a) $\det A = 0 \rightarrow$ not intervable

$$\det A = \begin{vmatrix} 1 & -x & 0 & 0 \\ 0 & 1 & -x & 0 \\ 0 & 0 & 1 & -x \\ 0 & 1 & 0 & -1 \end{vmatrix} =$$

$$= 1 \begin{vmatrix} 1 & -x & 0 \\ 0 & 1 & -x \\ 1 & 0 & -1 \end{vmatrix} + x \begin{vmatrix} 0 & -x & 0 \\ 0 & 1 & -x \\ 0 & 0 & -1 \end{vmatrix}$$

0

$$= 1 \begin{vmatrix} 1 & -x & 0 \\ 0 & 1 & -x \\ 1 & 0 & -1 \end{vmatrix} = 1 \begin{vmatrix} 1 & -x \\ 0 & -1 \end{vmatrix} + x \begin{vmatrix} 0 & -x \\ 1 & -1 \end{vmatrix}$$

$$= -1 + x + x(0 + x) = x^2 + x - 1$$

$x^2 + x - 1 = 0$

$\Delta = 1 + 4 = 5 \qquad x = \dfrac{-1 \pm \sqrt{5}}{2} \begin{cases} \dfrac{-1+\sqrt{5}}{2} \\ \dfrac{-1-\sqrt{5}}{2} \end{cases}$

b) $\det A = 81$

$$\det(2A^T A^{-1}) \xrightarrow{\det(AB) = \det A \det B} \det 2A^t \det A^{-1}$$

$$\xrightarrow[\substack{\det 2A = 2\det A \\ \det 2A = \cancel{8}\, 2^n \det A}]{\det A^{-1} = \frac{1}{\det A}} 2^4 \det A^t \times \frac{1}{\det A} \xrightarrow{\det A^T = \det A}$$

$$2^4 \det A \times \frac{1}{\det A} = \boxed{2^4}$$

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