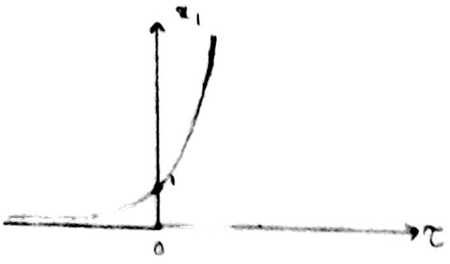
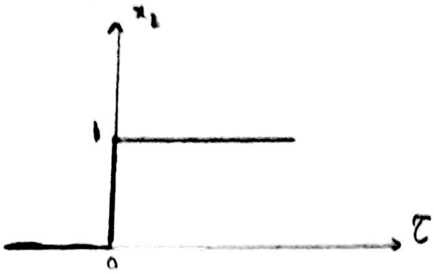
Question 1.

(a) $\begin{cases} x_1(t) = e^t \\ x_2(t) = u(t) \end{cases}$

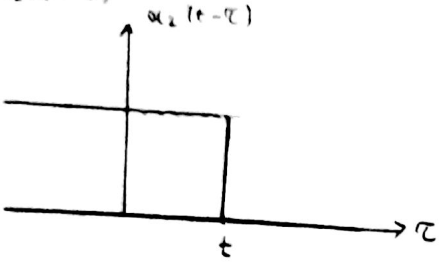
$x_1(\tau)$:



$x_2(\tau)$



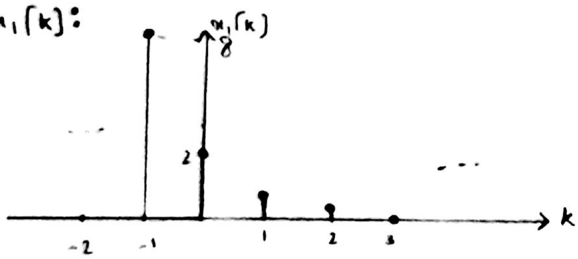
$x_2(t-\tau)$



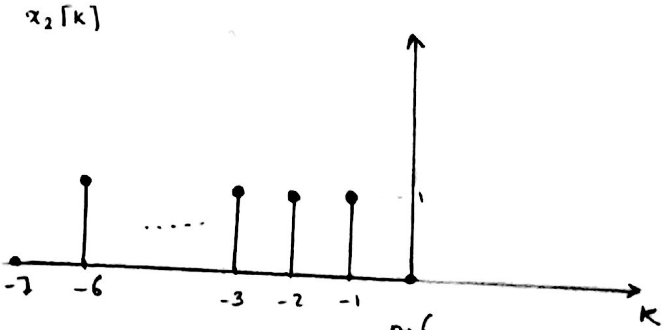
$$x_1 * x_2 = \int_{-\infty}^{+\infty} x_1(\tau)\, x_2(t-\tau)\, d\tau = \int_{-\infty}^{t} e^{\tau} d\tau = e^{\tau}\Big|_{-\infty}^{t} = \boxed{e^t}$$

(b) $\begin{cases} x_1[n] = \left(\frac{1}{2}\right)^{2n-1} \\ x_2[n] = u[-n-1] - u[-n-7] \end{cases}$

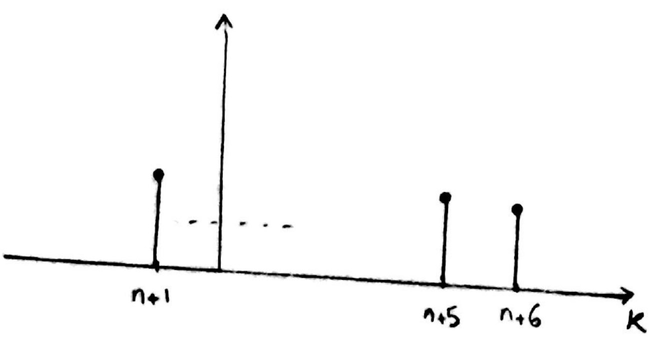
$x_1[k]$:



$x_2[k]$



$x_2[n-k]$:



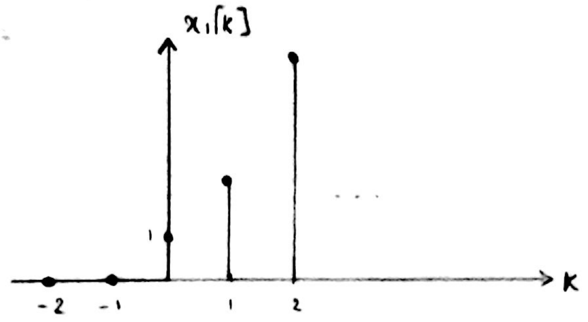
$$x_1 * x_2 = \sum_{k=-\infty}^{+\infty} x_1[k]\, x_2[n-k] = \sum_{k=n+1}^{n+6} \left(\frac{1}{2}\right)^{2k-1}$$

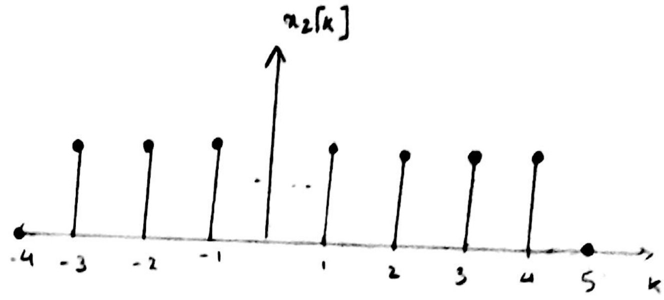
$$= 2 \sum_{k=n+1}^{n+6} \left(\frac{1}{4}\right)^k = 2 \times \frac{1 - \left(\frac{1}{4}\right)^{n+6-(n+1)+1}}{1 - \frac{1}{4}} \times \left(\frac{1}{4}\right)^{n+1}$$

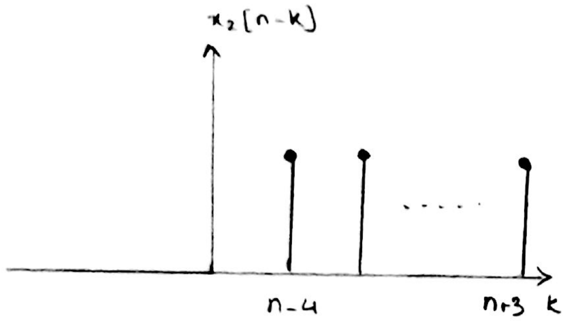
$$= 2 \times \frac{1 - \left(\frac{1}{4}\right)^6}{\frac{3}{4}} \times \left(\frac{1}{4}\right)^{n+1}$$

$$= \frac{8}{3}\left(1 - \left(\frac{1}{4}\right)^6\right) \times \left(\frac{1}{4}\right)^{n+1}$$

(c) $\begin{cases} x_1[n] = 2^n u[n] \\ x_2[n] = u[n+3] - u[n-5] \end{cases}$







$n < -3$:

$$x_1 * x_2 = 0$$

$-3 \leq n < 4$:

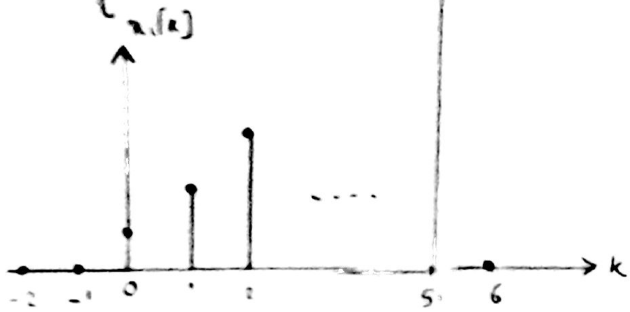
$$x_1 * x_2 = \sum_{k=0}^{n+3} 2^k \overset{1}{\cancel{u[k]}} = \sum_{k=0}^{n+3} 2^k \overset{\text{دنباله هندسی}}{=\!=} \frac{1-2^{n+3+1}}{1-2} = 2^{n+4} - 1$$

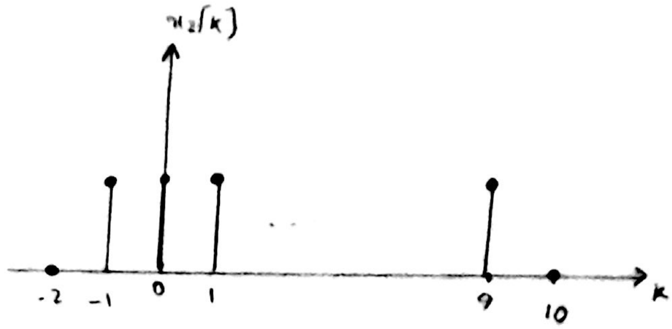
$4 \leq n$:

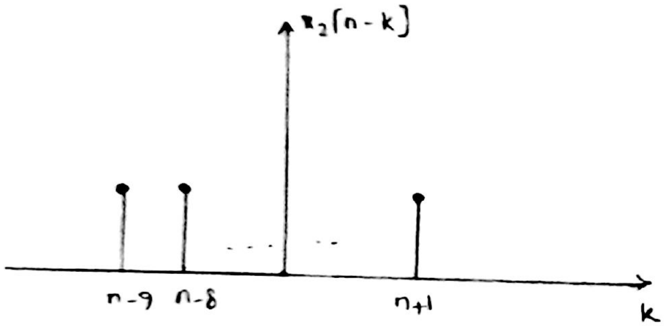
$$x_1 * x_2 = \sum_{k=n-4}^{n+3} 2^k \overset{1}{\cancel{u[k]}} = \sum_{k=n-4}^{n+3} 2^k \overset{\text{دنباله هندسی}}{=\!=} \frac{1-2^{(n+3)-(n-4)+1}}{1-2} \times 2^{n-4} = \frac{255}{16} \cdot 2^n$$

$\Longrightarrow$ جواب نهایی : $x_1[n] * x_2[n] = \begin{cases} 0 & n < -3 \\ 2^{n+4} - 1 & -3 \leq n < 4 \\ \frac{255}{16} \times 2^n & 4 \leq n \end{cases}$

(d) $\begin{cases} x_1[n] = 3^n\left(u[n] - u[n-6]\right) \\ x_2[n] = u[n+1] - u[n-10] \end{cases}$







$n < -1$:

$$x_1 * x_2 = 0$$

$-1 \leqslant n < 5$

$$x_1 * x_2 = \sum_{k=0}^{n+1} 3^k \left(u[k] - u[k-6]\right) = \sum_{k=0}^{n+1} 3^k \overset{\text{دنباله هندسی}}{=} \frac{1-3^{n+2}}{1-3} = \frac{3^{n+2}-1}{2}$$

$5 \leqslant n < 9$:

$$x_1 * x_2 = \sum_{k=0}^{5} 3^k \overset{\text{دنباله هندسی}}{=} \frac{1-3^6}{1-3} = \frac{3^6-1}{2} = 364$$

$9 \leqslant n < 15$:

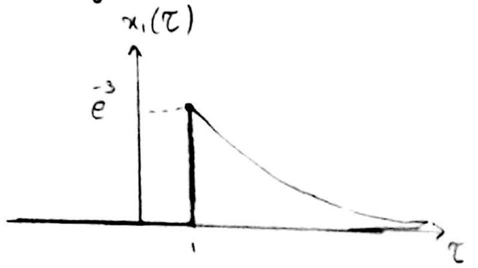
$$x_1 * x_2 = \sum_{k=n-9}^{5} 3^k \overset{\text{دنباله هندسی}}{=} \frac{1-3^{5-(n-9)+1}}{1-3} \times 3^{n-9} = \frac{3^6 - 3^{n-9}}{2}$$

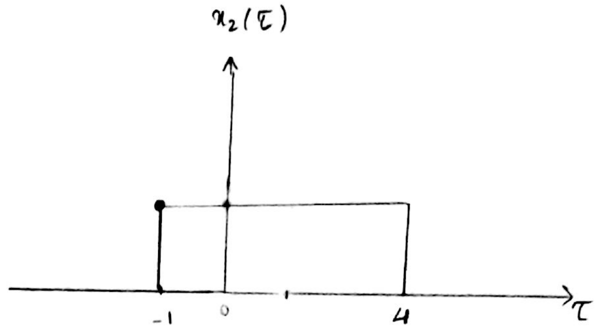
$n \geqslant 15$:

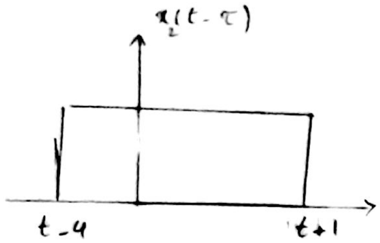
$$x_1 * x_2 = 0$$

$$\Longrightarrow \text{جواب نهایی}: \quad x_1[n] * x_2[n] = \begin{cases} 0 & n < -1 \\ \dfrac{3^{n+2}-1}{2} & -1 \leqslant n < 5 \\ 364 & 5 \leqslant n < 9 \\ \dfrac{3^6 - 3^{n-9}}{2} & 9 \leqslant n < 15 \\ 0 & 15 \leqslant n \end{cases}$$

(e) $\begin{cases} x_1(t) = e^{-3t}\, u(t-1) \\ x_2(t) = u(t+1) - u(t-4) \end{cases}$







$t < 0:$

$$x_1 * x_2 = 0$$

$0 \leqslant t < 5$

$$x_1 * x_2 = \int_{1}^{t+1} e^{-3\tau}\, \cancel{u(\tau-1)}\, d\tau = \int_{1}^{t+1} e^{-3\tau}\, d\tau = -\frac{1}{3} e^{-3\tau}\Big|_{1}^{t+1} = \frac{1}{3}\left(e^{-3} - e^{-3(t+1)}\right)$$

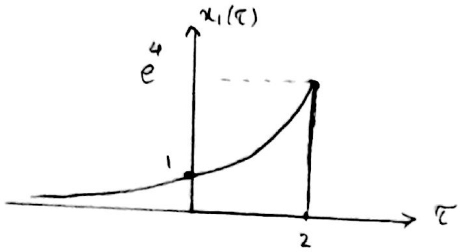
$5 \leqslant t:$

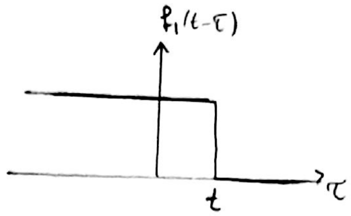
$$x_1 * x_2 = \int_{t-4}^{t+1} e^{-3\tau}\, \cancel{u(\tau-1)}\, d\tau = \int_{t-4}^{t+1} e^{-3\tau}\, d\tau = -\frac{1}{3} e^{-3\tau}\Big|_{t-4}^{t+1} = \frac{1}{3}\left(e^{12-3t} - e^{-3-3t}\right)$$

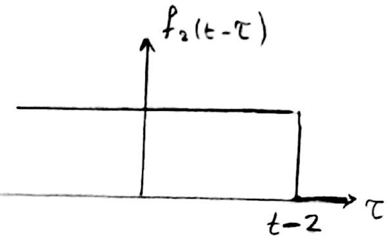
$\Longrightarrow$ جواب نهایی: $x_1(t) * x_2(t) = \begin{cases} 0 & t < 0 \\ \frac{1}{3}\left(e^{-3} - e^{-3(t+1)}\right) & 0 \leqslant t < 5 \\ \frac{1}{3}\left(e^{12-3t} - e^{-3-3t}\right) & t \geqslant 5 \end{cases}$

(f) $\begin{cases} x_1(t) = e^{2t}\,u(2-t) \\ x_2(t) = u(t) + u(t-2) \end{cases}$

$$x_1 * x_2 = x_1 * (\underbrace{u(t)}_{f_1(t)} + \underbrace{u(t-2)}_{f_2(t)}) = x_1(t) * f_1(t) + x_1 * f_2(t)$$







$t < 2$:

$$x_1 * f_1 = \int_{-\infty}^{t} e^{2\tau}\,\overset{1}{\cancel{u(2-\tau)}}\,d\tau = \int_{-\infty}^{t} e^{2\tau}d\tau = \frac{e^{2\tau}}{2}\Big|_{-\infty}^{t} = \frac{e^{2t}}{2}$$

$t \geq 2$:

$$x_1 * f_1 = \int_{-\infty}^{2} e^{2\tau}\,\overset{1}{\cancel{u(2-\tau)}}\,d\tau = \int_{-\infty}^{2} e^{2\tau}d\tau = \frac{e^{2\tau}}{2}\Big|_{-\infty}^{2} = \frac{e^4}{2}$$

$$x_1(t) * f_1(t) = \begin{cases} \dfrac{e^{2t}}{2} & t < 2 \\ \dfrac{e^4}{2} & t \geq 2 \end{cases}$$

$t < 4$:

$$x_1 * f_2 = \int_{-\infty}^{t+2} e^{2\tau}d\tau = \frac{e^{2\tau}}{2}\Big|_{-\infty}^{t-2} = \frac{e^{2t-4}}{2}$$

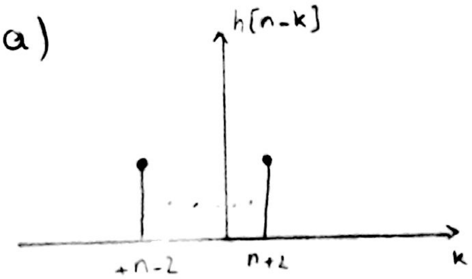
$t \geq 4$:

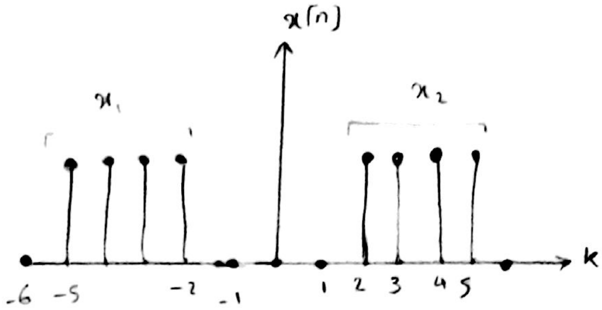
$$x_1 * f_2 = \int_{-\infty}^{2} e^{2\tau}d\tau = \frac{e^{2\tau}}{2}\Big|_{-\infty}^{2} = \frac{e^4}{2}$$

$$\Rightarrow x_1 * (f_1 + f_2) = x_1(t) * x_2(t) = \begin{cases} \dfrac{e^{2t} + e^{2t-4}}{2} & t < 2 \\ \dfrac{e^{2t-4} + e^4}{2} & 2 \leq t < 4 \\ e^4 & t \geq 4 \end{cases}$$

Question 2.

(a)





$$y[n] = x_1[n] * h[n] = (x_1[n] + x_2[n]) * h[n] = x_1 * h + x_2 * h$$

$n < -7$:

$$x_1 * h = 0$$

$-7 \leqslant n < -3$

$$x_1 * h = \sum_{k=-5}^{n+2} 1 = n+2-(-5)+1 = n+8$$

$-3 \leqslant n < 1$

$$x_1 * h = \sum_{k=n-2}^{-2} 1 = -2-(n-2)+1 = 1-n$$

$n \geqslant 1$

$$x_1 * h = 0$$

$n < 0$:

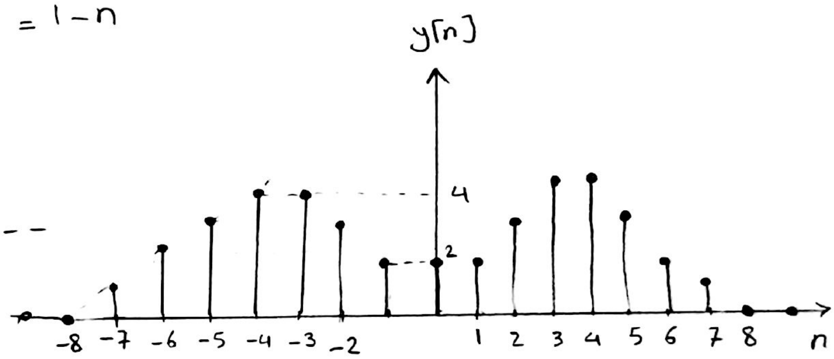
$$x_2 * h = 0$$

$0 \leqslant n < 4$:

$$x_2 * h = \sum_{k=2}^{n+2} 1 = n+\cancel{2}-\cancel{2}+1 = n+1$$

$4 \leqslant n < 8$

$$x_2 * h = \sum_{k=n-2}^{5} 1 = 5-(n-2)+1 = 8-n$$



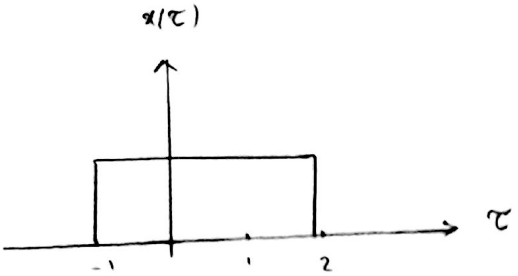
$n \geqslant 8$:

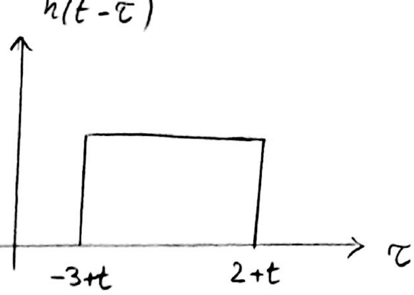
$$x_2 * h = 0$$

جواب نهایی: $x_1[n] * h[n] = \begin{cases} 0 & n < -7 \\ n+8 & -7 \leqslant n < -3 \\ 1-n & -3 \leqslant n < 0 \\ 2 & 0 \leqslant n < 1 \\ n+1 & 1 \leqslant n < 4 \\ 8-n & 4 \leqslant n < 8 \\ 0 & 8 \leqslant n \end{cases}$

Generated by CamScanner

(b)





$y(t) = x(t) * h(t)$

$t < -3$:

$x * h = 0$

$-3 \leq t < 0$:

$$x * h = \int_{-1}^{2+t} d\tau = 3 + t$$

$0 \leq t < 2$

$$x * h = \int_{-1}^{2} d\tau = 3$$

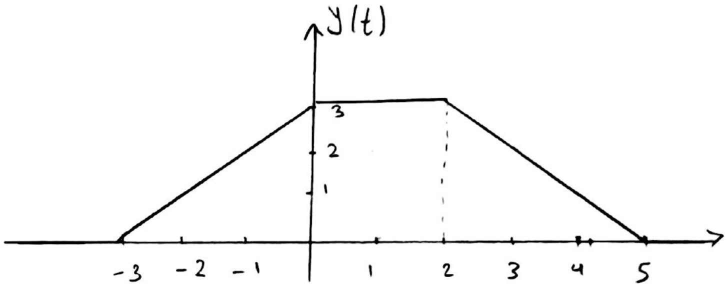
$2 \leq t < 5$

$$x * h = \int_{t-3}^{2} d\tau = 5 - t$$

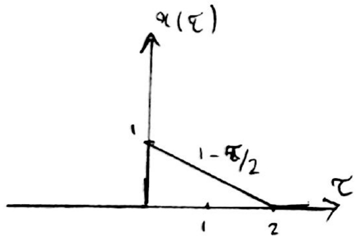
$t \geq 5$

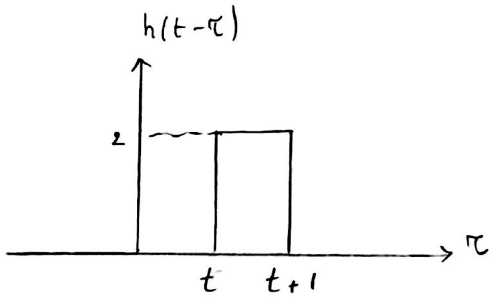
$x * h = 0$

$$y(t) = \begin{cases} 0 & t < -3 \\ 3+t & -3 \leq t < 0 \\ 3 & 0 \leq t < 2 \\ 5-t & 2 \leq t < 5 \\ 0 & t \geq 5 \end{cases}$$



(c)





$t < -1$:

$y(t) = x * h = 0$

$-1 \leq t < 0$

$$x * h = 2\int_{0}^{t+1} \left(1 - \frac{\tau}{2}\right) d\tau = 2\left(\tau - \frac{\tau^2}{4}\right)\Big|_0^{t+1} = 2(t+1) - \frac{(t+1)^2}{2}$$

$0 \leq t < 1$:

$$x * h = 2\int_{t}^{t+1} \left(1 - \frac{\tau}{2}\right) d\tau = 2 - \frac{2t+1}{2}$$

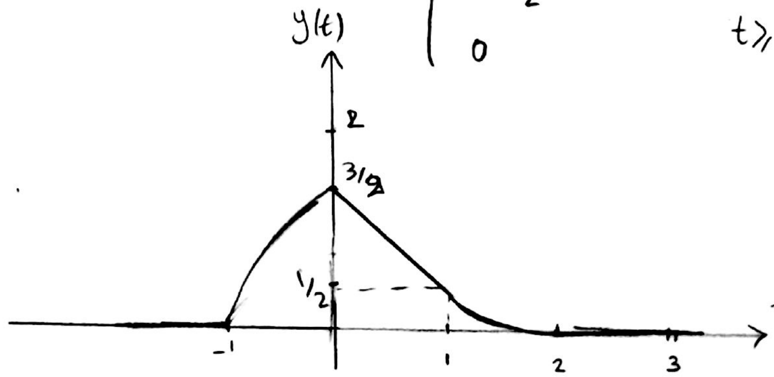
$1 \leq t < 2$:

$$x * h = 2\int_{t}^{2} \left(1 - \frac{\tau}{2}\right) d\tau = \frac{(t-2)^2}{2}$$

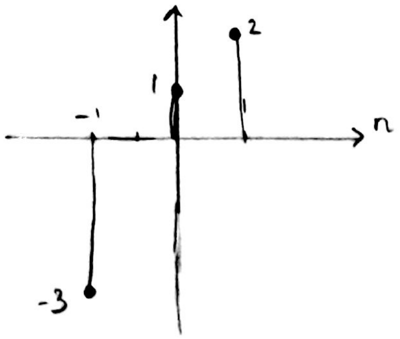
$t \geq 2$:

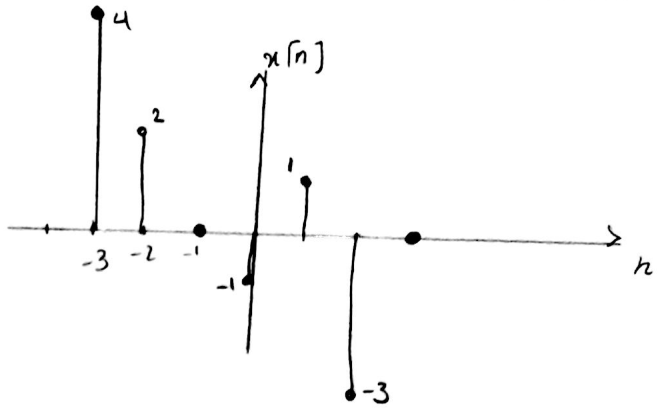
$x * h = 0$

$$y(t) = x * h = \begin{cases} 0 & t < -1 \\ \frac{(3-t)(t+1)}{2} & -1 \leq t < 0 \\ \frac{3-2t}{2} & 0 \leq t < 1 \\ \frac{(t-2)^2}{2} & 1 \leq t < 2 \\ 0 & t \geq 2 \end{cases}$$



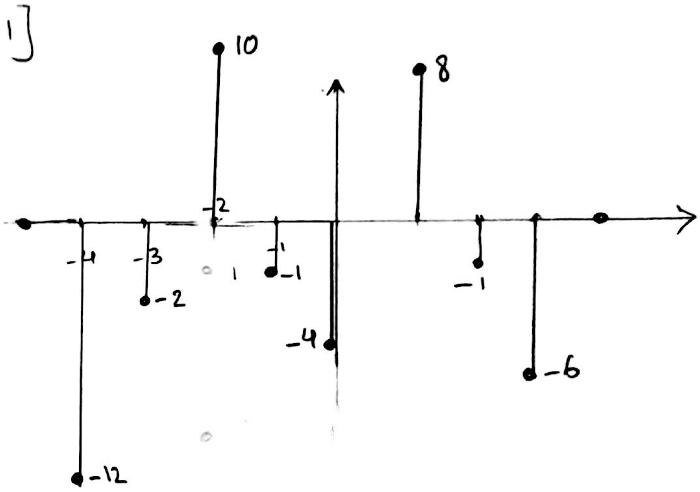
(d)



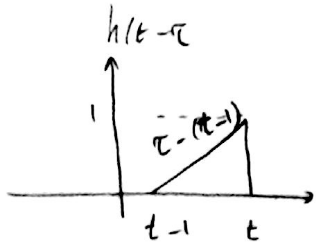


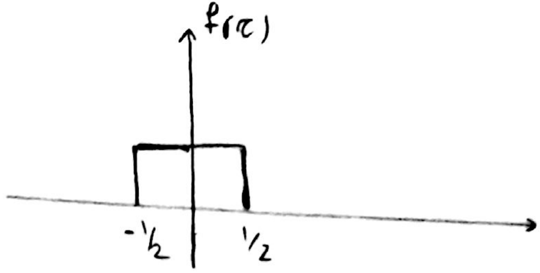
$h[n] = \delta[n] + 2\delta[n-1] - 3\delta[n+1]$

$x[n] * h[n] = x[n] + 2x[n-1] - 3x[n+1]$

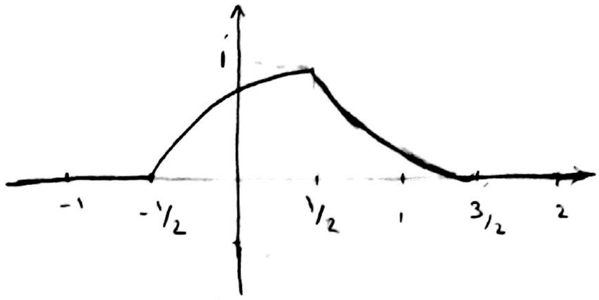


(ج)





$w(t) = h(t) * f(t)$:



$t < -1/2$:

$$h(t) * f(t) = 0$$

$-1/2 < t < 1/2$:

$$h(t) * f(t) = \int_{-1/2}^{t} (\tau - (t+1))\, d\tau = \frac{\tau^2}{2} - \tau(t-1)\Big|_{-1/2}^{t} = \frac{t^2}{2} - \frac{1}{8} - (t+\tfrac{1}{2})(t-1)$$

$$= -\frac{1}{2}t^2 + \frac{1}{2}t + \frac{3}{8}$$

$\frac{1}{2} < t < 3/2$

$$h(t) * f(t) = \int_{t-1}^{1/2} (\tau - (t-1))\, d\tau = \frac{\tau^2}{2} - \tau(t-1)\Big|_{t-1}^{1/2} = \frac{1}{8} - \frac{(t-1)^2}{2} + (t-1)(t-3/2)$$

$$= \frac{t^2}{2} - \frac{3}{2}t + \frac{9}{8}$$

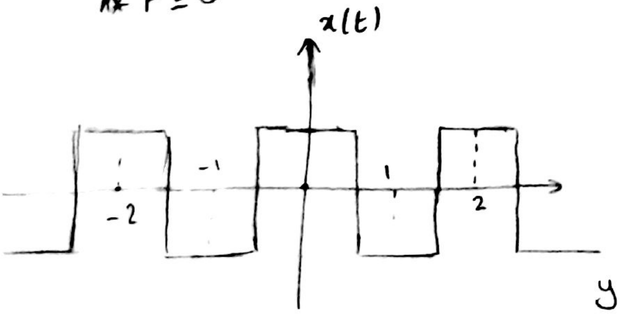
$t \geq 3/2$:

$$h * f = 0$$

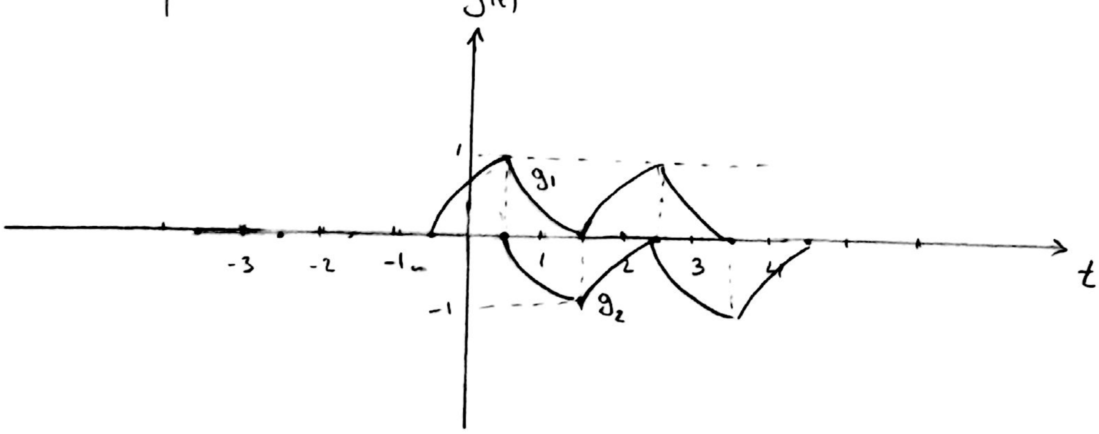
جواب نهایی: $w(t) = h * f = \left(-\frac{t^2}{2} + \frac{1}{2}t + \frac{3}{8}\right) u(t+\tfrac{1}{2})$

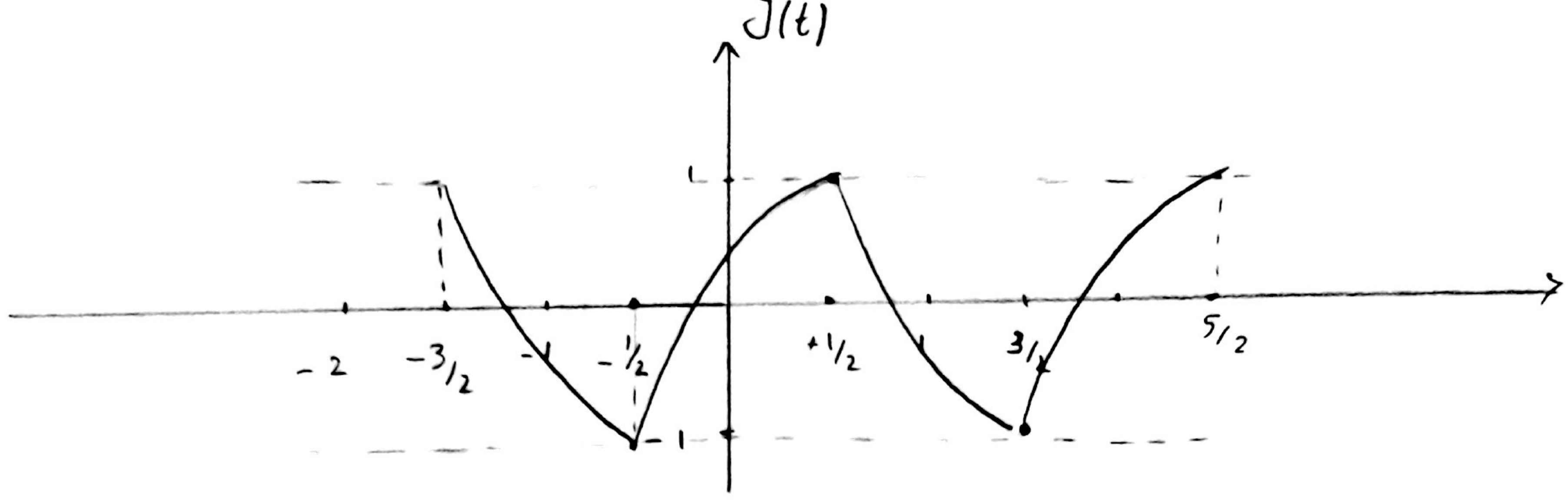
$$+\left(t^2 - 2t + \frac{3}{4}\right) u(t - \tfrac{1}{2})$$

$$-\left(\frac{t^2}{2} - \frac{3}{2} + \frac{9}{8}\right) u(t - \tfrac{3}{2})$$

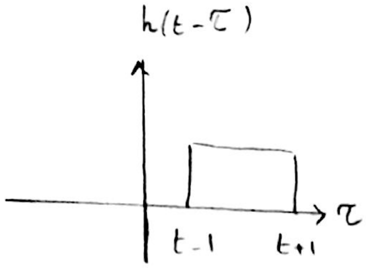


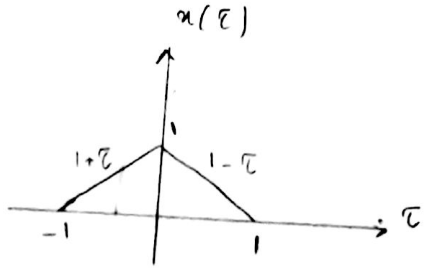
$$y(t) = \underbrace{\sum_{k=-\infty}^{+\infty} w(t-2k)}_{g_1(t)} - \underbrace{\sum_{k=-\infty}^{+\infty} w(t-(2k+1))}_{g_2(t)}$$




y(t)
1
-1
-2
-3/2
-1
-1/2
+1/2
1
3/2
5/2

(f)





$t < -2$:

$$x*h = 0$$

$-2 \leq t < -1$

$$x*h = \int_{-1}^{t-1} (1+\tau)\, d\tau = \tau + \frac{\tau^2}{2}\Big|_{-1}^{t+1} = 2+t + \frac{(t+1)^2-1}{2} = 2+t+\frac{(t+2)t}{2} = \frac{(t+2)^2}{2}$$

$-1 \leq t < 0$:

$$x*h = \int_{-1}^{0} (1+\tau)\, d\tau + \int_{0}^{t+1} (1-\tau)\, d\tau = \tau+\frac{\tau^2}{2}\Big|_{-1}^{0} + \tau - \frac{\tau^2}{2}\Big|_{0}^{t+1} = +\frac{1}{2} + t+1 - \frac{(t+1)^2}{2}$$

$$= \frac{1}{2} + \frac{(t+1)(1-t)}{2}$$

$0 \leq t < 1$:

$$x*h = \int_{t-1}^{0} (1+\tau)\, d\tau + \int_{0}^{1} (1-\tau)\, d\tau = \tau+\frac{\tau^2}{2}\Big|_{t-1}^{0} + \frac{1}{2} = -t+1-\frac{(t-1)^2}{2} + \frac{1}{2} = \frac{(t-1)(-1-t)}{2} + \frac{1}{2}$$

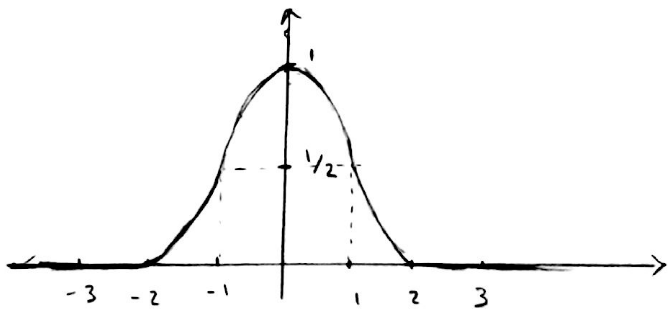
$1 \leq t < 2$:

$$x*h = \int_{t-1}^{1} (1-\tau)\, d\tau = \tau - \frac{\tau^2}{2}\Big|_{t-1}^{1} = 2-t-\frac{1}{2}+\frac{(t-1)^2}{2} = \frac{(t-2)^2}{2}$$

$t \geq 2$:

$$x*h = 0$$

$$y(t) = x*h = \begin{cases} 0 & t < -2 \\ \frac{(t+2)^2}{2} & -2 \leq t < -1 \\ \frac{1}{2} + \frac{(t+1)(1-t)}{2} & -1 \leq t < 0 \\ \frac{1}{2} + \frac{(1-t)(t+1)}{2} & 0 \leq t < 1 \\ \frac{(t-2)^2}{2} & 1 \leq t < 2 \\ 0 & 2 \leq t \end{cases}$$



Question 3.

(a) $h[n] = 2^{-3|n|}$

(1) System is with memory: $h[0] = 2^{-3\times 0} = 2^0 = 1 \neq 0$

(2) System is not causal: $n=-1 \Rightarrow h[-1] = 2^{-3\times 1} = \frac{1}{8} \neq 0$

(3) System is stable:

$$\sum_{k=-\infty}^{+\infty} |h[k]| = \sum_{k=-\infty}^{+\infty} |2^{-3|k|}| = \sum_{k=0}^{+\infty} 2^{-3k} + \sum_{k=-\infty}^{-1} 2^{3k} = \left(1 + \frac{1}{8} + \left(\frac{1}{8}\right)^2 + \cdots\right) + \left(\frac{1}{8} + \left(\frac{1}{8}\right)^2 + \cdots\right)$$

$$= \frac{1}{1-\frac{1}{8}} + \frac{\frac{1}{8}}{1-\frac{1}{8}} = \frac{8}{7} + \frac{1}{7} = \frac{9}{7} < \infty$$

(b) $h(t) = te^{-t}u(t)$

(1) System is memoryless: $h(0) = 0 \times e^0 \times 1 = 0$.

(2) System is causal: $\forall t<0.\ u(t)=0 \Rightarrow h(t)=0$

(3) System is stable:

$$\int_{-\infty}^{+\infty} |h(\tau)|\,d\tau = \int_{-\infty}^{+\infty} |\tau| e^{-\tau} u(\tau) d\tau = \int_0^{+\infty} \tau e^{-\tau} d\tau = -\tau e^{-\tau} - e^{-\tau}\Big|_0^{+\infty} = \lim_{\tau \to +\infty} \frac{\tau}{e^{\tau}} - 1$$

$$= -1 < \infty$$

(c) $h(t) = \cos(t)u(t+1)$

(1) System is with memory: $h(0) = \cos(0)u(1) = 1 \neq 0$

(2) System is not causal: $t = -1/2 \Rightarrow h(-1/2) = \cos(-1/2)u(1/2) = \cos(-\frac{1}{2}) \neq 0$

(3) System is not stable:

$$\int_{-\infty}^{+\infty} |h(\tau)|\,d\tau = \int_{-\infty}^{+\infty} |\cos(\tau)|\,u(\tau+1)\,d\tau = \int_{-1}^{+\infty} |\cos(\tau)|\,d\tau = +\infty$$

(d) $u(t+1) * \frac{d}{dt} u(t-2)$

$$\frac{d}{dt} u(t-2) = \delta(t-2) \Rightarrow h(t) = u(t+1) * \delta(t-2) = u(t-1)$$

(1) System is memoryless: $h(0) = u(-1) = 0$

(2) System is causal: $\forall t < 0 .\ u(t-1) = 0 \Rightarrow h(t) = 0$

(3) System is not stable: $\int_{-\infty}^{+\infty} |h(\tau)| d\tau = \int_{-\infty}^{+\infty} u(\tau-1) d\tau = \int_{1}^{+\infty} d\tau = \infty$

(e) $h(t) = \tan^{-1}(\cos(t))$

(1) System is with memory: $h(0) = \tan^{-1}(\cos(0)) = \tan^{-1}(1) = \frac{\pi}{4}$

(2) System is not causal: $t = -2\pi \Rightarrow h(-2\pi) = \tan^{-1}(\cos(-2\pi)) = \tan^{-1}(1) = \frac{\pi}{4}$

(3) System is not stable:

پاسخ ضربه $h(t)$ یک سیگنال متناوب است با $T = 2\pi$.

$$\int_{0}^{T} |h(\tau)| d\tau = \int_{0}^{2\pi} |\tan^{-1}(\cos t)| d\tau > 0 \Rightarrow \int_{-\infty}^{+\infty} |h(\tau)| d\tau = +\infty$$

(f) $h(t) = (t^2+1)\delta(t)$

(1) System is with memory: $h(0) = (0+1)\delta(0) = 1 \neq 0$

(2) System is causal: $\forall t < 0 .\ \delta(t) = 0 \Rightarrow h(t) = (t^2+1)\delta(t) = 0$

(3) System is stable:

$$\int_{-\infty}^{+\infty} |h(\tau)| d\tau = \int_{-\infty}^{+\infty} (\tau^2+1)\delta(\tau) d\tau = \tau^2+1 \Big|_{\tau=0} = 1 < \infty$$

(g) $h[n] = \cos(\frac{\pi}{2} n) u[n+2] + \delta[n+2]$

(1) System is with memory: $h[0] = \cos(0) u[2] + \delta[2] = 1 \neq 0$

(2) System is causal: if $n = -2 \Rightarrow h[-2] = \cos(\pi) u[0] + \delta[0] = -1 + 1 = 0$

else if $n = -1 \Rightarrow h[-1] = \cos(-\pi/2) u[1] + \delta[1] = 0$

else $(n < -2) \Rightarrow h[n] = \cos(\frac{\pi}{2} n) \overset{0}{u[n+2]} + \overset{0}{\delta[n+2]} = 0$

(3) System is not stable:

$$\sum_{k=-\infty}^{+\infty} |h[k]| = \sum_{k=-2}^{+\infty} |\cos(\pi/2\, k) u[k+2] + \delta[k+2]| \overset{h[-2]=0}{=} \sum_{k=-1}^{+\infty} |\cos(\frac{\pi}{2} k)| = +\infty$$

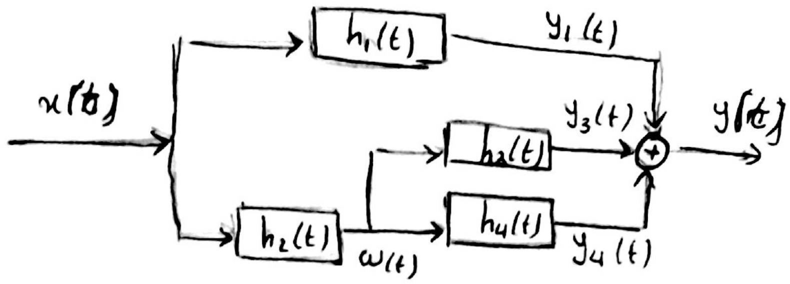
Question 4.

$h_1(t) = \sin\left(\frac{\pi}{3}t\right)$

$h_2(t) = e^{2t}u(1-t)$

$h_3(t) = u(t)$

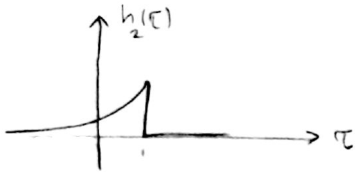
$h_4(t) = \delta(t+1)$

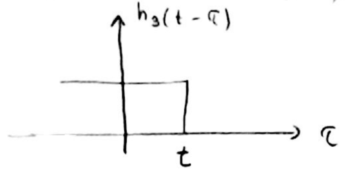


(a) $x(t) = \delta(t)$

$y_1(t) = \delta(t) * h_1(t) = h_1(t)$

$y_3(t) = \delta(t) * h_2(t) * h_3(t) = h_2(t) * h_3(t)$





$t<1: \quad h_2 * h_3 = \int_{-\infty}^{t} e^{2\tau} d\tau = \frac{e^{2\tau}}{2}\Big|_{-\infty}^{t} = \frac{e^{2t}}{2}$

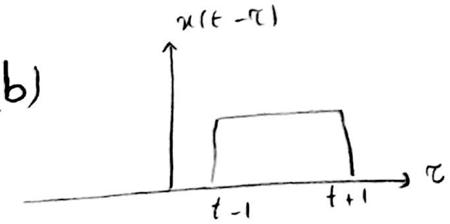
$t \geq 1: \quad h_2 * h_3 = \int_{-\infty}^{1} e^{2\tau} d\tau = \frac{e^{2\tau}}{2}\Big|_{-\infty}^{1} = \frac{e^{2}}{2}$

$\Rightarrow h_2 * h_3 = \frac{e^2}{2} + \left(\frac{e^{2t} - e^2}{2}\right)u(1-t)$

$y_4(t) = \delta(t) * h_2(t) * h_4(t) = h_2(t) * h_4(t) = h_2(t) * \delta(t+1) = h_2(t+1) = e^{2(t+1)}u(-t)$

$\Rightarrow h_{eq} = y_1 + y_3 + y_4 = \sin(\pi/3\, t) + \frac{e^2}{2} + \left(\frac{e^{2t} - e^2}{2}\right)u(1-t) + e^{2(t+1)}u(-t)$

(b)



$y_1 = h_1(t) * x_1(t) = \int_{t-1}^{t+1} \sin(\pi/3\,\tau)d\tau = -\frac{3}{\pi}\cos\left(\frac{\pi}{3}\tau\right)\Big|_{t-1}^{t+1} = -\frac{3}{\pi}\left[\cos\left(\pi/3(t+1)\right) - \cos\left(\pi/3(t-1)\right)\right]$

$y_2 = h_2 * x$

$t<0:$

$h_2 * x = \int_{t-1}^{t+1} e^{2\tau} d\tau = \frac{e^{2\tau}}{2}\Big|_{t-1}^{t+1} = \frac{e^{2(t+1)} - e^{2(t-1)}}{2}$

$0 \leqslant t < 2$:

$$x * h_2 = \int_{t-1}^{1} e^{2\tau} d\tau = \frac{e^{2\tau}}{2}\Big|_{t-1}^{1} = \frac{e^2 - e^{t-1}}{2}$$

$2 \leqslant t$:

$$x * h_2 = 0$$

$$\omega_{(t)} = \left(\frac{e^{2(t+1)} - e^{-2(t-1)}}{2}\right) u(-t) + \left(\frac{e^2 - e^{t-1}}{2}\right)\left(u(t) - u(t-2)\right)$$

$$y_4(t) = \omega_{(t)} * h_4(t) = \omega_{(t)} * \delta(t+1) = \omega(t+1)$$

$$\Rightarrow y_4(t) = \left(\frac{e^{2(t+2)} - e^{-2t}}{2}\right) u(-t-1) + \frac{e^2 - e^t}{2}\left(u(t+1) - u(t-1)\right)$$

$y_3(t) = \omega(t) * u(t)$:

$t < 0$

$$y_3(t) = \int_{-\infty}^{t} \frac{e^{2(\tau+1)} - e^{-2(\tau-1)}}{2} d\tau = \frac{e^2}{2}\int_{-\infty}^{t} e^{2\tau} - \frac{e^2}{2}\int_{-\infty}^{t} e^{-2\tau} d\tau$$

$$= \frac{e^2}{2} \times \frac{e^{2t}}{2} + \frac{e^2}{2}\left(\frac{e^{-2t}}{2}\right) = \frac{e^2}{4}\left(e^{2t} - e^{-2t}\right)$$

$0 \leqslant t < 2$

$$y_3(t) = \int_{-\infty}^{0} \frac{e^{2(\tau+1)} - e^{-2(\tau-1)}}{2} d\tau + \int_{0}^{t} \frac{e^2 - e^\tau}{2} d\tau$$

$$= 0 + \frac{e^2}{2}t - \frac{e^\tau}{2}\Big|_0^t = \frac{e^2}{2}t - \left(\frac{e^t}{2} - \frac{1}{2}\right) = \frac{e^2 t - e^t + 1}{2}$$

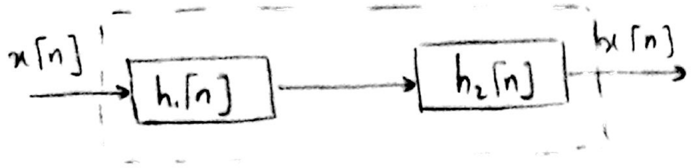
$t \geqslant 2$:

$$y_3(t) = \int_{-\infty}^{0} \frac{e^{2(\tau+1)} - e^{-2(\tau-1)}}{2} d\tau + \int_{0}^{2} \frac{e^2 - e^\tau}{2} d\tau = 0 + \frac{2e^2 - e^2 + 1}{2}$$

$$= \frac{e^2 + 1}{2}$$

Question 5.

(a) $h_1[n] = (\frac{1}{5})^n u[n]$, $h_2[n] = \delta[n] - \frac{1}{5}\delta[n-1]$



Inverse System: $h_1[n] * h_2[n] = \delta[n]$

$$h_1[n] * h_2[n] = (\frac{1}{5})^n u[n] * \left(\delta[n] - \frac{1}{5}\delta[n-1]\right) = (\frac{1}{5})^n u[n] * \delta[n] - (\frac{1}{5})^{n+1} u[n] * \delta[n-1]$$

$$= (\frac{1}{5})^n u[n] - (\frac{1}{5})^n u[n-1] = (\frac{1}{5})^n (u[n] - u[n-1]) = \delta[n]$$

این دو سیستم معکوس یکدیگر هستند

(b) $h_1(t) = e^{-t} u(t)$, $h_2(t) = \delta(t) + \delta'(t)$

$$h_1 * h_2 = e^{-t} u(t) * \delta(t) + e^{-t} u(t) * \delta'(t) = e^{-t} u(t) + \frac{d}{dt}(e^{-t} u(t))$$

$$\frac{d}{dt}(e^{-t} u(t)) = e^{-t} \frac{d(u(t))}{dt} + u(t) \frac{d}{dt}(e^{-t}) = e^{-t}\delta(t) - e^{-t} u(t)$$

$$\Rightarrow h_1 * h_2 = \cancel{e^{-t} u(t)} + e^{-t}\delta(t) - \cancel{e^{-t} u(t)} = e^{-t}\delta(t) = \delta(t)$$

این دو سیستم معکوس یکدیگر هستند

$$e^{-t}\delta(t) = \begin{cases} e^{0}\delta(0) = \delta(0) & t = 0 \\ 0 & t \neq 0 \end{cases} = \delta(t)$$

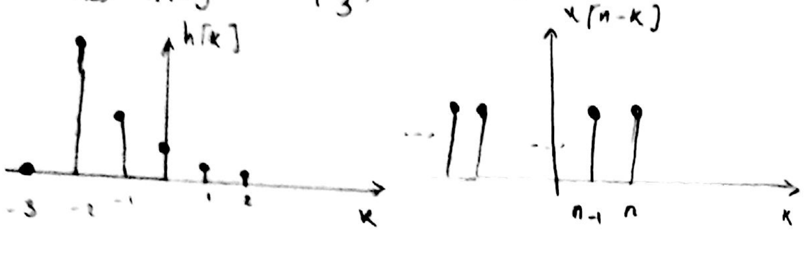
Question 6.

(a) $h(t) = \delta(t+1) - \delta(t-1) \qquad x(t) = u(t)$

$$y(t) = x * h = u(t) * (\delta(t+1) - \delta(t-1)) = u(t) * \delta(t+1) - u(t) * \delta(t-1)$$

$$= u(t+1) - u(t-1)$$

(b) $h[n] = e^n \left(\frac{1}{3}\right)^n u[n+2]$



$n < -2$:

$y[n] = 0$

$-2 \leqslant n$:

$$y[n] = \sum_{k=-2}^{n} \left(\frac{e}{3}\right)^k = \left(\frac{e}{3}\right)^{-2} \times \frac{1 - \left(\frac{e}{3}\right)^{n+3}}{1 - \frac{e}{3}}$$

$$\Rightarrow y[n] = \left(\frac{e}{3}\right)^{-2} \times \frac{1 - \left(\frac{e}{3}\right)^{n+3}}{1 - e/3} u[n-2]$$

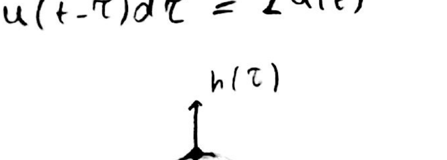
(c) $h(t) = 2\delta^2(t)$

$t < 0$:

$y(t) = 0$

$t \geqslant 0$

$$y(t) = \int_{-\infty}^{t} 2\delta^2(\tau) u(t-\tau) d\tau = 2u(t)$$



(d) $h(t) = \frac{1}{1+t^2}$

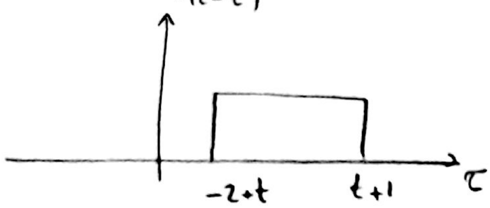
$$y(t) = x * h = \int_{-\infty}^{t} \frac{1}{1+\tau^2} d\tau = \tan^{-1}(\tau) \Big|_{-\infty}^{t} = \tan^{-1}(t) - \frac{\pi}{2}$$

Question 7.

(a) $$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau-2)\, d\tau \underset{d\tau' = d\tau}{\overset{\tau' = \tau - 2}{=\!=}} \int_{-\infty}^{t} e^{-(t-\tau'-2)} x(\tau')\, d\tau'$$

$$= \int_{-\infty}^{+\infty} e^{-(t-\tau'-2)} u(t-\tau')\, x(\tau')\, d\tau' \Rightarrow h(t) = e^{-(t-2)} u(t)$$

(b)



$t < -1$:

$x * h = 0$

$-1 \leq t < 2$

$$x * h = e^2 \int_0^{t+1} e^{-\tau}\, d\tau = e^2 \times -e^{-\tau}\Big|_0^{t+1} = e^2\left(1 - e^{-(t+1)}\right) = e^2 - e^{1-t}$$

$t \geq 2$:

$$x * h = e^2 \int_{t-2}^{t+1} e^{-\tau}\, d\tau = e^2 \times -e^{-\tau}\Big|_{t-2}^{t+1} = e^2 \times \left(e^{2-t} - e^{-1-t}\right)$$

$$y(t) = \begin{cases} 0 & t < -1 \\ e^2 - e^{1-t} & 1 \leq t < 2 \\ e^2(e^{2-t} - e^{-1-t}) & t \geq 2 \end{cases}$$