فرمت گزارش:

گزارش نهایی می‌بایست به زبان فارسی یا انگلیسی در قالب فایل PDF باشد. در کنار آن پوشه‌ای با نام Code قرار داشته باشد و همه فایل‌های یاد شده، در یک فایل فشرده با فرمت مشخص جهت ارزیابی ارسال گردند. در گزارش نیاز است روی خروجی‌ها و نمودارهای درج شده **حتما** گزارش تحلیلی داشته باشید و استدلال خود را از تحلیل ها نشان دهید. نتیجه‌گیری خود راجع به هر تمرین را به صورت حداقل یک پاراگراف بیان نمایید (البته اگر در مسائل ساده تحلیل خوبی صورت گرفته باشد نتیجه‌گیری در قالب همان تحلیل کافیست ولی برای مسائلی که چند قسمتی بوده و موارد مرتبط به هم هستند حتما نتیجه‌گیری کلی سوال را نیز داشته باشید).

فایل گزارش را به فرمت SML1_report_StdNum.pdf نام گذاری نمایید (همانند SML1_report_98131.pdf).

فرمت کد نویسی:

برای هر تمرین باید فایل کد جداگانه با زبان‌های MATLAB یا Python نوشته شود. علاقه‌مندان به پایتون حتما از محیط Jupyter notebook استفاده نمایند تا خروجی در فایل مذکور نیز مشهود باشد. کامنت گذاری در حد لازم نیز انجام پذیرد. فرمت نام‌گذاری فایل اصلی مربوط به هر بخش از تمرین متناسب با فرمت SML1_ProblemNum_StdNum و در پوشه Code ذخیره شده باشد.

نحوه تحویل:

فایل‌های کد و گزارش خود را مطابق فرمت‌های فوق آماده و در قالب یک فایل فشرده با نام SML1_StdNum.zip تهیه نمایید.

Theoretical (part A)

1. Two computers are connected to a password-protected wireless network. When the password is temporarily removed, a virus can attack the first computer with probability 0.5, the second computer with probability 0.7, and it can attack both computers with probability 0.4.

   a. The first computer appears infected with a virus. What is the probability that the second computer was also attacked?
   b. The first computer was not attacked at all. What is the probability that the second computer was attacked?

2. There are three cards. The first is green on both sides, the second is red on both sides and the third is green on one side and red on the other. We choose a card at random and we see one side (also chosen at random). If the side we see is green, what is the probability that the other side is also green?

3. Let $X,Y \sim Unif(0,1)$ be independent. Find the density $f_Z(z)$ for following Z:
   a. $Z = X - Y$.
   b. $Z = X/Y$.

4. Show that $\mathbb{V}(x) = 0$, if and only if there is a constant c such that $P(X = c) = 1$.

5. Let X and Y have joint PDF $f(x,y) = \frac{3}{16}xy^2$ for $0 < x < 2$ and $0 < y < 2$
   a. What is the $\mathbb{E}(X)$?
   b. What is the $\mathbb{V}(Y)$?

6. Let $X_1,X_2, \dots . X_n \sim Uniform(0,1)$ and let $Y_n = \max\{X_1,X_2, \dots . X_n\}$. Find $\mathbb{E}(Y_n)$.

7. Let $X \sim N(0,1)$ and let $Y = e^x$. Find $\mathbb{E}(Y)$ and $\mathbb{V}(Y)$.

Programming (part B)

1. Suppose a coin has probability p of falling heads. If we flip coin many times, we would expect the proportion of heads to be near p. Take $p = 0.4$ and $n = 1000$ and simulate n coin flips. Plot the proportion of heads as a function of n. Repeat for $p = 0.04$.
2. Suppose we flip a coin n times and let p denote the probability of heads. Let X be the number of heads. Intuition suggests that X will be close to np. To see if this is true, we can repeat this experiment many times and average the X values. Carry out a simulation and compare the average of the X's to np. Try this for p=0.4 and n =10, 100, 1000.
3. Consider tossing a fair die. Let $A = \{2,3,6\}$ and $B = \{1,2,3,4\}$. Simulate draws from the sample space and verify that $P(AB) = P(A)\,P(B)$. Now consider tow **disjoint** events A and B that are not independent. Compute $P(A)$, $P(B)$ and $P(AB)$. Compare the calculated values to their theoretical values. Draw van diagram and report the results.
4. Use computer simulation to show that changing selected door in "Monty Hall Problem" leads to higher winning probability. Simulate game for 2000 rounds and change the door in first 1000 rounds. Calculate winning probability in two conditions and compare them.
5. Let $X \sim N(5,18)$. Solve the following parts using computer simulation.
   a. Find $P(X < 9)$.
   b. Find $P(X > -3)$.
   c. Find x such that $P(X > x) = 0.05$.
   d. Find $P(0 \leq X < 4)$.
   e. Find x such that $P(|X| > |x|) = 0.05$.
6. Simulate the PDF for problem 3 of the theoretical part and compare results.
7. Write a program to generate a pair of Gaussian random numbers (X_1, X_2) with zero mean and covariance $E(X_1^2) = 1$, $E(X_2^2) = \frac{1}{3}$, $E(X_1 X_2) = \frac{1}{2}$. Generate 1000 pairs of such numbers, evaluate their sample averages and sample covariance.
8. A large circular dartboard is set up with a "bullseye" at the center of the circle, which is at the coordinate (0,0). A dart is thrown at the center but lands at (X,Y), where X and Y are two different Gaussian random variables $N(0,1)$. What is the average distance of the dart from the bullseye?
9. Consider $X1 \sim Binomial(1000, 0.3)$, $X2 \sim Binomial(1000, 0.5)$ and $X3 \sim Binomial(2000, 0.5)$. Show $X1 + X2$ and $X2 + X3$ have binomial distributions and find their parameters.

10. Let $X_1, X_2, \ldots . X_n$ be $N(0,1)$ random variables and $\bar{X}_n$is sample mean of first n samples. Plot $\bar{X}_n$ versus n for 1,...,10000 . Repeat for $X_1, X_2, \ldots . X_n$ be $Cauchy$. Explain why there is such a difference.

11. Consider $X_i \sim N(0, \frac{1}{i})$. Draw CDF of X_i for $i = 1{,}100{,}1000{,}10000$. Consider an arbitrary very small number(epsilon). Show X_i converges to 0 in probability and distribution.

12. Try to simulate the following questions:

    a. Generate 1000 samples to estimate $P(|\bar{X} - p| > \varepsilon$ in Example 4.3 of textbook for n=100, $\varepsilon = 0.2$ and $p = 0.3$. Estimate $\bar{X}$ after each $n = 100$ sample generation. Compare the result with boundaries in Example 4.3, Example 4.6 of course textbook[1].

    b. Repeat part (a) for $p = 0.5$ and report results.

[1] All of Statistics by Larry A. Wasserman