3)

$$A = \sqrt{2}\begin{bmatrix} 1 & 1/2 \\ 0 & 1/2 \\ 1 & 1/2 \\ 0 & 1/2 \end{bmatrix} \qquad b = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

$A = QR \longrightarrow$ Q یک orthonormal برای colA

R یک بالا مثلثی است

+ بدون $\sqrt{2}$ حل می‌کنیم و جواب نهایی را در $\sqrt{2}$ ضرب می‌کنیم. و منظور A را در 2 ضرب می‌کنیم و نهایتا بر 2 تقسیم می‌کنیم که عدد اعشاری از بین رود

$$A' = \begin{bmatrix} 2 & 1 \\ 0 & 1 \\ 2 & 1 \\ 0 & 1 \end{bmatrix} \Rightarrow v_1 = \begin{bmatrix} 2 \\ 0 \\ 2 \\ 0 \end{bmatrix} \qquad v_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$u_1 = v_1 = \begin{bmatrix} 2 \\ 0 \\ 2 \\ 0 \end{bmatrix} \qquad u_2 = v_2 - \frac{\overset{4}{v_2 u_1}}{\underset{8}{u_1 u_1}} u_1 = v_2 - \frac{1}{2}u_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

normalize

$$u_1' = \frac{u_1}{\|u_1\|} = \frac{u_1}{\underset{2\sqrt{2}}{\sqrt{8}}} = \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \\ 0 \end{bmatrix}$$

$$u_2' = \frac{u_2}{\|u_2\|} = \frac{u_2}{\sqrt{2}} = \begin{bmatrix} 0 \\ 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix} \Rightarrow Q = [u_1 \ u_2] = \begin{bmatrix} 1/\sqrt{2} & 0 \\ 0 & 1/\sqrt{2} \\ 1/\sqrt{2} & 0 \\ 0 & 1/\sqrt{2} \end{bmatrix}$$

$$A' = QR' \rightarrow Q^{-1}A' = R' \xrightarrow[Q^{-1} = Q^T]{QQ^T = I} Q^T A' = R'$$

$$\begin{bmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 0 & 1/\sqrt{2} & 0 & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & 1 \\ 2 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2\sqrt{2} & \sqrt{2} \\ 0 & \sqrt{2} \end{bmatrix} = R'$$

اثر $\frac{\sqrt{2}}{2}$ باید در R ضرب شود زیرا Q یک orthonormal Basis است R

$$A = Q \frac{\sqrt{2}}{2} R' \Rightarrow A = \underbrace{\begin{bmatrix} 1/\sqrt{2} & 0 \\ 0 & 1/\sqrt{2} \\ 1/\sqrt{2} & 0 \\ 0 & 1/\sqrt{2} \end{bmatrix}}_{Q} \underbrace{\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}}_{R}$$

$$\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$$

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$$A^T A x = b x \Rightarrow x = (A^T A)^{-1} A^T b$$

$$\Rightarrow x = ((QR)^T QR)^{-1} (QR)^T b = (R^T \underbrace{Q^T Q}_{I} R)^{-1} R^T Q^T b$$

$$= (R^T R)^{-1} R^T Q^T b = \underbrace{R^{-1} R^{T^{-1}} R^T}_{I} Q^T b = R^{-1} Q^T b$$

$$\Rightarrow x = R^{-1} Q^T b$$

$$R^{-1} = \frac{1}{\det R} \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1/2 & -1/2 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow R^{-1} Q^T = \begin{bmatrix} 1/2 & -1/2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 0 & 1/\sqrt{2} & 0 & 1/\sqrt{2} \end{bmatrix} =$$

$$= \begin{bmatrix} 1/2\sqrt{2} & -1/2\sqrt{2} & 1/2\sqrt{2} & -1/2\sqrt{2} \\ 0 & 1/\sqrt{2} & 0 & 1/\sqrt{2} \end{bmatrix}$$

$$R^{-1} Q^T b = \begin{bmatrix} 1/2\sqrt{2} & -1/2\sqrt{2} & 1/2\sqrt{2} & -1/2\sqrt{2} \\ 0 & 1/\sqrt{2} & 0 & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1/2\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$