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External Acceleration Elimination for Complementary Attitude Filter*

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***Abstract* - Generalized complementary filter (GCF) is an attractive solution for data fusion in attitude and heading reference system (AHRS) and inertial measurement unit (IMU), mainly because its computational efficiency. However, since the accelerometer in AHRS and IMU is sensitive to both the gravity and external acceleration, the performance of GCF will be degraded under dynamic circumstances, just as other types of attitude estimators. This paper introduce a novel approach to eliminate the external acceleration, which is computationally efficient and flexible. Numerical simulation and experiment results are presented to illustrate the effectiveness of the proposed algorithm. Furthermore, the proposed method can cooperate with not only GCF, but various attitude estimators, such as extended Kalman filter (EKF) and unscented Kalman filter (UKF).**

Index Terms - Attitude estimation, complementary filter, vector observation, external acceleration

I. INTRODUCTION

Attitude estimator is extensively used in various fields, including advanced robot, unmanned aerial vehicle (UAV), virtual reality (VR), etc. It can provide orientation information by fusing raw data from accelerometer, magnetometer, and gyroscope. Among the massive solutions for attitude estimation in the existing literature, Kalman filter is considered to be the standard framework [1]. Its most prominent variant is the extended Kalman filter (EKF). EKF is frequently encountered in attitude and heading reference systems (AHRS) and inertial measurement units (IMU) due to its lower computational burden, although its numerical accuracy is inferior to some other algorithms, such as unscented Kalman filter (UKF) and particle filter (PF).

Complementary filter (CF) is another workhorse for attitude estimation [2]-[7]. Compared with EKF and other variants of Kalman filter, CF does not involve matrix multiplications or inversions, and thus it has much higher computational efficiency. In the case of limited hardware resources, e.g. applications on micro control unit (MCU), CF is particularly attractive.

Despite different frameworks for attitude estimation, most of them are based on observations of three vectors. The gravity vector (denoted as $\boldsymbol{g}$) measured by a tri-axial accelerometer is always vertical downward, while the geomagnetic field (denoted as $\boldsymbol{h}$) measured by 3-axis magnetometer can indicate the magnetic north. Besides, three-dimensional angular velocity $\boldsymbol{\omega}$ provided by gyroscope can be integrated to obtain the change of orientation.

However, the accelerometer is sensitive to both the gravitational and motional acceleration. In another word, the measurement provided by accelerometer is actually the sum of gravity and motional acceleration (also referred to as external acceleration) of the carrier that the sensor is attached to. As a result, the accuracy of attitude estimator will be affected under dynamic conditions.

Approaches to address the above problem can be classified into two categories. The first is threshold-based, which judges the existence of external accelerometer according to one or more thresholds, and makes adjustments to the attitude estimator. This approach is also referred to as switching filter [8] [9]. The other way is to estimate and compensate the external acceleration, which is called model-based method [10] [11]. Both the above methods have been applied to EKF. But in the research of CF, the problem of external acceleration is seldom considered.

In subsequent sections, after a brief review of previous research on complementary filter and elimination of external acceleration, we will introduce a novel approach to enhance the performance of CF under dynamic circumstances, and illustrate its effectiveness by simulation and experiment results.

II. OVERVIEW OF PREVIOUS WORK

A. Complementary Filter

The principle of classical complementary filter is to fuse data from different sensors according to their frequency characteristics, i.e. data fusion is implemented by CF in frequency domain.

The fundamentals of CF are straight forward, but W. T. Higgins pointed out that CF is inherently related to fixed-gain Kalman filter [12].

Early studies of complementary attitude filter were confined to the architecture presented in Fig.1. Two or more estimations of attitude are made according to different sensors' data, and then merged in frequency domain. R. Mahony suggested a modified CF for attitude estimation [2], which is

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named generalized complementary filter (GCF) by K. J. Jensen later [3].

The typical transfer function of GCF can be described as

$$\hat{x} = \frac{H(s)\cdot x_A + s\cdot x_B}{H(s)+s}. \tag{1}$$

In (1), different forms of $H(s)$ can be chosen. If we set $H(s)$ to a proportional coefficient k_P, a first-order GCF is acquired. Meanwhile, second-order GCF can be implemented with the transfer function in (2), which has been adopted in [2].

$$H(s) = k_p + \frac{k_I}{s} \tag{2}$$

It is also possible to use higher order forms of $H(s)$, as presented in [13]. It should be noticed that different representations of attitude will lead to different forms of GCF as well. For instance, direction cosine matrix (DCM) was used in [2] and [3] to represent the attitude, while attitude quaternion was adopted in [5] and [6]. A better choice is to use the gravity and geomagnetic vectors, i.e. the vector-based (or sensor-based) filter. More details about the vector-based attitude estimation can be found in [13]-[15].

B. Elimination of External Acceleration

As mentioned in the above section, there are two approaches to eliminate the external acceleration, i.e. the threshold-based and model-based methods.

In a threshold-based attitude estimator, the external acceleration is detected by one or more criteria, such as the norm of gravity vector (denoted as $|\boldsymbol{g}|$). Once the norm $|\boldsymbol{g}|$ differs from 9.8m/s^2 and the difference exceeds a pre-defined threshold ε, it implies that the external acceleration arises. In that case, the output of accelerometer is no longer a reliable measurement of gravity vector $\boldsymbol{g}$, and its weighting factor should be reduced. In EKF, we can enlarge the measurement noise covariance, or even set the covariance to infinity [8] [9], in order to discard the output of accelerometer.

On the other hand, the model-based method attempts to dissociate the external acceleration from the accelerometer's output, instead of discarding it. This method is named after the frequently used Gauss-Markov (GM) model [10] [11], as shown in (3).

$$\boldsymbol{a}_k = c_a \boldsymbol{a}_{k-1} + \boldsymbol{w}_a \tag{3}$$

In (3), $\boldsymbol{a}_k$ denotes the external acceleration at the kth moment, and c_a is a fading factor that satisfied $0<c_a<1$, while $\boldsymbol{w}_a$ is zero-mean Gaussian white noise.

Model-based attitude estimators can be further divided into two different types. If the external acceleration is included in the state vector and is estimated along with the attitude, it can be referred to as an intrinsic estimator [11]. On the other hand, if the external acceleration is computed outside the attitude estimator, it forms an extrinsic estimator [10].

The threshold-based method relies on pre-set criteria and thresholds, but complicated dynamic conditions may give rise to erroneous judgments. The intrinsic estimator brings in supplementary dimensions of the state vector, and thus the computational burden will considerably increase. The extrinsic estimator can estimate and compensate external acceleration without changing or modifying the architecture of attitude estimator, hence it is more flexible and convenient.

It is noteworthy that the studies of external acceleration and its elimination are generally based on Kalman filter. Among literature on GCF, the above methods were rarely applied, partially because the architecture and implementation of GCF is quite different from that of Kalman filter. In the next section, we will consider the estimation and compensation of external acceleration in GCF.

III. Methodology

A. Basic Design of GCF

We use the vector-based GCF proposed in [15], which can be described by (4) and (5):

$$\boldsymbol{\omega}_{err} = k_g\left(\boldsymbol{g}^* \times \hat{\boldsymbol{g}}\right) + k_h\left(\boldsymbol{h}^* \times \hat{\boldsymbol{h}}\right) \tag{4}$$

$$\begin{cases} s\hat{\boldsymbol{g}} = \hat{\boldsymbol{g}} \times \left[\boldsymbol{\omega}^* + H(s)\boldsymbol{\omega}_{err}\right] \\ s\hat{\boldsymbol{h}} = \hat{\boldsymbol{h}} \times \left[\boldsymbol{\omega}^* + H(s)\boldsymbol{\omega}_{err}\right] \end{cases} \tag{5}$$

In (4), $\boldsymbol{g}^*$ and $\boldsymbol{h}^*$ denote the measurements of $\boldsymbol{g}$ and $\boldsymbol{h}$, respectively, while $\hat{\boldsymbol{g}}$ and $\hat{\boldsymbol{h}}$ are the corresponding estimations. Besides, k_g and k_h are weighting factors for $\boldsymbol{g}$ and $\boldsymbol{h}$. In (5), $\boldsymbol{\omega}^*$ is the measurement of angular velocity $\boldsymbol{\omega}$. Figure 1 shows the architecture of the proposed GCF.

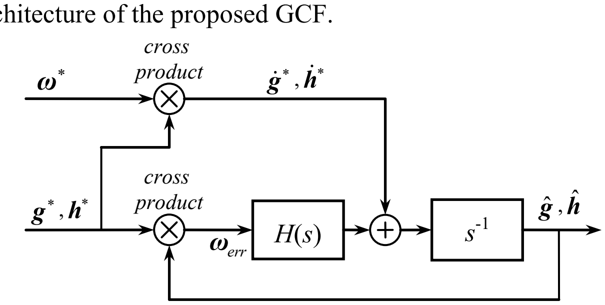

Fig. 1 Vector-based generalized complementary filter.

B. Estimation and Compensation of External Acceleration

We use the extrinsic estimator to estimate and compensate external acceleration in GCF, due to its flexibility stated in Section II.

It has been proved in previous studies that the GCF is asymptotically stable [2] [3] [13]. Thus we can assume that the above mentioned GCF is able to provide reliable estimation of $\boldsymbol{g}$ and $\boldsymbol{h}$. Let $\boldsymbol{f}_k$ denotes the raw output of accelerometer (i.e. the specific force) at the kth moment, we can estimate the external acceleration $\boldsymbol{a}_k$ by (6):

$$\hat{\boldsymbol{a}}_k = \boldsymbol{f}_k - \hat{\boldsymbol{g}}_k. \tag{6}$$

Next, at the $(k+1)$th moment, since the different between $\boldsymbol{a}_k$ and $\boldsymbol{a}_{k-1}$ is boundless, we can use the estimation of $\boldsymbol{a}_k$ to modify the accelerometer's output:

$$\boldsymbol{g}_{k+1}^* = \boldsymbol{f}_{k+1} - \hat{\boldsymbol{a}}_k. \tag{7}$$

Equation (7) gives a corrected measurement of $\boldsymbol{g}$, which can be input into the GCF. Repeatedly using (6) and (7) at each time step, continual estimation and compensation of external acceleration can be implemented, as illustrated in Fig. 2.

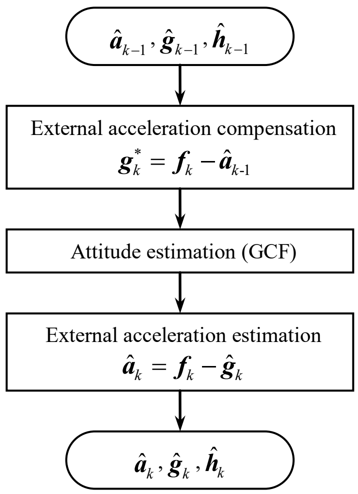

Fig. 2 Flowchart of external acceleration's estimation and compensation.

It can be seen from Fig. 2 that the extrinsic estimation of external acceleration is entirely outside GCF. Hence it can be applied to not only GCF, but all categories of attitude estimators (e.g. EKF, UKF, PF, etc).

C. Discussions and Remarks

Since GCF has been proved to be stable, if the external acceleration $\{\boldsymbol{a}_k\}$ can be treated as a zero-mean random process, it can be handled by GCF itself. Otherwise, the above proposed algorithm can extract the non-zero-mean component of $\{\boldsymbol{a}_k\}$ and accommodate it in the estimation $\hat{\boldsymbol{a}}_k$. This is the essence of the proposed algorithm.

Actually, the estimation $\hat{\boldsymbol{a}}_k$ in (6) is *a posteriori*, while in (7) it is used as *a priori* estimation of $\boldsymbol{a}_{k+1}$. In other words, we essentially assume that

$$\hat{\boldsymbol{a}}_{k+1}^{-} = \hat{\boldsymbol{a}}_k^{+}. \tag{8}$$

In (8), the superscript '+' indicates *a posteriori* estimation, while the superscript '−' stands for *a priori* estimation. The assumption described by (8) causes that the difference between $\boldsymbol{a}_{k+1}$ and $\boldsymbol{a}_k$ transfers into the measurement noise of accelerometer. Then it will be eliminated along with the measurement noise.

We can further deduce that higher stability of the attitude estimator will lead to better performance under dynamic conditions. This deduction will be illustrated by simulations in the following section.

Nevertheless, if certain error of $\hat{\boldsymbol{a}}_k$ has already arisen, it cannot be corrected by GCF, since the extrinsic estimator is completely outside (and independent of) GCF. Worse yet, this error will be retained in $\{\boldsymbol{a}_k\}$, and continuously affects attitude estimation.

The fading factor c_a in (3) is actually used for solving this problem. According to (3), the assumption in (8) can be modified into

$$\hat{\boldsymbol{a}}_k^{-} = c_a \hat{\boldsymbol{a}}_{k-1}^{+}. \tag{9}$$

Since $0<c_a<1$, the error in $\{\boldsymbol{a}_k\}$ will gradually decrease. In a word, the fading factor c_a is used to attenuate the error of $\hat{\boldsymbol{a}}_k$, rather than $\hat{\boldsymbol{a}}_k$ itself. Smaller c_a leads to faster decline, but also results in poorer compensation of external acceleration. We will see the impacts of c_a via the experiment in Section V.

IV. Simulations

A. Basic Settings

Assuming that the gravity vector is $\boldsymbol{g}_0=(0\ \ 0\ \ 9.8)^{\mathrm{T}}$ m/s^2 in north-east-down (NED) reference frame, and the geomagnetic vector is $\boldsymbol{h}_0=(40\ \ 0\ \ 30)^{\mathrm{T}}$ μT (micro-Tesla). The carrier of the sensor triads moves along *x*-axis of its front-right-down body frame, while its orientation remains unchanged (i.e. the angular velocity keeps zero). The external acceleration on *x*-axis during the movement is shown in Fig. 3.

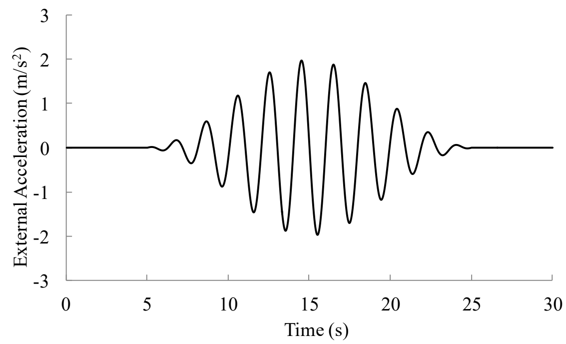

Fig. 3 External acceleration along *x*-axis in simulation.

The measurement noise is set to Gaussian white noise. The accelerometer's noise is $\boldsymbol{\sigma}_a \sim N(0,\ 0.01\text{m/s}^2)$, the magnetometer's noise is $\boldsymbol{\sigma}_m \sim N(0,\ 0.1\mu\text{T})$, and the gyro's noise is $\boldsymbol{\sigma}_\omega \sim N(0,\ 0.05°/\text{s})$. Moreover, the gyro bias is a first-order random walk driven by the noise $\boldsymbol{\sigma}_b \sim N(0,\ 0.05°/\text{s})$. The sampling rate is 50Hz for all sensors.

The transfer function in (2) is adopted for GCF, with k_P=0.5, k_I=0.1, k_g=0.5, and k_h=0.5.

B. Simulation Result of Proposed Algorithm

Figure 4 shows the simulation result of GCF without compensation of external acceleration. It can be seen that the pitch angle has significant error due to external acceleration on *x*-axis.

The simulation result of GCF with proposed algorithm (c_a=1) is shown in Fig. 5. The pitch error in Fig. 5 is greatly reduced compared to Fig. 4. Therefore, the proposed method is effective for eliminating external acceleration.

C. Simulation Results of Other Attitude Estimators

As stated in the last section, the proposed algorithm for external acceleration elimination can cooperate with different types of attitude estimators. For instance, simulation results of EKF and UKF are listed in Table I. All the settings are the same as those for GCF.

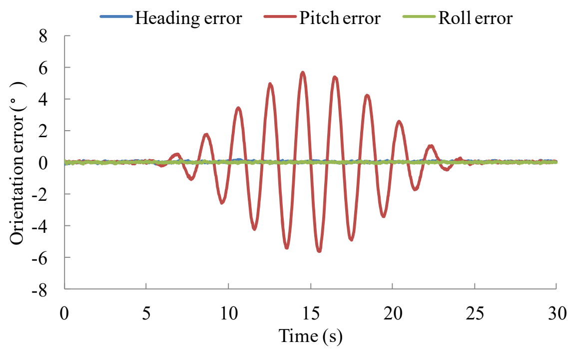

Fig. 4 Simulation result of GCF without compensation of external acceleration.

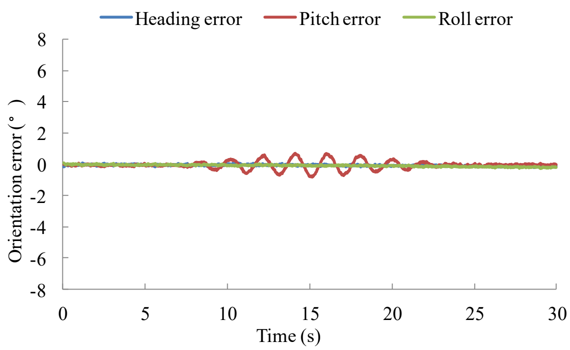

Fig. 5 Simulation result of GCF with compensation of external acceleration.

TABLE I
SIMULATION RESULTS OF DIFFERENT ALGORITHM

Attitude estimation algorithm		Pitch error (root-mean-square error)
GCF	External acceleration uncompensated	2.036°
	External acceleration compensated	0.252°
EKF	External acceleration uncompensated	1.225°
	External acceleration compensated	0.123°
UKF	External acceleration uncompensated	1.178°
	External acceleration compensated	0.113°

As illustrated in Table I, the proposed method is effective for different attitude estimators. Furthermore, the sequence of three different attitude estimators with respect to pitch error is GCF>EKF>UKF, either with or without compensation of external acceleration. That is to say, more accurate attitude estimator can better cooperate with the proposed method under dynamic conditions.

V. EXPERIMENTS

The proposed method along with GCF are applied to a 9-DOF (degrees of freedom) AHRS module MPU9250, which incorporates 3-axis accelerometer, magnetometer, and gyroscope in a single package.

We use a rate table that can rotate around vertical axis. The angular rate generated by the rate table can be up to ±300°/s, and its resolution is 0.0001°/s. The MPU9250 is horizontally mounted on the rate table, with its *x*-and *y*-axis pointed to the tangential and centripetal directions, respectively. The distance between MPU9250 and the rotation axis is approximately 0.5m.

In the experiment, the angular rate increases from zero to 90°/s in 3s (i.e. the angular acceleration is 30°/s^2), then keeps for 40s, and return to zero in 3s. The corresponding tangential and centripetal acceleration endured by MPU9250 is shown in Fig. 6. Orientation errors without and with compensation of external acceleration are shown in Fig. 7 and 8, respectively.

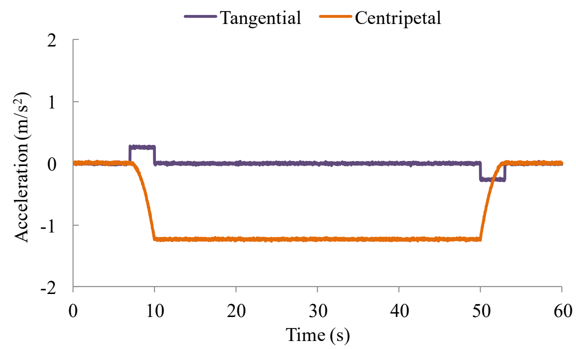

Fig. 6 Tangential and centripetal acceleration in experiment.

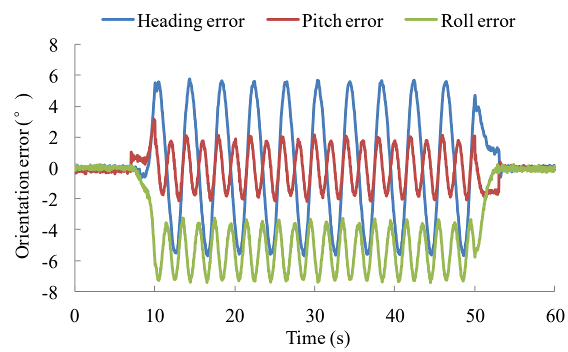

Fig. 7 Experiment result without compensation of external acceleration.

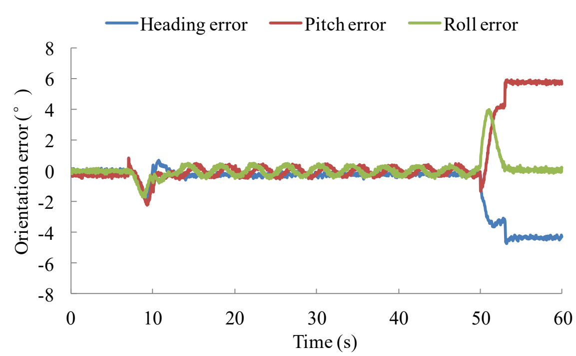

Fig. 8 Experiment result with compensation of external acceleration.

In Fig. 7, significant orientation errors arise due to the external acceleration. On the other hand, it can be seen in Fig. 8 that the impact of external acceleration has been substantially eliminated, but certain errors arise when the angular rate returns to zero. To address this issue, we introduce the fading factor c_a and set c_a=0.995. The corresponding results are shown in Fig. 9.

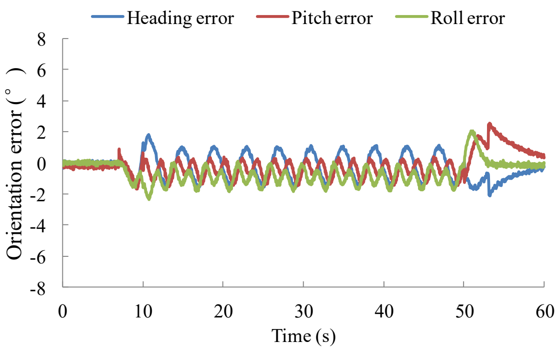

Fig. 9 Experiment result with c_a=0.995.

As shown in Fig. 9, the fading factor c_a can make the orientation error converge. Nonetheless, we can also see that the orientation errors throughout rotation slightly increase. In one sense, the fading factor is double-edged. Thus it may be necessary to adjust the fading factor c_a experimentally under different circumstances.

VI. Conclusions

An extrinsic estimator of external acceleration is introduced in this paper, which can help improve dynamic performance of generalized complementary filter. Moreover, the proposed algorithm can cooperate with various attitude estimators, and it is computationally efficient. Simulation and experiment results illustrate the effectiveness of the proposed algorithm.

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