$$A = \begin{bmatrix} 1 & 0 & -5 & 110 \\ 0 & 1 & -\sqrt{2} & e^2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\det A = 1\times1\times1\times1 = 1 \longrightarrow \det A = 1 \neq 0 \Rightarrow$ معکوس پذیر

بالا مثلثی

b)

$$A = \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{13} \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} A_{11}^{-1} & -A_{11}^{-1}A_{12}A_{22}^{-1} \\ 0 & A_{22}^{-1} \end{bmatrix}$$

$$A = \left[\begin{array}{cc|cc} 1 & 0 & -5 & 110 \\ 0 & 1 & -\sqrt{2} & e^2 \\ \hline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right] = \begin{bmatrix} I_2 & A_{12} \\ 0 & I_2 \end{bmatrix}$$

$$A_{11}^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A_{12} = \begin{bmatrix} -5 & 110 \\ -\sqrt{2} & e^2 \end{bmatrix}^{-1} = \frac{1}{-5e^2 + 110\sqrt{2}} \begin{bmatrix} e^2 & -110 \\ \sqrt{2} & -5 \end{bmatrix}$$

$$AA^{-1} = \begin{bmatrix} I_2 & 0 \\ 0 & I_2 \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} A_{11}^{-1} & A_{12}^{-1} \\ A_{21}^{-1} & A_{22}^{-1} \end{bmatrix}$$

$$AA^{-1} = \begin{bmatrix} A_{11}^{-1} & A_{12}^{-1} \end{bmatrix}$$

$$AA^{-1} = \begin{bmatrix} I_2 & A_{12} \\ 0 & I_2 \end{bmatrix} \begin{bmatrix} A_{11}^{-1} & A_{12}^{-1} \\ A_{21}^{-1} & A_{22}^{-1} \end{bmatrix}$$

$$= \begin{bmatrix} A_{11}^{-1} + A_{12}A_{21}^{-1} & A_{12}^{-1} + A_{12}A_{22}^{-1} \\ A_{21}^{-1} & A_{22}^{-1} \end{bmatrix} = \begin{bmatrix} I_2 & 0 \\ 0 & I_2 \end{bmatrix}$$

$A_{21}^{-1} = 0$ $\qquad A_{12}^{-1} + A_{12} A_{22}^{-1} = 0$

$A_{22}^{-1} = I_2 \qquad A_{11}^{-1} + \underbrace{A_{12} A_{21}^{-1}}_{0} = I_2 \longrightarrow A_{11}^{-1} = I_2$

$\Rightarrow A_{12}^{-1} + A_{12} \times I_2 = 0 \longrightarrow A_{12}^{-1} = -A_{12}$

$$\Rightarrow A^{-1} = \begin{bmatrix} I_2 & -A_{12} \\ 0 & I_2 \end{bmatrix} = \left[\begin{array}{cc|cc} 1 & 0 & 5 & -11 \\ 0 & 1 & \sqrt{2} & -e^2 \\ \hline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$