Sabject: حل تمرین سری 1 سیگنال
Year: Month: Date:

1) (a) $x(t) = \frac{1}{2}^{-3|t|} \Rightarrow E_\infty = \int_{-\infty}^{\infty} \left(\left|\frac{1}{2}\right|^{-3|t|}\right)^2 dt = \int_{-\infty}^{\infty} \left(\frac{1}{2}\right)^{-6|t|} dt$

$= \int_{-\infty}^{0} \frac{1}{2}^{6t} dt + \int_{0}^{\infty} \frac{1}{2}^{-6t} dt = \infty$

$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \left(\frac{1}{2}\right)^{-6|t|} dt = \infty$

1) (b) $x[n] = 3^n u[-n] \Rightarrow E_\infty = \sum_{-\infty}^{\infty} (3^n u[-n])^2$

$= \sum_{-\infty}^{0} 9^n = \frac{1}{1-\frac{1}{9}} = \boxed{\frac{9}{8}} \Rightarrow \boxed{P_\infty = 0}$

1) (c) $x[n] = e^{j\pi n + \frac{\pi}{4}}$

$\Rightarrow E_\infty = \sum_{-\infty}^{\infty} \left|e^{j\pi n} \times e^{\frac{\pi}{4}}\right|^2$

$= \sum_{-\infty}^{\infty} e^{\frac{\pi}{2}} = \boxed{\infty}, \quad P_\infty = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{-N}^{N} e^{\frac{\pi}{2}} = \boxed{e^{\frac{\pi}{2}}}$

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1) (d) $x(t) = \cos(wt) \Rightarrow E_\infty = \int_{-\infty}^{\infty} \cos^2(wt)\, dt = \infty$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \frac{1+\cos(2wt)}{2}\, dt = \lim_{T\to\infty} \frac{1}{2T} \left[\frac{t}{2} \Big|_{-T}^{T} + \underbrace{\frac{1}{4w} \sin(2wt) \Big|_{-T}^{T}}_{\text{finite}} \right]$$

$$= \lim_{T\to\infty} \frac{1}{2T} \left[T + \frac{\sin(2wT)}{2w} \right] = \boxed{\frac{1}{2}}$$

P_∞ مستقل از w است.

علت این است که $\cos(wt)$ یک سیگنال متناوب است. برای سیگنال های متناوب P_∞ همان توان یک دوره تناوب است. یعنی انرژی سیگنال تقسیم بر دوره تناوب. برای این سیگنال متناوب خاص، یعنی $\cos(wt)$ با تغییر دوره تناوب، انرژی سیگنال (در یک دوره تناوب) به همان نسبت تغییر می‌کند و اثر تغییرات دوره تناوب را خنثی می‌کند. برای همین P_∞ نسبت به w مستقل است. علت تغییرات متناسب انرژی با دوره تناوب هم این است که طبق این فرمول: $\frac{1+\cos(2wt)}{2} = \cos^2(wt)$ توان 2 $\cos(wt)$ را می‌توان به صورت ترکیبی خطی از سیگنال‌ها با همان دوره تناوب پایه نوشت (یعنی $\cos(2wt)$)

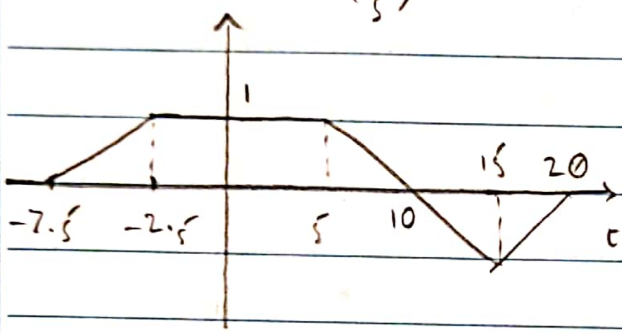
(1)(e) $x(t) = \cos(2t) + j\cos(t) \Rightarrow E_\infty = \int_{-\infty}^{\infty} \cos^2(2t) + \cos^2(t)\, dt = \infty$

طبق تمرین حل $\longrightarrow P_\infty = \frac{1}{2} + \frac{1}{2} = 1$

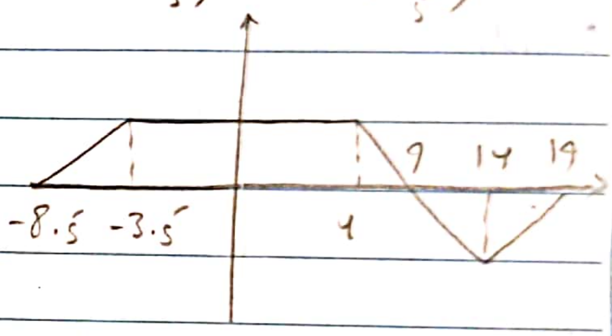
1) f) $x[n] = e^{j\pi n + n} \Rightarrow E_\infty = \sum_{-\infty}^{\infty} |e^{j\pi n + n}|^2 = \sum_{-\infty}^{\infty} e^{2n} = \infty$

$P_\infty = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{-N}^{N} e^{2n} = \infty$

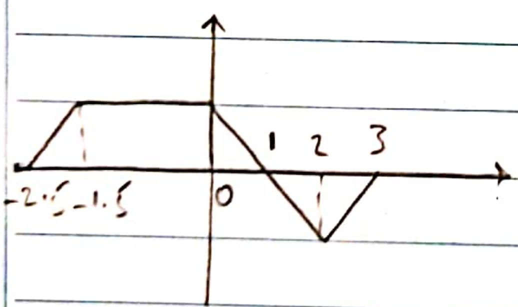
2)(a) $x(t) \longrightarrow x(\frac{t}{5})$



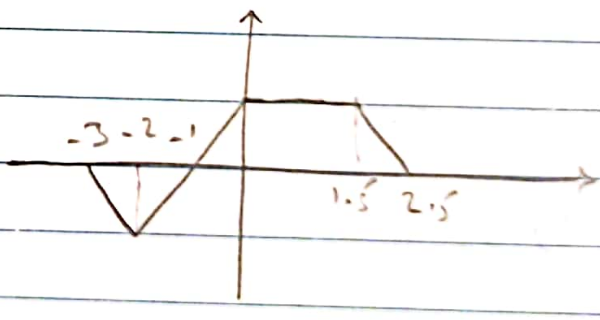
$x(\frac{t}{5}) \longrightarrow x(\frac{t+1}{5})$



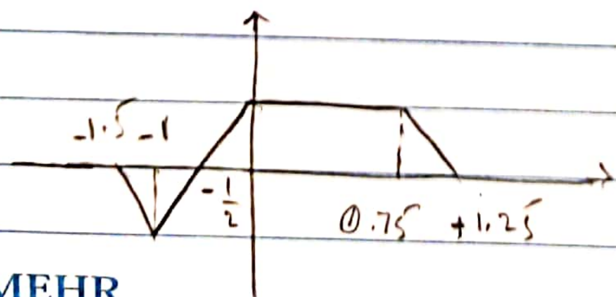
2)(b) $x(t) \longrightarrow x(t+1)$



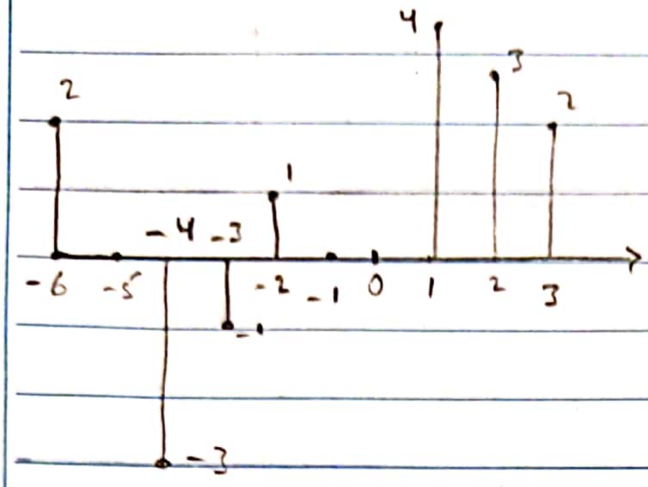
$x(t+1) \longrightarrow x(-t+1)$

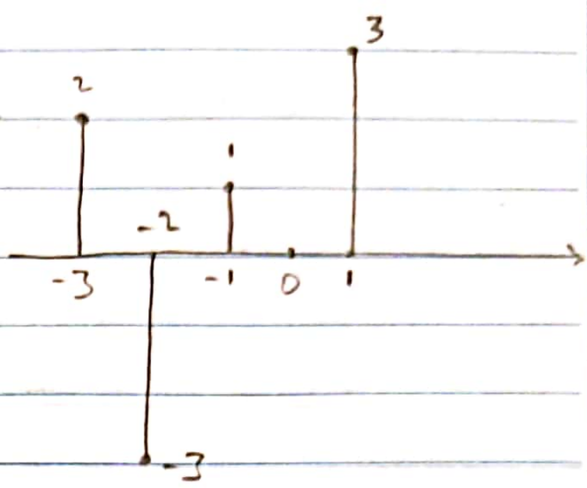


$x(-t+1) \longrightarrow x(-2t+1)$

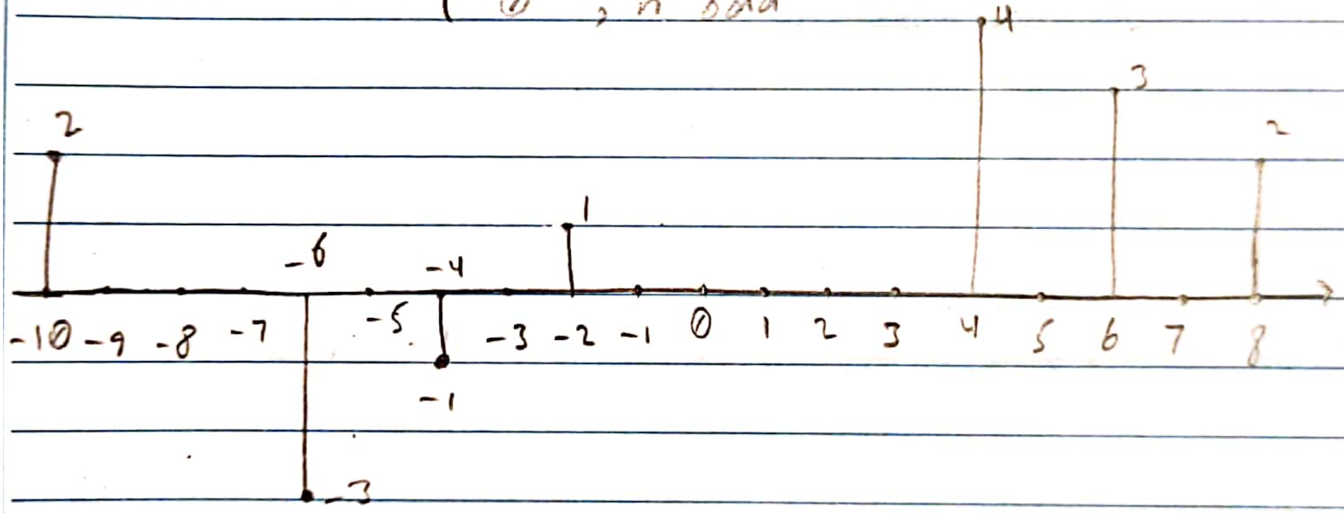


2) (c) $x[n] \longrightarrow x[n+1]$ $x[n+1] \longrightarrow x[2n+1]$





2) (d) $x[n] \longrightarrow \begin{cases} x[\frac{n}{2}], & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$

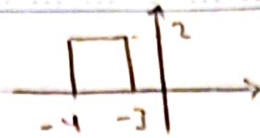


3) (a) $x_e(t) = \frac{\cos(t)u(t) + \cos(t)u(-t)}{2} = \cos(t)$

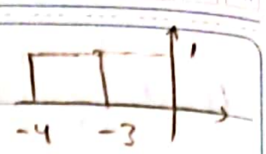
$x_o(t) = \frac{\cos(t)u(t) - \cos(-t)u(-t)}{2}$

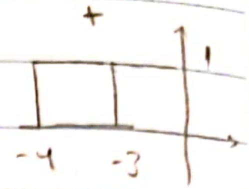
3) (b) $x_e(t) = \frac{e^{-2|t|}\sin(t) + e^{-2|t|}\sin(-t)}{2} = 0$

$x_o(t) = x(t)$

3) (c) $x(t) =$ 

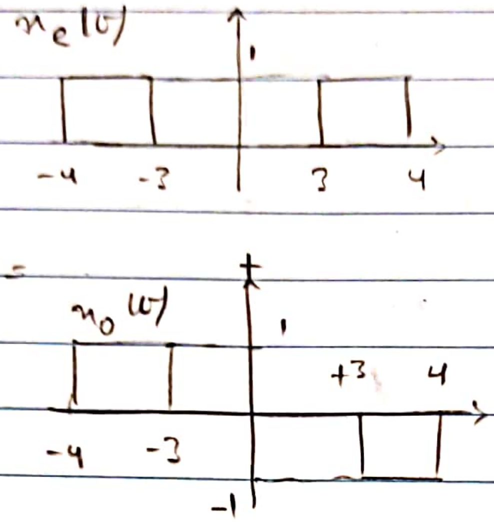
اولین کاری که به ذهن
می‌رسد و ساده است این
است که سیگنال را به
دو قسمت مساوی تقسیم کنیم.

$x(t) =$ 



اما هیچ یک از این دو قسمت نه
فرد هستند نه زوج، باید به نحوی
زوج و فرد بودنشان را تعیین کنیم
که سیگنال را عوض نکنند.

$x(t) =$



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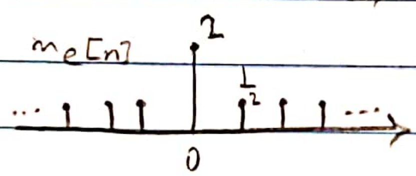
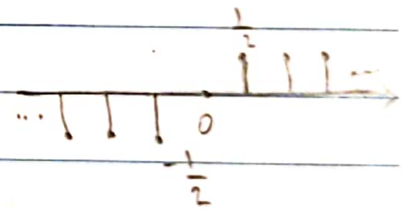
3) (d) $x[n] = \delta[n] + u[n] \Rightarrow x_e[n] = \dfrac{\delta[n]+u[n]+\delta[-n]+u[-n]}{2}$



$= 2\delta[n] + \dfrac{1}{2}$

$x_o[n] = \dfrac{\delta[n]+u[n]-\delta[-n]-u[-n]}{2} =$ 

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3) (e) $x_e[n] = \dfrac{(n+1)^2+(n-1)^2}{2} = n^2+1$

$x_o[n] = \dfrac{(n+1)^2-(n-1)^2}{2} = 2n$

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4) (a) $x(t) = e^{j\pi t} \Rightarrow \omega_0 = \pi \ , \ T_0 = \frac{2\pi}{\pi} = 2$

4) (b) $x(t) = e^{(1+\pi j)t} = \underbrace{e^{t}}_{\text{نامتناوب}} \times \underbrace{e^{\pi j t}}_{\text{متناوب}}$, نامتناوب

4) (c) $x(t) = e^{jt} \Rightarrow \omega_0 = 1 , T_0 = 2\pi$

4) (d) $x[n] = e^{j\frac{\pi}{3}n} \Rightarrow \omega_0 = \frac{\pi}{3} \Rightarrow N_0 = \frac{2\pi}{\frac{\pi}{3}} = 6$

4) (e) $x[n] = e^{j4n} \Rightarrow \omega_0 = 4 \neq k\pi \Rightarrow$ نامتناوب

4) (f) $x(t) = \int_{-\infty}^{t} \delta(\tau)\,d\tau + \int_{-\infty}^{-t} \delta(\tau)\,d\tau = u(t) + u(-t) = 1$

متناوب با دوره تناوب پایه تعریف نشده

4) (g) $x(t) = \underbrace{\cos(6\pi t)}_{\omega_1} + \underbrace{\sin(4\pi t)}_{\omega_2}$ جمع دو متناوب خود متناوب است

$$\left.\begin{array}{l} \omega_1 = 6\pi \Rightarrow T_1 = \frac{1}{3} \quad \times 3 = 1 \\ \omega_2 = 4\pi \Rightarrow T_2 = \frac{1}{2} \quad \times 2 = 1 \end{array}\right\} \Rightarrow T_0 = 1$$

4) (h) $x[n] = \cos\left(\frac{2\pi}{7}n\right) + e^{j\frac{5\pi}{3}n}$

$$\left.\begin{array}{l} \omega_1 = \frac{2\pi}{7} \Rightarrow N_1 = 7 \\ \omega_2 = \frac{5\pi}{3} \Rightarrow N_2 = \frac{6}{5} \rightarrow 6 \end{array}\right\} \Rightarrow N_0 = 6 \times 7 = 42$$

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4) (i) $x[n+5] = \sum_{k=-\infty}^{\infty} \delta[n+5-5k] + \delta[n-1+5-5k] + 2\delta[n+2+5-5k]$

$= \sum_{k=-\infty}^{\infty} \delta[n-5(k-1)] + \delta[n-1-5(k-1)] + 2\delta[n+2-5(k-1)]$

$k-1=m \longrightarrow x[n+5] = \sum_{m=-\infty}^{\infty} \delta[n-5m] + \delta[n-1-5m] + \delta[n+2-5m]$

$= x[n] \Rightarrow \boxed{N_0 = 5}$

4) (j) $x[n] = \cos^2\left(\frac{2\pi}{9}n + \frac{\pi}{6}\right) + \sin\left(\frac{5\pi}{3}\right) + 2\sin\left(\frac{3\pi}{5}n\right)$

$= \frac{1+\cos\left(\frac{4\pi}{9} + \frac{\pi}{3}\right)}{2} + \sin\left(\frac{5\pi}{3}\right) + 2\sin\left(\frac{3\pi}{5}n\right)$

$$\left.\begin{aligned} \omega_1 = \frac{4\pi}{9} &\Rightarrow N_1 = \frac{9}{2} \longrightarrow 9 \\ \omega_2 = \frac{3\pi}{5} &\Rightarrow N_2 = \frac{10}{3} \longrightarrow 10 \end{aligned}\right\} \Rightarrow \boxed{N_0 = 90}$$

4) (k) $x(t) = Od\{\cos(\pi t)u(t)\} = \frac{\cos(\pi t)u(t) - \cos(-\pi t)u(-t)}{2}$

$= \frac{\cos(\pi t)u(t) - \cos(\pi t)u(-t)}{2} = \begin{cases} \cos(\pi t), & t > 0 \\ -\cos(\pi t), & t < 0 \\ 0, & t = 0 \end{cases}$

نا متناوب

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4) (l) $x[n] = \cos\left(\frac{\pi}{8} n^2\right)$

$\cos\left(\frac{\pi}{8}(n+N)^2\right) = \cos\left(\frac{\pi}{8} n^2\right) = \cos\left(\frac{\pi}{8} n^2 + \frac{\pi}{8} N^2 + \frac{\pi}{4} N n\right)$

$\Rightarrow \begin{cases} \frac{\pi}{8} N^2 = 2k\pi \Rightarrow N^2 = 16k \\ \frac{\pi}{4} N = 2k'\pi \Rightarrow N = 8k' \end{cases} \Rightarrow \boxed{N_0 = 8}$

5) (a) $y(t) = \cos(x(t))$ memoryless ✓ causal ✓

$y(t-t_0) = \cos(x(t-t_0)) \Rightarrow$ time-invariant ✓

$-1 \leqslant \cos(x(t)) \leqslant 1 \Rightarrow$ stable ✓

$\cos(x_1(t) + x_2(t)) \neq \cos(x_1(t)) + \cos(x_2(t)) \Rightarrow$ linear ✗

5) (b) $y[n] = x[n-1]$ memoryless ✗ causal ✓

$y[n-n_0] = x[n-n_0-1] \Rightarrow$ time-invariant ✓

$\underbrace{|x[n]| < K}_{\text{bounded input}} \Rightarrow \underbrace{|x[n-1]| < k}_{\text{bounded output}} \Rightarrow$ stable ✓

$y_1 = x_1[n-1]$

$y_2 = x_2[n-1]$

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5) (c) $y[n] = \sum_{k=-K}^{K} x[n-k]$, memoryless X   causal X

$y[n-n_0] = \sum_{-K}^{K} x[n-n_0-k]$ time-invariant ✓

$\underbrace{|x[n]| < N}_{\text{bounded input}} \Rightarrow \underbrace{\sum_{-k}^{K} |x[n-k]| \leq (2K+1)N}_{\text{bounded output}}$ stable ✓

$$\left.\begin{array}{l} y_1[n] = \sum_{-k}^{K} x_1[n-k] \\ y_2[n] = \sum_{-k}^{K} x_2[n-k] \end{array}\right\} \Rightarrow y_1[n] + y_2[n] = \sum_{-K}^{K} x_1[n-k] + x_2[n-k]$$

$\Rightarrow$ linear ✓

5) (d) $y(t) = \cos(t)\,x(t)$ memoryless ✓   causal ✓

$\cos(t)\,x(t-t_0) \neq y(t-t_0) \Rightarrow x(t-t_0) \not\to y(t-t_0) \Rightarrow$ TI X

$\underbrace{|x(t)| < k}_{\text{bounded input}} \Rightarrow \underbrace{|\cos(t)\,x(t)| < k}_{\text{bounded output}} \Rightarrow$ stable ✓

$\cos(t)[x_1(t) + x_2(t)] = \cos(t)\,x_1(t) + \cos(t)\,x_2(t) \Rightarrow$ linear ✓

5) (e) $y(t) = e^{t\,x(t)}$ memoryless ✓ causal ✓

$e^{t\,x(t-t_0)} \neq y(t-t_0) \Rightarrow x(t-t_0) \not\rightarrow y(t-t_0) \Rightarrow$ time-invariant X

$\underbrace{x(t)=1}_{\text{bounded input}} \longrightarrow \underbrace{e^{t}}_{\text{unbounded output}} \rightarrow$ stable X

$e^{t[x_1(t)+x_2(t)]} \neq e^{t x_1(t)} + e^{t x_2(t)} \rightarrow$ linear X

5) (f)

$y(t) = \int_{-\infty}^{t+1} x(\tau)\,d\tau$ memoryless X $y(0) = \int_{-\infty}^{1} x(\tau)\,d\tau \Rightarrow$ causal X

$$\left.\begin{array}{l}\displaystyle\int_{-\infty}^{t+1} x(\tau-t_0)\,d\tau \\ u=\tau-t_0 \Rightarrow du = d\tau\end{array}\right\} = \int_{-\infty}^{t+1-t_0} x(u)\,du = y(t-t_0) \Rightarrow \text{time-invariant ✓}$$

stable X $\int_{-\infty}^{t+1} x_1(\tau)+x_2(\tau)\,d\tau = \int_{-\infty}^{t+1} x_1(\tau)\,d\tau + \int_{-\infty}^{t+1} x_2(\tau)\,d\tau$

$\Rightarrow$ linear ✓

5) (g) $y[n] = x[3n+2]$ memoryless X causal X

stable ✓ linear ✓

$x_1[n] \longrightarrow x_1[3n+2] = y_1[n]$

$x_2[n] = x_1[n-n_0]$ $x_2[n] \longrightarrow x_2[3n+2] = y_2[n] = x_1[3n+2-n_0]$

$\Rightarrow$ time-invariant X

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5) (h) $y[n] = \sin(x[n])$ memoryless ✓ causal ✓

time-invariant ✓ stable ✓ $\sin(x_1[n] + x_2[n]) \neq \sin(x_1[n]) + \sin(x_2[n])$

linear ✗

5) (i) $y(t) = x(\cos(t) - 1)$ $y(-2\pi) = x(0)$

$\Rightarrow$ memoryless ✗

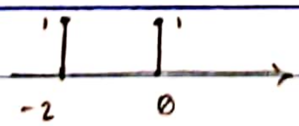
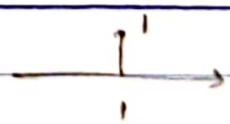
Causal ✗

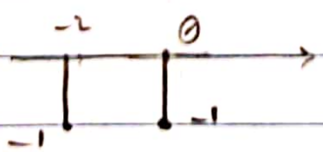
$y(t-t_0) = x(\cos(t-t_0) - 1) \neq x(\cos(t) - t_0)$ $\Rightarrow$ time-invariant ✗

stable ✓ linear ✓

6) (a) $y^{-1}(t) = x(\frac{t}{2})$ invertible ✓

6) (b) $x[n] = 1 \Rightarrow y[n] = 1$
$x[n] = -1 \Rightarrow y[n] = 1$ $\Rightarrow$ invertible ✗

6) (c) $x_1 =$  $\Rightarrow y_1[n] =$ 

$x_2 =$  $\Rightarrow y_2[n] =$ 

$\Rightarrow$ invertible ✗

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6) (d) چون بخشی از سیگنال ورودی بین (۱- تا ۱) از دست می‌رود، معکوس پذیر نیست

6) (e) $y[n] = \begin{cases} x[n] & , n \geq 0 \\ x[1] & , -2 \leq n < 0 \\ x[n+2] & , n < -2 \end{cases}$ invertible ✓

$y^{-1}[n] = \begin{cases} x[n] & , n \geq 0 \\ x[n-2] & , n < 0 \end{cases}$

6) (f) invertible ✗