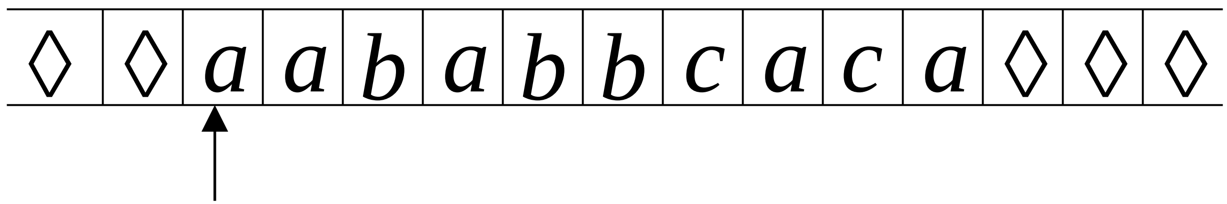
CHAPTER 10

Other Models Of Turing Machines

By R.Ameri

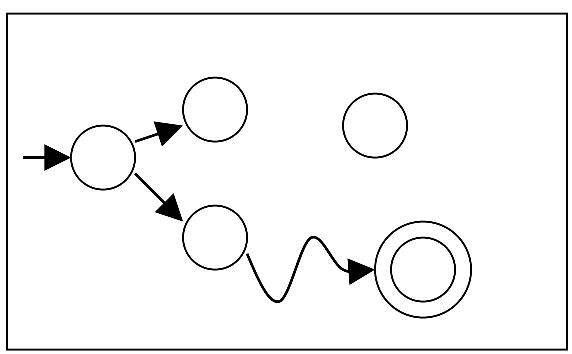
The Standard Model

Infinite Tape



Read-Write Head (Left or Right)

Control Unit



Deterministic

Variations of the Standard Model

Turing machines with:

- Stay-Option
- Multiple Track Tape
- Semi-Infinite Tape
- Off-Line
- Multitape
- Multidimensional
- Nondeterministic

The variations form different Turing Machine **Classes**

We want to prove:

Each **Class** has the same power with the Standard Model

Same Power of two classes means:

Both classes of Turing machines accept the same languages

Same Power of two classes means:

For any machine M_1 of first class

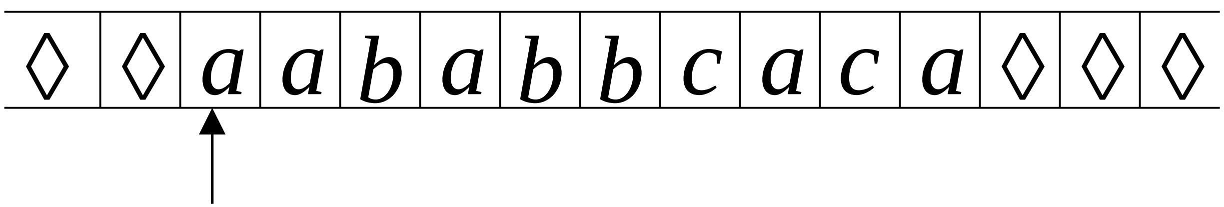
there is a machine M_2 of second class

such that: $L(M_1) = L(M_2)$

And vice-versa

Turing Machines with Stay-Option

The head can stay in the same position



Left, Right, Stay

L,R,S: moves

$$\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R, S\}$$

Example:

Time 1

◊	◊	a	a	b	a	b	b	c	a	c	a	◊	◊	◊
		↑ q_1												

Time 2

◊	◊	b	a	b	a	b	b	c	a	c	a	◊	◊	◊
		↑ q_2												

$$q_1 \xrightarrow{a \rightarrow b, S} q_2$$

Theorem: Stay-Option Machines have the same power with Standard Turing machines

Proof:

Part 1: Stay-Option Machines
are at least as powerful as
Standard machines

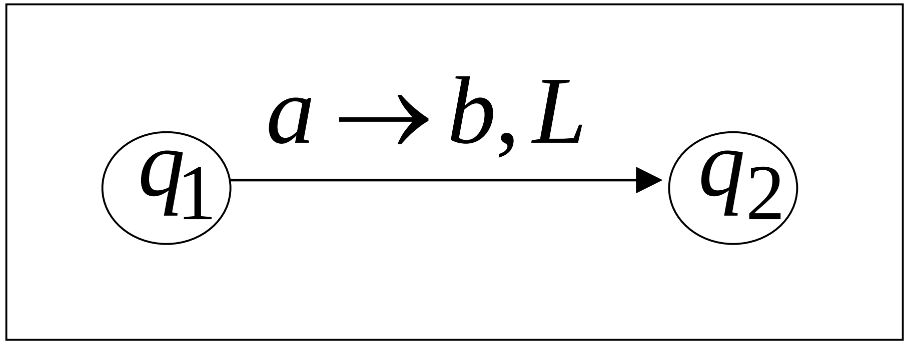
Proof: a Standard machine is also
a Stay-Option machine
(that never uses the S move)

Proof:

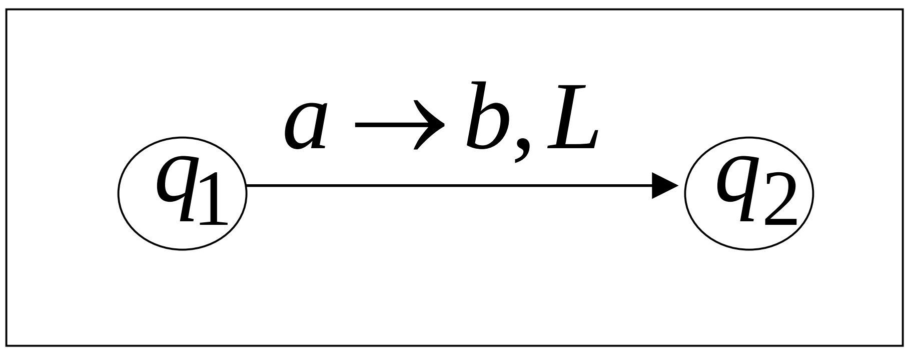
Part 2: Standard Machines are at least as powerful as Stay-Option machines

Proof: a standard machine can simulate a Stay-Option machine

Stay-Option Machine

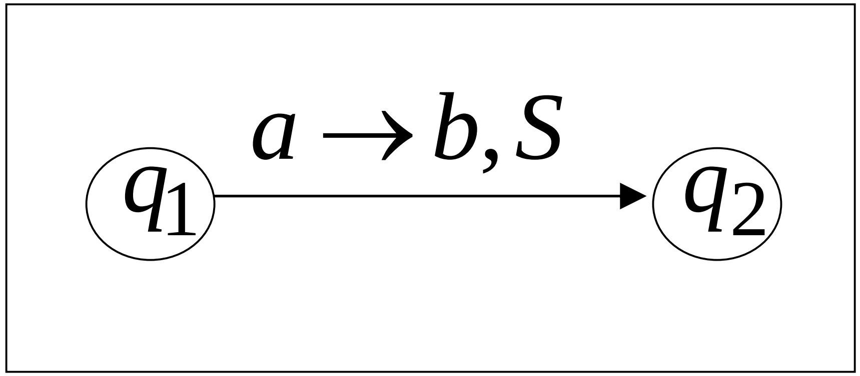


Simulation in Standard Machine

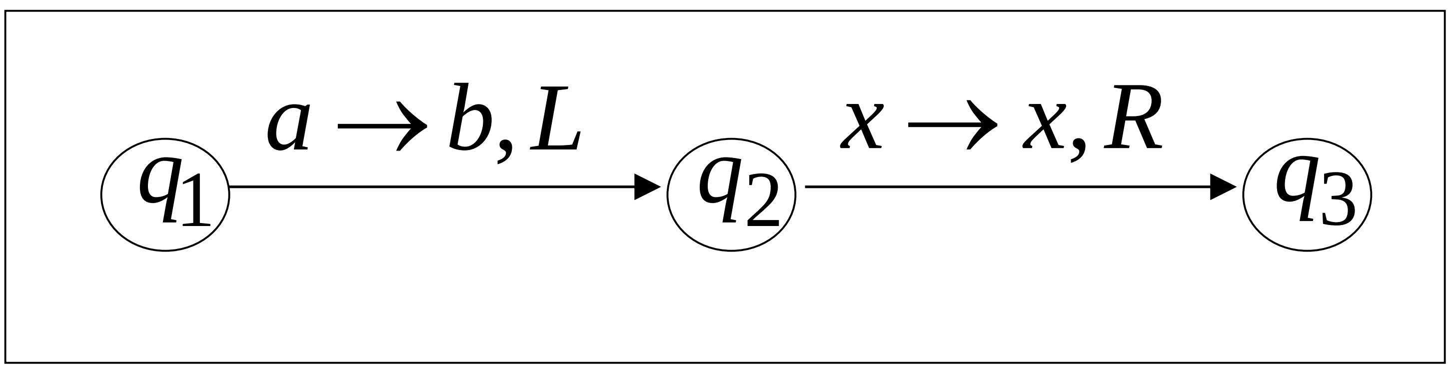


Similar for Right moves

Stay-Option Machine



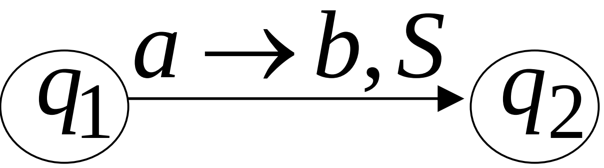
Simulation in Standard Machine

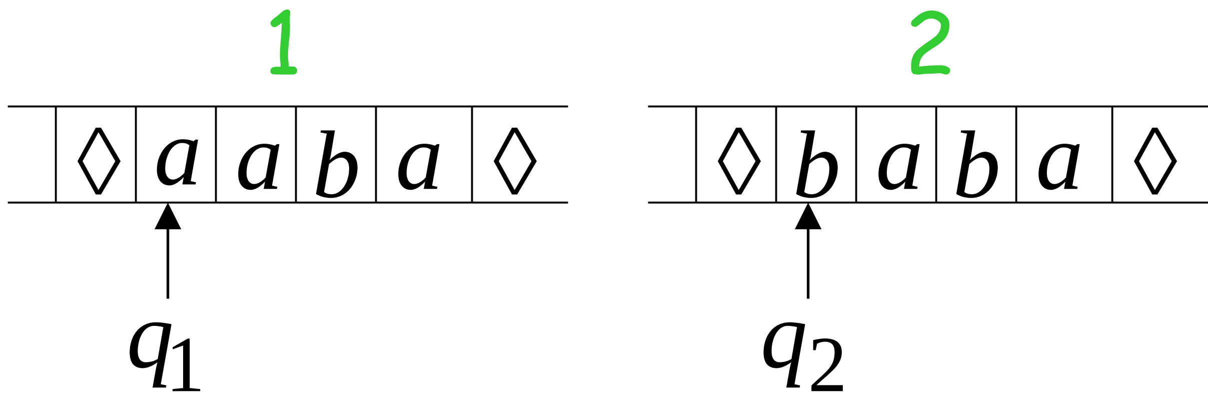


For every symbol x

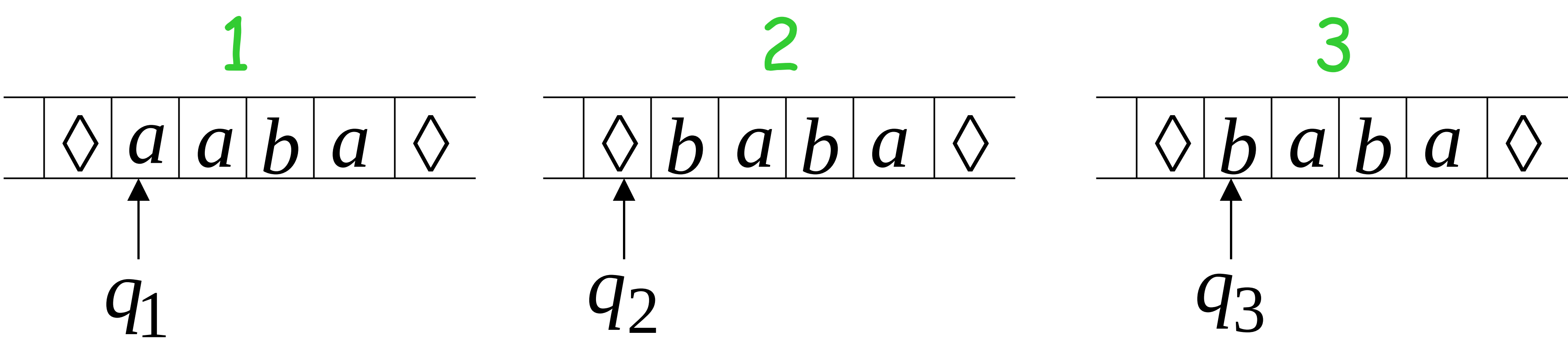
Example

Stay-Option Machine:

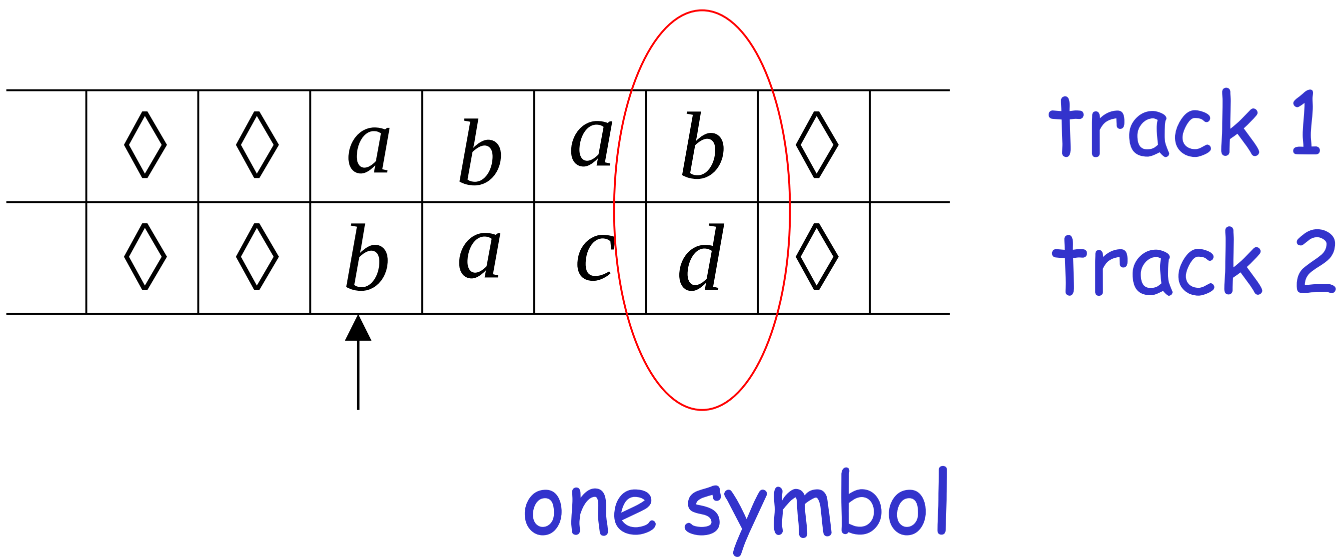




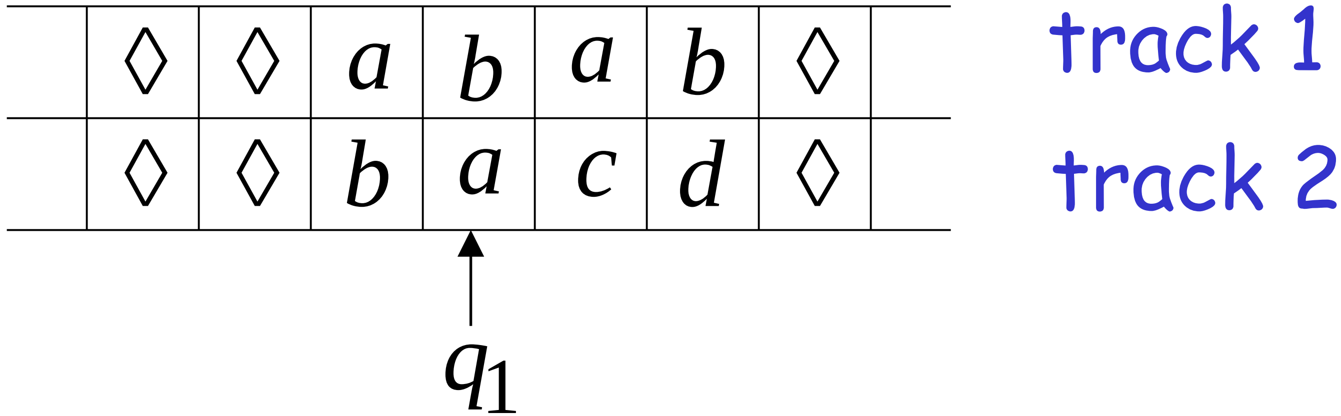
Simulation in Standard Machine:

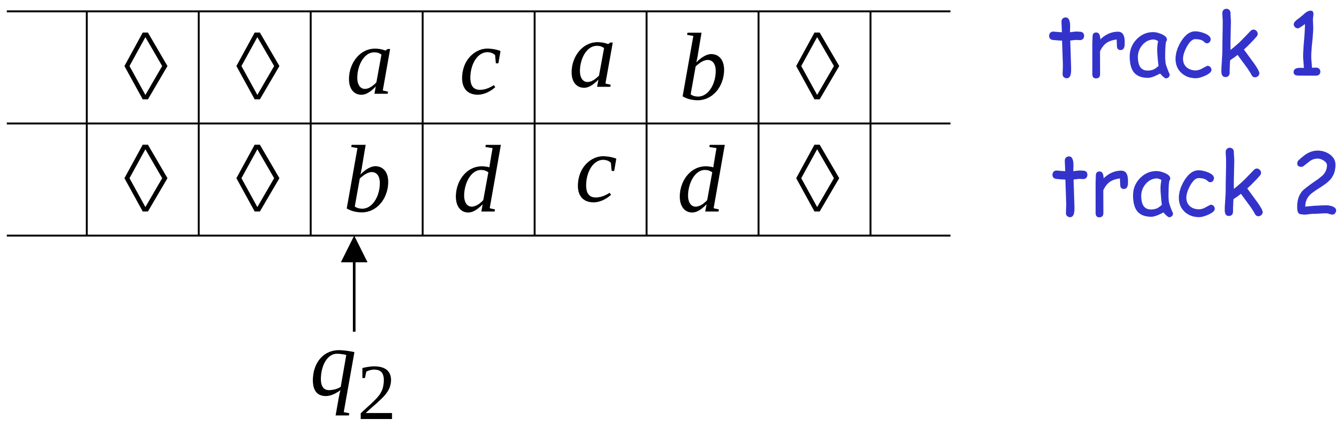


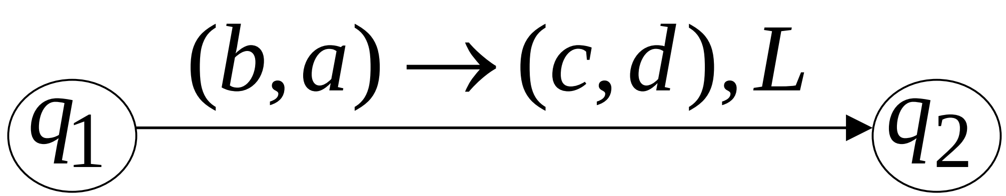
Standard Machine--Multiple Track Tape



$$\delta : Q \times \Gamma^{n} \rightarrow Q \times \Gamma^{n} \times \{L, R\}$$

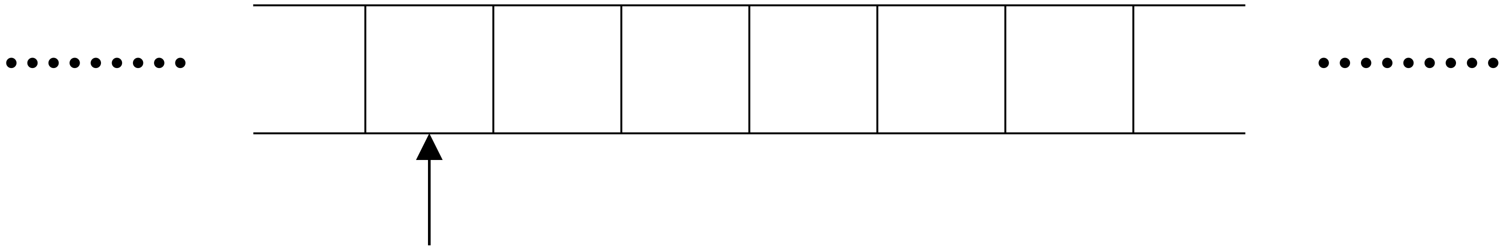

◊ ◊ a b a b ◊
◊ ◊ b a c d ◊
track 1
track 2
q_1


◊ ◊ a c a b ◊
◊ ◊ b d c d ◊
track 1
track 2
q_2

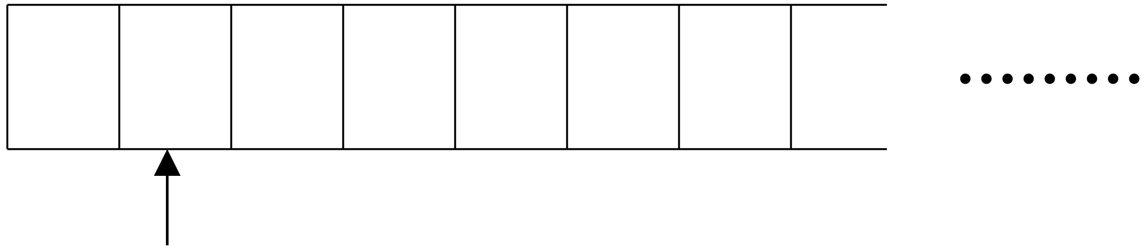

q_1
$(b,a) \to (c,d), L$
q_2

Semi-infinite tape machines simulate Standard Turing machines:

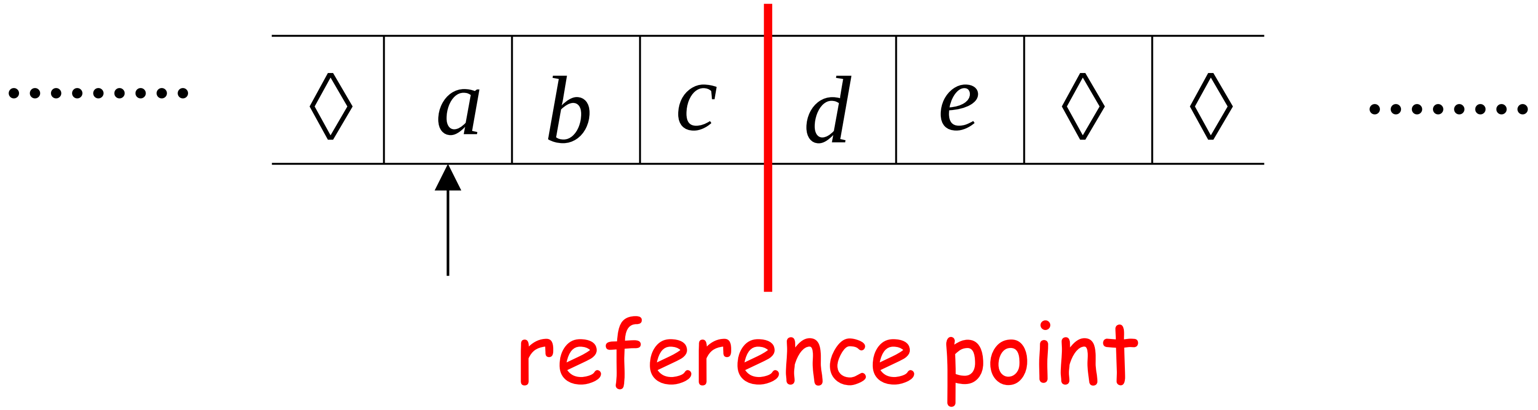
Standard machine



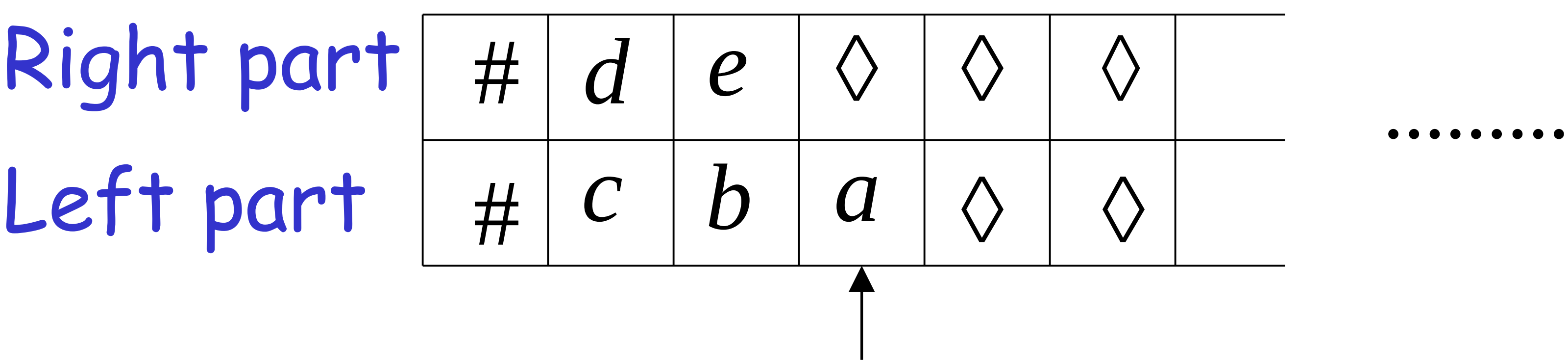
Semi-infinite tape machine



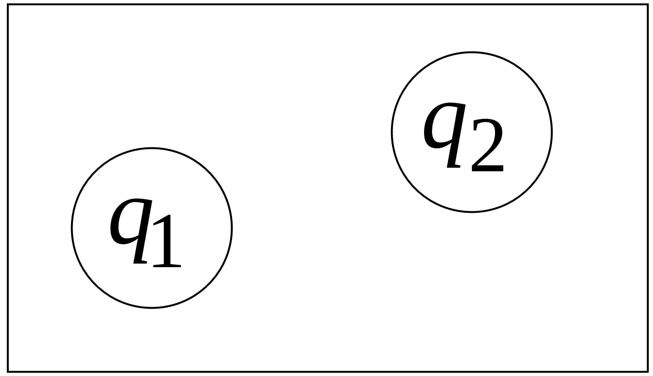
Standard machine



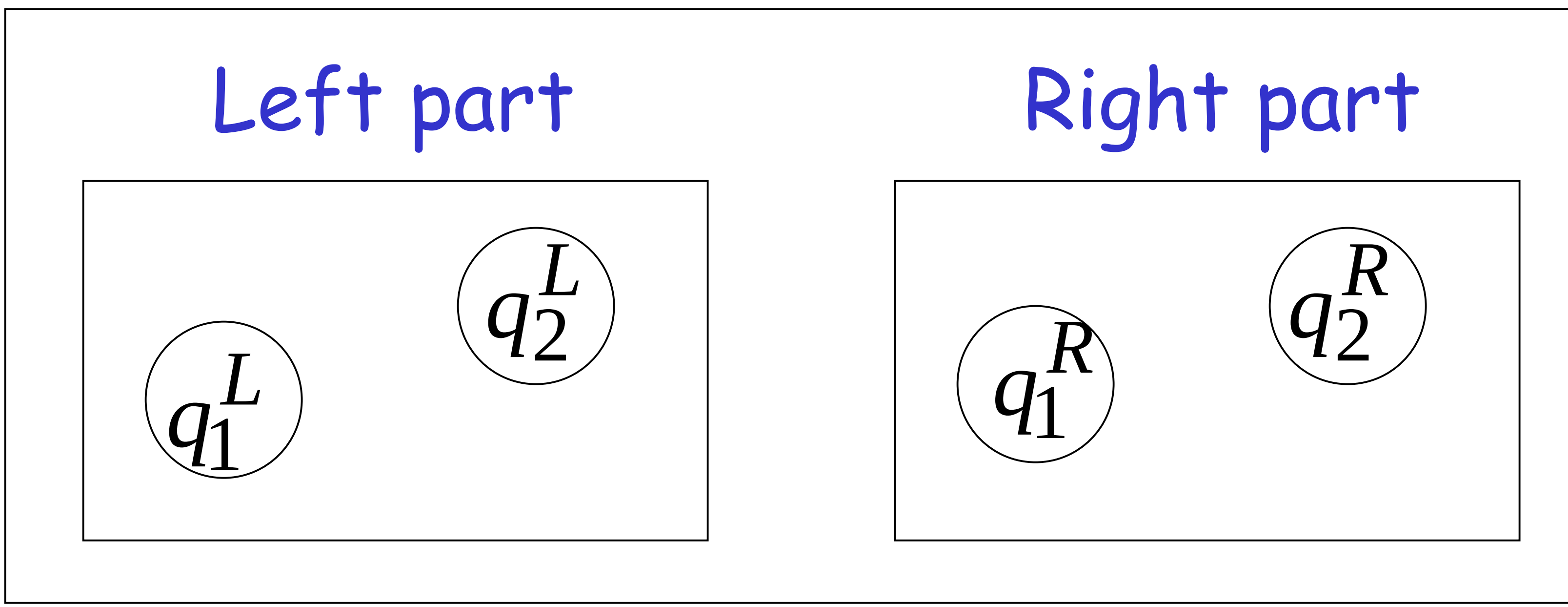
Semi-infinite tape machine with two tracks



Standard machine



Semi-infinite tape machine



Standard machine

$$q_1 \xrightarrow{a \rightarrow g, R} q_2$$

Semi-infinite tape machine

Right part

$$q_1^R \xrightarrow{(a, x) \rightarrow (g, x), R} q_2^R$$

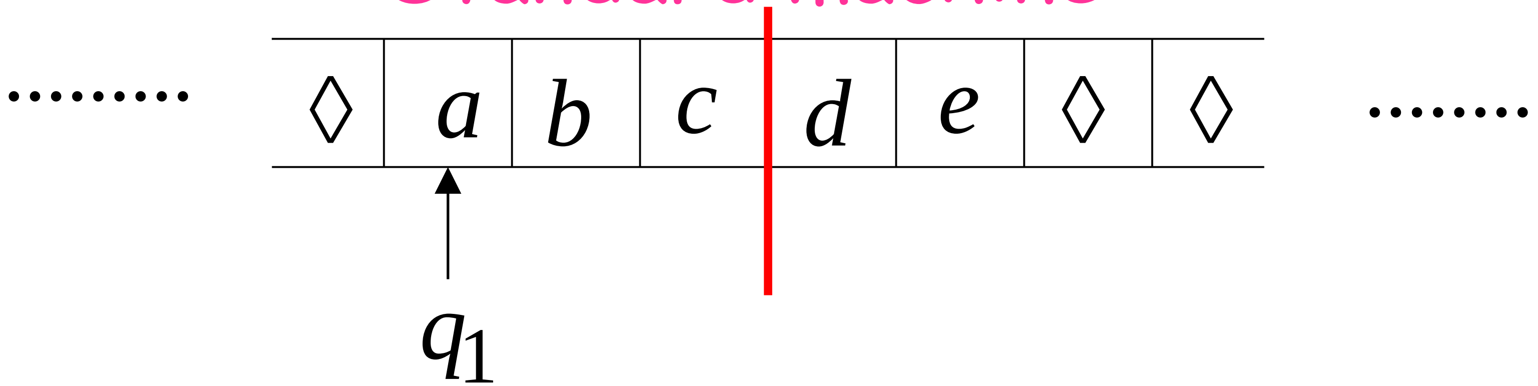
Left part

$$q_1^L \xrightarrow{(x, a) \rightarrow (x, g), L} q_2^L$$

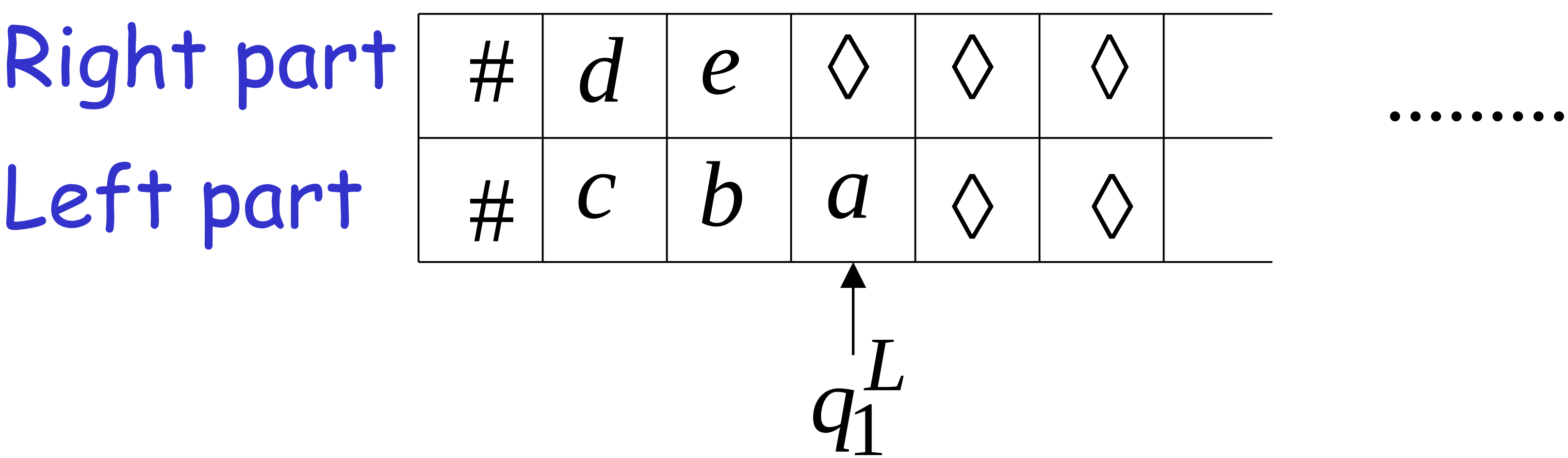
For all symbols x

Time 1

Standard machine

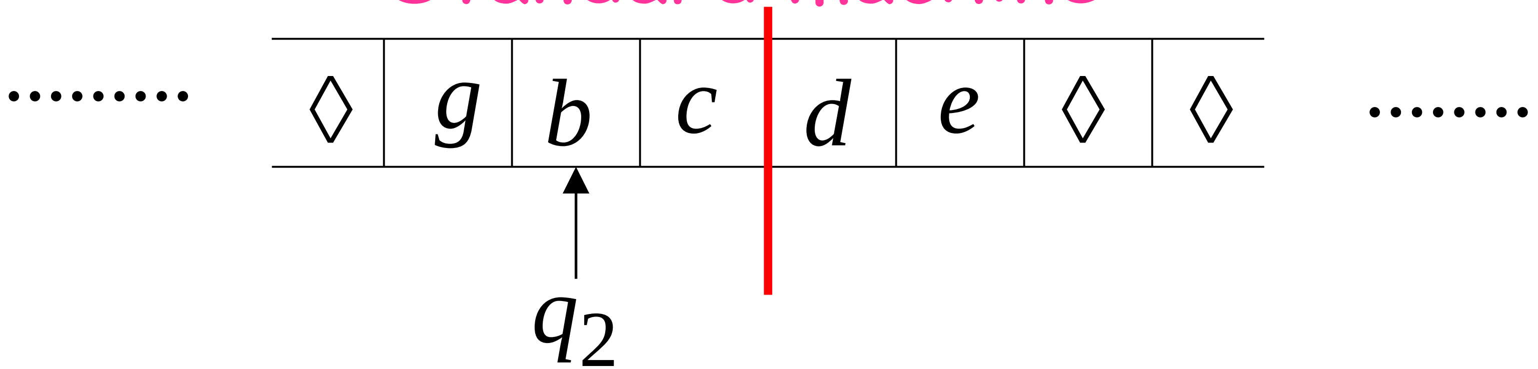


Semi-infinite tape machine

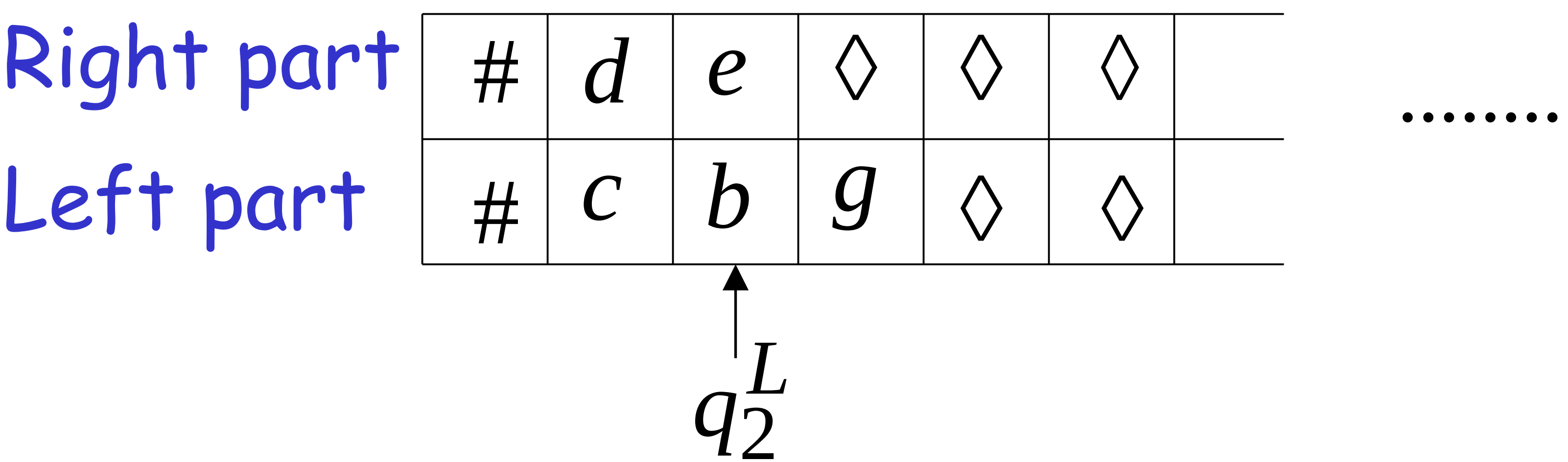


Time 2

Standard machine

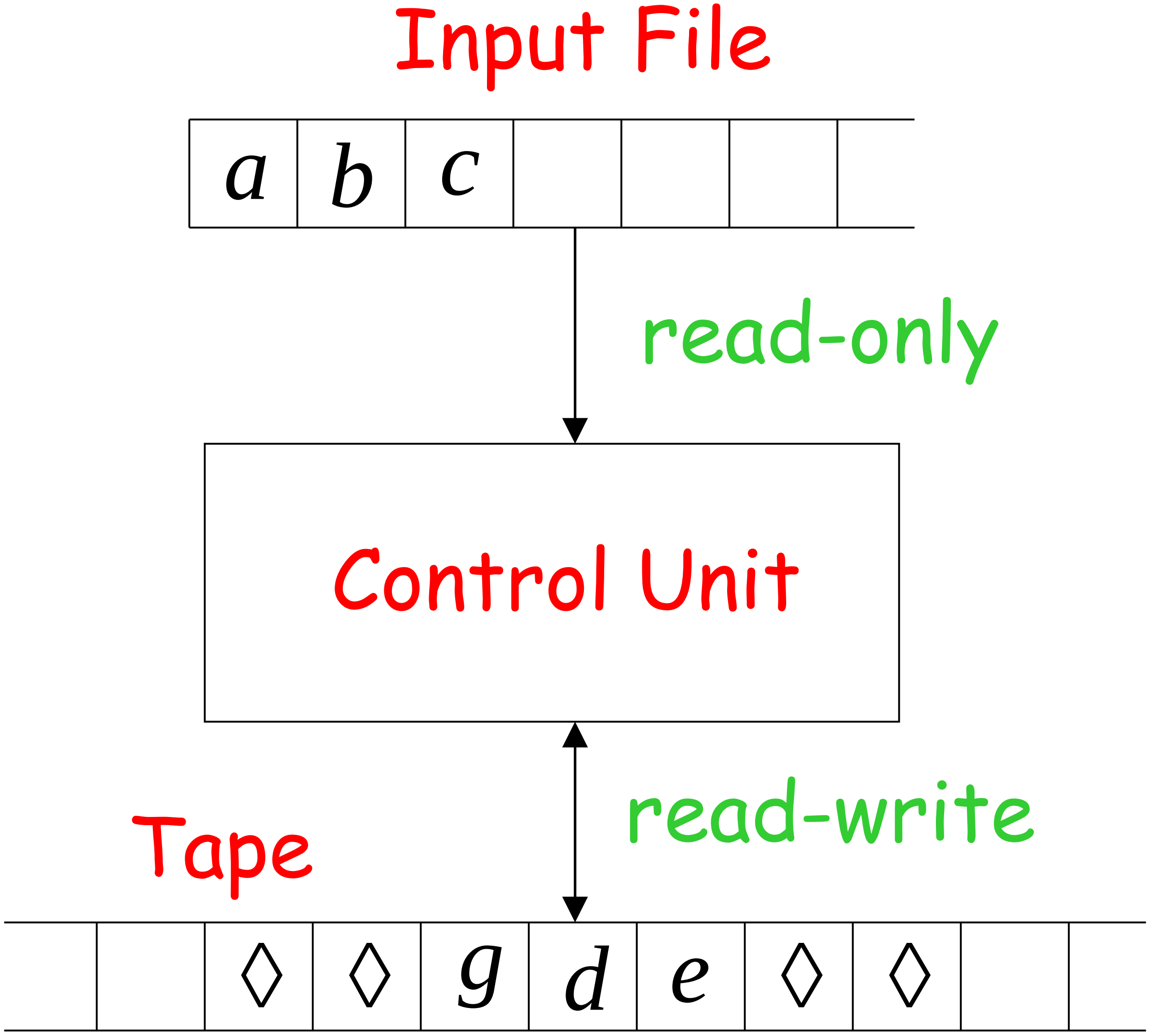


Semi-infinite tape machine



Theorem: Semi-infinite tape machines have the same power with Standard Turing machines

The Off-Line Machine



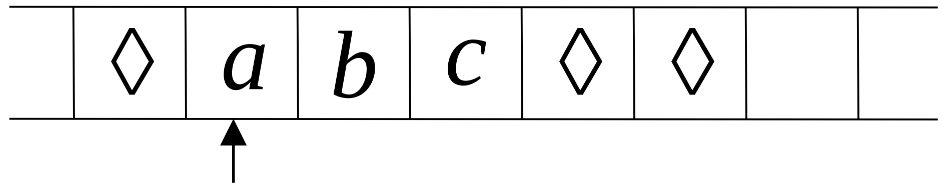
Off-line machines simulate Standard Turing Machines:

Off-line machine:

1. Copy input file to tape

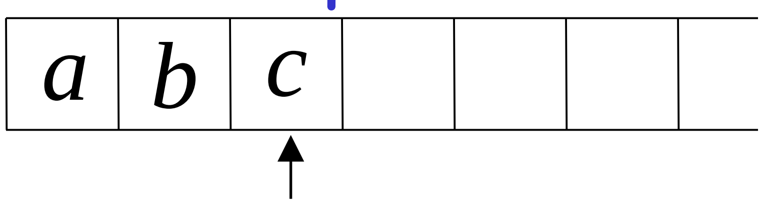
2. Continue computation as in Standard Turing machine

Standard machine

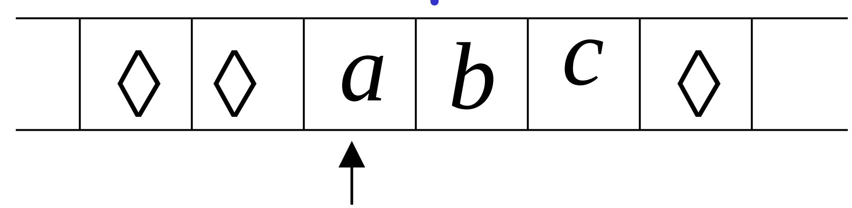


Off-line machine

Input File

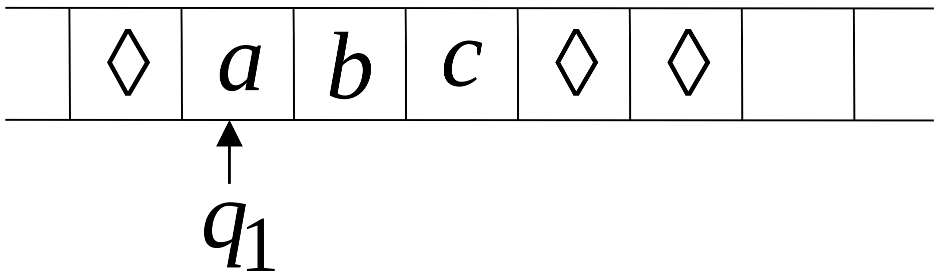


Tape



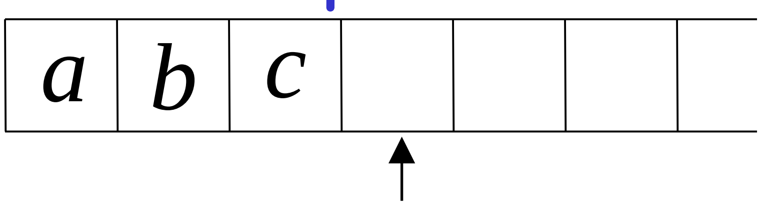
1. Copy input file to tape

Standard machine

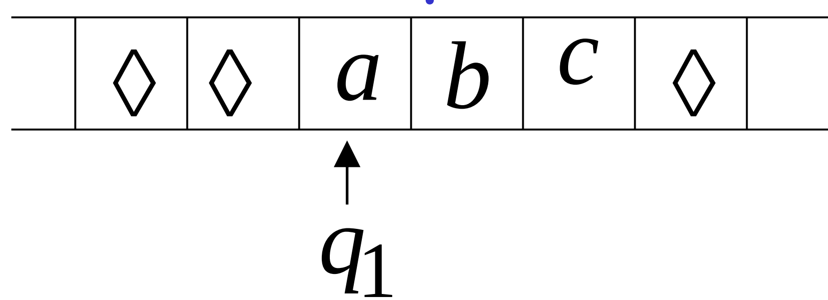


Off-line machine

Input File



Tape



2. Do computations as in Turing machine

Standard Turing machines simulate
Off-line machines:

Use a Standard machine with four track tape
to keep track of
the Off-line input file and tape contents

Off-line Machine

Input File

a	b	c	d			

Tape

	◊	◊	e	f	g	◊	

Four track tape -- Standard Machine

	#	a	b	c	d			Input File
	#	0	0	1	0			head position
		e	f	g				Tape
		0	1	0				head position

Reference point

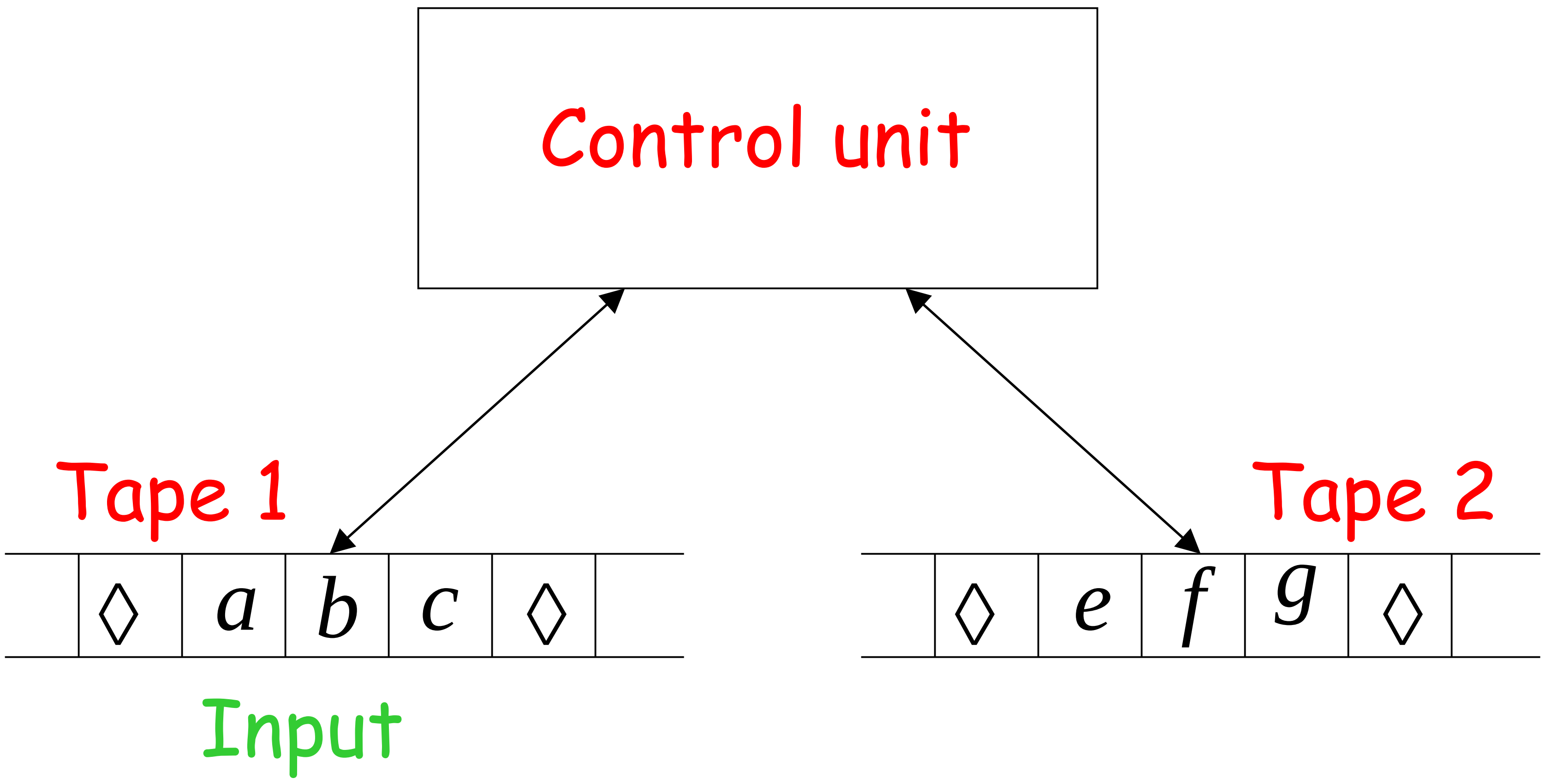
	#	a	b	c	d				Input File
	#	0	0	1	0				head position
		e	f	g					Tape
		0	1	0					head position

Repeat for each state transition:

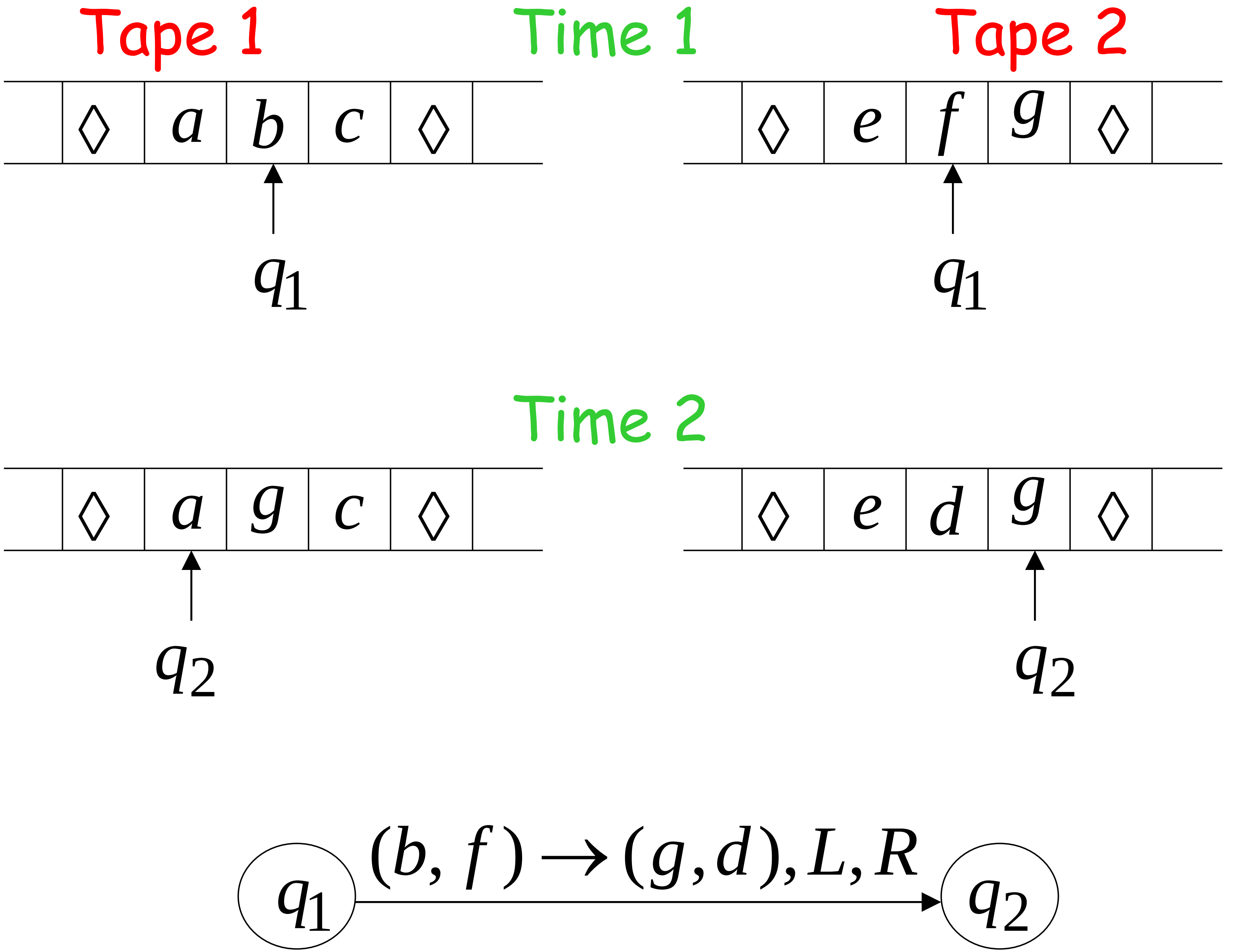
- Return to reference point
- Find current input file symbol
- Find current tape symbol
- Make transition

Theorem: Off-line machines have the same power with Stansard machines

Multitape Turing Machines



$$\delta : Q \times \Gamma^n \to Q \times \Gamma^n \times \{L, R\}^n$$


Tape 1
Time 1
Tape 2
◊ a b c ◊
◊ e f g ◊
q_1
q_1
Time 2
◊ a g c ◊
◊ e d g ◊
q_2
q_2
q_1
$(b, f) \rightarrow (g, d), L, R$
q_2

Multitape machines simulate Standard Machines:

Use just one tape

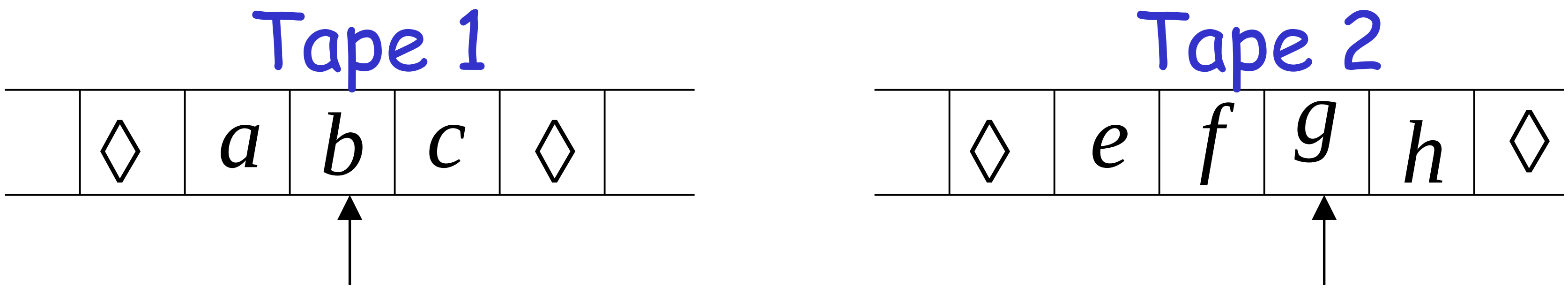
Standard machines simulate
Multitape machines:

Standard machine:

- Use a multi-track tape

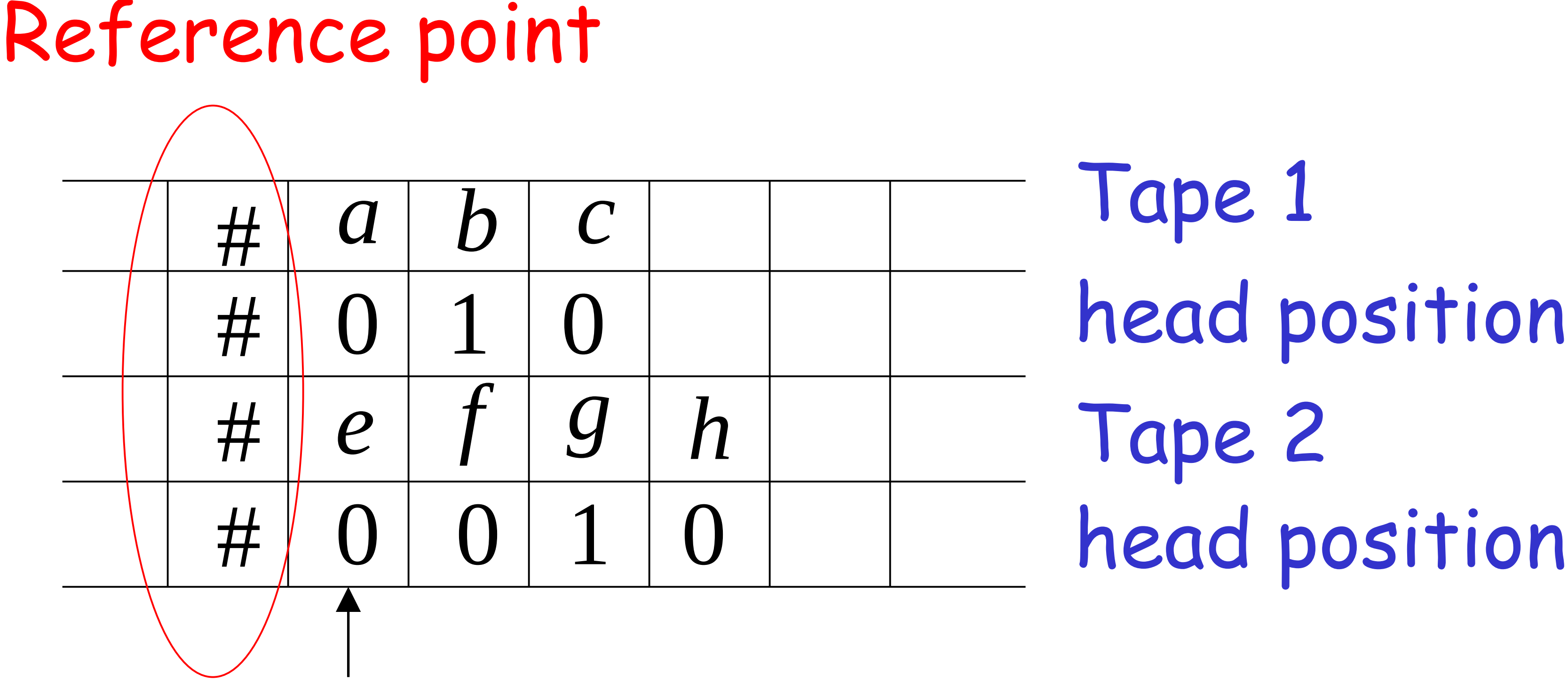
- A tape of the Multiple tape machine corresponds to a pair of tracks

Multitape Machine



Standard machine with four track tape

		a	b	c				Tape 1
		0	1	0				head position
		e	f	g	h			Tape 2
		0	0	1	0			head position



Repeat for each state transition:

- Return to reference point
- Find current symbol in Tape 1
- Find current symbol in Tape 2
- Make transition

Theorem: Multi-tape machines
have the same power with
Standard Turing Machines

Same power doesn't imply same speed:

Language $L = \{a^n b^n\}$

	Acceptance Time
Standard machine	n^2
Two-tape machine	n

$$L = \{a^n b^n\}$$

Standard machine:

Go back and forth n^2 times

Two-tape machine:

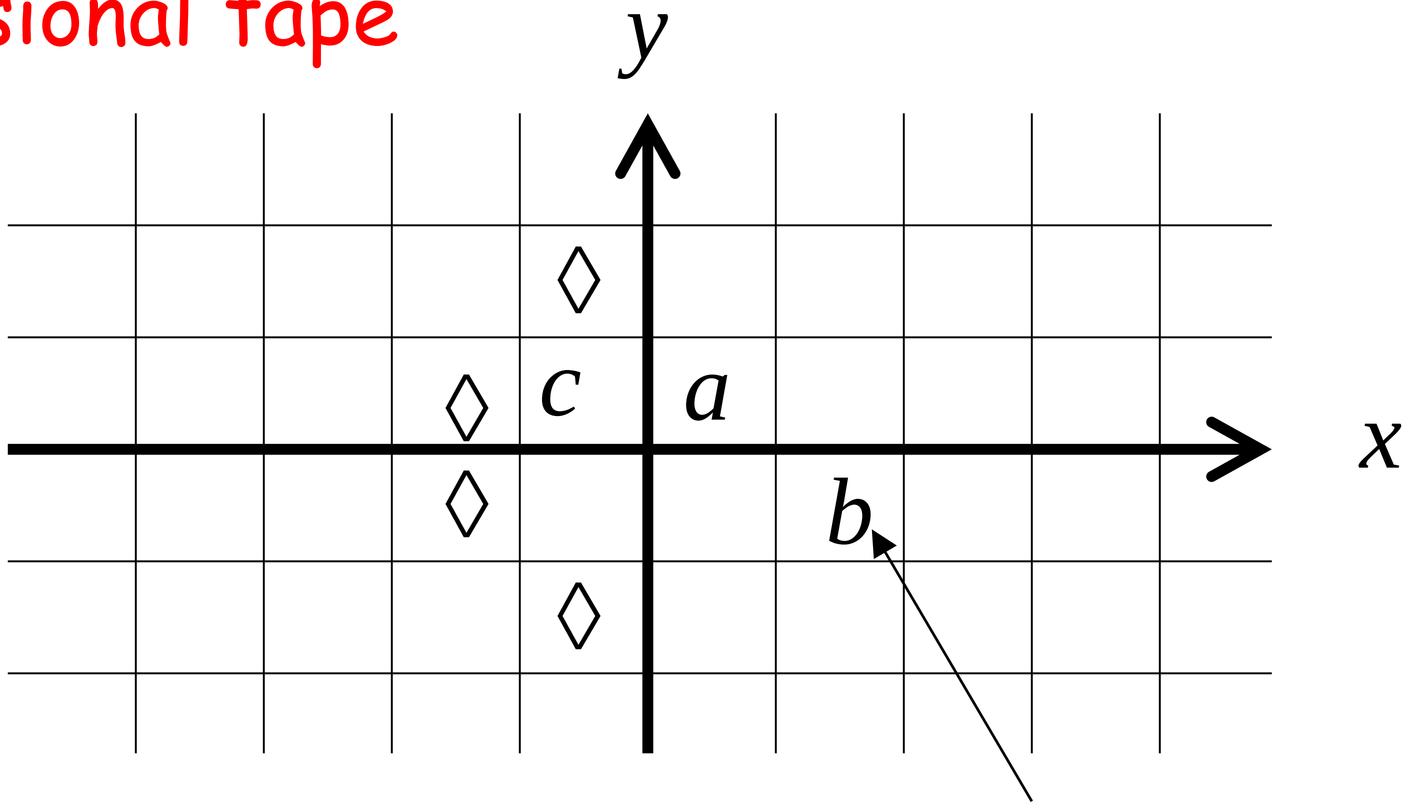
Copy b^n to tape 2 (n steps)

Leave a^n on tape 1 (n steps)

Compare tape 1 and tape 2 (n steps)

MultiDimensional Turing Machines

Two-dimensional tape



MOVES: L,R,U,D

U: up   D: down

HEAD

Position: +2, -1

$$\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R, U, D\},$$

Multidimensional machines simulate Standard machines:

Use one dimension

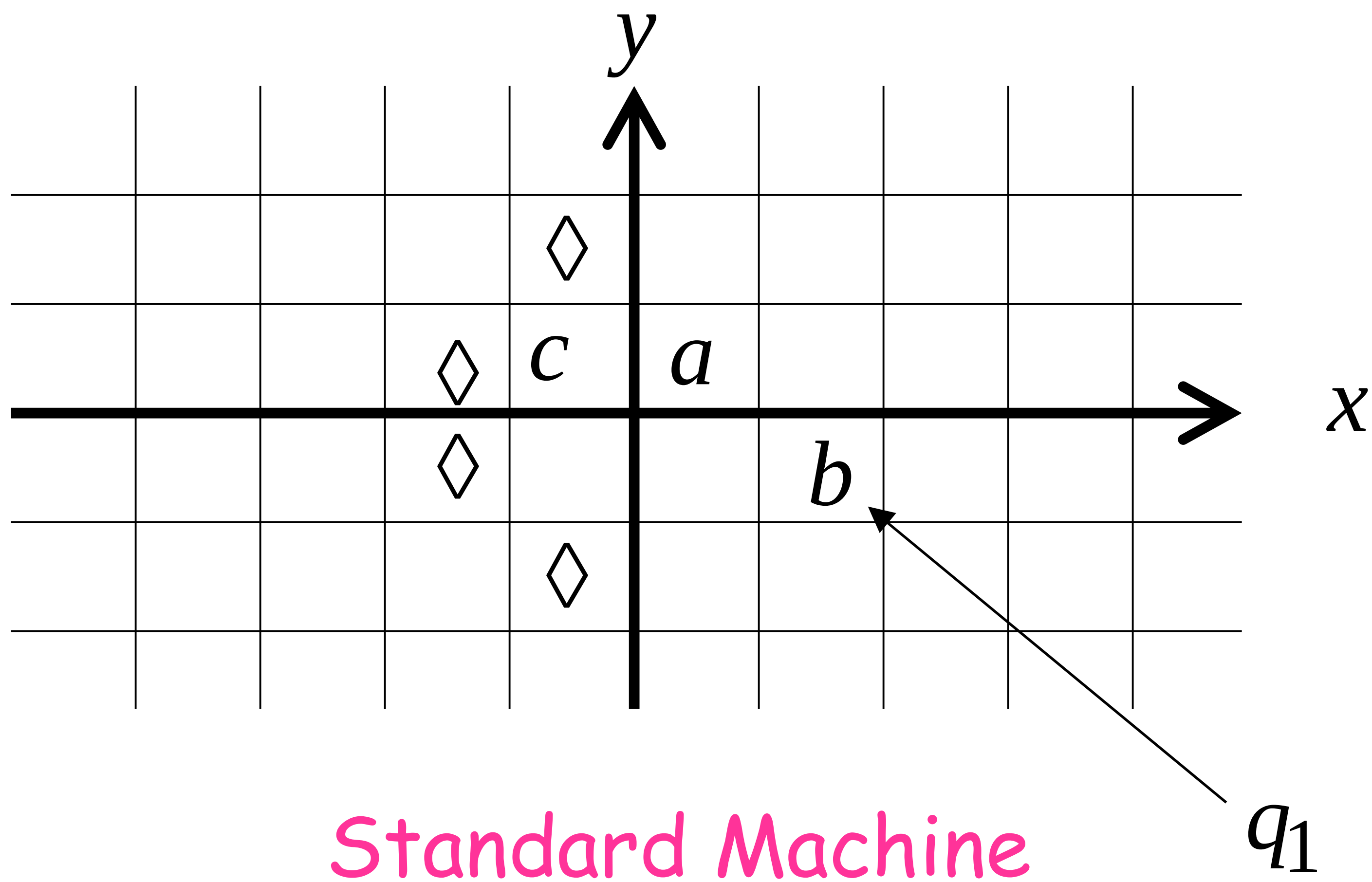
Standard machines simulate Multidimensional machines:

Standard machine:

- Use a two track tape

- Store symbols in track 1
- Store coordinates in track 2

Two-dimensional machine



Standard Machine

a				b					c	
1	#	1	#	2	#	−	1	#	−	1

q_1

symbols

coordinates

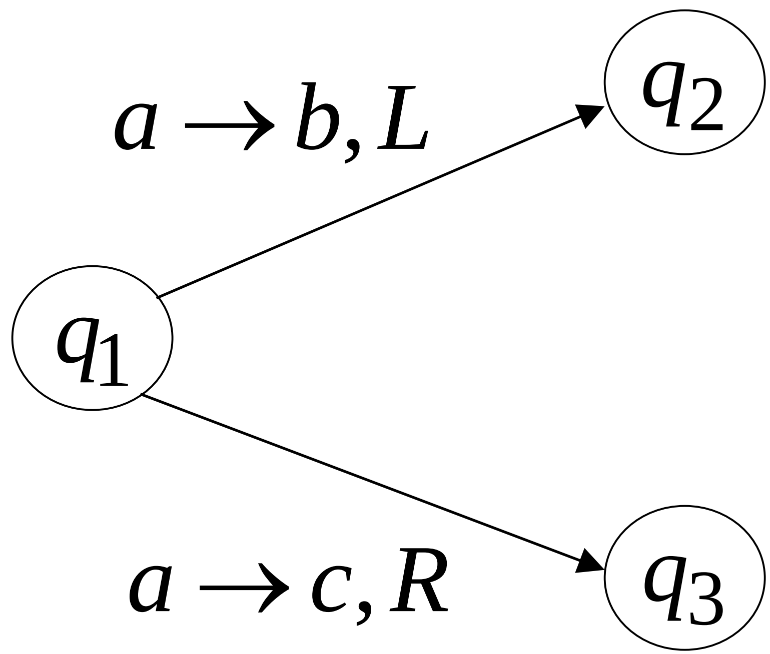
Standard machine:

Repeat for each transition

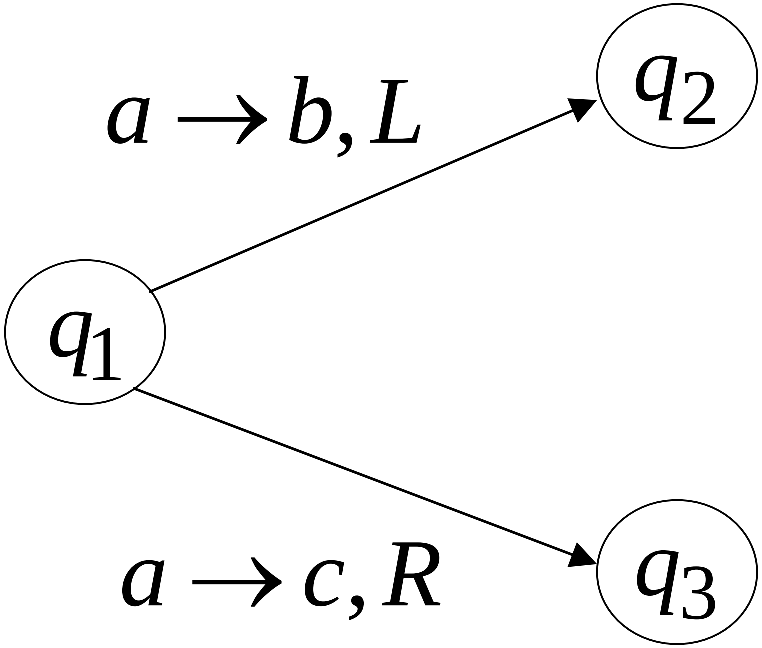
- Update current symbol
- Compute coordinates of next position
- Go to new position

Theorem: MultiDimensional Machines have the same power with Standard Turing Machines

NonDeterministic Turing Machines



Non Deterministic Choice



Time 0

	$\Diamond$	a	b	c	$\Diamond$	
		$\uparrow$ q_1				

Time 1

Choice 1

	$\Diamond$	b	b	c	$\Diamond$	
	$\uparrow$ q_2					

Choice 2

	$\Diamond$	c	b	c	$\Diamond$	
			$\uparrow$ q_3			

Input string w is accepted if this a possible computation

$$q_0 w \overset{*}{\succ} x\, q_f\, y$$

Initial configuration

Final Configuration

Final state

$$\delta : Q \times \Gamma \rightarrow 2^{Q \times \Gamma \times \{L, R\}}.$$

NonDeterministic Machines simulate Standard (deterministic) Machines:

Every deterministic machine is also a nondeterministic machine

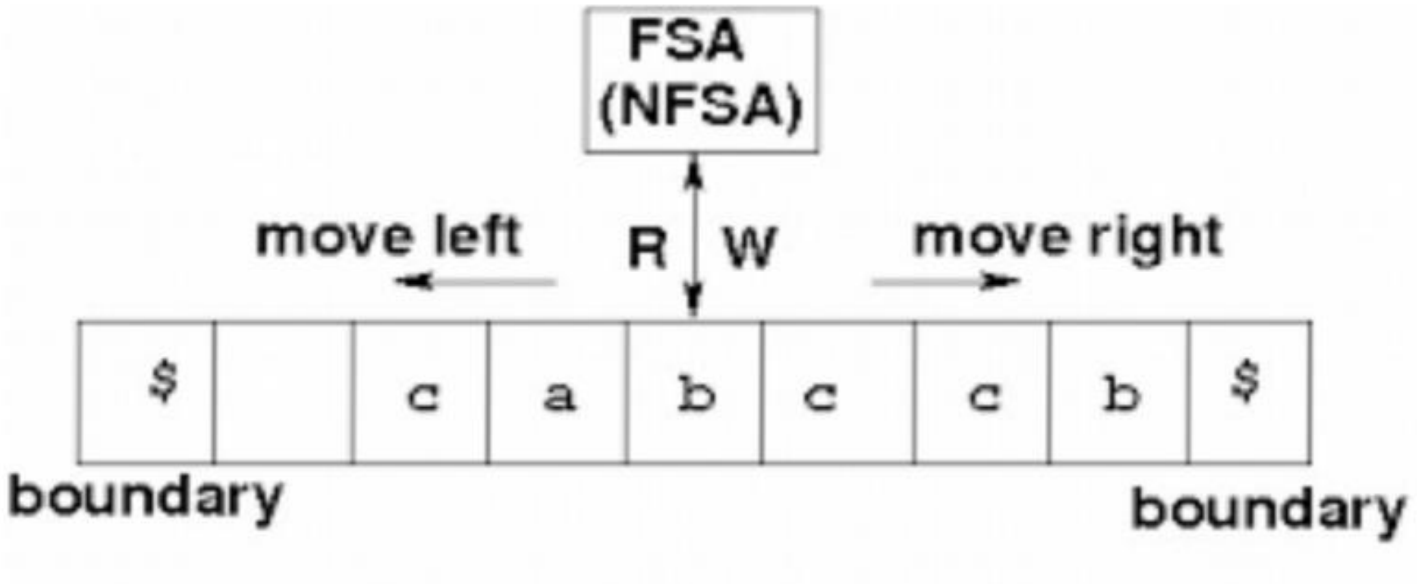
Theorem: NonDeterministic Machines have the same power with Deterministic machines

Remark:

The simulation in the Deterministic machine takes time exponential time compared to the NonDeterministic machine

Linear Bounded Automata (LBA)

- A non-deterministic Turing machine that uses only the tape space occupied by the input.



Linear Bounded Automata

- Its input alphabet includes two special symbols, serving as left and right endmarkers.
- Linear bounded automata are acceptors for the class of context-sensitive languages.
- linear bounded automata are more powerful than pushdown automata, since neither of the languages is context free.

Linear Bounded Automata

The language $L = \{a^n b^n c^n : n \geq 1\}$

is accepted by some linear bounded automaton. The computation outlined there does not require space outside the original input.

~~a~~ bbaab|~~a~~ bbaab

~~a~~ ~~b~~ baab|~~a~~ ~~b~~ baab

a bbaab|abbaab

a bbaababbaab

Recursively Enumerable and Recursive Languages

Definition:

A language is **recursively enumerable or Turing-recognizable** if some Turing machine accepts it

Let L be a recursively enumerable language

and M the Turing Machine that accepts it

For string w :

if $w \in L$ then M halts in a final state

if $w \notin L$ then M halts in a non-final state

or loops forever

Definition:

A language is **recursive or decidable** if some Turing machine accepts it and halts on any input string

In other words:

A language is recursive if there is a membership algorithm for it

Let L be a recursive language

and M the Turing Machine that accepts it

For string w:

if $w \in L$ then M halts in a final state

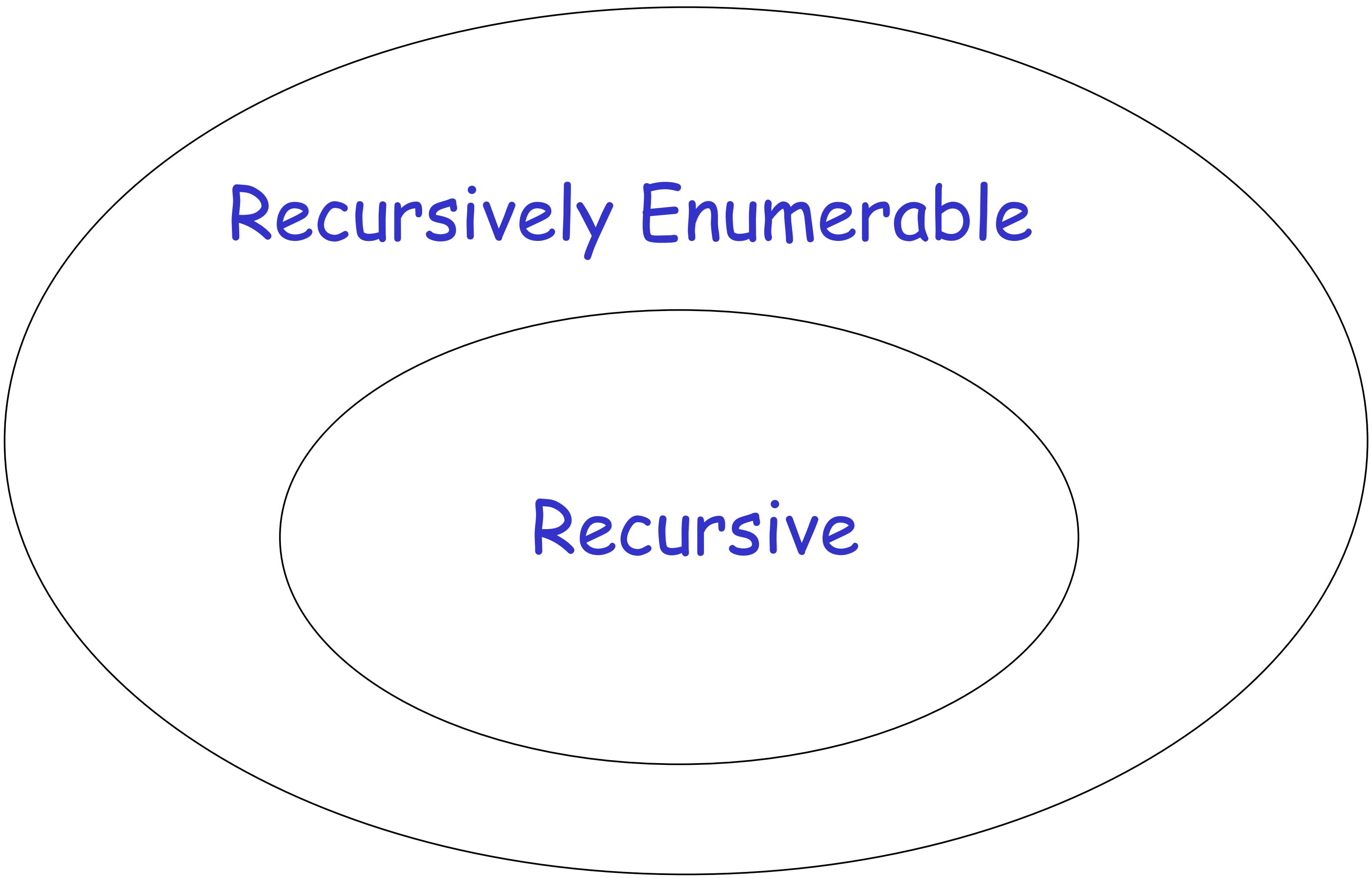
if $w \notin L$ then M halts in a non-final state

A Turing Machine **decides** a language if it accepts all strings in the language and rejects all strings not in the language

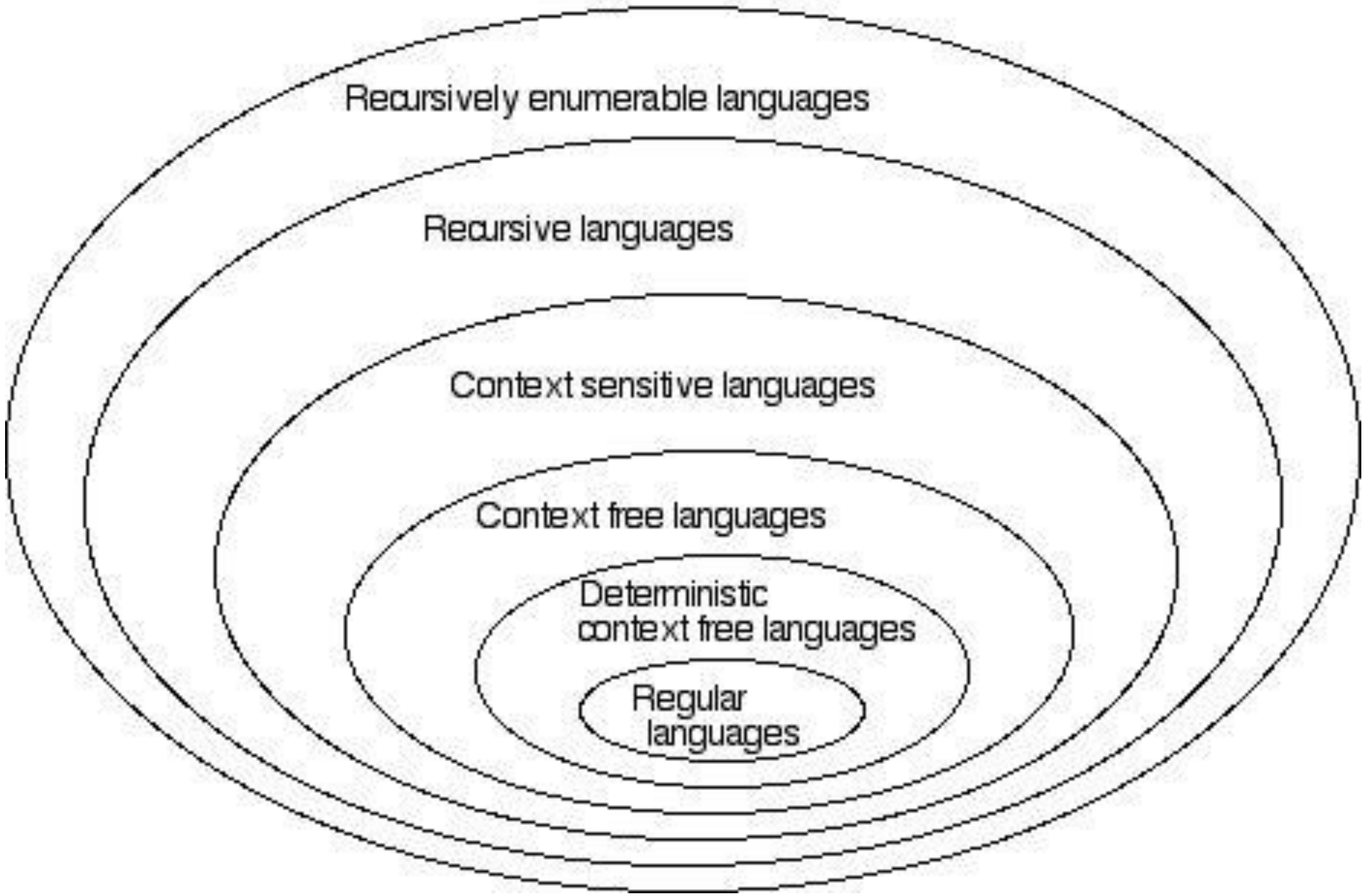
- There is a specific language which is not recursively enumerable (not accepted by any Turing Machine)

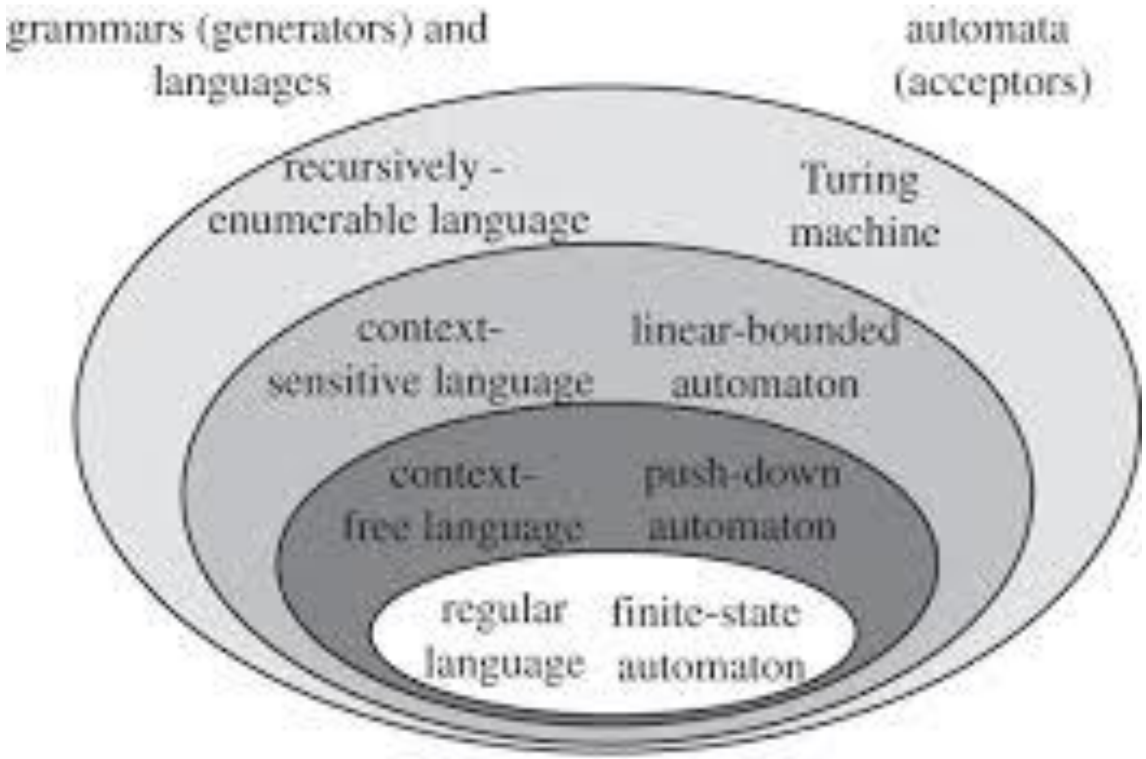
- There is a specific language which is recursively enumerable but not recursive

Non Recursively Enumerable



Elements of the Chomsky Hierarchy





the traditional Chomsky hierarchy