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Q1: a) $x(t) = 3\cos\left(\frac{2\pi t}{3} + \frac{\pi}{3}\right) + 5\sin\left(\frac{2\pi t}{18}\right)$

$\omega_1 = \frac{2\pi}{3} \rightarrow T_1 = 3 \qquad \omega_2 = \frac{2\pi}{18} \rightarrow T_2 = 18 \qquad T = \text{lcm}(3,18) = 18$

$\omega = \frac{2\pi}{18} = \frac{\pi}{9}$

$x(t) = \frac{3}{2}\left(e^{j\left(\frac{2\pi}{3}t + \frac{\pi}{3}\right)} + e^{-j\left(\frac{2\pi}{3}t + \frac{\pi}{3}\right)}\right) + \frac{5}{2j}\left(e^{j\frac{2\pi}{18}t} - e^{-j\frac{2\pi}{18}t}\right) = \sum a_k e^{jk\frac{\pi}{9}t}$

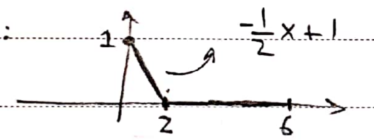
$a_6 = \frac{3}{2}e^{j\frac{\pi}{3}} \qquad a_{-6} = \frac{3}{2}e^{-j\frac{\pi}{3}} \qquad a_1 = \frac{5}{2j} \qquad a_{-1} = \frac{-5}{2j}$

b) $x(t) = 2\sin\left(\frac{2\pi}{3}t + \frac{\pi}{6}\right)$

$x(t) = \frac{2}{2j}\left(e^{j\left(\frac{2\pi}{3}t + \frac{\pi}{6}\right)} - e^{-j\left(\frac{2\pi}{3}t + \frac{\pi}{6}\right)}\right) = \sum a_k e^{jk\frac{2\pi}{3}t}$

$a_1 = \frac{1}{j}e^{j\frac{\pi}{6}} \qquad a_{-1} = \frac{-1}{j}e^{-\frac{\pi}{6}}$

c) one period:



$T = 6 \Rightarrow \omega_0 = \frac{2\pi}{6} = \frac{\pi}{3}$

$a_0 = \frac{1}{6}\int_0^6 x(t)\,dt = \frac{1}{6}$

$a_k = \frac{1}{6}\int_0^6 x(t)\, e^{-jk\omega_0 t} = \frac{1}{6}\int_0^1 \left(\frac{-1}{2}t + 1\right) e^{-jk\frac{\pi}{3}t}\,dt$

$= \frac{-1}{12}\int_0^1 t\, e^{-jk\frac{\pi}{3}t}\,dt + \frac{1}{6}\int_0^1 e^{-jk\frac{\pi}{3}t}\,dt = \frac{-1}{12}I_1 + \frac{1}{6}I_2$

I_1: $\int_0^1 u\,dv = uv\big|_0^1 - \int_0^1 v\,du$

$u = t \Rightarrow du = dt \qquad dv = e^{-jk\frac{\pi}{3}t}dt \Rightarrow v = \frac{-3}{jk\pi}e^{-jk\frac{\pi}{3}t}$

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$$uv\Big|_0^1 = \frac{-3t}{jk\pi} e^{-jk\frac{\pi}{3}t}\Big|_0^1 = \frac{-3}{jk\pi} e^{-jk\frac{\pi}{3}}$$

$$\int_0^1 v\,du = \frac{-3}{jk\pi}\left(\frac{-3}{jk\pi} e^{-jk\frac{\pi}{3}t}\Big|_0^1\right) = \frac{-9}{k^2\pi^2}\left(e^{-jk\frac{\pi}{3}} - 1\right)$$

$$I_1 = \frac{-3}{jk\pi} e^{-jk\frac{\pi}{3}} + \frac{9}{k^2\pi^2}\left(e^{-jk\frac{\pi}{3}} - 1\right)$$

I_2: $$I_2 = \int_0^1 e^{-jk\frac{\pi}{3}t}\,dt = \frac{-3}{jk\pi}\left(e^{-jk\frac{\pi}{3}} - 1\right)$$

$$a_k = \frac{-1}{12} I_1 + \frac{1}{6} I_2 = \frac{1}{6}\left(\frac{-1}{2} I_1 + I_2\right) \qquad e^{-jk\frac{\pi}{3}} = m$$

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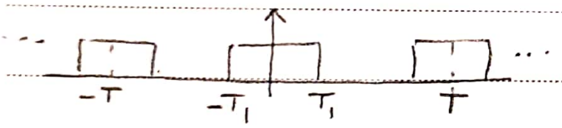
$$a_k = \frac{1}{6}\left(\frac{-1}{2}\left(\frac{-3}{jk\pi} m + \frac{9}{k^2\pi^2}(m-1)\right) + \frac{-3}{jk\pi}(m-1)\right)$$

Q2:

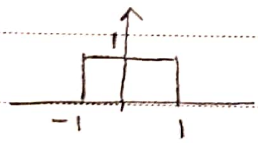
a) Oppenheim pages 193 - 194



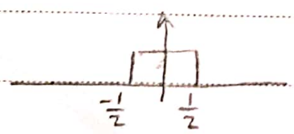
$$\overset{FS}{\longleftrightarrow} \quad \frac{\sin(k\omega_0 T_1)}{k\pi} \qquad k \neq 0$$

$$\frac{2T_1}{T} \qquad k = 0$$

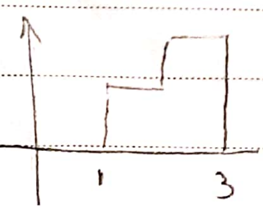
b)



$T_1 = 1$, $T = 5$ $\overset{FS}{\longleftrightarrow}$ $a_k = \dfrac{\sin(k\frac{2\pi}{5})}{k\pi} \xrightarrow[(2)]{\text{shift right}} C_k = e^{-jk\frac{4\pi}{5}} a_k$



$T_1 = \frac{1}{2}$, $T = 5$ $\overset{FS}{\longleftrightarrow}$ $b_k = \dfrac{\sin(k\frac{\pi}{5})}{k\pi} \xrightarrow[(2.5)]{\text{shift right}} d_k = e^{-jk\pi} b_k$



$$\longrightarrow E_k = C_k + d_k = e^{-jk\frac{4\pi}{5}} a_k + e^{-jk\frac{\pi}{5}} b_k$$

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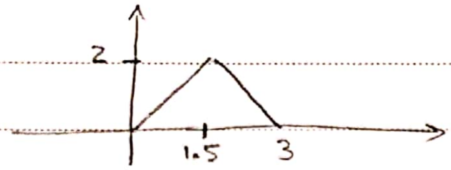
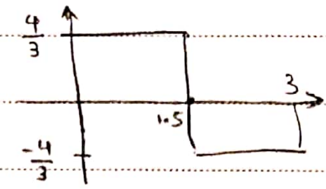
c.)



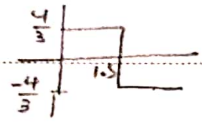
derivative $\longrightarrow$



FS properties: $x(t) \longleftrightarrow a_k$

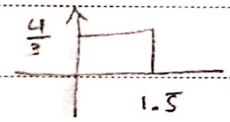
$$\int_{\infty}^{t} x(t)\,dt \longleftrightarrow \frac{1}{jk\omega_0} a_k \quad (\text{if } a_0 = 0)$$

in this case, $x(t) =$



$\xleftrightarrow{FS} a_k$ and we want to compute FS coeffs for $\int_{\infty}^{t} x(t)\,dt$. Since $a_0 = \frac{1}{3}(2-2) = 0$, it is possible to find a_k and then compute $\frac{1}{jk\omega_0} a_k$ $\left(\omega_0 = \frac{2\pi}{3}\right)$

$x(t) =$



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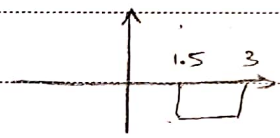

$T_1 = \frac{3}{4}$ $\qquad$ $T_1 = \frac{3}{4}$

$T = 3$ $\qquad$ $T = 3$

$$\Rightarrow a_k = \frac{4}{3} e^{-jk\frac{2\pi}{3}\left(\frac{3}{4}\right)} \frac{\sin\left(k\frac{2\pi}{3}\frac{3}{4}\right)}{k\pi} - \frac{4}{3} e^{-jk\frac{2\pi}{3}\left(\frac{9}{4}\right)} \frac{\sin\left(k\frac{2\pi}{3}\frac{3}{4}\right)}{k\pi}$$

$$\text{ANS} = \frac{1}{jk\frac{2\pi}{3}} a_k$$

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Q3: $T = 4 \Rightarrow w_0 = \frac{\pi}{2}$

$$x(t) = \sum_{k=-2}^{2} a_k e^{jk\frac{\pi}{2}t} = a_{-2} a^{j(-2)\frac{\pi}{2}t} + a_{-1} e^{-j(-1)\frac{\pi}{2}t} + a_1 e^{j\frac{\pi}{2}t} + a_2 e^{j2(\frac{\pi}{2})t}$$

a) $$y(t) = \sum_{k=-2}^{2} \underbrace{a_k H(jkw_0)}_{=b_k} e^{jkw_0 t}$$

$$\rightarrow b_{-2} = a_{-2} H(j(-2(\tfrac{\pi}{2}))) = a_{-2} \left| H(j(-\pi)) \right| e^{j\angle H(j(-2\pi))}$$

$b_{-1} = \dots$

$b_1 = \dots$

$b_2 = \dots$

b) Avg power of $y(t) = \sum_{k=-2}^{2} |b_k|^2$

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