جواب سوالات Homwork 7

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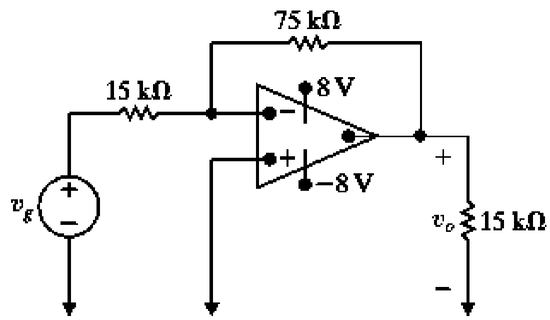
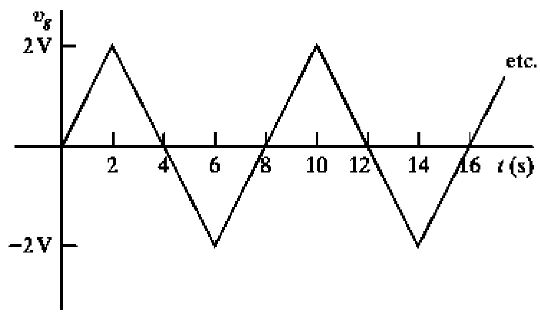




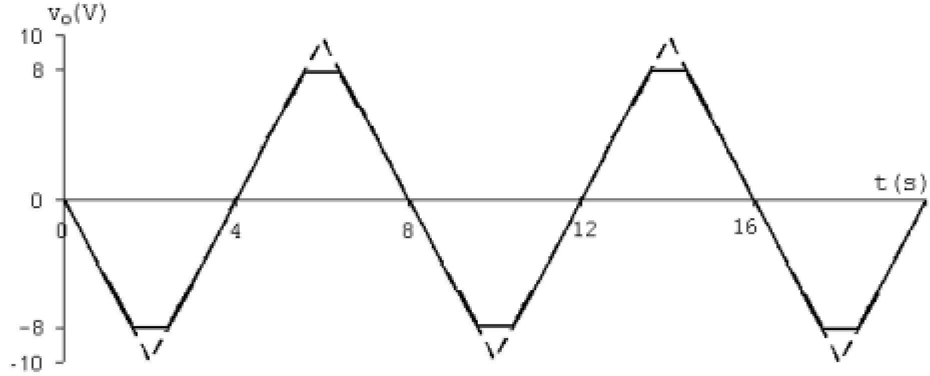
It follows directly from the circuit that $v_o = -(75/15)v_g = -5v_g$
From the plot of v_g we have $\quad v_g = 0, \quad t < 0$

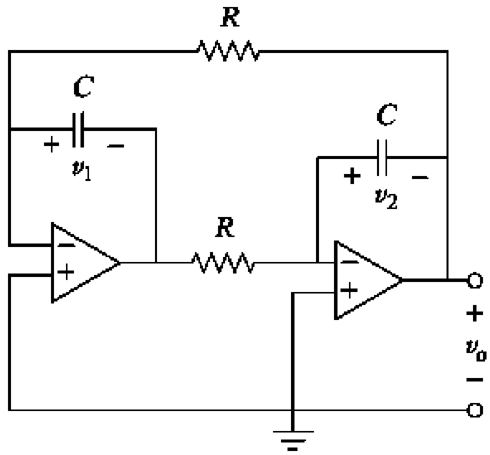
$$\begin{aligned}
v_g &= t && 0 \le t \le 2 \\
v_g &= 4 - t && 2 \le t \le 6 \\
v_g &= t - 8 && 6 \le t \le 10 \\
v_g &= 12 - t && 10 \le t \le 14 \\
v_g &= t - 16 && 14 \le t \le 18, \quad \text{etc.}
\end{aligned}$$

Therefore

$$\begin{aligned}
v_o &= -5t && 0 \le t \le 2 \\
v_o &= 5t - 20 && 2 \le t \le 6 \\
v_o &= 40 - 5t && 6 \le t \le 10 \\
v_o &= 5t - 60 && 10 \le t \le 14 \\
v_o &= 80 - 5t && 14 \le t \le 18, \quad \text{etc.}
\end{aligned}$$

These expressions for v_o are valid as long as the op amp is not saturated. Since the peak values of v_o are ± 9, the output is clipped at ± 9. The plot is shown below.





At the input of the first op amp,

$$(v_o - 0)/R = Cd(v_1 - 0)/dt \qquad (1)$$

At the input of the second op amp,

$$(-v_1 - 0)/R = Cdv_2/dt \qquad (2)$$

Let us now examine our constraints. Since the input terminals are essentially at ground, then we have the following,

$$v_o = -v_2 \text{ or } v_2 = -v_o \qquad (3)$$

Combining (1), (2), and (3), eliminating v_1 and v_2 we get,

$$\frac{d^2v_o}{dt^2} - \left(\frac{1}{R^2C^2}\right)v_o = \frac{d^2v_o}{dt^2} - 100v_o = 0$$

Which leads to $s^2 - 100 = 0$

Clearly this produces roots of –10 and +10.

And, we obtain,

$$v_o(t) = (Ae^{+10t} + Be^{-10t})V$$

$$\text{At } t = 0,\; v_o(0+) = -v_2(0+) = 0 = A + B,\; \text{thus } B = -A$$

This leads to $v_o(t) = (Ae^{+10t} - Ae^{-10t})V$. Now we can use $v_1(0+) = 2V$.

$$\text{From (2)},\; v_1 = -RCdv_2/dt = 0.1dv_o/dt = 0.1(10Ae^{+10t} + 10Ae^{-10t})$$

$$v_1(0+) = 2 = 0.1(20A) = 2A \text{ or } A = 1$$

$$\text{Thus},\quad v_o(t) = \underline{\mathbf{(e^{+10t} - e^{-10t})V}}$$

It should be noted that this circuit is unstable (clearly one of the poles lies in the right-half-plane).

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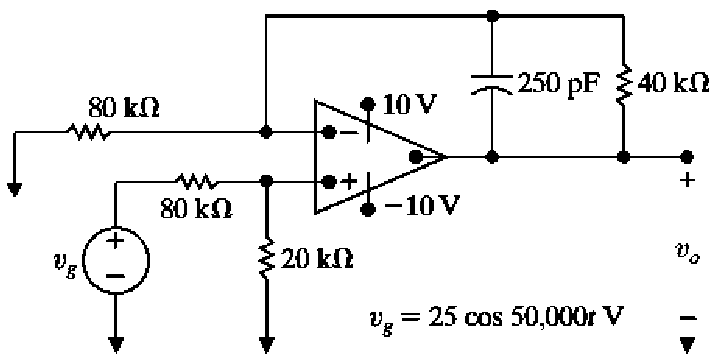

[a] $\mathbf{V}_g = 25\underline{/0^\circ}$ V

$$\mathbf{V}_\mathrm{p} = \frac{20}{100}\mathbf{V}_g = 5\underline{/0^\circ}; \qquad \mathbf{V}_\mathrm{n} = \mathbf{V}_\mathrm{p} = 5\underline{/0^\circ}\text{ V}$$

$$\frac{5}{80{,}000} + \frac{5 - \mathbf{V}_o}{Z_\mathrm{p}} = 0$$

$$Z_\mathrm{p} = -j80{,}000\|40{,}000 = 32{,}000 - j16{,}000\,\Omega$$

$$\mathbf{V}_o = \frac{5Z_\mathrm{p}}{80{,}000} + 5 = 7 - j = 7.07\underline{/-8.13^\circ}$$

$$v_o = 7.07\cos(50{,}000t - 8.13^\circ)\text{ V}$$

[b] $\mathbf{V}_\mathrm{p} = 0.2V_m\underline{/0^\circ}; \qquad \mathbf{V}_\mathrm{n} = \mathbf{V}_\mathrm{p} = 0.2V_m\underline{/0^\circ}$

$$\frac{0.2V_m}{80{,}000} + \frac{0.2V_m - \mathbf{V}_o}{32{,}000 - j16{,}000} = 0$$

$$\therefore \quad \mathbf{V}_o = 0.2V_m + \frac{32{,}000 - j16{,}000}{80{,}000}V_m(0.2) = V_m(0.28 - j0.04)$$

$$\therefore \quad |V_m(0.28 - j0.04)| \leq 10$$

$$\therefore \quad V_m \leq 35.36\text{ V}$$

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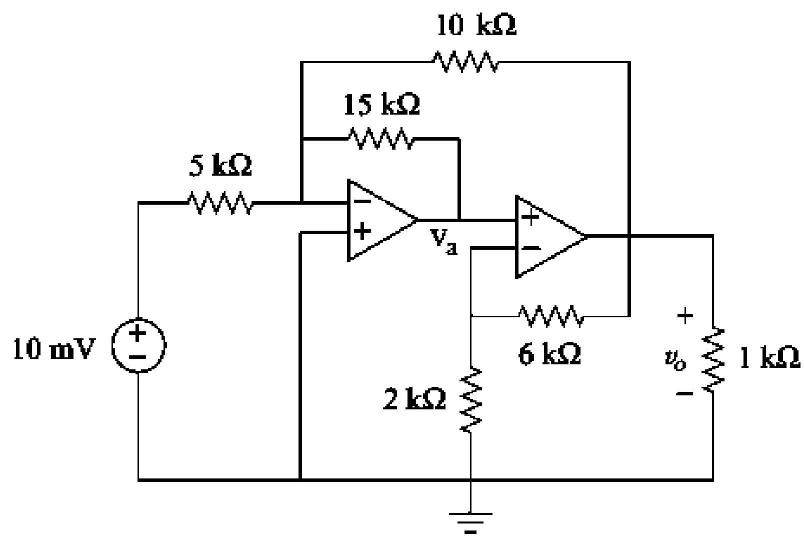


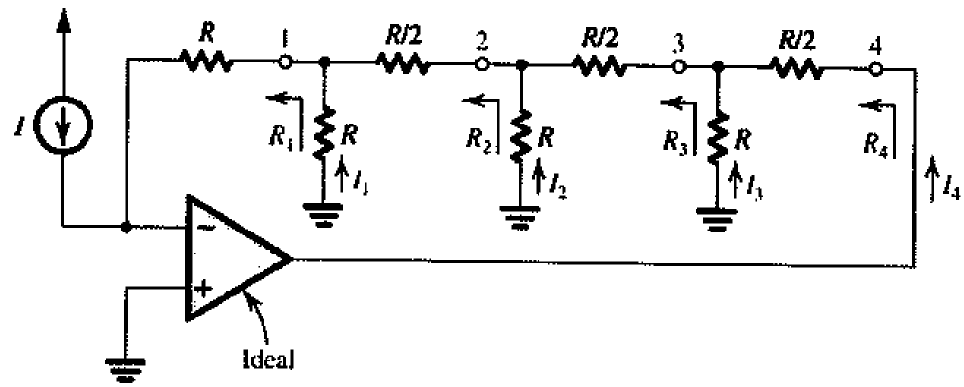
In this case, the first stage is a summer

$$v_a = -\frac{15}{5}(10) - \frac{15}{10}v_o = -30 - 1.5v_o$$

For the second stage,

$$v_o = \left(1 + \frac{6}{2}\right)v_a = 4v_a = 4(-30 - 1.5v_o)$$

$$7v_o = -120 \longrightarrow v_o = -\frac{120}{7} = \underline{\mathbf{-17.143mV}}$$



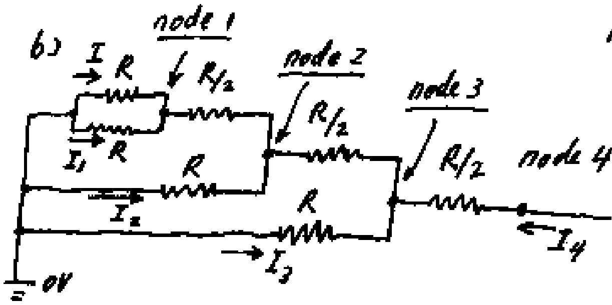
a) looking into node 1 : $R_1 = R$

node 2 : $R_2 = (R//R) + \frac{R}{2} = R$

node 3 : $R_3 = (R_2 // R) + \frac{R}{2} = R$

node 4 : $R_4 = (R_3 // R) + \frac{R}{2} = R$

b)



$I_1 = I$

based on current division,

$I_2 = 2I$

$I_3 = 4I$

$I_4 = -8I$

c)

$V_1 = 0 - IR = -IR$

$V_2 = -I_2 R = -2IR$

$V_3 = -I_3 R = -4IR$

$V_4 = V_3 + \frac{R}{2}(I_4) = -4IR + \frac{R}{2}(-8I) = -8IR$

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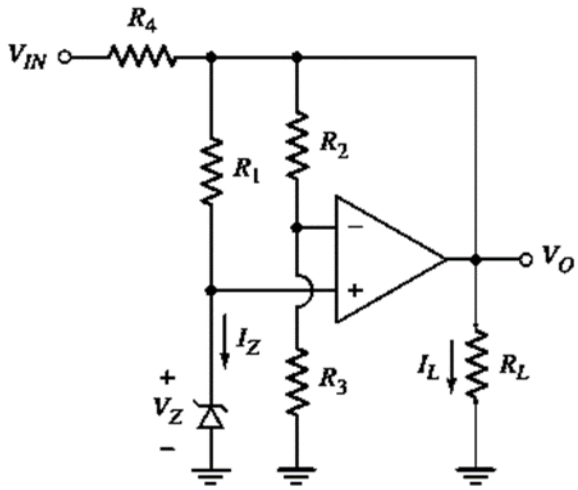

$$R_1 = \frac{V_O - V_Z}{I_Z} = \frac{12-5.6}{2} = 3.2\,\mathrm{k}\Omega$$

$$\frac{V_O}{V_Z} = \left(1+\frac{R_2}{R_3}\right) = \frac{12}{5.6} \Rightarrow \frac{R_2}{R_3} = 1.143$$

Let $I_R = 2\,\mathrm{mA}, \;\Rightarrow R_2 + R_3 = \dfrac{V_O}{I_R} = \dfrac{12}{2} = 6\,\mathrm{k}\Omega$

Then $1.143R_3 + R_3 = 6\,, \;\Rightarrow R_3 = 2.8\,\mathrm{k}\Omega$ and $R_2 = 3.2\,\mathrm{k}\Omega$

Let $I_{R4} = 4\,\mathrm{mA}, \; R_4 = \dfrac{V_{IN} - V_O}{I_{R4}} = \dfrac{15-12}{4} = 0.75\,\mathrm{k}\Omega$