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1- $x(t) = 3t^2 e^{-3t} u(t)$ سوال ۱-

$-t\,x(t) \xrightarrow{L} \frac{dX(s)}{ds}$ $\quad ROC = R_x$

$\frac{t^{n-1}}{(n-1)!} e^{-\alpha t} u(t) \xrightarrow{L} \frac{1}{(s+\alpha)^n}$ $\quad Real\{s\} > -\alpha$

$e^{-3t} u(t) \xrightarrow{L} \frac{1}{s+3}$ $\quad Real\{s\} > -3$

$te^{-3t} u(t) \xrightarrow{L} \frac{1}{(s+3)^2}$ $\quad$ "

$t^2 e^{-3t} u(t) \xrightarrow{L} \frac{2}{(s+3)^2}$ $\quad$ "

$\Rightarrow X(s) = \frac{6}{(s+3)^3}$ $\quad Real\{s\} > -3$

2. $x(t) = 3|t|e^{-4t} = 3te^{-4t}u(t) - 3te^{-4t}u(-t)$

$\Rightarrow d\{x(t)\} = 3\left[-\frac{d}{ds}\left[\underbrace{\mathcal{L}\{e^{-4t}u(t)\}}_{\frac{1}{s+4}\ \text{for } s>-4}\right] + 3\frac{d}{ds}\left[\underbrace{\mathcal{L}e^{-4t}u(-t)}_{\frac{-1}{s+4}\ s<-4}\right]\right.$

$= \begin{cases} \frac{3}{(s+4)^2} & \text{for } s>-4 \\ \frac{3}{(s+4)^2} & \text{for } s<-4 \end{cases}$

3. $x(t) = (t-3)e^{-2t}u(t-3) = \underbrace{e^{-2t}}_{x_1(t)} \cdot \underbrace{(t-3)u(t-3)}_{x_2(t)}$

$tu(t) \xrightarrow{L} -\frac{d}{ds}\underbrace{(\mathcal{L}\{u(t)\})}_{\frac{1}{s},\ s>0} = \frac{1}{s^2}$ $\quad s>0$

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$t \to t-3$: $(t-3)u(t-3) \xrightarrow{\mathcal{L}} e^{-3s} \times \frac{1}{s^2} = X_2(s)$ $\quad s>0$

$x(t) = e^{-2t} x_2(t) \xrightarrow{\mathcal{L}} X_2(s-2) = \frac{e^{-3(s-2)}}{(s-2)^2}$ $\quad s-2>0$

4. $x(t) = \begin{cases} 1 & \text{for } 0 \le t \le 1 \\ 0 & \text{o.w.} \end{cases}$

$x(t) = u(t) - u(t-1) \xrightarrow{\mathcal{L}} \frac{1}{s} - e^{-s}\frac{1}{s}$ $\quad ROC: s$

سوال 2 -

1. $X(s) = \frac{s}{s^2+9}$ $\quad Real\{s\} > 0$ ← سگنال دست راستی

$\Rightarrow x(t) = \cos 3t \cdot u(t)$

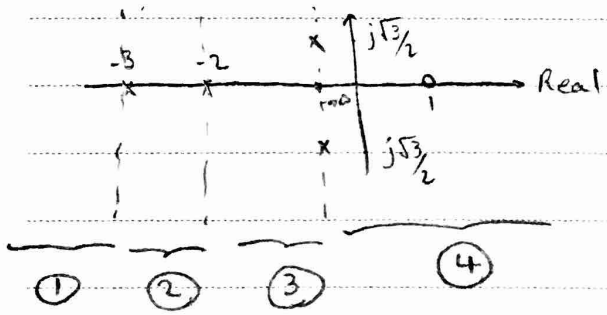
2. $X(s) = \frac{s+2}{s^2+7s+12}$ $\quad -4 < Real\{s\} < -3$

$X(s) = \frac{-1}{s+3} + \frac{2}{s+4} \Rightarrow x(t) = \mathcal{L}^{-1}\{\frac{1}{s+3}$ for $s<-3\}$

$\Rightarrow x(t) = e^{-3t}u(t) + 2e^{-4t}u(t)$ $\quad + \mathcal{L}^{-1}\{\frac{2}{s+4}$ for $s>-4\}$

3. $X(s) = \frac{s-1}{(s+2)(s+3)(s^2+s+1)} = \frac{s-1}{(s+2)(s+3)(s^2+s+1)}$

↳ $\pm j\sqrt{3}/2 - 1/2$



نواحی همگن

$X(s) = \frac{-1}{s+2} + \frac{\frac{4}{7}}{s+3} + \frac{\frac{3}{7}s + \frac{1}{7}}{s^2+s+1}$

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سؤال 3 -

4 قطب داریم و صفر ندارد $\Rightarrow X(s) = \dfrac{K}{(s-p_1)(s-p_2)(s-p_3)(s-p_4)}$

یکی از قطب‌ها را می‌دانیم: $p_1 = 0.5\, e^{j\pi/4}$ ①

$x(t)$ حقیقی $\Rightarrow X(s) = X^*(s^*)$ ②

چون: $x(t) = x^*(t)$ ، $\mathcal{L}\{x^*(t)\} = X^*(s^*)$

①، ②: $p_2 = p_1^* = 0.5\, e^{-j\pi/4}$ ③

$x(t)$ زوج: $X(s) = X(-s)$ ④

چون: $x(t) = x(-t)$ ، $\mathcal{L}\{x(-t)\} = X(-s)$

$p_3 = -p_1 = -0.5\, e^{j\pi/4}$

$p_4 = -p_2 = -0.5\, e^{-j\pi/4}$

$$X(s) = \int_{-\infty}^{+\infty} x(t)\, e^{-st}\, dt \Rightarrow X(0) = \int_{-\infty}^{+\infty} x(t) \times 1\, dt$$

$$\Rightarrow 4 = X(0) = \frac{K}{(0-p_1)(0-p_2)(0-p_3)(0-p_4)} = \frac{K}{p_1 p_2 p_3 p_4}$$

$$= \frac{K}{\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times e^{0}} \Rightarrow \boxed{K = 4}$$

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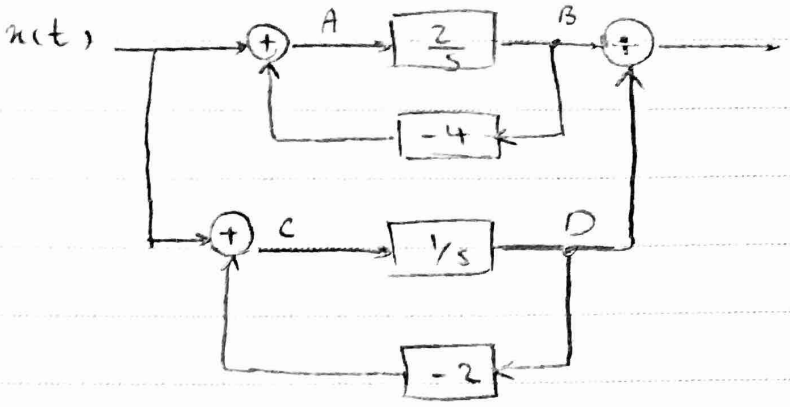
سوال 4 -



$A = X(s) + (-4B)$

$B = A \times \frac{2}{s}$

$\Rightarrow B = X(s) \times \frac{2}{s} + B(-4 \times \frac{2}{s})$

$\Rightarrow B(1 + \frac{8}{s}) = X(s) \times \frac{2}{s} \Rightarrow B = X(s) \times \frac{2}{s+8}$ (1)

$C = X(s) - 2D$
$D = C \times \frac{1}{s}$
$\Rightarrow D = X(s) \times \frac{1}{s} + D(-2 \times \frac{1}{s})$

$\Rightarrow D = X(s) \times \frac{1}{s+2}$

$Y(s) = B + D = X(s)\left(\frac{2}{s+8} + \frac{1}{s+2}\right) = X(s) \times \frac{3s+12}{s^2+10s+12} = Y(s)$

$\Rightarrow s^2 Y(s) + 10s\,Y(s) + 12\,Y(s) = 3s\,X(s) + 12\,X(s)$

$\mathcal{L}^{-1} \hookrightarrow \frac{d^2}{dt^2} y(t) + 10 \frac{d}{dt} y(t) + 12\,y(t) = 3 \frac{d}{dt} x(t) + 12\,x(t)$