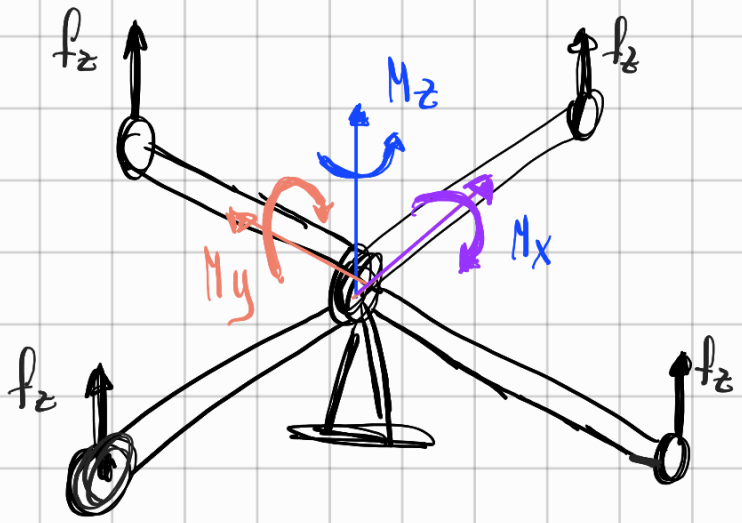
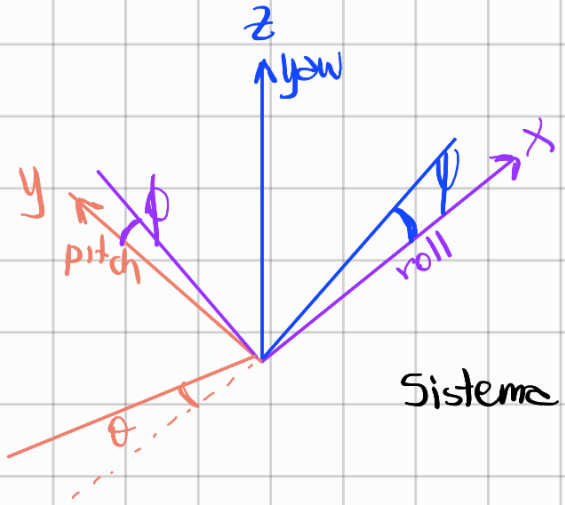


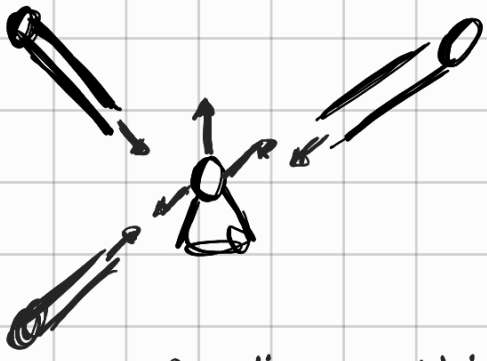
6 dof -
3 doc

3dof : ϕ, θ, ψ

$$\eta = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix}$$



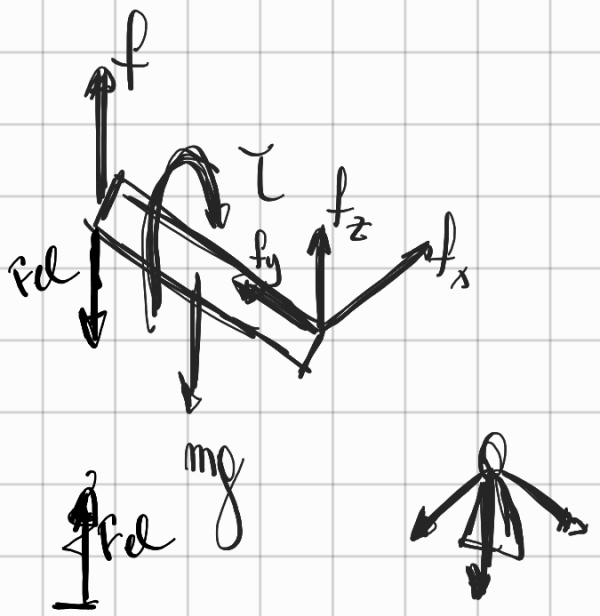
Sistema di rif inerziale

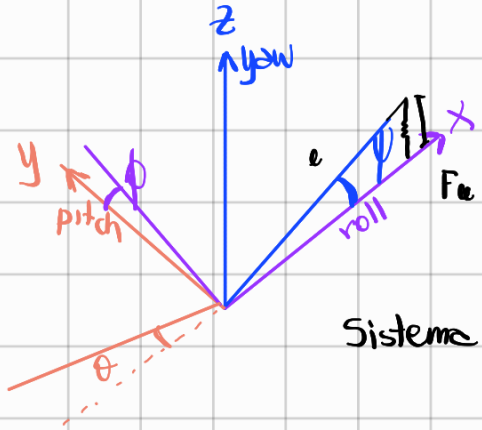


$$\tau^{ext} + \tau = J\ddot{\eta} + C(\eta, \dot{\eta})\dot{\eta}$$

$$f^{ext} + f = Ma$$

$$\tau^{ext} + \tau = I\dot{\omega}$$





$F_{el} = k_{el} \cdot \Delta x = k_{el} \cdot l \cdot \sin(\psi)$

Sistema di rif inerziale

$$m \begin{bmatrix} \cancel{\ddot{x}} \\ \cancel{\ddot{y}} \\ \ddot{z} \end{bmatrix} = -mg \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + R T_B + \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix}$$

$\to 0$

with $T_B = \begin{bmatrix} 0 \\ 0 \\ k \sum_{i=1}^{4} w_i^2 \end{bmatrix}$

$\hookrightarrow R \cdot \begin{bmatrix} f_{x_B} \\ f_{y_B} \\ f_{z_B} \end{bmatrix} = F$

input: velocità angolari motori (w_1, w_2, w_3, w_4)

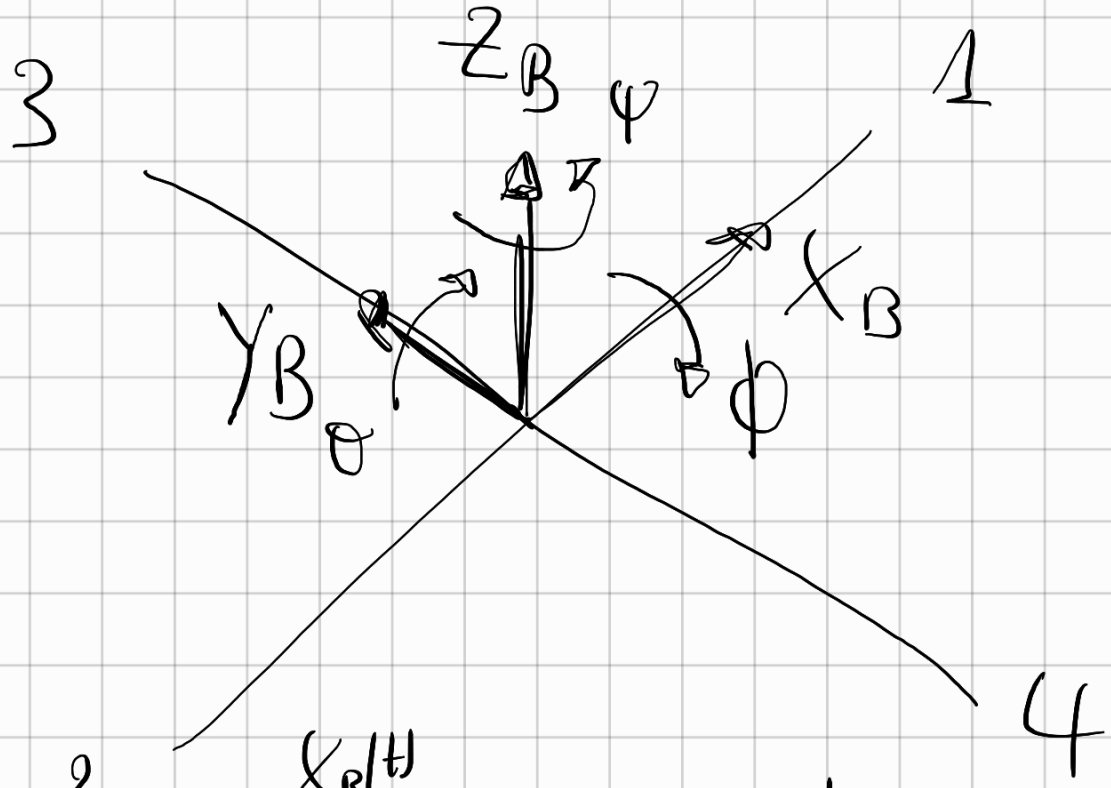
output: angoli eulero: (φ, θ, ψ)

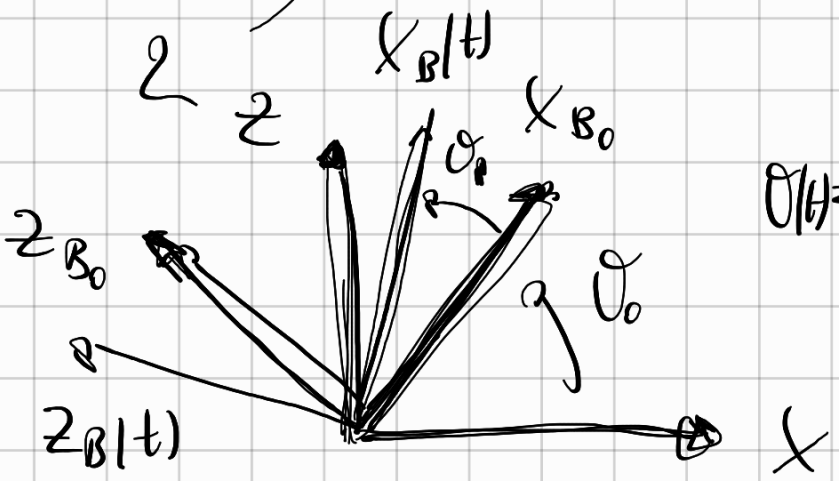
$$\to F + mg \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = R \cdot T_B$$

3x1

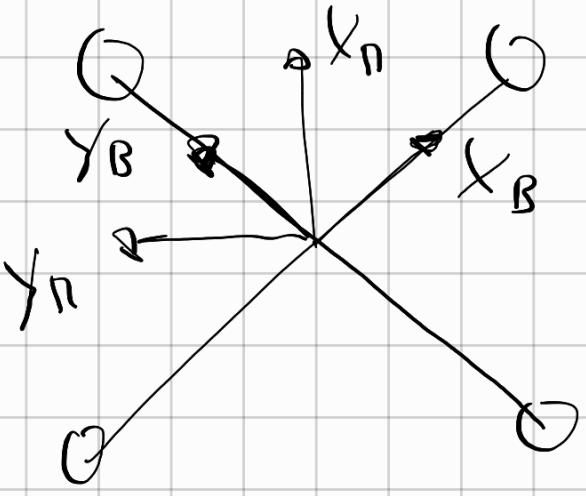
- $\underbrace{\tau}_{\text{input}} = J \ddot{\eta} + C(\underbrace{\eta}_{\text{output}}, \dot{\eta}) \dot{\eta}$

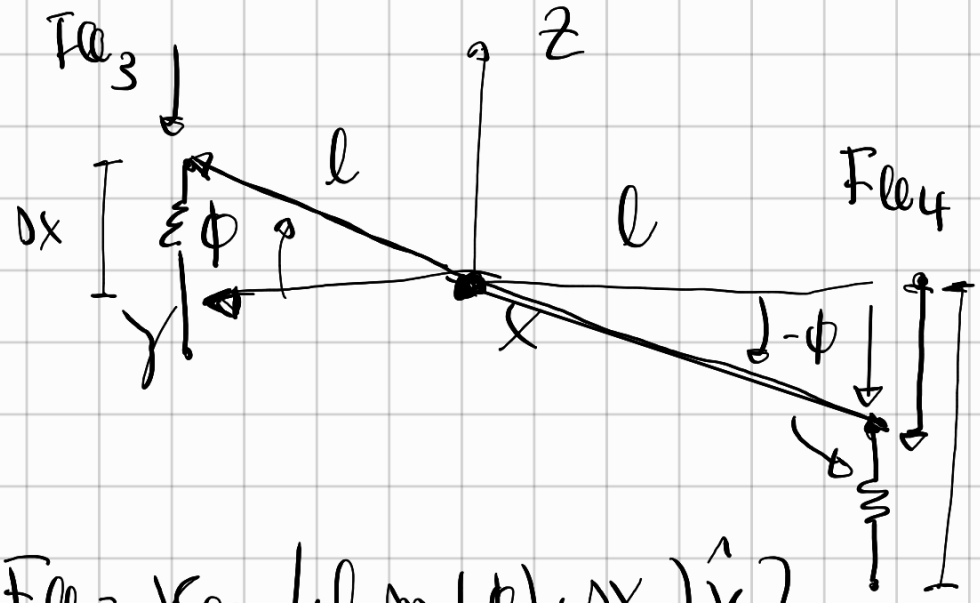
$\hookrightarrow \ddot{\eta} = J^{-1}(\tau - C(\eta, \dot{\eta}) \dot{\eta}) = \begin{bmatrix} \ddot{\varphi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix}$





$$\theta(t) = \theta_n + \theta_0$$





$$\left.\begin{aligned} Fel_3 &= -Kel \cdot \left(+l\sin(\phi) + \Delta X_3\right)\hat{k} \\ Fel_4 &= -Kel\left(-l\sin(\phi) + \Delta X_4\right)\hat{k} \end{aligned}\right\} z$$

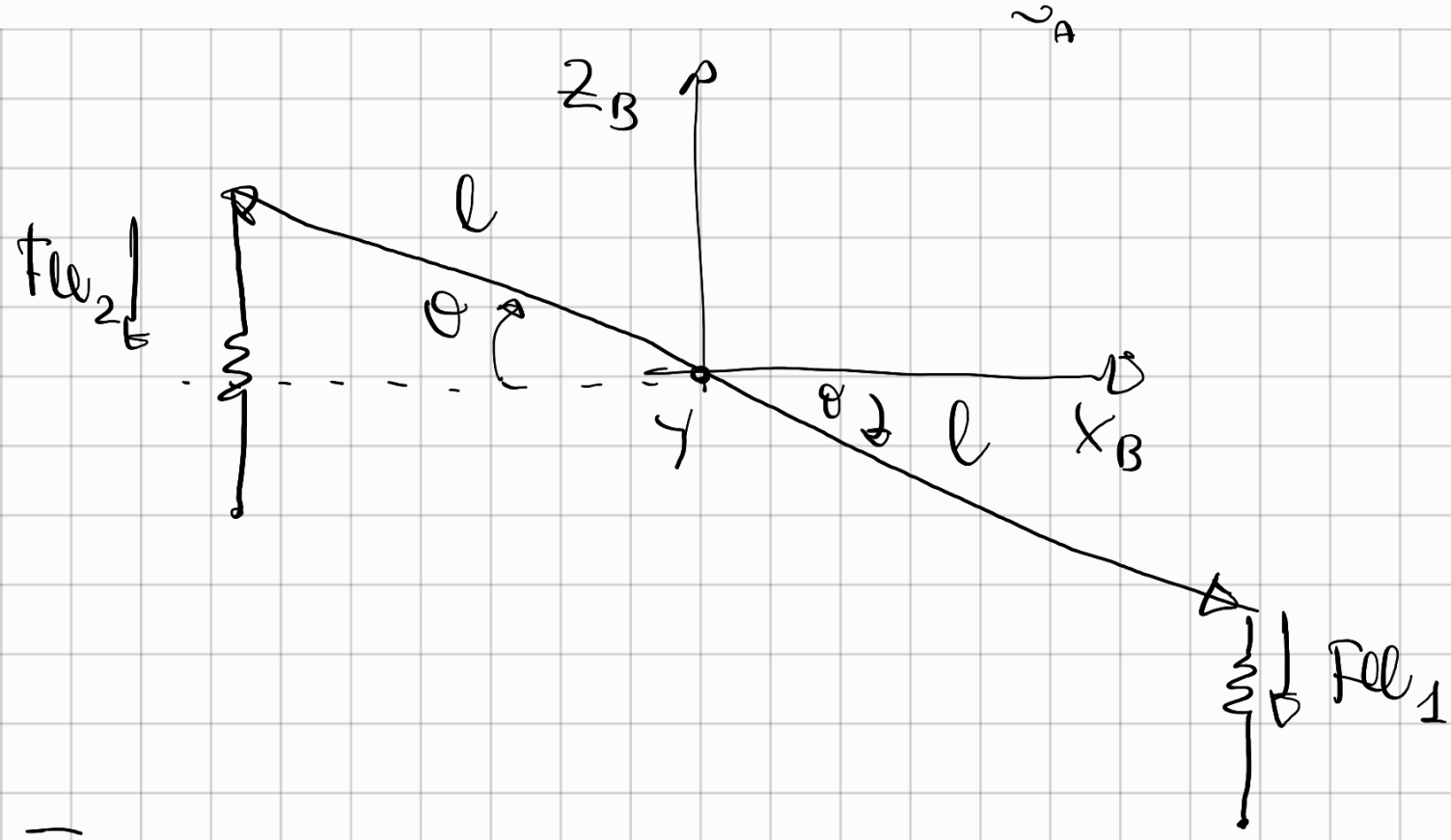
$$Tel_3 = \bar{l} \times Fel_3 = -Kel_3 \cdot l\left(+l\sin(\phi) + \Delta X_3\right) \cdot \sin\left(\frac{\pi}{2} - \phi\right)$$

$$= \begin{bmatrix} 0 \\ l \cdot \cos(\phi) \\ l \cdot \sin(\phi) \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ Fel_3 \end{bmatrix} = \begin{bmatrix} i & j & k \\ 0 & l\cos(\phi) & l\sin(\phi) \\ 0 & 0 & Fel_3 \end{bmatrix}$$

$$= \begin{bmatrix} Fel_3 \cdot l \cdot \cos(\phi) \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -Kel_3\, l\left(l\sin(\phi) + \Delta X_3\right) \cdot \cos(\phi) \\ 0 \\ 0 \end{bmatrix}$$

$$Tel_4 = \bar{l} \times Fel_4 = \begin{bmatrix} 0 \\ -l\cos(\phi) \\ -l\sin(\phi) \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ Fel_4 \end{bmatrix}$$

$$= \begin{bmatrix} Fel_4 l \cdot (-\cos(\phi)) \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} Kel_4 l\left(-l\sin(\phi) + \Delta X_4\right) \cdot \cos(\phi) \\ 0 \\ 0 \end{bmatrix}$$



$$\bar{F}el_2 = -Kel_2\left(l\sin(\theta) + \Delta x_2\right)\hat{k}$$

$$\bar{F}el_1 = -Kel_1\left(-l\sin(\theta) + \Delta x_1\right)\hat{k}$$

$$T_{el_2} = \bar{l} \times F_{el_2} = \begin{bmatrix} -l\cos(\theta) \\ 0 \\ +l\sin(\theta) \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ F_{el_2} \end{bmatrix}$$

$$= \begin{bmatrix} i & j & k \\ -l\cos\theta & 0 & +l\sin\theta \\ 0 & 0 & F_{el_2} \end{bmatrix} = \begin{bmatrix} 0 \\ F_{el_2} \cdot l\cos(\theta) \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -Kel_2\, l\left(l\sin(\theta) + \Delta x_2\right)\cos(\theta) \\ 0 \end{bmatrix}$$

$$\tau_{el_1} = \bar{l} \times F_{el_1} = \begin{bmatrix} l\cos(\theta) \\ 0 \\ -l\sin(\theta) \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ F_{el_1} \end{bmatrix}$$

$$= \begin{bmatrix} i & j & k \\ l\cos\theta & 0 & -l\cos\theta \\ 0 & 0 & F_{el_1} \end{bmatrix} = \begin{bmatrix} 0 \\ -F_{el_1} \cdot l\cos\theta \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ k_{el_1}\, l\left(-l\sin(\theta) + \Delta x_1\right) \cdot \cos(\theta) \\ 0 \end{bmatrix}$$

$$\tau = \tau_M + \tau_K = J \cdot \ddot{\eta} + C(\eta) \cdot \dot{\eta}$$

USO METODO ROCCO (INDUSTRIAL ROBOTS)

→ Era una prova di inverse dynamic control + feedback linearization + PD control con robustness (ma non so se si possa applicare nel nostro caso)

Model is:

$$\tau = J\ddot{\eta} + C(\eta, \dot{\eta})\dot{\eta} \qquad \eta = \begin{bmatrix} \varphi \\ \theta \\ \psi \end{bmatrix}$$

Inverse dynamic control is:

$$\tau = \hat{J}y + \underbrace{\hat{C}(\eta, \dot{\eta})\dot{\eta}}_{\hat{n}(\eta, \dot{\eta})}$$

$$= \hat{J}y + \hat{n}(\eta, \dot{\eta})$$

Comparing we have:

$$J\ddot{\eta} + n(\eta, \dot{\eta}) = \hat{J}y + \hat{n}(\eta, \dot{\eta})$$

$$J\ddot{\eta} = \hat{J}y + \underbrace{\hat{n}(\eta, \dot{\eta}) - n(\eta, \dot{\eta})}_{\tilde{n}(\eta, \dot{\eta})}$$

Since J is invertible we can write:

$$\ddot{\eta} = J^{-1}\hat{J}y + J^{-1}\tilde{n}(\eta, \dot{\eta}) + y - y$$

$$= y + (J^{-1}\hat{J} - I)y + J^{-1}\tilde{n}(\eta, \dot{\eta})$$

We now apply the robust control law:

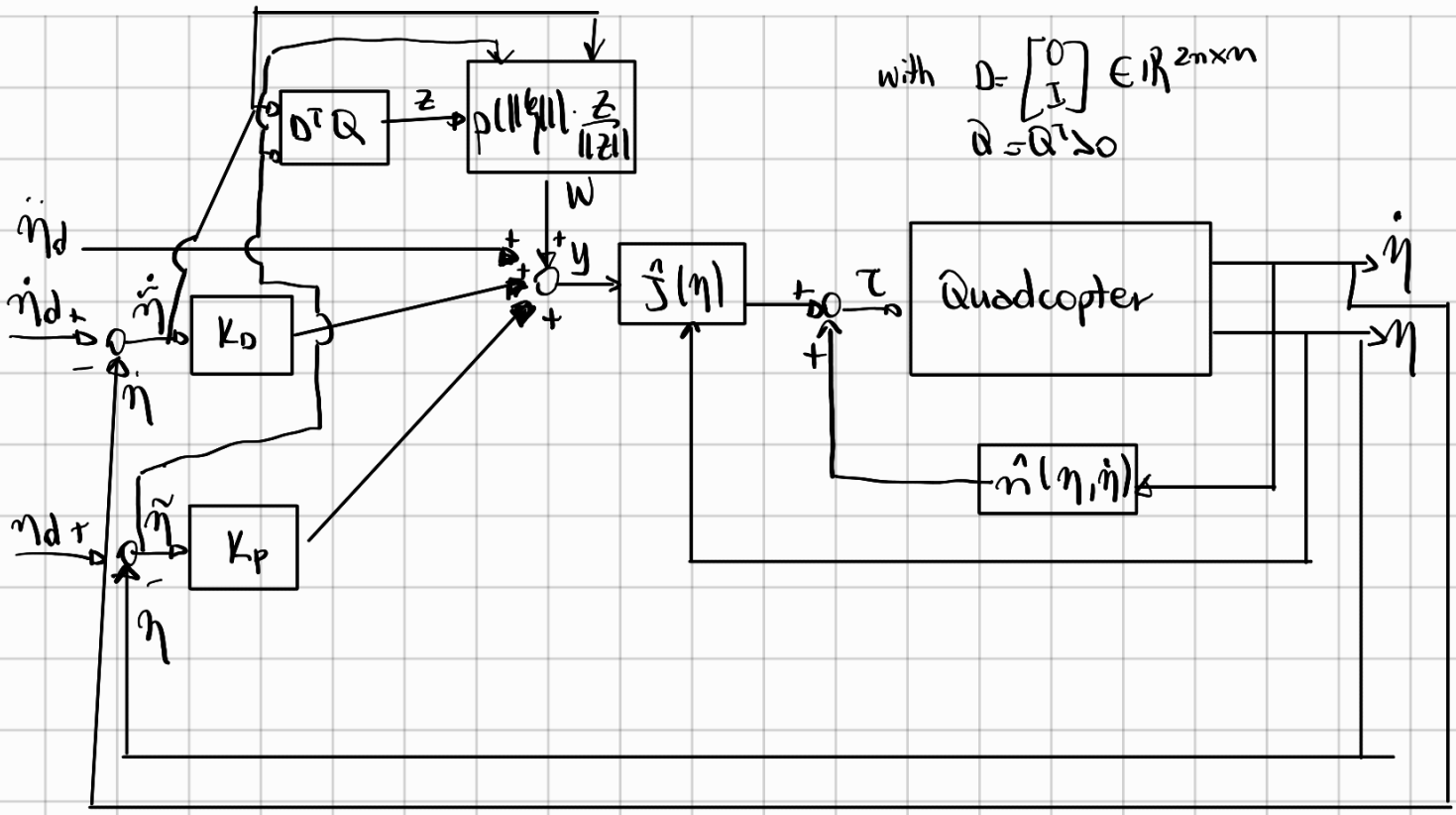
$$y = K_p\tilde{\eta} + K_D\dot{\tilde{\eta}} + \ddot{\eta}_d + w \quad \text{with} \quad w = \rho(\|\xi\|)\frac{z}{\|z\|}$$

where $\xi = \begin{bmatrix} \tilde{\eta} \\ \dot{\tilde{\eta}} \end{bmatrix}$ state of the sys

$\rho(\|\xi\|) > \|\eta\|$

positive function:

$\rho(\|\xi\|) = \beta_0 + \beta_1\|\xi\| + \beta_2\|\xi\|^2$



Nota: Lo stato del nostro sistema è stimato con KF (state observer against noise)

Trovo spazio di stato nella forma: $\dot{x} = a(x) + b(x) \cdot u$

$$\eta = \begin{bmatrix} \varphi \\ \theta \\ \psi \end{bmatrix}$$

$$\tau = J\ddot{\eta} + C(\eta, \dot{\eta})\,\dot{\eta}$$

$$\begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{bmatrix} \begin{bmatrix} \ddot{\varphi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} + \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \begin{bmatrix} \dot{\varphi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

Setto:

$x_1 = \varphi \qquad x_4 = \dot{\varphi}$

$x_2 = \theta \qquad x_5 = \dot{\theta}$

$x_3 = \psi \qquad x_6 = \dot{\psi}$

Esplicitamente:

$$\begin{cases} \tau_1 = J_{11}\ddot{\varphi} + J_{12}\ddot{\theta} + J_{13}\ddot{\psi} + C_{11}\dot{\varphi} + C_{12}\dot{\theta} + C_{13}\dot{\psi} = J_{11}\dot{x}_4 + J_{12}\dot{x}_5 + J_{13}\dot{x}_6 + \overbrace{C_{11}x_4 + C_{12}x_5 + C_{13}x_6}^{A} \\ \tau_2 = J_{21}\ddot{\varphi} + J_{22}\ddot{\theta} + J_{23}\ddot{\psi} + C_{21}\dot{\varphi} + C_{22}\dot{\theta} + C_{23}\dot{\psi} = J_{21}\dot{x}_4 + J_{22}\dot{x}_5 + J_{23}\dot{x}_6 + \overbrace{C_{21}x_4 + C_{22}x_5 + C_{23}x_6}^{B} \\ \tau_3 = J_{31}\ddot{\varphi} + J_{32}\ddot{\theta} + J_{33}\ddot{\psi} + C_{31}\dot{\varphi} + C_{32}\dot{\theta} + C_{33}\dot{\psi} = J_{31}\dot{x}_4 + J_{32}\dot{x}_5 + J_{33}\dot{x}_6 + \underbrace{C_{31}x_4 + C_{32}x_5 + C_{33}x_6}_{C} \end{cases}$$

Nel codice è M (x non confondersi con Condis matrix)

$\dot{x}_1 = x_4$

$\dot{x}_2 = x_5$

$\dot{x}_3 = x_6$

$\left.\begin{array}{l} \dot{x}_4 = \alpha \\ \dot{x}_5 = \beta \\ \dot{x}_6 = \gamma \end{array}\right\}$ Momentaneamente per alleggerire i calcoli

$$\begin{cases} \text{I} & (\tau_1 = J_{11}\alpha + J_{12}\beta + J_{13}\gamma \quad -A) \\ \text{II} & (\tau_2 = J_{21}\alpha + J_{22}\beta + J_{23}\gamma \quad -B) \\ \text{III} & (\tau_3 = J_{31}\alpha + J_{32}\beta + J_{33}\gamma \quad -C) \end{cases} \rightarrow \alpha = \frac{\tau_1 - J_{12}\beta - J_{13}\gamma - A}{J_{11}}$$

• N.B.: Mi ero dimenticata ABC ma basta sottrarli sempre dove ci sono le τ

$$\text{II}\quad \tau_2^{-B} = J_{22}\beta + J_{23}\gamma + \frac{J_{21}}{J_{11}}\left(\tau_1^{-A} - J_{12}\beta - J_{13}\gamma\right)$$

$$\text{III}\quad \tau_3^{-C} = J_{32}\beta + J_{33}\gamma + \frac{J_{31}}{J_{11}}\left(\tau_1^{-A} - J_{12}\beta - J_{13}\gamma\right)$$

$$\hookrightarrow J_{11}(\tau_3^{-C}) = \beta(J_{32}J_{11} - J_{31}J_{12}) + \gamma(J_{33}J_{11} - J_{31}J_{13}) + J_{31}(\tau_1^{-A})$$

$$\text{II}\quad (\tau_2^{-B})J_{11} = \beta(J_{11}J_{22} - J_{21}J_{12}) + \gamma(J_{11}J_{23} - J_{21}J_{13}) + J_{21}(\tau_1^{-A})$$

$$\beta = \frac{(\tau_2^{-B})J_{11} - (\tau_1^{-A})J_{21} - (J_{11}J_{23} - J_{21}J_{13})\gamma}{(J_{11}J_{22} - J_{21}J_{12})} \quad \text{insert in III}$$

$$\text{III}\quad \underbrace{(J_{33}J_{11} - J_{31}J_{13})}_{L}\gamma = (J_{11}(\tau_3^{-C}) - J_{31}(\tau_1^{-A})) - (J_{32}J_{11} - J_{31}J_{12})\left[\frac{(\tau_2^{-B})J_{11} - (\tau_1^{-A})J_{21} - (J_{11}J_{23} - J_{21}J_{13})\gamma}{(J_{11}J_{22} - J_{21}J_{12})}\right]$$

$$= \underbrace{(J_{11}(\tau_3^{-C}) - J_{31}(\tau_1^{-A}))}_{H} - \underbrace{\frac{(J_{32}J_{11} - J_{31}J_{12})}{(J_{11}J_{22} - J_{21}J_{12})}}_{E}\left[\underbrace{(\tau_2^{-B})J_{11} - (\tau_1^{-A})J_{21}}_{D} - \underbrace{\gamma(J_{11}J_{23} - J_{21}J_{13})}_{F}\right]$$

$$L\gamma = H - E[D - \gamma F]$$

$$= H - ED + E\gamma F$$

$$\gamma(L - EF) = H - ED$$

$$\boxed{\gamma = \frac{H - ED}{L - EF}}$$

$$\text{II}\quad (\tau_2^{-B})J_{11} = \beta(J_{11}J_{22} - J_{21}J_{12}) + \gamma(J_{11}J_{23} - J_{21}J_{13}) + J_{21}(\tau_1^{-A})$$

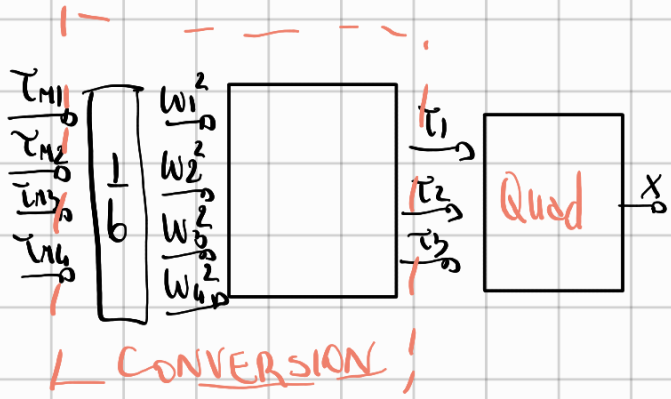
$$\boxed{\beta = \frac{\underbrace{(J_{11}(\tau_2^{-B}) - J_{21}(\tau_1^{-A}))}_{D} - \underbrace{(J_{11}J_{23} - J_{21}J_{13})}_{F}\left(\frac{H - ED}{L - EF}\right)}{J_{11}J_{22} - J_{21}J_{12}}}$$

$$\text{I}\quad \alpha = \frac{(\tau_1^{-A}) - J_{12}\beta - J_{13}\gamma}{J_{11}}$$

$$\begin{cases} w_1^2 = \dfrac{T}{4K} - \dfrac{\tau_2}{2Kl} - \dfrac{\tau_3}{4b} \\ w_2^2 = \dfrac{T}{4K} - \dfrac{\tau_1}{2Kl} - \dfrac{\tau_3}{4b} \\ w_3^2 = \dfrac{T}{4K} + \dfrac{\tau_2}{2Kl} - \dfrac{\tau_3}{4b} \\ w_4^2 = \dfrac{T}{4K} + \dfrac{\tau_1}{2Kl} - \dfrac{\tau_3}{4b} \end{cases}$$

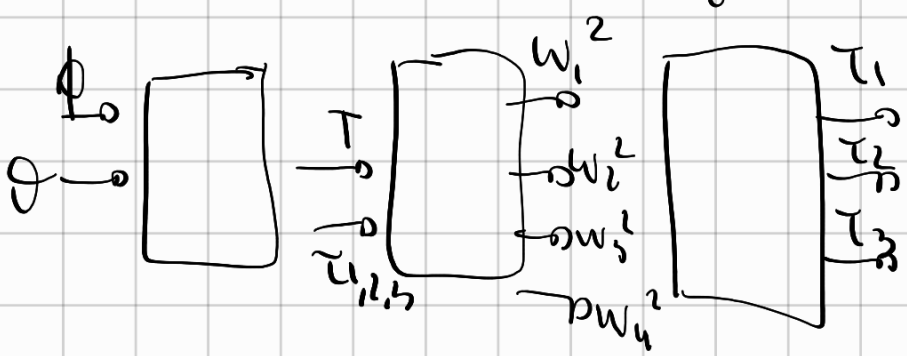
with $T = \sum_{i=1}^{4} f_i$

$$\eta = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix}$$



$\tau_{Mi} = b\, w_i^2 \quad \forall i$

$w_1^2 = \dfrac{\tau_{M1}}{b}$



$T = K \sum w_i^2$

$T = \dfrac{mg}{c_\theta c_\phi}$

$\tau_\phi = PD$

$\tau_\theta = PD$

$\tau_\psi = PD$

$\tau_\phi = l \cdot K \left(-w_2^2 + w_4^2\right)$

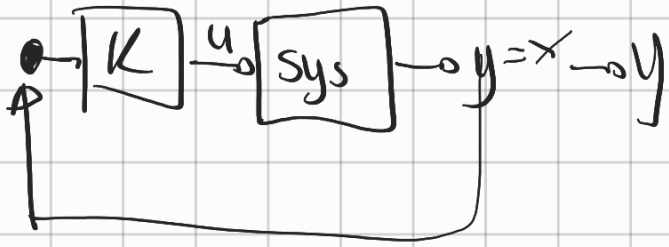
$T = K \sum_{i=1}^{4} w_i^2$

$w_1^2 = \dots \propto T, \tau_\phi, \tau_\psi, \tau_\theta$

$w_2^2 =$

$w_3^2 =$

H_2 control



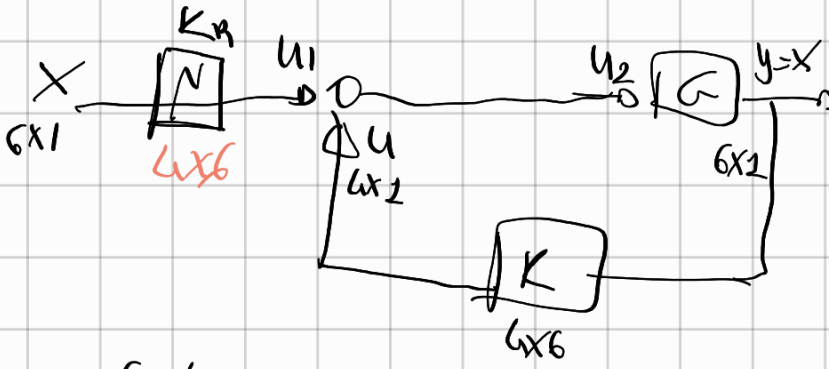
$$\begin{cases} \dot{x} = Ax + Bu \\ u = Kx \end{cases}$$

$$\dot{x} = Ax + BKx$$
$$\dot{x} = (A+BK)x$$

$$\underset{4\times1}{u} = \underset{4\times6}{K} \times \underset{6\times1}{x}$$

$$\begin{bmatrix} \frac{1}{N} & \frac{1}{N} & \frac{1}{N} \end{bmatrix}$$

T



$$DC\ GAIN = \frac{y}{u} =$$

$$\underset{6\times1}{y} = \underset{6\times4}{N}\ \underset{4\times1}{u}$$

$N = 6\times4$

K_R

$\cancel{4\times6}\ \cancel{6}\times1 = 4\times1$

$$\underset{6\times4}{K_{DC}} \cdot \underset{\boxed{4\times6}}{K_R} = \underset{6\times6}{I}$$

$2\times1 \quad 1\times2 = 2\times2$

$$\underset{A}{\begin{bmatrix} a \\ b \end{bmatrix}} \underset{B}{\begin{bmatrix} c & d \end{bmatrix}} = \underset{C}{\begin{bmatrix} ac & ad \\ bc & bd \end{bmatrix}}$$

$$\frac{1}{C(A-BK)^{-1}B}$$

$[6\times6] - [(6\times4)(4\times6)]$

$(6\times6)[6\times6](6\times4)$

$\boxed{6\times4}$

$A \cdot B = C$

$\underset{1\times2}{B} = \underset{1\times2}{A^{-1}} \cdot \underset{2\times2}{C}$ [

$$K_p\Big(\underbrace{1}_{D}+\underbrace{\frac{1}{sT_I}}_{I}\Big)e = K\left(\frac{sT_I+1}{sT_I}\right).e$$

$$K_p e + \frac{K_p e}{sT_I} = K_p e + \frac{K_I e}{s}$$

$$\tau_1 = \underbrace{K_D e + K_p \cdot e}_{A} + \frac{K_p e}{s}^{B}$$

$$s\tau_1 = As + B$$

$p = 6$

$m = 4$

POLE PLACEMENT

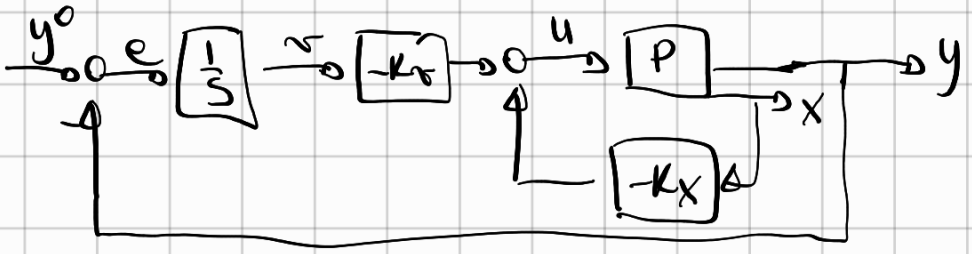
$$\begin{bmatrix}\dot{x}\\ \dot{v}\end{bmatrix} = \begin{bmatrix}A & 0\\ -C & 0\end{bmatrix}\begin{matrix}6\\ 6\end{matrix}\left\{\begin{bmatrix}\overbrace{6\times 6}^{6} & \overbrace{6\times 6}^{6}\\ 6\times 6 & 6\times 6\end{bmatrix}\right\}12\begin{bmatrix}x\\ v\end{bmatrix}$$

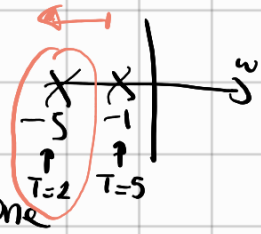
$$\begin{bmatrix}B\\ 0\end{bmatrix}\begin{bmatrix}6\times 4\\ 6\times 4\end{bmatrix}\Big\}12$$

$$u = -K\overbrace{\begin{bmatrix}x\\ v\end{bmatrix}}^{1}\begin{matrix}\}6\\ \}6\end{matrix}$$

We enlarge the system

$$\begin{cases} \dot{x} = Ax + Bu \\ \dot{v} = \underbrace{-Cx}_{y} + y^o \\ \vdots \\ \rightarrow v = \frac{1}{s} \cdot e \end{cases} \qquad x_e = \begin{bmatrix} x \\ v \end{bmatrix} \qquad \begin{cases} \dot{x}_e = \underbrace{\begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix}}_{A_e} x_e + \underbrace{\begin{bmatrix} B \\ 0 \end{bmatrix}}_{B_e} u + \begin{bmatrix} 0 \\ I \end{bmatrix} y^o \\ u = -Kx_e = -[K_x \;\; K_v] \begin{bmatrix} x \\ v \end{bmatrix} \end{cases}$$





<u>C.N.S</u> (A,B) reachable

<u>obs.</u> close loop sys must be slower than the open loop one

Nel nostro caso:

$$\dot{x} = Ax + Bu = \underbrace{\begin{bmatrix} 6\times 6 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_6 \end{bmatrix} + \underbrace{\begin{bmatrix} 6\times 4 \end{bmatrix}}_{B} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}$$

$$\vdots$$

$$6 \left\{ \begin{bmatrix} b_1 & b_2 & b_3 & b_4 \end{bmatrix} \right.$$

$$\underset{4\times 1}{u} = -\underset{4\times 6}{K}\,\underset{6\times 1}{x} = -\underbrace{\begin{bmatrix} K_1 & K_2 & K_3 & K_4 & K_5 & K_6 \end{bmatrix}}_{4} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}$$

Bisogna controllare reachability di (A,b_1), (A,b_2), (A,b_3), (A,b_4).
Se sono tutti reachable setto $u_1 = -K_1 x$ e $u_2, u_3, u_4 = 0$

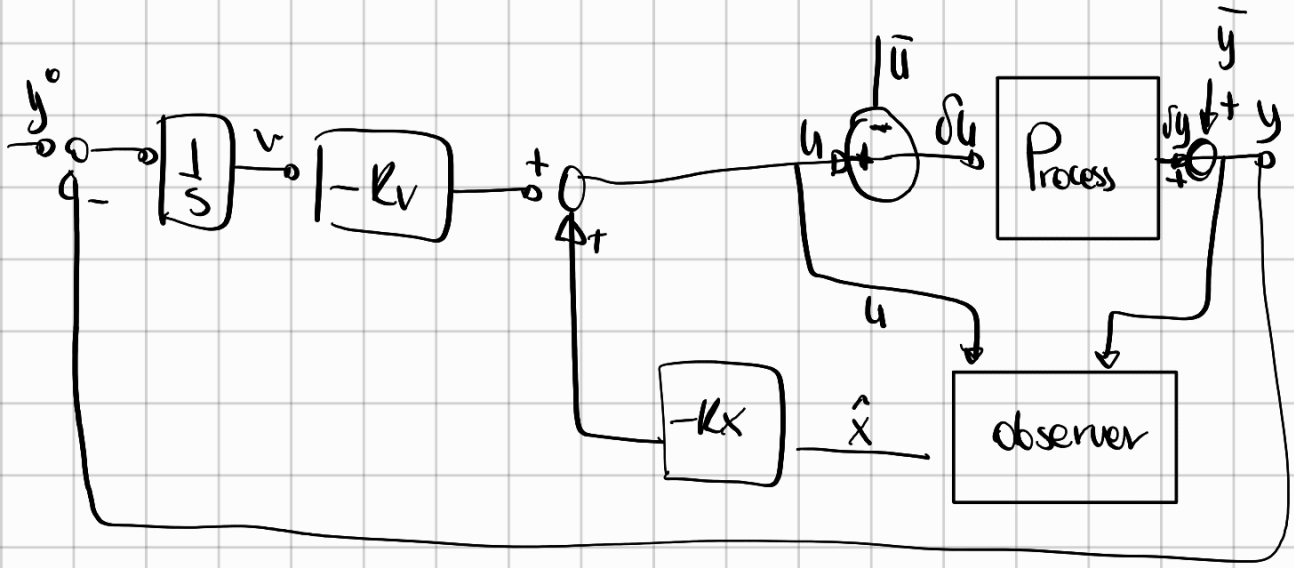
$$\Rightarrow K = [K_1 \;\; 0 \; 0 \; 0 \; 0 \; 0]$$

Observer

Ip (A,C) osservabile

→ use place with $(A \to A', B \to C')$

$L' = \text{place}(A', C', [\underbrace{\text{new poles}}_{\text{+ veloci di } (A-BK)\ (+ \to sx)}])$ → $\dot{e} = (A - LC) \cdot e$



$\delta u = u - \bar{u}$

$\delta y = y - \bar{y}$

↳ Sistema Linearizzato

PD control

$$J(q)\ddot{q} + C(q,\dot{q})\cdot\dot{q} = \tau \qquad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}$$

$$\dot{x}_1 = x_2$$

$$J(x_1)\dot{x}_2 + C(x_1,x_2)\cdot x_2 = \tau \qquad \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = J^{-1}(x_1)\left[\tau - C(x_1,x_2)x_2\right] \end{cases}$$

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = v \end{cases} \qquad \underline{\underline{\tau = J(x_1)\cdot v + C(x_1,x_2)\cdot x_2}}$$

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ & 0 & & & 0 & \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} I & 0 \end{bmatrix} \qquad D = 0$$

$$\Phi \downarrow \qquad \text{GYRO}$$

$$\underline{\dot{\Phi} = T\cdot\omega}$$

$$\begin{cases} \dot{x} = Ax + Bv \\ y = C\cdot x \end{cases} \qquad \text{POLE}(A,B)$$