(1)

$B = \{x, y, z\}$

$$A \times B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \rightarrow \det(A \times B) = 1 - 0 + (-1) = 0$$

$$\det(A \times B) = \det(A) \times \det(B) \rightarrow 0 = \det(A) \cdot \det(B)$$

$x, y, z \longrightarrow$ مستقل خطی $\Rightarrow \det(B) \neq 0$

$$\Rightarrow \boxed{\det(A) = 0}$$

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(2)

$$u + v = \begin{bmatrix} 4 \\ 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

$$\Rightarrow S = \left| \det\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \end{bmatrix}\right) \right| \Rightarrow \det\left(\begin{bmatrix} 1 & 3 \\ 2 & 0 \end{bmatrix}\right) = 1 - 6 = -5$$

$$\Rightarrow S = |-5| \Rightarrow \boxed{S = +5}$$

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(3)

$$LI \Rightarrow \det([v_1 \ v_2 \ v_3]) \neq 0$$

$$\Rightarrow \begin{bmatrix} 1 & x & 1 \\ 0 & 1 & 2x \\ 0 & -x & 3x+1 \end{bmatrix} = A$$

$$\det(A) = 3x + 1 + 2x^2 - (x)(0) + 0 \neq 0$$

$$\rightarrow 2x^2 + 3x + 1 \neq 0$$

$2x^2 + 3x + 1 = 0$

$\Delta = 9 - 8 = 1$

$$\Rightarrow x = \frac{-3 \pm 1}{4} \begin{cases} -1 \\ \frac{-1}{2} \end{cases}$$

$$\Rightarrow x \in \mathbb{R} - \left\{-1, \frac{-1}{2}\right\}$$

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a) $B = A^{-1}$ I

(4)

$\det(A \times B) = \det(A) \times \det(B)$

$AB = I$

$\rightarrow \det(AB) = \det(I) = 1$

$\rightarrow \det(A) \times \det(B) = 1 \implies \det(A) = \dfrac{1}{\det(B)}$

$\xrightarrow{\mathrm{I}} \det(A) = \dfrac{1}{\det(A^{-1})}$

b) $\det((A^4)^T B^{-1} A^{-4} (B^3)^T)$

$\det(A^4)^T \times \det(B^{-1}) \times \det(A^{-4}) \times \det((B^3)^T)$

$\xrightarrow[\det A = \det A^t]{\text{scalars}} \cancel{\det(A)^4} \times \dfrac{1}{\cancel{\det B}} \times \dfrac{1}{\cancel{\det(A)^4}} \times \underbrace{\det \cancel{(B)}^3}_{\det B^2}$

$= \det B^2 = 24^2 = \boxed{576}$

$B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix} \rightarrow \begin{vmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{vmatrix} = 24$

$C_{11} = I^2$ $\quad C_{12} = -AI$ $\quad C_{13} = AD - IB$

$C_{21} = 0$ $\quad C_{22} = I^2$ $\quad C_{23} = ID$

$C_{31} = 0$ $\quad C_{32} = 0$ $\quad C_{33} = I^2$

$$M = \begin{bmatrix} I & 0 & 0 \\ A & I & 0 \\ B & D & I \end{bmatrix}$$

$$\Rightarrow adj(M) = \begin{bmatrix} I^2 & 0 & 0 \\ -AI & I^2 & 0 \\ AD-IB & ID & I^3 \end{bmatrix}$$

$\det(A) = I^3$

$$\rightarrow A^{-1} = \begin{bmatrix} \frac{1}{I} & 0 & 0 \\ \frac{-A}{I^2} & \frac{1}{I} & 0 \\ \frac{AD-IB}{I^3} & \frac{D}{I^2} & \frac{1}{I} \end{bmatrix}$$

$$I^2 A^{-1} = \begin{bmatrix} I & 0 & 0 \\ -A & I & 0 \\ \frac{AD-IB}{I} & D & I \end{bmatrix} = \begin{bmatrix} I & 0 & 0 \\ P & I & 0 \\ Q & R & I \end{bmatrix}$$

$P = -A \qquad R = D$

$Q = \frac{AD}{I} - B$

$$x_i = \frac{\det A_i(b)}{\det A} \qquad i = 1, 2, 3$$

$$\det A = \begin{vmatrix} 3 & 0 & 2 \\ 2 & 0 & -2 \\ 0 & 1 & 1 \end{vmatrix} = -\begin{vmatrix} 3 & 2 \\ 2 & -2 \end{vmatrix} = -6 - 4 = \quad +10$$

$$x_1 = \frac{\det\begin{vmatrix} 4 & 0 & 2 \\ 5 & 0 & -2 \\ 6 & 1 & 1 \end{vmatrix}}{10} = \frac{-(-8-10)}{10} = \frac{18}{10}$$

$$x_2 = \frac{\det\begin{vmatrix} 3 & 4 & 2 \\ 2 & 5 & -2 \\ 0 & 6 & 1 \end{vmatrix}}{10} = \frac{\overset{3\times17}{3\,\cancel{+}(5+12)} - \overset{-8}{\cancel{4}(2)} + \overset{24}{2\cancel{(12)}}}{10} = \frac{67}{10}$$

$$x_3 = \frac{\det\begin{vmatrix} 3 & 0 & 4 \\ 2 & 0 & 5 \\ 6 & 1 & 6 \end{vmatrix}}{10} = \frac{-\begin{vmatrix} 3 & 4 \\ 2 & 5 \end{vmatrix}}{10} = \frac{-(\overset{7}{\cancel{15-8}})}{10} = \frac{-7}{10}$$

$a = x_1 \longrightarrow a = 1.8$

$b = x_2 \longrightarrow b = 6.7 \quad \Rightarrow \quad x = \begin{bmatrix} 1.8 \\ 6.7 \\ -0.7 \end{bmatrix}$

$c = x_3 \longrightarrow c = -.7$

a) $V = \{v_1, v_2, \ldots, v_n\}$

$H = \operatorname{span}\{v_1, v_2, \ldots, v_n\}$

$K = \operatorname{span}\{v_1, v_2, \ldots, v_n\}$

$\mathrm{I} = 0(H) + 0(K) = 0$

$\mathrm{II} =$ $u = s_1 h + s_2 k$ $w = s_2 h + s_2 k$

$\Rightarrow u + w = h(s_1 + s_2) + k(s_1 + s_2)$

$h \in H$ $k \in K$ $\Rightarrow u + w$ is in V

$\mathrm{III} =$ $Cu = C(s_1 h + s_2 k) = h(cs_1) + k(cs_2)$

$\Rightarrow Cu$ is in V

بدیهی است

b) $h \in H \to \begin{cases} h(0) = 0 \\ h + 0 \in H + K \to h \in H + K \\ Ch \in H + K \end{cases}$

به همین ترتیب می‌شود برای K هم اثبات کرد.

a) if $x > 0$ then we cant prove

we have $\{0\}$ in subset S_1, thus $S_1 \not\subset \mathbb{R}^3$

b) $x_1 - 4x_2 + 5x_3 - 2 = 0$

$x_1 = x_2 = x_3 \longrightarrow 0 - 0 + 0 = 2$ ✗.

we dont have $\{0\}$ in this subset

c) $u = \begin{bmatrix} -x_1 \\ x_1^2 \end{bmatrix}$ $w = \begin{bmatrix} x_1 \\ x_1^2 \end{bmatrix}$

$u + w = \begin{bmatrix} 0 \\ 2x_1^2 \end{bmatrix} \notin H$

$x_1 = 0$ $y_1 \neq 0 \Rightarrow$ S_3 is not a subset of $\mathbb{R}^2$

a)

$$T(A+B) = (A+B) + (A+B)^T = A+B+A^T+B^T$$

$$= \underbrace{A+A^T}_{T(A)} + \underbrace{B+B^T}_{T(B)}$$

$$T(KA) = KA + (KA)^T = K(A+A^T) = KT(A)$$

$\Rightarrow$ خطی است linear Transformation

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b) $B = B^T$, $T(A) = A + A^T$

$$\left.\begin{aligned} T(B) &= 2B \\ T(cB) &= 2cB \end{aligned}\right\} \quad c = \frac{1}{2} \rightarrow T(A) = B$$

$$T\left(\tfrac{1}{2}B\right) = B$$

---

c) $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\quad A^T = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$

$$T(A) = \begin{bmatrix} 2a & b+c \\ b+c & 2d \end{bmatrix} = B$$

$$\rightarrow B^T = \begin{bmatrix} 2a & b+c \\ b+c & 2d \end{bmatrix} \qquad B^T = B$$

---

d) $T(A) = 0 \rightarrow \begin{bmatrix} 2a & b+c \\ b+c & 2d \end{bmatrix} = 0$

$\rightarrow a = 0 \quad d = 0$

$b = -c$

$$\begin{bmatrix} a & b & 0 \\ c & d & 0 \end{bmatrix} \xrightarrow[c=-b]{a=d=0} \begin{bmatrix} 0 & b & 0 \\ -b & 0 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 0 & 1 & 0 \\ -b & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \qquad x_1 = 0 \quad x_2 = 0$$

$$\text{Ker}\,(T(A)) = \{ a, b, c, d \in \mathbb{R} \mid a = d = 0 \,,\; b = -c \}$$

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 6 & 4 \end{bmatrix}$$

$$a = \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix} \quad b = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} \quad c = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ 3 & 6 & 4 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$A a = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 6 & 4 \end{bmatrix} \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \rightarrow a \notin Nul\, A$$

$$A b = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 6 & 4 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow b \in Nul\, A$$

$C \notin Nul\, A \qquad Nul\, A \in \mathbb{R}^3 , \; C \in \mathbb{R}^2$

$A, B \notin Col\, A \qquad A, B \in \mathbb{R}^3 , \; Col\, A \in \mathbb{R}^2$

$$[A \; c] = \begin{bmatrix} 1 & 2 & 1 & 1 \\ 3 & 6 & 4 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

consistance

$\rightarrow C \in Col\, A$

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 6 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow A \text{ spans } \mathbb{R}^3$$

$\Rightarrow$ A, B in Range A

C is not in Range A $\rightarrow$ C is in $\mathbb{R}^2$

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$\dim(U+W) = \dim U + \dim W - \dim U \cap W$

$\{v_1, — v_p\}$ is a basis for $U \cap W$

$\{v_1, — v_p, u_1, — u_q\}$ " " for U

$\{v_1, — v_p, w_1, — w_r\}$ " " for W

$a_1 v_1 + — a_p v_p + B_1 u_1 + — B_q v_q + c_1 w_1 + — + c_r w_r = 0$

$\rightarrow \quad x = \underbrace{a_1 v_1 + — + a_p v_p + B_1 u_1 + — + B_q v_q}_{\in U} = \underbrace{-(c_1 w_1 + — c_r w_r)}_{\in W}$

belongs $U \cap W$:

$x = s_1 v_1 + — + s_p v_p$

$\Rightarrow \quad s_1 v_1 + — + s_p v_p = -(c_1 w_1 + — + c_r w_r)$

$\Rightarrow \quad s_1 v_1 + — + s_p v_p + c_1 w_1 + — + c_r w_r = 0$

$- \{v_1, — v_p, w_1, —, w_r\}$ is linear independant

$\Rightarrow \quad s_1 = 0, -, s_p = 0, \; c_1 = 0, —, c_r = 0$

$\Rightarrow \quad a_1 v_1 + — a_p v_p + B_1 u_1 + — B_q v_q = 0$

$- \{v_1, — v_p, W_1, —, W_q\}$ is linear independant

$\Rightarrow a_1 = 0, — a_p = 0, B_1 = 0 — B_q = 0$

$\Rightarrow \quad \dim(U+W) = \dim(U) + \dim(W) - \dim(U \cap W)$

$\dim V+W + \dim U \cap W = \dim V + \dim W \Rightarrow \quad \dim(V+W) \leq \dim(U) + \dim(W)$

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \\ -1 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 \\ 0 & 2 \\ 0 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

2 non-zero row $\rightarrow$ rank $A = 2$

$$A^t = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 2 \end{bmatrix} \rightarrow A^tA = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 2 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -1 & 9 \end{bmatrix}$$

$$A^tA = \begin{bmatrix} 2 & -1 \\ -1 & 9 \end{bmatrix} \sim \begin{bmatrix} 2 & -1 \\ 0 & 8.5 \end{bmatrix} \sim \begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Thank $(A^tA) = 2$

$$AA^t = \begin{bmatrix} 1 & 1 \\ 0 & 2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 1 \\ 2 & 4 & 4 \\ 1 & 4 & 5 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1/2 \\ 2 & 4 & 4 \\ 1 & 4 & 5 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & \frac{1}{2} \\ 0 & 2 & 3 \\ 0 & 3 & 4.5 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 3 & \frac{9}{2} \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \text{Thank}(AA^t) = 2$$

rank A^tA = rank A for any $A \in M_{m\times n}$

this is also true for AA^t

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a)

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 3 & -1 & 0 & 0 \\ -5 & 1 & \lambda & 0 \end{array}\right] \sim \begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & -4 & -6 & 0 \\ 0 & 6 & 10+\lambda & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 1 & \frac{3}{2} & 0 \\ 0 & 6 & 10+\lambda & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & \frac{1}{2} & 0 \\ 0 & 1 & \frac{3}{2} & 0 \\ 0 & 0 & \lambda+1 & 0 \end{bmatrix}$$

$$\det \begin{bmatrix} 1 & 1 & 2 \\ 3 & -1 & 0 \\ -5 & 1 & \lambda \end{bmatrix} \neq 0 \rightarrow -\lambda - (3\lambda) + \underbrace{2(\overset{-2}{\cancel{3}} - 5)}_{-4} = 0$$

$$-4\lambda - 4 = 0 \rightarrow \lambda = -1$$

$$\rightarrow \lambda \in \mathbb{R} - \{-1\}$$

b)

$$\det \begin{bmatrix} 1 & 1 & 2 \\ 3 & -1 & 0 \\ -5 & 1 & 1 \end{bmatrix} = -1 - (3) + 2(\overset{-4}{\cancel{3}} - 5) = -8 \neq 0$$

$\Rightarrow$ basis for $\mathbb{R}^3$

$$\det \begin{bmatrix} 1 & 1 & 2 \\ 3 & -1 & 0 \\ -5 & 1 & 3 \end{bmatrix} = -3 - (9) + 2(\overset{-4}{\cancel{3}} - 5) = -12 - 4 = -16 \neq 0$$

$\Rightarrow$ basis for $\mathbb{R}^3$

c)

$$P_{B \to S} = \left[[b_1]_S \;\; [b_2]_S \;\; [b_3]_S \right]$$

$$B = [v_1, v_2, b] \qquad S = [v_1, v_2, s]$$

$$[b_1]_S = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \quad [b_2]_S = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \quad [b_3]_S = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

$$\left[s_1 \;\; s_2 \;\; s_3 \mid b_1 \;\; b_2 \;\; b_3 \right] = \left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 1 & 2 \\ 3 & -1 & 0 & 3 & -1 & 0 \\ -5 & 1 & 3 & -5 & 1 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 1 & 2 \\ 0 & -4 & -6 & 0 & -4 & -6 \\ 0 & 6 & 13 & 0 & 6 & 11 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 1 & 2 \\ 0 & 1 & \frac{3}{2} & 0 & 1 & \frac{3}{2} \\ 0 & 6 & 13 & 0 & 6 & 11 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & \frac{1}{2} & 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} & 0 & 1 & \frac{3}{2} \\ 0 & 0 & 4 & 0 & 0 & 2 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & \frac{1}{2} & 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} & 0 & 1 & \frac{3}{2} \\ 0 & 0 & 1 & 0 & 0 & \frac{1}{2} \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & \frac{1}{4} \\ 0 & 1 & 0 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & 1 & 0 & 0 & \frac{1}{2} \end{array}\right] \Rightarrow [b_1]_S = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$[b_2]_S = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$[b_3]_S = \begin{bmatrix} \frac{1}{2} \\ \frac{3}{2} \\ \frac{1}{2} \end{bmatrix}$$

$$\Rightarrow \underset{C \leftarrow B}{P} = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$$

$V = M_2(\mathbb{R}) \qquad v = \begin{bmatrix} -3 & -2 \\ -1 & 2 \end{bmatrix}$

$B = \left\{ \begin{bmatrix} 1 & 0 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} -2 & -4 \\ 0 & 0 \end{bmatrix} \right\}$

$C = \left\{ \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \right\}$

$$B = \begin{bmatrix} 1 & 0 & 0 & -1 & 3 & 5 & -2 & -4 \\ -1 & -2 & 3 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & -1 & 3 & 5 & -2 & -4 \\ 0 & 1 & -3/2 & 1/2 & -3/2 & -5/2 & 1 & 2 \end{bmatrix}$$

$$\Rightarrow B = \text{span}\Big\{ \underbrace{\begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \end{bmatrix}}_{\text{Basis of } B} \Big\}$$

$$C = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$\Rightarrow C = \text{span}\Big\{ \underbrace{\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{\text{Basis of } C} \Big\}$$

$$\left[\begin{array}{cc|cc} -3 & -2 & 1 & 0 \\ -1 & 2 & -1 & -2 \end{array}\right] \sim \begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{-1}{2} & \frac{-3}{4} \end{bmatrix}$$

$\Rightarrow \begin{bmatrix} 0 & \frac{1}{2} \\ \frac{-1}{2} & \frac{-3}{4} \end{bmatrix} \longrightarrow [v_1 \ v_2]_B$ $\qquad$ v در پایه B

$$\left[\begin{array}{cc|cc} -3 & -2 & 1 & 1 \\ -1 & 2 & 1 & 0 \end{array}\right] \sim \begin{bmatrix} 1 & 0 & \frac{-1}{2} & \frac{-1}{4} \\ 0 & 1 & \frac{1}{4} & \frac{-1}{8} \end{bmatrix}$$

$\Rightarrow \begin{bmatrix} \frac{-1}{2} & \frac{-1}{4} \\ \frac{1}{4} & \frac{-1}{8} \end{bmatrix} \longrightarrow [v_1 \ v_2]_C$ $\qquad$ v در پایه C b)

$\underset{C \leftarrow B}{P}$

$$\left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 1 & 0 & -1 & -2 \end{array}\right] \sim \left[\begin{array}{cccc} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 2 \end{array}\right]$$

$$\Rightarrow \underset{C \leftarrow B}{P} = \left[[b_1]_C , [b_2]_C \right] = \begin{bmatrix} -1 & -2 \\ 2 & 2 \end{bmatrix}$$

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