ORDINARY LEAST SQUARE METHOD

** Cost function = It is refers to entire error. that means total errors. It is denoted with 'J'.

$$\left[J = \sum_{i=1}^{n} \left(y_{actual\,i} - y_{predicted\,i} \right)^2 \right]$$

$y_{actual\,i}$ = Actual input

$y_{prediction\,i}$ = Prediction value.

⇒ Differentiation of J makes equals to zero, partial differentiate of J with respect to b_0 and b_1.

⇒ There are two cases ⇒

(1) Case I = $\dfrac{\partial J}{\partial b_0} = 0$

(2) Case II = $\dfrac{\partial J}{\partial b_1} = 0$

$$\left\{ \begin{array}{l} B_0 = b_0 \\ B_1 = b_1 \\ y_{ai} = y_{actual\,i} \end{array} \right\}$$

** Case I -

$$\Rightarrow \frac{\partial J}{\partial \beta_0} = \frac{\partial \sum_{i=1}^{n} \left(y_{actual\,i} - \beta_0 - \beta_1 x_i \right)^2}{\partial \beta_0}$$

$$\Rightarrow 2 \sum_{i=1}^{n} \left(y_{actual\,i} - \beta_0 - \beta_1 x_i \right) \times \frac{\partial}{\partial \beta_0} \left(y_{actual\,i} - \beta_0 - \beta_1 x_i \right)$$

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$$\Rightarrow 2\times\left(\frac{1}{n}\right)\sum_{i=1}^{n}\left(y_{actual\,i}-\beta_0-\beta_1 x_i\right)\times(0-1-0)$$

$$\Rightarrow -2\times\left(\frac{1}{n}\right)\sum_{i=1}^{n}\left(y_{actual\,i}-\beta_0-\beta_1 x_i\right)$$

$$\Rightarrow -2\times\left(\frac{1}{n}\sum_{i=1}^{n} y_{ai}-\frac{1}{n}\sum_{i=1}^{n}\beta_0-\frac{1}{n}\sum_{i=1}^{n}\beta_1 x_i\right)$$

$$\Rightarrow -2\times\left(\frac{1}{n}\sum_{i=1}^{n} y_{ai}-\frac{1}{n}\beta_0\sum_{i=1}^{n}1-\frac{1}{n}\beta_1\sum_{i=1}^{n}x_i\right) \quad ---\ \text{eq. 1}$$

$$\Rightarrow \frac{1}{n}\sum_{i=1}^{n} y_{ai}=\frac{y_{a1}+y_{a2}+y_{a3}----+y_{an}}{n}=\bar{y}_a$$

$$\Rightarrow \frac{1}{n}\sum_{i=1}^{n}1=\frac{1+1+1+------n}{n}=n/n=1$$

** Substitute this 3 points in above eq. 1 =

$$\Rightarrow -2\times\left(\bar{y}_a-\beta_0-\beta_1\bar{x}\right) \quad ----\ \text{eq. 2}$$

Now equating eq. 2 with equal to zero =

$$\Rightarrow \frac{\partial J}{\partial \beta_0}=0$$

$$\Rightarrow \frac{\partial J}{\partial \beta_0}=-2\times\left(\bar{y}_a-\beta_0-\beta_1\bar{x}\right)=0$$

$$\Rightarrow \bar{y}_a-\beta_0-\beta_1\bar{x}=0$$

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* $\boxed{\beta_0 = \bar{y}_a - \beta_1 \bar{x}}$

* This β_0 is called as bias formula; here -

  $\bar{y}_a$ = actual output

  β_0 and β_1 = constants.

  $\bar{x}$ = coefficient value

* Now we want complete about equation in order to find β_0 value completely we need to find β_1.

* from case-II we have to find that as shown below:

$$\Rightarrow \underline{\text{Case-I}}: \frac{\partial J}{\partial \beta_1} = \frac{\partial \sum_{i=1}^{m} (y_{actual\,i} - \beta_0 - \beta_1 x_i)^2}{\partial \beta_1}$$

$$\Rightarrow 2 \sum_{i=1}^{m} (y_{actual_i} - \beta_0 - \beta_1 x_i) \times \frac{\partial}{\partial \beta_1} (y_{actual_i} - \beta_0 - \beta_1 x_i)$$

$$\Rightarrow 2 \times \frac{1}{m} \sum_{i=1}^{m} (y_{actual_i} - \beta_0 - \beta_1 x_i) \times (0 - 0 - x_i)$$

$$\Rightarrow -2 (1/m) \sum_{i=1}^{m} (y_{ai} x_i - \beta_0 x_i - \beta_1 x_i x_i)$$

$$\Rightarrow -2 \left(\frac{1}{m}\right) \sum_{i=1}^{m} (x_i (y_a - \bar{y}_a) - \beta_1 x_i (\bar{x} - x_i)$$

$$\Rightarrow -2 \left(\frac{1}{m}\right) \left(\sum_{i=1}^{m} x_i (y_{ai} - \bar{y}_a) + \sum_{i=1}^{m} (\beta_1 x_i (\bar{x} - x_i))\right)$$

--- eq. 3

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Now equaling eq.3 with equal to zero -

$$\Rightarrow \frac{\partial J}{\partial \beta_0} = 0$$

$$\Rightarrow \frac{\partial J}{\partial \beta_0} = -2\left(\frac{1}{m}\right)\left(\sum_{i=1}^{n} x_i(y_{ai} - \bar{y}_a) + \sum_{i=1}^{n}(\beta_1 x_i(\bar{x} - x_i) = 0\right.$$

$$\Rightarrow -\sum_{i=1}^{n} x_i(y_{ai} - \bar{y}_a) = \sum_{i=1}^{n} \beta_1 x_i(\bar{x} - x_i)$$

$$\Rightarrow \sum_{i=1}^{n} (\bar{y}_a - y_{ai}) = \sum_{i=1}^{n} \beta_1 (\bar{x} - x_i)$$

$$\Rightarrow \beta_1 = \sum_{i=1}^{n} (\bar{y}_a - y_{ai}) \Big/ \sum_{i=1}^{n} \beta_1 (\bar{x} - x_i) \text{ ---- eq.4}$$

Multiply eq.4 with there denominator -

$$\Rightarrow \beta_1 = \frac{\sum_{i=1}^{n} (\bar{y}_a - a_i) \times \sum_{i=i}^{n} (\bar{x} - x_i)}{\sum_{i=1}^{n} (\bar{x} - x_i) \times \sum_{i=1}^{n} (\bar{x} - x_i)}$$

$$\Rightarrow \boxed{\beta_1 = \frac{\sum_{i=1}^{n} (\bar{y}_a - y_{ai}) \times (\bar{x} - x_i)}{\sum_{i=1}^{n} (\bar{x} - x_i)^2}} = \boxed{\frac{\mathrm{cov}(x, y)}{\mathrm{Var}(x)}}$$

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Now substitute above β_1 values in case-I equation inorder to get β_0 value.

$$\beta_0 = \bar{y}_a - \bar{x} \frac{\text{cov}(x, y)}{\text{Var}(x)} \quad \& \quad \beta_1 = \frac{\text{cov}(x, y)}{\text{Var}(x)}$$

* This equation is called as ordinary least square method (OLS).

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