①

a) $x(t) = \sin(t) + \cos(\frac{\pi}{2}t + \frac{\pi}{4})$ $\xrightarrow{①}$ $\boxed{\frac{\pi}{j}[\delta(w-1) - \delta(w+1)] + \pi[\delta(w-\frac{\pi}{2}) + \delta(w+\frac{\pi}{2})]}$

Fourier pairs ① $\begin{cases} \sin w_0 t = \frac{\pi}{j}[\delta(w-w_0) - \delta(w+w_0)] \\ \cos w_0 t = \pi(\delta(w-w_0) + \delta(w+w_0)] \end{cases}$

b) $x(t) = \frac{2\sin^2(3t)}{t}$ $\xrightarrow{①}$ $X(jw) = \int_{-\infty}^{\infty} \frac{2\sin^2(3t)}{t} \cdot e^{-jwt} dt = \boxed{-\ln(jw) + \ln(-jw)}$

Fourier Transform: $X(jw) = \int_{-\infty}^{\infty} x(t) e^{-jwt} dt$ ①

c) $x(t) = \begin{cases} 0 & t < 0 \\ 2 & 0 < t < 1 \\ 2e^{-(t-1)} & t \geq 1 \end{cases}$

$X(jw) = \cancel{\int_{-\infty}^{\infty} x(t) e^{-jwt} dt} = \underbrace{\int_{-\infty}^{0} x(t) e^{-jwt} dt}_{0} + \int_{0}^{1} x(t) e^{-jwt} dt + \int_{1}^{\infty} 2e^{-(t-1)} dt$

$\Rightarrow \boxed{0 - \frac{e^{-jw} + 1}{jw} + 2ee^{-1}}$

d) $x(t) = \begin{cases} 0 & t < -2 \\ x+2 & -2 \leq t < 0 \\ -x+2 & 0 \leq t < 2 \\ 0 & t \geq 2 \end{cases}$

$x(jw) = \int_{-\infty}^{\infty} x(t) e^{-jwt} dt = \int_{-2}^{0} (x+2) e^{-jwt} dt + \int_{0}^{2} (2-x) e^{-jwt} dt$

$\Rightarrow \boxed{\frac{e^{2jw} x + 2e^{2jw} - x - 2}{jw} + \frac{x - 2e^{-2jw} - x + 2}{jw}}$

e)

$x(t) = \begin{cases} -1 & t < -3 \\ 0 & -3 \leq t < -2 \\ -1 & -2 \leq t < -1 \\ x+1 & -1 \leq t < 1 \\ 1 & 1 \leq t < 2 \\ 2 & 2 \leq t < 3 \\ 3 & t \geq 3 \end{cases}$

$X(jw) = \int_{-\infty}^{\infty} x(t) e^{-jwt} = \int_{-\infty}^{-3} -e^{-jwt} + \int_{-2}^{-1} -e^{-jwt} + \int_{-1}^{1} (x+1) e^{-jwt}$

$+ \int_{1}^{2} e^{-jwt} + \int_{2}^{3} 2e^{-jwt} + \int_{3}^{\infty} 3e^{-jwt}$

$= \boxed{\lim_{t \to -\infty}\left(\frac{e^{3jw} - e^{-jwt}}{jw}\right) + \frac{e^{jw} - e^{2jw}}{jw} - \frac{(x+1)(e^{-jw} - e^{jw})}{jw} + \frac{e^{-jw} - e^{-2jw}}{jw} + 2\frac{e^{-2jw} - e^{-3jw}}{jw} + \lim_{t \to \infty}\left(\frac{-3(e^{-jwt} - e^{-3jw})}{jw}\right)}$

②

a) $X(j\omega) = \dfrac{7j\omega+32}{-\omega^2+9\omega j+20} = \dfrac{7j\omega+32}{(j\omega+4)(j\omega+5)} = \dfrac{A}{j\omega+4} + \dfrac{B}{j\omega+5} = \dfrac{5A+j\omega A+4B+j\omega B}{(j\omega+4)(j\omega+5)}$

Reverse Fourier: $\boxed{x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} x(j\omega)e^{j\omega t}\,d\omega}$

$7 = A+B$
$32 = 5A+4B$ $\Big\}$ $A=4$, $B=3$

b)

$X(j\omega) = \dfrac{4}{j\omega+4} + \dfrac{3}{j\omega+5} \longleftrightarrow (4e^{-4t}+3e^{-5t})u(t)$

③

$X(j\omega) = \dfrac{1}{\omega^2+a^2}$

$\Downarrow$

$F^{-1} = \frac{1}{2a}(e^{-at})$

پارسوال: $\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$ ①

$\xRightarrow{①} 2\pi\int_{-\infty}^{\infty} \left((\tfrac{1}{2a})(e^{-at})\right)dt = \frac{\pi}{a}\int e^{-at(2)}dt = \frac{\pi}{a}\int_0^{\infty} e^{-2at}dt$

$= \frac{\pi}{a^2}$

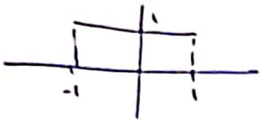
④

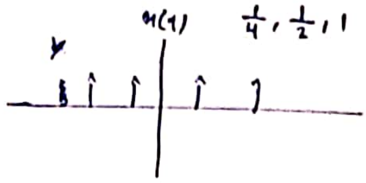
a) $x(t) = \cos(3t) + 2\cos(2t) + \frac{1}{2}\cos(\frac{1}{2}t) \Rightarrow \frac{1}{2}(e^{j3t} + e^{-j3t}) + (e^{j2t} + e^{-j2t}) + \frac{1}{4}(e^{j\frac{1}{2}t} + e^{-j\frac{1}{2}t})$

$h(t) = \frac{\sin(t)}{\pi t} \Rightarrow \frac{1}{\pi}\operatorname{sinc}(\frac{t}{\pi})$

$X(j\omega) = x(t) * h(t)$

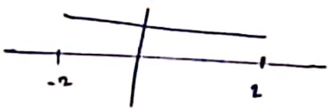
h(t)



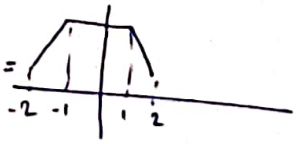


$\frac{\sin(t)}{\pi t} \xleftrightarrow{FT} X(\omega j) \begin{cases} 1 & |\omega| < W \\ 0 & |\omega| > W \end{cases}$

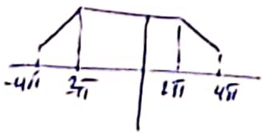
b) $h(t) = \frac{\sin(t) \cdot \sin(2t)}{\pi t^2} = \frac{2\pi}{\pi}\underbrace{\left(\frac{\sin t}{\pi t}\right)}_{A}\underbrace{\left(\frac{\sin(2t)}{2t\pi}\right)}_{B} = 2\pi\left(\frac{1}{\pi}\operatorname{sinc}\left(\frac{t}{\pi}\right)\right)\left(\frac{2}{\pi}\operatorname{sinc}\left(\frac{2t}{\pi}\right)\right)$



$\Rightarrow A * B =$



$2\pi^{A * B} \Rightarrow$



⑤

۳ a)

ب b)

$x(t), S(t) \Rightarrow y(t), h(t)$

$$\Rightarrow -\frac{d^2}{dt^2}h(t) - 7\frac{d}{dt}h(t) - 10h(t) = 2\frac{d}{dt}S(t) + 13S(t)$$

$$\overset{FT}{\Rightarrow} -j^2\omega^2 H(j\omega) - 7j\omega H(j\omega) - 10H(j\omega) = 2j\omega + 13$$

$$\Rightarrow H(j\omega) = \frac{-(2j\omega+13)}{+j^2\omega^2+7j\omega+10} = \frac{-(2j\omega+13)}{(j\omega+2)(j\omega+5)} = \frac{A}{j\omega+2} + \frac{B}{j\omega+5} \quad \begin{cases} A = +1 \\ B = -3 \end{cases}$$

$$\Rightarrow h(t) = e^{-2t} \cancel{+ie^{3}} - 3e^{-5t}$$

ج)

د)

نوشتیم: $h(t) * g(t) = S(t) \Rightarrow H(j\omega) \times G(j\omega) = 1 \Rightarrow G(j\omega) = \frac{1}{H(j\omega)}$

$$G(j\omega) = \frac{(j\omega+2)(j\omega+5)}{-(2j\omega+13)} = \frac{j^2\omega^2+7j\omega+10}{-2j\omega+13}$$

ه)