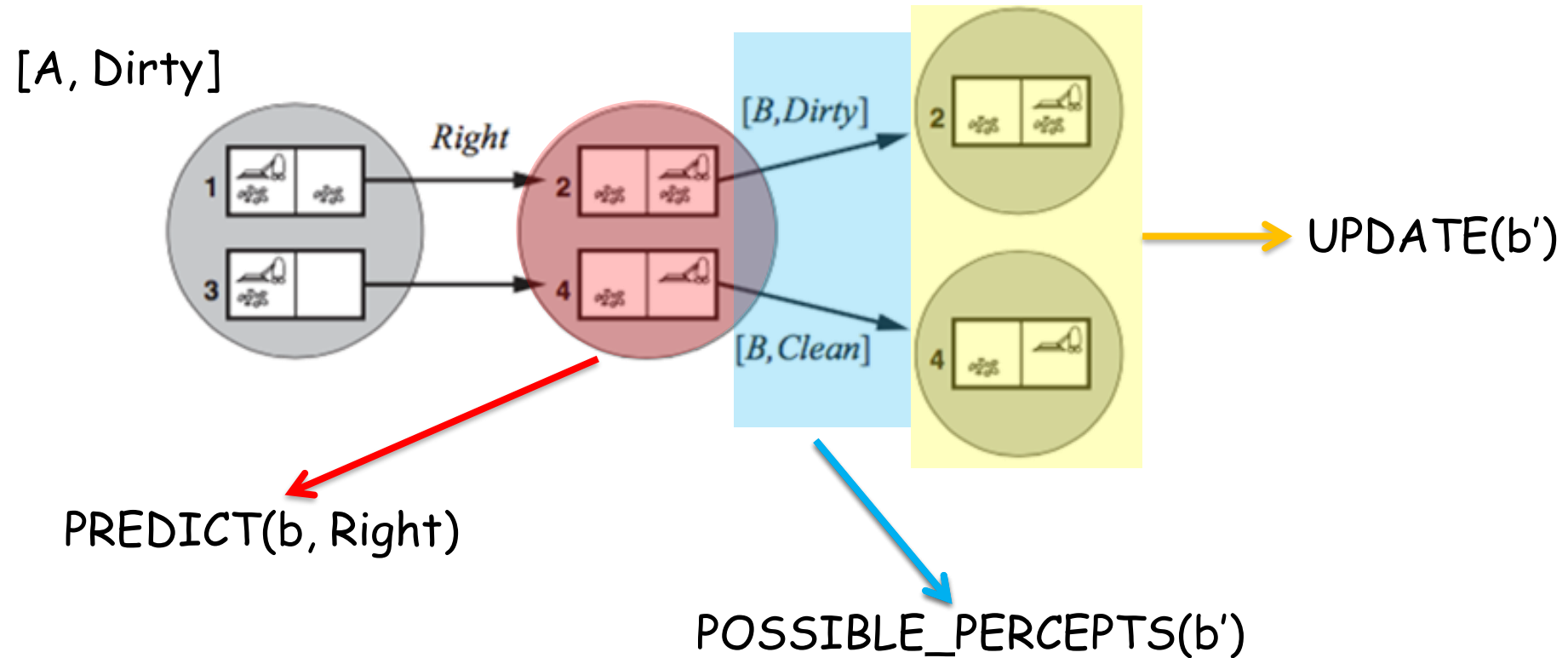
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- Deterministic world



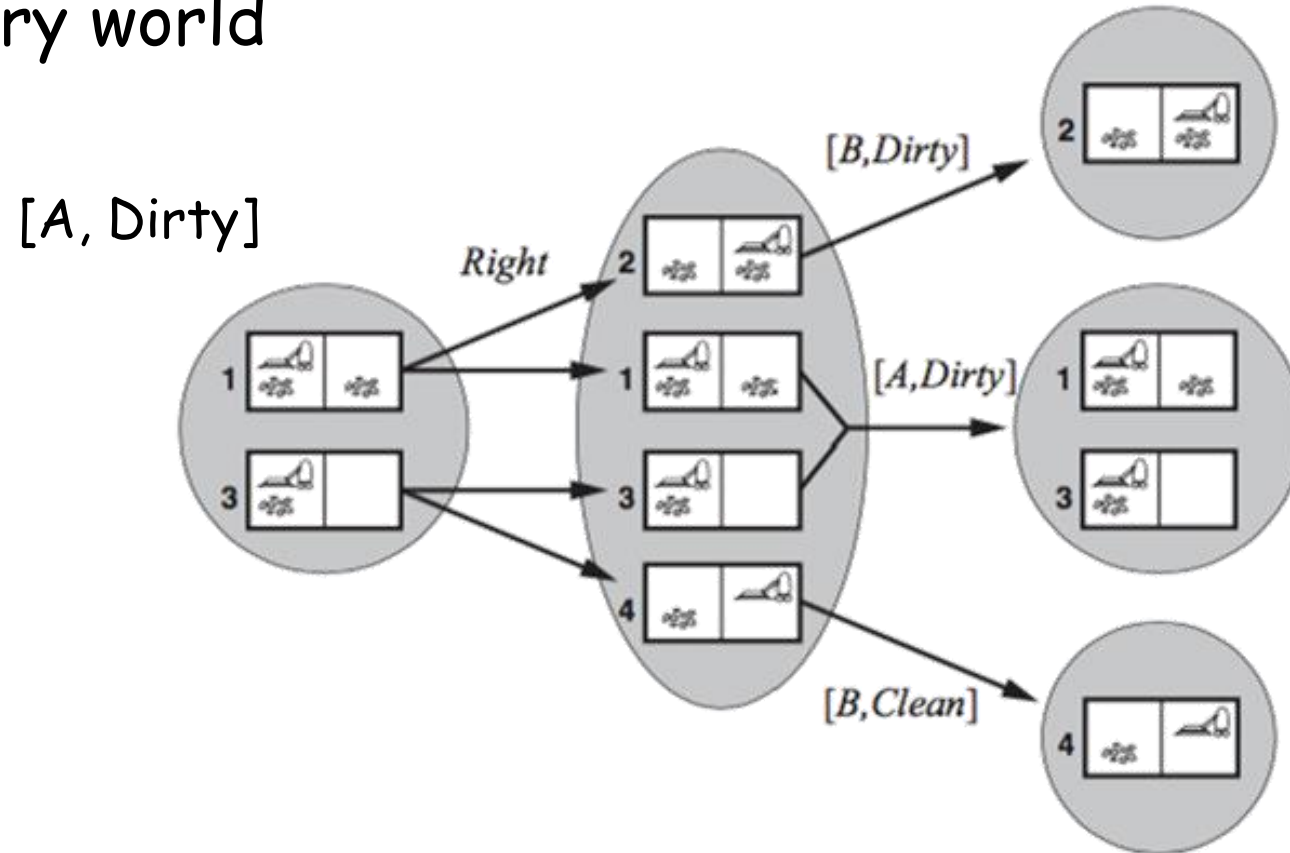
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- Deterministic world



# Vacuum world with A position sensor and local dirt sensor

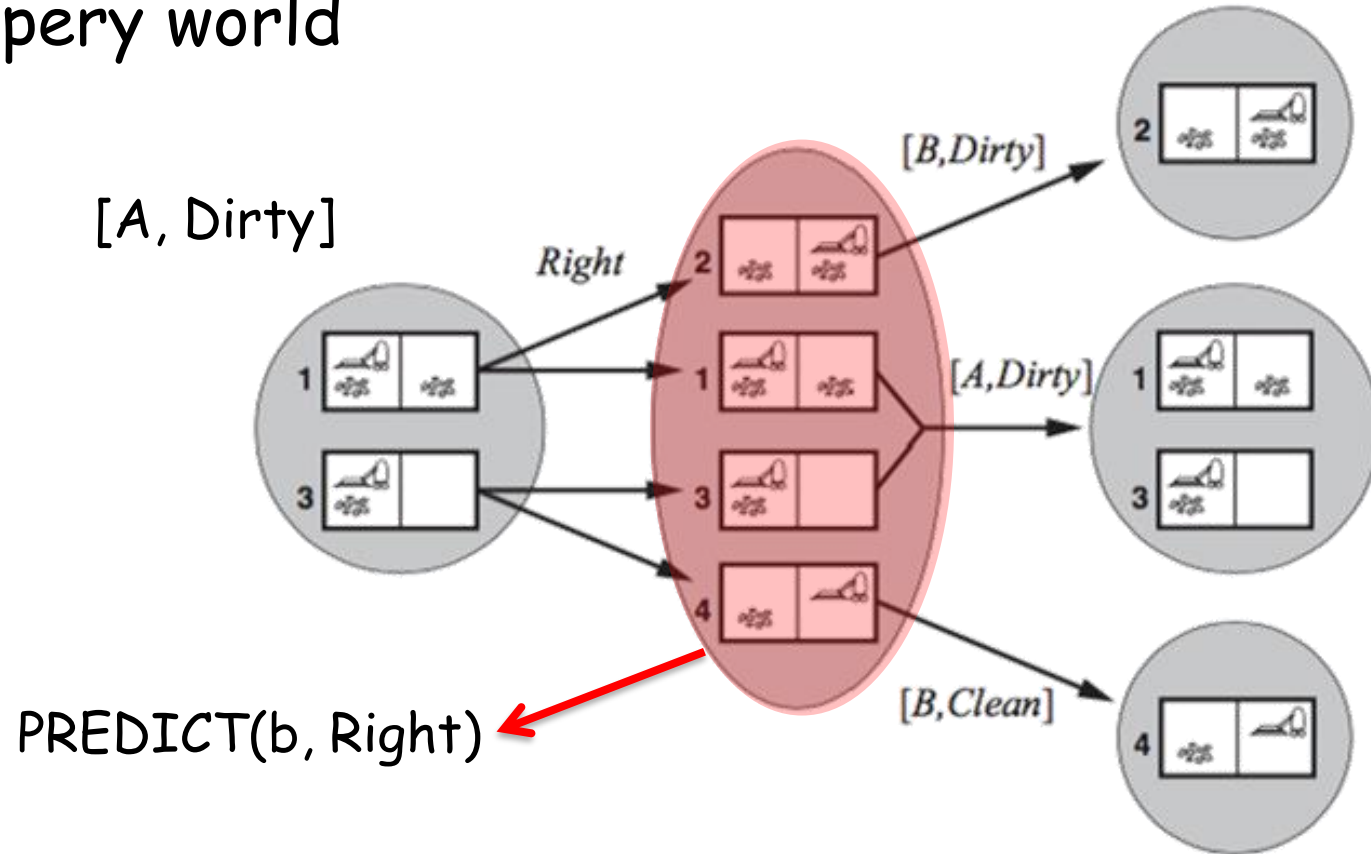
- Slippery world





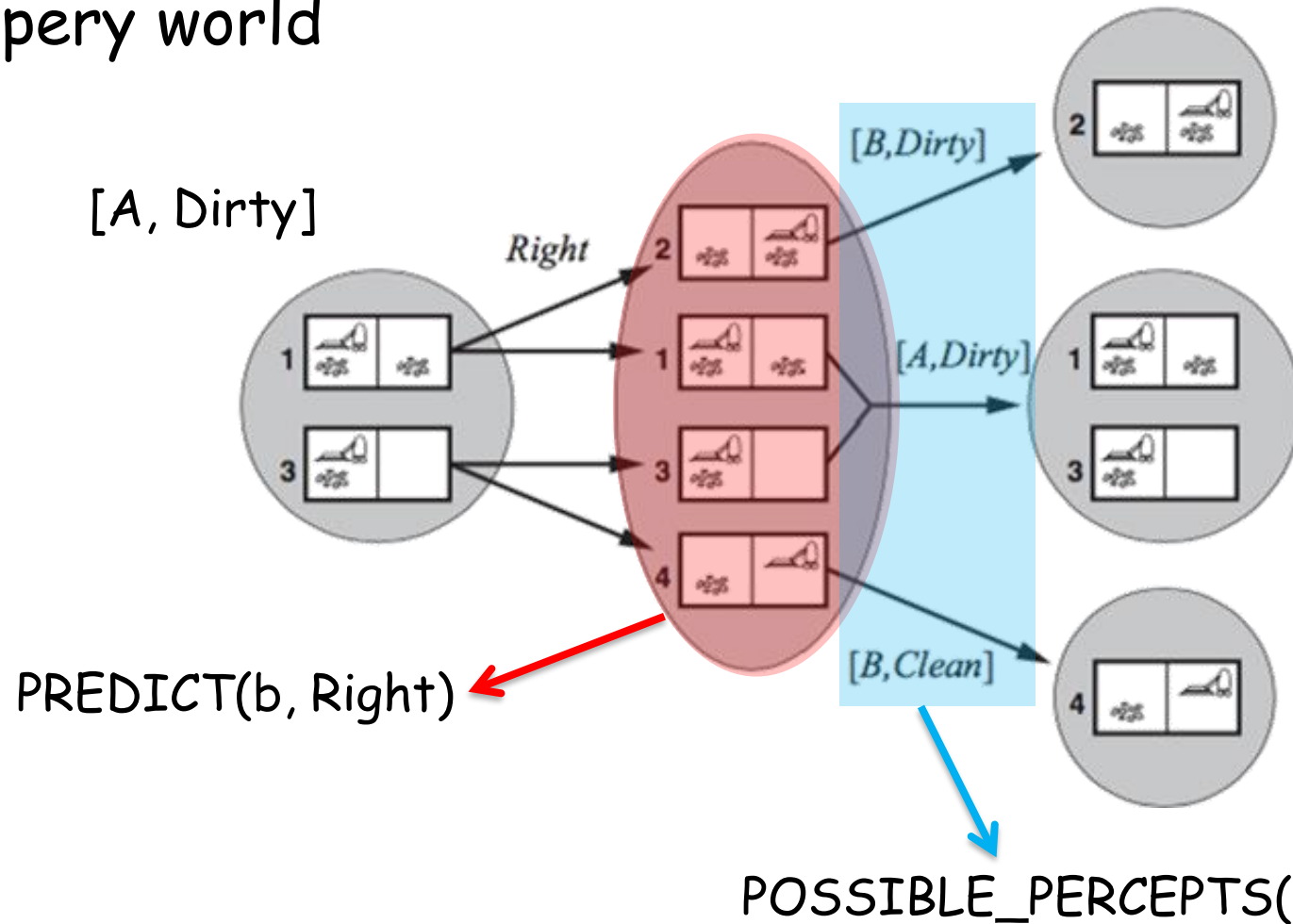
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- Slippery world



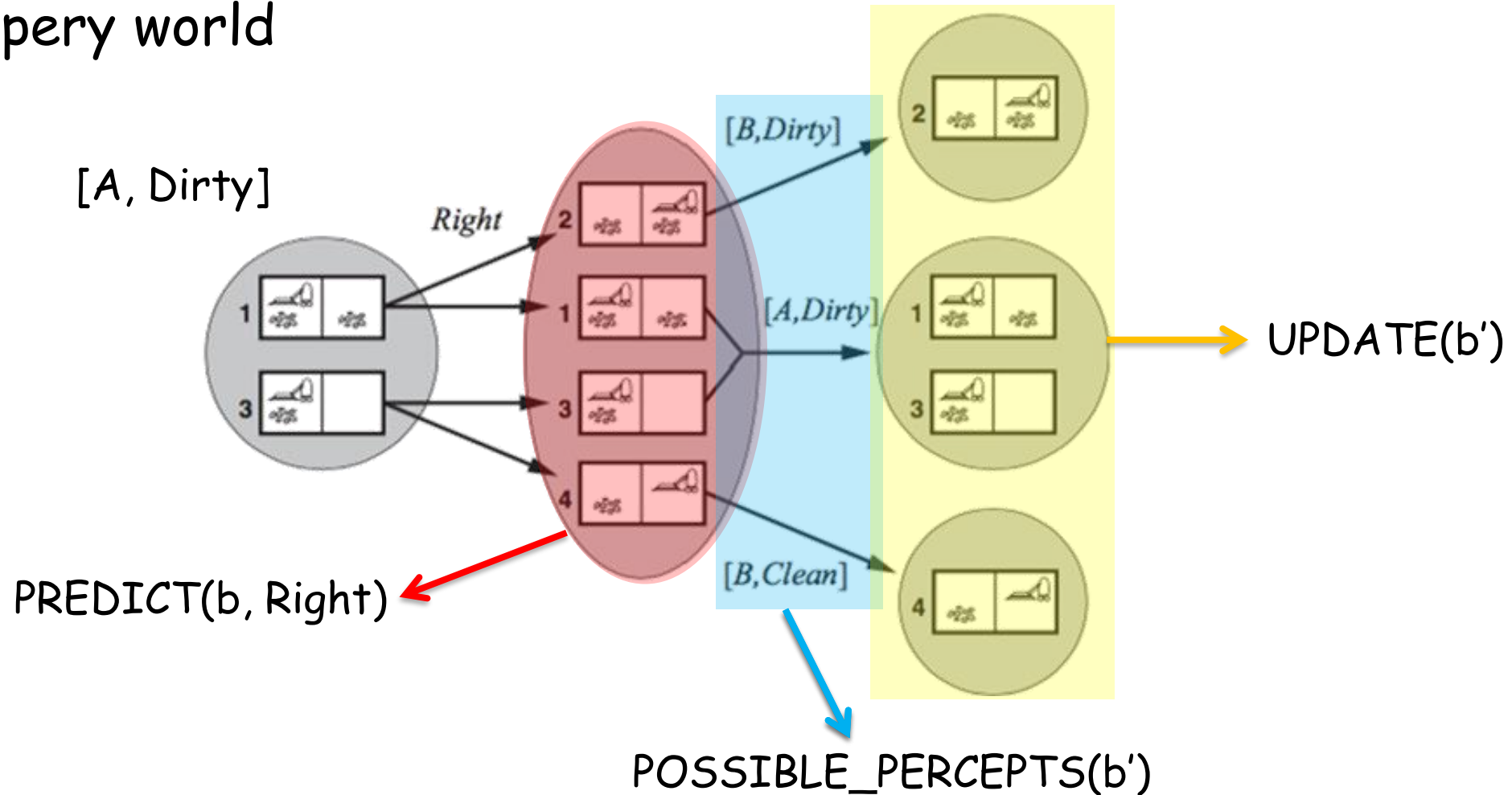
# Vacuum world with A position sensor and local dirt sensor

- Slippery world



# Vacuum world with A position sensor and local dirt sensor

- Slippery world



# Transition model (partially observable env.)

## ■ Prediction stage

- How does the belief state change after doing an action?

$$b' = \text{PREDICT}(b, a)$$

- Deterministic actions

$$b' = \{s' : s' = \text{RESULTS}_p(s, a) \text{ and } s \in b\}$$

- Nondeterministic actions

$$b' = U_{s \in b} \text{RESULTS}_p(s, a)$$

## ■ Possible Perceptions

- What are the possible perceptions in a belief state?

$$\text{POSSIBLE\_PERCEPTS}(b') = \{o : o = \text{PERCEPT}(s) \text{ and } s \in b'\}$$

# Transition model (partially observable env.)

- Update stage

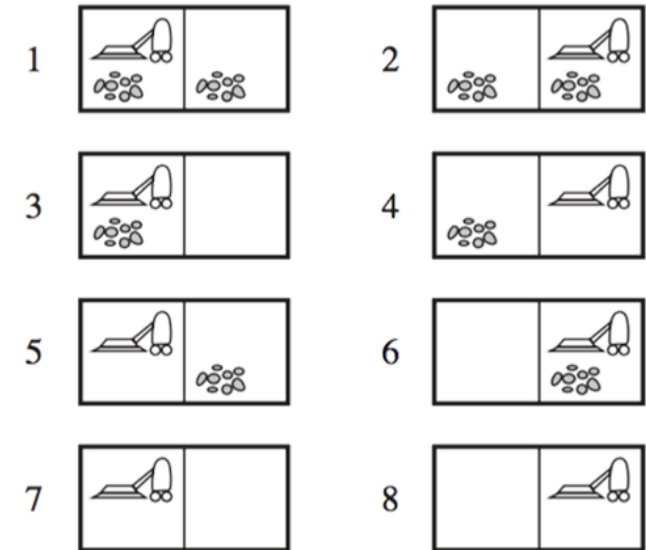
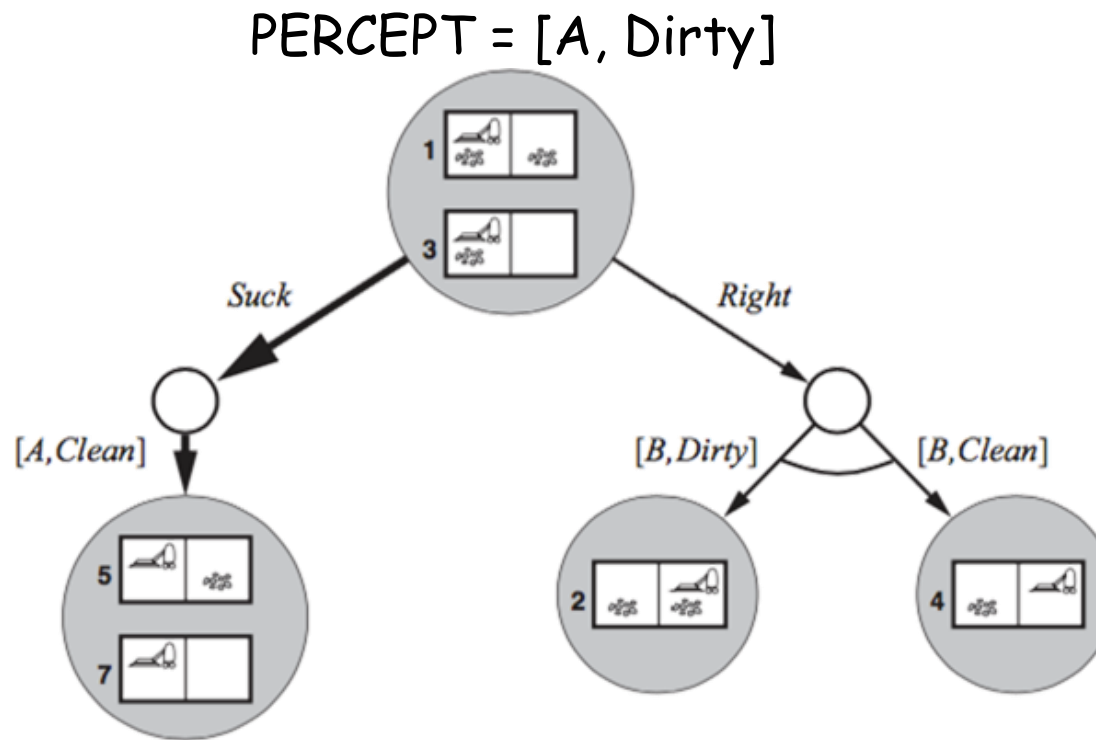
- How is the belief state updated after a perception?

$$b_o = \text{UPDATE}(b', o) = \{s : o = \text{PERCEPT}(s) \text{ and } s \in b'\}$$

$$\text{RESULTS}(b, a) = \{b_o : b_o = \text{UPDATE}(\text{PREDICT}(b, a), o) \text{ and } o \in \text{POSSIBLE-PERCEPTS}(\text{PREDICT}(b, a))\}$$

# Local sensing vacuum world

- AND-OR search tree on **belief states**



[Suck, Right, if Bstate={6} then Suck else []]

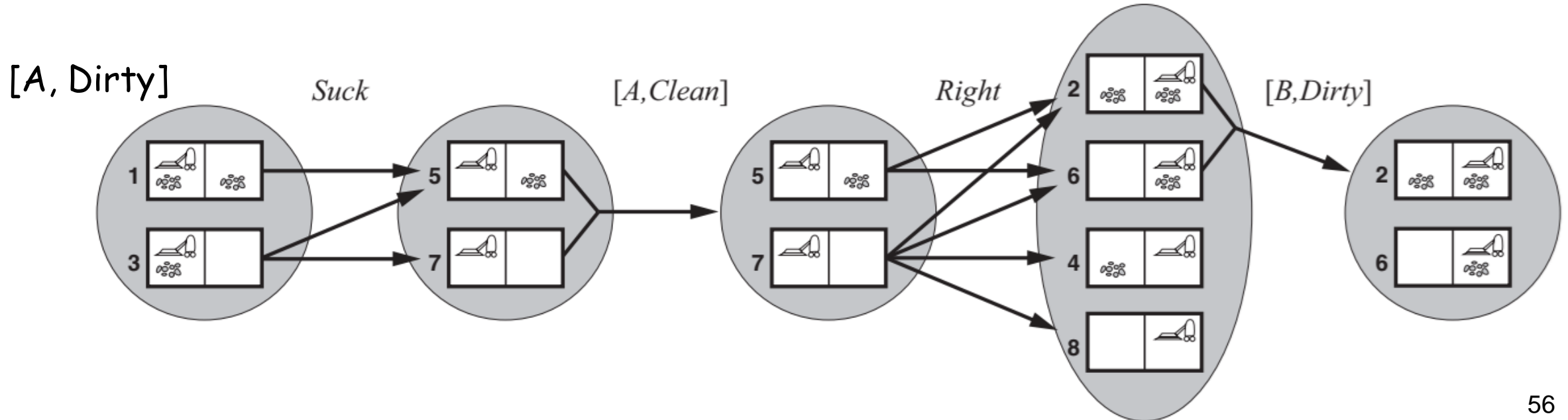
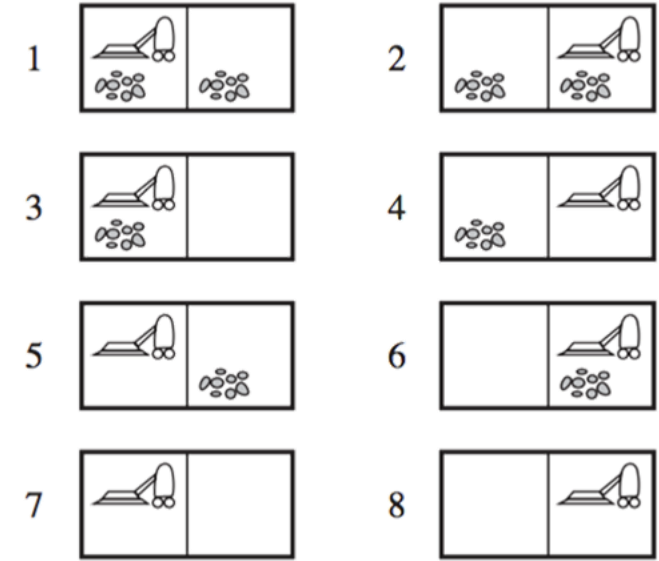
# Solving partially observable problems

- AND-OR graph search
- Execute the obtained contingency plan
  - Based on the achieved perception either then-part or else-part of a condition is run
  - Agent's belief state is updated when performing actions and receiving percepts
    - Maintaining the belief state is a core function of any intelligent system

$$b' = \text{UPDATE}(\text{PREDICT}(b, a), o)$$

# Vacuum world example

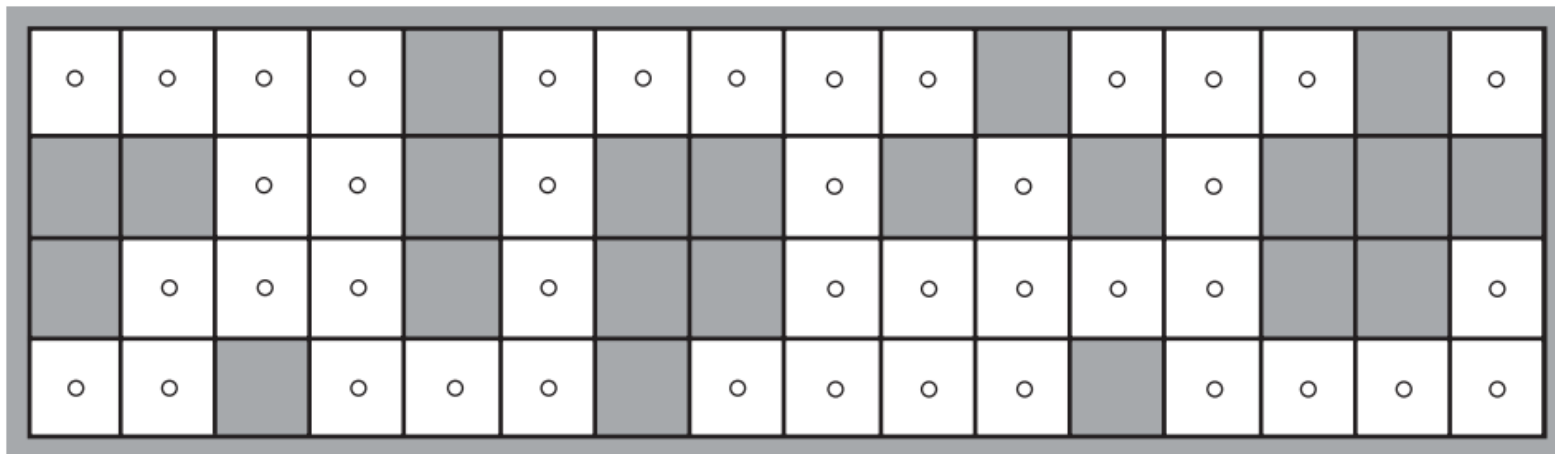
- Local sensing
- Any square may be dirty at any time (unless the agent is now cleaning it)





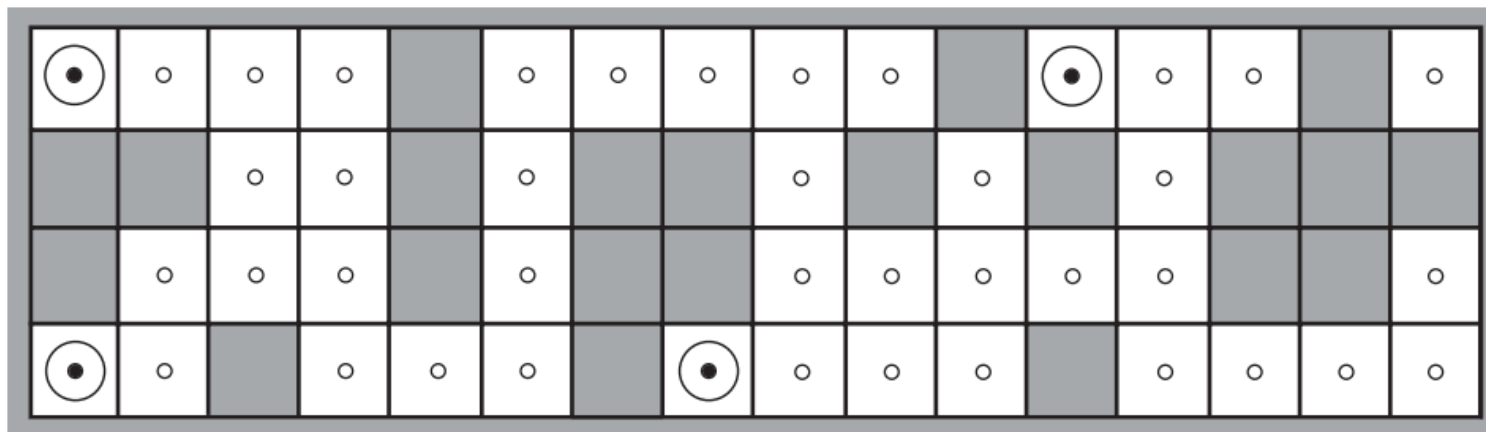
# Robot localization example

- **Determining current location** given a map of the world and a sequence of percepts and actions
  - Perception
    - one sonar sensor in each direction (telling obstacle existence)
      - E.g., percepts=NW means there are obstacles to the north and west
  - Broken navigational system
    - Move action randomly chooses among {Right, Left, Up, Down}

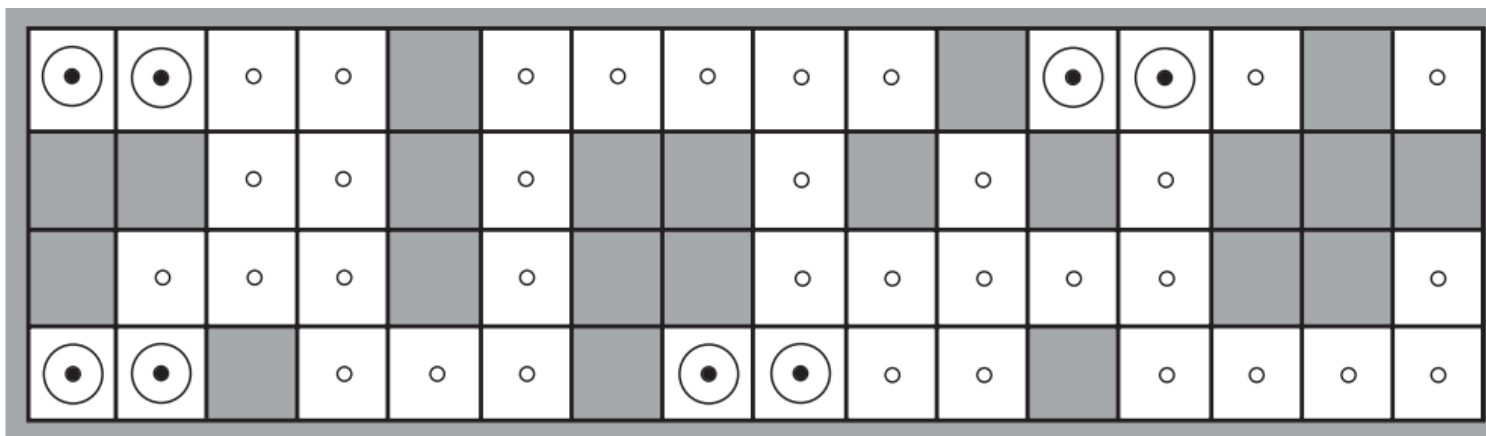


# Robot localization example

- $b^0$  : 0 squares
- Percept: NSW
- $b^1 = \text{UPDATE}(b^0, \text{NSW})$

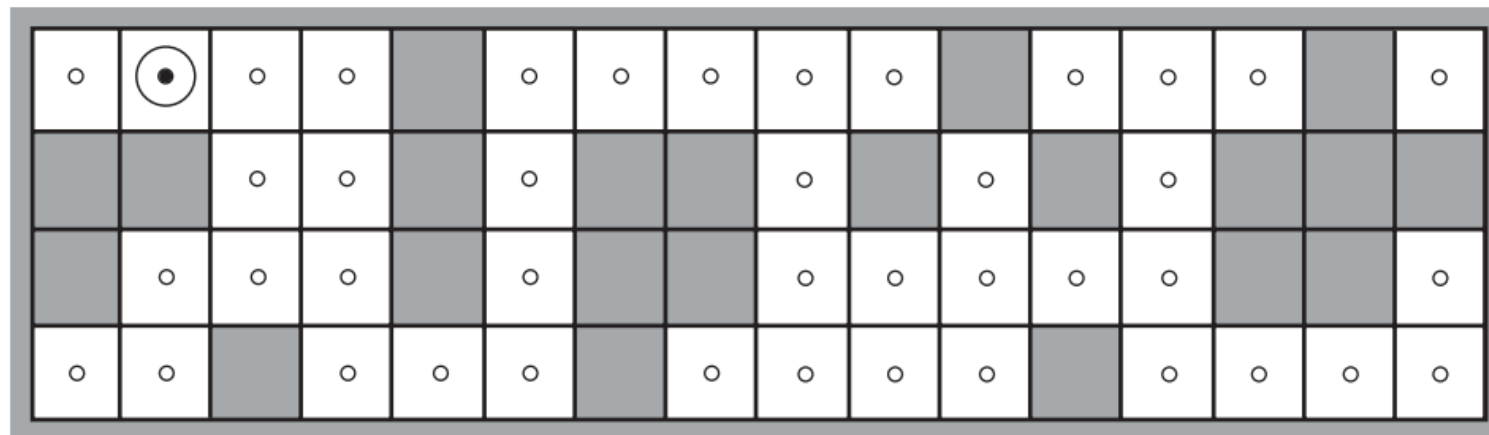


- Execute action  $a = \text{Move}$
- $b^1_a = \text{PREDICT}(b^1, a)$



# Robot localization example

- Percept: NS
- $b^2 = \text{UPDATE}(b^1_a, \text{NS})$



- This is the only location that could be the result of

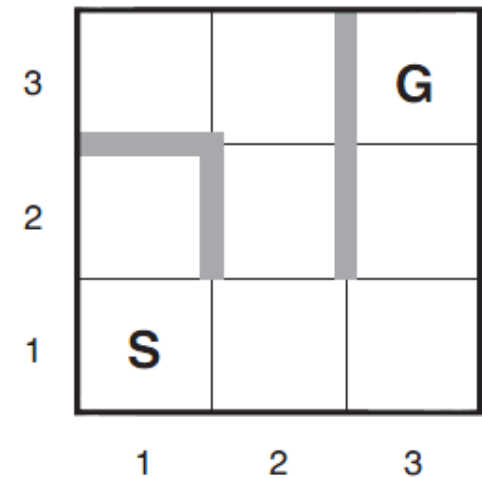
$\text{UPDATE}(\text{PREDICT}(\text{UPDATE}(b, \text{NSW}), \text{Move}), \text{NS})$

# Online search

- Off-line Search
  - Solution is found before the agent starts acting in the real world
- On-line search
  - Interleaves search and acting
  - Necessary in unknown environments
  - Useful in dynamic and semi-dynamic environments
  - Saves computational resource in non-deterministic domains (focusing only on the contingencies arising during execution)
    - Tradeoff between finding a guaranteed plan (to not get stuck in an undesirable state during execution) and required time for complete planning ahead
- Examples
  - Robot in a new environment must explore to produce a map
  - Autonomous vehicles

# Online search problems

- We assume deterministic & fully observable environment
  - we assume the agent knows
    - $ACTIONS(s)$
    - $c(s, a, s')$  (can be used after knowing  $s'$  as the outcome)
    - $GOAL\_TEST(s)$
- Agent must perform an action to determine its outcome
  - $RESULTS(s, a)$  is found by actually being in  $s$  and doing  $a$
  - By filling RESULTS map table, the map of the environment is found
- Agent may access to a heuristic function

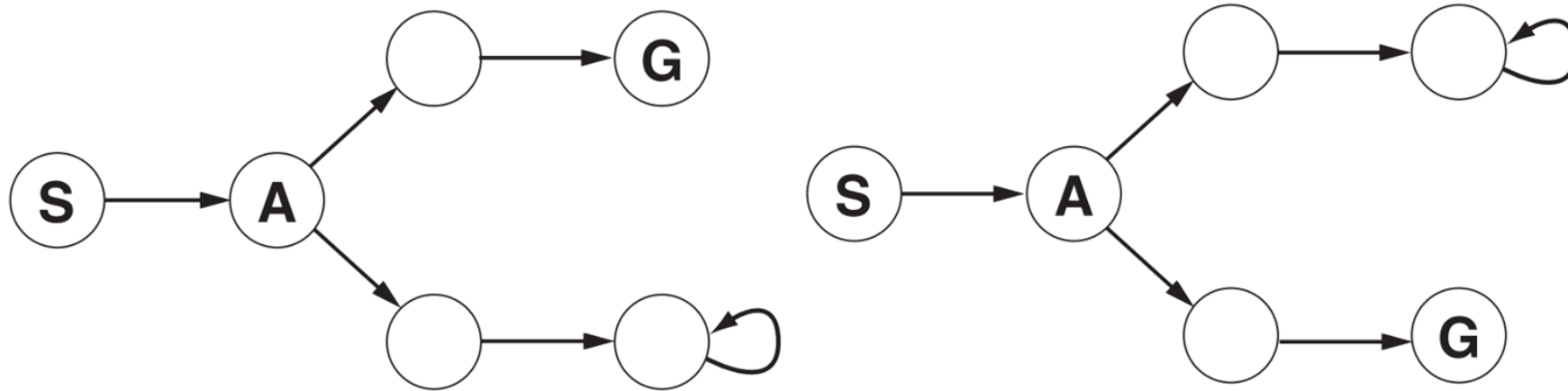


# Competitive ratio

- Typically, the agent's objective is to reach a goal state while minimizing cost
  - Online path cost
    - Total cost of the path that the agent actually travels
  - Best cost
    - Cost of the shortest path "if it knew the search space in advance"
- $\text{Competitive ratio} = \text{Online path cost} / \text{Best path cost}$ 
  - Smaller values are more desirable
- Competitive ratio may be infinite
  - Dead-end state: no goal state is reachable from it
    - irreversible actions can lead to a dead-end state

# Infinite Competitive ratio (Dead-end)

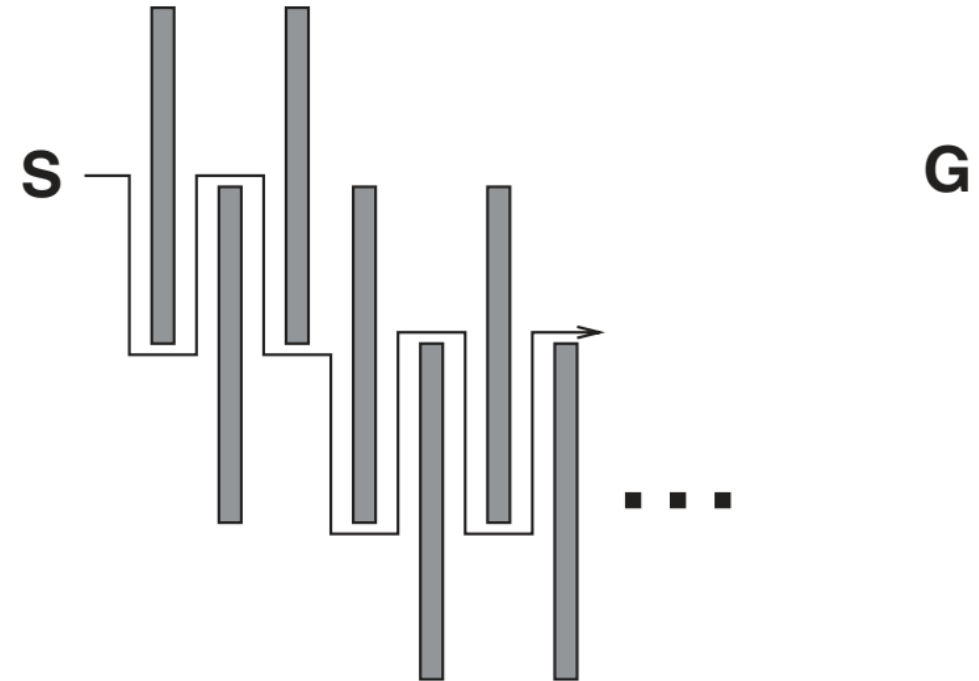
- No algorithm can avoid dead-ends in all state spaces



- Simplifying assumption: **Safely explorable** state space
  - A goal state is achievable from every reachable state
    - Example: State spaces with reversible actions

# Infinite Competitive ratio (Unbounded cost)

- A two-dimensional environment that can cause an online search agent to follow an arbitrarily inefficient route to the goal.
  - Whichever choice the agent makes, the adversary blocks that route with another long, thin wall, so that the path followed is much longer than the best possible path

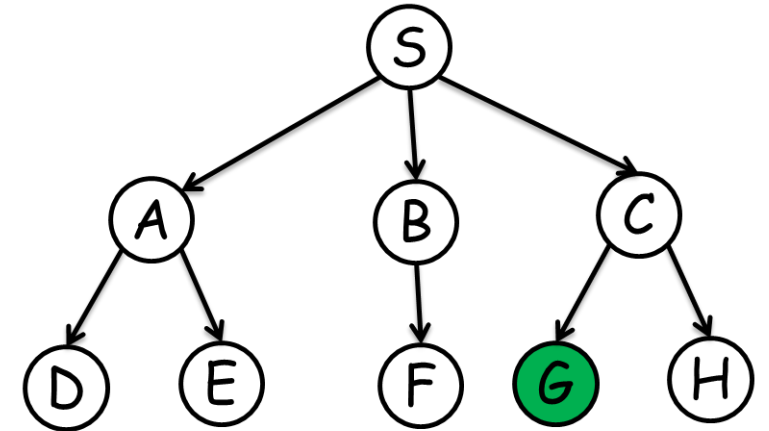




# Online search agents

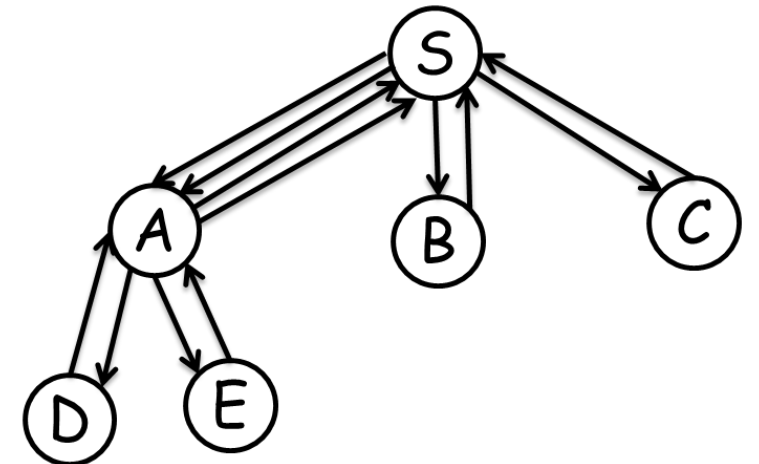
## ■ Offline search

- Node expansion involves **simulated rather than real actions**
- Can expand a node in one part of the space and then immediately expand a node in another part of the space



## ■ Online search

- Can discover successors only for a node that it physically occupies
- To avoid traveling all the way across the tree to expand the next node, it seems better to **expand nodes in a local order**

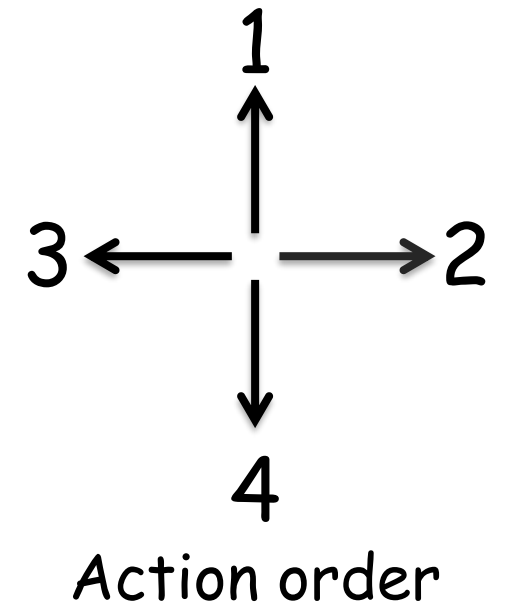
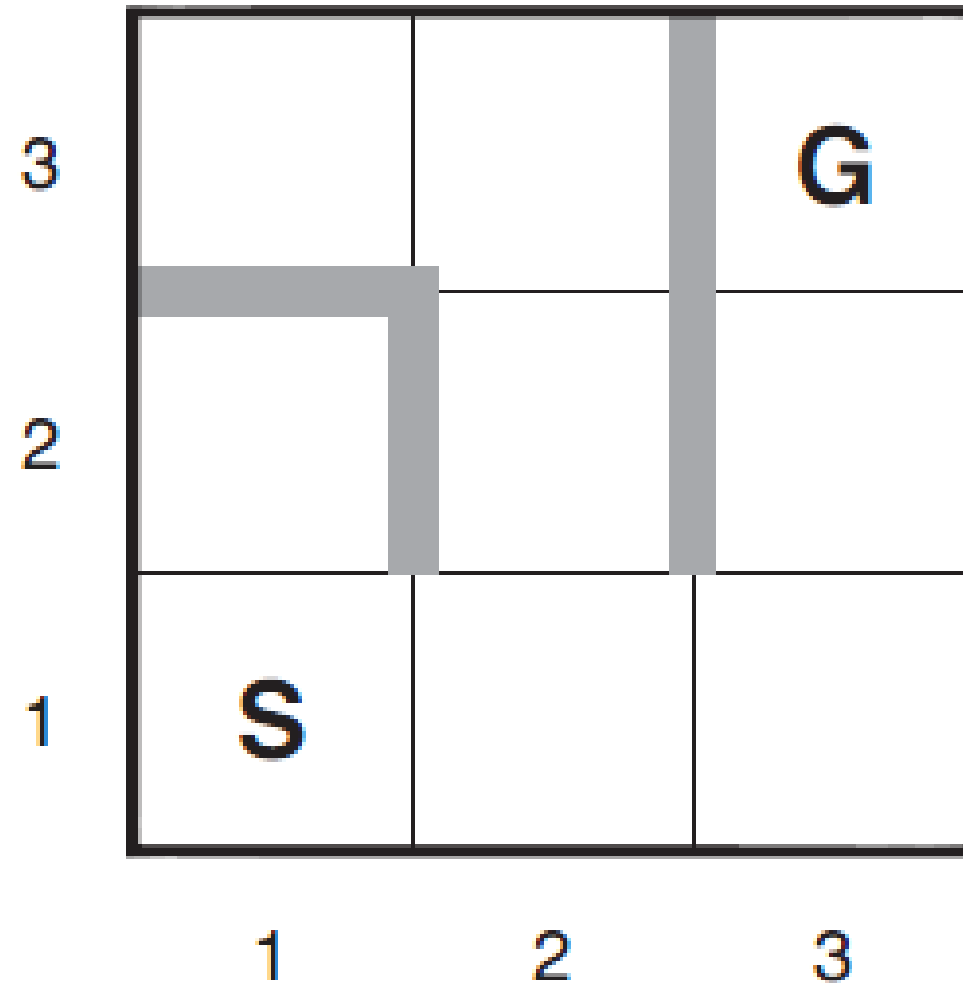


# Online search agents

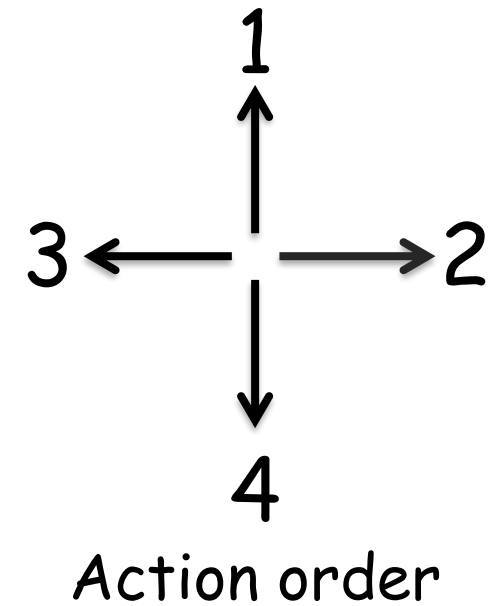
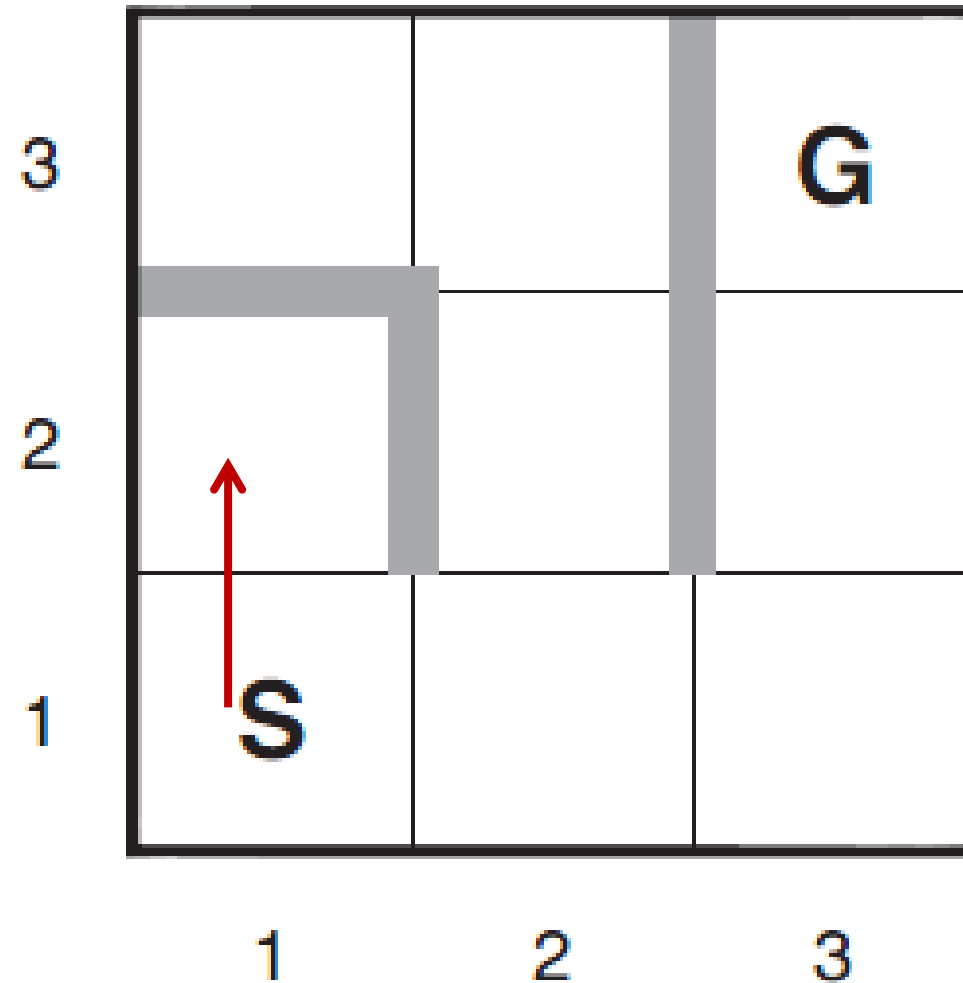
- Online DFS

- Whenever an action from the current state has not been explored, the agent tries that action.
- Physical backtrack
  - When the agent has tried all the actions in a state
  - Goes back to the state from which the agent most recently entered the current state
  - Works only for state spaces with reversible actions

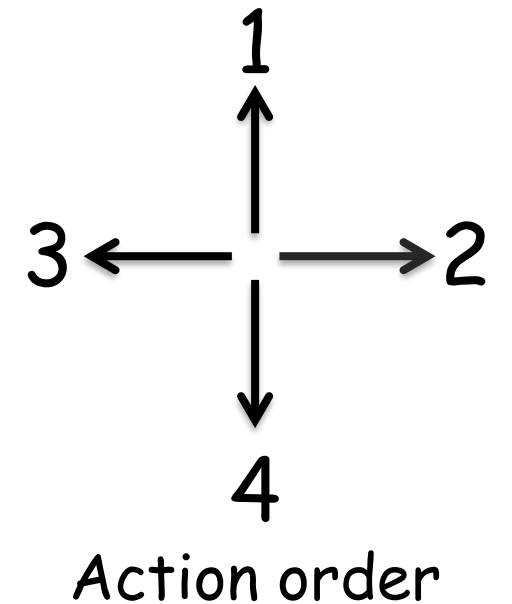
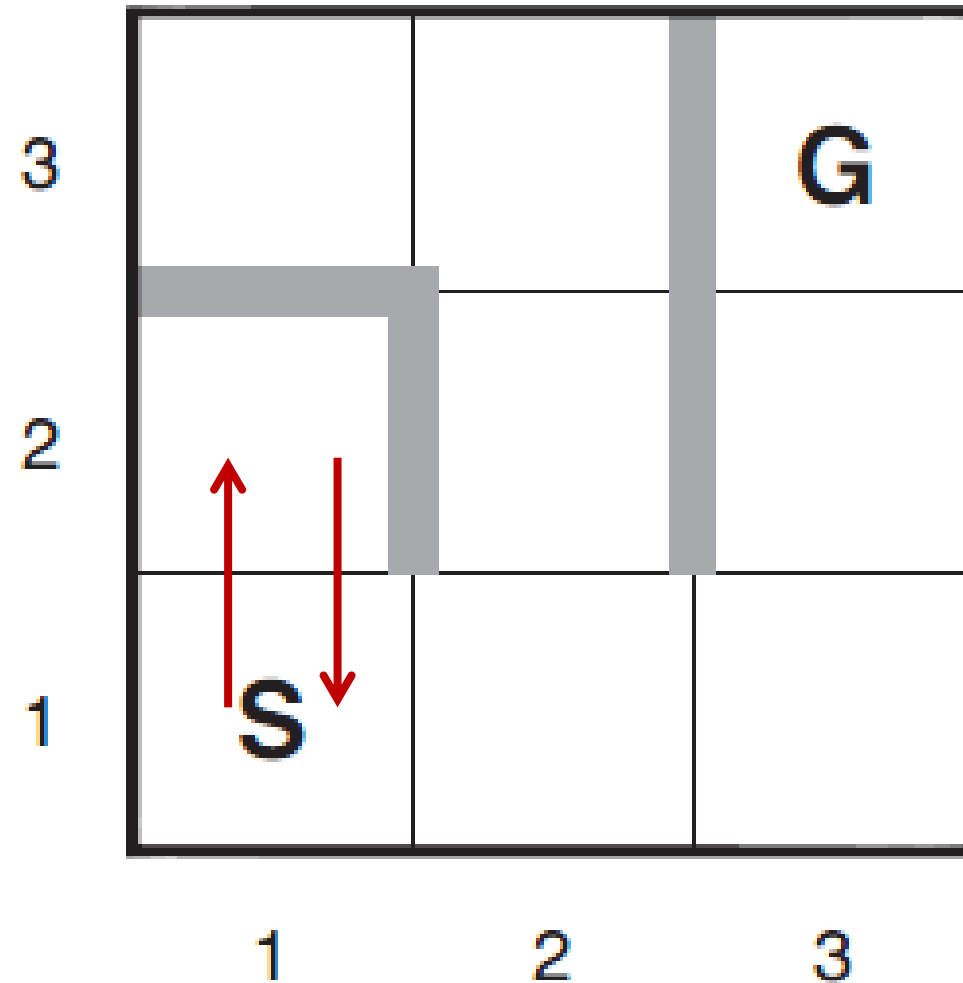
# Online DFS (Example)



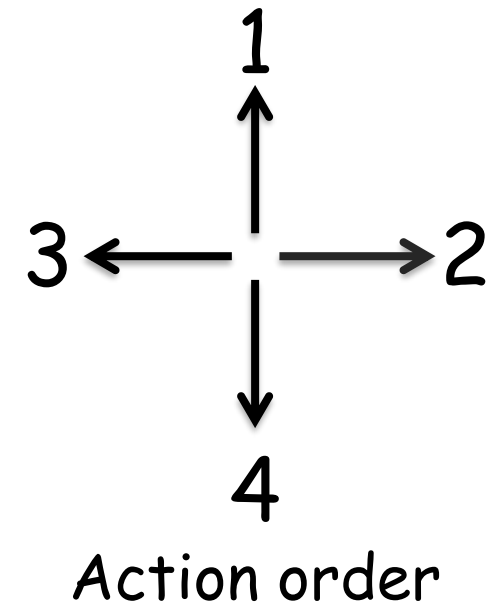
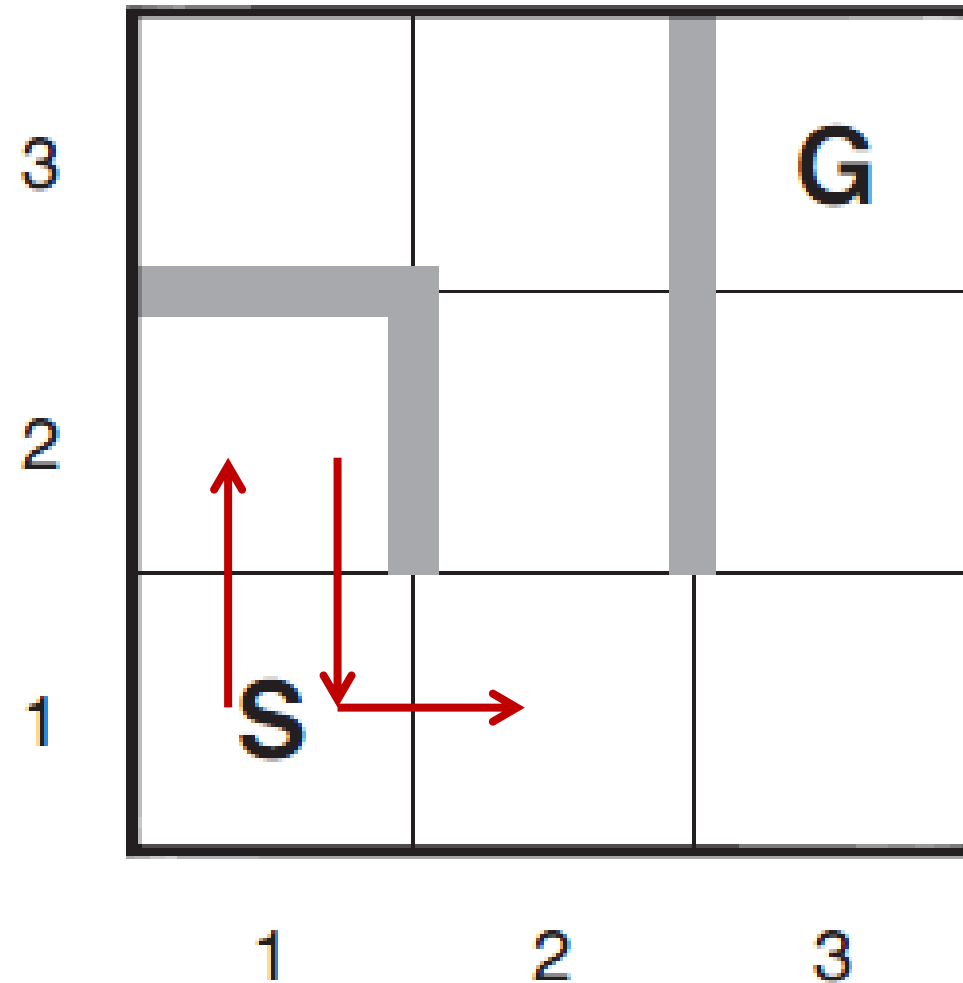
# Online DFS (Example)



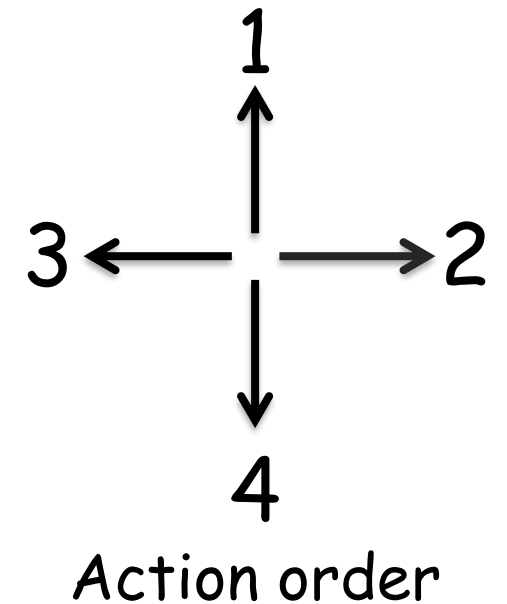
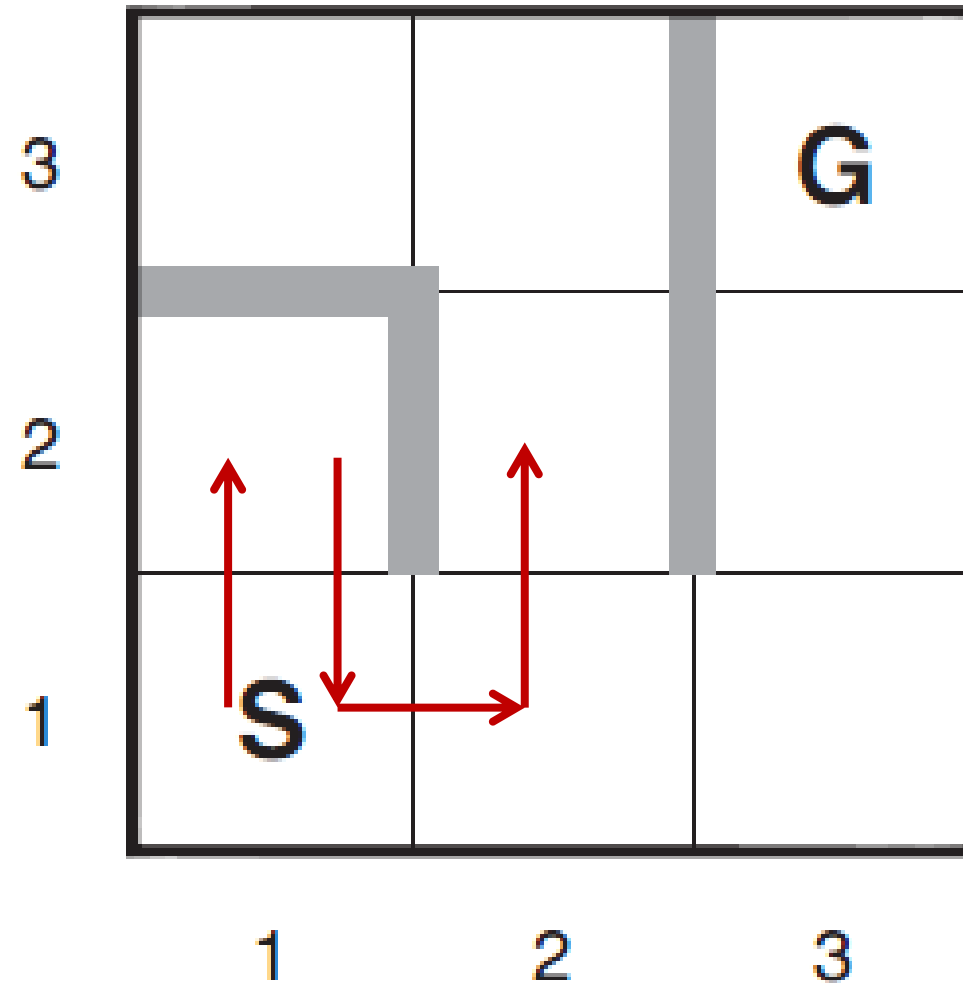
# Online DFS (Example)



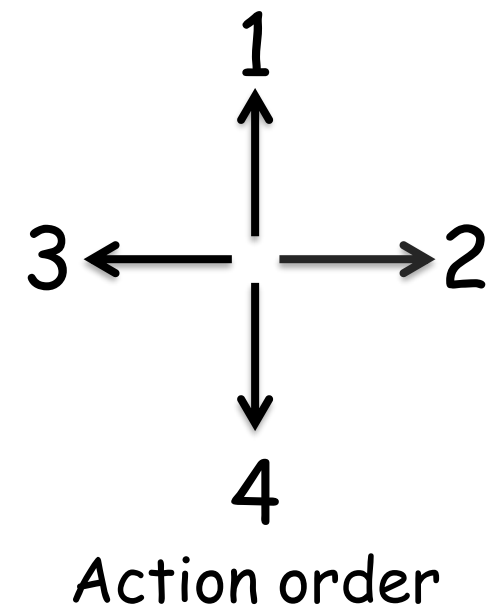
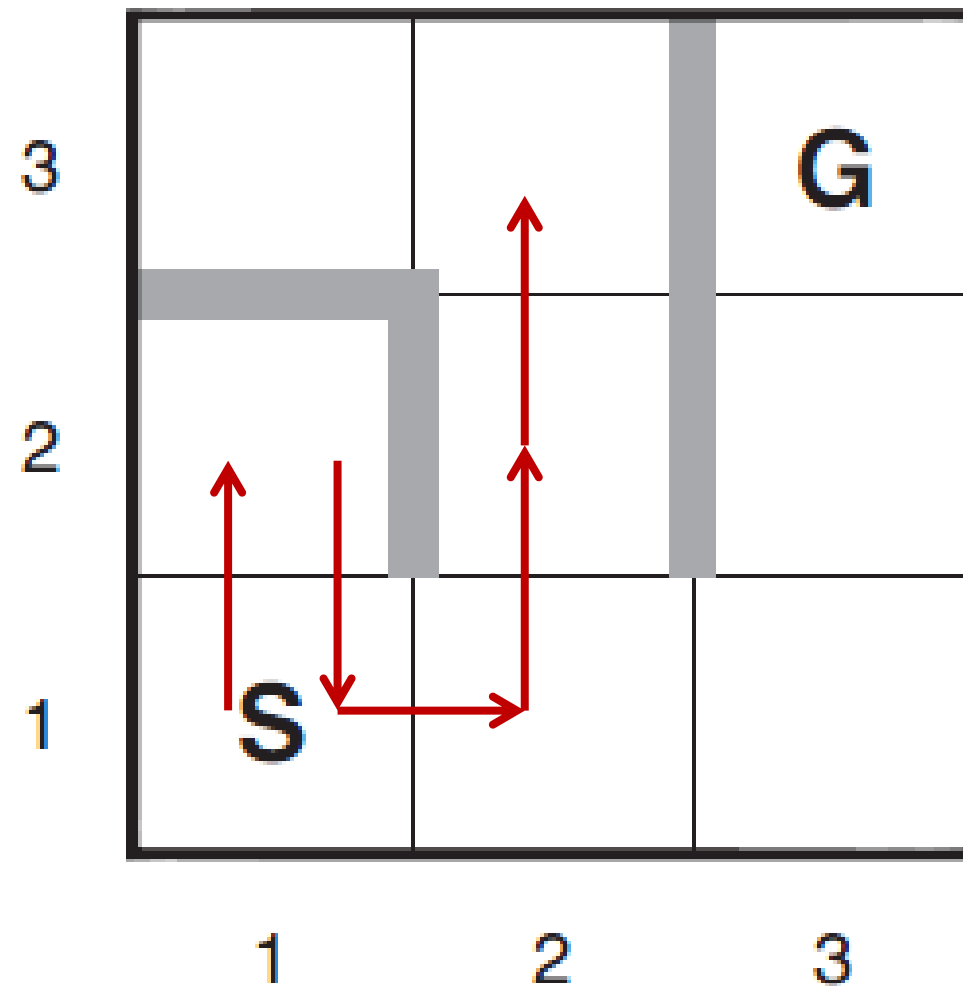
# Online DFS (Example)



# Online DFS (Example)

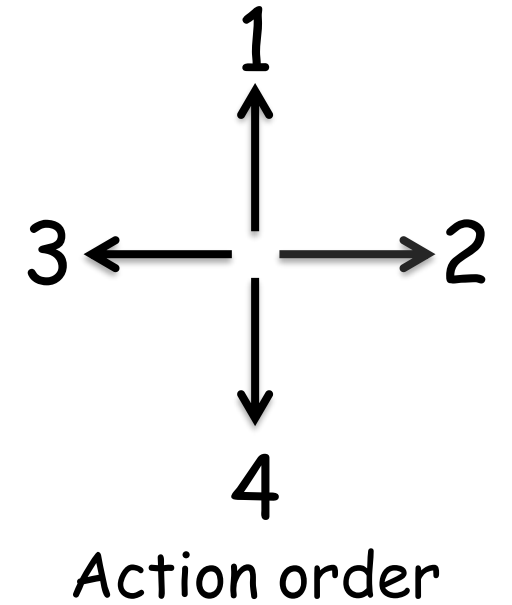
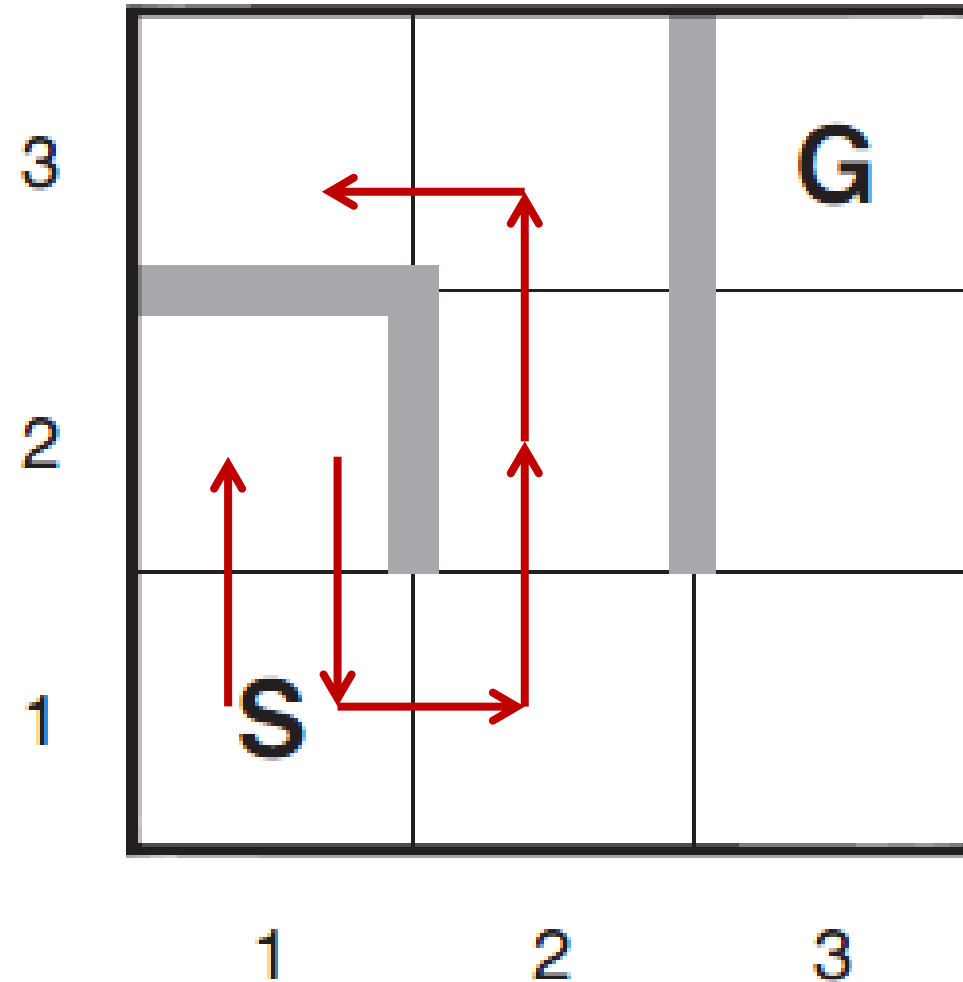


# Online DFS (Example)

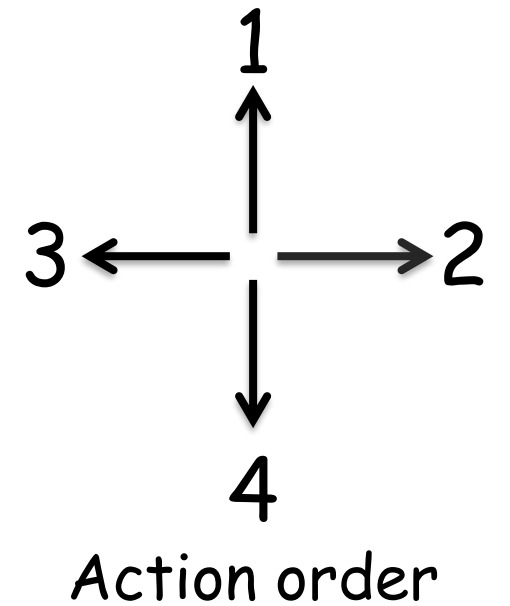
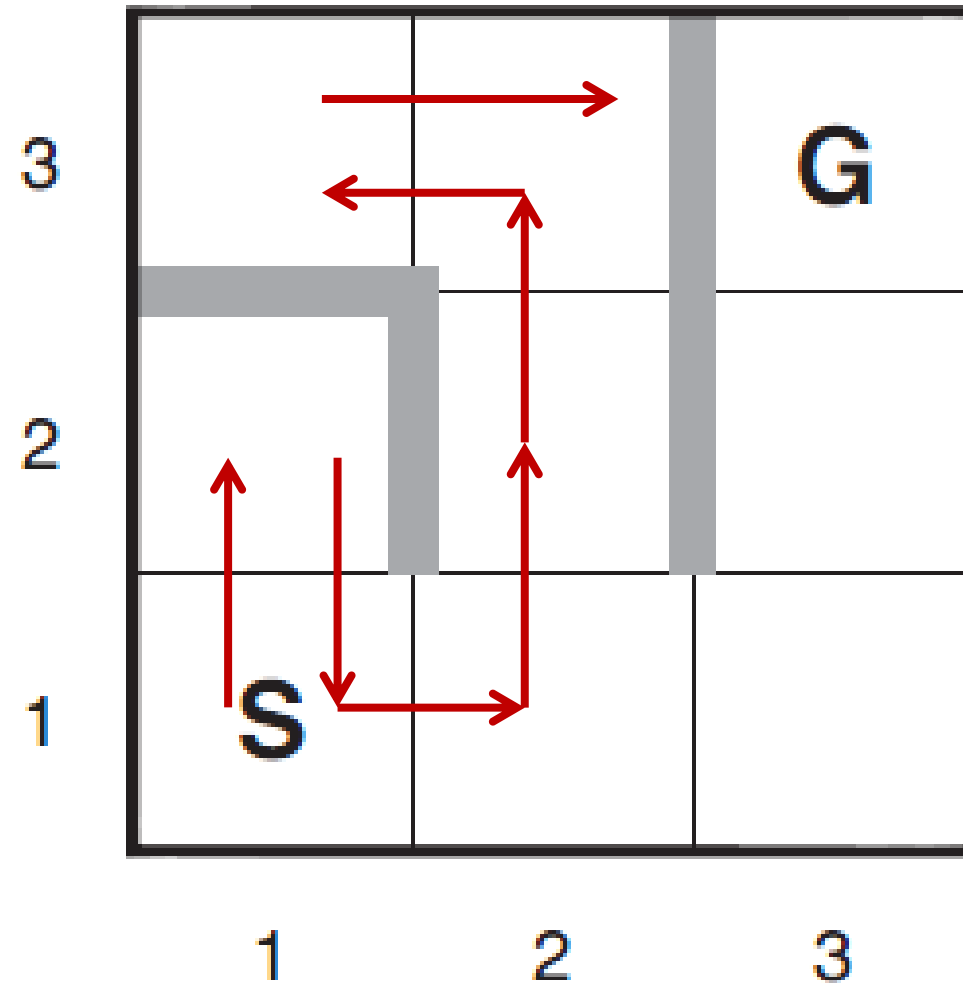




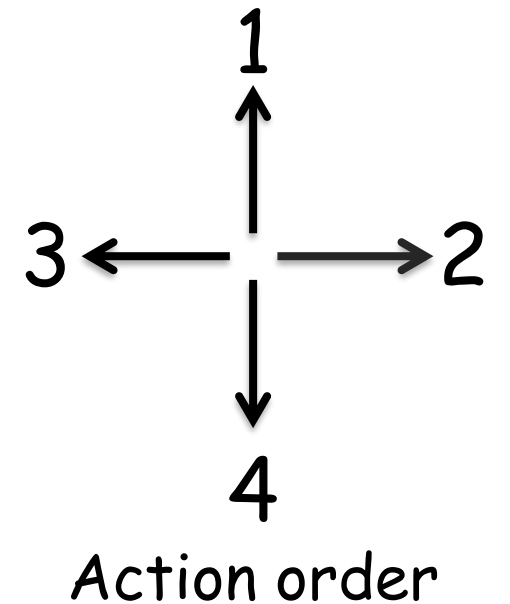
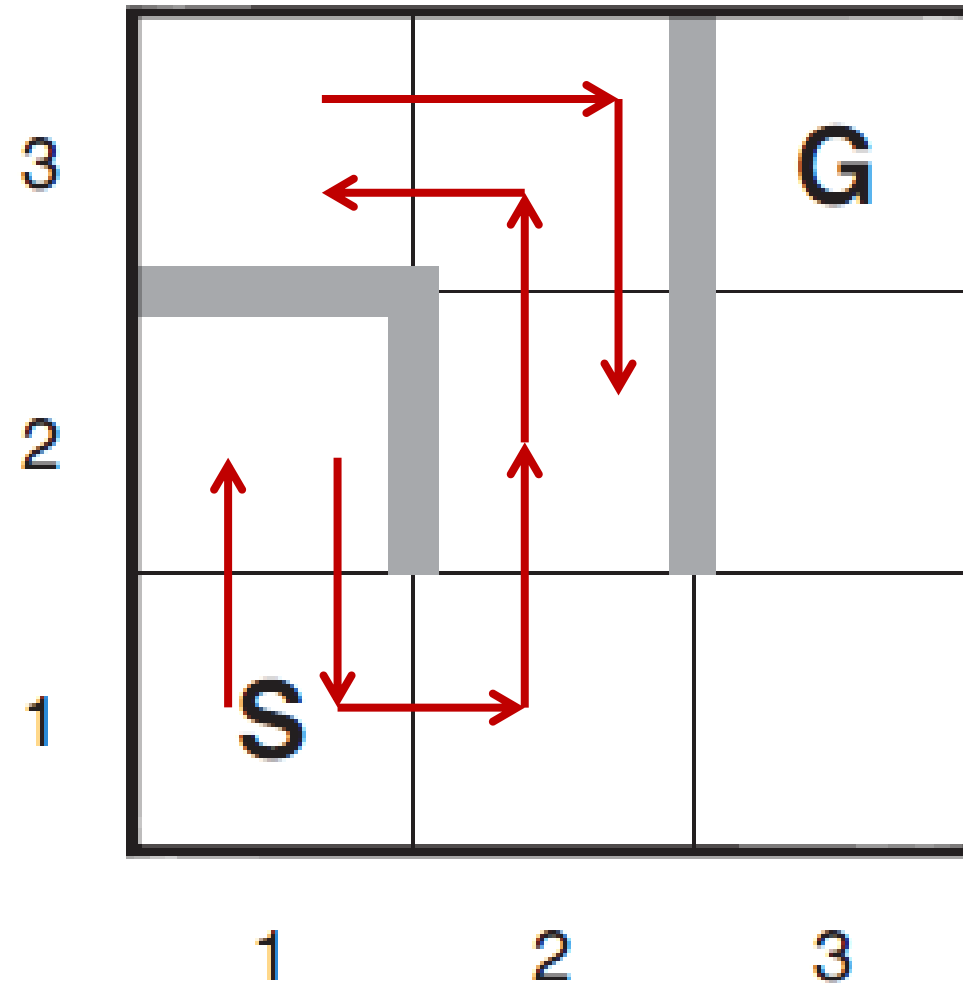
# Online DFS (Example)



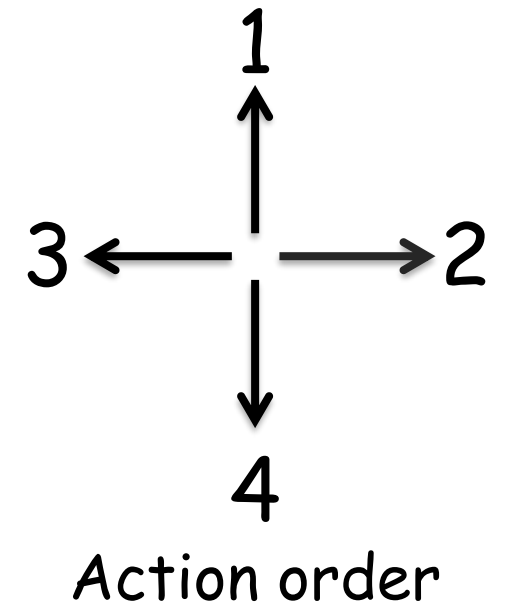
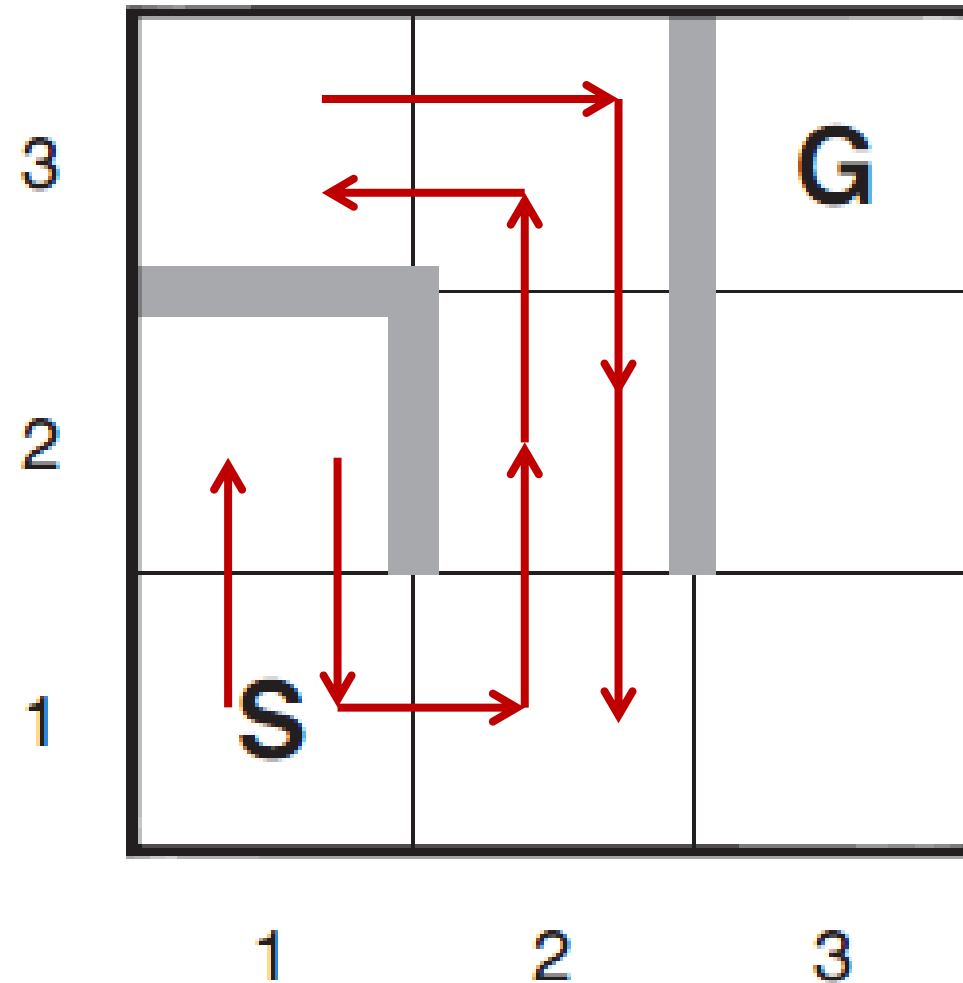
# Online DFS (Example)



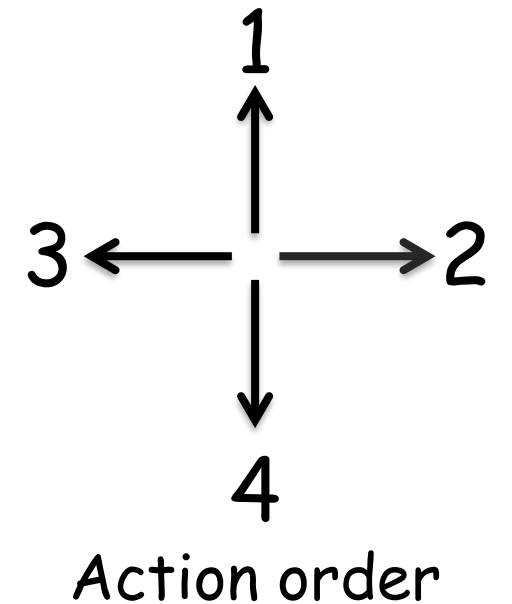
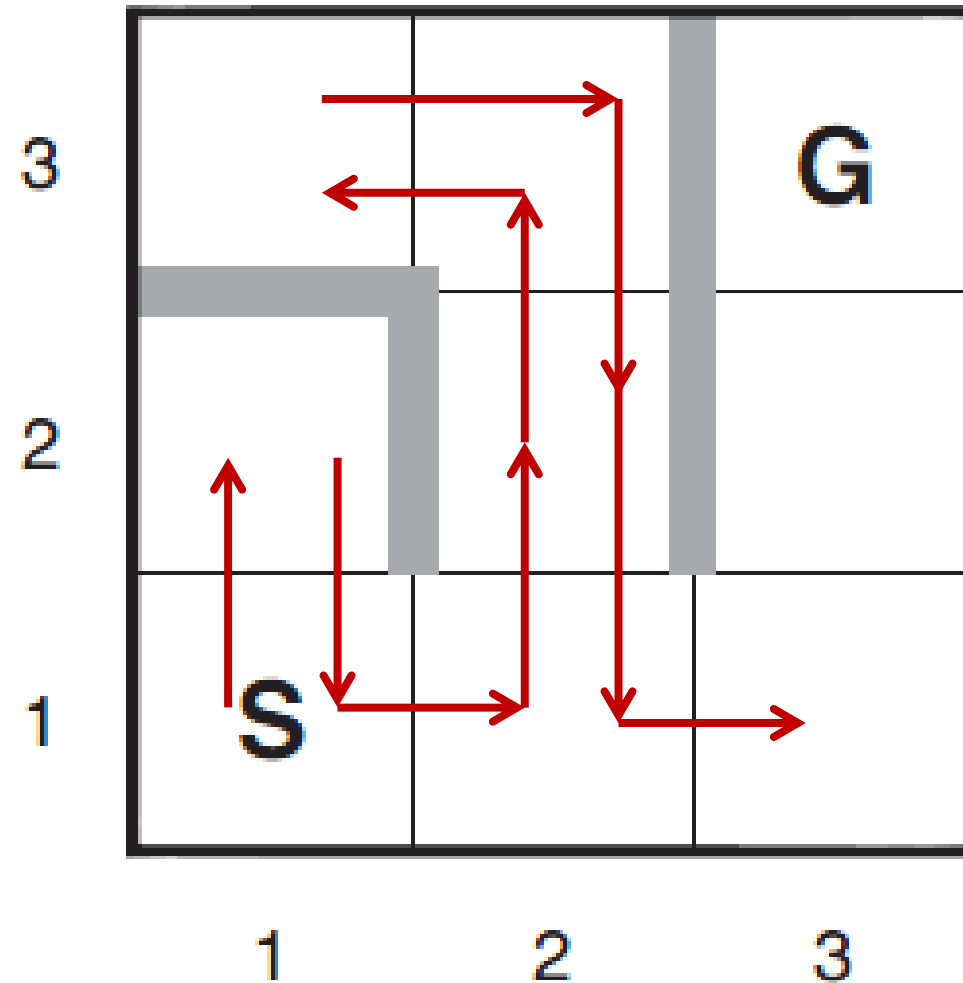
# Online DFS (Example)



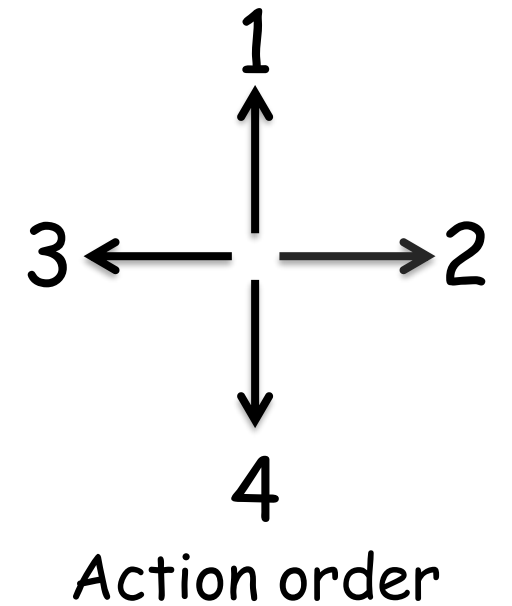
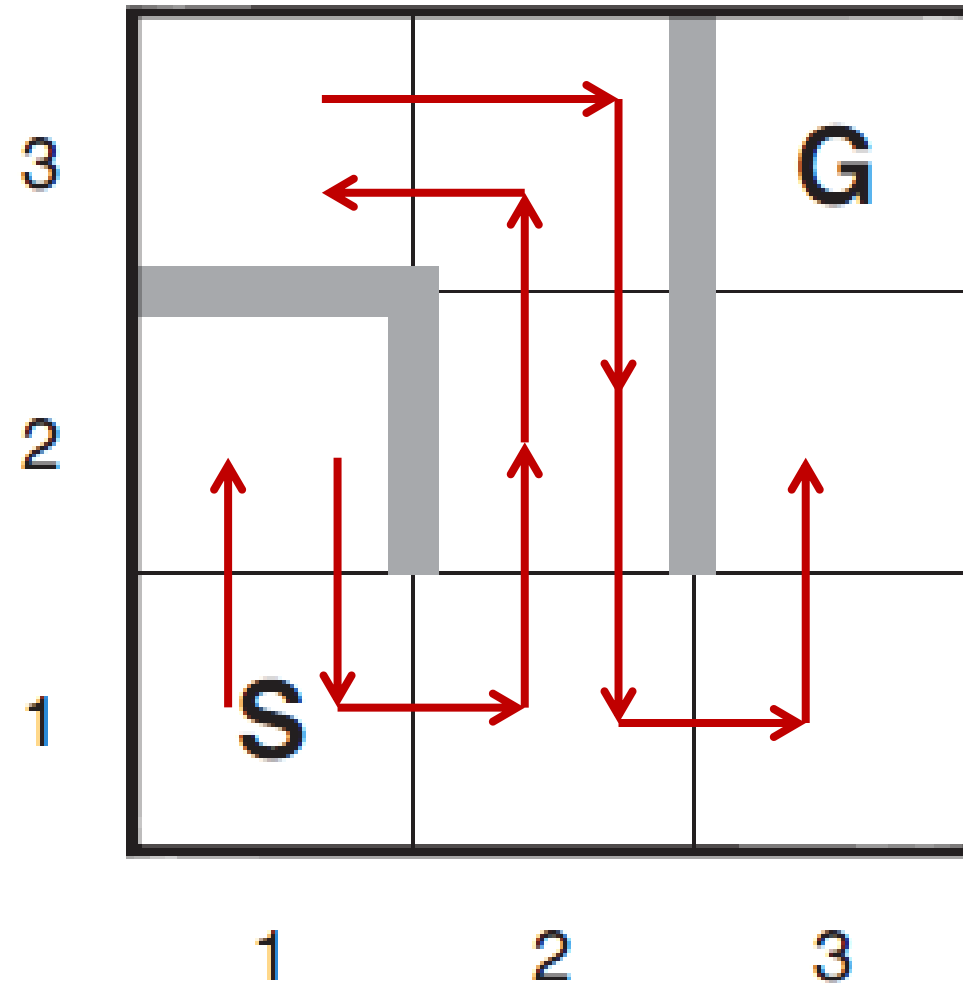
# Online DFS (Example)



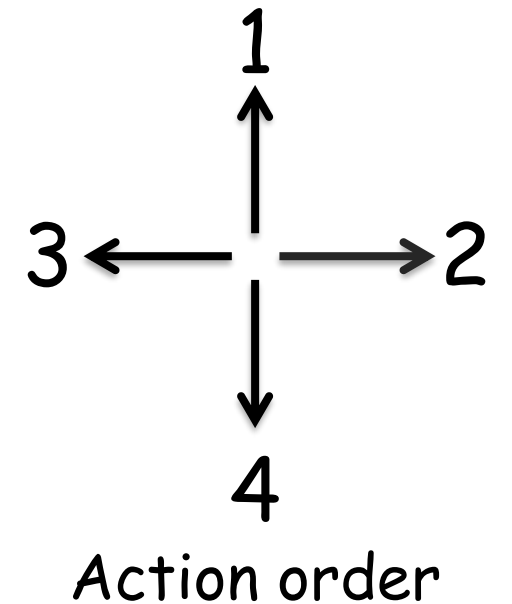
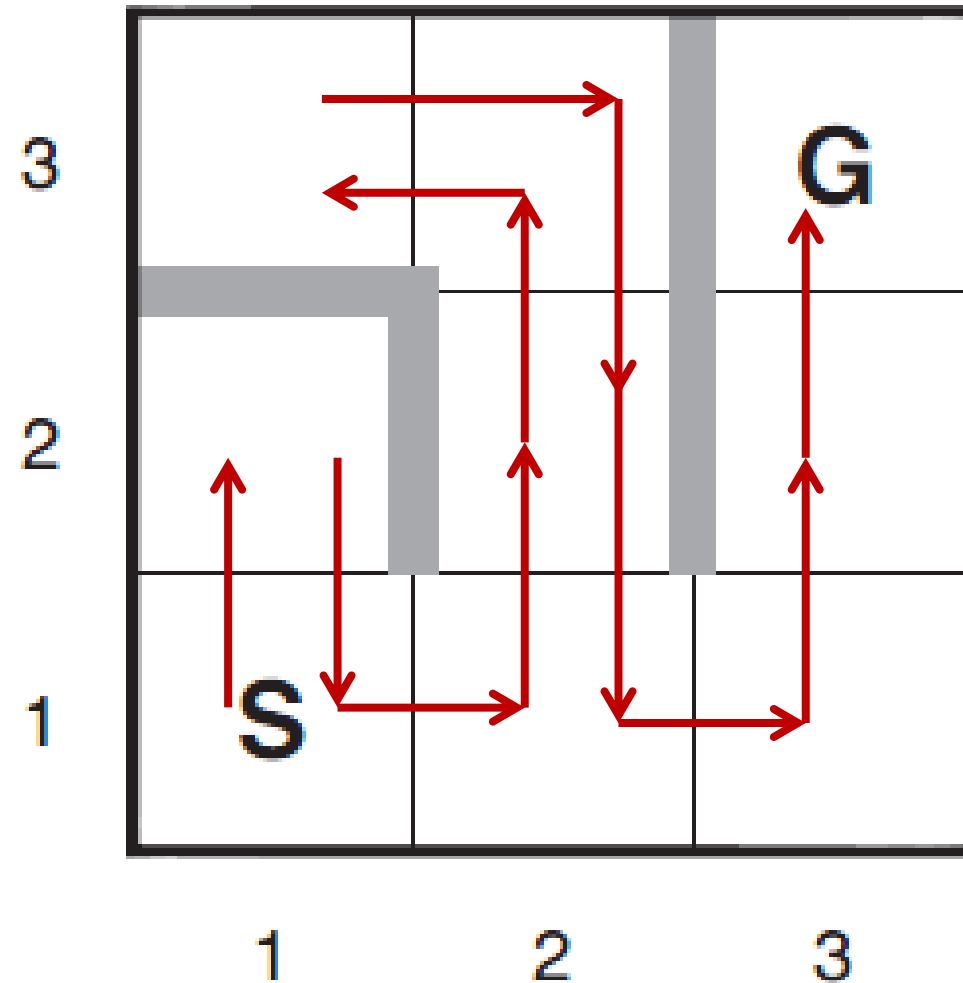
# Online DFS (Example)



# Online DFS (Example)



# Online DFS (Example)

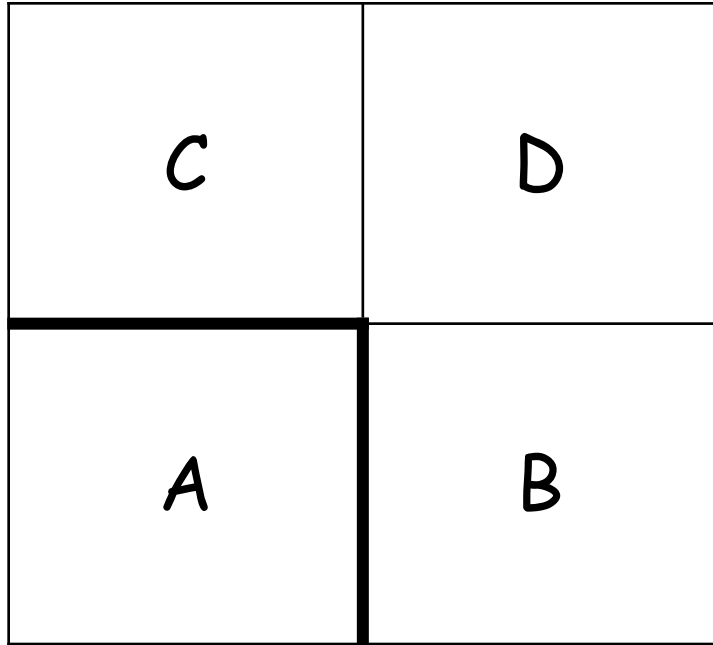


# An online search agent that uses depth-first exploration

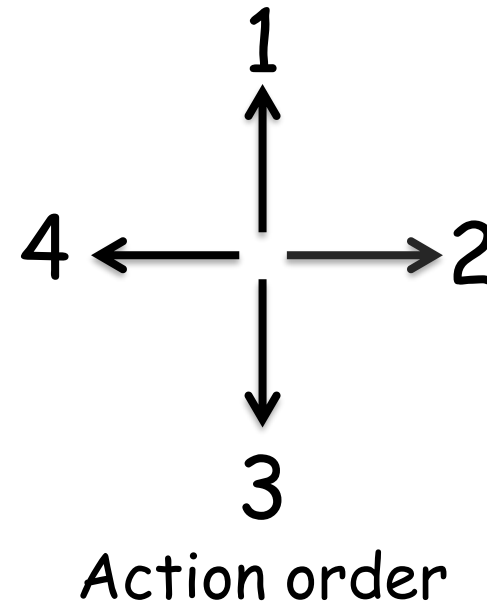
```
function ONLINE-DFS-AGENT(s') returns an action
  inputs: s', a percept that identifies the current state
  persistent: result, a table indexed by state and action, initially empty
               untried, a table that lists, for each state, the actions not yet tried
               unbacktracked, a table that lists, for each state, the backtracks not yet tried
               s, a, the previous state and action, initially null
  if GOAL-TEST(s') then return stop
  if s' is a new state (not in untried) then untried[s'] ← ACTIONS(s')
  if s is not null then
    result[s, a] ← s'
    add s to the front of unbacktracked[s']
  if untried[s'] is empty then
    if unbacktracked[s'] is empty then return stop
    else a ← an action b such that result[s', b] = POP(unbacktracked[s'])
  else a ← POP(untried[s'])
  s ← s'
  return a
```



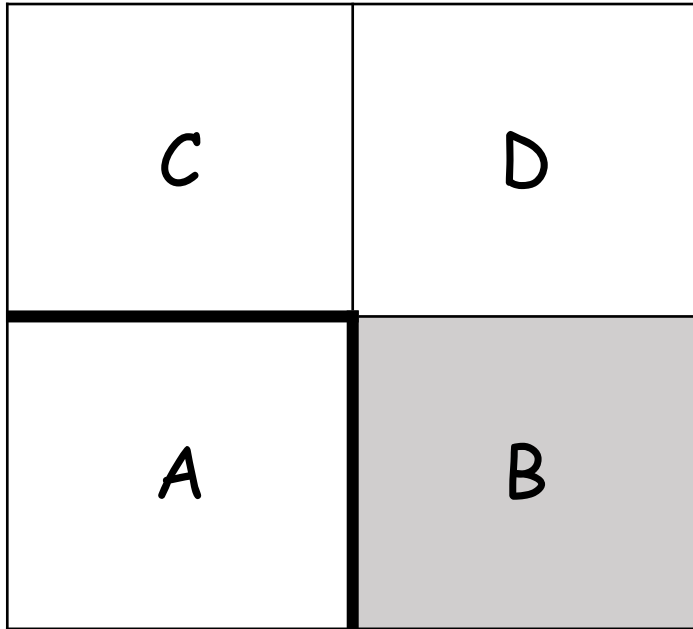
# Online DFS (Example)



Start: B  
Goal: C



# Online DFS (Example)



## Step 1

$s' = B$

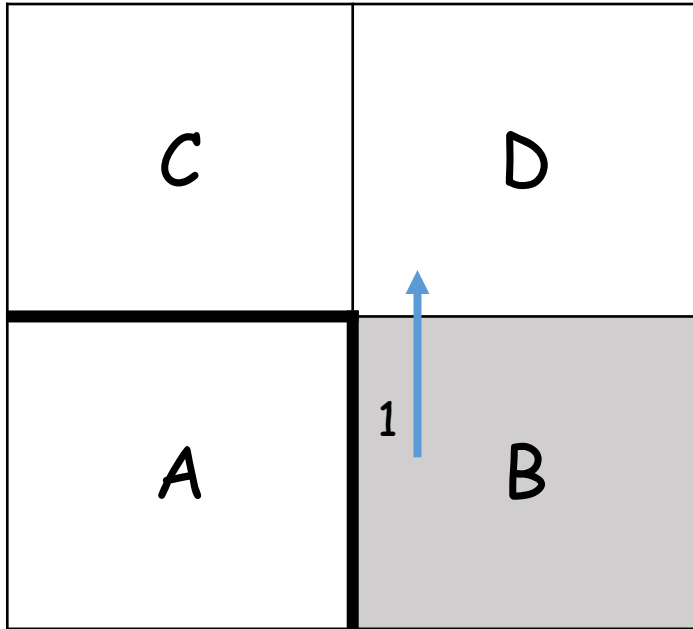
$s = \text{null}$

$\text{untried}[B] = \{u, r, d, l\}$

$a = u \rightarrow \text{untried}[B] = \{r, d, l\}$

$s = B$

# Online DFS (Example)



Step 1

$s' = B$

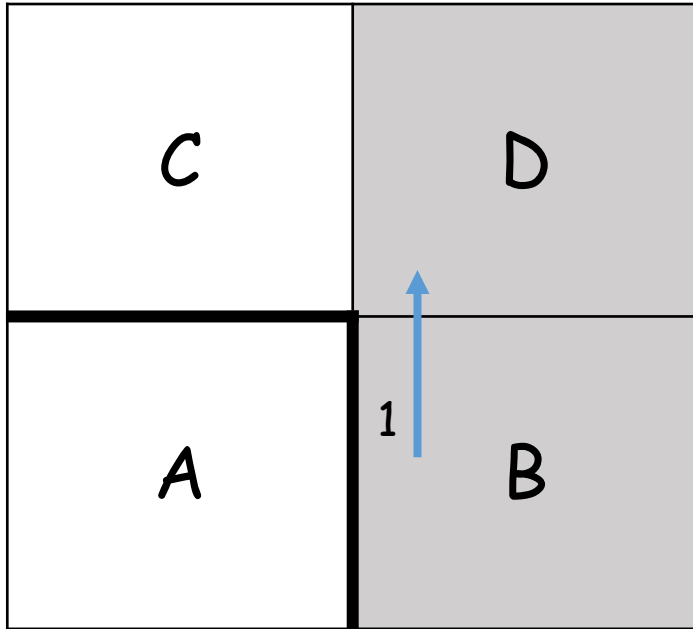
$s = \text{null}$

$\text{untried}[B] = \{u, r, d, l\}$

$a = u \rightarrow \text{untried}[B] = \{r, d, l\}$

$s = B$

# Online DFS (Example)



## Step 2

$s' = D$

$s = B$

$\text{untried}[D] = \{u, r, d, l\}$

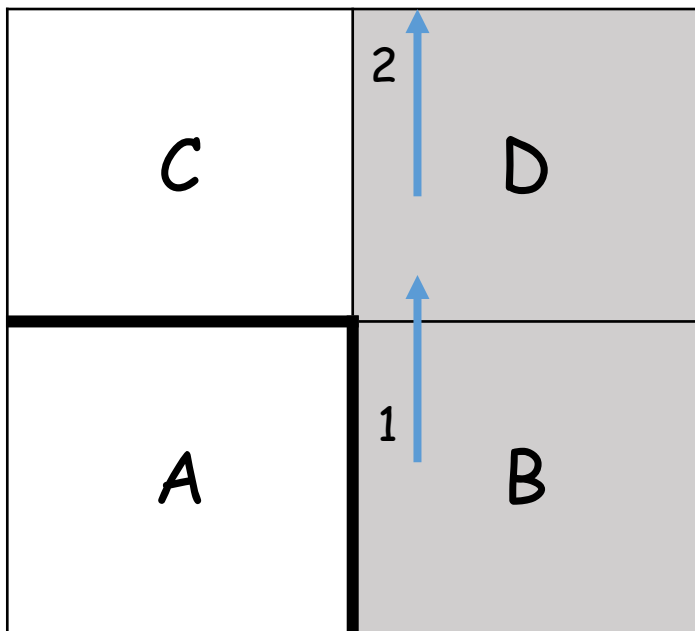
$\text{result}[B, u] = D$

$\text{unbacktracked}[D] = \{B\}$

$a = u \rightarrow \text{untried}[D] = \{r, d, l\}$

$s = D$

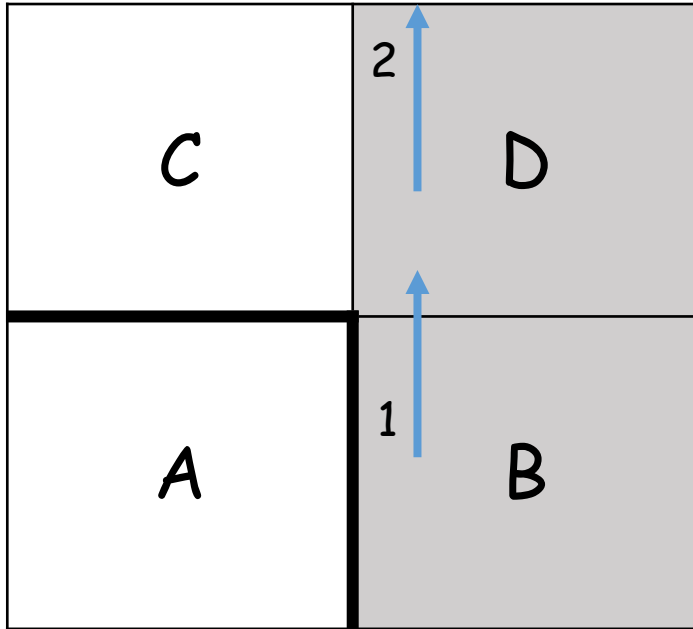
# Online DFS (Example)



## Step 2

$s' = D$                        $s = B$   
 $\text{untried}[D] = \{u, r, d, l\}$   
 $\text{result}[B, u] = D$   
 $\text{unbacktracked}[D] = \{B\}$   
 $a = u \rightarrow \text{untried}[D] = \{r, d, l\}$   
 $s = D$

# Online DFS (Example)



## Step 3

$s' = D$

$s = D$

$\text{untried}[D] = \{r, d, l\}$

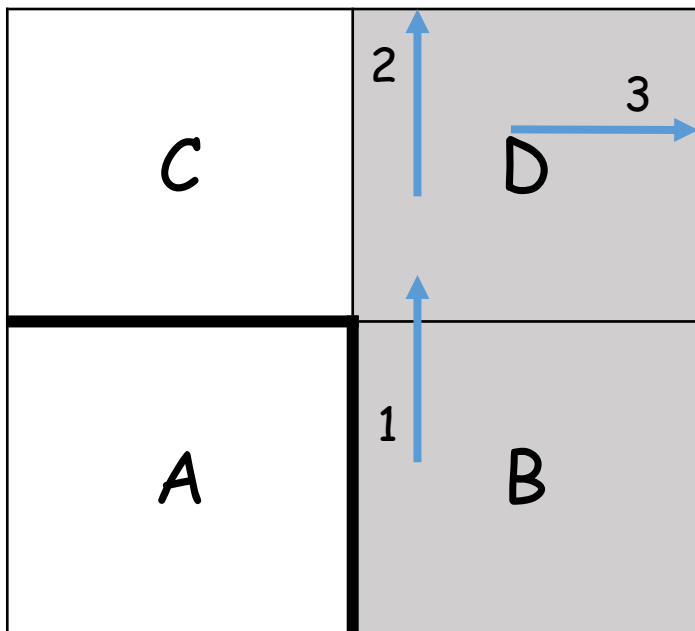
$\text{result}[D, u] = D$

$\text{unbacktracked}[D] = \{B\}$

$a = r \rightarrow \text{untried}[D] = \{d, l\}$

$s = D$

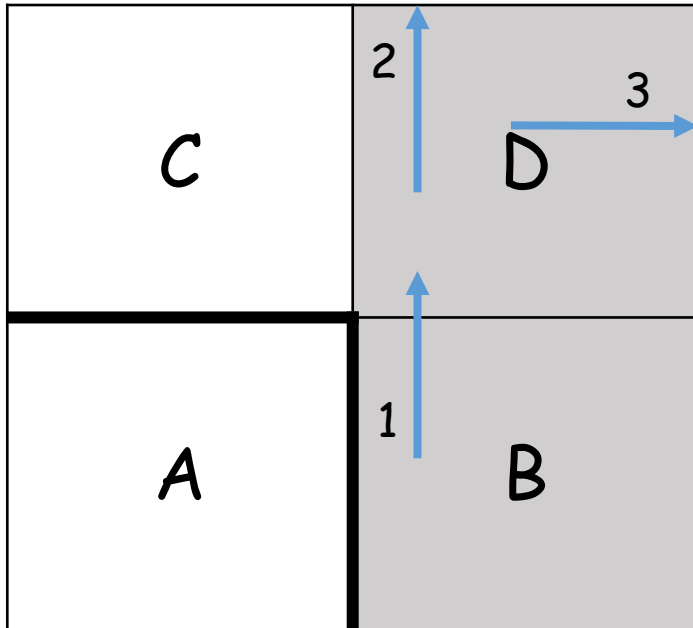
# Online DFS (Example)



## Step 3

$s' = D$                        $s = D$   
 $\text{untried}[D] = \{r, d, l\}$   
 $\text{result}[D, u] = D$   
 $\text{unbacktracked}[D] = \{B\}$   
 $a = r \rightarrow \text{untried}[D] = \{d, l\}$   
 $s = D$

# Online DFS (Example)

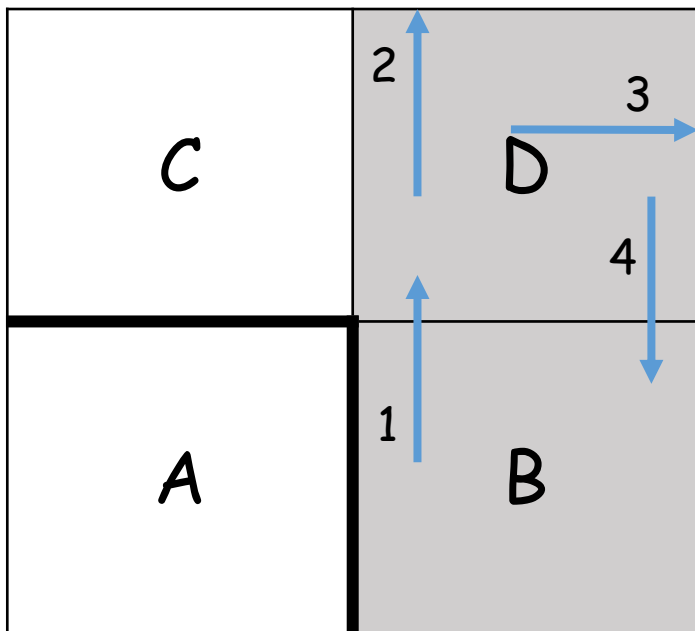


## Step 4

$s' = D$                        $s = D$   
 $\text{untried}[D] = \{d, l\}$   
 $\text{result}[D, r] = D$   
 $\text{unbacktracked}[D] = \{B\}$   
 $a = d \rightarrow \text{untried}[D] = \{l\}$   
 $s = D$



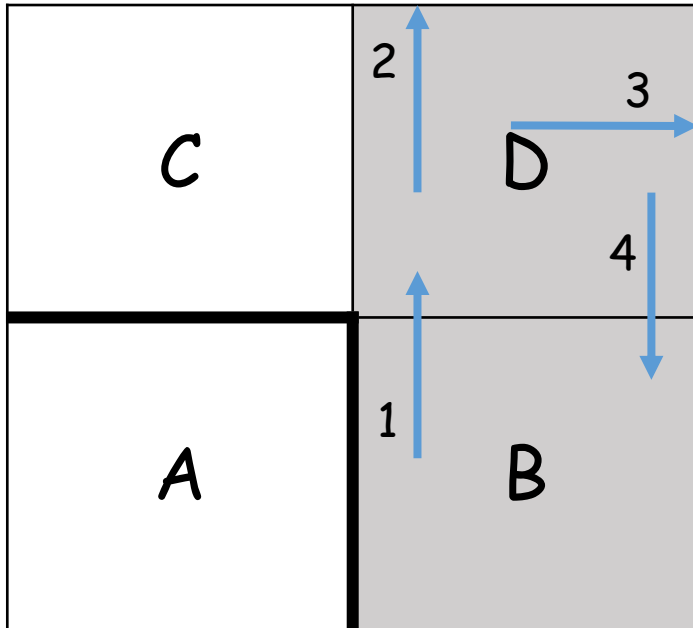
# Online DFS (Example)



## Step 4

$s' = D$                        $s = D$   
 $\text{untried}[D] = \{d, l\}$   
 $\text{result}[D, r] = D$   
 $\text{unbacktracked}[D] = \{B\}$   
 $a = d \rightarrow \text{untried}[D] = \{l\}$   
 $s = D$

# Online DFS (Example)



## Step 5

$s' = B$

$s = D$

$\text{untried}[B] = \{r, d, l\}$

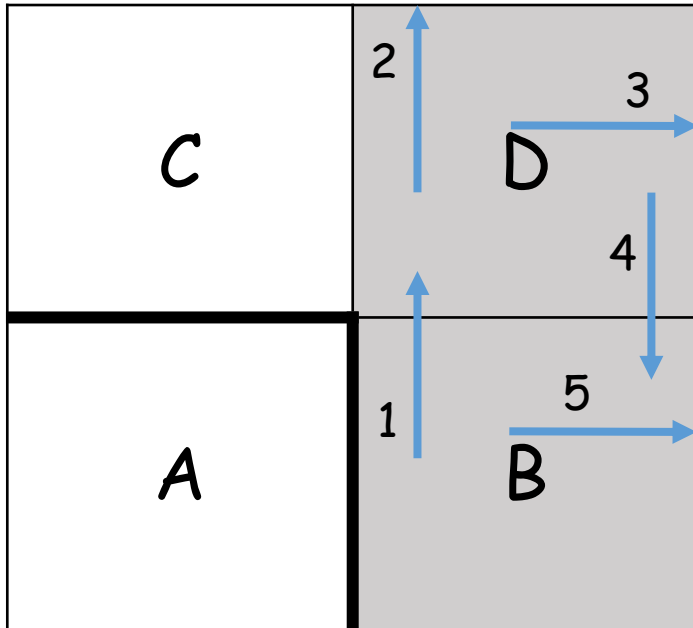
$\text{result}[D, d] = B$

$\text{unbacktracked}[B] = \{D\}$

$a = r \rightarrow \text{untried}[B] = \{d, l\}$

$s = B$

# Online DFS (Example)



## Step 5

$s' = B$

$s = D$

$\text{untried}[B] = \{r, d, l\}$

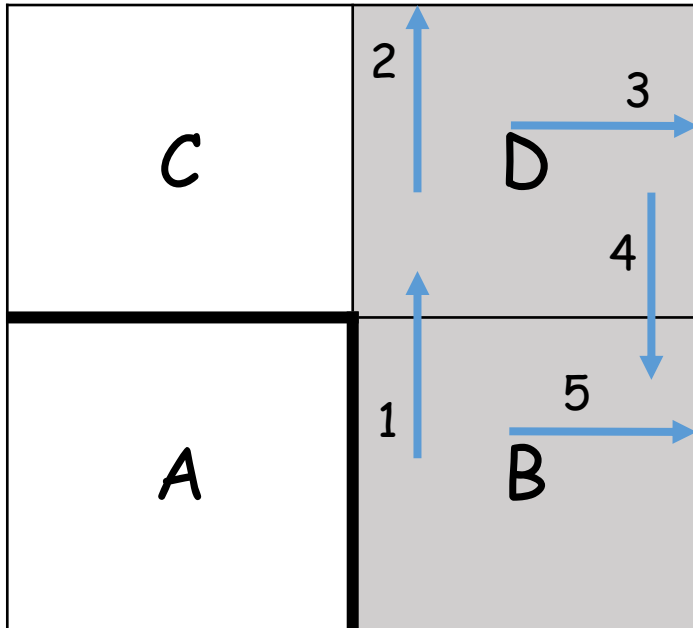
$\text{result}[D, d] = B$

$\text{unbacktracked}[B] = \{D\}$

$a = r \rightarrow \text{untried}[B] = \{d, l\}$

$s = B$

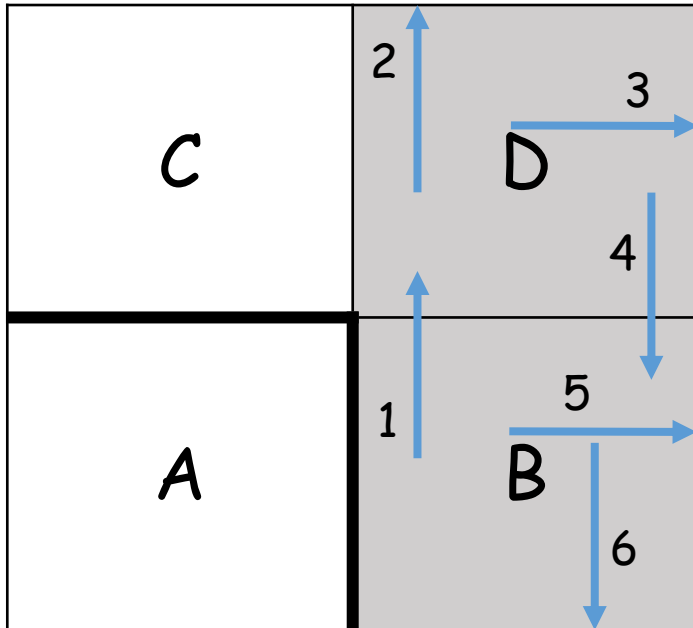
# Online DFS (Example)



## Step 6

$s' = B$                        $s = B$   
 $\text{untried}[B] = \{d, l\}$   
 $\text{result}[B, r] = B$   
 $\text{unbacktracked}[B] = \{D\}$   
 $a = d \rightarrow \text{untried}[B] = \{l\}$   
 $s = B$

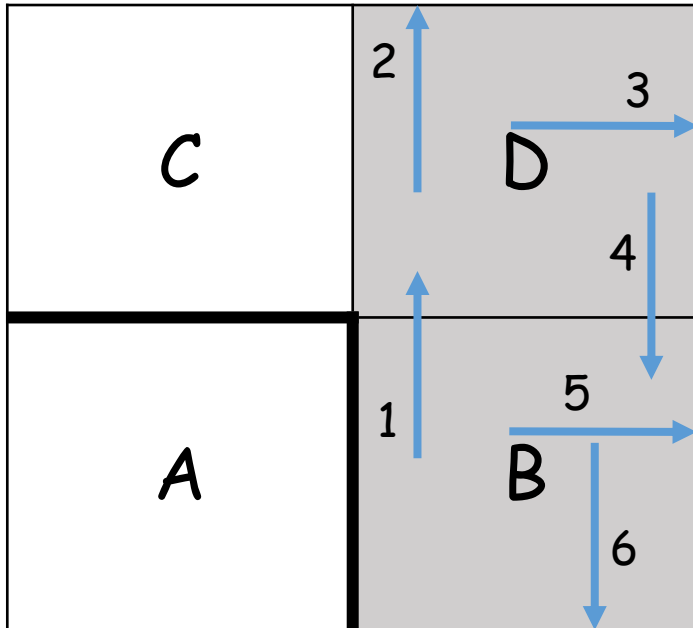
# Online DFS (Example)



## Step 6

$s' = B$                        $s = B$   
 $\text{untried}[B] = \{d, l\}$   
 $\text{result}[B, r] = B$   
 $\text{unbacktracked}[B] = \{D\}$   
 $a = d \rightarrow \text{untried}[B] = \{l\}$   
 $s = B$

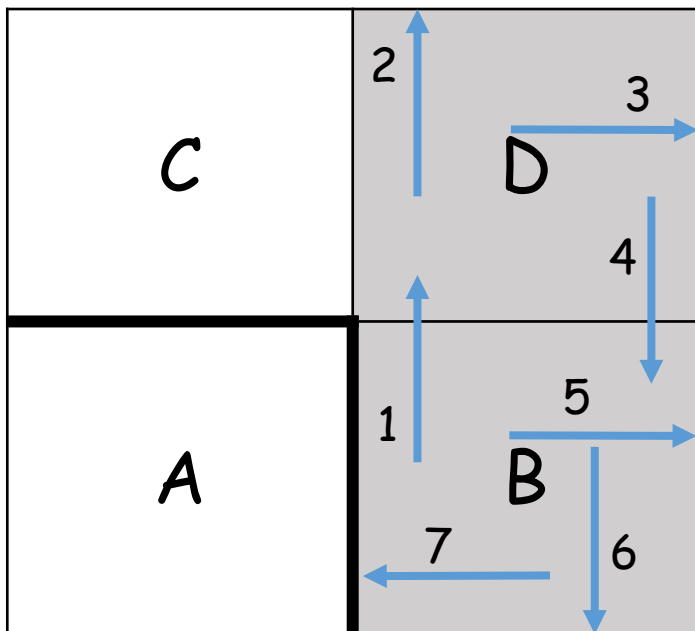
# Online DFS (Example)



## Step 7

$s' = B$                        $s = B$   
 $\text{untried}[B] = \{I\}$   
 $\text{result}[B, d] = B$   
 $\text{unbacktracked}[B] = \{D\}$   
 $a = I \rightarrow \text{untried}[B] = \{\}$   
 $s = B$

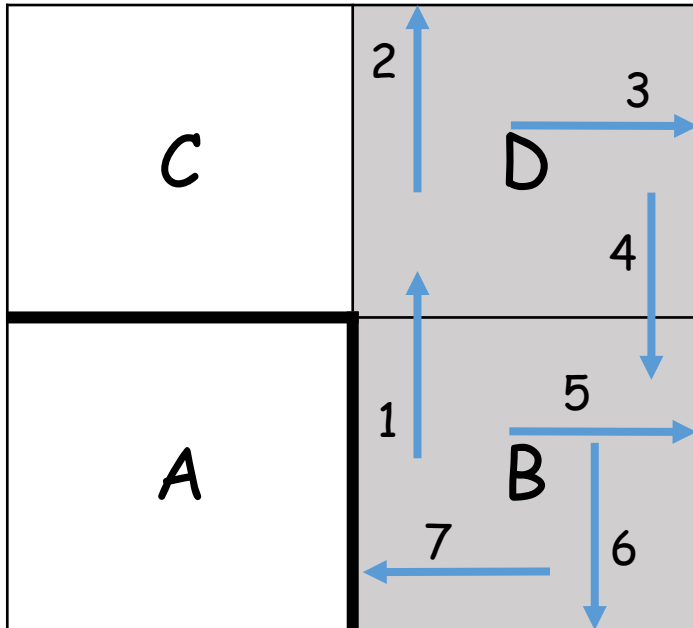
# Online DFS (Example)



## Step 7

$s' = B$                        $s = B$   
 $\text{untried}[B] = \{I\}$   
 $\text{result}[B, d] = B$   
 $\text{unbacktracked}[B] = \{D\}$   
 $a = I \rightarrow \text{untried}[B] = \{\}$   
 $s = B$

# Online DFS (Example)



## Step 8

$s' = B$

$s = B$

$\text{untried}[B] = \{\}$

$\text{result}[B, l] = B$

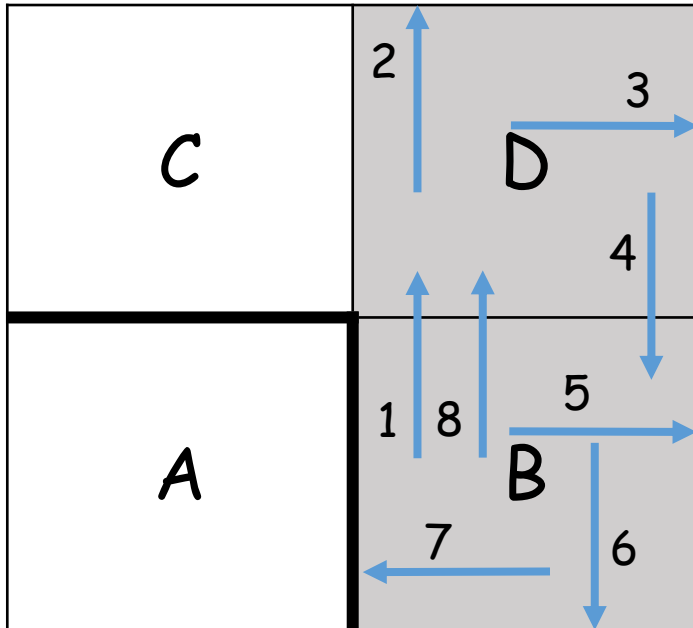
$\text{unbacktracked}[B] = \{D\}$

$a = u \rightarrow \text{unbacktracked}[B] = \{\}$

$s = B$



# Online DFS (Example)



## Step 8

$s' = B$

$s = B$

$\text{untried}[B] = \{\}$

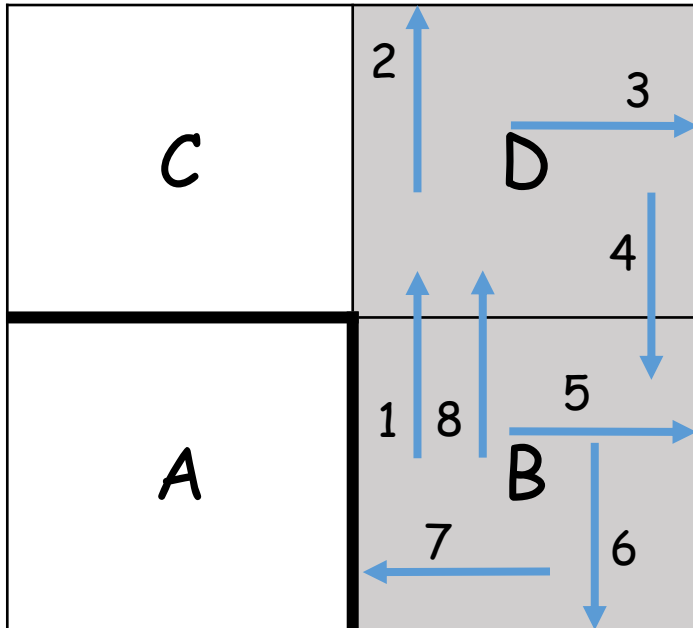
$\text{result}[B, 1] = B$

$\text{unbacktracked}[B] = \{D\}$

$a = u \rightarrow \text{unbacktracked}[B] = \{\}$

$s = B$

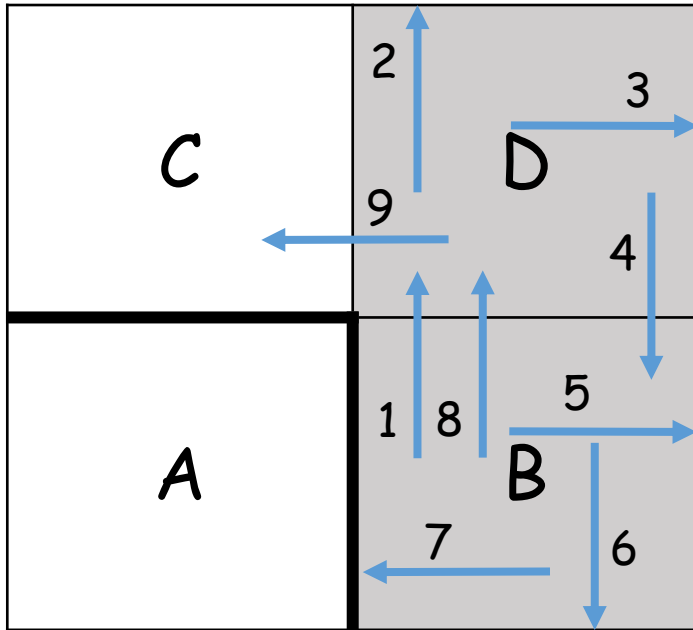
# Online DFS (Example)



## Step 9

$s' = D$                        $s = B$   
 $\text{untried}[D] = \{I\}$   
 $\text{result}[B, u] = D$   
 $\text{unbacktracked}[D] = \{B\}$   
 $a = I \rightarrow \text{untried}[D] = \{\}$   
 $s = D$

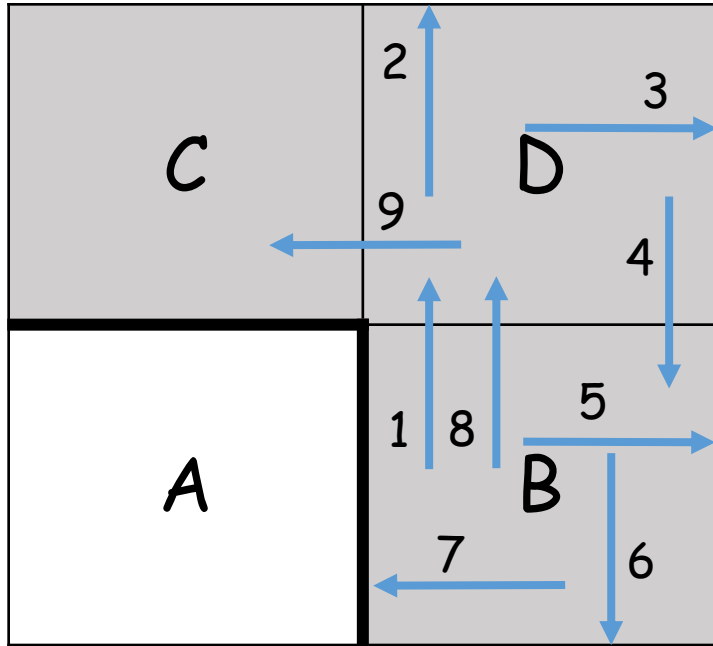
# Online DFS (Example)



## Step 9

$s' = D$                        $s = B$   
 $\text{untried}[D] = \{I\}$   
 $\text{result}[B, u] = D$   
 $\text{unbacktracked}[D] = \{B\}$   
 $a = I \rightarrow \text{untried}[D] = \{\}$   
 $s = D$

# Online DFS (Example)



Step 10

$s' = C$

$s = D$

Goal-Test( $C$ ) = true  $\rightarrow$  STOP

# Online local search

## ■ Hill-climbing

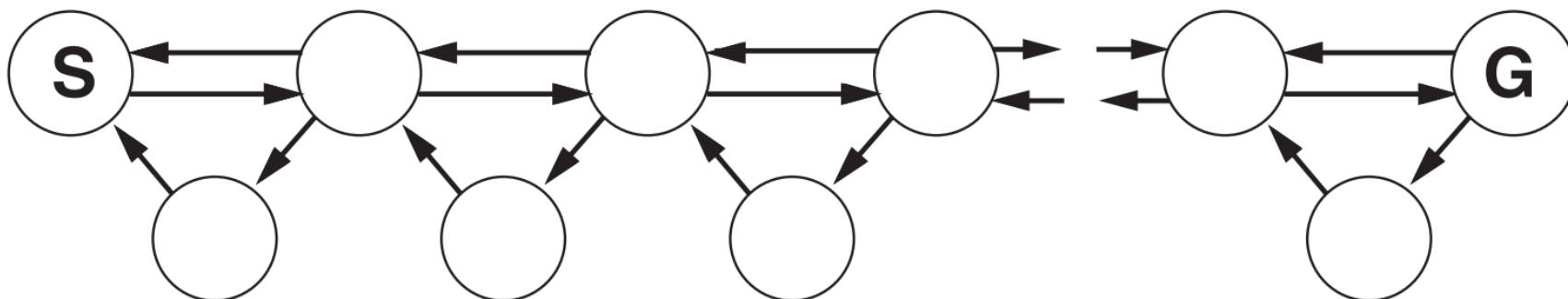
- Has the property of locality in its node expansions
- Because it keeps just one current state in memory, **hill-climbing search is already an online search algorithm!**
- It leaves the agent sitting at local maxima with nowhere to go
  - Random restarts cannot be used, because the agent cannot transport itself to a new state

## ■ Solution

- Random walk instead of random restart
  - Randomly selecting one of available actions (preference to untried actions)
- Adding Memory (Learning Real Time  $A^*$ ): more effective
  - To remember and update the costs of all visited nodes

# Random walk

- A random walk simply selects at random one of the available actions from the current state
  - Preference can be given to actions that have not yet been tried
- A random walk will eventually find a goal or complete its exploration, provided that the space is finite.



# Learning real-time A\* (LRTA\*)

- Augmenting hill climbing with memory rather than randomness turns out to be a more effective approach
  - Store a "current best estimate"  $H(s)$  of the cost to reach the goal from each state that has been visited
  - Initially  $H(s)$  is a heuristic estimate  $h(s)$
  - $H(s)$  is updated by experience (More accurate estimates are acquired using local updating rules)
$$H(s) \leftarrow \min_{a \in \text{ACTIONS}(s)} H(s') + c(s, a, s')$$
  - Assumption: Untried actions in a state  $s$  lead to the goal with the least possible cost  $h(s)$ 
    - Encouraging to explore new (possibly promising) paths

# Learning real-time A\* (LRTA\*)

**function** LRTA\*-AGENT( $s'$ ) **returns** an action

**inputs:**  $s'$ , a percept that identifies the current state

**persistent:** *result*, a table, indexed by state and action, initially empty

*H*, a table of cost estimates indexed by state, initially empty

*s*, *a*, the previous state and action, initially null

**if** GOAL-TEST( $s'$ ) **then return** stop

**if**  $s'$  is a new state (not in *H*) **then**  $H[s'] \leftarrow h(s')$

**if** *s* is not null

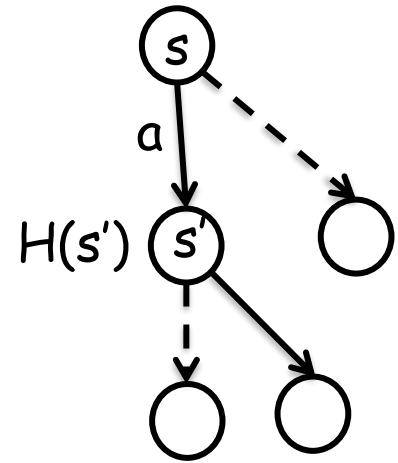
$result[s, a] \leftarrow s'$

$H[s] \leftarrow \min_{b \in ACTIONS(s)} LRTA^*_COST(s, b, result[s, b], H)$

$a \leftarrow$  an action  $b$  in  $ACTIONS(s')$  that minimizes  $LRTA^*_COST(s', b, result[s', b], H)$

$s \leftarrow s'$

**return**  $a$



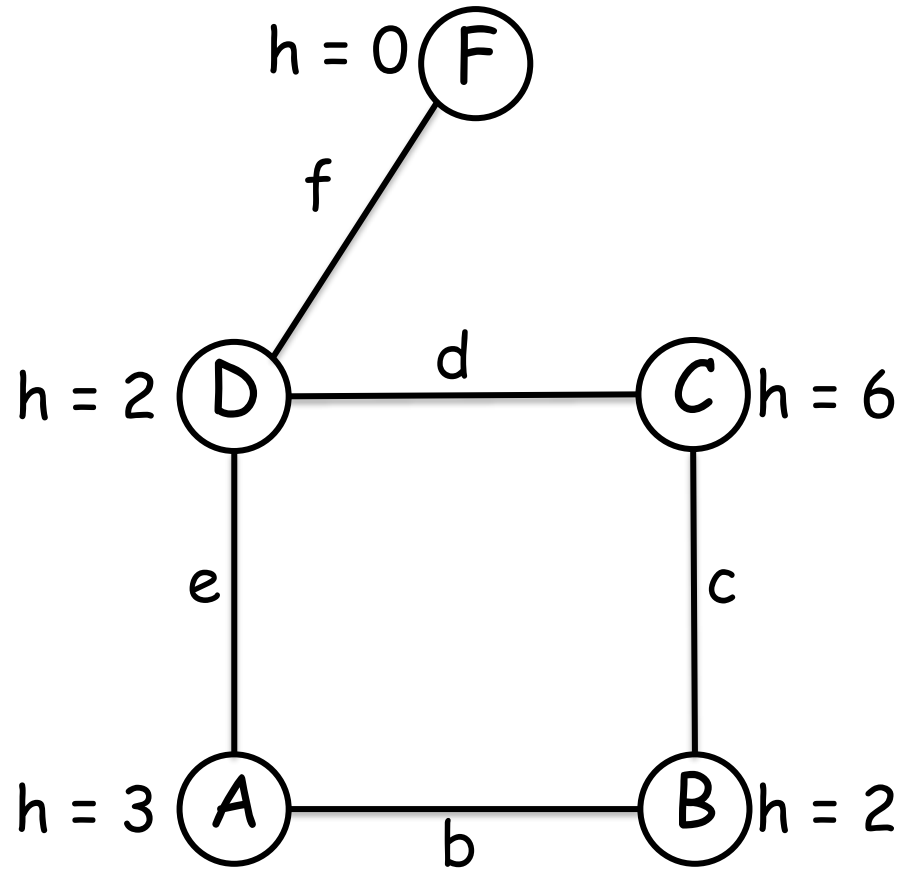
**function** LRTA\*\_COST( $s, a, s', H$ ) **returns** a cost estimate

**if**  $s'$  is undefined **then return**  $h(s)$

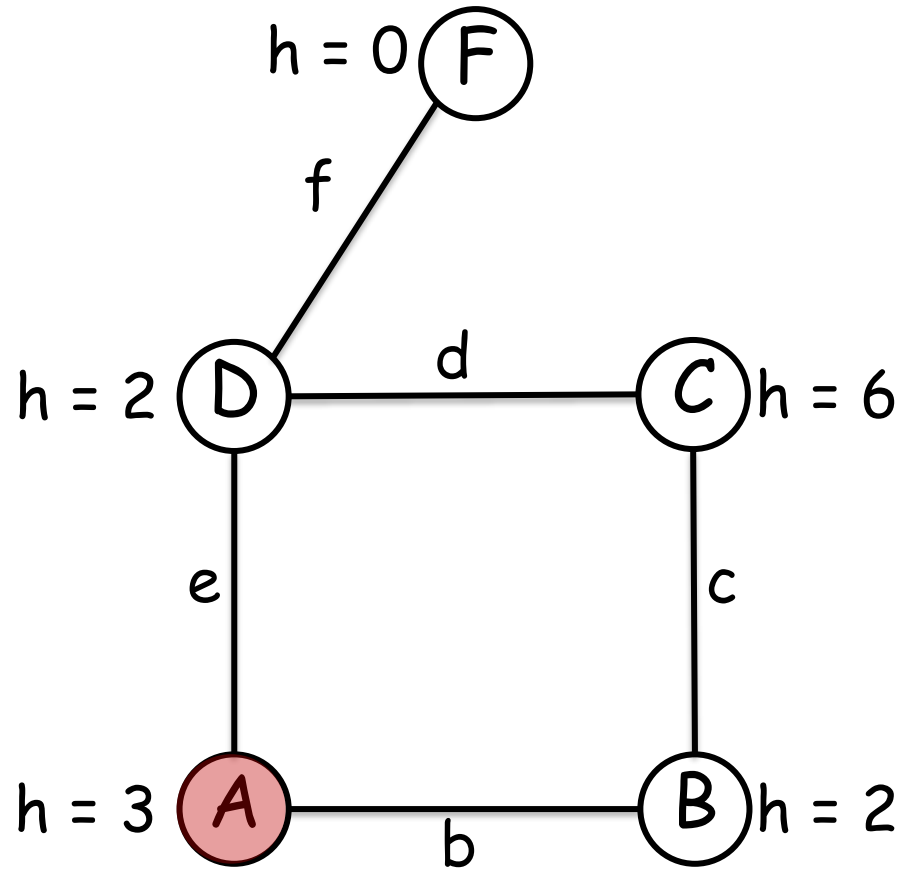
**else return**  $c(s, a, s') + H[s']$



# LRTA\* (Example)



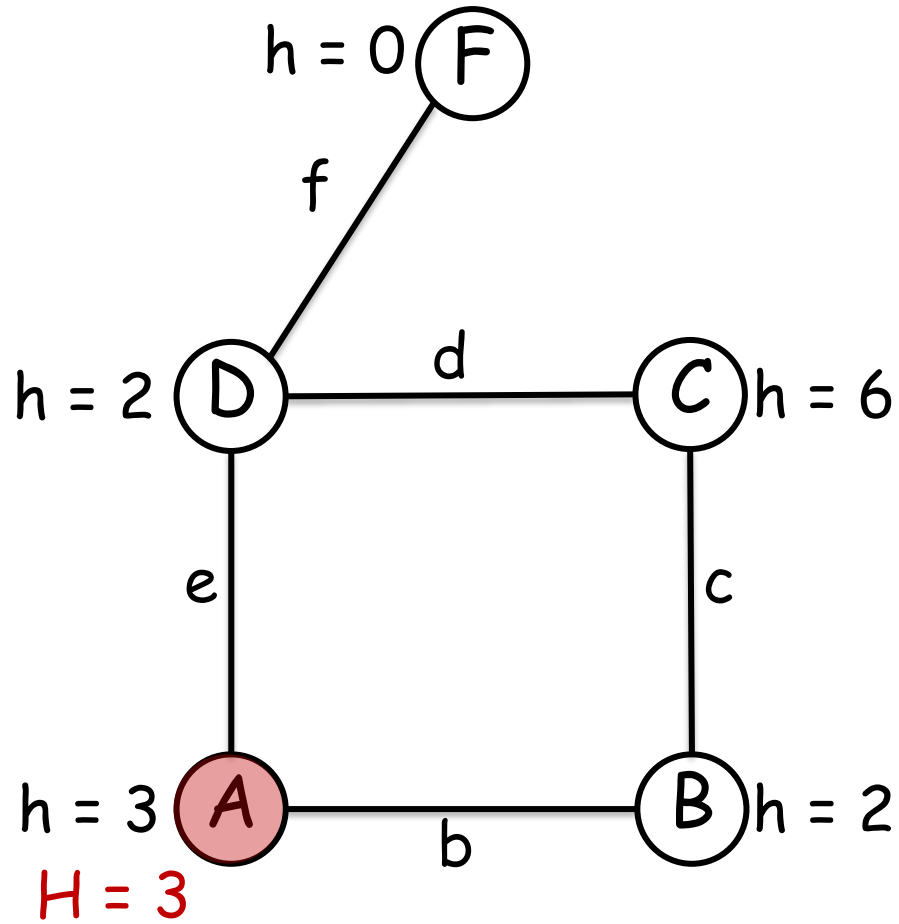
# LRTA\* (Example)



$s' = A$

$s = \text{null}$

# LRTA\* (Example)

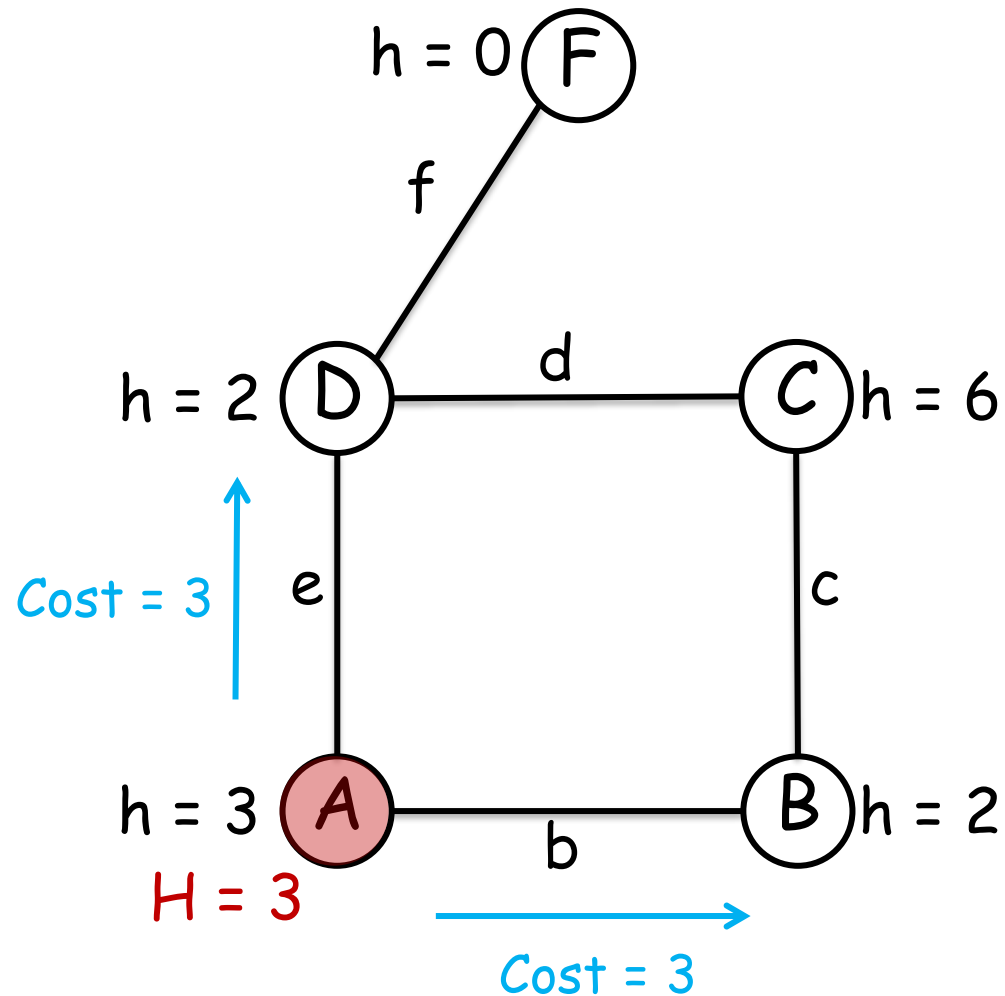


$s' = A$

$s = \text{null}$

$H(A) = h(A) = 3$

# LRTA\* (Example)



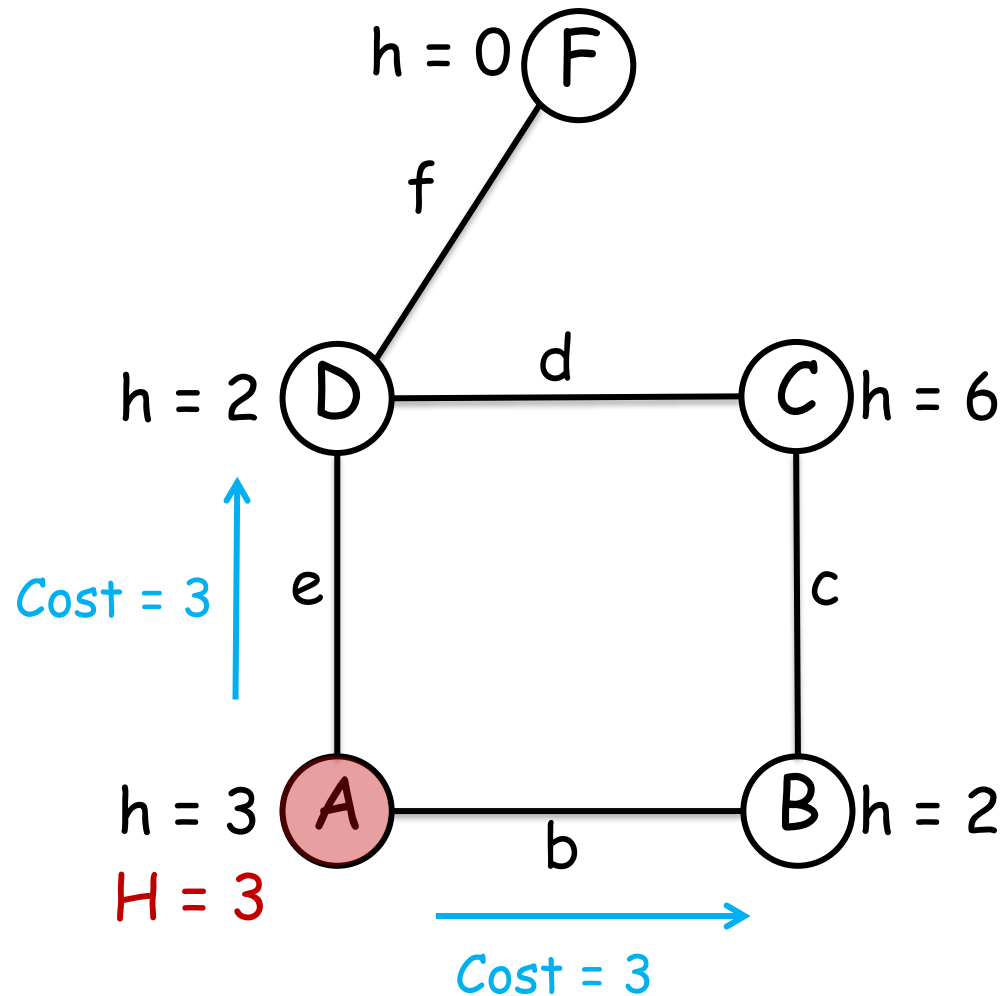
$s' = A$

$s = \text{null}$

$H(A) = h(A) = 3$

$a = b$

# LRTA\* (Example)



$s' = A$

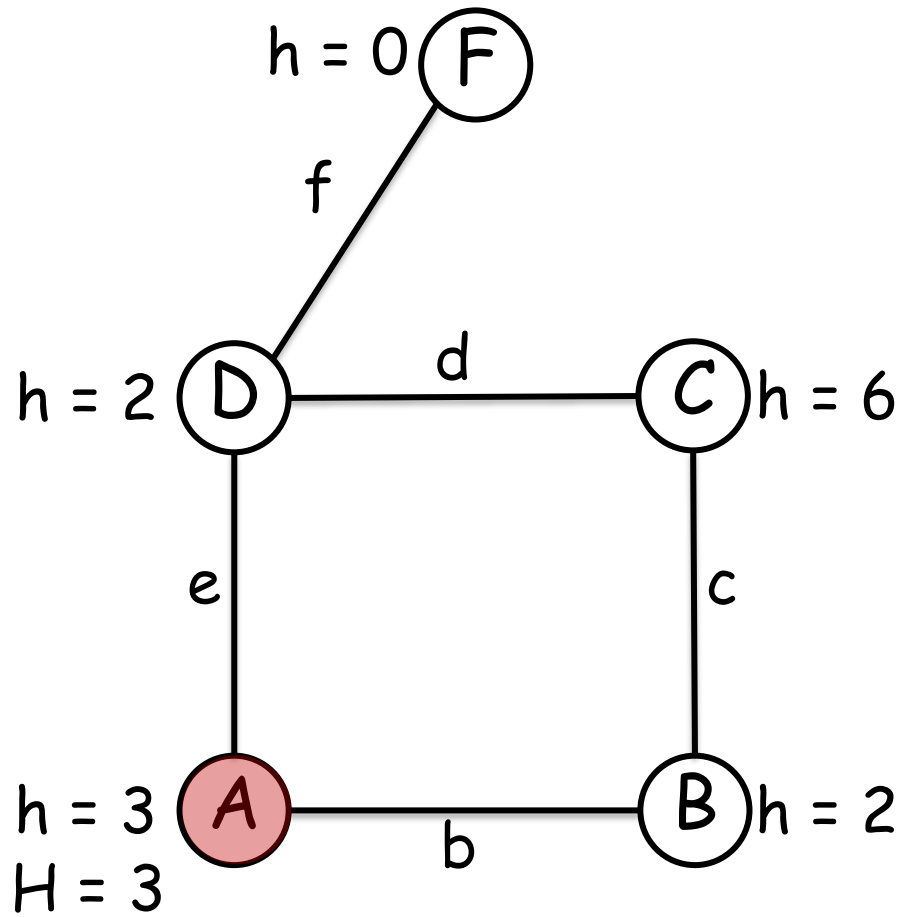
$s = \text{null}$

$H(A) = h(A) = 3$

$a = b$

$s = A$

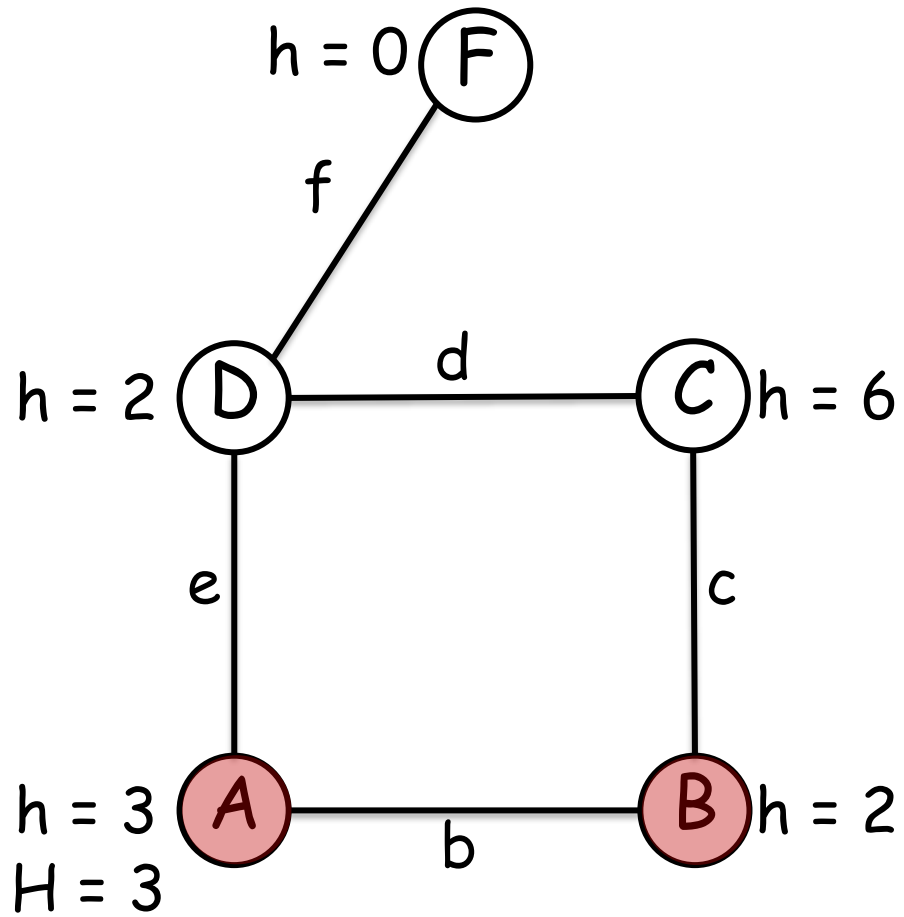
# LRTA\* (Example)



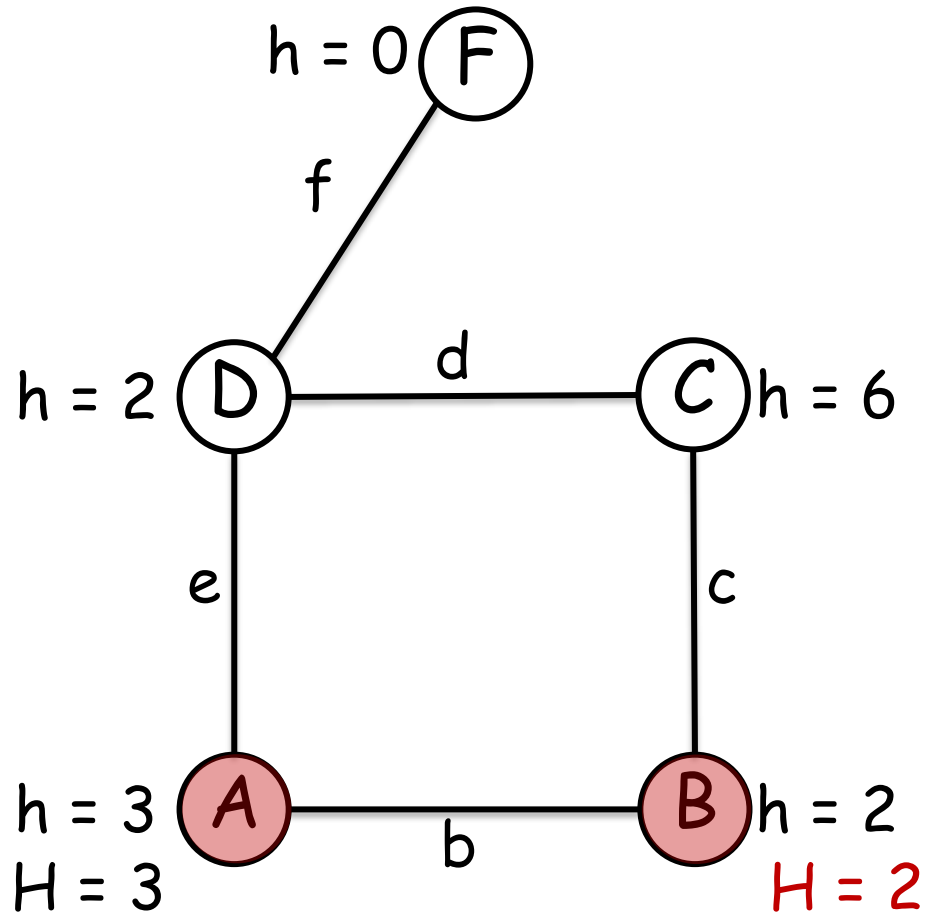
# LRTA\* (Example)

$s' = B$

$s = A$



# LRTA\* (Example)



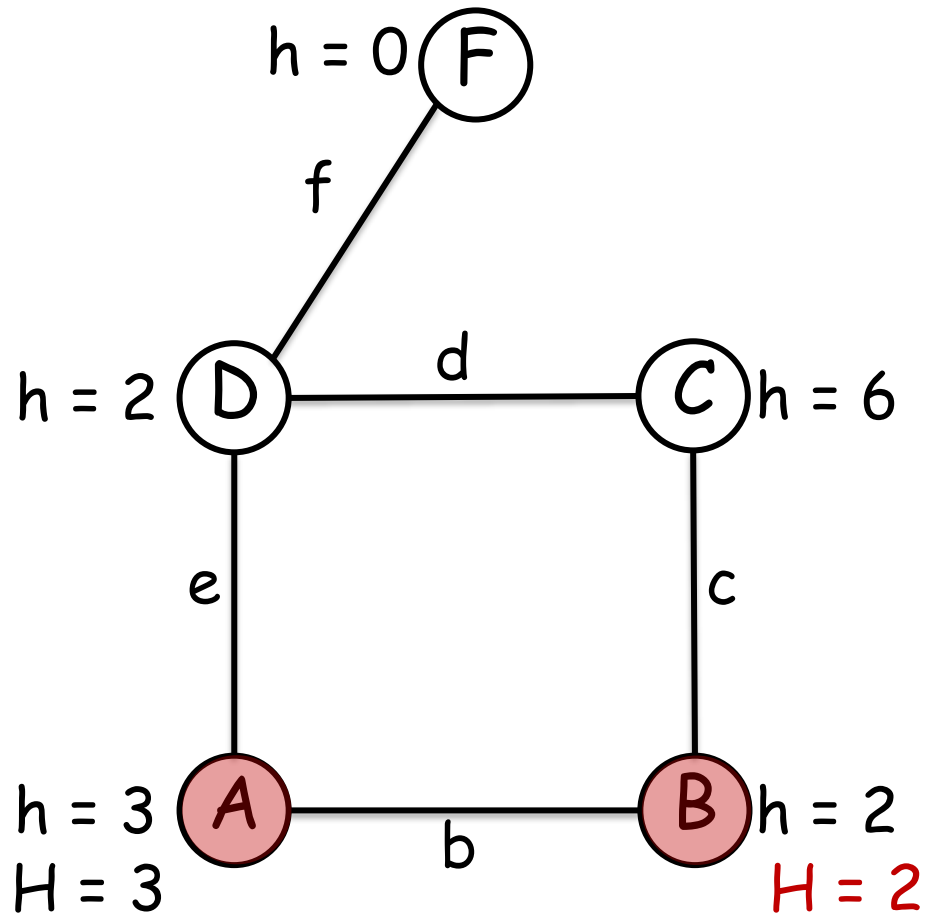
$s' = B$

$s = A$

$H(B) = h(B) = 2$



# LRTA\* (Example)



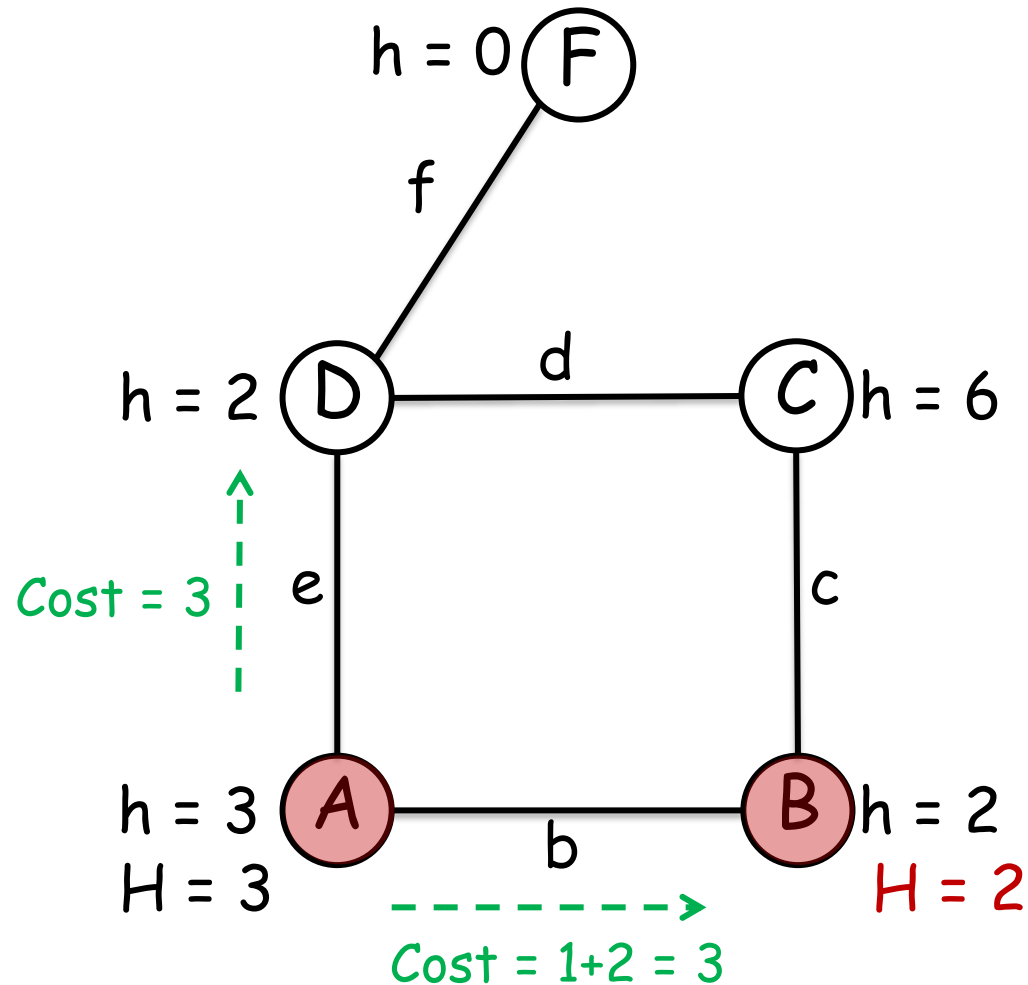
$s' = B$

$s = A$

$H(B) = h(B) = 2$

$\text{result}(A, b) = B$

# LRTA\* (Example)



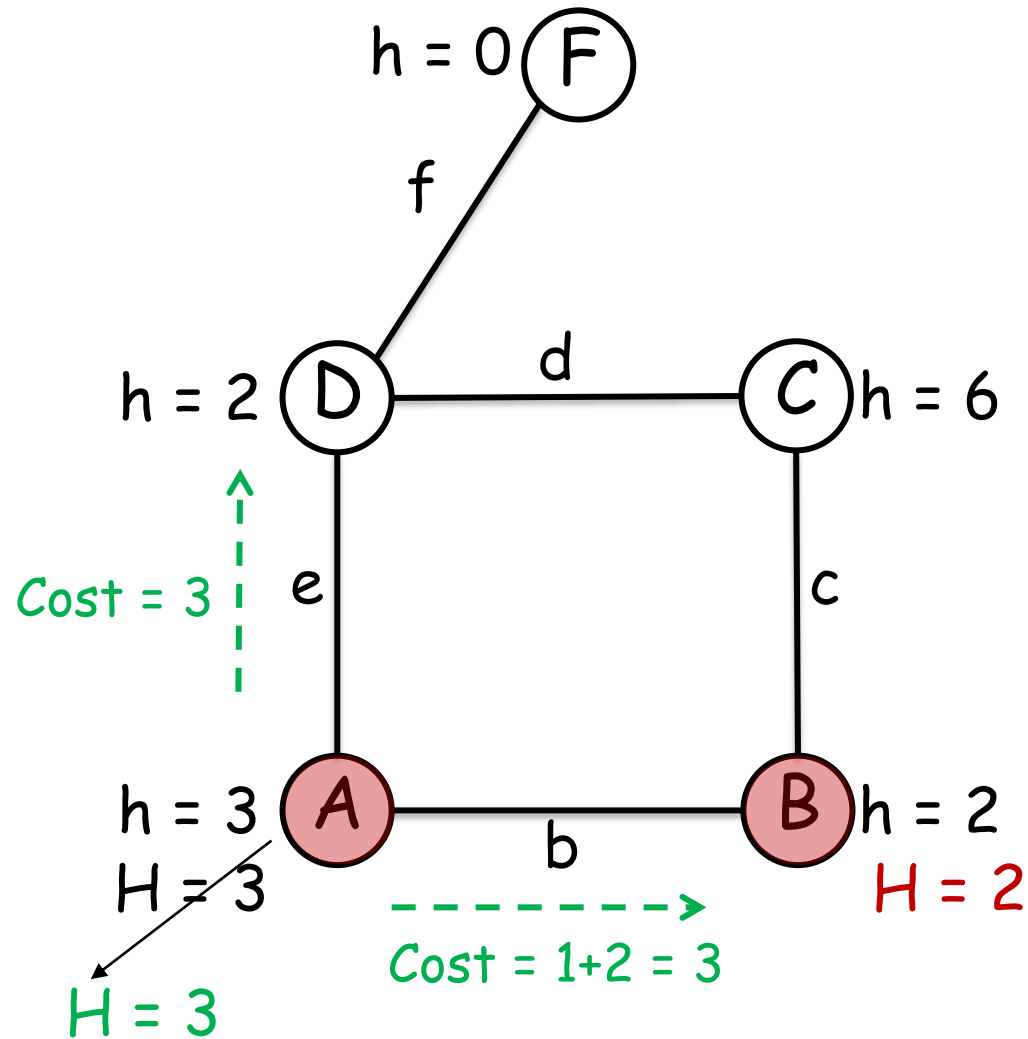
$s' = B$

$s = A$

$H(B) = h(B) = 2$

result(A, b) = B

# LRTA\* (Example)



$s' = B$

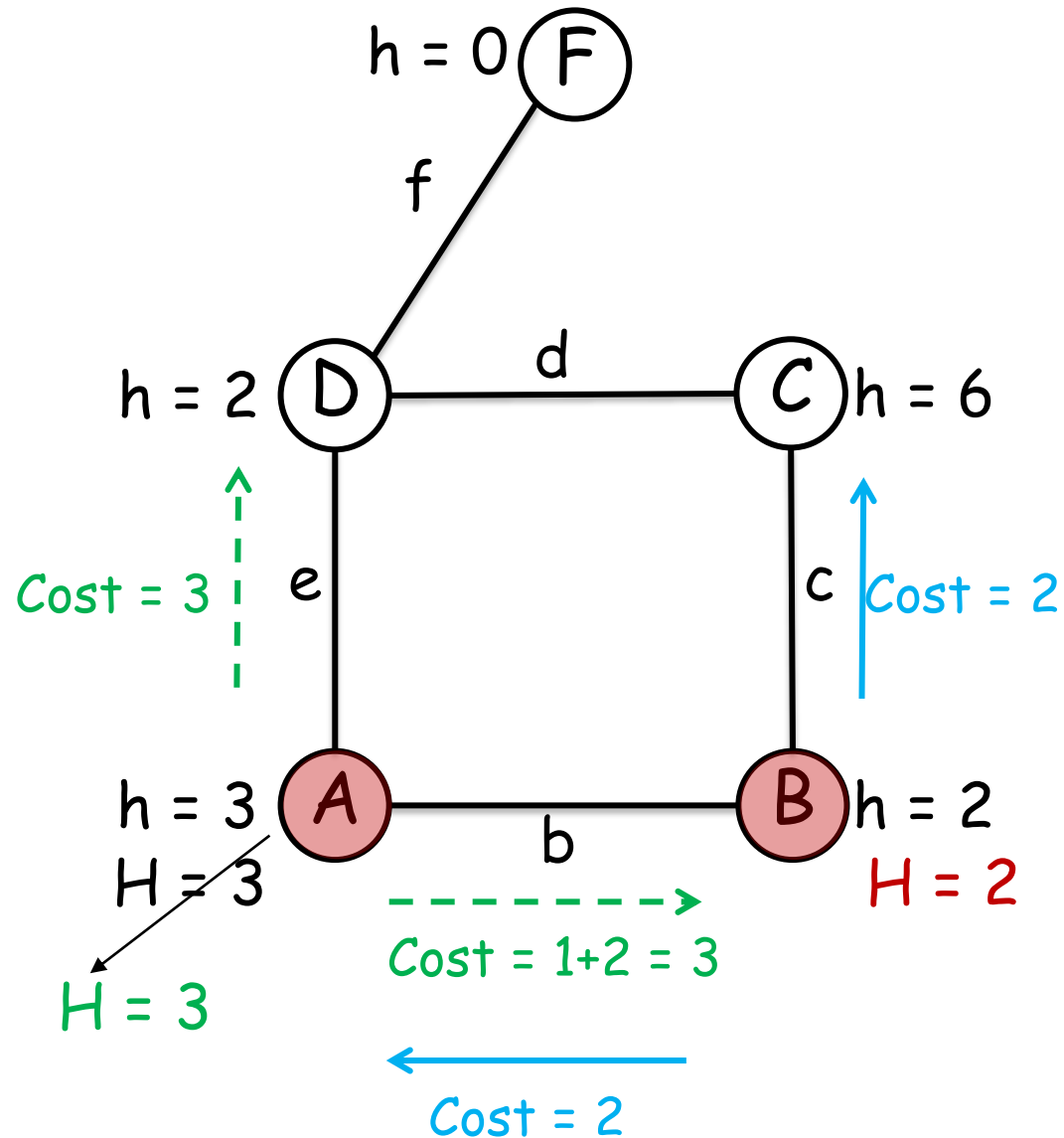
$s = A$

$H(B) = h(B) = 2$

$\text{result}(A, b) = B$

$H(A) = 3$

# LRTA\* (Example)



$s' = B$

$s = A$

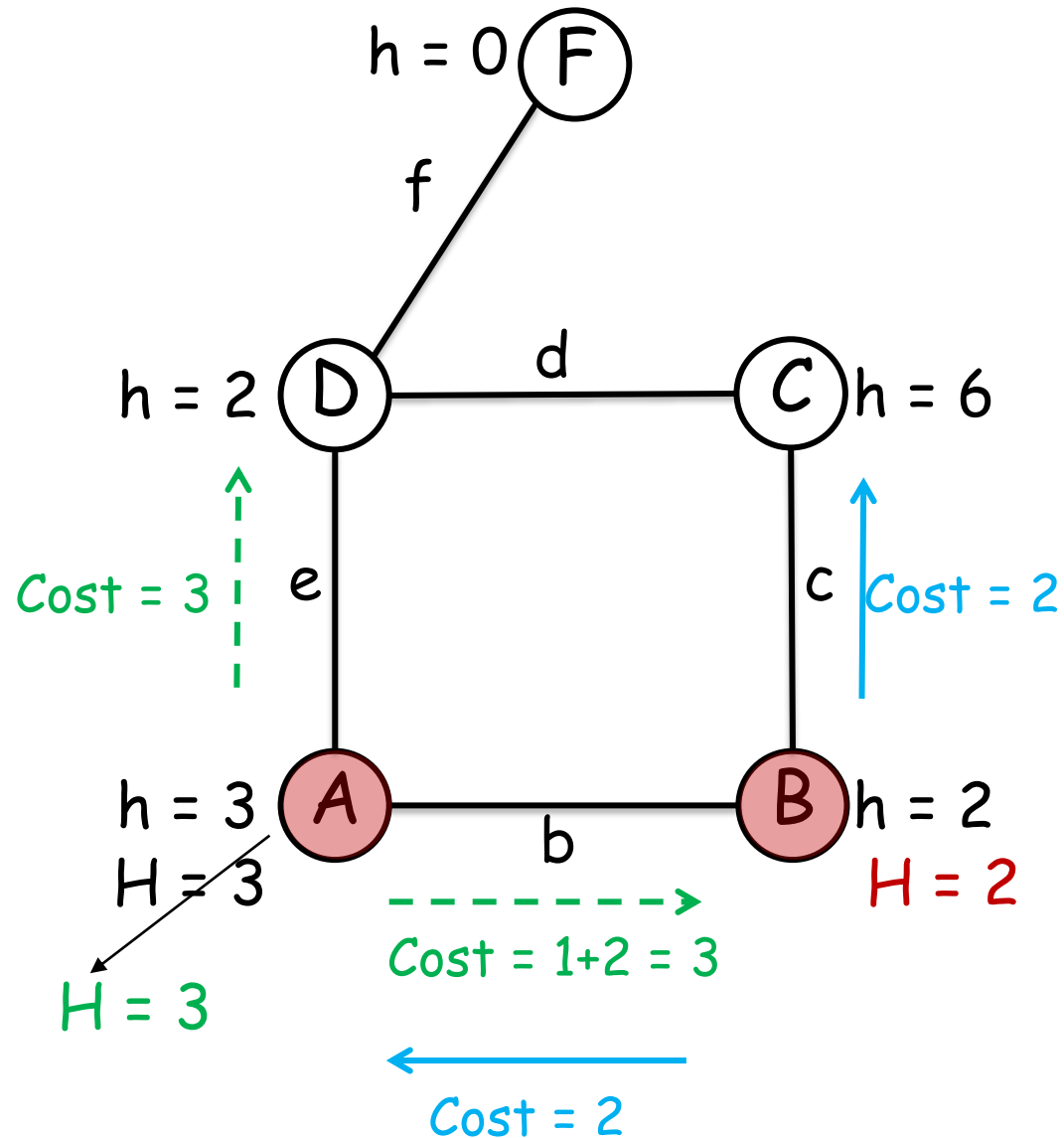
$H(B) = h(B) = 2$

$\text{result}(A, b) = B$

$H(A) = 3$

$a = c$

# LRTA\* (Example)



$s' = B$

$s = A$

$H(B) = h(B) = 2$

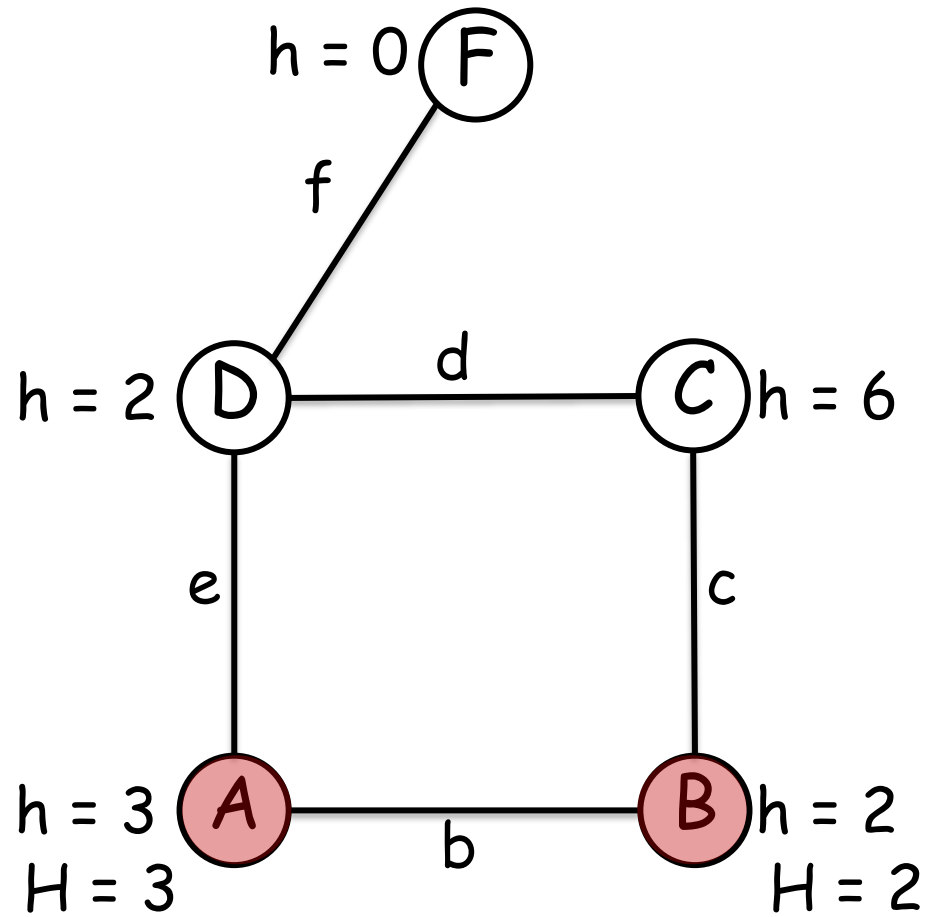
$\text{result}(A, b) = B$

$H(A) = 3$

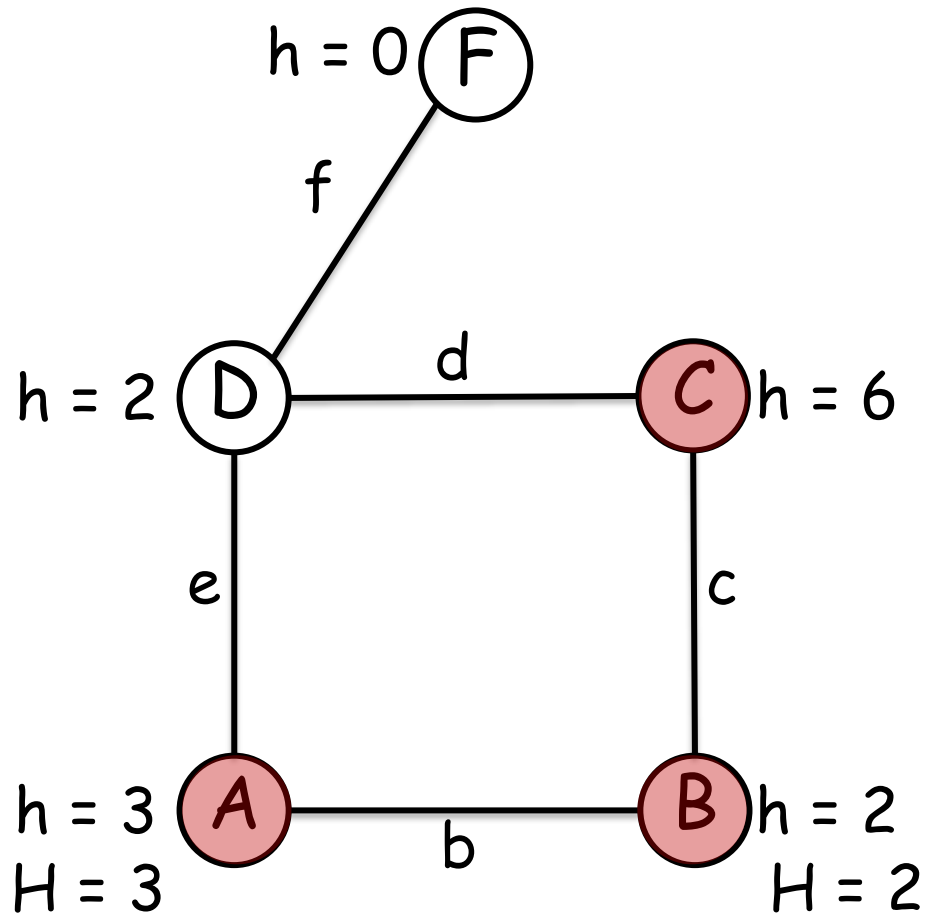
$a = c$

$s = B$

# LRTA\* (Example)



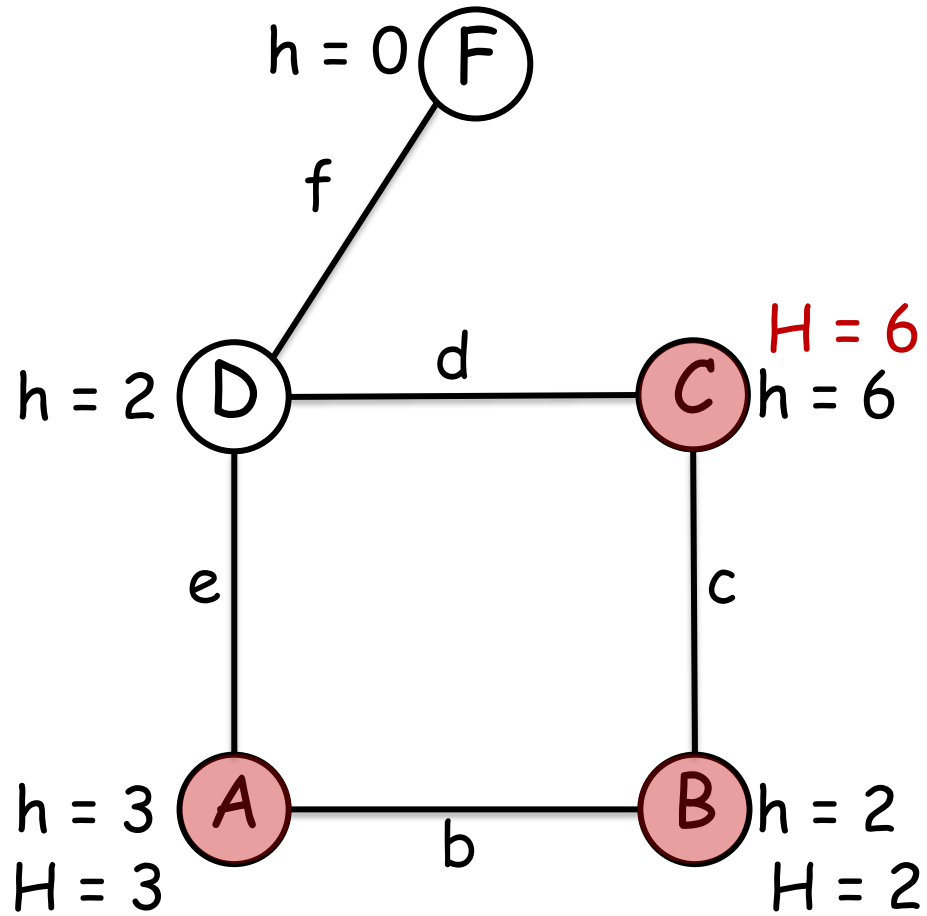
# LRTA\* (Example)



$s' = C$

$s = B$

# LRTA\* (Example)



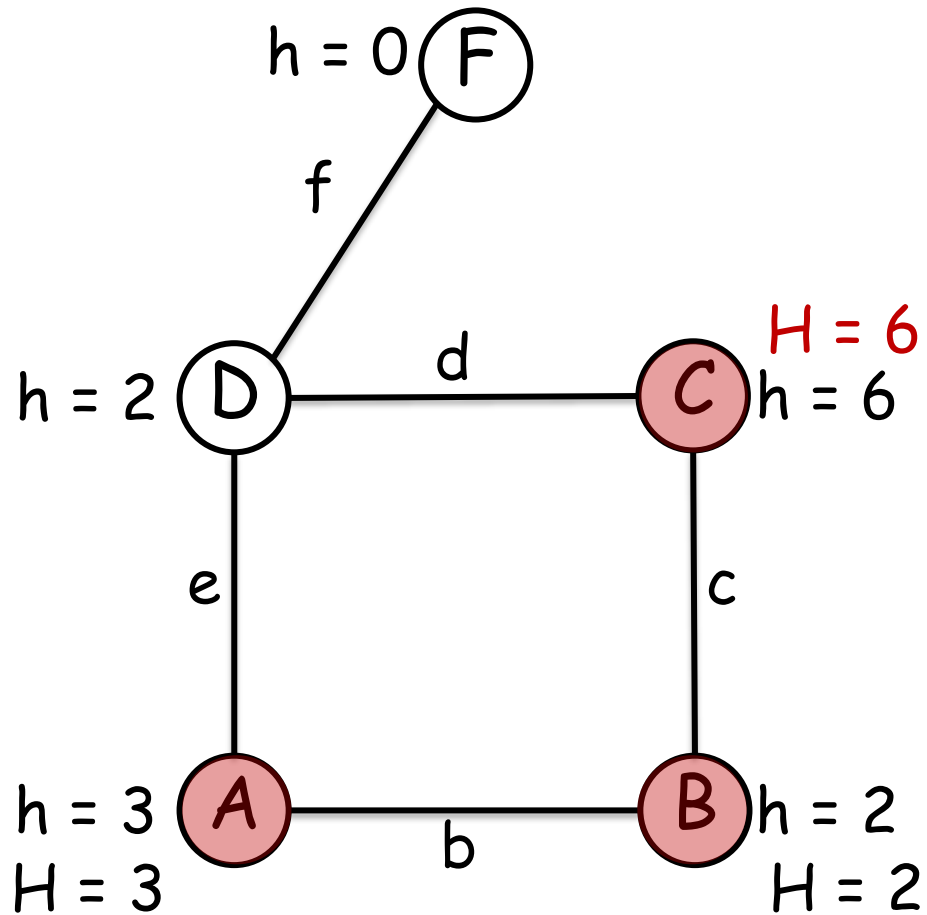
$$s' = C$$

$$s = B$$

$$H(C) = h(C) = 6$$



# LRTA\* (Example)



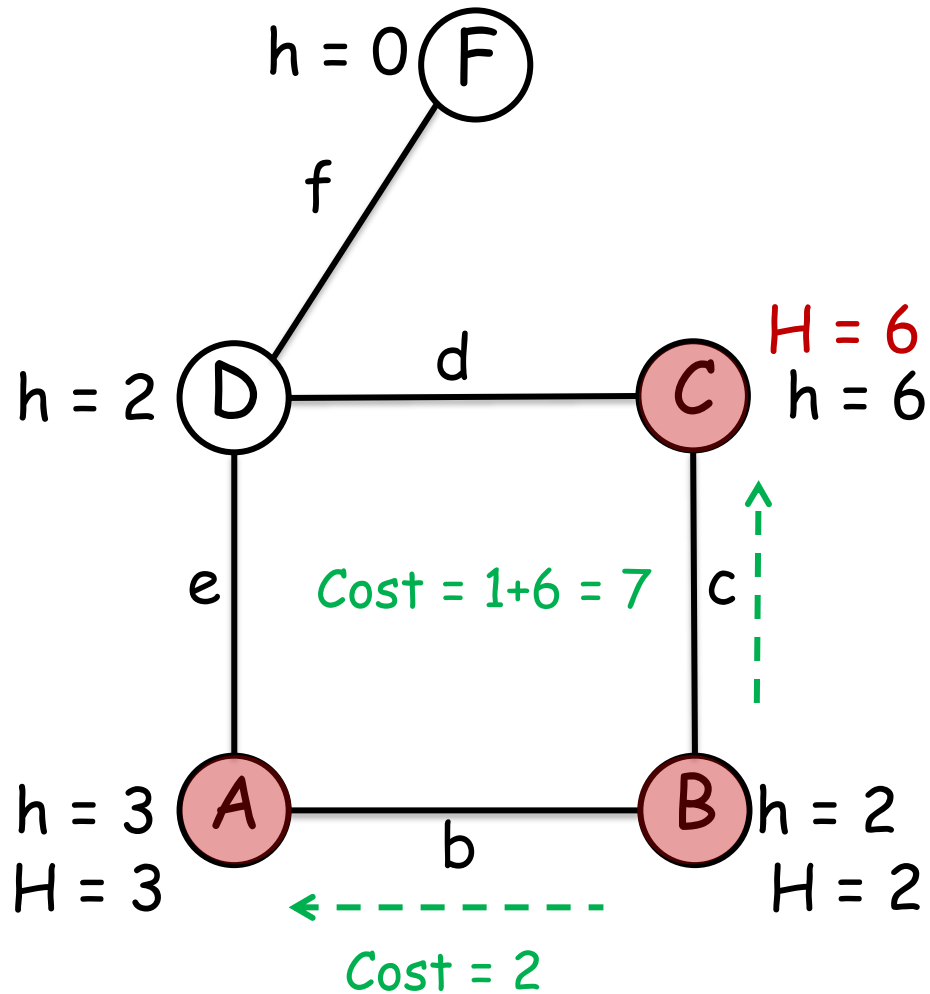
$$s' = C$$

$$s = B$$

$$H(C) = h(C) = 6$$

$$\text{result}(B, c) = C$$

# LRTA\* (Example)



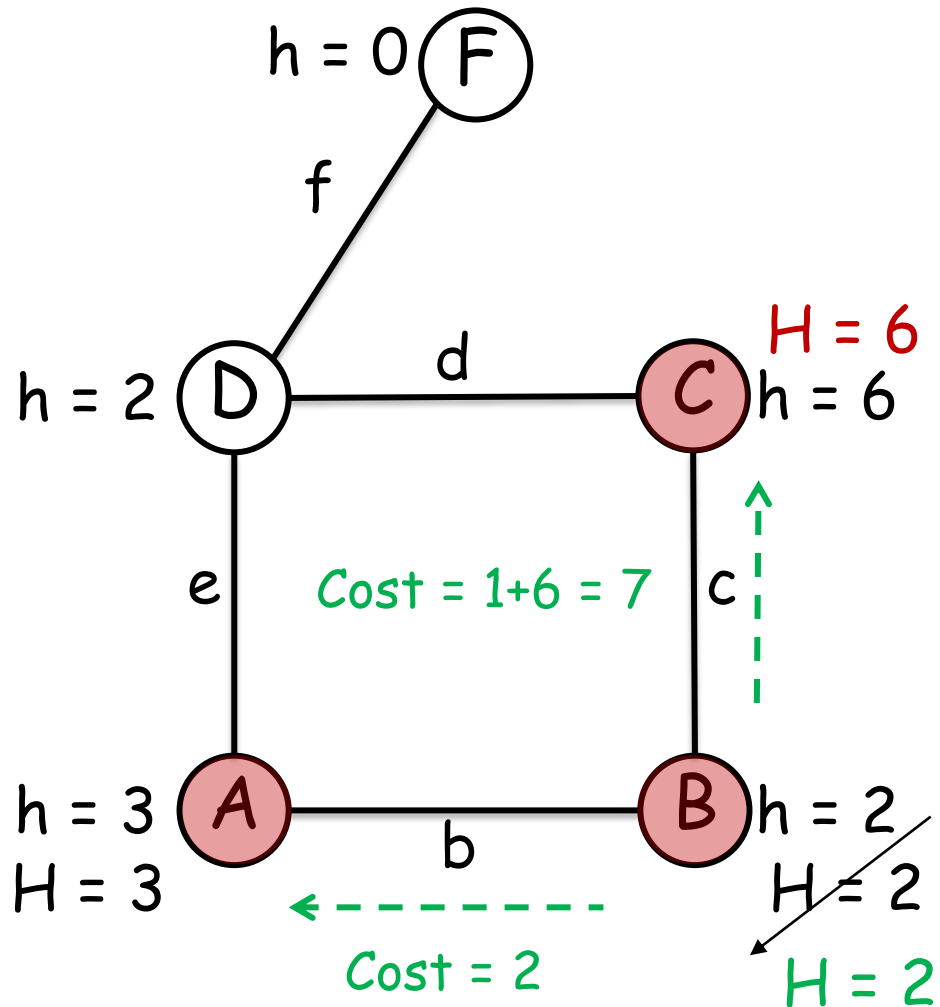
$$s' = C$$

$$s = B$$

$$H(C) = h(C) = 6$$

$$\text{result}(B, c) = C$$

# LRTA\* (Example)



$s' = C$

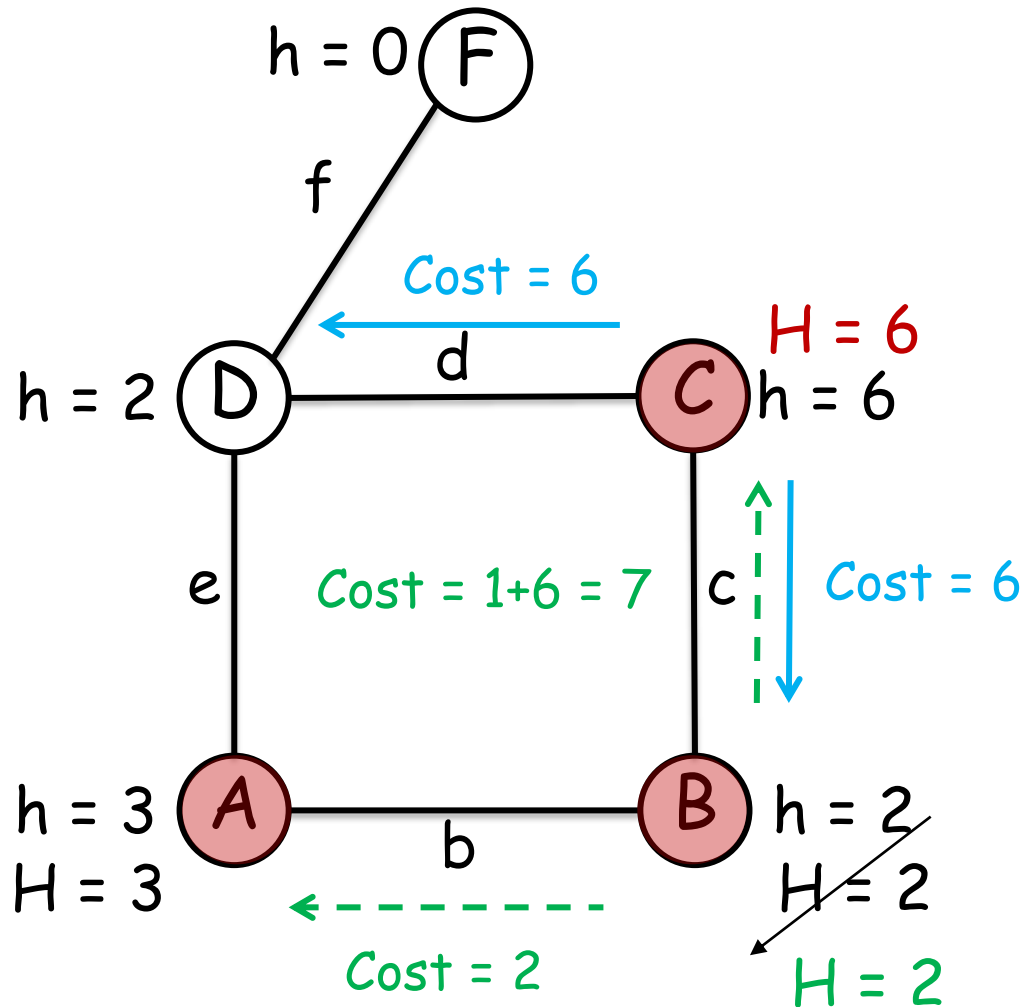
$s = B$

$H(C) = h(C) = 6$

$\text{result}(B, c) = C$

$H(B) = 2$

# LRTA\* (Example)



$s' = C$

$s = B$

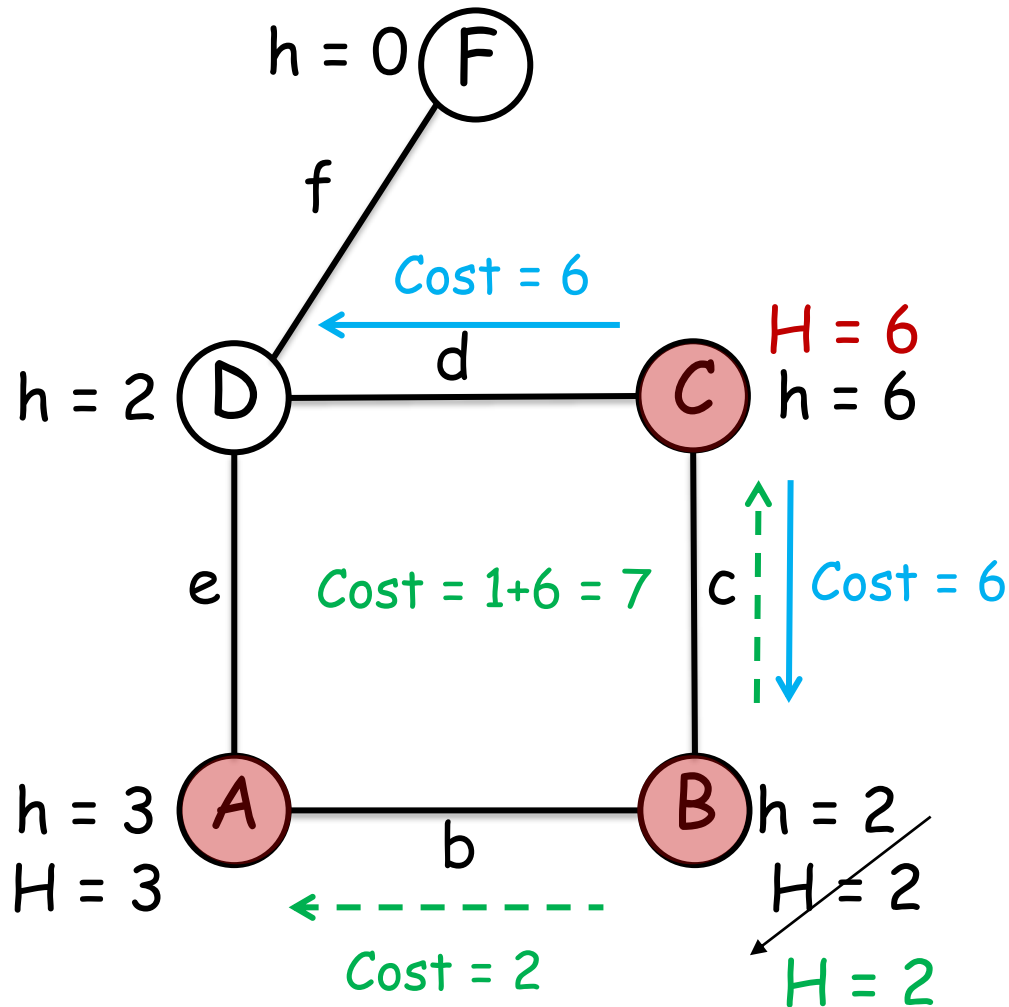
$H(C) = h(C) = 6$

$\text{result}(B, c) = C$

$H(B) = 2$

$a = c$

# LRTA\* (Example)



$s' = C$

$s = B$

$H(C) = h(C) = 6$

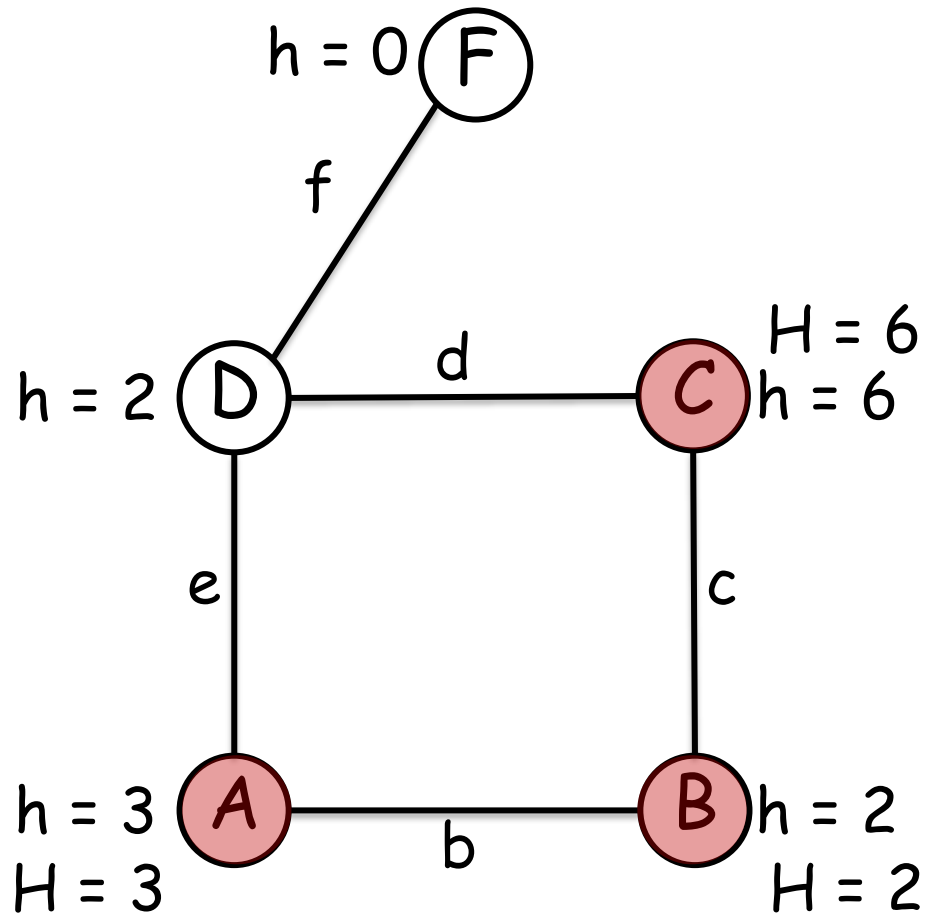
$\text{result}(B, c) = C$

$H(B) = 2$

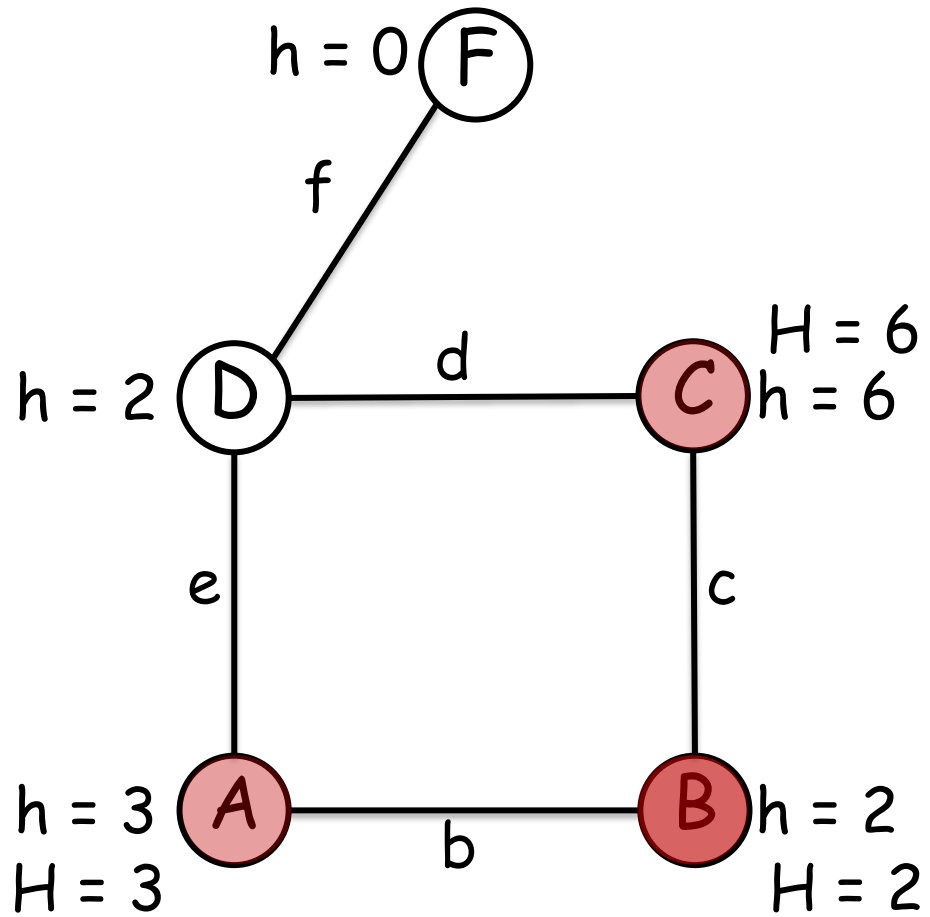
$a = c$

$s = C$

# LRTA\* (Example)



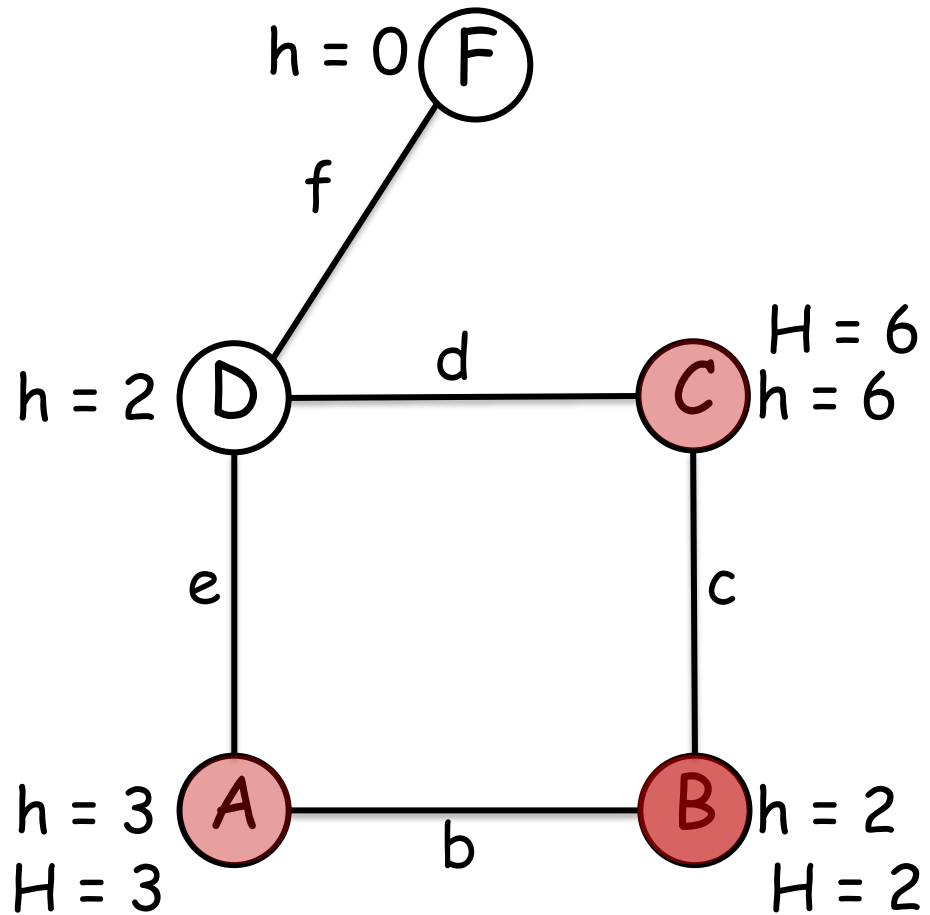
# LRTA\* (Example)



$s' = B$

$s = C$

# LRTA\* (Example)



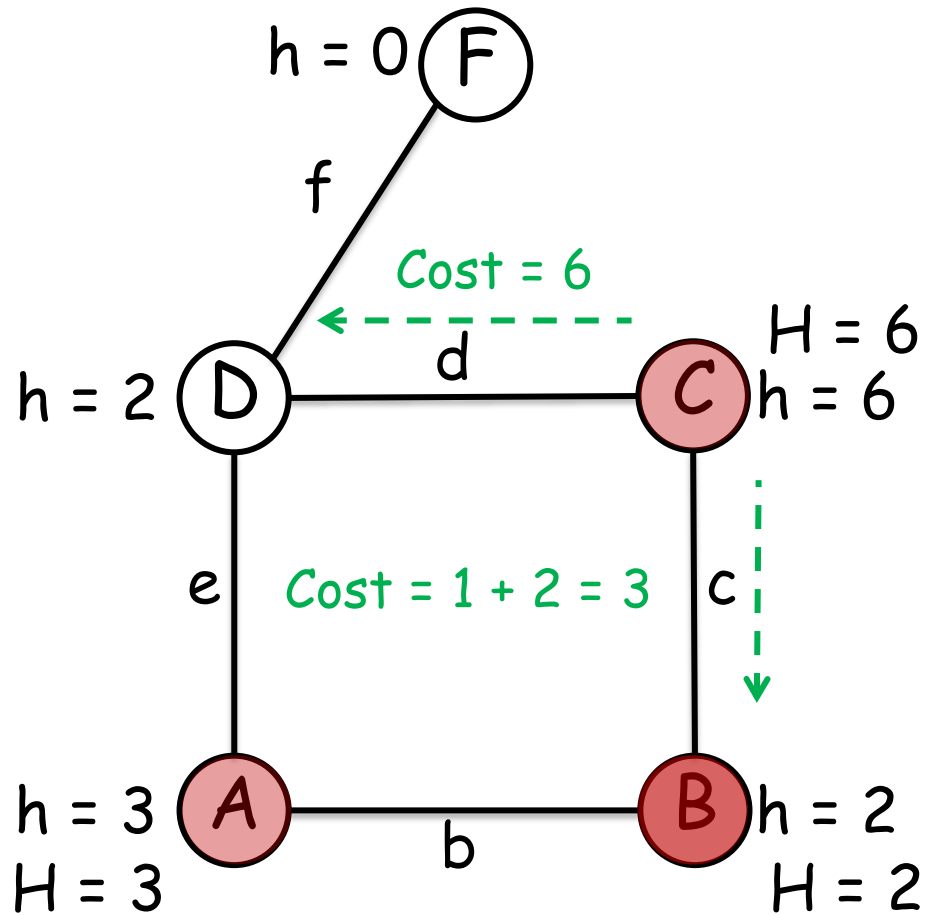
$s' = B$

$s = C$

$\text{result}(C, c) = B$



# LRTA\* (Example)

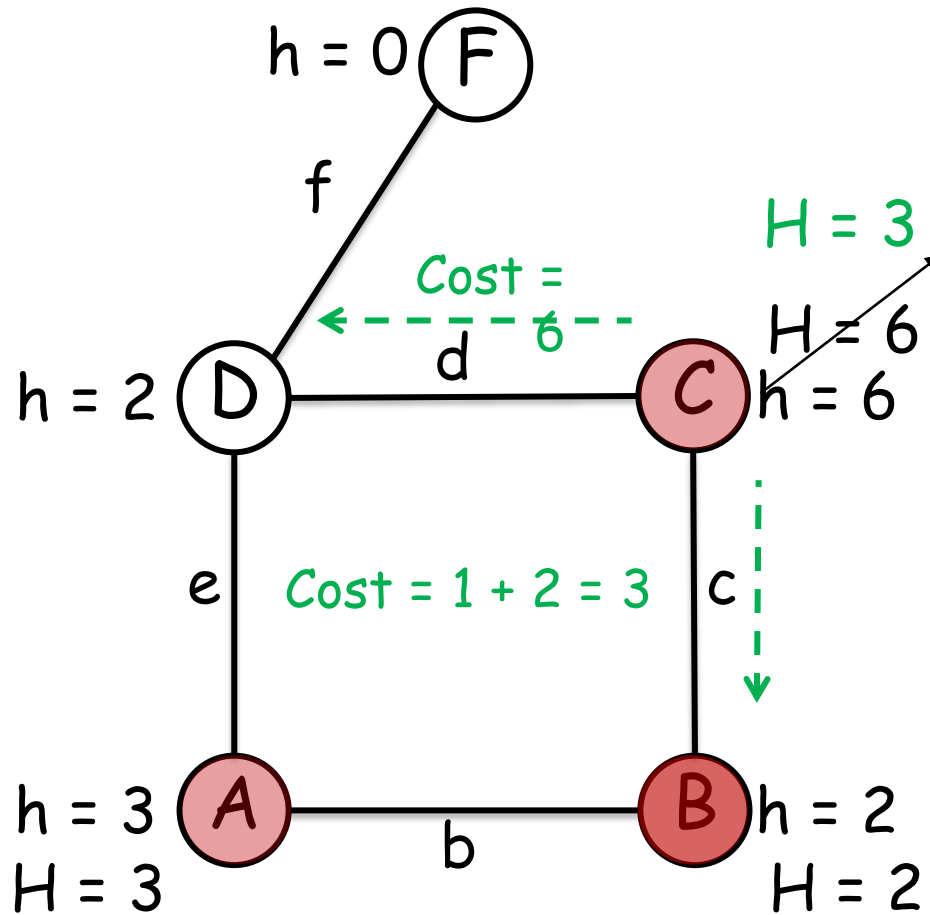


$s' = B$

$s = C$

$\text{result}(C, c) = B$

# LRTA\* (Example)



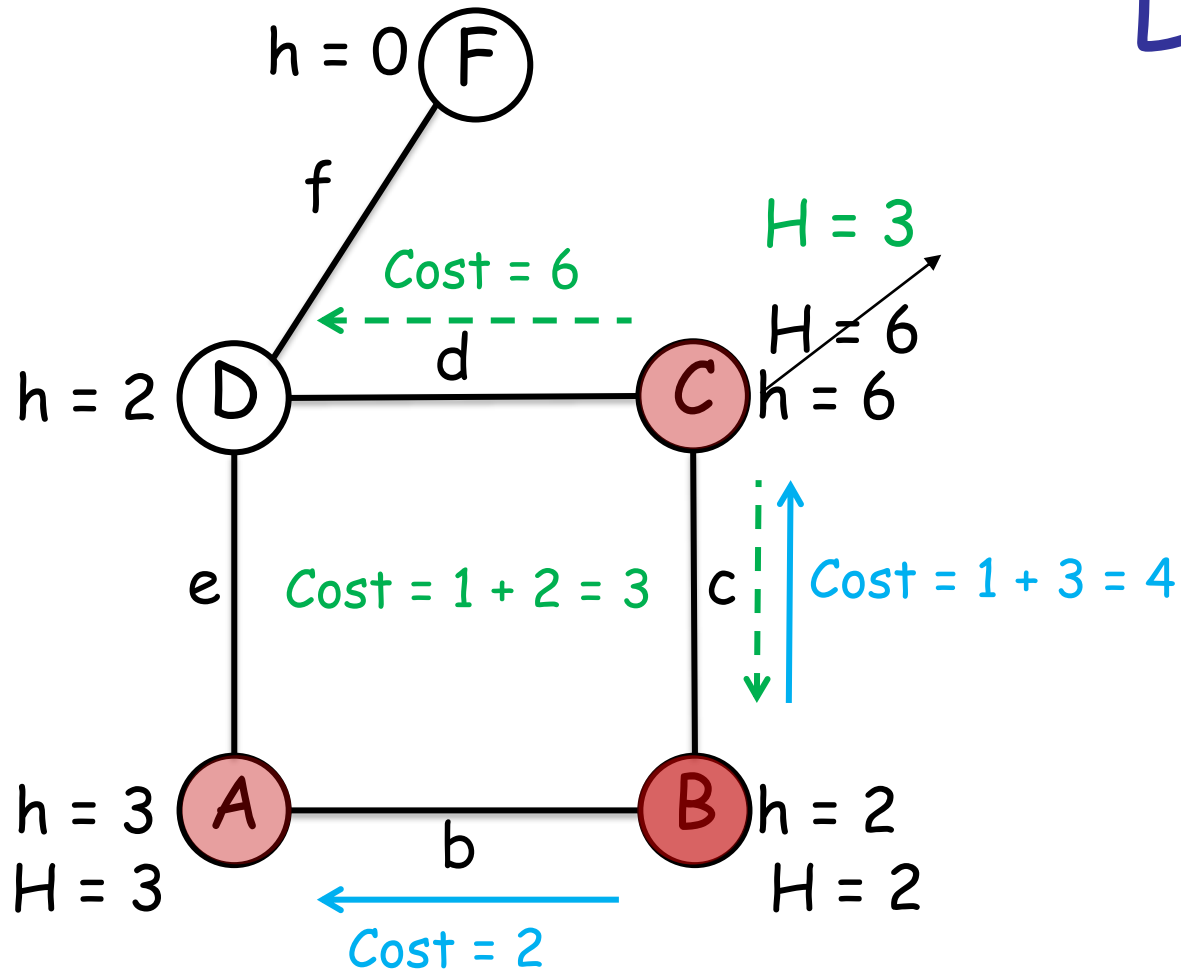
$s' = B$

$s = C$

$\text{result}(C, c) = B$

$H(C) = 3$

# LRTA\* (Example)



$s' = B$

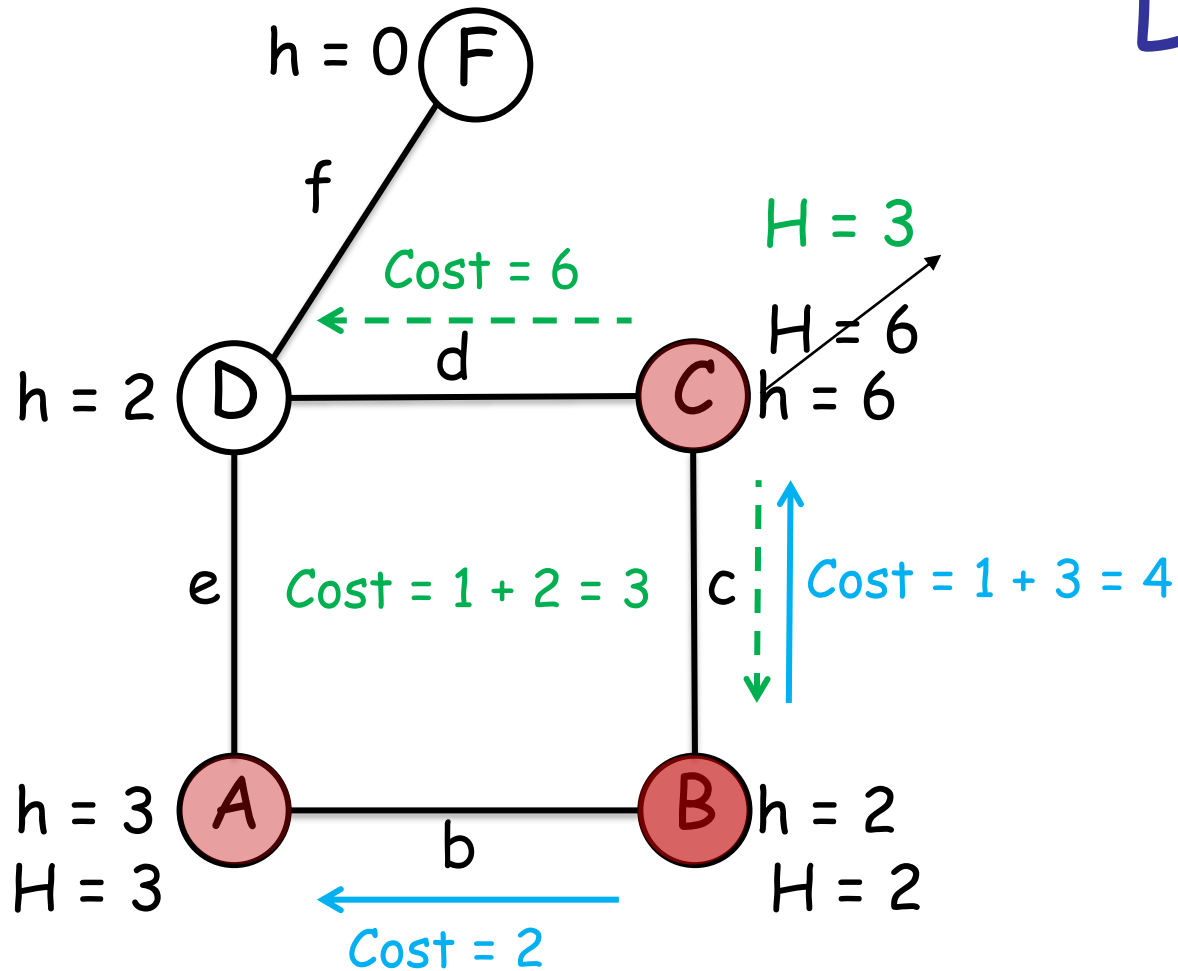
$s = C$

$\text{result}(C, c) = B$

$H(C) = 3$

$a = b$

# LRTA\* (Example)



$s' = B$

$s = C$

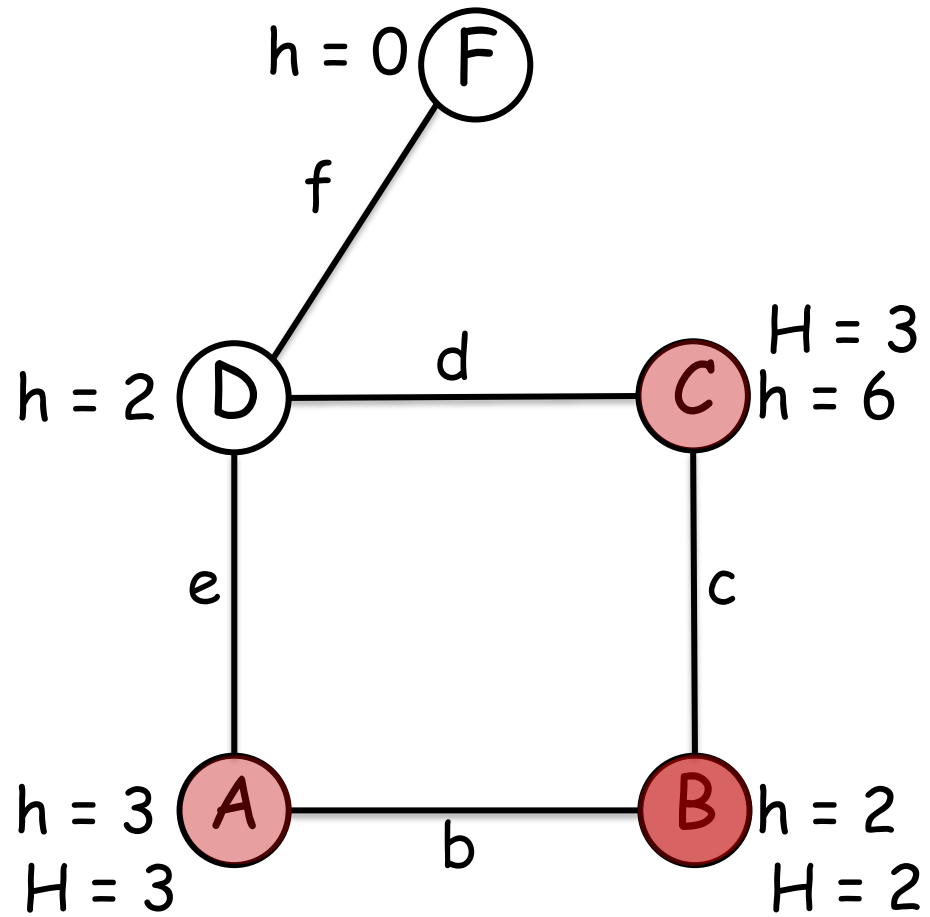
$\text{result}(C, c) = B$

$H(C) = 3$

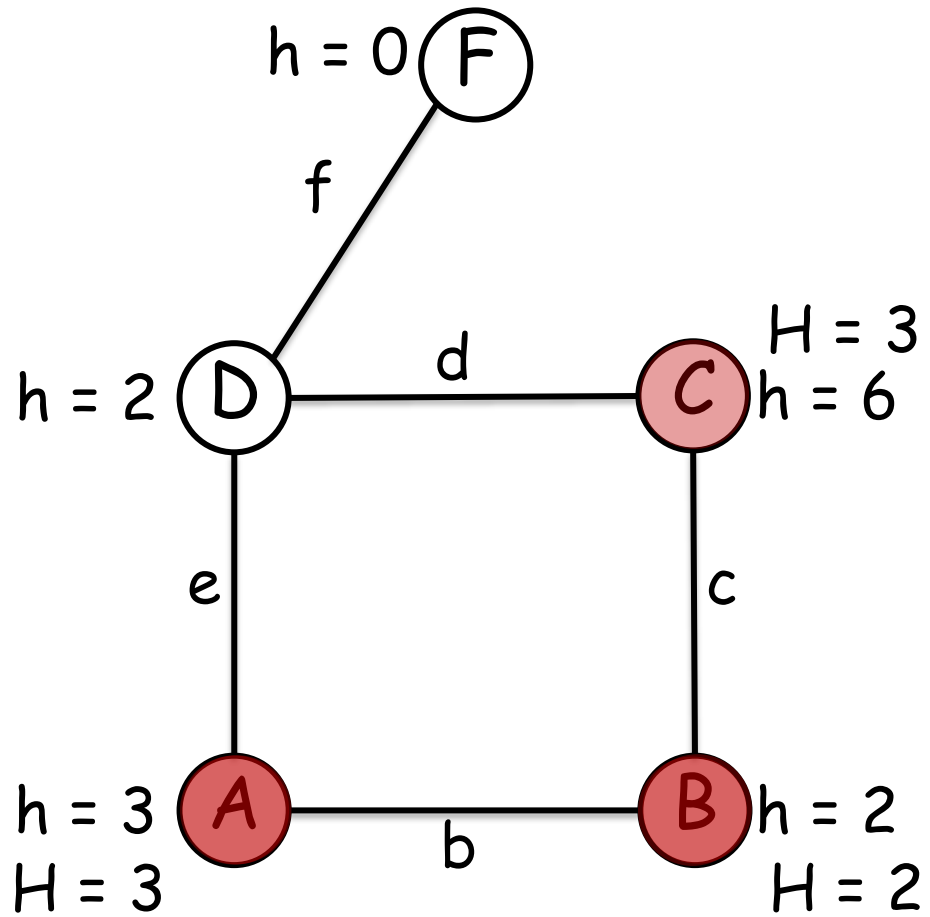
$a = b$

$s = B$

# LRTA\* (Example)



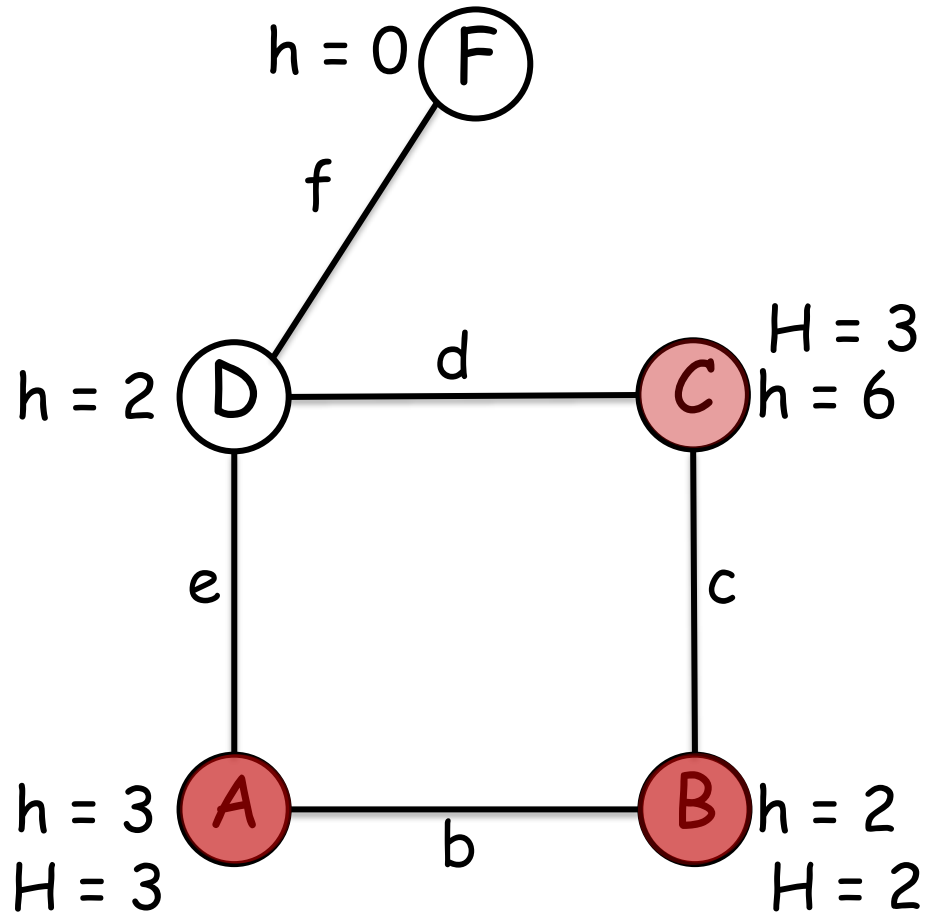
# LRTA\* (Example)



$s' = A$

$s = B$

# LRTA\* (Example)

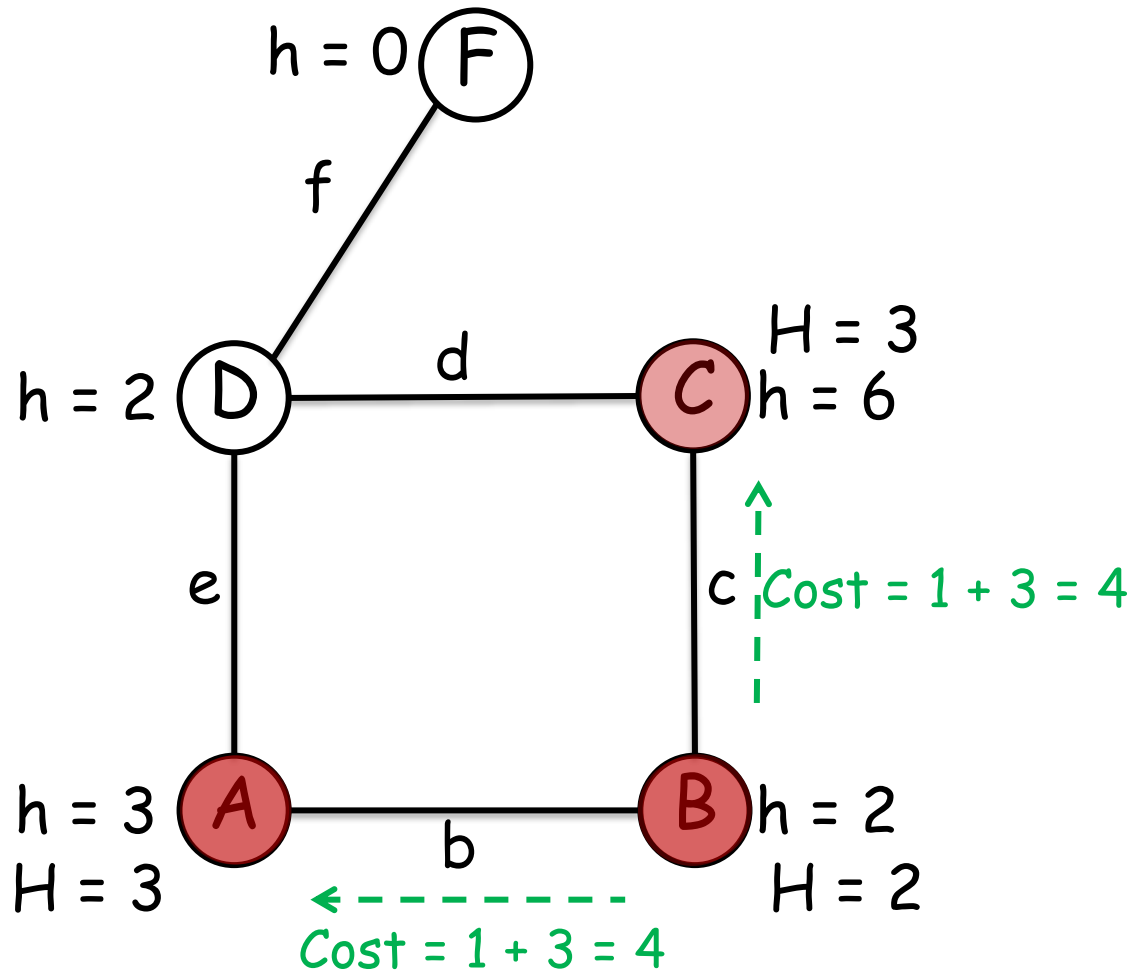


$s' = A$

$s = B$

$\text{result}(B, b) = A$

# LRTA\* (Example)



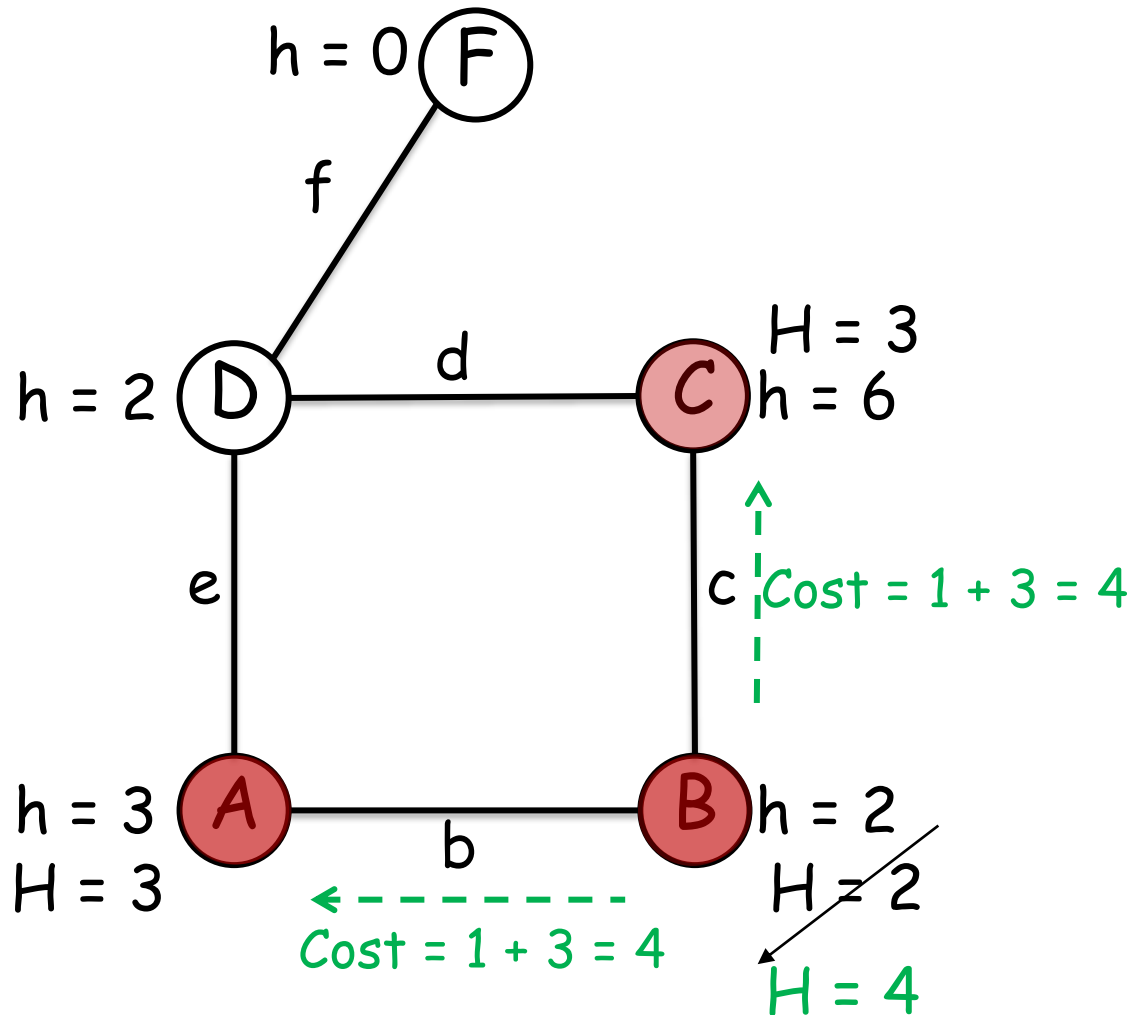
$s' = A$

$s = B$

$\text{result}(B, b) = A$



# LRTA\* (Example)



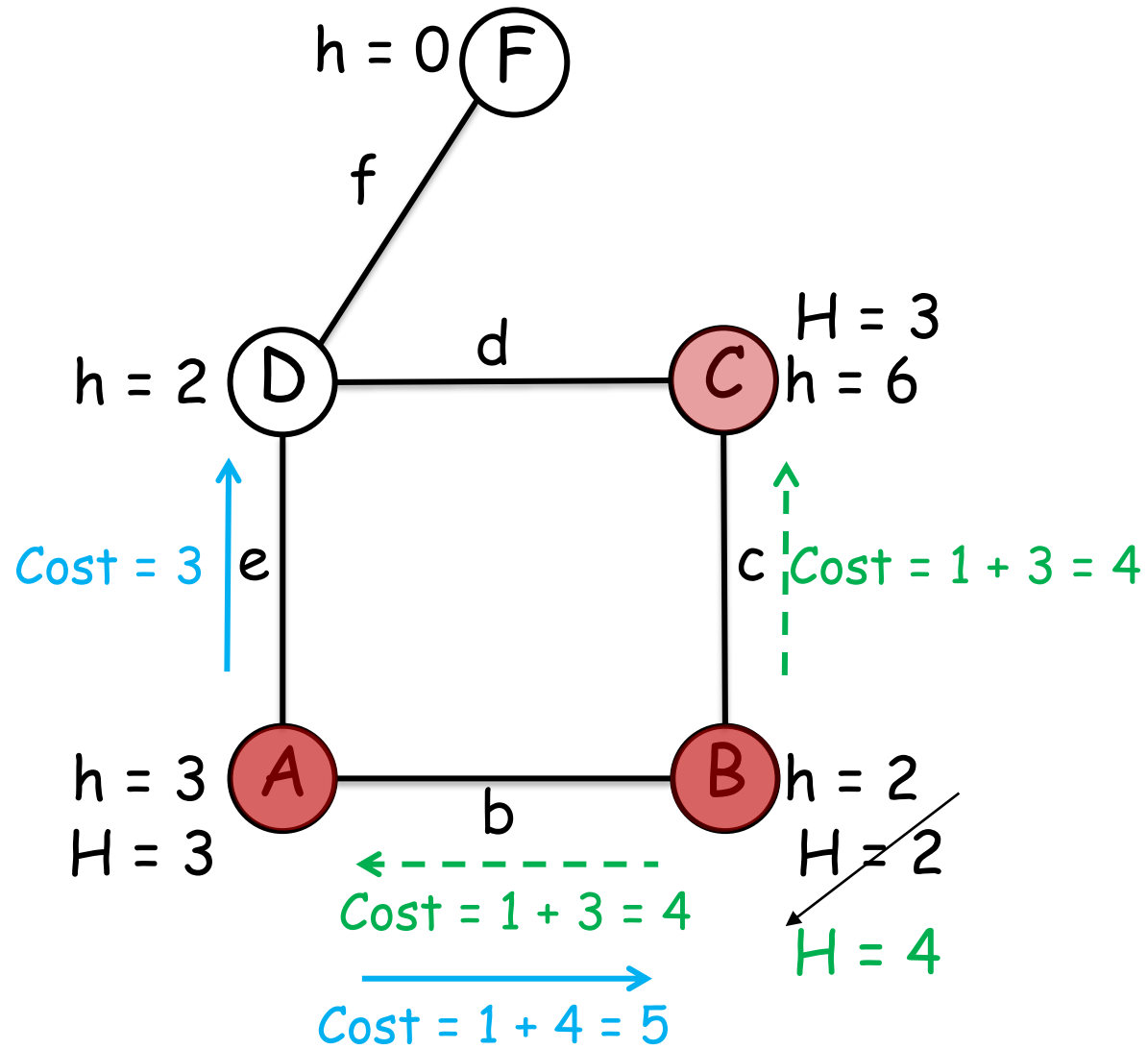
$s' = A$

$s = B$

$\text{result}(B, b) = A$

$H(B) = 4$

# LRTA\* (Example)



$s' = A$

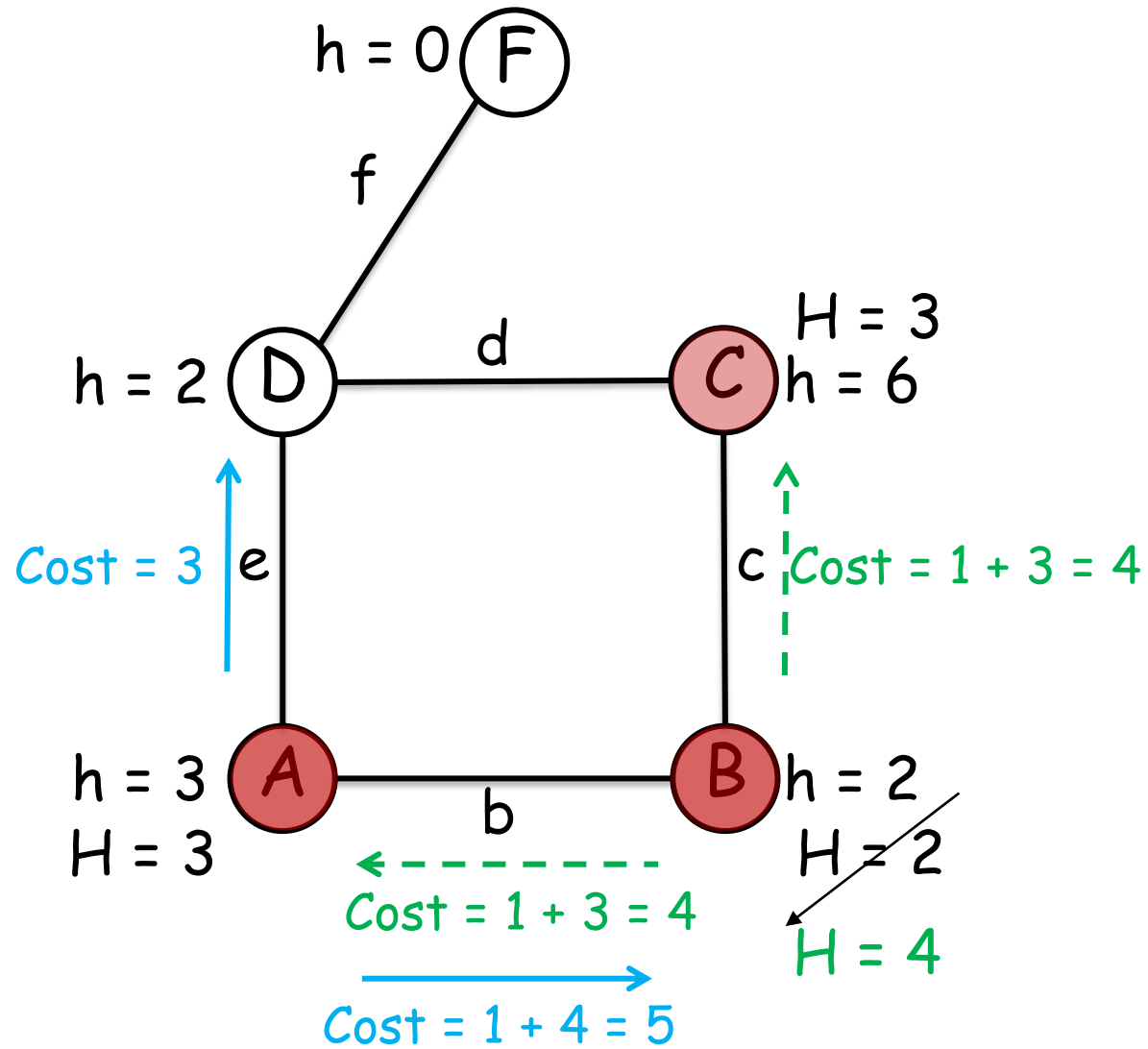
$s = B$

$\text{result}(B, b) = A$

$H(B) = 4$

$a = e$

# LRTA\* (Example)



$s' = A$

$s = B$

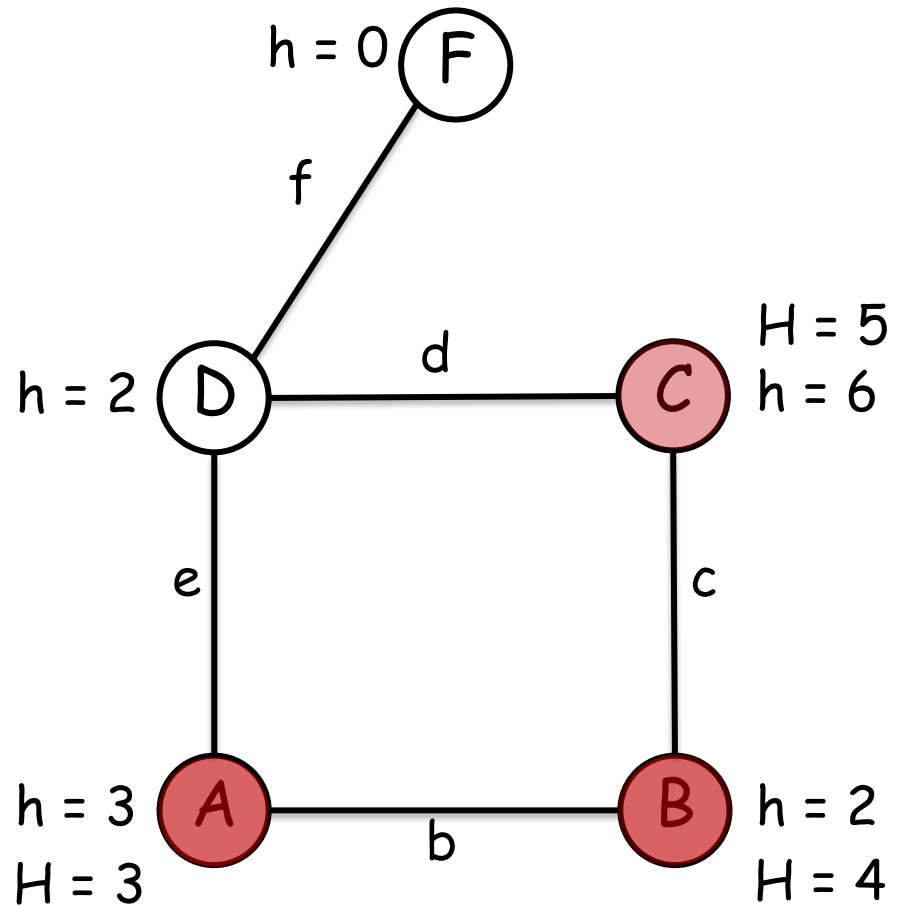
$\text{result}(B, b) = A$

$H(B) = 4$

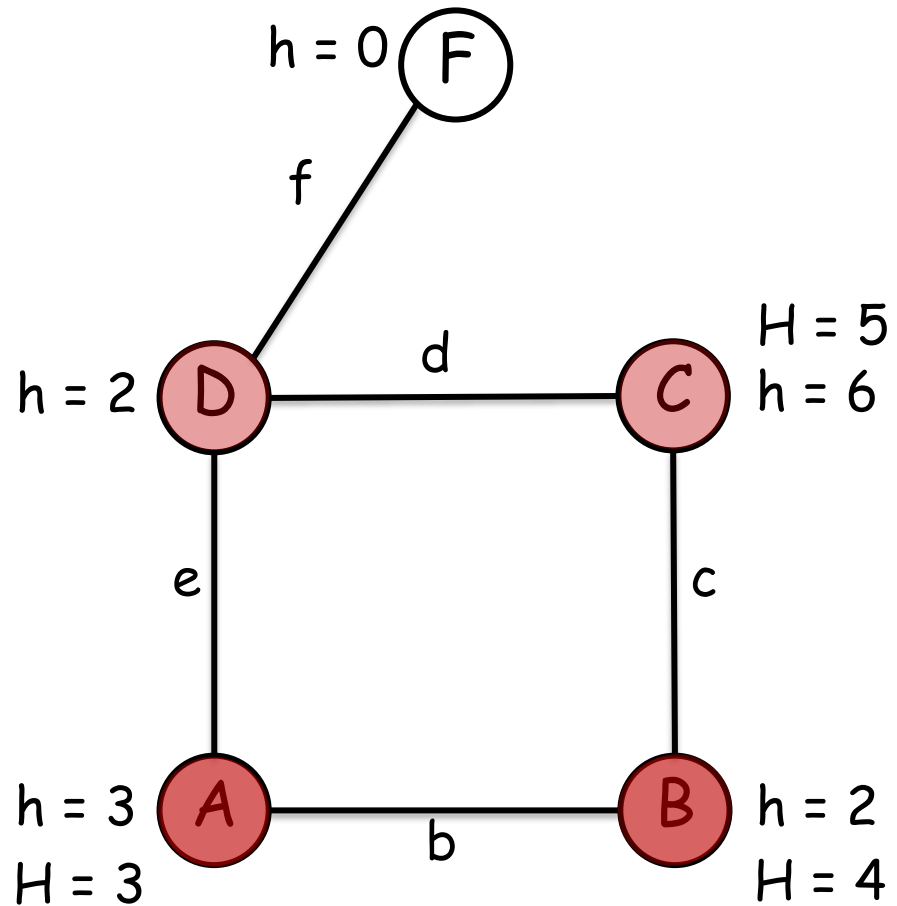
$a = e$

$s = A$

# LRTA\* (Example)



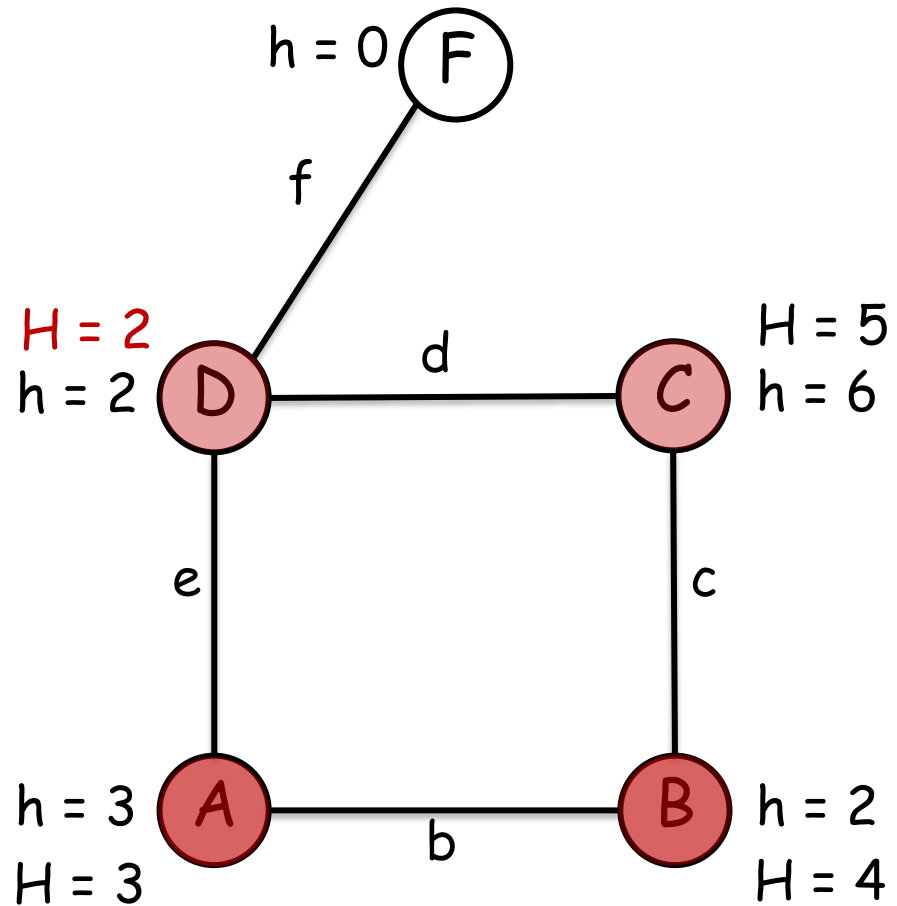
# LRTA\* (Example)



$s' = D$

$s = A$

# LRTA\* (Example)

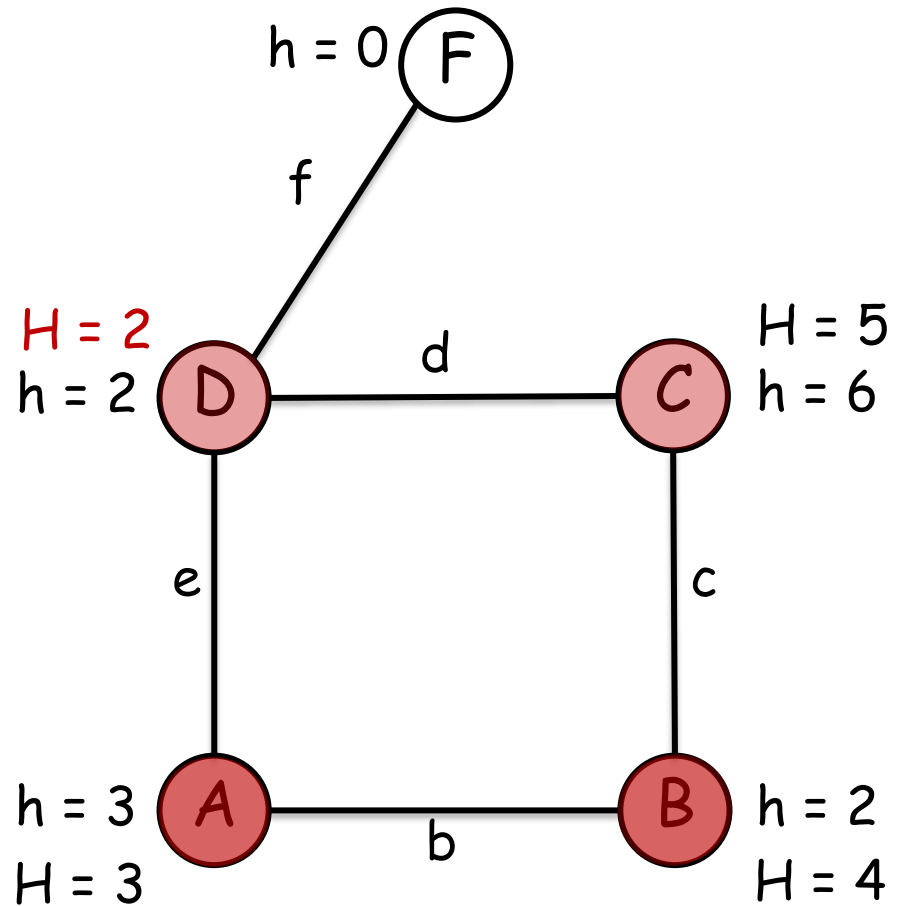


$s' = D$

$s = A$

$H(D) = h(D) = 2$

# LRTA\* (Example)

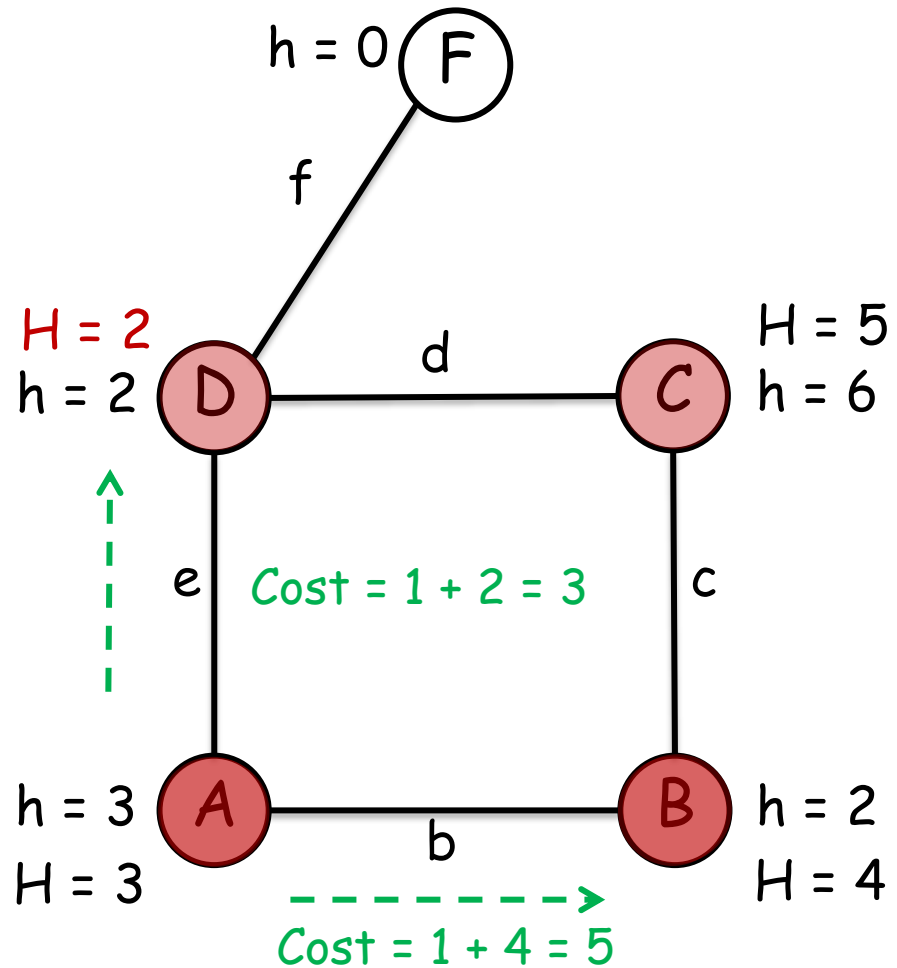


$s' = D$                        $s = A$

$H(D) = h(D) = 2$

$\text{result}(A, e) = D$

# LRTA\* (Example)



$s' = D$

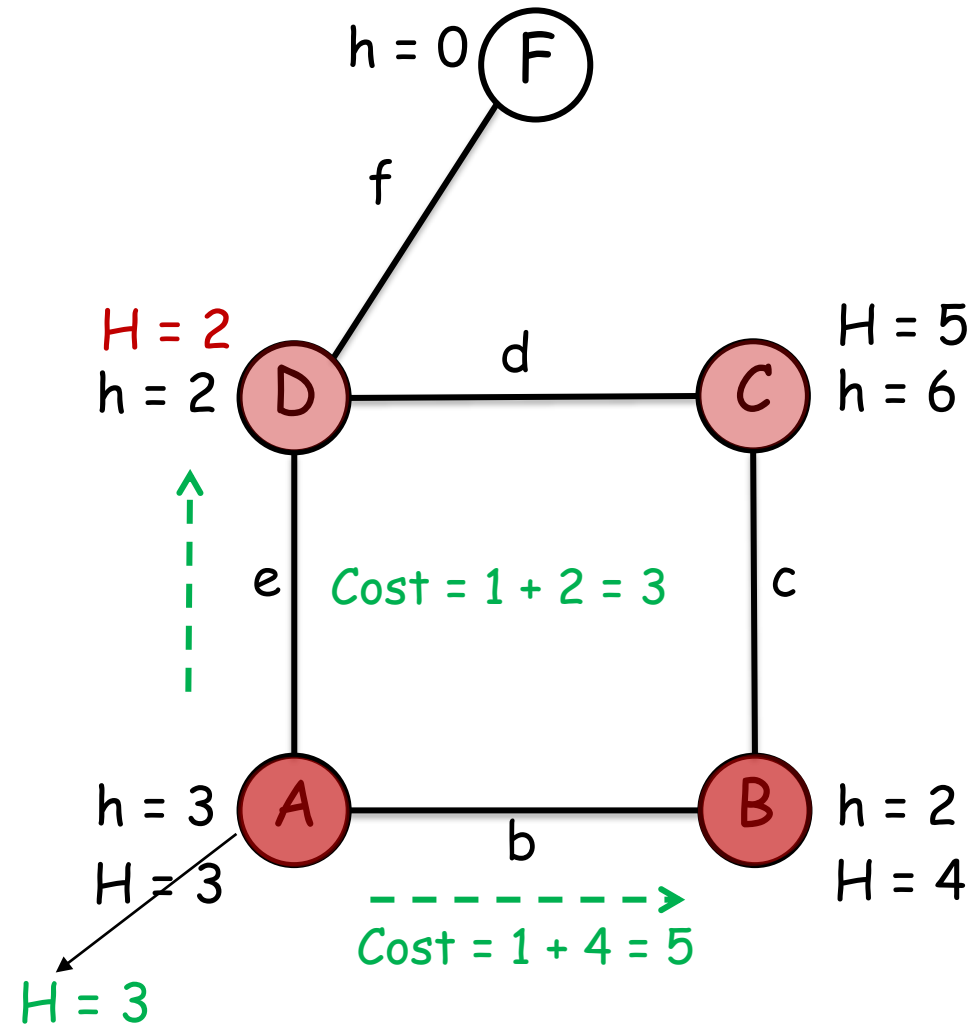
$s = A$

$H(D) = h(D) = 2$

$\text{result}(A, e) = D$



# LRTA\* (Example)



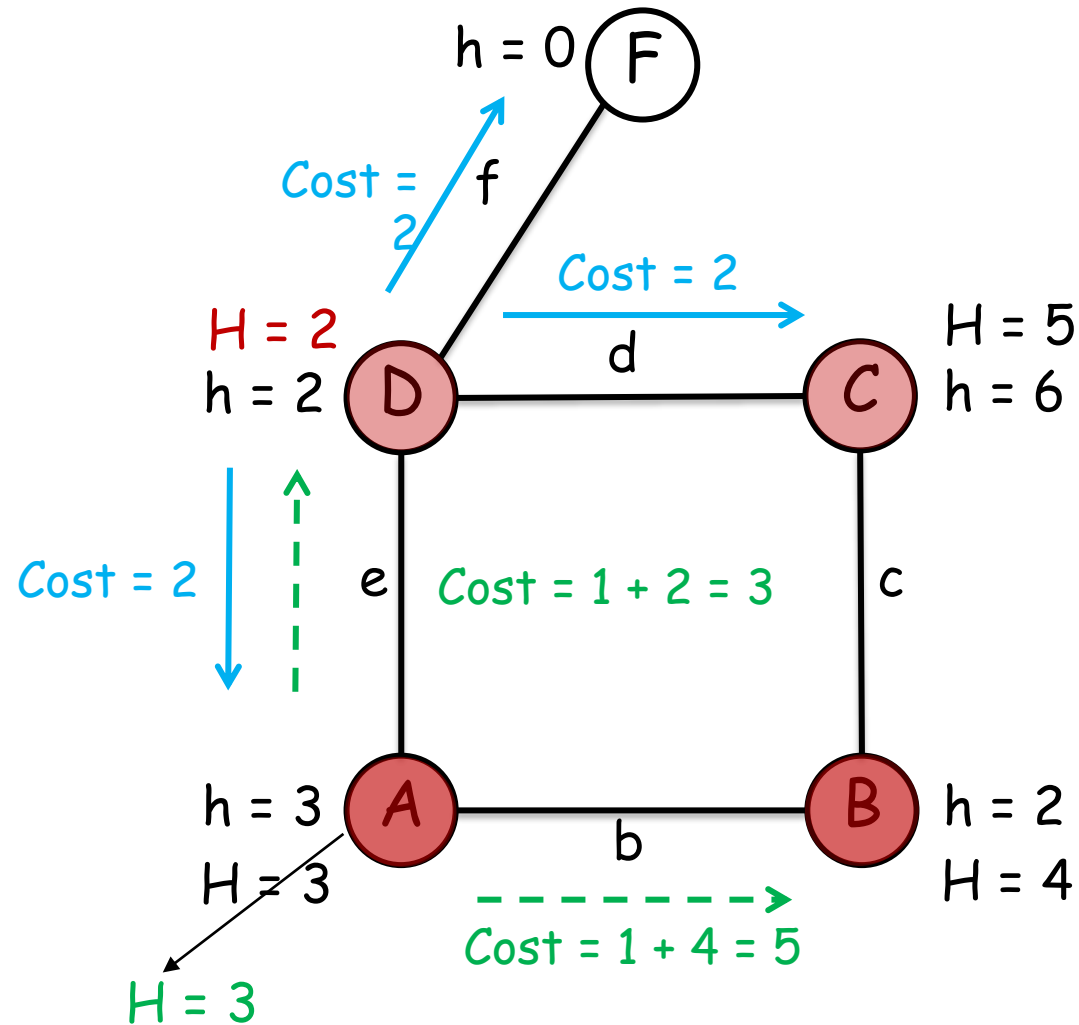
$s' = D$        $s = A$

$H(D) = h(D) = 2$

$\text{result}(A, e) = D$

$H(A) = 3$

# LRTA\* (Example)



$s' = D$        $s = A$

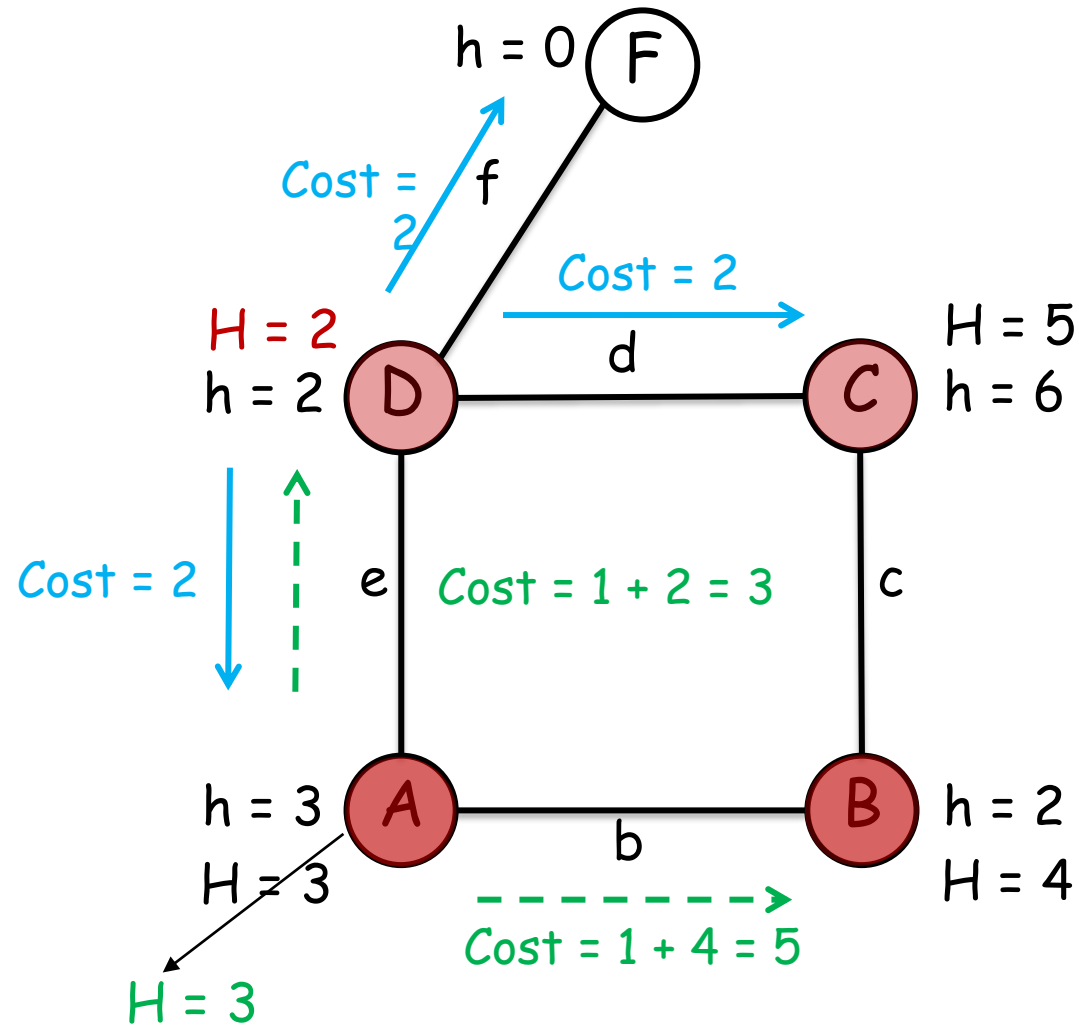
$H(D) = h(D) = 2$

$\text{result}(A, e) = D$

$H(A) = 3$

$a = f$

# LRTA\* (Example)



$s' = D$        $s = A$

$H(D) = h(D) = 2$

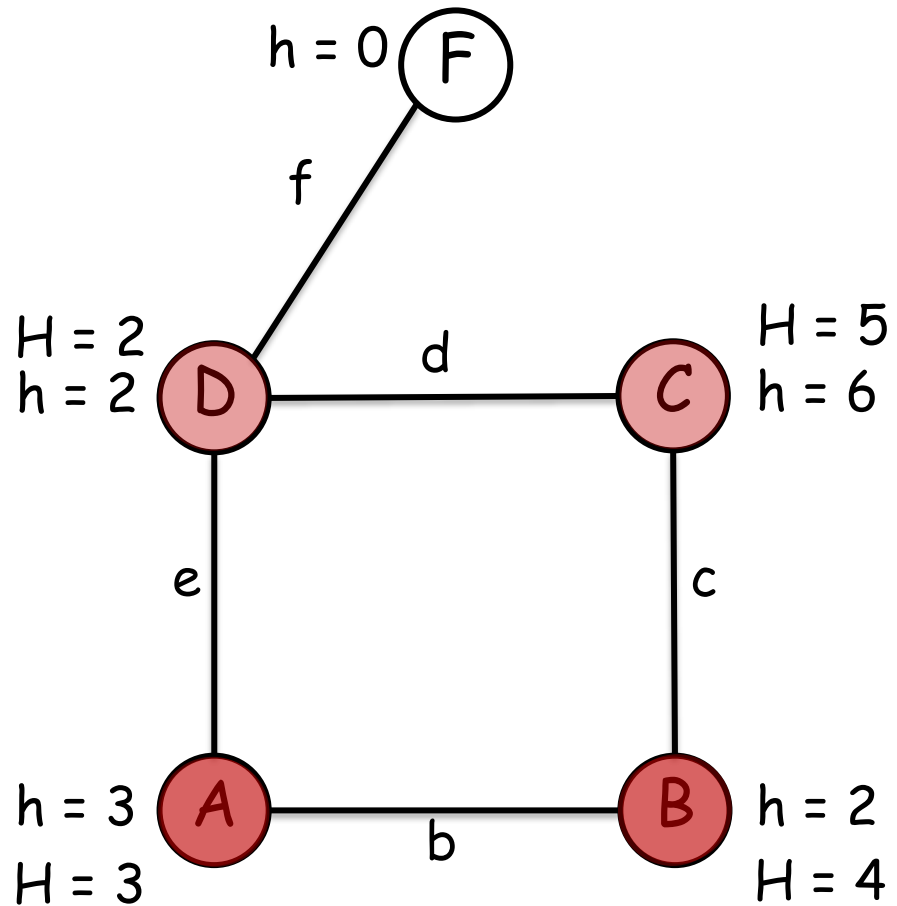
$\text{result}(A, e) = D$

$H(A) = 3$

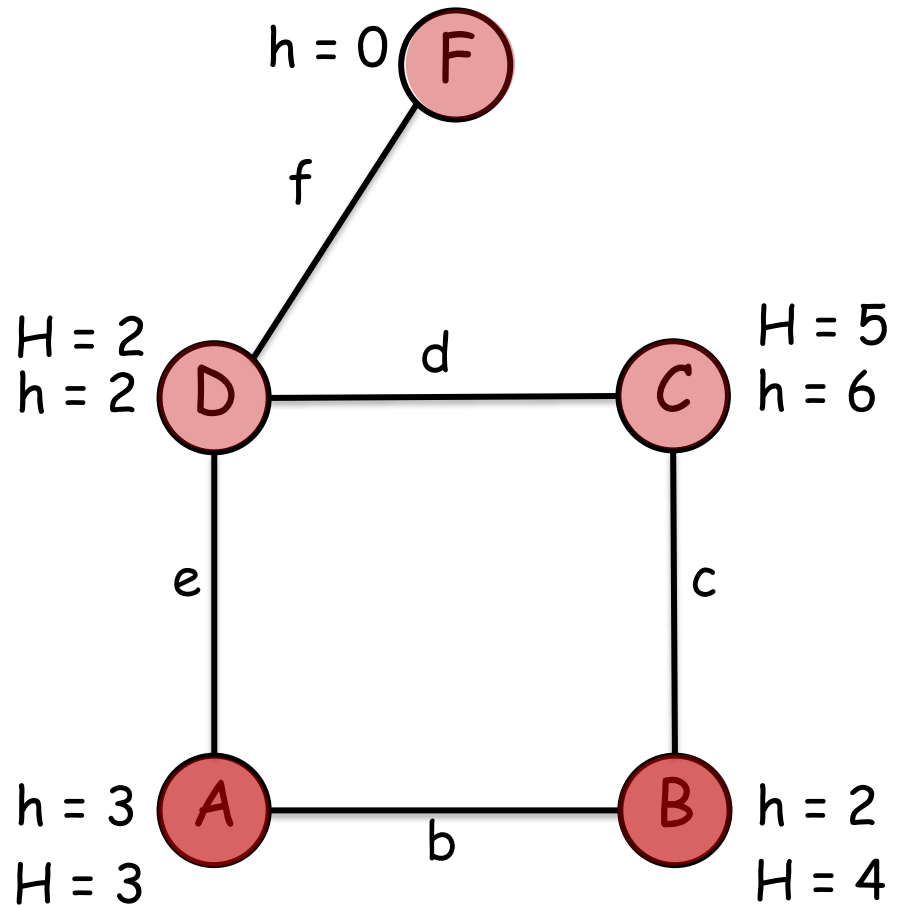
$a = f$

$s = D$

# LRTA\* (Example)



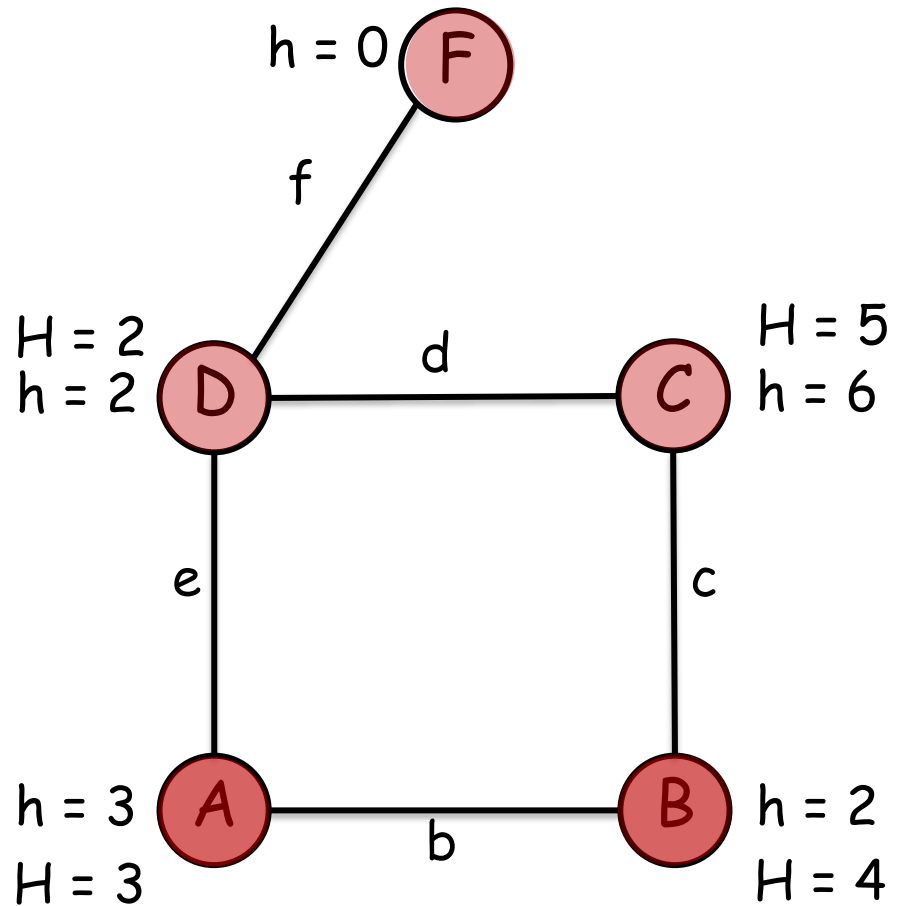
# LRTA\* (Example)



$s' = F$

$s = D$

# LRTA\* (Example)

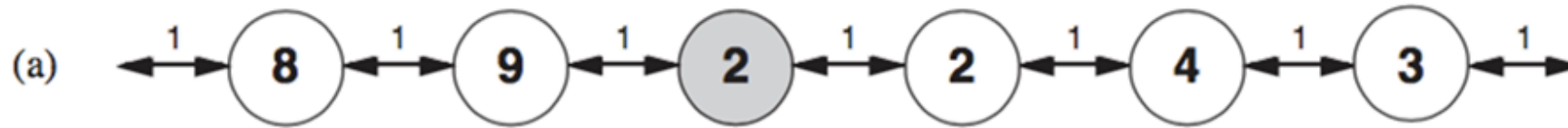


$s' = F$

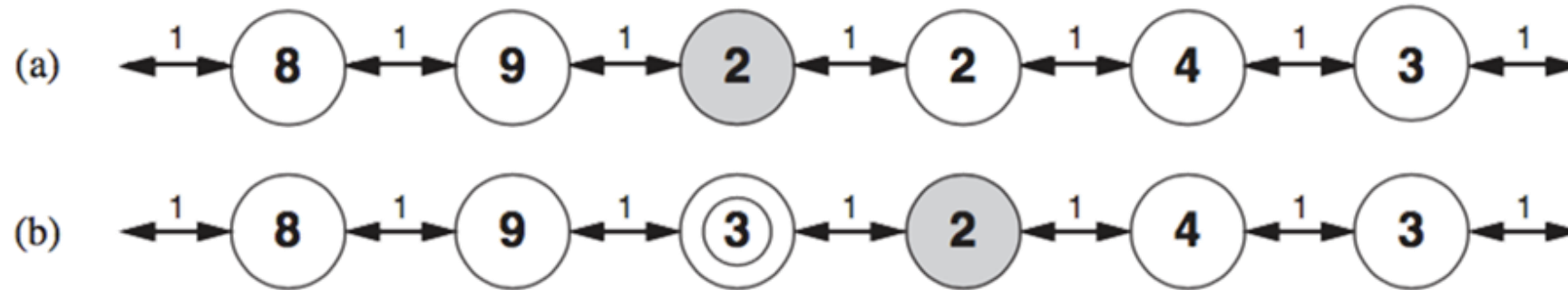
$s = D$

F is Goal return STOP

# LRTA\* (Example)

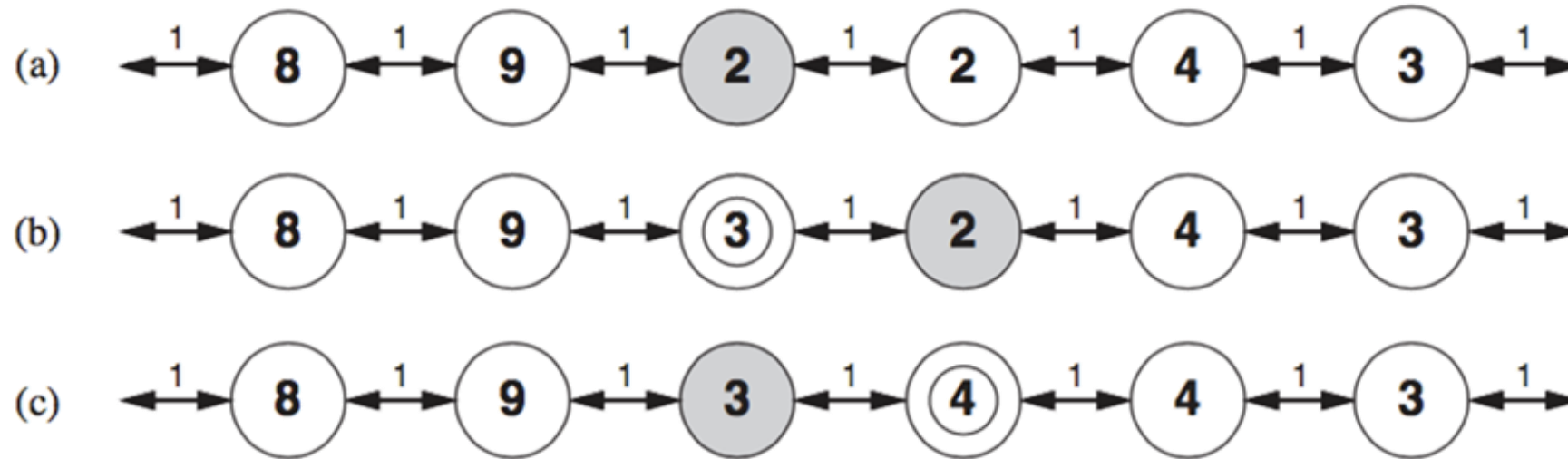


# LRTA\* (Example)

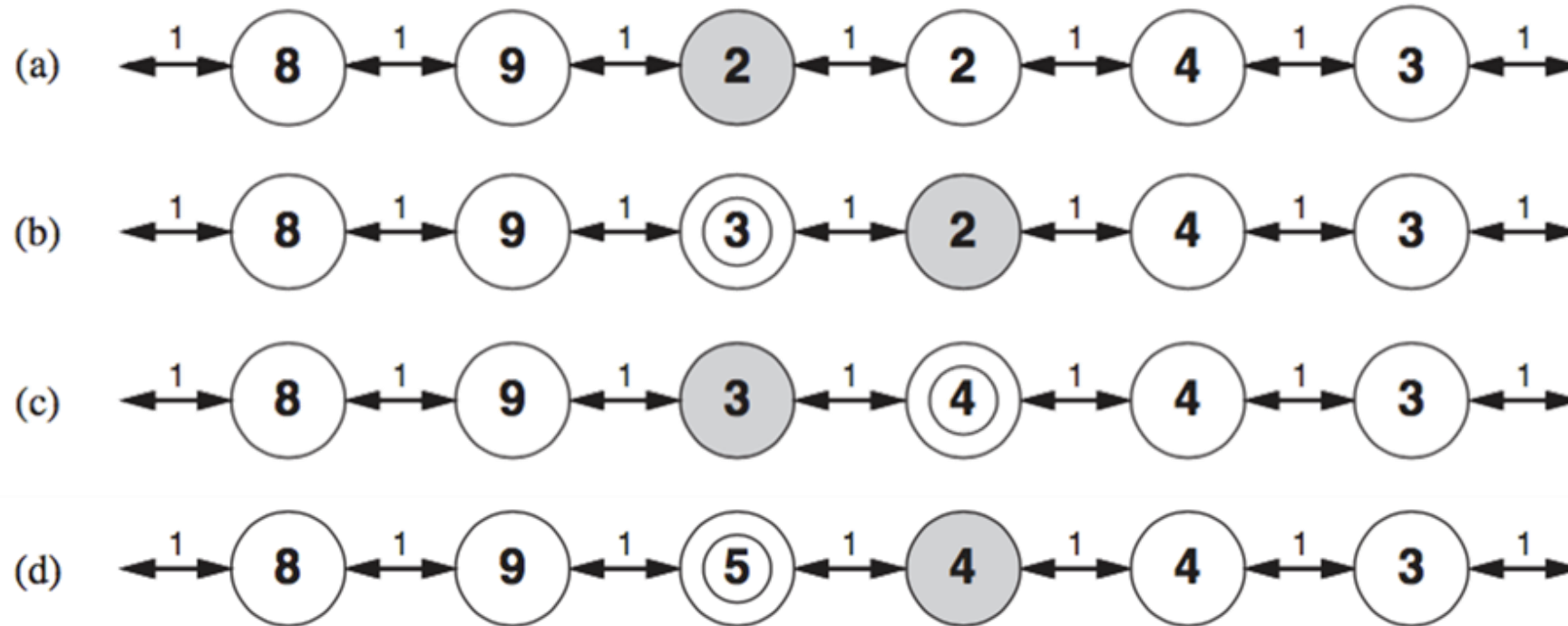




# LRTA\* (Example)



# LRTA\* (Example)



# LRTA\* (Example)

