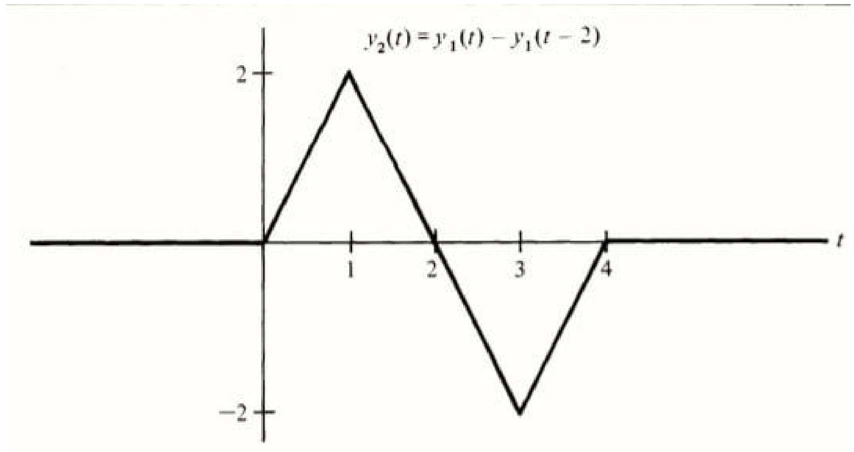
دانشگاه صنعتی امیرکبیر
دانشکده مهندسی کامپیوتر

پاسخ تمرین دوم درس «سیگنال‌ها و سیستم‌ها»

اساتید درس: دکتر راستی، دکتر آقائیان

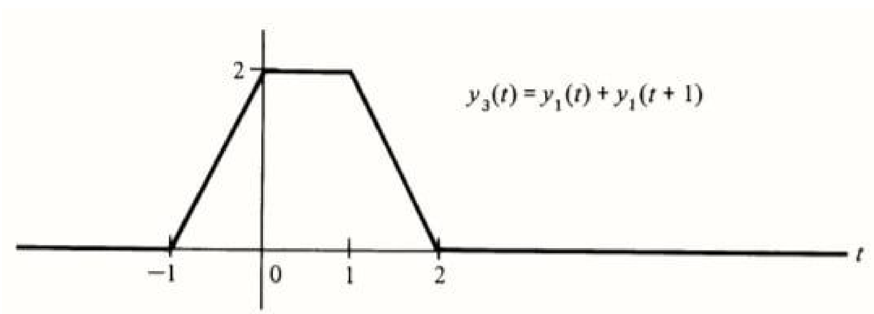
۱- الف)

$$x_2(t) = x_1(t) - x_1(t-2)$$



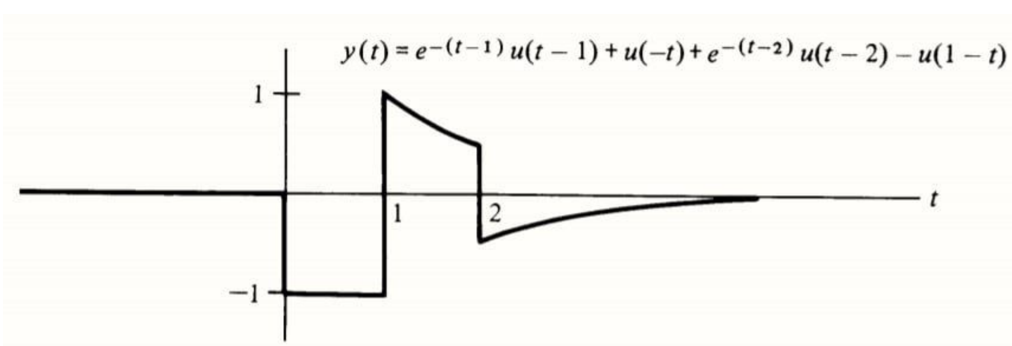
ب)

$$x_3(t) = x_1(t) + x_1(t+1)$$



ج)

$$x(t) = u(t-1) - u(t-2)$$



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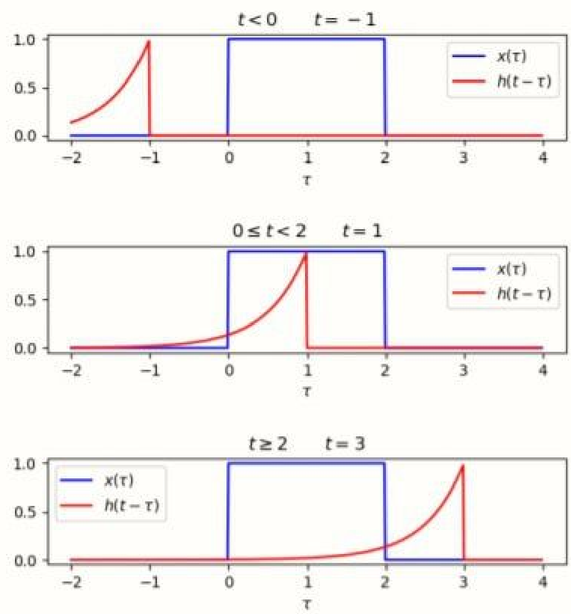
(a) $x(t) = u(t) - u(t-2)$, $h(t) = e^{-2t}u(t)$



- $t < 0$

$$y(t) = 0$$

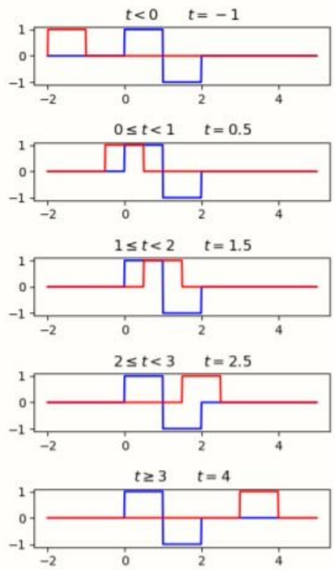
- $0 \le t < 2$

$$y(t) = \int_0^t e^{-2(t-\tau)} d\tau = \frac{1}{2}\left(e^{2(\tau-t)}\right)\Big|_0^t = \frac{1-e^{-2t}}{2}$$

- $t \ge 2$

$$y(t) = \int_0^2 e^{-2(t-\tau)} d\tau = \frac{1}{2}\left(e^{2(\tau-t)}\right)\Big|_0^2 = \frac{e^{2(2-t)}-e^{-2t}}{2}$$

(b) $x(t) = \Pi(t - \frac{1}{2}) - \Pi(t - \frac{3}{2})$, $h(t) = u(t) - u(t-1)$



- $t < 0$ or $t \geq 3$

$$y(t) = 0$$

- $0 \leq t < 1$

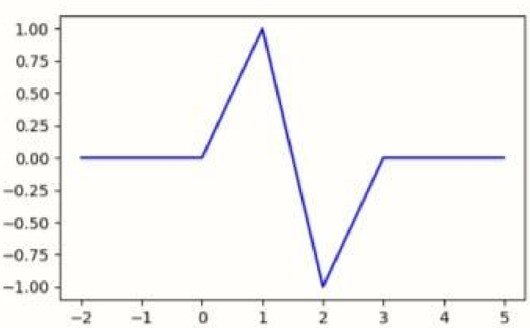
$$y(t) = \int_0^t d\tau = t$$

- $1 \leq t < 2$

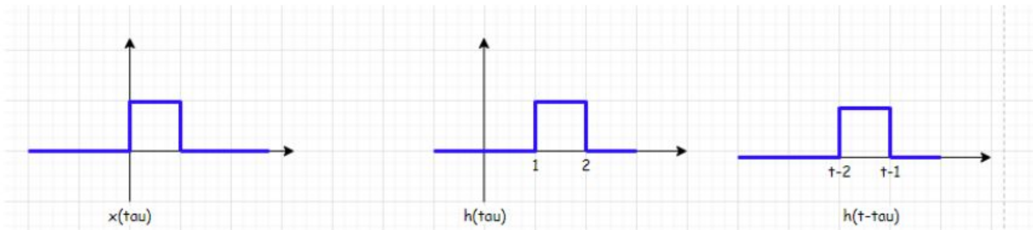
$$y(t) = \int_{t-1}^1 d\tau + \int_1^t -d\tau = 2 - t + 1 - t = 3 - 2t$$

- $2 \leq t < 3$

$$y(t) = \int_{t-1}^2 -d\tau = -(2 - (t-1)) = t - 3$$



c)



$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,\mathrm{d}\tau =$$

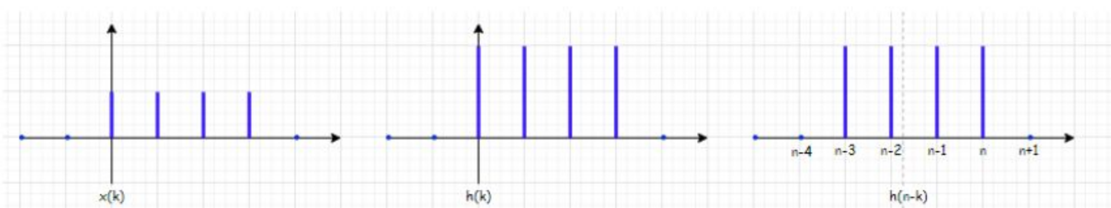
if $t < 1$: $y(t) = 0$.

if $1 < t < 2$: $y(t) = \int_{0}^{t-1} 1 \times 1 \,\mathrm{d}\tau = t - 1$.

if $2 < t < 3$: $y(t) = \int_{t-2}^{1} 1 \times 1 \,\mathrm{d}\tau = 3 - t$.

if $t > 3$: $\quad y(t) = 0$.

d)



$y[n] = \Sigma_{k=-\infty}^{\infty} x[k]h[n-k] =$

if $n < 0$: $y[n] = 0$.

if $0 \le n < 4$: $y[n] = \Sigma_{k=0}^{n} 2$.

if $4 \le n < 8$: $y[n] = \Sigma_{k=n-3}^{3} 2$.

if $n \ge 8$: $y[n] = 0$.

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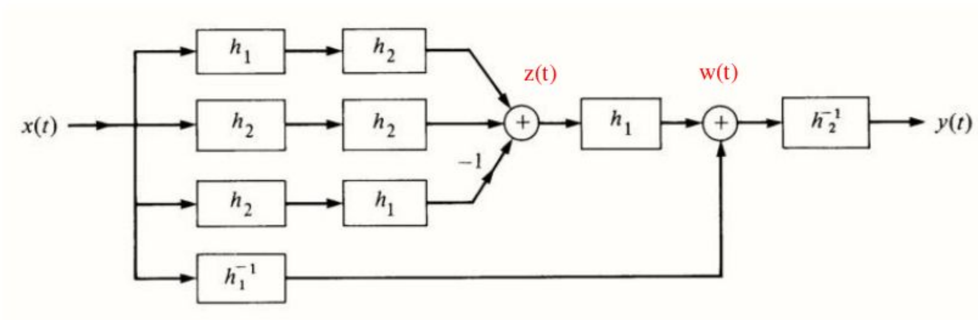

$$z(t) = (h_1 * h_2) + (h_2 * h_2) - (h_2 * h_1) = (h_1 * h_2) + (h_2 * h_2) - (h_1 * h_2)$$
$$= (h_1 + h_2 - h_1) * h_2 = h_2 * h_2.$$
$$w(t) = z(t) * h_1 + h_1^{-1} = h_2 * h_2 * h_1 + h_1^{-1}.$$
$$h(t) = w(t) * h_2^{-1} = h_2 * h_1 + h_1^{-1} * h_2^{-1}.$$

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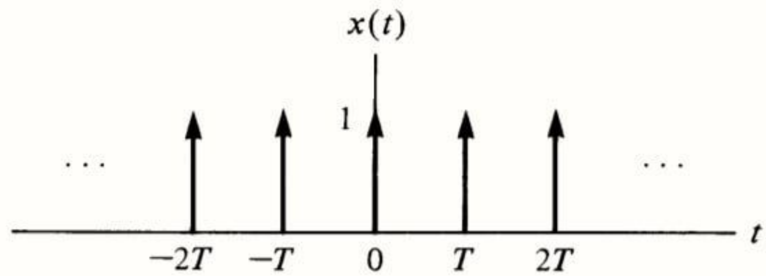
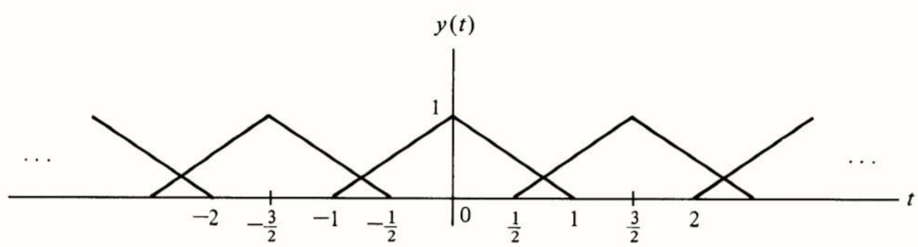
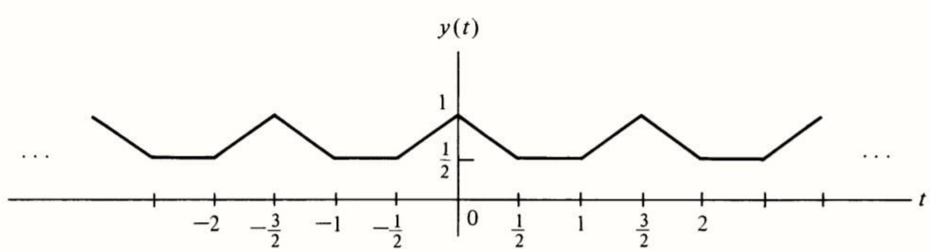
(a) $x(t) = \Sigma_{k=-\infty}^{\infty}\delta(t - kT)$ is a series of impulses spaced T apart.



(b) Using the result $x(t) * \delta(t - t_0) = x(t_0)$, we have



So $y(t) = x(t) * h(t)$ is



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(a) False. Counterexample: Let $g[n] = \delta[n]$. Then

$$x[n] * \{h[n]g[n]\} = x[n] \cdot h[0],$$
$$\{x[n] * h[n]\}g[n] = \delta[n] \cdot [x[n] * h[n]]\Big|_{n=0}$$

and $x[n]$ may in general differ from $\delta[n]$.

(b) True.

$$y(2t) = \int_{-\infty}^{\infty} x(2t - \tau)h(\tau)d\tau$$

Let $\tau' = \tau/2$. Then

$$\begin{aligned} y(2t) &= \int_{-\infty}^{\infty} x(2t - 2\tau')h(2\tau')2\, d\tau' \\ &= 2x(2t) * h(2t) \end{aligned}$$

(c) True.

$$\begin{aligned} y(t) &= x(t) * h(t) \\ y(-t) &= x(-t) * h(-t) \\ &= \int_{-\infty}^{\infty} x(-t + \tau)h(-\tau)\, d\tau = \int_{-\infty}^{\infty} [-x(t - \tau)][-h(\tau)]\, d\tau \\ &= \int_{-\infty}^{\infty} x(t - \tau)h(\tau)\, d\tau \qquad \text{since } x(\cdot) \text{ and } h(\cdot) \text{ are odd functions} \\ &= y(t) \end{aligned}$$

Hence $y(t) = y(-t)$, and $y(t)$ is even.

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a) $h(t) = t\left(e^{-t}u(t)\right)$

حافظه‌دار است، زیرا:

$$\text{if } t = 1 \rightarrow h(1) \neq 0.$$

علی است، زیرا:

$$\forall n < 0 \rightarrow h(n) \overset{u(n)=0}{=} 0.$$

پایدار است، زیرا:

$$\int_{-\infty}^{\infty} |h(t)|\,dt = \int_{-\infty}^{\infty} |t(e^{-t})u(t)\,dt \overset{\text{for } t<0 \rightarrow \text{u}(t)=0}{=} \int_{0}^{\infty} |t\,e^{-t}|\,dt \overset{\text{I}}{=} [-(t+1)e^{-t}]_0^{\infty}$$
$$= [-(\infty+1)e^{-\infty} + (0+1)e^{0}] = [0(\infty) + 1] < \infty.$$

I: $\int UV = U\int V - \int dU \int V.$

b) $h[n] = (0.8)^n u[n+2]$

حافظه‌دار است، زیرا:

$$h[1] = 0.8 \neq 0.$$

علی نیست، زیرا:

$$h[-1] = 0.8^{-1}u[1] = 0.8^{-1} \neq 0.$$

پایدار است، زیرا:

$$\Sigma_{n=-\infty}^{\infty} h[n] = \Sigma_{n=-2}^{\infty} 0.8^n = 0.8^{-2} + 0.8^{-1} + \Sigma_{n=0}^{\infty} 0.8^n \cong 2.8 + \frac{1}{0.2} = 7.8 < \infty.$$

c) $h(t) = e^{-6t}u(t+2)$

حافظه‌دار است، زیرا:

$$h(1) = e^{-6}u(3) \neq 0.$$

علی نیست، زیرا:

$$h(-1) = e^{6t} \neq 0.$$

پایدار است، زیرا:

$$\int_{-\infty}^{\infty} |e^{-6t}u(t+2)|\, dt = \int_{-2}^{\infty} e^{-6t}\, dt = -\frac{1}{6}(e^{-6t})|_{-2}^{\infty} = -\frac{1}{6}(0 - e^{12}) < \infty.$$

d) $h[n] = 5^n u[3-n]$

حافظه‌دار است، زیرا:

$$h[3] = 5^3 \neq 0.$$

علی نیست، زیرا:

$$h[-1] = \frac{1}{5} \neq 0.$$

پایدار است، زیرا:

$$\Sigma_{n=-\infty}^{\infty} 5^n u[3-n] = \Sigma_{n=-\infty}^{3} 5^n = 5^3 + 5^2 + 5 + \Sigma_{n=-\infty}^{0} 5^n = 155 + \frac{1}{\frac{4}{5}}$$

$$= 155 + \frac{5}{4} \neq \infty.$$

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$y[n] + ۲y[n-1] = x[n]$

ورودی $x[n] = \delta[n]$ ، خروجی $y[n]$ $\xrightarrow{\text{سکون ابتدایی}}$ $\forall n<0 ,\ \delta[n]=0 \Rightarrow y[n]=0$

$n=0 \Rightarrow y[0] + ۲\overset{0}{y[-1]} = \overset{1}{\delta[0]} \Rightarrow y[0]=1$

$n=1 \Rightarrow y[1] + ۲\overset{1}{y[0]} = \overset{0}{\delta[1]} \Rightarrow y[1]=-۲$

$n=۲ \Rightarrow y[۲] + ۲\overset{-۲}{y[1]} = \overset{0}{\delta[۲]} \Rightarrow y[۲]=۴$

$\Rightarrow \dots \Rightarrow \forall n\geq 0 ,\ y[n] = (-۲)^n$

$$\Rightarrow y[n] = \begin{cases} 0 & n<0 \\ (-۲)^n & n\geq 0 \end{cases}$$