Compiler Design

Lecture 5:
Syntax Analysis
Top-Down Parsing

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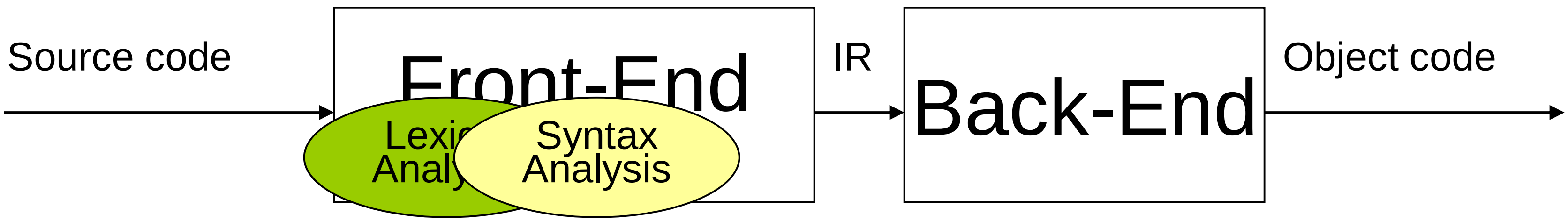
based on the slides of the course book

Parsing (Syntax Analysis)



- Syntax Analysis:
  - Derivation and parse trees
  - Top-down parsing
  - Bottom-up parsing

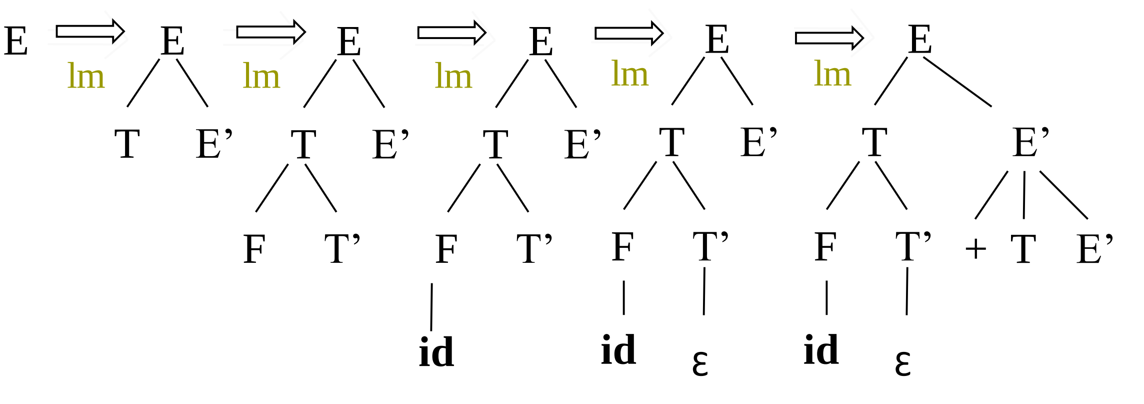
Outline

- **Introduction**
- Parsing as a Search
- Predictive Parsing

Introduction

- A Top-down parser tries to create a parse tree from the root towards the leafs scanning input from left to right
- It can be also viewed as finding a leftmost derivation for an input string
- Example: id+id*id

E -> TE'
E' -> +TE' | ε
T -> FT'
T' -> *FT' | ε
F -> (E) | **id**

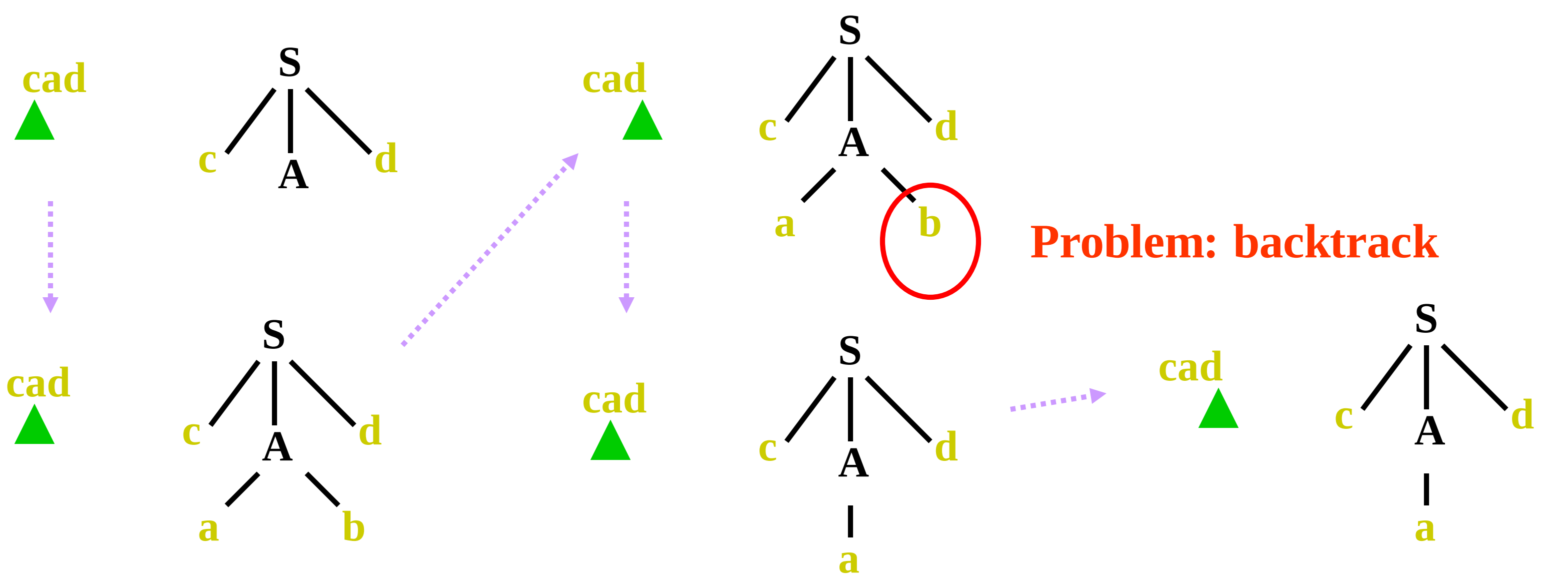


Top-Down Parsing

- Choose production rule based on input symbol
- May require backtracking to correct a wrong choice.
- Example:

$$\left.\begin{array}{l} S \rightarrow c\ A\ d \\ A \rightarrow ab \mid a \end{array}\right\}$$

input: cad



Top-Down Recursive-Descent Parsing

- Construct the root with the starting symbol of the grammar.
- Repeat until the fringe of the parse tree matches the input string:
  - Assuming a node labelled A, select a production with A on its left-hand-side and, for each symbol on its right-hand-side, construct the appropriate child.
  - When a terminal symbol is added to the fringe and it doesn't match the fringe, backtrack.
  - Find the next node to be expanded.

The key is picking the right production in the first step: that choice should be guided by the input string.

Example: Top-Down Recursive-Descent Parsing

Example:

1. *Goal* $\rightarrow$ *Expr*
2. *Expr* $\rightarrow$ *Expr* + *Term*
3. | *Expr* – *Term*
4. | *Term*
5. *Term* $\rightarrow$ *Term* * *Factor*
6. | *Term* / *Factor*
7. | *Factor*
8. *Factor* $\rightarrow$ *number*
9. | *id*

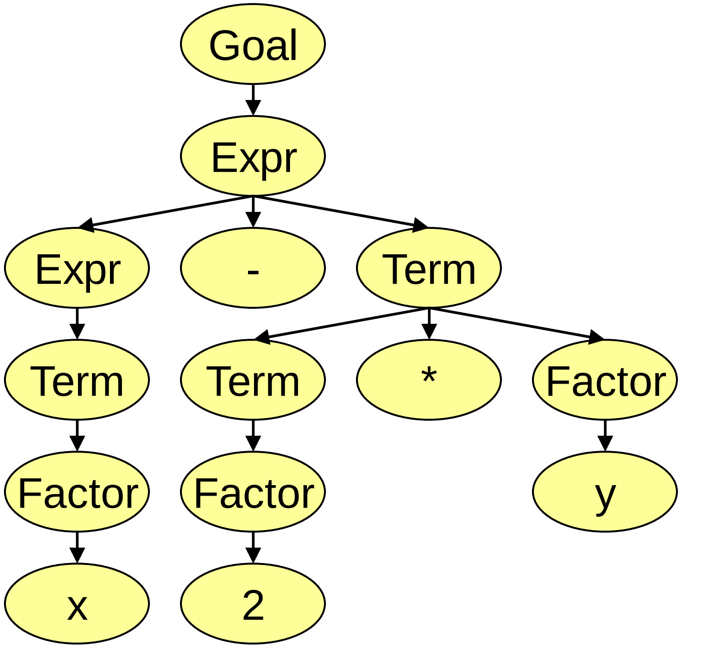
Parse *x-2*y*

Steps (one scenario from many)

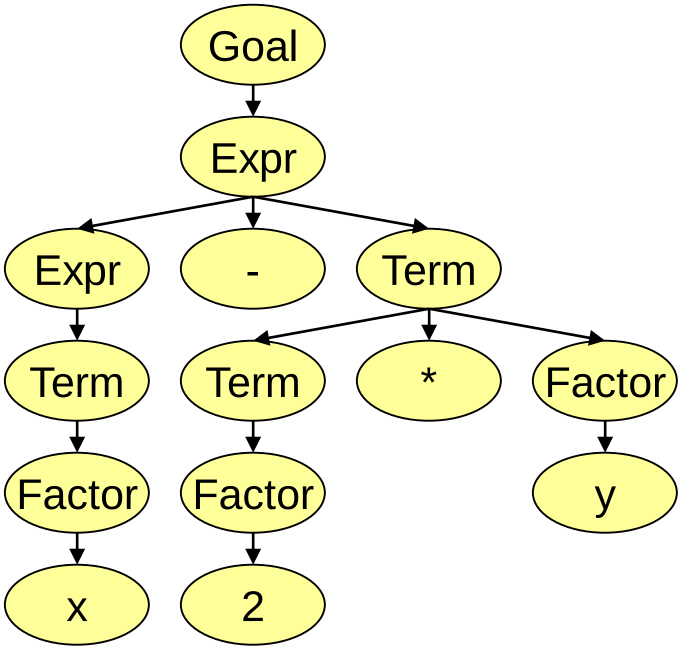
Rule	**Sentential Form**	**Input**
-	*Goal*	| x – 2*y
1	*Expr*	| x – 2*y
2	*Expr + Term*	| x – 2*y
4	*Term + Term*	| x – 2*y
7	*Factor + Term*	| x – 2*y
9	*id + Term*	| x – 2*y
Fail	*id + Term*	x | – 2*y
Back	*Expr*	| x – 2*y
3	*Expr – Term*	| x – 2*y
4	*Term – Term*	| x – 2*y
7	*Factor – Term*	| x – 2*y
9	*id – Term*	| x – 2*y
Match	*id – Term*	x – | 2*y
7	*id – Factor*	x – | 2*y
9	*id – num*	x – | 2*y
Fail	*id – num*	x – 2 | *y
Back	*id – Term*	x – | 2*y
5	*id – Term * Factor*	x – | 2*y
7	*id – Factor * Factor*	x – | 2*y
8	*id – num * Factor*	x – | 2*y
match	*id – num * Factor*	x – 2* | y
9	*id – num * id*	x – 2* | y
match	*id – num * id*	x – 2*y |

Example: Top-Down Recursive-Descent Parsing

Rule	Sentential Form	Input
-	*Goal*	| x – 2*y
1	*Expr*	| x – 2*y
2	*Expr + Term*	| x – 2*y
4	*Term + Term*	| x – 2*y
7	*Factor + Term*	| x – 2*y
9	*id + Term*	| x – 2*y
Fail	*id + Term*	x | – 2*y
Back	*Expr*	| x – 2*y
3	*Expr – Term*	| x – 2*y
4	*Term – Term*	| x – 2*y
7	*Factor – Term*	| x – 2*y
9	*id – Term*	| x – 2*y
Match	*id – Term*	x – | 2*y
7	*id – Factor*	x – | 2*y
9	*id – num*	x – | 2*y
Fail	*id – num*	x – 2 | *y
Back	*id – Term*	x – | 2*y
5	*id – Term * Factor*	x – | 2*y
7	*id – Factor * Factor*	x – | 2*y
8	*id – num * Factor*	x – | 2*y
match	*id – num * Factor*	x – 2* | y
9	*id – num * id*	x – 2* | y
match	*id – num * id*	x – 2*y |



Example: Top-Down Recursive-Descent Parsing



Other choices for expansion are possible:

Rule	Sentential Form	Input
-	*Goal*	| x – 2*y
1	*Expr*	| x – 2*y
2	*Expr + Term*	| x – 2*y
2	*Expr + Term + Term*	| x – 2*y
2	*Expr + Term + Term + Term*	| x – 2*y
2	*Expr + Term + Term + ... + Term*	| x – 2*y

- Wrong choice leads to non-termination!
- This is a bad property for a parser!
- Parser must make the right choice!

Challenges in Top-Down Parsing

- How can we know which productions to apply?
- In general, we can't.
  - There are some grammars for which the best we can do is guess and backtrack if we're wrong.
  - If we have to guess, how do we do it?

Outline

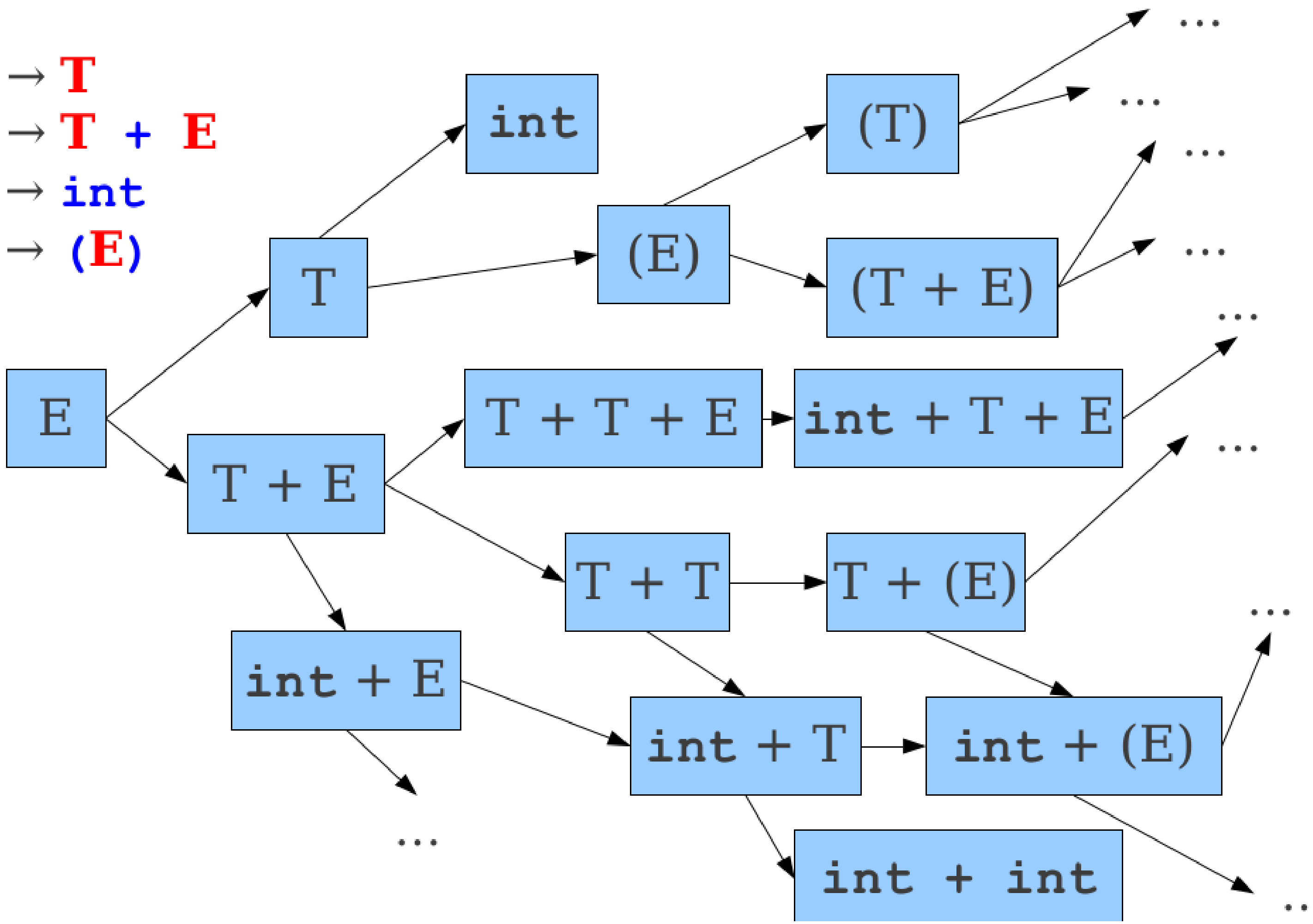
- Introduction
- **Parsing as a Search**
- Predictive Parsing

Parsing as a Search

- An idea: treat parsing as a graph search.
- Each node is a sentential form (a string of terminals and nonterminals derivable from the start symbol).
- There is an edge from node α to node β iff $\alpha \Rightarrow \beta$.

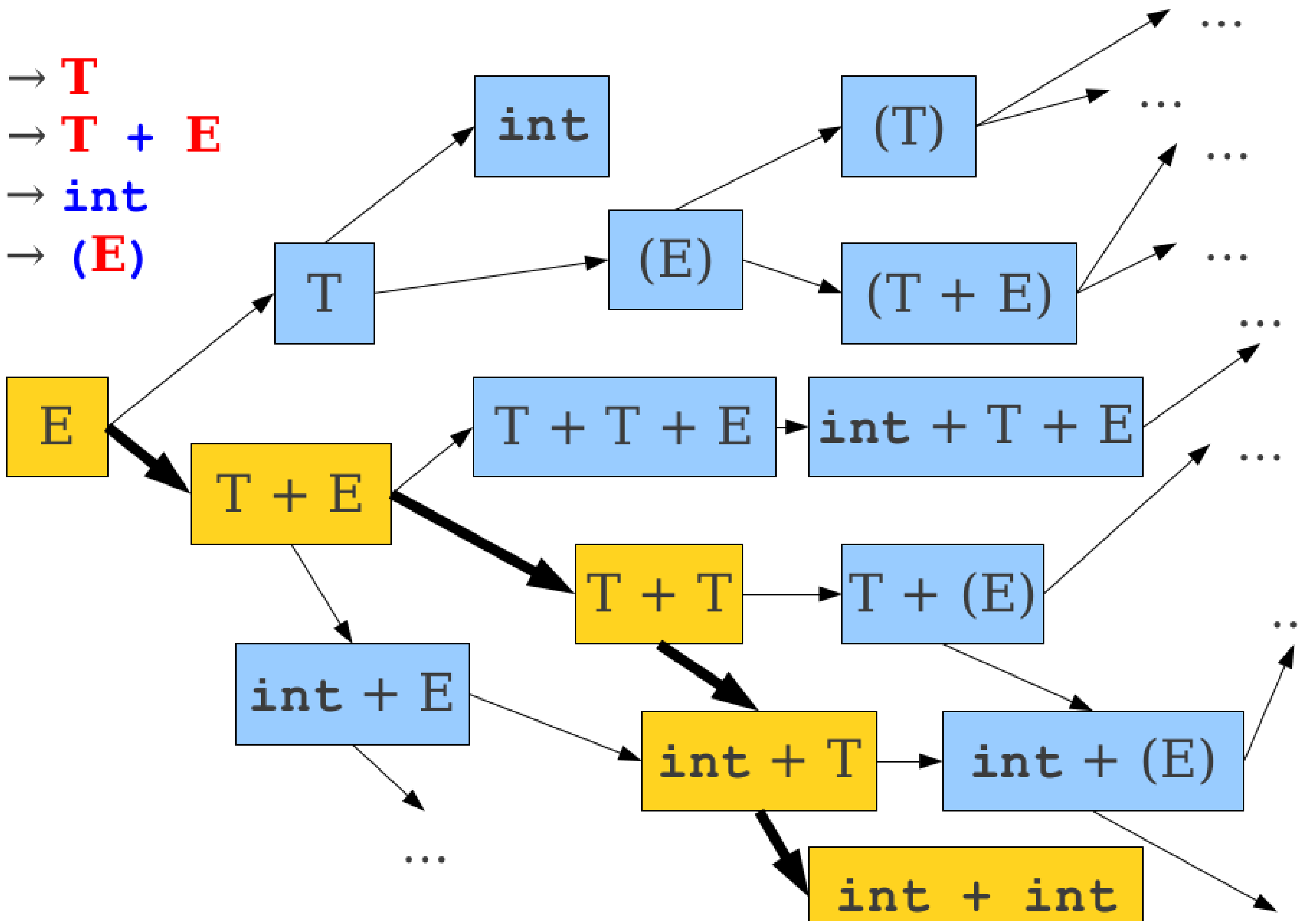
Parsing as a Search

E → T
E → T + E
T → int
T → (E)



Parsing as a Search

$E \rightarrow T$
$E \rightarrow T + E$
$T \rightarrow \texttt{int}$
$T \rightarrow (E)$



Our First Top-Down Algorithm

- Breadth-First Search
- Maintain a worklist of sentential forms, initially just the start symbol S.
- While the worklist isn't empty:
  - Remove an element from the worklist.
  - If it matches the target string, you're done.
  - Otherwise, for each possible string that can be derived in one step, add that string to the worklist.
- Can recover a parse tree by tracking what productions we applied at each step.

Breadth-First Search Parsing

- Example:

$E \rightarrow T$
$E \rightarrow T + E$
$T \rightarrow \texttt{int}$
$T \rightarrow (E)$

`int + int`

BFS is Slow

- Enormous time and memory usage:
  - Lots of wasted effort:
    - Generates a lot of sentential forms that couldn't possibly match.
    - But in general, extremely hard to tell whether a sentential form can match – that's the job of parsing!
  - High branching factor:
    - Each sentential form can expand in (potentially) many ways for each nonterminal it contains.

Reducing Wasted Effort

- Suppose we're trying to match a string γ.
- Suppose we have a sentential form $\tau = \alpha\omega$, where α is a string of terminals and ω is a string of terminals and nonterminals.
- If α isn't a prefix of γ, then no string derived from τ can ever match γ.
- If we can find a way to try to get a prefix of terminals at the front of our sentential forms, then we can start pruning out impossible options.

Reducing the Branching Factor

- If a string has many nonterminals in it, the branching factor can be high.
  - Sum of the number of productions of each nonterminal involved.
- If we can restrict which productions we apply, we can keep the branching factor lower.

Leftmost Derivations

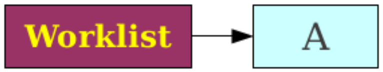
- Recall: A leftmost derivation is one where we always expand the leftmost symbol first.
- Updated algorithm:
  - Do a breadth-first search, only considering leftmost derivations.
    - Dramatically drops branching factor.
    - Increases likelihood that we get a prefix of nonterminals.
  - Prune sentential forms that can't possibly match.
    - Avoids wasted effort.

Leftmost BFS

- Substantial improvement over naïve algorithm.
- Will always find a valid parse of a program if one exists.
- Can easily be modified to find if a program can't be parsed.
- But, there are still problems.

Leftmost Breadth-First Search Parsing

- Example:



$A \rightarrow Aa \mid Ab \mid c$

caaaaaaaaa

Problems with Leftmost BFS

- Grammars like this can make parsing take exponential time.
- Also uses exponential memory.
- What if we search the graph with a different algorithm?

Leftmost DFS

- Idea: Use depth-first search.
- Advantages:
  - Lower memory usage: Only considers one branch at a time.
  - High performance: On many grammars, runs very quickly.
  - Easy to implement: Can be written as a set of mutually recursive functions.

Leftmost DFS

- Example:

E

$$E \rightarrow T$$
$$E \rightarrow T + E$$
$$T \rightarrow \texttt{int}$$
$$T \rightarrow (E)$$

`int + int`

Problems with Leftmost DFS

- Example:

$A \rightarrow Aa \mid c$

A
Aa
Aaa
Aaaa
Aaaaa

c

Problems with Leftmost DFS

- Left Recursion
- A grammar is left-recursive if it has a left-recursive nonterminal symbol with the following derivation

$$A \Rightarrow^{+} A\omega$$

for some string ω.

- A left-recursive grammar can cause a recursive-descent parser to go into an infinite loop.
- Leftmost DFS may fail on left-recursive grammars.

Left-Recursive Grammars

- Fortunately, in many cases it is possible to eliminate left recursion.

- **Eliminating left-recursion**:
  - In many cases, it is sufficient to replace

$$A \rightarrow Aa \mid b \quad \text{with} \quad A \rightarrow bA' \quad \text{and} \quad A' \rightarrow aA' \mid \varepsilon$$

- Example:

$$Sum \rightarrow Sum + number \mid number$$

would become:

$$Sum \rightarrow number \;\; Sum'$$

$$Sum' \rightarrow +number \;\; Sum' \mid \varepsilon$$

Eliminating Left Recursion

Example:

1. $Goal \rightarrow Expr$
2. $Expr \rightarrow Expr + Term$
3. $\quad\quad | \ Expr - Term$
4. $\quad\quad | \ Term$
5. $Term \rightarrow Term * Factor$
6. $\quad\quad | \ Term \ / \ Factor$
7. $\quad\quad | \ Factor$
8. $Factor \rightarrow number$
9. $\quad\quad | \ id$

Eliminating Left Recursion

Example:

1. $Goal \rightarrow Expr$
2. $Expr \rightarrow Expr + Term$
3. $\quad | \; Expr - Term$
4. $\quad | \; Term$
5. $Term \rightarrow Term * Factor$
6. $\quad | \; Term \,/\, Factor$
7. $\quad | \; Factor$
8. $Factor \rightarrow number$
9. $\quad | \; id$

$$
\begin{aligned}
Expr &\rightarrow Term\ Expr' \\
Expr' &\rightarrow +Term\ Expr' \\
&\quad |\ -Term\ Expr' \\
&\quad |\ \varepsilon \\
Term &\rightarrow Factor\ Term' \\
Term' &\rightarrow *Factor\ Term' \\
&\quad |\ /\ Factor\ Term' \\
&\quad |\ \varepsilon
\end{aligned}
$$

remain unchanged:

$$
\begin{aligned}
Goal &\rightarrow Expr \\
Factor &\rightarrow number \\
&\quad |\ id
\end{aligned}
$$

Eliminating Left Recursion

- Problem: If left recursion is two-or-more levels deep, this is not enough

$$\left.\begin{array}{l} S \rightarrow Aa \mid b \\ A \rightarrow Ac \mid Sd \mid \epsilon \end{array}\right\} \quad S \Rightarrow Aa \Rightarrow Sda$$

Left-Recursive Grammars

- General algorithm

1. Arrange the non-terminal symbols in order: $A_1, A_2, A_3, \ldots, A_n$

2. For i=1 to n do

for j=1 to i-1 do

I) replace each production of the form $A_i \rightarrow A_j\gamma$ *with*

the productions $A_i \rightarrow \delta_1\ \gamma \mid \delta_2\ \gamma \mid \ldots \mid \delta_k\ \gamma$

where $A_j \rightarrow \delta_1 \mid \delta_2 \mid \ldots \mid \delta_k$ *are all the current* A_j *productions*

II) eliminate the immediate left recursion among the A_i

Summary of Leftmost BFS/DFS

- Leftmost BFS works on all grammars.
- Worst-case runtime is exponential.
- Worst-case memory usage is exponential.
- Rarely used in practice.

- Leftmost DFS works on grammars without left recursion.
- Worst-case runtime is exponential.
- Worst-case memory usage is linear.
- Often used in a limited form as recursive descent.

Outline

- Introduction
- Parsing as a Search
- **Predictive Parsing**
  - LL(1) Parser

Predictive Parsing

- The leftmost DFS/BFS algorithms are backtracking algorithms.
  - Guess which production to use, then back up if it doesn't work.

- There is another class of parsing algorithms called predictive algorithms.
  - Based on remaining input, predict (without backtracking) which production to use.

Tradeoffs in Prediction

- Predictive parsers are fast.
  - Many predictive algorithms can be made to run in linear time.
  - Often can be table-driven for extra performance.

- Predictive parsers are weak.
  - Not all grammars can be accepted by predictive parsers.

- Trade expressiveness for speed.

Exploiting Lookahead

- Given just the start symbol, how do you know which productions to use to get to the input program?

- Idea: Use lookahead tokens.

- When trying to decide which production to use, look at some number of tokens of the input to help make the decision.

Implementing Predictive Parsing

- Predictive parsing is only possible if we can predict which production to use given some number of lookahead tokens.
  - Increasing the number of lookahead tokens increases the number of grammars we can parse, but complicates the parser.
  - Decreasing the number of lookahead tokens decreases the number of grammars we can parse, but simplifies the parser.

Predictive Parsing

- Example

E

$$\mathbf{E} \rightarrow \mathbf{T}$$
$$\mathbf{E} \rightarrow \mathbf{T} + \mathbf{E}$$
$$\mathbf{T} \rightarrow \texttt{int}$$
$$\mathbf{T} \rightarrow (\mathbf{E})$$

int	+	(	int	+	int	)

Predictive Parsing

- Example

$E \rightarrow T$
$E \rightarrow T + E$
$T \rightarrow$ int
$T \rightarrow (E)$

E
T + E
int + E
int + T
int + (E)
int + (T + E)
int + (int + E)
int + (int + T)
int + (int + int)

int	+	(	int	+	int	)

Parsing – Top-Down & Predictive

- Top-Down Parsing:

Parse tree / derivation of a token string occurs in a top down fashion.

type → *simple* Start symbol
| ↑ **id**
| **array** [*simple*] **of** *type*
simple → **integer**
| **char**
| **num dotdot num**

- Example:

input:

array [**num dotdot num**] **of integer**

Parsing would begin with

type → **???**

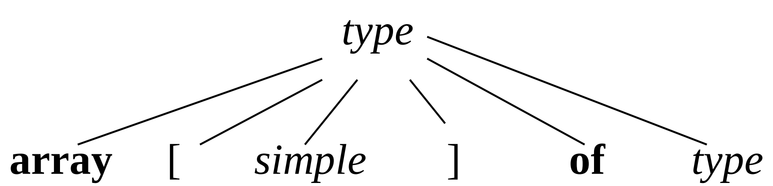
Top-Down Parse

Lookahead symbol

Input : **array** [**num dotdot num**] **of integer**

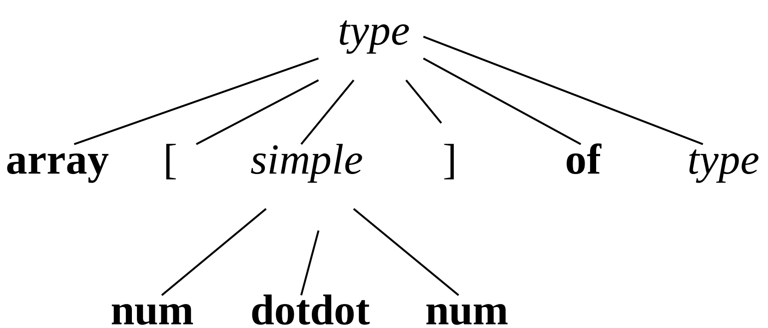
type

?



Lookahead symbol

Input : **array** [**num dotdot num**] **of integer**



type → *simple* Start symbol

| ↑ **id**

| **array** [*simple*] **of** *type*

simple → **integer**

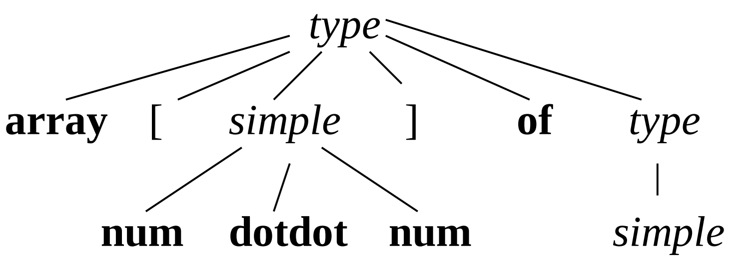
| **char**

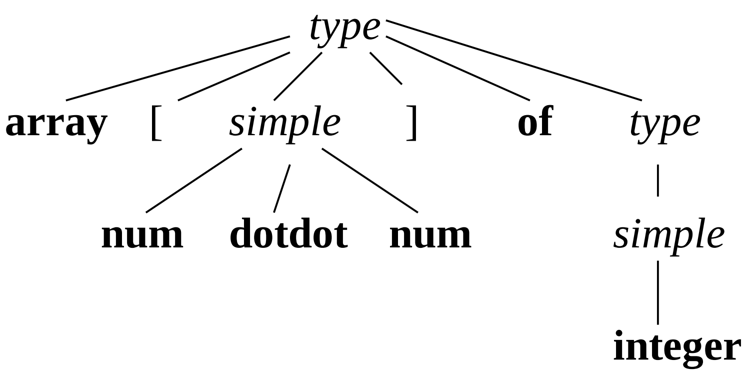
| **num dotdot num**

Top-Down Parse

Lookahead symbol

Input : **array** [**num dotdot num**] **of** **integer**





type → *simple* Start symbol

| ↑ **id**

| **array** [*simple*] **of** *type*

simple → **integer**

| **char**

| **num dotdot num**

Recursive Descent Top-Down Parsing

- Parser operates by attempting to match tokens in the input stream

type → *simple* **Start symbol**
  | ↑ **id**
  | **array** [*simple*] **of** *type*
simple → **integer**
  | **char**
  | **num dotdot num**

array [**num dotdot num**] **of integer**

procedure *match* (*t* : token) ;
begin
  if *lookahead* = *t* **then**
    lookahead := *nexttoken*
  else *error*
end ;

Recursive Descent Top-Down Parsing

array [**num dotdot num**] **of** **integer**

type → *simple* **Start symbol**
| ↑ **id**
| **array** [*simple*] **of** *type*
simple → **integer**
| **char**
| **num dotdot num**

```
procedure simple ;
begin
    if lookahead = integer then match ( integer );
    else if lookahead = char then match ( char );
        else if lookahead = num then begin
                match (num); match (dotdot); match (num)
            end
            else error
end ;
```

Recursive Descent Top-Down Parsing

array [**num dotdot num**] **of** **integer**

type → *simple* — **Start symbol**
  | ↑ **id**
  | **array** [*simple*] **of** *type*
simple → **integer**
  | **char**
  | **num dotdot num**

procedure *type* ;
begin
  if *lookahead* is in { **integer, char, num** } **then** *simple*
  else if *lookahead* = '↑' **then** **begin** *match* ('↑') ; *match*(**id**) **end**
   else if *lookahead* = **array** **then** **begin**
    match(**array**); *match*('['); simple; *match*(']'); *match*(**of**); *type*
    end
    else *error*
end ;

Outline

- Introduction
- Parsing as a Search
- Predictive Parsing
  - LL(1) Parser

A Simple Predictive Parser: LL(1)

- Top-down, predictive parsing:
  - L: Left-to-right scan of the tokens
  - L: Leftmost derivation.
  - (1): One token of lookahead
- Construct a leftmost derivation for the sequence of tokens.
- When expanding a nonterminal, we predict the production to use by looking at the next token of the input. The decision is forced.

LL(1) Parse Table

$E \rightarrow$ `int`
$E \rightarrow$ `(`E Op E`)`
$Op \rightarrow$ `+`
$Op \rightarrow$ `*`

	int	(	)	+	*
E	int	(E Op E)			
Op				+	*

LL(1) Parsing

- Predict step
  - If the first symbol of our is a nonterminal. We then look at our parsing table to see what production to use.

- Match step
  - If the first symbol of our guess is now a terminal symbol. We thus match it against the first symbol of the string to parse.

Example: LL(1) Parsing

E$	(int + (int * int))$

(1) E → int
(2) E → (E Op E)
(3) Op → +
(4) Op → *

	int	(	)	+	*
E	1	2			
Op				3	4

Example: LL(1) Parsing

E$	(int + (int * int))$

(1) E → int
(2) E → (E Op E)
(3) Op → +
(4) Op → *

	int	(	)	+	*
E	1	2			
Op				3	4

The $ symbol is the end-of-input marker and is used by the parser to detect when we have reached the end of the input. It is not a part of the grammar.

(1) E → `int`
(2) E → `(`E Op E`)`
(3) Op → `+`
(4) Op → `*`

	int	(	)	+	*
E	1	2			
Op				3	4

E$	(int + (int * int))$
(E Op E)$	(int + (int * int))$
E Op E)$	int + (int * int))$
int Op E)$	int + (int * int))$
Op E)$	+ (int * int))$
+ E)$	+ (int * int))$
E)$	(int * int))$
(E Op E))$	(int * int))$
E Op E))$	int * int))$
int Op E))$	int * int))$
Op E))$	* int))$
* E))$	* int))$
E))$	int))$
int))$	int))$
))$	))$
)$	)$
$	$

LL(1) Error Detection

(1) E → int
(2) E → (E Op E)
(3) Op → +
(4) Op → *

E$	int + int$

	int	(	)	+	*
E	1	2			
Op				3	4

LL(1) Error Detection

(1) E → int
(2) E → (E Op E)
(3) Op → +
(4) Op → *

E$	int + int$
int $	int + int$
$	+ int$

	int	(	)	+	*
E	1	2			
Op				3	4

LL(1) Error Detection

(1) E → int
(2) E → (E Op E)
(3) Op → +
(4) Op → *

E$	(int (int))$

	int	(	)	+	*
E	1	2			
Op				3	4

LL(1) Error Detection

(1) E → int
(2) E → (E Op E)
(3) Op → +
(4) Op → *

	int	(	)	+	*
E	1	2			
Op				3	4

E$	(int (int))$
(E Op E)$	(int (int))$
E Op E)$	int (int))$
int Op E)$	int (int))$
Op E)$	(int))$

The LL(1) Algorithm

- Suppose a grammar has start symbol S and LL(1) parsing table T. We want to parse string ω
- Initialize a stack containing S$.
- Repeat until the stack is empty:
  - Let the next character of ω be t.
  - If the top of the stack is a terminal r:
    - If r and t don't match, report an error.
    - Otherwise consume the character t and pop r from the stack.
  - Otherwise, the top of the stack is a nonterminal A:
    - If T[A, t] is undefined, report an error.
    - Replace the top of the stack with T[A, t].

A Simple LL(1) Grammar

$$
\begin{aligned}
\textbf{STMT} \;&\rightarrow\; \texttt{if}\ \textbf{EXPR}\ \texttt{then}\ \textbf{STMT} \\
&\mid\; \texttt{while}\ \textbf{EXPR}\ \texttt{do}\ \textbf{STMT} \\
&\mid\; \textbf{EXPR}\ \texttt{;} \\
\textbf{EXPR} \;&\rightarrow\; \textbf{TERM}\ \texttt{->}\ \texttt{id} \\
&\mid\; \texttt{zero?}\ \textbf{TERM} \\
&\mid\; \texttt{not}\ \textbf{EXPR} \\
&\mid\; \texttt{++}\ \texttt{id} \\
&\mid\; \texttt{--}\ \texttt{id} \\
\textbf{TERM} \;&\rightarrow\; \texttt{id} \\
&\mid\; \texttt{constant}
\end{aligned}
$$

A Simple LL(1) Grammar

```
STMT  → if EXPR then STMT
      | while EXPR do STMT
      | EXPR ;

EXPR  → TERM -> id
      | zero? TERM
      | not EXPR
      | ++ id
      | -- id

TERM  → id
      | constant
```

```
id -> id;

while not zero? id
   do --id;

if not zero? id then
   if not zero? id then
       constant -> id;
```

Constructing LL(1) Parse Tables

STMT	→	`if` **EXPR** `then` **STMT**	(1)
	|	`while` **EXPR** `do` **STMT**	(2)
	|	**EXPR** `;`	(3)
EXPR	→	**TERM** `-> id`	(4)
	|	`zero?` **TERM**	(5)
	|	`not` **EXPR**	(6)
	|	`++ id`	(7)
	|	`-- id`	(8)
TERM	→	`id`	(9)
	|	`constant`	(10)

	if	then	while	do	zero?	not	++	--	→	id	const	;
STMT												
EXPR												
TERM												

Constructing LL(1) Parse Tables

STMT	→	`if` EXPR `then` STMT	(1)
	|	`while` EXPR `do` STMT	(2)
	|	EXPR `;`	(3)
EXPR	→	TERM `->` `id`	(4)
	|	`zero?` TERM	(5)
	|	`not` EXPR	(6)
	|	`++` `id`	(7)
	|	`--` `id`	(8)
TERM	→	`id`	(9)
	|	`constant`	(10)

	if	then	while	do	zero?	not	++	--	→	id	const	;
STMT												
EXPR												
TERM										9	10	

Constructing LL(1) Parse Tables

STMT	→	if EXPR then STMT	(1)
	|	while EXPR do STMT	(2)
	|	EXPR ;	(3)
EXPR	→	TERM -> id	(4)
	|	zero? TERM	(5)
	|	not EXPR	(6)
	|	++ id	(7)
	|	-- id	(8)
TERM	→	id	(9)
	|	constant	(10)

	if	then	while	do	zero?	not	++	--	→	id	const	;
STMT												
EXPR					5	6	7	8				
TERM										9	10	

Constructing LL(1) Parse Tables

STMT	→	`if` **EXPR** `then` **STMT**	**(1)**
	|	`while` **EXPR** `do` **STMT**	**(2)**
	|	**EXPR** `;`	**(3)**
EXPR	→	**TERM** `-> id`	**(4)**
	|	`zero?` **TERM**	**(5)**
	|	`not` **EXPR**	**(6)**
	|	`++ id`	**(7)**
	|	`-- id`	**(8)**
TERM	→	`id`	**(9)**
	|	`constant`	**(10)**

	if	then	while	do	zero?	not	++	--	→	id	const	;
STMT												
EXPR					5	6	7	8		4	4	
TERM										9	10	

Constructing LL(1) Parse Tables

STMT	→	`if` EXPR `then` STMT	(1)
	|	`while` EXPR `do` STMT	(2)
	|	EXPR `;`	(3)

EXPR	→	TERM `-> id`	(4)
	|	`zero?` TERM	(5)
	|	`not` EXPR	(6)
	|	`++ id`	(7)
	|	`-- id`	(8)

TERM	→	`id`	(9)
	|	`constant`	(10)

	if	then	while	do	zero?	not	++	--	→	id	const	;
STMT	1		2									
EXPR					5	6	7	8		4	4	
TERM										9	10	

Constructing LL(1) Parse Tables

STMT	→	`if` EXPR `then` STMT	(1)
	|	`while` EXPR `do` STMT	(2)
	|	EXPR `;`	(3)
EXPR	→	TERM `->` `id`	(4)
	|	`zero?` TERM	(5)
	|	`not` EXPR	(6)
	|	`++` `id`	(7)
	|	`--` `id`	(8)
TERM	→	`id`	(9)
	|	`constant`	(10)

	if	then	while	do	zero?	not	++	--	→	id	const	;
STMT	1		2		3	3	3	3				
EXPR					5	6	7	8		4	4	
TERM										9	10	

Constructing LL(1) Parse Tables

STMT	→	`if` EXPR `then` STMT	(1)
	|	`while` EXPR `do` STMT	(2)
	|	EXPR `;`	(3)
EXPR	→	TERM `-> id`	(4)
	|	`zero?` TERM	(5)
	|	`not` EXPR	(6)
	|	`++ id`	(7)
	|	`-- id`	(8)
TERM	→	`id`	(9)
	|	`constant`	(10)

	if	then	while	do	zero?	not	++	--	→	id	const	;
STMT	1		2		3	3	3	3		3	3	
EXPR					5	6	7	8		4	4	
TERM										9	10	

Constructing LL(1) Parse Tables

- Can we find an algorithm for constructing LL(1) parse tables?

Filling in Table Entries

- Intuition: The next character should uniquely identify a production, so we should pick a production that ultimately starts with that character.
- T[A, t] should be a production $A \rightarrow \omega$ iff ω derives something starting with t.

Selecting the Appropriate Production Rule

- Basic Tools:
  - First: Let α be a string of grammar symbols. First(α) is the set that includes every terminal that appears leftmost in α or in any string originating from α.
    NOTE: If $\alpha \Rightarrow \in$, then $\in$ is First(α).
  - Follow: Let A be a non-terminal. Follow(A) is the set of terminals *a* that can appear directly to the right of A in some sentential form. ($S \Rightarrow \alpha A a \beta$, for some α and β).
    NOTE: If $S \Rightarrow \alpha A$, then \$ is Follow(A).

FIRST Sets

- If a particular nonterminal A derives a string starting with a particular terminal t, we can formalize this with FIRST sets.

$$\text{FIRST}(A) = \{\ t \mid A \Rightarrow^* t\omega \text{ for some } \omega\ \}$$

- Intuitively, FIRST(A) is the set of terminals that can be at the start of a string produced by A.
- If we can compute FIRST sets for all nonterminals in a grammar, we can efficiently construct the LL(1) parsing table.

Computing FIRST Sets

- Initially, for all nonterminals A, set

$$\text{FIRST}(A) = \{\ t \mid A \rightarrow t\omega \text{ for some } \omega\ \}$$

- Then, repeat the following until no changes occur:
  - For each nonterminal A, for each production $A \rightarrow B\omega$, set

$$\text{FIRST}(A) = \text{FIRST}(A) \cup \text{FIRST}(B)$$

- This is known a fixed-point iteration or a transitive closure algorithm.

Iterative FIRST Computations

```
STMT  →  if EXPR then STMT
      |  while EXPR do STMT
      |  EXPR ;

EXPR  →  TERM -> id
      |  zero? TERM
      |  not EXPR
      |  ++ id
      |  -- id

TERM  →  id
      |  constant
```

STMT	EXPR	TERM

Iterative FIRST Computations

STMT	→	`if` **EXPR** `then` **STMT**
	|	`while` **EXPR** `do` **STMT**
	|	**EXPR** `;`
EXPR	→	**TERM** `-> id`
	|	`zero?` **TERM**
	|	`not` **EXPR**
	|	`++ id`
	|	`-- id`
TERM	→	`id`
	|	`constant`

STMT	EXPR	TERM
`if` `while`		

Iterative FIRST Computations

```
STMT → if EXPR then STMT
     | while EXPR do STMT
     | EXPR ;

EXPR → TERM -> id
     | zero? TERM
     | not EXPR
     | ++ id
     | -- id

TERM → id
     | constant
```

STMT	EXPR	TERM
if while	zero? not ++ --	

Iterative FIRST Computations

```
STMT  →  if EXPR then STMT
      |  while EXPR do STMT
      |  EXPR ;

EXPR  →  TERM -> id
      |  zero? TERM
      |  not EXPR
      |  ++ id
      |  -- id

TERM  →  id
      |  constant
```

STMT	EXPR	TERM
if while	zero? not ++ --	id constant

Iterative FIRST Computations

```
STMT → if EXPR then STMT
     | while EXPR do STMT
     | EXPR ;

EXPR → TERM -> id
     | zero? TERM
     | not EXPR
     | ++ id
     | -- id

TERM → id
     | constant
```

STMT	EXPR	TERM
if while	zero? not ++ --	id constant

Iterative FIRST Computations

```
STMT → if EXPR then STMT
     | while EXPR do STMT
     | EXPR ;

EXPR → TERM -> id
     | zero? TERM
     | not EXPR
     | ++ id
     | -- id

TERM → id
     | constant
```

STMT	EXPR	TERM
if	zero?	id
while	not	constant
zero?	++	
not	--	
++		
--		

Iterative FIRST Computations

```
STMT  →  if EXPR then STMT
      |  while EXPR do STMT
      |  EXPR ;

EXPR  →  TERM -> id
      |  zero? TERM
      |  not EXPR
      |  ++ id
      |  -- id

TERM  →  id
      |  constant
```

STMT	EXPR	TERM
if while zero? not ++ --	zero? not ++ --	id constant

Iterative FIRST Computations

```
STMT  →  if EXPR then STMT
      |  while EXPR do STMT
      |  EXPR ;

EXPR  →  TERM -> id
      |  zero? TERM
      |  not EXPR
      |  ++ id
      |  -- id

TERM  →  id
      |  constant
```

STMT	EXPR	TERM
if	zero?	id
while	not	constant
zero?	++	
not	--	
++	id	
--	constant	

Iterative FIRST Computations

```
STMT → if EXPR then STMT
     | while EXPR do STMT
     | EXPR ;

EXPR → TERM -> id
     | zero? TERM
     | not EXPR
     | ++ id
     | -- id

TERM → id
     | constant
```

STMT	EXPR	TERM
if	zero?	id
while	not	constant
zero?	++	
not	--	
++	id	
--	constant	

Iterative FIRST Computations

```
STMT → if EXPR then STMT
     | while EXPR do STMT
     | EXPR ;

EXPR → TERM -> id
     | zero? TERM
     | not EXPR
     | ++ id
     | -- id

TERM → id
     | constant
```

STMT	EXPR	TERM
if	zero?	id
while	not	constant
zero?	++	
not	--	
++	id	
--	constant	
id		
constant		

From FIRST Sets to LL(1) Tables

STMT	→	if EXPR then STMT	(1)
	|	while EXPR do STMT	(2)
	|	EXPR ;	(3)
EXPR	→	TERM -> id	(4)
	|	zero? TERM	(5)
	|	not EXPR	(6)
	|	++ id	(7)
	|	-- id	(8)
TERM	→	id	(9)
	|	constant	(10)

STMT	EXPR	TERM
if while zero? not ++ -- id constant	zero? not ++ -- id constant	id constant

	if	then	while	do	zero?	not	++	--	→	id	const	;
STMT												
EXPR												
TERM												

From FIRST Sets to LL(1) Tables

STMT	→	`if` EXPR `then` STMT	(1)
	|	`while` EXPR `do` STMT	(2)
	|	EXPR `;`	(3)
EXPR	→	TERM `->` `id`	(4)
	|	`zero?` TERM	(5)
	|	`not` EXPR	(6)
	|	`++` `id`	(7)
	|	`--` `id`	(8)
TERM	→	`id`	(9)
	|	`constant`	(10)

STMT	EXPR	TERM
if while zero? not ++ -- id constant	zero? not ++ -- id constant	id constant

	if	then	while	do	zero?	not	++	--	→	id	const	;
STMT	1											
EXPR												
TERM												

From FIRST Sets to LL(1) Tables

STMT	→	if EXPR then STMT	(1)
	|	while EXPR do STMT	(2)
	|	EXPR ;	(3)
EXPR	→	TERM -> id	(4)
	|	zero? TERM	(5)
	|	not EXPR	(6)
	|	++ id	(7)
	|	-- id	(8)
TERM	→	id	(9)
	|	constant	(10)

STMT	EXPR	TERM
if while zero? not ++ -- id constant	zero? not ++ -- id constant	id constant

	if	then	while	do	zero?	not	++	--	→	id	const	;
STMT	1		2									
EXPR												
TERM												

From FIRST Sets to LL(1) Tables

STMT	→	if EXPR then STMT	(1)
	|	while EXPR do STMT	(2)
	|	EXPR ;	(3)
EXPR	→	TERM -> id	(4)
	|	zero? TERM	(5)
	|	not EXPR	(6)
	|	++ id	(7)
	|	-- id	(8)
TERM	→	id	(9)
	|	constant	(10)

STMT	EXPR	TERM
if	zero?	id
while	not	constant
zero?	++	
not	--	
++	id	
--	constant	
id		
constant		

	if	then	while	do	zero?	not	++	--	→	id	const	;
STMT	1		2		3	3	3	3		3	3	
EXPR												
TERM												

From FIRST Sets to LL(1) Tables

STMT	→	if EXPR then STMT	(1)
	|	while EXPR do STMT	(2)
	|	EXPR ;	(3)
EXPR	→	TERM -> id	(4)
	|	zero? TERM	(5)
	|	not EXPR	(6)
	|	++ id	(7)
	|	-- id	(8)
TERM	→	id	(9)
	|	constant	(10)

STMT	EXPR	TERM
if while zero? not ++ -- id constant	zero? not ++ -- id constant	id constant

	if	then	while	do	zero?	not	++	--	→	id	const	;
STMT	1		2		3	3	3	3		3	3	
EXPR										4	4	
TERM												

From FIRST Sets to LL(1) Tables

STMT	→	if EXPR then STMT	(1)
	|	while EXPR do STMT	(2)
	|	EXPR ;	(3)
EXPR	→	TERM -> id	(4)
	|	zero? TERM	(5)
	|	not EXPR	(6)
	|	++ id	(7)
	|	-- id	(8)
TERM	→	id	(9)
	|	constant	(10)

STMT	EXPR	TERM
if while zero? not ++ -- id constant	zero? not ++ -- id constant	id constant

	if	then	while	do	zero?	not	++	--	→	id	const	;
STMT	1		2		3	3	3	3		3	3	
EXPR					5					4	4	
TERM												

From FIRST Sets to LL(1) Tables

STMT	→	if EXPR then STMT	(1)
	|	while EXPR do STMT	(2)
	|	EXPR ;	(3)
EXPR	→	TERM -> id	(4)
	|	zero? TERM	(5)
	|	not EXPR	(6)
	|	++ id	(7)
	|	-- id	(8)
TERM	→	id	(9)
	|	constant	(10)

STMT	EXPR	TERM
if while zero? not ++ -- id constant	zero? not ++ -- id constant	id constant

	if	then	while	do	zero?	not	++	--	→	id	const	;
STMT	1		2		3	3	3	3		3	3	
EXPR					5	6				4	4	
TERM												

From FIRST Sets to LL(1) Tables

STMT	→	`if` **EXPR** `then` **STMT**	**(1)**
	|	`while` **EXPR** `do` **STMT**	**(2)**
	|	**EXPR** `;`	**(3)**
EXPR	→	**TERM** `->` `id`	**(4)**
	|	`zero?` **TERM**	**(5)**
	|	`not` **EXPR**	**(6)**
	|	`++` `id`	**(7)**
	|	`--` `id`	**(8)**
TERM	→	`id`	**(9)**
	|	`constant`	**(10)**

STMT	EXPR	TERM
if while zero? not ++ -- id constant	zero? not ++ -- id constant	id constant

	if	then	while	do	zero?	not	++	--	→	id	const	;
STMT	1		2		3	3	3	3		3	3	
EXPR					5	6	7			4	4	
TERM												

From FIRST Sets to LL(1) Tables

```
STMT → if EXPR then STMT      (1)
     | while EXPR do STMT     (2)
     | EXPR ;                 (3)

EXPR → TERM -> id             (4)
     | zero? TERM             (5)
     | not EXPR               (6)
     | ++ id                  (7)
     | -- id                  (8)

TERM → id                     (9)
     | constant               (10)
```

STMT	EXPR	TERM
if	zero?	id
while	not	constant
zero?	++	
not	--	
++	id	
--	constant	
id		
constant		

	if	then	while	do	zero?	not	++	--	→	id	const	;
STMT	1		2		3	3	3	3		3	3	
EXPR					5	6	7	8		4	4	
TERM												

From FIRST Sets to LL(1) Tables

STMT	→	if EXPR then STMT	(1)
	|	while EXPR do STMT	(2)
	|	EXPR ;	(3)
EXPR	→	TERM -> id	(4)
	|	zero? TERM	(5)
	|	not EXPR	(6)
	|	++ id	(7)
	|	-- id	(8)
TERM	→	id	(9)
	|	constant	(10)

STMT	EXPR	TERM
if while zero? not ++ -- id constant	zero? not ++ -- id constant	id constant

	if	then	while	do	zero?	not	++	--	→	id	const	;
STMT	1		2		3	3	3	3		3	3	
EXPR					5	6	7	8		4	4	
TERM										9		

From FIRST Sets to LL(1) Tables

STMT	→	if EXPR then STMT	(1)
	|	while EXPR do STMT	(2)
	|	EXPR ;	(3)
EXPR	→	TERM -> id	(4)
	|	zero? TERM	(5)
	|	not EXPR	(6)
	|	++ id	(7)
	|	-- id	(8)
TERM	→	id	(9)
	|	constant	(10)

STMT	EXPR	TERM
if while zero? not ++ -- id constant	zero? not ++ -- id constant	id constant

	if	then	while	do	zero?	not	++	--	→	id	const	;
STMT	1		2		3	3	3	3		3	3	
EXPR					5	6	7	8		4	4	
TERM										9	10	

ε-Free LL(1) Parse Tables

- The following algorithm constructs an LL(1) parse table for a grammar with no ε-productions.
- Compute the FIRST sets for all nonterminals in the grammar.
- For each production $A \to t\omega$, set $T[A, t] = t\omega$.
- For each production $A \to B\omega$, set $T[A, t] = B\omega$

  for each $t \in \mathrm{FIRST}(B)$.

Expanding our Grammar

STMT	→	`if` EXPR `then` STMT	(1)
	|	`while` EXPR `do` STMT	(2)
	|	EXPR `;`	(3)
EXPR	→	TERM `-> id`	(4)
	|	`zero?` TERM	(5)
	|	`not` EXPR	(6)
	|	`++ id`	(7)
	|	`-- id`	(8)
TERM	→	`id`	(9)
	|	`constant`	(10)

```
id → id;

while not zero? id do --id;

if not zero? id then
   if not zero? id then
       constant → id;
```

Expanding our Grammar

```
STMT  →  if EXPR then STMT        (1)
      |  while EXPR do STMT       (2)
      |  EXPR ;                   (3)

EXPR  →  TERM -> id               (4)
      |  zero? TERM               (5)
      |  not EXPR                 (6)
      |  ++ id                    (7)
      |  -- id                    (8)

TERM  →  id                       (9)
      |  constant                 (10)

BLOCK →  STMT                     (11)
      |  { STMTS }                (12)

STMTS →  STMT STMTS               (13)
      |  ε                        (14)
```

```
id → id;

while not zero? id do --id;

if not zero? id then
    if not zero? id then
        constant → id;
```

Expanding our Grammar

STMT	→	`if` EXPR `then` BLOCK	(1)
	|	`while` EXPR `do` BLOCK	(2)
	|	EXPR `;`	(3)
EXPR	→	TERM `-> id`	(4)
	|	`zero?` TERM	(5)
	|	`not` EXPR	(6)
	|	`++ id`	(7)
	|	`-- id`	(8)
TERM	→	`id`	(9)
	|	`constant`	(10)
BLOCK	→	STMT	(11)
	|	`{` STMTS `}`	(12)
STMTS	→	STMT STMTS	(13)
	|	ε	(14)

```
id → id;
```

```
while not zero? id do --id;
```

```
if not zero? id then
    if not zero? id then
        constant → id;
```

Expanding our Grammar

STMT	→	`if` EXPR `then` BLOCK	(1)
	|	`while` EXPR `do` BLOCK	(2)
	|	EXPR `;`	(3)
EXPR	→	TERM `-> id`	(4)
	|	`zero?` TERM	(5)
	|	`not` EXPR	(6)
	|	`++ id`	(7)
	|	`-- id`	(8)
TERM	→	`id`	(9)
	|	`constant`	(10)
BLOCK	→	STMT	(11)
	|	`{` STMTS `}`	(12)
STMTS	→	STMT STMTS	(13)
	|	ε	(14)

```
id → id;

while not zero? id do --id;

if not zero? id then
   if not zero? id then
       constant → id;

if zero? id then
    while zero? id do {
        constant → id;
        constant → id;
    }
```

LL(1) with ε-Productions

- Computation of FIRST is different.
  - What if the first nonterminal in a production can produce ε?

- Building the table is different.
  - What action do you take if the correct production produces the empty string?

FIRST Sets with ε

$$
\begin{aligned}
\text{Num} &\rightarrow \text{Sign Digits} \\
\text{Sign} &\rightarrow + \mid - \mid \varepsilon \\
\text{Digits} &\rightarrow \text{Digit More} \\
\text{More} &\rightarrow \text{Digits} \mid \varepsilon \\
\text{Digit} &\rightarrow 0 \mid 1 \mid 2 \mid \ldots \mid 9
\end{aligned}
$$

Num	Sign	Digit	Digits	More

FIRST Sets with ε

Num → Sign Digits
Sign → + | – | ε
Digits → Digit More
More → Digits | ε
Digit → 0 | 1 | 2 | … | 9

Num	Sign	Digit	Digits	More
	+ –	0 5 1 6 2 7 3 8 4 9		

FIRST Sets with ε

Num → **Sign Digits**
Sign → + | – | ε
Digits → Digit More
More → Digits | ε
Digit → 0 | 1 | 2 | … | 9

Num	Sign	Digit	Digits	More
+ –	+ –	0 5 1 6 2 7 3 8 4 9		

FIRST Sets with ε

Num → Sign Digits
Sign → + | – | ε
Digits → Digit More
More → Digits | ε
Digit → 0 | 1 | 2 | … | 9

Num	Sign	Digit	Digits	More
+ –	+ –	0 5 1 6 2 7 3 8 4 9	0 5 1 6 2 7 3 8 4 9	

FIRST Sets with ε

Num → Sign Digits
Sign → + | – | ε
Digits → Digit More
More → **Digits** | ε
Digit → 0 | 1 | 2 | … | 9

Num	Sign	Digit	Digits	More
+ –	+ –	0 5 1 6 2 7 3 8 4 9	0 5 1 6 2 7 3 8 4 9	0 5 1 6 2 7 3 8 4 9

FIRST Sets with ε

Num	→	**Sign Digits**
Sign	→	+ | – | ε
Digits	→	**Digit More**
More	→	**Digits** | ε
Digit	→	0 | 1 | 2 | … | 9

Num	Sign	Digit	Digits	More
+ –	+ – ε	0 5 1 6 2 7 3 8 4 9	0 5 1 6 2 7 3 8 4 9	0 5 1 6 2 7 3 8 4 9 ε

FIRST Sets with ε

Num → **Sign Digits**
Sign → + | – | ε
Digits → Digit More
More → Digits | ε
Digit → 0 | 1 | 2 | … | 9

Num	Sign	Digit	Digits	More
+ –	+ –	0 5	0 5	0 5
0 5	ε	1 6	1 6	1 6
1 6		2 7	2 7	2 7
2 7		3 8	3 8	3 8
3 8		4 9	4 9	4 9
4 9				ε

FIRST and ε

- When computing FIRST sets in a grammar with ε-productions, we often have to “look through” nonterminals.

- Rationale: Might have a derivation like this:

$$A \Rightarrow Bt \Rightarrow t$$

- So $t \in$ FIRST(A).

FIRST Computation with ε

- Initially, for all nonterminals A, set

$$\text{FIRST}(A) = \{ t \mid A \rightarrow t\omega \text{ for some } \omega \}$$

- For all nonterminals A where $A \rightarrow \varepsilon$ is a production, add ε to FIRST(A).
- Repeat the following until no changes occur:
  - For each production $A \rightarrow \alpha$, where α is a string of nonterminals whose FIRST sets contain ε, set $\text{FIRST}(A) = \text{FIRST}(A) \cup \{ \varepsilon \}$.
  - For each production $A \rightarrow \alpha t\omega$, where α is a string of nonterminals whose FIRST sets contain ε, set

$$\text{FIRST}(A) = \text{FIRST}(A) \cup \{ t \}$$

  - For each production $A \rightarrow \alpha B\omega$, where α is string of nonterminals whose FIRST sets contain ε, set

$$\text{FIRST}(A) = \text{FIRST}(A) \cup (\text{FIRST}(B) - \{ \varepsilon \}).$$

A Notational Diversion

- Once we have computed the correct FIRST sets for each nonterminal, we can generalize our definition of FIRST sets to strings.
- Define FIRST(ω) as follows:
  - FIRST(ε) = { ε }
  - FIRST(tω) = { t }
  - If ε ∉ FIRST(A):
    - FIRST(Aω) = FIRST(A)
  - If ε ∈ FIRST(A):
    - FIRST(Aω) = (FIRST(A) - { ε }) ∪ FIRST(ω)

FIRST Computation with ε

- Initially, for all nonterminals A, set

$$FIRST(A) = \{ t \mid A \to t\omega \text{ for some } \omega \}$$

- For all nonterminals A where $A \to \varepsilon$ is a production, add ε to FIRST(A).
- Repeat the following until no changes occur:
  - For each production $A \to \alpha$, set

$$FIRST(A) = FIRST(A) \cup FIRST(\alpha)$$

LL(1) Tables with ε

Num → Sign Digits
Sign → + | - | ε
Digits → Digit More
More → Digits | ε
Digit → 0 | 1 | … | 9

When constructing LL(1) tables with ε-productions, we need to have an extra column for $.

	+	-	#	$
Num				
Sign				
Digits				
More				
Digit				

LL(1) Tables with ε

Num	→	Sign Digits
Sign	→	+ | – | ε
Digits	→	Digit More
More	→	Digits | ε
Digit	→	0 | 1 | … | 9

Num	Sign	Digit	Digits	More
+ – 0 1 2 3 4 5 6 7 8 9	+ – ε	0 1 2 3 4 5 6 7 8 9	0 1 2 3 4 5 6 7 8 9	0 1 2 3 4 5 6 7 8 9 ε

	+	–	#	$
Num	Sign Digits	Sign Digits		
Sign				
Digits				
More				
Digit				

LL(1) Tables with ε

Num → Sign Digits
Sign → + | - | ε
Digits → Digit More
More → Digits | ε
Digit → 0 | 1 | … | 9

Num	Sign	Digit	Digits	More
+ − 0 5 1 6 2 7 3 8 4 9	+ − ε	0 5 1 6 2 7 3 8 4 9	0 5 1 6 2 7 3 8 4 9	0 5 1 6 2 7 3 8 4 9 ε

	+	−	#	$
Num	Sign Digits	Sign Digits		
Sign	+	-		
Digits				
More				
Digit				

LL(1) Tables with ε

Num → Sign Digits
Sign → + | – | ε
Digits → Digit More
More → Digits | ε
Digit → 0 | 1 | … | 9

Num	Sign	Digit	Digits	More
+ – 0 5 1 6 2 7 3 8 4 9	+ – ε	0 5 1 6 2 7 3 8 4 9	0 5 1 6 2 7 3 8 4 9	0 5 1 6 2 7 3 8 4 9 ε

	+	–	#	$
Num	Sign Digits	Sign Digits		
Sign	+	-		
Digits			Digits More	
More				
Digit				

LL(1) Tables with ε

Num → Sign Digits
Sign → + | − | ε
Digits → Digit More
More → Digits | ε
Digit → 0 | 1 | … | 9

Num	Sign	Digit	Digits	More
+ − 0 5 1 6 2 7 3 8 4 9	+ − ε	0 5 1 6 2 7 3 8 4 9	0 5 1 6 2 7 3 8 4 9	0 5 1 6 2 7 3 8 4 9 ε

	+	−	#	$
Num	Sign Digits	Sign Digits		
Sign	+	-		
Digits			Digits More	
More			Digits	
Digit				

LL(1) Tables with ε

Num → Sign Digits
Sign → + | - | ε
Digits → Digit More
More → Digits | ε
Digit → 0 | 1 | … | 9

Num	Sign	Digit	Digits	More
+ − 0 5 1 6 2 7 3 8 4 9	+ − ε	0 5 1 6 2 7 3 8 4 9	0 5 1 6 2 7 3 8 4 9	0 5 1 6 2 7 3 8 4 9 ε

	+	−	#	$
Num	Sign Digits	Sign Digits		
Sign	+	-		
Digits			Digits More	
More			Digits	
Digit			#	

LL(1) Tables with ε

Num → Sign Digits
Sign → + | - | ε
Digits → Digit More
More → Digits | ε
Digit → 0 | 1 | … | 9

Num	Sign	Digit	Digits	More
+ − 0 5 1 6 2 7 3 8 4 9	+ − ε	0 5 1 6 2 7 3 8 4 9	0 5 1 6 2 7 3 8 4 9	0 5 1 6 2 7 3 8 4 9 ε

	+	−	#	$
Num	Sign Digits	Sign Digits	Sign Digits	
Sign	+	-		
Digits			Digits More	
More			Digits	
Digit			#	

LL(1) Tables with ε

Num → Sign Digits
Sign → + | - | ε
Digits → Digit More
More → Digits | ε
Digit → 0 | 1 | … | 9

Num	Sign	Digit	Digits	More
+ – 0 5 1 6 2 7 3 8 4 9	+ – ε	0 5 1 6 2 7 3 8 4 9	0 5 1 6 2 7 3 8 4 9	0 5 1 6 2 7 3 8 4 9 ε

	+	–	#	$
Num	Sign Digits	Sign Digits	Sign Digits	
Sign	+	-	ε	
Digits			Digits More	
More			Digits	
Digit			#	

LL(1) Tables with ε

Num → **Sign Digits**
Sign → + | – | ε
Digits → **Digit More**
More → **Digits** | ε
Digit → 0 | 1 | … | 9

Num	Sign	Digit	Digits	More
+ – 0 5 1 6 2 7 3 8 4 9	+ – ε	0 5 1 6 2 7 3 8 4 9	0 5 1 6 2 7 3 8 4 9	0 5 1 6 2 7 3 8 4 9 ε

	+	–	#	$
Num	Sign Digits	Sign Digits	Sign Digits	
Sign	+	-	ε	
Digits			Digits More	
More			Digits	
Digit			#	

LL(1) Tables with ε

Num → Sign Digits
Sign → + | – | ε
Digits → Digit More
More → Digits | ε
Digit → 0 | 1 | … | 9

Num	Sign	Digit	Digits	More
+ – 0 5 1 6 2 7 3 8 4 9	+ – ε	0 5 1 6 2 7 3 8 4 9	0 5 1 6 2 7 3 8 4 9	0 5 1 6 2 7 3 8 4 9 ε

	+	–	#	$
Num	Sign Digits	Sign Digits	Sign Digits	
Sign	+	-	ε	
Digits			Digits More	
More			Digits	ε
Digit			#	

LL(1) Tables with ε

Num → Sign Digits
Sign → + | - | ε
Digits → Digit More
More → Digits | ε
Digit → 0 | 1 | … | 9

Num	Sign	Digit	Digits	More
+ – 0 5 1 6 2 7 3 8 4 9	+ – ε	0 5 1 6 2 7 3 8 4 9	0 5 1 6 2 7 3 8 4 9	0 5 1 6 2 7 3 8 4 9 ε

	+	–	#	$
Num	Sign Digits	Sign Digits	Sign Digits	
Sign	+	-	ε	
Digits			Digits More	
More			Digits	ε
Digit			#	

FOLLOW Sets

- With ε-productions in the grammar, we may have to "look past" the current nonterminal to what can come after it.
- The FOLLOW set represents the set of terminals that might come after a given nonterminal.
- Formally:

$$\text{FOLLOW}(A) = \{\ t \mid S \Rightarrow^{*} \alpha A t \omega \text{ for some } \alpha, \omega\ \}$$

where S is the start symbol of the grammar.

- Informally, every nonterminal that can ever come after A in a derivation.

Computation of FOLLOW Sets

- Initially, for each nonterminal A, set

  $$\text{FOLLOW}(A) = \{ t \mid B \rightarrow \alpha A t \omega \text{ is a production } \}$$

- Add \$ to FOLLOW(S), where S is the start symbol.
- Repeat the following until no changes occur:
  - If $B \rightarrow \alpha A \omega$ is a production, set

    $$\text{FOLLOW}(A) = \text{FOLLOW}(A) \cup \text{FIRST}(\omega) - \{ \varepsilon \}$$

  - If $B \rightarrow \alpha A \omega$ is a production and $\varepsilon \in \text{FIRST}(\omega)$, set

    $$\text{FOLLOW}(A) = \text{FOLLOW}(A) \cup \text{FOLLOW}(B).$$

The Final LL(1) Table Algorithm

- Compute FIRST(A) and FOLLOW(A) for all nonterminals A.
- For each rule $A \to \omega$, for each terminal $t \in \text{FIRST}(\omega)$, set $T[A, t] = \omega$.
  - Note that ε is not a terminal.
- For each rule $A \to \omega$, if $\varepsilon \in \text{FIRST}(\omega)$, set $T[A, t] = \omega$ for each $t \in \text{FOLLOW}(A)$.

An Egregious Abuse of Notation

- Compute FIRST(A) and FOLLOW(A) for all nonterminals A
- For each rule $A \rightarrow \omega$, for each terminal $t \in \mathrm{FIRST}(\omega\mathrm{FOLLOW}(A))$, set $T[A, t] = \omega$

LL(1) Construction

- The Limits of LL(1)
  - Some grammars are Not LL(1)
- The Strengths of LL(1)
  - LL(1) is straightforward
  - LL(1) is fast

LL(1) Limitation

- Left-recursive grammars:
- Example:

  $A \rightarrow Ab \mid c$

  - FIRST(A) ?
  - Parse table ?

LL(1) Limitation

- Left-recursive grammars:
- Example:

  $A \rightarrow Ab \mid c$

  - FIRST(A) = {c}
  - Parse table:

	b	c
A		$A \rightarrow Ab$ $A \rightarrow c$

    - Cannot uniquely predict production!
    - This is called a FIRST/FIRST conflict.

LL(1) Limitation

- Left-recursive grammars:
- Example:

$A \rightarrow Ab \mid c$

- FIRST(A) = {c}
- Parse table:

	b	c
A		$A \rightarrow Ab$ $A \rightarrow c$

  - Cannot uniquely predict production!
  - This is called a FIRST/FIRST conflict.

Solution: Eliminating Left Recursion

LL(1) Limitation

- Left-recursive grammars:
- Example:

$E \rightarrow T$

$E \rightarrow T + E$

$T \rightarrow int$

$T \rightarrow (E)$

- FIRST(E) = { int, (}
- FIRST(T) = { int, (}

LL(1) Limitation

- Left-recursive grammars:
- Example:

$E \rightarrow T$

$E \rightarrow T + E$

$T \rightarrow int$

$T \rightarrow (E)$

Can not predict which rule to use

- FIRST(E) = { int, (}
- FIRST(T) = { int, (}

Solution: Left Factoring

LL(1) Limitation

- Left-recursive grammars:
- Example:

$E \rightarrow T$

$E \rightarrow T + E$

$T \rightarrow int$

$T \rightarrow (E)$

LL(1) Limitation

- Left-recursive grammars:
- Example:

$E \longrightarrow T\ \varepsilon$

$E \longrightarrow T + E$

$T \longrightarrow int$

$T \longrightarrow (E)$

LL(1) Limitation

- Left-recursive grammars:
- Example:

  $E \longrightarrow T\ \mathbf{Y}$

  $T \longrightarrow \text{int}$

  $T \longrightarrow (E)$

LL(1) Limitation

- Left-recursive grammars:
- Example:

$E \rightarrow T\ \mathbf{Y}$

$T \rightarrow int$

$T \rightarrow (E)$

$\mathbf{Y \rightarrow + E}$

$\mathbf{Y \rightarrow \varepsilon}$

Example: LL(1) with Left Factoring

$E \rightarrow TY$	1
$T \rightarrow$ int	2
$T \rightarrow (E)$	3
$Y \rightarrow + E$	4
$Y \rightarrow \varepsilon$	5

FIRST		
E	T	Y

FOLLOW		
E	T	Y

Example: LL(1) with Left Factoring

E → TY		1
T → int		2
T → (E)		3
Y → + E		4
Y → ε		5

FIRST		
E	T	Y
	int (	

FOLLOW		
E	T	Y

Example: LL(1) with Left Factoring

$E \rightarrow TY$		1
$T \rightarrow$ int		2
$T \rightarrow$ (E)		3
$Y \rightarrow + E$		4
$Y \rightarrow \varepsilon$		5

FIRST		
E	T	Y
	int (	+ ε

FOLLOW		
E	T	Y

Example: LL(1) with Left Factoring

E → TY		1
T → int		2
T → (E)		3
Y → + E		4
Y → ε		5

FIRST		
E	T	Y
int (	int (	+ ε
FOLLOW		
E	T	Y

Example: LL(1) with Left Factoring

E → TY		1
T → int		2
T → (E)		3
Y → + E		4
Y → ε		5

FIRST		
E	T	Y
int (	int (	+ ε

FOLLOW		
E	T	Y
$)		

Example: LL(1) with Left Factoring

E	→ TY	1
T	→ int	2
T	→ (E)	3
Y	→ + E	4
Y	→ ε	5

FIRST		
E	T	Y
int (	int (	+ ε
FOLLOW		
E	T	Y
\$)	+	

Example: LL(1) with Left Factoring

$E \rightarrow TY$		1
$T \rightarrow$ `int`		2
$T \rightarrow$ `(E)`		3
$Y \rightarrow$ `+` E		4
$Y \rightarrow \varepsilon$		5

FIRST		
E	T	Y
int (	int (	+ ε

FOLLOW		
E	T	Y
$)	+	$)

Example: LL(1) with Left Factoring

E → TY		1
T → int		2
T → (E)		3
Y → + E		4
Y → ε		5

FIRST		
E	T	Y
int (	int (	+ ε

FOLLOW		
E	T	Y
$)	+ $)	$)

Example: LL(1) with Left Factoring

E → TY		1
T → int		2
T → (E)		3
Y → + E		4
Y → ε		5

FIRST		
E	T	Y
int (	int (	+ ε

FOLLOW		
E	T	Y
\$)	+ \$)	\$)

	int	(	)	+	\$
E					
T					
Y					

Example: LL(1) with Left Factoring

E → TY		1
T → `int`		2
T → `(`E`)`		3
Y → `+` E		4
Y → ε		5

FIRST		
E	T	Y
int (	int (	+ ε

FOLLOW		
E	T	Y
$)	+ $)	$)

	int	(	)	+	$
E	1	1			
T	2	3			
Y			5	4	5

The Strengths of LL(1)

- LL(1) is Straightforward
  - Can be implemented quickly with a table-driven design.
  - Can be implemented by recursive descent:
    - Define a function for each nonterminal.
    - Have these functions call each other based on the lookahead token.

The Strengths of LL(1)

- LL(1) is Fast
- Both table-driven LL(1) and recursive descent-powered LL(1) are fast.
- Can parse in O(n |G|) time, where n is the length of the string and |G| is the size of the grammar.

Summary

- Top-down parsing tries to derive the user's program from the start symbol.
- Leftmost BFS is one approach to top-down parsing; it is mostly of theoretical interest.
- Leftmost DFS is another approach to top-down parsing that is uncommon in practice.
- LL(1) parsing scans from left-to-right, using one token of lookahead to find a leftmost derivation.
- FIRST sets contain terminals that may be the first symbol of a production.
- FOLLOW sets contain terminals that may follow a nonterminal in a production.
- Left recursion and left factorability cause LL(1) to fail and can be mechanically eliminated in some cases.

Reading

- Aho2, Sections 4.1; 4.2; 4.3.1; 4.3.2; (see also pp.56-60)
- Aho1, pp. 160-175
- Hunter, pp. 21-44
- Grune pp.34-40; 110-115
- Cooper, pp.73-89.

Question?