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سؤال ۱- سیگنال متناوب است $T = 2\pi$ $f_1(t) = e^{-jt}$

$$f_1(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t} = a_{k_0} e^{jk_0\omega_0 t} = e^{-jt}$$

$$\Rightarrow a_{k_0} = 1 \quad , \quad k_0\omega_0 = -1 \quad , \quad \omega_0 = \frac{2\pi}{2\pi} = 1$$

$$\Rightarrow k_0 = -1 \quad , \quad a_{-1} = 1$$

$$f_2(t) = \begin{cases} 1 & t \in [-1, 0) \\ -1 & t \in [0, 1) \end{cases} \qquad T = 2 \Rightarrow \omega_0 = \frac{2\pi}{2} = \pi$$

$$a_k = \frac{1}{T} \oint_T f_2(t) e^{-jk\omega_0 t} dt = \frac{1}{2} \int_{-1}^{1} x(t) e^{-jk\omega_0 t} dt$$

$$= \frac{1}{2} \int_{-1}^{0} e^{-jk\omega_0 t} dt - \frac{1}{2} \int_{0}^{1} e^{-jk\omega_0 t} dt$$

$$= \frac{1}{2} \left[\frac{-1}{jk\omega_0 t} e^{-jk\omega_0 t} \right]_{-1}^{0} - \frac{1}{2} \left[\frac{-1}{jk\omega_0 t} e^{-jk\omega_0 t} \right]_{0}^{1}$$

$$= \frac{1}{2jk\omega_0 t} \left(-(1 - e^{+jk\omega_0}) + (e^{-jk\omega_0} - 1) \right)$$

$$= \frac{\overbrace{e^{jk\omega_0} + e^{-jk\omega_0}}^{\to 2\cos(k\omega_0)} - 2}{2jk\omega_0 t} = \frac{\cos(k\omega_0) - 1}{jk\omega_0} \quad \text{for } k \neq 0$$

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$$a_0 = \frac{1}{T} \oint_T x(t)\, dt = \frac{1}{2} \left(\int_{-1}^{0} 1\, dt + \int_0^1 -1\, dt \right) = 0 \text{ for } k=0$$

$$f_3(t) = \begin{cases} t-1 & t \in [0,2) \\ 3-t & t \in [2,4) \end{cases} , \quad T = 4 \Rightarrow \omega = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$a_k = \frac{1}{4} \int_0^2 (t-1)\, e^{-jk\omega_0 t}\, dt + \frac{1}{4} \int_2^4 (3-t)\, e^{-jk\omega_0 t}\, dt$$

$= \ldots$ ✓

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سوال 2 - الف) اگر $x(t)$ حقیقی باشد، آنگاه داریم:

$$\begin{cases} Re\{a_k\} = Re\{a_{-k}\} \\ Im\{a_k\} = -Im\{a_{-k}\} \end{cases} \qquad a_k = \begin{cases} 2 & k=0 \\ j\left(\frac{1}{2}\right)^{|k|} & o.w \end{cases}$$

$\Leftarrow$ شرط دوم Im برقرار نیست و $x(t)$ حقیقی نیست.

ب) آیا $x(t)$ زوج است؟ اگر $x(t)$ زوج باشد داریم: $a_k = a_{-k}$

که به وضوح برقرار است و $x(t)$ زوج است.

ج) آیا $\frac{dx(t)}{dt}$ زوج است؟

$x(t) \rightarrow a_k$

$\frac{dx(t)}{dt} \rightarrow b_k = jk\omega_0 a_k$

$$\Rightarrow b_k = \begin{cases} 0 & k=0 \\ -k\omega_0 \left(\frac{1}{2}\right)^{|k|} & o.w \end{cases}$$

$\Rightarrow b_k \neq b_{-k} \Rightarrow$ فرد است.

سوال ۳)

$x_1(t) = \sin(10\pi t + \frac{\pi}{6})$ $\qquad a(t) \rightarrow a_K \qquad x_1(t) \rightarrow b_K$

$a(t) = \sin \underbrace{\omega_0}_{10\pi} t = \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j} \Rightarrow a_1 = \frac{1}{2j}, \quad a_{-1} = \frac{-1}{2j}, \quad a_K = 0$

$x_1(t) = a(t - t_0) \longrightarrow b_K = a_K e^{-jK\omega_0 t_0}$

$a(t + \frac{1}{60}) = \sin(10\pi(t + \frac{1}{60})) = \sin(10\pi t + \frac{\pi}{6}) = a_K e^{-jK(10\pi)(\frac{-1}{60})}$

$x_2(t) = 1 + \cos(2\pi t)$ $\qquad a(t) \rightarrow a_K$

$a(t) = \cos \omega_0 t = \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2} \Rightarrow a_1 = \frac{1}{2}, \quad a_{-1} = \frac{1}{2} \qquad \omega_0 = 2\pi \qquad a_K = 0 \quad \text{o.w}$

$x_2(t) = e^{j(0)\omega_0 t} + \cos(2\pi t) \Rightarrow b_K = \begin{cases} a_K & K \neq 0 \\ 1 & K = 0 \end{cases}$

سوال ۳، ج)

$$x_3(t) = \left(1 + \cos(2\pi t)\right)\left(\sin\left(10\pi t + \frac{\pi}{6}\right)\right)$$

$$\omega_{0_1} = 2\pi \qquad \omega_{0_2} = 10\pi$$

$$T_{0_1} = 1 \qquad T_{0_2} = \frac{1}{5} \qquad \Rightarrow T_0 = 1 \ , \ \omega_0 = 2\pi$$

$$\left(1 + \frac{1}{2} e^{j2\pi t} + \frac{1}{2} e^{-j2\pi t}\right)\left(e^{+j\frac{\pi}{6}} \times \frac{e^{j10\pi t}}{2j} + \frac{-e^{-j10\pi t}}{2j} \times e^{-j\frac{\pi}{6}}\right)$$

$$\frac{e^{j\frac{\pi}{6}}}{2j} e^{j10\pi t} + \frac{-e^{-j\frac{\pi}{6}}}{2j} e^{-j10\pi t} + \frac{1}{2} \times e^{j\frac{\pi}{6}} \cdot e^{j2\pi t(1+5)} + \cdots$$

همین جوری ضرب رو می‌بریم جلو.

در مرحله بعد باید $e^{jk\omega_0 t}$ بسازیم و ضریبش هر چی باشه می‌شه جواب سوال

$$\omega_0 = 2\pi$$

سوال ۴) الف)

$T_0 = 6 \qquad a_k = \frac{1}{T_0}\int_{T_0} x(t)\, e^{-jk\omega_0 t}\, dt \qquad \omega_0 = \frac{2\pi}{T_0} = \frac{\pi}{3}$

دامنه‌ی انتگرال را از 3- تا 3 در نظر بگیرید

$$a_k = \frac{1}{6}\int_{-2}^{-1} e^{-jk\frac{\pi}{3}t}\, dt - \frac{1}{6}\int_{1}^{2} e^{-jk\frac{\pi}{3}t}\, dt$$

$$= \frac{1}{6}\cdot\frac{1}{-jk\frac{\pi}{3}}\, e^{-jk\frac{\pi}{3}t}\Big|_{-2}^{-1} - \frac{1}{6}\cdot\frac{1}{-jk\frac{\pi}{3}}\, e^{-jk\frac{\pi}{3}t}\Big|_{1}^{2}$$

$$= \frac{1}{-j2\pi k}\left[e^{j\frac{\pi}{3}k} - e^{j\frac{2\pi}{3}k} - e^{-j\frac{2\pi}{3}k} + e^{-j(\frac{\pi}{3})k}\right]$$

$$\Rightarrow x(t) = \sum_k a_k e^{jk\omega_0 t} \qquad \omega_0 = \frac{\pi}{3} \qquad a_k = \frac{\cos(\frac{2\pi}{3})k - \cos(\frac{\pi}{3})k}{j\pi k}$$

$$a_0 = \left(\frac{1}{T_0}\right)\int_{T_0} x(t)\, dt$$

ب) بازه‌ی انتگرال را از $-\frac{1}{2}$ تا $\frac{3}{2}$ در نظر می‌گیریم.

$T_0 = 2 \longrightarrow \omega_0 = \frac{2\pi}{T_0} = \pi$

$$a_k = \frac{1}{2}\int_{-\frac{1}{2}}^{\frac{3}{2}} x(t)e^{-jk\omega_0 t}\,dt = \frac{1}{2}\int_{-\frac{1}{2}}^{\frac{3}{2}} [\delta(t) - 2\delta(t-1)]e^{-jk\omega_0 t}\,dt$$

$$= \frac{1}{2} - e^{-jk\omega_0} = \frac{1}{2} - e^{-jk\pi} \longrightarrow a_0 = -\frac{1}{2},\quad a_k = \frac{1}{2} - (-1)^k$$

سوال ۵)

1. $x(t+t_0) \longrightarrow a_k e^{jk\omega_0 t_0}$

$\Rightarrow b_k = a_k\left(e^{-jk\omega_0 t_0} + e^{jk\omega_0 t_0}\right)$

2. $Even\{x(t)\} = \frac{1}{2}(x(t) + \underbrace{x(-t)}_{\to a_{-k}}) \Rightarrow b_k = \frac{1}{2}(a_k + a_{-k})$

3. $\frac{dx(t)}{dt} \longrightarrow a_k \cdot jk\omega_0 \Rightarrow b_k = jk\omega_0(jk\omega_0 a_k) = -k^2\omega_0^2 a_k$

4. $x(\alpha t) \longrightarrow a_k$ (for $\alpha > 0$) $\quad x(4t) \longrightarrow a_k \quad x(4t+1) \longrightarrow a_k e^{jk\omega_0}$