$1-a)$ $x_{(t)} = e^{-r|t|} \sin(rt)$

$\downarrow FT$ $\qquad$ $\downarrow FT$

$\dfrac{4}{9+\omega^r}$ $\qquad$ $\dfrac{\pi}{j}(\delta(\omega-r) - \delta(\omega+r))$

$x_{(t)} \, y_{(t)} \xrightarrow{FT} \dfrac{1}{r\pi} X(j\omega) * Y(j\omega)$

$\Rightarrow X(j\omega) = \dfrac{1}{r\pi} \times \dfrac{\pi}{j} \left(\dfrac{4}{9+(\omega-r)^r} - \dfrac{4}{9+(\omega+r)^r} \right)$

$1-b)$ $x_{(t)} = \begin{cases} 1-t^r & 0 < t < 1 \\ 0 & o.w \end{cases}$

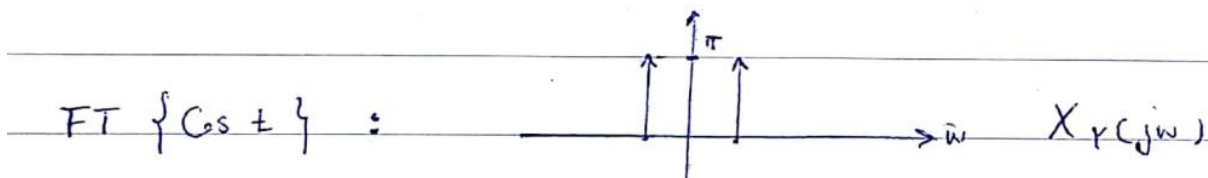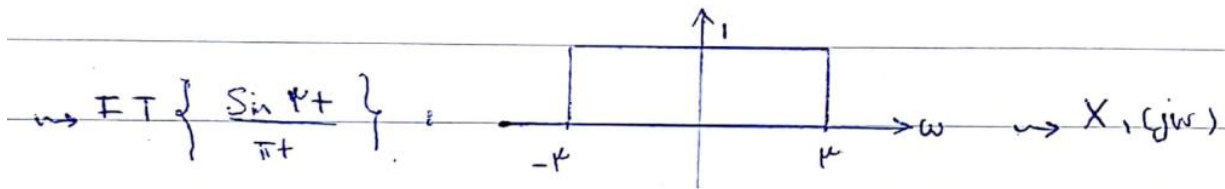$\Rightarrow X(j\omega) = \int_o^1 (1-t^r) e^{-j\omega_o t} \, dt$

$= \left[(1-t^r) \times \dfrac{-1}{j\omega_o} e^{-j\omega_o t} \right]\Big|_o^1 - \int_o^1 (-rt) \times \dfrac{-1}{j\omega_o} e^{-j\omega_o t} \, dt$

$o - \left(\dfrac{-1}{j\omega_o} \right) \qquad + \dfrac{1}{j\omega_o} \left(\left[(-rt) \times \dfrac{-1}{j\omega_o} e^{-j\omega t} \right]\Big|_o^1 - \int_o^1 \dfrac{r}{j\omega} e^{-j\omega_o t} \right)$

$\dfrac{1}{j\omega_o} + \dfrac{1}{j\omega_o} \left(\dfrac{r}{j\omega_o} e^{-j\omega_o} + \dfrac{r}{j^r\omega_o^r} \left[e^{-j\omega_o} - 1 \right] \right)$
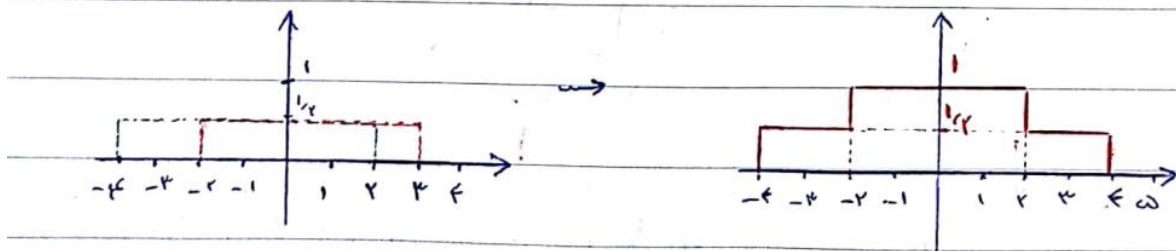
1-C) $x(t) = \dfrac{\sin^2 t \cdot \cos t}{\pi t}$

$\dfrac{\sin \omega_0 t}{\pi t} \xrightarrow{\;FT\;} X(j\omega) = \begin{cases} 1 & |\omega| < \omega_0 \\ 0 & |\omega| > \omega_0 \end{cases}$

$\rightsquigarrow FT\left\{ \dfrac{\sin 2t}{\pi t} \right\}$:

$\rightsquigarrow X_1(j\omega)$

$FT\left\{ \cos t \right\}$:

$X_2(j\omega)$

خاصیت ضرب در حوزه زمان : $X(j\omega) = \dfrac{1}{2\pi} X_1(j\omega) \; * \; X_2(j\omega)$

1-d) $te^{-\Upsilon|t-1|}$

$\leadsto e^{-a|t|} \xrightarrow{FT} \dfrac{\Upsilon a}{a^{\Upsilon}+\omega^{\Upsilon}} \Rightarrow e^{-\Upsilon|t|} \xrightarrow{FT} \dfrac{F}{F+\omega.^{\Upsilon}}$

$\leadsto x(t+t_0) \xrightarrow{FT} e^{j\omega_0 t_0} X(j\omega) \Rightarrow e^{-\Upsilon|t-1|} \xrightarrow{FT} e^{-j\omega_0} \times \dfrac{F}{F+\omega_0^{\Upsilon}}$

$\leadsto t\,x(t) \xrightarrow{FT} -\dfrac{1}{j}\dfrac{dX(j\omega)}{d\omega} \Rightarrow te^{-\Upsilon|t-1|} \xrightarrow{FT} -\dfrac{1}{j}\cdot\dfrac{d\left(\dfrac{Fe^{-j\omega_0}}{F+\omega_0^{\Upsilon}}\right)}{d\omega}$

سوال ۲)

1. $X(\omega) = \omega e^{-|\omega|}$

$e^{-\alpha|t|} \xrightarrow{\alpha>0} \dfrac{2\alpha}{\alpha^2+\omega^2} \Rightarrow e^{-|t|} \xrightarrow{\quad} \dfrac{2}{1+\omega^2}$

خاصیت $\begin{cases} x(t) \to X(\omega) \\ t\,x(t) \to -\dfrac{1}{j}\dfrac{dX(\omega)}{d\omega} \end{cases}$

$te^{-|t|} \xrightarrow{\quad} -\dfrac{1}{j}\cdot\dfrac{4\omega}{(1+\omega^2)^2} = \dfrac{4j\omega}{(1+\omega^2)^2}$

$te^{-|t|} \xrightarrow{\quad} \dfrac{4j\omega}{(1+\omega^2)^2}$

تقارن $\begin{cases} f(t) \to g(\omega) \\ g(t) \to 2\pi f(-\omega) \end{cases}$

$? \xrightarrow{\quad} 2\pi(-\omega)e^{-|-\omega|} \qquad |-\omega| = |\omega|$

$\underbrace{\qquad\qquad}_{-2\pi\omega e^{-|\omega|}}$

$\varsigma = \dfrac{4jt}{(1+t^2)^2}$

$\Rightarrow x(t) = \dfrac{-1}{2\pi}\cdot\dfrac{4jt}{(1+t^2)^2}$

b. $X(\omega) = \begin{cases} e^{-\omega} & \omega > 0 \\ -e^{\omega} & \omega < 0 \end{cases}$ $X(\omega) = e^{-\omega} u(\omega) - e^{-(-\omega)} u(-\omega)$

$\xrightarrow{\text{گردد}} e^{-t} u(t) \longrightarrow \dfrac{1}{j\omega + 1} \xRightarrow{\text{معکوس زمانی}} e^{t} u(-t) \longrightarrow \dfrac{1}{-j\omega + 1}$

$\xrightarrow{\text{دوگان}} f(t) \longrightarrow g(\omega) = e^{-\omega} u(\omega) \Rightarrow g(t) = e^{-t} u(t)$ $\textcircled{1}$

$\quad g(t) \longrightarrow 2\pi f(-\omega) = \dfrac{1}{j\omega + 1} \Rightarrow f(t) = \dfrac{1}{2\pi} \cdot \dfrac{1}{-jt + 1}$

$f(t) \longrightarrow g(\omega) = e^{\omega} u(-\omega) \Rightarrow g(t) = e^{t} u(-t)$

$g(t) \longrightarrow 2\pi f(-\omega) = \dfrac{1}{-j\omega + 1} \Rightarrow f(t) = \dfrac{1}{2\pi} \cdot \dfrac{1}{jt + 1}$ $\textcircled{2}$

$\Rightarrow x(t) = \dfrac{1}{2\pi} \left(\dfrac{1}{1 - jt} - \dfrac{1}{jt + 1} \right)$

c. $X(\omega) = \dfrac{2a - j\omega}{2a + j\omega} = \dfrac{2a}{2a + j\omega} - \dfrac{j\omega}{2a + j\omega}$ سوال ۲)

$\xrightarrow{\text{داریم}} e^{-at} u(t) \longrightarrow \dfrac{1}{j\omega + a} \Rightarrow e^{-2at} u(t) \longrightarrow \dfrac{1}{j\omega + 2a}$

$\qquad\qquad \underbrace{\dfrac{d}{dt}\left(e^{-2at} u(t)\right) \longrightarrow j\omega\left(\dfrac{1}{j\omega + 2a}\right)}_{}$

$\qquad\qquad -2a e^{-2at} u(t) + \delta(t) e^{-2at}$

$\Rightarrow x(t) = 2a e^{-2at} u(t) - \left(-2a e^{-2at} u(t) + \delta(t) e^{-2at}\right)$

$$d. \quad X(\omega) = \frac{d}{d\omega}\left(\underbrace{\frac{\sin 2\omega - j\cos 2\omega}{1 + \frac{j\omega}{3}}}_{g(\omega)} \right)$$

$$g(\omega) = \frac{3}{j}\left(\frac{\cos 2\omega + j\sin 2\omega}{3+j\omega} \right) = \frac{e^{2\omega}}{3+j\omega} \cdot \frac{3}{j}$$

$$f(t) \longrightarrow g(\omega)$$

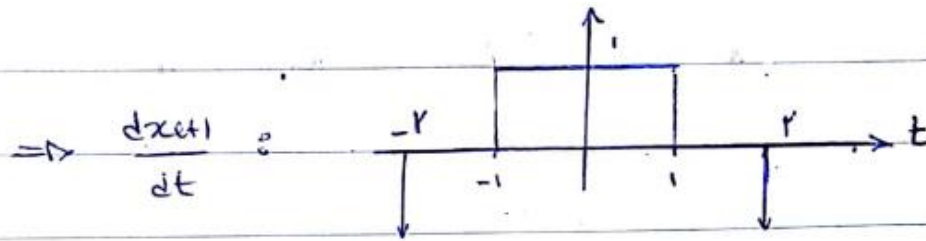$$-jt\, f(t) \longrightarrow \frac{dg(\omega)}{d\omega}$$

$$a(t) \rightarrow b(\omega)$$

$$a(t-t_0) \rightarrow e^{-j\omega t_0}\, b(\omega) = e^{2\omega}\frac{1}{3+j\omega}$$

$$\Rightarrow a(t) = e^{-3t}\, u(t) \quad , \quad t_0 = 2j$$

$$\Rightarrow f(t) = \frac{3}{j} \cdot e^{-3(t-2j)}\, u(t-2j)$$

سوال ۳)

$$\Rightarrow \frac{dx(t)}{dt} \circ$$

$$\delta(t-t_0) \xrightarrow{FT} e^{-jwt_0}$$

و

$$x(t) = \begin{cases} 1 & |t| < T \\ 0 & |t| > T \end{cases} \xrightarrow{FT} \frac{2 \sin wT}{w}$$

$$\Rightarrow \frac{dx(t)}{dt} \xrightarrow{FT} -e^{-jw \times 2} - e^{-jw \times (-2)} + \frac{2 \sin w}{w}$$

ارضي ، $\dfrac{dx(t)}{dt} \xrightarrow{FT} jw\, X(jw)$

$$\Rightarrow X(jw) = \frac{1}{jw}\left(-e^{2jw} - e^{-2jw} + \frac{2\sin w}{w}\right)$$

$$= \frac{1}{jw}\left(-2\cos 2w + \frac{2\sin w}{w}\right)$$

$$= -\frac{2\cos 2w}{jw} - \frac{2j\,\sin w}{w^2}$$

<div dir="rtl">

سوال ٤)

</div>

$$H(\omega) = \frac{j\omega + 2}{6 - \omega^2 + 5j\omega}$$

$$6H(\omega) - \omega^2 H(\omega) + 5j\omega H(\omega) = j\omega + 2$$

F^{-1}

$$6h(t) + \frac{d^2 h(t)}{dt^2} + 5\frac{d h(t)}{dt} = \frac{d \delta(t)}{dt} + 2\delta(t)$$

فرض می‌کنیم

$$6y(t) + \frac{d^2 y(t)}{dt^2} + 5\frac{d y(t)}{dt} = \frac{d x(t)}{dt} + 2x(t)$$

$x(t) = \delta(t)$

$y(t) = h(t)$

ب) $h(t) = F^{-1}\{H(\omega)\}$

$$H(\omega) = \frac{j\omega + 2}{(j\omega + 2)(j\omega + 3)} = \frac{1}{j\omega + 3} \longrightarrow e^{-3t}u(t)$$

$$\text{2) } x(t) = \overbrace{e^{-4t} u(t)}^{x_1(t)} - \overbrace{te^{-4t} u(t)}^{x_2(t)} \longrightarrow X(\omega) = ?$$

چون ($y_1(t)$ و $y_2(t)$) را به ترتیب

فروض سیستم به x_1 و x_2 می دانیم

و هرکدام را جداگانه می یابیم کرده و در آخر

طبق خاصیت خطی بودن سیستم‌های LTI

$y(t)$ را بدست می آوریم.

$$e^{-4t} u(t) \longrightarrow \frac{1}{j\omega+4} = X_1(\omega)$$

$$-jte^{-4t} u(t) \longrightarrow \frac{d}{d\omega}\left(\frac{1}{j\omega+4}\right) = \frac{-j}{(j\omega+4)^2}$$

$$\Rightarrow te^{-4t} u(t) \longrightarrow \frac{1}{(j\omega+4)^2} = X_2(\omega)$$

$$Y_1(\omega) = X_1(\omega) \cdot H(j\omega) = \frac{1}{j\omega+4} \cdot \frac{1}{j\omega+3}$$

$$= \frac{A}{j\omega+4} + \frac{B}{j\omega+3}$$

$$A = \lim_{j\omega \to -4} (j\omega+4) \frac{1}{(j\omega+4)(j\omega+3)} = -1$$

$$B = \lim_{j\omega \to -3} (j\omega+3) \frac{1}{(j\omega+4)(j\omega+3)} = 1$$

$$= \frac{-1}{j\omega+4} + \frac{1}{j\omega+3}$$

$$y_1(t) = \left(-e^{-4t} + e^{-3t}\right) u(t)$$

$$Y_2(\omega) = \frac{1}{(j\omega+4)^2} \cdot \frac{1}{j\omega+3} = \frac{A}{j\omega+4} + \frac{B}{(j\omega+4)^2} + \frac{C}{j\omega+3}$$

$$A = \frac{1}{-1} = \boxed{-1}$$

$$B = ?$$

$$\boxed{C = 1}$$

$$\Rightarrow j\omega = 0 \Rightarrow \frac{1}{16 \times 3} = \frac{-1}{4} + \frac{B}{16} + \frac{1}{3} = \frac{-12+3B+16}{16 \times 3}$$
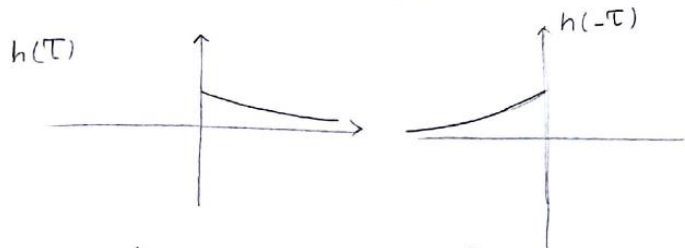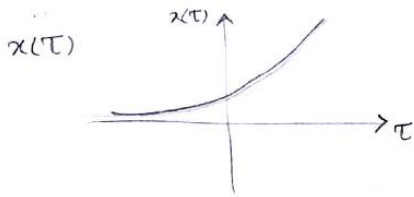
$$\Rightarrow 4+3B = 1 \Rightarrow \boxed{B = -1}$$

$$\Rightarrow Y_2(\omega) = \frac{-1}{j\omega+4} + \frac{-1}{(j\omega+4)^2} + \frac{1}{j\omega+3}$$

$$y_2(t) = \left(-e^{-4t} - te^{-4t} + e^{-3t}\right) u(t)$$

$$y(t) = y_1(t) - y_2(t)$$

>) $x_1(t) = e^{2t}$, $h(t) = e^{-3t} u(t)$ $\qquad y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) \, d\tau$

$x(\tau)$

$h(\tau)$

$h(-\tau)$

$h(t-\tau)$

$$\Rightarrow y(t) = \int_{-\infty}^{t} e^{2\tau} \cdot e^{-3(t-\tau)} \, d\tau = \int_{-\infty}^{t} e^{2\tau} \cdot e^{3\tau - 3t} \, d\tau$$

$$= e^{-3t} \int_{-\infty}^{t} e^{5\tau} \, d\tau = e^{-3t} \cdot \frac{1}{5} e^{5\tau} \Big|_{-\infty}^{t} = \frac{e^{-3t}}{5} \cdot e^{5t}$$

$$= \frac{e^{2t}}{5}$$

اول) $\quad x(t) = \delta(t) \Rightarrow y(t) = h(t)$

$\Rightarrow \quad \dfrac{d^2 h(t)}{dt^2} + 4\dfrac{dh(t)}{dt} + 9h(t) = \dfrac{d^2 \delta(t)}{dt^2} + 3\dfrac{d\delta(t)}{dt} + 2\delta(t)$

$\boxed{\text{FT}}$

$\Rightarrow \quad j^2\omega^2 H(j\omega) + 4j\omega H(j\omega) + 9H(j\omega) = j^2\omega^2 + 3j\omega + 2$

$\Rightarrow H(j\omega) = \dfrac{j^2\omega^2 + 3j\omega + 2}{j^2\omega^2 + 4j\omega + 9} = 1 - \dfrac{3j\omega + 7}{(2+j\omega)^2}$

$= 1 - \dfrac{3}{j\omega + 2} + \dfrac{3}{(j\omega + 2)^2}$

$$h(t) = \delta(t) - 4e^{-3t}u(t) + 3t\,e^{-3t}u(t)$$

ب عوض کردن دارنک جای x و y در معادلهی سیستم در واقع معادلهی

سیستم دارون را خواهیم دانست.

$$\frac{d^2 y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 4y(t) = \frac{d^2 x(t)}{dt^2} + 9\frac{dx(t)}{dt} + 9x(t)$$

برای کلاسیک $g(t)$ ؛ → من سیست این $x(t) = \delta(t)$ علاقهی تبدیل فورس بگیریم و بعد F^{-1}.

راه دوم: → در مورد پاسخ ضربهی سیستم معکوس می دانیم:

$$h(t) \ast g(t) = \delta(t) \Rightarrow H(jw) \times G(jw) = 1$$
$$\Rightarrow G(jw) = \frac{1}{H(jw)}$$

$$\Rightarrow G(jw) = \frac{jw^2 + 9jw + 9}{jw^2 + 3jw + 2} = 1 + \frac{4jw + 7}{(2+jw)(1+jw)}$$

$$= 1 + \frac{4}{1+jw} + \frac{-1}{2+jw} \xrightarrow{F^{-1}} \delta(t) + 4e^{-t}u(t) - e^{-2t}u(t)$$

الف ,

$\overline{\text{فرض}}$ $x(t)$: Even $\{x(t)\}$ $\xrightarrow{\text{FT}}$ Real $\{X(j\omega)\}$ ——— ①

ب , $\frac{1}{2\pi} \int_{-\infty}^{+\infty}$ Real $\{X(j\omega)\} e^{j\omega t} \, d\omega = |t| e^{-|t|}$

ب , ١ : Even $\{x(t)\} = |t| e^{-|t|}$

ج , $t < 0$: $x(t) = 0$ $\implies$ $x(t) =$ Even $\{x(t)\} +$ Odd $\{x(t)\} = 0$ for $t < 0$

$\implies$ Odd $\{x(t)\} = -|t| e^{-|t|} = t e^{t}$ $t < 0$

$\underset{t < 0}{\}$

Odd $\{x(t)\}$: زوج $\implies$ $-$ odd $\{x(-t)\} =$ odd $\{x(t)\}$

$\implies -(-t) e^{-t} = t e^{-t} =$ Odd $\{x(t)\}$

$\rightsquigarrow$ $x(t) =$ Odd $\{x(t)\} +$ Even $\{x(t)\}$

$$H(\omega) = \frac{Y(\omega)}{X(\omega)}$$

$$e^{-2t} u(t) \longrightarrow \frac{1}{2+j\omega}$$

$$\frac{1}{2} e^{-2t} u(t) \longrightarrow \frac{1}{2} \cdot \frac{1}{2+j\omega}$$

$$\Rightarrow Y(\omega) = \frac{1}{2}\left(\frac{1}{2+j\omega} - \frac{1}{4+j\omega} \right)$$

$$\frac{-1}{2} e^{-4t} u(t) \longrightarrow \frac{-1}{2} \cdot \frac{1}{4+j\omega}$$

$$Y(\omega) = \frac{1}{2}\left(\frac{2}{(2+j\omega)(4+j\omega)} \right) = \frac{1}{(2+j\omega)(4+j\omega)}$$

$$\Rightarrow H(\omega) = \frac{\frac{1}{(2+j\omega)(4+j\omega)}}{\frac{1}{2+j\omega}} = \frac{1}{4+j\omega}$$

$$x_2(t) = e^{-5t} u(t) \longrightarrow \frac{1}{5+j\omega} \qquad \Rightarrow Y_2(\omega) = H(\omega) \cdot X(\omega) = \frac{1}{(5+j\omega)(4+j\omega)}$$

$$Y_2(\omega) = \frac{A}{j\omega+5} + \frac{B}{j\omega+4} \qquad A = \frac{1}{4-5} = -1 \qquad B = \frac{1}{5-4} = 1$$

$$Y_2(\omega) = \frac{-1}{j\omega+5} + \frac{1}{j\omega+4} \qquad \Rightarrow y_2(t) = -e^{-5t} u(t) + e^{-4t} u(t)$$

سوال ۸)

1. $x(1-t) + x(-1-t)$

$$\begin{cases} x(t) \longrightarrow X(\omega) \\ x(-t) \longrightarrow X(-\omega) \\ x(t-t_0) \longrightarrow e^{-j\omega t_0} X(\omega) \end{cases}$$

$$\Rightarrow \quad X_{F_1}(\omega) = e^{j\omega} X(-\omega) + e^{-j\omega} X(-\omega)$$

2. $x(3t - 6) = x(3(t-2))$

$$\begin{cases} x(t) \longrightarrow X(\omega) \\ x(t-t_0) \longrightarrow e^{-j\omega t_0} X(\omega) \\ x(at) \longrightarrow \dfrac{1}{|a|} X\left(\dfrac{\omega}{a}\right) \end{cases}$$

$$\Rightarrow \quad X_{F_2}(\omega) = \frac{1}{3} e^{-j\frac{\omega}{3} \times 2} X\left(\frac{\omega}{3}\right)$$

3. $\dfrac{d^2}{dt^2} x(t-1)$

$$\Rightarrow X_{F_3}(\omega) = (j\omega)^2 e^{-j\omega} X(\omega)$$

$$\begin{cases} x(t) \longrightarrow X(\omega) \\ x(t-t_0) \longrightarrow e^{-j\omega t_0} X(\omega) \\ \dfrac{dx(t)}{dt} \longrightarrow j\omega X(\omega) \end{cases}$$

سوال ۹)

$$a(t) = x(t) \times p(t) \quad \leadsto \quad A(w) = X(jw) * P(jw)$$

$$B(jw) = A(jw) \times H_1(jw)$$

$$C(jw) = A(jw) \times H_1(jw) \times H_2(jw)$$

$$x(t) = Sinc(t) \Rightarrow X(jw) = \begin{cases} 1 & |w| < \pi \\ 0 & |w| > \pi \end{cases}$$

$$P(t) = Cos\, 4\pi t \Rightarrow P(jw) = \pi\left(\delta(w - 4\pi) + \delta(w + 4\pi)\right)$$

$$h_2(t) = \frac{Sin\, 5\pi t}{\pi t} \Rightarrow H_2(jw) = \begin{cases} 1 & |w| < 5\pi \\ 0 & |w| > 5\pi \end{cases}$$

$$\Rightarrow C(jw) = A(jw) = X(jw) * P(jw)$$

$$\Rightarrow C(t) = a(t) = x(t) \times P(t) = \frac{Sin\, \pi t}{\pi t} \times Cos\, 4\pi t$$

$$Y(\omega) = \frac{4}{\omega^2} \sin^2\omega$$

$$X(\omega) = \frac{2\sin\omega}{\omega}$$

$$h(\omega) = \frac{2\sin\omega}{\omega}$$