①

a) $T=1 \Rightarrow w_0 = 2\pi$

$$\left.\begin{aligned} a_k &= \frac{1}{T}\int_0^1 x(t)e^{-jw_0kt}dt \\ x(t) &= e^{-t} \end{aligned}\right\} a_k = \int_0^1 e^{-t(1+jw_0k)}dt \Rightarrow a_k = \left(\frac{-1}{1+jw_0k}\right)e^{-t(1+jkw_0)}\Big|_0^1$$

$$\xrightarrow{w_0=2\pi} \boxed{\frac{1-e^{-(1-jk(2\pi))}}{1+2k\pi j}}$$

b) $LCM(2\pi, 10\pi) = 2\pi$

$\hookrightarrow w_0$

$$\Rightarrow \frac{1}{2j}\left(e^{j(10\pi t+\frac{\pi}{6})} - e^{-j(10\pi t+\frac{\pi}{6})}\right)\left(1+\frac{1}{2}\left(e^{j2\pi t} + e^{-j(2\pi)t}\right)\right)$$

$$\Rightarrow -\frac{j}{2}e^{j(10\pi t+\frac{\pi}{6})} + \frac{j}{2}e^{-j(10\pi t+\pi/6)} - \frac{j}{4}e^{j(12\pi t+\pi/6)} + \frac{j}{4}e^{j(-8\pi t+\pi/6)} - \frac{j}{4}e^{j(8\pi t+\pi/6)} + \frac{j}{4}e^{-j(12\pi t+\pi/6)}$$

$$\underbrace{-\frac{j}{2}e^{j\pi/6}, \frac{j}{2}e^{-j\pi/6}}_{a_5, a_{-5}}, \underbrace{\frac{-j}{4}e^{j\pi/6}, \frac{j}{4}e^{j\pi/6}}_{a_6, a_{-6}}, \underbrace{\frac{-j}{4}e^{j\pi/6}, \frac{j}{4}e^{\pi/6}}_{a_4, a_{-4}}$$

c)

$$\left.\begin{aligned} a_k &= \frac{1}{T}\int_T x(t)e^{-jkw_0t} \\ x(t) &= \delta(t) + \delta(t-1) \\ T &= 2 \end{aligned}\right\} \Rightarrow \frac{1}{2}\int_0^2 (\delta(t)+\delta(t-1))e^{-jw_0kt} = \boxed{\frac{1}{2}\int_0^2 e^{-jw_0kt}dt + \frac{1}{2}\int_0^2 e^{-jw_0k(t-1)}dt}$$

②

a) $a_k = \frac{1}{T}\int_T x(t)e^{-jw_0kt}dt \Rightarrow a_k = \frac{1}{T}\int_{-T_1}^{T_1} e^{-jw_0t}dt = \frac{-1}{Tjw_0}e^{-jw_0t}\Big|_{-T_1}^{T_1} = \frac{-(e^{-jw_0T_1} - e^{jw_0T_1})}{Tjw_0}$ $\frac{\sin(jw_0T_1)}{}$

$$\left.\begin{aligned} &\Rightarrow \frac{\sin(jw_0T_1)}{Tjw_0} \\ &T_1 = \tfrac{1}{2}T \end{aligned}\right\} \boxed{\frac{\sin(jw_0\frac{T}{2})}{Tjw_0}}$$

b)

?!

(3)

الف) $x(t) \xleftrightarrow{FS} a_k$

$x(-t) \xleftrightarrow{FS} a_{-k}$

$x(1-t) = a_{-k} e^{j\omega_0 t_0} \xleftrightarrow{FS} 2(a_{-k} e^{j\omega_0 t_0}) + 1$

ب)

$x(t), a_k \xleftrightarrow{FS} x^*(t) = a^*_{-k} \xleftrightarrow{FS} x^*(2t) = a^*_{-k}$

$x^*(-\frac{t}{2}), a^*_{-k}$

$a^*_{-k} + a^*_{-k}$

ج)

$x(t) = a_k \xleftrightarrow{FS} x(t + \frac{T_0}{2}) = a_k \cdot e^{j\omega_0 \frac{T_0}{2}}$

$T_0 = \frac{2\pi}{\omega_0} \to \omega_0 = \frac{2\pi}{T_0}$

$a_k e^{j\pi}$

(4)

$x(t) = 3e^{j3t} + \frac{1}{2} e^{j2t} + \frac{1}{2} e^{-j2t} + e^{j4t}$

$\omega_0 = 1 \to T = 2\pi$

$a_{-2} = \frac{1}{2}$

$a_2 = \frac{1}{2}$

$a_3 = 3$

$a_4 = 1$

Total $p = \sum |a_k|^2 = \frac{1}{2} + 9 + 1 = 10.5$

(5)

$T = 6 \to \omega_0 = \frac{\pi}{3}$

Real $\to a_k = a^*_{-k}$

$k \in \{\pm 2, \pm 1, 0\}$

$a_{-1} = a_1$

$x(t) = -x(t-3) \Rightarrow a_k = -e^{-j\omega_0 k 3} a_k$

$k=0 \to a_0 = 0$

$k = \pm 1 \to a_k = a_k$

$k = \pm 2 \to a_k = 0$

$a_0 = 0$, $a_{-2} = 0$, $a_2 = 0$, $a_1 = a_{-1}$

$$\int_{-3}^{3} \left| a_{-1} e^{-j\frac{\pi}{3}t} + a_1 e^{j\frac{\pi}{3}t} \right| dt = |a_1| \int_{-3}^{3} \left| e^{-j\frac{\pi}{3}t} + e^{j\frac{\pi}{3}t} \right| dt$$

$$\Rightarrow |2a_1| \int_{-3}^{3} \left| \cos(\frac{\pi}{3}t) \right| dt \Rightarrow \frac{24}{\pi} |a_1| = 12\pi \Rightarrow a_1 = a_{-1} = \frac{\pi^2}{2}$$

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