Signals and systems - HW5

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①

a) x_1: $N = \frac{2\pi}{w} = 2\pi \times \frac{10}{2\pi} = 10$

x_2: $N = \frac{2\pi}{w} = 2\pi \times \frac{12}{20\pi} = 1.2 = 12$ DT

b) $\tilde{x}[n] = \sum_{k=\langle N\rangle} a_k e^{jwkn}$

$a_k = \frac{1}{N} \sum_{k=\langle N\rangle} \tilde{x}[n] e^{-jwkn}$

x_1: $1 + \sin(\frac{2\pi}{10}) = 1 + \frac{1}{2j}\left(e^{j(\frac{2\pi}{10})n} - e^{-j(\frac{2\pi}{10})n}\right)$

$a_{k_1} = \frac{1}{10} \sum \left(1 + \frac{e^{j(\frac{2\pi}{10})n} - e^{-j(\frac{2\pi}{10})n}}{2j}\right) e^{-j(\frac{2\pi}{10})kn}$

x_2: $1 + \sin(\frac{20\pi}{12}n + \frac{\pi}{2}) = 1 + \cos(\frac{20\pi}{12}) = 1 + \frac{1}{2}\left(e^{j\frac{20\pi}{12}n} + e^{-j\frac{20\pi}{12}n}\right)$

$a_{k_2} = \frac{1}{12} \sum \left(1 + \frac{e^{j(\frac{20\pi}{12})n} + e^{-j(\frac{20\pi}{12})n}}{2}\right) e^{-j(\frac{2\pi}{12})kn}$

c) a_{k_1}: $a_0 =$

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② $N = 3 \Rightarrow w = \frac{2\pi}{3}$

$a_k = \frac{1}{N} \sum_{k=\langle N\rangle} x[n] e^{-jwkn} = \frac{1}{3} \sum_{k=\langle N\rangle} x[n] e^{-jwkn}$

$a_0 = 1$

$a_1 = \frac{1}{3}\left(e^{-j(\frac{2\pi}{3})} + e^{-j2(\frac{2\pi}{3})}\right)$ ①

$a_{-1} = \frac{1}{3}\left(e^{j(\frac{2\pi}{3})} + e^{j2(\frac{2\pi}{3})}\right)$

LTI: $b_k = a_k H(e^{jw})$

$k=-1$: $b_{-1} = a_{-1} H(e^{jw}) \xRightarrow{①} \frac{1}{3}\left(e^{j(\frac{2\pi}{3})} + 2e^{j2(\frac{2\pi}{3})}\right)\left(\frac{1}{3}\right) = \boxed{\frac{1}{9}} e^{j\frac{2\pi}{3}} + \dots$

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③ $N = 10$ ②

$a_{11} = 5 \xRightarrow{①} a_{-11} = 5 \xrightarrow{②} a_1 = 5, a_9 = 5$

$x(e^{jw}) \to$ Even & Real $\Rightarrow a_k = a_{-k}$ for $k \in \{1, 2, \dots\}$ ①

①, ②: $a_{11} = a_1 = a_{-1} = a_9 = 5$

$\frac{1}{10} \sum_0^9 |x[n]|^2 = 50 \xRightarrow[\text{Periodic}]{\text{Parseval}} \sum_0^9 a_k^2 = 50$

$a_1 = a_9 = 5$

$a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 = 0$ } All are 0

$x[n] = a_1 e^{jwn} + a_9 e^{j9wn} = 5e^{j(\frac{2\pi}{10})n} + 5e^{j9(\frac{2\pi}{10})n}$

$A = \frac{1}{5}$ $C = 0$

$B = \frac{2\pi}{10}$

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$T = \frac{2\pi}{4}$

④

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⑤

b) $x[n] = (n+1)a^n u[n] = \underset{\xrightarrow{FT}}{na^n u[n]} + \underset{\xrightarrow{FT}}{a^n u[n]} = \frac{ae^{-j\omega} + 1 - ae^{-j\omega}}{(1-ae^{-j\omega})^2} \overset{a=\frac{1}{4}}{=} \boxed{\frac{1}{(1-(\frac{1}{4})e^{-j\omega})^2}}$

$\frac{ae^{-j\omega}}{(1-ae^{-j\omega})^2} \qquad \frac{1}{1-ae^{-j\omega}}$

a) $x[n] = (\frac{3}{4})^n u[n] \overset{FT}{\longleftrightarrow} \boxed{\frac{(\frac{3}{4})e^{-j\omega}}{(1-(\frac{3}{4})e^{-j\omega})^2}}$

c) ?!

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⑥

a) $x[n] \simeq \delta[n] \underset{\text{فرض}}{\overset{FT}{\Rightarrow}} X(j\omega) = 1$

Then: $x[n] = \boxed{\delta[n] + 3\delta[n-1] + 2\delta[n-2] \mp 4\delta[n-3] + \delta[n-10]}$

b)

فرض: $(7-e^{-j\omega})(5+e^{-j\omega}) \Rightarrow$ Then: $\frac{A}{5+e^{-j\omega}} + \frac{B}{7-e^{-j\omega}} = X(j\omega)$ $\begin{cases} A = -25 \\ B = 49 \end{cases}$

$\frac{-25}{5+e^{-j\omega}} + \frac{49}{7-e^{-j\omega}} \overset{F\text{-pairs}}{\Longrightarrow} \boxed{-(\frac{1}{5})^{n-1} u[n] + (\frac{1}{7})^{n-1} u[n]}$

c) فرض: $\boxed{\sum_{k=0}^{+\infty} \delta[n-k]}$

d) FT^{-1}: $x[n] = \frac{1}{2\pi}\int_{-\frac{3\pi}{4}}^{\frac{\pi}{4}} e^{j\omega n} d\omega + \frac{1}{2\pi}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} e^{j\omega n} d\omega$

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(7)

(8)

آ) $y - \frac{1}{2}y[n-1] = x[n]$

$$H(e^{jw}) = \frac{Y(e^{jw})}{X(e^{jw})} = \frac{\sum b_k e^{-jkw}}{\sum a_k e^{-jkw}} = \frac{1}{1-\frac{1}{2}e^{-jw}} \xRightarrow{\text{F-pairs}} \boxed{h[n] = (\tfrac{1}{2})^n u[n]}$$

ب) $Y(e^{jw}) = H(e^{jw})X(e^{jw})$

i) $\left(\frac{1}{1-\frac{1}{2}e^{-jw}}\right)(1) = \boxed{(\tfrac{1}{2})^n u[n]}$

ii) مانند بالا دی : $\boxed{y[n] = (\tfrac{1}{2})^{n-n_0} u[n-n_0]}$

iii) $Y(e^{jw}) = \left(\frac{1}{1-\frac{1}{2}e^{-jw}}\right)\left(\frac{1}{1-\frac{3}{4}e^{-jw}}\right) = \frac{A}{1-\frac{1}{2}e^{-jw}} + \frac{B}{1-\frac{3}{4}e^{-jw}}$, $\begin{cases} A = -2 \\ B = 3 \end{cases}$ $\boxed{y[n] = -2(\tfrac{1}{2})^n u[n] + 3(\tfrac{3}{4})^n u[n]}$

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