# Mining Association Rules in Large Databases

# Association rules

- Given a set of transactions **D**, find rules that will predict the occurrence of an item (or a set of items) based on the occurrences of other items in the transaction
- **Unsupervised Learning**

**Market-Basket transactions**

| *TID* | *Items* |
|---|---|
| 1 | Bread, Milk |
| 2 | Bread, Cookies, Butter, Eggs |
| 3 | Milk, Cookies, Butter, Coke |
| 4 | Bread, Milk, Cookies, Butter |
| 5 | Bread, Milk, Cookies, Coke |

**Examples of association rules**

{Cookies} $\rightarrow$ {Butter},
{Milk, Bread} $\rightarrow$ {Cookies, Coke},
{Butter, Bread} $\rightarrow$ {Milk},

# An even simpler concept: frequent itemsets

- Given a set of transactions **D**, find combination of items that occur frequently

**Market-Basket transactions**

| *TID* | *Items* |
|---|---|
| 1 | Bread, Milk |
| 2 | Bread, Cookies, Butter, Eggs |
| 3 | Milk, Cookies, Butter, Coke |
| 4 | Bread, Milk, Cookies, Butter |
| 5 | Bread, Milk, Cookies, Coke |

**Examples of frequent itemsets**

{Cookies, Butter},
{Milk, Bread}
{Butter, Bread, Milk},

# Lecture outline

- **Task 1:** Methods for finding all frequent itemsets efficiently

- **Task 2:** Methods for finding association rules efficiently

# Definition: Frequent Itemset

- **Itemset**
  - A set of one or more items
    - E.g.: {Milk, Bread, Cookies}
  - k-itemset
    - An itemset that contains k items
- **Support count (σ)**
  - Frequency of occurrence of an itemset (number of transactions it appears)
  - E.g. **σ({Milk, Bread, Cookies}) = 2**
- **Support**
  - Fraction of the transactions in which an itemset appears
  - E.g. **s({Milk, Bread, Cookies}) = 2/5**
- **Frequent Itemset**
  - An itemset whose support is greater than or equal to a ***minsup*** threshold

| *TID* | *Items* |
|---|---|
| 1 | Bread, Milk |
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| 5 | Bread, Milk, Cookies, Coke |

# Why do we want to find frequent itemsets?

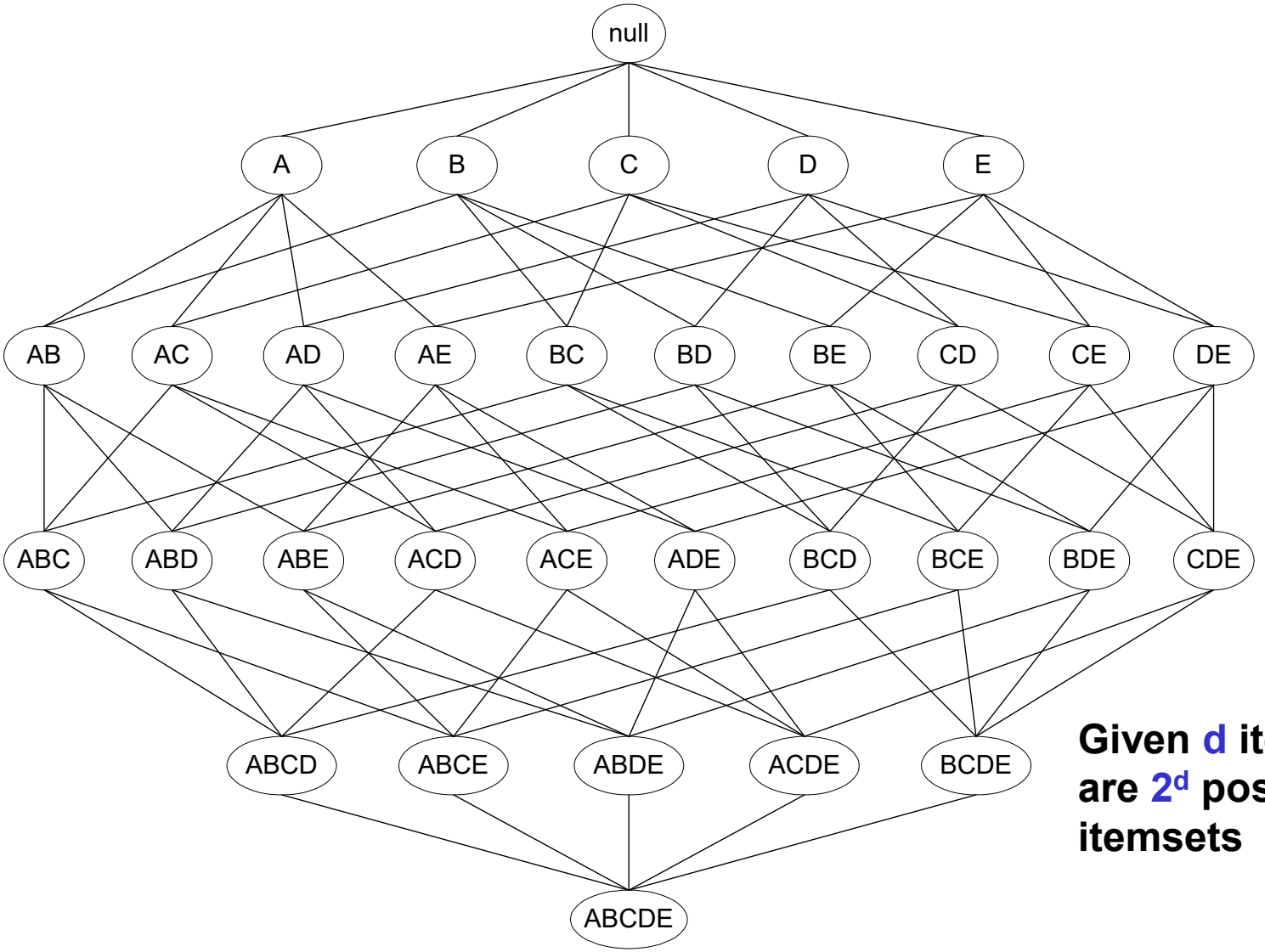
- Find all combinations of items that occur together
- They might be interesting (e.g., in placement of items in a store)
- Frequent itemsets are only positive combinations (we do not report combinations that do not occur frequently together)
- Frequent itemsets aims at providing a summary for the data

# Finding frequent sets

- **Task:** Given a transaction database **D** and a **minsup** threshold find all frequent itemsets and the frequency of each set in this collection

- **Stated differently:** Count the number of times combinations of attributes occur in the data. If the count of a combination is above **minsup** report it.

# How many itemsets are there?

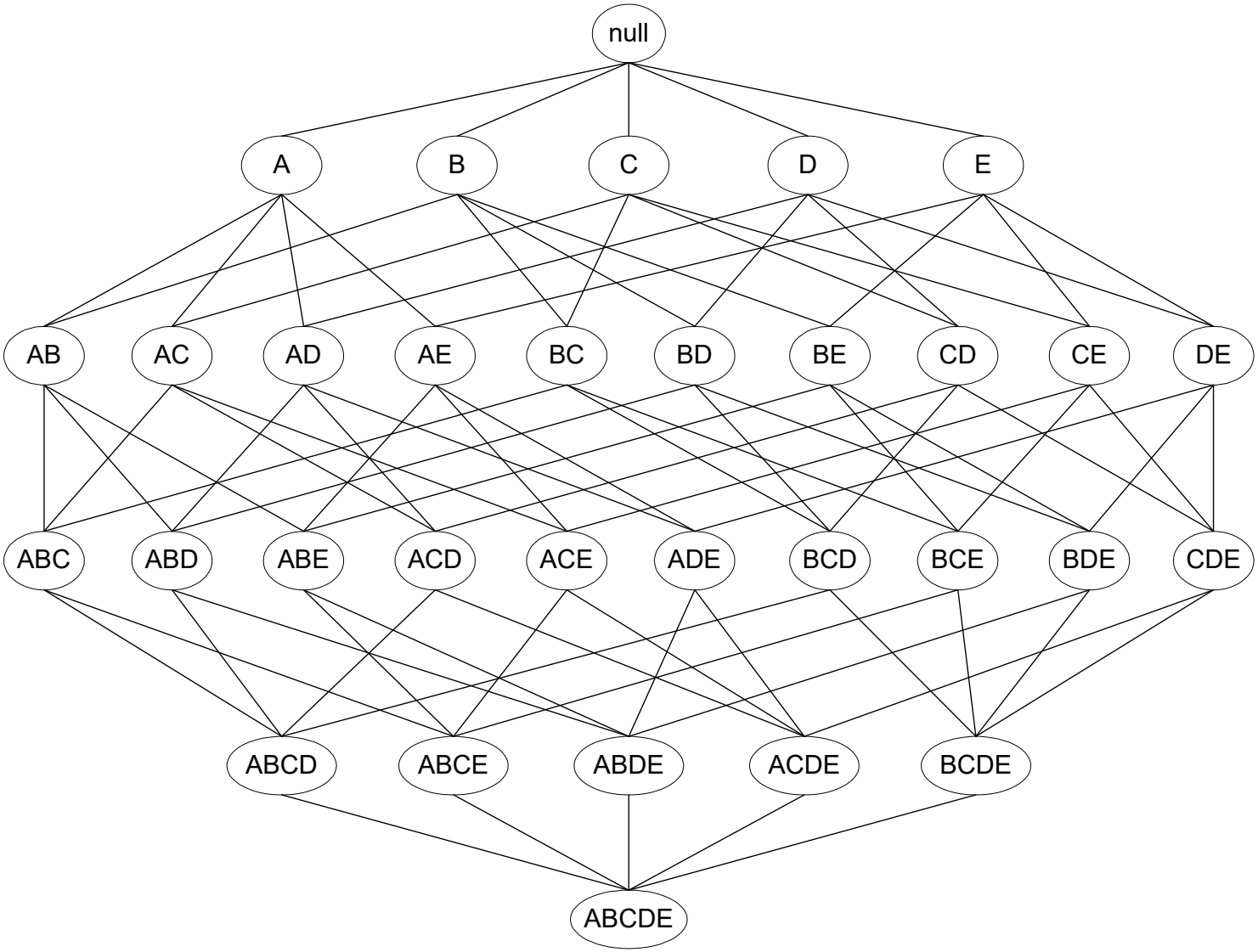



**Given d items, there are $2^d$ possible itemsets**

# When is the task sensible and feasible?

- If **minsup = 0**, then all subsets of ***I*** will be frequent and thus the size of the collection will be very large
- This summary is very large (maybe larger than the original input) and thus not interesting
- The task of finding all frequent sets is interesting typically only for relatively large values of **minsup**

# A simple algorithm for finding all frequent itemsets ??



# Brute-force algorithm for finding all frequent itemsets?

- Generate all possible itemsets (lattice of itemsets)
  - Start with 1-itemsets, 2-itemsets,...,d-itemsets
- Compute the frequency of each itemset from the data
  - Count in how many transactions each itemset occurs
- If the support of an itemset is above **minsup** report it as a frequent itemset

# Brute-force approach for finding all frequent itemsets

- Complexity?
  - Match every candidate against each transaction
  - For **M** candidates and **N** transactions, the complexity is~ **O(NMw)** => Expensive since $M = 2^d$ !!!

# Reduce the number of candidates

- Apriori principle (Main observation):
  - If an itemset is frequent, then all of its subsets must also be frequent

- Apriori principle holds due to the following property of the support measure:

$$\forall X, Y : (X \subseteq Y) \Rightarrow s(X) \geq s(Y)$$

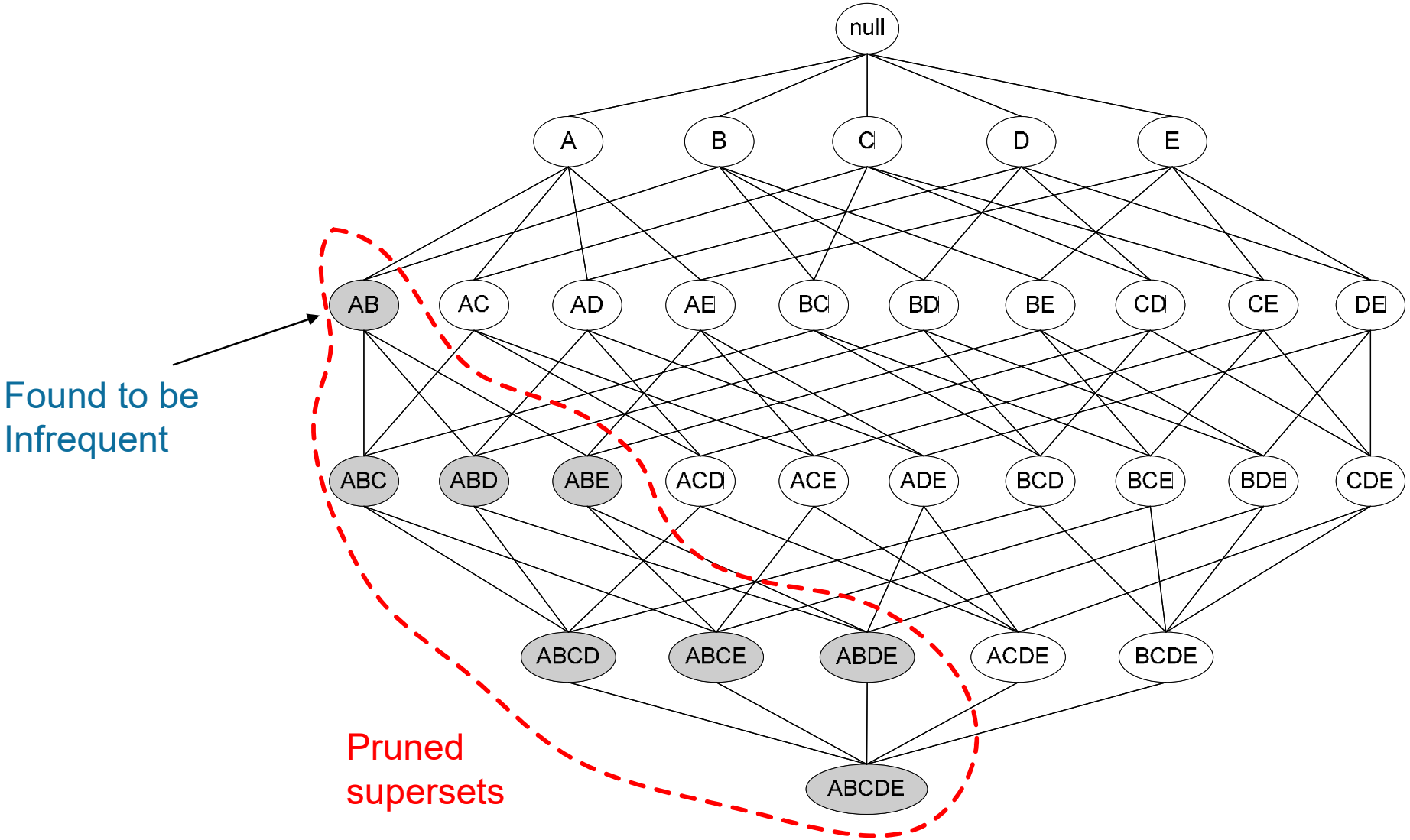
  - The support of an itemset ***never exceeds*** the support of its subsets
  - This is known as the ***anti-monotone*** property of support

# Example

| *TID* | *Items* |
|---|---|
| 1 | Bread, Milk |
| 2 | Bread, Cookies, Butter, Eggs |
| 3 | Milk, Cookies, Butter, Coke |
| 4 | Bread, Milk, Cookies, Butter |
| 5 | Bread, Milk, Cookies, Coke |

s(Bread) > s(Bread, Butter)
s(Milk) > s(Bread, Milk)
s(Cookies, Butter) > s(Cookies, Butter, Coke)

# Illustrating the Apriori principle



# Illustrating the Apriori principle

| Item | Count |
|---|---|
| Bread | 4 |
| Coke | 2 |
| Milk | 4 |
| Butter | 3 |
| Cookies | 4 |
| Eggs | 1 |

Items (1-itemsets)

| Itemset | Count |
|---|---|
| {Bread,Milk} | 3 |
| {Bread,Butter} | 2 |
| {Bread,Cookies} | 3 |
| {Milk,Butter} | 2 |
| {Milk,Cookies} | 3 |
| {Butter,Cookies} | 3 |

Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)

minsup = 3/5

Triplets (3-itemsets)

| Itemset | Count |
|---|---|
| {Bread,Milk,Cookies} | 3 |

# Exploiting the Apriori principle

1. Find frequent 1-items and put them to $L_k$ ($k=1$)
2. Use $L_k$ to generate a collection of *candidate* itemsets $C_{k+1}$ with size ($k+1$)
3. Scan the database to find which itemsets in $C_{k+1}$ are frequent and put them into $L_{k+1}$
4. If $L_{k+1}$ is not empty
   - $k=k+1$
   - Goto step 2

R. Agrawal, R. Srikant: "Fast Algorithms for Mining Association Rules",
*Proc. of the 20th Int'l Conference on Very Large Databases*, 1994.

# The Apriori algorithm

$C_k$: Candidate itemsets of size k

$L_k$ : frequent itemsets of size k

$L_1$ = {frequent 1-itemsets};

**for** ($k$ = 2; $L_k$ !=∅; $k$++)

$C_{k+1}$ = GenerateCandidates($L_k$)

**for** each transaction ***t*** in database do

increment count of candidates in $C_{k+1}$ that are contained in **t**

**endfor**

$L_{k+1}$ = candidates in $C_{k+1}$ with support ≥***min_sup***

**endfor**

**return** $\cup_k L_k$**;**

# Discussion of the Apriori algorithm

- Much faster than the Brute-force algorithm
  - It avoids checking all elements in the lattice
- The running time is in the worst case **$O(2^d)$**
  - Pruning really prunes in practice
- It makes multiple passes over the dataset
  - One pass for every level **k**
- Multiple passes over the dataset is inefficient when we have thousands of candidates and millions of transactions

# Lecture outline

- **Task 1:** Methods for finding all frequent itemsets efficiently

- **Task 2:** Methods for finding association rules efficiently

# Definition: Association Rule

Let **D** be database of transactions

– e.g.:

| Transaction ID | Items |
|---|---|
| 2000 | A, B, C |
| 1000 | A, C |
| 4000 | A, D |
| 5000 | B, E, F |

- Let ***I*** be the set of items that appear in the database, e.g., ***I={A,B,C,D,E,F}***
- A **rule** is defined by **X → Y**, where **X⊂*I***, **Y⊂*I***, and **X∩Y=∅**
  – e.g.: **{B,C} → {A}** is a rule

# Definition: Association Rule

- **Association Rule**
  - An implication expression of the form **X** → **Y**, where **X** and **Y** are non-overlapping itemsets
  - Example:
    ***{Milk, Cookies}*** → ***{Butter}***

- **Rule Evaluation Metrics**
  - **Support (s)**
    - Fraction of transactions that contain both **X** and **Y**
  - **Confidence (c)**
    - Measures how often items in **Y** appear in transactions that contain **X**

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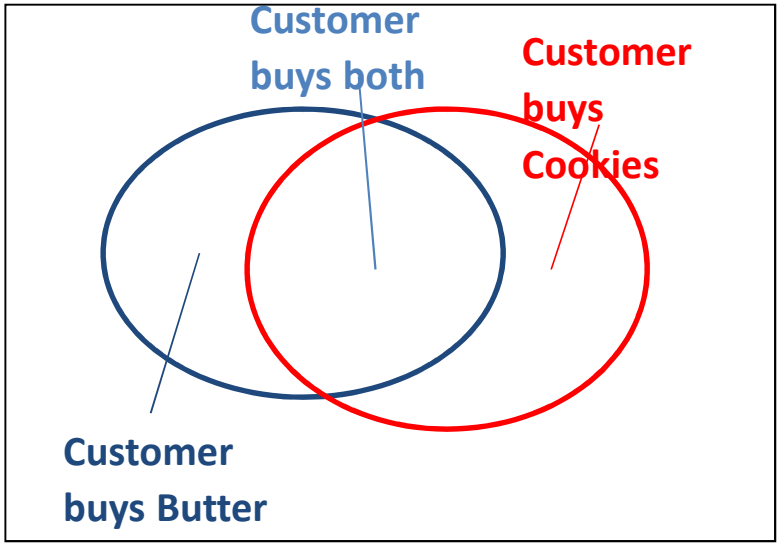
**Example:**

$$\{\text{Milk}, \text{Cookies}\} \rightarrow \text{Butter}$$

$$s = \frac{\sigma(\text{Milk}, \text{Cookies}, \text{Butter})}{|\text{T}|} = \frac{2}{5} = 0.4$$

$$c = \frac{\sigma(\text{Milk}, \text{Cookies}, \text{Butter})}{\sigma(\text{Milk}, \text{Cookies})} = \frac{2}{3} = 0.67$$

# Rule Measures: Support and Confidence




Find all the rules $X \rightarrow Y$ with minimum confidence and support

- support, *s*, probability that a transaction contains $\{X \cup Y\}$
- confidence, *c*, ***conditional probability*** that a transaction having **X** also contains **Y**

| TID | Items |
|---|---|
| 100 | A,B,C |
| 200 | A,C |
| 300 | A,D |
| 400 | B,E,F |

*Let minimum support 50%, and minimum confidence 50%, we have*

- $A \rightarrow C$ (50%, 66.6%)
- $C \rightarrow A$ (50%, 100%)

# Example

| TID | date | items bought |
|---|---|---|
| 100 | 10/10/99 | {F,A,D,B} |
| 200 | 15/10/99 | {D,A,C,E,B} |
| 300 | 19/10/99 | {C,A,B,E} |
| 400 | 20/10/99 | {B,A,D} |

What is the ***support*** and ***confidence*** of the rule: {B,D} → {A}

- Support:
  - percentage of tuples that contain {A,B,D} = 75%
- Confidence:

$$\frac{\text{number of tuples that contain } \{A,B,D\}}{\text{number of tuples that contain } \{B,D\}} = 100\%$$

# Association-rule mining task

- Given a set of transactions **D**, the goal of association rule mining is to find **all** rules having
  - support ≥ ***minsup*** threshold
  - confidence ≥ ***minconf*** threshold

# Mining Association Rules

| TID | Items |
|---|---|
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## Example of Rules:

{Milk,Cookies} → {Butter} (s=0.4, c=0.67)
{Milk,Butter} → {Cookies} (s=0.4, c=1.0)
{Cookies,Butter} → {Milk} (s=0.4, c=0.67)
{Butter} → {Milk,Cookies} (s=0.4, c=0.67)
{Cookies} → {Milk,Butter} (s=0.4, c=0.5)
{Milk} → {Cookies,Butter} (s=0.4, c=0.5)

## Observations:

- All the above rules are binary partitions of the same itemset: {Milk, Cookies, Butter}
- Rules originating from the same itemset have identical support but can have different confidence
- Thus, we may decouple the support and confidence requirements

# Mining Association Rules

- Two-step approach:
  - Frequent Itemset Generation
    - Generate all itemsets whose support $\geq$ minsup
  - Rule Generation
    - Generate high confidence rules from each frequent itemset, where each rule is a binary partition of a frequent itemset

# Efficient rule generation

- How to efficiently generate rules from frequent itemsets?
  - In general, confidence does not have an anti-monotone property

    **c(ABC →D)** can be larger or smaller than **c(AB →D)**

  - ***But confidence of rules generated from the same itemset has an anti-monotone property***
  - Example: X = {A,B,C,D}:

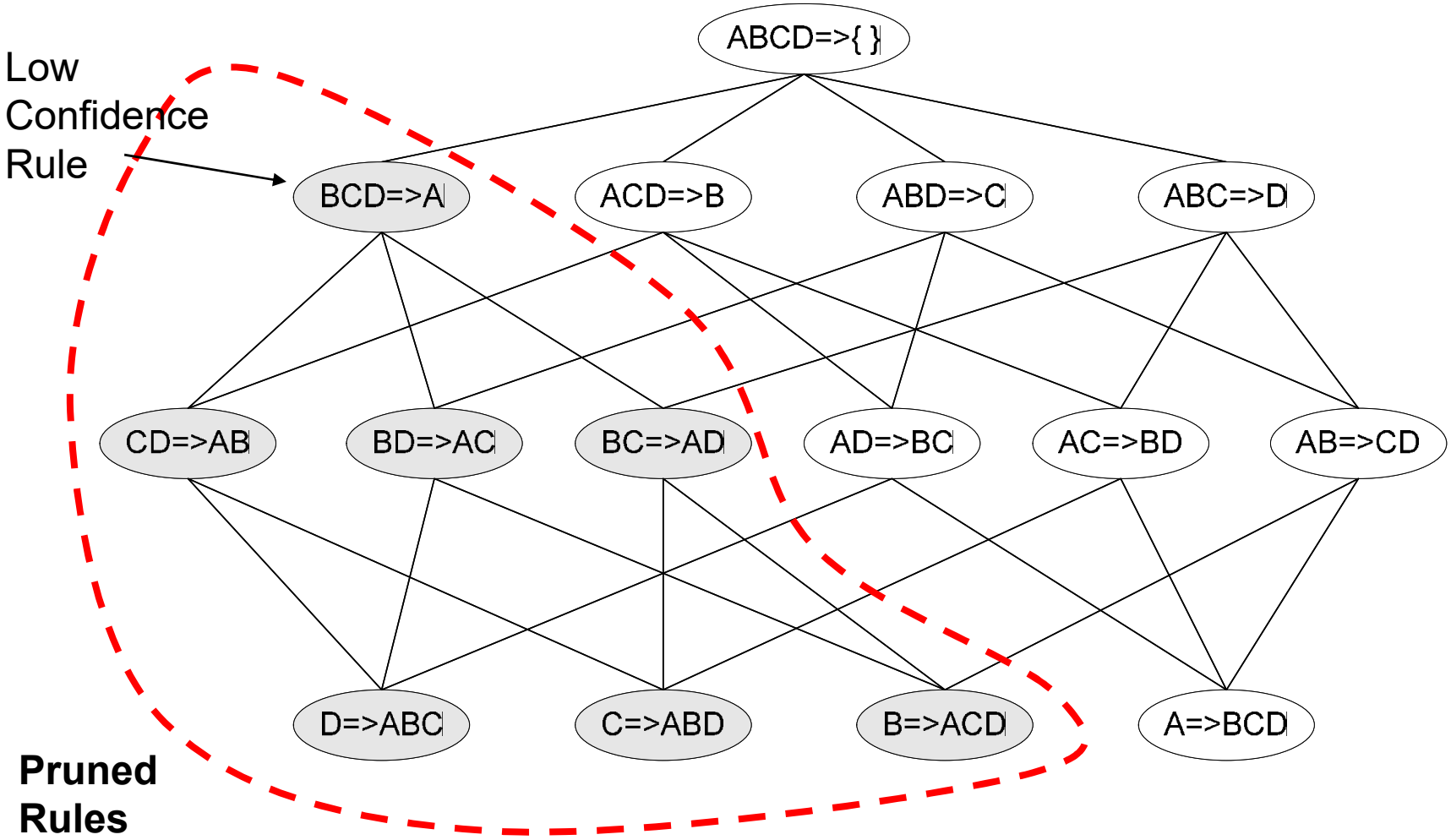
$$c(ABC \rightarrow D) \geq c(AB \rightarrow CD) \geq c(A \rightarrow BCD)$$

  - **Why?**

**Confidence is anti-monotone w.r.t. number of items on the RHS of the rule**

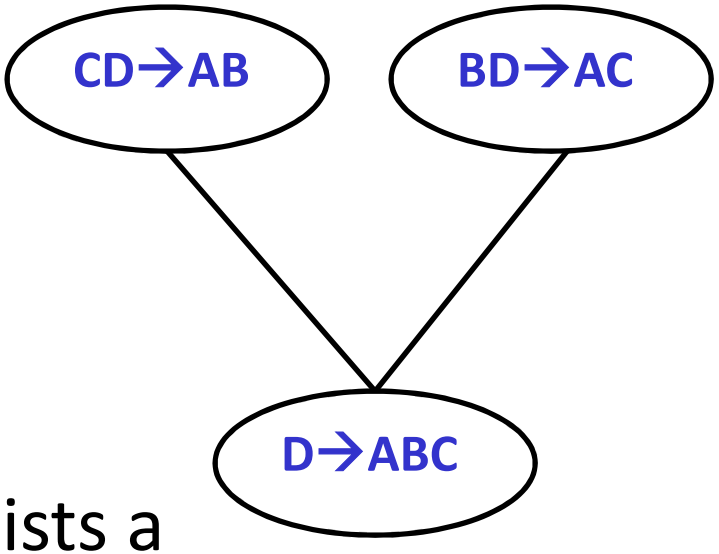
# Rule Generation for Apriori Algorithm

Lattice of rules



# Apriori algorithm for rule generation

- Candidate rule is generated by merging two rules that share the same prefix in the rule consequent




- **join(CD→AB,BD—>AC)** would produce the candidate rule **D →ABC**

- **Prune** rule **D→ABC** if there exists a subset (e.g., **AD→BC**) that does not have high confidence

| FP growth algorithm | Apriori algorithm |
| --- | --- |
| FP growth algorithm is faster than Apriori algorithm. | It is slower than FP growth algorithm. |
| FP growth algorithm is an array based algorithm. | Apriori algorithm is a tree-based algorithm. |
| FP growth algorithm required only two database scan. | It requires multiple database scan to generate a candidate set. |
| It uses depth-first search | It uses breadth-first search. |