2)

$$A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{bmatrix}$$

$$A^TA = \begin{bmatrix} 3 & 2 \\ 2 & 3 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{bmatrix} = \begin{bmatrix} 13 & 12 & 2 \\ 12 & 13 & -2 \\ 2 & -2 & 8 \end{bmatrix} = M$$

$$\det M - \lambda I = \begin{bmatrix} 13-\lambda & 12 & 2 \\ 12 & 13-\lambda & -2 \\ 2 & -2 & 8-\lambda \end{bmatrix} = (13-\lambda)\begin{vmatrix} 13-\lambda & -2 \\ -2 & 8-\lambda \end{vmatrix} - 12\begin{vmatrix} 12 & -2 \\ 2 & 8-\lambda \end{vmatrix} + 2\begin{vmatrix} 12 & 13-\lambda \\ 2 & -2 \end{vmatrix}$$

$$= -\lambda^3 + 34\lambda^2 - 255\lambda = -\lambda(\lambda^2 - 34\lambda + 225) = -\lambda(\lambda-9)(\lambda-25) = 0$$

$$\Rightarrow \lambda_3 = 0 \qquad \lambda_2 = 9 \qquad \lambda_1 = 25$$

$$\lambda_1 = 25 \Rightarrow M - \lambda I = \begin{bmatrix} -12 & 12 & 2 \\ 12 & -12 & -2 \\ 2 & -2 & -17 \end{bmatrix} \Rightarrow (M-\lambda I)v = 0$$

$$\begin{bmatrix} -12 & 12 & 2 & 0 \\ 12 & -12 & -2 & 0 \\ 2 & -2 & -17 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & -\frac{1}{6} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-50}{3} & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & -\frac{1}{6} & 0 \\ 0 & 0 & \frac{-50}{3} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -1 & -\frac{1}{6} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow x = x_2\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$x_2 = 1 \longrightarrow v_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\lambda_2 = 9 \qquad M - \lambda I = \begin{bmatrix} 4 & 12 & 2 \\ 12 & 4 & -2 \\ 2 & -2 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 12 & 2 & 0 \\ 12 & 4 & -2 & 0 \\ 2 & -2 & -1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & \frac{1}{2} & 0 \\ 0 & -32 & -8 & 0 \\ 0 & -8 & -2 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & \frac{1}{2} & 0 \\ 0 & 1 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -\frac{1}{4} & 0 \\ 0 & 1 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow x = x_3\begin{bmatrix} \frac{1}{4} \\ -\frac{1}{4} \\ 1 \end{bmatrix} \xrightarrow{x_3 = 4} v_2 = \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}$$

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$\lambda_3 = 0 \Rightarrow M - \lambda I = \begin{bmatrix} 13 & 12 & 2 \\ 12 & 13 & -2 \\ 2 & -2 & 8 \end{bmatrix}$

$$\begin{bmatrix} 13 & 12 & 2 & 0 \\ 12 & 13 & -2 & 0 \\ 2 & -2 & 8 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 12/13 & 2/13 & 0 \\ 0 & 25/13 & -50/13 & 0 \\ 0 & -50/13 & 100/13 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 12/13 & 2/13 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow x = x_3 \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix} \xrightarrow{x_3=1} v_3 = \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}$$

$$\left(\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix} \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix} \right) \rightarrow \text{orthogonal set}$$

$\sqrt{2}$ $\sqrt{18} = 3\sqrt{2}$ $\sqrt{9} = 3$

نرمال می کنیم

$$V = \begin{bmatrix} 1/\sqrt{2} & 1/3\sqrt{2} & -2/3 \\ 1/\sqrt{2} & -1/3\sqrt{2} & 2/3 \\ 0 & 4/3\sqrt{2} & 1/3 \end{bmatrix} \rightarrow V^T = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1/3\sqrt{2} & -1/3\sqrt{2} & 4/3\sqrt{2} \\ -2/3 & 2/3 & 1/3 \end{bmatrix}$$

$$\Sigma = \begin{bmatrix} \sqrt{25} & 0 & 0 \\ 0 & \sqrt{9} & 0 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 3 & 0 \end{bmatrix}$$

$$u_1 = \frac{1}{\sigma_1} A v_1 = \frac{1}{5} \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 5/\sqrt{2} \\ 5/\sqrt{2} \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

$3 - 1/2 = 2$ $\quad 2 - 1/3 = 8/3$

$$u_2 = \frac{1}{\sigma_2} A v_2 = \frac{1}{3} \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{bmatrix} \begin{bmatrix} 1/3\sqrt{2} \\ -1/3\sqrt{2} \\ 4/3\sqrt{2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 3/\sqrt{2} \\ -3/\sqrt{2} \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$$

$$\Rightarrow U = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$$

$$U \Sigma V^T = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 5 & 0 & 0 \\ 0 & 3 & 0 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1/3\sqrt{2} & -1/3\sqrt{2} & 4/3\sqrt{2} \\ -2/3 & 2/3 & 1/3 \end{bmatrix}$$

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